POWER FACTOR MEASUREMENT

A source of alternating current (ac) delivers electric energy to a receiver at an average rate, over the period of the current, called an active power. The mathematical product of the active power and the time of energy delivery provides the amount of energy delivered. Customers are billed for this energy.

Electric energy receivers operate at a voltage specified by its root mean square (*rms*) value and frequency. The supply source has to provide this voltage. Electrical power systems are built in such a way that this supply voltage has a near to sinusoidal waveform and is almost independent of receivers. While the supply source delivers energy to the receiver, a supply current flows through the source. Similar to the voltage, this current has an rms value.

The mathematical product of the voltage rms value and the supply current rms value is called the apparent power. This power has no physical meaning. The apparent power cannot be lower than the active power, and it is usually higher. When it is, the supply current rms value is greater than that needed for active power delivery.

The ratio of the active power to the apparent power is known as the power factor (PF). Since the active power cannot be higher than the apparent power, the PF cannot be higher than one. A power factor value lower than one indicates that the receiver is loading the supply source with a current of higher rms value than that needed for active power delivery.

The power factor value is a matter of concern for electric power utilities, which provide electric energy to customers. Power system equipment has to have the capability to handle the needed current rms value. At a low power factor, the supply current rms value is higher than that at high power factor. Thus, a low power factor when supplying receivers has several inconveniences for power utilities: (1) To handle currents with higher rms values, distribution system equipment has to be more expensive. (2) When energy is delivered to a customer, a part of it is lost in the distribution system equipment. This loss increases with the square of the supply current rms value, that is, with the decline of the power factor. (3) The supply current causes a voltage drop at the impedance of the distribution system, which reduced the voltage provided to customers. In a case of inductive loads and a sinusoidal supply voltage, the lower the PF, the higher the voltage drop.

Consequently, a decline of the PF increases the investment and operating cost of electric power utilities. Therefore the power factor affects the financial accounts for energy between utilities and customers, especially the large ones. This is the reason for the measurement of power factor. Also, this is the reason for developing methods of power factor improvement.

The power factor is a feature of energy receivers. The PF of single-phase receivers that, when supplied with sinusoidal voltage, do not cause current distortion, depends on the phase-shift between the supply voltage and the supply current. Such receivers are referred to as linear. This phase-shift is the only cause of the power factor decline for linear receivers. The supply current, which is shifted with respect to the voltage, contains two components. One of them is in phase with the voltage. Another is shifted by a quarter of the current's period. Only the component in phase with the voltage contributes to the active power delivery. The remaining one only increases the rms value of the supply current and the apparent power. The PF of linear receivers is equal to the cosine value of the phase-shift angle between the supply voltage and current.

When supplied with nonsinusoidal voltage, linear receivers may cause supply current distortion. Also, nonlinear receivers cause current distortion. In both such situations the supply current cannot be referred to as shifted with respect to the voltage. The power factor of such receivers does not equal the cosine value of any angle. Harmonics of the supply current contribute to an increase of the current rms value and consequently to the decline of the power factor. Only the ratio of the active and apparent powers provides the value of the power factor, as defined by Eq. (1).

The power factor of linear three-phase receivers supplied with a sinusoidal voltage is specified similarly as for single-phase receivers only if the supply voltage is symmetrical and the receivers are balanced. In such a case, the PF is equal to the cosine value of the phase-shift angle between the line current and the line-to-ground voltage.

When impedances of particular phases of a 3-phase system are mutually different, the system is unbalanced. Line currents of a three-phase system with unbalanced receivers contain components that do not contribute to the active power of the load. These components contribute only to an increase of the supply current rms value. Therefore load imbalance reduces the power factor. In asymmetrical systems, only the ratio of the active and apparent powers provides the value of the power factor. Unfortunately, a few different quantities are considered to be the apparent power in three-phase systems. These apparent powers are mutually equivalent only when a three-phase system is symmetrical. When the system is asymmetrical, these apparent powers differ. Consequently, the PF depends on the chosen definition of the apparent power.

Measurements of the power factor in systems where it is equal to the cosine value of the voltage and current phase-shift angle are a relatively simple task. The first power factor meters for such systems were built in the last years of the nineteenth century. Construction of the power factor meters for systems where the power factor can be expressed only as the ratio of the active and apparent powers had to wait, however, until sufficient progress in electronics was achieved.

A power factor meter is a direct-reading instrument for measuring power factor. It is provided with a scale graduated in PF or with a digital display. There are two different types of PF meters: (1) electrodynamic meters (iron-vane or crossed-coil), and (2) electronic (analog or digital) meters.

Single-phase, electrodynamic meters of the power factor, known as cross-coil meters, have two moving coils rigidly connected on a shaft located in the center of the third, a fixed coil carrying the load current. The current in one of the moving coils is proportional to the load voltage. The current in the second coil is shifted by 90° with respect to the current in the first moving coil. The currents flowing through the three coils produce magnetic fields, and consequently, torques are exerted on moving coils. One torque increases with the deflection angle of the coils: the remaining torque declines with that angle. They balance each other at the deflection angle equal to the phase-shift angle between the supply voltage and current. To have the scale of the meter graduated in power factor, the values of the cosine function are allocated to particular deflection angles.

Analog meters can measure the power factor when it is equal to the cosine value of the phase-shift angle as well as when it is only the ratio of the active and apparent powers. In the first case, the difference in the zero crossings of the supply voltage and the supply current provide the phase-shift angle as a fraction of the period, 2π. This fraction is next converted electronically to a cosine value. When the power factor is only the ratio of the active and apparent powers, these powers are converted to dc voltage with electronic multipliers, integrators and rms value to dc voltage converters. These two dc voltages proportional to the active and apparent powers are divided electronically. The output voltage of the divider is proportional to the power factor. It can be measured by a voltmeter with a scale graduated in power factor or with a digital display.

Digital meters calculate the PF based on sequences of digital samples of the supply voltage and current. When sufficiently dense sequences of digitized instantaneous values of the voltage and current are available, all powers, thus also the power factor, can be calculated. Sampling circuits and analog-to-digital (ADCs) converters are the main components of such meters. Also, digital hardware capable of performing calculating algorithms is needed. This can be done by a separate microcontroller or a microcomputer. Digital meters can measure PF if it is equal to the cosine value of the phase-shift angle, or if it is only the ratio of the active and reactive powers.

POWER FACTOR AND ITS MEANING

Power factor λ of a receiver of electric energy is the ratio of active power P of the receiver to its apparent power S, that is, the power factor is defined as

$$\lambda = \frac{P}{S} \tag{1}$$

Active power P is the average rate of energy flow from the supply source to the energy receiver, calculated or measured over one period T of the voltage. In single-phase circuits, the apparent power S is the product of rms values of voltage and current, U and I, at the terminals of the supply source, as shown in Fig. 1. This is the maximum value of active power P that can be delivered at voltage and current rms values U and I. The PF has similar meaning in 3-phase balanced systems. However, various quantities are considered to be the apparent power in three-phase unbalanced systems, and consequently, there is a controversy regarding the meaning of the power factor.

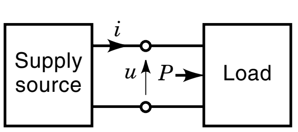

Figure 1. A single-phase load with a supply source and quantities that specify power factor at the supply terminals.

Various features of energy receivers, such as current phase shift, generation of current harmonincs, and current asymmetry of 3-phase equipment cause active power P to be lower than the apparent power S. Therefore, power factor can be interpreted as a measure of the supply source utilization.

