

## VOLTAGE MEASUREMENT

By international agreement the units for physical quantities (i.e., measurable attributes of physical phenomena) have been coordinated into the International System of Units (*SI*) (1), which consists of seven base units, assumed to be independent of one another, and derived units, which can be expressed as combinations of the base units. For electrical quantities, electric current has been assigned as the base quantity (unit: ampere), and voltage is one of the derived quantities. There is little significance for practical measurements whether a quantity is a base quantity or derived, and in some cases derived quantities are more easily measured than base quantities.

The *SI* unit for voltage is the volt, named after Alessandro Volta (1745–1827). Voltage is sometimes also referred to as electromotive force (*emf*) or as electrical potential, the latter particularly in the context of electrostatics. In electrochemistry and for electrothermal phenomena the use of “electromotive force” is more common, while in electric power and electronic application the term “voltage” is usually preferred.

The *SI* definition of the volt is: “The volt (unit of electric potential difference and electromotive force) is the difference of electric potential between two points of a conductor carrying a constant current of one ampere when the power dissipated between these points is equal to one watt” (2).

## APPLICATIONS OF VOLTAGE MEASUREMENT

Of all the electrical quantities, voltage is probably the one measured most often, routinely and for diagnostic purposes. It can be measured conveniently with a meter connected externally, in parallel with the electric circuit being tested, and the measurement usually causes minimal disturbance of the operating conditions. In contrast, current measurements require the circuit to be reconfigured physically so that the meter can be inserted in series with the branch of the circuit carrying the unknown current.

Voltage also is used widely in specifying operating conditions of equipment or appliances—household electricity supply (generally 120 V in the United States), batteries, etc. In most countries electric utilities supply the end user with a voltage controlled within narrow limits, which are set by administrative entities. Countries within certain economic communities (e.g., Europe, North America) generally have international agreements to furnish the same supply voltage to all member nations. However, there are exceptions to this rule, and voltages may vary, in some cases, even between different districts of the same country (3).

Being able to manufacture electric and electronic equipment for a common specified supply voltage has obvious economic advantages and allows the use of such equipment wherever appropriate standard electric power is available. This is only possible by having voltage-measuring equipment and standards that provide consistent and accurate results.

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### AC AND DC VOLTAGES

Although the basic concept of voltmeters for ac and dc is the same, specific design considerations differ, and they are treated separately below.

A dc voltage is associated with direct current, that is, an electricity supply that does not reverse its polarity. Dc voltages are typically found as the output of batteries and power supplies of electronic equipment, including computer and logic circuits; in electrochemical applications; in certain electric motors; and in some high-voltage power transmission lines.

### MEASUREMENT OF DC VOLTAGE

Instruments to measure voltage, commonly known as voltmeters, use several different methods to determine the voltage applied to the input terminals of the voltmeter. Whatever the method, the instrument must first be calibrated to give a correct reading (see also the section “Voltmeter Calibration Methods” below). Depending on the degree of accuracy desired, such calibrations are performed, manually or automatically, at intervals ranging from once only (at manufacture) to every time a reading is taken. For calibration, internal or external voltage reference standards are used in a comparison measurement procedure. These are devices that have an output voltage that is stable over time and relatively insensitive to external influences. Once calibrated, the meter is ready to determine unknown voltages, within its range, until recalibration is deemed necessary. The calibration interval depends on the application.

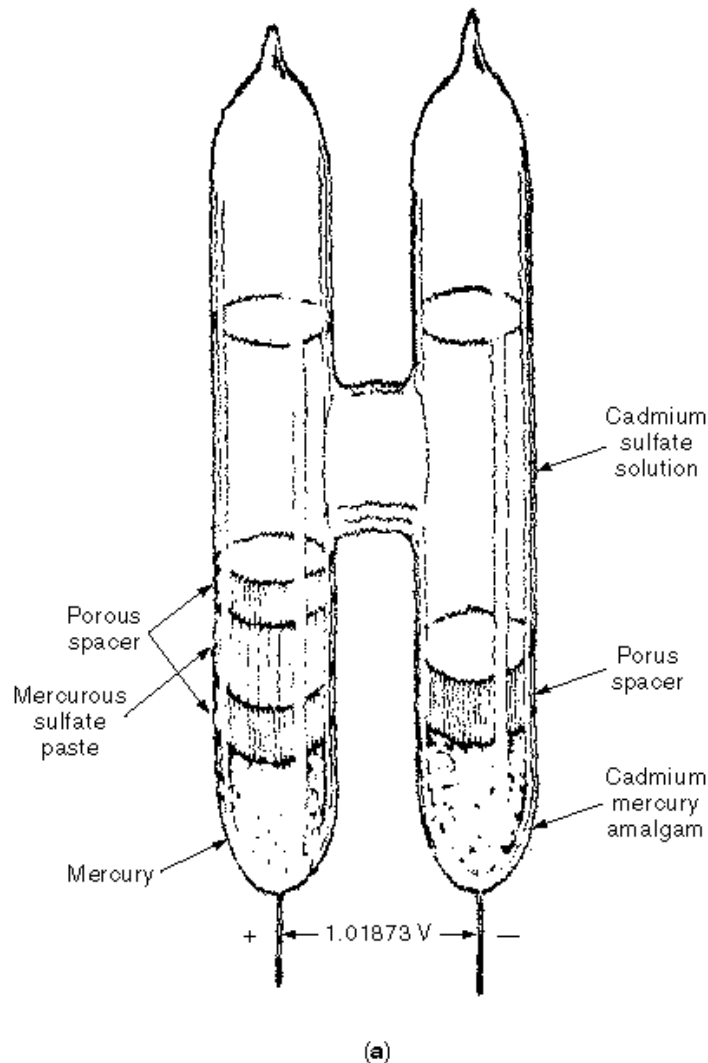
**Voltage References.** The theoretical concept of the *SI* definition is rather difficult to translate into an experimental realization of the volt. In practice, the unit of voltage is obtained by comparison with a physical reference standard. Several such standards exist, differing in their accuracies and ease of use. The value assigned to a reference standard is ultimately based on experiments carried out by various national laboratories and the international agreements derived from them.

**Josephson Volt Standard.** A very stable dc reference standard has been developed that was first described by Brian Josephson in 1962. As a primary voltage reference standard, it has largely replaced the special types of electrochemical batteries—the so-called standard cells that commonly were used earlier. Although a standard cell’s output voltage is fairly stable, it does exhibit some drift even under controlled conditions.

