ACCELERATION MEASUREMENT

When we use the term *acceleration,* we usually implicitly mean *instantaneous acceleration.* However, when acceleration is measured, we actually obtain an estimate of *average acceleration* within some time window. The average acceleration a_a during a certain time interval Δt is equal to the change in velocity Δv per unit time during that interval. The instantaneous acceleration (or just acceleration) a is the limiting value of the average acceleration, when the observation interval approaches zero (1). This can be expressed mathematically as

$$a = \lim_{\Delta t \to 0} a_a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \tag{1}$$

Acceleration measurements are frequently needed for automatic control, protective supervision, and condition monitoring in applications like vertical and horizontal transportation, robotics, aerospace navigation, and technical diagnostics (2,3). Acceleration belongs to the important chain of kinematic quantities: position, velocity, acceleration, and jerk. They all have a linear connection to their neighbor quantities; for example, acceleration is obtained by differentiating the corresponding velocity or integrating the jerk. Therefore, in principle, all the kinematic quantities could be derived from a single quantity. In practice, however, only integration is widely used to process these kinematic quantities, since integration always provides advantageous noise attenuation. Differentiation, on the other hand, is noise amplifying by its nature. Therefore it is seldom utilized in practical applications when the input signal contains noise or other disturbances.

This possible noise results from various measuring and processing errors, as well as external disturbances like electromagnetic interferences (EMI). The measuring errors are caused by sensor nonidealities and by cumulative effects of the entire measuring instrumentation (4). Quantization, finite-precision computations, sampling, and approximative algorithms are typical sources of processing errors. External disturbances may enter a measuring system due to inadequate grounding, shielding, isolation, or poor cabling.

REFERENCE AXES AND DIFFERENT TYPES OF MOTION

Measurement of acceleration, as well as all kinematic quantities, are made with respect to some system of reference axes. The basic frame of reference used in mechanics is known as the *primary inertial system* (or astronomical frame of reference). It consists of an imaginary set of rectangular axes that neither translate nor rotate in space. Measurements made with respect to this primary inertial system are said to be *absolute.* In most earth-bound engineering applications, measurements made relative to the earth can also be considered absolute (at least the introduced error is negligible).

There are many engineering problems for which the analysis of motion is simplified by measuring kinematic quantities with respect to a moving coordinate system. These measurements, when combined with the observed motion of the moving coordinate system, permit the determination of the absolute motion. This approach is known as a *relative motion analysis* (2).

Direct and Indirect Measuring Techniques

There exist two classes of acceleration measurement techniques: *direct* measurements by special sensors or accelerometers and *indirect* measurements where velocity is differentiated using some differentiator circuitry or a computational algorithm (2). The applicability of these techniques depends on whether one is measuring rectilinear, angular, or curvilinear motion or equilibrium-centered vibration, which is a special class of acceleration.

Direct measuring using an accelerometer is usually preferred when the motion is either rectilinear or curvilinear. Techniques used to measure linear unidirectional acceleration include spring mass, stretched wire, pendulum, piezoelectric, strain gauge, and force balance (3). Vibration measurements are based almost solely on direct techniques because there usually exists no actual velocity signal that can be differentiated successfully. Angular acceleration, on the other hand, is usually measured indirectly because the rotation range of existing angular accelerometers is severely limited due to their mechanical structures (5). Angular velocity, a necessary basis for indirect measurement of angular acceleration, can easily be measured by ac or dc tachogenerators or pulse encoders with some postprocessing electronics (6–8). Besides, rectilinear motion is often first converted mechanically into angular motion, and the corresponding angular acceleration is then measured using some indirect technique.

Differentiators needed in indirect measuring have two important requirements: They must provide simultaneously *adequate noise attenuation* and *appropriate delay characteristics.* The delay properties are particularly critical in real-time control and protective supervision applications. Even a small delay in an acceleration signal that is used for feedback control can reduce the control performance drastically. Besides, a considerably delayed acceleration curve is not a sufficient basis for time-critical actions.

RECTILINEAR, ANGULAR, AND CURVILINEAR ACCELERATION

Before we go into specific measuring techniques, we need to formulate the different types of acceleration in the corresponding coordinate systems. The applicable measuring techniques are closely connected to these formulations.

Rectilinear Acceleration

Rectilinear motion is illustrated in Fig. 1. This kind of motion occurs, for example, in vertical and horizontal transportation

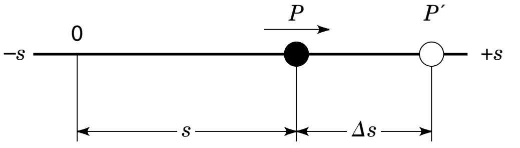

Figure 1. Rectilinear motion of a particle. Its original position is $s(P)$, and the new position is $s + \Delta s(P')$. *Instrumentation for Engineering Measurements,* J. W. Dally, W. F. Riley, and K. G. McConnell, Copyright © 1984, by John Wiley & Sons, Inc. Reproduced by permission of John Wiley & Sons, Inc.

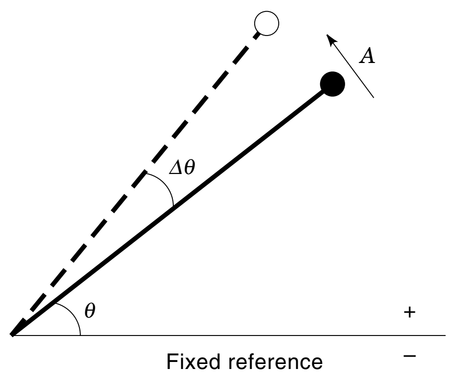

Figure 2. Angular motion of a rotating line. Its original angle is θ, and the new angle is $\theta + \Delta\theta$. *Instrumentation for Engineering Measurements,* J. W. Dally, W. F. Riley, and K. G. McConnell, Copyright © 1984, by John Wiley & Sons, Inc. Reproduced by permission of John Wiley & Sons, Inc.

as well as in various positioning servo applications. The average velocity v_a during a time interval Δt is the displacement Δs divided by the time interval. The instantaneous velocity (or just velocity) v can be defined in a similar way as the instantaneous acceleration of Eq. (1):

$$v = \lim_{\Delta t \to 0} v_a = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \tag{2}$$

Rectilinear acceleration can now be written as the time derivative of velocity or the double time derivative of the corresponding distance

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d(ds/dt)}{dt} = \frac{d^2 s}{dt^2} \tag{3}$$

Equation (3) is used frequently in indirect acceleration measurement. However, the double differentiation is seldom implemented in practice due to the noise amplification problem discussed earlier. Instead, the time derivative of measured velocity is somehow approximated. By combining Eqs. (2) and (3), we obtain another important formula to be potentially used in indirect acceleration measurement:

$$a = v\frac{dv}{ds} \tag{4}$$

Depending on the applied velocity measuring technique, it is sometimes more natural to compute the displacement derivative instead of the time derivative.

The roughness of motion is widely described by jerk. This roughness is directly related to ride comfort in vertical and horizontal transportation systems. Acceleration difference Δa at the beginning and end of a diminishing time interval Δt can be used to derive the instantaneous jerk k as

$$k = \lim_{\Delta t \to 0} \frac{\Delta a}{\Delta t} = \frac{da}{dt} \tag{5}$$

All the kinematic quantities can be either positive or negative. Throughout the preceding equations, the sign of a kinematic quantity follows the base convention of defining the rectilinear displacement Δs positive or negative.