POWER FACTOR IN SINGLE-PHASE, 2-PHASE, AND 3-PHASE BALANCED CIRCUITS WITH SINUSOIDAL VOLTAGE AND CURRENT

In single-phase circuits with sinusoidal voltage and sinusoidal current, that is, equal to

$$u = \sqrt{2}U \sin \omega t \qquad i = \sqrt{2}I \sin(\omega t - \varphi) \tag{2}$$

where U and I denote the rms values of the voltage and current and φ denote their phase-shift angle, the power factor is equal to the cosine value of the voltage and current phase-shift angle φ, since

$$\lambda = \frac{P}{S} = \frac{UI\cos\varphi}{UI} = \cos\varphi \tag{3}$$

The phase-shift between supply voltage and current is the only cause for the power factor's decline in single-phase circuits with sinusoidal voltages and currents. The same is true with balanced 2-phase, 3-wire systems, where

$$\lambda = \frac{P}{S} = \frac{\sqrt{2}UI\cos\varphi}{\sqrt{2}UI} = \cos\varphi \tag{4}$$

where U denotes the rms value of the line-to-line voltage, I is the rms value of the line current, and φ is the phase-shift angle between line-to-ground voltage and the line current. Similarly, in balanced 3-phase systems

$$\lambda = \frac{P}{S} = \frac{\sqrt{3}UI\cos\varphi}{\sqrt{3}UI} = \cos\varphi \tag{5}$$

The power factor in single-phase circuits with sinusoidal voltages and currents can be expressed not only in terms of powers but also in terms of rms values of the active and reactive components of the supply current. These rms values, I_a and I_r, are equal to the real and imaginary parts of the complex rms (crms) value of the supply current $\boldsymbol{I}$ at the voltage crms value $\boldsymbol{U} = U\, e^{j0}$. At such a voltage, the supply current crms value is equal to

$$\boldsymbol{I} = Ie^{-j\varphi} = I\cos\varphi - jI\sin\varphi = I_\mathrm{a} - jI_\mathrm{r} \quad \text{with} \quad I = \sqrt{I_\mathrm{a}^2 + I_\mathrm{r}^2} \tag{6}$$

The PF can be expressed as

$$\lambda = \frac{P}{S} = \frac{UI\cos\varphi}{UI} = \frac{I_\mathrm{a}}{\sqrt{I_\mathrm{a}^2 + I_\mathrm{r}^2}} = \frac{1}{\sqrt{1 + (I_\mathrm{r}/I_\mathrm{a})^2}} \tag{7}$$

This result means that the presence of the reactive component in the supply current is the cause of the PF decline.

The power factor in single-phase circuits with sinusoidal voltages and currents can also be expressed in terms of the load parameters. If $\boldsymbol{Y} = Y\, e^{-j\varphi}. = G + jB$ is the load admittance, then the rms values of the active and reactive components of the supply current are equal to $I_a = GU$ and $I_r = |B|U$, respectively, and consequently, power factor can be expressed as

$$\lambda = \frac{1}{\sqrt{1 + (B/G)^2}} \tag{8}$$

If $\mathbf{Z} = Z\, e^{j\varphi} = R + jX$ is the load impedance, then the load conductance G and susceptance B are equal to

$$G = \frac{R}{R^2 + X^2} \qquad B = -\frac{X}{R^2 + X^2} \tag{9}$$

and the power factor can be expressed as

$$\lambda = \frac{1}{\sqrt{1 + (X/R)^2}} \tag{10}$$

Thus, a nonzero reactance (a nonzero susceptance) of the load is the cause of the power factor's decline. The Eqs. (6)–(9) are also valid for balanced 3-phase circuits with sinusoidal voltages and currents.

EFFECTS OF THE POWER FACTOR ON THE SUPPLY SOURCE

Differences in active power delivery at different power factors are illustrated with Fig. 2. Figure 2(a) shows the single-phase equivalent circuit of a supply source and an inductive load. Figs. 2(b) and 2(c) show phasor diagrams of the voltage, current, and voltage drop on the inductance and resistance of the supply for two different PF, $\lambda = 0.95$ and $\lambda = 0.5$, but with the same load active power P. Active power P delivery at a low PF requires a higher rms value I of the supply current as compared with active power delivery at a higher PF. Consequently, the active power loss in the supply source, ΔP_s, equal to

$$\Delta P_s = R_s I^2 \tag{11}$$

increases as the PF declines. The same is true for the voltage drop on the internal impedance of the source, ΔU_s. Moreover, as can be seen in Fig. 2(c), the voltage drop on the reactance $\omega_1 L_s$ at a low PF contributes mainly to the magnitude of the difference between the load voltage u and internal voltage e of the supply. The voltage drop on the resistance R_s contributes to a phase-shift between these two voltages. It is almost opposite at a high PF, namely, as shown in Fig. 2(b), the voltage drop on the resistance contributes mainly to the magnitude of the voltage difference, while the voltage drop on the reactance contributes to the phase shift. The reactance of distribution systems is usually a few times higher than the resistance, and therefore the load voltage declines more strongly when the PF is low than when it is high. Consequently, a low PF has a number of harmful effects on distribution systems:

1. It increases the needed power ratings of the distribution equipment, thus increasing the investment cost.
2. It increases the active power loss in the distribution system, thus increasing the fuel cost.
3. It generally reduces the load voltage. This may require the elevation of the distribution voltage, thus increasing the operational cost needed for voltage regulation. Also, at higher values of the distribution voltage e the active power loss in the distribution system is higher.

These detrimental effects of a low PF, λ, are the reason for which it is measured for energy accounts between power utilities and customers. It is important to observe that all extra costs caused by low power factors are on the power utilities' side. Therefore, PF may affect the energy tariff, or customers may be obliged to pay penalities if the power factor of their loads is too low. This is also the cause for developing various methods of power factor improvement.

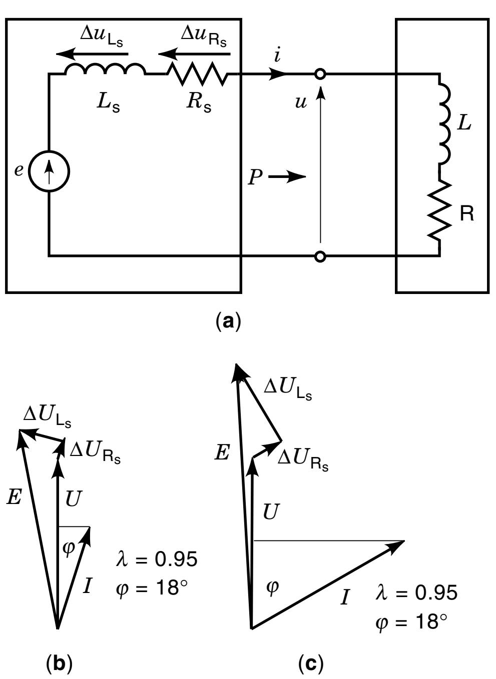

Figure 2. (a) Equivalent circuits of the load and the source, (b) phasor diagram of the circuit with a high power factor, equal to $\lambda = 0.95$, (c) phasor diagram of the circuit with a low power factor, equal to $\lambda = 0.5$. Active power P in both situations is the same but the supply current rms value I and voltage drop ΔU in the supply source is higher at a low power factor.

POWER FACTOR IN 3-PHASE UNBALANCED SYSTEMS WITH SINUSOIDAL VOLTAGES AND CURRENTS

Power factor is the ratio of active to apparent power. The apparent power in 3-phase systems can be defined, however, in various ways. When the system is balanced, these definitions result in the same value; differences appear only when the system is unbalanced. Consequently, the power factor in unbalanced systems depends on the chosen definition of apparent power.