The Josephson standard is a quantum standard (based on the ratio of fundamental constants  $e/h$ ) and is linked to the phenomenon of electron tunneling. The Josephson voltage is essentially invariant, independent of the material properties of a particular Josephson junction, and depends only on an excitation frequency applied to the junction. Although it is not an absolute standard in the sense of the *SI* definition, the Josephson effect has been adopted by international agreement as a reproducible embodiment of the volt. The appropriate excitation frequency has been determined empirically, and its value has been agreed upon internationally (4). (Note that frequency reference standards inherently have much greater stability than other electrical reference standards by several orders of magnitude.)

Because of its excellent reproducibility, the Josephson volt is considered the top echelon in a hierarchy of voltage standards. However, the experimental implementation of the Josephson experiment, at the current state of the art, is too complex for use in other than large laboratories. The junction must be in a superconducting state, necessitating cooling to liquid-helium temperatures, and a calibrated radio-frequency (*RF*) signal source and controlled dc bias source for the junction must be provided. A single Josephson junction has an output voltage of the order of 5 mV, but by connecting many junctions in series, reference voltages as high as 10 V have been obtained (5). For a 10 V array, the experimental uncertainty is of the order of 1 nV.

**Standard Cells.** For many routine laboratory measurements the complexity and expense of a Josephson voltage standard is not necessary. Traditionally, standard cells (Fig. 1) have been used as the reference in



**Fig. 1.** Weston standard cell. By courtesy of Weston Electrical Instrument Corp.

the laboratory, particularly at low voltages. Standard cells are relatively inexpensive and can provide good accuracy when used with care. They are sensitive, however, to temperature and mechanical shock, both of which disturb the equilibrium of the electrochemical system that provides their stable output voltage. Once disturbed, recovery can be a matter of days or weeks, depending on the severity of the disturbance. Recalibration and reassignment of a new value of the cell voltage is then always advisable.

Normally, in the laboratory, one or more standard cells are kept permanently in a temperature-controlled environment, such as an oil or an air bath. Even under these conditions, their voltage output tends to drift with time. Periodic intercomparisons with groups of other standard cells help to detect such drifts and to keep the reference voltage assigned to a particular cell in statistical control by determining corrections to it.

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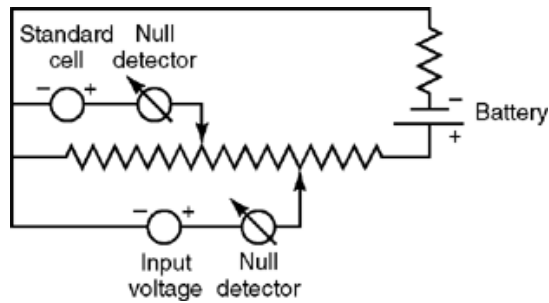


Fig. 2. Laboratory potentiometer.

Carrying out intercomparisons on a regular schedule helps to maintain the uncertainty of the cell voltage at the microvolt level.

If standard cells have to be transported from one laboratory to another, a group of them (usually four) can be enclosed in a battery-energized, temperature-controlled, portable air bath. With such enclosures, which maintain a very stable environment, voltage intercomparisons between distant laboratories has been successfully performed. Several cells are included in each air bath, so that relative shifts in voltage between individual cells can be checked before and after transport and help identify possible cell failures.

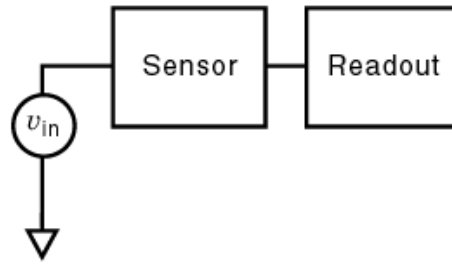
There are two types of standard cells: Saturated cells are somewhat more robust, but subject to greater drift. Unsaturated cells are preferred for their greater precision, but require more careful handling. Either type must be used so that no appreciable current is drawn from the cell. For that purpose, special instrumentation has been developed, known as *laboratory potentiometers*. (These should not to be confused with the variable-resistance dividers used in electronic equipment, such as audio volume controls, which are frequently also called potentiometers.)

**Potentiometers and Null-Current Measurements.** Because only very small currents can be drawn from standard cells, a special circuit, a *potentiometer* (6) (Fig. 2) is needed for voltage measurements that use standard cells as a reference. A potentiometer is a laboratory instrument designed to determine the ratio of the unknown (input) voltage to the known standard-cell reference voltage. The instrument is essentially a calibrated resistive voltage divider with special provisions that limit the current drawn from the standard-cell voltage reference. The divider has two adjustable taps, one at the low end for the standard-cell connection, and the other for connection to the unknown voltage.

The principle of operation is the adjustment of an auxiliary (battery) current through the divider until the voltage drop to the first tap is exactly equal to the standard-cell voltage. A second adjustment is then made by positioning the input voltage tap until the voltage drop along the divider is equal to the unknown voltage. A null detector (galvanometer) between the standard cell and the first tap and between the unknown voltage and the input tap is used to establish balance (no-current) conditions. These adjustments prevent currents from flowing through the standard cell and the voltage source.

The resistances of the divider of the potentiometer are arranged so that the ratio of the unknown voltage to the standard-cell voltage is equal to the ratio of the resistances of the corresponding sections of the divider. The potentiometer usually has calibrated dials so that the unknown voltage can be read off directly. Laboratory potentiometers are generally operated manually, using sensitive galvanometers as null detectors. While the accuracy can be very good, taking readings is laborious and slow. In some industrial versions the adjustments are performed automatically by a servomechanism, with greater speed but somewhat lower accuracy.

In all measurements involving standard cells the currents are very small, and great care must be taken to avoid leakage paths across the insulation. Also, thermoelectric *emfs* at metal-to-metal junctions and induced voltages from external circuits can affect the accuracy of the measurement adversely.



**Fig. 3.** Schematic representation of a voltmeter.

**Solid-State Voltage References.** For accuracy requirements that do not justify the complexity of a Josephson standard or the inconvenience of a standard-cell setup, using an electronic voltage reference is a more practical alternative. In many industrial applications standard cells have been replaced by solid-state reference modules because of their greater ruggedness, smaller size, and easier temperature compensation. Such circuits are based on Zener avalanche diodes or integrated-circuit reference amplifiers. Zener diodes have reverse-current breakdown characteristics that make them useful as voltage references. The reverse voltage changes only over a narrow range when the current varies over a wide range. In combination with solid-state amplifiers these voltage standards have a low output impedance and do not have to be used under null-current conditions as do standard cells.

For instance, 10 V Zener diodes with compensating circuitry typically have a short-term stability of  $1 \mu\text{V}$ . Under carefully controlled conditions, Zener diodes even can be used as transfer standards with uncertainties of the order of 30 nV (7, 8).

