J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*

Q-METERS

The concept of resonance is one that is well known to all students of science. There is hardly a field of science that does not have a prominent example of the resonant phenomenon. Resonance is used in popular examples for solving differential equations in mathematics texts, linear systems, and electrical circuits. Resonance is found from the most microscopic atomic structure to the orbits of celestial bodies and orbiting satellites.

Resonance requires at least two energy storage elements that are coupled so that the energy stored in one element may be transferred to the other. An example of this is the pendulum. The pendulum swings from left to right at a constant frequency. When the pendulum has reached the highest point of its swing, the velocity of the pendulum bob reaches zero and the velocity reverses. Thus, the pendulum begins to swing in the reverse direction and will continue to swing until it reaches the opposite side and the process is repeated.

What causes this periodic activity to take place? A pendulum hanging straight down will not move (as Newton showed). A mass at rest will remain at rest unless acted upon by an external force. The pendulum bob hanging at its lowest position is in the lowest energy state, which is a stable condition for a mechanical system. If the pendulum were pulled to one side by a small amount, the pendulum would be raised slightly in the vertical direction. This slight raising of the pendulum mass would constitute an additional energy in the system. Certainly, pulling the pendulum mass to one side requires a force to be exerted through a distance, which is energy.

At the point when the pendulum is pulled to one side and before the bob is released, the pendulum system has a certain amount of potential energy. The actual amount of potential energy is not important at this time.

To keep the pendulum mass at the displaced position, it is necessary to retain a hold of the mass. Once the mass is released, the natural force of gravity will tend to pull the pendulum bob to its original position, hanging straight down. This is the lowest energy position, where the potential energy is zero. Mechanical systems attempt to assume the lowest energy level if there are no restraints, such as holding the pendulum mass.

The natural forces of nature—gravity in this example—tug at the pendulum bob; and when the bob has reached the lowest point of its swing, because forces have been acting on a mass, there has been acceleration of the bob and it now has a velocity. The pendulum will not stop at the lowest kinetic energy state because there is nothing to stop the swinging mass. The system is now an isolated system.

This situation can be further understood if the energy content of the system is investigated. One of the most powerful tools for analyzing physical systems is the concept of conservation of energy. All energy is conserved: mechanical, electrical, heat, and so on. Energy is a scalar so the complexity of vector calculations may be avoided. In the process of conservation of energy, the form of energy may be changed. Thus electric energy may become heat energy, and mechanical energy may be converted to electric energy. Even mass can be converted into energy and vice versa.

In the pendulum example, the energy conversion is simple; it is the conversion from potential energy to kinetic energy.

When the pendulum mass was pulled to its maximum deflection, the added energy was potential. Since the bob was not moving and thus had no velocity, the entire energy of the system was potential. When the mass was released and accelerated to the lowest point of its travel, the energy was all kinetic because this was the original point where the bob hung before energy was added to the system by pulling the bob to one side.

Therefore, as the pendulum swings, the energy is transferred from potential to kinetic but the total energy remains constant.

The pendulum would swing for all eternity if the energy did, in fact, remain constant. However, a swinging pendulum eventually comes to rest. The pendulum must give up some of its energy in moving the air to make way for the swinging bob. This energy expended in moving the air is not replaced and must come from the total energy of the pendulum. The bearings that the pendulum swings about will have some friction; and this, too, will subtract from the total energy content of the pendulum.

A pendulum can be made to have the least amount of friction so that it will swing for a long time. The pendulum clock is enclosed in a cabinet to reduce the effects of air friction due to moving air, and very low friction knife edge supports are used. The typical "grandfather" clock can run more than a week with just the energy from a weight.

In addition to reducing friction, a pendulum can be made to swing for a long time by increasing the mass of the bob. Foucalt pendulums, found in a number of science museums, will swing for days with no external energy sources due to their bob mass of thousands of kilograms.

The pendulum swings with a constant frequency regardless of amplitude, provided that it is not swinging at a very great amplitude. Even though the amplitude of the pendulum's oscillations is constantly decreasing, the period of oscillation remains the same until the very last oscillation.

Electric resonant circuits operate in a fashion similar to that of the pendulum. Two energy storage elements are required to create an electric resonant circuit. This would appear, at first, to be quite different from the example of the mechanical pendulum which had only one storage element, the bob at the end of the pendulum. Although there is, physically, only one bob, energy is stored two ways, kinetic and potential. Therefore, there are two energy storage "means," to borrow a term from the patent attorneys. In the electrical circuit, two different physical devices are required to store energy two different ways.

A capacitor stores energy in an electric field that resides in the dielectric between the conducting plates of a capacitor. This energy storage means could be likened to potential energy because the charges that create the electric field can be static, with no motion—that is, no current. The energy stored in a capacitor is

$$E(t) = \tfrac{1}{2}CV^2(t) \qquad (1)$$

where $E(t)$ is the energy contained in the capacitor as a function of time in joules, $V(t)$ is the voltage across the capacitor, and C is a parameter called capacitance, with the units of farads which could actually be defined by this very equation. C defines how much energy is stored per volt of electric potential difference.

Opposite the capacitor, a magnetic field is the storage means for energy in an inductor. A magnetic field is created by current, or moving charges. Thus, the energy storage in an inductor could be viewed as kinetic energy. The energy stored in an inductor is

$$E(t) = \tfrac{1}{2}LI^2(t) \qquad (2)$$

where $E(t)$ is energy, as before, $I(t)$ is the current within the inductor, and L is a parameter called inductance, with the units of henrys, which could, as in the case of the capacitor, be defined by this equation.

