J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*

INDUCTANCE MEASUREMENT

Along with resistors and capacitors, inductors which are basically components with the property of inductance, are commonly used for building circuits for operating a wide range of electrical and electronic apparatus and instruments. The inductance parameter also inevitably enters the equivalent circuits of various electromagnetic devices and machines. Consequently, the need for inductance measurement arises in the characterization, design evaluation, and production tests of the relevant components and in the circuit modeling of electrical apparatus. Inductance measurement is also important in instrumentation systems where a physical quantity is sensed through its effect on the inductance of a transducer.

Basic Concepts, Definitions, and Units

The Inductance Parameter. Inductance is the property whereby an electrical device sets up magnetic flux when current passes through it. In principle, any conductor of current has this property. However it is more pronounced in a coil of wire or any electrical device whose construction is akin to a coil. Taking the solenoid of N turns shown in Fig. 1(a) as representing a general situation, let a current of i amperes through it produce a magnetic flux of ϕ webers. For simplicity, if we assume that the closed path of every flux line is interlinked with all N turns, then the coil has flux linkages of $N\phi$ weber-turns. The inductance L of the coil, defined as the flux linkages produced in it per unit current, is measured in *henrys.* One henry corresponds to one weber-turn per ampere.

$$L = \frac{N\phi}{i} \tag{1}$$

Any physical device, such as the coil considered, which has the property of inductance, is called an *inductor.* It may also have other properties such as resistance and capacitance to a less consequential degree. If these other parasitic properties are absent, the device is referred to as an *ideal* or *pure inductor.* If the current i in an inductor and the flux ϕ produced by it are proportionally related, then the value of inductance as defined by Eq. (1) is constant and the inductor is *linear.* A *nonlinear* inductor does not have a linear $N\phi$ versus i relationship and its inductance L, defined by Eq. (1), is a function of the current i.

An inductor is built in the form of a coil of good conducting material, usually copper, wound around a core of a nonmagnetic medium like air or a magnetic material such as iron. For the same core geometry, the inductance of a linear inductor is proportional to the square of the number of turns in the coil. Use of iron core permits larger values of inductance for a given inductor size but makes the inductor nonlinear because of the nonlinear magnetization characteristic of iron.

When the current i in an inductor varies, there is an induced voltage e in the inductor, which, according to Faraday's law of induction, is equal to the rate of charge of its flux linkages, that is, $d(N\phi)/dt = [d(N\phi)/di](di/dt)$. This voltage acts in a direction opposing the change in i. For the coil in Fig. 1(a), if di/dt were positive, the

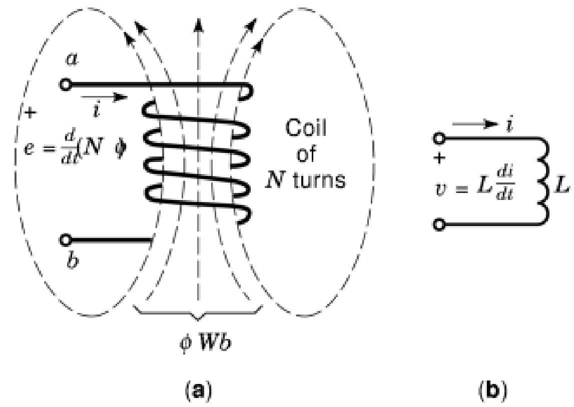


Fig. 1. An inductor. (a) Physical configuration of a typical inductor. (b) Representation of a pure inductor as a circuit element.

induced voltage makes the coil behave like a voltage source driving a current in the local circuit in a direction opposite to that of i, that is, in a sense so as to arrest the growth in i. Terminal a then acquires positive polarity with respect to terminal b. For a decreasing i and hence a negative di/dt, the polarity is reversed. In either event, with the reference polarity shown in Fig. 1(a), the induced voltage in the inductor is given by

$$e = \frac{d(N\phi)}{di}\frac{di}{dt} = L\frac{di}{dt} \tag{2}$$

For a linear inductor, the value of L in the foregoing equation is the constant value of inductance given by Eq. (1). If the inductor is nonlinear, the value of L in Eq. (2) is the incremental inductance, that is, the slope of the $N\phi$ versus i characteristic at the particular operating current.

Inductor as a Circuit Element. Figure 1(b) is a representation of a pure inductor as a circuit element. In a pure inductor free from resistive and capacitive parasitic effects, the terminal voltage v equals the induced voltage e and the same current i passes through every section of the conductor. As a consequence, its terminal v-i relationship is given by

$$v = L\frac{di}{dt} \tag{3}$$

where L is the incremental inductance if the inductor is nonlinear. In the sequel, we take all inductors as linear, unless specifically stated otherwise.

As seen from Eq. (3), the inductance parameter quantifies the opposition offered by an inductive device to a change in its current. The higher the rate of change of current, the more the applied voltage needed to support this change. The relation between v and i in Eq. (3) is a statement of the cardinal property of an ideal inductor from the circuit viewpoint and is applicable to general time-varying current and voltage. In ac circuits at steady state, where all currents and voltages are sinusoids of the same frequency, the phasors of v and i are related by the algebraic expression,

$$\overline{V} = j(2\pi fL)\overline{I} = j\omega L\overline{I} = jX_L\overline{I} = \overline{Z}_L\overline{I} \tag{4}$$

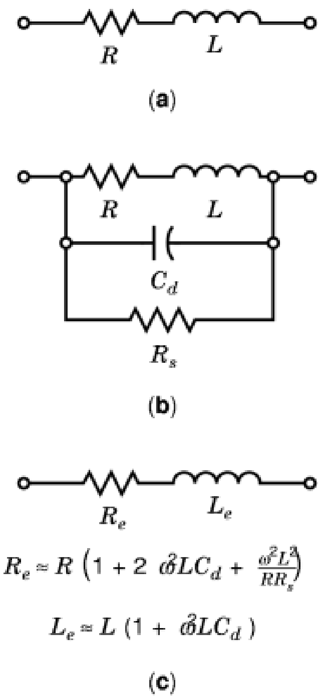


Fig. 2. Circuit representations of a practical inductor. (a) Equivalent circuit suitable for an air-cored inductor at dc and low frequencies. (b) Equivalent circuit valid for a wide range of frequencies. (c) Simplified equivalent circuit valid for $R \ll \omega L \ll R_s$ and $\omega^2 LC_d \ll 1$.

where f is the frequency in Hz and $\omega = 2\pi f$ is the angular frequency in radians/sec, $X_L = \omega L$ and $\bar{Z}_L = jX_L$ are the reactance and complex impedance of the inductor, both expressed in ohms. Equation (4) embodies two important properties of an ideal inductor in an ac circuit. First, the impedance (opposition) offered to a sinusoidal current varies proportionately with frequency. Secondly, the voltage has a phase angle of $\pi/2$ radians more than the current.

Practical inductors do not occur in pure form. They include incidental resistive and capacitive properties to a degree which depends on their design. These impurities arise from the resistance of the conductor used to wind the inductor, the distributed capacitance between turns and between layers of the winding, and the imperfection of the insulation used. With dc and low frequencies, the effects of the latter two factors are negligible, and an inductor can be represented by the simple series circuit of Fig. 2(a), where L is the inductance and R the dc resistance of its winding. A more accurate representation valid over a wide range of frequencies is the circuit of Fig. 2(b). Here, C_d is the lumped-element approximation of the distributed capacitance of the winding. The shunt resistance R_s accounts for the dielectric power losses in the insulating material of the winding and the eddy-current power losses caused in the neighbouring conductors and the shield if any, by an ac current in the inductor. For iron-cored inductors, R_s also includes the effects of eddy-current and hysteretic power losses in the core and consequently has a lower value than the corresponding air-cored inductor. Furthermore, with magnetic cores, R_s and L are nonlinear. Their values depend on the applied voltage at any given frequency.

In ac circuit applications of an inductor, the parasitic elements R, C_d, R_s restrict the useful frequency range over which the device has a predominantly inductive character. This range extends over frequencies which are well below the self-resonant frequency of the inductor ($\omega^2 LC_d \ll 1$) and at which the inductive reactance ωL is much larger than the series resistance R and much smaller than the shunt resistance R_s.

