WAVELENGTH METER

BASIC INFORMATION

Definition of Wavelength for a Monochromatic Source

Wavelength meters are instruments designed to measure the wavelength of an electromagnetic wave. Electromagnetic waves are the classical way to represent modifications of the space surrounding a moving electrically charged body. Time-varying electric and magnetic fields are produced and can be sensed. Wavelength is a well-established parameter when considering a monochromatic (single-frequency) plane wave propagating in vacuum. It is the distance between two planes, measured along the direction of propagation of energy, where the phase and its derivatives have the same values. For the purpose of electromagnetic wave propagation, vacuum is a homogeneous, linear, isotropic, and nondispersive medium. This implies that the reaction of the medium to the electromagnetic wave's presence is instantaneous and does not depend on the location of the observation or on the amplitude and direction of propagation of the wave.

Nonconducting materials, like gases, ceramics, or polymers, are called dielectrics. They have electric charges but the charges are not allowed to move freely within the medium. When an electromagnetic wave moves through such a medium, it stimulates the charges to oscillate around their equilibrium positions. These oscillating charges radiate an electromagnetic field that adds up to the impinging field. The resulting field is the field propagating in the medium. The reaction of the medium will then change the conditions of propagation when compared to those of vacuum. In general, a real medium is nonhomogeneous, nonlinear, nonisotropic, and dispersive. This implies that the reaction will be dependent on location in the medium and the amplitude and direction of propagation of the field, that it will not be instantaneous, and that some of the wave energy will be transferred to other spectrum bands (harmonic generation). In a first approximation, it is useful to reduce the complexity of the medium reaction to something close to that of the vacuum, taking only into account a reduction of the propagation speed associated with the summing action. The speed of the wave in a perfect dielectric material is less than that in vacuum, and the ratio of the speed value in vacuum to the speed value in the medium is the index of refraction. It is then possible to reconsider the propagation of infinite plane waves and use the same definition for the wavelength. Since the speed of propagation is less in the medium than in vacuum, the distance between the planes will be less, which implies that the wavelength in the material is less than that for vacuum. Consequently, the wavelength value is dependent on the medium in which the wave is propagating.

Wavelength or Frequency?

Wavelength is measured along the space (position) axis. But if we measure the field associated with a monochromatic electromagnetic wave at a fixed position, we observe that this field oscillates regularly with time. The rate of this oscillation per unit of time is the frequency. The frequency is then measured along the time axis. The wavelength (λ) times the frequency (ν) is equal to the speed of light in the medium (v).

The two measurements, wavelength and frequency, are dual quantities. They reveal two aspects of the same physical interaction. The wavelength is related to the "wave" aspect of electromagnetic energy while the frequency is related to the "corpuscular" aspect, the photon or quantum of energy. The wavelength is helpful for evaluating the probability of finding photons, which are the unit of exchange of electromagnetic energy with the surroundings during an interaction. The energy E of each photon is $E = h\nu$, where h is the Planck constant. The frequency ν is not dependent on the medium of propagation. The measurement of frequency is preferred by signal engineers, while wavelength is the preferred choice for opticists.

Spectral Distribution

Another important aspect of wavelength measurement is the distribution of optical energy as a function of the wavelength. When a pure monochromatic wave is considered, there is no problem with the definition of wavelength because all the energy is concentrated on a single value. But this is not realistic. Such a distribution would be generated by a noise-free source that evidently does not exist. This brings up a point related to the spectral distribution of the electromagnetic energy for which wavelength has to be measured.

Generally speaking the energy emitted by a source is distributed over a given range of wavelengths or frequencies. This is called the *spectral distribution* of the source. If the spectral distribution is a symmetrically decreasing function around a central wavelength value, this value is defined as the nominal wavelength. If the distribution is not symmetrical and/or reveals many intense peaks, the wavelength definition is more complex. Giving a specific value of nominal wavelength is then somewhat arbitrary and must be justified by stating the prevailing conditions for the selection. In the next section, wavelength (or frequency) definitions will be given more in detail.

Particular cases that will require specifications are modulated and pulsed sources. According to the Fourier analysis of such signals, a periodically modulated source will produce a number of sidelobes, each being the replica of the unmodulated original source while a pulse has a broad spectrum whose width is related to the reciprocal of its time duration.

Instruments Used for Wavelength Measurement

In the subsequent sections, we describe different instruments performing wavelength measurements. First, we present a brief overview of the principle of operation of the two most commonly used instruments, which are the spectrometer-based and the interferometer-based wavelength meters.

If the source has a broad spectrum, spectrometers are the instruments commonly used. The spectrum of the source is first analyzed using a dispersive element such as a prism or

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a grating. The intensity and central wavelength are then recorded by an appropriate detector for a number of dispersive element settings. This method results in a spectral analysis from which a nominal wavelength can be identified if the instrument has been previously calibrated.

Since wavelength is defined in the space domain, its accurate measurement will be done essentially through interferometric phenomena. Let us first consider a monochromatic source. The source signal is first split into space, allowing the various parts to travel different trajectories and recombine to produce interference patterns. Calibrated measurement of the periodicity revealed in the pattern gives an indication on the wavelength. We will later describe in detail the principle of operation of the most commonly used type of interferometers used to measure wavelengths: Michelson, Fizeau, and Fabry–Perot interferometers.

Before we proceed with the description of the different wavelength meters encountered in research laboratories and in the industry, the next section gives some information regarding basic electromagnetic wave theory as well as useful wavelength-measurement definitions.

BASIC ELECTROMAGNETIC WAVE THEORY AND USEFUL WAVELENGTH-MEASUREMENT DEFINITIONS

Theoretical Background

The resolution of Maxwell's equations in vacuum gives rise to plane electromagnetic waves. Such waves are characterized by an electric field (units of volts per meter) and a magnetic field (units of amperes per meter), mutually oriented in perpendicular directions, which are themselves perpendicular to the direction of propagation of the plane wave, given by the direction of the Poynting vector (units of watts per square meter). The latter is defined as the cross-product between the electric and magnetic fields. The modulus of the Poynting vector represents the instantaneous intensity of the light beam. Since both fields are perpendicular to the direction of propagation, such waves are called transverse electromagnetic (TEM) waves. Practical collimated laser beams are often well described by plane waves.

Fluctuating Electric Field

We define a Cartesian component of the electric field labeled $E^{(r)}(\boldsymbol{r}, t)$, which is a real field variable at a point represented by a position vector $\boldsymbol{r}$, at a time t. For any real light beam $E^{(r)}(\boldsymbol{r}, t)$, will be a fluctuating function of time. For example, laser fields exhibit intensity noise as well as phase noise.

In order to simplify the present analysis, we do not use the real field variable $E^{(r)}(\boldsymbol{r}, t)$. We rather consider its corresponding analytic signal $E(\boldsymbol{r}, t)$, which has only spectral components for the positive frequency part of the spectrum. This complex field is defined as

$$E(\boldsymbol{r}, t) = \tfrac{1}{2}E^{(r)}(\boldsymbol{r}, t) + j\tfrac{1}{2}\mathscr{H}\{E^{(r)}(\boldsymbol{r}, t)\} \tag{1}$$

where $\mathscr{H}\{E^{(r)}(\boldsymbol{r}, t)\}$ denotes the Hilbert transform of the real field.

The field intensity at the position $\boldsymbol{r}$ and at time t is defined as

$$I(\boldsymbol{r}, t) = E^{*}(\boldsymbol{r}, t)E(\boldsymbol{r}, t) \tag{2}$$

The field intensity gives the beam power distribution over the beam shape.

We are usually concerned with stationary fields, in which case the statistical properties are independent of the origin of time. Such a field could be characterized by the correlations of the field at two space-time points,

$$\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau) = \langle E^{*}(\boldsymbol{r}_1, t)E(\boldsymbol{r}_2, t + \tau)\rangle \tag{3}$$

where $\langle\ \rangle$ denotes an ensemble average. Since the field is stationary, this ensemble average does not depend on t. This function is also called the mutual coherence function (1, Ch. 4). When evaluated at the same position, it gives the autocorrelation function of the field $\Gamma(\boldsymbol{r}, \boldsymbol{r}, \tau)$ at position $\boldsymbol{r}$. Moreover, it follows that the autocorrelation function evaluated for a zero delay gives the ensemble average of the field intensity at position $\boldsymbol{r}$,

$$\Gamma(\boldsymbol{r}, \boldsymbol{r}, 0) = \langle I(\boldsymbol{r}, t)\rangle \tag{4}$$

The intensity is constant over time for stationary fields.

According to the generalized Wiener–Khintchine theorem, the cross-spectral density is defined as the Fourier transform of the mutual coherence function,

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2, \nu) = \mathscr{F}\{\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau)\} \tag{5}$$

The spectral density, also called the spectrum of the light beam at position $\boldsymbol{r}$, is then the Fourier transform of its autocorrelation function,

$$W(\boldsymbol{r}, \boldsymbol{r}, \nu) = \mathscr{F}\{\Gamma(\boldsymbol{r}, \boldsymbol{r}, \tau)\} \tag{6}$$

So the spread of the spectral density is directly related to the decorrelation time. Short decorrelation time gives rise to broad spectrum and vice versa. Upon proper normalization, the field spectral density could be related to the measured power spectrum.

Usually the spectral density of the field has important components only over a frequency range much narrower than the frequencies of interest. This case is called quasimonochromatic light. For such situations the spectral density of the field is often characterized by two global parameters: one giving the position of the spectrum and the other one estimating its width.