In the case of the pendulum, since the bob stored both the potential and kinetic energies there was no need to connect the two energy means. With two separate energy storage devices in the electric resonant circuit it is necessary to connect the two together. This can be done in two ways: a parallel connection and a series connection. The circuit shown in Fig. 1 is a resonant circuit that could be either parallel or series, because there

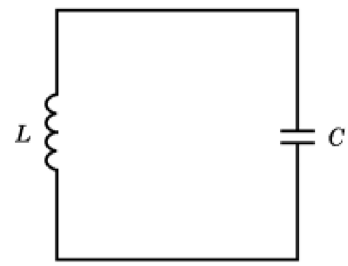

Fig. 1. A simple resonant circuit with no energy dissipation.

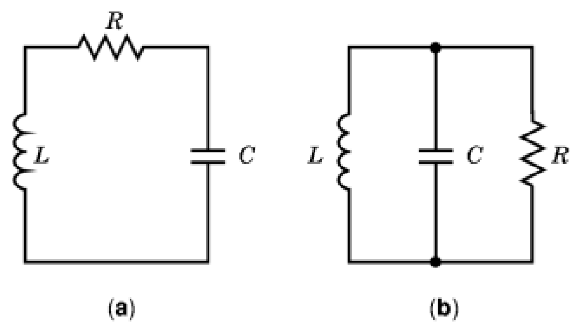

Fig. 2. Series (a) and parallel (b) resonant circuits with energy dissipation.

is only one way to connect the inductor and the capacitor. Later in the discussion of electric circuit resonance the differences between series and parallel will become evident.

The circuit shown in the figure will oscillate if there is any energy in the circuit, and the oscillations will continue infinitely because the circuit shows no method of converting the internal energy of the circuit to any other form. The frequency of the oscillations will be

$$F_0 = \frac{1}{2\pi (LC)^{1/2}} \qquad (3)$$

where F_0 is the resonant frequency in Hz or 1/s.

No resonant circuit can oscillate for eternity and the circuit in Fig. 1 must include a resistance, which is the circuit element that represents the conversion of energy to another form or represents a loss of energy from the circuit.

Now, it is clear that series and parallel resonant circuits are different. Figure 2 shows a series and a parallel resonant circuit with an energy loss element, a resistor.

As an aid to the understanding of resonance, a parameter is desired that would quantify how quickly or how slowly energy is lost from a resonant circuit. If the fraction of the energy that is lost for each cycle of oscillation were the parameter used, it would provide a good indicator of how quickly the energy was lost from the circuit. As an example, if one-tenth of the energy were lost each cycle, the energy would decay exponentially and could be easily predicted. Based on this premise, a factor called Q is defined as

$$Q = 2\pi \left(\frac{E_s}{E_d} \right) \qquad (4)$$

where E_s is the total energy stored in a system and E_d is the energy dissipated per cycle of oscillation. Notice that Q has no units because it is a ratio of energies. Q factor can be used to describe resonance that involves electric circuits, mechanical devices, and combinations of electric and mechanical devices such as control systems.

In the discussion of the pendulum, the actual mechanisms responsible for energy loss were discussed and included air resistance and friction. It would be useful to investigate the loss mechanisms in the electric resonant circuit. First, consider a resonant circuit that receives no external energy and provides no energy to any external circuits. The resonant circuit has a finite initial energy and simply oscillates until the internal energy is totally exhausted. Of course an isolated resonant circuit has no use and must be eventually connected to other circuits to be of use, and this will be discussed further. A resonant circuit that has no external energy source or sink loses energy due to its inductor or capacitor.

The Q of a circuit that has no external energy paths, either source or sink, is called the unloaded Q. When other circuits are coupled to the resonant circuit, the Q for this situation is called the loaded Q. In the example of the clock pendulum, some energy is required to be removed from the swinging pendulum to drive the escapement of the clock, and energy must be replaced by the weights to keep the pendulum in motion. Both the extraction and addition of energy to the pendulum will reduce the Q.

In the case of the capacitor, there are several loss mechanisms. First is dielectric heating. Second are resistive losses due to the finite resistance of the conducting plates of the capacitor. Finally, there is radiation, which occurs when the electric field is not totally contained in the dielectric of the capacitor and can radiate. Generally, capacitors are not the major factor in resonant circuit losses. Even at the highest frequencies, capacitors can be manufactured to have very low losses.

The inductor loses energy in three major ways. First is the resistive loss due to the wire used to make the inductor. At higher frequencies, this can become significant because of the increase of resistance due to the skin effect. Second are the losses due to energy dissipated in the core material of the inductor. When air or some other nonmagnetic material is used in the core, losses are virtually zero. However, when ferromagnetic material such as ferrite is used, the losses can be quite significant. The final major loss mechanism in an inductor is radiation. Unlike the capacitor, an inductor can have considerable radiation.

The solenoid inductor is the worst form factor when it comes to radiation. Solenoids are actually used as H-field antennas at low frequencies. By bending the solenoid around on itself to create a toroidal shape the radiation from the end of the solenoid can be significantly reduced. This type of inductor is expensive to make and difficult to work with and is avoided for many designs.

The Q of an inductor is a function of the frequency of measurement. The loss mechanisms for inductor loss are functions of frequency. Skin effect causes increased resistance with increasing frequency, and hence increased resistance losses. Radiation also increases with increasing frequency because the physical dimensions of the inductor become a more significant part of a wavelength. When ferromagnetic material is used in the core of an inductor, the frequency dependence of the inductor's Q can become quite complex. Shielded enclosures are often used to reduce the amount of radiation from an inductor, and this adds even more complexity to the Q factor calculation.

For most resonant circuits, the losses of the inductor are the major losses of the circuit, and the quality of the inductor will set the quality of the resonant circuit.

Because the inductor is the major loss element in a resonant circuit, it is common to attach a Q value to the inductor, alone. The very same definition of Q is applied except the resonant circuit is the inductor for which the Q value will be associated and a perfect capacitor. Therefore, the losses in the resonant circuit are those of only the inductor.

