NOISE, LOW-FREQUENCY

Low-frequency noise, containing fluctuations of current or voltage with frequency components below 10 kHz, is mainly fundamental $1/f$ noise and sometimes nonfundamental $1/f$ noise. In addition, shot noise, generation–recombination (GR) noise, and thermal noise, which are important at higher frequencies, also extend to the low-frequency domain. All these forms of electronic noise, each defined below, are characterized by the mean squared current fluctuation $\langle(\delta I)^2\rangle$ [or $\langle(\delta V)^2\rangle \equiv$ (rms $\delta V)^2$ for voltage], measured in series with [or across] the device or sample under test, when a constant voltage [or current] is applied, except for thermal noise, which is present even in thermal equilibrium, with no bias applied. All the other forms of noise present in addition to thermal noise are also known as current noise and are absent in thermal equilibrium. Nevertheless, $1/f$ noise and GR noise also modulate the rms level of the thermal noise currents (or voltages) in thermal equilibrium, while the available power remains constant at k_BT per hertz, where $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant and T the absolute temperature. These two forms of current noise are also called modulation noise, because they modulate the resistance. If a bandpass filter is inserted between the measuring device (usually a quadratic meter) and the noise source, then the spectral density of the fluctuations, $\langle(\delta I)^2\rangle_f \equiv S_I(f)$ [or $S_V(f)$] is obtained by dividing the measured mean square by the bandwidth Δf of the filter.

SHOT NOISE

With the notable exception of $1/f$ noise, also known as excess noise, the various types of noise mentioned above were well known and understood in the third decade of the twentieth century through the works of J. B. Johnson, H. Nyquist, and W. Schottky. For instance, shot noise is caused, in vacuum tubes, electron beams, Schottky diodes, p–n junctions, and any other device carrying a current, by the discrete, atomistic nature of electricity. It is easily described as a Poisson process, and is given at low frequencies by

$$S_I(f) = 2eI_0 \tag{1}$$

where e is the electric charge of the charge carriers and I_0 the average electrical current in the direction of their motion. For electrons both e and I_0 are negative. The mean squared current in a frequency interval Δf is thus $2eI_0\ \Delta f$. The general formula is given by Carson's theorem, which gives the spectral density of a random uncorrelated repetition of identical processes with spectrum $\phi(f)$ and repetition rate λ as

$$2\lambda|\phi(f)|^2 \tag{2}$$

The case with arbitrary correlations present between the moments t_0 of passage was treated by C. Heiden (1) and is usually not called shot noise. The elementary process in shot noise is the current $i(t - t_0)$ caused by the passage of a single carrier. Therefore

$$\phi(0) = \int_{-\infty}^{\infty} i(t - t_0)\,dt = e \tag{3}$$

is the total charge e transported by a single carrier. With $e\lambda = I_0$, Carson's theorem then gives Eq. (1). The name "shot noise" recalls the noise caused by small shot (or raindrops) falling on a drum.

$1/f$ noise, however, remained shrouded in mystery, and fundamental $1/f$ noise was understood only after the advent of the quantum $1/f$ theory (2) in 1975. It turns out there is always fundamental $1/f$ noise (3,4) caused by the quantum $1/f$ effect (Q1/fE), a new aspect of quantum mechanics as fundamental as space and time or existence itself. But there is also nonfundamental $1/f$ noise, characterized by accidental $1/f$-like spectra arising from a fortuitous superposition of GR noise spectra. Both fundamental and nonfundamental $1/f$ noise types are important in practice, as we show below.

We briefly consider first GR and thermal noise here before tackling $1/f$ noise at an elementary level. Armed with an understanding of the basic low-frequency noise processes, we then proceed to practical device applications. Next, we delve into the quantum $1/f$ theory and finally consider briefly the epistemological and ontological origin of the $1/f$ spectra in general, trying to understand their much wider presence in nature, their ubiquitous character.

GENERATION–RECOMBINATION NOISE

GR noise is caused by the random generation and recombination or trapping and detrapping of current carriers in semiconductors, being described by the (always one-sided) spectral density

$$S_I(f) = \frac{8\langle(\delta I)^2\rangle\pi\tau}{1+\omega^2\tau^2} \tag{4}$$

Here τ is the lifetime of the carriers and $\omega = 2\pi f$. According to the Wiener–Kinchine theorem, the spectral density is the Fourier transform of the autocorrelation function

$$A(\tau) \equiv \langle I(t)I(t+\tau)\rangle \tag{5}$$

and is given by

$$S(f) = 4\int_0^\infty A(\tau)\cos(2\pi f\tau)\,d\tau \tag{6}$$

Equation (3) is obtained by Fourier transformation from the exponential autocorrelation function $A(\tau) = \langle(\delta I)^2\rangle e^{-t/\tau}$, which describes for instance the exponential decay of the number of carriers that have not yet recombined at the time t. There is a term similar to Eq. (4) present in the spectral density of current noise in semiconductors, for each type of carriers.

Let N be the number of carriers of a certain type in a semiconductor sample in stationary conditions. In terms of the generation rate $g(N)$ and of the recombination rate $r(N)$, the general formulas for both the lifetime τ and the mean square entering in Eq. (4) are:

$$\tau = \frac{1}{r'(N_0)-g'(N_0)}, \qquad \langle(\delta I)^2\rangle \equiv \frac{I_0^2}{N_0^2}\langle(\delta N)^2\rangle = \frac{I_0^2}{N_0^2}\tau g(N_0) \tag{7}$$

Here the prime denotes a derivative w.r.t. N. The derivatives are taken for $N = N_0 \equiv \langle N\rangle$, and we have denoted $\langle I\rangle$ by I_0. The following special cases are highlighted:

1. For a n-type semiconductor with N_d deep donors, the generation rate $g(N) = \gamma(N_d - N)$ is proportional to the number of neutral donors, $N_d - N$, while $r(N) = \rho N^2$, with constant γ and ρ, because there are N free electrons and N ionized donors. Therefore, one obtains

$$\begin{aligned}\tau &= \frac{1}{\gamma+2\rho N_0} = \frac{N_d-N_0}{\rho N_0(2N_d-N_0)}\\ \langle(\delta N^2)\rangle &= \frac{N_0(N_d-N_0)}{2N_d-N_0}\end{aligned} \tag{8}$$

2. For a near-intrinsic n-type semiconductor with N electrons, N_d donors (all ionized), and $P = N - N_d$ holes, we write g = const because the fluctuations are due to the thermal generation of electron–hole pairs. In this case the recombination rate is $r = \rho NP = \rho N(N - N_d)$. Therefore,

$$\tau = \frac{1}{\rho(N_0+P_0)}, \qquad \langle(\delta N)^2\rangle = \langle(\delta P)^2\rangle = \frac{N_0P_0}{N_0+P_0} \tag{9}$$

3. For a semiconductor with N_t traps and N trapped electrons, the trapping rate is proportional to the number $N_t - N$ of empty traps, while the release rate is proportional to N. Therefore, $g(N) = a(N_t - N)$ and $r(N) = bN$. The constants a and b are determined by the equilibrium condition $a(N_t - N_0) = bN_0$, which yields $N_0 = [a/(a + b)]N_t$. Therefore,

$$\tau = \frac{1}{a+b}, \qquad \langle(\delta N)^2\rangle = \frac{bN_0}{a+b} = \frac{abN_t}{(a+b)^2} \tag{10}$$

In this special case the rates g and r are not nonlinear functions of N and Eqs. (10) are therefore independent of N_0. In this case, the fluctuation of N obeys the binomial distribution law.

THERMAL NOISE

Also known as Johnson (or Nyquist) noise, thermal equilibrium noise has a white (frequency-independent) spectrum at not too high frequencies. It is given in general, for a circuit component of impedance $Z = 1/Y$ of conductance $G = \mathrm{Re}\, Y$ and resistance $R = \mathrm{Re}\, Z$, by the Planck–Nyquist formula

$$\begin{aligned}S_I(f) &= 4G\frac{hf}{e^{hf/kT}-1} \approx 4kTG\\ S_V(f) &= 4R\frac{hf}{e^{hf/kT}-1} \approx 4kTR\end{aligned} \tag{11}$$

Here $h = 6.62 \times 10^{-34}$ J·s is Planck's constant, and $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant. The thermal noise power available (for a matched load) is

$$S_a(f) = \frac{hf}{e^{hf/kT}-1} \approx kT \tag{12}$$

With the exception of ultrahigh frequencies at very low temperatures, only the approximate forms are used in practice and are known as equivalent forms of the Nyquist formula. The amplitude distribution of thermal noise is Gaussian, with small deviations of fundamental origin caused by the Q1/fE noise even in thermal equilibrium (4).

GENERAL INTRODUCTION TO 1/f NOISE

At low frequencies, the observed noise spectrum, in general, is roughly proportional to the reciprocal frequency, as Johnson first observed in 1925 in vacuum tubes. This $1/f$ noise accounts for most of the low-frequency noise. Low-frequency noise is therefore often considered synonymous with $1/f$ noise in practice. Schottky first called the $1/f$ noise "flicker noise" in 1926 and blamed it on a random flickering process on the surface of the cathode. In 1937 Schottky observed that flicker noise is suppressed by space charge in vacuum tubes to a larger extent than shot noise.

Nonfundamental 1/*f* Noise

A. L. McWhorter suggested in 1954 that 1/*f* noise in semiconductor samples and devices might arise from transitions of electrons to and from traps in the oxide at the surface. The superposition of many Lorentzian spectra [Eq. (4)] resulting from traps with different exponential relaxation times τ in the interval $\tau_1 < \tau < \tau_2$ can yield a 1/*f*-like spectrum in a limited frequency domain if two conditions are satisfied, causing a nonfundamental, or accidental, 1/*f* noise, as mentioned above. The two conditions are: (1) an electron is not allowed to interact with many traps at the same time, and (2) the distribution of the characteristic times has a probability density c/τ. Indeed, one expects then an addition of power spectra

$$\begin{aligned} S_I(f) &= 8(\langle\delta I\rangle^2)\pi c \int_{\tau_1}^{\tau_2} \frac{\tau}{1+\omega^2\tau^2}\frac{d\tau}{\tau} \\ &= 8(\langle\delta I\rangle^2)\frac{\pi c}{\omega}(\arctan \omega\tau_2 - \arctan \omega\tau_1) \\ &\approx 8(\langle\delta I\rangle^2)\frac{\pi^2 c}{\omega} \end{aligned} \tag{13}$$

The last approximation is valid only for $1/\tau_2 \ll \omega \ll 1/\tau_1$. There is strong evidence favoring a major contribution of this mechanism in MOSFETs from studies of the relaxation of slow surface states, particularly since the observed spectrum often differs slightly from 1/*f*. The slow states are distributed uniformly in the oxide volume, which serves as gate insulation, at the surface of the semiconductor. This nonfundamental contribution is usually larger in MOSFETs than the fundamental 1/*f* noise. The constant *c* is proportional to the superficial density of slow surface states, which can in principle be determined from the slow relaxation of the surface charges, but is hard to determine in practice without measuring the 1/*f* noise. Therefore, Eq. (13) is difficult to apply in practice.

General Aspects of Fundamental 1/*f* Noise

In fact, 1/*f* noise was found in carbon resistors and microphones, in all semiconductors and semiconductor devices, in contacts (contact noise), in infrared detectors, in bolometers, in photodetectors, in piezoelectric transducers and sensors, in mixers, in thin metallic sheets, in Josephson junctions and SQUIDs, in electron beams in vacuum, in the rate of electron tunneling and cold emission, in the recombination and generation rates for current carriers in the bulk and on the surface of semiconductors, in the frequency fluctuations of quartz resonators and SAW devices and arrays, and so on. It is always observed when a bottleneck is present, causing an electrical current to be carried by only a few current carriers. This ubiquity of 1/*f* noise indicates that 1/*f* noise is "the way of life" for electric currents.

The ubiquitous character of 1/*f* noise inspired the development of a turbulence theory of it (5,6), which generalized Heisenberg's hydrodynamic turbulence theory to the hydromagnetic plasma turbulence case. This theory yielded for the first time a universal 1/*f* spectrum from postulated instabilities of the laminar flow in the plasma of current carriers (electrons and holes in semiconductors). This physical theory was limited to homogeneous isotropic turbulence in an infinite, randomly stirred-up plasma of current carriers, and could therefore not be applied in practice. Nevertheless it demonstrated the fundamental nature of the 1/*f* spectrum caused by the universal feedback reaction of the electric current on itself via the electromagnetic field.

F. N. Hooge recognized that the turbulence theory (5,6) was the only physical theory of fundamental 1/*f* noise available at the time. This sparked experiments trying to verify 1/*f*-noise universality in the laboratory. In 1969 Hooge claimed, on the basis of his measurements, that the known inverse proportionality of 1/*f* noise with the volume of the sample under test becomes universal (i.e., with the same coefficient α_0 for any semiconductor, metal, or electrolyte) if the number of carriers, *N*, rather than the volume of the sample (assumed to be homogeneous), is used in the denominator:

$$S_I(f) = \alpha_0 |^2/Nf \tag{14}$$

This relation was known long before Hooge's work, but the coefficient α_0 was considered dependent on the material, and the volume of the sample was used with preference in the denominator, instead of *N*. This was thought to be equivalent, because the volume is proportional to *N*. Early experiments seemed to support Hooge's hypothesis, with a universal α_0 of 2 to 3 times 10^{-3}, but later experiments with smaller samples showed that Hooge was wrong, because any α_0 value from 10^3 down to 10^{-10} was shown to be possible. Although Hooge was proven wrong in his suggestion of a universal constant, his initial optimism helped accredit the notion of the fundamental nature of 1/*f* noise. The experimental "constant" α_0 was called the "Hooge parameter" (7) by A. van der Ziel, although it was in fact material-dependent, as had been assumed before Hooge. In 1974 the quantum 1/*f* theory derived Eq. (10) for the first time from first principles, allowing the exact calculation of α_0, and explaining why small devices have α_0 values ranging from 10^{-5} to 10^{-10}. In 1982 it also explained why large devices have α_0 values close to 4×10^{-3} and why ferroelectric substances have α_0 values as large as 10^3. This quantum-electrodynamic (QED) theory is presented here, first at an over simplified elementary level and with practical applications to devices, in order to clarify the physical basis and the new notions it introduces.

Elementary Introduction to Fundamental 1/*f* Noise

The main form of fundamental 1/*f* noise known at the present time is quantum 1/*f* noise, which is a manifestation of the coherent and conventional Q1/*f*E, representing a little-known new aspect of quantum mechanics. It can be obtained from a straightforward QED calculation of fundamental quantum fluctuations in cross sections, process rates, and electric currents, resulting from the author's attempts to quantize the earlier turbulence theory. These attempts were necessitated by the absence of instabilities with zero threshold, which could otherwise trigger the turbulence. They resulted in the discovery first of the conventional (1–4) and then of the coherent (3,8–10) quantum 1/*f* effect. The Q1/*f*E was proven to be responsible for most of the 1/*f* noise observed in electronic devices, thereby allowing for a unified presentation of noise in electronic devices (7,11–22).

Other forms of fundamental 1/*f* noise are found in nature beyond the realm of electrophysics. Like the Q1/*f*E, these other forms of fundamental 1/*f* noise have been proven (23) to arise from a coincidence of nonlinearity and homogeneity in physical systems. Just as in the case of the ontologically more fundamental Q1/fE, these other forms occur in systems that satisfy a universal sufficient criterion.

Simplified Derivation of the Conventional Quantum 1/f Effect

Definition. The Q1/fE is a fundamental quantum fluctuation of all physical cross sections σ, process rates Γ, and currents j given by the universal formula $S(f) = 2\alpha A/fN$ [conventional quantum 1/f equation (16)] for small devices, and $S(f) = 2\alpha/\pi fN$ [coherent quantum 1/f equation (16,23,24)] for large devices. These two forms can be combined into a single general formula, as we show below. Here $S(f)$ is the spectral density of fractional fluctuations in current, $\delta j/j$, in the scattering or recombination cross section $\delta\sigma/\sigma$, or in any other process rate $\delta\Gamma/\Gamma$. The number $\alpha \equiv e^2/\hbar c \approx 1/137$ is Sommerfeld's fine structure constant, a basic number of our world depending only on Planck's constant $\hbar$, the charge of the electron, e, and the speed of light in vacuum, c. The quantity $A \equiv 2(\Delta\boldsymbol{v}/c)^2/3\pi$ is essentially the square of the vector velocity change $\Delta\boldsymbol{v}$ of the scattered particles in the scattering process whose fluctuations we are considering, in units of c. Finally, N is the number of particles used to define the notion of current j, of cross section σ or of process rate Γ.