Angular Acceleration

Angular motion is illustrated in Fig. 2. Here the counterclockwise direction is defined to be positive, and the signs of the other motion quantities are selected correspondingly. This type of motion exists, for example, in electric machines and industrial robots. Therefore the measurement of angular motion quantities is of great practical importance. In angular motion the observed displacement is an angle $\Delta\theta$ instead of a linear distance. Thus the SI unit of angular acceleration is rad/s^2 instead of m/s^2.

The average angular acceleration α_a during a defined time interval Δt is equal to the change in angular velocity $\Delta\omega$ per unit time during that interval. Now the instantaneous angular acceleration (or just angular acceleration) α can be expressed as

$$\alpha = \lim_{\Delta t \to 0} \alpha_a = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} \tag{6}$$

The equation of angular velocity is analogous to the rectilinear velocity of Eq. (2) and can be written as a time derivative of the angular displacement

$$\omega = \lim_{\Delta t \to 0} \omega_a = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \tag{7}$$

By combining Eqs. (6) and (7), we obtain another useful formula for indirect measurement of angular acceleration:

$$\alpha = \omega\frac{d\omega}{d\theta} \tag{8}$$

Angular jerk is a seldom used quantity, but it can be computed indirectly using an equation analogous to Eq. (5).

Curvilinear Acceleration

When a particle is moving along a curved path, the motion is curvilinear. Curvilinear motion can be two-dimensional, plane curvilinear or three-dimensional, space curvilinear. Next we discuss the curvilinear acceleration, referring to Figs. 3 and 4.

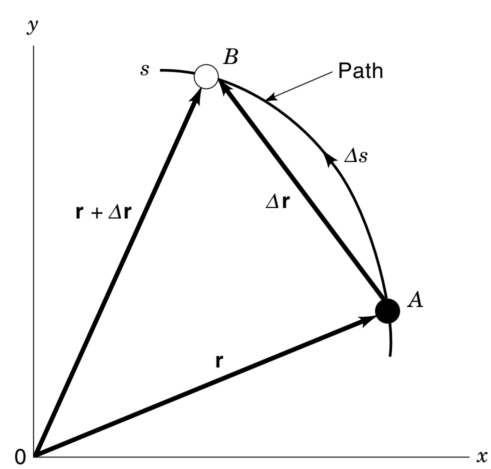

Figure 3. Plane curvilinear motion of a particle. Its original position is A, and the new position is B. The moving path is denoted by s, and Δs is the traveled distance. Now the particle can be located by a position vector $\mathbf{r}$ (and $\mathbf{r} + \Delta\mathbf{r}$). *Instrumentation for Engineering Measurements,* J. W. Dally, W. F. Riley, and K. G. McConnell, Copyright © 1984, by John Wiley & Sons, Inc. Reproduced by permission of John Wiley & Sons, Inc.

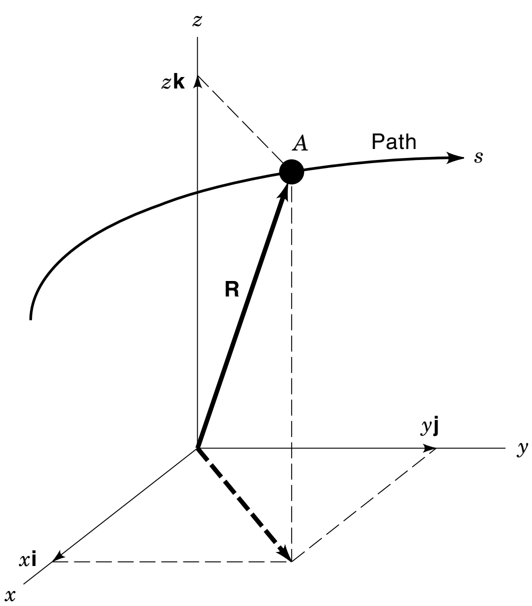

Figure 4. Space curvilinear motion of a particle along the path s. The position is determined by rectangular coordinates (x, y, z). The position vector $\mathbf{R}$ can be expressed as $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. *Instrumentation for Engineering Measurements,* J. W. Dally, W. F. Riley, and K. G. McConnell, Copyright © 1984, by John Wiley & Sons, Inc. Reproduced by permission of John Wiley & Sons, Inc.

Plane Curvilinear Acceleration. Plane curvilinear motion equations are very similar to those of rectilinear motion. Now we only need a two-dimensional vector instead of a scalar to define the position $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ of a particle. All the other kinematic quantities are correspondingly of vector value. Figure 3 illustrates such a motion type. The instantaneous velocity $\boldsymbol{v}$ can be expressed as the limiting value of the average velocity $\boldsymbol{v}_a$ when the observation interval Δt approaches zero:

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \boldsymbol{v}_a = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} \tag{9}$$

The vector displacement $\Delta\boldsymbol{r}$ is called *linear displacement,* while the scalar displacement Δs is the traveled distance. The magnitude of the instantaneous velocity is called the *speed* of the particle. Next we define the instantaneous acceleration $\boldsymbol{a}$ using the limiting value of average acceleration $\boldsymbol{a}_a$ as the observation interval approaches zero:

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \boldsymbol{a}_a = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{d\boldsymbol{v}}{dt} = \frac{d^2x}{dt^2}\boldsymbol{i} + \frac{d^2y}{dt^2}\boldsymbol{j} \tag{10}$$

Different coordinate systems (rectangular, normal, tangential, and polar) are commonly used to describe plane curvilinear motion. A detailed discussion of these is given by Dally et al. (2).

Space Curvilinear Acceleration. Rectilinear acceleration of Eq. (3) is a special case of plane curvilinear acceleration of Eq. (10). Further, plane curvilinear acceleration is a special case of space curvilinear acceleration (see Fig. 4). The most general form of motion, space curvilinear motion occurs in a three-dimensional space. We use the rectangular coordinates in defining the kinematic quantities.

The spatial position $\boldsymbol{R}$ can be expressed as

$$\boldsymbol{R} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \tag{11}$$

The corresponding spatial velocity $\boldsymbol{v}$ is a time derivative of the spatial position

$$\boldsymbol{v} = \frac{d\boldsymbol{R}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k} \tag{12}$$

Finally, we can write the spatial acceleration $\boldsymbol{a}$ as a time derivative of the spatial velocity

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d^2x}{dt^2}\boldsymbol{i} + \frac{d^2y}{dt^2}\boldsymbol{j} + \frac{d^2z}{dt^2}\boldsymbol{k} \tag{13}$$

Cylindrical and spherical coordinates are considered in Dally et al. (2). The choice of coordinates (rectangular, cylindrical, or spherical) for a particular application, involving space curvilinear motion, depends on the nature of the motion and the choice of measurement techniques.

VIBRATORY ACCELERATION

Vibratory acceleration is a distinct form of planar or spatial acceleration. It can be one-, two-, or three-dimensional. The principal characteristic of vibratory motion is that it occurs around some equilibrium position. Therefore the longer-term time average of the vibratory displacement is zero.

Vibratory motion occurs, for example, in all kinds of machines, as well as in almost all natural and artificial structures. The considerable interest in vibration analysis has three main motivations: (1) persistent vibration causes long-term wearing and possible breakdown of vibrating structures, (2) even slight vibration can degrade the performance of vibrating machines, and (3) medium-frequency vibration can also cause disturbing audible noise. For monitoring the condition of machines and installations, diagnostic procedures based on frequency-domain vibration analysis are in everyday use. Piezoelectric accelerometers have become widely used in machine vibration acquisition. Small dimensions and rigid design allow their utilization in various fields of technology. In contrast to displacement pickup and velocity pickup, these acceleration sensors can be applied into a wider range of frequencies (9).