If the loss of a resonant circuit is considered to be that of the inductor alone, the resistance shown in the series resonant circuit of Fig. 2(a) or 2(b) is associated with the inductor.To find a relationship between Q, the equivalent resistance, and the inductance of an inductor, the current of the series circuit is given as

$$i(t) = I(t)\cos(2\pi F_0 t) \qquad (5)$$

where $i(t)$ is the circuit current and $I(t)$ is the peak amplitude that is decreasing in some unspecified manner. A cosine function is assumed for the circuit current. A sine function could be employed as well as the inclusion of a phase angle. The solution to the circuit current of Fig. 1(a) is a sinusoid, which is either a sine or cosine function. The initial conditions will determine which of the two functions or a possible phase angle will solve the equation. For the purpose of understanding Q, the cosine function with a phase angle of zero will suffice.

Assume that a relatively high Q is involved such that $I(t)$ does not change much during the time of one cycle. The root-mean-square (*rms*) value of $i(t)$ is (0.707) $I(t)$, where $I(t)$, is treated as a constant. The power during this cycle is $(I_{\rm rms})^2R$. The energy dissipated during the cycle is simply the time duration of one cycle times the power dissipated during the cycle, which is then

$$E_{\rm d} = \frac{(I_{\rm rms})^2 R}{F_0} = \frac{\frac{1}{2}(I^2(t)R)}{F_0} \qquad (6)$$

Returning to the definition of Q and using the equations for $E_{\rm d}$ and $E_{\rm s}$, the following relationship is obtained:

$$Q = \frac{2\pi \mathrm{E_s}}{\mathrm{E_d}} = \frac{2(\frac{1}{2}LI^2(t))}{\frac{1}{2}I^2(t)R/F_0} = \frac{2pF_0L}{R} = \frac{X_L}{R} \qquad (7)$$

where $X_{\rm L}$ is the inductive reactance.

The entire concept of Q is applied to circuits and systems where only a small fraction of the energy is dissipated per cycle or typically for Q values greater than about 10. Circuits with Q values significantly less than 10 do not oscillate but have a more exponential decay. Q values between 1 and 10 represent highly damped circuits where the normal assumptions concerning the calculation and measurement of Q may not produce accurate predictions of circuit behavior.

A similar analysis may be applied to the parallel resonant circuit as shown in Fig. 2(b). In this example, the voltage is the common parameter to the two circuit elements and may be represented by

$$v(t) = V(t)\cos(2\pi F_0 t) \qquad (8)$$

where $v(t)$ is the circuit voltage and $V(t)$ is the peak amplitude. The same reasoning as was applied to the series circuit is used here to justify the form of the equation for $v(t)$. Likewise the same analysis will be applied to determine the relationship between Q and the circuit elements:

$$E_{\rm d} = \frac{(V_{\rm rms}(t))^2}{RF_0} = \frac{\frac{1}{2}V^2(t)}{RF_0} \qquad (9)$$

For the stored energy to calculate Q, the energy stored in the capacitor will be used because this is a voltage equation and the energy stored in the capacitor is a function of voltage. Therefore,

$$E_{\rm s}(t) = \tfrac{1}{2}CV(t)^2 \qquad (10)$$

$$Q = \frac{2\pi E_s}{E_d} = \frac{2\pi\left(\frac{1}{2}CV^2(t)\right)}{\frac{1}{2}V^2(t)/RF_0} = 2\pi F_0 CR = \frac{R}{X_C} = \frac{R}{X_L} \tag{11}$$

where X_C is the capacitive reactance.

Measuring *Q*

The next consideration is measuring Q. From the discussion thus far, it would appear that if a resonant circuit were set into oscillation along with the number of cycles required to reach half-amplitude or other criterion, then Q could be determined in this fashion. This technique would work and produce the value of Q. However, there are other methods of measuring Q that are more practical. In order to understand the measurement of Q, consider the impedance of a series resonant circuit at near resonance:

$$Z(f) = R + j(X_L - X_C) = R + j\left(2\pi fL - \frac{1}{2\pi fC}\right) \tag{12}$$

Using the substitutions

$$L = \frac{1}{4\pi^2 F_0^2 C} \quad \text{and} \quad C = \frac{1}{4\pi^2 F_0^2 L} \tag{13}$$

the impedance may be rewritten as

$$\begin{aligned} Z(f) &= R + j\left(\frac{2\pi fF_0 L}{F_0} - \frac{4\pi^2 F_0^2 L}{2\pi f}\right) \\ &= R + j(2\pi F_0 L)\left(\frac{f}{F_0} - \frac{F_0}{f}\right) \\ &= R\left[1 + j\left(\frac{2\pi F_0 L}{R}\right)\left(\frac{f}{F_0} - \frac{F_0}{f}\right)\right] \\ &= R\left[1 + jQ\left(\frac{f}{F_0} - \frac{F_0}{f}\right)\right] \end{aligned} \tag{14}$$

From this equation, the real part of the resonant circuit is a constant at R while the imaginary part is zero at the resonant frequency. Also, notice that if the frequency at which the impedance is measured is greater than the resonant frequency, then the sign of the imaginary part of the impedance is positive, indicating that the impedance becomes more inductive. Below the resonant frequency sign is negative, indicating that the impedance is more capacitive.

For the case of the parallel resonant circuit a similar equation is derived for the admittance of the circuit:

$$Y(f) = \frac{1}{R[1 + jQ(f/F_0 - F_0/f)]} \tag{15}$$

In this situation, when the frequency is greater than the resonant frequency the admittance becomes positive, indicating an inductive circuit. When the frequency is less than the resonant frequency, the admittance is capacitive.

