COLOR GRAPHICS

Color plays an important role in most visual tasks. We continuously make decision processes driven by color information input (for example, crossing an intersection, selecting a vegetable at the market, deciding whether the bread in the oven is ready).

Imagine a world in which the colors seen are not consistent and predictable. Imagine an orange carefully selected in the market having a completely different color once out of the shopping bag at home. Imagine colors of traffic lights different from one block to the next one. It is hard to imagine functioning in such a world, but at the computer that is the most likely experience—although computer professionals' awareness of color has grown orders of magnitude compared with just a few years ago.

We should expect—and can have—the same color consistency, predictability, and accuracy we take for granted in our day-to-day life in computer graphics based applications. We need such color reliability for our ease and comfort of use, as well as for our ability to perform various tasks accurately. Color has an important role in distinguishing among a small number of categories (typically less than 10) and in highlighting items found in a search. Color plays an important symbolic role, one that varies among different cultures. Color also has an assumed—but as yet unsubstantiated—role in visualizing larger numbers of data items (magnitudes of hundreds, thousands, and more) for easier analysis of these data.

This article discusses the basic concepts of the steps necessary to accomplish accurate and reliable colors on computer-generated displays, and it points readers to sources for further exploration. In addition, this article identifies outstanding issues that need to be further researched and resolved and outlines the challenges involved in addressing those issues.

The steps necessary for accurate and reliable color computer-generated displays are as follows:

1. Study and understand human color vision and perception.
2. Develop models of human color vision and perception.
3. Develop models of color graphics display devices.
4. Apply and implement modeling approaches.

HUMAN COLOR VISION AND PERCEPTION

> The rays, to speak properly, are not coloured. In them there is nothing else than a certain Power and Disposition to stir up a Sensation of this or that Colour.
>
> (*Sir Isaac Newton,* Opticks, *1704)*

What is color? Is it a property of the object that we see? Is it a property of our visual system? Of light? It is all of these. Color is our response to the combination of light, object, and observer. Remove any one, and there is no perception of color. Thus, to use color fully, we must understand all of these. And we have to make sure that visual technologies are matched to human visual capabilities.

The Human Interface

Print technology has evolved over centuries, and this evolution has given us time to learn how to present information in print so that the reader will get the most out of it, with the least amount of effort. It is no coincidence that most books have similar formats; the type of fonts used, their size, the number of words per line, and the number of lines per page have all been optimized for the reader. It took centuries to learn this.

On the other hand, the electronic display revolution has happened in just 30 years, from the first utilization of CRT (cathode ray tube) displays to today's virtual-reality head-mounted displays. Display technology has not reached the point where we can deliver sufficient spatial and color resolution to provide the same image quality that photography offers. Moreover, different color display devices provide rather incompatible color gamuts. (The *gamut* of a color display device is the set of all colors that the device is capable of displaying.) Thus, it is not guaranteed that a color image will appear the same (or even similar) on two displays. Color modeling that will provide cross-device color compatibility is still a topic of research (1).

Requirements for Image Quality

Adequate image quality requires:

Sufficient Luminance and Contrast.

No Flicker. Most current display devices utilize *refreshed* technology. The continuous update ("refresh") of contents causes periodic bright-dark changes of the display. Flicker is perceived at refresh frequencies of less than 50 cycles per second, but also depends on other conditions of the display device and the observer.

Minimized Effects of Spatial Sampling. All display devices sample continuous images and discretize them. The size of the smallest picture element, *pixel,* determines how sampling affects images (*aliasing*). Aliasing and antialiasing techniques have been discussed in the computer graphics literature (2).

Perceptually Lossless Image Compression. As the use of images and their size increase, compression becomes essential. High compression rates are possible only with lossy compression algorithms. The loss incurred should not affect critical decision making, such as that needed in medical visualization.

Convincing Impression of Depth. The display of three-dimensional objects on a two-dimensional display requires techniques to create the illusion of depth. These have been discussed in the computer graphics literature (2).

Effective Use of Color. The remainder of this article is dedicated to this aspect.

Technology Questions Hinge on Visual Perception

To provide the correct technological answers to accomplish image quality, we need to understand how the human observer perceives visual information. More specifically, we must be able to answer the following questions:

- How do we process luminance, contrast, color, and motion?

J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*. Copyright © 1999 John Wiley & Sons, Inc.

- How do these mechanisms constrain our choice of how to capture, sample, compress, and display information?

Some Definitions

We now define a few terms we will be using.

Physical Stimulus. Measurable properties of the physical world, such as luminance, sound pressure, wavelength

Sensation. The immediate effect on the human of the physical stimulus

Perception. The effect of sensory phenomena that are also mediated by higher-level processes, such as memory, attention, and experience

Psychophysics. The study of the sensations and perceptions that physical energies—such as brightness, loudness, and color—produce

Color as a Tristimulus Medium

Color is a sensation produced in the brain in response to the incidence of light on the retina of the eye. The sensation of color is caused by differing qualities of the light emitted by light sources or reflected by objects. Color may be defined in subjective, observer-based perceptual terms, or in terms of the physical characteristics of light by which the individual is made aware of objects or light sources.

The two definitions are related to each other. Light projected on the retina is composed of a spectrum of energies in different wavelengths. At the eye, and further at the brain, this spectrum energy profile is translated to the experience of a particular color. Moreover, many different spectra are perceived as the same color. This phenomenon—*metamerism*—was first reported by Newton in his 1704 book *Opticks* (3,4).

Both specifications require the three components of a vector in a three-dimensional space.

Color as Perceived by the Observer. The three components that specify color as perceived by the observer are as follows:

Hue. The actual "color" that we see (red, yellow, etc.)

Lightness. The achromatic (luminance) component, which is the amount of light emitted or reflected by the color. Lightness usually refers to a color reflected from an object, while *brightness* is typically used for light emitted from a source. Other terms have been also used (e.g., intensity, value).

Saturation. The purity, or vividness, of hue as a ratio of the amount of white mixed in the color. It is the degree of difference from a gray of the same lightness or brightness. Saturation is colorfulness relative to the color's lightness; *chroma* is colorfulness compared with white. Increased lightness causes a perceived decreased saturation, and vice versa. Chroma does not change with lightness.