The apparent power in 3-phase systems is defined according to the conclusion (1) of the joint committee of AIEE and NELE (presently, Edison Institute) in 1920. Thus, the apparent power can be defined as

$$S_G = \sqrt{P^2 + Q^2} \tag{12}$$

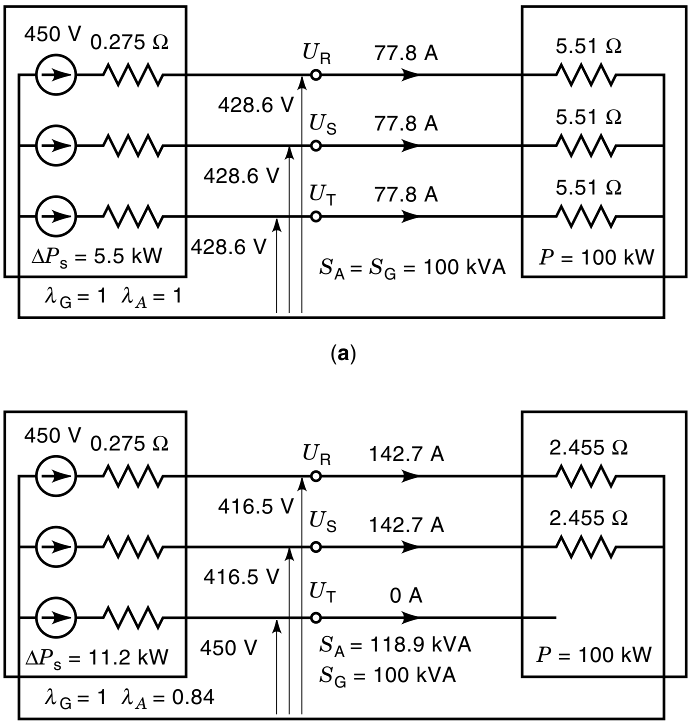

Figure 3. Two 3-phase circuits with the same load active power P = 100 kW, but circuit (a) is balanced while circuit (b) is unbalanced. The load imbalance causes an increase of the active power loss. Arithmetic and geometric apparent powers and power factors in the balanced circuit are the same, but different in the unbalanced circuit.

where Q is the reactive power of the load. This quantity is known as geometric apparent power. It can also be defined as

$$S_A = U_R I_R + U_S I_S + U_T I_T \tag{13}$$

where U_R, U_S, and U_T are the rms values of the line-to-ground voltages, and I_R, I_S, and I_T are the rms values of the line currents. This quantity is known as arithmetical apparent power. The controversy and confusion caused by the presence of different definitions of apparent power with respect to PF measurement is discussed in Ref. 2 and remain unsolved. The latest edition of the *Standard Dictionary of Electrical and Electronic Terms* (3) provides both definitions.

The controversy regarding apparent power and power factor is illustrated here with a numerical example. Figure 3 shows a 3-phase symmetrical source that supplies two customers with the same active power, P = 100 kW. Thus they are billed for the same energy. However, one load is balanced, and the second is unbalanced. The unbalanced load requires higher current ratings of the supply source (142.7 A) than the source that supplies the balanced load (77.8 A). Moreover, the active power loss in the supply source is higher when supplying the unbalanced load (11.2 kW) than when supplying the balanced one (5.5 kW). The increase of the active power loss is comparable to the delivery of the same active power P = 100 kW in the balanced circuit with the power factor of the order of λ = 0.7. However, the geometric apparent power S_G is not affected by the load imbalance; S_G = 100 kVA and $\lambda_G = P/S_G = 1$. The arithmetic apparent power amounts to S_A = 118.9 kVA and $\lambda_A = P/S_A = 0.84$, that is, the arithmetic PF is higher than the PF of an RL load, which causes the same increase of the active power loss. Thus, geometric and arithmetic apparent powers do not have the same properties as the apparent power in single-phase circuits $S = UI$ which is the product of quantities that are responsible for the line and shunt active power loss, $R_s I^2$ and $G_s U^2$.

Three-phase distribution equipment is manufactured as symmetrical devices, thus the line resistance $R_R = R_S = R_T = R_s$. It was shown in Ref. 4 that the active power loss in such a device caused by line currents i_R, i_S, and i_T arranged in a vector

$$\boldsymbol{i} = \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix}$$

can be expressed as

$$\Delta P_s = R_s \frac{1}{T}\int_0^T (i_R^2 + i_S^2 + i_T^2)\,dt = R_s \frac{1}{T}\int_0^T \boldsymbol{i}^t \boldsymbol{i}\,dt = R_s \|\boldsymbol{i}\|^2 \tag{14}$$

where $\boldsymbol{i}^t$ denotes a transposed vector, and

$$\|\boldsymbol{i}\| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{i}^t \boldsymbol{i}\,dt} = \sqrt{\|i_R\|^2 + \|i_S\|^2 + \|i_T\|^2} \tag{15}$$

is the rms value of the 3-phase current vector. A symmetrical 3-phase resistive device with the line currents, i_R, i_S, and i_T, is shown in Fig. 4(a). It is equivalent, with respect to active power loss, to a single-phase device shown in Fig. 4(b) with the current rms value $\|\boldsymbol{i}\|$. Similarly, the active power loss caused by line-to-ground voltages, arranged in the vector

$$\boldsymbol{u} = \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}$$

can be expressed as

$$\Delta P_s = G_s \frac{1}{T}\int_0^T \boldsymbol{u}^t \boldsymbol{u}\,dt = G_s \|\boldsymbol{u}\|^2 \tag{16}$$

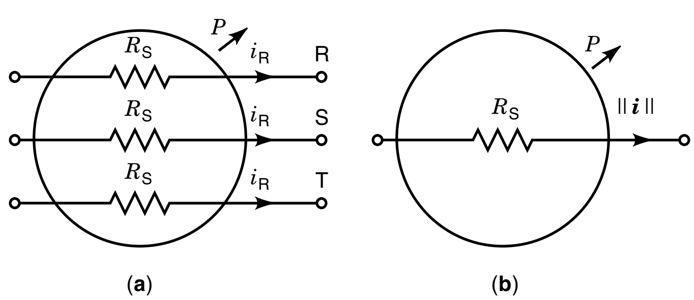

Figure 4. (a) A 3-phase symmetrical device with asymmetrical currents i_R, i_S, and i_T and (b) a single-phase device equivalent with respect to the active power P at the current rms value $\|\boldsymbol{i}\|$.

where

$$\|\boldsymbol{u}\| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{u}^t\boldsymbol{u}\,dt} = \sqrt{\|u_\mathrm{R}\|^2 + \|u_\mathrm{S}\|^2 + \|u_\mathrm{T}\|^2} \tag{17}$$

is the rms value of a 3-phase voltage vector.

Since the rms values $\|\boldsymbol{i}\|$ and $\|\boldsymbol{u}\|$ are related to the active power loss in 3-phase supply sources similarly as the rms values U and I are related to active power loss in single-phase sources, then the apparent power in 3-phase systems with symmetrical distribution equipment should be defined as

$$S = \|\boldsymbol{u}\|\,\|\boldsymbol{i}\| \tag{18}$$

Such a definition was assumed in Ref. 4 for 3-phase systems with nonsinusoidal voltages and currents. For 3-phase systems with sinusoidal waveforms, such a definition was suggested by Buchholz (5) in 1922, but this definition is not referenced by the IEEE Standard (3).

The rms values $\|\boldsymbol{i}\|$ and $\|\boldsymbol{u}\|$ in the previous example are equal to $\|\boldsymbol{i}\| = 201.8$ A, and $\|\boldsymbol{u}\| = 741.2$ V; and apparent power $S = 149$ kVA. The active power loss in the supply source is equal to $\Delta P_\mathrm{s} = R_\mathrm{s}\,\|\boldsymbol{i}\|^2 = 0.275 \times (201.8)^2 = 11.2$ kW. The power factor using such a definition of apparent power is equal to $\lambda = 0.67$.

The increase of the supply current rms value and decline of the PF in 3-phase systems with sinusoidal voltages and currents is caused by the presence of the reactive current. The example considered above shows that PF may also decline in the absence of the reactive current only because of the load imbalance. This conclusion was reported by Lyon in 1920 (6). Reference 7 shows that this is because the current vector $\boldsymbol{i}$ contains not only the active and reactive currents, $\boldsymbol{i}_\mathrm{a}$ and $\boldsymbol{i}_\mathrm{r}$, but also an unbalanced current, $\boldsymbol{i}_\mathrm{u}$. Therefore the supply current in 3-phase systems can be decomposed into three currents, namely

$$\boldsymbol{i} = \boldsymbol{i}_\mathrm{a} + \boldsymbol{i}_\mathrm{r} + \boldsymbol{i}_\mathrm{u} \tag{19}$$

Since the rms values of these currents fulfill the relationship

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}\|^2} \tag{20}$$

then, the power factor can be expressed as

$$\lambda = \frac{P}{S} = \frac{\|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{a}\|}{\|\boldsymbol{u}\|\,\|\boldsymbol{i}\|} = \frac{\|\boldsymbol{i}_\mathrm{a}\|}{\sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}\|^2}} \tag{21}$$

where the rms values of these currents depends on the load equivalent conductance G_e, equivalent susceptance B_e, and unbalanced admittance A; expressed

$$\|\boldsymbol{i}_\mathrm{a}\| = G_\mathrm{e}\|\boldsymbol{u}\|, \qquad \|\boldsymbol{i}_\mathrm{r}\| = |B_\mathrm{e}|\,\|\boldsymbol{u}\|, \qquad \|\boldsymbol{i}_\mathrm{u}\| = A\|\boldsymbol{u}\| \tag{22}$$

Equation (21) shows that the unbalanced current $\boldsymbol{i}_\mathrm{u}$ contributes to the decline of the PF in a manner similar to the reactive current $\boldsymbol{i}_\mathrm{r}$.

