DISPLACEMENT MEASUREMENT

The measurement of displacement is an important aspect of many industrial, scientific, and engineering systems. The displacement is associated with the motion or the change in the position of an object. It is basically the vector representing a change in position of a body or from one stable point to another with respect to a reference point. In measurement systems, when the word *displacement* is used, the change in the relative position is considered to be very small—within few millimeters or less. Because displacement is a fundamental component of many instrumentation and measurements systems, many different devices have been developed to measure displacements from a few nanometers to a few meters. A list of manufacturers is given in Table 1.

Displacement is closely associated with acceleration and velocity. Double integration of acceleration and single integration of velocity yield the displacement of an object. Therefore, some devices resembling acceleration and velocity sensors are modified for displacement measurements. Displacement sensors are manufactured in different dimensions and sizes, and they are based on many different physical prin-

Table 1. List of Manufacturers

Adsen Tech, Inc.
18310 Bedford Circle
La Puente, CA 91744
Fax: 818-854-2776

ANALITE, Inc.
24-T Newtown Plaza
Plainview, NY 11803
Tel: 1-800-229-3357

Analog Devices, Inc.
1 Technology Way
P.O. Box 9106
Norwood, MA 02062-9106
Tel: 800-262-5643
Fax: 781-326-8703

Dynalco Controls
3690 N. W. 53rd Street
Ft. Lauderdale, FL 33309
Tel: 305-739-4300 & 800-368-6666
Fax: 305-484-3376

Electro Corporation
1845 57th Street
Sarasota, FL 34243
Tel: 813-355-8411 & 800-446-5762
Fax: 813-355-3120

FSI/FORK Standards, Inc.
668 Western Avenue
Lombard, IL 60148-2097
Tel: 708-932-9380

Gordon Engineering Corp.
67 Del Mar Drive
Brookfield, CT 06804
Tel: 203-775-4501

Honeywell
Dept. 722
11 West Spring Street
Freeport, IL 61032
Tel: 800-537-6945
Fax: 815-235-5988

Hecon Corporation
15-T Meridian Road
Eatontown, NJ 07724
Tel: 1-800-524-1669

Kaman Instrument Company
1500 Garden of the Gods Road
Colorado Springs, CO 80907
Tel: 719-599-1132 & 800-552-6267
Fax: 719-599-1823

Kavlico Corporation
14501 Los Angeles Avenue
Moorpark, CA 93021
Tel: 805-523-2000
Fax: 805-523-7125

Kistler Instrumentation Corporation
Amherst, NY 14228-2171
Tel: 716-691-5100
Fax: 716-691-5226

Locon Sensor Systems, Inc.
1750 S. Eber Road
P.O. Box 789
Holland, OH 43526
Tel: 419-865-7651
Fax: 419-865-7756

Lucas
1000 Lucas Way
Hampton, VA 23666
Tel: 800-745-8008
Fax: 800-745-8004

Motion Sensors, Inc.
786 Pitts Chapel Road
Alizabeth City, NC 27909
Tel: 919-331-2080
Fax: 919-331-1666

Rechner Electronics Industries, Inc.
8651 Buffalo Avenue
Niagara Falls, NY 14304
Tel: 800-544-4106
Fax: 716-283-2127

RDP Electrosense, Inc.
2216-Dept. B
Pottstown, PA
Tel: 1-800-334-5838

Reed Switch Developments
Company, Inc.
P.O. Drawer 085297
Racine, WI 53408
Tel: 414-637-8848
Fax: 414-637-8861

Smith Research & Technology, Inc.
205 Sutton Lane, Dept. TR-95
Colorado Springs, CO 80907
Tel: 719-634-2259
Fax: 719-634-2601

Smith Systems, Inc.
6 Mill Creek Drive
Box 667
Brevard, NC 28712
Tel: 704-884-3490
Fax: 704-877-3100

Standex Electronics
4538 Camberwell Road
Dept. 301L
Cincinnati, OH 45209
Tel: 513-871-3777
Fax: 513-871-3779

Turck, Inc.
3000 Campus Drive
Minneapolis, MN 55441
Tel: 612-553-7300 & 800-544-7769
Fax: 612-553-0708

Xolox Sensor Products
6932 Gettysburg Pike
Ft. Wayne, IN 46804
Tel: 800-348-0744
Fax: 219-432-0828

ciples (e.g., capacitance, magnetism, optics, piezoelectricity, resistivity). Here, the most frequently used displacement sensors will be explained.

CAPACITIVE DISPLACEMENT SENSORS

Capacitive displacement sensors satisfy the requirements of applications where high linearity and wide ranges from a few centimeters to a couple of nanometers are needed. Precision signal processing allows sensing of capacitance changes down to a few femtofarads. Capacitive displacement sensors enjoy wide application in industry.

The basic sensing element of a typical displacement sensor consists of two simple electrodes with capacitance C. The capacitance is a function of the distance d (meters) between the electrodes of a structure, the surface area A (square meters) of the electrodes, and the permittivity ϵ (8.85×10^{-12} F/m for air) of the dielectric between the electrodes; therefore,

$$C = f(d, A, \epsilon) \tag{1}$$

There are three basic methods for realizing a capacitive displacement sensor (i.e., by varying d, A, or ϵ, as discussed next).

Variable Plate-Distance Sensors

A variable-distance capacitive displacement sensor, made from two flat coplanar plates with a variable distance x apart, is illustrated in Fig. 1. Ignoring fringe effects, the capacitance of this arrangement may be expressed by

$$C(x) = \epsilon A/x = \epsilon_r \epsilon_0 A/x \tag{2}$$

where ϵ is the dielectric constant or permittivity, ϵ_r is the relative dielectric constant (in air and vacuum $\epsilon_r \approx 1$), $\epsilon_0 = 8.854188 \times 10^{-12}$ F/m is the dielectric constant of vacuum, x is the distance of the plates in meters, and A is effective area of the plates in square meters.

The capacitance of this transducer is nonlinear with respect to distance x, which has hyperbolic transfer function characteristics. The sensitivity of capacitance to changes in plate separation may be found as

$$dC/dx = -\epsilon_r \epsilon_0 A/x^2 \tag{3}$$

Equation (3) indicates that the sensitivity increases as x decreases. Nevertheless, from these equations, it follows that

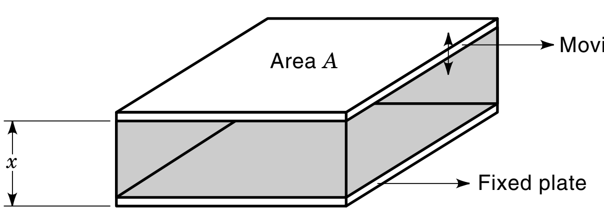

Figure 1. A variable plate-distance capacitive displacement sensor. In response to a physical stimuli, one of the plates of the capacitor moves; this varies the distance between plates. The outputs of these sensors are nonlinear with respect to distance x, having an hyperbolic transfer function characteristics. Appropriate signal processing must be employed for linearization.

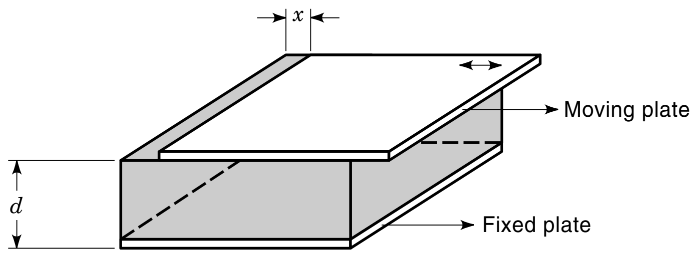

Figure 2. A variable-area capacitive displacement sensor. The sensor operates on the variation of the effective area between plates. The output is linear with displacement x. This type of sensor is normally implemented as a rotating capacitor for measuring angular displacement.

the percent change in C is proportional to the percent change in x, which can be expressed as

$$dC/C = -dx/x \tag{4}$$

This type of sensor is often used for measuring small incremental displacements without making contact with the object.

Variable-Dielectric Displacement Sensors

The displacement may also be sensed by the relative movement of the dielectric material, attached to the object under investigation, between the plates. The corresponding equations will be

$$C = \epsilon_0 w[\epsilon_2 l - (\epsilon_2 - \epsilon_1)x] \tag{5}$$

where ϵ_1 is the relative permittivity of the dielectric material and ϵ_2 is the permittivity of the displacing material (e.g., liquid).

In this case, the output of the transducer is linear. This type of transducer is mostly used in the form of two concentric cylinders for measuring the level of nonconducting fluids in tanks.