A simple vibration can be modeled as a periodically repeated motion about the position of equilibrium (2). Figure 5 illustrates a rotating line representation of a simple type of vibratory motion that occurs commonly in various physical systems. This two-dimensional vibration can be separated into vertical (y-axis) and horizontal (x-axis) vibration components. The $x(y)$ position of the vibrating particle $P(Q)$ can be expressed as a function of time as

$$\begin{cases} x_P = A_0 \cos \omega t \\ y_Q = A_0 \sin \omega t \end{cases} \tag{14}$$

By differentiating the separate xy-position components of Eq. (14), we obtain the corresponding horizontal and vertical ve-

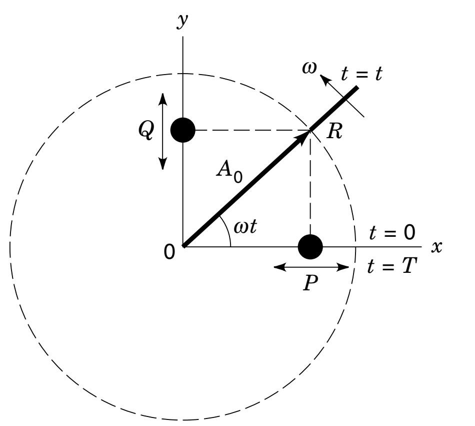

Figure 5. Rotating line representation of a simple type of vibratory motion. A_0 is the amplitude of vibration, T is the vibration period, and ω is the angular velocity of the rotating line $0R$. *Instrumentation for Engineering Measurements,* J. W. Dally, W. F. Riley, and K. G. McConnell, Copyright © 1984, by John Wiley & Sons, Inc. Reproduced by permission of John Wiley & Sons, Inc.

locities of the vibratory motion:

$$\begin{cases} v_\mathrm{P} = \dfrac{dx_\mathrm{P}}{dt} = -A_0\omega\sin\omega t \\ v_\mathrm{Q} = \dfrac{dy_\mathrm{Q}}{dt} = A_0\omega\cos\omega t \end{cases} \tag{15}$$

Finally, after differentiating the vibratory velocity components, we can write the vibratory acceleration components as

$$\begin{cases} a_\mathrm{P} = \dfrac{dv_\mathrm{P}}{dt} = -A_0\omega^2\cos\omega t \\ a_\mathrm{Q} = \dfrac{dv_\mathrm{Q}}{dt} = -A_0\omega^2\sin\omega t \end{cases} \tag{16}$$

Given the basic trigonometric equivalencies of Eq. (17), we can easily see that the vibratory velocities, Eq. (15), can be calculated straightforwardly by multiplying the corresponding displacement, Eq. (14), by ω and increasing the angle by $\pi/2$ radians:

$$\begin{cases} \sin\omega t = -\cos\left(\omega t + \dfrac{\pi}{2}\right) \\ \cos\omega t = \sin\left(\omega t + \dfrac{\pi}{2}\right) \end{cases} \tag{17}$$

Similarly, given the equivalencies of Eq. (18), we obtain the vibratory accelerations, Eq. (16), by multiplying the displacements, Eq. (14), by ω^2 and increasing the angle by π radians:

$$\begin{cases} \cos\omega t = -\cos(\omega t + \pi) \\ \sin\omega t = -\sin(\omega t + \pi) \end{cases} \tag{18}$$

Now we can write the vibratory acceleration as a simple function of the vibratory position:

$$\begin{cases} a_\mathrm{P} = -\omega^2 x_\mathrm{P} \\ a_\mathrm{Q} = -\omega^2 y_\mathrm{Q} \end{cases} \tag{19}$$

Any motion for which the acceleration is proportional to the displacement from a fixed point on the path of motion and always directed toward that point is defined as *simple harmonic motion* (2). Periodic motion that is not simple harmonic motion can be modeled as a sum of several simple harmonic motions with different displacement amplitudes and harmonically related angular frequencies. This natural generalization is discussed further by Dally et al. (2).

ACCELERATION MEASUREMENT TECHNIQUES

After defining the different types of motion, we are ready to go into the details of the available measurement techniques. The main emphasis of our discussion is in the indirect measuring methods of acceleration. The direct methods, based on commercially available linear or angular accelerometers, are usually straightforward instrumentation applications where low absolute or differential voltages or currents are measured from a specific acceleration sensor. For a comprehensive presentation of accelerometers see ACCELEROMETERS.

Indirect Acceleration Measurement

Indirect acceleration measuring techniques are based on analog or digital postprocessing of position or velocity signals. These are typically measured by pulse encoders or tachogenerators. Therefore some kind of differentiator needs to be constructed to provide the acceleration signal. As discussed earlier, this complete differentiator is not trivial because the differentiation operator is noise amplifying by nature. To loosen the noise attenuation requirements of the final differentiator that produces the acceleration signal, the measuring noise problem must be tackled already in all the functional blocks of the preceding measuring chain. High-performance velocity measuring techniques are presented, for example, by Brown et al. (9), Pasanen et al. (8), and Laopoulos and Papageorgiou (10). The articles contain velocity measuring techniques that can be parametrized to provide low output noise. The applied sensors are pulse encoders. Thus the proposed methods can be used to measure both angular and rectilinear velocities, which can then be postprocessed by a differentiator.

In principle, it is easy to attenuate any noise when the primary signal and the disturbing noise are clearly separated in the frequency domain. However, the filtering task becomes considerably more difficult if there also are strict delay constraints for the filtering process. Simple nonrecursive and recursive smoothing techniques to enhance the quality of the differentiator output were suggested by Jaritz and Spong (11). The z-domain transfer functions of their smoothers are given as

$$S_\mathrm{n}(z) = \frac{1}{N}\sum_{i=0}^{N-1} z^{-i} \tag{20}$$

$$S_\mathrm{r}(z) = \frac{1/N}{1 - \sum_{i=1}^{N-1} z^{-i}/N} \tag{21}$$

These smoothers still suffer from a notable tracking error or lag. This forces the designer to use a small value of N that, on the other hand, leads to poor noise attenuation capabilities.

Ultimately we would prefer a delayless lowpass filter because any additional delay degrades the overall performance when the filtered acceleration signal is used for feedback control or time-critical supervision. Unfortunately, there exists no general-purpose lowpass filter that does not delay the frequencies on its passband. At present we can choose between two approaches to solve this difficult problem: predictive filtering or state observing. In the first case, an application-specific, predictive lowpass filter is cascaded with a differentiator, and in the second, a linear (or nonlinear) stochastic model to represent acceleration is developed and used as an acceleration estimator. Under greatly time-varying conditions, the predictive filter as well as the stochastic estimator should be adaptive to maximize the noise attenuation capabilities and to minimize the harmful lag of the primary acceleration curve.