Under these conditions, the equivalent circuit of Fig. 2(b) is approximated by the simpler series equivalent circuit of Fig. 2(c), where the effective inductance L_e and series resistance R_e are functions of frequency given by

$$L_e = L(1 + \omega^2 LC_d) \tag{5}$$

and

$$R_e = R\left(1 + 2\omega^2 LC_d + \frac{\omega^2 L^2}{RR_s}\right) \tag{6}$$

In the frequency range under consideration, the effect of self-capacitance C_d, thus, is increased effective inductance and series resistance over their dc values. A larger C_d leads to larger increases. The dielectric and other losses represented by R_s contribute to an increased R_e with negligible effect on L_e. A further contributing factor to the dependence of R_e on frequency is that the winding resistance R is a function of frequency which increases above its dc value at higher frequencies because of skin and proximity effects. The resulting modified current distribution over the cross section of the conductors also lowers the value of L at higher frequencies.

The quality factor Q is commonly used as a figure of merit for a practical inductor in ac applications. A measure of the effectiveness of an energy storage device in performing its role with small power dissipation, it has the general definition

$$Q = 2\pi \frac{\text{peak value of energy stored}}{\text{energy dissipated per cycle}} \tag{7}$$

For the series equivalent circuit of Fig. 2(c), its value is equal to the ratio of its reactance to resistance, that is,

$$Q_e = \frac{\omega L_e}{R_e} \tag{8}$$

The Q-factor of a practical inductor exhibits a characteristic variation. Increasing linearly with frequency at first, then more gradually, it attains a peak in middle range of frequencies before it tapers off to low values at higher frequencies well below the self-resonant frequency. The self-capacitance of the winding, the skin and proximity effects on the conductor resistance, and the various power losses represented by R_s all contribute to reducing Q at high frequencies. The midrange of frequencies over which Q is fairly large and nearly constant is important in applications of the inductor as a circuit element in filter circuits. Inductors with iron cores have larger values of Q but a smaller usable frequency range. The maximum attainable value of Q with practical inductors is limited to a few hundred.

The important parameters of an inductor in circuit design are L_e and Q_e. The latter is preferred to R_e because it shows a smaller variation over the frequency band of interest. Because the theoretical deduction of the values of these parameters at any specified frequency has limited scope, it becomes necessary to measure them experimentally. Measurements on inductors imply the measurement of L_e and Q_e (or alternatively R_e) at different frequencies over a specified range.

The Mutual Inductance Parameter. The inductance of a single isolated coil discussed so far is called its self-inductance. The property of mutual inductance arises when two inductors are in proximity so that some or all the flux lines set up by the current in one inductor also link with the other inductor. Because of this magnetic coupling, when the current in the first inductor varies, its self-flux linkages and also the flux linkages created by it in the other inductor (called mutual flux linkages) vary, thereby inducing voltage in the latter.

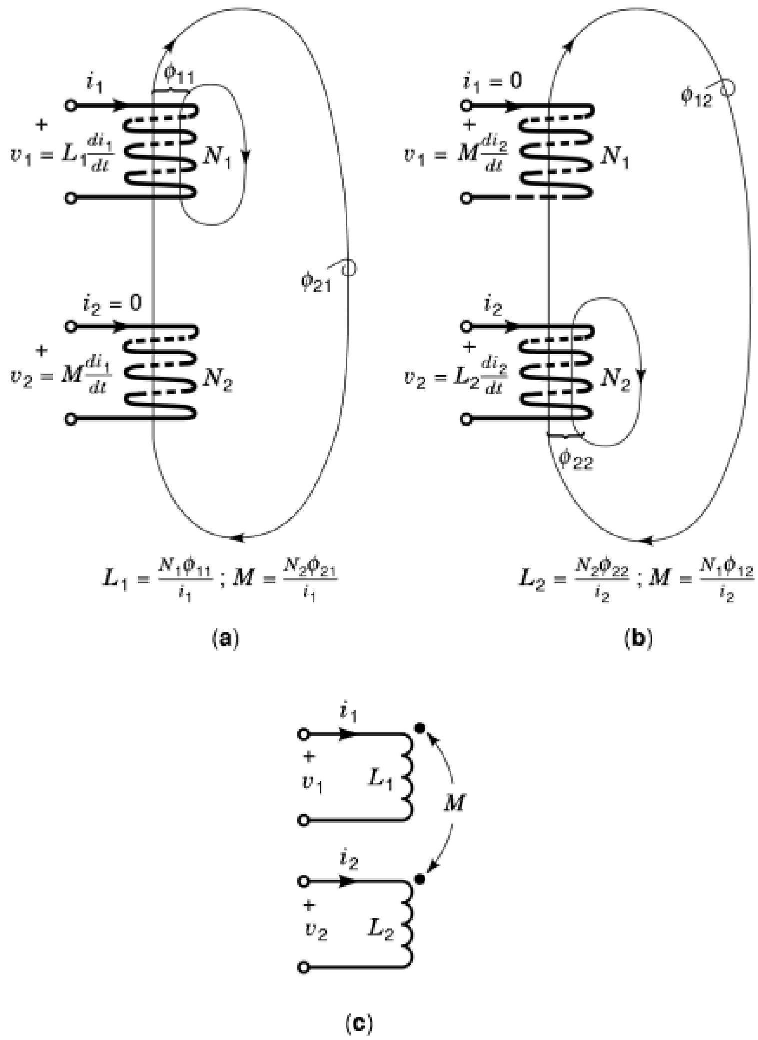


Fig. 3. Mutual inductance between two coils. (a) Voltages induced when coil 1 alone carries a current. ϕ_{11} and ϕ_{21} represent the self and mutual fluxes. (b) Voltages induced when coil 2 alone carries a current. ϕ_{22} and ϕ_{12} represent the self and mutual fluxes. (c) Circuit representation of two coupled coils.

To illustrate the concept of magnetic coupling, we consider two ideal linear inductors constituted by coils of N_1 and N_2 turns, as shown in Fig. 3(a), and with self-inductances of L_1 and L_2. As a simplified description, let current i_1 passing through coil 1 establish a self-flux of ϕ_{11} webers which links with all of its N_1 turns and let ϕ_{21}, a part of this flux, link with all the N_2 turns of coil 2. The mutual inductance M between the coils is defined as the flux linkages in coil 2 per unit current in coil 1, that is, $M = N_2\phi_{21}/i_1$ and is measured in henrys, the same units adopted for self-inductances. It is also defined as the induced voltage in coil 2 per unit rate of change of current in coil 1. The self- and mutually induced voltages in the two coils, when only one of them carries a current, are specified in Figs. 3(a) and 3(b). The equality of $\phi_{21}N_2/i_1$ in Fig. 3(a) and $\phi_{12}N_1/i_2$ in Fig. 3(b) is established from energy considerations and is termed the reciprocity property of mutual inductances. The circuit representation of two coupled ideal inductors is given in Fig. 3(c). When they carry currents i_1 and

i_2 simultaneously, their terminal voltages are given by Eqs. 9(a) and 9(b):

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{9a}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \tag{9b}$$

One terminal of each inductor is specially marked, often with a *dot,* as in Fig. 3(c). If the two currents, in their reference directions, both enter (or both leave) their respective inductors through the dotted terminals, then the self- and mutual fluxes are in the same direction in each coil. In this event, the signs of both the self- and mutually induced voltages in each terminal equation are the same. They are opposite otherwise. In Fig. 3(c), the reference directions for i_1 and i_2 both enter the inductors through dotted terminals and hence Eqs. 9(a) and (b) have the same signs for both self- and mutually induced voltages. The dot notation avoids the necessity of indicating explicitly the winding sense of each coil as done in Figs. 3(a) and (b).