Color as Produced by Light. The three components that specify color as produced by light are as follows:

Dominant Wavelength. The actual "color" seen. Corresponds to the subjective notion of hue.

Luminance. The amount of light or reflection. Corresponds to the subjective notion of lightness or brightness.

Purity. The spectral distribution that produces a certain color of light. It is the proportion of pure light of the dominant wavelength and white light needed to define the color. Corresponds to the perceptual notion of saturation.

The Human Visual System

The Eye. The front-end interface of our visual system is the eye with its several components. The pupil controls the amount of light admitted to the eye. Two lenses, the fixed cornea and a variable-focus lens, provide distance adaptation. The retina, located at the back of the eye, provides the first layer of "image processing."

The Retina. The retina contains five layers of cells, in charge of several early image processing tasks. The first layer contains four types of light-sensitive photoreceptors, grouped to filter different light phenomena. Approximately 120 million *rods* are responsible for achromatic ("black and white") night and dark vision. Daytime color vision is provided by approximately 8 million *cones* of three types, operating as filters peaking at three wavelengths:

S-Type. Short wavelength, peak sensitivity 440 nm (violet, erroneously called "blue")

M-Type. Medium wavelength, peak sensitivity 550 nm (yellowish-green, called "green")

L-Type. Long wavelength, peak sensitivity 570 nm (yellow, called "red")

Cones are mostly concentrated in the central vision center of the retina, the *fovea* (pit—the central one degree of vision); rods are mostly concentrated in the periphery.

Four other classes of cells in the retina, handling image compression and lateral inhibition, are beyond the scope of our discussion. See the vision literature (e.g., Refs. 5–9).

Sensitivity versus Resolution. The human visual system provides trade-offs between sensitivity (to low levels of luminance) and resolution (the ability to resolve small spatial detail). A one-to-one mapping of cones in the fovea to ganglion cells provides the highest spatial resolution (*acuity*), but only at sufficient luminance levels. A many-to-one mapping of rods in the periphery of the retina provides the highest luminance sensitivity, but only at lower resolution. See the cited vision literature for more details.

Basic Visual Mechanisms: Luminance and Contrast Perception

The human visual system comprises many different mechanisms. The earliest and most basic ones involve luminance and contrast perception.

Early Vision: Luminance Perception. Luminance perception is accomplished by a sensitivity to a range of 14 log-units, from the luminance of a dim star to that of the bright sunlight. However, at any given moment, the sensitivity is limited to a window of two log-units, matched to the ambient illumination. Luminance levels below the lower level of the window are perceived as darkest, while those above the upper level are perceived as brightest. This provides the dynamics of light-dark adaptation. Typical to all psychometric functions, the apparent brightness is a logarithmiclike relationship: Equal steps in perceived brightness require geometric increases in values of luminance.

Contrast and Spatial Resolution. Next to luminance perception is contrast perception. One definition of contrast is the ratio between the luminance levels of the object and the background.

Contrast sensitivity depends on the spatial distribution of light and dark regions (10). The minimum modulation required for detecting a grating pattern is a tuned function of the spatial frequency, described by the *contrast sensitivity function* (CSF). Its peak sensitivity is at two to four cycles per degree, decreasing for lower (coarser) and higher (finer) spatial frequency patterns.

Image Applications of the Contrast Sensitivity Function. In image coding, efficiency can be gained by devoting the greatest bandwidth of data to regions of the greatest spatial-frequency sensitivity, as defined by the CSF. In digital halftoning, one can hide sampling noise (dotted patterns) using regions of the lowest contrast sensitivity. Measuring display quality, one can evaluate the display modulation transfer function (MTF) with the CSF.

Introduction to Human Color Vision

Trichromacy (having three cone mechanisms in the retina) does not resolve color ambiguities completely. The sensation of any color can be created by exposing the three cones to many different three-dimensional vectors (metamerism). With fewer cone mechanisms, color perception degrades or disappears. For example, equal excitations of the medium- and long-wavelength mechanisms by a bichromatic light at 530 and 630 nm will produce the sensation of "yellow," the same as a monochromatic light at 550 nm. Thus, any hue can be matched by the combination of three primaries and can be produced by an infinite number of wavelength combinations. This is the basis for color video display technology: One needs only three primaries to produce millions of colors. For example, there is no need for a separate "yellow" gun.

Primaries and Color Mixing. Any three linearly independent colors can be used as *primaries* to generate other colors.

In *additive mixing* colors are generated by mixing the emissions of light sources covering different parts of the spectrum. Black is the result of no colors mixed in, and white is the result of mixing the maximum amounts of the three primaries. A color monitor is an example of additive mixing. Red, green, and blue (RGB) are the most common additive primaries.

In *subtractive mixing* colors are generated by filtering the reflection of parts of the spectrum. White is the result of no mixing (the entire spectrum is reflected); mixing the maximum amounts of the three primaries yields no reflection (i.e., black). Color printing is an example of subtractive mixing. Cyan, magenta, and yellow (CMY) are the most common subtractive primaries.

Second Stage: Opponent Processes. The photoreceptor outputs go through a recombination process in the optic nerve, where they are converted to three *opponent channels:*

1. $R + G$. The achromatic contents of the color (lightness/brightness). Blue is excluded because it does not contribute to the perception of lightness. Thus, changes in blue only are not sufficient to convey perceived changes in color.
2. $R - G$. Humans do not perceive the color relationships "reddish-green" or "greenish-red" (compare to "yellowish-green," "greenish-yellow," "greenish-blue," "bluish-green," "reddish-blue," "bluish-red").
3. $Y - B$. Humans do not perceive the color relationships "yellowish-blue" or "bluish-yellow."

Color Deficiencies. Approximately 8% of males and slightly less than 1% of females suffer from some genetic color deficiency. (These are the people we incorrectly call "color blind." Truly color-blind people are very rare. They have no color perception—they see the world in shades of gray—usually as a result of a head trauma.)

