DYNAMOMETERS

The dictionary defines a dynamometer as (1) a device for measuring mechanical force, as a balance, and (2) a device for measuring mechanical power, especially one that measures the output or drive torque of a rotating machine. The prefix “dynamo-” itself is a variation of “dyna-,” which is a learned borrowing from Greek meaning “power,” and is used in the formulation of compound words, such as dynamometer. According to the US Patent Office, however, dynamometers include a very wide range of measuring instruments. Some manufacturers refer to only “complex” devices, which incorporate load sensors or transducers in their structure as dynamometers.

The classical usage of the word dynamometer is primarily for power measurement. In such a dynamometer, power is determined as the product of the measured torque and angular velocity. Currently, and more generally, the term dynamometer and load transducer are used synonymously, and they cover load measurement instruments ranging from the simple to the more complex. For example, a simple device measuring grip strength of the human hand is known as a grip dynamometer. Also, terms such as load sensor, load cell, force transducer, and force platform (plate) are being used to cover the same range of instruments. For completeness, a brief description of specialized dynamometers as power measurement instruments is given first, followed by a more detailed discussion of dynamometers as load transducers in general.

POWER MEASUREMENT

Two types of mechanical power measuring devices exist: (1) those absorbing power and dissipating it as heat and (2) those transmitting the measured power. These are called, respectively, absorption dynamometers and transmission dynamometers. Common absorption dynamometers are the Prony brake, the electric generator, the water brake, and the magnetic or eddy–current brake. Figure 1 schematically depicts a Prony brake. A Prony brake is typically used for wheels smaller than 0.6 m (24 in.) in diameter, where the power to be absorbed is relatively small. As the brake is applied, the force, f, perpendicular to the cantilever beam is measured. Simultaneously, the angular velocity of the wheel is also monitored. The product of the angular velocity, the measured torque (i.e., force, f, times moment arm, L), and the appropriate unit conversion constants gives the instantaneous

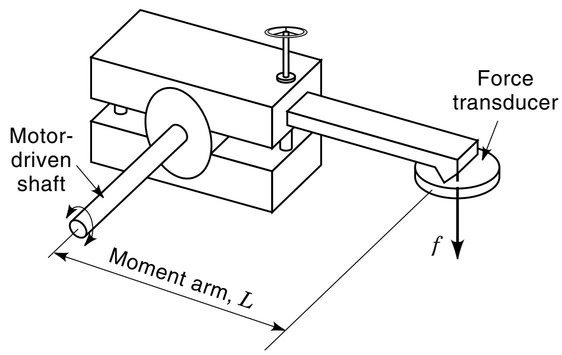

Figure 1. Schematic of a Prony brake.

power. The other absorption dynamometers work in a similar way, where a load is applied to impede the rotating shaft while both the load applied and the angular velocity of the shaft are measured.

Many types of transmission dynamometers have been used in the past, but currently only the torsion dynamometer and the cradle-mounted electric motor (i.e., chassis dynamometer) are in use. Torsion dynamometers directly measure the torque transmitted through a shaft using a torque transducer placed in series with that shaft. When the power output of an electric motor is in question, this can also be determined by measuring the reaction forces transmitted to the floor at the motor supports. Naturally, in both systems the angular velocity of the shaft also needs to be measured.

FORCE AND TORQUE MEASUREMENT

The force applied to a body is measured by measuring its physical effect on that body. This can take the form of a balance, where the unknown force is balanced against the known gravitational force of a standard mass, or against a magnetic force developed by interaction of a current-carrying coil and a magnet. Alternatively, the physical effect measured can be the elastic deformation of a body. Relating force to an acceleration via a known mass, or relating it to fluid pressure, is a variation of the latter method and will not be discussed separately. A simple example of an instrument where force is measured by relating it to a deformation is a spring scale, where the spring deforms proportional to the applied force. All of the above methods can be successfully used if the force to be measured is static or slowly varying. However, if the force is dynamic, the method of measuring deformations is used. These measured deformations are extremely small, and such a measuring device is generally referred to as a force transducer or a dynamometer.