POWER FACTOR OF LINEAR LOADS WITH NONSINUSOIDAL VOLTAGE

An increase of the supply current rms value and decline of the PF in linear single-phase circuits with nonsinusoidal voltage is caused by (1) a phase shift of the voltage and current harmonics and (2) by a varying of the load conductance with harmonic frequency. The supply current i in such circuits may contain, according to Ref. 8, not only active and reactive currents i_a and i_r, but also a scattered current i_s, namely

$$i = i_\mathrm{a} + i_\mathrm{s} + i_\mathrm{r} \tag{23}$$

The rms values of these currents fulfill the relationship

$$\|i\| = \sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{s}\|^2 + \|i_\mathrm{r}\|^2} \tag{24}$$

Therefore, the power factor can be expressed as

$$\lambda = \frac{P}{S} = \frac{\|i_\mathrm{a}\|}{\sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{s}\|^2 + \|i_\mathrm{r}\|^2}} \tag{25}$$

The scattered current i_s contributes to a PF decline in a manner similar to the reactive current i_r. However, reactive shunt compensators are incapable of compensating scattered current.

POWER FACTOR OF NONLINEAR LOADS WITH SINUSOIDAL SUPPLY VOLTAGE

The conclusion that the PF can decline because of current distortion, without any phase shift, was drawn for the first time by Steinmetz (9) in 1892 and discussed in Ref. 10.

If the supply voltage is sinusoidal, then current distortion occurs when the load is nonlinear, and/or its parameters change periodically because of periodic switching. The supply current i of such a load may contain not only active and reactive currents i_a and i_r, but also a load generated harmonic current i_g, namely

$$i = i_\mathrm{a} + i_\mathrm{r} + i_\mathrm{g} \tag{26}$$

Since the rms values of these currents fulfill the relationship

$$\|i\| = \sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{r}\|^2 + \|i_\mathrm{g}\|^2} \tag{27}$$

the power factor can be expressed as

$$\lambda = \frac{P}{S} = \frac{\|i_\mathrm{a}\|}{\sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{r}\|^2 + \|i_\mathrm{g}\|^2}} \tag{28}$$

The load generated harmonic current i_g contributes to the decline of the PF in a manner similar to the reactive current i_r.

Active and reactive currents i_a and i_r are sinusoidal currents of the fundamental frequency, that is, $i_\mathrm{a} + i_\mathrm{r} = i_1$. Their rms values are commonly denoted by I_a, I_r, and I_1. Hence, the PF can be expressed as

$$\lambda = \frac{\|i_\mathrm{a}\|}{\sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{r}\|^2}}\,\frac{1}{\sqrt{1 + \dfrac{\|i_\mathrm{g}\|^2}{\|i_\mathrm{a}\|^2 + \|i_\mathrm{r}\|^2}}} = \frac{\lambda_1}{\sqrt{1 + \delta_\mathrm{i}^2}} \tag{29}$$

where

$$\lambda_1 = \frac{\|i_\mathrm{a}\|}{\sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{r}\|^2}} = \frac{I_\mathrm{a}}{\sqrt{I_\mathrm{a}^2 + I_\mathrm{r}^2}} = \cos\varphi_1 \tag{30}$$

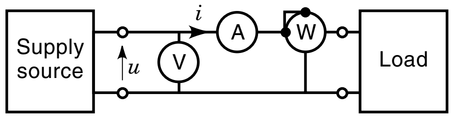

Figure 5. Meter arrangement for power factor calculation in single-phase circuits.

is known as a *displacement power factor*. The angle φ_1 is the phase-shift angle between the supply voltage and the current fundamental harmonic i_1. The coefficient

$$\delta_\mathrm{i} = \sqrt{\frac{\|i_\mathrm{g}\|^2}{\|i_\mathrm{a}\|^2 + \|i_\mathrm{r}\|^2}} = \sqrt{\sum_{n=2}^{\infty}\left(\frac{I_n}{I_1}\right)^2} \tag{31}$$

where I_n denotes the rms value of the n-th order harmonic, specifies the supply current harmonic distortion.

The power factor is affected, according to Eqs. (29) and (30), by the phase shift φ_1 of the voltage and current fundamental harmonics and the current harmonic distortion δ_i. Therefore, the power factor λ alone does not provide (26) sufficient information relative to the loading conditions and the possibility of the power factor improvement. The displacement power factor λ_1 and the current harmonic distortion δ_i have to be known for this. Methods of improvement of the power factor have to take into account their effect both on the displacement power factor λ_1 and on the current distortion δ_i. In particular, capacitive compensation may be ineffective, since an improvement in the displacement power factor is accompanied usually with an increase in the current distortion.

POWER FACTOR MEASUREMENT

To measure the power factor, occasionally it is enough to measure the active power P, the supply voltage and current rms values U and I, and then calculate the apparent power S. The ratio of powers P and S provides the power factor. Such a measurement in single-phase circuits, shown in Fig. 5, provides the power factor whether or not the voltage and current waveforms are sinusoidal. However, such a measurement may provide an incorrect value of the power factor in 2-phase, 3-wire circuits and in 3-phase circuits when the voltages and currents are nonsinusoidal. When waveforms are sinusoidal and the load is balanced, then the meters connected as shown in Fig. 6 and Fig. 7 provide data needed for calculating the power factor in 2-phase and in 3-phase, 3-wire circuits. The PF in a balanced 2-phase circuit with meters connected as shown in Fig. 6 can be calculated as in a single-phase circuit,

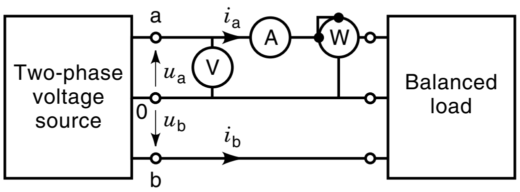

Figure 6. Meter arrangement for power factor calculation in balanced 2-phase, 3-wire circuits.

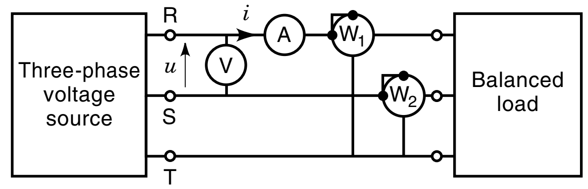

Figure 7. Meter arrangement for power factor calculation in balanced 3-phase, 3-wire circuits.

that is,

$$\lambda = \frac{P}{UI} \tag{32}$$

In the case of a 3-phase balanced circuit with meters connected as shown in Fig. 7, the power factor is equal to

$$\lambda = \frac{W_1 + W_2}{\sqrt{3}UI} \tag{33}$$

where W_1 and W_2 are the values measured by wattmeters. In 2-phase, 3-wire balanced circuits, the power factor can also be calculated from data provided by two wattmeters connected as shown in Fig. 8. Since the voltage coil of the wattmeter W_2 is supplied with a voltage shifted by $\pi/2$, this wattmeter measures the value

$$W_2 = -UI\cos(\varphi + \pi/2) = UI\sin\varphi \tag{34}$$

the ratio

$$\frac{W_2}{W_1} = \frac{UI\sin\varphi}{UI\cos\varphi} = \tan\varphi \tag{35}$$

Hence

$$\lambda = \cos\left(\mathrm{atan}\left\{\frac{W_2}{W_1}\right\}\right) \tag{36}$$

Similarly, the power factor can be measured with only two wattmeters in 3-phase balanced systems. As shown in Ref. 11, wattmeters connected for active power measurement also provide data sufficient for the PF calculation. Wattmeters

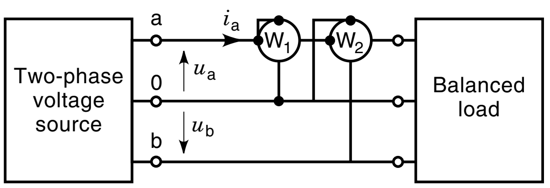

Figure 8. Wattmeter connection that enables PF calculation without measuring voltage and current rms values in 2 phase, 3-wire balanced circuits.