Plan. We will present here first a back-of-the-envelope derivation of the conventional Q1/fE. After presenting some practical applications to devices, we present below first an elementary derivation and later a more rigorous derivation of both the conventional and coherent Q1/fE.

Origin. The physical origin of electrodynamic conventional quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of current carriers (e.g., electrons) on a center of force, keeping in mind that electrons are described as probability amplitude waves ψ according to quantum mechanics. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by de Broglie waves ψ of a frequency corresponding to their energy. However, the electrons have energy loss amplitudes in the scattering process, due to the emission of bremsstrahlung into low-frequency photon modes. Therefore, part of the outgoing de Broglie waves are shifted to slightly lower frequencies. When we calculate the probability density $|\psi|^2$ in the scattered beam, we obtain also cross terms, linear in both the parts of ψ scattered with and without bremsstrahlung. These cross terms oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons. The emission of photons at all frequencies results therefore in probability density fluctuations at all frequencies. The corresponding quantum fluctuations of the current density $\boldsymbol{v}|\psi|^2$ are obtained by multiplying the probability density fluctuations by the velocity $\boldsymbol{v}$ of the scattered current carriers. Finally, these current fluctuations, present in the scattered beam, will be noticed at the detector as low-frequency current fluctuations, and will be interpreted as fundamental fluctuations in the physical scattering cross section of the scatterer.

Derivation. For a simple semiclassical calculation of the conventional Q1/fE along these lines, we start from the classical (Larmor) formula

$$P = 2q^2\boldsymbol{a}^2/3c^3 \tag{15}$$

for the power P radiated by a particle of charge q in the scattering process. The acceleration $\boldsymbol{a}$ can be approximated by a delta function $\boldsymbol{a}(t) = \Delta\boldsymbol{v}\ \delta(t)$ whose Fourier transform $\Delta\boldsymbol{v}$ is constant and is the change in the velocity vector of the particle during the almost instantaneous scattering process. The one-sided spectral density P_{f} of the emitted bremsstrahlung power,

$$P_{\mathrm{f}} = 4q^2(\Delta\boldsymbol{v})^2/3c^3 \tag{16}$$

is therefore also constant, independent of frequency, but goes to zero for frequencies larger than the reciprocal duration of the scattering process. The number $4q^2(\Delta\boldsymbol{v})^2/3hfc^3$ of emitted photons per unit frequency interval is obtained by dividing by the energy hf of one photon. The probability amplitude of photon emission,

$$A_f = \left(\frac{4q^2(\Delta\boldsymbol{v})^2}{3hfc^3}\right)^{1/2} e^{i\gamma} \tag{17}$$

is given by the square root of this photon number spectrum, including also a phase factor $e^{i\gamma}$. The Schrödinger wave function Ψ of the scattered outgoing charged particles can be constructed from products of single-particle wave functions ψ. The beat term in the probability density $|\psi|^2$ is linear both in this bremsstrahlung amplitude A_{f} and in the nonbremsstrahlung amplitude, which does not depend on f. The spectral density of this beat term will therefore be given by the product of the squared probability amplitude $|A_{\mathrm{f}}|^2 \ll 1$ of photon emission (proportional to 1/f) with the squared nonbremsstrahlung amplitude $1 - |A_{\mathrm{f}}|^2 \approx 1$, which is practically independent of f. The resulting spectral density of fractional probability density fluctuations is obtained by dividing by $|\psi|^4$ and is therefore

$$\begin{aligned} |\psi|^{-4} S^2_{\delta|\psi|}(f) &= \frac{8q^2(\Delta\boldsymbol{v})^2}{3hfNc^3} \equiv \frac{2\alpha A}{fN} \\ &= j^{-2}S_j(f) = S_{\delta j/j}(f) = S_{\delta\sigma/\sigma}(f) \end{aligned} \tag{18}$$

where $\alpha = e^2/\hbar c$ is the fine structure constant if $q = e$ is the elementary charge, $\hbar = h/2\pi$, and $\alpha A = 2e^2(\Delta\boldsymbol{v})^2/3\pi\hbar c^3$ is the bremsstrahlung coefficient, also known as the infrared exponent in quantum field theory. It is derived here as the quantum 1/f-noise coefficient in electrophysics.

The spectral density S_j of current density fluctuations $\delta j = v\delta|\psi|^2$ is obtained by multiplying the probability density fluctuation spectrum with the velocity $v = p/m$ of the outgoing particles. When we calculate the spectral density of fractional fluctuations in the scattered current j, the outgoing velocity simplifies or drops out, and therefore Eq. (18) also gives the spectrum of current fluctuations $S_j(f)$, as indicated above. Finally, the scattered particle current j per unit incoming flux is what we shall call the *physical scattering cross section* σ. This allows for the first equality in Eq. (18) after the identity sign.

The quantum 1/f noise contribution of each carrier is independent, and therefore the quantum 1/f noise from N carriers is N times larger; however, the current j will also be N times larger, and therefore in Eq. (1) a factor N was included in the denominator for the case in which the cross-section fluctuation is observed on N carriers simultaneously. Finally, note that the simplified back-of-the-envelope derivation which led to Eq. (18) is similar to considering diffraction of a single photon in Young's diffraction experiment and then estimating the autocorrelation function and the spectral density in the probability fringes obtained on the screen, claiming it should apply to the diffraction pattern generated by a large number of photons. The correct way is based on the two-particle wave function, which is a product of two single-particle functions in the noninteracting case considered here. This yields the same result, replacing $|\psi|^4$ in the calculation with the physically reasonable squared absolute value of the two-particle wave function (see below).

Discussion. We have defined the physical cross section as the quantum-mechanical cross section plus the corresponding quantum fluctuations, which were eliminated in the calculation of the quantum-mechanical expectation value, which is usually defined as the cross section. Our new notion of *physical process rate* is defined in the same way. The physical quantities are the directly observed ones, because in the Q1/*f*E the quantum fluctuations become macroscopic—observable at low frequencies—due to the $1/f$ dependence.

Although the wave function φ of each carrier is split into a bremsstrahlung part and a nonbremsstrahlung part, no quantum $1/f$ noise can be observed from a single carrier. A single carrier will only provide a pulse in the detector. Many carriers are needed to produce the quantum $1/f$ noise effect, just as in the case of electron diffraction patterns, where each individual particle is diffracted, but unless we repeat the experiment many times, or use many particles, no diffraction pattern can be seen. A single particle only yields a point of impact on the photographic plate in diffraction, or a pulse in the detector in $1/f$ noise. While incoming carriers may have been Poisson-distributed, the scattered beam will exhibit super-Poissonian statistics, or *bunching,* due to this new effect, the Q1/*f*E. The Q1/*f*E is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.

In conclusion, the conventional Q1/*f*E (1–4,6,8–15) is a fundamental fluctuation of physical cross sections and process rates, caused by the infrared-divergent coupling of current carriers to low-frequency photons (electrodynamic Q1/*f*E) and to other infraquanta, such as transverse phonons with piezoelectric coupling (lattice-dynamic Q1/*f*E), or electron–hole pairs on the Fermi surface of metals (electronic Q1/*f*E).

Application. The fundamental quantum $1/f$ fluctuations of physical cross sections σ and process rates Γ are reflected in the collision frequency $\nu = 1/\tau$ and collision time τ of the carriers, and in various kinetic coefficients in condensed matter, such as the mobility μ and the diffusion constant D, the surface and bulk recombination speeds s, and recombination times τ_r, the rate of tunneling j_t, and the thermal diffusivity in semiconductors. Specifically, neglecting the energy distribution of the carriers or using appropriate averages, $\delta\sigma/\sigma = \delta\Gamma/\Gamma = \delta\nu/\nu = -\delta\tau/\tau = -\delta\mu/\mu = -\delta D/D$. Therefore, the spectral density of fractional fluctuations in all these coefficients is given also by Eq. (18) in a first approximation that neglects the statistical effects of the momentum distribution of the current carriers. This is true in spite of the fact that each carrier will undergo many consecutive scattering transitions in the diffusion process. The resulting quantum $1/f$ noise in the mobility and in the diffusion coefficient is most often practically the same as (and can never be smaller than) the quantum $1/f$ noise in a single representative scattering event that limits the mobility or the diffusion coefficient.

Coherent Effect. For large devices the concept of coherent-state Q1/*f*E was introduced by the author (23,24). In this case the $1/f$ noise parameter α_0 as derived in the theory section below is given by

$$\alpha_0 = (\alpha_0)_{\mathrm{coh}} = 2\alpha/\pi = 4.6 \times 10^{-3} \tag{19}$$

where $\alpha \approx 1/137$ is the fine structure constant as mentioned above. This is of the same order of magnitude as the empirical value $\alpha_0 = 2$ to 3 times 10^{-3} that Hooge and others found for large devices. It is obvious that Hooge's empirical value for α_0 is due to the coherent Q1/*f*E and has a fundamental origin.

Conventional Effect. For small samples or devices we consider conventional quantum $1/f$ noise (1–4,5,6,8–15), which is just the cross-section fluctuation introduced above in Eq. (18). In that case α_0 may be written

$$\alpha_0 = (\alpha_0)_{\mathrm{conv}} = \frac{4\alpha}{3\pi}\frac{(\Delta \boldsymbol{v})^2}{c^2} \tag{20}$$

This general principle is now illustrated on practical examples of materials and devices. The exact meaning of large and small is explained below and also in the theory section in terms of the parameter s (24–26).

Simplified Application to Homogeneous Semiconductor Samples. In a homogeneous sample of length L, cross section A, volume $V = AL$, carrier mobility μ, carrier concentration n, and total number of carriers $N = nAL$, the conductance $C = n\mu eA/L$ and the resistance $R = 1/C$ will exhibit quantum $1/f$ fluctuations with a spectral density $S_{\delta C/C}$ of fractional fluctuations $\delta C/C = -\delta R/R$ given by

$$S_{\delta C/C}(f) = S_{\delta R/R}(f) = S_{\delta\mu/\mu}(f) = \alpha_0/fN \tag{21}$$

Size Dependence. To calculate α_0 we first evaluate the parameter $s = nA \times 5.5 \times 10^{-13}$ cm introduced in the theory section below. If $s \geq 1$, coherent quantum $1/f$ noise is observed with $\alpha_0 = 2\alpha/\pi$.

If $s < 1$, Eq. (21) requires knowledge of $(\alpha_0)_{\mathrm{conv}}$. The latter is calculated from Eq. (18) or (20) for each type of scattering that limits the mobility $\mu = e\tau/m^*$ of the carriers. Here τ is the mean collision time or scattering time of the carriers and m^* is their effective mass. In terms of the mean frequency of collisions $\nu = 1/\tau = \sigma v n_i$, one obtains $\mu = e/\nu m^* = e/\sigma v n_i m^*$. Here v is the mean speed of the carriers between collisions, σ a scattering cross section, and n_i the concentration of scatterers.

Conventional Quantum 1/f Effect in the Mobility. In general, Matthiessen's rule allows us to write, in terms of mobility,

$$\frac{1}{\mu} = \sum_j \frac{1}{\mu_j} \tag{22}$$

where μ_j is the mobility that would be obtained if only the jth scattering mechanism were present and limited the mobility. Applying a quantum $1/f$ fluctuation to Eq. (22), squaring, and averaging quantum-mechanically and statistically, we obtain as a reasonable first approximation

$$\frac{\delta\mu}{\mu^2} = \sum_j \frac{\delta\mu_j}{\mu_j^2}, \qquad S_{\delta\mu/\mu}(f) = \sum_j \left(\frac{\mu}{\mu_j}\right)^2 S_{\delta\mu/\mu_j}(f) \tag{23}$$

Equation (20) yields the strongest conventional quantum $1/f$ noise for umklapp scattering, followed by the f and g forms of intervalley scattering or intervalley with umklapp scattering (in indirect bandgap semiconductors such as Si and Ge only), followed by normal-phonon scattering, by neutral-impurity scattering, and by ionized-impurity scattering. The corresponding terms in Eq. (23) reflect this hierarchy only partially, because of the factors $(\mu/\mu_j)^2$, which gauge the importance of each of the scattering processes in limiting the resultant mobility. To gain physical insight, the conventional Q1/fE present in the various scattering processes is only estimated below and is actually calculated in the theory section in the second half of this article, taking into account the corrections introduced by the momentum distribution of the carriers and by the phonon distribution function at the temperature T.

Impurity Scattering. For instance, in the case of impurity scattering, n_i is. One obtains $S_{\delta\mu/\mu}(f) = S_{\delta\sigma/\sigma}(f)$. The physical scattering cross section σ, in turn, exhibits the Q1/fE with the spectral density given by Eqs. (18) and (20):

$$S_{\delta\sigma/\sigma}(f)_{\rm i} = \frac{4\alpha}{3\pi fN}\left\langle\left(\frac{\Delta \boldsymbol{v}}{c}\right)^2\right\rangle \approx \frac{3\times 10^{-3}}{fN}\left(\frac{\hbar\,\Delta \boldsymbol{k}}{m^* c}\right)^2 \tag{24}$$

The average quadratic velocity change of the electrons in a scattering process is smaller in impurity scattering than in lattice scattering, which includes normal-phonon scattering, intervalley scattering in indirect-bandgap semiconductors with several valleys, and umklapp scattering, as well as optical-phonon scattering. The Coulomb–Rutherford or Conwell–Weisskopf scattering cross section is proportional to $1/|\Delta \boldsymbol{k}|^4$, which favors small-angle scattering. Nevertheless, there are a few larger-angle scattering events, which are most effective in limiting the mobility and which therefore are decisive in the exact evaluation of the Q1/fE coefficient as a slow function of n_i, the concentration of impurities, given in the theory section below. This corresponds approximately to assuming randomizing collisions,

$$\langle(\Delta \boldsymbol{v})^2\rangle_{\rm i} = 2\langle v^2\rangle = 6k_{\rm B}T/m^* \tag{25}$$

although impurity scattering is not randomizing. With m_0 representing the free-electron mass, we obtain this way

$$S_{\delta\mu/\mu}(f)_{\rm i} = S_{\delta\sigma/\sigma}(f)_{\rm i} = \frac{4\alpha}{3\pi fN}\left\langle\frac{6k_{\rm B}T}{m^* c^2}\right\rangle \approx \frac{10^{-9}}{fN}\,\frac{Tm_0}{4m^*(100\,{\rm K})} \tag{26}$$

The quantum $1/f$ noise power present in impurity scattering is therefore proportional to T.

Normal-Acoustic-Phonon Scattering. For normal-phonon scattering, the product $\sigma v n_i$ has to be replaced by the lattice scattering rate Γ, given by an effective number of phonons times the squared phonon scattering matrix element, $|M|^2$. The latter exhibits quantum $1/f$ fluctuations, because the carriers emit bremsstrahlung photons in the scattering process. Therefore, if the mobility μ of electrons (of random velocity $v = \hbar k/m^*$) is limited by phonon scattering, we get

$$S_{\delta\mu/\mu}(f)_{\rm ap} = S_{\delta\Gamma/\Gamma}(f)_{\rm ap} = \frac{4\alpha}{3\pi fN}\left\langle\left(\frac{\hbar\,\Delta \boldsymbol{k}}{m^* c}\right)^2\right\rangle = \frac{3\times10^{-3}}{fN}\left\langle\left(\frac{\hbar\,\Delta \boldsymbol{q}}{m^* c}\right)^2\right\rangle \tag{27}$$

Here $\Delta\boldsymbol{q}$ is the acoustic-phonon momentum transfer in the scattering process, and the brackets indicate the average value. Using the linear approximation of the acoustic-phonon dispersion relation $E_q = v_s q\hbar$, with v_s denoting the speed of sound, we obtain for a thermal phonon with $E_q = k_{\rm B}T/2$

$$\langle(\hbar\,\Delta\boldsymbol{q}/m^* c)^2\rangle = \left(\frac{k_{\rm B}T}{2v_s m^* c}\right)^2 = 1.25\times10^{-5}\left(\frac{m_0}{m^*}\right)^2 \tag{28}$$

We finally obtain

$$S_{\delta\mu/\mu}(f)_{\rm ap} = \frac{3.75\times10^{-8}}{fN}\left(\frac{m_0}{m^*}\right)^2 \tag{29}$$

The mean squared momentum change and the $1/f$ noise are much larger (e.g., 50 times; see Fig. 1 below) for acoustic-phonon scattering, because impurity scattering is mainly small-angle scattering. A more rigorous treatment for the many types of scattering present in semiconductors, taking into account the corrections introduced by the momentum distribution of the carriers and the phonon distribution function at the temperature T, is given in the theory section in the second half of this article for the case of silicon.