Forward diode voltage drops can also be used as voltage references in conjunction with operational amplifiers. Such *reference amplifiers* are available commercially as integrated circuits. By proper choice of circuitry and components, they can be made less sensitive to external temperature fluctuations over a limited range of temperatures and currents. Such solid-state voltage-reference modules are incorporated in most digital voltmeters.

**Voltmeters.** A voltmeter, like many other measuring instruments, can be represented schematically (Fig. 3) by two major components blocks:

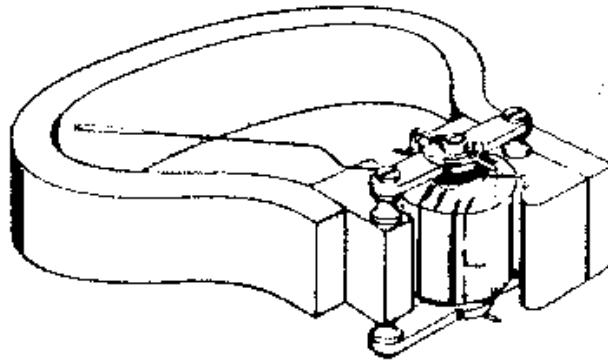
A sensor that converts the input voltage into an internal signal

A readout module that makes use of that internal signal to display the measured voltage

The display can be either a visible indication, a printout, or an electronic record.

**Electromechanical Voltmeters.** In the simplest type of voltmeter (Fig. 4), the sensor is a resistor (range resistor) in series with a D'Arsonval movement (moving coil) to which a pointer is attached that indicates the input voltage. The coil is suspended in a constant, stationary magnetic field, usually provided by a permanent magnet. When a voltage is applied across the combination of the range resistor and the moving coil, a current flows through the coil and produces a magnetic field proportional to the input voltage. The interaction of the stationary field with that generated by the current in the coil results in a mechanical torque that rotates the coil about an axis perpendicular to the axis of the coil. The rotation is opposed by the torque of a hairspring or torsion wire attached to the coil and the meter frame. A pointer fixed to the coil is deflected in proportion to the input voltage and points to the appropriate value marked on the scale.

By proper shaping of the field of the permanent magnet and shielding of external magnetic fields, the voltage scale can be made essentially linear, so that the angle of the pointer deflection is proportional to the input voltage. The pivot and jewel in the coil suspension can sometimes be subject to undesired friction, causing



**Fig. 4.** Permanent-magnet moving-coil voltmeter. By courtesy of Weston Electrical Instrument corp.

the meter to stick. To overcome this problem, newer designs use a *taut-band suspension* where the pivots and hairspring are replaced by metal bands (ligaments) attached so that the coil is at the midpoint of the suspension while the ends are anchored to the meter frame. The suspension is kept under tension so as to limit lateral movement of the moving coil. As the band is twisted by the magnetically induced rotation of the coil, the elastic deformation of the band provides the countertorque to return the coil and pointer to the rest position.

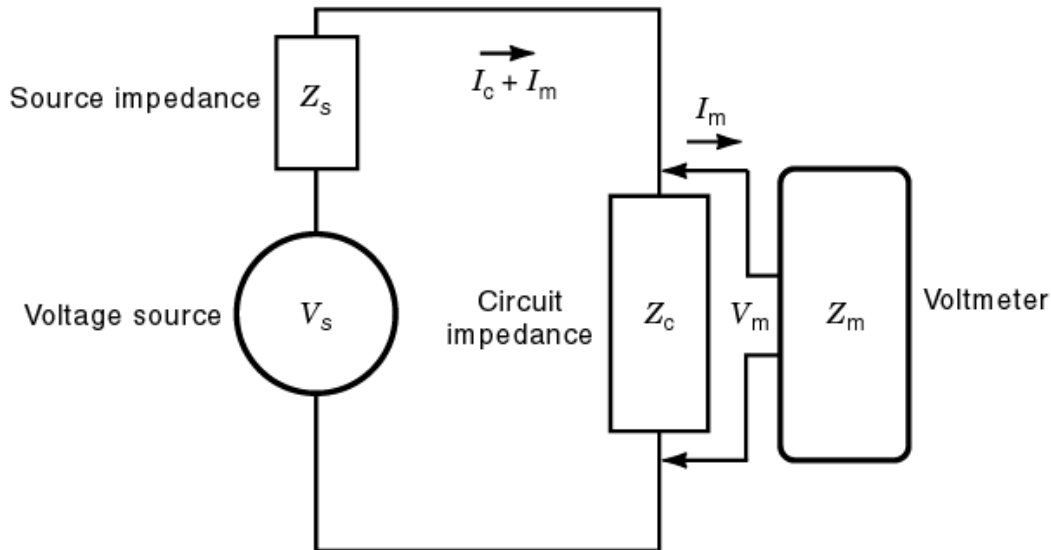
**Amplifier-Aided Measurements.** In the simple scheme described above, the voltage source must supply appreciable current to the D'Arsonval movement to deflect the pointer. The magnitude of the current needed will depend on the electromechanical efficiency of the movement. Unless the impedance of the voltage source is sufficiently low, connecting the voltmeter as a current path in parallel with the circuit will disturb normal operation noticeably (Fig. 5). The additional current flowing through the meter movement increases the total current from the voltage source and causes a voltage drop due to the internal resistance of the source. This will adversely affect the desired measurement, because the meter no longer indicates the unperturbed circuit voltage.

A modification of this simple electromechanical arrangement can reduce the circuit loading by the meter. Using the high-impedance input of a feedback amplifier (typically up to  $100\text{ M}\Omega$ ) as the voltage sensor, the current drawn from the voltage source by the amplifier-indicator combination is greatly reduced, so that the source voltage will not decrease appreciably. The current needed to deflect the D'Arsonval movement is then furnished by the output of the amplifier and is derived from the amplifier power supply and not from the voltage source. (Before the advent of transistors, this arrangement was known as a "vacuum-tube voltmeter".)

**Integrating Voltage Measurements.** While the amplifier-aided meter is an improvement over the directly connected instrument, it still suffers from limitations. The D'Arsonval movement, the amplifier, and its associated circuitry each contribute to the electrical noise and nonlinear response and therefore affect resolution and linearity. Disturbing random voltages can be induced by external electric or magnetic fields or can be generated by thermoelectric effects in some circuit components. Such disturbances may mask the voltage to be determined, particularly when measuring small dc voltages.

Although, theoretically, ac voltages have no effect on dc measurements (provided that the effective time constant is long with respect to the ac signal period), some rectification of the ac can occur at partially oxidized copper junctions and may result in dc offset voltages. To reduce the effects of nearby ac power lines or *RF* circuits, careful shielding and grounding are of great importance, especially when signal levels are low. (9).