Variable Plate-Area Displacement Sensors

The displacements may be sensed by varying the surface area of the electrodes of a flat plate capacitor, as illustrated in Fig. 2. In this case, the capacitance can be written as

$$C = \epsilon_r \epsilon_0 \frac{A - wx}{d} \tag{6}$$

where w is the width and wx is the reduction in the area caused by the movement of the plate.

The transducer output is linear with displacement x. This type of sensor is normally implemented as a rotating capacitor for measuring angular displacement. The rotating capacitor structures are also used as an output transducer for measuring electric voltages.

Differential Capacitive Sensors

Some of the nonlinearity can be eliminated by using differential capacitive sensors. These sensors are basically three terminal capacitors, as shown in Fig. 3. Slight variations of these

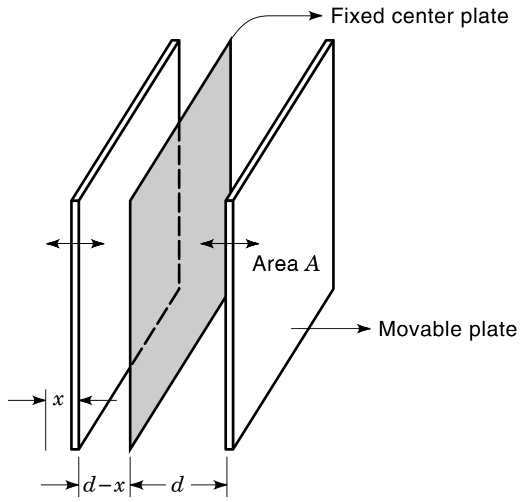

Figure 3. A differential capacitive sensor is three terminal capacitors with one fixed center plate and two outer plates. The response to motion is linear. In some versions, the center plate moves in response to a physical variable with respect to the two outer plates; in others, the center plate is fixed, and the outer plates are allowed to move.

sensors find many applications including differential pressure measurements. In some versions, the center plate responds to physical variables while the other two plates are fixed. In the others, the center plate is fixed, and the outer plates are allowed to move. The output from the center plate is zero at the center position, and it increases as it moves left or right. The range is equal to twice the separation d. For a small displacement d, the change in capacitance will be

$$2\delta C = C_1 - C_2 = \frac{\epsilon_r \epsilon_0 lw}{(d - \delta d)} - \frac{\epsilon_r \epsilon_0 lw}{(d + \delta d)} = \frac{2\epsilon_r \epsilon_0 lw \delta d}{d^2 + \delta d^2} \tag{7}$$

and

$$C_1 + C_2 = 2C = \frac{\epsilon_r \epsilon_0 lw}{d - \delta d} + \frac{\epsilon_r \epsilon_0 lw}{d + \delta d} = \frac{2\epsilon_r \epsilon_0 lwd}{d^2 + \delta d^2} \tag{8}$$

Giving approximately

$$\delta C/C = \delta d/d \tag{9}$$

This indicates that the response of the device is more linear than the response of the two parallel plate types. However, some nonlinearity is still observed in practice as a result of defects in the structure. Therefore, the outputs of these types of sensors may still need to be processed carefully by appropriate signal-processing techniques.

MAGNETIC DISPLACEMENT SENSORS

Magnetic displacement sensors are commonly used in industry, particularly in manufacturing. The majority of the magnetic displacement sensors are inductive types based on Faraday's Law of Induction. They can be classified as reluctance, transformative, tachogenerator, induction potentiometer, eddy current, and magnetometer types. They are designed to measure linear as well as rotary displacements.

Linear Variable-Reluctance Types

The variable-reluctance transducers are based on change in the reluctance of a magnetic flux path. These types of transducers find applications particularly in acceleration measurements. However, they can be configured for sensing displacements and velocities. They are constructed in many different forms as described later.

Single-Coil Linear Variable-Reluctance Sensors. A typical single-coil variable-reluctance displacement sensor is illustrated in Fig. 4. The sensor consists of three elements: a ferromagnetic core in the shape of a semicircular ring, a variable air gap, and a ferromagnetic plate. The total reluctance of the magnetic circuit is the sum of the individual reluctances:

$$\mathscr{R}_T = \mathscr{R}_C + \mathscr{R}_G + \mathscr{R}_A \tag{10}$$

where $\mathscr{R}_C$, $\mathscr{R}_G$, and $\mathscr{R}_A$ are the reluctances of the core, air gap, and armature, respectively.

Each one of these reluctances can be determined from its material properties and dimensions. In this particular case, $\mathscr{R}_T$ can be approximated as

$$\mathscr{R}_T = \frac{R}{\mu_C \mu_0 r^2} + \frac{2d}{\mu_0 \pi r^2} + \frac{R}{\mu_A \mu_0 rt} \tag{11}$$

In Eq. (11), the length of the flux path in the core is taken as πR, and the cross-sectional area is assumed to be uniform with a value of πr^2. The total length of the flux path in air is $2d$, and it is assumed that there is no fringing or bending of the flux through the air gap, so that the cross-sectional area of the flux path in air will be close to the cross section of the core. The length of an average central flux path in the armature is $2R$. The calculation of an appropriate cross section of the armature is difficult, but it may be approximated to $2rt$, where t is the thickness of the armature.

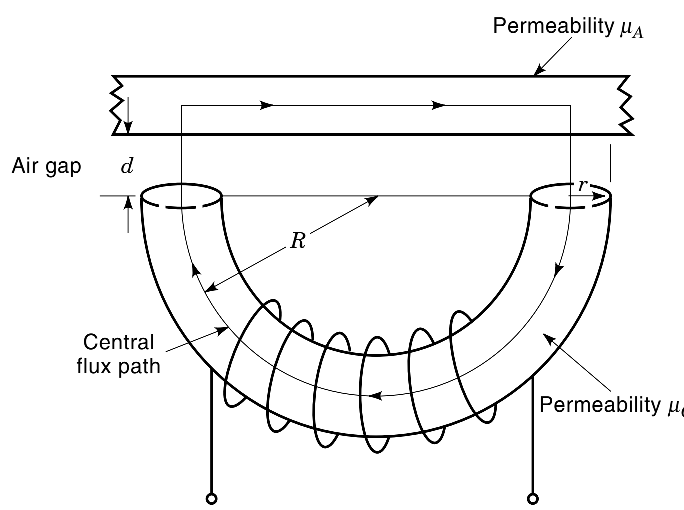

Figure 4. A typical single-coil variable-reluctance displacement sensor. The sensor consists of three elements: a ferromagnetic core in the shape of a semicircular ring, a variable air gap, and a ferromagnetic plate. The reluctance of the coil is dependent on the air gap. The air gap is the single variable, and the reluctance increases nonlinearly with the increasing gap.

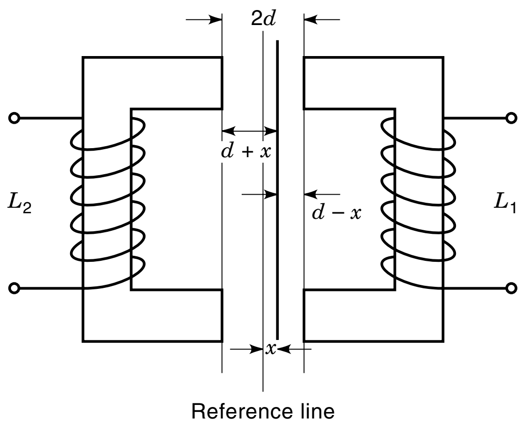

Figure 5. A variable-differential reluctance sensor consists of an armature moving between two identical cores separated by a fixed distance. The armature moves in the air gap in response to a mechanical input. This movement alters the reluctance of coils 1 and 2, thus altering their inductive properties. This arrangement overcomes the problem of nonlinearity inherent in single-coil sensors.

Equation (11) may be rewritten by fixing all the parameters except the air gap, which is the independent variable. Hence,

$$\mathscr{R}_{\mathrm{T}} = \mathscr{R}_0 + kd \tag{12}$$

where $\mathscr{R}_0 = R/(\mu_0 r)[1/(\mu_C r) + 1/(\mu_A t)]$ and $k = 2/(\mu_0 \pi r^2)$. Now, the inductance can be written as

$$L = \frac{n^2}{\mathscr{R}_0 + kd} = \frac{L_0}{1 + \alpha d} \tag{13}$$

where L_0 represents the inductance at zero air gap and

$$\alpha = k/\mathscr{R}_0$$

The values of L_0 and α can be determined mathematically; they depend on the core geometry, permeability, and the like, as explained previously. From Eq. (13), it can be seen that the relationship between L and α is nonlinear. Despite this nonlinearity, this type of single-coil sensor finds application in such areas as force measurements and telemetry. In force measurements, the resultant change in inductance can be made to be a measure of the magnitude of the applied force (say on a spring). A coil usually forms one of the components of an LC oscillator whose output frequency varies with the applied force. Hence, the coil modulates the frequency of the local oscillator.