Q was shown to be a parameter that described how the oscillations of a resonant circuit decayed in time. This was the time-domain picture of Q applied to a resonant circuit. When analyzed in the frequency domain, a circuit with a high Q has a narrow bandwidth. To determine the relationship between Q and the resonant circuit bandwidth the impedance of a series resonant circuit is considered:

$$Z(f) = R\left[1 + jQ\left(\frac{f}{F_0} - \frac{F_0}{f}\right)\right] \tag{16}$$

The series resonant circuit has a minimum impedance of, simply, R at the resonant frequency. The half-power level of a system is the usual way of specifying bandwidth of a circuit. In the case of a series circuit driven from a current source, half-power in the resonant circuit would occur when the magnitude of the impedance is $1.414R$.

Therefore,

$$|Z(F_{3\text{dB}})| = 1.414R \tag{17}$$

$$Z^2(F_{3\text{dB}}) = 2R^2 = R^2\left[1 + Q^2\left(\frac{F_{3\text{dB}}}{F_0} - \frac{F_0}{F_{3\text{dB}}}\right)^2\right] \tag{18}$$

Rearranging, we obtain

$$F_{3\text{dB}}^2 - \frac{F_0 F_{3\text{dB}}}{Q} - F_0^2 = 0 \tag{19}$$

Solving this equation for $F_{3\text{dB}}$, we obtain

$$F_{3\text{dB}} = \left(\frac{F_0}{2Q}\right)(1 \pm (1 + 4Q^2)^{1/2}) \tag{20}$$

For $Q \gg 1$ the equation above reduces to

$$F_{3\text{dB}} = \left(\frac{F_0}{2Q}\right)(1 \pm 2Q) \tag{21}$$

Again, by rearranging we obtain

$$F_{3\text{dB}} = F_0 \pm \left(\frac{F_0}{2Q}\right) \tag{22}$$

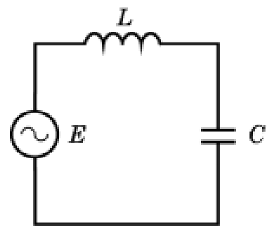

Fig. 3. A driven resonant circuit.

There are two 3 dB frequencies, one below the resonant frequency and a second above the resonant frequency. The difference between these two frequencies is the 3 dB bandwidth, BW, and is

$$\mathrm{BW} = \frac{F_0}{Q} \tag{23}$$

Because a resonant circuit is highly frequency-selective, it is the major component of filters. The bandwidth of a resonant circuit is dependent on the Q of the circuit. When a resonant circuit is used in a filter the Q of the circuit is set by adding resistors to the basic resonant circuit to decrease the loaded Q and achieve the necessary bandwidth required for the filter design.

The unloaded Q will determine the bandwidth of the basic resonant circuit. To understand the relationship between Q and bandwidth, consider the driven resonant circuit in Fig. 3. In this example a 0 Ω generator is shown providing the input driving function to the circuit. Thus, the loaded Q and the unloaded Q are the same. If a real generator were used, some finite amount of impedance must be added to the voltage generator and the loaded Q will be less than the unloaded Q. In this circuit the maximum circuit current will be found at the resonant frequency and is V_{in}/R.

Generally, the unloaded Q of a resonant circuit is significantly higher than the desired loaded Q. This is done so that the loaded Q can be controlled. Unloaded Qs are of a function of parameters that are not easily controlled.

The discussion thus far is of Q as associated with a resonant circuit. It was also shown that the Q of a resonant circuit is mainly set by the inductor because the inductor tends to have significantly more loss than capacitors. The Q of an inductor is the Q of a resonant circuit that is constructed with the inductor with a perfect capacitor. The Q of a capacitor is that of a resonant circuit with a perfect inductor. A resonator such as a quartz crystal or ceramic resonator is often described using a Q factor; this would be the Q of the resonator, alone.

For the case of Q-factor for inductors and capacitors, the Q of these components is a function of frequency. Generally, the higher the frequency the lower the Q because high frequencies exacerbate the loss mechanisms such as skin effect and radiation. Because of this frequency dependence, Q-factor must be measured and specified at a frequency near that which the inductor will operate.

Q-factor is also used for devices that are, in themselves, a resonant circuit. One example of this is the quartz crystal. This resonator has very high values of Q extending from tens of thousands to over 1 million. Ceramic resonators are similar to quartz crystals except the Q-factors are considerably lower and the ceramic devices do not have the superb frequency stability found in quartz crystals.

There are other devices such as ultrasonic transducers that are essentially resonators that are characterized by a Q-factor.

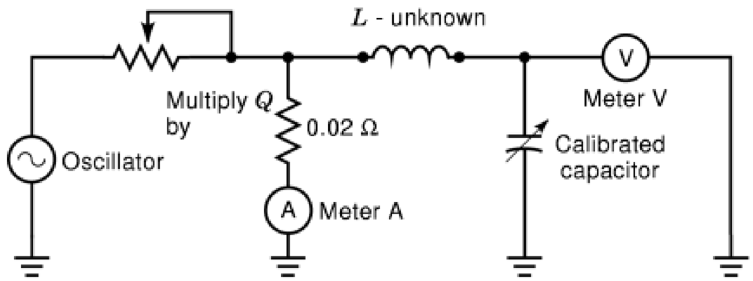

Fig. 4. The basic schematic of a Q-meter.

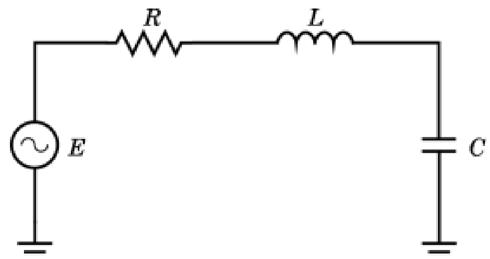

Fig. 5. A driven resonant circuit with finite Q.