In steady-state ac circuits with sinusoidal currents and voltages of angular frequency ω, the terminal equations of the coupled inductors in terms of the respective phasors take the form

$$\overline{V}_1 = (j\omega L_1)\bar{I}_1 + (j\omega M)\bar{I}_2 \tag{10a}$$

and

$$\overline{V}_2 = (j\omega M)\bar{I}_1 + (j\omega L_2)\bar{I}_2 \tag{10b}$$

The quantity ωM measured in ohms is the mutual reactance between the inductors.

Standard Inductors

The term "standard" refers to an object built to realize a physical quantity or parameter very accurately. Inductance is the parameter of concern in this section, and we deal with two groups of standards, namely, standards of self-inductance and those of mutual inductance. Either type is further classified as fixed or variable, depending on whether the standard has a single definite value of the inductance parameter or whether it permits a continuous variation of the value over a certain range through an appropriate adjustment.

Another classification of standards is absolute or secondary. An absolute standard of inductance is one whose inductance is computed from accurately measured dimensions of the device. Such standards serve as the basis for calibrating secondary standards. The latter are either reference standards (which are directly compared with absolute standards) or laboratory working standards (which are calibrated with respect to the reference standards). Both the reference and working standards of inductance have essentially the same constructional features. These are discussed after the following section on absolute standards.

Absolute Standards. Attaining the highest possible accuracy rather than economy is the chief concern in designing absolute standards. These standards of inductance incorporate coil configurations that permit applying the relevant formula for inductance with negligible error. The choice of materials and details of construction ensure precise and permanent positioning of the conductor on the coil former. The coil configuration generally adopted is a single layer solenoid of copper wire uniformly wound on a truly cylindrical former made of glass, ceramic, or marble and with a precisely machined helical groove to accommodate the conductor. The cost involved in their construction and maintenance and the sophisticated optical methods used to measure their dimensions render these standards unsuitable for use in a general laboratory. The tasks of maintaining absolute standards and of employing them to calibrate secondary standards are undertaken only at the national

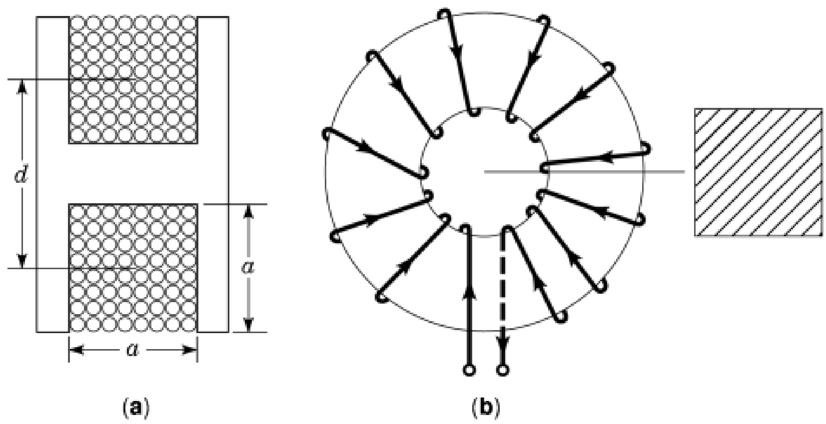


Fig. 4. Windings for standard self-inductors. (a) Solenoidal coil provides larger Q values. Optimum design requires $d = 3a$. (b) Toroidal coil provides a nearly astatic arrangement.

standards laboratories in each country. Even here, absolute standards of mutual inductance are preferred to absolute standards of self-inductance because formulas of higher theoretical accuracy are available for mutual inductance than for self-inductance and because skin effect and self-capacitance of windings are of smaller consequence. Details of representative absolute standards of mutual inductance are available in Refs. 1,2,3.

The cylindrical cross-capacitor, whose capacitance is determined by a single length measurement has become a fundamental electrical standard of choice in national laboratories, because of its extremely high accuracy. As an analog, a calculable mutual inductance standard in the form of a cage of five parallel wires is proposed by Page (4) for calibrating mutual inductor reference standards. The mutual inductance of this calculable standard is given by

$$\frac{1}{10}\ln\frac{3+\sqrt{5}}{2}\,\mu\mathrm{H/m}$$

Fixed Inductance Standards. As a rule, standard inductors are made without iron cores to obtain an inductance value independent of current. Marble, glass, porcelain or wood impregnated with paraffin are some of the materials used to construct the coil former. They are nonmagnetic and provide good insulation and adequate dimensional stability. Litz wire is employed for the coil to reduce the consequences of skin effect in the conductor.

There are two basic coil forms, illustrated in Fig. 4, in common use for self-inductance standards. The flat cylindrical coil of square winding cross section shown in Fig. 4(a) provides larger inductance values for a given length and diameter of the conductor than the toroidal configuration of Fig. 4(b). The maximum inductance is obtained when $d = 3a$. However, the arrangement is significantly affected by external magnetic fields from other sources and produces a large interfering field of its own. The uniformly wound toroidal inductor, on the other hand, has smaller Q values but provides a nearly astatic arrangement. As far as generation of and coupling with external fields is concerned, the toroidal winding is equivalent to a single turn with the mean radius of the torus. Figure 5 illustrates two ways by which this “single-turn effect” is effectively overcome. An auxiliary circular turn carrying a current in the opposite direction is provided on the surface of the toroid in the arrangement of Fig. 5(a). In the duplex winding of Fig. 5(b), the current from a lead does not take a path around the complete toroid, but splits equally into two parallel paths, each around half the toroid, and

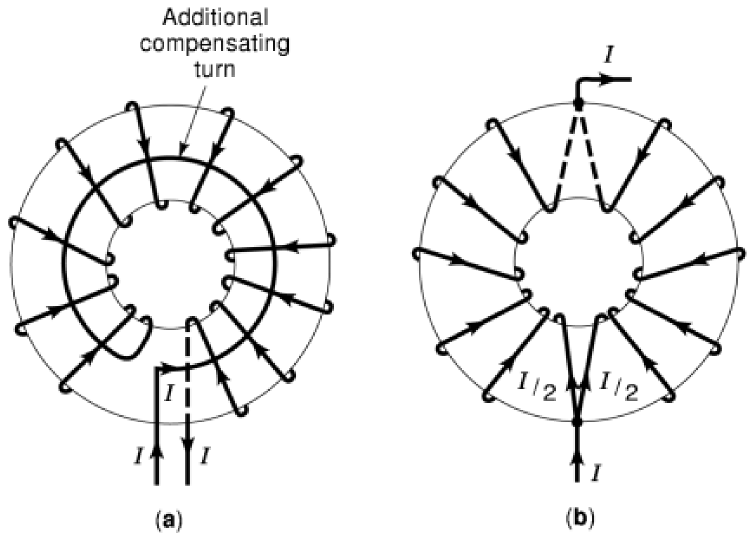


Fig. 5. Methods of compensation for "single-turn effect." (a) An additional circular turn carries current in the opposite direction. (b) In the duplex winding, current flows clockwise in one-half of the effective single turn and anticlockwise in the other half.

recombines at the diametrically opposite point. The currents in the two parallel paths have a cumulative action in establishing the magnetic flux around the toroid but act in opposite directions in setting up an interfering field in the axial direction.

Due attention is necessary in designing and constructing standard inductors to the size and placement of terminals and to the effects of temperature on the inductance value. See Refs. 1,2, and 5. The manufacturer of a standard inductor certifies its effective inductance at a particular frequency and suitable corrections are needed for use at other frequencies. In any event, the operating frequency must be well below the self-resonant frequency of the inductor. The typical accuracy of a standard inductor is 0.1%.

Fixed mutual-inductance standards have design features similar to those of self-inductance standards except that they consist of two windings instead of one. Two circular coils of the same radius are fixed on the same bobbin axially displaced from one another, or they are concentric coils of different radii.

Variable Inductance Standards. Decade units of self- and mutual inductances are realized by an appropriate series connection of uncoupled fixed self-inductors or of sections of the secondary winding of a mutual inductor. The stepwise variation available with such units may not be sufficiently fine in several applications. Furthermore, in bridge measurements, the need exists for an inductor whose inductance value can be continuously changed without changing its resistance. Such a variable inductor is called a variometer or an inductometer. As a rule, these devices are less accurate than fixed standards. A typical value is 1%.