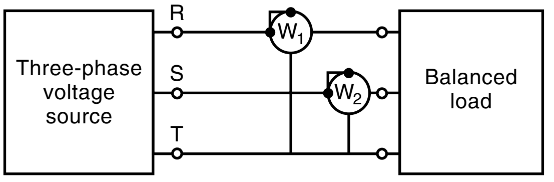

Figure 9. Wattmeter connection that enables PF calculation without measuring voltage and current rms values in 3-phase, 3-wire balanced circuits.

connected as shown in Fig. 9 measure the values

$$W_1 = UI\cos(\varphi - 30^\circ) \text{ and } W_2 = UI\cos(\varphi + 30^\circ) \tag{37}$$

respectively, where U is the rms value of the line-to-line voltage. Hence, the power factor is equal to

$$\lambda = \cos\left(\operatorname{atan}\left\{\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right\}\right) \tag{38}$$

POWER FACTOR METERS

Until a sufficient progress in electronics was made so that the measurement of the power factor according to definition (1) was possible, all PF meters were built only as meters of the phase-shift angle φ between voltage and current. Such meters are scaled in the cosine of this angle, that is, in power factor, or this angle is electronically converted to the power factor.

There are two main types of such power factor meters:

1. Electrodynamic PF meters (iron-vane or crossed-coil PF meters),
2. Electronic PF meters, built as analog or digital instruments.

IRON-VANE PF METERS

An iron-vane PF meter was the first PF meter built in the USA, patented in 1899 (12). There are a number of varieties of construction of iron-vane PF meters. Essentially, such meters are 3-phase instruments; one of them is shown in Fig. 10. It is built of three stationary coils supplied with 3-phase voltage, a stationary coil with the line current, and a movable soft-iron vane with a pointer to a PF-scale. The resistors connected in series with the 3-phase coils are sufficiently large so that the coil currents are approximately in phase with the line-to-ground voltages. The conductors in the coil are distributed such that the radial component of the magnetic field intensity in Θ direction changes as the sine of direction angle Θ, that is, for the coil supplied with the voltage $u_R = U_m \cos(\omega t + \alpha)$, the radial component of the magnetic field intensity is equal to

$$H_R(t,\Theta) = ku_R \sin\Theta = H_{vm}\cos(\omega t + \alpha)\sin\Theta \tag{39}$$

where k is a dimensional coefficient. The voltage coils are distributed around axes rotated by 120° and are supplied with the voltage shifted by 120°. Hence

$$H_S(t,\Theta) = H_m\cos(\omega t + \alpha - 120^\circ)\sin(\Theta + 120^\circ) \tag{40}$$

$$H_T(t,\Theta) = H_m\cos(\omega t + \alpha - 120^\circ)\sin(\Theta + 120^\circ) \tag{41}$$

The resultant radial component in Θ direction, $H_v(t,\Theta)$, of the magnetic field intensity created by the voltage coils is equal to

$$\begin{aligned} H_v(t,\Theta) &= H_R(t,\Theta) + H_S(t,\Theta) + H_T(t,\Theta) \\ &= \frac{3}{2}H_{vm}\sin(\omega t + \alpha + \Theta) \end{aligned} \tag{42}$$

The maximum of this radial component of the magnetic field intensity occurs at direction

$$\Theta = \pi/2 - \alpha - \omega t \tag{43}$$

that is, it rotates with the radial frequency ω. Because of this, these meters are often referred to as PF meters with a rotating field. This term does not distinguish them from crossed-coil PF meters, however, since the crossed coils in those meters also induce a rotating magnetic field.

The current coil induces a magnetic field in the soft-iron vane. Since the vane forms an asymmetrical magnetic path, the maximum field intensity is along the vane. If the current in line R is equal to $i_R = I_m\cos(\omega t + \alpha - \varphi)$, the magnetic field intensity along the vane changes as

$$H_c(t) = H_{cm}\cos(\omega t + \alpha - \varphi) \tag{44}$$

The vane positions in the rotating field in such a direction Θ that the maximum of its magnetic field H_c, which occurs at

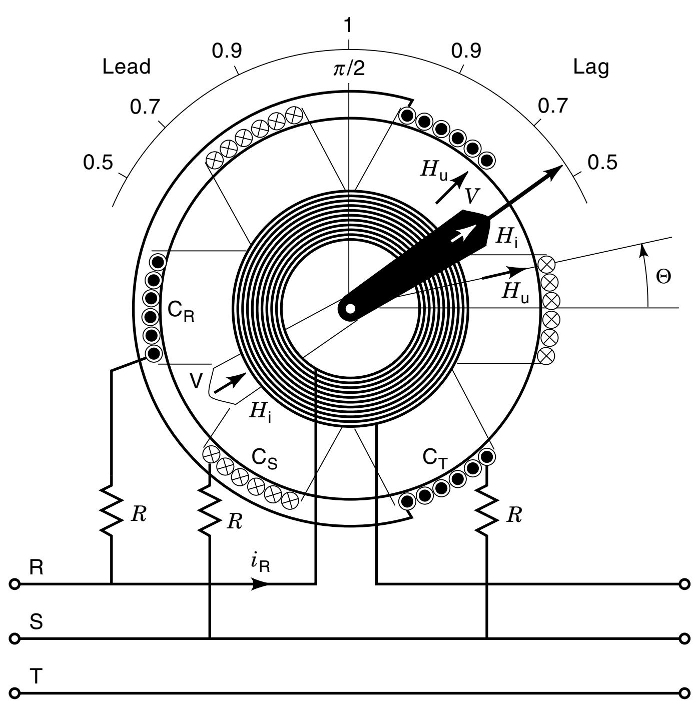

Figure 10. Iron-vane PF meter and its connection to a 3-phase system. Coils C_R, C_S, and C_T create a rotating magnetic field H_u, which deflects the iron vane V with magnetic field H_i, which is created by stationary coil by the angle Θ equal to the phase-shift angle φ.

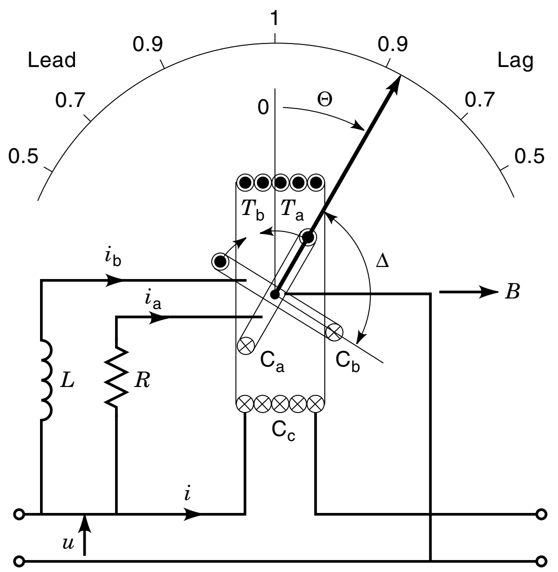

Figure 11. Crossed-coil PF meter and its connection to a single-phase circuit. Movable coils with currents i_a and i_b are deflected in the magnetic field of the stationary coil. Torques T_a and T_b are in equilibrium at deflection angle Θ equal to the phase-shift angle φ.