The Q-meter

A method of measuring Q, primarily of inductors, is the appropriately named Q-meter. This device was invented in the mid-1930s by the Boonton Radio Company in Boonton, NJ. These instruments were manufactured for more than 40 years, mostly by Boonton Radio. There were a few other manufacturers such as The Marconi Corporation in England, and there was also a do-it-yourself kit by the venerable Heathkit Company that was a copy of the classic Boonton meter. When Boonton Radio was purchased by Hewlett-Packard in the 1960s a new, solid-state Q-meter was designed and was available for another two decades or so.

The Q-meter, as shown in Fig. 4, consists of an oscillator with a very low output impedance which was used to excite a resonant circuit which was made up of the unknown inductance and an internal calibrated capacitor. This instrument not only provides the ability to measure Q at a variety of frequencies, but could be used to measure inductance as well.

An oscillator feeds the resonant circuit which is in series resonance. A high-impedance alternating-current (*ac*) voltmeter is connected to the inductor and the actual inductor voltage is measured. The calibrated capacitor resonates the unknown inductor, and resonance is indicated by the maximum voltage indication of the voltmeter.

Historically, the output resistance of the generator was 0.02 Ω and the voltage was 0.02 V.

For a resonant circuit with an applied voltage from a 0 Ω generator, the voltage across the inductor or capacitor is Q times the applied voltage. The voltage-increasing phenomenon gives rise to the English name for a Q-meter, namely, "circuit magnification meter." To derive this relationship, consider the circuit shown in Fig. 5. In this example the generator has exactly 0 Ω internal resistance, which is, of course, only theoretically possible. The actual 0.02 Ω source resistance of the Q-meter causes only small errors for medium- or high-Q inductors.

At resonance the rms circuit current is

$$I = \frac{E_{\text{gen}}}{R} \tag{24}$$

where E_{gen} is the rms generator voltage.

The inductor rms voltage is

$$E_{\text{ind}} = IX_{\text{L}} = \frac{E_{\text{gen}} X_{\text{L}}}{R} = E_{\text{gen}} Q \tag{25}$$

The inductor voltage is Q times the generator, which shows how the input voltage is "magnified" by Q.

The voltage source in the Q-meter is not 0 Ω but 0.02 Ω. The actual source voltage is determined by measuring the current in the 0.02 Ω resistor using a thermocouple ammeter. From the previous equation, the inductor voltage, which is the actual indication of Q, is directly proportional to the generator voltage. The accuracy of the meter depends on the precision of the generator voltage. The Q-meter provided a control to accurately set the generator voltage using a meter. Also, the generator voltage could be reduced to increase the scale factor of the meter. The meter was labeled "multiply Q by" with a prominent 1 on the scale along with other convenient values such as 2, 5, and 10.

The capacitor must not contribute significantly to the energy loss in the resonant circuit formed by the unknown inductor and the Q-meter's capacitor. Generally, it is easy to make a capacitor with significantly lower losses than the inductor under test, and the losses of the resonant circuit are those of the inductor. In the Q-meter, the resistance of the driving generator is very low and does not contribute to the total losses. Also, a high-impedance voltmeter is used to measure the voltage across the inductor and is the indicator of Q.

The vacuum tube circuit used as a voltmeter in the early Q-meters was called the infinite impedance detector. Although the input impedance was not infinite, the actual impedance was quite high. This was due to the fact that vacuum tubes are inherently high-impedance devices and the infinite impedance detector used a high level of feedback in the circuit, which further increased the impedance. For a modern Q-meter, a field-effect transistor (*FET*) would be an acceptable replacement for the high-impedance vacuum tube. The input impedance of the vacuum tube voltmeter circuit is a high resistance in parallel with a small capacitor.

The capacitive part of the input impedance of the voltmeter can be compensated by including the fixed capacitance of the voltmeter in the calibration of the calibrated variable capacitor. The resistive part of the voltmeter would cause errors for extremely high Q. The typical Q-meter is capable of measuring Q values as high as 400 or so with accuracy. Very few conventional inductors would have Q values exceeding this range.

The resistive component of the variable capacitor is due to losses in the capacitor. In order to keep the losses low, the variable capacitor of the Q-meter is carefully constructed of the highest-quality materials. The capacitor is a conventional mechanically adjusted parallel-plate capacitor. The plates are either silver- or gold-plated, and the insulators supporting the fixed plates are a low-loss ceramic or glass. The shaft supporting the rotating plates has a large rotating contact to ensure a low-loss path to ground. The capacitor is mounted to the top of the Q-meter so that the "high" terminal of the inductor is directly connected to the stator plates of the calibrated capacitor. This reduces the amount of residual inductance in the tuned circuit.

In spite of all of the cautions taken to create a low-loss capacitor, the variable capacitor represents exposed plates that can radiate and lose energy. For measurements made at low frequencies, below about 1 MHz, the wavelengths involved are so long relative to the dimensions of the capacitor that no significant radiation occurs. This is not the case when the desired measuring frequency is in the very high frequency (*VHF*) range and higher. Because of this reason and other factors, Q-meters were made in several frequency ranges.

Special small geometry capacitors were made for the higher-frequency Q-meter to reduce the amount of radiation. Since the amount of capacitance required of a VHF range Q-meter was significantly less the lower-frequency counterparts, the capacitor loss was mostly achieved by simply scaling down a conventional parallel-plate variable capacitor.