Variable-Differential Reluctance Sensors. The problem of the nonlinearity may be overcome by modifying a single-coil system into a variable-differential reluctance sensor (also known as a push-pull sensor), as shown in Fig. 5. This sensor consists of an armature moving between two identical cores separated by a fixed distance $2d$. Equation (13) can now be modified to consider the effect of the two coils, as

$$L_1 = \frac{L_{01}}{1 + \alpha(d - x)} \qquad L_2 = \frac{L_{02}}{1 + \alpha(d + x)} \tag{14}$$

Although the relationship between L_1 and L_2 is still nonlinear, the sensor can be incorporated into an ac deflection bridge to give a linear output for small movements. The hysteresis errors of these transducers is almost entirely limited to the mechanical components. These sensors respond to static and dynamic displacements. They have continuous resolution and high outputs, but they may give an erratic performance in response to external magnetic fields. A typical sensor of this type has an input span of 1 cm, a coil inductance of 25 mH, and a coil resistance of 75 Ω. The resistance of the coil must be carefully considered when designing oscillator circuits. The maximum nonlinearity is 0.5%.

A typical commercially available variable differential sensor is shown in Fig. 6. The iron core is located half way between the two E-shaped frames. The flux generated by primary coils depends on the reluctance of the magnetic path, the main reluctance being the air gap. Any motion of the core increases the air gap on one side and decreases it on the other side. Consequently, reluctance changes in accordance with the principles already explained, thus inducing more voltage on one of the coils than the other. Motion in the other direction reverses the action with a 180° phase shift occurring at null. Depending on the requirements in signal processing, the output voltage can be modified by means of rectification, demodulation, and filtering. In these instruments, full-scale motion may be extremely small, on the order of few thousandths of a centimeter.

In general, variable-reluctance transducers have small ranges and are used in specialized applications such as pressure transducers. Magnetic forces imposed on the armature are quite large, and this can limit the application severely.

Linear Variable-Inductive Sensors

A typical linear-variable inductor consists of a movable iron core, which provides the mechanical input, and the two coils

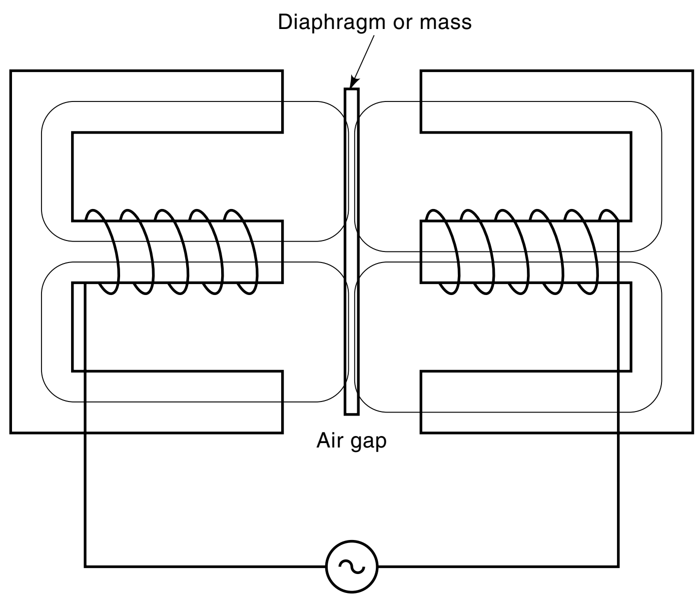

Figure 6. A typical commercial variable-differential sensor. The iron core is located half way between the two E frames. Motion of the core increases the air gap for one of the E frames while decreasing the other. This causes reluctances to change, thus inducing more voltage on one side than the other. Motion in the other direction reverses the action, with a 180° phase shift occurring at null.

forming two legs of a bridge network. A typical example of such a transducer is the variable coupling transducer.

Variable-Coupling Sensors. Variable-coupling sensors consist of a former holding a center-tapped coil and a ferromagnetic plunger. The plunger and the two coils have the same length l. As the plunger moves, the inductances of the coils change. The two inductances are usually placed to form two arms of a bridge circuit with two equal balancing resistors. The bridge is excited with 5 V to 25 V ac at a frequency of 50 Hz to 5 kHz. At the selected excitation frequency, the total transducer impedance at null is set to the 100 Ω to 1000 Ω range. The resistors are chosen to have about the same value as the transducer impedances. The load for the bridge output must be at least ten times the resistance R value. When the plunger is in the reference position, each coil will have equal inductances of value L. As the plunger moves by δl, changes in inductances $+\delta L$ and $-\delta L$ create a voltage output from the bridge. By constructing the bridge carefully, the output voltage may be made a linear function of the displacement of the moving plunger within a rated range.

It is easy to construct transducers of this types, by simply winding a center-tapped coil on a suitable former. Variable inductance-transducers are commercially available in strokes from about 2 mm to 500 cm. The sensitivity ranges between 1% full scale to 0.02% in long-stroke special constructions. These devices are also known as linear displacement transducers (LDTs) and are available in various shape and sizes.

In addition to linear variable inductors, rotary types are available. Their cores are specially shaped for rotational motion applications. Their nonlinearity can vary between 0.5% and 1% full scale over a range of 90° rotation. Their sensitivity can be up to 100 mV/1° of rotation.

Linear Variable-Differential Transformers

The linear variable-differential transformer (LVDT) is a passive inductive transducer finding extensive applications; therefore, they deserve lengthy discussion here. An LVDT consists of a single primary winding positioned between two identical secondary windings wound on a tubular ferromagnetic former, as shown in Fig. 7. The primary winding is energized by a frequency of 50 Hz to 20 kHz ac. The two secondaries are made identical by having an equal number of turns. They are connected in series opposition so that the induced output voltages oppose each other.

The ferromagnetic core or plunger moves freely inside the former, thus altering the mutual inductance between the primary and secondaries. With the core in the center, or at the reference position, the induced electromagnetic fields (emfs) in the secondaries are equal; because they oppose each other, the output voltage is zero. When the core moves, say to the left, from the center, more magnetic flux links with the left-hand coil than the right-hand coil. The voltage induced in the left-hand coil is, therefore, larger than the induced emf on the right-hand coil. The magnitude of the output voltage is then larger than at the null position and is equal to the difference between the two secondary voltages. The net output voltage is in phase with the voltage of the left-hand coil. The output of the device is then an indication of displacement of the core. Similarly, movement in the opposite direction to the right from the center reverses this effect, and the output voltage is now in phase with the emf of the right-hand coil.

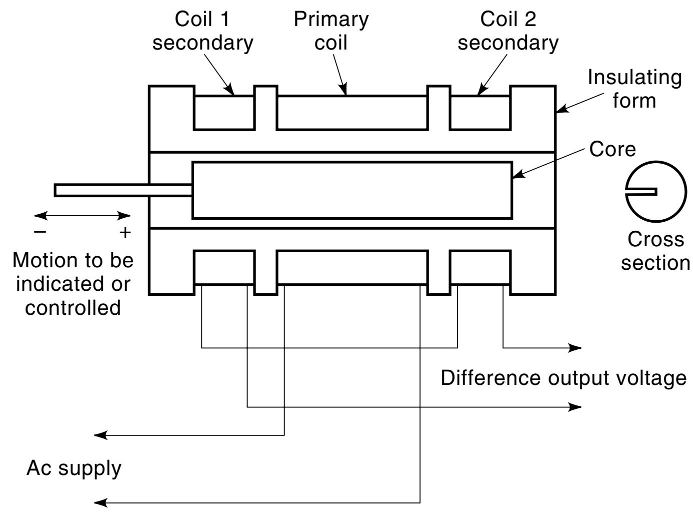

Figure 7. An LVDT is a passive inductive transducer consisting of a single primary winding positioned between two identical secondary windings wound on a tubular ferromagnetic former. As the core inside the former moves, the magnetic paths between primary and secondaries alters, thus producing secondary outputs proportional to the movement. The two secondaries are made to be as identical as possible by having equal sizes, shapes, and number of turns.