$\omega t = -\alpha + \varphi$, coincides with the maximum of the rotating field H_v, in that direction, hence,

$$\Theta = \pi/2 - \alpha - (-\alpha + \varphi) = \pi/2 - \varphi \tag{45}$$

Thus, the pointer deflects from the $\pi/2$ angle by the phase-shift angle φ. Cosine values are allocated on the scale to the phase-shift angles φ, so that the device measures the power factor of the load, both for lagging and leading current. Due to properties of the cosine function, the scale of such meters cannot be uniform. It is denser for lower power factors than for higher ones.

Iron-vane PF meters are essentially 3-phase devices, though they also can be built for 2-phase systems, and, equipped with additional phase-shift circuits, even for single-phase systems.

However, accuracy of iron-vane PF meters is low because the space distribution of magnetic fields created by voltage coils may differ substantially from the required sinusoidal distribution. Also, the sine function varies slowly around its maximum. Consequently, because of the friction of the movable vanes, fields H_v and H_c may not align accurately. Iron-vane PF meters for 2-phase and for single-phase systems are even less accurate than 3-phase PF meters, since the magnitude of the magnetic field intensity H_v cannot be kept constant during the field rotation (because it is induced with only two coils). In the case of PF meters for single-phase circuits, a capacitor or an inductor is connected instead of the series resistor to obtain a phase-shift between the magnetic fields produced by these two coils.

CROSSED-COIL PF METERS

Crossed-coil power factor meters are developments from the phase-shift meter developed by Tuma (13) in 1897. Detailed analyses can be found in Refs. 14 and 15.

A single-phase crossed-coil PF meter is shown in Fig. 11. It is built of two movable coils, C_a and C_b, pivoted to rotate freely with a pointer inside of a stationary coil, C_c. Movable coin planes are deflected by a fixed angle Δ, hence these devices are known as *crossed-coil* meters. Movable coils are supplied with the load voltage in such a way that no mechanical torque is exerted on the movable part. Their currents i_a and i_b are mutually shifted, since one coil is supplied through a series resistor R and the other through a series inductor L. The stationary coil induces a magnetic field proportional to the load current. The stationary coil is constructed in such a way that the magnetic field around the movable coils is uniform.

At the supply voltage $u = U_m \sin \omega t$, the currents in coils C_a and C_b are equal to

$$i_a = I_{am} \sin(\omega t - \alpha) \tag{46}$$

$$i_b = I_{bm} \sin(\omega t - \beta) \tag{47}$$

The instantaneous torque exerted on the coil C_a, deflected from the coil C_c plane by angle Θ, at the load current $i = I_m \sin(\omega t - \varphi)$, is equal to

$$\tau_a = k_a i_a i \sin\Theta = k_a I_{am} I_m \sin(\omega t - \alpha)\sin(\omega t - \varphi)\sin\Theta \tag{48}$$

where k_a is a coefficient dependent on the windings and geometry of coils C_a and C_c. The average torque in one cycle T of the supply voltage is equal to

$$T_a = \frac{1}{T}\int_0^T \tau_a\,dt = T_{am}\sin\Theta\cos(\varphi - \alpha), \qquad T_{am} = \frac{1}{2}k_a I_{am} I_m \tag{49}$$

Similarly, the average torque exerted on the coil C_b deflected with respect to coil C_a by angle Δ is equal to

$$T_b = T_{bm}\sin(\Delta + \Theta)\cos(\varphi - \beta), \qquad T_{bm} = \frac{1}{2}k_b I_{bm} I_m \tag{50}$$

If the currents in coils C_a and C_b are such that maximum torques T_{am} and T_{bm} are mutually equal, which requires that $k_a I_{am} = k_b I_{bm}$, then the movable part with the pointer is in equilibrium when

$$T_a = T_b \tag{51}$$

at such a deflection angle Θ that

$$\sin\Theta\cos(\varphi - \alpha) - \sin(\Delta + \Theta)\cos(\varphi - \beta) = 0 \tag{52}$$

In an ideal case, when $\alpha = 0$ and $\beta = 90°$, coil C_b should be deflected with respect to coil C_a by the angle $\Delta = 90°$. Then, Eq. (52) simplifies to the form

$$\sin\Theta\cos\varphi - \cos\Theta\sin\varphi = 0 \tag{53}$$

and results in the deflection angle

$$\Theta = \varphi \tag{54}$$

equal to the phase-shift angle between the supply voltage and current.

Because of some inductance in the coil C_a circuit there is, however, a phase shift between the supply voltage and the coil current i_a, thus $\alpha > 0$. Also, because of a resistance in the coil C_b circuit, the phase-shift angle β between the coil current i_b and the supply voltage is lower than 90°. In such a case, even at $\varphi = 0$, the movable part deflects by an angle Θ_0 which satisfies the equation

$$\sin\Theta_0 \cos\alpha - \sin(\Delta + \Theta_0)\cos\beta = 0 \tag{55}$$

Hence, it is equal to

$$\Theta_0 = \tan^{-1}\left(\frac{\sin\Delta\cos\beta}{\cos\alpha - \cos\Delta\cos\beta}\right) \tag{56}$$

When the angle between coils C_a and C_b is chosen such that

$$\Delta = \beta - \alpha \tag{57}$$

thus the initial deflection Θ_0 becomes independent of Δ and it is equal to

$$\Theta_0 = 90^\circ - \beta \tag{58}$$

If condition (57) is fulfilled, the equilibrium Eq. (53) has the solution

$$\Theta = \Theta_0 + \varphi \tag{59}$$

The meter can be scaled in power factor with respect to the initial deflection angle Θ_0, which can be used for calibrating the meter rotating the stationary coil by that angle, that is, to $\lambda = 1$ at a resistive load.

Figure 12 shows the structure of a 3-phase crossed-coil PF meter. Since there are voltages shifted mutually in 3-phase circuits, the inductor is not needed for shifting the current in the coil C_b. Such a device measures the power factor in balanced circuits with sinusoidal and symmetrical supply voltage. In such a case, the phase-shift angles of the coil currents are $\alpha = -30°$, $\beta = 30°$, respectively. Consequently, coil C_b should be deflected with respect to coil C_a by the angle $\Delta = \beta - \alpha = 60°$. The initial deflection angle, Θ_0, according to Eq. (58), amounts to $\Theta_0 = 60°$. To simplify the scale and the meter design, the pointer is deflected by this angle from the coil C_a plane, as shown in Fig. 12.

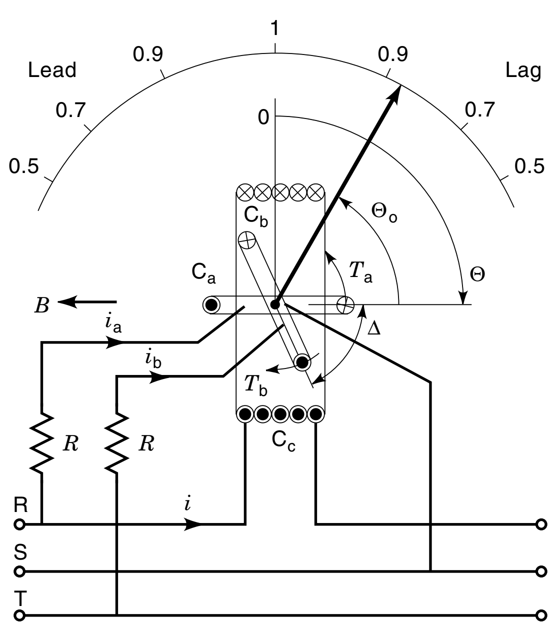

Figure 12. Three-phase crossed-coil PF meter and its connection to the circuit. The pointer is permanently deflected from the coil C_a plane by 60°.