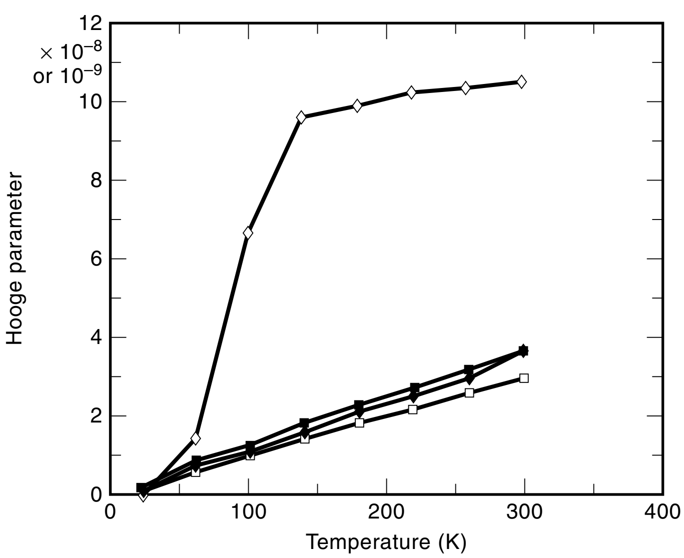

Figure 1. Acoustic Hooge parameter in units of 10^{-8} (open diamonds) and impurity Hooge parameter in units of 10^{-9} for three doping concentrations: 10^{21} (open squares), 10^{23} (solid diamonds), and 10^{24} (solid squares).

Umklapp Scattering. In this case the momentum change is close to the smallest reciprocal lattice vector approximated by $\hbar G = 2\pi\hbar/a$, where a is the lattice constant. Therefore, Eq. (20) yields

$$(\alpha_{0\mathrm{u}})_{\mathrm{conv}} = \frac{4\alpha}{3\pi}\left(\frac{2\pi\hbar}{am^*c}\right)^2 = \frac{6\times 10^{-8}}{fN}\left(\frac{m_0}{m^*}\right)^2 \tag{30}$$

for umklapp scattering.

Intervalley Scattering. In indirect-bandgap semiconductors the location of the energy minima of the conduction band in $\boldsymbol{k}$ space is different from the location of the valence-band energy maxima. In thermal equilibrium electrons and holes are present close to the minima of the conduction and valence bands. Scattering processes carrying electrons from one minimum (or valley) to the other are known as intervalley scattering. This is large-angle scatteing, compared with normal (intravalley) scattering, and it is therefore affected by larger conventional quantum $1/f$ noise, almost as large as umklapp scattering. Indeed, for example, in Si the eight minima are located at $0.85G$ from the origin, where G is the smallest reciprocal lattice vector magnitude. For *g processes,* which scatter an electron to the valley symmetrically located on the other side of the origin, Eq. (30) remains valid with a correction factor of $(0.85)^2$. On the other hand, for *f processes* in Si, scattering electrons between neighboring valleys, the factor is 2 times smaller. There is also the possibility of intervalley scattering with umklapp, which requires a correction factor of $(1 - 0.85)^2$. Equation (20) thus yields for intervalley scattering with umklapp

$$(\alpha_{0\mathrm{iu}})_{\mathrm{conv}} = 0.0225\frac{4\alpha}{3\pi}\left(\frac{2\pi\hbar}{am^*c}\right)^2 = \frac{1.35\times 10^{-9}}{fN}\left(\frac{m_0}{m^*}\right)^2 \tag{31}$$

While this appears to indicate a lower contribution from these intervalley umklapp processes, the corresponding factor $(\mu/\mu_j)^2$ in Eq. (23) ensures a larger contribution to the resulting spectral density of quantum $1/f$ noise, $S_{\delta\mu/\mu}(f)$. Physically, this is caused by the scarcity of high-energy phonons able to bridge the momentum gap of $0.85G$.

CALCULATION OF THE CONVENTIONAL QUANTUM 1/*f* EFFECT IN HOMOGENEOUS SEMICONDUCTOR MATERIALS

A first-principles calculation of quantum $1/f$ cross-correlations, performed for the first time in 1987 by Handel (27), has yielded a slightly different result from earlier expectations. This same new form of the quantum $1/f$ cross-correlations was rederived with a different method by Van Vliet (15) in 1989. It differs from the old form used in the 1985 calculation of Kousik et al. (28) by a correction that is zero when the momentum changes of the two current carriers involved in the cross correlation are identical, but increases when the momentum differences caused by the scattering process are different. The correction is proportional to the squared difference of the two momentum changes. Handel and Chung (29) have repeated all calculations in the original paper by Kousik et al. (28), obtaining both for impurity scattering and for the various types of phonon scattering new analytical expressions that show a considerable increase of the final quantum $1/f$ noise. The results obtained are in general applicable both to direct- and to indirect-bandgap semiconductors.

Introduction

Handel and Chung (29) have performed an analytical calculation of mobility fluctuations in silicon and gallium arsenide, using the new quantum $1/f$ cross-correlations formula. This calculation is of major importance for the $1/f$-noise-related optimization of the two types of materials, and of the many devices constructed with them for military and civilian applications in the electronic and optoelectronic industries.

The new cross-correlation formula gives the cross-spectral density, which describes the way in which simultaneous quantum $1/f$ scattering rate fluctuations ΔW observed in the direction of the outgoing scattered wave vector $\boldsymbol{K}'$ are correlated with those in the $\boldsymbol{K}''$ direction when the two corresponding incoming current carriers have the wave vectors $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$:

$$S_{\Delta W}(\boldsymbol{K}_1,\boldsymbol{K}';\boldsymbol{K}_2,\boldsymbol{K}'';f) = \frac{2\alpha}{3\pi f}\left(\frac{\hbar}{m^*c}\right)^2 W_{\boldsymbol{K}_1,\boldsymbol{K}'}W_{\boldsymbol{K}_2,\boldsymbol{K}''}[(\boldsymbol{K}'-\boldsymbol{K}_1)^2+(\boldsymbol{K}''-\boldsymbol{K}_2)^2]\delta_{\boldsymbol{K}_1,\boldsymbol{K}_2} \tag{32}$$

The form conjectured by us earlier had $2(\boldsymbol{K}' - \boldsymbol{K}_1)(\boldsymbol{K}'' - \boldsymbol{K}_2)$ in place of the rectangular bracket. The difference between the rectangular bracket and $2(\boldsymbol{K}' - \boldsymbol{K}_1)(\boldsymbol{K}'' - \boldsymbol{K}_2)$ is the perfect square $[(\boldsymbol{K}' - \boldsymbol{K}_1) - (\boldsymbol{K}'' - \boldsymbol{K}_2)]^2$. Therefore we expect the new results to be always larger than the results obtained on the basis of the previously conjectured form.

Impurity Scattering

For impurity scattering of electrons in solids, fluctuations $\Delta\tau$ of the collision times τ will cause mobility fluctuations

$$\Delta\mu_{\mathrm{band}}(t) = \frac{e}{m^*\langle\langle v^2\rangle\rangle}\sum_{\boldsymbol{K}} v_{\boldsymbol{K}}^2\,\Delta\tau(t)\,n_{\boldsymbol{K}} \tag{33}$$

where $\langle\langle v^2\rangle\rangle$ is both the average over all states of wave vectors $\boldsymbol{K}$, with occupation numbers $n_{\boldsymbol{K}}$, in the conduction band, and the thermal equilibrium average, of the squared carrier velocities. With the help of the relation

$$\frac{1}{\tau(\boldsymbol{K})} = \frac{V}{8\pi^3}\int\left(1-\frac{\cos\theta'}{\cos\theta}\right)W_{\boldsymbol{K},\boldsymbol{K}'}\,d^3K' \tag{34}$$

the mobility fluctuations are reduced to fluctuations of the elementary scattering rates $W_{\boldsymbol{K},\boldsymbol{K}'}$, governed by Eq. (32). Here V is the volume of the normalization box, which disappears in the final result, and θ and θ' respectively the angles $\boldsymbol{K}$ and $\boldsymbol{K}'$ form with the direction of the applied field. One finally obtains, after tedious multiple integrations,

$$\mu^{-2}S_{\Delta\mu}(f) = \frac{256\pi\alpha\kappa^2\epsilon^4\hbar^{12}}{3m^{*8}Z^4e^8N_i^2}\frac{1}{f} \times\sum_K K^{10}\left(\ln(1+a^2)-\frac{a^2}{1+a^2}\right)^{-3} \left(\frac{2a^2+a^4}{1+a^2}-2\ln(1+a^2)\right)F(E_K) \times\left(\sum_K v_K^2\tau(K)F(E_K)\right)^{-2} \tag{35}$$

where $a = 2K/\kappa$, $\kappa^2 = e^2 n(T)/\epsilon k_B T$, $n(T)$ is the electron concentration, $F(E_K) = \exp(E_F - E_K)$ for nondegenerate semiconductors, N_i is the concentration of impurities of charge Ze, and ϵ is the dielectric constant. The corresponding partial Hooge parameter for impurity scattering is thus

$$\alpha_i = \frac{4\sqrt{2\pi}\alpha\kappa\hbar^5 N_c}{3m^{*7/2}(k_B T)^{3/2}c^2} \int_0^\infty dx\, x^{11/2}e^{-x}\left(\ln(bx+1) - \frac{bx}{bx+1}\right)^{-3} \left(\frac{2bx+b^2x^2}{bx+1} - 2\ln(bx+1)\right) \times \left[\int_0^\infty dx\, x^3 e^{-x}\left(\ln(bx+1) - \frac{bx}{bx+1}\right)^{-1}\right]^{-2} \quad (36)$$

This result is graphed in Fig. 1 for three different values of the donor concentration N_d and is compared with old results obtained by simply recalculating the old analytical expression (28). As expected, the new cross-correlation formula leads to slightly higher α_i values than the previously conjectured expression. This was mentioned in connection with Eq. (32) above.

Electron–Acoustic-Phonon Scattering

In this case the calculation is similar, and leads to the result

$$\begin{aligned}\alpha_{ac} = \frac{32\pi\alpha N_c m^* C^7\hbar^3}{(3c^2 k_B T)^4}\Bigg[\frac{1}{R^2}\int_1^\infty dx\, x^{-4} &\times \left(\frac{(x-1)^7}{7} + (R+1)\frac{(x-1)^6}{6} + R\frac{(x-1)^5}{5}\right)\\ &\times \left(\frac{(x-1)^5}{5} + (R+1)\frac{(x-1)^4}{4} + R\frac{(x-1)^3}{3}\right)\exp\left(-\frac{x^2}{4R}\right)\\ &+ \int_0^1 dx\, x^{-4}\left(\frac{(x+1)^5}{5} - \frac{(x+1)^6}{6} + \frac{(x-1)^5}{5} + \frac{(x-1)^6}{6}\right)\\ &\times \left(\frac{(x+1)^3}{3} + \frac{(x-1)^4}{4} + \frac{(x-1)^3}{3} - \frac{(x+1)^4}{4}\right)\\ &\exp\left(-\frac{x^2}{4R}\right) + \int_1^\infty dx\, x^{-4}\left(\frac{(x+1)^5}{5} - \frac{(x+1)^6}{6}\right)\\ &\left(\frac{(x+1)^3}{3} - \frac{(x+1)^4}{4}\right)\exp\left(-\frac{x^2}{4R}\right)\Bigg] \end{aligned} \quad (37)$$

where $R = k_B T/2m^* C_1^2$, C_1 is the deformation potential, and N_c is the effective density of states for the conduction band.

Nonpolar Optical-Phonon Scattering

This time one obtains

$$\begin{aligned}\alpha_{noph} = \frac{8\pi\sqrt{2\hbar\omega_o}\alpha N_c\hbar^2}{3m^{*5/2}c^2\omega_o}\Bigg[\int_0^\infty dx\, x^{5/2} &\times [(F+1)(x-1)^{1/2}\theta(x-1) + F(x+1)^{1/2}]^{-4}\\ &\times [(F+1)^2(x-1)(2x-1)\theta(x-1)\\ &+ F^2(x+1)(2x+1)]\exp\left(-\frac{\hbar\omega_o x}{k_B T}\right)\Bigg]\\ &\times\Bigg[\int_0^\infty dx\, x^{3/2}[(F+1)(x-1)^{1/2}\theta(x-1)\\ &+ F(x+1)^{1/2}]^{-1}\exp\left(-\frac{\hbar\omega_o x}{k_B T}\right)\Bigg]^{-2}\end{aligned} \quad (38)$$

where $F = [\exp(\hbar\omega_o/k_B T) - 1]^{-1}$, and ω_o is the optical-phonon frequency.

Polar Optical-Phonon Scattering

Proceeding as in the cases of impurity and nonpolar optical-phonon scattering, we obtain

$$\begin{aligned}\alpha_{poph} = \frac{8\pi\sqrt{2\hbar\omega_l}\alpha N_c\hbar^2}{3m^{*5/2}c^2\omega_l}\Bigg(\int_0^\infty dx\, x^4 &\times \{F^2(x+1)^{1/2}\ln[2x^{1/2} + 2(x+1)^{1/2}]\\ &+ (F+1)^2(x-1)^{1/2}\ln[2x^{1/2}\\ &+ (x-1)^{1/2}]\theta(x-1)\}\exp(-\hbar\omega_l x/k_B T)\\ &\times\{(F+1)\operatorname{arcsinh}[(x-1)^{1/2}\theta(x-1)]\\ &+ F\operatorname{arcsinh}(x^{1/2})\}^{-4}\Bigg)\end{aligned} \quad (39)$$

Here ω_l is the longitudinal phonon frequency.

Intervalley Scattering

This type of scattering, present in indirect-bandgap semiconductors, transfers electrons from one of the six minima (or valleys) of the conduction-band energy in $\boldsymbol{k}$ space to one of the other five minima. Transitions between a valley and the nearest valley, which is along the same $\boldsymbol{k}$-space direction in the next copy of the first Brillouin zone in the periodic zone scheme, are of the umklapp type, and are called *g processes.* Transitions to the four valleys present in the same zone along the other two $\boldsymbol{k}$-space directions are called *f processes.* Repeating a previous calculation (31) on the basis of the new cross-correlation formula Eq. (32), we obtain for g processes

$$\begin{aligned}\alpha_g = \frac{8\pi\sqrt{2\hbar\omega_{ij}}\alpha N_c\hbar^2}{3m^{*5/2}c^2\omega_{ij}}\Bigg[\int_0^\infty dx\, x^{5/2} &\times [(F+1)(x-1)^{1/2}\theta(x-1) + F(x+1)^{1/2}]^{-4}\\ &\times [(F+1)^2(x-1)(2x-1)\theta(x-1)\\ &+ F^2(x+1)(2x+1)]\exp\left(-\frac{\hbar\omega_{ij}x}{k_B T}\right)\Bigg]\\ &\times\Bigg[\int_0^\infty dx\, x^{3/2}[(F+1)(x-1)^{1/2}\theta(x-1)\\ &+ F(x+1)^{1/2}]^{-1}\exp\left(-\frac{\hbar\omega_{ij}x}{k_B T}\right)\Bigg]^{-2}\end{aligned} \quad (40)$$

where $\hbar\omega_{ij}$ is the phonon energy corresponding to the momentum difference required by the intervalley transition. For the corresponding f process we obtain (30)

$$\alpha_f = \left(\frac{k_0}{q_0}\right)^2 \alpha_g \frac{\hbar\omega_{ijf}}{k_B T} \quad (41)$$

where k_0/q_0 is the ratio between length of the position vector of a conduction-band energy minimum in $\boldsymbol{k}$ space, and twice the distance of the same minimum from the Brillouin zone boundary, 0.85/0.3 for silicon. $\alpha_g(\hbar\omega_{ijf}/k_B T)$ is calculated with the f momentum difference. There are three g-type alphas, α_{g1}, α_{g2}, and α_{g3} (from LA, TA, and LO phonons respectively), and three f-type ones, α_{f1}, α_{f2}, and α_{f3} (from TA, LA, and TO

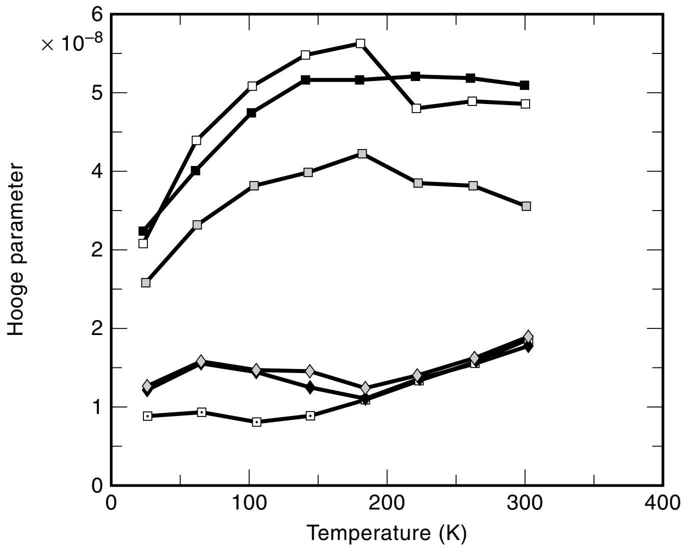

Figure 2. Hooge parameters for intervalley scattering in units of 10^{-8} for g processes (solid) and f processes (shaded).

phonons). Their values are given in Fig. 2 and are a few times larger than the old values.