In many applications, the outputs are connected in opposing form, as shown in Fig. 8(a). The output voltages of the individual secondaries v_1 and v_2 at the null position are illustrated in Fig. 8(b). However, in opposing connection, any displacement in the core position x from the null point causes the amplitude of the voltage output v_o and the phase difference α to change. The output waveform v_o in relation to the core position is shown in Fig. 8(c). When the core is positioned in the middle, there is an equal coupling between the primary and the secondaries, thus giving a null point or reference point of the sensor. As long as the core remains near the center of the coil arrangement, the output is very linear. The linear ranges of commercial differential transformers are clearly specified, and the devices are seldom used outside this linear range.

For mathematical analysis of the operation of LVDTs, Fig. 8(a) may be used. The voltages induced in the secondary coils are dependent on the mutual inductance between the primary and individual secondary coils. Assuming that there is no cross coupling between the secondaries, the induced voltages may be written as

$$v_1 = M_1 s i_p \quad \text{and} \quad v_2 = M_2 s i_p \tag{15}$$

where M_1 and M_2 are the mutual inductances between the primary and secondary coils for a fixed core position, s is the Laplace operator, and i_p is the primary current.

In the case of opposing connection, no load output voltage v_o without any secondary current may be written as

$$v_o = v_1 - v_2 = (M_1 - M_2) s i_p \tag{16}$$

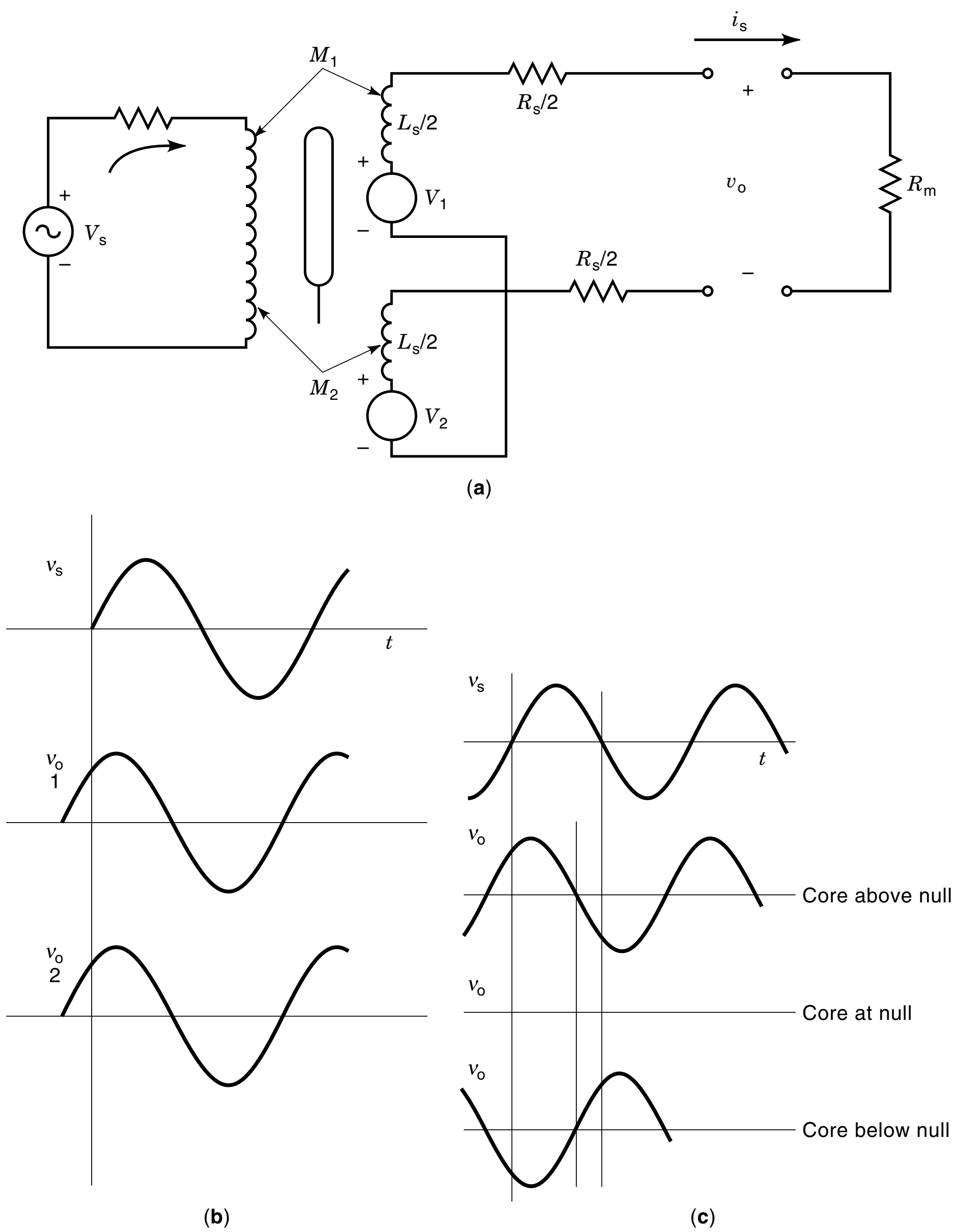

Figure 8. The voltages induced in the secondaries of a linear variable-differential transformer (a) may be processed in a number of ways. The output voltages of individual secondaries v_1 and v_2 at the null position are illustrated in (b). In this case, the voltages of individual coils are equal and in phase with each other. Sometimes the outputs are connected opposing each other, and the output waveform v_o becomes a function of core position x and phase angle α as in (c). Note the phase shift of 180° as the core position changes above and below the null position.

writing

$$v_s = i_p(R + sL_p) \tag{17}$$

Substituting i_p in Eq. (16) gives the transfer function of the transducer as

$$\frac{v_o}{v_s} = \frac{(M_1 - M_2)s}{R + sL_p} \tag{18}$$

However, if there is a current resulting from output signal processing, then the equations may be modified as

$$v_o = R_m i_s \tag{19}$$

where

$$i_s = \frac{(M_1 - M_2)si_p}{R_s + R_m + sL_s}$$

and

$$v_\mathrm{s} = i_\mathrm{p}(R + sL_\mathrm{p}) - (M_1 - M_2)si_\mathrm{s} \tag{20}$$

Eliminating i_p and i_s from Eqs. (19) and (20) results in a transfer function

$$\frac{v_\mathrm{o}}{v_\mathrm{s}} = \frac{R_\mathrm{m}(M_1 - M_2)s}{[(M_1 - M_2)^2 + L_\mathrm{s}L_\mathrm{p}]s^2 + [L_\mathrm{p}(R + R_\mathrm{m}) + RL_\mathrm{s}]s + (R_\mathrm{s} + R_\mathrm{m}) + R} \tag{21}$$

This is a second-order system, which indicates that with the effect of the numerator, the phase of the system changes from $+90°$ at low frequencies to $-90°$ at high frequencies. In practical applications, the supply frequency is selected such that at the null position of the core the phase angle of the system is 0°.

The amplitudes of the output voltages of the secondary coils are dependent on the position of the core. These outputs may directly be processed from each individual secondary coil for slow movements of the core, if the direction of the movement of the core does not bear any importance. However, for fast movements of the core, the signals may be converted to dc, and the direction of the movement from the null position may be detected. There are many options to do this; however, a *phase-sensitive demodulator* and filter are the most popular methods.

Phase-sensitive demodulators are extensively used in differential-type inductive sensors. They basically convert the ac outputs to dc values and also indicate the direction of movement of the core from the null position. A typical phase-sensitive demodulation circuit may be constructed, based on the diodes as shown in Fig. 9(a). This arrangement is useful for very slow displacements, usually less than 1 or 2 Hz. In Fig. 9(a), bridge 1 acts as a rectification circuit for secondary 1, and bridge 2 acts as a rectifier for secondary 2. The net output voltage is the difference between the outputs of two bridges as in Fig. 9(b). The position of the core is given by the amplitude of the dc output, and the direction of the movement of the core can be determined from the polarity of the dc voltage. For rapid movements of the core, the output of the diode bridges is filtered. This filtered output passes only the frequencies of the movement of the core but filters all the other frequencies produced by the modulation process. For this purpose, a suitably designed simple *RC* filter may be sufficient.