A simple mathematical model is used to establish the input–output relationship of a force transducer. Figure 2 shows

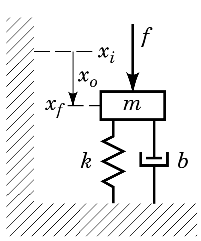

Figure 2. Idealized model of an elastic force transducer.

a physical model of a force transducer, where the system is idealized as a known mass, m, carried by a spring of known stiffness, k, and a damper with a known constant, b. This relationship, along with the resulting transfer function in the Laplace domain, is seen as a simple second-order relationship as follows:

$$\begin{aligned} m\ddot{x}_{\mathrm{o}} &= -kx_{\mathrm{o}} - b\dot{x}_{\mathrm{o}} + f \\ \frac{x_{\mathrm{o}}}{f}(s) &= \frac{K}{\dfrac{s^2}{\omega_{\mathrm{n}}^2} + \dfrac{2\zeta s}{\omega_{\mathrm{n}}} + 1} \end{aligned} \tag{1}$$

The dynamic response characteristic parameters of natural frequency, damping ratio, and sensitivity are found to be the following:

$$\begin{aligned} \omega_{\mathrm{n}} &= \sqrt{\frac{k}{m}} \quad \text{(natural frequency)} \\ \zeta &= \frac{b}{2\sqrt{km}} \quad \text{(damping ratio)} \\ K &= \frac{1}{k} \quad \text{(sensitivity)} \end{aligned} \tag{2}$$

As can be seen in Eq. (2), the stiffness, k, is used to determine each of the above parameters, because it is inversely proportional to the damping ratio and sensitivity and is directly proportional to the natural frequency. Therefore, the stiffness is very important in transducer behavior and design.

So far, only a force acting in a known direction has been discussed. More generally, all six components of a load acting on a body can be considered, where the force vector and the moment vector are each expressed with their three orthogonal components. A transducer which measures more than one component of the load is called a multicomponent load transducer or is referred to by the number of its components (i.e., a six-component load transducer).

Torque is a special case of a moment acting about the long axis of an elongated member. It causes shear deformation of this member, and thus it requires a somewhat different measurement method. Because torque measurement is usually carried out as an independent measurement on a shaft, torque transducers are commonly looked upon as a separate class of single-component load transducers.

Different sensing technologies are used to measure the elastic deformations of a transducer element. These include strain gauges, piezoelectric crystals, piezoresistive elements, and capacitive transducers. Of these, the former two are the most commonly used methods in load measurement technology, and they will be discussed here.

Strain-Gauged Transducers

Strain-gauged transducers are used to measure both static and dynamic loads. These devices use either metal-foil gauges or semiconductor gauges. Metal-foil strain gauges are produced by etching the gauge pattern into a thin sheet of metal foil deposited onto a plastic backing. Typical sizes vary from a few millimeters to several centimeters. Semiconductor strain gauges are generally smaller in size and have a higher sensitivity. However, they are fragile and relatively brittle, and they are more temperature-sensitive than metal-foil gauges. Semiconductor gauges have mainly been used in miniature

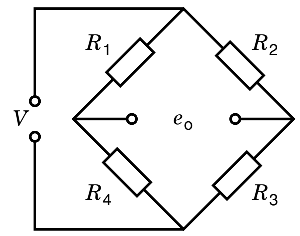

Figure 3. Wheatstone bridge.

transducers, the availability of which are increasing as technology advances.

Strain gauges are applied to the body of an elastic member of a transducer using adhesive so that when the elastic member deforms, the length of the strain gauge, and therefore its electrical resistance, changes proportionally. When a constant current is applied across the gauge, the change in length is seen as a change in voltage proportional to the strain induced. Strain gauges are made to be sensitive to changes in length along a principal direction, but insensitive in the transverse direction. Whether a transducer is designed to measure a single load component or multiple load components, strain gauges used for each load channel are wired in a Wheatstone bridge configuration. This not only allows measurement isolation of the desired load component, but also compensates for deformations resulting from temperature changes. A Wheatstone bridge is depicted in Fig. 3, where each resistance represents one of the strain gauges used in the transducer. Also shown are the input and output voltages. When the bridge is balanced (i.e., $R_1 = R_2$ and $R_3 = R_4$) the output voltage is zero. In a typical transducer the nominal gauge resistances are equal, so $R_1 = R_2 = R_3 = R_4 = R$, where standard values for R are 120 Ω, 350 Ω and 1000 Ω; the most commonly used value is 350 Ω. The bridge output voltage, as a function of individual gauge strains, is given as

$$e_o = \frac{V \cdot G_f}{4}[(\epsilon_1 - \epsilon_2) + (\epsilon_3 - \epsilon_4)] \tag{3}$$

where, the ϵ's are the individual strains, V is the excitation or supply voltage, and G_f is a material property of the gauge called the gauge factor. The gauge factor is the relationship of the strain to resistance change (i.e., the sensitivity of the gauge) and is expressed as

$$G_f = \frac{\frac{\Delta R}{R}}{\epsilon} \tag{4}$$

Typical foil gauges would have a gauge factor of approximately 2, while the gauge factor for semiconductor strain gauges can be as high as 100.