With the advent of inexpensive integrated circuits, there is less economic incentive to continue using electromechanical voltmeters even in less demanding applications. Instead, the trend is to electronic, digital



**Fig. 5.** Circuit loading by a voltmeter. The voltage indicated by the voltmeter,  $V_m$ , is smaller than the unperturbed circuit voltage,  $V_c$ . The difference is due to the additional component of the voltage drop across the source impedance,  $Z_s$ , caused by the current through the meter,  $I_m$ , where  $V_m = V_c - I_m Z_s$ .

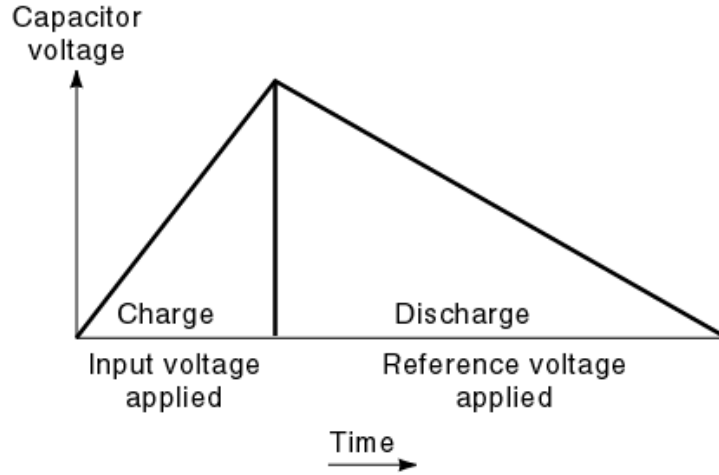
instruments. Digital voltmeters have evolved from early potentiometric types using mechanical stepping switches, to meters using discrete-component electronic analog-to-digital converters, and eventually to the use of custom integrated circuits. Developments in the 1960s brought the voltmeter technology an important step forward by combining measurements integrated over a time interval with digital techniques (10).

*Integrating Digital Voltmeters.* The effect of residual periodic and nonperiodic small disturbances that interfere with dc measurements can be reduced by integrating the measured voltage. The time interval for integration should be chosen to be a multiple of the period of the interfering fluctuating signal. (Often, a time interval of several periods of the power-line frequency is selected, because signals at line frequency are a likely source of interference.) Under these conditions, the positive and negative excursions of periodic disturbances tend to cancel, and will do so completely if the interfering signal is symmetrical. Even the effect of nonperiodic signals, such as sporadic pulses, will be effectively reduced because they contribute less energy than the integrated voltage (11).

The voltmeter can be further improved by combining Integration with digitization (i.e., quantization of the analog value) and using a numerical display that provides better resolution and linearity than a pointer and scale. Voltages can be digitized by several methods, such as using successive approximations, frequency-to-voltage converters, or dual-slope integration.

*Dual-Slope Integration.* The widely used technique for digital dc voltmeters based on *dual-slope integration* was first described by S. K. Ammann (12, 13). The method is popular because it is simple and because it does not require precision circuit components as long as the components are stable during the measurement period (normally a fraction of a second). Only the voltage reference source directly affects the instrument accuracy. This requirement can be met by a built-in Zener diode or reference amplifier module. Essentially, the method determines the ratio of the unknown voltage to that of the reference by integrating both these voltages using the same circuit paths.

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**Fig. 6.** Principle of dual-slope integration.

The basic principle of the dual-slope method (Fig. 6) briefly is the following: The voltage to be measured is applied to the input of an amplifier and converted to a current, which is nominally proportional to the voltage. The current charges a capacitor for a predetermined time at a rate given by the resistance–capacitance ( $RC$ ) time constant of the circuit. After a given number of timing pulses provided by an auxiliary oscillator, a switching circuit disconnects the input voltage  $V_i$  to be measured from the input of the amplifier and substitutes the internal reference voltage, which is of opposite polarity.

The application of the reference voltage  $V_r$  causes the capacitor to be discharged through the same  $RC$  network until the capacitor voltage reaches its original value (threshold). Again, the time taken for the capacitor to discharge is determined by counting pulses from the oscillator. The final pulse count indicates the result of the measurement.

During the charging up cycle, lasting for  $N_u$  counts, the change in capacitor voltage,  $\Delta V_C$ , is given by

$$\Delta V_C = V_i N_u / RCf \quad (1)$$

and during discharge for  $N_d$  counts,

$$\Delta V_C = V_r N_d / RCf, \quad (2)$$

where  $f$  is the pulse frequency and  $RC$  is the charge and discharge time constant. Dividing Eq. (1) by Eq. (2) and transposing, we get

$$N_d = V_i / -V_r N_u \quad (3)$$

The number of pulses that determines the length of the capacitor up charging cycle,  $N_u$ , can be chosen such that  $N_d$  is numerically equal to  $V_i$ . This choice makes the pulse counter (i.e., the voltmeter display) direct-reading. The parameters  $R$ ,  $C$ , and  $f$  drop out of Eq. (3), so that the accuracy of their values will not influence the final result, provided they are stable while the reading is being taken (a few milliseconds). (Note that for the voltage-to-frequency conversion method both the circuit time constant and the measurement time affect the result. Their long-term stability is thus of critical importance.)



Several enhancements have been added to the original design of the dual-slope meter to compensate for zero offset, resolution and range calibration, and to speed up readings. For instance, the zero offset can be determined by making an additional dual-slope measurement with short-circuited input and subtracting the value determined from the voltage measurement. The resolution can be increased by using a multislope technique where, for a given  $RC$  time constant, charge and discharge cycles are alternated so that the physical voltage limitation of the integrator is not exceeded (14).

## MEASUREMENT OF AC VOLTAGE

**Methods Based on Comparison with.** Industrial power systems predominantly use alternating (ac) voltage to supply industrial and residential customers. The voltage reverses polarity periodically, generally 50 or 60 times a second, resulting (ideally) in a sinusoidal waveform. The great advantage of this form of electricity is that the voltage can be raised or lowered easily and efficiently, to suit particular operating conditions and for distribution, with only passive components called *transformers*. The latter have efficiencies of typically 95% or better. In contrast, resistive voltage dividers, the usual means to step down dc voltage levels, always dissipate an appreciable fraction of the total power.

To be able to measure ac voltage, the unit of measurement, the volt, has to be defined for alternating current. Since the *SI* definition of the volt applies strictly only to direct (dc) voltage, where the polarity of the voltage does not change, the definition cannot be applied directly to an alternating voltage. This is because the instantaneous ac voltage varies so that the time average over one or more periods of the waveform is zero. Thus measuring ac voltage using a dc method will not yield a meaningful result. To extend the applicability of the *SI* definition, the basis for equivalence of ac and dc voltages has to be established.