In addition to measuring inductance and Q of an inductor, the Q-meter can measure capacitors. The method of measuring capacitors is to resonate an inductor using the internal calibrated capacitor and adding the unknown capacitor in parallel with the calibrated capacitor. The additional capacitance of the external capacitor

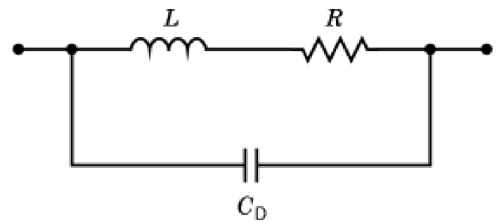

Fig. 6. The equivalent circuit for an inductor showing the distributed capacitance.

will detune the resonant circuit. The circuit can be returned to resonance by adjusting the calibrated capacitor. The amount of capacitance removed from the calibrated capacitor is equal to the amount of capacitance added by the external capacitor.

A set of inductors, called a "standard inductor set," sometimes called a "working inductor," was available from the Q-meter manufacturer to make these capacitor measurements. The actual value of the inductor was not important because it was necessary to resonate the inductor at the frequency of interest. The inductance needs to be such that the Q-meter will resonate with the external capacitor and without it.

What is important, however, concerning the standard inductors, is that the Q of the working inductor is as high as practical. The inductors were typically large because it was made from large-gauge wire, often Litz wire. The inductor is shielded to prevent radiation.

When an inductor is resonated using the calibrated capacitor in a Q-meter, the inductance of the unknown may be calculated using the familiar resonance equation. This is making the assumption, however, that the unknown is a pure inductance. This is not exactly the case.

Although there are exceptions, inductors are made from wire wound into a solenoid or toroid shape. If a single layer or wire will not produce the necessary inductance, additional layers of wire can be added. In many cases, dozens of layers may be involved with hundreds of turns. When wires lie side by side with only the thin enamel insulation of the wire to separate the wires, capacitance is added to the inductor. This added capacitance is called distributed capacitance and can have an effect on the measurement of inductance.

When an unknown inductor is resonated in a Q-meter, the value of capacitance that is in parallel with the inductance is the external capacitance plus the distributed capacitance. Therefore, the capacitance indicated on the calibrated capacitor is that which is required to resonate the inductor at the desired frequency.

The typical inductor has an equivalent circuit as shown in Fig. 6. If this inductor was to be used in an oscillator in a parallel resonant circuit, the external capacitance required to resonate the inductor in the oscillator needs to be known. The actual inductance is not a factor.

The circuit shown in the figure also has a resonant frequency of its own, with no external components called the self-resonant frequency. When the series impedance of the inductor is an important factor such as the three-pole low-pass filter shown in Fig. 7, the self-resonant frequency of the inductor is important. In this example, the distributed capacitance causes two zeros to appear in the transfer function of the filter which will affect the performance of the filter in the stop band.

The Q-meter can measure the distributed capacitance of an inductor. The inductor is resonated at a frequency near the frequency where the inductor will be operated. The capacitance required for resonance is recorded. Notice that the inductance is not measured; only the capacitance required to resonate the inductor at the set frequency. The inductor is resonated at a second frequency and the capacitance is noted.

Historically, the two frequencies at which the inductor is resonated are a ratio of two to one. In other words, the inductor is resonated at a frequency and then at twice that frequency. Using this convention the distributed capacitance, C_0, can be calculated by the following reasoning:

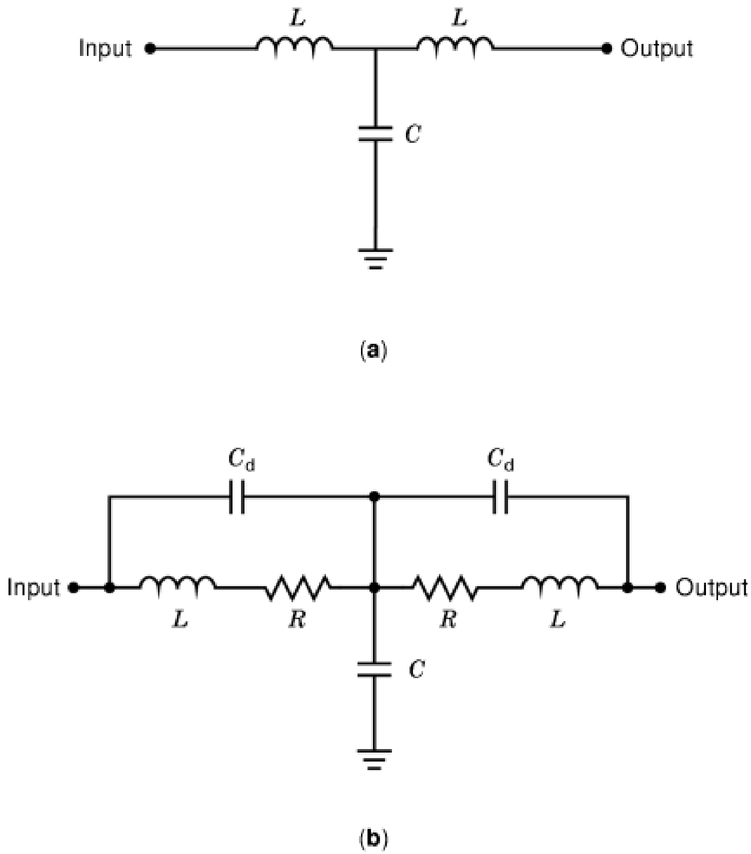

Fig. 7. (a) The theoretical schematic of a three-pole low-pass filter. (b) The actual schematic showing indicators with finite Q and distributed capacitance.

For resonance at a frequency F_0 the resonance equation is

$$F_0 = \frac{1}{2\pi[L(C_0 + C_d)]^{1/2}} \quad (26)$$

where C_0 is the capacitance set on the Q-meter and C_d is the distributed capacitance. The frequency of the Q-meter oscillator is set to twice the previous frequency and the inductor is resonated again. In this case the resonance equation is

$$2F_0 = \frac{1}{2\pi[L(C_1 + C_d)]^{1/2}} \quad (27)$$

where C_1 is the capacitance set on the Q-meter for this higher frequency.