The most common deficiency (5% of males, 0.5% of females) is *deuteranomaly,* an anomalous trichromacy caused by an abnormal M-type or L-type cone and resulting in abnormal matches and poor discrimination between colors in the medium (M) and long (L) range of wavelengths. The M-type and L-type cone peak sensitivities are much closer to each other than normally. The result of is an inability to discriminate reds and greens.

A more severe case causing red-green deficiency is *deuteronomy,* a complete lack of the M-type cone. Similar deficiency can be caused by a complete lack of the L-type cone. These and other deficiencies caused by a missing or an abnormal cone type are much less common (8).

Color Deficiencies and Visual Displays. Understanding the nature (and prevalance) of color deficiencies can help design displays that do not exclude color-deficient users.

A simple rule of thumb is always to code important distinctions in the image with a redundant luminance cue. For example, a World Wide Web search tool should highlight found words using both different color and different brightness. In addition, it is strongly recommended to code differences along neither of the main opponent-processes chromatic channels (in particular, the red-green channel, since red-green is the most common color deficiency). Instead, select an axis that is a combination of the red-green and the yellow-blue channels, and then code the information along that axis (11).

Color-Luminance Interactions

Color and luminance perception complement each other in the visual system, mostly with respect to resolution.

Luminance Versus Color Resolution. The luminance system can resolve very fine spatial variations. Its sensitivity peaks at two to four cycles per degree, with a cutoff frequency of 60 cycles per degree.

The color system can resolve only coarse spatial variations. Its peak sensitivity for isoluminant gratings is at the low end of the spatial frequency spectrum, with a cutoff frequency between 10 and 20 cycles per degree.

We can thus make these basic observations:

1. High spatial resolution depends on luminance, not on hue. High spatial-frequency contents (such as text and thin lines) are hard to discriminate without sufficient luminance contrast. For example, it is difficult to detect yellow text on a white background as there is little luminance difference between the text and the background.
2. Low spatial-frequency sensitivity is mediated by the color mechanism. Colors look more saturated and intense over large areas. Conversely, small color targets "lose" their color and look achromatic. This means that due to their size, small hue differences between windows of a graphical user interface (GUI) are sufficient to distinguish them; there is no need for saturated colors.
3. The luminance mechanism has a greater bandwidth. More bandwidth is required to encode spatial variations of luminance. Therefore, image compression schemes devoting most bandwidth to luminance achieve higher compression rates while maintaining higher image quality.

For detailed descriptions of the human visual system, see the general vision and color literature (e.g., Refs. 7–9,12,13).

COLOR ORGANIZATION AND MODELING

Color organization and modeling attempts to describe the set and order of colors perceived by an observer, or those a particular device can produce.

A *color model* (also *solid, space*) is a three-dimensional body that represents some color organization based on a set of three color axes. The attempts to organize colors in some order can be traced back to Leonardo da Vinci's *Notebooks* (ca. 1500). Since then, many have tried to organize colors in different solid shapes. Early models varied in shape from pyramids to cones to spheres, as well as some irregular shapes. Historical details can be found in Refs. 14–18. Most of the models in use today [e.g., the *Munsell* color system, Ref. 19; the *Ostwald* color system, Ref. 16; the *Natural Color System (NCS),* Ref. 20; the *Colorid* system, Ref. 21; the Optical Society of America (OSA) system, and several models used in computer graphics] are based on similar concepts. Their color solids can be continuously deformed into the color sphere proposed by Runge in 1810 in Die Farbenkugel (The Color Sphere) (22).

The basic concept of all these models is continuous variations along the three axes of the model (such as hue, saturation, lightness). Combined with upper and lower bounds, they yield a three-dimensional color solid.

To maximize the perceptual functionality of a color model, several considerations are necessary. Since most people think about colors in terms of (1) hue, (2) lightness, and (3) saturation, it is important to assess any model with respect to the following considerations:

- Sufficient separation of hues (typically specified in angular measures)
- Sufficient relative separation of saturation and lightness
- Useful coordinate systems, both perceptually (human terms) and computationally (machine terms)

Overview of Color Specification Systems

The aforementioned color models are part of a broader hierarchy of color specification systems that have been in use in a wide range of applications. In this subsection, we summarize color specification systems and discuss some of them in some detail. We dedicate the majority of the next section to systems that have direct applications in computer graphics and related disciplines.

Our goal is to order colors in a rational system while providing a model that is as close as possible to the way humans perceive colors. We examine several color ordering systems.

Process-Dependent Systems (Instrumental)

These are systems that model the gamuts of specific instruments, such as color monitors and printers. We divide them into two groups, additive and subtractive.

Additive Systems. Additive systems, such as stage lights and color monitors, generate colors by additive mixing of light from (typically) three light sources, such as individual lights or electron guns.

Such systems are typically modeled by the color cube (described in more detail in the next section), using its RGB (red, green, and blue) corners as primaries.

These models suffer from a number of shortcomings:

1. Modeling is limited to a specific device.
2. There are no perceptual relationships to human-driven models (usually hue, lightness, saturation).
3. There is no accurate perceptual color difference metric.
4. There are limited gamuts, and there is no treatment of out-of-gamut colors.

Subtractive Systems. Subtractive systems—such as most hard-copy devices—are based on subtractive mixing. The process applies pigments—filters that prevent the reflection of parts of the energy spectrum—thus "subtracting" certain colors from the white page and preserving the peception of the remaining colors.

These instruments are also modeled by the color cube, but using its CMY (cyan, magneta, and yellow) corners as primaries. (For details see the next section.)

Subtractive systems suffer from the same problems of additive ones. In addition, however, imperfect dyes and filters may cause crosstalk (which sometimes demonstrates itself as "bleeding" of colors). Furthermore, the perception of colors these devices produce depends heavily on illumination.

A typical problem in both additive and subtractive systems is the lack of an easy answer to the question, "What are the coordinates of a particular color, say, a mid-orange?" Pseudoperceptual systems partially alleviate this problem.

Pseudoperceptual Systems

The axes of these systems represent intuitive concepts—such as hue, lightness, and saturation (HLS)—but are not based on precise psychophysical metrics. Thus, they are only *pseu-*

doperceptual. We discuss some of these in greater detail in the next section.