An inductometer consists of two coils, which are rotatable with respect to each other, so that the mutual inductance between them can be varied continuously, say from $-M_0$ to $+M_0$. If the two coils are connected in series, then the self-inductance of the combination can be varied from $L_1 + L_2 - 2M_0$ to $L_1 + L_2 + 2M_0$ where L_1 and L_2 are the individual self-inductances. Thus the inductometer is used as a variable self-inductor or as a variable mutual inductor.

Figure 6 illustrates two forms of the inductometer. The Ayrton–Perry arrangement, in which the fixed and rotatable coils are wound on sections of spherical surfaces, has the disadvantages of lack of astaticism and a nonlinear variation of M with angular rotation. The Brooks–Weaver inductometer has a disk-shaped structure with two pairs of fixed coils and a rotatable third pair sandwiched between them. The left and right

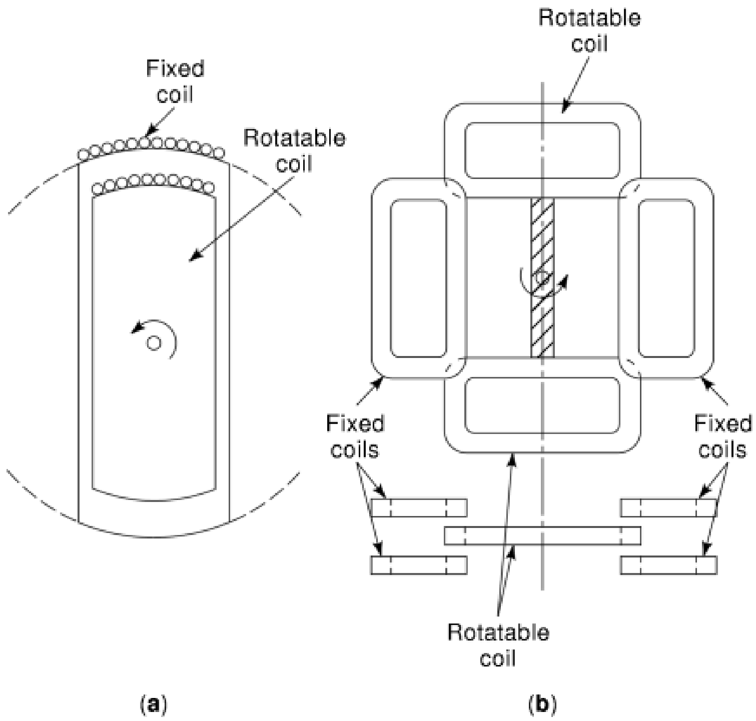


Fig. 6. Two forms of inductometer. (a) The Ayrton–Perry inductometer has a nonlinear scale and is not astatic. (b) The Brooks–Weaver inductometer provides a linear scale and is astatic.

groups of fixed coils are connected to form an astatic arrangement. When the flux direction is upward in one group, it is downward in the other group, and one side serves as the return path for the flux of the other group. The coils are shaped to provide a fairly linear variation of M with the angle of rotation. An advantage of the arrangement is that small axial displacements of the rotor with respect to the stator caused by mechanical wear of the bearings do not significantly alter the value of M at any angular position.

Methods of Measurement of Self-Inductance

The salient schemes of measurement, grouped under four heads, are detailed in the following. All measurements are made under sinusoidal steady-state conditions and the values of L_e and R_e (or equivalently Q_e) of the inductor at the test frequency are determined. The inductor current (voltage) in the measuring circuit should be kept at the same level as in the actual application of the device to obtain reliable results for nonlinear inductors.

Methods Employing Indicating Instruments. These methods are easy and convenient in implementation and yield results of moderate accuracy. Figure 7 illustrates two commonly adopted schemes.

In the three-voltmeter method of Fig. 7(a), a known standard resistance R_s is connected in series with the test inductor. The values of R_e and L_e are computed from the three voltmeter readings V_s, V_1, and V_2, as indicated in the figure. The large number of algebraic operations involved render the final results prone to gross accumulation of errors.

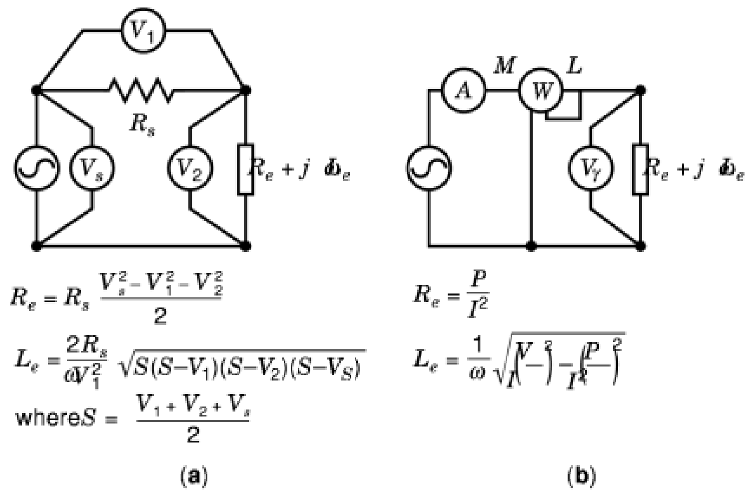


Fig. 7. Inductance measurement using indicating instruments. (a) Three voltmeter method. (b) Wattmeter method.

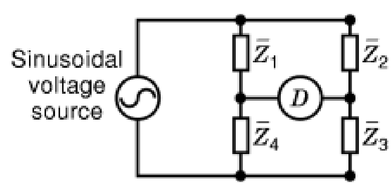


Fig. 8. Standard form of a four-arm bridge network.

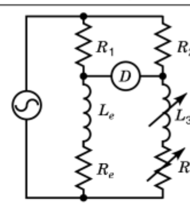
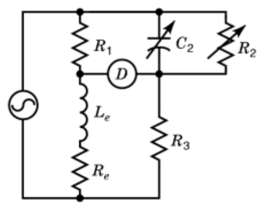
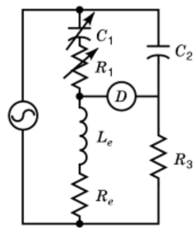
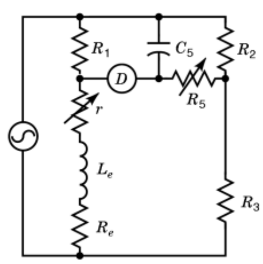
The wattmeter method of Fig. 7(b) yields the values of L_e and R_e from the readings V, I, and P of the voltmeter, ammeter, and wattmeter. This method is particularly suitable at power frequencies. Its frequency range is limited by that of the dynamometer wattmeter.

ac Bridge Measurements. The ac bridge method is one of the most accurate and sensitive means of determining the parameters of a circuit component such as an inductor. The accuracy stems from the fact that the unknown impedance is measured purely in terms of standard circuit components, which can be built to a much higher degree of accuracy than the best indicating instruments and are also less susceptible to deterioration in accuracy from ageing and environmental influences.

The Four-Arm ac Bridge. The four-arm ac bridge network has the general form shown in Fig. 8. The voltage source is typically a sinusoidal oscillator whose frequency is adjusted to the desired value. $\bar{Z}_4$ is the impedance to be measured, and the other three arms contain fixed and variable standard elements. The variable elements are adjusted until the detector D senses zero voltage across it. The bridge is then balanced, and $\bar{Z}_4$ is given by $\bar{Z}_4 = \bar{Z}_1\bar{Z}_3/\bar{Z}_2$.

The previous equation in complex numbers yields two real equations in terms of the bridge elements, which are called the balance conditions of the bridge. At least two variable elements should be provided to force bridge balance. Variable resistors are usually preferred for this purpose. If each variable element enters only one of the balance equations exclusively, then the balance adjustments are independent and the balancing process is faster. Furthermore, the balance conditions should preferably be independent of frequency to achieve a sharper balance detection when the source voltage contains harmonics. Detectors include vibration galvanometers, telephone receivers, tuned-amplifier detectors, oscilloscopes and radio receivers. References 1,2,6 give a comprehensive account of their relative characteristics and useful frequency ranges.