When selecting a material to be used for a transducer elastic element, we need to consider mechanical properties, thermal properties, and machinabilty of the material. Linear elastic load response with minimal hysteresis is one of the most desirable mechanical properties for a transducer material. As for thermal properties, the thermal conductivity contributes to the transducer accuracy and stability. The most commonly used materials include SAE 4340 steel, 631 (17-4 PH) stainless steel, and 2024-T4 and 7075-T6 aluminum. A comprehensive list can be found in Ref. 1.

Strain-gauged transducers involve a considerable design trade-off. From a natural frequency point of view, which influences the capability of measuring dynamic forces, the transducer element needs to have as high a degree of stiffness as possible. On the other hand, higher sensitivity requires greater deformations and thus a lower degree of stiffness. Bridging the gap in this design trade-off are high-end electronics. Today, electronics providing a high signal-to-noise ratio (SNR) allow for sensitive transducers which have relatively stiff elastic elements.

Considering single-component force transducers, a relatively low capacity transducer would typically utilize a bending beam member as the spring element. Figure 4(a) depicts such a member with gauges located at A and B; this member is loaded by the force to be measured, f. In Fig. 4(b) the Wheatstone bridge is shown, where the numbers correspond to the gauge designations in Fig. 4(a). Note that the bridge output is proportional to the difference between strains measured at levels A and B. Figure 4(c) shows the stress (or strain) distribution in the beam at the two levels, and Fig. 4(d) shows the bending moment diagram for the beam. Strain at A is proportional to the moment at the same location (i.e., $f \cdot a$), and similarly the strain at B is proportional to $f \cdot b$. The difference between the strains at the two levels, therefore, is proportional to $f \cdot (a - b)$. Thus, the output of the bridge is independent of the location of the force. This makes it possible to design a transducer as depicted in Fig. 5, where the force can act anywhere on the surface provided. To increase sensitivity without decreasing stiffness, very often the bending

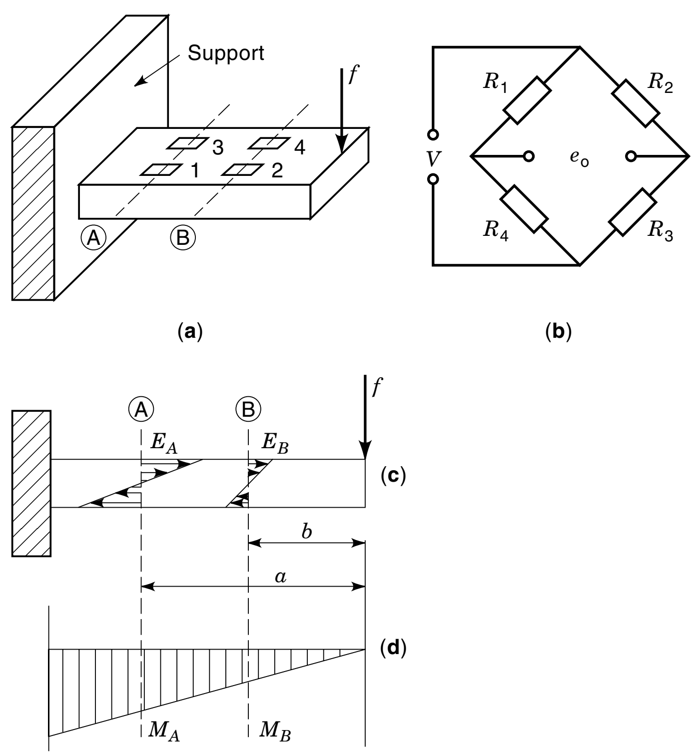

Figure 4. Single-component force transducer. (a) Strain gauge location and orientation, (b) Wheatstone bridge, (c) stress (strain) distribution in beam, and (d) bending moment diagram.

