PHOTOREFRACTIVE EFFECT

The photorefractive effect is a phenomenon in which the local index of refraction of some transparent crystals is changed by the illumination of a beam of light with a spatial variation of the intensity. Such an effect was first discovered in 1966 when researchers were studying the transmission of laser beams through electro-optic crystals, like lithium niobate. It was found that the presence of laser beams inside some electro-optic crystals leads to an index inhomogeneity that distorts the wavefront of the transmitted laser beam. Such an effect was referred to as "optical damage" (1). The photorefractive effect has since been observed in many electro-optic crystals, including $BaTiO_3$, $KNbO_3$, $LiNbO_3$, $LiTaO_3$, $Sr_{1-x}Ba_xNb_2O_6$ (SBN), $Ba_{2-x}Sr_xK_{1-y}Na_yNb_5O_{15}$ (BSKNN), $Bi_{12}SiO_{20}$ (BSO), $Bi_{12}GeO_{20}$ (BGO), GaAs, InP, and CdTe.

BAND TRANSPORT MODEL

In 1979, Vinetskii and Kukhtarev developed a model (2) that is very useful for explaining most of the observed photorefractive phenomena. In the model, photorefractive media are assumed to contain certain types of impurities or imperfections. For the sake of simplicity in explaining the concept, we assume that all donor impurities are identical and have exactly the same energy state somewhere in the middle of the bandgap (see Fig. 1). These donor impurities can be ionized by absorbing photons. As a result of the ionization, electrons are generated in the conduction band, leaving empty states behind. Such ionized impurities are capable of capturing electrons. For every impurity ionized, there is an electron generated. On the other hand, an electron is eliminated when a recapture occurs that fills an empty impurity state. The rate of generation of electrons is the same as that of the ionized impurities except that the electrons are mobile whereas the impurities are stationary. This is essential for photorefractive effect. The transport of electrons may affect the electron density distribution. These charge carriers (electrons) lead to a space-charge field, which in turn affects the transport of the carriers. The current density consists of contributions from the drift of charge carriers due to the electric field, the diffusion due to the gradient of carrier density, and the current due to the photovoltaic effect.

As indicated in Fig. 1, there are donor impurities and acceptor impurities. The density of donors is often much larger than that of the acceptor impurities. Here we assume that all acceptors are also identical. In the case that there is no electrons in the conduction band and no holes in the valence band, the density of ionized donor impurities is identical to that of the acceptor impurities. The neutral donor impurities are capable of donating electrons via photoexcitation, and the ionized ones are capable of capturing these photoelectrons. In this simplified model, the acceptor impurities are for the purpose of charge neutrality only. They do not directly participate in the photorefractive effect.

We now consider the incidence of two laser beams into a photorefractive medium. If the polarization states of the two beams are not orthogonal, the beams will form an interference pattern. In the bright regions, photoionized charges are generated by the absorption of photons. These charge carriers

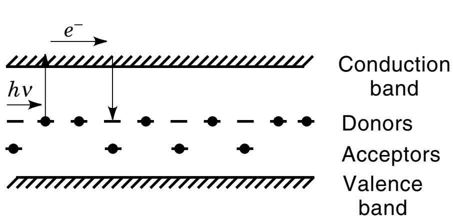

Figure 1. Band transport model.

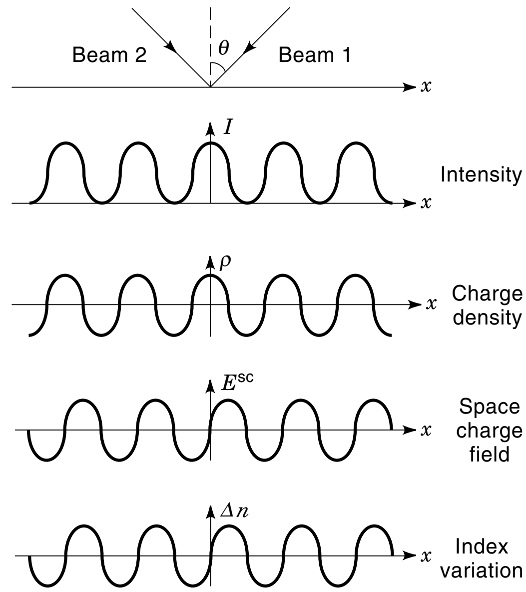

Figure 2. Photorefractive effect.