**Equivalence of AC and DC Voltages.** An ac voltage is considered equivalent to a corresponding dc voltage if the same energy is dissipated when the voltage is applied to a pure resistance. The voltage causes an electric current to flow through the resistance, which then dissipates power in the resistor and raises its temperature. When the heating effect is exactly the same for the ac voltage as it is for the dc voltage, the voltages are effectively equal, provided that the measurement extends over a time much longer than the period of the ac (so that any fractional period included has a negligible effect). Since the heat produced is proportional to the square of the voltage, and if  $\langle V_{ac}^2 \rangle = \langle V_{dc}^2 \rangle$ , where the angle brackets indicate a time average, the ac voltage is known as the *root-mean-square (rms)* voltage, or sometimes the *effective* voltage.

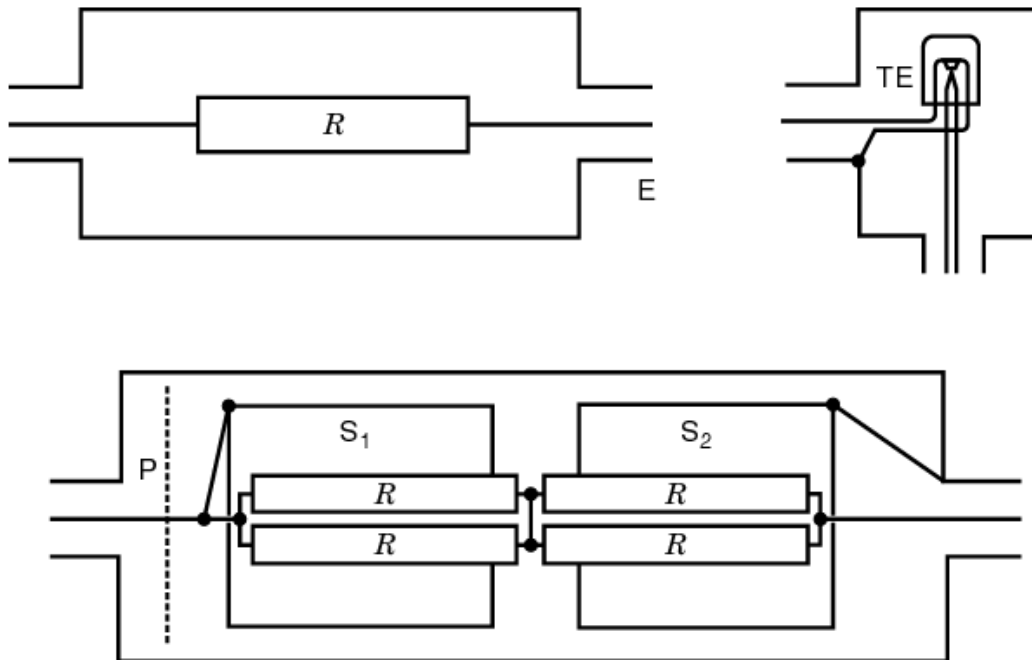
The power  $W$  dissipated in the resistance  $R$  is

$$W = V_{\text{rms}}^2/R = V_{\text{dc}}^2/R \quad (\text{in watts}) \quad (4)$$

$$V_{\text{rms}} = \left( \frac{1}{T} \int_0^T V^2(t) dt \right)^{1/2} \quad (5)$$

where  $T$  is the duration of an integer number of periods.

The *rms* value of an ac voltage can be measured directly with analog circuitry that effectively computes the integral in Eq. (5), or by sampling the signal at discrete intervals and approximating the integral numerically. These methods are discussed later. Traditionally, however, indirect methods are used that equate the heating effect due to the power dissipated by the ac with the equivalent heating effect due to the dc power, using thermal voltage converters.



**Fig. 7.** Thermal voltage converters. (Adapted from 15.) Top left: low-voltage resistor (1 V to 100 V); top right: single-junction thermoelement (5 mA); bottom: high-voltage resistor with shields [ $S_1$ ,  $S_2$ ] (200 V to 500 V). Resistors are joined to the thermoelement by means of coaxial connectors.

**Thermal Voltage Converters.** Based on the heating effect mentioned above, ac voltages can be measured by means of a thermal voltage converter (Fig. 7) (15). Such determinations can be carried out in the laboratory with an uncertainty better than  $1 \mu\text{V/V}$  at audio frequencies. The thermal converter is a combination of a thermoelement and a set of special, shielded detachable resistors, each selected for a particular voltage range. The thermoelement consists of a heater with one or several temperature measuring devices (thermocouples) that are thermally connected to the heater but electrically insulated from it.

Thermoelements are often designed for currents of either 10 mA or 5 mA, and the resistance of the special resistor in series with the thermoelement is chosen to limit the current at either of those values for the voltage range selected (15, 16).

Thermoelements are designed either with one (single-junction) or several (multijunction) thermocouples. Single-junction thermoelements are usually constructed with a straight heater wire at the center of which a single thermocouple is attached with a small glass bead. The bead provides electrical insulation to isolate the thermoelement output (a few millivolts) from the voltage input circuit, but it has sufficient heat conductivity to allow measuring the rise in temperature of the heater. For single-junction thermoelements the whole assembly is usually enclosed in an evacuated glass bulb.

Multijunction thermoelements can be assembled from separate thermocouples attached to the heater wires (17), or they can be fabricated using thin-film thermocouples deposited on semiconductor chips (1819–20).

**AC–DC Transfer Measurements.** A high-accuracy ac–dc laboratory measurement is a multistep process often carried out manually when the best accuracy is desired. For somewhat lower accuracies, fully automated systems can be used, and commercial instruments are available that perform such measurements. In any case,

great care must be taken because the very small differences in thermocouple output that are obtained can easily be masked by electrical noise.

In a typical measurement sequence, the unknown ac voltage is applied first; then a dc voltage is connected to the input and adjusted until the same thermocouple output is obtained as with the ac voltage. The polarity of the dc voltage is then reversed, and again adjusted to obtain the same thermocouple output. Often, a second ac measurement is then made to compensate for small drifts in the measurement setup. The measurements are then combined using the known ac–dc difference  $\delta$  of the thermal converter, and the ac voltage is determined with

$$V_{ac} = V_{dc}(1 + \delta) \quad (6)$$

The dc voltage has to be applied with both polarities because the current through the heater causes two different thermal phenomena in addition to Joule heating: the Thomson and Peltier effects. The latter two are sensitive to the direction of the current and produce or absorb heat along temperature gradients in the heater and where the heater is attached to dissimilar metals. This distorts the temperature distribution along the heater wire so that the thermocouple does not measure the correct temperature. With ac, at frequencies above roughly 20 Hz, the thermal time constants of the heater structure are too long for that temperature change to be detected by the thermocouple before the ac current reverses polarity. However, when applying voltage from a dc supply, enough time elapses to affect the temperature distribution along the heater and the temperature sensed by the thermocouple.

