HIGH-FIELD EFFECTS

Electrons and holes contribute to the charge transport in semiconductors, while ordinary charge transport in metals is restricted to electrons only. Another distinctive feature of semiconductor transport is its inherent nonlinearity in high electric fields. It is difficult to generate high electric fields in metals because of their high electrical conductivity and the necessity of the presence of high currents according to Ohm's law. In semiconductors, high fields can exist with current densities of the order of 10^5 A/cm^2, or even much lower, depending on their conductance, which at low temperatures may approach zero. Ryder and Shockley noticed (1,2) that semiconductor electron transport was extremely nonlinear in high electric fields. Strong deviations from the proportionality of current density to fields were measured in germanium at room temperature for fields as small as 1000 V/cm.

High field transport in semiconductors became then an area of considerable research. It was found that the nonlinear behavior (the deviation from Ohm's law) was due to an elevation of the energy of the charge carriers caused by the accelerating force of the electric field. Interactions with the lattice vibrations lower the energy of the charge carriers as Joule heat is transferred to the crystal lattice. However, electric fields always cause a finite rise of charge carrier energy above the equilibrium energy (corresponding to the temperature of the crystal lattice). This excess energy can sometimes also be described by a temperature, the temperature T_c of the charge carriers, which is larger than the temperature T_L of the crystal lattice. One speaks therefore of *hot carrier transport* in semiconductors.

The rise in charge carrier energy changes the conductance for two reasons. For one, a higher energy gives rise to significant changes in the interactions of the charge carriers with crystal imperfections that form scattering centers. The scattering rate influences the conductance directly. Increases or decreases of this rate lead to decreases or increases of the conductance, respectively. The second reason for variations in conductance with charge carrier energy arise from changes in the $E(\boldsymbol{k})$ relation. This function gives the connection between the energy E and the wave vector $\boldsymbol{k}$ of the charge carriers, which corresponds in classical mechanics to the energy–momentum relation. Since conductance is a sensitive function of $\partial E(\boldsymbol{k})/\partial k$, it changes with the energy of the electrons (or with T_c whenever a temperature of the carriers is well defined).

The most complete existing theory of high field transport in semiconductors involves the solution of a Boltzmann type equation (3). The use of this equation can be justified by invoking the dephasing of quantum coherence over distances that correspond to the feature sizes of semiconductor structures and devices. Consequently, the charge carriers do behave classically and can, in a way, be understood from the principles of classical mechanics. Quantum mechanics has then only a background role and determines, for example, the effective mass or the velocity of the electrons or gives justification to the existence of holes and hole transport. Fermi's "golden rule" of quantum mechanics is used to calculate the scattering rates and thus represents another quantum contribution. It is also easy to include the Pauli principle in such an equation and thus to approach some properties of a Fermi liquid. However, the assumption of weak perturbational interaction of the particles, which is basic to Boltzmann's derivation, must remain true, and indeed is a good approximation for most of the important semiconductor materials.

The basic phenomena of high field transport are nonlinear conductance (3); changed responses to magnetic fields (4); changed high-frequency response, including the dielectric function (5); and changed confinement in potential wells or at heterojunctions between different semiconductors (6).

In semiconductor devices, the high field effects are based on the same phenomena. For example, in transistors of the field effect family, the electric fields can be as high as 10^5 V/cm^2 and the corresponding carrier temperatures may reach $T_c = 10^4$ K (7). Such temperatures cause changes in the conductivity by orders of magnitude (mostly reductions) (3,5). Remember that T_c is the temperature of the charge carriers and

not of the crystal lattice, which would melt at such temperatures. T_L, the temperature of the crystal lattice, can also be raised, but is usually raised orders of magnitude less. This difference is possible because the systems of electrons and crystal lattice are only weakly coupled. Under certain circumstances the conductance can also be raised by hot electron phenomena, leading to speed advantages in devices (e.g. velocity overshoot phenomena) (8). Devices made of semiconductor heterojunctions also exhibit various forms of hot electron transfer (e.g. real-space transfer) between the different materials. These effects often give rise to a device performance degradation, but also can be used advantageously (6).

In the following, a general theory of high field transport is developed. The next section describes high field transport in bulk semiconductors (theory and experimental results), and finally an overview of major effects in devices is given.

THE BOLTZMANN EQUATION FOR HIGH FIELD SEMICONDUCTOR TRANSPORT

The following is a top-down approach to the theory of high field phenomena. The next section derives a modern Boltzmann equation including the most important quantum effects as derived from the energy band structure. The relevance of the various terms of this equation to hot electron phenomena are discussed and general ways to obtain solutions are briefly reviewed and referenced. Subsequently a more phenomenological discussion and analytical approximation of several high field effects is given.

Derivation of a Modern Boltzmann Equation

The Boltzmann equation derived here is more general than the original equation derived by Boltzmann. The only cases of weakly interacting charge carriers in solids that cannot be understood from this modified equation are those that involve macroscopic feature sizes (e.g., device boundaries) that are smaller than the quantum dephasing length under the given conditions. This dephasing length in silicon at room temperature is of the order of several hundred angstrom, but can be much smaller in high electric fields, approaching 20 Å for electron energies of about 1.5 eV. This means that for conventional devices operating with high fields, hot electron transport can be understood extremely well with the theory developed below even if the feature sizes are below 100 Å. Quantum effects such as tunneling can often be added to this theory by the Bardeen transfer Hamiltonian formalism (3,9).

The following derivation lacks complete mathematical rigor but is valid under widely varying circumstances. For example, we need not assume conservation of the number of particles, which is important in that electrons and holes can annihilate each other or can be created by light. We follow Boltzmann, but replace the velocity in seven-dimensional phase space by the wave vector $\boldsymbol{k}$ in the definition of a distribution function $f(\boldsymbol{k}, \boldsymbol{r}, \boldsymbol{t})$, meaning as usual the probability of finding charge carriers at $\boldsymbol{k}$, $\boldsymbol{r}$ and at time t in the volume elements $d\boldsymbol{k}$, dr, dt. Consider then a cube in $\boldsymbol{r}$ space and in $\boldsymbol{k}$ space as shown in Fig. 1.