Parameter Definitions

The center frequency could be defined in many ways. We could use the median frequency ν_{m}, which is the frequency for which half the power of the signal is distributed on each side, so

$$\int_0^{\nu_{\mathrm{m}}} W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu = \int_{\nu_{\mathrm{m}}}^{\infty} W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu \tag{7}$$

A common way of specifying the position of the spectrum is the most probable frequency ν_{mp}, which is the frequency at which the spectral density of the field is maximum. Finally the usual frequency value used and measured is the mean frequency, which is

$$\overline{\nu} = \frac{\int_0^{\infty} \nu W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu}{\int_0^{\infty} W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu} \tag{8}$$

The other parameter is the linewidth of the spectral density of the field. We could define the half-power bandwidth $\Delta\nu_{\text{hp}}$ as the symmetrical range around the median frequency containing half the power of the light,

$$\int_{\nu_{\text{m}}-\Delta\nu_{\text{hp}}/2}^{\nu_{\text{m}}+\Delta\nu_{\text{hp}}/2} W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu = \tfrac{1}{2}\int_0^{\infty} W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu \tag{9}$$

The root mean square (rms) width of the spectrum is also often encountered,

$$\Delta\nu_{\text{rms}}^2 = \frac{\int_0^{\infty} (\nu - \overline{\nu})^2 W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu}{\int_0^{\infty} W(\boldsymbol{r}, \boldsymbol{r}, \nu)\, d\nu} \tag{10}$$

Usually we use a more experimentally convenient definition, which is the full width at half maximum (FWHM) width. This width gives the frequency range over which the spectral density exceeds half its maximum value,

$$\Delta\nu = \nu_2 - \nu_1, \quad W(\boldsymbol{r}, \boldsymbol{r}, \nu_{1,2}) = \tfrac{1}{2} W(\boldsymbol{r}, \boldsymbol{r}, \nu_{\text{mp}}) \tag{11}$$

For quasimonochromatic light, the effective width of its power spectrum is much smaller than the mean frequency,

$$\frac{\Delta\nu}{\overline{\nu}} \ll 1 \tag{12}$$

All these definitions also apply to wavelength and to wave number ($\sigma = \nu/v$).

Interference

Let us introduce the concept of interference through a simple experiment. Consider a light beam incident on an opaque screen having two distinct pinholes at positions r_1 and r_2. The light emerging from these pinholes is observed on a second screen distant from the first one. The electric field at a position $\boldsymbol{r}$ on the second screen is the sum of two components,

$$E_s(\boldsymbol{r}, t) = K_1 E(\boldsymbol{r}_1, t - t_1) + K_2 E(\boldsymbol{r}_2, t - t_2) \tag{13}$$

where K_1 and K_2 are constant factors and t_1 and t_2 are the propagation delays from the pinholes,

$$t_1 = |\boldsymbol{r}_1 - \boldsymbol{r}|/c \quad \text{and} \quad t_2 = |\boldsymbol{r}_2 - \boldsymbol{r}|/c \tag{14}$$

The average intensity observed on the second screen is

$$\begin{aligned}\langle I_s(\boldsymbol{r}, t)\rangle &= |K_1|^2 \langle I(\boldsymbol{r}_1, t - t_1)\rangle + |K_2|^2 \langle I(\boldsymbol{r}_2, t - t_2)\rangle \\ &\quad + 2\,\text{Re}[K_1^* K_2 \Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, t_1 - t_2)]\end{aligned} \tag{15}$$

This intensity differs from the sum of the intensities of the distinct components. A supplementary term arises, which takes into account the "correlation" between the two beams. This phenomenon is called interference.

For quasi-monochromatic beams, the mutual coherence function could be expressed as

$$\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau) = g(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau) e^{j2\pi\overline{\nu}\tau} \tag{16}$$

where $g(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau)$ is a slowly varying function of τ, since its bandwidth is much smaller than the mean frequency of the field. In this case, the observed intensity on the second screen is

$$\begin{aligned}\langle I_s(r, t)\rangle &= |K_1|^2 \langle I(r_1, t - t_1)\rangle + |K_2|^2 \langle I(r_2, t - t_2)\rangle \\ &\quad + 2|K_1 K_2||g(r_1, r_2, t_1 - t_2)| \cos[2\pi\overline{\nu}(t_1 - t_2) \\ &\quad + \angle(K_1^*, K_2) + \angle g(r_1, r_2, t_1 - t_2)]\end{aligned} \tag{17}$$

where $\angle$ is defined as the angle. Thus, as we move along the second screen, the intensity varies sinusoidally at the spatial frequency $\overline{\nu}/c$. This variation is the interference fringe pattern. Fringe contrast is measured through their visibility, defined as

$$V(\boldsymbol{r}) = \frac{\langle I_{\text{s}}(\boldsymbol{r}, t)\rangle_{\max} - \langle I_{\text{s}}(\boldsymbol{r}, t)\rangle_{\min}}{\langle I_{\text{s}}(\boldsymbol{r}, t)\rangle_{\max} + \langle I_{\text{s}}(\boldsymbol{r}, t)\rangle_{\min}} \tag{18}$$

where the extreme values are evaluated in the neighborhood of the position $\boldsymbol{r}$.

When the mutual coherence function vanishes, the visibility is 0. This represents complete incoherence. The other extreme case is called complete coherence. In this case,

$$|\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau)| = \sqrt{\langle I(\boldsymbol{r}_1, t)\rangle \langle I(\boldsymbol{r}_2, t)\rangle} \tag{19}$$

which corresponds to the following visibility:

$$V(\boldsymbol{r}) = \frac{2|K_1 K_2| \sqrt{\langle I(\boldsymbol{r}_1, t)\rangle \langle I(\boldsymbol{r}_2, t)\rangle}}{|K_1|^2 \langle I(\boldsymbol{r}_1, t)\rangle + |K_2|^2 \langle I(\boldsymbol{r}_2, t)\rangle} \tag{20}$$

The visibility is 1 if the intensities associated with each beam are equal. Otherwise, perfect contrast is not achieved for complete coherence. In the general case, we experiment partial coherence, so the resulting visibility does not reach 1.

SPECTROMETER-BASED WAVELENGTH METERS

In the next section, we describe instruments based on spectrometers using prism or diffraction gratings. We discuss their performances and limitations and explain their use as wavelength meters.

Basic Properties of Spectrometers

Spectrographs or Monochromators? The difference between the two types of instrument is related to the detection of diffracted/dispersed light: in a spectrograph, a photographic plate or a charge-coupled device (CCD) array is placed in the output focal plane and records the entire spectrum. In a monochromator, a slit and a photodetector are used so that only a portion of the output spectrum is recorded. It is therefore necessary to move the detection system in the output focal plane, or to rotate the dispersive/diffractive element, to ob-

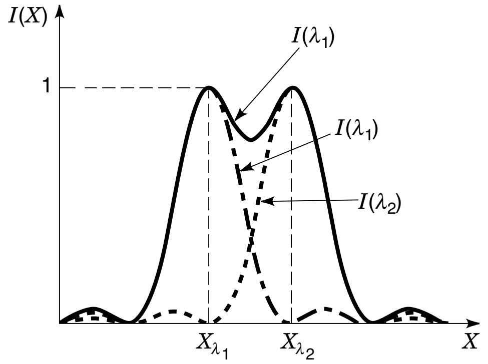

Figure 1. Rayleigh criterion for the definition of the spectral resolving power: two lines are resolved when the intensity profile corresponding to λ_1 has its maximum in coincidence with the minimum of the intensity profile corresponding to λ_2. The interval $\Delta\lambda$ between two resolved wavelengths is the resolution limit.

tain the complete spectrum. Both instruments are often referred to as spectrometers in the literature.

Spectral Resolving Power and Rayleigh's Criterion. The spectral resolving power R is defined by

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\nu}{\Delta\nu} \tag{21}$$

where $\Delta\lambda = \lambda_1 - \lambda_2$ is the smallest separation between two wavelengths λ_1 and λ_2 that the instrument is able to resolve. According to Rayleigh's criterion, illustrated in Fig. 1, two lines are resolved when the intensity profile corresponding to λ_1 has its maximum in coincidence with the minimum of the intensity profile corresponding to λ_2. The interval $\Delta\lambda$ between two resolved wavelengths, known as the resolution limit, can also be expressed in terms of frequency $\Delta\nu$ or wave number $\Delta\sigma$.

The achievable resolving power of a spectrometer is in direct relation with the slit width and its linear dispersion $dx/d\lambda$, which is given by

$$\frac{dx}{d\lambda} = f\frac{d\theta}{d\lambda} \tag{22}$$

where f is the lens focal length and $d\theta/d\lambda$ the angular dispersion. If the entrance slit width is ϵ_1, the slit image in the focal plane of the focusing lens will be

$$\epsilon_2 = \frac{f_2}{f_1}\epsilon_1 \tag{23}$$

where f_1 and f_2 are, respectively, the focal length of the collimating and focusing lenses of the spectrometer. If we want to be able to resolve two lines at λ_1 and λ_2, the separation of their image

$$\Delta x = f_2\frac{d\theta}{d\lambda}\Delta\lambda = \frac{dx}{d\lambda}\Delta\lambda \tag{24}$$

must be greater than $2\epsilon_2$. It is therefore possible to increase the resolving power by decreasing the input slit width (at the expense of decreasing the amount of light to be detected). However, it will be limited because of diffraction due to the finite aperture a of the dispersive element. When the collimated beam is incident on the prism or grating, it experiences Fraunhofer diffraction resulting in an intensity distribution in $\sin^2(\gamma)(\sin(\pi\gamma)/\pi\gamma)$ with γ the diffraction angle. The central lobe of the distribution has a width equal to $2\lambda/a$ and contains 90% of the incident intensity. The distance between the two maxima in the output plane will be $f_2\lambda/a$ so that the condition for resolving λ_1 and λ_2 becomes

$$\Delta x \geq f_2\frac{\lambda}{a} \qquad \text{or} \qquad \frac{\lambda}{\Delta\lambda} \leq a\frac{d\theta}{d\lambda} \tag{25}$$

giving the corresponding resolving power. One must notice that the spectral resolving power is limited by diffraction effects due to the finite aperture a (determined by the size of the prism or diffraction grating) and not by the entrance slit width. When taking into account the entrance finite slit width ϵ_1, the condition is

$$\Delta x \geq \left(f_2\frac{\lambda}{a} + \epsilon_1\frac{f_2}{f_1}\right) \qquad \text{or} \qquad \Delta\lambda \geq \left(\frac{\lambda}{a} + \frac{\epsilon_1}{f_1}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1} \tag{26}$$

However, there is a lower limit for ϵ_1 (2) resulting in a practical resolving power

$$R = \frac{\lambda}{d\lambda} = \frac{1}{3}a\frac{d\theta}{d\lambda} \tag{27}$$

Free Spectral Range. In the case of a spectrometer, the free spectral range is the wavelength region where the instrument has a one-valued relation between the wavelength and the position $x(\lambda)$ in the focal plane of the focusing lens. For prism spectrometers, it corresponds to the whole wavelength range of the instrument, while for grating spectrometers it is related to the diffraction order m.