The various quantum $1/f$ contributions derived here can be approximately superposed to yield the resultant quantum $1/f$ coefficient according to the rule

$$\alpha_{\rm H} = \sum_j \left(\frac{\mu}{\mu_i}\right)^2 \alpha_i \tag{42}$$

In the next section we illustrate the application of these results to inhomogeneous semiconductor devices on the simplest case of pn junctions. The case of transistors and other junction devices, as well as the cases of field-effect transistors, HEMTs, PBTs, and other devices, is presented in the literature (see, e.g., Ref. 16).

DERIVATION OF MOBILITY QUANTUM 1/*f* NOISE IN n^+p DIODES AND METAL–INSULATOR–SEMICONDUCTOR DEVICES

Mobility Quantum 1/*f* Noise in n^+p Diodes

For a diffusion limited n^+p junction the current is controlled by diffusion of electrons into the p region over a distance of the order of the diffusion length $L = (D_n\tau_n)^{1/2}$, which is shorter than the length w_p of the p region in the case of a long diode; τ_n is the lifetime of the electrons. Quantum $1/f$ fluctuations of the scattering rates, discussed in the previous section, will cause fluctuations in the local carrier mobility μ and diffusion constant $D = \mu kT/e$. If $N(x)$ is the number of electrons per unit length and D_n their diffusion constant, the electron current at x is

$$I_{n\rm d} = -eD_n \frac{dN}{dx} \tag{43}$$

where we have assumed a planar junction and taken the origin $x = 0$ in the junction plane. Diffusion constant fluctuations, given by kT/e times the mobility fluctuations, will lead to local current fluctuations in the interval Δx:

$$\delta\Delta I_{n\rm d}(x,t) = I_{n\rm d}\,\Delta x \frac{\delta D_n(x,t)}{D_n} \tag{44}$$

The normalized weight with which these local fluctuations representative of the interval Δx contribute to the total current $I_{\rm d}$ through the diode at $x = 0$ is determined by the appropriate Green function and can be shown to be $(1/L)$ $\exp(-x/L)$ for $w_p/L \gg 1$. Therefore the contribution of the section Δx is

$$\delta\Delta I_{\rm d}(x,t) = \frac{\Delta x}{L}\exp\left(-\frac{x}{L}\right) I_{n\rm d}\frac{\delta D_n(x,t)}{D_n} \tag{45}$$

with the spectral density

$$S_{\Delta I_{\rm d}}(x,f) = \left(\frac{\Delta x}{L}\right)^2 \exp\left(-\frac{2x}{L}\right)\frac{I_{n\rm d}^2 S_{D_n}(x,f)}{D_n^2} \tag{46}$$

For mobility and diffusion fluctuations the fractional spectral density is given by $\alpha_{{\rm H}nd}/(fN\,\Delta x)$, where the quantum $1/f$ coefficient $\alpha_{{\rm H}nd}$ for electronic diffusion or mobility is determined from quantum $1/f$ theory according to Eq. (42). With Eq. (43) we obtain then

$$S_{\Delta I_{\rm d}}(x,f) = \frac{\Delta x}{L^2}\exp\left(-\frac{2x}{L}\right)\left(eD_n\frac{dN}{dx}\right)^2 \frac{\alpha_{{\rm H}n\rm d}}{fN} \tag{47}$$

The electrons are distributed according to the solution of the diffusion equation:

$$\begin{aligned} N(x) &= [N(0) - N_p]\exp\left(-\frac{x}{L}\right) \\ \frac{dN}{dx} &= -\frac{N(0)-N_p}{L}\exp\left(-\frac{x}{L}\right) \end{aligned} \tag{48}$$

Substituting into Eq. (47) and simply summing over the uncorrelated contributions of all intervals Δx, we obtain

$$S_{I_{\rm d}}(f) = \alpha_{{\rm H}nd}\left(\frac{eD_n}{L^2}\right)^2 \int_0^{W_p} \frac{[N(0)-N_p]^2 e^{-4x/L}\,dx}{[N(0)-N_p]e^{-x/L} + N_p} \tag{49}$$

We note that $eD_n/L^2 = e/\tau_n$. With the expression for the saturation current $I_0 = e(D_n/\tau_n)^{1/2}N_p$ and of the current $I = I_0[\exp(eV/kT) - 1]$, we can carry out the integration:

$$S_{I_{\rm d}}(f) = \alpha_{{\rm H}n\rm d}\frac{eI}{f\tau_n}\int_0^1 \frac{a^2u^3\,du}{au+1} = \alpha_{{\rm H}n\rm d}\frac{eI}{f\tau_n}F(a) \tag{50}$$

Here we have introduced the notation

$$\begin{gathered} u = \exp(-x/L), \qquad a = \exp(eV/kT) - 1 \\ F(a) = \frac{1}{3} - \frac{1}{2a} + \frac{1}{a^2} - \frac{1}{a^3}\ln(1+a) \end{gathered} \tag{51}$$

Equation (50) gives the diffusion noise as a function of the quantum $1/f$ noise parameter $\alpha_{{\rm H}n\rm d}$. A similar result can be derived for the quantum $1/f$ fluctuations of the recombination rate in the bulk of the p region. The result is the same, with $\alpha_{{\rm H}n\rm d}$ replaced by $\alpha_{{\rm H}nr}$.

Mobility Quantum 1/*f* Noise in Metal–Insulator–Semiconductor Devices

As an example of results on quantum $1/f$ noise in high-tech devices, we provide here without proof the results obtained by Handel for the $1/f$-limited performance of metal–insulator–semiconductor (MIS) HgCdTe infrared detectors. The current density I in the detector contains a diffusion term I_d, a term I_r caused by recombination in the space charge region, a surface recombination term I_s, a tunneling term I_t, and a photovoltaic term caused by the creation of electron–hole pairs by photons:

$$\begin{aligned} I &= I_d + I_r + I_s + I_t + q\eta\Phi \\ &= qn_i\left[\frac{n_i}{n_0}\left(\frac{D_n}{\tau_n}\right)^{1/2}(e^{qV/kT}-1)+\frac{W}{\tau}(e^{qV/2kT}-1)+s\right] \\ &\quad + I_t + q\eta\Phi \end{aligned} \tag{52}$$

Here n_i is the intrinsic concentration, n_0 the concentration of acceptors on the p side, D_n and τ_n the diffusion constant and lifetime of minority carriers on the p side, W the width of the depletion region, $\tau = \tau_{p^0+n^0}$ the Shockley–Hall–Read lifetime, V the applied voltage, s the surface recombination speed, η the quantum efficiency, and Φ the incident flux of photons. With the exception of the last term, the terms in Eq. (52) are known as dark-current components.

We write the total dark-current fluctuation in the form

$$\delta I_d = \delta I_d + \delta I_r + \delta I_s + \delta I_{tb} + \delta I_{tc} + \delta I_{tsc} \tag{53}$$

and the spectral density of current fluctuations will be

$$S_{I_d} = S_{I_d} + S_{I_r} + S_{I_s} + S_{I_{tb}} + S_{I_{tc}} + S_{I_{tsc}} \tag{54}$$

Here we have lumped the recombination current on the back surface I_b together with the surface recombination (generation) current I_S. If we denote all the corresponding spectral densities of fractional fluctuations by a prime ($S'_{I_i} = S_{I_i}/I_i^2$), we obtain

$$\begin{aligned} S'_{I_d} &= (I_{dif}/I_d)^2 S'_{I_{dif}} + (I_{dep}/I_d)^2 S'_{I_{dep}} + (I_s/I_d)^2 S'_{I_s} \\ &\quad + (I_{tb}/I_d)^2 S'_{I_{tb}} + (I_{tc}/I_d)^2 S'_{I_{tc}} + (I_{tsc}/I_d)^2 S'_{I_{tsc}} \end{aligned} \tag{55}$$

This equation was obtained by dividing the previous equation through I_d^2, and shows that the biggest contribution will not necessarily come from the process with the highest fractional quantum $1/f$ noise, that is, with the highest $1/f$ noise coefficient. The weight of each type of noise is determined by the corresponding squared current ratio.

The detectivity of infrared detectors is limited in general by three types of noise: (1) current noise in the detector, (2) noise due to background photons (photon noise), (3) noise in the electronic system following the detector. We shall neglect here the background photon noise and the noise in the electronic system. The detectivity is defined as

$$D^*(\lambda, f) = \frac{(A\,\Delta f)^{1/2}}{\text{NEP}} \qquad (\text{cm}\cdot\text{Hz}^{1/2}/\text{W}) \tag{56}$$

where A is the area of the detector; NEP is the noise equivalent power, defined as the rms optical signal of wavelength λ required to produce an rms noise voltage (current) equal to the rms noise voltage (current) in a bandwidth Δf, and f is the frequency of modulation. The noise equivalent power is given by

$$\text{NEP} = \frac{hv}{\eta q}[S_{I_d}(f)\,\Delta f]^{1/2} \tag{57}$$

Therefore we obtain for the detectivity

$$D^*(\lambda, f) = \frac{q\eta\lambda}{hc}\left(\frac{A}{S_{I_d}(f)}\right)^{1/2} = \frac{q\lambda}{hc}[S_{I_d}(f)]^{-1/2} \tag{58}$$

We notice that $D^*(\lambda, f)$ is proportional to λ up to the peak wavelength λ_c. For $\lambda > \lambda_c$ we have $\eta = 0$ and thus $D^*(\lambda, f) = 0$. By substituting our result for S_{I_d}, we obtain the general expression for the detectivity as a function of various parameters of the MIS device.

Let us now evaluate the spectral density $S'(f)$ of fractional fluctuations in the various dark-current noise contributions per square centimeter of transversal detector (or gate) area, including also a numerical example for a MIS infrared detector. For a given detector, this needs to be divided by the area A of the detector to yield the corresponding fractional spectral densities: $S(f) = S'(f)/A$. Fractional fluctuations are dimensionless, so $S'(f)$ will have the dimension of a reciprocal frequency times a squared length unit, which simplifies when we divide by the area of the detector at hand. Let S'_{I_d} be the spectral density of fractional fluctuations in the noise caused by quantum $1/f$ fluctuations in diffusion, S'_{I_r} in bulk recombination, S'_{I_s} in surface recombination, and S'_{I_t} in tunneling. With $m_p^* = 0.55m_0$, $m_n^* = 0.02m_0$, $\tau_n = 10^{-6}$ s, $E_g = 0.1$ eV, $3kT/2 = 0.01$ eV, we obtain for a p-type MIS device with $w_p \gg L_d$

$$\begin{aligned} S'_{I_d} &= (\alpha_{Hnd} + \alpha_{Hnr})\frac{e}{f\tau_n I_d}F(a) = \alpha_{coh}\frac{e^{1/2}}{f(kT\mu\tau_n)^{1/2}N_p}\frac{F(a)}{a} \\ &= \frac{4.6\times10^{-3}}{4fN_p}\frac{4\times10^{-10}\,\text{C}^{1/2}}{[(10^{-6}\,\text{s})(1.5\times10^5\,\text{cm}^2/\text{V}\cdot\text{s})(4\times10^{-21}\,\text{J})]^{1/2}} \\ &= \frac{1.8\times10^{-6}\,\text{cm}^2}{f} \end{aligned} \tag{59}$$

$$\begin{aligned} S'_{I_r} &= \frac{\alpha_{He}e}{f(\tau_{no}+\tau_{po})I_r}\tanh x = \frac{\alpha_{He}e}{feAwn\tanh x}\tanh x \\ &= \frac{\alpha_{He}}{fAwn_i} = \frac{4.6\times10^{-9}\,\text{cm}^2}{f} \qquad (x = eV/2kT) \end{aligned} \tag{60}$$

$$\begin{aligned} S'_{I_s} &= \frac{4\alpha}{3\pi}\frac{2}{m^*c^2}\left(\frac{3kT}{2}+\frac{eU}{2}+0.1Ve\right)\frac{e\tanh x}{f(\tau_{no}+\tau_{po})I_s} \\ &= \frac{4\alpha}{3\pi\times0.02}\frac{2}{500{,}000}(0.025+0.5+0.5)\frac{e\tanh x}{feAwn_i(e^x-1)} \\ &= \frac{7\times10^{-8}\,\text{cm}^2}{f} \approx S_{I_b} \end{aligned} \tag{61}$$

$$\begin{aligned} S'_{I_{tb}} &= \frac{4\alpha}{3\pi}\frac{E_g+3kT/2}{m^*c^2} = \frac{4}{9.5\times137\times0.02}\frac{0.11}{500{,}000} \\ &= \frac{3.3\,10^{-8}\,\text{cm}^2}{f} \end{aligned} \tag{62}$$

$$\begin{aligned} S'_{I_{tc}} &= \frac{4\alpha}{3\pi}\frac{E_g+3kT}{2m^*c^2} = \frac{4}{9.5\times137\times0.02}\frac{0.12}{10^6} \\ &= \frac{1.8\times10^{-8}\,\text{cm}^2}{f} = S'_{I_{tsc}} \end{aligned} \tag{63}$$

S'_{I_d} was calculated in the small-bias limit for $w_p \gg L$, but $w_p = 0.25L$ gives the same result; the incoherent case with a lattice constant of 0.65 nm and $\Theta = 320$ K was also listed above (because a 10 μm thick device is very small, so it may be applicable), and would give $(1.8 \times 10^{-10}\ \mathrm{cm}^2)/f$ for a n-type device. Equations (60)–(63) would be reduced $m_p^*/m_n^* = 27.5$ times for n-type devices. We mention that S'_{I_s} has been calculated with the inclusion of a term of 10% of the applied gate voltage V in the kinetic energy of the carriers at the surface, and that for the back-surface recombination current this term has to be dropped in the similar expression for S'_{I_b}. However, we have neglected this here, because the surface recombination terms will not turn out to be important, as we will see below. The applied gate voltage was taken to be $V = 5$ V. Calculating the fraction of each current, we obtain

$$\begin{aligned}(1\,\mathrm{cm}^{-2})fS'_I(f) &= (20/132)^2 \times 1.8 \times 10^{-6} + (10/132)^2 \\ &\quad \times 4.6 \times 10^{-9} + (3.6/132)^2 \times 7 \times 10^{-8} \\ &\quad + (0.01/132)^2 \times 3.3 \times 10^{-8} + (80/132)^2 \times 1.8 \times 10^{-8} \\ &\quad + (17.5/132)^2 \times 1.8 \times 10^{-8} \\ &= 3.67 \times 10^{-8} + 2.6 \times 10^{-11} + 5.2 \times 10^{-11} \\ &\quad + 1.9 \times 10^{-16} + 6.61 \times 10^{-9} + 3.17 \times 10^{-10} \\ &= 4.37 \times 10^{-8} \end{aligned} \tag{64}$$

or for incoherent $1/f$ noise, 7.1×10^{-9} (p) and 3×10^{-10} (n). This value can be used in order to estimate the detectivity of the device in our example. Substituting into Eq. (57), we obtain with a quantum efficiency $\eta = 0.7$ and wavelength of $\lambda = 10\ \mu$m:

$$\begin{aligned} D^*(\lambda, f) &= \frac{\eta q \lambda}{hc}[S_{I_d}(f)]^{-1/2} \\ &= \frac{(0.7 \times 1.6 \times 10^{-19}\,\mathrm{C})(10^{-5}\,\mathrm{m})}{(6.6 \times 10^{-34}\,\mathrm{J\cdot s})(3 \times 10^8\,\mathrm{m/s})} \\ &\quad \left(\frac{f}{(4.37 \times 10^{-8}\,\mathrm{cm}^2)(1.74 \times 10^{-6}\,\mathrm{A}^2/\mathrm{cm}^4)}\right)^{1/2} \\ &= (2 \times 10^7\,\mathrm{cm\cdot Hz}^{1/2}/\mathrm{W}) \times f^{1/2} \end{aligned} \tag{65}$$

or for incoherent $1/f$ noise, 5×10^7 (p) and 2.5×10^8 (n).