There are phase-sensitive demodulator chips available, such as AD598 offered by Analog Devices, Inc., in the marketplace. These chips are highly versatile and flexible in order to satisfy particular application requirements. These chips offer many advantages over conventional phase-sensitive demodulation devices. For example, frequency of excitation may be adjusted to any value between 20 Hz and 20 kHz by connecting an external capacitor between two pins. The amplitude of the excitation voltage can be set up to 24 V. The internal filters may be set to the required values by external capacitors. Connections to analog-to-digital converters are made easy by converting the bipolar output to a unipolar scale.

The frequency response of LVDTs is primarily limited by the mechanical inertia characteristics of the device. In general, the frequency of the applied voltage should be at least ten times the desired frequency to be measured. Commercial LVDTs are available in a broad range of sizes, and they are widely used for displacement measurements in a variety of applications. The displacement sensors are available to cover ranges from ± 0.25 mm to ± 7.5 cm. They are sensitive enough to respond to displacements well below 0.5 μm. They can have an operational temperature range from $-265°$ to 600°C. They are also available in radiation-resistant designs for operation in nuclear reactors. For a typical sensor in the range of ± 25 mm, the recommended supply voltage is 4 V to 6 V, with a nominal frequency of 5 kHz and a maximum nonlinearity of 1% full scale. Several commercial models that can produce a voltage output of 300 mV for a 1 mm displacement of the core are available.

One important advantage of the LVDT is that there is no physical contact between the core and the coil form; hence, there is no friction or wear. Nevertheless, there are radial and longitudinal magnetic forces on the core at all times. These magnetic forces may be regarded as magnetic springs that try to displace the core from its null position. This may be a critical factor in some applications.

One problem with LVDTs is that it may not be easy to make the two halves of the secondary identical; their inductance, resistance, and capacitance may be different, causing a large unwanted quadrature output in the balance position. Precision coil-winding equipment may be required to reduce this problem to an acceptable level.

Another problem is associated with null-position adjustments. The harmonics in the supply voltage and stray capacitances result in small null voltages. The null voltage may be reduced by center-tapped voltage source arrangements and proper grounding, which reduces the capacitive effects. In center-tapped supplies, a potentiometer may be used to obtain a minimum null reading.

The LVDTs find a variety of applications including controlling jet engines and measuring roll positions in the thickness of materials in hot-slab steel mills. Force and pressure measurements may also be made by LVDTs after some mechanical modifications.

Eddy Current Displacement Sensors

Inductive transducers based on eddy currents are mainly probe types, containing two coils. One of the coils, known as the active coil, is influenced by the presence of the conducting target. The second coil, known as the balance coil, completes the bridge circuit and provides temperature compensation. The magnetic flux from the active coil passes into the conductive target by means of a probe. When the probe is brought close the target, the flux from the probe links with the target, producing eddy currents within the target.

The eddy current density is greatest at the target surface and becomes negligibly small about three skin depths below the surface. The skin depth depends on the type of material used and the excitation frequency. Even though thinner targets can be used, a minimum of three skin depths may often be necessary to minimize temperature effects. As the target comes closer to the probe, the eddy currents become stronger, causing the impedance of the active coil to change, altering the balance of the bridge in relation to target position. This unbalance voltage of the bridge may be demodulated, filtered, and linearized to produce a dc output proportional to the target displacement. The bridge oscillation frequency may be as high as 1 MHz. High frequencies allow the use of thin targets and provide good system frequency response.

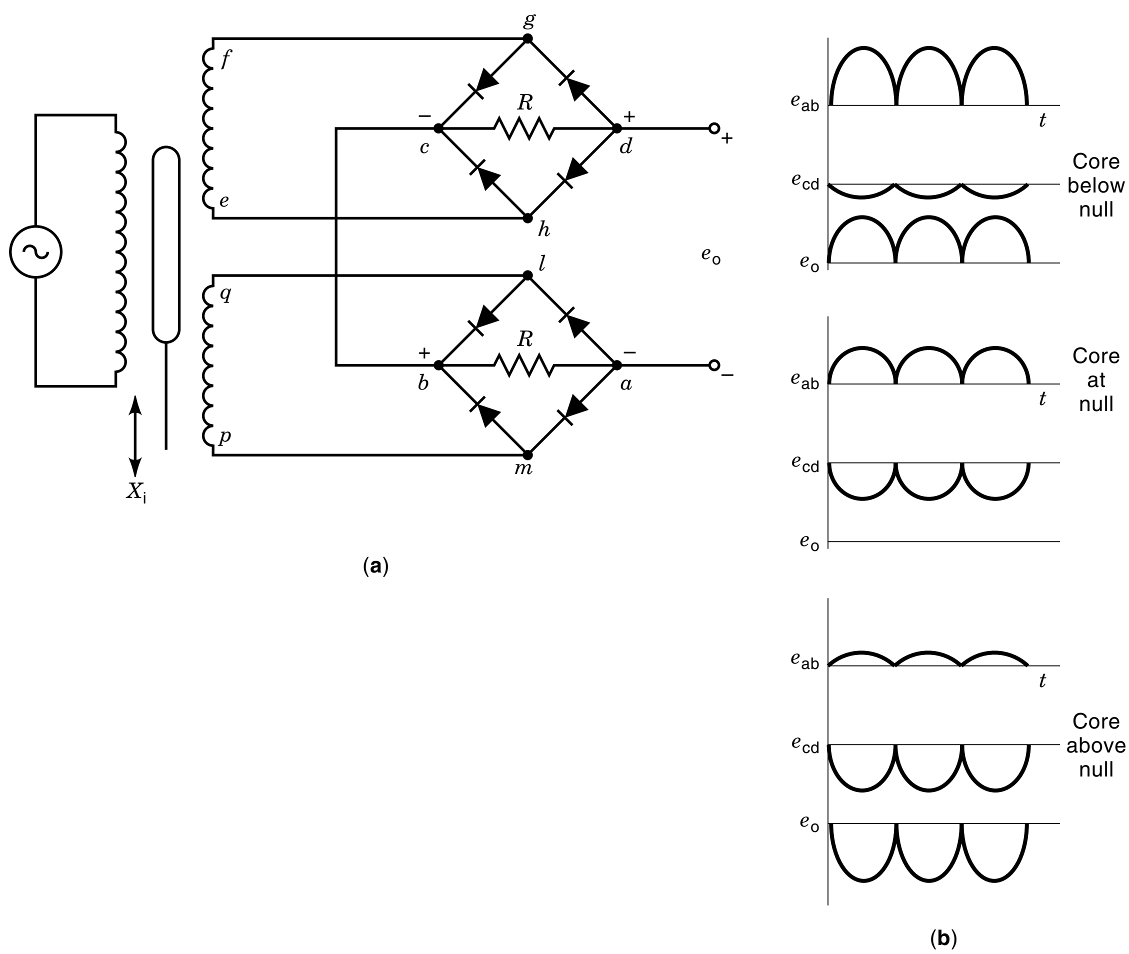

Figure 9. A typical phase-sensitive demodulation circuit based on diode bridges as in (a). Bridge 1 acts as a rectification circuit for secondary 1, bridge 2 acts as a rectifier for secondary 2, and the net output voltage is the difference between the two bridges as in (b). The position of the core can be found from the amplitude of the dc output, and the direction of the movement of the core can be determined from the polarity of the voltage. For rapid movements of the core, the output of the diode bridges need to be filtered. For filters, a suitably designed simple *RC* filter may be sufficient.

Probes are commercially available with full-scale ranges from 0.25 mm to 30 mm with a nonlinearity of 0.5% and a maximum resolution of 0.1 μm. Targets are usually supplied by the user, and often for noncontact measurements of machine parts. For nonconductive targets, conductive materials of sufficient thickness must be attached to the surface by means of commercially available adhesives. Because the target material, shape, and the like influence the output, it is necessary to calibrate the system statistically for a specific target. The recommended measuring range of a given probe begins at a standoff distance equal to about 20% of the stated range of the probe. In some cases, a standoff distance of 10% is recommended and so is calibrated as the standard for the system. A distance greater than 10% of measuring range can be used as long as the calibrated measuring range is reduced by the same amount.

Flat targets must be the same diameter as the probe or larger. If the target diameter is smaller than the probe diameter, the output drops considerably, thus becoming unreliable. Curved-surface targets may behave similarly to flat surfaces if the diameter exceeds about three or four probe diameters. In this case, the target essentially becomes an infinite plane. This also allows some cross-axis movement, which is the movement parallel to the axis of rotation, without affecting the system output. Target diameter comparable in size to the sensor can result in detrimental effects from cross-axis movements.