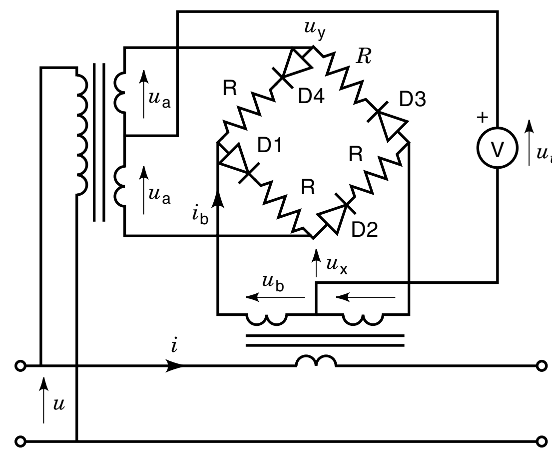

Figure 13. Electronic PF meter with synchronous switch. Diodes D1 and D2 are conducting at positive current i_b. Diodes D3 and D4 are conducting at negative current i_b.

ELECTRONIC POWER FACTOR METERS BASED ON PHASE-SHIFT ANGLE MEASUREMENT

Electronic PF meters are built as analog, digital, or hybrid devices. Analog devices measure the phase-shift angle as an interval of time by detecting the zero crossings of the supply voltage and current. This interval of time, referenced to the voltage period T, is converted next to the cosine value, hence, to the power factor. It can be done with continuous or with digital signals, consequently, the PF meter is referred to as an analog or as a hybrid device. Digital PF meters provide the power factor as a result of a digital algorithm on sequences of digital samples of the supply voltage and current.

The structure of one of the first analog electronic PF meter, described in Ref. 16, is shown in Fig. 13. A synchronous switch, built of four diodes and resistors, is the main component of the device. When current i_b is positive, diodes D1 and D2 are conducting and the voltage $u_x = 0$. The voltage at the voltmeter is equal to

$$u_v = u_x + u_a = u_a \tag{60}$$

When current i_b is positive, diodes D3 and D4 are conducting and the voltage $u_y = 0$. The voltage at the voltmeter is equal to

$$u_v = u_y - u_a = -u_a \tag{61}$$

and has the waveform shown in Fig. 14. The voltmeter measures the average value of this voltage. It is equal to

$$\begin{aligned}\overline{u}_v &= \frac{1}{\pi}\int_{\varphi}^{\pi+\varphi} u_a \,d(\omega t) = \frac{1}{\pi}\int_{\varphi}^{\pi+\varphi} U_{am}\sin\omega t \,d(\omega t)\\ &= \frac{2U_{am}}{\pi}\cos\varphi = k\lambda\end{aligned} \tag{62}$$

Thus, the voltmeter can be calibrated directly in power factor. It has a linear scale. Unfortunately, changes in the voltage amplitude, U_{am}, affect the measurement result. Also, lagging and leading PF cannot be distinguished. Frequency division is suggested in Refs. (17–19) to eliminate the dependence of the measurement result on the voltage. A voltage to frequency converter (VFC) converts the mean value of the voltage u_v into sequence of binary pulses of frequency

$$f_1 = k_1 U_{\mathrm{am}} \cos\varphi \tag{63}$$

At the same time, voltage u_a is rectified and its mean value converted to binary pulses of frequency

$$f_2 = k_2 U_{\mathrm{am}} \tag{64}$$

by another VFC. The device has two binary counters, C1 and C2. Counter C2 specifies the time interval T_2 needed for counting N_2 pulses of frequency f_2, which is expressed

$$T_2 = \frac{N_2}{f_2} \tag{65}$$

The number of pulses of frequency f_1 counted in the interval T_2 is equal to

$$N_1 = T_2 f_1 = N_2 \frac{f_1}{f_2} = \frac{k_1}{k_2} \cos\varphi = k\lambda \tag{66}$$

Such a device can be directly equipped with a digital display for reading the PF value.

Figure 15 shows the structure of another analog PF meter, described in Ref. 20 with the measuring results independent of the supply voltage. A reference dc voltage U_{ref} is applied to a voltmeter through two electronically controlled switches. The switch S_A turns dc voltage U_{ref} ON when the supply voltage u changes to a positive value, and turns it OFF when the supply current i changes to a negative value, as shown in Fig. 16. The switch S_B turns the voltage U_{ref} ON when the supply current i changes to a positive value, and turns it OFF when the supply voltage u changes to a negative value. The average value of the voltmeter voltage, $u_v = u_A - u_B$, is equal to

$$\overline{u}_v = U_{\mathrm{ref}} \frac{\varphi}{90} = k\varphi;\ -90^\circ < \varphi < 90^\circ \tag{67}$$

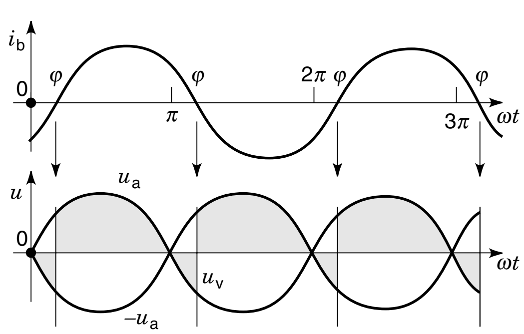

Figure 14. Voltage at the voltmeter provided by the synchronous switch, (shown in Fig. 13) that is controlled by the load current. The average value of voltage u_a is proportional to the power factor of the load.

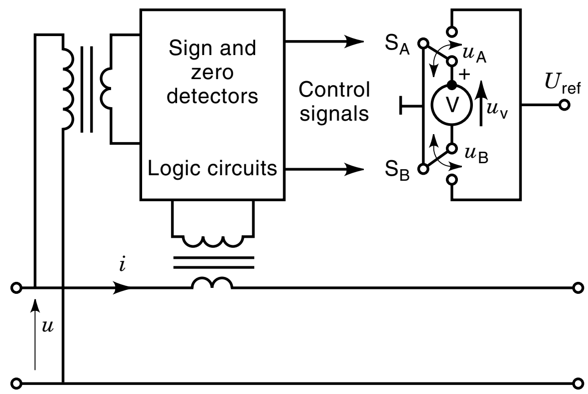

Figure 15. Electronic PF meter with sign and zero detectors of the supply voltage and current. A dc voltmeter measures the value proportional to the phase-shift angle φ both for lagging and leading power factor.

and a voltmeter, scaled in cosine value of the angle φ, with zero in the middle, provides both the lagging and leading power factor. However, its scale is not uniform. It becomes denser as the phase-shift angle approaches $\pm 90°$. It is inconvenient in the case of an analog meter, that is, with a voltmeter scaled in PF. However, the power factor cannot be displayed digitally. A nonlinear converter of the voltage to its cosine value is needed in such a case. Reference 21 describes such a converter for power factor measurement applications.

ELECTRONIC POWER FACTOR METERS BASED ON THE MEASUREMENT OF THE RATIO OF ACTIVE TO APPARENT POWER

When the voltage and current waveforms are nonsinusoidal and/or the 2-phase or 3-phase system is asymmetrical, then data provided by PF meters, that are based on phase-shift angle measurement are meaningless. The PF meter has to operate in such a case according to power factor definition (1).

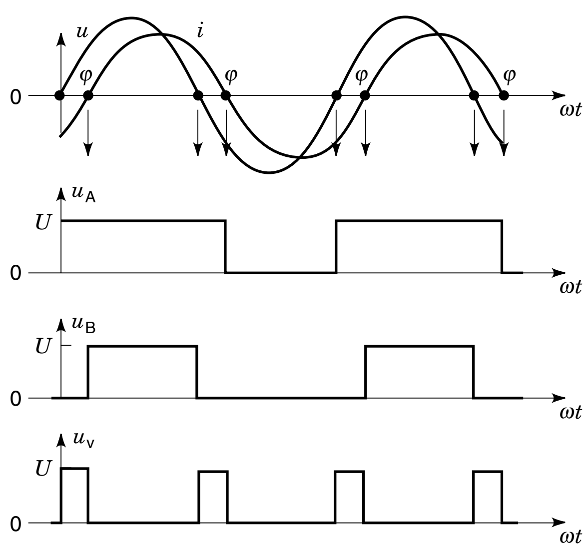

Figure 16. Generating switching pulses by the sign and zero detectors and logic circuits in the PF meter shown in Fig. 15.

It has to measure the ratio of the active and apparent power. Analog, binary, and digital signal technology can be employed for this. There is, however, the problem of selection of the apparent power definition in asymmetrical 3-phase systems. When the system is symmetrical, PF can be measured with a single-phase power factor meter.