can diffuse away from the bright regions, leaving behind positively charged ionized donor impurities. If these charge carriers are trapped in the dark regions, they will remain there because there is no light to reexcite them. This leads to a charge separation, as depicted in Fig. 2. As a result of the illumination with periodic intensity in the photorefractive medium, the dark regions are negatively charged and the bright regions are positively charged (see Fig. 2). The buildup of the space-charge separation will continue until the diffusion current is counterbalanced by the drift current. In general, the space-charge field is shifted in space relative to the intensity pattern. This space-charge field will induce a change in the index of refraction via the Pockel's effect. Figure 2 illustrates the spatial variation of the light intensity, space-charge density, space-charge field, and the induced index grating.

In summary, the photorefractive effect consists of five fundamental processes that occur in electro-optic crystals: (1) photoionization of impurities and the generation of charge carriers, (2) transport of the charge carriers, (3) trapping of charge carriers and the formation of space-charge density, (4) formation of the space-charge electric field, and (5) formation of index grating via the linear electro-optic effect.

Wave Mixing

When two beams of coherent electromagnetic radiation intersect inside a photorefractive medium, the periodic variation of the intensity due to the beam interference will induce a volume index grating. The grating wave vector is given by $\boldsymbol{K} = \pm(\boldsymbol{k}_2 - \boldsymbol{k}_1)$, where $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are wave vectors of the beams. The presence of such an index grating will affect the propagation of these two beams. In fact, these waves are strongly diffracted by the induced index grating because Bragg scatterings are perfectly phase matched. Thus beam 1 is scattered by the index grating and the diffracted beam is propagating along the direction of beam 2. Similarly, beam 2 is scattered by the same grating and the diffracted beam is propagating along the direction of beam 1. This leads to en-

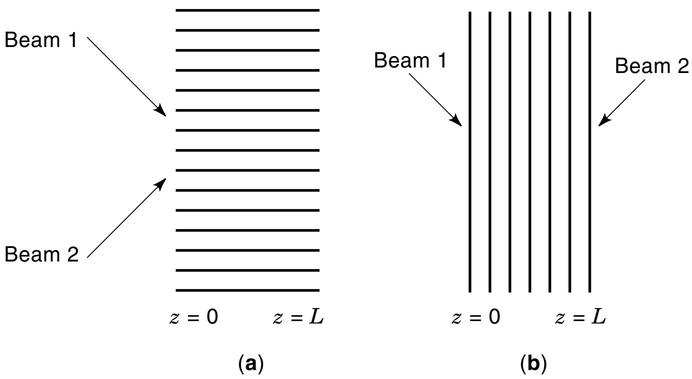

Figure 3. Two-wave mixing configurations: (a) codirectional two-wave mixing, (b) contradirectional two-wave mixing.

ergy coupling, and the scattering is known as self-diffraction (3). This process is referred to as two-wave mixing.

We consider the interaction of two laser beams inside a photorefractive medium. If the two beams are of the same frequency, a stationary interference pattern is formed, which will induce a stationary index grating. The scattering of the laser beams through the stationary index grating is referred to as degenerate two-wave mixing. When the frequencies of the laser beams are different, the interference fringe pattern is no longer stationary. A volume index grating can still be induced provided the fringe pattern does not move too fast. The amplitude of the index modulation decreases as the speed of the fringe pattern increases. This is related to the finite time needed for the formation of the index grating. The scattering of the laser beams through the moving index grating is referred to as nondegenerate two-wave mixing.

Referring to Fig. 3, two-wave mixing can also be divided into the following two categories. When the two laser beams enter the medium from the same side, the configuration is referred to as codirectional two-wave mixing. When the two beams enter the medium from opposite sides, the configuration is referred to as contradirectional two-wave mixing. In codirectional two-wave mixing, the sum of the beam power is a constant of position provided the medium is lossless, whereas in contradirectional two-wave mixing, the difference of the beam power is a constant. There exist qualitative differences in the energy exchange between the two waves in the two cases.