For curved or irregularly shaped targets, the system needs to be calibrated using the exact target that may be seen in the operation. This tends to eliminate any errors caused by the curved surfaces. However, special multiprobe systems are available for orbital motions of rotating shafts. If the curved (shaft) target is about ten times greater than the sensor diameter, it acts as an infinite plane; hence, it does not need special calibration. Special care must be exercised to deal with electrical runout caused by such factors as inhomegeneities in hardness, and are particularly valid for ferrous materials.

ANGULAR DISPLACEMENT SENSORS

Angular displacement sensors generally involve rotational capacitive or magnetic devices. The most commonly used de-

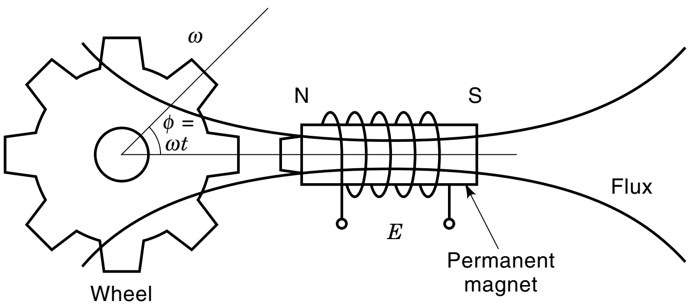

Figure 10. A variable-reluctance tachogenerator is a sensor based on Faraday's Law of Electromagnetic Induction. It consists of a ferromagnetic toothed wheel attached to the rotating shaft and a coil wound onto a permanent magnet extended by a soft iron pole piece. The wheel rotates in close proximity to the pole piece, thus causing the flux linked by the coil to change. The change in flux causes an output in the coil similar to a square waveform whose frequency depends on the speed of the rotation of the wheel and the number of teeth.

vices involve tachogenerators, microsyns, synchros, and induction potentiometers.

Tachogenerators

An example of an angular displacement sensor is the variable-reluctance tachogenerator shown in Fig. 10. These sensors are based on Faraday's Law of Electromagnetic Induction. Basically, the induced emf in the sensor depends on the linear or angular velocity of the motion.

The variable-reluctance tachogenerator consists of a ferromagnetic toothed wheel attached to the rotating shaft and a coil wound onto a permanent magnet, extended by a soft iron pole piece. The wheel moves in close proximity to the pole piece, causing the flux linked by the coil to change, thus inducing an emf in the coil. The reluctance of the circuit depends on the width of the air gap between the rotating wheel and the pole piece. When the tooth is close to the pole piece, the reluctance is minimal, and it increases as the tooth moves away from the pole. If the wheel rotates with a velocity ω, the flux may mathematically be expressed as

$$\Psi(\theta) = A + B\cos m\theta \tag{22}$$

where A is the mean flux, B is the amplitude of the flux variation, and m is the number of teeth.

The induced emf is given by

$$E = \frac{-d\Psi(\theta)}{dt} = -\frac{d\Psi(\theta)}{d\theta}\, x\, \frac{d\theta}{dt} \tag{23}$$

or

$$E = bm\omega \sin n\omega t \tag{24}$$

Both amplitude and frequency of the generated voltage at the coil are proportional to the angular velocity of the wheel. In principle, the angular velocity ω can be found from either the amplitude or the frequency of the signal. In practice, the amplitude measured may be influenced by loading effects and electrical interference. In signal processing, the frequency is the preferred option because it can be converted into digital signals easily.

Microsyn

Another commonly used example of a variable-reluctance transducer is the microsyn illustrated in Fig. 11. In this arrangement, the coils are connected in such a manner that, at the null position of the rotary element, the voltages induced in coils 1 and 3 are balanced by voltages induced in coils 2 and 4. The motion of the rotor in the clockwise direction increases the reluctance of coils 1 and 3 while decreasing the reluctance of coils 2 and 4, thus giving a net output voltage e_o. The movement in the counterclockwise direction causes a similar effect in coils 2 and 4 with a 180° phase shift. A direction-sensitive output can be obtained by using phase-sensitive demodulators, as explained in the LVDT section of this article.

Microsyn transducers are extensively used in applications involving gyroscopes. By using microsyns, very small motions can be detected, giving output sensitivity as low as 0.01° of changes in angles. The output of the device can be made as high as 5 V/1° of rotation. The nonlinearity may vary from 0.5% to 1.0% of full scale. The main advantage of these transducers is that the rotor does not have windings and sliprings. The magnetic reaction torque is also negligible.

Synchros

The term *synchro* is associated with a family of electromechanical devices, which could be discussed under different headings. They are primarily used in angle measurements and are commonly applied in control engineering as parts of servomechanisms, machine tools, antennas, and the like.

The construction of synchros is similar to that of wound-rotor induction motors. The rotation of the motor changes the mutual inductance between the rotor coil and the three stator coils. The three voltage signals from these coils define the angular position of the rotor. Synchros are used in connection

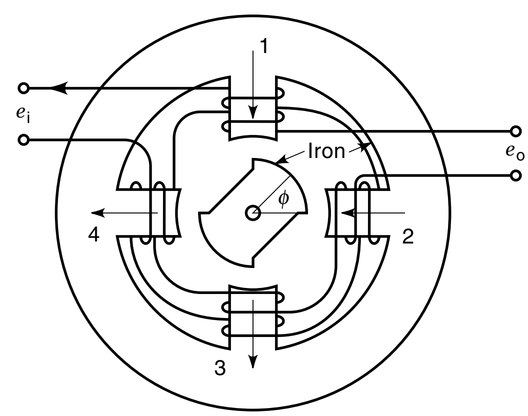

Figure 11. A microsyn is a variable-reluctance transducer, which consists of a ferromagnetic rotor and a stator carrying four coils. The stator coils are connected such that at the null position, the voltages induced in coils A and C are balanced by voltages induced in coils B and D. The motion of the rotor in one direction increases the reluctance of two opposite coils while decreasing the reluctance in others resulting in a net output voltage e_o. The movement in the opposite direction reverses this effect with a 180° phase shift.

with a variety of devices, including control transformers, Scott T transformers, resolvers, phase-sensitive demodulators, and analog-to-digital (AD) converters.

In some cases, a control transformer is attached to the outputs of the stator coils such that the output of the control transformer produces a resultant magnetomotive force (mmf) aligned in the same direction as that of the rotor of the synchro. In other words, the synchro rotor acts as a search coil in detecting the direction of the stator field of the control transformer. When the axis of this coil is aligned with the field, the maximum voltage is supplied to the transformer.

In other cases, ac signals from the synchros are first applied to a Scott T transformer, which produces ac voltages with amplitudes proportional to the sine and cosine of the synchro shaft angle. It is also possible to use phase-sensitive demodulation to convert the output signals to make them suitable for digital signal processing.

Induction Potentiometers

A version of the rotary-type linear inductor is the induction potentiometer. Two concentrated windings are wound on stator and rotor. The rotor winding is excited with alternating current, thus inducing voltage in the stator windings. The amplitude of the output voltage is dependent on the mutual inductance between the two coils, where the mutual inductance itself is dependent on the angle of rotation. For concentrated coil-type induction potentiometers, the variation of the amplitude is sinusoidal, so linearity is restricted to the region of the null position. A linear variation over an angle of $\pm 180°$ may be obtained by carefully designed distributed coils.

Standard commercial induction pots operate in the 50 Hz to 400 Hz frequency range. They are small in size, from 1 cm to 6 cm, and their sensitivity can be in the order of 1 V/1° of rotation. Although the range of induction pots is limited to less than 60° of rotation, it is possible to measure displacement in angles from 0° to full rotation by suitable arrangements of a number of induction pots. As in the case of most inductive sensors, the output of the induction pots may need phase-sensitive demodulators and suitable filters. In many inductive pots, additional dummy coils are used to improve linearity and accuracy.

Rotary Variable-Differential Transformers

A variation of the linear variable-differential transformer is the rotary core differential transformer shown in Fig. 12. Here the primary winding is wound on the center leg of an E core, and the secondary windings are wound on the outer legs of the E core. The armature is rotated by an externally applied force about a pivot point above the center leg of the core. When the armature is displaced from its reference or balance position, the reluctance of the magnetic circuit through one secondary coil is decreased; simultaneously the reluctance through the other coil is increased. The induced emfs in the secondary windings, which are equal in the reference position of the armature, are now different in magnitude and phase as a result of the displacement. The induced emfs in the secondary coils are made to oppose each other, and the transformer operates in the same manner as an LVDT. The rotating variable transformers may be sensitive to vibrations. If a dc output is required, a demodulator network may be used as in the case of LVDTs.