Prism Spectrometers

Principle of Operation. A typical arrangement for a prism spectrometer is given in Fig. 2. The entrance slit S illuminated by the light source is placed in the focal plane of a collimating lens L_c. The prism P diffracts the incident collimated beam with an angle dependent on the wavelength. As shown

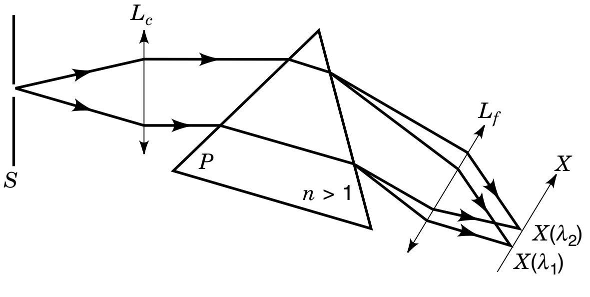

Figure 2. Typical configuration for a prism spectrometer: the light source illuminates the entrance slit S which is placed in the focal plane of a collimating lens L_C. The prism P diffracts the incident collimated beam and a focusing lens L_f images the entrance slit so that the position $x(\lambda)$ of the focal point in the x plane is wavelength dependent.

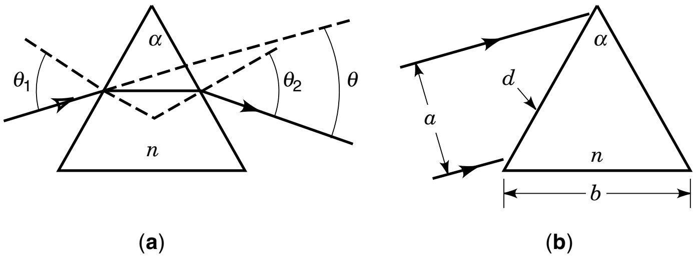

Figure 3. (a) Illustration of a diffraction prism used at minimum deviation: the beam exits the prism with an angle θ verifying the equation $\frac{1}{2}\sin(\theta + \alpha) = n \sin(\alpha/2)$, where α is the prism angle and n is its refraction index. (b) The limiting aperture of the prism a depends on the height of the prism d and on the incident angle θ_1 following the equation $a = d \cos \theta_1 = b[(\cos\theta_1)/2 \sin(\alpha/2)]$.

in Fig. 3(a), the angle θ, relative to the incident direction, depends on the prism angle α, the angle of incidence of the beam θ_1, and the refractive index of the prism material $n(\lambda)$. A focusing lens L_f is used after the prism to image the entrance slit so that the position $x(\lambda)$ of the focal point in the x plane is a function of λ. A wavelength scan can be done either by rotating the prism while keeping the output slit fixed, or the contrary.

Angular and Linear Dispersion. When the prism is used at the minimum deviation [Fig. 3(a)], we have

$$\tfrac{1}{2}\sin(\theta + \alpha) = n \sin(\alpha/2) \tag{28}$$

Then, it is possible to derive the angular dispersion $d\theta/d\lambda$ by first evaluating $d\theta/dn$. We have

$$\frac{d\theta}{dn} = \frac{2\sin(\alpha/2)}{\cos[(\theta+\alpha)/2]} = \frac{2\sin(\alpha/2)}{\sqrt{1 - n^2 \sin^2(\alpha/2)}} \tag{29}$$

The angular dispersion is given by

$$\frac{d\theta}{d\lambda} = \frac{2\sin(\alpha/2)}{\sqrt{1 - n^2 \sin^2(\alpha/2)}} \frac{dn}{d\lambda} \tag{30}$$

We see that dispersion increases with the prism angle α (and so does the size of the prism) but does not depend on the prism size. Therefore, small prisms can be used for small laser beams while keeping the same angular dispersion. However, the prism must be chosen large enough to prevent diffraction problems and to achieve a large spectral resolving power. An equilateral prism with $\alpha = 60°$ is usually chosen as the best compromise. In that case

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{1 - (n/2)^2}} \frac{dn}{d\lambda} \tag{31}$$

For values of n around 1.4 to 1.6, the angular dispersion reduces to

$$\frac{d\theta}{d\lambda} \cong n \frac{dn}{d\lambda} \tag{32}$$

The linear dispersion $dx/d\lambda$ depends directly on the prism material dispersion $dn/d\lambda$ and on the focal length f of L_f. It is given by

$$\frac{dx}{d\lambda} = f \frac{d\theta}{d\lambda} \tag{33}$$

Spectral Transmission and Dispersion of Prism Materials. The transparent spectral range for fused silica prisms is 180 nm to 3000 nm. For shorter wavelengths (VUV region), lithium or calcium fluoride can be used, while for longer wavelengths CaF_2, KBr, or NaCI are transparent up to 30 μm.

The spectral dispersion $dn/d\lambda$ increases greatly near a region of absorption, making glass (e.g., BK7) an attractive material in the visible and near-ultraviolet regions while quartz is more interesting for the ultraviolet region.

Performance and Limitations. If the collimating and focusing lenses L_c and L_f are not achromatic, their focal length will decrease with wavelength. This effect can be compensated by tilting the image plane so that it matches the focal point for every wavelength. If the plane is tilted by an angle φ,the linear dispersion will be increased by a factor 1/cos φ.

The resolving power is

$$R = \tfrac{1}{3} a \frac{d\theta}{d\lambda} \tag{34}$$

where the limiting aperture of the prism a is given by [see Fig. 3(b)]

$$a = d \cos\theta_1 = b \frac{\cos\theta_1}{2\sin(\alpha/2)} \tag{35}$$

with d the height of the input face of the prism and b the length of its base. At minimum deviation, we have

$$n \sin(\alpha/2) = \sin\theta_1 \tag{36}$$

so that

$$\frac{\lambda}{d\lambda} = b \frac{dn}{d\lambda} \tag{37}$$

is only limited by the size of the prism base and the dispersion. For example, if the prism material has a dispersion $dn/d\lambda$ of 500 cm^{-1} at 633 nm and the prism has a base b = 50 mm, then $R \leq \frac{1}{3} \times 2500$. A spectrometer with such a prism is then able to resolve two lines separated by $\Delta\lambda$ = 0.75 nm at 633 nm.

A prism spectrometer is interesting because it allows an unambiguous determination of wavelengths due to the nature of the equation $x(\lambda)$. Its cost can be low when using a small prism and it is simple to make and to adjust. However, as its spectral resolution is limited, it is mainly used for wavelengths scans over large spectral regions as a preliminary survey work.

Grating Spectrometers

Principle of Operation. Many configurations have been demonstrated for grating spectrometers. One possibility is to use an arrangement similar to the prism spectrometer (Fig.

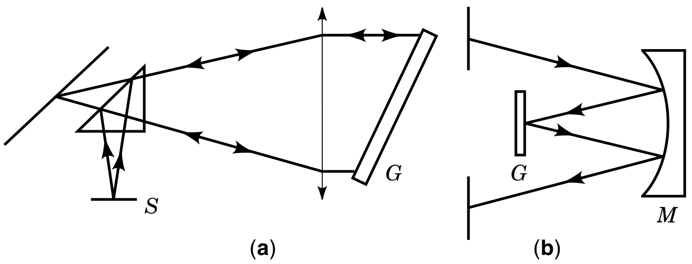

Figure 4. (a) Grating spectrometer using a Littrow mounted grating: the light exiting the input slit is first reflected by the prism and diffracted by the grating. The diffracted light is imaged on the output plane. Note that the slit and the prism are placed in a different plane from the grating so that the light reflected by the grating does not pass twice in the prism. (b) Grating spectrometer using the Ebert mounting configuration: the light exiting the slit is collimated by a concave mirror, diffracted by the grating and imaged in the output plane after a second reflection on the concave mirror.

2) and replace the prism by a diffraction grating used in transmission. Another way is to use the diffraction grating in reflection. Plane diffraction gratings can be used in a Littrow mounting [Fig. 4(a)] or in the Ebert mounting with a concave mirror [Fig. 4(b)]. A variation of the Ebert mounting is the Czerny–Turner mounting where two adjacent concave mirrors are used. Many configurations have also been demonstrated using concave diffraction gratings (3).

The input light is first collimated by a lens or a concave mirror. It is then diffracted by the grating that separates the different wavelengths. The refracted light is focused in the detection plane. If an aperture is used, its width determines the wavelength resolution of the spectrometer.