Predictive Postfiltering. Predictive filters (8) as well as state observers (12) are usually implemented in the discrete time domain either because they have their theoretical origins in digital signal processing or because an analog implementation would be unfeasible due to the mathematical operations required. This assumption is valid throughout the following discussion. Predictive (or phase-advancing) lowpass filtering is possible if we can place easing constraints for the incoming signal characteristics. These constraints may be explicit either in the time domain or in the frequency domain. There are many practical applications where the kinematic quantities can be approximated with sufficient accuracy by piecewise low-degree polynomials, as illustrated in the experimental sections of Refs. 12, 8, and 5. This is due to the mechanical inertia that necessarily smoothens the movements of masses. Typical rectilinear position curves, such as those related to vertical and horizontal transportation, can be approximated piecewise by third- and fourth-degree polynomials, velocity curves by second- and third-degree polynomials, acceleration curves by first- and second-degree polynomials, and jerk curves by zeroth- and first-degree polynomials. For instance, a third-degree polynomial model can be expressed as

$$f(k) = \beta_0 + \beta_1 k + \beta_2 k^2 + \beta_3 k^3 \tag{22}$$

where k is the discrete time index, and β_i, $i = 0, 1, 2, 3$, are the signal-dependent curve-fitting parameters. A thorough discussion of polynomial modeling is given by Williams (13).

By using the polynomial signal model, we can easily design predictive filters that provide the desired forward prediction behavior with the mandatory lowpass characteristics (14). One-step-ahead prediction is typically adequate to compensate for the delay caused by the differentiation algorithm as well as the data acquisition and processing delays. Thus we are performing on-line curve fitting. For this we only need to select an appropriate polynomial degree; the signal-dependent parameters of the polynomial model are handled implicitly.

The difference equation of a general n-step-ahead predictive filter can be expressed as

$$\hat{u}(k+n) = \sum_{i=1}^{N} \chi_i \hat{u}(k+n-i) + \sum_{j=0}^{M} \delta_j u(k-j) \tag{23}$$

where the coefficients χ_i and δ_j are real-valued constants, $\hat{u}(k + n)$ is the n-step-ahead output of the predictive filter, and $u(k)$ is the corresponding input sample. The prediction step n is an application-specific parameter, and it depends on the cumulative measuring and processing delays of the entire instrumentation system. An important consequence of the polynomial model is naturally that the result of predictive filtering is less satisfactory for other signal classes, like vibratory acceleration. This is due to a narrow prediction bandwidth (a frequency range where the group/phase delay of the filter is negative) that is a principal restricting characteristic of polynomial predictive filters. There exist several finite impulse response (FIR) and infinite impulse response (IIR) polynomial predictors as reviewed by Ovaska (15).

Recursive linear smoothed Newton (RLSN) predictors form a class of computationally efficient IIR predictors (14), which are particularly attractive for postprocessing of the noisy output of a differentiator. Their applicability is an immediate consequence of the simple design process and efficient noise attenuation capabilities. The z-domain transfer functions of the first- and second-degree [$H_1(z)$ and $H_2(z)$, respectively], one-step-ahead RLSN polynomial predictors are given as

$$H_1(z) = \frac{[c + (1/N)] - (z^{-N}/N)}{1 - (1-c)z^{-1}} \tag{24}$$

$$H_2(z) = \frac{[2c + (1/N)] + [c^2 - 2c - (1/N)]z^{-1} - (z^{-N}/N) + (z^{-(N+1)}/N)}{1 - (2-2c)z^{-1} + (1-2c+c^2)z^{-2}} \tag{25}$$

The only adjustable parameters, c and N, control primarily the stopband attenuation and the passband gain peak, respectively. In acceleration measurement applications, typical values for these parameters are $c = 0.01$–0.05 and $N = 16$–64.

Hence we can postfilter the noisy output of the simple backward-difference differentiator of Eq. (26) by a polynomial predictive filter without harmfully delaying the low-degree polynomial component. This polynomial component approximates our desired acceleration signal. The delay introduced by the difference operation of Eq. (26) is half a sampling period.

$$a(k) = \frac{v(k) - v(k-1)}{T_s} \tag{26}$$

In Eq. (26), $a(k)$ is the average acceleration, $v(k)$ the instantaneous velocity, and T_s the constant sampling period. The accuracy of Eq. (26) can be improved by decreasing the sampling period as suggested in the original definition, Eq. (3), of this time derivative. In practical applications the selection of the sampling period is based on the bandwidth requirements of the closed acceleration control loop, the latency requirements of some supervision action such as emergency shutdown, and the computational capacity of the implementation environment. Figure 6 depicts the indirect acceleration measuring scheme based on differentiation and predictive postfiltering. Three alternative implementation techniques are presented: fully analog, analog-digital, and fully digital. Analog polynomial predictors needed in the fully analog alternative were introduced in Ref. 16.

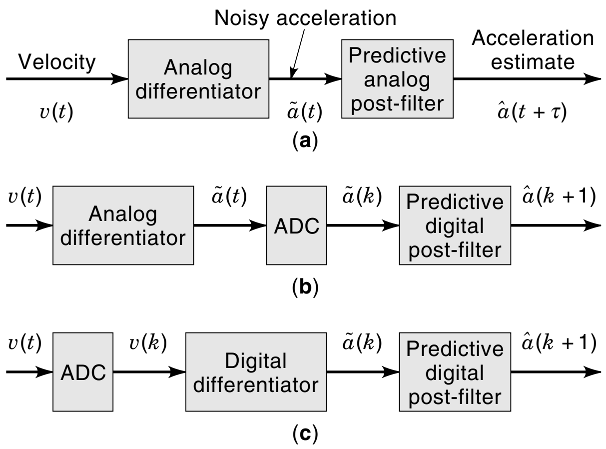

Figure 6. Alternative implementations of indirect acceleration measurement using a cascade of a differentiator and a polynomial predictive postfilter. The prediction step is τ with the analog predictive filter and one sampling period with the digital ones.

Linear State Observing. Instead of differentiation and (predictive) postfiltering, an optimized Kalman filter-based state observer can be used for estimating acceleration. An angular acceleration estimator is proposed by Bélanger (12), but the same principle can be used in estimating rectilinear acceleration. First, a stochastic state-space model is developed to represent the rotation angle $\theta(t)$:

$$\begin{cases} \dfrac{d\boldsymbol{x}}{dt} = A\boldsymbol{x}(t) + \Gamma w(t) \\ y(t) = \theta(t) = \boldsymbol{C}\boldsymbol{x}(t) + e(t) \end{cases} \tag{27}$$

where $\boldsymbol{x}(t)$ is the state vector containing the angle, angular velocity, and angular acceleration; $w(t)$ is zero mean, white Gaussian noise with covariance q. When the modeled motion is not characterized by such a stochastic process but is merely deterministic, the parameter q may be considered as a pure filter parameter to be adjusted empirically. Further A is a (3×3) matrix, Γ is a (3×1) vector, and $\boldsymbol{C}$ is a (1×3) vector. The scalar output $y(t)$ is the actual angle measurement, $\theta(t)$, and $e(t)$ is the additive quantization error. The versatile Kalman filter (11,17) provides an optimal (minimum variance) solution to this observing problem (12). Before the model of Eq. (27) can be implemented by some digital processor, it must be discretized using one of the available continuous-time to discrete-time transformations (18).

For estimating the angular acceleration, Bélanger (12) postulates the fixed numerical model:

$$\begin{cases} \dfrac{d\boldsymbol{x}(t)}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w(t) \\ y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t) + e(t) \end{cases} \tag{28}$$

The application discussed in that article is a single robot joint under proportional-derivative (PD) control with all the evaluated estimates corresponding to one-step-ahead prediction. Although only a marginal improvement of the angle estimates is reported over those provided directly by the pulse encoder, the velocity estimate standard deviations are improved by a factor of 2 to 4 over the standard deviations of the plain backward-difference differentiation. In the estimate of the angular acceleration, there is an order of magnitude improvement. Hence even the fixed Kalman filter approach offers clear benefits by remarkably improving the accuracy of acceleration measurements. On the other hand, the computational complexity of the Kalman filter may be a limiting factor when the estimator must be adapted on-line with a high sampling rate. Nevertheless, such an adaptive approach offers satisfactory accuracy even under unknown and time-varying conditions.