In conclusion we note that for the relatively large devices which we have considered, most of the quantum $1/f$ noise comes from fluctuations in diffusion and in the rate of tunneling via impurity centers in the bandgap. The effective mass of the carriers is present in the denominator of all quantum $1/f$ noise contributions except the coherent quantum $1/f$ fluctuation present in the diffusion current of large devices. In smaller devices the diffusion current will also be given by the conventional quantum $1/f$ formula, which contains the effective mass of the carriers in the denominator. For umklapp scattering the mass of the carriers in the denominator is squared. Consequently we expect lower quantum $1/f$ noise from n-type devices, in which the minority carriers are holes, particularly if the devices are very small, say, below 10 μm. We are now in a position to explain how "smart" ultralow-noise materials can be designed for specific classes of device applications (see the section "Development of Special Materials for Ultralow-Noise FET and Junction Devices").

DERIVATION OF THE CONVENTIONAL QUANTUM 1/*f* EFFECT

The simplified description of quantum $1/f$ noise was presented above in the elementary terms of Schrödinger's statistical catalog model, without using second quantization. This approach is natural in view of the close connection between this new effect and diffraction, which is usually treated without second quantization, in the statistical catalog model based on the single-particle solution of the Schrödinger equation, normalized to the number of particles, N. Just as the superposition of elementary phase-shifted waves allows for the simplest and most intuitive description of diffraction through a slit, the description of quantum $1/f$ noise in terms of interference beats between slightly frequency-shifted scattered partial waves with bremsstrahlung energy losses will always provide the simplest and most elementary quantitative derivation of the Q1/*f*E, easily accessible even at the undergraduate level.

Below we now present the derivation of the Q1/*f*E in a general form that determines the scattered current j from the observation of a sample of N outgoing particles. The minimal outgoing sample for defining particle–particle correlations in the scattered wave consists of two particles, and therefore the effect can be calculated for the case of two outgoing particles. Since the general derivation also yields a factor $1/N$ for bosons and a factor $1/(N-1)$ for fermions, and since the simplifying restriction to $N = 2$ has given rise to some misinterpretations, a presentation of the general case of N bosons or N fermions will be of interest. We consider the case of bosons.

We start with the expression of the Heisenberg representation state $|S\rangle$ of N identical bosons of mass M emerging at an angle θ from some scattering process with various undetermined bremsstrahlung energy losses reflected in their one-particle waves $\varphi_i(\xi_i)$:

$$|S\rangle = (N!)^{-1/2}\prod_i d^3\xi_i\,\varphi_i(\xi_i)\psi^\dagger(\xi_i)|0\rangle = \prod_i d^3\xi_i\,\varphi_i(\xi_i)|S^o)\rangle \tag{66}$$

where $\psi^\dagger(\xi_i)$ is the field operator creating a boson with position vector ξ_i, $\psi(\xi_i)$ is the field operator annihilating a particle, and $|0\rangle$ is the vacuum state, while $|S^o\rangle$ is the state with N bosons of position vectors ξ_i with $i = 1, \ldots, N$. All products and sums in this section run from 1 to N, unless otherwise stated.

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

$$O(\boldsymbol{x}_1, \boldsymbol{x}_2) = \psi^\dagger(\boldsymbol{x}_1)\psi^\dagger(\boldsymbol{x}_2)\psi(\boldsymbol{x}_2)\psi(\boldsymbol{x}_1) \tag{67}$$

known as the operator of the pair correlation. This operator corresponds to a density autocorrelation function. The presence of two-particle coordinates in the operator O does not mean that we are considering two-particle interactions; it only means that the expectation value that we are calculating depends on the relative position of the particles. Using the well-known commutation relations for boson field operators,

$$\psi(\boldsymbol{x})\psi^\dagger(\boldsymbol{y}) - \psi^\dagger(\boldsymbol{y})\psi(\boldsymbol{x}) = \delta(\boldsymbol{x} - \boldsymbol{y}) \tag{68}$$

$$\psi(\boldsymbol{x})\psi(\boldsymbol{y}) - \psi(\boldsymbol{y})\psi(\boldsymbol{x}) = 0 \tag{69}$$

$$\psi^\dagger(\boldsymbol{x})\psi^\dagger(\boldsymbol{y}) - \psi^\dagger(\boldsymbol{y})\psi^\dagger(\boldsymbol{x}) = 0 \tag{70}$$

we first calculate the matrix element:

$$N|\langle S^o|O|S^o\rangle = \sum_{\mu v}{}' \sum_{mn}{}' \delta(\eta_v - \boldsymbol{x}_1)\delta(\eta_\mu - \boldsymbol{x}_2)\delta(\xi_n - \boldsymbol{x}_1)\delta(\xi_m - \boldsymbol{x}_2) \sum_{(i,j)} \prod_{ij}{}' \delta(\eta_j - \xi_i) \tag{71}$$

where $|S^o\rangle$ is the state with well-defined particle coordinates. Here the prime excludes $\mu = \nu$ and $m = n$ in the summations and excludes $i = m$, $i = n$, $j = \mu$, and $j = \nu$ in the product. The summation $\Sigma_{(i,j)}$ runs over all permutations of the remaining $N - 2$ values of i and j. On the basis of this result we now calculate the complete matrix element

$$\begin{aligned}\langle S|O|S\rangle &= \frac{1}{N(N-1)} \sum_{\mu v}{}' \sum_{mn}{}' d^3\eta_\mu\, d^3\eta_v\, d^3\xi_m\, d^3\xi_n \\ &\times \varphi_\mu^*(\eta_\mu)\varphi_v^*(\eta_v)\varphi_m(\xi_m)\varphi_n(\xi_n)\delta(\eta_v - \boldsymbol{x}_1) \\ &\delta(\eta_\mu - \boldsymbol{x}_2)\delta(\xi_n - \boldsymbol{x}_1)\delta(\xi_m - \boldsymbol{x}_2) \\ &= \frac{1}{N(N-1)} \sum_{\mu v}{}' \sum_{mn}{}' \varphi_\mu^*(\boldsymbol{x}_2)\varphi_v^*(\boldsymbol{x}_1)\varphi_m(\boldsymbol{x}_1)\varphi_n(\boldsymbol{x}_2)\end{aligned} \tag{72}$$

The one-particle states are spherical waves emerging from the scattering center located at $x = 0$:

$$\varphi(\boldsymbol{x}) = \frac{C}{x} e^{iKx} \left(1 + \sum_{\boldsymbol{k}l} b(\boldsymbol{k}, l) e^{-iqx} a_{\boldsymbol{k}l}^\dagger\right) \tag{73}$$

Here C is an amplitude factor, K the boson wave vector magnitude, and $b(\boldsymbol{k}, l)$ the bremsstrahlung amplitude for photons of wave vector $\boldsymbol{k}$ and polarization l, while $a_{kl}^\dagger$ is the corresponding photon creation operator, allowing the emitted photon state to be created from the vacuum if Eq. (73) is inserted into Eq. (72). The momentum magnitude loss $\hbar q = Mck/K \equiv 2\pi Mf/K$ is necessary for energy conservation in the bremsstrahlung process. Substituting Eq. (73) into Eq. (72), we obtain

$$\langle S|O|S\rangle = \left|\frac{C}{x}\right|^4 \left(N(N-1) + 2(N-1)\sum_{\boldsymbol{k},l} |b(\boldsymbol{k}, l)|^2[1 + \cos q(x_1 - x_2)]\right) \tag{74}$$

where we have neglected a small term of higher order in $b(\boldsymbol{k}, l)$. To perform the angular part of the summation in Eq. (74), we calculate the current expectation value of the state in Eq. (73) and compare it with the well-known cross sections without and with bremsstrahlung:

$$\boldsymbol{j} = \frac{\hbar \boldsymbol{K}}{Mx^2}\left[1 + \sum_{kl} |b(k, l)|^2\right] = \boldsymbol{j}_0 \left(1 + \alpha A \frac{df}{f}\right) \tag{75}$$

where the quantum fluctuations have disappeared; $\alpha A = (2\alpha/3\pi)(\Delta \boldsymbol{v}/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent; and the $1/f$ dependence of the bremsstrahlung part displays the well-known infrared catastrophe, that is, the emission of a logarithmically divergent number of photons in the low-frequency limit. Here $\Delta \boldsymbol{v}$ is the velocity change $\hbar(\boldsymbol{K} - \boldsymbol{K}_0)/M$ of the scattered boson, and $f = ck/2\pi$ the photon frequency. Equation (74) thus gives

$$\langle S|O|S\rangle = \left|\frac{C}{x}\right|^4 \left(N(N-1) + 2(N-1)\alpha A[1 + \cos q(x_1 - x_2)]\frac{df}{f}\right) \tag{76}$$

which is the pair correlation function, or density autocorrelation function, along the scattered beam with $df/f = dq/q$. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency f. According to the Wiener–Khintchine theorem, we obtain the spectral density of fractional scattered particle density ρ, (or current j, or cross section σ) fluctuations in frequency f or wave number q by dividing the coefficient of the cosine by the constant term $N(N - 1)$:

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = \frac{2\alpha A}{fN} \tag{77}$$

where N is the number of particles or current carriers used to define the current j whose fluctuations we are studying. Quantum $1/f$ noise is thus a fundamental $1/N$ effect. The exact value of the exponent of f in Eq. (77) can be determined by including the contributions from all real and virtual multiphoton processes of any order (infrared radiative corrections), and turns out to be $\alpha A - 1$ rather than -1, which is important only philosophically, since $\alpha A \ll 1$. The spectral integral is thus convergent at $f = 0$.

For fermions we repeat the calculation, replacing in the derivation of Eq. (10) the commutators of field operators by anticommutators, which finally yields in the same way

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = \frac{2\alpha A}{f(N-1)} \tag{78}$$

which causes no difficulties, since $N \geq 2$ for particle correlations to be defined, and which is practically the same as Eq. (77), since usually $N \gg 1$. Equations (77) and (78) suggest a new notion of physical cross sections and process rates that contain $1/f$ noise and express a fundamental law of physics, important in most high-technology applications (16).

We conclude that the conventional quantum $1/f$ effect can be explained in terms of interference beats between the part of the outgoing de Broglie waves scattered without bremsstrahlung energy losses above the detection limit (given in turn by the reciprocal duration T of the $1/f$ noise measurement) on one hand, and the various parts scattered with bremsstrahlung energy losses; but there is more to it than that: exchange between identical particles is also important. This, of course, is just one way to describe the reaction of the emitted bremsstrahlung back on the scattered current. This reaction thus reveals itself as the cause of the quantum $1/f$ effect, and implies that the effect cannot be obtained with the independent-boson model. The effect, just like the classical turbulence-generated $1/f$ noise, is a result of the scale-invariant nonlinearity of the equations of motion describing the coupled system of matter and field. Ultimately, therefore, this nonlinearity is the source of the $1/f$ spectrum in both the classical and the quantum form of the author's theory. We can say that the quantum $1/f$ effect is an infrared divergence phenomenon, this divergence being the result of the same nonlinearity. The new effect is, in fact, the first time-dependent infrared radiative correction. Finally, it is also deterministic in the sense of a well-determined wave function, once the initial phases γ of all field oscillators are given. In quantum-mechanical correspondence with its classical turbulence analog, the new effect is therefore a quantum manifestation of classical chaos, which we can take as the definition of a certain type of quantum chaos.

PHYSICAL DERIVATION OF THE COHERENT QUANTUM 1/*f* EFFECT

This effect arises in a beam of electrons (or other charged particles propagating freely in vacuum) from the definition of the physical electron as a bare particle plus a coherent state of the electromagnetic field. It is caused by the energy spread characterizing any coherent state of the electromagnetic field oscillators, an energy spread that spells nonstationarity, that is, fluctuations. To find the spectral density of these inescapable fluctuations, which are known to characterize any quantum state that is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger's definition of coherent states, followed by a rigorous derivation from a well-known quantum-electrodynamical propagator. The chaotic character of these fluctuations is discussed in the last subsection of the next section.

The coherent quantum $1/f$ effect will be derived in three steps: First we consider a hypothetical world with just a single mode of the electromagnetic field coupled to a beam of charged particles; considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle density (or concentration) that arise from the nonstationarity of the coherent state. Then we calculate the amplitude with which this one mode is represented in the field of an electron, according to electrodynamics. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the electromagnetic field be characterized by the wave vector q, the angular frequency $\omega = cq$ and the polarization λ. Denoting the variables q and λ simply by q in the labels of the states, we write the coherent state (25,31,32) of amplitude $|z_q|$ and phase arg z_q in the form

$$|z_q\rangle = \exp(-\tfrac{1}{2}|z_q|^2)\exp(z_q a_q^\dagger)|0\rangle = \exp(-\tfrac{1}{2}|z_q|^2)\sum_{n=0}^{\infty}\frac{z_q^n}{n!}|n\rangle \tag{79}$$

Here $a_q^\dagger$ is the creation operator that adds one energy quantum to the energy of the mode. Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$,

$$|n\rangle = (2^n n!\sqrt{\pi})^{-1/2}\exp(-x^2/2)\,H_n(x)e^{in\omega t} \tag{80}$$

This yields for the coherent state $|z_q\rangle$ the representation

$$\begin{aligned}\psi_q(x) &= \exp(-\tfrac{1}{2}|z_q|^2)\exp\left(-\frac{x^2}{2}\right)\sum_{n=0}^{\infty}\frac{(z_q e^{i\omega t})^n}{[n!(2^n\sqrt{\omega})]^{1/2}}H_n(x)\\ &= \exp(-\tfrac{1}{2}|z_q|^2)\exp\left(-\frac{x^2}{2}\right)\exp(-z_q^2e^{-2i\omega t}+2xz_qe^{i\omega t})\end{aligned} \tag{81}$$

In the last form the generating function of the Hermite polynomials was used. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of z_q, is, for $|z_q| \ll 1$,

$$\begin{aligned}P_q(\tau,x) &= \langle|\psi_q|_t^2|\psi_q|_{t+\tau}^2\rangle\\ &= [1+8x^2|z_q|^2(1+\cos\omega\tau)-2|z_q|^2]\exp(-x^2/2)\end{aligned} \tag{82}$$

Integrating over x from $-\infty$ to ∞, we find the autocorrelation function

$$A^1(\tau) = 2^{-1/2}(1+2|z_q|^2\cos\omega\tau) \tag{83}$$

This result shows that the probability distribution contains a constant background with small superposed oscillations of frequency ω. Physically, the small oscillations in the total probability describe self-organization, or bunching, of the particles in the beam. They are thus more likely to be found in a measurement at some times and places than at others along the beam. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator, which is also an energy eigenstate, and therefore stationary and free of oscillations.