In every case, the diffraction grating acts as a wavelength-selective reflector, by reflecting light into well-defined directions for each incident wavelength. The different reflected beams correspond to the orders of the grating. The general equation for the grating is

$$d(\sin\alpha \pm \sin\beta) = m\lambda \tag{38}$$

where d is the groove dimension, α and β are the incidence and reflection angles relative to the grating normal, m is the grating order, and λ the wavelength [see Fig. 5(a)]. The amplitude of the wave reflected in the detection β is the superposition of the amplitudes reflected by all the grooves illuminated. The corresponding intensity is

$$I = R_g I_0 \frac{\sin^2(N\varphi/2)}{\sin^2(\varphi/2)} \tag{39}$$

where R_g is the reflectivity of the grating, I_0 the intensity of the incident wave on each groove, N the number of grooves illuminated, and

$$\varphi = \frac{2\pi}{\lambda} d(\sin\alpha \pm \sin\beta) \tag{40}$$

is the phase difference between two adjacent grooves. Figure 5(b) shows I as a function of φ for $N = 10$. Each maximum occurs when $\varphi = 2k\pi$. In terms of β, the line profile has a base half width of

$$\Delta\beta = \frac{\lambda}{Nd\cos\beta} \tag{41}$$

We can evaluate the angular dispersion from the grating equation to obtain

$$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta} = \frac{\sin\alpha \pm \sin\beta}{\lambda\cos\beta} \tag{42}$$

which only depends on α and β and not on the number of grooves. The angular dispersion is reduced to

$$2\tan\alpha/\lambda \tag{43}$$

in the case of a Littrow mounted grating ($\alpha = \beta$).

Possible Monochromator Configurations

A double monochromator is a combination of two single monochromators placed in cascade. This configuration improves the dynamic range, but the wavelength span is reduced and the losses are increased, degrading the sensitivity. Another possible configuration is the double-pass monochromator, which provides the dynamic range of a double monochromator while keeping the sensitivity and compactness of a single one.

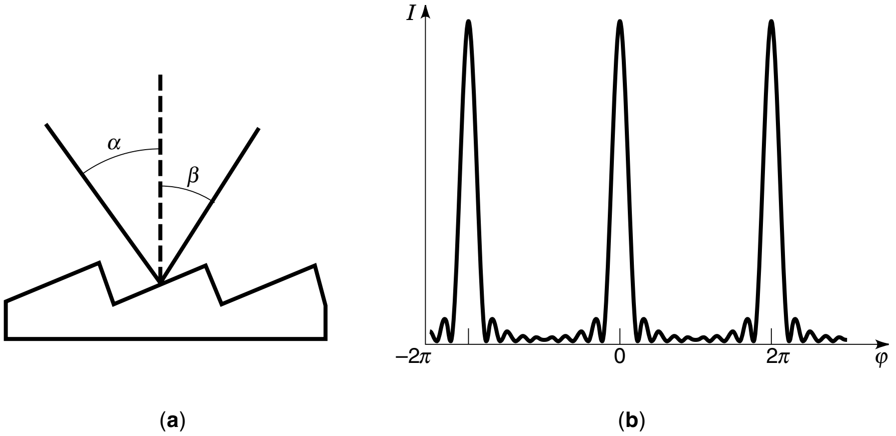

Figure 5. (a) The diffraction of a beam by a diffraction grating follows the equation $d(\sin\alpha \pm \sin\beta) = m\lambda$, where d is the groove dimension, α and β are the incidence and reflection angles relative to the grating normal, m is the grating order and λ the wavelength. (b) Intensity distribution of the reflected beam for $N = 10$. The amplitude of the wave reflected in the direction β results from the interference of the beams reflected by all the grooves illuminated. The corresponding intensity is $I = R_g I_0 \sin^2(N\varphi/2)/\sin^2(\varphi/2)$.

Performance and Limitations. The resolving power R can be expressed from

$$\Delta\lambda = \Delta\beta \left(\frac{d\beta}{d\lambda}\right)^{-1} \tag{44}$$

so that

$$R = \frac{Nd(\sin\alpha \pm \sin\beta)}{\lambda} = mN \tag{45}$$

R depends on the number of illuminated grooves N and on the diffraction order m used. If the finite slit width and diffraction effects are considered we have:

$$R = \tfrac{1}{3}mN \tag{46}$$

R is limited physically by the dimensions of the grating. It is advantageous to use a high-order m but the grating reflectivity might decrease drastically. For example, a 5 cm × 5 cm grating with 1200 grooves/mm used to first order has a practical resolving power of 2×10^4. When used at 1550 nm, it is capable of resolving $\Delta\lambda = 0.075$ nm.

The main disadvantage of grating spectrometers is the possible ambiguity in interpreting the output spectrum as the different orders will overlap for different wavelengths. A solution consists in using wavelength-selective filters in order to isolate one particular order of diffraction. However, the resolving power is much better than that obtained with a prism spectrometer. It can be increased by using a double-pass configuration where the beam is diffracted once by the grating, then passes into an aperture and diffracted again before being detected.

Wavelength Measurement Using Spectrometers. Both prism and grating spectrometers can be used as wavelength meters. To do so, it is necessary to tune the instrument over the unknown light-source spectrum. Another important aspect is to perform an accurate calibration for the wavelength determination.

One possibility for tuning monochromators is to rotate either the prism or the grating, which causes a different wavelength to be focused on the output slit, placed in front of a detector. Another possibility is to keep the prism or grating fixed and move the slit and photodetector in the output plane. In both cases, the prism or grating angle or the slit–photodetector movement must be controlled precisely and in a repeatable way to allow accurate tuning from measurement to measurement (4).

In the case of spectrographs (no moving parts), the spectrum is recorded at once using a photodetector array, and it is only necessary to know the wavelength calibration along the photodetector array. In order to make sure that the wavelength is correctly assigned, a calibration procedure is required for spectrometers as they do not have an internal wavelength reference. It is necessary to have external reference sources such as spectral lamps or lasers stabilized to atomic or molecular absorption lines. For an accurate calibration, the use of several wavelength references (at least three) is recommended and they should be distributed over the whole spectral range of the instrument. Some commercial instruments now include internal references based on the use of molecular resonances, such as acetylene, to perform internal calibrations (see Table 1).

INTERFEROMETER-BASED WAVELENGTH METERS

The next section describes wavelength meters using interference phenomenon, including Michelson, Fizeau, and Faby–Perot interferometers.

Table 1. Comparative Table of Commercial Instruments

Model	MS9710A	HP 71451B	WA-1500	HP 86120B	7711	LMW-6500B
Company	Anritsu	Hewlett Packard	Burleigh	Hewlett Packard	New Focus	ILX Lightwave
Configuration	Grating spectrometer	Double-pass grating spectrometer	Michelson interferometer	Michelson interferometer	Fizeau interferometer	Colored glass filter
Wavelength range (nm)	600–1750 nm	600–1700 nm	200–650 nm (UV) 600–1800 nm (NIR) 1500–4000 nm (IR)	700–1650 nm	400–1000 nm	400–1100 nm (OMH6722B) 950–1650 nm (OMH6727B)
Absolute accuracy (nm)	±0.05 nm	±0.3 nm	$\pm 1 \times 10^{-7}$	±0.005 nm @ 1550 nm	±0.01 nm	±0.5 nm
Resolving power (nm)	0.07 nm	0.08 nm	NA	0.16 nm @ 1550 nm	NA	NA
Sensitivity	−90 dBm (1.25–1.6 μm)	−90 dBm (1.2–1.6 μm)	−10 dBm (UV) −17 dBm (other)	−25 dBm (0.8–1.2 μm) −40 dBm (1.2–1.6 μm)	−24 dBm (400 nm) −7 dBm (1 μm)	−20 dBm
Maximum input power (dBm)	20	30	20	Not available	11	30
Dynamic range (dB)	70 $\Delta\lambda = \pm 1$ nm	50 $\Delta\lambda > \pm 1$ nm	NA	25 $\Delta\lambda = 0.8$ nm	NA	NA
Meas. rate	0.5 s/500 nm	≥50 ms/40 nm	1 Hz	1 Hz	1–10 Hz	1, 4, or 16 Hz
Input laser requirements	CW or pulsed	CW or pulsed	CW single mode	CW up to 100 wavelengths	CW or pulsed (pulse length ≥ 30 ps)	CW single-mode or pulsed
Calibration source	Internal wavelength Reference (C_2H_2)	External wavelength references	Internal stabilized He-Ne laser	Internal He-Ne laser	Plant calibration	Plant calibration

NA: not applicable.

Classification of Interferometers

Optical interferometers can be used to perform a wide variety of precision measurements such as measurement of length, studies of surface structure, measurement of temperature, pressure, and particle velocities. Those measurements are based on the interference phenomena produced by light waves. Wavelength measurement can also be realized with very high precision. In fact, interferometers are the most accurate instruments for measuring the wavelength of an optical source.

Interferometers can be classified as two-beam interferometers or multiple-beam interferometers according to the number of interfering beams. The most common type of two-beam interferometers used to measure wavelength are the Michelson and Fizeau interferometers. The Fabry–Perot interferometer is the most usual type of multiple-beam interferometers.

Wavelength meters based on interference phenomena can be subdivided in two categories; dynamic and static wavelength meters. The former relies on the displacement of an element. It is the most accurate type of instrument but it can only perform wavelength measurement on continuous wave (CW) sources. On the other hand, static wavelength meters have no moving parts and can be used to measure wavelength of CW or pulsed sources.

Michelson-Based Wavelength Meters

The principle of operation of the Michelson interferometer is presented in Fig. 6. The incident beam is first divided by a 50-50 beam splitter that can be either a partially reflecting metal mirror or dielectric film on a transparent substrate. The two beams are next recombined by the same beam splitter on a detector (or a screen). Since the second beam passes only one time through the beam splitter, a compensating plate may be used in the second arm to equalize the optical path followed by the two beams, but is not always necessary for small collimated beams.