We first calculate how many electrons arrive from the left and enter the cube through the left $dy\ dz$ plane, and how many leave at the corresponding plane on the right, all during a time period dt. Since the x-direction travel distance of electrons with velocity $\boldsymbol{v}$ is $v_x\ dt$, we have

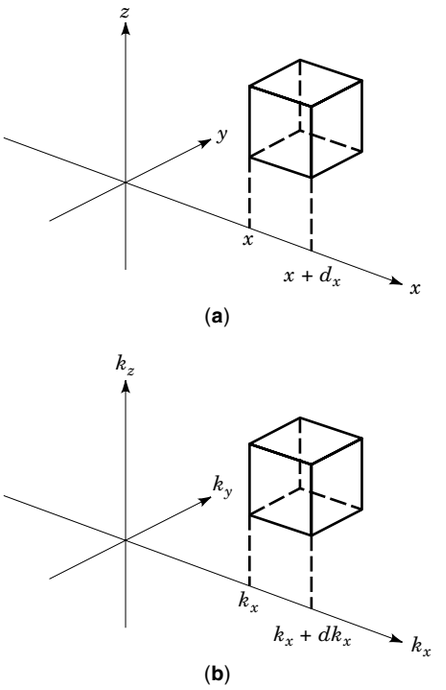

Figure 1. Cubes in (a) $\boldsymbol{r}$ space and (b) $\boldsymbol{k}$ space to illustrate the balance of incoming (e.g. into the y–z face) and outgoing (e.g. at $x + dx$) electrons (or charge carriers in general). The changes in $\boldsymbol{r}$ space are due to carrier velocity, those in $\boldsymbol{k}$ space due to acceleration by a force.

$$\text{incoming:} \qquad f(\boldsymbol{k}, \boldsymbol{r}, t)\, d\boldsymbol{k}\, dy\, dz\, v_x\, dt \tag{1}$$

$$\text{outgoing:} \qquad f(\boldsymbol{k}, (x + dx, y, z), t)\, d\boldsymbol{k}\, dy\, dz\, v_x\, dt \tag{2}$$

and the net particle gain is

$$\begin{aligned} &- v_x[f(\boldsymbol{k}, (x + dx, y, z), t) - f(\boldsymbol{k}, \boldsymbol{r}, t)]\, dy\, dz\, d\boldsymbol{k}\, dt \\ &= -v_x \frac{\partial f}{\partial x} dx\, dy\, dz\, d\boldsymbol{k}\, dt \\ &= -\boldsymbol{v} \cdot \nabla f\, d\boldsymbol{k}\, d\boldsymbol{r}\, dt \end{aligned} \tag{3}$$

in 3 dimensions.

Note that the velocity $\boldsymbol{v}$ is here equal to the group velocity and is related to $\boldsymbol{k}$ by $\boldsymbol{v} = \nabla_k E(\mathbf{k})/\hbar$ (3).

This balance depends on the velocity and $\boldsymbol{k}$ vector of the electrons and therefore on how "hot" the electrons are. It also describes diffusion, since concentration gradients enter this term. The relation between diffusion and electron heating is actually a complicated one and will be discussed below phenomenologically.

In analogous manner, we obtain the change of the number of electrons at $\boldsymbol{k}$ in $\boldsymbol{k}$ space because of accelerations. Replacing dx by dk_x etc. as illustrated in Fig. 1, and replacing $dx/dt = v_x$ as used in Eq. (1) by dk_x/dt, one gets

$$-\frac{d\boldsymbol{k}}{dt} \cdot \nabla_{\boldsymbol{k}} f\, d\boldsymbol{k}\, d\boldsymbol{r}\, dt \tag{4}$$

where $d\hbar/dt = -e\boldsymbol{F}$ (3) and $\boldsymbol{F}$ is the electric field. This term represents the major energy supply to the electrons and

therefore the root cause of the heating and hot electron effects.

There is still another possibility to change the number of electrons with wave vector $\boldsymbol{k}$ at $\boldsymbol{r}$. The electrons can be scattered and change their wave vector from $\boldsymbol{k}$ to $\boldsymbol{k}'$ at a given point $\boldsymbol{r}$ in real space. Figure 2 shows the two infinitesimal volumes in $\boldsymbol{k}$ space to illustrate the scattering events. The outgoing (out of state $\boldsymbol{k}$) electrons are

$$\text{out} = -\sum_{\boldsymbol{k}'} S(\boldsymbol{k}, \boldsymbol{k}') f(\boldsymbol{k}, \boldsymbol{r}, t)\, d\boldsymbol{k}\, d\boldsymbol{r}\, dt \tag{5}$$

The factor $f(\boldsymbol{k}, \boldsymbol{r}, t)$ is necessary because an electron has first to be in the $\boldsymbol{k}$ state to be scattered out. In degenerate systems (Fermi statistics), an additional factor $1 - f(\boldsymbol{k}', \boldsymbol{r}, t)$ arises from the Pauli principle. The incoming (into the $\boldsymbol{k}$ state) electrons are

$$\text{in} = \sum_{\boldsymbol{k}'} S(\boldsymbol{k}', \boldsymbol{k}) f(\boldsymbol{k}', \boldsymbol{r}, t)\, d\boldsymbol{k}\, d\boldsymbol{r}\, dt \tag{6}$$

Again, the Pauli principle will call for a factor $1 - f(\boldsymbol{k}, \boldsymbol{r}, \boldsymbol{t})$. The in and out scatterings lead to a (more or less) random distribution of the $\boldsymbol{k}$ vector. The energy gained from the field is therefore distributed in $\boldsymbol{k}$ space and receives thus a random temperaturelike component. The interaction with lattice vibrations is inelastic and also changes the energy (magnitude of $\boldsymbol{k}$). This energy loss represents Joule heat and determines, together with the force, the actual average electron energy (temperature).

The Boltzmann equation describes all of these heating–cooling dynamics and is obtained by balancing the particle numbers and the change in f given by the net change of incoming and outgoing particles. Therefore, we have

$$\begin{aligned}\frac{\partial f(\boldsymbol{k}, \boldsymbol{r}, t)}{\partial t} &= -\boldsymbol{v} \cdot \nabla f(\boldsymbol{k}, \boldsymbol{r}, t) - \frac{1}{\hbar} \boldsymbol{F}_0 \cdot \nabla_{\boldsymbol{k}} f(\boldsymbol{k}, \boldsymbol{r}, t) \\ &\quad + \sum_{\boldsymbol{k}'} [f(\boldsymbol{k}', \boldsymbol{r}, t) S(\boldsymbol{k}', \boldsymbol{k}) - f(\boldsymbol{k}, \boldsymbol{r}, t) S(\boldsymbol{k}, \boldsymbol{k}')]\end{aligned} \tag{7}$$

where $\boldsymbol{F}_0$ is the force ($-e\mathbf{F}$ for an electric field $\boldsymbol{F}$).

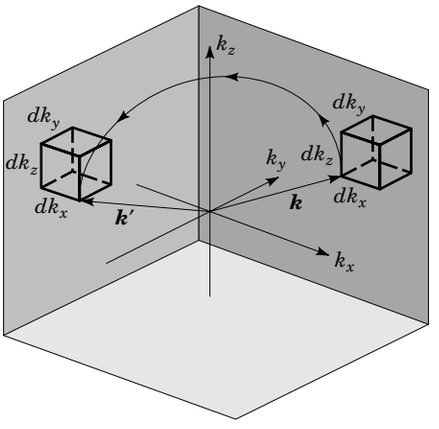

Figure 2. Schematic of a scattering process. The charge carrier scatters from a volume $dk_x\, dk_y\, dk_z$ at $\boldsymbol{k}$ to another such volume at $\boldsymbol{k}'$.