Coordinate Systems Based on Human Visual Models

Opponent-Process Models. Many models are based on opponent-process formulations, where the achromatic component ($R + G$) represents lightness or brightness, and the "ratio" of the chromatic components ($R - G$, $Y - B$) defines hue. The combination of chromatic and achromatic components defines saturation (23).

One difficulty in such systems is that they do not embody experimentally observed perceptual attributes of color discrimination as a primary consideration.

CIE Coordinate Systems. The CIE (*Commission Internationelle l'Eclerage*) coordinate systems have been accepted by color scientists as the standard for objective, device-independent color specifications. These coordinates were derived through color matching experiments that yielded color matching functions to describe an "average" observer with normal color vision, utilizing the CIE tristimulus values X, Y, and Z.

The initial result was the CIE 1931 Standard Observer, which describes "average" normal color vision for a field of view of 2°. Additional development yielded the CIE 1964 Standard Supplementary Observer, extending results to a field of view of 10°. Together they provide a reliable, reproducible, precise specification of normal color vision, but not a useful indication of *color difference* sizes or intuitive perceptual interpretation (that is, in tems of HLS). In addition, these specifications do not incorporate surround adaptation conditions or other perceptual effects.

Perceptually Uniform Systems

Perceptually uniform systems have two basic characteristics. They provide

1. perceptual (HLS) ordering and addressing; and
2. uniformity (i.e., distance in the coordinate system indicates the size of perceived color differences uniformly over the whole color "space").

Two approaches have been taken to develop perceptually uniform systems:

1. Experimental (e.g., the *Munsell Book of Color,* Ref. 19).
2. Analytical formulation based on discrimination experiments (e.g., CIELAB, CIELUV; see Refs. 24–26).

Uniform Color Spaces

In a uniform color space (UCS), equal metric steps correspond to equal perceptual steps.

The *Munsell Book of Color.* The *Munsell Book of Color* is an empirical organization of colors based on human perception. It was derived empirically by Munsell—based on visual judgment—to be uniform (19,27,28). Its approximately 1600 color chips are organized in equal perceptual steps along its three numerically labeled axes HVC: hue, value (lightness), and chroma (saturation). Each color is specified by a unique HVC vector.

The resulting solid is a distorted color sphere (22). A total of 40 hue values are arranged in a circle, divided into 10 sectors (red, yellow-red, yellow, green-yellow, green, blue-green, blue, purple-blue, purple, and red-purple). Each sector is subdivided into 10 sections, for a total of 100 equal parts. Values are arranged vertically from 0 (black) to 10 (white). Chroma values are arranged in a radial direction horizontally from the achromatic (Value) axis. The number of chroma steps varies for different hues and values.

Table 1. Hue, Lightness, and Saturation in CIELUV and CIELAB

CIELUV	CIELAB
$H = \arctan(v^*/u^*)$ $S = \sqrt{u^{*2} + v^{*2}}$	$H = \arctan(a^*/b^*)$ $S = \sqrt{a^{*2} + b^{*2}}$

The CIELUV and CIELAB uniform color spaces. These two UCSs were developed by the CIE. It is claimed that perceived differences between colors are well represented by the Euclidean (square norm) distance in the coordinates of these spaces (24–26).

CIELUV is recommended for modeling additive light source stimuli. Each color $\mathbf{c} = (r, g, b)$ has a unique representation, $(L^*(\mathbf{c}), u^*(\mathbf{c}), v^*(\mathbf{c}))$ in CIELUV. CIELAB is recommended for modeling reflected light conditions. Here, each color $\mathbf{c} = (r, g, b)$ has a unique representation $(L^*(\mathbf{c}), a^*(\mathbf{c}), b^*(\mathbf{c}))$.

For formulas to compute CIELUV and CIELAB coordinates, see, Refs. 24–26,29.

Hue, Saturation, and Lightness in CIELUV and CIELAB. While CIELUV and CIELAB are claimed to provide an accurate representation of color, as perceived by humans, they do not provide a very intuitive one. It is not trivial to find common color locations as it is not immediately clear that their axes are actually organized after opponent-processes models: L^* is the lightness (achromatic) axis, while u^*/a^* ($R - G$) and v^*/b ($Y - B$) are the chromatic opponent-processes axes, where positive u^*/a^* values represent reds, negative ones represent greens, positive v^*/b^* values represent yellows, and negative ones represent blues. Table 1 shows the relationship between these spaces' coordinates and lightness, hue, and saturation. For more in-depth discussions on color organization and modeling see Refs. 29–33.

COLOR MODELING IN COMPUTER GRAPHICS

In computer graphics we need color specifications that are compatible with the hardware and also comprehensible to the user. These *hardware-* and *user-oriented* requirements are difficult to fulfill in a single model.

The red, green, and blue (RGB color cube) model is used to model color monitors; the lightness, hue, and saturation (LHS) models are better suited for human interaction. Previously developed LHS models have been derived by coordinate transformations of the RGB color cube (34). The Generalized Lightness, Hue, and Saturation (GLHS) family of models generalizes and unifies all LHS models under a single framework, using a single pair of transformation algorithms between LHS and RGB models. We discuss these models in this section.

The Color Monitor, The Color Cube, and The RGB Model

Color monitors produce colors by additive mixtures of the three primaries red (R), green (G), and blue (B). Three electron guns and corresponding color-emitting phosphors on the screen surface together produce the three primaries and their mixtures. Independent inputs of R, G, and B control the colors displayed on the screen with gun intensities ranging between zero and a maximum voltage M. The resulting gamut can be represented by a cube, the *RGB Colorcube,* or just the *Colorcube*. Mathematically, the color cube consists of all points (r, g, b), such that $0 \leq r, g, b \leq M$. Although not every perceivable color can be mixed from nonnegative amounts of red, green, and blue (35), this gamut is sufficiently large for most practical purposes.

Note that the RGB model does not provide a standard for exact color specification, since the color produced by a particular RGB specification depends on the spectral distribution of the primaries and the gamma characteristics of the display (35). The relationship between a typical RGB gamut and the collection of all perceivable colors can be seen in the *CIE Chromaticity Diagram;* see Ref. 2, p. 585 and color plate II.2.