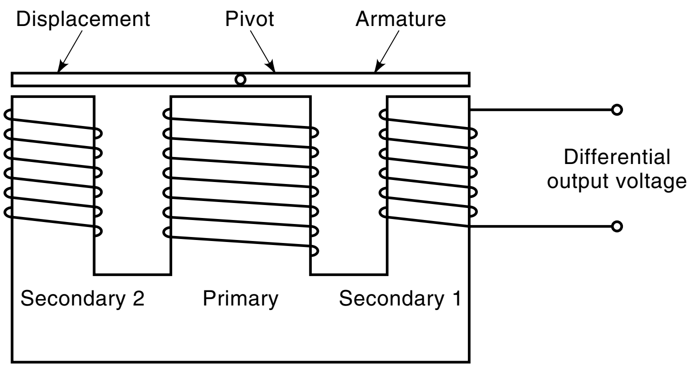

Figure 12. A rotary-core differential transformer has an E-shaped core, carrying the primary winding on the center leg and the two secondaries on the outer legs. The armature is rotated by an externally applied force about a pivot point above the center leg of the core. When the armature is displaced from its reference or balance position, the reluctance of the magnetic circuit through one secondary coil is decreased; simultaneously the reluctance through the other coil is increased. The induced emfs in the secondary windings are different in magnitude and phase as a result of the applied displacement.

In most rotary linear-variable differential transformers, the rotor mass is very small, usually less than 5 g. The nonlinearity in the output ranges between $\pm 1\%$ and $\pm 3\%$, depending on the angle of rotation. The motion in the radial direction produces a small output signals that can effect the overall sensitivity. But this transverse sensitivity is kept less than 1% of the longitudinal sensitivity.

LINEAR SENSORS

In this section, a number of displacement sensors, based on different principles such as resistance or piezoelectric principles, are discussed.

Resistive Displacement Sensors

In these displacement sensors, the sliding or rotational arm of a potentiometer is connected to the moving object while the main body of the potentiometer is kept in a fixed position, as illustrated in Fig. 13(a,b). Essentially, these are passive sensors requiring a stable external power source. By careful design of the potentiometers, a resolution of less than 20 μm may be obtained. They are also manufactured from conductive plastic materials, which covers a displacement span from few millimetres to 250 mm. Typical resistances are in the range of 500 Ω to 100 kΩ. In ideal operations, the relation between position and output voltage is linear. If the output devices draw excessive currents, the output and input relation becomes nonlinear as shown in Fig. 13(c).

A commonly used resistive displacement sensor is manufactured as a hybrid track potentiometer by depositing a conductive plastic coating on a precision wire-wound resistance track. This enables the best use of wire-wound and film technology. Although a linearity exist between the voltage and displacement, this linearity may be disturbed by the presence of load currents in the output. In many cases, the resistance of the load used with the potentiometer must be several times greater than the potentiometer resistance to avoid severe nonlinearity effects.

Strain Gage Sensors

Strain gage displacement sensors are based on resistance properties of electrical conductors. If a conductor is stretched or compressed, its resistance alters because of dimensional changes and changes in the fundamental property of material called *piezoresistance.* This indicates that the resistivity ρ of the conductor depends on the mechanical strain applied onto it. The dependence is expressed as the gage factor

$$\frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{dL/L} \tag{25}$$

where l indicates the resistance change due to length, 2ν indicates resistance change due to area, and $(d\rho/\rho)/(dL/L)$ indicates the resistance change due to piezoresistivity.

In displacement measurements, the resistance strain gages can be selected from different types, such as unbonded metal-wire gages, bonded metal-wire gages, bonded metal-foil gages, vacuum deposited thin-metal-film gages, bonded semiconductor gages, and diffused semiconductor gages. But, generally, bonded and unbonded metal-wire gages find wider applications. Occasionally bonded semiconductor gages, known as piezoresistive transducers, are used, but they suffer from high-temperature sensitivities, nonlinearity, and some mounting difficulties.

Unbonded strain-gage displacement sensors use the strain wires as spring elements and as the motion transducer, using arrangements similar to those in Fig. 14. They are useful for general-purpose motion and vibration measurements from low- to medium-frequency displacements. They are often manufactured in the form of piezoelectric ceramic cartridges giving inexpensive but highly reliable displacement measurements.

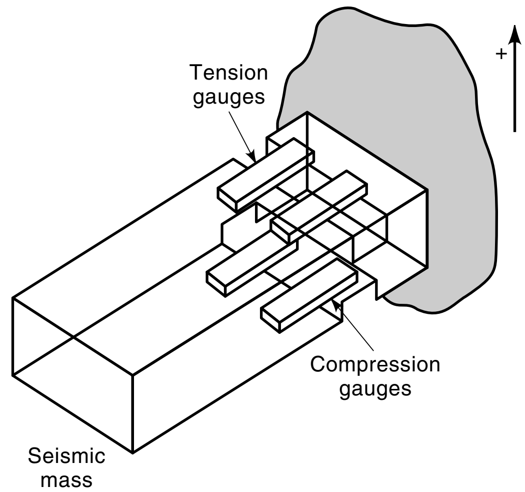

Figure 14. Bonding of piezoelectric and piezoresistive elements onto an inertial system. As the inertial member is displaced, deformation of the tension and compression gages causes the resistance to change. The change in resistance is picked up and processed further.

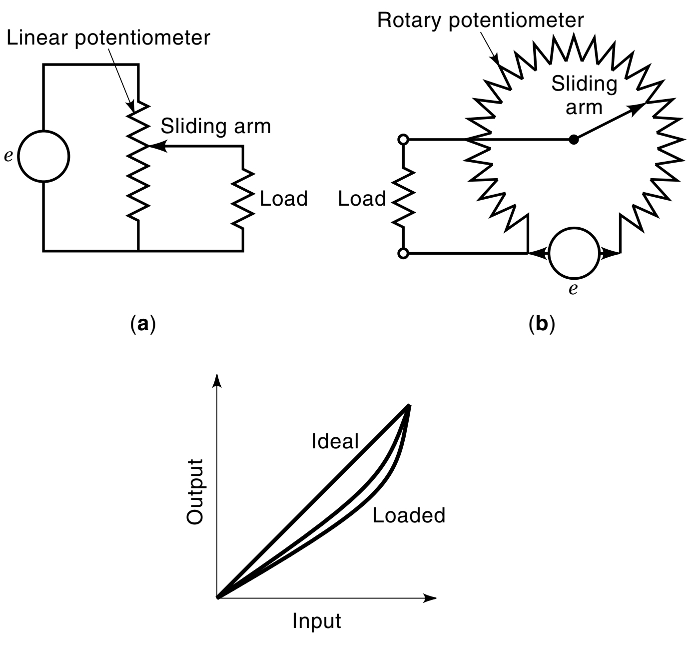

Figure 13. Resistive displacement sensors. The object under investigation is connected to the sliding arm of the potentiometer. They can measure linear motion or rotary motion as shown in (a) and (b), respectively. A loading effect occurs if the output devices have low impedance (c).

Piezoresistive Displacement Sensors

Piezoresistive displacement sensors are essentially semiconductor strain gages with large gage factors. High gage factors are obtained because the material resistivity depends primarily on the stress, not on dimensions only. Most piezoresistive sensors use two or four active gages arranged in a Wheatstone bridge. Extra precision resistors are used (as part of the circuit) in series with the input to control the sensitivity, balancing and offsetting temperature effects. The mechanical construction of a piezoresistive displacement sensors is similar to the installation in Fig. 14.

In some applications, overload stops are necessary to protect the gauges from high-amplitude inputs. These instrument is also useful for acquiring vibration information at low frequencies (e.g., below 1 Hz).

OPTICAL AND LASER DISPLACEMENT SENSORS

Optical methods are commonly used in displacement sensing particularly where high precision and small displacements are involved. There are many different techniques available, some of which are optical fibers, laser methods, encoders, and interferometric types. Here, some common methods will be described.

Optical Fibers

Optical fibers are suitable to measure linear and angular displacements in a noncontacting manner. Figure 15(a,b) illustrates two typical arrangements in which the intensity of the radiation incident on the detector is dependent on the relative displacement of the optical fibers. In Fig. 15(a) one of the fibers is fixed and the other is allowed to move with respect to displacement. The maximum intensity is detected when the fibers are at closest proximity and axially aligned. As the distance x between fibers increases, the light entering the second fiber decreases. Hence, the output of the detector can be related to displacement. In this arrangement, the displacement must be strictly in one direction, in a straight line of x or in y directions only. Some of the problems of using a two-fiber arrangement can be eliminated by using multiple fibers as exemplified in Fig. 16(a). In this case, the emitting fiber is fixed, and the distance between the emitting fiber and the receiving fibers is kept constant. As the fibers move in the y and z directions, the intensity of light received by each fiber varies. These variations are related to the displacement of the object. A slight modification of this arrangement can lead to reflection-type arrangements in which case the emitting and receiving fibers are bunched together, as shown in Fig. 16(b). As the reflecting surface is displaced, the detected intensity of light is varied.