Thus using both polarities of the dc voltage as part of the measurement sequence yields a better approximation, but does not completely eliminate the distortion of the temperature distribution. It is customary to assign the residual discrepancies from this and other causes as a characteristic of the thermal converter (*ac–dc difference*). In many cases the characteristic ac–dc difference is based on a theoretical evaluation of the properties of the materials and design of the thermal converter. Confidence in the assignment is gained by intercomparing various thermal converters of different design and manufacture. It is also possible to evaluate these thermoelectric effects experimentally with a technique using very rapid dc reversals (21).

*Frequency and Voltage Effects in Thermal Converters.* As pointed out in the preceding sub-subsection, at frequencies below approximately 20 Hz, the thermal response of most converters follows the input signal and therefore does not have the desired averaging effect. At the high end of the audio-frequency band, the ac–dc difference becomes progressively larger as the frequency increases because of circuit reactances and skin effect in the conductors. The behavior of the converter is also noticeably voltage-dependent at voltages above 100 V, and, again, the ac–dc difference increases with voltage. The converter properties, in general, depend on the geometric configuration and materials of construction (22, 23).

A given resistor–thermocouple combination can accommodate only a limited voltage amplitude ratio of about 2 : 1. At the low end there is a serious loss of sensitivity because of the quasi-square-law temperature characteristic of the thermoelement, and at the high end, if the rated current is exceeded, the device will burn out. Typical thermoelements used with thermal converters operate at a voltage drop of at least 0.5 V and therefore cannot measure voltage less than that. However, thermoelements can be used in devices (micropots) (24, 25) that generate small ac voltages accurately, using a similar ac–dc transfer. The voltages generated can then be used to calibrate ac voltmeters down to signal levels as low as 1  $\mu$ V.

**Analog Methods.** Unless the highest accuracy is desired, most ac voltage measurements are carried out with instrumentation that does not rely on thermal response, is less complicated, responds more quickly, and is much simpler to use than laboratory thermal-converter measuring systems. However, there are some special applications where voltage measured using thermal sensing is the preferred method. For example, some RF voltmeters use the expansion, caused by self-heating, of a current-carrying wire to move an indicating pointer. Similarly, in automotive applications where ruggedness is of importance, the self-heating effect in a

**Table 1. AC Voltmeter Classification**

Instrument Type	Operating Principle	
	For Approximate Response <sup>a</sup>	For True-RMS Response
Electromechanical	Rectifier and moving coil	Electrodynamometer; thermal and mechanical
Electronic/mechanical	Rectifier, amplifier, and moving coil	Thermal converters
Electronic	Rectifier, amplifier, and digital readout	Analog multiplier
Electronic/Sampling		Sampling digital voltmeter

<sup>a</sup>Accurate for sinusoidal wave forms.

wire of bimetal strip is used to measure current or voltage. However, for the vast majority of measurements of ac voltage are based on electromechanical or electronic principles (Table 1).

**Electromechanical AC Voltmeters.** A modification of the dc moving-coil instrument can be used to measure  $V_{\text{rms}}^2$  by replacing the (stationary) permanent magnet of the movement with an electromagnet connected in series with the moving coil (electrodynamometer voltmeter). The arrangement acts as a torque multiplier and therefore produces a deflection proportional to the square of the voltage. This type of instrument has a limited frequency range ( $\approx 500$  Hz) and a nonlinear (nearly quadratic) voltage scale.

For waveforms that are almost sinusoidal, rectifier instruments offer a simpler solution. A rectified ac voltage, either half-wave or full-wave, is a varying waveform that is always unipolar (either positive or negative). When averaged over one or more periods of the ac, a dc voltage is obtained that can be measured with a conventional dc voltmeter, except that the voltmeter scale is calibrated in terms of *rms* values with the implicit assumption that the original ac waveform is sinusoidal.

For waveforms that are not necessarily sinusoidal, the *rms* value can be approximated with electronic analog multiplication or digital computation from sampled instantaneous voltage amplitudes. In these applications there are usually frequency and amplitude limitations to prevent overloading of the electronic circuitry involved. For some special applications peak-reading meters that determine the highest instantaneous amplitude in a signal are appropriate.

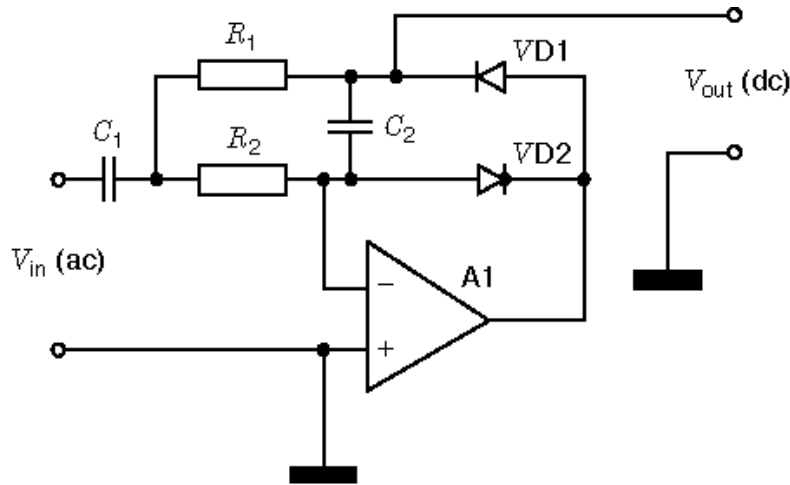
**Approximate Method Using Rectifiers.** In the ideal case, when the voltage to be measured is a pure sinusoid, it is possible to compute the *rms* ac voltage from a determination of the peak voltage (maximum amplitude of the sinusoidal waveform) or the dc average of the rectified (half-wave or full-wave) voltage. For ac waveforms that are close to sinusoidal, this is often a convenient approximation to the *rms* voltage, especially when the accuracy requirements are not stringent (2% to 5%). Average-reading instruments are widely used in low-priced test or panel meters. The method is simple, requiring only rectifiers and resistance-capacitance (*RC*) filtering, but it may lead to large errors when the waveform departs appreciably from a sinusoid. The approximations used are

$$V_{\text{rms}} \approx V_{\text{peak}}/\sqrt{2} \quad (7)$$

$$V_{\text{rms}} \approx \frac{V_{\text{av}} \pi}{2\sqrt{2}} \quad (\text{full-wave rectification}) \quad (8)$$

A full-wave rectifier circuit is shown in Fig. 8.