Table 1. Bridge Networks for Measurements on Self-Inductors

Type	Configuration	Balance Equations	Remarks
Maxwell		$L_e = L_3 \frac{R_1}{R_2}$ $R_e = R_3 \frac{R_1}{R_2}$	Requires an inductometer.
Maxwell–Wien		$L_e = C_2R_1R_3$ $R_e = R_3 \frac{R_1}{R_2}$ $Q_e = \omega C_2R_2$	For measurement of L_e and Q_e, the adjustable elements are R_1 (or R_3) and R_2. Suitable for low Q ($Q < 10$) inductors.
Owen		$L_e = C_2R_1R_3$ $R_e = \frac{C_2}{C_1} R_3$	Suitable for a wide range of inductance values. Determination of L_e in terms of a fixed capacitor is an advantage.
Anderson		$L_e = C_5R_1 \left[R_3 + R_5 + \frac{R_3R_5}{R_2} \right]$ $R_e = R_1 \frac{R_3}{R_2} - r$	Suitable for a wide range of inductance. Only resistors are used as balancing elements.

With the background of the foregoing general remarks, let us now look at a few popular ac bridge networks used for measuring inductors. Table 1 gives the configurations and balance equations of these networks. For measuring high-Q inductors, impractically large values of R_2 are needed in the Maxwell–Wien configuration. This difficulty is overcome by reconnecting R_2 and C_2 in series in arm 2. The resulting configuration, known as the Hay bridge, is quite suitable for use with high-Q inductors. The Owen bridge yields good accuracy but needs a large-valued adjustable capacitor C_1 for high-Q inductors. The Anderson bridge, a modification of the four-arm configuration, is quite suitable for a wide range of inductances and Q-values. A merit of this network is that preliminary balance is obtained with dc excitation and a suitable dc detector by adjusting r and the final balance with ac excitation effected essentially with R_5 accompanied by a slight readjustment of r.

Several important issues common to all bridge measurements are the rapidity with which balance adjustments converge, the sensitivity of detection, the effects of residuals (impurities) in the bridge elements, and the adoption of screening and other strategies to combat the effects of stray admittances and parasitic coupling between elements. The reader may consult Refs. 1,2, and 5,6,7,8,9,10,11 for good coverage of these topics.

Typical commercial bridges for inductance measurements have accuracies ranging from 0.05% to 1%. Besides measurements on conventional coils used as circuit components, inductance bridges are also employed for such purposes as determining inductive output impedances of amplifiers, inductance and Q-values of loudspeaker voice coils and television deflection yokes.

T-networks for RF Measurements. To combat the effects of stray capacitances, which are more pronounced at higher frequencies, it is desirable to ground one end of both the voltage source and the detector. The four-arm bridge does not permit this without using isolating transformers. Two networks that permit this condition are the bridged-T and the twin-T measuring networks, which are useful for measuring inductances, especially at radio frequencies. These have three-terminal, two-port configurations, and the balance equations necessarily involve the frequency. References 2,5,6, and 8 may be consulted for further details.

Transformer Ratio Bridges. A very precise and stable voltage or current ratio standard is established by a suitably designed transformer. The voltage or current ratio is determined by the ratio of the number of turns in two windings of a transformer and is unaffected by temperature, ageing and other influences. The imperfections of the materials used for the core and conductor and in the magnetic coupling between the various turns cause only second-order effects, which can be minimized by careful design and choice of materials. Transformer ratio bridges that take advantage of this property have been developed over the last four decades, which incorporate variable-ratio standards of extraordinarily fine resolution (typically 1 in 10^7) and compatible accuracy. The transformer ratio bridges effectively solve the problem of stray admittances in measuring impedances and have the further advantage of needing only fixed-value standard R and C elements. The principle of a transformer ratio bridge scheme of inductor measurement is shown in Fig. 9. The transformer ratios n_1, n_2, k_1, and k_2 are adjusted to secure a null response in the detector D. The flux in the core of T_2 is zero under these conditions. Hence, the primary winding of T_2 presents only a very small impedance equal to its resistance and the currents in C_s and R_s are controlled by $n_1\,\bar{V}_s$ and $n_2\,\bar{V}_2$, respectively. The ampere-turn balance at this condition is given by

$$N(\overline{Y}_e\overline{V}_s) + k_1N(j\omega C_s n_1\overline{V}_s) - k_2N(n_2\overline{V}_s/R_s) = 0$$

or

$$\overline{Y}_e = \frac{k_2 n_2}{R_s} - j\omega k_1 n_1 C_s \qquad (11)$$

In practice, T_1 consists of six to eight decade dividers connected in cascade, and hence the ratios n_1 and n_2 can be fixed in steps of 10^{-6} to 10^{-8}. The details are omitted in Fig. 9 for simplicity. Transformer ratio bridges have been made for use at frequencies up to 200 MHz. References 2,8,10, and 11 may be consulted for additional details on transformer ratio bridges.

Resonance Methods. If a variable capacitor is employed to resonate with the inductor under test, then the inductance value is deduced from the values of the capacitance at resonance and source frequency. A bridge method and a meter method using this principle are illustrated in Fig. 10. Both methods are particularly suited to measurements in the RF range, which permits the use of small capacitors.

In the resonance bridge of Fig. 10(a), C_4 and a resistor in one of the other three arms is varied to secure the balance. At balance, arm 4 of the bridge is equivalent to a pure resistance, and an accurate value of R_e can be deduced. The frequency f must be known accurately, because it enters the balance equation for L_e. Furthermore, the waveform of the source must be pure for sharp detection of balance.

The Q-meter is a versatile and useful instrument for measuring inductors and capacitors at radio frequencies. Referring to the circuit of Fig. 10(b), a small voltage V_s of fixed value (of the order of 20 mV) is introduced into the series-resonant circuit formed by the test inductor and a variable standard capacitor C. R_s

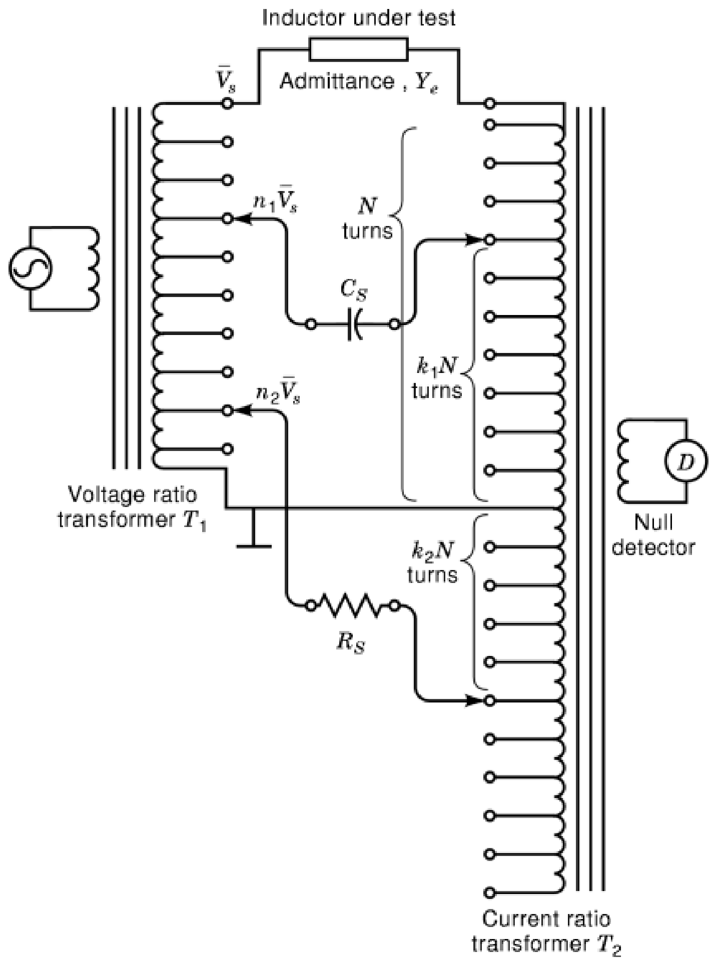


Fig. 9. The double-ratio, transformer-bridge circuit measures the admittance $\bar{Y}_e$ of the inductor in terms of fixed standards C_s and R_s and transformer ratios n_1, n_2, k_1, and k_2.

is a small, fixed, noninductive resistance (of the order of 40 mΩ) and the current I' through it is sensibly equal to I, which is read by an RF ammeter and adjusted to a predetermined value to yield the required value of V_s. Now the capacitance of C is adjusted to result in the maximum value $(V_0)_m$ of the voltage across it read by the high impedance voltmeter V. For all practical purposes, this is also the resonant condition of the circuit and the voltage magnification $(V_0)_m/V_s$ is equal to the Q-factor of the coil. Accordingly, we deduce L_e and Q_e at frequency f from

$$L_e = 1/(4\pi^2 f^2 C_0)$$

and (12)

$$Q_e = (V_0)_m/V_s$$

where C_0 is the value of the capacitance for maximum V_0.