The Lightness, Hue, and Saturation Family of Models

LHS models make it easier for humans to estimate colors. These models correspond to artists' mixing and specifying colors using *tints, shades,* and *tones.* An artist mixes white into a pure hue to get a tint (reduce saturation), or black to get a shade (reduce lightness). Mixing both produces a tone.

We now briefly describe several LHS models. All are derived from the color cube by coordinate transformations from RGB to LHS. (Note that the terms *lightness* and *saturation* have been defined less colloquially by color scientists; see Ref. 36.) In all of these models

1. An approximate cylindrical coordinate system is used. The lightness ℓ is the distance along the axis perpendicular to the "polar" coordinate plane, and the "polar" coordinates are the saturation s (proportional to radial distance) and the hue h (a function of the angle).
2. Points in the color cube for which $r = g = b$ (the grays, also called *achromatic colors*) are assigned zero saturation ($s = 0$); their hue h is undefined. The lightness ℓ of these colors is given the common value of r, g, and b. Geometrically, picture this by considering the color cube being stood on its black point vertex (**Bk**: $r = g = b = 0$) with the main diagonal of the cube (**Bk** to **W**: $r = g = b = M$, the white point) corresponding to the positive lightness axis from 0 to M.
3. The lightness ℓ assigned to an arbitrary point (r, g, b) in the color cube is defined such that
   a. ℓ is always between 0 and M, and
   b. the set of points (a, g, b) in the color cube that are assigned a common value of ℓ form a *constant-lightness surface* with the special property that any line parallel to the main diagonal of the color cube meets the surface at no more than one point. (The members of the LHS family differ from each other in the actual shapes of these surfaces. Since we restrict these surfaces to be subsets of the color cube, in a few "pathological" cases a "surface" contains a single point or is the union of three line segments.)

   Each one of these constant-lightness surfaces is projected onto a plane perpendicular to the lightness axis intersecting it at the origin. This projection defines a shape (e.g., a triangle or hexagonal disk) that depends on the lightness function chosen and the specific lightness value. The projected constant-lightness surface is then "moved back" so that it intersects the lightness axis at its lightness value. Repeating the process for all lightness values stacks all the projected constant-lightness surfaces in the order of their lightness values ($\ell = 0$ at the bottom, $\ell = M$ at the top). This yields the three-dimensional color solid of the model. Note that since the entire process of projecting constant-lightness surfaces and color vectors is done in RGB space, the shape of the resulting color solid varies as a function of the lightness function.

   Mathematically, the *projected color vector* of a color (r, g, b) [the projection of (r, g, b) onto the plane through the origin—the black point—perpendicular to the lightness axis] is the vector $(2r - g - b)/3$, $(2g - b - r)/3$, $(2b - r - g)/3$. This implies that the location in the color solid of the point that corresponds to (r, g, b) in the color cube is, in (r, g, b)-coordinates, $(2r - g - b)/3 + \ell$, $(2g - b - r)/3 + \ell$, $(2b - r - g)/3 + \ell$, where ℓ is the lightness of (r, g, b). So the shape of the color solid (and the transformation of the color cube into it) depends only on the definition of lightness (and not on the definition of hue and saturation).
4. The hue h of a chromatic color $(r, g, , b)$ is defined by a function of the angle between its projected color vector and a predefined vector (traditionally the projected color vector of a pure red). Typically, this function is chosen so that
   a. it maps 0 into 0 and its whole domain [0, 360) onto [0, 360);
   b. it is continuous and monotonically increasing; and
   c. its value for any argument is an approximation of that argument.

   The angle between the projected color vectors of any two chromatic colors is independent of the particular choice of the lightness function. Hence, in all LHS models in which the same function satisfying the aforementioned conditions is used to specify hue, the hue assigned to a particular color (r, g, b) will be unchanged by the addition or subtraction of an achromatic color (i.e., by tinting and shading). This is a valuable property for some applications, as discussed in Ref. 29.
5. The saturation s of a color (r, g, b) is defined as the ratio of the length of its projected color vector to the length of the longest projected color vector in the same direction, in the same constant-lightness surface. Thus, for the vectors of any fixed constant-lightness surface, a color that has the longest projected color vector (in any particular direction) has maximum saturation ($s = 1$), and the achromatic color has minimum saturation ($s = 0$).

The essential choice in selecting a particular LHS model is made in the definition of the lightness function, which in turn determines the constant-lightness surfaces (and hence the shape of the color solid that represents the model). An independent secondary choice is made in selecting the hue function. Once the lightness function is chosen, saturation is completely defined by item 5. (In particular, it does not depend on the choice of the hue function.)

The LHS-Triangle Model. The simplest constant-lightness surfaces are planes. The triangle model defines the lightness $\ell(\mathbf{c})$ of a color $\mathbf{c} = (r, g, b)$ as

$$\ell(\mathbf{c}) = \frac{r+g+b}{3} \tag{1}$$

(where the division by 3 serves only to normalize the lightness into the range [0, M]). A constant-lightness surface with lightness ℓ is the plane

$$\{(r,g,b) : (r+g+b)/3 = \ell\} \tag{2}$$

For $0 \leq \ell \leq M$, these planes are perpendicular to the main diagonal of the color cube and parallel to each other. Thus, in this case, the constant-lightness surfaces are "projected" onto themselves and so the color solid is still the cube. The LHS-triangle model has been introduced in Ref. 34 as a variant of Smith's HSL-triangle model (37). The colors in the LHS-triangle model are (by definition) exactly those in the color cube, whereas in the original model many colors are unrealizable within the color cube (38).

The HSV-Hexcone Model. The HSV-hexcone model is derived from the color cube by defining the lightness (called *value* by users of this model) of a given color $\mathbf{c} = (r, g, b)$ as

$$\ell(\mathbf{c}) = \max\{r,g,b\} \tag{3}$$

With constant-lightness surfaces,

$$\{(r,g,b) : \max\{r,g,b\} = \ell\} \tag{4}$$

Several selection and projection steps, which we omit, yield for each lightness level a corresponding hexagonal disk. The disks are stacked vertically, bottom to top, yielding a hexagonal cone (*hexcone*). Details and the complete derivation of the model and the transformation algorithms between the RGB and the HSV models are given in Refs. 30 and 37. Additional discussions can be found in Refs. 2 and 39.