An advantageous characteristic of such a linear state observer is that it provides estimates of all the state variables *simultaneously*. Therefore no explicit cascade processing structure is needed as in the case of differentiation and predictive postfiltering. When direct measurements of the velocity signal are available, the number of states, namely the dimension of the state vector $\boldsymbol{x}(t)$, is reduced from three to two. This simplifies the implementation of the optimized Kalman filter.

Conclusions of the Indirect Methods. As a conclusion, we can state that backward-difference differentiation cascaded with a polynomial predictive filter provides a computationally efficient indirect method for acceleration measurement. However, the method is applicable solely when the acceleration curve can be approximated piecewise by a low-degree polynomial. Practical degrees of the polynomial model are less than three or four because the available polynomial predictive filters for higher degree polynomials can offer only marginal noise attenuation capabilities (14).

Linear state observing is an attractive method when no such assumption on the polynomial nature of the acceleration curve can be made. By using an adaptive Kalman filter, estimators for time-varying acceleration behavior can be developed if only the required computational complexity can be supported by the available implementation hardware. Also, nonlinear state observers can be developed to handle the possible nonlinearities of the underlying dynamic system, as shown in the case of velocity estimation by Jaritz and Spong (11).

Direct Acceleration Measuring

Direct acceleration measuring is based on sensors that transform either linear or angular acceleration into an electrical variable: charge, voltage, or current. Acceleration measurement without a fixed reference requires the use of a seismic transducer (2). Those transducers detect relative motion between a fixed mounting base and a moving seismic mass. The seismic mass tends, due to inertia, to resist any changes in the movement. Acceleration sensors require an extremely small mass, which is connected to the frame through a stiff spring. This makes it possible to provide a wide operating bandwidth.

The currently available accelerometers usually need an external high-sensitivity preamplifier with a high-input impedance to amplify the weak primary signal (proportional to acceleration) to a suitable level for the following signal processing or data acquisition electronics. In recent designs, however, the critical preamplifier is often incorporated into the transducer housing. This is an obvious advantage because an application engineer can concentrate on higher-level instrumentation electronics instead of highly sensor-dependent solutions. Now a high-level voltage output with moderate or high signal-to-noise ratio (SNR) is obtainable. On the other hand, some design flexibility is always lost with such integrated components.

Linear and Vibratory Acceleration. Compact piezoelectric accelerometers are widely applied to the measuring of linear acceleration due to their wide operating bandwidth, usually from a few Hz to several kHz. This wide bandwidth is particularly useful in precise inertial navigation and the measurement of vibratory acceleration. In inertial navigation the spatial acceleration of Eq. (13) is measured with three accelerometers, one accelerometer for each of the three dimensions (x, y, z). Vibratory acceleration may also have more than one dimension, but all these spatial dimensions are usually represented by individual linear components.

Piezoelectric accelerometers are charge-generating devices, and after the necessary charge-to-voltage conversion and preamplification, they can produce typical output voltages of 10 mV/g to 30 mV/g (where g is the acceleration of gravity, 9.8 m/s^2) with accuracies of a few percent (2,3). Hence the electrical measuring task is necessarily more demanding with low acceleration levels than it is with moderate or high accelerations. The operating range of such sensors is typically from zero to a few hundred or thousand g : s. Manufacturers of piezoelectric accelerometers include, e.g., the following companies (in alphabetic order): Endevco Corporation, PCB Piezotronics, Kistler, and Murata. For a comprehensive presentation of various types of accelerometers see ACCELEROMETERS.

We can model a piezoelectric accelerometer as a charge generator, which is connected in parallel with an internal capacitor C_T. The terminals of this capacitor are the actual output pins of the transducer. Therefore we can measure the voltage difference over these pins. When we connect a measuring cable to the terminals of the piezoelectric accelerometer, the cable and possible connectors introduce an additional capacitance C_C, which is summed to the internal capacitance. Further the input capacitance of the preamplifier C_I is an additional component of the total capacitance $C_\Sigma = C_T + C_C + C_I$. An ac-coupled voltage follower or an instrumentation amplifier is preferred to amplify the low voltage that is observed over the total input capacitance. Figure 7 illustrates the voltage follower-based measuring circuitry. Typical values

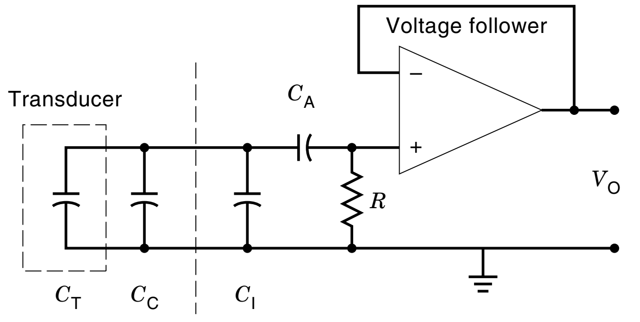

Figure 7. Voltage follower-based acceleration measuring scheme using a piezoelectric transducer.

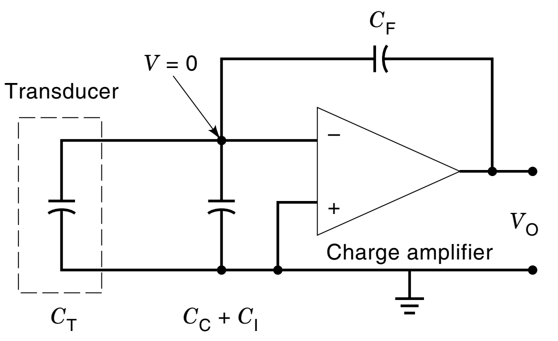

Figure 8. Charge-to-voltage converter-based acceleration measuring scheme using a piezoelectric transducer.

of the total capacitance C_Σ range from 300 pF to 10 nF, and the value of the ac-coupling capacitor C_A is usually about 100 nF (2).

Unfortunately, the voltage sensitivity of such a straightforward measuring system is inversely proportional to the total capacitance C_Σ (2). This may cause accuracy problems because, in an industrial operating environment, the cable capacitance C_C can change remarkably due to varying environmental conditions, such as humidity and dirt. Besides, the variation of C_C causes changes in the transfer function of the dynamical system formed by the sensor, cable, and preamplifier. These changes necessarily affect both the magnitude and phase (or delay) responses of the acceleration sensor.

Thus there is a natural demand toward integrated sensor modules, where the critical wiring would be of minimal length and the entire circuitry could even be hermetically sealed. Also the critical component values would be trimmed by the manufacturer during fabrication of the accelerometer module to provide constant voltage sensitivity and dynamical characteristics.

Charge amplifiers are widely used to preamplify the output of a piezoelectric acceleration transducer. They make use of an operational amplifier having a high open loop gain. A complete charge amplifier consists of two cascaded sections: a charge-to-voltage (C/V) converter and a trimmed voltage amplifier. Figure 8 illustrates a basic inverting C/V-converter, which is formed by using a capacitive feedback C_F. The voltage gain of this circuit is proportional to C_F/C_T. Here the possibly varying cable capacitance C_C appears between the summing point and circuit common. Because the voltage at the summing point is zero, C_C does not affect the provided voltage gain. However, the cable capacitance naturally affects the noise gain of the C/V-converter, which is proportional to $(C_T + C_C)/C_F$. Morrison (19) gives a short practical presentation of the implementation aspects of charge amplifiers. His discussion concentrates on the noise characteristics and the required component tolerances of charge-to-voltage converters. Hence the voltage sensitivity is subject solely to negligible variations due to changes in environmental conditions. On the other hand, a complete charge amplifier (C/V-converter cascaded with a standardization amplifier) is more complex than a simple voltage follower. Charge amplifiers can handle frequencies down to about 1 Hz (3-dB point). A detailed presentation and analysis of charge amplifier-based measurement circuits is given by Dally et al. (2).