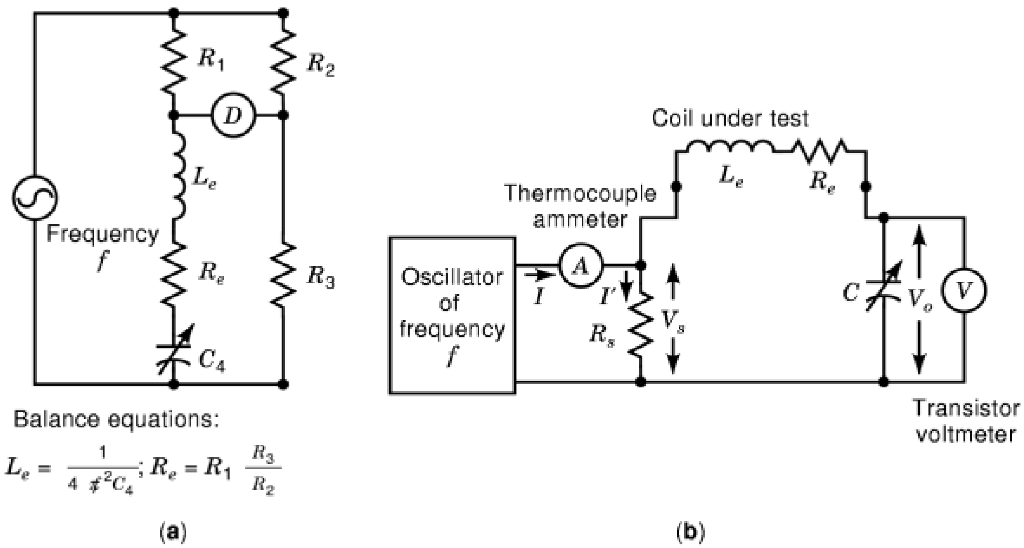


Fig. 10. Resonance methods employ a variable capacitor to resonate with the inductor at the test frequency. (a) Resonance bridge. (b) Q-meter.

Because V_s has a predetermined value, the output voltmeter is calibrated directly in terms of Q_e. By repeating the above measurement at a revised frequency of $2f$, the self-capacitance C_d of the inductor is deduced as $(C_0 - 4C'_0)/3$, where C'_0 is the value of the variable capacitor needed for resonance at the frequency $2f$.

Digital Methods. Modern electronics has brought a variety of instruments to the art of impedance measurement which provide the convenience of a direct digital readout and accuracies comparable to commercial measuring bridges. The underlying principle of measurement in most of these instruments is based on one or the other of the two schemes shown in Fig. 11.

The inductor under test and a standard resistor R_s of known value are connected in series in the scheme of Fig. 11(a). The components of the phasor of the inductor voltage in phase and in quadrature with the phasor of the resistor voltage are measured by suitable analog and digital electronics. The second scheme shown in Fig. 11(b) employs an automatic balancing arrangement to null the detector voltage $\bar{V}_d$. This is effected by varying the in-phase and quadrature components of the controlled voltage $\bar{V}_z$. In both schemes, the parameters α and β are given by

$$\alpha = R_e/R_s \tag{13a}$$

$$\beta = \omega L_e/R_s \tag{13b}$$

and form the basis of the digital display of the values of R_e and L_e. The details of the circuit implementation in a few representative designs are given in Refs. 11,12,13.

Modern digital *RLC* meters measure a wide range of impedances at frequencies from a few hertz to megahertz with a basic accuracy of 0.1% to 1.0%. They are being increasingly used for routine measurements in the laboratory and for production and inspection tests.

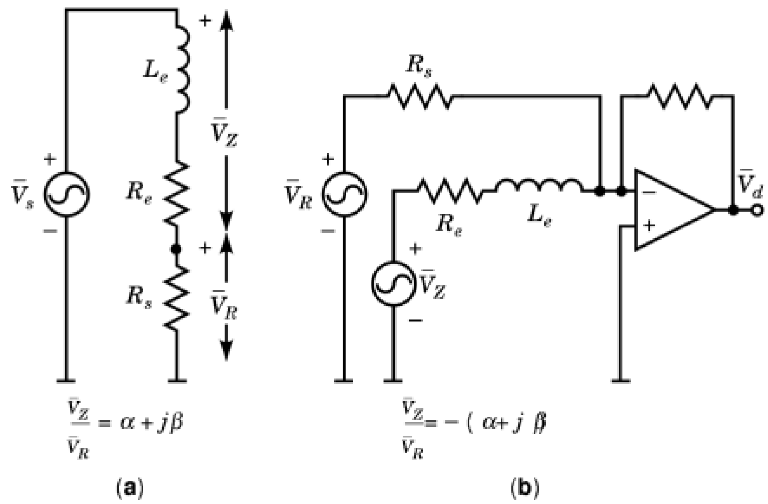


Fig. 11. Schemes adopted for digital measurement of impedances. (a) The measurement of α and β enables the determination of R_e and L_e. (b) Feedback forces the controlled voltage $\bar{V}_z$ to a value that makes $\bar{V}_d = 0$.

Measurement of Mutual Inductance

The measurement of mutual inductance is called for in testing mutual inductors and when dealing with mutual inductance transducers for measuring nonelectrical quantities. We shall look at a few representative methods.

Voltmeter-Ammeter Method. In the voltmeter-ammeter method, Fig. 12(a), the current in the primary winding and the induced emf in the secondary are measured by an ammeter and a high-resistance voltmeter, such as an electrostatic voltmeter or an electronic voltmeter. M is calculated as $V_2/\omega I_1$. The value of the supply frequency is needed for this measurement.

Measurement in Terms of Self-Inductances. This is a popular method of measuring mutual inductance. If two coils whose mutual inductance is to be measured are connected in series, the self-inductance of the combination is given by $L_1 + L_2 \pm 2M$, depending on whether the magnetic fluxes set up by a common current in the two coils are additive or not [Fig. 12(b)]. Thus, M is deduced from the measurement of the two resulting values of self-inductance. Any one of the methods of measurement of self-inductance can be used to determine L_a and L_b. If $M \ll L_1 + L_2$, the method suffers from inaccuracy because then the value of M is deduced from the difference between two nearly equal measured values.

Mutual Inductance Bridges. There are several bridge methods for measuring M (1,2,6,7,9). The Carey–Foster bridge shown in Fig. 12(c) is representative. It measures M in terms of a fixed capacitor. At balance, $(R_1 - j/\omega C_1)\bar{I}_1 = R_2\bar{I}_2$, and $(R_4 + j\omega L_4)\bar{I}_1 - j\omega M(\bar{I}_1 + \bar{I}_2) = 0$. Solution of these equations yields

$$M = C_1 R_2 R_4$$

and (14)

$$L_4 = C_1 R_4 (R_1 + R_2)$$

L_4 must be larger than M. Further, the resistance R_4 associated with L_4 must be known accurately.