The HLS-Double-Hexcone Model. In this model, the lightness $\ell(\mathbf{c})$ of a color $\mathbf{c} = (r, g, b)$ is defined as

$$\ell(\mathbf{c}) = \frac{\max\{r,g,b\} + \min\{r,g,b\}}{2} \tag{5}$$

For each ℓ, the constant-lightness surface is the locus of points

$$\{(r,g,b) : \max\{r,g,b\} + \min\{r,g,b\} = 2\ell\} \tag{6}$$

As with the hexcone model, selection and projection steps omitted here yield a hexagonal disk, similar to the one in the HSV-hexcone, for each lightness level. However, the largest disk now corresponds to $\ell = M/2$, and the disks for both $\ell =$ 0 (black) and $\ell = M$ (white) are single points. Stacking the disks vertically in their lightness order yields a double hexagonal cone (*double hexcone*) with primaries located on the largest disk ($\ell = M/2$) in the same way they are organized in the HSV-hexcone model.

Table 2. Comparison of the Lightness of Pure Hues, Secondary Colors, and White in the Three LHS Models

	Pure Hues	Secondary	White
HSL triangle	$M/3$	$2M/3$	M
HSV hexcone	M	M	M
HLS double hexcone	$M/2$	$M/2$	M

Table 2 summarizes the lightness values of pure hues (primaries), secondary colors, and white in the three LHS colors. Note that the property of the triangle model—different lightness for each one of these groups—provides an advantage in color applications on monochrome displays: The three groups of colors can be distinguished on a monochrome display based on their lightness alone (compare with the hexcone model, where they are the same).

GLHS: A Generalized Lightness, Hue, and Saturation Model

Definition and Basic Properties. The LHS models presented in the previous section belong to a general class of models. GLHS, the generalized lightness, hue, and saturation color model, provides a first-order mathematical framework for that class of models. The models described previously are special cases of GLHS, realized by special parameter values (30,34,40).

The first-order generalization uses piecewise planar constant-lightness surfaces. We define three nonnegative weights $w_{\min}$, w_{mid}, $w_{\max}$, such that $w_{\max} > 0$ and $w_{\min} + w_{\text{mid}} + w_{\max} = 1$. The lightness function is defined as

$$\ell(\mathbf{c}) = w_{\min} \cdot \min(\mathbf{c}) + w_{\text{mid}} \cdot \text{mid}(\mathbf{c}) + w_{\max} \cdot \max(\mathbf{c}) \tag{7}$$

where min($\mathbf{c}$), mid($\mathbf{c}$), and max($\mathbf{c}$) are defined as

$$\begin{aligned} \min(\mathbf{c}) &= \min\{r,g,b\} \\ \text{mid}(\mathbf{c}) &= \text{mid}\{r,g,b\} \\ \max(\mathbf{c}) &= \max\{r,g,b\} \end{aligned} \tag{8}$$

and a constant-lightness surface for a given lightness ℓ is given by the locus of points:

$$\{\mathbf{c} : w_{\min} \cdot \min(\mathbf{c}) + w_{\text{mid}} \cdot \text{mid}(\mathbf{c}) + w_{\max} \cdot \max(\mathbf{c}) = \ell\} \tag{9}$$

Generally, this consists of the six planar polygons corresponding to the six combinations of the order of the magnitudes of

Table 3. The Values of the Three Weights That Realize the Computer-Graphics Color Models

	$w_{\min}$	w_{mid}	$w_{\max}$
HSL triangle	1/3	1/3	1/3
HSV hexcone	0	0	1
HLS double hexcone	1/2	0	1/2

r, g, and b. Pathological cases arise when some of the six planes intersect the color cube in a point or a line. The mathematical derivation of the family is valid for the pathological cases as well as for the general case.

Different values of $w_{\min}$, w_{mid}, and $w_{\max}$ give rise to different color models. Table 3 gives the values of the weights for the models discussed in the preceding subsection. By changing the values of the three weights, a continuum of models can be achieved. To complete the definition of a GLHS model, we need to define the hue $h(\mathbf{c})$ and the saturation $s(\mathbf{c})$ of a color $\mathbf{c}$.

The hue $h(\mathbf{c})$ of a chromatic color $\mathbf{c} = (r, g, b)$, $0 \le h(\mathbf{c}) < 360$ is defined as

$$h(\mathbf{c}) = (k(\mathbf{c}) + f(\mathbf{c})) \cdot 60 \tag{10}$$

where $k(\mathbf{c}) \in \{0, 1, \ldots, 5\}$ is the number of the *sector* defined by the order of the magnitudes of the r, g, and b values:

$$k(\mathbf{c}) = \begin{cases} 0, & \text{if } r > g \ge b \\ 1, & \text{if } g \ge r > b \\ 2, & \text{if } g > b \ge r \\ 3, & \text{if } b \ge g > r \\ 4, & \text{if } b > r \ge g \\ 5, & \text{if } r \ge b > g \end{cases} \tag{11}$$

and $f(\mathbf{c}) \in [0, 1)$, the *hue-fraction* within the sector, is calculated as

$$f(\mathbf{c}) = \begin{cases} \dfrac{\mathrm{mid}(\mathbf{c}) - \min(\mathbf{c})}{\max(\mathbf{c}) - \min(\mathbf{c})}, & \text{if } k(\mathbf{c}) \text{ is even,} \\ \dfrac{\max(\mathbf{c}) - \mathrm{mid}(\mathbf{c})}{\max(\mathbf{c}) - \min(\mathbf{c})}, & \text{if } k(\mathbf{c}) \text{ is odd} \end{cases} \tag{12}$$

This is a modified representation of one of the hue functions presented in Ref. 37. It satisfies all the properties specified for hue in item 4 of the preceding subsection; for a proof see Refs. 29 and 40. This definition of hue is independent of the definition of lightness; for any chromatic color $\mathbf{c}$, $h(\mathbf{c})$ is the same in all the GLHS models.