Taking the ratio of the two frequencies, the following result is obtained:

$$2 = (C_0 + C_d)^{1/2}/(C_1 + C_d)^{1/2} \quad (28)$$

Solving for C_d the result is

$$C_\mathrm{d} = \frac{C_0 - 4C_1}{3} \tag{29}$$

In many cases it is not desirable to use a frequency ratio as high as two to one, particularly for a measuring frequency that is near the upper frequency region of the meter. Almost any ratio can be used; but if the ratio is not sufficient, the difference in resonating capacitance for the two frequencies will be small, making it difficult to read an accurate difference. Therefore, the calculated distributed capacitance will be prone to error. Some models of Q-meters had a precise ΔC dial to increase the accuracy of these measurements. One ratio that produces good accuracy is 1.414, or the square root of two. Using this ratio the equation for the distributed capacitance is

$$C_\mathrm{d} = (C_0 - 2C_1) \tag{30}$$

Although the Q-meter has been around for more than 60 years and is no longer available as a new instrument, there are thousands of the units in use. The older meters are doomed because their vacuum tubes are no longer available. There is lasting evidence of the Q-meter's long influence on inductors. The variable capacitor dial on the Q-meter was calibrated not only in microfarads and picofarads, but also in inductance. The inductance scale was only accurate if the frequency of the oscillator was set to one of several specific frequencies (250 kHz, 790 kHz, 2.5 MHz, etc.) which were etched on the front panel of the meter. These steps are in multiples of the square root of 10 so that the inductance scale on the variable capacitor may be used in multiples of 10. A new inductor of the latest modern design specifies a Q at a specific frequency which is, more often than not, one of the frequencies etched on the front panel of a 50-year-old Q-meter.

The Q-meter can measure reactances that are low Q, low values of inductance, or high values of capacitance. The difficulty in measuring low inductance is that large capacitance is required to resonate the inductance. For the case of a capacitor, the usual technique of measuring capacitance is to resonate a "working inductor," place the unknown capacitor across the calibrated capacitor, and note how much capacitance is to be removed from the calibrated capacitor to restore resonance. For capacitor values greater than the range of the calibrated capacitor, this could not be done because it would not be possible to remove an amount of capacitance equal to the unknown. To measure large capacitors and small inductors, the unknown circuit element is placed in series with a working inductor; the change in Q and capacitance is determined to find the capacitance, inductance, or Q of the unknown.

Assume that a working inductor is resonated with the Q-meter's calibrated capacitor, C_1. This capacitance can be represented as

$$L = \frac{1}{4\pi^2 C_1 F_0^2} \tag{31}$$

The unknown is added in series as shown in Fig. 8. If the unknown is a capacitor, C_x, the calibrated capacitor will have to be increased in value because the unknown is in series with the calibrated capacitor and will reduce the total capacitance across the working inductor. The dial capacitance is changed to bring the inductor to resonance. The new capacitance is the combination of the dial and the unknown, and together they are equal to the original capacitance as resonance is reestablished. Therefore,

$$\frac{1}{C_1} = \frac{1}{C_x} + \frac{1}{C_2} \tag{32}$$

where C_x is the unknown capacitance and C_2 is the dial capacitance required to establish resonance with the unknown in place. Solving for C_x, we obtain

$$C_x = \frac{C_1 C_2}{C_2 - C_1} \tag{33}$$

If the unknown is an inductor, L_x, and there is no mutual inductance between the working inductor (this is the reason for shielding the working inductor), the circuit inductance will increase by an amount equal to the added inductance. Therefore, the calibrated capacitance must be reduced to account for the increase in inductance. The new inductance is

$$L_2 = L + L_x = \frac{1}{4\pi^2 C_1 F_0^2} + L_x = \frac{1}{4\pi^2 C_2 F_0^2} \tag{34}$$

Solving for L_x, we obtain

$$L_x = \frac{C_1 - C_2}{4\pi^2 C_1 C_2 F_0^2} \tag{35}$$

To derive the relationship of the measured Q and the Q of the unknown, consider the situation at resonance with just the working inductor. The indicated Q_1 is

$$Q_1 = 2\pi fL/R = \frac{1}{2\pi fC_1 R} \tag{36}$$

When an unknown is added to the circuit, the equivalent series resistances add but the additional reactance is removed by either increasing or decreasing the calibrated capacitance until resonance is restored. The Q is measured by the voltage across the calibrated capacitor and the measured Q is relative to that circuit element. With both the unknown and the working inductor present in the circuit the indicated Q will be Q_2 and the setting of the calibrated capacitor will be C_2. Thus the total circuit resistance for this situation is

$$R + R_x = \frac{1}{2\pi f C_2 Q_2} \tag{37}$$

Solving for R_x we obtain

$$R_x = \left(\frac{1}{2\pi f}\right)\left(\frac{1}{C_2 Q_2} - \frac{1}{C_1 Q_1}\right) \tag{38}$$

The highest Q that can be measured by the Q-meter is under 1000. Conventional inductors are seldom manufactured with Q values as high as 1000. Ceramic resonators and other electromechanical resonators can have Q values as high as several thousand. Theoretically, the Q-meter could measure very high Q values if the "multiply Q by" control were reduced sufficiently to permit the Q to fall within the range of the meter. Even if this were possible, there is another problem with the use of the Q-meter for very high Q-circuits. The bandwidth of a circuit is inversely proportional to Q, and for very high Q-circuits the bandwidth can be so narrow that it becomes nearly impossible to adjust the frequency of the oscillator in the Q-meter to exact resonance.