Angular Acceleration. Although the angular acceleration can be measured indirectly using either a rotating angle sensor or a velocity sensor, the cumbersome noise-amplification problem associated with differentiators has motivated the efforts to develop transducers for direct sensing of angular acceleration. Direct measuring of linear acceleration is in wide everyday use, but the angular acceleration sensors, particularly those with unlimited rotation angle, can still be considered as emerging devices. Therefore the measuring techniques to be discussed below are not yet used widely in practical applications.

Godler et al. (5) proposed a rotary acceleration transducer that senses the angular acceleration independently of the rotation velocity and has an unlimited rotation angle. This *mechanic-opto-electronic* sensor is intended for motor control and vibration control in robotics applications. It has an obvious advantage when compared with indirect measuring techniques: The bandwidth of the output filter can be made wider than that of the (predictive) postfilter attenuating the noise of the differentiator. This is due to the more advantageous shape of the introduced noise spectrum. Therefore the new sensor offers a wider acceleration bandwidth that could directly improve the performance of an acceleration control loop. Considering the structural details and laboratory experiments presented in Ref. 5, it can be concluded that the mechanic-opto-electronic angular acceleration sensor can provide comparable accuracy with the indirect time derivative methods when the base velocity ω of Eq. (6) is measured, for example, by a low-ripple dc tachogenerator. The sensor can provide a buffered voltage output, and hence its usage is very simple. No demanding preamplifiers are needed as with the piezoelectric accelerometers. Unfortunately, only evaluation prototypes were available when Ref. 5 was published in 1995.

Furukawa et al. (20) proposed a *piezoresistive* angular acceleration sensor for robotics applications. This sensor is manufactured by micromachining a small wafer of silicon. The micro acceleration sensor manufactured by integrated circuits (IC) technology has the advantage that the necessary measuring electronics can be integrated in the same chip design as the sensor itself. Therefore a compact, low-noise, and possibly even an intelligent accelerometer unit could be developed. The final accelerometer is so small that it can be mounted at most required places and will not disturb the primary motion of a robot arm. From the instrumentation point of view, the electronics required simply measures the change of sensor resistance, which is proportional to the angular acceleration. This could be accomplished using the Wheatstone bridge (21). In contrast to the proposed sensor of Godler et al. (5), this sensor operates solely within 0° to 360° angles. This accelerometer was still in the prototype phase when Ref. 20 was published.

Conclusions of the Direct Methods. Direct methods to measure linear acceleration are widely used because of the availability of accurate, compact, and low-cost accelerometers. They are far more accurate than indirect methods for wide acceleration bandwidths. However, in cases where a narrow bandwidth is adequate, either a direct or an indirect method can be applied with comparable results. The selection of an appropriate technique depends on the presence of distance or velocity sensors; if no other sensor type is needed in the specific application, it is natural to use only an accelerometer.

Angular acceleration sensors are not yet widely available as commercial products. The demand for such transducers will rise steadily as the performance requirements of various motion control applications increase. The basic components are presented in Refs. 5 and 20. Therefore, the field of direct angular acceleration measurement will become an important area of future research and development activities.

ACCELERATION MEASUREMENT ERRORS

Acceleration measurement errors are due to three primary sources: sensors, acquisition electronics, and signal processing algorithms. For an economical and robust measurement system, the total measurement error should preferably be evenly distributed into different parts of the entire instrumentation chain.

Errors of the Direct Acceleration Measurement

In direct acceleration measurement, the primary error sources are usually the acceleration sensor and the immediate acquisition electronics. This combined error is typically less than 2% to 5% with piezoelectric accelerometers when the measurement electronics design is based on the application notes of the corresponding sensor manufacturer. Within this moderately accurate range the errors related to sampling and analog-to-digital (A/D) conversion are easily controlled by careful selection of multiplexers, sample-and-hold (S&H) circuits, and A/D-converters. Garrett gives a thorough presentation of the different error types of computerized instrumentation systems in Ref. 4. Those error components are not acceleration measurement-specific but exist in all computerized data acquisition systems.

Errors of the Indirect Acceleration Measurement

In indirect acceleration measurement the approximative differentiation is the main source of noiselike error. On the other hand, when a state observer is used instead of differentiation and postfiltering, the accuracy and bandwidth of the state-space model of Eq. (27) become especially important. Pulse encoders and tachogenerators provide the base quantities, linear or angular displacement and velocity, for indirect acceleration measurement. The measurement error of such a base sensor is typically no more than 1%. However, there is a complicated dependence between the error of the base sensor and the final acceleration error, particularly when the velocity estimate is first calculated from encoder pulse information. This is due to the usual nonlinearity of velocity estimation algorithms (8,10).

To quantify the noise problem of the backward-difference differentiator, Eq. (26), we consider a simple numerical example. Let us assume that we are measuring linear motion with constant velocity of 1.00 m/s, and our dc tachogenerator has a 1% ripple in its output voltage. The output is sampled with a constant 1-ms sampling period T_s. Let $v(k) = 0.995$ m/s and $v(k-1) = 1.005$ m/s be two consecutive measurements of the linear velocity containing sensor-originated ripple. Now the simple estimate of the instantaneous acceleration $a(k) = -10\ \mathrm{m/s^2}$. Although the base velocity with the 1% ripple could be considered adequate for many velocity control applications, even this small error makes the direct utilization of the differentiator output impossible. Therefore postfiltering is needed to reduce the acceleration ripple down to an acceptable level. In off-line data acquisition a conventional analog or digital lowpass filter can be used to achieve a suitable SNR. However, in real-time applications a predictive filter can offer indisputable benefits as discussed earlier.

To get insight into the required bandwidth of the lowpass postfilter, let us consider the simple case where the constant velocity signal v_c is deteriorated by an additive sinusoid ripple $A \sin 2\pi ft$. The available measurement signal $v_r(t)$ can be expressed as

$$v_r(t) = v_c + A \sin 2\pi ft \tag{29}$$

This signal is sampled using a sampling period of T_s. When we apply the differentiator of Eq. (26), the maximum erroneous acceleration a_m has the value

$$a_m = \frac{2A}{T_s} \sin \pi f T_s \tag{30}$$

If the amplitude A of the sinusoid ripple is 0.005 m/s, and the sampling period is the same as in the example above, we can calculate the maximum acceleration value a_m for different sinusoid frequencies. With $f = 500$ Hz, $a_m = 10.00$ m/s^2; with $f = 50$ Hz, $a_m = 1.56$ m/s^2; with $f = 5$ Hz, $a_m = 0.16$ m/s^2; and with $f = 0.5$ Hz, $a_m = 0.02$ m/s^2. This shows clearly the tradeoff between the bandwidth and the corresponding erroneous acceleration level.

Hence a very narrow-band lowpass filter is needed to keep the disturbing acceleration ripple moderate or low. This is an obvious motivation for the usage of polynomial predictive filters in real-time applications because they can provide an adequate prediction bandwidth even when a narrowband magnitude response is required.