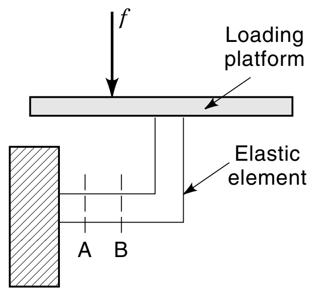

Figure 5. Adapted single-component force transducer.

beam is weakened at the locations where the strain gauges are attached. Two examples of such a design are shown in Fig. 6.

Using a shear beam, as shown in Fig. 7(a), would be an appropriate solution when the force to be measured is relatively high and the above design is not satisfactory. The gauges are positioned at 45° angles to the long axis of the beam (on both sides). Figure 7(b) shows the shear stress distribution on the gauged surface, and Fig. 7(c) is the same shear stress acting on a small area where the gauges are located. If the element of Fig. 7(c) is rotated by 45°, the compressive and tensile stresses of Fig. 7(d) are obtained. Therefore, gauge 1 would be subjected to compression, while gauge 2 would be subjected to tension, and the difference between the two gauge readings would still be proportional to the applied force. When we combine all four gauges (both sides), a full Wheatstone bridge with an output proportional to the force is obtained. It is again possible to modify the cross-sectional geometry by reducing the beam thickness at the gauge location, thus increasing the sensitivity without significantly affecting the stiffness of the beam. References 1 and 2 provide design variations based on the above principles.

The beam of either Fig. 4 or Fig. 7 can be instrumented on the currently blank adjacent sides to measure the force perpendicular to the one depicted. Furthermore, another set of gauges, as shown in Fig. 8 (gauges on the opposite side of

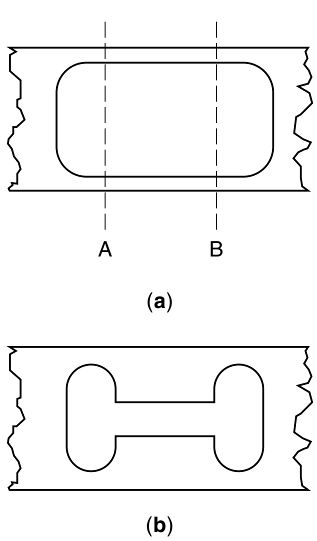

Figure 6. Example cross-sections to increase sensitivity without decreasing stiffness.

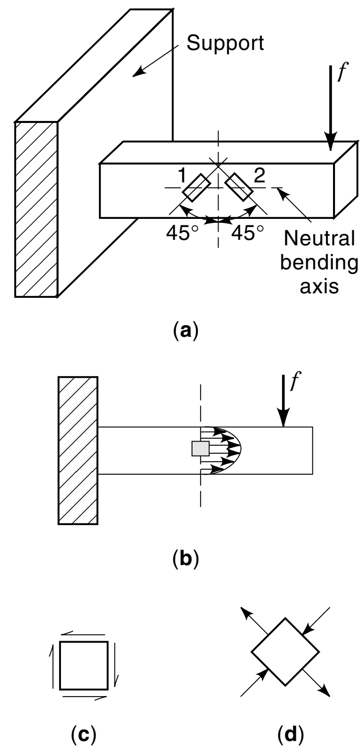

Figure 7. Shear beam single-component force transducer. (a) Strain gauge location and orientation, (b) shear stress (strain) distribution in beam, (c) unit area shear stress, and (d) rotation of unit area to obtain normal stresses.

the beam are not shown), can be used to measure the force along the long axis of the beam. It should be noted, however, that the configuration of Fig. 8 usually provides the highest stiffness and the least sensitivity. One way of increasing sensitivity in this direction would naturally be reducing the cross-sectional area. This could be achieved without significantly lowering the stiffness in the transverse direction by using a hollow geometry such as a tube. By selecting a suitable ratio between the wall thickness and the diameter of the tube, it is usually possible to find a satisfactory solution to measure all three force components with the desired resolution.

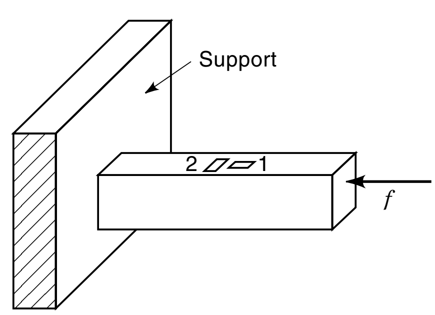

Figure 8. Gauge location and orientation for normal stress measurement.