We now determine the amplitude z_q with which the field mode q is represented in the physical electron. One way to do this is to let a bare particle dress itself through its interaction with the electromagnetic field, by performing first-order perturbation theory with the interaction Hamiltonian

$$H' = A_\mu j^\mu = -\frac{e}{c}\boldsymbol{v}\cdot\boldsymbol{A}+e\phi \tag{84}$$

where $\boldsymbol{A}$ is the vector potential and ϕ the scalar electric potential. Another way is to Fourier-expand the electric potential e/r of a charged particle in a box of volume V. In both ways we obtain

$$|z_q|^2 = \pi(e/q)^2(\hbar cqV)^{-1} \tag{85}$$

Considering now all modes of the electromagnetic field, we obtain from the single-mode result of Eq. (83)

$$\begin{aligned}A(\tau) &= C\prod_q(1+2|z_q|^2\cos\omega_q\tau) = C\left(1+\sum_q 2|z_q|^2\cos\omega_q\tau\right)\\ &= C\left(1+\frac{4V}{2^3\pi^3}\,d^3q\,|z_q|^2\cos\omega_q\tau\right)\end{aligned} \tag{86}$$

Here we have again used the smallness of z_q, and we have introduced a constant C. Using Eq. (85) we obtain

$$\begin{aligned}A(\tau) &= C\left(1+4\pi\frac{V}{2^3\pi^3}\frac{4\pi}{V}\frac{e^2}{\hbar c}\frac{dq}{q}\cos\omega_q\tau\right)\\ &= C\left(1+2\frac{\alpha}{\pi}\cos\omega\tau\frac{d\omega}{\omega}\right)\end{aligned} \tag{87}$$

Here $\alpha = e^2/\hbar c$ is the fine structure constant $\approx 1/137$. The first term in the large parentheses is unity and represents the constant background, or the dc part of the current carried by the beam of particles through vacuum. The autocorrelation function for the relative (fractional) density fluctuations, or for the current density fluctuations in the beam of charged particles, is obtained therefore by dividing the second term by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener–Khintchine theorem, the coefficient of cos $\omega\tau$ is the spectral density of the fluctuations, $S^2_{|\psi|}$ for the particle concentration, or S_j for the current density $j = e(k/m)|\psi|^2$:

$$S^2_{|\psi|}\langle|\psi|^{-2}\rangle = S_j\langle j\rangle^{-2} = 2\frac{\alpha}{\pi fN} = 4.6\times 10^{-3} f^{-1}N^{-1} \tag{88}$$

Here we have included in the denominator the total number N of charged particles that are observed simultaneously, because the noise contributions from each particle are independent. This result is related to the conventional Q1/fE considered in the next section. A similar calculation yields the gravidynamical quantum 1/f effect (QGD 1/f effect) by substituting gravitons for the photons considered so far as infraquanta.

RIGOROUS DERIVATION OF THE COHERENT QUANTUM 1/*f* EFFECT

The present derivation is based on the well-known new propagator $G_s(x' - x)$ derived relativistically (33,34) in 1975 in a new picture required by the infinite range of the Coulomb potential. The corresponding nonrelativistic form (35) was provided by Zhang and Handel (see the last subsection under "Recent Results" below):

$$\begin{aligned} &-i\langle\Phi_0|T\psi_{s'}(x')\psi_s^\dagger(x)|\Phi_0\rangle \\ &\quad\equiv \delta_{ss'}G_s(x'-x) \\ &\quad= \frac{i}{V}\sum_{\boldsymbol{p}}\left(\exp i\frac{\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}') - \boldsymbol{p}^2(t-t')/2m}{\hbar}\right)n_{\boldsymbol{p}},s \\ &\qquad\times\left(-i\frac{\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}{\hbar} + i(m^2c^2+\boldsymbol{p}^2)^{1/2}(t-t')\frac{c}{\hbar}\right)^{\alpha/\pi} \end{aligned} \tag{89}$$

Here $\alpha = e^2/\hbar c \approx 1/137$ is Sommerfeld's fine structure constant, $n_{\boldsymbol{p},s}$ the number of electrons in the state of momentum $\boldsymbol{p}$ and spin s, m the rest mass of the fermions, $\delta_{ss'}$ the Kronecker symbol, c the speed of light, $x = (\boldsymbol{r}, t)$ any space–time point, and V the volume of a normalization box. T is the time-ordering operator, which orders the operators in the order of decreasing times from left to right and multiplies the result by $(-1)^P$, where P is the parity of the permutation required to achieve this order. For equal times, T normal-orders the operators, that is, for $t = t'$ the left-hand side of Eq. (89) is $i\langle\Phi_0|\psi_s^\dagger(x)\psi_{s'}(x')|\Phi_0\rangle$. The state Φ_0 of the N electrons is described by a Slater determinant of single-particle orbitals.

The resulting spectral density coincides with the result $2\alpha/\pi fN$, derived directly in the section above from the coherent state of the electromagnetic field of a physical charged particle. The connection with the conventional quantum 1/f effect is discussed in the section.

To calculate the current autocorrelation function we need the density correlation function, which is also known as the two-particle correlation function, and is defined by

$$\begin{aligned} &\langle\Phi_0|T\psi_s^\dagger(x)\psi_s(x)\psi_{s'}^\dagger(x')\Phi^{s'}(x')|\Phi_0\rangle \\ &\quad= \langle\Phi_0|\psi_s^\dagger(x)\psi_s(x)|\Phi_0\rangle\langle\Phi_0|\psi_{s'}^\dagger(x')\psi_{s'}(x')|\Phi_0\rangle \\ &\qquad- \langle\Phi_0|T\psi_{s'}(x')\psi_s^\dagger(x)|\Phi_0\rangle\langle\Phi_0|T\psi_s(x)\psi_{s'}^\dagger(x')|\Phi_0\rangle \end{aligned} \tag{90}$$

The first term can be expressed in terms of the particle density of spin s, $n/2 = N/2V = \langle\Phi_0|\psi_s^\dagger(x)\psi_s(x)|\Phi_0\rangle$, while the second term can be expressed in terms of the Green function Eq. (89) in the form

$$\begin{aligned} A_{ss'}(x-x') &\equiv \langle\Phi_0|\psi_s^\dagger(x)\psi_{s'}^\dagger(x')\psi_{s'}(x')\psi_s(x)|\Phi_0\rangle \\ &= (n/2)^2 + \delta_{ss'}G_s(x'-x)G_s(x-x') \end{aligned} \tag{91}$$

The *relative* autocorrelation function $A(x - x')$ describing the normalized pair correlation independent of spin is obtained by dividing by n^2 and summing over s and s':

$$\begin{aligned} &A(x-x') \\ &\quad= 1 - \frac{1}{n^2}\sum_s G_s(x-x')G_s(x'-x) \\ &\quad= 1 - \frac{1}{N^2}\sum_s\sum_{\boldsymbol{p}\boldsymbol{p}'} \\ &\qquad\left(\exp i\frac{(\boldsymbol{p}-\boldsymbol{p}')\cdot(\boldsymbol{r}-\boldsymbol{r}') - (E_{\boldsymbol{p}}-E_{\boldsymbol{p}'})(t-t')}{\hbar}\right)n_{\boldsymbol{p},s}n_{\boldsymbol{p}',s} \\ &\qquad\times\left(\frac{\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}{\hbar} - (m^2c^2+\boldsymbol{p}^2)^{1/2}(t-t')\frac{c}{\hbar}\right)^{\alpha/\pi} \\ &\qquad\times\left(\frac{\boldsymbol{p}'\cdot(\boldsymbol{r}'-\boldsymbol{r})}{\hbar} - (m^2c^2+\boldsymbol{p}'^2)^{1/2}(t'-t)\frac{c}{\hbar}\right)^{\alpha/\pi} \end{aligned} \tag{92}$$

Here we have used Eq. (89).

We now consider a beam of charged fermions (e.g., electrons), represented in momentum space by a sphere of radius p_F, centered on the momentum $\mathbf{p}_0$, which is the average momentum of the fermions. The energy and momentum differences between terms of different $\mathbf{p}$ are large, leading to rapid oscillations in space and time, which contain only high-frequency quantum fluctuations. The low-frequency and low-wave-number part A_l of this relative density autocorrelation function is given by the terms with $\boldsymbol{p} = \boldsymbol{p}'$:

$$A_l(x-x') = 1 - \frac{1}{N^2}\sum_s\sum_{\boldsymbol{p}} n_{\boldsymbol{p},s} \times\left|\frac{\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}{\hbar} - (m^2c^2+\boldsymbol{p}^2)^{1/2}(t-t')\frac{c}{\hbar}\right|^{2\alpha/\pi} \tag{93}$$

$$\approx 1 - \frac{1}{N}\left|\frac{\boldsymbol{p}_0\cdot(\boldsymbol{r}-\boldsymbol{r}')}{\hbar} - \frac{mc^2\tau}{\hbar}\right|^{2\alpha/\pi} \quad \text{for} \quad p_F \ll \left|p_{03} - \frac{mc^2\tau}{z}\right| \tag{94}$$

Here we have used the mean value theorem, considering the $2\alpha/\pi$ power as a slowly varying function of $\boldsymbol{p}$ and neglecting $\boldsymbol{p}_0$ in the coefficient of $\tau \equiv t - t'$, with $z \equiv |\boldsymbol{r} - \boldsymbol{r}'|$. The correlations propagate along the beam with a group velocity given by the average velocity $\boldsymbol{p}_0$/m of the particles in the beam, and with the phase velocity c^2/v. Using an identity in Ref. 36, we obtain from Eq. (94) with $\theta \equiv |\tau - \boldsymbol{p}_0 \cdot (\boldsymbol{r} - \boldsymbol{r}')/mc^2|$ the form

$$\begin{aligned} A_1(x-x') &= 1 - \frac{1}{N}\left|\frac{mc^2\theta}{\hbar}\right|^{2\alpha/\pi} = 1 - \frac{1.25}{N}|\Theta|^{2\alpha/\pi} \\ &= 1 - \frac{1.25}{N} e^{(2\alpha/\pi)\ \ln\ \Theta} \\ &\approx 1 - \frac{1.25}{N}\left(1 + \frac{2\alpha}{\pi}\ln\ \Theta\right) \\ &= 1 - \frac{2.5}{N} + \frac{1.25}{N}\left(1 - \frac{2\alpha}{\pi}\ln\ \Theta\right) \\ &\approx \frac{N-2.5}{N} + \frac{1.25}{N} e^{-(2\alpha/\pi)\ \ln\ \Theta} \\ &= \frac{1}{N}\left(N - 2.5 + \frac{2.5\alpha}{\pi\cos\alpha}\int_0 \frac{\cos\ \omega\Theta\ d\omega}{\omega^{1-2\alpha/\pi}}\right) \end{aligned} \tag{95}$$

This indicates a $\omega^{-1+2\alpha/\pi}$ spectrum and a $1/(N - 2.5)$ dependence of the spectrum of fractional fluctuations in density n and current j. The total error corresponding to the two linear approximations of exponentials in Eq. (95) is less than 1%, provided $|\ln\ \Theta| < 20$, or $(250{,}000)^{-1}$ h $< |\theta| <$ 250,000 h. Here $\Theta = \theta/(1\ \mathrm{s})$, and ω is the circular Fourier frequency in radians per second. We have used $[(1\ \mathrm{s})\ mc^2/\hbar]^{2\alpha/\pi} \approx 1.25$; this accounts also for the presence of the number 2.5 instead of the more usual number 2 in the final form. The form we have chosen here is more convenient for applications. The equivalent normal form would have been

$$A_1(x-x') \approx \frac{1}{N}\left(N - 2 + \frac{2\alpha}{\pi\cos\alpha}\int_0 \left(\frac{mc^2}{\hbar\omega}\right)^{2\alpha/\pi}\cos\ \omega\theta\frac{d\omega}{\omega}\right) \tag{96}$$

in which the error caused by the two linear approximations of exponentials would have been of the order of 20%, and in which the fractional power would also have been neglected in the integrand for all purposes except for the theoretical question of the integrability of the $1/\omega$ spectrum.

The fractional autocorrelation of current fluctuations δj is obtained by multiplying Eq. (92) on both sides by $(ep_0/m)^2$ and dividing by $(enp_0/m)^2$, which is the square of the average current density j, instead of just dividing by n^2. So it is the same as the fractional autocorrelation for quantum density fluctuations. The last form of Eq. (95) for the coherent quantum-electrodynamical chaos process in electric currents becomes

$$S_{\delta j/j}(k) \approx \frac{2.5\alpha}{\pi\omega(N-2.5)}\omega^{2\alpha\pi} \approx \frac{2.5\alpha}{\pi\omega N} = \frac{0.0058}{\omega N} \tag{97}$$

Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations. Most of the conventional quantum $1/f$ fluctuations in physical cross sections and process rates are also mobility fluctuations, but some are also in the recombination speed or tunneling rate.

RECENT RESULTS

Six recent developments are reported. They include (1) a first-principles proof of the absence of the Q1/*f*E in the process of photogeneration of carriers in photodetectors, (2) the verification of the quantum $1/f$ noise theory in quartz resonators, (3) the application of quantum $1/f$ noise to explain the anisotropy observed for conventional quantum $1/f$ noise in monocrystal silicon, (4) the derivation of the nonrelativistic propagator of QED, which predicts the presence of the coherent quantum $1/f$ effect, and (5) a clear formulation of the problem of transition between the coherent and conventional quantum $1/f$ effects. In addition we have improved our universal sufficient criterion for $1/f$ spectra in chaotic nonlinear systems, and (6) we have applied it to QED, obtaining the quantum $1/f$ effect as a consequence of the nonlinearity of the system formed by the charged particles together with the electromagnetic field.

Method Used

The derivation of the coherent nonrelativistic propagator of QED was performed in the picture introduced by Dollard in 1964, and uses the branch-point propagator introduced later by Zwanziger and Kibble (33,34). The derivation of the anisotropy of $1/f$ noise in monocrystalline silicon is based on the conventional quantum $1/f$ noise theory and the known structure of the conduction band of silicon in the Brillouin zone.

Results

Below we report the main results of the four recent achievements mentioned above.

First-Principles Proof of the Absence of the Quantum 1/*f* Effect in the Photogeneration of Carriers in Photodetectors. Quantum $1/f$ noise is a fundamental aspect of quantum mechanics, representing universal fluctuations of physical process rates R and cross sections σ given by the fractional (or relative) spectral density $S(f) = 2\alpha A/fN$. Therefore it is present in the process rates generating the dark current observed in junction photodetectors, such as *diffusion* (scattering cross sections fluctuate) in diffusion-limited junctions, and *recombination* in the recombination-limited regime. One is therefore tempted to expect similar fluctuations in the *photogeneration* of electron–hole pairs. However, as we show below, the corresponding quantum $1/f$ coefficient is zero, precluding the existence of quantum $1/f$ fluctuations in the photogeneration rate. Here N is the number of carriers used to define or measure the process rate or cross section considered.

For an arbitrary process involving a total of n incoming and outgoing charged particles, the nonrelativistic quantum $1/f$ coefficient is given (37) by

$$2\alpha A = \frac{4\alpha}{3\pi c^2}\sum_{i,j=1}^{n}\eta_i\eta_j q_i q_j(\boldsymbol{v}_i - \boldsymbol{v}_j)^2 \tag{98}$$

where the summation runs over the charges q_i and velocities $\boldsymbol{v}_i$ of all incoming ($\eta_i = -1$) and outgoing ($\eta_i = 1$) particles (altogether n of them) in the process whose Q1/*f*N we want to find, and α is Sommerfeld's fine structure constant, $e^2/\hbar c \approx 1/137$. In a photogeneration process a photon ($q = 0$) is absorbed, and a pair of oppositely charged particles is generated ($\eta = 1$) with velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, which either are zero or quickly decay to zero in a time negligible with respect to the reciprocal frequency at which we calculate the quantum $1/f$ noise. Thus in our case there are no incoming charged particles, and $n = 0 + 2 = 2$. The coefficient αA of a photogeneration process is therefore zero:

$$\alpha A_{\text{ph}} = (1,1) + (2,2) + (1,2) + (2,1)$$
$$= 0 + 0 + \frac{4\alpha}{3\pi c^2}(\boldsymbol{v}_1 - \boldsymbol{v}_2)^2 \approx 0 \qquad (99)$$

All photogenerated carriers of the right sign are collected in the well of the charge-coupled device, although they may generate quantum $1/f$ voltage fluctuations on their way. Since usually only the number of carriers collected at readout matters, no quantum $1/f$ noise will be observed in a photoelectric CCD as long as the dark current is negligible with respect to the photocurrent. This is in agreement with the experiments performed by Mooney (38). The same considerations apply to MIS photodetectors.