If we include the factors arising from the Pauli principle as discussed above, we arrive then at

$$\begin{aligned}\frac{\partial f(\boldsymbol{k}, \boldsymbol{r}, t)}{\partial t} &= -\frac{1}{\hbar} \nabla_{\boldsymbol{k}} E(\boldsymbol{k}) \cdot \nabla f(\boldsymbol{k}, \boldsymbol{r}, t) - \frac{1}{\hbar} \boldsymbol{F}_0 \cdot \nabla_{\boldsymbol{k}} f(\boldsymbol{k}, \boldsymbol{r}, t) \\ &\quad + \sum_{\boldsymbol{k}'} \{f(\boldsymbol{k}', \boldsymbol{r}, t)[1 - f(\boldsymbol{k}, \boldsymbol{r}, t)] S(\boldsymbol{k}', \boldsymbol{k}) \\ &\quad - f(\boldsymbol{k}, \boldsymbol{r}, t)[1 - f(\boldsymbol{k}', \boldsymbol{r}, t)] S(\boldsymbol{k}, \boldsymbol{k}')\}\end{aligned} \tag{8}$$

This equation is very general and includes automatically, for example, effects of the energy band structure as described by $E(\boldsymbol{k})$. Of course, it describes transport in one band only (3). The functional form of this band, however, can be arbitrary. Effects of strain in the solid, for example, need only be included in the band structure and then are automatically taken into account in the distribution function f once the Boltzmann equation is solved. From the distribution function, one can in turn obtain macroscopic quantities such as the electronic current in the usual prescribed way.

To summarize, all terms of this equation have special significance for hot electron effects. The second term on the right-hand side (RHS) of Eq. (8) represents the driving force, the electric field $\boldsymbol{F}$, and signifies the heating of the electrons. The third term on the RHS describes the scattering and how momentum and energy are distributed. This term signifies the influence of hot electrons on the classical conductivity. Electrons are scattered out of a given range of the wave vector $\boldsymbol{k}$ or scattered into that range from all other possible values of $\boldsymbol{k}$, denoted by $\boldsymbol{k}'$. This scattering term makes the Boltzmann equation an integrodifferential equation. Therefore, to solve it, one needs a tenfold integral over time, space $\boldsymbol{r}$, and $\boldsymbol{k}$ as well as $\boldsymbol{k}'$. The tenfold numerical integration can probably be done best by Monte Carlo methods, though approximations in lower dimensions may permit the use of computationally more efficient numerics. The first term on the RHS represents space-dependent effects such as diffusion and indicates that the hot electron diffusion not only is dependent on the heating of the electrons but in turn influences the heating. This term is relevant to the question of how the force heats the electrons. For example, a confining force that does not give rise to a current (as, e.g., at a potential minimum) may not heat the electrons, since for this case the accelerating drift (second term on RHS) and diffusion (first term on RHS) can cancel exactly (10,3,11).

The energy band structure enters the Boltzmann equation through the $E(\boldsymbol{k})$ relation in the first term on the RHS. It also enters indirectly through the sum over $\boldsymbol{k}$, since this sum includes the density of states. Finally, if one wants to calculate a current, one needs to integrate the product of velocity and distribution function over all $\boldsymbol{k}$ space, where the velocity is given by $\boldsymbol{v} = \nabla_{\boldsymbol{k}} E(\boldsymbol{k})/\hbar$.

A full solution of the Boltzmann equation as derived above does describe all hot electron phenomena currently known in semiconductors. It can be and has been achieved numerically in bulk semiconductors (12) and in devices (13). We refer the reader to these references and to software and explanations available on Web sites (14). Here we continue with discussions of important experimental results in terms of approximate concepts and solutions.

Approximate Solutions of the Boltzmann Equation and Hot Electrons

Since the Boltzmann equation is an integrodifferential equation, precise and explicit solutions can be found only under

very special circumstances. The best-known example is the time-independent solution for homogeneous systems (no space-dependent terms) in the relaxation time approximation. Under the assumption of weak forces (electric fields), one can write the distribution function as a sum of a function f_0 that is even in the wave vector $\boldsymbol{k}$ and an odd function f_1. Assuming that the Pauli terms are negligible, as they are for not too high carrier concentrations, the whole collision integral of Eq. (8) reduces then to

$$\text{collision integral} = f_1/\tau_{\text{tot}} \tag{9}$$

with

$$\frac{1}{\tau_{\text{tot}}} = \sum_{\boldsymbol{k}'} S(\boldsymbol{k}, \boldsymbol{k}') \tag{10}$$

and f_0 being equal to the equilibrium Boltzman distribution given by

$$f_0 = \exp(E_{\text{F}}/kT_{\text{L}}) \exp(-E/kT_{\text{L}}) \tag{11}$$

The odd part of the distribution function that determines the electric current is then

$$f_1 = -\tau_{\text{tot}} \frac{\boldsymbol{F}_0}{\hbar} \cdot \nabla_{\boldsymbol{k}} e^{(E_{\text{F}}-E)/kT} \tag{12}$$

with E_{F} being the Fermi level. The electric current density j is then obained from

$$j = -\frac{e}{4\pi^3} \int \boldsymbol{v} f_1 \, d\boldsymbol{k} \tag{13}$$

The high field or hot electron term is neglected in this approach, which describes only low fields and ohmic behavior. The reason is the approximation of f_0 by the equilibrium distribution. To allow for high electric fields, one needs to solve the full Boltzmann equation or at least the coupled equations resulting for both f_0 and f_1 in the relaxation time approximation. As described above, the force accelerates the electrons (holes), and the scattering randomizes, thus causing f_0 to contain more energetic electrons and therefore changing its form away from the equilibrium.

As mentioned, the complete solution of Eq. (8) has been achieved by so-called Monte Carlo methods, which are related to the Monte Carlo integration known from numerical mathematics (3,5). These solutions have been described at length in the literature and form a vast field (5,12,13,14).

Approximate solutions of the Boltzmann equation for high electric fields are also well known. Of particular importance is the electron temperature approximation, which is described in the next section together with typical experimental results for high field transport in bulk (homogeneous) semiconductors.

HOT ELECTRON EFFECTS IN SEMICONDUCTORS AND APPROXIMATE THEORY

Electron Temperature and Scattering Rate

A first glimpse of how important hot electrons would be in semiconductor transport was given by the work of Shockley (1951) (2) and Ryder (1953) (1), who found a saturation of the electron current in germanium at electric fields around 1000 V/cm as shown in Fig. 3. This current saturation is basic to hot electron phenomena and has been shown to arise from the increase of average electron energy $\langle E \rangle$. Under certain assumptions that are approximately satisfied in n-type silicon and germanium for intermediate electric fields (3), one can represent the average energy by a temperature T_{c}, which, for not too high electron densities, can be approximated by a Boltzmann type formula:

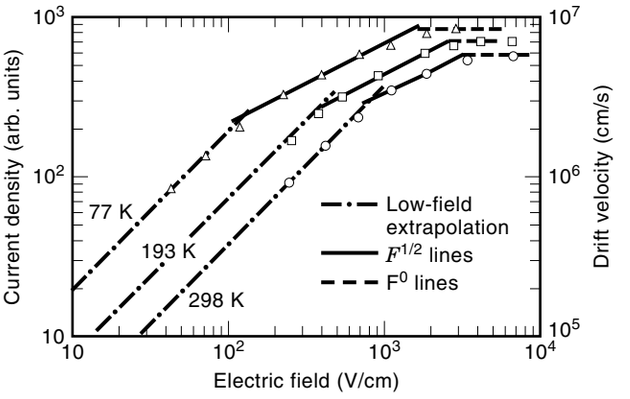

Figure 3. Current density (or drift velocity) versus electric field in bulk n-type germanium for three temperatures (as indicated). Notice the saturation at electric fields above 10^3 V/cm. (After Ref. 1.) Reprinted with permission, © 1953 by the American Physical Society.

$$\langle E \rangle = \tfrac{3}{2} k_{\text{B}} T_{\text{c}} \tag{14}$$

where k_{B} is Boltzmann's constant. The actual solution of the Boltzmann equation to arrive at the electron temperature concept is involved, and the reader is referred to Ref. 3 for detailed information.