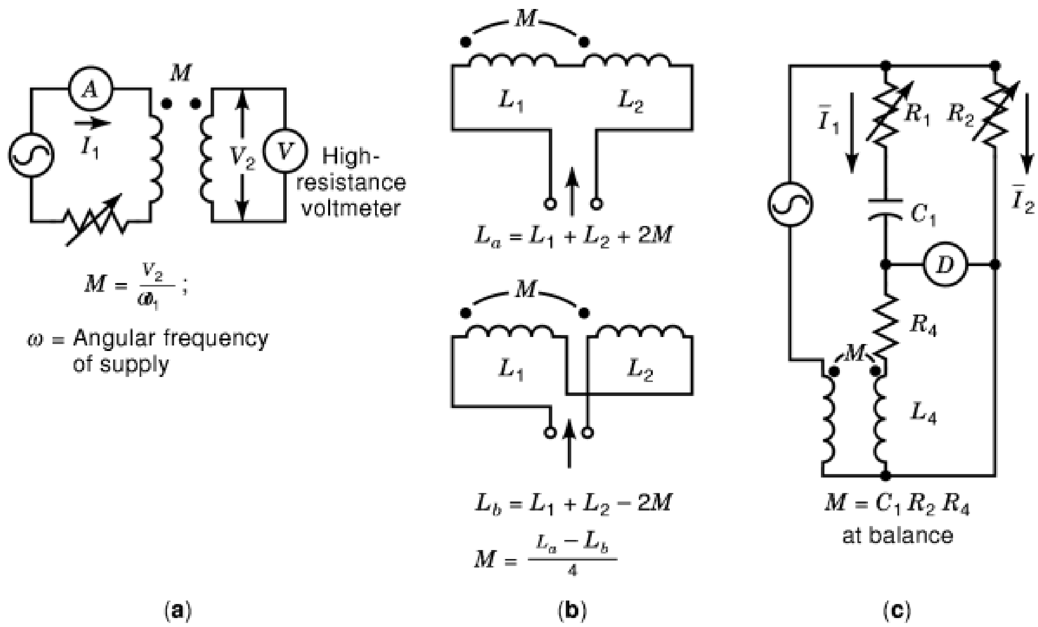


Fig. 12. Methods of measuring mutual inductance. (a) The voltmeter-ammeter method is direct and simple but requires the frequency value. (b) M is deduced from the measured self-inductance values L_a and L_b. (c) The Carey–Foster bridge measures M in terms of a capacitance and two resistances.

Special Techniques and Measurements

Quasi-Balanced Bridges. Unlike in a conventional bridge measurement where the repetetive adjustment of two elements in succession is needed, the quasi-balanced bridge requires adjusting only one element, leading to a rapid convergence to what is termed the *quasi-balance* condition. This condition is the detection of the minimum or maximum value of a voltage or even null detection in some forms. The quasi-balance, in principle, yields only one component of an unknown complex impedance. The second component, if needed, is obtained by adjusting another variable element for a different quasi-balance condition. The principles and some examples of quasi-balanced bridges are given in Ref. 9.

The microcontroller-based bridge described in Ref. 14 illustrates the application of the quasi-balance technique in conjunction with modern electronic hardware. Two independent quasi-balance conditions are obtained in succession in the bridge circuit of Fig. 13. The phasor $\bar{V}_d$ is brought into quadrature with the phasor $\bar{V}_R$ in the first adjustment and with the phasor $\bar{V}_s$ in the second. Both adjustments are effected by the potentiometer setting x. The quadrature condition is sensed by the zero output of a phase-sensitive detector (*PSD*). If m and n are the required values of the potentiometer setting x in the two adjustments, then R_e and L_e are computed from the equations

$$R_e = \frac{(1-m)}{m} R_s \tag{15a}$$

and

$$L_e = \frac{\sqrt{m-n}}{\omega m \sqrt{n}} R_s \tag{15b}$$

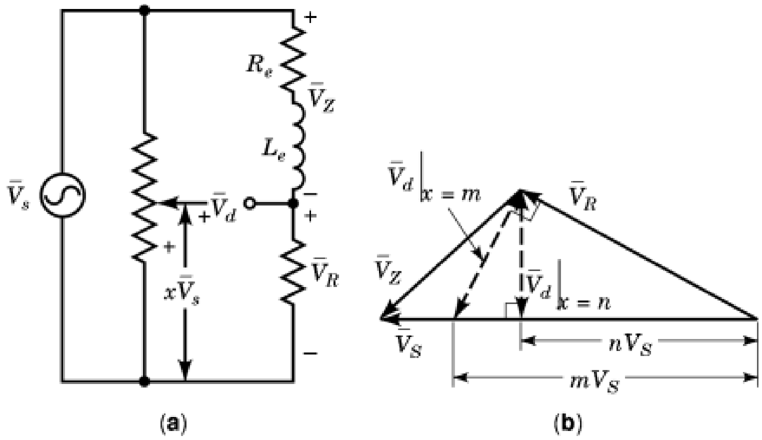


Fig. 13. A quasi-balanced bridge measurement of the unknown impedance $R_e + j\omega\, L_e$. (a) Circuit configuration. The potentiometer setting x is used to obtain quasi-balance. (b) Phasor diagram. $\bar{V}_d$ is brought into quadrature with $\bar{V}_R$ and $\bar{V}_S$ for $x = m$ and $x = n$, respectively.

A multiplying digital-to-analog converter (*MDAC*) is used to implement the potential divider action. A micro-controller senses the PSD output, adjusts the inputs to MDAC to secure quasi-balance conditions, measures the source frequency, and finally displays digital values of R_e and L_e computed from the measurement.

Measurement of Incremental Inductance. A measurement of practical importance is the effective resistance and incremental inductance of iron-cored inductors when they carry a comparatively large direct current with a small superimposed alternating current, such as occurs with filter chokes and transformer windings in electronic circuits. The variation of incremental inductance as a function of the dc bias current needs to be determined. The Owen and Hay bridge circuits are well suited to introduce the dc current into the inductor branch, because the capacitors block the dc in two of the branches. Figure 14 shows the Hay bridge adapted to measure incremental inductance. The resistor in series with the dc supply is first varied to set up the required dc current in the inductor, as read by the dc ammeter A. This bias current also flows through R_1, whose power rating should be appropriately chosen. The detector responds only to the ac signal because of the blocking capacitor in series with it. Now the bridge is balanced for ac signals by adjusting R_2 and C_2. The following balance equations, which yield the values of incremental parameters, are the same as for the conventional Hay bridge:

$$L_i = \frac{C_2}{1 + \omega^2 C_2^2 R_2^2} R_1 R_3 \tag{16a}$$

and

$$R_i = \frac{\omega^2 C_2^2 R_2}{1 + \omega^2 C_2^2 R_2^2} R_1 R_3 \tag{16b}$$

where L_i is the incremental inductance and R_i the effective ac resistance at the chosen dc bias current. See Ref. 2 for further details.

Residual Inductance and Its Measurement. The balance equations of the bridge circuits in Table 1 are derived on the assumption that the standard elements used therein are pure. The impurities in these elements, called residuals, are no doubt small in practice, but must nevertheless be considered in high-precision

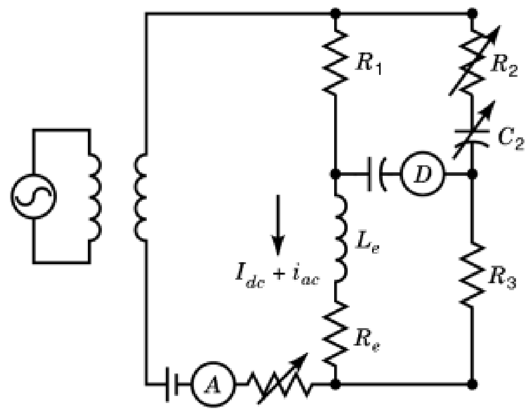


Fig. 14. Hay's bridge circuit adapted for the measurement of incremental inductance.

measurements. Although the residuals of standard capacitors normally used can be ignored as negligibly small, the residual inductances associated with resistors have to be considered, particularly when low-valued inductances are to be measured.