If the two waves are of the same frequency, the electric field of the two waves can be written as

$$E_j = A_i \exp(i\omega t - i\boldsymbol{k}_j \cdot \boldsymbol{r}) \qquad j = 1, 2$$

where A_1, A_2 are the wave amplitudes, ω is the angular frequency, and $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ are the wave vectors. The coupled-wave equations describing codirectional (contradirectional) two-wave mixing can be written as

$$\frac{d}{dz}A_1 = -\frac{1}{2I_0}\Gamma|A_2|^2 A_1 - \frac{\alpha}{2}A_1$$
$$\frac{d}{dz}A_2 = \pm\frac{1}{2I_0}\Gamma^*|A_1|^2 A_2 \mp \frac{\alpha}{2}A_2$$

where the upper sign in the last equation is for codirectional mixing, α is the bulk absorption coefficient, Γ is the complex coupling coefficient, and $I_0 = |A_1|^2 + |A_2|^2$ is the total intensity of the two beams. Let γ be the real part of Γ. By examining the coupled equations, we note that in the absence of material absorption ($\alpha = 0$), the intensity of beam 1 ($I_1 = |A_1|^2$) is a decreasing function of z provided γ is positive. This indicates that the energy is flowing from beam 1 to beam 2. The direction of energy flow is determined by the sign of γ, which depends on the orientation of the crystal axes. For a positive γ, beam 1 is referred to as the pump beam, and beam 2 as the signal beam. In the nondepleted pump regime, where the intensity of the pump beam ($I_1 = |A_1|^2$) is much larger than the intensity of the signal beam ($I_2 = |A_2|^2$), the intensity of the signal beam is an exponential function of position z. For a positive γ, the intensity of the signal beam can be written as $I_2(z) = \exp[(\gamma - \alpha)z]$ for codirectional two-wave mixing and $I_2(z) = \exp[(\gamma - \alpha)(L - z)]$ for contradirectional two-wave mixing. Therefore, the parameter γ is referred to as the intensity coupling constant.

When four laser beams intersect inside a photorefractive medium, each pair of the four beams will induce a volume index grating. In general, there will be six index gratings. Referring to Fig. 4, we denote these index gratings as (beam 1, beam 2), (beam 1, beam 3), (beam 1, beam 4), (beam 2, beam 3), (beam 2, beam 4). The four laser beams will be scattered into each other through these index gratings. This process is referred to as four-wave mixing.

In typical experimental setup of four-wave mixing, the four beams come in two oppositely directed pairs. If we further assume that the four beams enter the medium symmetrically,

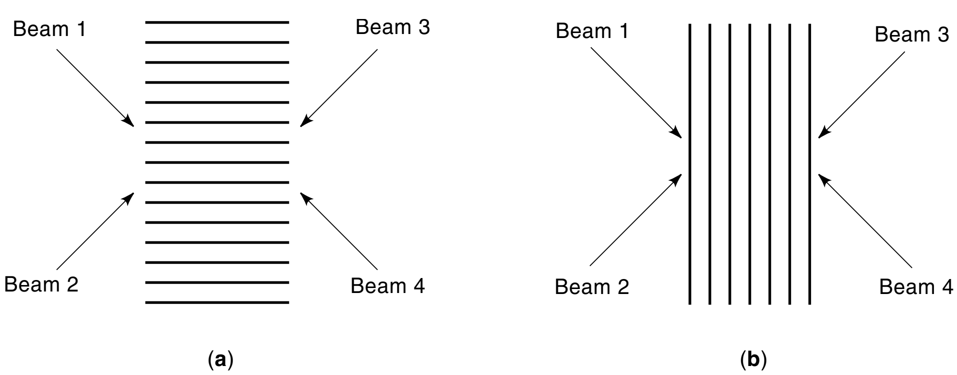

Figure 4. Four-wave mixing configurations: (a) four-wave mixing via transmission grating, (b) four-wave mixing via reflection grating.

then there are four different gratings. These gratings are represented by (beam 1, beam 2) + (beam 3, beam 4), (beam 1, beam 3) + (beam 2, beam 4), (beam 1, beam 4), and (beam 2, beam 3). The first one is a transmission grating and the second one is a reflection grating. The remaining ones are $2k$ gratings (i.e., grating with wave vector $2k$). The predominance of one grating is common in practical situations due to the dependence of the space-charge field on the grating periods, grating orientations, and the coherence between the beams. This is the so-called one grating approximation. When only the transmission grating gives rise to strong interaction among the beams, this interaction is referred to as four-wave mixing via transmission grating [see Fig. 4(a)]. When only the reflection grating gives rise to strong interaction among the beams, this interaction is referred to as four-wave mixing via reflection grating [see Fig. 4(b)].