An approximate formula for T_{c} is (3)

$$T_c \approx T_L \left[1 + \left(\frac{F}{F_{\text{c}}} \right)^2 \right] \tag{15}$$

F_{c} is a critical electric field that is around 10^4 V/cm for silicon at room temperature. The carrier temperature T_{c} can therefore become extremely high. For example, at the field $F = 2 \times 10^4$ V/cm, which can easily be reached and indeed is routinely reached in modern transistors, we have $T_{\text{c}} = 1500$ K for $T_{\text{L}} = 300$ K. Such temperatures have indeed been measured by various methods that can sense the electron energy inside the semiconductor (15). An outside touch does not reveal a temperature increase of the electrons, because of the large work function that the electrons would need to overcome in order to propagate out of the semiconductor (16). Equation (11) is not valid for much higher electric fields than 3×10^4 V/cm for silicon because of band structure effects such as nonparabolicity (5). For very high fields, a full band Monte Carlo approach is again necessary to calculate the average energy and distribution functions of the electrons reliably.

The rise of the electron temperature (or energy) causes a change in the scattering rate that enters the mobility μ of the

charge carriers and therefore the conductivity $\sigma = en\mu$ (n being the carrier concentration). The mobility derives from the relaxation time as defined in Eq. (10) if the scattering is randomizing (independent of the wave vector). Otherwise τ_{tot} needs to be replaced by the so-called momentum relaxation time (3). This relaxation time usually exhibits a significant energy dependence. For phonon scattering it decreases typically with increasing energy, while for scattering by weakly screened Coulomb charges it increases. The mobility μ is proportional to a weighted energy average of the relaxation time, and the conductivity therefore increases or decreases proportionally.

A typical dependence of the mobility on the electron temperature is then (for the case of scattering by phonons) (3)

$$\mu \approx \mu_0 \left(\frac{T_L}{T_c}\right)^{1/2} \tag{16}$$

where μ_0 is the mobility at zero electric field, i.e. for $T_L = T_c$. Using, then, the equation for the current density of a homogeneous semiconductor, $j = en\mu$, one gets from Eq. (16) and Eq. (15)

$$j = en\mu_0 \left(\frac{F}{1 + (F/F_c)^2}\right)^{1/2} \tag{17}$$

This equation shows immediately the essential feature of current saturation (nonlinear transport) at high electric fields and is quite general. For example, the square root in Eq. (16) derives from the density of states. Equation (17) is valid even for a more general density of states; it can be proven easily for any density of states that is proportional to E^p for any $p > 0$. The dependence of the electron phonon scattering rate on the density of states is illustrated below (Fig. 9). Note, however, that in devices other scattering mechanisms such as surface roughness scattering are of great importance. Some of these mechanisms are reviewed in Ref. 16.

Effects of Band Structure and the Gunn Effect

The derivation of Eq. (17) contains only changes in the scattering rate with carrier temperature. As mentioned above, however, the band structure influences not only the scattering rate (via the density of states) but also the velocity $\boldsymbol{v}$ of the electrons, since in the one-band approximation we have (3)

$$\boldsymbol{v} = \frac{1}{\hbar}\nabla_{\boldsymbol{k}} E(\boldsymbol{k}) \tag{18}$$

For the typical band structure of some III–V compounds, this leads to pronounced effects in the current–voltage characteristics. For GaAs, the effective mass is small at low energies ($m^* = 0.067m_0$) and $E(\boldsymbol{k}) = \hbar^2k^2/2m^*$. Correspondingly, the velocity of electrons becomes very high at moderate energies. However, the band structure changes drastically only 0.3 eV above the conduction band edge, exhibiting there additional minima with much higher effective mass (close to the germanium type conduction), and above 0.5 eV, even higher effective masses (close to the silicon conduction mass). In simple terms, GaAs contain three "highways," each one becoming available as the energy increases and each one exhibiting higher mass. If the electrons are heated by electric fields, they initially are on the fastest highway, exhibiting a high mobility and conductance. At higher fields they are heated enough to transfer to the germaniumlike conduction band minimum (highway 2) and at still higher fields to the siliconlike minimum (highway 3). This decreases their speed so drastically that GaAs actually shows a range of negative differential resistance, i.e., the current drops as the electric field is increased. This phenomenon leads to the Gunn effect, which manifests itself by high-frequency current oscillations, since the situation of negative differential resistance is not stable (7). These phenomena have received considerable attention, and a large framework of research exists (7,18).

The current–field characteristic of GaAs is shown in Fig. 4 and compared with the characteristics of silicon (both for electron transport). It is evident that the low field current behavior of GaAs transforms into siliconlike behavior at high electric fields, with a region of negative differential resistance in between. The reader is referred to Refs. (3,5,7) for more information.

A word of caution should be added here. There is a lack of direct experimental verification of band structure and related semiconductor parameters for high-field transport. Optical measurements and femtosecond spectroscopy (19) do give detailed information on the density of states.

Real–Space Transfer

The negative differential resistance of GaAs and the Gunn effect are mostly determined by the structure of the $E(\boldsymbol{k})$ relation, i.e. by effects in $\boldsymbol{k}$ space. The terms of the Boltzmann equation signifying real-space operations have not been discussed yet. From the discussions following Eq. (8) it is clear, however, that real-space effects complicate hot electron transport significantly. The prime reason for this complication is the real-space transfer effect (6), which is described here.

The transfer of electrons between two different solids is known from Bethe's thermionic emission theory (3). This type of transport includes only electric fields perpendicular to the different layers of semiconductors. However, electric fields parallel to semiconductors can energize the charge carriers (hot electron effect) and lead to a redistribution of them in

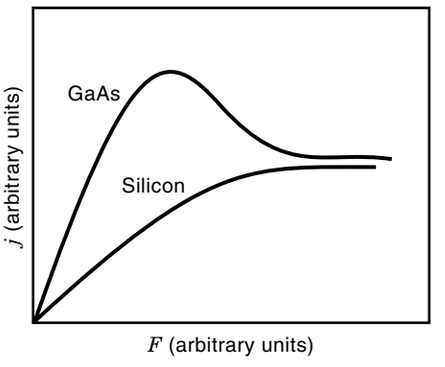

Figure 4. Schematic of current density versus electric field for homogeneous (bulk) GaAs and silicon. Notice the negative differential resistance of GaAs [which leads to instabilities (7) and inhomogeneities of the carrier density]. Also note that at very high fields the GaAs and silicon curves approach each other. Depending on temperature and electric field they can even cross. However, they are always close at very high fields.