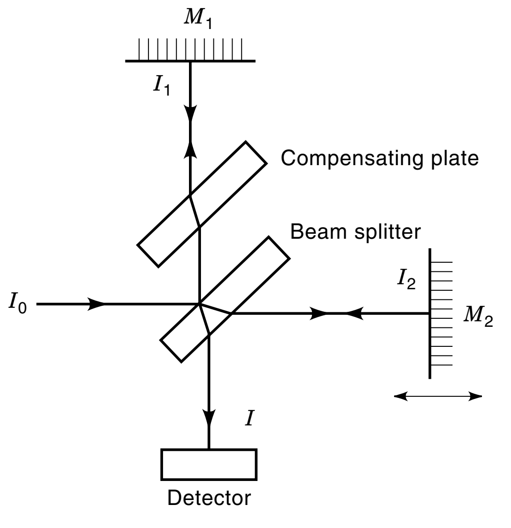

Figure 6. Principle of operation of a Michelson interferometer. An incident beam is divided by a 50–50 beam splitter. The two beams are next recombined by the same beam splitter on a detector. If the mirror M_2 is moved, the intensity of the interference pattern on the detector changes sinusoidally. The wavelength of the incident light beam is deduced if the interference maxima frequency is measured when the mirror M_2 is displaced at constant speed.

If the mirror M_2 is moved by Δx, the intensity of the interference pattern on the detector changes sinusoidally. A complete cycle is observed when the optical path difference between the two arms of the interferometer corresponds to one wavelength of the incident light beam. Thus, when a mirror displacement of $\lambda/2$ is performed, a complete cycle is observed.

The wavelength of the incident light beam can be deduced if the interference maxima frequency is measured when the mirror M_2 is displaced at constant speed

$$\lambda = \frac{2v_{\mathrm{m}}}{f} \tag{47}$$

where v_{m} is the displacement speed of moving mirror M_2 and f is the frequency of apparition of interference maxima.

The main problem with the previously described Michelson-based wavelength meter resides in the determination and the accuracy of the displacement speed of the moving mirror. For that reason, Michelson interferometers are rarely used in that configuration unless low accuracy ($\sim 1 \times 10^{-4}$) is sufficient.

The most common type of Michelson-based wavelength meter uses a two-beam interferometry process (5). In that configuration, a two-beam scanning Michelson interferometer compares the number of fringes obtained by both a reference laser for which the wavelength is known very accurately and an unknown laser. Absolute wavelength measurement of CW sources can then be performed. For the two beams, the optical path d is changed by the same and known amount. A typical optical configuration for that type of interferometer is presented in Fig. 7.

With this type of wavelength meter, the wavelength of the unknown laser can be determined by the following relation:

$$\lambda_{\mathrm{U}} = \frac{N_{\mathrm{R}} n_{\mathrm{U}} \lambda_{\mathrm{R}}}{N_{\mathrm{U}} n_{\mathrm{R}}} \tag{48}$$

where N_{R} and N_{U} are the number of fringes counted on the reference and input photodetectors (corresponding to the reference and unknown laser), n_{R} and n_{U} are the refractive index of the media at the reference and unknown wavelengths, and λ_{R} is the wavelength of the reference laser. From that equation, it is clear that all parameters in Eq. (48) must be known very accurately. The maximum relative uncertainty of the unknown wavelength λ_{U} is the sum of the relative uncertainty of each relevant parameter, which are the wavelength reference, number of fringes, the ratio of the refractive indexes, beam misalignment, and wave-front distortion (2, Ch. 4).

The measurement accuracy can be greatly enhanced by improving the wavelength reference's own accuracy. Wavelength references found in current laboratory instruments are relatively simple He–Ne gas lasers with about 10^{-7} absolute accuracy. A more accurate I_2-stabilized He–Ne laser is often used in laboratory installations where size is of minor concern but is somewhat difficult to implement in a portable instrument. Those type of lasers are calibrated with an uncertainty smaller than 10^{-10}. Recently, some progress has been made in replacing He–Ne lasers at 633 nm by semiconductor lasers,

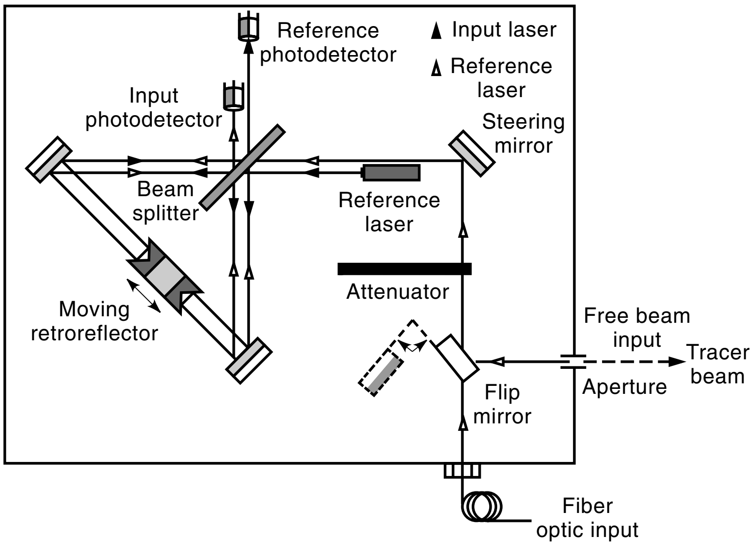

Figure 7. Commercially available two-beam scanning Michelson-based wavemeter™ (courtesy of Burleigh).

providing a more compact and reliable source for the reference laser (6).

Other sources of error for wavelength determination come from the number of fringes seen by the input and reference detectors. One way to improve the accuracy is to count as many fringes as possible; this can be done by increasing the retroreflector mirror displacement. Unfortunately, this also increases the size of the instrument which for a commercial instrument is not suitable. Usually, this technique is implemented in a laboratory environment where space in not a concern. The maximum retroreflector mirror displacement is limited by the coherence length l_c of the reference and unknown laser sources. After a displacement corresponding to one coherence length, no interference pattern can be observed (7). The coherence length is related to the spectral width of the source and is given by

$$l_c = c/\Delta\nu \tag{49}$$

where c is the speed of light in vacuum and $\Delta\nu$ is the spectral width of the source. From that relation, it is clear that the frequency of a source emitting in a very narrow band of frequency can be determined with greater precision than a broad-band source.

Another way to improve the wavelength-measurement accuracy without increasing the path displacement of the retroreflector mirror is to determine a fractional order number of interference fringes. Techniques such as phase-locking an oscillator to an exact multiple of the frequency of the ac signal from the reference laser (8) or using a vernier method in which the counting cycle starts and stops when the two signals coincide (9) have been proposed. With these techniques, fringe fractions can be determined with an uncertainty of $\frac{1}{500}$ of a fringe (10). Moreover, vibrations during the measurement must be reduced to a minimum in order to eliminate the frequency jitter on the fringe signal (11).

To eliminate the dispersion of air, the wavelength meter is often operated in a vacuum chamber. If the instrument is operated in air, the index of refraction depends on the wavelength, the total air pressure, the partial pressures of H_2O and CO_2, and the temperature. All those parameters must be controlled and/or recorded with great precision. Moreover, the ratio of the refractive indices depends on the wavelength difference $\Delta\lambda = \lambda_R - \lambda_U$. The relative uncertainty on the index of refraction depends on the wavelength range. Typically for visible wavelengths, the uncertainty ranges from 10^{-11} ($\Delta\lambda = 1$ nm) to 5×10^{-9} ($\Delta\lambda = 200$ nm).

There are other sources of systematic errors that can influence the achievable accuracy of a two-beam scanning Michelson-based wavelength meter. One of them is the misalignment of the two beams that causes them to travel slightly different path lengths. As an example, if the two beams are slightly tilted against each other by 10^{-4} rad, the systematic relative error becomes 5×10^{-9}. In the wavelength meter proposed in Fig. 7, the corner cube retroreflector guarantees that the incoming light beam is reflected exactly parallel to its incident direction regardless of a slight misalignment.

Finally, the quality of the optical components can also limit the measurement accuracy. With a surface quality of $\lambda/10$, wavefront distortions are already visible in the interference pattern (2). Moreover, to minimize diffraction effects particularly important in the infrared region, a large beam diameter should be used. The uncertainty due to diffraction is inversely proportional to the square of the beam diameter (11).

Laboratory and Commercial Instrument. In a laboratory experiment, Ishikawa, Ito, and Morinaga demonstrated a wavelength meter with an accuracy of 4×10^{-10} (10). In that experiment, the main limitation was caused by a slight optical misalignment between the two beams. An improved version of that wavelength meter was later designed (11). That time the wavelength uncertainty was evaluated at 7×10^{-11} and was limited by the accuracy of the I_2-stabilized He–Ne laser.

For the most accurate currently available commercial instrument (Burleigh WA-1500), the wavelength of the unknown laser source can usually be determined to an accuracy of 10^{-7}. In that case, the accuracy is limited by the reference laser, which is a He–Ne gas laser stabilized on its gain curve. Temperature and pressure sensors are used to evaluate the index of refraction of air as the interferometer is not evacuated. Figure 8 presents a picture of that instrument.

Figure 8. Front view of the Burleigh WA-1500 Michelson-based wavemeter (courtesy of Burleigh).

Multiwavelength Measurement. One of the disadvantages of the previously described wavelength meter is that it can only perform a single wavelength measurement. However, the previous configuration can be modified to perform multiwavelength measurement. This task is particularly suited to dense-wavelength division multiplexing (DWDM) communication systems. The fringe pattern produced by the unknown laser can be sampled at regular intervals corresponding to the period of the refererence-laser interference pattern. A Fourier transform on those data yields the complete spectrum. The resolution or the minimum frequency spacing $\Delta\lambda$ that can be measured by the instrument is directly related to the number of counted fringes (12)

$$\frac{\lambda}{\Delta\lambda} = 2N \tag{50}$$

As for the accuracy, the resolution can be improved by counting as many fringes as possible; this can be done by increasing the retroreflector mirror displacement.