PHOTOREFRACTIVE BEAM FANNING

When a laser beam passes through a photorefractive crystal, significant scattering often occurs. The scattered light appears to be asymmetrical with respect to the beam except for propagation along the c axis. For laser beams of moderate power, the scattered light appears to grow in time and eventually reaches a steady-state scattering pattern. This is known as photorefractive beam fanning.

The beam fanning originates from the energy coupling between the incident beam and the scattered beam. Scattering due to surface roughness, index inhomogeneity, impurity in solids, etc. occurs when a beam of light passes through a photorefractive crystal. If the incident beam has a finite transverse cross section, the scattered beam will have an overlap with the incident beam. Since the scattered beam has a large number of spatial components, the overlap leads to the formation of a large number of photo-induced gratings. Depending on the orientation of the crystal, some of the spatial components may be amplified by the incident beam via the energy transfer in two-beam coupling. The amplified scattering can be quite significant, as many of the photorefractive crystals exhibit strong coupling strength (i.e., $\gamma L \gg 1$) even though the initial scattering is quite small.

Beam fanning is inevitable, even if the material is near perfect. Nature phenomena, such as Rayleigh scattering and Brillouin scattering, are always present in optical media. Even if the initial scattering is as small as 10^{-8}, the amplified scattering can be very significant, considering the photorefractive gain length product of $\gamma L = 20$ or more. Photorefractive beam fanning plays an important role in the initiation of many phase conjugators and resonators, even though the fanning itself can be a source of noise in many experimental measurements.

PHASE CONJUGATION

Phase conjugators are optical devices that can generate a time-reserved replica of an incident electromagnetic wave (4). These devices play an important role in many optical systems that require the transmission of optical waves through scattering media such as atmosphere. Nonlinear optics offers techniques that can be used to generate phase conjugate waves in real time. Photorefractive crystals such as $BaTiO_3$, SBN, and BSO are by far the most efficient media for the generation of phase conjugate waves.

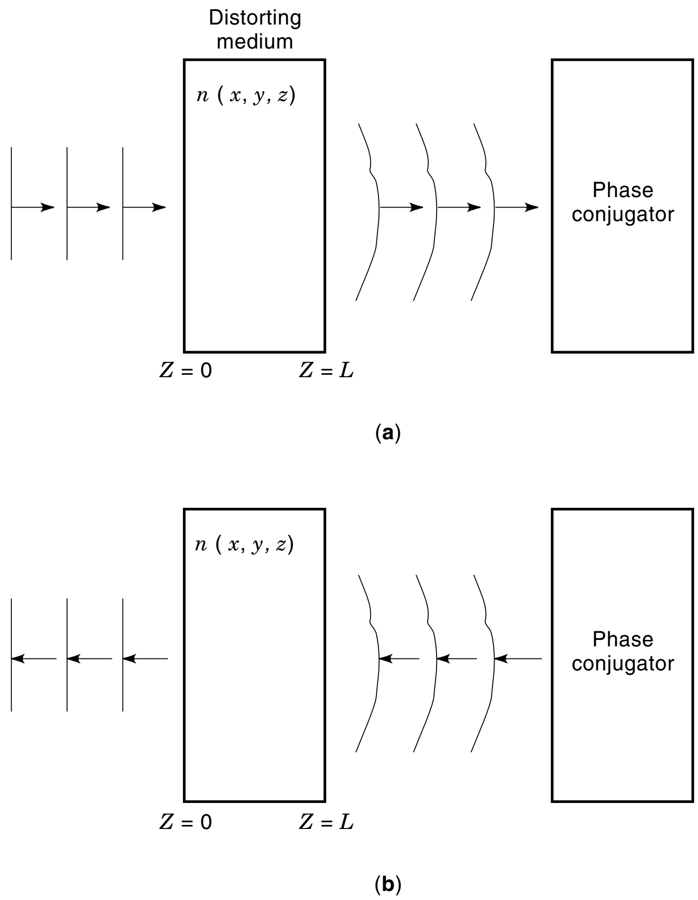

Figure 5. Optical phase conjugation: (a) incident wave, (b) phase conjugate wave.