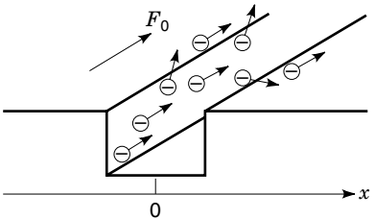

Figure 5. Electrons in a material with lower conduction band edge (e.g., GaAs) neighboring two layers with higher conduction band edge (e.g., AlAs) accelerated by a force $\boldsymbol{F}_0$. The electrons gain energy and are scattered and then transfer out of the central material layer, thus exhibiting real-space transfer.

the different layers that depends sensitively on the difference of the conduction band edges of the various materials. (Readers not used to solid state concepts should think of the conduction band edge as the minimum kinetic energy of conduction electrons, which is different in different materials, the electrons thus having different potential energy in each material). The effect of redistribution of electrons or holes due to electric fields parallel to different layers of semiconductors is called real-space transfer (RST) and is shown schematically in Fig. 5. This transfer of electrons (heated by parallel fields) over barriers is more complicated and more difficult to understand than other effects basic to nonlinear semiconductor transport and device operation. The reason is that RST can be visualized only by the combination of two concepts related to the energy distribution of electrons. The first concept is that of quasi-Fermi levels (3), and the second is the concept of a charge carrier temperature T_c as already discussed above. For RST problems, both concepts matter, and both the carrier temperature and the quasi-Fermi levels are a function of space coordinate and time.

Imagine, for example, electrons residing in a layer of high-mobility GaAs neighboring, on either side, two layers of low-mobility AlAs (Fig. 5). The GaAs equilibrium distribution function f_0 is

$$f_0 = \exp(-E/kT_L) \tag{19}$$

while in the AlGaAs we have

$$f_0 \propto \exp\left(-\frac{\Delta E_c + E}{kT_L}\right) \tag{20}$$

Here the energy is measured from the GaAs conduction band edge, and ∇E_c is the band edge discontinuity between AlAs and GaAs.

If now the electrons are heated by an external field parallel to the layers, we have to replace T_L in Eqs. (19) and (20) by a space-dependent carrier temperature T_c. It is clear that for $T_c \to \infty$ the difference between the AlAs and the GaAs population densities vanishes. In other words, the electrons will spread out into the AlAs layers. This also means that even perpendicular to the layers (z direction) a constant Fermi level cannot exist, and E_F has to be replaced by the quasi-Fermi level $E_{QF}(z)$ as the density of electrons becomes a function of $T_c(z)$. This is unusual, since commonly the quasi-Fermi levels differ only in the direction of the applied external voltage V_{ext} (by the amount eV_{ext}). In the present case, a voltage is applied parallel to the layers, the electrons redistribute themselves perpendicularly to the layers, and a field (and voltage perpendicular to the layers) develops owing to the carrier redistribution. Basic to the calculation of this process are the thermionic emission currents (3) of hot electrons from one layer to the other. Since the external voltage is applied parallel to the layers, we have in steady state a precise balance of currents flowing from left to right and right to left, which determines the z-dependent carrier population.

A complication of the theory is presented by the necessity (in most cases) of having to solve Poisson's equation as charge is transferred. For typical parameters of the GaAs–AlAs material system and electric fields of the order of 10^3 to 10^4 V/cm parallel to the layers, one obtains time constants of the order of picoseconds for the transfer, which gives the RST effect importance for device applications [RST transistors as developed by Luryi and Kastalsky (6)].

The real-space transfer effect is also of general importance in all situations when electrons are confined in potential wells and parallel fields are applied (and accelerate the charge carriers), even if the electrons do not propagate out of the wells but merely redistribute themselves within each well. This is of relevance for the understanding of the influence of transverse fields (such as the gate field) in a transistor (10). The RST effect and the spreading of the electrons are then determined by the transverse field. The quantum analog of this classical picture is the redistribution of hot electrons in the different quantized subbands of a quantum well.

Time Dependences, Velocity Overshoot, and Ballistic Transport

As complex as the considerations of nonlinear transport in $\boldsymbol{r}$ space and $\boldsymbol{k}$ space are already, time dependences add to the richness of hot electron effects. All the above discussions were essentially valid for the steady state only. On short time scales, however, the mode of transport changes its type due to transitions from the ballistic to the overshoot regime and to diffusive transport.

Diffusive transport is the well-known mode for which the mobility is proportional to the average of τ_{tot}. This proportionality implies the validity of the Boltzmann equation, which, as discussed at the beginning, involves the dephasing of the wave function. For times much shorter than τ_{tot}, the transport is essentially *ballistic,* which means that in crystalline semiconductors the accelerations are described by the equation

$$\hbar\frac{d\boldsymbol{k}}{dt} = -e\boldsymbol{F} \tag{21}$$

where $\boldsymbol{F}$ is the electric field and $\hbar\boldsymbol{k}$ is the crystal momentum. This equation becomes invalid at extremely short times, as described in Ref. 20, but is applicable under most circumstances. Neglecting the basic thermal motion of the electrons (due to T_L), this means that all electrons are accelerated in the direction of the force and gain speed according to Eq. (18) and Eq. (21). Since all electrons move in the same direction, the average velocity is very large compared to the thermal case where electrons move in all directions and have just a small average drift velocity on top of the thermal motion. In high electric fields, and with high values of T_c, the thermal velocity (pointing randomly in all directions) can be as high as 10^8 cm/s, while the saturated drift velocity is (in silicon) a

factor of ten below this value. A visualization of these facts is given in Fig. 6.

The range between the high-velocity ballistic transport and the low-velocity diffusive transport is called the range of velocity overshoot. Imagine transport in a semiconductor switched on at a time $t_0 = 0$ by application of a high field F. The electrons are then accelerated ballistically for about 100 fs and may reach a velocity well above 10^7 cm/s (the value of the saturated velocity in silicon and other important semiconductors). Then, as time goes on, scattering events randomize the velocity to all directions, which leads, typically after a picosecond or so, to the saturated average velocity. At the times in between, the velocity is higher and exhibits the overshoot. In GaAs these effects are accentuated by the change in effective mass at high energy, and the overshoot can be considerable (8). This is shown in Fig. 7.

A similar effect is also achieved for transport in short semiconductor sections sandwiched in between contacts. As soon as the electrons enter the semiconductor from the contact, they are accelerated by high electric fields into the overshoot or even the ballistic regime and leave the semiconductor, entering the second contact, before scattering can randomize their motion. Thus the velocity in short semiconductor diodes can overshoot the saturated value (now for all times), which in some devices gives a speed advantage (22).