Verification of the Quantum 1/f Noise Theory in Quartz Resonators. According to the general quantum $1/f$ formula (2), $\Gamma^{-2}S_\Gamma(f) = 2\alpha A/f$, where $\alpha = e^2/\hbar c \approx 1/137$ and $A = 2(\Delta\boldsymbol{J}/ec)^2/3\pi$ is the quantum $1/f$ effect in any physical process rate Γ. Setting

$$\boldsymbol{J} = \frac{d\boldsymbol{P}}{dt} = \dot{\boldsymbol{P}} \qquad (100)$$

where $\boldsymbol{P}$ is the vector of the dipole moment of the quartz crystal, we obtain for the fluctuations in the rate Γ of phonon removal from the main resonator oscillation mode of the crystal (by scattering on a phonon from any other mode of average frequency (ω), or via a two-phonon process at a crystal defect or impurity, involving a phonon of average frequency (ω')) the spectral density

$$S_\Gamma(f) = \Gamma^2 4\alpha(\Delta\dot{\boldsymbol{P}})^2/3\pi e^2 c^2 \qquad (101)$$

where $(\Delta\dot{\boldsymbol{P}})^2$ is the square of the dipole moment rate change associated with the process causing the removal of a phonon from the main oscillator mode. To calculate it, we write the energy W of the interacting resonator mode (ω) in the form

$$W = n\hbar(\omega) = 2\frac{Nm}{2}\left(\frac{dx}{dt}\right)^2 = \frac{Nm}{e^2}\left(e\frac{dx}{dt}\right)^2 = \frac{m}{Ne^2}\epsilon^2\dot{\boldsymbol{P}}^2 \qquad (102)$$

The factor 2 includes the potential energy contribution. Here m is the reduced mass of the elementary oscillating dipoles, e their charge, ϵ a polarization constant, and N their number in the quartz crystal. Applying a variation $\Delta n = 1$, we get

$$\frac{\Delta n}{n} = 2\frac{|\Delta\dot{\boldsymbol{P}}|}{|\dot{\boldsymbol{P}}|}, \qquad \text{or} \quad \Delta\dot{\boldsymbol{P}} = \frac{\dot{\boldsymbol{P}}}{2n}$$

Solving Eq. (102) for $\dot{\boldsymbol{P}}$ and substituting, we obtain

$$|\Delta\dot{\boldsymbol{P}}| = \left(\frac{N\hbar(\omega)}{n}\right)^{1/2}\frac{e}{2\epsilon}$$

Substituting $\Delta\dot{\boldsymbol{P}}$ into Eq. (3), we get

$$\Gamma^{-2}S_\Gamma(f) = N\alpha\hbar(\omega)/3n\pi mc^2 f\epsilon^2 \equiv \Lambda/f \qquad (103)$$

This result is applicable to the fluctuations in the loss rate Γ of the quartz.

The corresponding resonance frequency fluctuations of the quartz resonator are given by (39)

$$\omega^{-2}S_\omega(f) = \frac{1}{4Q^4}\frac{\Lambda}{f} = \frac{N\alpha\hbar(\omega)}{12n\pi mc^2 f\epsilon^2 Q^4} \qquad (104)$$

where Q is the quality factor of the single-mode quartz resonator considered, and (ω) is not the circular frequency of the main resonator mode, ω_0, but rather the practically constant frequency of the average interacting phonon, considering both three-phonon and two-phonon processes. The corresponding $\Delta\dot{\boldsymbol{P}}$ in the main resonator mode has to be also included in principle, but is negligible because of the very large number of phonons present in the main resonator mode.

Equation (6) can be written in the form

$$S(f) = \beta V/fQ^4 \qquad (105)$$

where, with a moderate value $(\omega) = 10^8\ \text{s}^{-1}$ and with $n = kT/\hbar(\omega)$, $T = 300$ K, and $kT = 4 \times 10^{14}$,

$$\beta = \frac{N}{V}\frac{\alpha\hbar(\omega)}{12n\pi\epsilon^2 mc^2} = 10^{22}\frac{(1/137)(10^{-27}\times 10^8)^2}{12kT\times\pi\times 10^{-27}\times 9\times 10^{20}} = 1$$

This is in very good agreement with experiment (40).

Application of Quantum 1/f Noise to Explain the Anisotropy of Conventional Quantum 1/f Noise in Monocrystalline Silicon. The conduction band of silicon has six equivalent energy minima along the six $\langle 100\rangle$ directions in the reciprocal lattice, which is bcc. These directions correspond to [111] in the direct lattice, which is fcc. If an electric field is applied along the [111] direct lattice axis, along which the energy minima are located, a lot of easy umklapp intervalley scattering processes (g processes) will take place along the direction opposing the applied field, because in the reciprocal lattice the minima are at $0.85K$ from the center of the first Brillouin zone, so there is only $0.3K$ to the next minimum in the neighboring zone. Here K is the distance between the center and the edge of the Brillouin zone. But umklapp processes are associated with the largest conventional Q1/fE, because in the expression $(4\alpha/3\pi)(\hbar\,\Delta k/mc)^2$ we have $\Delta k = G = 2\pi/a$ for umklapp, while normal scattering processes have smaller Δk. Therefore, the [111] direction will yield the highest quantum $1/f$ noise for identical currents applied in different directions in a Si monocrystal. Experimentally it is well known that devices built on (100) silicon surfaces have lower $1/f$ noise than those built on (111) surfaces (41).

Derivation of the Nonrelativistic Propagator of Quantum Electrodynamics. The derivation of the coherent Q1/fE by us (42) in second quantization was done on the basis of a new picture of QED introduced by Dollard, Zwanziger, and Kibble (29,31,33,34,43,44). This new picture includes the long-range part of the Coulomb potential in the unperturbed Hamiltonian H_0. The result is a more complicated free particle and a new propagator with a branch point instead of a pole. We used a nonrelativistic form of this new propagator and obtained the universal spectral density of fractional current fluctuations $S_{\delta j/j}(f) = 2\alpha/3\pi fN$, which we called the coherent Q1/fE. The purpose here is to derive this nonrelativistic propagator from the well-known relativistic propagator based on Dollard's picture.

Our derivation is similar to the derivation of the nonrelativistic equation from Dirac's theory of the electron. It is based on the distinction between the large and small components of the Dirac spinor.

The relativistic propagator $S(x' - x)$ in the equation

$$\theta(t'-t)\psi^{\dagger}(x') = i\int S(x'-x)\gamma_0\psi^{\dagger}(x)\,d^3\boldsymbol{x} \tag{106}$$

is

$$S(x) = i(2\pi)^{-3}\int \frac{d^3\boldsymbol{p}}{2E}e^{ipx}(-ipx)^{\alpha/\pi}(i\gamma p - m) \tag{107}$$

and valid for very large time t'. In the nonrelativistic limit, the Dirac spinor can be written in the form

$$\psi^{\dagger}(x) = e^{-imc^2/h}\begin{bmatrix}\varphi(\boldsymbol{x})\\ \chi(\boldsymbol{x})\end{bmatrix} \tag{108}$$

So we get

$$\begin{aligned}\vartheta(t'-t)\psi^{\dagger}(x') &= i(-i)\iint \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - E(t'-t) - mc^2t}{h}\right)\\ &\quad\times(-ipx)^{\alpha/\pi}\frac{E\gamma_0 - ic\boldsymbol{p}\cdot\boldsymbol{\gamma} + mc^2}{2E}\beta\begin{bmatrix}\varphi(\boldsymbol{x})\\ \chi(\boldsymbol{x})\end{bmatrix}d^3\boldsymbol{x}\\ &= \iint \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - E(t'-t) - mc^2t}{h}\right)\\ &\quad\times(-ipx)^{\alpha/\pi}\frac{E + c\boldsymbol{p}\cdot\boldsymbol{\alpha} + \beta mc^2}{2E}\beta\begin{bmatrix}\varphi(\boldsymbol{x})\\ \chi(\boldsymbol{x})\end{bmatrix}d^3\boldsymbol{x}\end{aligned} \tag{109}$$

and then we have

$$\begin{aligned}\vartheta(t'-t)\begin{bmatrix}\varphi(\boldsymbol{x})\\ \chi(\boldsymbol{x})\end{bmatrix} &= \vartheta(t'-t)\psi^{\dagger}(x')e^{imc^2t'/h}\\ &\quad\iint \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - (E-mc^2)(t'-t)}{h}\right)\\ &\quad\times(-ipx)^{\alpha/\pi}\frac{E + c\boldsymbol{p}\cdot\boldsymbol{\alpha} + \beta mc^2}{2E}\beta\begin{bmatrix}\varphi(\boldsymbol{x})\\ \chi(\boldsymbol{x})\end{bmatrix}d^3\boldsymbol{x}\\ &= \iint \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - (E-mc^2)(t'-t)}{h}\right)\\ &\quad\times(-ipx)^{\alpha/\pi}\left(\frac{1}{2}\begin{bmatrix}\varphi\\ \chi\end{bmatrix} + \frac{c\boldsymbol{p}}{2E}\cdot\begin{bmatrix}\boldsymbol{\sigma}\chi\\ \boldsymbol{\sigma}\varphi\end{bmatrix} + \frac{mc^2}{2E}\begin{bmatrix}\varphi\\ -\chi\end{bmatrix}\right)d^3\boldsymbol{x}\end{aligned} \tag{110}$$

Furthermore, after using the nonrelativistic-limit spinor component relation

$$\chi \approx \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2mc} \tag{111}$$

we get

$$\begin{aligned}\vartheta(t'-t)\varphi(\boldsymbol{x}') &= \int d^3\boldsymbol{x}\left[\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - (\boldsymbol{p}^2/2m)(t'-t)}{h}\right)\right.\\ &\quad\left.(-ipx)^{\alpha/\pi}\right]\varphi(\boldsymbol{x})\end{aligned} \tag{112}$$

If we compare this with the equation

$$\vartheta(t'-t)\varphi(\boldsymbol{x}') = i\int d^3\boldsymbol{x}\,G(x'-x)\varphi(\boldsymbol{x}) \tag{113}$$

which defines the nonrelativistic propagator, we get for the latter

$$G(x'-x) = -i\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - (\boldsymbol{p}^2/2m)(t'-t)}{h}\right)(-ipx)^{\alpha/\pi} \tag{114}$$

The propagator with a phase factor is

$$\begin{aligned}G(x'-x) &= -i\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\exp\left(i\frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x}) - (\boldsymbol{p}^2/2m)(t'-t)}{h}\right)\\ &\quad\times(-i)^{\alpha/\pi+i\gamma}\left(-\frac{(m^2c^2+p^2)(t'-t)c}{h} + \frac{\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{x})}{h}\right)^{\alpha/\pi+i\gamma}\end{aligned} \tag{115}$$

This is just the nonrelativistic propagator used by us in the preceding section. It has a branch point instead of a pole. For $\boldsymbol{x} = \boldsymbol{x}'$,

$$G = -i\left(i\frac{mc^2}{h}(t'-t)\right)^{\alpha/\pi+i\gamma}\left(\frac{m}{2\pi i(t'-t)}\right)^{3/2} \tag{116}$$

This propagator expresses the essence of our coherent Q1/*f*E.

Formulation of the Problem of Transition between the Coherent and Conventional Quantum 1/*f* Effects. From the beginning of the theory of fundamental $1/f$ noise in semiconductors and metals two situations were distinguished (45). The first, applicable to small semiconductor samples and very small (mesoscopic) metallic samples, has most of the energy excess $Nmv_d^2/2$ present in the stationary state carrying a finite current through the sample (excess over the energy of the equilibrium state) contained in the sum of the individual kinetic energies of the N current carriers, $\sum_i mv_i^2/2$. Here the velocities v_i of the carriers of mass m contain a small drift term v_d. The second, applicable in larger semiconductor or metal samples, has most of that energy excess contained in the collective magnetic energy of the current carrying state, $(B^2/8\pi)\,d^3x = LI^2/2$. The ratio s of this magnetic energy to the kinetic energy excess is roughly equal (45,46) to the number of carriers N' per unit length of the sample, multiplied by the classical radius of the electron, $r_0 = e^2/mc^2$: $s = N'r_0$. This situation was considered already in our classical magnetic turbulence theory (45,47).

In the first situation conventional quantum $1/f$ noise is applicable for fluctuations in physical scattering cross sections σ, in physical process rates Γ, and in the mobility μ or diffusion coefficient D (the latter two only if exclusively limited by σ or Γ):

$$\sigma^{-2}S_\sigma(f) = \Gamma^{-2}S_\Gamma(f) = \mu^{-2}S_\mu(f) = 2\alpha A/fN \qquad (s \ll 1) \tag{117}$$

because in this case the coherent, collective term in the Hamiltonian is negligible. In the second case, however, the coherent Q1/fE (26) is dominant:

$$j^{-2}S_j(f) = \mu^{-2}S_\mu(f) = 2\alpha/\pi fN \qquad (s > 1) \tag{118}$$

because the incoherent kinetic term can be neglected.

For the intermediate case, an interpolation formula was proposed (46):

$$j^{-2}S_j(f) = \mu^{-2}S_\mu(f) = \frac{2\alpha}{fN}\left(\frac{A}{s+1} + \frac{s}{\pi(s+1)}\right) \tag{119}$$

which is heuristic. The main purpose of Ref. 48 is to discuss various avenues to derive the correct form for the intermediary situation, and to consider initially the problem of coherent quantum $1/f$ noise in the $s \le 1$ case.

For a finite sample or device Eq. (115) should be replaced by a propagator that approaches the classical free-particle propagator of the Schrödinger equation when the transverse sample size, or the number of particles per unit length of the sample, approaches zero. This would cause the coherent Q1/fE to become very small compared with the conventional quantum $1/f$ noise present in the beam, due to the particular way in which the beam was generated. A formula like the interpolation in Eq. (119) would then express the fact that conventional quantum $1/f$ is always present, but is masked in larger samples by the coherent Q1/fE. However, a formula with a size-dependent infrared parameter intermediate between the coherent and conventional limits of α/π and αA, present both in the coefficient and in the exponent, would express the same transition in a slightly different, physically more meaningful form:

$$j^{-2}S_j(f) = \mu^{-2}S_\mu(f) = \frac{2\beta}{f^{1-\beta}N} \qquad \text{with} \quad \beta = \frac{\alpha A}{s+1} + \frac{\alpha s}{\pi(s+1)} \tag{120}$$

So far we have not derived an expression equivalent to Eq. (120) in any way. However, the physical unity of coherent and conventional Q1/fEs speaks in favor of a more sophisticated relation than Eq. (119). This same physical content can be expressed in a slightly different way by noting that Eq. (115) is equivalent to a energy–momentum relation that is not sharp, allowing for quantum fluctuations of the rest mass of the charged particle, or of any other particle with infrared divergent coupling to a group of massless infraquanta. Describing these quantum fluctuations of the rest mass μ with the help of a distribution function $\rho(\mu)$ peaked at the measured rest mass m, we could attempt to write Eq. (115) in the form

$$\begin{aligned}
&-i\langle\Phi_0|T\psi_{s'}(x')\psi_s^\dagger(x)|\Phi_0\rangle \\
&\quad \equiv \delta_{ss'}G_s(x'-x) \\
&\quad = \frac{i}{V}\sum_{\boldsymbol{p}}\left(\exp\frac{\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}') - \boldsymbol{p}^2(t-t')/2m}{\hbar}\right)n_{\boldsymbol{p},s} \\
&\quad\quad \times\left(-i\frac{\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}{\hbar} + i(m^2c^2+\boldsymbol{p}^2)^{1/2}(t-t')\frac{c}{\hbar}\right)^{\alpha/\pi} \\
&\quad = \frac{i}{V}\int d\mu\,\rho(\mu)\sum_{\boldsymbol{p}}\left(\exp i\frac{\boldsymbol{p}(\boldsymbol{r}-\boldsymbol{r}') - \boldsymbol{p}^2(t-t')/2m}{\hbar}\right)n_{\boldsymbol{p},s}
\end{aligned} \tag{121}$$

The distribution function $\rho(\mu)$ can be used to transform various classical results calculated simply with the Schrödinger propagator into the corresponding quantum $1/f$ results.