Quantization of the velocity signal during the sampling process is also a potential source of remarkable acceleration error because the quantization noise is amplified similarly as the sensor-originated ripple by the differentiator. There are two basic alternatives to alleviate this problem: Either perform the differentiation in the analog domain or use high-resolution quantization. In practice, however, the analog alternative is less attractive due to the large dynamic range required. If an analog differentiator is employed, a scaling amplifier with a programmable gain is suggested to be placed in front of the A/D-converter. Besides the sensor ripple and quantization noise, the coefficient word length and precision of arithmetic operations have effects on the noise level which is observed in the output of the composite differentiator. All these are very critical in real-time applications because of the greatly limited filtering freedom.

Since there are many potential sources of significant acceleration error in indirect acceleration measurement, the total error (or signal-to-error ratio, SER) is largely dependent on implementation. Thus a careful error analysis is important when any indirect technique is used. Typically even in well-designed systems the true maximum errors are between 5% and 10% depending on the required acceleration bandwidth and the allowed delay. In some applications this error is mainly due to the overshoot of a postfilter output under transient conditions. Such error percentages are obtainable in vertical and horizontal transportation as well as in robotics applications.

SPECIFIC ACCELERATION MEASUREMENT METHODS AND APPLICATIONS

Our presentation has, for the present, concentrated on the main principles of the available acceleration measurement methods. The characteristics and implementation considerations were presented mostly for the pure "main-line" techniques: direct measuring using accelerometers, as well as indirect measuring based on either differentiating and (predictive) postfiltering, or state-observing. Besides these straightforward and multipurpose methods, there exist a vast number of application-specific and application-tailored techniques that can provide an excellent base for future development and engineering activities.

Selective Review of the Advanced Literature

A collection of diverse acceleration measurement methods will be reviewed briefly here. The assortment consists solely of indirect measuring techniques, particularly techniques for indirect measuring of angular acceleration. Our emphasis is set intentionally on the angular acceleration measurement. Since there exist only a few commercial (and economical) angular accelerometers with unlimited rotation range, the indirect techniques are naturally of utmost importance.

Hoffmann de Visme (22) introduced a purely digital method for obtaining acceleration information of rotating shafts in 1968. His method is based on a rotating pulse encoder, and it gives good sensitivity and accuracy when a high pulse rate N (pulses per revolution) is available and a long measuring interval T can be tolerated. The estimated angular acceleration α (rad/s^2) is given by

$$\alpha = \frac{2\pi(n_1 - n_2)}{NT^2} \tag{31}$$

where n_1 and n_2 pulses are counted in two successive intervals of duration T. This method suits well both for hardware and software implementations.

Dunworth (6) proposed a sophisticated digital instrument for the measurement of angular acceleration, in which a rotating pulse encoder is used. This method comprises a digital frequency register controlling a variable-rate pulse generator that tracks the incoming pulse rate in a closed-frequency control loop. The rate at which the pulse generator frequency needs to be corrected (incremented or decremented) corresponds to the present acceleration if the control loop is just locked on the input pulse rate. Acceleration resolution of 1% is easily achievable. Although Dunworth's instrument shares the possibly troublesome long measuring interval of Ref. 22, it still provides an interesting measuring procedure that is particularly well suited for application-specific integrated circuit (ASIC) implementation.

Smith et al. (23) reported on a direct software implementation of Eq. (31) in 1973. A pulse encoder with 10,800 pulses per revolution was used. They utilized the angular acceleration estimate for computing the electromagnetic torque of an electric motor. When the inertia of the rotating loaded component is known and assumed to stay constant, the torque can be calculated as the product of acceleration and inertia. The presented experimental results are in close agreement with the corresponding theoretically computed quantities, the maximum errors being no more than 5% to 10%.

Hancke and Viljoen (7) presented an acceleration measurement technique for a condition-monitoring application of a turbo generator where only one pulse per shaft revolution is available. This naturally causes serious constraints for the obtainable performance, namely for resolution and measuring interval. The time interval between two consecutive pulses is measured using a counter that is clocked by a high-frequency square wave. An interesting characteristic of the developed method is that it can estimate the angular acceleration between pulse instants satisfactorily by extrapolation of the past two acceleration samples. This extrapolation is based on a ramp assumption, namely a first-degree polynomial model. Here the applicability of the first-degree polynomial model is justified by the obvious fact that the inertia of the rotating components is so high that the angular velocity cannot vary greatly between any two pulses. If the angular acceleration estimates corresponding to the latest two pulse instants are $\alpha(k-1)$ and $\alpha(k)$, the next acceleration sample, $\alpha(k+1)$, can be extrapolated as

$$\alpha(k+1) = \alpha(k) + [\alpha(k) - \alpha(k-1)] \tag{32}$$

Now linear interpolation can simply be used to compute the acceleration estimates at arbitrary instants between the discrete time indices k and $k+1$.

Kadhim et al. (24) presented a straightforward method for the measurement of steady-state and transient acceleration of a rotating motor shaft. They suggested the use of a slight modification of Eq. (31); three total pulse counts (c_1, c_2, c_3) are used instead of the two pulse count differences (n_1, n_2):

$$\alpha = \frac{2\pi(c_3 - 2c_2 + c_1)}{NT^2} \tag{33}$$

Also here the increase of the measuring interval T improves the accuracy of average acceleration. However, it increases as well the harmful delay between the instant when a measurement becomes available and the moment at which it applies. Kadhim et al. have suggested that for off-line applications, low-degree polynomial curve-fitting techniques can improve the signal-to-error ratio. Also on-line curve-fitting techniques are readily available for such a purpose, as discussed above in “Predictive Postfiltering.”

Laopoulos and Papageorgiou (10) proposed an angular acceleration measurement instrument based on a 24-bit signal processor in 1996. They computed the difference of the pulse encoder’s output frequency ΔF for two consecutive pulse cycles with the measuring interval of ΔT. Hence the angular acceleration is directly proportional to $\Delta F/\Delta T$. In this division both the frequency difference and the measuring interval should have a high resolution to keep the amplified quantization noise low. To provide this necessity, a 24-bit binary counter was applied to count the period of an encoder pulse cycle using a 20 MHz clock. The angular acceleration is calculated from

$$\alpha = \frac{2\pi\,\Delta F}{N\Delta T} \tag{34}$$

Notice the varying length of the measuring interval ΔT which depends on the availability rate of fresh encoder pulses. With this method the accuracy of the (nonuniformly sampled) acceleration estimate is determined largely by the random jitter corrupting the nominal encoder pulse length. A high-performance, uniformly sampled acceleration measurement method can be constructed by combining the predictive synchronization and restoration method of Pasanen et al. (8) with the accurate acceleration measurement procedure of Ref. 10.

All the methods discussed above are based on the noise-amplifying difference function: either a pulse count difference or a frequency difference. Schmidt and Lorenz (25) use another possible indirect approach; they propose a computationally efficient acceleration observer for a dc servo drive. Their acceleration observer uses two measured input quantities: the shaft angle and the actual current of the dc motor. The entire state-observer contains a velocity observer block that provides the estimate of the angular velocity for the following acceleration observer. It should be noted that this state-observer is intimately connected to the overall controller structure. A sampling period of 1.25 ms was successfully applied, and all the computations were performed using high-precision floating-point arithmetic. The observer-generated acceleration feedback was shown to improve the control performance remarkably.