Changes of Carrier Concentration and Hot Electrons: Impact Ionization

Up to now, all the conductivity changes we have discussed have arisen from changes in the carrier velocity and a redistribution of charge carriers in energy. The total number of electrons or holes was not affected by hot electron effects. There exists, however, an important phenomenon that causes considerable changes of conductance because of changes in the total number of charge carriers. This is the occurrence of

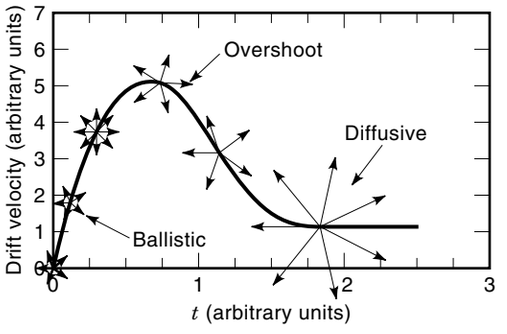

Figure 6. Schematic of the development of average drift velocity versus time, assuming that a high electric field F is suddenly applied at $t = 0$. The electrons are accelerated to high velocities and high average drift velocities. Their random velocity (indicated by arrows pointing in all directions) is at first small but is steadily increased due to randomizing scattering. At a certain point the average drift velocity approaches a maximum. From here on the random component of the velocity increases due to the very strong scattering at the high energies that the charge carriers now have, and the average drift velocity decreases. Finally, steady state is reached, where the scatterings and accelerations balance each other. The large random arrows indicate then a high electron temperature, and the drift velocity is saturated (at 10^7 cm/s in silicon).

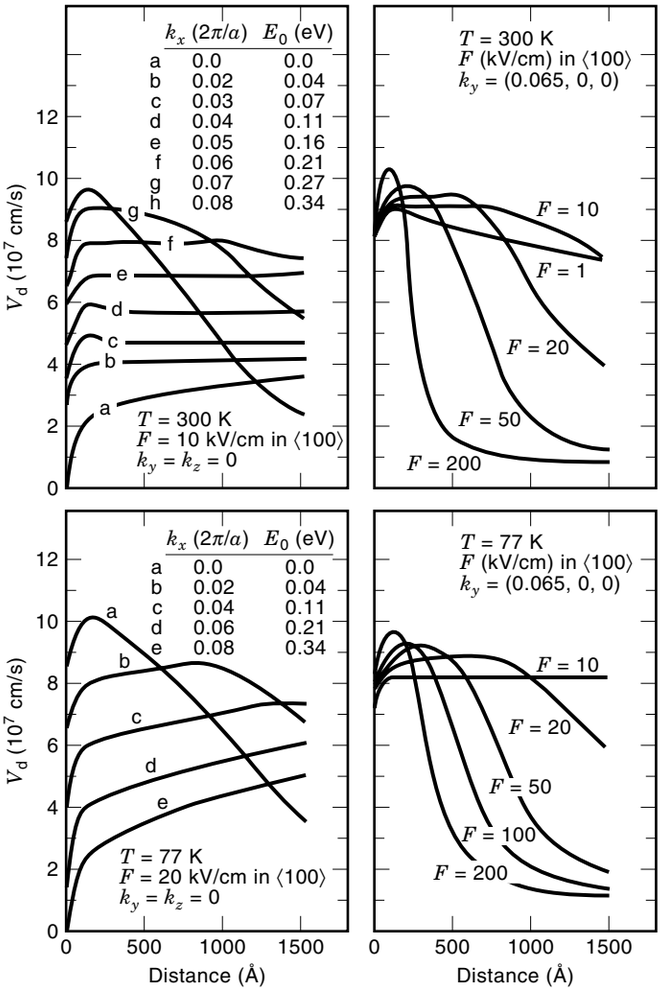

Figure 7. Overshoot of electrons injected with certain wave vectors k_x in the x direction and with certain energies E_0 into GaAs. A field F subsequently accelerates them and leads to velocity overshoot (19).

impact ionization, the exact inverse of the Auger effect (3). A heated electron in the conduction band (the same can be argued for holes) gains energy from the applied electric field and then collides with an electron in the valence band, lifting this electron up to the conduction band, with the net result of two conduction electrons and a hole. The primary electron and the secondary electron plus hole all contribute now to the conduction, and the electronic current density $\boldsymbol{j}$ therefore increases with time according to the equation

$$\frac{d\boldsymbol{j}}{dt} = \alpha_t \boldsymbol{j} \tag{22}$$

This increase of current with time also gives rise to a spatial increase of the current as

$$\nabla \boldsymbol{j} = \alpha_r \boldsymbol{j} \tag{23}$$

The coefficient α_r is in general a matrix. However, for a constant electric field impact ionization is mostly isotropic and α_r is a scalar (12,23).

We will concentrate, in the following on the theory of α_t. The theory of α_r proceeds very similarly, and one can almost always use $\alpha_t v_d = \alpha_r$, where v_d is the average (drift) velocity of the electrons. The multiplication of electrons (and holes) with time depends physically on two quantities. One is the actual ionization probability, that is, the probability for an electron with given energy to create an electron–hole pair. This probability per unit time is called the impact ionization rate R_I. The second quantity is the actual probability of having such an electron at a given energy, which is, of course, given by the distribution function. α_t is then the average product of these two probabilities:

$$\alpha_t = \frac{\int_{-\infty}^{\infty} d\boldsymbol{k}\, R_I f}{\int_{-\infty}^{\infty} d\boldsymbol{k}\, f} \tag{24}$$

The calculation of R_I involves the quantum mechanics of three particles (two electrons, one hole) in two different bands (conduction and valence) with different (Bloch) wave functions and $E(\boldsymbol{k})$ relations. Even using the approximate "golden rule" of time-dependent perturbation theory involves then a difficult numerical simulation with multiple integrations. Most of the theories of the past are therefore oversimplified and of very limited use, including the formula derived by Keldysh (24). The only theory in reasonably close agreement with experiments is the Monte Carlo integration of the "golden rule" formulae by Kane (25), the result of which is shown in Fig. 8.

An even greater numerical problem is the calculation of the distribution function entering Eq. (24). The reason is that R_I starts to become appreciably large only at relatively high energies, as shown in Fig. 8. Typically, in silicon, ionization becomes important only for electron energies of 3 eV above

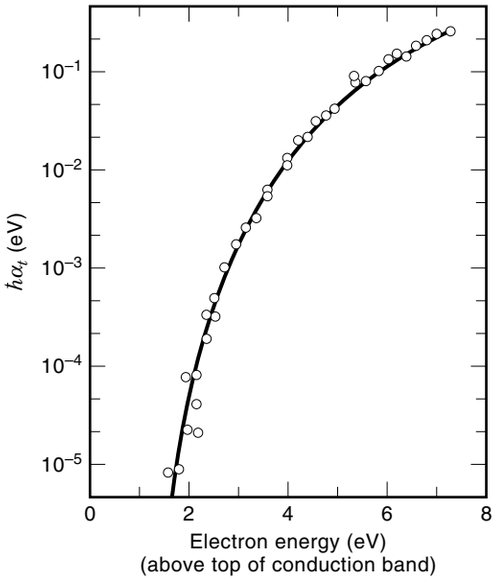

Figure 8. Impact ionization rate α_t (multiplied by $\hbar$) as a function of conduction electron energy according to the theory of Kane (25). Reprinted with permission, © 1967 by the American Physical Society.