To determine $\rho(\mu)$, we represent the nonrelativistic form (38) of the new QED propagator as a superposition of classical propagators, defined by an unknown mass distribution $\rho(\mu)$ that describes the fuzzy mass shell:

$$\begin{aligned}
&\exp\left\{\frac{im}{\hbar}\left[\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{r}') - \left(c^2+\frac{v^2}{2}\right)(t-t')\right]\right\} \\
&\quad\cdot\left\{\frac{im}{\hbar}\left[\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{r}') - \left(c^2+\frac{v^2}{2}\right)(t-t')\right]\right\}^{\alpha/\pi} \\
&\quad = \int_0^\infty d\mu\,\rho(\mu)\exp\left\{\frac{i\mu}{\hbar}\left[\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{r}') - \left(c^2+\frac{v^2}{2}\right)(t-t')\right]\right\}
\end{aligned} \tag{122}$$

Let $u = (1/\hbar)\,[\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{r}') - (c^2 + v^2/2)(t-t')]$. This allows us to simplify the above equation:

$$\int_0^\infty d\mu\,\rho(\mu)e^{i\mu u} = e^{imu}(imu)^{\alpha/\pi} \tag{123}$$

When we use $\mu' = \mu - m$, the equation becomes

$$\int_{-m}^\infty d\mu'\,\rho'(\mu')e^{i\mu' u} = (imu)^{\alpha/\pi} \tag{124}$$

Because $\rho'(\mu')$ is different from zero only around $\mu' = 0$ or $\mu = m$, we can extend the domain of integration:

$$\int_{-\infty}^\infty d\mu'\,\rho'(\mu')e^{i\mu' u} = (imu)^{\alpha/\pi} \tag{125}$$

Let us take the derivative with respect to u. This yields

$$\int_{-\infty}^\infty d\mu'\,\rho'(\mu')e^{i\mu' u}\cdot i\mu' = \frac{(\alpha/\pi)(im)^{\alpha/\pi}}{u^{1-\alpha/\pi}} \tag{126}$$

We can further simplify the above equation with the notation $\mu'\rho'(\mu') = X(\mu')$ and get

$$\int_{-\infty}^\infty d\mu'\,X(\mu')e^{i\mu' u} = \frac{(\alpha/\pi)m^{\alpha/\pi}}{(iu)^{1-\alpha/\pi}} \tag{127}$$

We can determine $X(\mu')$ by taking the Fourier transformation of the right-hand side,

$$
\begin{aligned}
X(\mu') &= \int_{-\infty}^{\infty} du \frac{(\alpha/2\pi^2)m^{\alpha/\pi}}{(iu)^{1-\alpha/\pi}} e^{-iu\mu'} = \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \int_{-\infty}^{\infty} du \frac{e^{-iu\mu'}}{u^{1-\alpha/\pi}} \\
&= \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \left(\int_{-\infty}^{0} du \frac{\cos(\mu' u) + i\sin(\mu' u)}{u^{1-\alpha/\pi}} \right. \\
&\quad \left. + \int_{0}^{\infty} du \frac{\cos(\mu'\mu) + i\sin(\mu' u)}{u^{1-\alpha/\pi}} \right) \\
&\underset{(u'=-u)}{=} \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \left(\int_{0}^{\infty} d(-u') \frac{\cos(-\mu' u') + i\sin(-\mu' u')}{(-u')^{1-\alpha/\pi}} \right. \\
&\quad \left. + \int_{0}^{\infty} du \frac{\cos(\mu' u) + i\sin(\mu' u)}{u^{1-\alpha/\pi}} \right) \qquad (128) \\
&= \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \left(\int_{0}^{\infty} du' \frac{-\cos(\mu' u') + i\sin(\mu' u')}{u'^{1-\alpha/\pi}} (-1)^{1-\alpha/\pi} \right. \\
&\quad \left. + \int_{0}^{\infty} du \frac{\cos(\mu' u) + i\sin(\mu' u)}{u^{1-\alpha/\pi}} \right) \\
&= \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \left([1-(-1)^{1-\alpha/\pi}] \int_{0}^{\infty} du \frac{\cos(\mu'\mu)}{u^{1-\alpha/\pi}} \right. \\
&\quad \left. + i[1+(-1)^{1-\alpha/\pi}] \int_{0}^{\infty} du \frac{\sin(\mu' u)}{u^{1-\alpha/\pi}} \right) \\
&= \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \left([1-(-1)^{1-\alpha/\pi}] \frac{\Gamma(\alpha/\pi)}{\mu'^{\alpha/\pi}} \cos\frac{\alpha}{2} \right. \\
&\quad \left. + i[1+(-1)^{1-\alpha/\pi}] \frac{\Gamma(\alpha/\pi)}{\mu'^{\alpha/\pi}} \sin\frac{\alpha}{2} \right) \\
&\qquad \text{(for } \mu' > 0) \qquad (129) \\
&= \frac{\alpha m^{\alpha/\pi}}{2\pi^2 i^{1-\alpha/\pi}} \left([1-(-1)^{1-\alpha/\pi}] \frac{\Gamma(\alpha/\pi)}{\mu'^{\alpha/\pi}} \cos\frac{\alpha}{2} \right. \\
&\quad \left. - i[1+(-1)^{1-\alpha/\pi}] \frac{\Gamma(\alpha/\pi)}{\mu'^{\alpha/\pi}} \sin\frac{\alpha}{2} \right) \\
&\qquad \text{(for } \mu' < 0) \qquad (130)
\end{aligned}
$$

Because both $1 + (-1)^{1-\alpha/\pi}$ and $\sin(\alpha/2)$ are much smaller than $1 - (-1)^{1-\alpha/\pi}$ and $\cos(\alpha/2)$, we can just use

$$X(\mu') = [1-(-1)^{1-\alpha/\pi}] \frac{\alpha\Gamma(\alpha/\pi)\cos(\alpha/2)}{2\pi^2 i^{1-\alpha/\pi}} \left(\frac{m}{\mu'}\right)^{\alpha/\pi} \qquad (131)$$

for all practical purposes. We thus conclude that the mass distribution function has to be

$$\rho(\mu) = \frac{\alpha\Gamma(\alpha/\pi)\cos(\alpha/2)}{\pi^2 i^{1-\alpha/\pi}} \frac{m^{\alpha/\pi}}{(\mu - m)^{1+\alpha/\pi}} \qquad (132)$$

This is a remarkable result. It allows us to approximate the effect of infrared radiative corrections on any electronic propagator by multiplying it by $\rho(\mu)$ and integrating over μ as was done with the free-particle propagator on the right-hand side of our first equation above. The result will represent an approximation of the physical electron's propagator corresponding to the problem at hand, that is, an approximation of the physical propagator including the infrared radiative corrections, which corresponds to the given potential in which the electron has to move, and which satisfies the given boundary conditions.

Application of the Universal Sufficient Criterion for 1/*f* Spectra in Chaotic Nonlinear Systems to Quantum Electrodynamics. The nonlinearity causing the $1/f$ spectrum of turbulence in both semiconductors and metals is caused by the reaction of the field generated by charged particles and their currents back on themselves. The same nonlinearity is present in QED, where it causes the infrared divergence, the infrared radiative corrections for cross sections and process rates, and the quantum $1/f$ effect. We shall prove this on the basis of our sufficient criterion for $1/f$ spectral density in chaotic systems.

Consider a beam of charged particles propagating in a well-defined direction, so that the Schrödinger equation describes the longitudinal fluctuations in the concentration of particles. Considering the nonrelativistic case, which is encountered in most quantum $1/f$ noise applications, we write in second quantization the equation of motion for the Heisenberg field operators ψ of the particles in the form

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}\boldsymbol{A}\right)^2\psi \qquad (133)$$

With the nonrelativistic form $\boldsymbol{J} = -i\hbar\psi^*\nabla\psi/m$ + (Hermitian conjugate), and with

$$\boldsymbol{A}(x,y,z,t) = \frac{\hbar}{2cmi} \cdot \frac{[\psi^*\nabla\psi - \psi\nabla\psi^*]}{|\boldsymbol{x}-\boldsymbol{x}'|} d^3x' \qquad (134)$$

where the small rectangular brackets are defined to include retardation, we obtain

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-i\hbar\nabla - \frac{e\hbar}{2c^2mi}\frac{[\psi^*\nabla\psi - \psi\nabla\psi^*]}{|\boldsymbol{x}-\boldsymbol{x}'|}d^3x'\right)^2\psi \qquad (135)$$

At very low frequencies or wave numbers the second term in the large parentheses is dominant on the right-hand side, being of order λ, while the first term is of order λ^{-1} when $\boldsymbol{x}$ is replaced by $\lambda\boldsymbol{x}$, giving

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\left(\frac{e\hbar}{2c^2m}\frac{[\psi^*\nabla\psi - \psi\nabla\psi^*]}{|\boldsymbol{x}-\boldsymbol{x}'|}d^3x'\right)^2\psi \qquad (136)$$

For $\boldsymbol{x}$ replaced by $\lambda\boldsymbol{x}$, and $\boldsymbol{x}'$ formally replaced by $\lambda\boldsymbol{x}'$, we obtain

$$
\begin{aligned}
i\hbar\frac{\partial\psi}{\partial t} &= -\frac{1}{2m}\left(\frac{e\hbar}{2c^2mi}\frac{[\psi^*(\nabla/\gamma)\psi - \psi(\nabla/\gamma)\psi^*]}{\lambda|\boldsymbol{x}-\boldsymbol{x}'|}\lambda^3 d^3x'\right)^2\psi \\
&= \lambda^2 H\psi = \lambda^{-p}H\psi \qquad (137)
\end{aligned}
$$

This satisfies our homogeneity criterion with $p = -2$, because if we also replace t with $\lambda^{-2}t$ on the left-hand side, λ drops out altogether, and the equation is invariant. Our sufficient criterion only requires homogeneity, with any value of the weight p, for the existence of a $1/f$ spectrum in chaos. Therefore, we expect partial self-ordering of the current carriers with long-range correlations leading to a universal $1/f$ spectrum of fundamental quantum current fluctuations (coherent quantum $1/f$ effect) and of fluctuations in physical cross sections and process rates, as derived in detail above. This is in agreement with the experimentally verified results of the quantum $1/f$ theory.

In conclusion, we realize that, both in classical and in quantum-mechanical nonlinear systems, the limiting behavior at low wave numbers is usually expressed by homogeneous functional dependences, leading to fundamental $1/f$

spectra on the basis of our criterion. This explains the ubiquity of the $1/f$ spectrum.

DEVELOPMENT OF SPECIAL MATERIALS FOR ULTRALOW-NOISE FET AND JUNCTION DEVICES

FET Devices

Consider, for example, the class of devices that are homogeneous in the direction of the current flow, such as FETs, including JFETs, MODFETs or HEMTs, and photoconductive detectors, as opposed to bipolar transistors, HJBTs, *pn* diodes, junction photodetectors, and other junction devices. The mobility quantum $1/f$ noise is determined in this class of devices by Eq. (57), with the various quantum $1/f$ coefficients α_i given by the results presented earlier for impurity scattering, acoustic-phonon scattering in indirect-bandgap semiconductors, various kinds of intervalley scattering with or without umklapp, and polar and nonpolar optical-phonon scattering. Ionized-impurity scattering consists of many small-angle scattering events, all with small velocity changes $\Delta\boldsymbol{v}$, and therefore also with a small value of the quantum $1/f$ coefficient α_i. On the other hand, intervalley scattering with or without umklapp causes large velocity changes, corresponding to wave-vector changes of the order of the fundamental reciprocal lattice vector $\boldsymbol{G}$, and a large quantum $1/f$ coefficient of the order $(4\alpha/3\pi)(\hbar G/mc)^2 = (4\alpha/3\pi)(\hbar\, 2\pi/amc)^2$, where a is the lattice constant and m the effective mass of the carriers. To reduce the $1/f$ noise of the resulting devices, one is interested in materials practically free of intervalley and umklapp scattering, even if this comes at the expense of a shorter lifetime of the carriers. One designs materials in which the mobility is limited mainly by ionized-impurity scattering. If this is not practicable due to other constraints, one takes advantage of the inverse square dependence of the intervalley- and umklapp-scattering quantum $1/f$ coefficients and chooses the conduction type (n or p) and the host material in order to maximize m. Finally, the $1/N$ dependence also favors materials with a large concentration of ionized impurities.

Junction Devices

On the other hand, for materials designed for use in junction devices, the last form of Eq. (65) requires a large lifetime of the minority carriers in the low-doping part of the device. In this case, the material must have particularly low concentrations of recombination centers, of point defects, of dislocations, and of other lattice defects. For this class of devices the elimination of surface recombination currents through surface passivation is very important, because volume recombination is much less noisy according to our equations.

DEVICE OPTIMIZATION FOR ULTRALOW $1/f$ NOISE

After the design of optimal materials for each class of solid-state devices, the next objective is the use of these materials and of the quantum $1/f$ theory for practical device optimization. The following is the present list of our principles of optimal quantum $1/f$ noise design, which we currently use in creating new technological prototypes of devices:

1. Avoid coherent-state quantum $1/f$ noise by device size reduction below the coherent limit. This size limit is concentration-dependent, as seen from the expression for the coherence parameter $s = 2e^2N'/mc^2 = 5 \times 10^{-13}$ $\text{cm}^{-1} \times N'$ defined in Eq. (27). $N' = nA$ is the number of carriers per unit length of the device in the direction of current flow. A is the cross-sectional area of the current-carrying device, and n is the concentration of carriers. For $s \ll 1$ we expect conventional quantum $1/f$ noise, while for $s \gg 1$ the much larger coherent-state quantum $1/f$ noise is to be expected. In conclusion: think submicron, think transversely ultrasmall.
2. Avoid control of the device current or voltage by elementary cross sections or process rates tested by a small number of carriers only. Indeed, the number of carriers interrogating the cross section or process rate appears in the denominator of both the conventional and coherent quantum $1/f$ noise formulae. In particular, avoid current concentrations in bottlenecks, and current inhomogeneities. In junction devices higher lifetimes of the carriers lead to an increase in the number of carriers present in the sample that have tested the current-controlling cross sections, and therefore lead to lower quantum $1/f$ noise.
3. Avoid control of a device exhibiting conventional quantum $1/f$ noise through elementary processes which involve large accelerations of the current carriers, or large velocity changes. The squared vector velocity change appears as a factor in the conventional quantum $1/f$ noise formula. For example, umklapp, intervalley, and lattice scattering are respectively worst, very bad, and bad, compared with ionized-impurity scattering, in terms of the fractional mobility fluctuations they yield. For a given scattering mechanism, choosing current carriers with a large effective mass will in general reduce the conventional quantum $1/f$ noise, because for the same momentum transfers this results in smaller accelerations. Bulk recombination control of the current through a *pn* junction will lead to lower quantum $1/f$ noise than having the current controlled even in part by surface recombination, because the surface recombination centers are in a high-localized-field region at the interface between bulk and the passivation layer. Therefore, the best passivation is one that reduces the number and the cross section of the surface recombination centers, while also providing the smallest surface potential jump.

On this basis a detailed identification and analysis of the various noise sources can be performed. In a next step, a figure of merit can be defined on the basis of the mission specification for the devices in the focal-plane array. Finally, material and design improvements are calculated and suggested, which optimize the figure of merit defined in the previous step. A similar sequence is applicable for quartz crystal resonators and SAW devices.

Use of these principles leads to lower $1/f$ device noise. The quantum $1/f$ theory can consequently be used for CAD optimization of $1/f$ device noise suppression.

DISCUSSION

We now have a clear understanding both of the general origin of fundamental $1/f$ spectra and of my practical engineering formulae $2\alpha A/fN$ and $2\alpha/\pi fN$ applicable to high-technology devices. No matter which device is concerned, if it is a high-technology device all trivial forms of instability and fluctuations have been eliminated, and the device will be limited in its performance by the fundamental quantum $1/f$ effect present in the elementary cross sections and process rates controlling the kinetics of the device. A Q$1/f$E research institute is needed to translate these fundamental discoveries into valuable practical breakthroughs in modern high-technology applications.

Many contributions to this field are included in the Proceedings of the IV to VI International Symposia on Quantum $1/f$ Noise (1990, 1992, and 1994) and in the session on "$1/f$ Noise and Quantum Chaos" at the March 1992 Meeting of the American Physical Society in Indianapolis. Continuing progress was reported in the Proceedings of the International Conference on Noise in Physical Systems and $1/f$ Noise, which were published 1975–1996; see the recent reviews (26,49).

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NOISE, RADIO. See RADIO NOISE.