Single-phase analog PF meters can be built of two integrated analog multipliers, a divider, two rms-to-dc converters, and a low-pass filter. The supply voltage and current have to be conditioned to voltage signals, usually bounded by ± 10 V with voltage and current converters. Figure 17 shows the basic structure of an analog PF meter. Conditioning and supply circuits are not shown. Arrows denote voltage signals proportional to various quantities in the device. Dimensional coefficients are omitted. The output voltage can be used for PF control or measured by a voltmeter scaled in PF. Such a meter cannot have PF value $\lambda = 1$ in the middle of the scale, since definition (1) does not specify whether the PF is lagging or leading. Also, in circuits with nonsinusoidal waveforms these terms are meaningless. Analog PF meters can also be built with a digital display, as discussed in Refs. (22,23).

An operation principle of a hybrid, that is, an analog and digital PF meter, is presented in Ref. 24. Two converters of the supply voltage and current provide voltages $k_u u$ and $k_i i$, where k_u and k_i are conversion coefficients, to a rms value meter. It measures the rms values $k_u \|u\|$ and $k_i \|i\|$, as well as, the rms value of the sum $u_s = k_u u + k_i i$ and the difference $u_d = k_u u - k_i i$. These four rms values are transferred to a computer. The difference of the squares of the rms values of voltages u_s and u_d, calculated by the computer gives

$$\|u_s\|^2 - \|u_d\|^2 = 4k_u k_i P \tag{68}$$

This value divided by $4k_u\|u\|k_i\|i\|$ is the PF of the load.

Figures 18 and 19 show structures of an analog-to-binary (A/B) signal converter and a PF meter based on binary signal technology, described in Ref. 25. A binary voltage signal x_B is a signal that has only two values, U and $-U$. Information on the magnitude of the analog input quantity $x(t)$ is coded in the duty factor $d(t)$, actually the ratio of time interval τ_+ (when $x_B = U$) to the switching cycle time $\tau = \tau_+ + \tau_-$, where

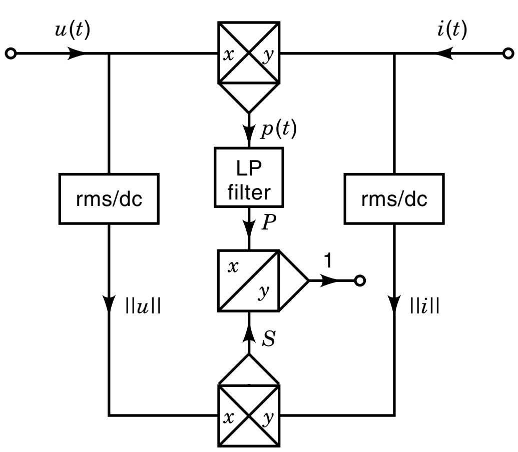

Figure 17. Structure of PF meter built of rms/dc converters, analog multipliers, divider, and a low-pass filter.

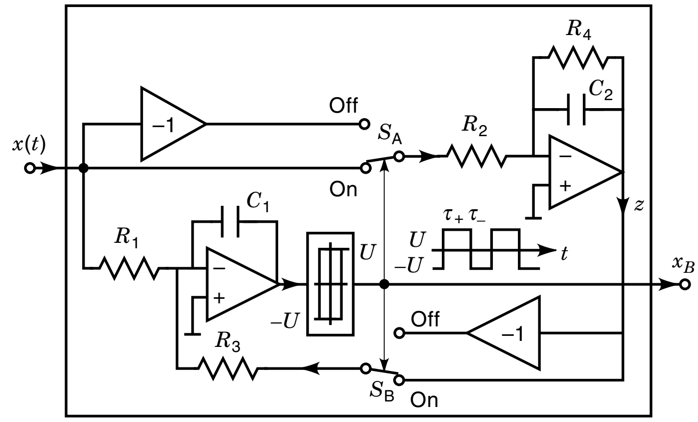

Figure 18. Converter of analog signal $x(t)$ to its binary representation x_B. Information on the analog signal's instantaneous value is coded in the duty factor of the binary signal.

τ_- is the time interval when $x_B = -U$, namely

$$d(t) = \frac{\tau_+}{\tau_+ + \tau_-} \tag{69}$$

Electronically controlled switches S_A and S_B are turned ON when $x_B = U$ and turned OFF when $x_B = -U$, consequently, capacitors C_1 and C_2 are charged with positive or negative voltages x and z. The charges on these capacitors must remain unchanged during each operation cycle, that is, time interval τ. The charging and discharging equilibrium occurs at the duty factor

$$d(t) = \frac{1}{2} - \frac{1}{2}k\frac{x(t)}{\|x\|} \qquad k = \sqrt{\frac{R_2 R_3}{R_1 R_4}} \tag{70}$$

and is dependent on the instantaneous value $x(t)$ of analog signal normalized with respect to its rms value $\|x\|$.

Binary representations u_B and i_B of the supply voltage and current are applied to a logical EXCLUSIVE OR gate as shown in Fig. 19. The duty factor $d_E(t)$ of the output signal of this gate is equal to

$$d_E(t) = d_u(t)\overline{d}_i(t) + d_i(t)\overline{d}_u(t) \tag{71}$$

where $\overline{d}$ denotes the duty factor of the negative binary signal x_B. Since $d(t) + \overline{d}(t) = 1$, then

$$\overline{d}(t) = \frac{1}{2} + \frac{1}{2}k\frac{x(t)}{\|x\|} \tag{72}$$

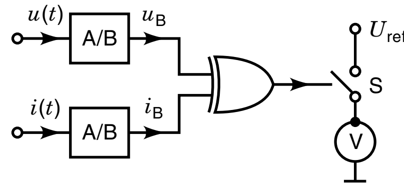

Figure 19. Electronic PF meter built of two analog-to-binary (A/B) converters, an EXCLUSIVE OR gate with an electronically controlled switch, and a dc voltmeter supplied from a source of reference voltage U_{ref}.

When the duty factors $d_u(t)$ and $d_i(t)$ of binary representations of the supply voltage and current are expressed in the form of Eq. (69), then Eq. (70) results in

$$d_E(t) = 1 - k_u k_i \frac{u(t)i(t)}{\|u\|\,\|i\|} = 1 - \frac{p(t)}{S} \tag{73}$$

since the coefficients k_u and k_i should be chosen such that $k_u k_i = 1$. Thus, a dc voltmeter supplied from a reference voltage U_{ref} through a switch S can be scaled in power factor. If the EXCLUSIVE OR gate ON signal opens the switch, the value $\lambda = 1$ is at the right end of the scale; if the gate ON signal closes the switch, the value $\lambda = 1$ is at the left end.

A single-phase, digital PF meter provides the power factor calculated from sequences of uniformly distributed digital samples u_n and i_n of the supply voltage and current. The meter is built of voltage and current converters that provide conditioned voltages in the range of, usually, ± 10 V; sample and hold (S&H) circuits, analog to digital converter (ADC), and an arithmetic unit, usually in the form of a microcontroller or microcomputer with a digital display or a printer.

The sampling frequency should be more than twice the frequency of the highest harmonic order of the supply voltage and current. If N denotes the number of samples in one cycle of the voltage, then the power factor in a single-phase or symmetrical 3-phase circuit is calculated from the formula

$$\lambda = \frac{P}{S} = \frac{\frac{1}{N}\sum_{n=0}^{N-1} u_n i_n}{\sqrt{\frac{1}{N}\sum_{n=0}^{N-1} u_n^2}\sqrt{\frac{1}{N}\sum_{n=0}^{N-1} i_n^2}} = \frac{\sum_{n=0}^{N-1} u_n i_n}{\sqrt{\sum_{n=0}^{N-1} u_n^2 \sum_{n=0}^{N-1} i_n^2}} \tag{74}$$

Such a digital PF meter can be very accurate but expensive. Such a measurement is inexpensive, however, taking into account that the PF measurement is usually accompanied by a variety of other measurements, recordings, and analysis.

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