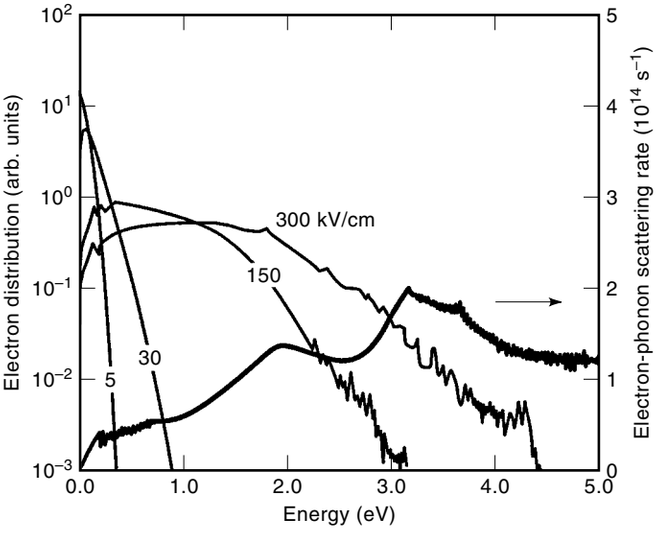

Figure 9. Electron energy distribution as a function of electron energy (in the conduction band), according to Ref. 24, for various constant electric fields. Also shown is the electron–phonon scattering.

the conduction band edge. If this were the average electron energy, it would correspond to a temperature of 24,000 K, which is seldom reached before catastrophic damage occurs. Therefore it is the high-energy tail of the distribution function that matters for impact ionization. This high-energy tail depends sensitively on the band structure and also on spatial and temporal changes of the electric field and can only be reliably obtained from a full band solution of the Boltzmann equation such as the full band Monte Carlo (12). An example of the distribution at high energies is given in Fig. 9 for a constant electric field. Figure 9 also shows the phonon scattering rate in silicon at room temperature as a function of conduction band energies. Note that this rate becomes of the order of 10^{14} s^{-1} at energies above 1.5 eV.

A discussion of all of these complexities and the corresponding results for α_r are given in Refs. 15, 27. The best results are obtained for theories of R_I á la Kane and for distributions from full band Monte Carlo integrations of the Boltzmann equation (25). Typical theoretical results of α_r are shown in Fig. 10.

The presence of layers of different semiconductors, as occurs in quantum well structures and superlattices, adds another dimension to the science and engineering of impact ionization and has been discussed extensively in the literature. Examples are given in Refs. (29) and (30).

A simplified approach of great usefulness and parametrized expressions for α_r have been derived by Baraff and others using his theory (7,31). These are excellent for estimates and for an appreciation of contributory physical parameters. Note, however, that all the simplified analytical expressions given in most textbooks for the threshold, phonon scattering mean free path, etc. are quantitatively incorrect and cannot replace the integration of Boltzmann's equation including the band structure.

HOT ELECTRONS IN DEVICES

The special status of hot electrons in semiconductor devices arises from the time and space dependences of electric field

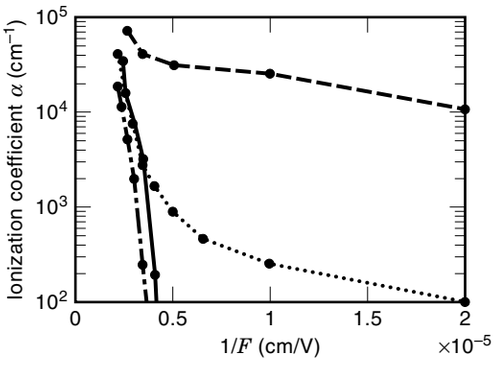

Figure 10. Theoretical electron impact ionization coefficients α_r versus 1/F for constant electric fields F and various materials after Bude and Hess (Ref. 25). Solid line: GaAs; dashed line: InAs; dot-dashed line: InP; and dotted line: $Ga_{0.43}In_{0.57}As$. The electron ionization coefficient for silicon is roughly a factor of 2 above the coefficient of GaAs. The hole ionization coefficients of GaAs and InP are close to the electron ionization coefficients while the hole ionization coefficient for silicon is much below the curve for electrons (factors of more than 10). Reprinted with permission, © 1992 by the American Physical Society.

and carrier concentrations. These dependences require special care in the theory of average velocity (overshoot, real-space transfer, transport including abrupt interfaces) and impact ionization. The nonlocality of these effects deserves special attention (32). Consistency with Gauss's law also needs to be satisfied. In fact, numerical solutions of Poisson's equation are a fixed part of any predictive device simulation. The field of hot electrons in devices is therefore a vast one and cannot be reviewed within this limited space. For a general appreciation the reader is referred to discussions of Gunn devices, IMPATT diodes, hot electron diodes and transistors, real-space transfer devices, and avalanche photodiodes in Ref. 18. All of these devices are based on hot electron effects. Many of the most important devices, such as metal–oxide–semiconductor field effect transistors (MOSFETs), involve hot electrons in their operation. Often, hot electrons are felt to cause great disadvantage because they reduce speed (current and velocity saturation), lead to degradation and aging of devices (33), and can cause various instabilities (e.g., through negative differential resistance). However, there are two counts in favor of hot electrons in devices that have led to the continual involvement of hot electrons in chip technology. For one, hot electrons lead to a large scattering rate by phonons as discussed above. This large rate gives rise to dephasing of the wave function on the length scale of 0.003 μm. Therefore it is possible to reach feature sizes down to 0.1 μm without major transitions from classical to quantum transport. The second point in favor of hot electrons arises from the need for aggressive designs, particularly with respect to switching speed. One needs to use the highest possible current densities, and this means in semiconductors also high electric fields. Frequent predictions that hot electron effects will be scaled away soon have therefore often not come true. Since hot electron effects are important in so many devices and of such variable consequences, we will discuss here only major effects that appear in many variations.

Hot Electrons in Field Effect Devices

Field effect transistors exhibit a large number of hot electron effects. It has been known since the sixties (34) that velocity saturation is important in these devices. The saturation was at first only encountered close to the drain, in the so-called pinchoff region (7). As the device sizes decreased, the velocity saturation spread all over the channel, at least for the highest drain voltages used (7). This effect reduces some of the figures of merit of the transistors (as, e.g., the transconductance) and was therefore seen as an unwelcome side effect; it was simulated by use of Eq. (16) and Eq. (15) or similar equations but with space-dependent electric fields. This local dependence on a varying electric field is, of course, only valid if the field varies so slowly that the transport is always diffusive without the nonlocal velocity overshoot or ballistic components. These latter effects become important for transistor channel length of the order of 0.1 μm (20). Then the overshoot effects counteract and undo part of the velocity saturation and generally contribute to higher device speed. This has been proven in MOSFETs, as discussed in Ref. 20.