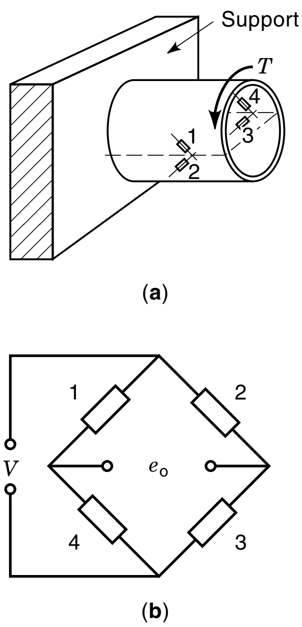

Figure 9. Torque transducer. (a) Gauge location and orientation. (b) Wheatstone bridge.

A tubular cross section is also very suitable for torque measurement. Referring to Fig. 9, note that the gauges are oriented to measure the shear due to the torque only, and any shear due to a transverse force (similar to the one in Fig. 7) is eliminated. Actually, a detailed analysis of the stresses caused by all possible loads applied to the end of the tube will show that the gauge configuration of Fig. 9 is only sensitive to the torque.

Piezoelectric Load Transducers

Certain materials respond to an applied electrical charge by mechanically deforming. Alternatively, an electrical charge is developed if the material is mechanically stressed. These actions are referred to as piezoelectric effects, and materials which exhibit this behavior are called piezoelectric materials. Quartz is a piezoelectric material with high stability and is normally used in building piezoelectric load transducers.

The piezoelectric properties of quartz crystal are directional, and axial and shear force components can be isolated by orienting the material in the transducer. Figure 10 shows a quartz crystal with disks cut out to be sensitive to compression (disk a) and shear (disk b). When a disk is placed between metal electrodes and subjected to a force, a charge, q, is produced. This charge is proportional to the deformation of

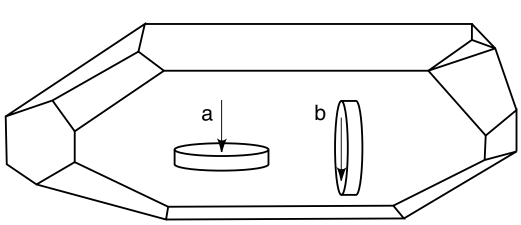

Figure 10. Quartz crystal. (a) Normal force orientation and (b) shear force orientation.

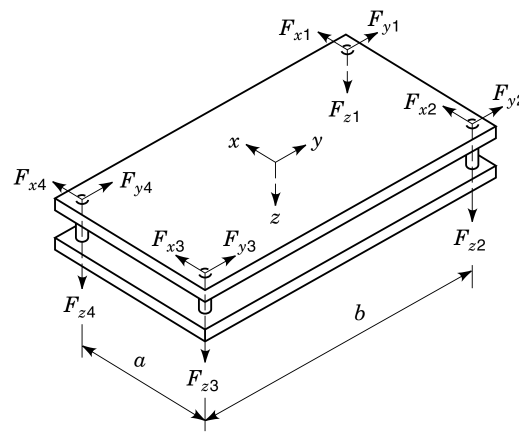

Figure 11. Six-component piezoelectric dynamometer made out of four three-component load transducers.

the crystal, and thus it is also proportional to the force applied. Simultaneously, a voltage is developed between the electrodes. Therefore, the voltage output can be amplified and calibrated to measure the force applied.

Normally, a pair of disks is used to measure each load component. Using the disks in pairs doubles the sensitivity and permits simple electrical contact by a central electrode. By stacking three pairs of disks, one pair for the axial load and one pair for each of the two orthogonal shears, a three-component force transducer can be obtained. This type of transducer must be assembled with a compressive preload, which prevents the disks from separating from the metal plate electrodes during tensile loading.

Six-component transducers can be built by placing four such three-component load transducers at four corners of a platform. Figure 11 (commonly called a piezoelectric dynamometer) shows such a configuration, where each load cell is numbered from 1 to 4, and the load direction is indicated by the direction of each coordinate axis. Thus, the two subscripts of each load component uniquely identify the force. The 12 outputs of the transducer are combined inside the transducer such that eight signals are actually output. These eight channels represent the four individual vertical force components (F_{z1}, F_{z2}, F_{z3} and F_{z4}): two shears in the x-direction ($F_{x1} + F_{x2}$ and $F_{x3} + F_{x4}$), and two shears in the y-direction ($F_{y1} + F_{y4}$ and $F_{y2} + F_{y3}$). In order to get the three force and the three moment components, the output needs to be further reduced as follows:

$$
\begin{aligned}
F_x &= (F_{x1}+F_{x2}) + (F_{x3}+F_{x4}) \\
F_y &= (F_{y1}+F_{y4}) + (F_{y2}+F_{y3}) \\
F_z &= (F_{z1}+F_{z2}) + (F_{z3}+F_{z4}) \\
M_x &= (F_{z1}+F_{z2}) - (F_{z3}-F_{z4})(b/2) \\
M_y &= (-F_{z1}+F_{z2}) + (F_{z3}-F_{z4})(a/2) \\
M_z &= [(F_{x1}+F_{x2}) - (F_{x3}+F_{x4})](b/2) \\
&\quad + [(F_{y2}+F_{y3}) - (F_{y1}+F_{y4})](a/2)
\end{aligned}
\tag{5}
$$

Piezoelectric transducers have the advantage of having high sensitivity and high rigidity. Typical stiffness properties of these crystals result in natural frequencies as high as 30,000 Hz. However, the governing natural frequency is normally that of the members supporting the transducer, which is usually much lower than that of the quartz alone. A disadvantage of piezoelectric transducers is that they are not inherently suited for static measurements. This is due to the charge developed in the quartz under load gradually leaking, causing signal decay eventually to zero. Then, if the applied load were released, the transducer will erroneously indicate a load in the opposite direction equal to the amount of the decay.

As stated previously, the applied force, f, to a piezoelectric transducer causes a deflection, x. A potential voltage, E, is produced from the resulting charge buildup. This can be expressed as

$$E = \frac{q}{C_{\mathrm{cr}}} \tag{6}$$

where C_{cr} is the crystal capacitance of the piezoelectric material and is defined as

$$C_{\mathrm{cr}} = \frac{\phi A}{t} \tag{7}$$

In Eq. (7), ϕ is the dielectric constant of the crystal, A is the area of the plates, and t is the distance between the plates. Figure 12 shows such a piezoelectric transducer. A piezoelectric constant, k, relates the applied force, f, to the resulting charge, q as shown:

$$q = kf \tag{8}$$

Using Eq. (8) in Eqs. (6) and (7) results in the following relation for the potential voltage, E:

$$E = \frac{kft}{\phi A} \tag{9}$$

In order to quantify the applied force the induced, or potential, voltage, E, must be measured. A charge amplifier is typically implemented due to the induced voltage's small magnitude. This leads to another complication in that the charge may leak back through the crystal. Figure 13 depicts the circuits required to determine the behavior of a piezoelectric transducer. Figure 13(a) shows the crystal itself, while Fig. 13(b) is a schematic of the cabling and amplifier. In Fig. 13 the charge generator is represented as a current generator, and the current source i_{s} is the time derivative of the charge generated, C_{cr} is again the crystal capacitance, C_{cb} is the capacitance of the connecting cables, C_{a} is the capacitance of the charge amplifier, R_{a} is the internal amplifier resistance, R_{l} is the sensor leakage resistance, and E_{o} the resulting output voltage to be measured.

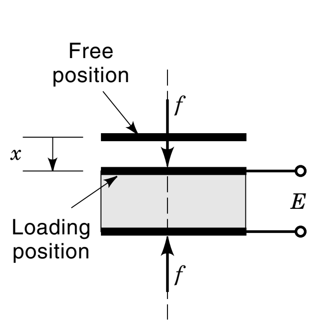

Figure 12. Piezoelectric transducer.

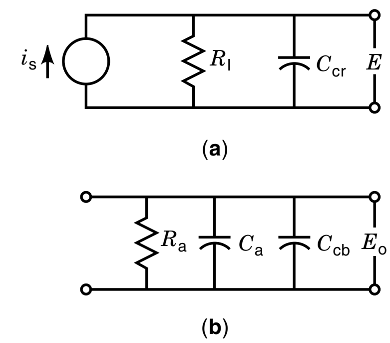

Figure 13. Equivalent circuits for (a) piezoelectric transducer and (b) amplifier and cables.