The saturation $s(\mathbf{c})$ of a color $\mathbf{c} = (r, g, b)$ is completely defined by the description in item 5 of the preceding subsection:

$$s(\mathbf{c}) = \begin{cases} \dfrac{\ell(\mathbf{c}) - \min(\mathbf{c})}{\ell(\mathbf{c})}, & \text{if } \ell(\mathbf{c}) \le \ell(\mathbf{q}(\mathbf{c})), \\ \dfrac{\max(\mathbf{c}) - \ell(\mathbf{c})}{M - \ell(\mathbf{c})}, & \text{if } \ell(\mathbf{c}) > \ell(\mathbf{q}(\mathbf{c})) \end{cases} \tag{13}$$

where the color $\mathbf{q}(\mathbf{c})$, which depends on $\mathbf{c}$, is

$$\mathbf{q}(\mathbf{c}) = \begin{cases} (M, f(\mathbf{c})M, 0), & \text{if } k(\mathbf{c}) = 0 \\ ((1 - f(\mathbf{c}))M, M, 0), & \text{if } k(\mathbf{c}) = 1 \\ (0, M, f(\mathbf{c})M), & \text{if } k(\mathbf{c}) = 2 \\ (0, (1 - f(\mathbf{c}))M, M), & \text{if } k(\mathbf{c}) = 3 \\ (f(\mathbf{c})M, 0, M), & \text{if } k(\mathbf{c}) = 4 \\ (M, 0, (1 - f(\mathbf{c}))M), & \text{if } k(\mathbf{c}) = 5 \end{cases} \tag{14}$$

[Note that $k(\mathbf{q}(\mathbf{c})) = k(\mathbf{c})$, $f(\mathbf{q}(\mathbf{c})) = f(\mathbf{c})$, and so $h(\mathbf{q}(\mathbf{c})) = h(\mathbf{c})$. In fact $\mathbf{q}(\mathbf{c})$ depends only on $f(\mathbf{c})$ and $k(\mathbf{c})$ and thus it is the same for all colors of the same hue.]

```
Algorithm RBG_TO_GLHS
Input: $\mathbf{c} = (r, g, b) \in [0, M^3]$,
        $w_{max}, w_{mid}, w_{min}$, such that $0 \le w_{max}, w_{mid}, w_{min} \le 1$, $w_{max} > 0$,
        and $w_{max} + w_{mid} + w_{min} = 1$.
Output: $(\ell, h, s)$, $\ell \in [0, M]$, $h \in [0, 360) \cup \{undefined\}$, $s \in [0, 1]$.
Auxiliary variables: the critical lightness $\ell(\mathbf{q})$, $k$, $f$.
begin
    $max := MAXIMUM(r, g, b)$;
    $mid := MID\_VALUE(r, g, b)$;
    $min := MINIMUM(r, g, b)$;
    if $max = min$
        then                                {achromatic}
            $(\ell, h, s) := (max, undefined, 0)$
        else begin                          {chromatic}
            $\ell := w_{max} * max + w_{mid} * mid + w_{min} * min$;
            begin case of                   {sector-number $k$}
                $r > g \ge b : k := 0$;
                $g \ge r > b : k := 1$;
                $g > b \ge r : k := 2$;
                $b \ge g > r : k := 3$;
                $b > r \ge g : k := 4$;
                $r \ge b > g : k := 5$;
            endcase
            begin case of                   {hue-within-sector $f$}
                $k$ even: $f := (mid - min)/(max - min)$;
                $k$ odd: $f := (max - mid)/(max - min)$:
            endcase
            $h := (k + f) * 60$:
            $\ell(\mathbf{q} = (w_{mid} * (mid - min)/(max - min) + w_{max}) * M$;
            if $\ell \le \ell(\mathbf{q})$
                then $s := (\ell - min)/\ell$;
                else $s := (max - \ell)/(M - \ell)$;
        end                                 {chromatic}
end; {RGB_TO_GLHS}
```

Figure 1. The RGB-TO-GLHS transformation algorithm.

Note that even though it appears that there is a potential for division by zero in Eq. (13), this cannot happen for a chromatic color. Details of this, as well as complete details of the derivation, are given in Refs. 29 and 40.

The omitted derivations show that the transformation from RGB coordinates to LHS coordinates is a one-to-one mapping onto its range. It therefore has an inverse defined on this range. Combinations of (ℓ, h, s) coordinates that are not in that range do not correspond to any color in the color cube. This guarantees that the transformation from LHS to RGB exists and is well defined.

Note that the generalization presented here affects the shape of the constant-lightness surfaces and the lightness ranges for which they hold, and thus the shape of the color solid. The shapes and ranges of the color solid for various cases are given in Refs. 29 and 40.

Algorithms to Transform Between GLHS and RGB. When using a color space other than RGB, it is necessary to transform color coordinates between that space and RGB for display and manipulation. We present a pair of algorithms to transform colors between RGB and any member of the GLHS family. For brevity, we skip detailed explanations and demonstrations; those can be found in Refs. 29 and 40.