The documentation of overshoot effects in silicon-based devices is a nontrivial task, since they always appear in connection with velocity degradation in other regions of the devices, and they are small (around a factor of 2 for T_L = 300 K). Overshoot effects are larger in III–V compound field effect transistors such as metal–semiconductor transistors (MESFETs). Ordinary MESFETs exhibit much scattering in the conducting channel due to the charged donors or acceptors, in addition to the always present scattering by the polar optical phonons (3,7). The overshoot effects are therefore more pronounced in modulation-doped field effect transistors (MODFET), which contain the dopants in a layer of a different semiconductor (e.g., AlGaAs) neighboring the channel (e.g., GaAs) as described in Ref. 35. These transistors exhibit, therefore, a significant speed advantage over MESFETs, as shown by numerous works (36,37). Typical values of the overshoot in such devices, as deduced from Monte Carlo simulations, correspond to those shown in Fig. 7. We note that real-space transfer can, of course, be also of importance in these devices and may reduce some of their advantages.

Impact ionization is an important limiting factor in field effect transistors, since its presence usually will distrurb device operation. The theory of impact ionization in devices proceeds along the lines discussed above. However, nonlocal effects are of great significance because of the presence of rapidly varying electric fields in the conducting channels of field effect devices. One then needs to allow for dead spaces of the ionization (32,38).

As discussed above, the probability of impact ionization is very small for electron energies below a certain energy in the conduction band. Of course, the minimum energy the electron needs is the energy of the gap, E_G. Even above this energy, the threshold for significant ionization is often not reached for energies of two or three times E_G, as can be seen from Kane's results in Fig. 8. This energy needs to be reached to start significant ionization.

With a space-dependent electric field $\boldsymbol{F}(z)$ in the z direction, the electron needs to traverse a certain distance d to reach effective threshold. Typically d can be obtained from an equation like

$$\int_0^d F(z)\,dz = cE_G \tag{25}$$

where c is a constant determining the effective threshold (normally of the order of 2). Over such a distance, ionization canot occur, even if the electric field is very high. Ionization in very short field effect transistors with highly peaked electric fields is therefore smaller than one would expect from an integration of the ionization probability over all fields not counting the dead space of length d. This explains the fact that ionization is not a function of the local electric field alone (nonlocality).

Anisotropies of impact ionization, which have not been found for constant electric fields (23), can occur when ballistic acceleration over short distances is important, mainly because of anisotropies of the effective threshold (39). These further complicate the simulation of impact ionization in devices. It is the conviction of this author that a quantitative understanding of impact ionization in devices is only possible by a full band solution of the Boltzmann equation consistent with the solution of Poisson's equation, as can be done with various simulation tools (13,14). Some of the controversies in the literature can be tracked to oversimplified simulation.

The hot electron effects discussed above are reversible in the sense that after turning off the electric fields, the hot electrons cool down to the ambient temperature within picoseconds without any structural changes of the crystal lattice. However, hot electrons can also cause structural changes. A particularly well-known hot electron degradation occurs at the MOSFET interface between the silicon and the silicon dioxide (33). This damage is linked to the breaking of silicon–hydrogen bonds that are always present at this interface. A clear proof was given by damage measurements involving the isotope deuterium (40). When the silicon–hydrogen complex was replaced by silicon–deuterium, a much-reduced hot electron degradation of MOSFETs was found, which has the beneficial effect of increasing the device lifetime (41). A large framework of experimental and theoretical contributions to this area exists in the literature and is reviewed in Ref. 42.

Hot Electrons in Quantum Well Laser Diodes

Hot electrons in quantum well laser diodes have a different origin and different consequences from what they have in field effect devices. The electrons are here not heated by the electric field but by other electrons propagating into the quantum well and exhibiting suddenly high kinetic energy as shown in Fig. 11. These electrons, originating from outside the well, can now transfer their energy in essentially two ways to the electrons in the quantum well. For one, they can transfer the energy through direct electron–electron interaction (electron–hole interactions can also be important in laser diodes because of the presence of both electrons and holes

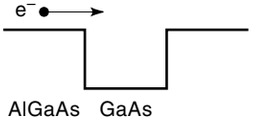

Figure 11. Electron propagating over a semiconductor heterojunction and acquiring significant kinetic energy (in the GaAs).

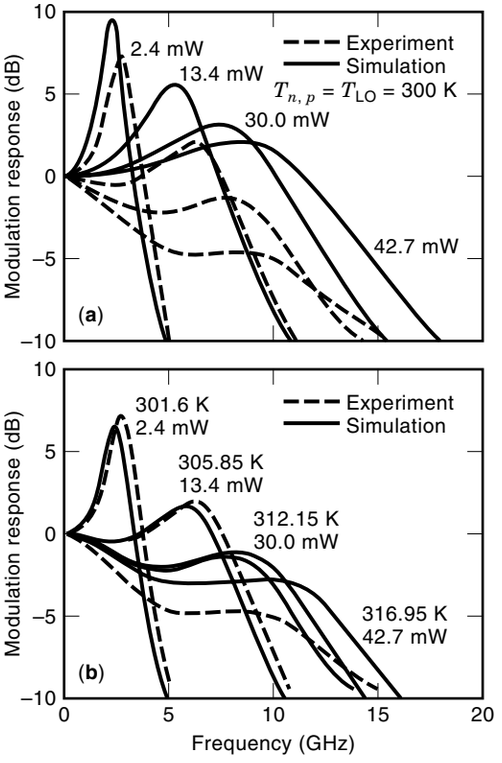

Figure 12. Modulation response of quantum well laser diodes for various driving currents corresponding to the power indicated. (a) Theory without hot electron effects. The agreement with experiments is weak, particularly at higher power levels. (b) Theory including hot electron effects (temperatures indicated above curves) shows excellent agreement with experiments (38).

with approximately equal density). The incoming electron beam therefore heats the quantum well electrons. Another pathway of energy transfer is by polar optical phonons. Electrons at high energies in the quantum wells emit polar optical phonons (within about 10^{-13} s). These phonons cannot propagate out of the quantum well and decay relatively slowly (typically within 3 to 10 ps). The phonons therefore accumulate, giving rise to a nonequilibrium (heated) phonon distribution. The quantum well electrons at low energy can then in turn absorb phonons and heat up themselves. This gives rise to a nonequilibrium electron temperature T_c.

A completely consistent calculation of these effects has been made (43) and shows that the modulation of electron density in quantum well laser diodes is always accompanied by a modulation of the electron temperature and therefore influences the laser performance and modulation response sensitively through very small temperature rises. While field effect devices operate at electron temperatures of several thousand kelvins caused by the heating of the electric field, laser diodes cease to operate for dynamic electron heating of a few kelvins, as shown in Fig. 12, which depicts the modulation response of a typical semiconductor laser diode. The static heating is also important in lasers and degrades their performance. For the static case electron temperatures around 100 K above room temperature may be tolerated—still much less than the T_c values reached in field effect devices.

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HIGH-FREQUENCY. See MILLIMETER WAVE MEASUREMENT.