Figure 14 is the equivalent circuit of the transducer, cabling and amplifier. From this circuit:

$$i_{\mathrm{s}} = \frac{dq}{dt} = k\frac{df}{dt} \tag{10}$$

$$R = \frac{R_{\mathrm{a}}R_{\mathrm{l}}}{R_{\mathrm{a}} + R_{\mathrm{l}}} \tag{11}$$

In a piezoelectric force transducer, R_{l} is usually on the order of $10^{11}\,\Omega$; however, there are commercially available systems with higher R_{a} and R_{l} values resulting in up to $10^{14}\,\Omega$ total resistance, R. Systems with this high a total resistance have a very slow leakage rate and therefore enable quasi-static force measurement.

Analyzing the equivalent circuit of Fig. 14 results in the following current loop equation:

$$i_{\mathrm{s}} = i_{\mathrm{c}} + i_{\mathrm{r}} \tag{12}$$

The output voltage is then expressed as

$$\begin{aligned} E_{\mathrm{o}} &= E_{\mathrm{c}} = \int \frac{i_{\mathrm{c}}}{C}\,dt = \frac{\int (i_{\mathrm{s}} - i_{\mathrm{r}})\,dt}{C} \\ CE_{\mathrm{o}} &= \int (i_{\mathrm{s}} - i_{\mathrm{r}})\,dt \end{aligned} \tag{13}$$

After differentiation and substitution we obtain

$$C\frac{dE_{\mathrm{o}}}{dt} = i_{\mathrm{s}} - i_{\mathrm{r}} = k\frac{df}{dt} - \frac{E_{\mathrm{o}}}{R} \tag{14}$$

This differential equation can be put in standard form and then expressed in the Laplace domain as

$$(RCs + 1)E_{\mathrm{o}} = (Rks)f \tag{15}$$

or as a first-order system input–output relationship, or trans-

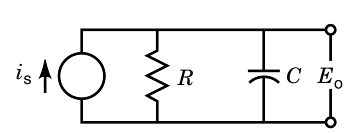

Figure 14. Equivalent circuit for piezoelectric transducer, amplifier, and cables.

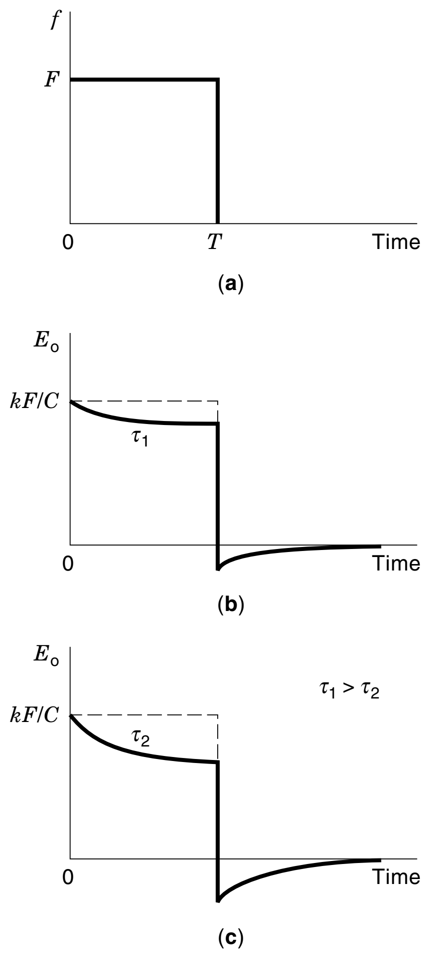

Figure 15. Response of a piezoelectric transducer to constant force input. (a) Input, (b) response for relatively high time constant, and (c) response for a relatively low time constant.

fer function:

$$\frac{E_o}{f}(s) = \frac{K\tau_s}{\tau_s + 1} \tag{16}$$

where

$\tau = RC$ (time constant)
$K = k/C$ (static sensitivity)

Figure 15(a) shows a constant force input, f, for a time of T. With these initial conditions for the time periods from 0 to T and from T to infinity, Eq. (16) can be solved:

$$E_o = \frac{kf}{C}e^{-t/\tau} \quad \text{and} \quad E_o = \frac{kf}{C}(e^{-T/\tau} - 1)e^{-(t-T)\tau} \tag{17}$$

The response of Eq. (17) is shown in Fig. 15(b) and 15(c). These figures show undershoot of the response and also show that as the time constant decreases, the signal response decay increases. This is the reason for the need of a high time constant in piezoelectric force transducers.