A motionless Michelson interferometer with a fixed path difference Δs can also be used to measure the wavelength of pulsed sources (13). In such a wavelength meter, the incident signal enters the interferometer polarized at 45°. A phase difference $\Delta\phi = \pi/2$ is next introduced between the two polarized components. The interference signal at the exit of the interferometer is recorded separately for both polarizations. From the two interference signals obtained, it is possible to deduce the wave number $\sigma = 1/\lambda$ modulo $1/\Delta s$ since all wave numbers $\sigma_m = \sigma_O + m/\Delta s$ (m =1, 2, 3, . . .) give the same interference signals. If similar interferometers having a path difference in geometric ratios are used, the wave number can be deduced without ambiguity. Since such an instrument measures the wave number it is called a *sigmameter*.

Fizeau-Based Wavelength Meters

Fizeau interferometers can be used to perform wavelength measurement (14). Those static wavelength meters have no moving parts and can measure the wavelength of CW and pulsed sources.

Fizeau interferometers consist of two plates inclined to form a wedge of angle α (see Fig. 9). When plane monochromatic light is incident on that type of interferometer, reflected light forms a series of uniformly spaced interference fringes

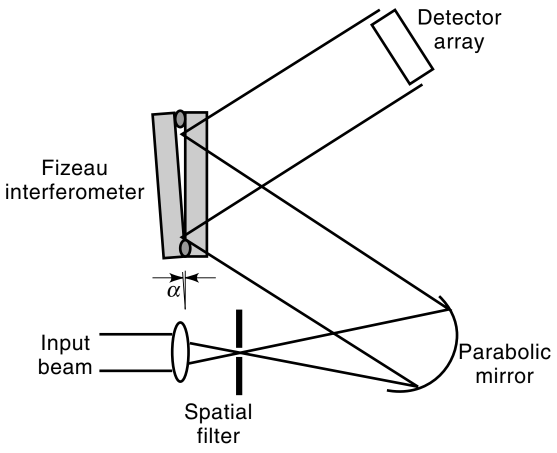

Figure 9. Principle of operation of a Fizeau-based wavelength meter. Fizeau interferometers consist of two plates inclined to form a wedge of angle α. When plane monochromatic light is incident on that type of interferometer, reflected light forms a series of uniformly spaced interference fringes for which the intensity will vary. The fringes produced are imaged on a detector array. The fringe period is computed and gives a wavelength measurement.

for which the intensity will vary as

$$I(x) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi x}{\Lambda} + \varphi\right)\right] \tag{51}$$

where $\Lambda = \lambda/(2\alpha)$, x is the distance along the detector array, and ϕ is the phase at $x = 0$. The fringes produced are imaged on a detector array. With a typical interferometer plate spacing of 1 mm and a wedge of 3 min, an accurate measurement of the fringe period can give a wavelength measurement accuracy of 10^{-4}. Moreover, if the phase of the interference pattern on the detector array is determined accurately (15), the accuracy can go as high as 10^{-7}.

Such an accurate measurement can only be accomplished if the wedge angle and spacing are previously calibrated. Also, the fringe pattern on the detector must approximate as much as possible the sinusoidal pattern of Eq. (51). This is done by ensuring that the incident light has plane-wave fronts. This is accomplished by spatially filtering the incident light and by using interferometer plates with excellent flatness. Dispersion effects must also be taken into account for very accurate wavelength measurement. The advantage of the Fizeau wavelength meter is its simple, compact, and robust design. However, the accuracy of such a wavelength meter is usually less than a scanning Michelson-based wavelength meter.

Figure 10 presents a commercially available Fizeau-based wavelength meter. It is worth mentioning that this instrument contains a thin fused silica etalon, a precision input aperture, and a silicon CCD array. Due to the type of detector array used, this instrument can perform wavelength measurement from 400 nm to 1000 nm. The accuracy of the instrument is 10^{-5}. The instrument is wavelength calibrated at the factory using five different stabilized lasers. The calibration process allows for effective determination of the etalon thickness and to linearize the detector array signal. All the components are housed in a thermally isolated enclosure. The index of refraction of air is mathematically compensated for temperature and pressure variations.

Figure 10. Front view of the New Focus Fizeau-based wavelength (courtesy of New Focus).

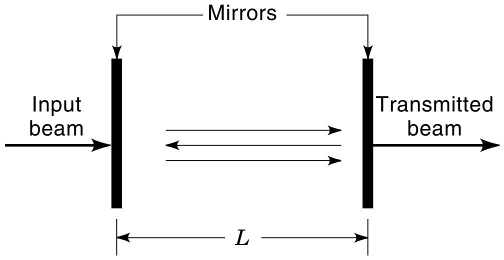

Figure 11. Principle of operation of a Fabry–Perot interferometer. This type of interferometer is made of two highly reflecting mirrors (flat or concave) uniformly spaced by air or a dielectric material. When coherent light is injected on one side of the interferometer, an interference phenomenon is created by the multiple reflections inside the cavity.

Fabry–Perot-Based Wavelength Meters

Scanning Fabry–Perot interferometers can be used to perform wavelength measurement although those instruments are usually used as optical spectrum analyzers. Figure 11 depicts the principle of operation of a Fabry–Perot interferometer. This type of interferometer is made of two highly reflecting mirrors (flat or concave) uniformly spaced by air or a dielectric material (16).

When coherent light is injected on one side of the interferometer, an interference phenomenon is created by the multiple reflections inside the cavity. For that reason, Fabry–Perot interferometers are called multiple-beam interferometers. The interference is constructive and a stationary wave is built when the distance between the two mirrors corresponds to an integer multiple of half wavelengths of the input beam. This condition is given by

$$L = \frac{m\lambda}{2n} \tag{52}$$

where L is the distance between the mirrors, m is an integer, and n is the refractive index of the material inside the cavity. When the condition in Eq. (52) is respected, light transmission through the interferometer is maximum. The frequency spacing between the transmission peaks is called the free spectral range (FSR) and is determined by the spacing between the mirrors and by the refractive index of the material inside the cavity. The FSR (in hertz) is given by

$$\mathrm{FSR} = \frac{c}{2nL} \tag{53}$$

The transmission coefficient of an ideal Fabry–Perot interferometer is described by an Airy function (16)

$$\tau(\lambda) = \frac{I_t}{I_0} = \left[1 + \frac{4R}{(1-R)^2}\sin^2\left(\frac{2\pi nL\cos(\theta)}{\lambda}\right)\right]^{-1} \tag{54}$$

where I_t and I_0 are the input and transmitted intensities, R and T are the mirror reflection and transmission intensity coefficients, and θ is the angle (relative to the mirror plane inside the cavity) of the input beam (usually the input beam

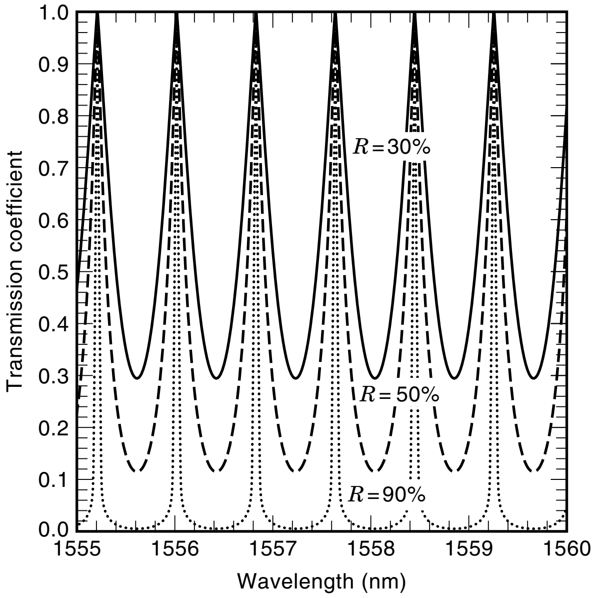

Figure 12. Transmission of an ideal Fabry–Perot interferometer for different mirror reflectivities ($\theta = 0°$, $L = 1.5$ mm, $n = 1$, FSR = 99.9 GHz).

is perpendicular to the mirrors and this angle is 0). Figure 12 presents the transmission of an ideal Fabry–Perot interferometer for various mirror reflectivities at $\theta = 0°$.

From Fig. 12, we see that another parameter, the finesse (F), gives information about the spectral width of each transmission peak. This parameter depends on the mirror reflectivities, optical quality, and alignment. The finesse is defined by

$$F = \frac{\mathrm{FSR}}{\Delta\nu} \tag{55}$$

where $\Delta\nu$ is the full width at half maximum of the transmission peaks. If the finesse is much larger than 1 (which is usually the case), F can be approximated by

$$F = \frac{\pi R^{1/2}}{1 - R} \tag{56}$$

Let us now return to the principle of operation of a scanning Fabry–Perot interferometer. If two signals are incident on the interferometer and if the distance L between the two mirrors is changed, the interference patterns of both signals appear one after the other. For a given FSR, a high finesse allows the resolution of very closely spaced signals. The resolution is given by (12)

$$\frac{\lambda}{\Delta\lambda} = mF \tag{57}$$

where m is the mode number. The resolution can be increased if a high-order mode is used. This can be done by using a large distance L between the mirrors. In that case, the FSR is reduced; this means that the spectral range is also reduced. Depending on the application, trade-offs have to be made. As we mentioned previously, that type of instrument is mainly used to perform high-resolution optical spectrum analysis over a relatively narrow spectral range.