**True-RMS Meters.** A meter is said to be *true rms* when it measures the average of the voltage squared over the entire frequency spectrum. In practice, voltmeters always have a limited frequency response, so that their frequency response must be taken into consideration. As mentioned above, in the simplest case, the squaring function can be obtained with a moving-coil instrument if both the moving and the stationary magnetic field are generated by the same alternating current (electrodynamometer). (In contrast, for a dc



**Fig. 8.** Full-wave rectifier circuit. From Olev Märtens and Toom Pungas, Precision average-sensing ac/dc converters, *IEEE Trans. Instrum. Meas.*, (1): 71, 1993.

movement the stationary field is constant.) Since the measured variable is  $V^2$ , the resulting deflection of the moving pointer produces a scale that is nonlinear in terms of voltage.

When electronic amplifier modules are combined with a dc meter, multiplication (squaring), averaging, and square-rooting can be performed electronically. The output is then a dc voltage (or current) nominally proportional to the *rms* value of the ac input voltage. The parameters of the amplifier(s) impose limitations on both amplitude and frequency response.

Using analog components, the squaring and square-root functions can be carried out by amplifiers with logarithmic and antilogarithmic characteristics. Doubling the scaled output of the logarithmic amplifier yields a signal proportional to the squares of the instantaneous input voltages  $V$ . A filtering circuit then averages the  $V^2$  outputs, and the resulting dc voltage is fed to another logarithmic amplifier connected as a feedback circuit so that the final output is a dc voltage proportional to  $V_{rms}$ . Such amplifier modules are commercially available. As with all operational amplifiers, the bandwidth (frequency range) and voltage range is limited. One has

$$V_{rms} = \log^{-1} \langle 2 \log V_{ac} \rangle \tag{9}$$

where the angle brackets indicate the time average.

**Sampling Measurements.** A great advantage of sampling methods is that an entire series of voltage values can be recorded, capturing the variations of instantaneous signal voltages with time. This process is treated more fully in the article on RECORDERS. This basic capability also makes it possible to use sampling methods for *rms* voltage measurements (26).

To obtain the *rms* voltage, the numerical values obtained for each sample of the instantaneous voltage are used to compute the *rms* value. If the sampling rate is synchronous with a multiple of the fundamental ac frequency, the algorithm is greatly simplified (2728–29):

$$V_{rms} = \left( \frac{1}{T} \int_0^T V_{inst}^2(t) dt \right)^{1/2} \tag{9a} \tag{1}$$

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If nonsynchronous sampling is employed, corrections are necessary to compensate for fractional cycles sampled (truncation error) (30). The integral of Eq. (9a) [see Eq. (5) above] is then replaced by a suitable summation formula, which depends on the distribution of samples in time and their relation to the fundamental period of the waveform. For synchronous, evenly spaced samples, the summation formula reduces to

$$V_{\text{rms}} = \left( \frac{1}{N} \sum_{j=1}^N V_j^2 \right)^{1/2} \quad (9b) \quad (2)$$

In practice it is prudent to collect more than the minimum number of samples given by the Nyquist criterion, because electrical noise and other disturbances tend to contaminate the data. If the original ac voltage waveform is sufficiently stable over time, the sampled values may be taken from several cycles instead of all from the same cycle. The sample times are then staggered so that when, for the purpose of calculation, the samples from various waveform cycles are superimposed on a single waveform, the samples appear to be uniformly distributed in time (equivalent-time sampling). This has the advantage that more time is available for the analog-to-digital conversion of each sample, which therefore can be carried out with greater precision.

The frequency response of sampling measurements is limited by the slew rate of the sample-and-hold circuit that captures the voltage at each sample point, as well as by the Nyquist condition, which requires that there be at least twice as many uniformly spaced samples per cycle (or per equivalent cycle) as the order of the highest frequency component contained in the waveform.

### OTHER VOLTAGE-MEASURING EQUIPMENT

**Multifunction Voltmeters.** The voltage-measuring function, especially in digital meters, is often combined with other electrical measurement capabilities such as current and resistance. Such multifunction meters, also called *multimeters*, are sometimes named more specifically for the functions performed, for example, “volt–ohmmeters”. They are common in test equipment, since a large part of their circuitry can be shared among the various measurement functions. The voltage measurement capabilities of these instruments are basically the same as those of stand-alone voltmeters. In diagrams they are often abbreviated MM for multimeter, DMM for digital multimeter, and VOM for volt–ohmmeter.

**Nanovoltmeters.** Measurement of very low voltages, particularly dc voltages, presents special problems because electrical noise, generated thermally or by electromagnetic induction, can greatly exceed the voltage to be detected (31). The general approach is

- To use integrated measurements, signal averaging, and filtering to increase the signal energy while reducing random noise
- To use high-impedance input circuitry designed to have high common-mode rejection
- To use input amplifier stages with low bias current, high gain, and feedback
- To use a physical layout and shielding to minimize thermal gradients and electrical interference

Commercial digital nanovoltmeters are available with a typical resolution of 1 nV.

**Vector Voltmeters.** A vector voltmeter measures two variables: the voltage and phase, or equivalently the in-phase and out-of-phase components, of the voltage signal with respect to a reference signal. To make this kind of measurement, a sinusoidal reference signal is required that is of the same frequency as the fundamental of the input. The process is similar to a power measurement, except that the reference-signal amplitude is constant instead of being equal to the current. The circuit can be implemented using an analog

multiplier to get the products of the instantaneous voltage and reference signals. The products are then integrated and averaged with suitable filters (32).

The same type of measurements can be performed by a different method. Sampling the input and reference signal waveforms, sets of numerical data are obtained, from which the in-phase and quadrature components of the amplitude can then be computed (33). For a voltage  $V_{\text{peak}}$  and phase angle  $\phi$ , the in-phase component is equal to  $V_{\text{peak}} \cos \phi$ , and the quadrature component is  $V_{\text{peak}} \sin \phi$ . Commercial vector voltmeters have measurement uncertainties in the range of 1% to 3%. Vector voltmeter measurements are part of network analysis, determining the magnitude and phase of the signal.

**Voltage Measurement with an Oscilloscope.** Another way to measure voltage is with a calibrated oscilloscope. In the simplest case, if the waveform is repetitive, the voltage amplitude is read off the screen using the grid pattern on the grating.

Readings are more convenient and more precise if the oscilloscope has a calibrated cursor with a digital readout. To adjust the cursor, the waveform is centered on the horizontal axis, and the cursor is lined up with the peak of the waveform. If the oscilloscope has a storage function, either analog or digital, a transient waveform can be captured and then measured. This permits the measurement of transient voltages, a task that is often difficult.

Commercial oscilloscopes have typical minimum sensitivities of 1 mV/div, bandwidth of 500 MHz, and gain accuracy of 1.5%, although higher bandwidths are available. Digital oscilloscopes typically have a resolution of 8 bits (1 in 256).