Consider the propagation of an electromagnetic wave predominately along the z axis. Let the electric field be written $E = A\cos(\omega t - kz - \phi)$, where the amplitude A and the phase ϕ are real functions of position (x, y, z). If the amplitude is a slowly varying function of z compared with $\cos(\omega t - kz - \phi)$, the propagation of the wave can be easily described in terms of the motion of the wavefronts. The phase conjugate of E is given by $E_c = A\cos(\omega t + kz + \phi)$. E and E_c form a conjugate pair. In other words, one is the phase conjugate of the other. If in some plane $z = z_0$ [say $z_0 > L$ in Fig. 5(a)], we generate, by some means, a wave E_c that is the phase conjugate of E, then E_c will propagate backward and remain everywhere the phase conjugate of E. Figure 5(b) illustrates the propagation of E_c.

Four-wave mixing is a convenient method for the generation of phase conjugate waves. In the method, a nonlinear medium is pumped by a pair of counterpropagating laser beams. When a signal beam is incident into the medium, a fourth beam is generated. This beam is propagating opposite to the signal beam and is a time-reversed replica of the signal beam.

In a self-pumped phase conjugation (SPPC), an incident wave will generate its own phase conjugate wave. There are several physical mechanisms that can yield self-pumped phase conjugation, such as SPPC with resonators, SPPC with

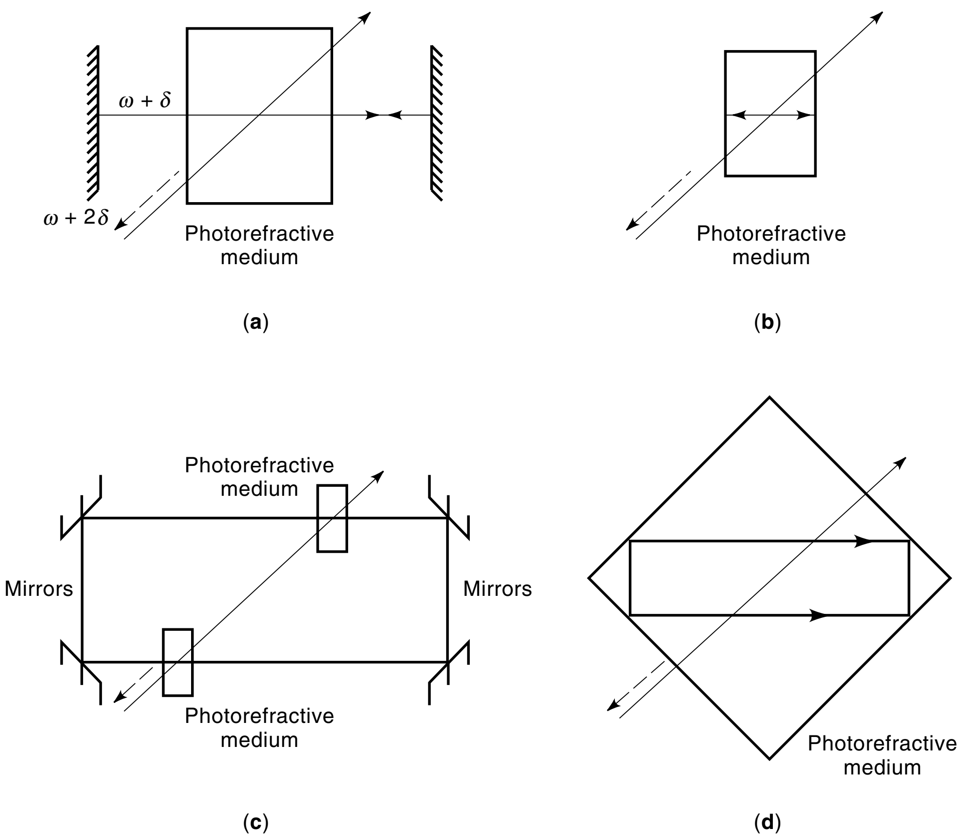

Figure 6. SPPC with resonators.

stimulated backward scattering, ring self-pumped phase conjugator, and SPPC with semiresonators. We will describe some of these phase conjugations separately.

Wave mixing in photorefractive media provides energy coupling between beams of coherent radiation. Significant beam amplification can be achieved under the appropriate conditions. Gain coefficients of $\Gamma L > 10$ are common in many samples of $BaTiO_3$ or SBN crystals. Such gain coefficients represent an enormous intensity amplification [exp(10)] for small signal beams. If such gains are introduced in resonators, oscillation can be easily achieved.