As an illustration of the methods used to tackle the problem of residuals, we consider the Maxwell–Wien bridge of Table I, modified by including a variable standard resistance R_4 in series with the test inductor. Let l_1, l_2, l_3 and l_4 be the residual inductance values of the resistors R_1, R_2, R_3 and R_4 and let C_2 be considered pure. Reworking the equations of the bridge and making appropriate approximations valid at medium frequencies, we obtain the following balance equation for L_e:

$$L_e = C_2 R_1 R_3 + \frac{1}{R_2}\left[l_1 R_3 + l_3 R_1 - l_2 \frac{R_1 R_3}{R_2} \right] - l_4 \qquad (17)$$

Now let the test inductor be removed from the bridge and replaced by a resistor with a resistance value approximately equal to R_e and with a calculable inductance L_s. Such resistors are readily constructed as a loop of two, parallel, high-resistance wires (see pp. 147–153 of Ref. 2). Now the bridge is rebalanced by adjusting C_2 to C'_2 and R_4 to R'_4. Because all other elements and their residuals remain unchanged,

$$L_s = C'_2 R_1 R_3 + \frac{1}{R_2}\left[l_1 R_3 + l_3 R_1 - l_2 \frac{R_1 R_3}{R_2} \right] - l'_4 \qquad (18)$$

where l'_4 is the residual inductance of R'_4. From Eqs. (17) and (18),

$$L_e = L_S + (C_2 - C'_2) R_1 R_3 + (l'_4 - l_4) \qquad (19)$$

Because the substituted resistor has approximately the same resistance as the test inductor, the resistance values R_4 and R'_4 should be approximately equal, and the quantity $(l'_4 - l_4)$ can be neglected. Thus the value L_e is given, independently of the residuals of the bridge elements, by

$$L_e = L_S + (C_2 - C'_2) R_1 R_3 \qquad (20)$$

For a detailed treatment of the solution of problem of residuals, consult Refs. 1,2, and 6.

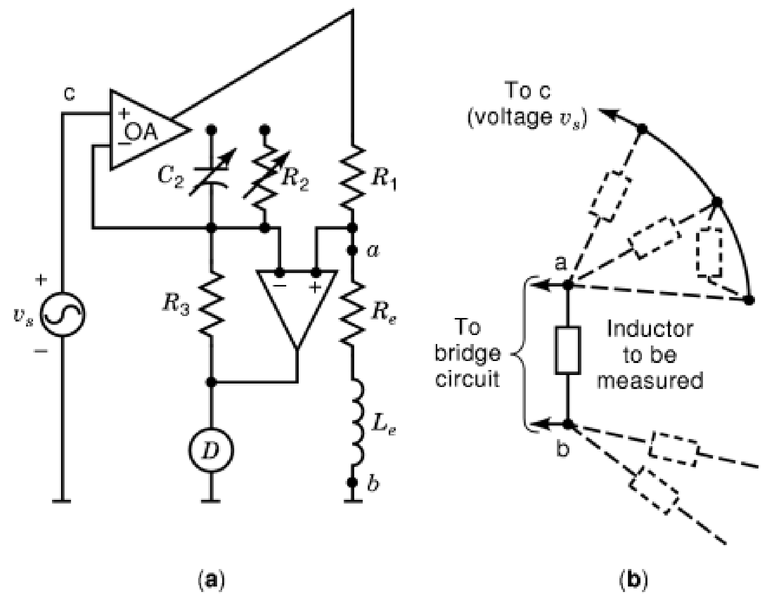


Fig. 15. In-circuit measurement of an inductor. (a) The active bridge circuit employed. $R_e = R_1R_3/R_2$ and $L_e = R_1R_3C_2$ at balance. (b) Network modifications for the in-circuit measurement of an inductor connected between nodes a and b in a network.

The substitution principle, which forms the basis of the foregoing procedure, is one of the most effective and accurate tools available, when the original measurement is beset by influencing factors of unknown magnitude. The method outlined in this section is also used to evaluate the residual inductance of a resistor, by treating the given resistor itself as the test inductor.

In-Circuit Measurement. It is useful to measure impedance values of elements already connected in a network without dismantling them. Such measurements, termed *in-circuit* or *in-situ* measurements, are called for in identifying, testing, and trouble-shooting electronic circuits. The shunting effect of other elements in the network has to be avoided in this evaluation.

In one approach (15), a current is injected into the element from an external sinusoidal voltage source. One of the leads to the element carrying this current and another wire carrying an adjustable opposing current derived from the same source through an auxiliary circuit are passed through a clamp-on type current probe. The circuit constants of the auxiliary circuit, adjusted to obtain a null output from the probe, are used to evaluate the parameters of the element under test.

A second approach is to block the currents in the disturbing network elements by forcing zero voltage on them or to make these currents inconsequential and then measure the desired impedance by a suitable method (16,17). The active bridge circuit of Fig. 15, discussed in Ref. 17, apart from other advantages, enables the in-circuit measurement of an inductor at any specified voltage. To this end, the source v_s is adjusted to the desired voltage and the test inductor is connected into the bridge with all nodes in the network adjacent to one terminal of the test inductor lumped together and connected to the source as shown in Fig. 15(b). This method is applicable only when no other element exists in the network, which is connected directly in parallel with the test inductor.

BIBLIOGRAPHY

1. F. K. Harris *Electrical Measurements*, New York: Wiley, 1966.
2. B. Hague T. R. Foord *Alternating Current Bridge Methods*, 6th ed. London: Pitman, 1971.
3. C. H. Dix A. E. Bailey Electrical standards of measurement—Part 1: D. C. and low frequency standards, *Proc. IEE*, **122**: 1018–1036, 1975.
4. C. H. Page A new type of computable inductor, *J. Res. Nat. Bur. Stand.*, **67B**: 31–39, 1963.
5. F. E. Terman J. M. Petit *Electronic Measurements*, 2nd ed. New York: McGraw-Hill, 1952.
6. M. B. Stout *Basic Electrical Measurements*, 2nd ed. New Delhi: Prentice-Hall of India, 1973.
7. E. W. Golding F. C. Widdis *Electrical Measurements and Measuring Instruments*, 5th ed. London: Pitman, 1963.
8. H. P. Hall Impedance measurement, in B. M. Oliver and J. M. Cage, eds., *Electronic Measurements and Instrumentation.* New York: McGraw-Hill, 1971, Chap. 9.
9. K. B. Karandeev *Bridge and Potentiometer Methods of Electrical Measurements* (English translation by B. Kuznetsov), Moscow: Peace Publishers, 1966.
10. B. A. Gregory *An Introduction to Electrical Instrumentation.* London: Macmillan, 1973.
11. M. T. Sanderson Electrical measurements, B. E. Noltingk, ed., *Instrumentation Reference Book.* Oxford: Butterworth-Heinemann, 1995, Part 3, Chap. 1.
12. K. M. Ibrahim M. A. H. Abdul Karim Digital impedance measurement by generating two waves, *IEEE Trans. Instrum. Meas.*, **34**: 2–5, 1985.
13. M. A. Atmanand V. Jagadeesh Kumar V. G. K. Murti A novel method of measurement of L and C, *IEEE Trans. Instrum. Meas.*, **44**: 898–903, 1995.
14. M. A. Atmanand V. Jagadeesh Kumar V. G. K. Murti A microcontroller—based quasi-balanced bridge for the measurement of L, C, and R. *IEEE Trans. Instrum. Meas.*, **45**: 757–761, 1996.
15. R. L. Forgacs In-circuit impedance measurement using current sensing. *IEEE Trans. Instrum. Meas.*, **34**: 6–14, 1985.
16. M. Rahman M. T. Ahmed V. G. K. Murti A TRA bridge technique for in- circuit impedance measurement, *IEEE Trans. Instrum. Meas.*, **33**: 252–256, 1984.
17. V. G. K. Murti Application of the generalised impedance converter in electrical instrumentation. *IEEE TENCON Conf. Proc.*, Singapore, 1984.

V. JAGADEESH KUMAR
Indian Institute of Technology
V. G. K. MURTI
Indian Institute of Technology
P. SANKARAN
Indian Institute of Technology, Madras