## VOLTMETER CALIBRATION METHODS

Voltmeters have to be calibrated after manufacture to ensure that they function correctly and within their specifications. For some less demanding measurement applications, the intrinsic stability of the instrument may be sufficient so that further calibrations, after the initial check, may not be required. However, for many other applications the accuracy of the voltage measurement is critically important, and periodic calibrations of the voltmeter are recommended to assure continued performance within specifications.

The process of calibration is the comparison of the voltmeter output (readings) with a standard of known accuracy. The standard may be an uncalibrated, adjustable voltage source in conjunction with a calibrated voltmeter, preferably one with better accuracy than the meter under test. Alternatively, programmable calibrated voltage sources may be used. If no calibrated ac voltage source is available, a thermal converter can be added to the dc system to carry out ac–dc transfers (34). Ac calibration can also be performed using a digitally synthesized voltage source, where the *rms* value of the ac voltage is calculated from measurements of the dc levels of each step of the synthesized sine wave (35).

**Calibration Procedure.** Calibration, i.e., comparing the unit under test with an appropriate standard, always involves taking experimental data and the inevitable concomitant uncertainty in the values recorded. A statistical approach to calibration, making use of the parameters of a *calibration curve*, can reduce the overall uncertainty of the process, and it can provide some insight into the extent to which corrections must be applied to individual readings.

For consistent results, several readings should be taken at each calibration test point, preferably in a nonsequential, random order to minimize bias due to drift, temperature change, and so on. Comparison of the variability of the calibration test data at each calibration point with the difference between the actual and ideal calibration curves can verify whether the instrument characteristic is modeled correctly; for voltmeters the straight-line model normally applies. If the model is correct, as confirmed by the statistical indices, then the ideal calibration curve should be used to estimate the corrections rather than the experimental data obtained during calibration. This follows because the ideal curve is less subject to random fluctuations introduced by the actual calibration measurements. Using the statistical indices, it can then be decided what corrections need

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to be applied to bring the instrument to within its specifications. The method is described in 36, where it is applied to phase meters; it is equally applicable to voltmeters.

### VOLTAGE DIVIDERS

Many voltmeters are not designed to withstand the application of high voltages directly to the input terminals. For measurement, such high voltages must first be stepped down using voltage dividers or transformers that have known and stable ratios for specified voltage and frequency ranges. Dc and (audio-frequency) ac voltages can be stepped down with resistance networks that divide the input voltage to an appropriate level. Such networks are known as *voltage dividers*. For ac voltages *inductive voltage dividers*, a specialized form of voltage transformers, serve the same purpose.

When ac or dc voltages are above 1000 V, special consideration must be given in the design and construction of voltage dividers to items such as insulation, arcing, corona discharge, and voltage coefficients of the resistors used (37, 38).

After construction and before use, the ratio must be verified by calibration. As with other instruments, periodic recalibration is recommended. Ratio accuracies of such high-voltage dividers can reach values of a few microvolts per volt (39).

**Resistive Voltage Dividers.** A resistive voltage divider in its simplest form consists of two or more resistors that are connected in series. Applying the input voltage to the entire network of resistors and selecting a suitable tap by means of a manual or automated range switch provides one or more fixed, known voltage ratios. Note that the resistance ratios are frequency-sensitive, because of stray circuit capacitances and because of small inductances in the leads and in the resistors themselves. For use at audio frequencies, ratio resistors can be compensated with small values of capacitance in parallel with the resistors, so that the *RC* time constant of each section of the divider is the same. This compensation reduces the frequency dependence of the divider over the design bandwidth. Temperature and voltage also can affect the resistance ratios unless high-stability resistors with similar temperature and voltage characteristics are selected for construction of the divider.

Dividers should be calibrated to verify their accuracy, and multistep dividers also should be calibrated for linearity. This is most easily accomplished by comparison with a laboratory standard divider of known accuracy and linearity. In the design of standard dividers, special consideration is given to current leakage by providing a *guard circuit network*, in effect a duplicate divider, arranged so that every node of the main divider is surrounded by a shield at the same potential as the node, preventing any leakage current from the divider. The assembly is immersed in a temperature-controlled bath filled with dry transformer oil (40).

Specialized dividers, such as Kelvin–Varley dividers (41), are convenient for checking calibrations. However, in the absence of calibrated standard dividers, it is possible to calibrate dividers by a bootstrap method in which higher ratios are derived from an initial 1 : 1 ratio by a step-up procedure (42, 43).

**Voltage Transformers.** Voltage transformers (instrument transformers) are devices that can change ac voltage levels either up or down by means of magnetic induction. The ac input voltage is fed to a primary winding (coil), producing an alternating magnetic field, which induces an output voltage in a secondary winding. The resulting voltage ratio ideally is equal to the turns ratio of the primary and secondary windings. To increase the efficiency at audio frequencies, the primary and secondary coils are wound on a core of magnetic material. Efficiencies of voltage transformers can be 95% or better.

The voltage ratio of a two-winding transformer is not quite equal to the turns ratio, because a small amount of the input power is used to magnetize the core. Adding a separate core and magnetizing winding [two-stage transformers (44)] permits the voltage ratio to equal the turns ratio to a high degree of accuracy within the specified frequency and voltage ranges of the design.

**Inductive Voltage Dividers.** A special class of transformers, the inductive voltage dividers (*IVDs*), are the ac equivalent of multiratio resistive voltage dividers, with a switching arrangement often similar to that of



Kelvin–Varley dividers. With great care, they can be built with ratio accuracies of the order of 100 nV/V. Ratio increments of inductive voltage dividers follow a decimal sequence or a binary sequence. Decimal dividers are more convenient in use, especially in a manually operated system, but binary dividers have the advantage of a symmetrical arrangement of the windings and therefore, are less prone to stray coupling, resulting in better accuracy (454647–48).

**Capacitive Dividers.** Voltage dividers for ac voltages at power-line frequencies can be built based on ratios of capacitive impedances instead of resistive or inductive impedances as in transformers. This a particularly suitable method for voltages above 10 kV. The high-voltage capacitor is usually a compressed-gas type in the picofarad range, and is equipped with a insulated bushing at the high-voltage end. These capacitors have low dielectric losses and do not produce undesirable phase shift in the divider. The low-voltage capacitor that forms the low end of the divider can be of the same type, but because the capacitance ratio is the reciprocal of the voltage ratio, a very large capacitor is required. Capacitors that are constructed with materials that have a higher dielectric constant than compressed gas are smaller in size. They can have reasonably low losses and are a less expensive alternative. Recent designs employ electronic feedback to stabilize the divider and correct any phase shift (49). Overall ratio accuracy of these devices can be expected to be 0.002%.

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