The amplifier properties, along with the dimensions and physical properties of the piezoelectric material, determine the actual time constant of the transducer system. However, there are a limited number of ways to increase the time constant. By the relation above, an increase in either R or C will increase the time constant. A shunt capacitor or a series resistor at the amplifier can achieve this; however, the sensitivity of the transducer would decrease if a shunt capacitor were used. Therefore, much like strain-gauged transducers, there is a design trade-off with piezoelectric transducers as well. This trade-off is between the high time constant required for static measurement and the desired quasi-static sensitivity.

Cross-talk

Generally, in the measuring range of a transducer, relationships between the loads and signals are linear, and calibration parameters are given by constant values. Ideally, each output channel of a transducer would only be sensitive to the load component that it is designed to measure. However, manufacturing tolerances as well as some design considerations cause a transverse sensitivity or a cross-talk between the channels. For example, in a six-component load transducer, the force in the x-direction might be sensitive to some or all of the other five outputs of the transducer. Thus, F_x would be expressed as a linear combination of these signals:

$$F_x = c_{11}\cdot s_1 + c_{12}\cdot s_2 + c_{13}\cdot s_3 + c_{14}\cdot s_4 + c_{15}\cdot s_5 + c_{16}\cdot s_6 \tag{18a}$$

or

$$F_x = c_{lj}\cdot s_j \tag{18b}$$

where c_{1j} (j = 1 to 6) are the calibration constants, and s_j (j = 1 to 6) are the six output signals. The relationship between the six signals and the six load components can be expressed as

$$\{F_x\,F_y\,F_z\,M_x\,M_y\,M_z\}^T = [c_{ij}]\{s_1\,s_2\,s_3\,s_4\,s_5\,s_6\}^T \tag{19}$$

where each load and signal vector is a column vector, and the calibration coefficients, c_{ij}, are given in a 6 × 6 matrix. Typically, the nondiagonal terms of the calibration matrix represent the cross-talk between the channels.

In most commercially available multicomponent transducers the cross-talk terms are less than 1% or 2% of the main calibration constants. This was common practice when data were primarily recorded by chart recorders and correcting for cross-talk usually involved a tedious process. Current means of data acquisition and processing has removed this requirement, thus providing designers with more possibilities of creative load transducer configurations.

Users may incorrectly assume that the load components which are not measured by a transducer do not affect the measured components. However, this is usually not guaranteed. For example, a single axis tension or compression transducer might give erroneous results in the presence of shear forces.

Calibration

Transducer calibration normally involves applying a known force, and relating the output of the transducer to the magnitude of this force. Applying methods of good experimental procedure, the applied load is incrementally increased up to the working limit of the transducer, then again incrementally decreased to zero. The linearity as well as any hysteresis present in the transducer should then be verified. If the transducer is to measure a dynamic load, the frequency response, or the first natural frequency of the transducer also needs to be determined. It is desirable to have the natural frequency of the transducer at least four or five times higher than the maximum frequency content of the applied load.

In calibrating a multicomponent transducer, at least as many independent load sets as the number of channels involved must be applied. If higher number of independent load sets are applied, then the calibration matrix can be obtained using a least-squares fit. If $\{F_k\}$ is the kth independent load set and $\{s_k\}$ is the corresponding output signal set, then

$$[F_1F_2\ldots F_k] = [c_{ij}][s_1s_2\ldots s_k] \tag{20}$$

where c_{ij} are the elements of the unknown calibration matrix. For example, for a six-component transducer where the number of output channels is also six, the calibration matrix would be a 6×6 matrix. In the case of a transducer having eight outputs representing the six loads (i.e., the three forces and three moments), like most piezoelectric six-component transducers, a 6×8 calibration matrix would be used.

Representing $[F_1F_2\ .\ .\ .\ F_k]$ by $[F]$, and similarly representing the signals by $[s]$, Eq. (20) can be written as

$$[F] = [c_{ij}][s] \tag{21}$$

Post-multiplying both sides by the transpose of $[s]$, $[s]^T$, and then post-multiplying by the inverse of $\{[s][s]^T\}$, the calibration matrix is obtained:

$$[c_{ij}] = [F][s]^T\{[s][s]^T\}^{-1} \tag{22}$$

In order to determine whether there is any advantage to using more calibration points than the minimum required, the authors used up to 50 independent load sets in calibrating six-component load transducers used in gait analysis (i.e., force plates). For these particular transducers, any number of load sets larger than 15 have not been found to improve the accuracy of the calibration matrix. Although this number of load sets does not necessarily apply to all six-component transducers, it provides a guideline for establishing a successful calibration procedure.

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