In the case of a linear resonator [see Fig. 6(a)], the gain due to two-wave mixing allows the oscillation to occur. As soon as oscillation starts, four-wave mixing happens simultaneously. This leads to the return of a phase conjugate beam. The oscillation between the two mirrors provides the counterpropagating beams, which are needed in phase conjugation via four-wave mixing. To achieve high fidelity in such a phase conjugator, it is important that a single mode (spatial mode) oscillation is maintained in the resonator. There is a frequency shift associated with such a phase conjugator. Let ω be the frequency of the incident beam. The frequency of oscillation in the resonator is determined by the cavity length. Let $\omega + \delta$ be the frequency of the oscillation. The phase conjugate wave will be shifted in frequency by 2δ. For photorefractive crystals at an operating intensity on the order of 1 W/cm^2, δ is in the range of 1 Hz. Under the appropriate conditions, oscillation can also occur between two parallel faces of a photorefractive crystal, provided the surfaces are polished. Photorefractive crystals such as $BaTiO_3$ and SBN have an index of refraction on the order of 2.5. Fresnel reflectivity at the crystal-air interface is approximately 18%. Oscillation in such a lossy resonator is still possible because of the high gain due to two-wave mixing. Small signal gains in these materials in a 1 cm length can be as high as $\exp(10) \sim \exp(20)$. Such oscillation and self-pumped phase conjugation have been observed in photorefractive SBN crystals. Figure 6(b) shows such a self-pumped conjugator.

SPPC can also be achieved by using bidirectional oscillation in ring oscillators. Referring to Fig. 6(c), we consider a ring oscillator in which gains for bidirectional oscillation are provided. The oppositely directed oscillation beams in ring oscillators provide the pumping needed in four-wave mixing. The process thus leads to self-pumped phase conjugation. Such self-pumped phase conjugation can occur in a single cube of photorefractive crystal under appropriate conditions. As indicated in Fig. 6(d), crystal surfaces provide the mirror reflection needed in bidirectional ring oscillation. In fact, total oscillations occur in many of the oscillations. With an index of refraction around $n = 2.5$, the incident angle for total internal reflection is $\theta > 21.8°$. Thus, ring oscillations in photorefractive crystal cubes involve total internal reflections. These internal ring oscillations only suffer loss due to material absorption and scattering. The frequency of these oscillations is determined by the path length of oscillation and can be shifted relative to that of the pump beam. This leads to a small frequency shift in the phase conjugate wave.

One of the most interesting phenomena in photorefractive media is the strong energy transfer in two-wave mixing. For beams with low intensities, the small signal gain can be as big as $g = \exp(\gamma L)$, where γ is the intensity coupling constant and L is the interaction length. For $\gamma L = 20$, this gain is on

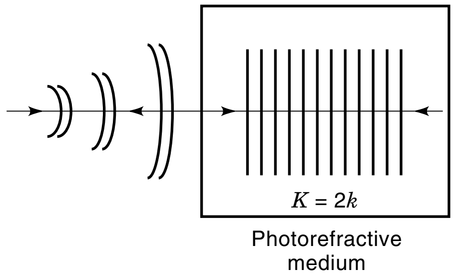

Figure 7. SPPC with stimulated backward scattering.

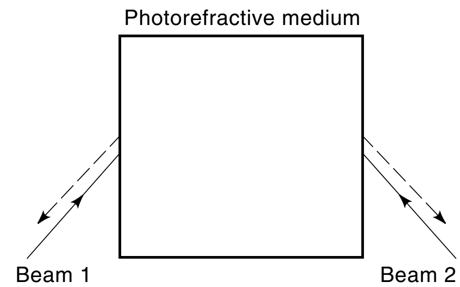

Figure 9. Mutually pumped phase conjugation.

the order of 10^9. In such a high-gain medium, scattering from an incident laser beam can get amplified. This is known as fanning. Under the appropriate conditions, the backward scattered light can get amplified. It has been known for some time that the backward scattered light in stimulated Brillouin scattering (SBS) exhibits phase conjugation. In other words, the wavefront of the stimulated backward scattering is identical to that of the incident wave. Such phase conjugation also occurs in photorefractive media. Phase conjugation via stimulated photorefractive backscattering is sometimes referred to as phase conjugation via $2k$ gratings. The photorefractive gratings in such phase conjugation have a grating wave vector of $\boldsymbol{K} = 2k$. Figure 7 depicts a self-pumped phase conjugation via $2k$ gratings. We also note that in practice, when a laser of finite width is used, backscattering with $2k$ gratings has the longest interaction length with the incident beam.