However, by counting the fringes obtained by one reference and one unknown laser while the mirrors are scanned over a relatively long displacement and by using a coincidence technique similar to the vernier technique, wavelength measurement with an accuracy of 1×10^{-7} can be obtained (17). Also, by carefully calibrating the position of the mirrors against wavelength and by carefully controlling parameters such as temperature, pressure and alignment, the instrument could be turned into a wavelength meter. Today, Fabry–Perot-based wavelength meters are widely used for measuring the wavelength of pulsed lasers. In that case, a plane-mirror Fabry–Perot interferometer is operated in a static configuration (etalons). Such etalons, when illuminating with diverging light, produce a characteristic bull's-eye fringe pattern corresponding to different angles of incidence. A CCD camera positioned across the fringe pattern is then used along with a computer to reconstruct the interferometer spectrum. The fringes of a Fabry–Perot etalon are unequally spaced; for that reason, Fizeau etalons, which produce equally spaced fringes, can be used instead of the Fabry–Perot etalon. However, Fabry–Perot etalons have better shaped fringes. Let us also mention that high-accuracy (1 ppm) wavelength measurement of pulsed laser sources can also be performed using two different fixed Fabry–Perot etalons and a reference laser for continuous calibration. In that case, the interference patterns generated by both etalons are recorded by two CCD cameras. The patterns are finally analyzed in a computer and the wavelength is deduced.

APPLICATIONS AND ADVANCED TOPICS

Applications

Scientific Measurements. With the now generalized use of the laser as a scientific investigative tool, wavelength meters have established a firm presence in research and industrial environments. Laser-based measurement techniques reach well beyond the optics field into general physics, chemistry, biology, and engineering. The extremely coherent signal emitted from a laser, whether semiconductor, solid-state, gas, or dye-based, is ideal for probing materials in order to determine their composition or properties, measure dimensions or distances accurately, and much more. In such measurements, the laser wavelength must generally be known precisely, and often the parameter of interest can actually be transformed into a reading of wavelength or wavelength offset. This is where the accuracy of modern wavelength meters as instruments plays a vital role. Wavelength meters in general can also be used for spectral analysis, which expands even further their applicability.

Optical spectroscopy, in its various forms, is useful in a vast number of applications, since it deals with the precise measurement of atomic and molecular absorption or emission features. These features reveal fundamental characteristics of virtually any material, e.g. metals, gases, organic tissue, and crystals (18). Possible wavelength ranges go from visible to infrared, which will guide the choice of a particular tunable laser, for example, semiconductor or dye (19), and of a particular wavelength meter. In a basic measurement, the frequency of an absorption feature is readily obtained by tuning the laser to it and measuring its wavelength with a wavelength meter. Simple spectral analysis can then be achieved by scanning the laser wavelength through the absorption feature and recording the amplitude versus wavelength for each step, as depicted in Fig. 13.

A different application of spectroscopy is the light detection and ranging (LIDAR), used for monitoring aerosols in the atmosphere. A pulsed (nanosecond time scale) laser signal is sent to a target zone, from which backscattered light is captured. Detailed optical frequency/spectrum measurement of this signal yields information about the target atmosphere. Since this is a pulsed application, the typical scanned Michelson interferometer is not suitable and instruments with faster response times using Fabry–Perot or Fizeau intenferometers are likely to be used. Calibrated with a reference He–Ne laser, such an instrument gives information about the absolute wavelength and spectral properties of the backscattered light, which in turn give information about the scattering medium properties.

DWDM Optical Communication. The laser has also enabled an area of significance for electrical engineering, that of optical communication, where wavelength meters find widespread use. After years of steady growth with single-wavelength systems at 0.8 μm, then 1.3 μm, the advent of the erbium-doped fiber amplifier (EDFA) has enabled a massive deployment of systems in the low-loss region at 1.5 μm (20). Accurate knowledge of the wavelength is generally not required in a single-wavelength system. However, technology advances coupled to the ever-increasing demand for bandwidth fueled by the Internet explosion have recently led to industrial implementation of dense wavelength division multiplexing (DWDM). In such systems, up to tens of optical channels are transmitted on a single fiber with frequency spacing currently as low as 100 GHz (about 0.8 nm at 1.55 μm). Typical bit rates for individual channels in current high-capacity transport applications are 2.488 Gbit/s (Sonet OC-48) and 9.952 Gbit/s (Sonet OC-192). (Sonet is a multiplexing format and is the acronym for Synchronous Optical NETwork.)

The 100 GHz standard channel spacing has been endorsed globally through a recommendation of the International Telecommunication Union (ITU). This recommendation actually defines precisely each channel frequency to be used: at this time, 40 channels from $\nu = 192\ 100$ GHz to $\nu = 196\ 100$ GHz applying to the individual transmitters and receivers. Corresponding vacuum wavelengths can be computed with $\lambda = c/\nu$, where $c = 299\ 792\ 458$ m/s, the speed of light. Each frequency (or wavelength) value is in essence absolute: no particular technique for obtaining it is specified. This allows various equipment manufacturers and users worldwide to work with a common set of specifications, easing interoperability while employing potentially different technologies. The frequency tolerance around the absolute value is not standardized rigidly and is evolving as manufacturers and users refine their designs. Current observed tolerances are in the ±10 GHz to ±20 GHz range for a 100 GHz channel spacing.

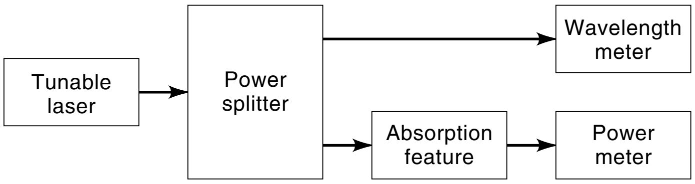

Figure 13. Use of the wavelength meter in a simple absorption spectroscopy experiment. Simple spectral analysis can be achieved by scanning the laser wavelength through the absorption feature and recording the amplitude versus wavelength for each step.

Since every transmitter and receiver must be designed, constructed, verified, and maintained in accordance with such tolerances, the need for accurate and easy wavelength measurements is clear. DWDM transmitters typically use distributed feedback (DFB) semiconductor lasers (21), which present a significant frequency dependence on operational parameters like temperature and current. Since typical coefficients are 10 GHz/°C and 1.5 GHz/mA, and given the tight-frequency tolerance required, it follows that transmitter emission wavelengths need to be calibrated in terms of temperature and current. These parameters also influence markedly the laser's output power, a critical parameter that must also be mapped out.

Wavelength meters as instruments now find widespread industrial use in optical telecommunication manufacturers production facilities at various instances. At the DFB fabrication and testing stage, they are used for selecting a laser operating point (temperature and current within an acceptable range) resulting in a wavelength situated within a desired channel tolerance, this at a predefined power level. Devices for which such a setpoint cannot be found are rejected. At the transmitter subsystem assembly stage, they are used for setting up and verifying the laser initial operating conditions, always ensuring an output wavelength within prescribed tolerances. At the DWDM transmission system setup stage, they are used for final verification of the multiple channel wavelengths. Meters based on FFT techniques providing a multiwavelength function are particularly useful at this point since their spectral analysis capability provides a quick view of the entire transmitted signal. See the HP 86120B instrument in Table 1.

DWDM systems must be designed for very high reliability over many years, and laser frequency aging must be taken into account in order to guarantee that transmitter wavelengths will remain within the specified tolerances. Frequency stabilization external to the laser itself is then required and various schemes have been developed (22–24). The basic approach is to compare the emitted wavelength with some active (reference laser) or passive (reference filter) wavelength standard, determine if the transmitter wavelength is adequate, and take corrective action if not. This will counteract the effect of laser wavelength aging and maintain a precise emission frequency over long periods of time. In a way, this comparison with a wavelength reference implements the wavelength meter function, although it does not necessarily return a reading of the wavelength itself. Of course, one possibility is to monitor the transmitter output

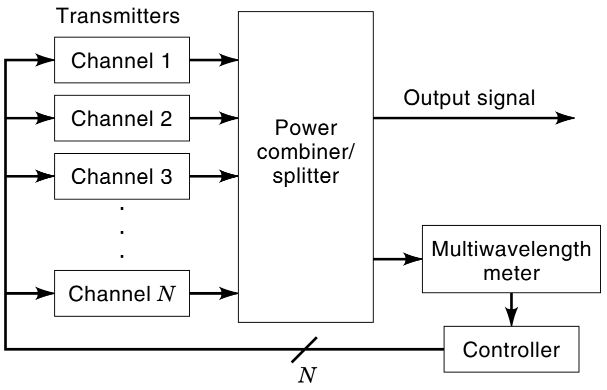

Figure 14. Use of a multiwavelength meter to control simultaneously the frequency of multiple laser transmitters in a DWDM communication system.

with an actual wavelength meter instrument, and instruct a controller unit to slightly modify the laser operating conditions over time so as to maintain a prescribed wavelength. As represented in Fig. 14, a multiwavelength meter allows this concept to be extended simultaneously to every channel in the system (25).

DWDM receivers generally use demultiplexing filters with well-defined transmission responses that must be measured precisely. As in the electrical domain, a simple way to achieve this measurement is to send the signal from a broadband noise source through the filter and look at the transmitted shape on an Optical Spectrum Analyzer (OSA). For the 1.55 μm communication band, an easily available noise source is the amplified spontaneous emission (ASE) found at the output of an EDFA (20). The OSA, typically a grating-based monochromator-type instrument, can measure wavelengths with less accuracy than a typical interferometer-based wavelength meter but offers more advanced spectral analysis capability. Again, wavelength meters with FFT-based spectral analysis capability can be used advantageously depending on the application. However, for very precise transmission versus wavelength measurements, one can revert to the fundamental spectroscopic technique of scanning the wavelength of a tunable laser (typically a laboratory instrument based on an external-cavity laser design (19, Ch. 8), recording simultaneously the transmitted power and the wavelength with, respectively, a power meter and a wavelength meter, exactly as depicted in Fig. 13.