RGB to GLHS. The algorithm presented in Fig. 1 computes the (ℓ, h, s) values of a color $\mathbf{c} = (r, g, b)$ in some

```
Algorithm GLHS_TO_RGB
Input: $w_{max}$, $w_{mid}$, $w_{min}$, such that $0 \leq w_{max}\ w_{mid}, w_{min} \leq 1$, $w_{max} > 0$,
    and $w_{max} + w_{mid} + w_{min} = 1$.
        $(\ell, h, s) \in R_A \cup R_B \cup R_C$, where
        $R_A = [0, M] \times \{undefined\} \times \{0\}$,
        $R_B = \emptyset$, if $w_{min} > 0$
        $R_B = \{M\} \times \{60, 180, 300\} \times (0, 1]$, if $w_{min} = 0$ and $w_{mid} > 0$,
        $R_B = \{M\} \times [0, 360) \times (0, 1]$, if $w_{min} = w_{mid} = 0$,
        and $R_C = (0, M) \times [0, 360) \times (0, 1]$.
Output: $\mathbf{c} = (r, g, b)$, $r, g, b \in [0, M]$.
Auxiliary variables: the critical lightness $\ell(\mathbf{q})$, $k$, $f$, and $f'$.
begin
    if $s = 0$
        then $r := g := b := \ell$;                          {achromatic}
        else begin                                           {chromatic}
            $k := FLOOR(h/60)$;                              {sector-number}
            $f := h/60 - k$;                                 {hue-within-the-sector}
            begin case of
                $k$ even: $f' := f$;
                $k$ odd: $f' := 1 - f$;
            endcase
            $\ell(\mathbf{q}) := (w_{mid} * f' + w_{max}) * M$;
            if $\ell \leq \ell(\mathbf{q})$
                then begin
                    $min := (1 - s) * \ell$;
                    $mid := (f' * \ell + min * ((1 - f') * w_{max} - f' * w_{min}))/$
                        $(w_{max} + f' * \mathrm{mid})$;
                    $max := (\ell - w_{mid} * mid - w_{min} * min)/w_{max}$;
                end
                else begin    $\{\ell > \ell(\mathbf{q})\}$
                    $max := s * M + (1 - s) * \ell$;
                    $mid := ((1 - f') * \ell - max * ((1 - f') * w_{max} - f' * w_{min}))/$
                        $((1 - f') * w_{mid} + w_{min})$;
                    if $w_{min} > 0$
                        then $min := (\ell - w_{max} * max - w_{mid} * mid)/w_{\mathrm{min}}$;
                        else $min := (mid - f' * max)/(1 - f')$;
                end
            case $k$ of
                0: $(r, g, b) := (max, mid, min)$;
                1: $(r, g, b) := (mid, max, min)$;
                2: $(r, g, b) := (min, max, min)$;
                3: $(r, g, b) := (min, mid, max)$;
                4: $(r, g, b) := (mid, min, max)$;
                5: $(r, g, b) := (max, min, mid)$;
            endcase
        end                                                  {chromatic}
end; {GLHS_TO_RGB}
```

Figure 2. The GLHS-TO-RGB transformation algorithm.

GLHS model defined by the weights w_{max}, w_{mid}, and w_{min}.

GLHS to RGB. The algorithm in Fig. 2 computes the color $\mathbf{c} = (r, g, b)$ in RGB coordinates for a color (ℓ, h, s) given in some GLHS model defined by the weights w_{max}, w_{mid}, w_{min}. Note that the input is restricted to those combinations of the (ℓ, h, s) coordinates that correspond to a color in the color cube.

Summary

The GLHS family introduces new LHS models that have not been explored previously. Such models may be advantageous to achieve better properties within an LHS model, such as uniformity (41,42). In addition, the GLHS model provides the potential for dynamic interactive color model changes, which may enhance the utility of color displays.

One must remember, though, that models within GLHS suffer the same drawbacks as most mathematical models of color, which usually do not describe human color perception adequately. For example, colors of the same mathematical lightness often do not appear so. These geometrical (pseudo-perceptual) models simplify color vision and are only perceptual in quality not in quantity.

APPLICATION AND IMPLEMENTATION

The modeling approaches described in this article can be applied to improve color graphics. Some have been applied. We mention three such applications briefly. We provide references for readers who wish to get more details. The graphics, visualization, imaging, and color literature can offer accounts of other applications.

Color Scales for Image Data

A *color scale* is a pictorial representation of a set of distinct categorical or numerical values in which each value is assigned its own color. Most scales are derived from some physical or mathematical behavior; sometimes they are selected based upon hardware capabilities. In most known cases, no consideration is made to the perceptual capabilities of the human observer, the ultimate "consumer" of the information delivered by the scale.

People have been seeking alternative—*pseudocolor*—scales for aesthetic, as well as functional or perceptual, reasons.

Levkowitz and Herman (43) state desirable properties of color scales. They introduce the notion of an optimal color scale and describe the development of a particular optimal color scale; their major criterion is the maximization of the scale's perceived dynamic range (the number of just noticeable differences). They state restrictions on colors in an optimal color scale and present an algorithm to search for scales that maximize their criteria while obeying their constraints. As a result, they present and evaluate the Linearized Optimized Color Scale (LOCS).

Levkowitz (44) presents a method and an algorithm to derive color scales, such that their perceptual properties—in particular, the perceptual steps between colors along the scale—can be controlled by the scale designer. This approach has been used to design the linearized gray scale and the Linearized Optimized Color Scale described in Ref. 43.

Uniform LHS Models

We have discussed the advantages of uniform color spaces previously. The GLHS family discussed previously offers the potential for finding a model that has the perceptual properties of a uniform model and the algorithmic properties of LHS models. Finding such a model requires a search among all GLHS models for one that is the closest approximation to the selected uniform model, subject to some predefined criteria for closeness.

Levkowitz and Herman (41) approximated the CIELUV space using this approach; Levkowitz and Xu (42) approximated the Munsell system. The details of such efforts have been described in Ref. 29.

Color Icons

Iconographic displays have been developed to help integrated multivariate image sets into a single integrated display. Such displays hold the promise of utilizing better the human analyst's perceptual capabilities while analyzing the images.

Levkowitz (45) and Erbacher et al. (46) have developed the *color icon,* which harnesses color and texture perception to create integrated displays of multiparameter distributions. The development of the color icon is based on many of the color graphics modeling concepts in this article. For more details, see Ref. 29.

CHALLENGES FOR THE FUTURE

Several color graphics topics are still open for development and improvements. Among them are (1) color appearance models, (2) cross-device gamut-matching algorithms, and (3) models for better color support on the World Wide Web.

Readers are invited to visit the Color Center World Wide Web site at the University of Massachusetts Lowell's Institute for Visualization and Perception Research (http://www.cs.uml.edu/~haim/ColorCenter), where they will find demonstrations, images, an extended color graphics bibliography, and an updated list of color graphics projects and challenges.

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COLOR IMAGE ANALYSIS. See IMAGE COLOR ANALYSIS.