When a laser beam passes through a high-gain photorefractive crystal, beam fanning will occur. If the fanning beam hits the corner of the crystal, it will be retroreflected. The forward-propagating fanning beam and the backward-propagating fanning beam provide the counterpropagating beams that are needed in phase conjugation via four-wave mixing. This leads to the return of a phase conjugate beam. This kind of SPPC, as is shown in Fig. 8, is often referred to as a two-interaction-region SPPC.

In mutually pumped phase conjugation, two incident laser beams (mutually pumped laser beams) pump the photorefractive medium and produce phase conjugation. Referring to Fig. 9, we consider the incidence of two beams, one at each face of the photorefractive medium. As a result of mutual phase conjugation, two phase conjugate waves are generated. The process can be explained in terms of self-oscillation in four-wave mixing, the resonator model, or hologram sharing.

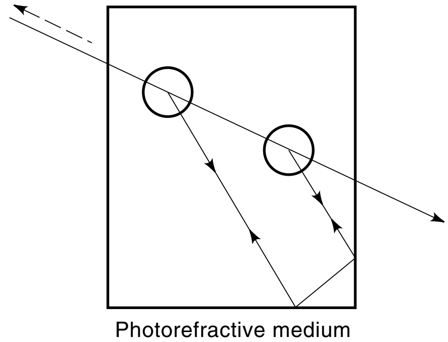

Figure 8. TIR-SPPC. The circles indicate the two interaction regions where four-wave mixing occurs.

APPLICATIONS

Photorefractive materials can be used as holographic recording media. When the complex amplitude of an object is recorded in the medium by interfering the object beam and a reference beam within the holographic recording medium, a photo-induced periodic index variation is produced. The recorded interference pattern is referred to as a hologram, meaning whole information or total recording. A three-dimensional image of the original object can be reconstructed by directing the reference beam onto the recorded hologram. Multiple two-dimensional images can be recorded in a common volume of the holographic recording medium. This is referred to as volume holographic memory. Volume holographic memory offers many desirable features, including compactness, high storage density, and fast parallel access. The information storage capacity of such a system is ultimately limited by the geometric factor $O(V/\lambda^3)$, where V is the volume of the holographic medium and λ is the wavelength of light (4). The method of recording and readout of multiple holograms in a common volume of the holographic recording medium is referred to as a multiplexing scheme. Examples of multiplexing schemes include wavelength multiplexing, angle multiplexing, phase multiplexing, shift multiplexing, and peristrophic multiplexing. In wavelength multiplexing, each hologram is recorded and read out with a unique wavelength (see Fig. 10). The optimal configuration for wavelength multi-

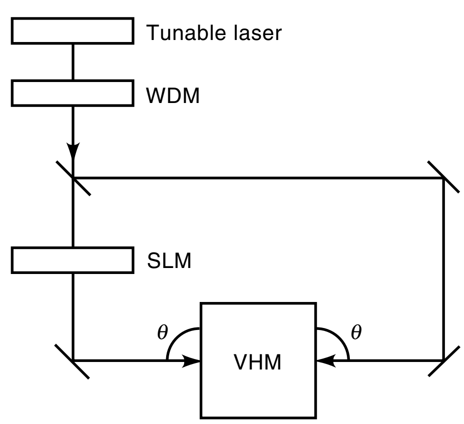

Figure 10. Wavelength multiplexing. WMD: wavelength multiplexing device (tunable filters, e.g., gratings); SLM: spatial light modulator; VHM: volume holographic memory; θ: incident beam angle. Light is wavelength multiplexed by the WMD, encoded by the SLM, and holographically recorded in VHM.

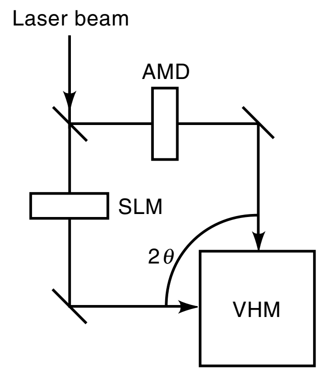

Figure 11. Angle multiplexing. AMD: angle multiplexing device; SLM: spatial light modulator; VHM: volume hologrpahic memory; 2θ: full beam angle. Light is angle multiplexed by the AMD, encoded by the SLM, and then holographically recorded in the VHM.

plexing is that the signal beam and the reference beam propagate in opposite directions within the holographic recording medium. The frequency separation between adjacent holograms is on the order of c/t in the optimal configuration, where t is the thickness of the medium and c is the speed of light in vacuum. In angle multiplexing, each hologram is recorded and read out with a unique angle between the object beam and the reference beam (see Fig. 11). The optimal configuration for angle multiplexing is that the object beam and the reference beam propagate perpendicularly within the holographic recording medium. The angle separation between adjacent holograms is on the order of λ/t in the optimal configuration.