Another common optical fiber application uses microbending principles, as illustrated in Fig. 17. As the displacement between the fixed and the displaced plates varies, the changes in the fiber dimensions between the two plates alters such fiber characteristics as attenuation and absorption. These changes are detected suitably as a measure of the displacement.

In all fiber optic displacement sensors, the variations in intensity and amplitudes are detected. For accurate operations, the source intensity must be stable.

Interferometric Sensors

There are many different types of interferometric sensors including optical fiber, laser, holographic, and monochromatic light types. In these sensors, Michelson, Sagnac, or Fizeau effects of interference of two or more light beams is used by suitable optical arrangements. The most commonly used optical arrangements are beam splitters, polarizing prisms, and diffraction gratings.

As two light beams interfere, interactions of light cause a series of bright and dark lines. If, for example, two interfering lights with wavelengths 1.5 μm are interfered. By suitable arrangements as in Fig. 18, the spacing of bright and dark lines will be 0.75 μm. In many cases, the dark and bright points are counted in relation to the displacement of an object located within the interference. The laser interferometric sensors operate on precise wavelength monochromatic light and are the most accurate in these family of sensors. An accuracy better than 1 nm can be obtained. Many different designs based on different displacement mechanisms are available. Some of these include fringe counting inteferometers, frequency modulation inteferometers, heterodyne interferome-

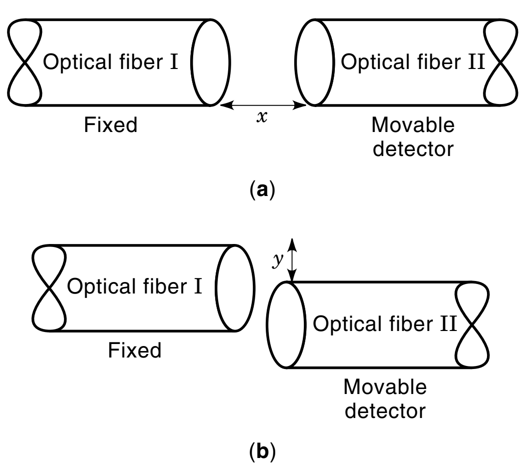

Figure 15. Examples of optical fiber displacement sensor. The intensity of the radiation incident on the detector is dependent on the relative displacement of the optical fibers. The maximum intensity of light is detected when the fibers are at closest proximity and axially aligned. Arrangement in (a) detects displacement in the x direction and that in (b) detects in the y direction.

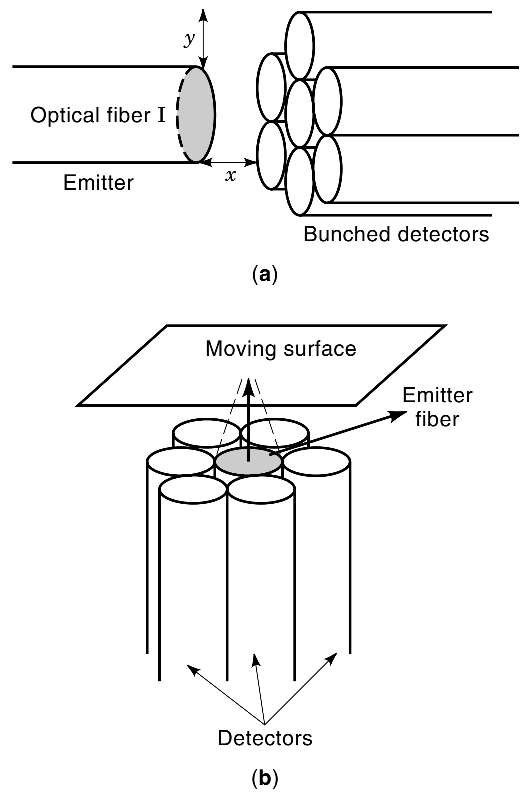

Figure 16. Multiple fiber displacement sensors. In some types, emitting fibers are fixed, and the distance between the emitting fiber and the receiving fibers is kept constant allowing vertical movements of the detectors. As the fibers move in the y and z directions, the intensity of light received by each fiber varies. In other types, the emitting and receiving fibers are bunched together, and the reflection from the moving surface are detected.

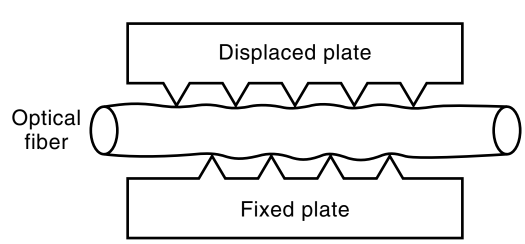

Figure 17. Microbend optical fiber displacement sensor. The displacement between the fixed plate and the displaced plate varies the fiber characteristics between the two plates, altering the attenuation and absorption. These changes are detected suitably as a measure of the displacement.

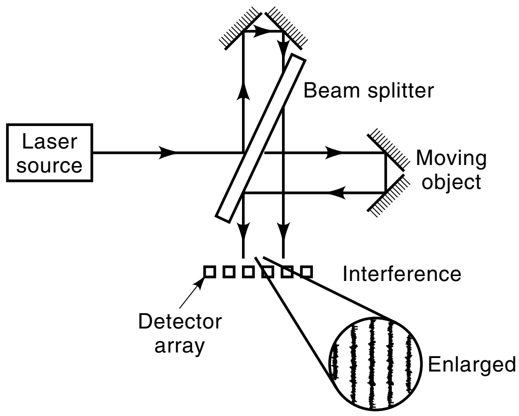

Figure 18. Interferometric displacement sensor. Interference of two light beams causes a series of bright and dark lines. In many cases, the dark and bright points are sensed by photodetectors in relation to the displacement of an object located within the interference area.

ters, phase-locked interferometers, laser Doppler and laser feedback interferometers, and fiber interferometers.

Encoders and Digital Displacement Sensors

Digital optical displacement sensors are used for both angular and linear measurements. An example of these sensors is shown in Fig. 19. As the wheel rotates, light from the source is transmitted and blocked, producing digital signal in the photodetectors. The output of the detector is counted and processed to give the angular displacement of the wheel. In another version, it is possible to use regularly positioned reflectors on the surface of the wheel rather than punching holes. In this case, light from the source is alternatively reflected or absorbed. Reflective-type encoders can be arranged to give natural binary-coded or Gray-coded digital signals. Some of these encoders are manufactured to give resolution better than 1 μm. In many cases, semiconductor laser light sources, which can give over 1 million samples per revolution, are used.

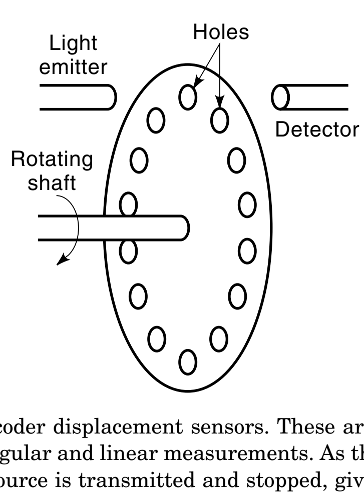

Figure 19. Encoder displacement sensors. These are digital sensors used for both angular and linear measurements. As the wheel rotates, light from the source is transmitted and stopped, giving a digital signal in the photodetectors. The output of the detector is counted and processed to measure the angular displacement of the wheel.

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DISPLAYS, CATHODE-RAY TUBE. See CATHODE-RAY TUBE DISPLAYS.

DISPLAYS, ELECTROLUMINESCENCE. See ELECTROLUMINESCENCE.

DISPLAYS, FLAT PANEL. See FLAT PANEL DISPLAYS.

DISPLAYS, THREE-DIMENSIONAL. See THREE-DIMENSIONAL DISPLAYS.

DISTORTION ANALYSIS. See VOLTERRA SERIES.

DISTORTION FACTOR. See POWER SYSTEM HARMONICS.

DISTORTION MEASUREMENT. See ELECTRIC DISTORTION MEASUREMENT.