Advanced Topics

Let us now address some foreseen or desirable avenues for improvements of the wavelength meter instrument, as well as discuss the current state-of-the art in wavelength measurement. In stand-alone instruments, one may wish to improve the resolution and accuracy of present units, in order to enable yet more precise measurements of all kinds. Improving the resolution means being able to sense a smaller wavelength change, and for the Michelson interferometer this requires a longer mirror travel. While this can be achieved easily in a laboratory interferometer setup, it is harder to realize in an actual instrument because of size, reliability and cost constraints. For example, a wavelength ratio uncertainty of 7×10^{-11} has been demonstrated in a laboratory configuration presenting a 60 cm mirror travel in vacuum and a 30 mm beam size (11). Mechanically scanned interferometers tend also to have slow update rates, which will likely be slowed down further by longer travels. Simultaneous improvements in resolution and measurement speed appear difficult, although compromises toward either parameter can certainly be achieved.

Improving the measurement accuracy rests first on improving the wavelength reference's own accuracy. Semiconductor lasers stabilized to atomic or molecular features are a foreseen development in future instruments, featuring reduced size, increased reliability, and an improvement in reference accuracy to the 10^{-10} level (23,24). Given an improved reference wavelength, one can then refine the interferometer's intrinsic design. This implies a minimization and a precise characterization of the sources of error inherent in the design, its tolerance to misalignment, and the ultimate effect on the wavelength reading. Again, in the case of a commercial instrument, cost and reliability issues must be addressed at the same time as these refinements.

For the DWDM field, improvement paths may be oriented slightly differently, since the resolution and accuracy of commercial wavelength meters are basically sufficient for the needs of current and near-future systems. Emphasis should then be put on realizing wavelength meters which are smaller, cheaper, and more robust, particularly by developing solid-state designs with no moving parts. Resolution and accuracy may well be traded off for these considerations, depending on the particular application. Such developments might result in the ability to integrate an accurate wavelength- meter function (as well as spectrum analysis) into optical transmission subsystems. Various possibilities for solid-state wavelength meters have been investigated (25,26).

Finally, let us touch on absolute wavelength measurement. Here, the term *absolute* means that the measurement must be related directly to the universal reference for frequency and time, the cesium primary standard. The SI unit of time, that is, the second, is defined with respect to a microwave transition from the ground state of cesium, around 9.192 GHz. Cesium primary standards, or "atomic clocks," exist as standalone instruments, but the most accurate and stable ones are fairly complex installations found at various national laboratories around the world. Their output usually consists of an RF signal in the 10 MHz range. One approach for absolute optical frequency measurement is to synthesize from the cesium reference, through frequency multiplication and heterodyning, phase-locked signals at progressively higher frequencies until the unknown wavelength has been reached (24, Ch. 5). This unknown wavelength can of course be measured with less accuracy prior to the absolute measurement with the help of a standard laboratory wavelength/ optical frequency meter. The final determination of frequency is done by measuring a specific RF or microwave transfer frequency at a particular point in the chain. Such an absolute optical frequency measurement can then be transformed into an absolute vacuum wavelength measurement through the basic relation $\lambda_0 = c_0/\nu$. This was enabled by the 1983 redefinition of a meter by the Bureau International des Poids et Mesures (BIPM) (27), which effectively defined the speed of light in vacuum as c_0 = 299 792 458 m/s exactly. Measurement accuracies of 10^{-12} have been obtained in the visible range and also at the well-established 3.39 μm and 10 μm wavelengths of metrological He–Ne and CO_2 lasers. This is to be compared with the 10^{-7} basic accuracy of the most accurate currently available commercial instrument, the Burleigh WA-1500 depicted in Fig. 8.

Optical comb generation (28), that is the generation of precisely controlled optical sidebands at RF or microwave intervals from a precise reference wavelength, can also be used for creating signals in the vicinity of the unknown signal. This technique has recently been employed for measuring accurately a series of molecular resonances of acetylene in the 1.5 μm communication band (29). An accuracy in the 10^{-9} range was achieved.

CONCLUSION

Wavelength measurement is a fundamental and versatile function in many aspects of science and technology, as optical waves are prevalent in our everyday lives, in the scientific laboratory and in technology-based commercial ventures. As we have seen, wavelength measurement can take various shapes depending on the particular application, and it calls upon diverse aspects of optics and both electrical and mechanical engineering.

One can use calibrated spectrometers based on prisms or gratings, or for better precision, Michelson, Fizeau, or Fabry–Perot interferometers. Each type of instrument presents advantages and limitations, so specific applications will dictate the choice of a particular configuration. Refined laboratory setups built for absolute measurements demand very accurate modeling and optical/mechanical design as well as construction, but size, complexity, and cost constraints are relaxed. Stand-alone laboratory instruments for scientific applications must be engineered so that a suitable level of performance is packaged in an easy to operate, robust, and cost-effective instrument. Finally, further improvements in the design of wavelength meters is leading the way toward the integration of this function into more and more compact subsystems, where size and cost are paramount.

BIBLIOGRAPHY

1. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics,* Cambridge, UK: Cambridge Univ. Press, 1995.
2. W. Demtröder, *Laser Spectroscopy,* 2nd enlarged ed., Berlin: Springer-Verlag, 1996.
3. A. P. Thorne, *Electrophysics,* London: Chapman Hall & Science Paperbacks, 1974.
4. J. N. West and J. D. Knight, A high-resolution direct-drive diffraction grating rotation system, *Hewlett-Packard J.,* **44**(6): 75–79, 1993.
5. F. V. Kowalski, R. T. Hawkins, and A. L. Schawlow, Digital wavemeter for cw lasers, *J. Opt. Soc. Amer.,* **66**: 965–966, 1976.
6. A. Zarka et al., Intracavity iodine cell spectroscopy with an extended-cavity laser diode around 633 nm, *IEEE Trans. Instrum. Meas.,* **46**: 145–148, 1997.
7. M. Born and E. Wolf, *Principles of Optics,* 6th ed., Oxford, UK: Pergamon, 1980.
8. J. L. Hall and S. A. Lee, Interferometric real-time display of cw dye laser wavelength with sub-Doppler accuracy, *Appl. Phys. Lett.,* **29**: 367–369, 1976.
9. A. Kahane et al., Vernier fringe counting device for laser wavelength measurements, *Rev. Sci. Instrum.,* **54**: 1138–142, 1983.
10. J. Ishikawa, N. Ito, and K. Tanaka, Accurate wavelength meter for cw lasers, *Appl. Opt.,* **25**: 639–643, 1986.
11. J. Ishikawa, N. Ito, and A. Morinaga, Accurate wavelength measurement of the intercombination line of calcium, *Jpn. J. Appl. Phys.,* **33**: 1652–1654, 1994.
12. K. D. Moller, *Optics,* Mill Valley, PA: University Science Books, 1988.
13. P. Jacquinot, P. Juncar, and J. Pinard, Motionless Michelson for high precision laser frequency measurement, in J. L. Hall and J. L. Carlsten (eds.), *Laser Spectroscopy III,* Berlin: Springer-Verlag, 1977.
14. J. J. Snyder, Fizeau wavemeter, *Proc. SPIE,* **288**: 258–262, 1981.
15. M. B. Morris, T. J. McIlrath, and J. J. Snyder, Fizeau wavemeter for pulsed laser wavelength measurement, *Appl. Opt.,* **23**: 3862–3868, 1984.
16. B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics,* New York: Wiley, 1991.
17. R. Salimbeni and R. V. Pole, Compact high-accuracy wavemeter, *Opt. Lett.,* **5**: 39–41, 1980.
18. L. J. Radziemski, R. W. Solarz, and J. A. Paisner (eds.), *Laser Spectroscopy and Its Applications,* New York: Dekker, 1987.
19. F. J. Duarte (ed.), *Tunable Lasers Handbook,* San Diego, CA: Academic Press, 1995.
20. I. P. Kaminow and T. L. Koch (eds.), *Optical Fiber Telecommunications III,* San Diego, CA: Academic, 1997.
21. G. P. Agrawal and N. K. Dutta, *Semiconductor Lasers,* 2nd ed., New York: Van Nostrand-Reinhold, 1993.
22. M. de Labachelerie et al., The frequency control of laser diodes, *J. Phys. (France), Sect. 3,* **2**: 1557–1589, 1992.
23. T. Ikegami, S. Sudo, and Y. Sakai, *Frequency Stabilization of Semiconductor Laser Diodes,* Norwood, MA: Artech House, 1995.
24. M. Ohtsu (ed.), *Frequency Control of Semiconductor Lasers,* New York: Wiley, 1996.
25. M. Guy et al., Simultaneous absolute frequency control of laser transmitters in both 1.3 and 1.55 μm bands for multiwavelength communication systems, *J. Lightw. Technol.,* **14**: 1136–1143, 1996.
26. J. C. Braasch, W. Holzapfel, and S. Neuschaefer-Rube, Wavelength determination of semiconductor laesrs: Precise but inexpensive, *Opt. Eng.,* **34**: 1417–1420, 1995.
27. Documents concerning the new definition of the metre, *Metrologia,* **19** (4): 163–178, 1984.
28. M. Kourogi, K. Nakagawa, and M. Ohtsu, Wide-span optical frequency comb generator for accurate optical frequency difference measurement, *IEEE J. Quantum Electron.,* **29**: 2693–2701, 1993.
29. K. Nakagawa et al., Accurate optical frequency atlas of the 1.5-μm bands of acetylene, *J. Opt. Soc. Amer. B,* **13**: 2708–2714, 1996.

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