Image processing has been a subject of considerable interest in signal processing. Electronic digital processing of images is slow because of its serial nature. Optical techniques offer the capability of parallel processing over the entire images. The parallel processing is versatile and inherently faster. The technique of optical image synthesis by the addition and subtraction of the complex amplitude of light was first described by Gabor et al. (6). The basic principle consists of holographically recording the two images such that they are mutually shifted by a phase of 180°. Although there are other techniques of image subtraction, interferometers such as the Mach-Zehnder or Michelson offer convenient ways for

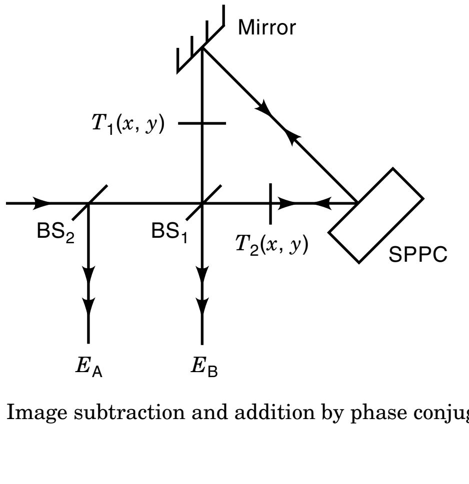

Figure 12. Image subtraction and addition by phase conjugate interferometry.

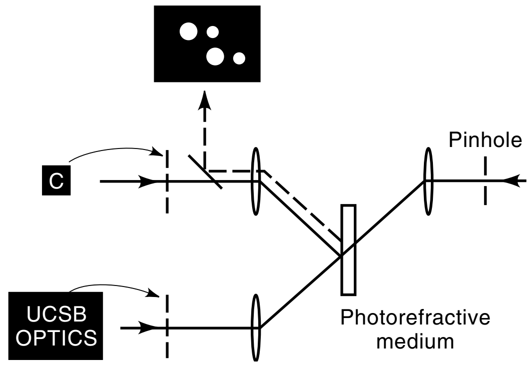

Figure 13. Real-time optical correlator.

the addition and subtraction of the complex amplitude of images. In the interferometric methods the subtraction is obtained by introducing the two images symmetrically in the two arms of the interferometer, where a path difference corresponding to π phase shift exists between them. It is known that the interferometers are extremely difficult to adjust and that they cannot easily maintain the fixed path differences. In addition, only the center fringe is useful for image subtraction and addition. In many cases, the center fringe is not large enough to cover all the images. The method of parallel image subtraction by phase conjugate interferometry eliminates these two problems (see Fig. 12). Referring to Fig. 12, $T_1(x, y)$ and $T_2(x, y)$ are the intensity transmittance of the input transparencies, and E_A and E_S are the electric fields at the output ports. E_A is the addition of the two input images, and E_S is the difference of the two input images. It is important to note that the use of SPPC is only a matter of convenience. Externally pumped phase conjugators work as well. Parallel subtraction and addition of two images have been demonstrated experimentally using $BaTiO_3$ SPPC (7).

Correlation is an efficient method for pattern recognition. Photorefractive crystals can be used to implement real-time optical correlators. An example is shown in Fig. 13. Two image-bearing input beams are Fourier transformed onto a photorefractive crystal. A third input beam is projected onto the photorefractive crystal in the opposite direction of one of the first two input beams. As a result of four-wave mixing, a fourth beam is generated in the opposite direction of the other of the first two input beams. The fourth beam bears the correlation pattern of the two input images. In the case that the two input images are C and UCSB OPTICS, the correlation pattern consists of two bright spots at the locations of C within UCSB OPTICS. The two small spots indicate a weak correlation between C and S. This kind of optical correlator is often referred to as a joint Fourier-transform optical correlator.

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