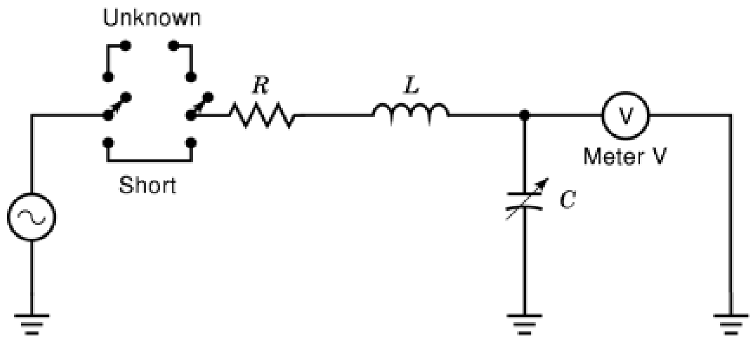

Fig. 8. Simplified schematic of the method of measuring small inductors or large capacitors.

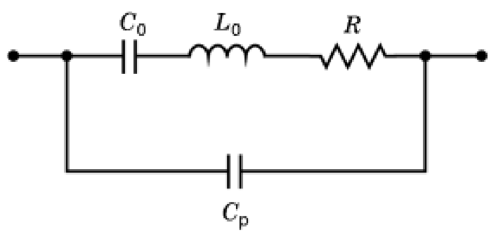

Fig. 9. The equivalent circuit of quartz or ceramic resonator.

An instrument (which is similar to the Q-meter) used for very-high-Q resonators is the crystal impedance meter or CI-meter. The first major difference between the CI-meter and the Q-meter is that the resonator is put in an oscillator circuit such that it is not necessary to adjust the signal generator with great accuracy.

The quality of a quartz or ceramic resonator is usually specified by the equivalent series resistance. Figure 9 shows the equivalent circuit of a quartz or ceramic resonator. The inductor L_0 is due to the motion of the quartz bar and is called the motional inductance. The capacitor C_0 is called the motional capacitance and is also due to the mechanical motion. The capacitance C_p is due to the electrodes plated on the quartz bar and the capacitance of the mechanical supports for the crystal and is called the parallel capacitance of the crystal. R is the equivalent series resistance and represents the losses in the crystal.

There are two resonant frequencies of the quartz resonator: the series and the parallel resonance. The series resonant frequency occurs when the circuit is at the minimum impedance, which is, essentially, the equivalent series resistance. There would also be a reactance due to the capacitance C_p; but because this reactance is so much higher than the relatively low resistance of R, its effect can be discounted.

In the parallel mode of oscillation, the quartz crystal resonates with its motional capacitance in series with the capacitor C_p, with the two capacitors in parallel with the motional inductance.

In a quartz resonator, the equivalent inductance is in the tens and hundreds of millihenrys while the motional capacitance is very small, on the order of femtofarads. Although the capacitance C_p is small, it is large compared to C_0. Therefore, the parallel resonant frequency of the quartz crystal is only slightly higher than the series resonant frequency, typically on the order of 0.1%.

The equivalent series resistance of a quartz crystal is measured in two ways. The first method is to place the quartz crystal in a resonant circuit and measure the input and output voltage of the crystal. Then the crystal is replaced with a variable resistor; and because of tuned circuits in the oscillator, the oscillator will continue to oscillate at nearly the resonant frequency of the crystal. The resistance value is adjusted until the input and output voltages and currents are the same as when the crystal was in the oscillator circuit. This

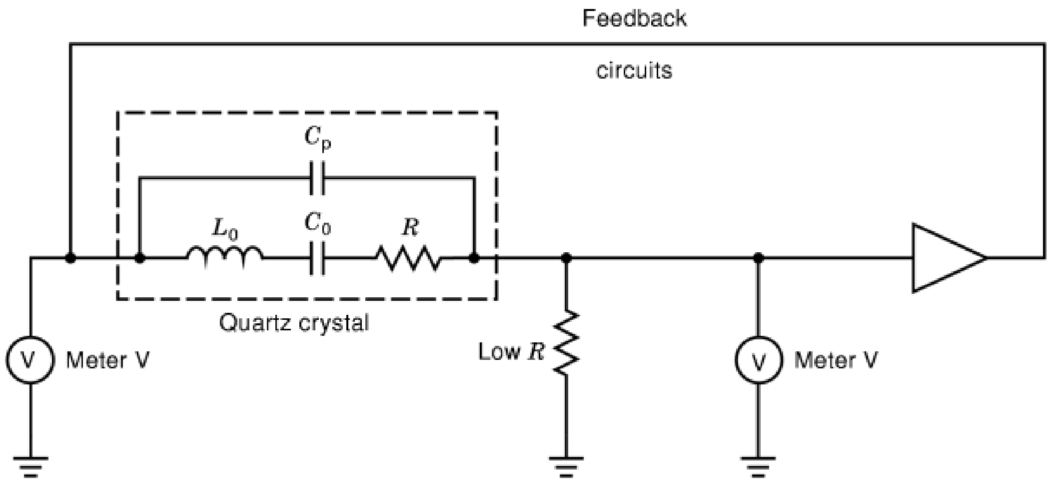

Fig. 10. A simplified diagram of a crystal impedance meter.

method requires manual manipulation; and although it has been used for many years, it cannot be applied to an all-electronic method.

A method that can be adapted to an all-electronic method is similar to a *Q*-meter in reverse. The oscillator is adjusted to operate in the series mode of the crystal, and the crystal current passes through a low resistance (Fig. 10). Measuring the voltage across the low resistance provides an indication of the crystal current while the voltage across the crystal provides the applied voltage. The resistance is simply the voltage divided by the current.

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