Evaluative Discussion of the Specific Methods

The development of indirect measuring instruments of angular acceleration began in the late 1960s. Then there were two important advancements that made the classic work of Hoffmann de Visme (22) and Dunworth (6) possible: the emerging availability of rotating encoders with high-pulse rates and the rapid development of solid-state digital electronics and integrated circuits. The early instrumentation techniques were strongly technology driven without a strict connection to any specific application.

At the beginning of the 1970s no outstanding innovations occurred in the field while available methods continued to evolve, such as that of Smith et al. (23) which was implemented in a general-purpose minicomputer. Nevertheless, with the development of high-resolution optical encoders came an increase in the number of successful industry applications, and acceleration feedback provided opportunities to increase functional performance.

In the late 1970s and during all of the 1980s, the availability of microprocessors and microcontrollers turned the instrumentation engineers’ interests almost entirely from pure hardware implementations to software implementation of acceleration measurement procedures. During that productive period the entire control and instrumentation industry began to introduce novel microprocessor-based products, pushing the need for acceleration feedback mostly to the background.

Finally, at the very beginning of the 1990s, it became evident that the performance of motion control systems in the well-established robotics and servo industry could be improved economically by using additional acceleration feedback. Acceleration feedback is advantageous because it allows higher overall stiffness without requiring higher bandwidths of the velocity and position control loops in servo applications.

Essentially, as is pointed out by Schmidt and Lorenz (25), the acceleration feedback acts like an “active inertia.” Along with this application-pull factor in the resurgence of interest in acceleration feedback, there were the traditional technology-push cases. The recent development of implementation techniques (signal processors and application-specific integrated circuits), signal-processing algorithms, and estimation methods has opened a new era of growth for indirect techniques in acceleration measurement. Therefore it is foreseeable that both the availability of high-performance methods (10,12,25), and their successful applications are going to increase steadily.

CONCLUSION

A comprehensive variety of direct and indirect techniques exist for acceleration measurement. Application-tailored measurement techniques currently predominate in real-time systems, and computerized instrumentation, in general, is becoming widespread. However, mixed analog-digital processing offers some benefits over pure digital signal processing in cases where the primary measurement quantity is available as an analog voltage or current.

The field of linear and vibratory acceleration measurement is stable and well developed. A complete assortment of instruments for vibration measurement is available, for example, from Bruel and Kjaer Instruments. The main focus of future development is in the intelligent postprocessing of accelerometer data and the automatic interpretation of measurement results. The performance and capabilities of integrated accelerometer modules will continue to increase as low-cost accelerometers and more novel applications are introduced such as is presently being seen in the field of technical diagnostics.

The whole field of angular acceleration measurement is expanding as both direct and indirect techniques are evolving to new applications. There exist two principal challenges for the research and development community: to develop economical and accurate angular accelerometers for unlimited rotation range and to create wideband indirect techniques with a low signal-to-error ratio. It is likely that the entire technology profile of acceleration measurement will advance in the early twenty-first century.

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BIBLIOGRAPHY

1. M. Alonso and E. J. Finn, *Fundamental University Physics, Volume One: Mechanics and Thermodynamics,* 2nd ed., Reading: Addison-Wesley, 1980, Ch. 5.
2. J. W. Dally, W. F. Riley, and K. G. McConnell, *Instrumentation for Engineering Measurements,* New York: Wiley, 1984, Ch. 7.
3. G. C. Barney, *Intelligent Instrumentation: Microprocessor Applications in Measurement and Control,* London: Prentice-Hall International, 1985, Ch. 6.7.
4. P. H. Garrett, *Advanced Instrumentation and Computer I/O Design: Real-Time System Computer Interface Engineering,* New York: IEEE Press, 1994, Ch. 7.
5. I. Godler et al., A novel rotary acceleration sensor, *IEEE Control Syst.,* **15:** 56–60, 1995.
6. A. Dunworth, Digital instrumentation for angular velocity and acceleration, *IEEE Trans. Instrum. Meas.,* **18:** 132–138, 1969.
7. G. P. Hancke and C. F. T. Viljoen, The microprocessor measurement of low values of rotational speed and acceleration, *IEEE Trans. Instrum. Meas.,* **39:** 1014–1017, 1990.
8. J. Pasanen, O. Vainio, and S. J. Ovaska, Predictive synchronization and restoration of corrupted velocity samples, *Measurement,* **13:** 315–324, 1994.
9. R. H. Brown, S. C. Schneider, and M. G. Mulligan, Analysis of algorithms for velocity estimation from discrete position versus time data, *IEEE Trans. Ind. Electron.,* **39:** 11–19, 1992.
10. T. Laopoulos and C. Papageorgiou, Microcontroller-based measurement of angular position, velocity, and acceleration, in *Proc. IEEE Instrumentation and Measurement Technology Conference,* 73–77, 1996.
11. A. Jaritz and M. W. Spong, An experimental comparison of robust control algorithms on a direct drive manipulator, *IEEE Trans. Control Syst. Technol.,* **4:** 627–640, 1996.
12. P. R. Bélanger, Estimation of angular velocity and acceleration from shaft encoder measurements, in *Proc. IEEE International Conference on Robotics and Automation,* 585–592, 1992.
13. C. S. Williams, *Designing Digital Filters,* Englewood Cliffs, NJ: Prentice-Hall, 1986, Ch. 7.
14. S. J. Ovaska and O. Vainio, Recursive linear smoothed Newton predictors for polynomial extrapolation, *IEEE Trans. Instrum. Meas.,* **41:** 510–516, 1992.
15. S. J. Ovaska, Predictive signal processing in instrumentation and measurement: A tutorial review, in *Proc. IEEE Instrumentation and Measurement Technology Conference,* 48–53, 1997.
16. O. Vainio and S. J. Ovaska, A class of predictive analog filters for sensor signal processing and control instrumentation, *IEEE Trans. Ind. Electron.,* **44**: 565–570, 1997.
17. P. S. Maybeck, *Stochastic Models, Estimation, and Control,* vol. 1, San Diego: Academic, 1979.
18. P. Katz, *Digital Control Using Microprocessors,* London: Prentice-Hall International, 1981.
19. R. Morrison, *Solving Interference Problems in Electronics,* New York: Wiley, 1995, Ch. 6.
20. N. Furukawa and K. Ohnishi, A structure of angular acceleration sensor using silicon cantilevered beam with piezoresistors, in *Proc. IEEE IECON,* 1524–1529, 1992.
21. M. U. Reissland, *Electrical Measurements: Fundamentals, Concepts, Applications,* New Delhi: Wiley Eastern, 1989, Ch. 7.3.
22. G. Hoffmann de Visme, Digital processing unit for evaluating angular acceleration, *Electronic Engineering,* **40:** 183–188, 1968.
23. I. R. Smith, M. J. Hajiroussou, and J. F. Miller, Precision digital tachometer, *IEEE Trans. Instrum. Meas.,* **22:** 278–279, 1973.
24. A. H. Kadhim, T. K. M. Babu, and D. O'Kelly, Measurement of steady-state and transient load-angle, angular velocity, and acceleration using an optical encoder, *IEEE Trans. Instrum. Meas.,* **41:** 486–489, 1992.

25. P. B. Schmidt and R. D. Lorenz, Design principles and implementation of acceleration feedback to improve performance of dc drives, *IEEE Trans. Ind. Appl.,* **28:** 594–599, 1992.

SEPPO J. OVASKA
Helsinki University of Technology

ACCELERATOR SUPERCONDUCTING CAVITY RESONATORS. See SUPERCONDUCTING CAVITY RESONATORS.