SIGNAL AMPLIFIERS

OPEN-LOOP AMPLIFIERS

Amplification is needed whenever a signal (coming from a transducer, an antenna, etc.) is too small to be efficiently processed. A signal amplifier is primarily intended to operate on very small input signals with the aim of increasing the signal energy. For instance, a voltage amplifier works with input signals in the range of millivolts or even microvolts, and has to provide a power gain. This last property distinguishes a voltage amplifier from a transformer. A transformer, in fact, can provide an output voltage greater than the input (primary) voltage, but the output power never exceeds the power supplied by the signal source. The smallest signal which can be detected and amplified is limited by the noise performance of the amplifier. In fact, noise masks the signal so that recovery may not be possible. Linearity is another fundamental re-

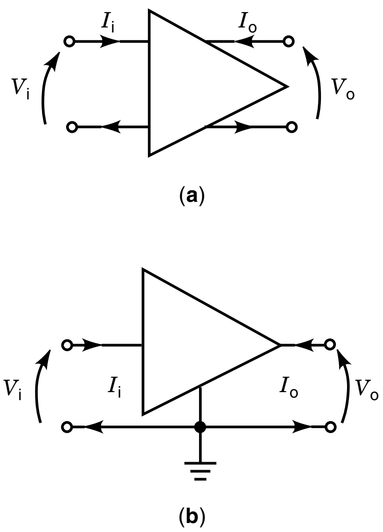


Figure 1. (a) Amplifier symbol; (b) amplifier with a common terminal (ground).

quirement for a signal amplifier, in order to ensure that the signal information is not changed and no new information is introduced. An amplifier providing an output signal linearly related to the input is characterized by the relationship

$$x_o = Ax_i \tag{1}$$

where x_i and x_o are the input and the output signals, respectively, which can be either voltage or currents, and A is a constant representing the magnitude of the amplification, usually termed the *amplifier gain.*

In general, an amplifier is a two-port network, which can be represented by the circuit symbol in Fig. 1(a), showing the input and output ports as well as the signal flow direction. The amplifier model considered is *unilateral* since the signal flow is unidirectional. This usually leads to a good approximation of real-life amplifiers which, however, also exhibit an undesired reverse transmission. Figure 1(b) illustrates a usual situation where a common terminal between the input and the output port exists and is used as a reference point called the *circuit ground.* Depending on the signal type to be amplified and on the desired type of output, amplifiers can be classified into four categories:

1. Voltage amplifiers, with an open-circuit voltage gain $A_{vo} = V_o/V_i$
2. Current amplifiers, with a short-circuit current gain $A_{io} = I_o/I_i$
3. Transresistance amplifiers, with an open-circuit transresistance gain $R_{to} = V_o/I_i$
4. Transconductance amplifiers, with a short-circuit transconductance gain $G_{to} = I_o/V_i$

In Fig. 2 circuit models for the four types of amplifier are illustrated, also accounting for finite input and output resistances. These models are independent of the complexity of the amplifier, which can be made up of a single stage or of several stages. Referring to a voltage amplifier connected at the input to a signal voltage source (V_s with a series resistance R_s) and connected at the output to a load resistance, R_L, the overall voltage gain is

$$A_v = \frac{r_i}{r_i + R_s} A_{vo} \frac{R_L}{R_L + r_o} \tag{2}$$

It is apparent that in order to preserve the gain, the input resistance r_i should be much greater than the source resistance R_s and the output resistance r_o should be much smaller than the load resistance R_L. For the other three types of amplifier a similar result is obtained, which can be summarized with the concept of *mismatched amplifiers.* In this case only one kind of variable (voltage or current) has to be processed and the other is reduced to its minimum possible value.

FEEDBACK AMPLIFIERS

Although open-loop amplifiers have their own specific range of applications (e.g., RF amplifiers are always open-loop circuits), an important class of amplifier is constituted by feedback stabilized amplifiers.

Negative feedback is widely used in the design of amplifiers, since it allows the gain to be stabilized with respect to

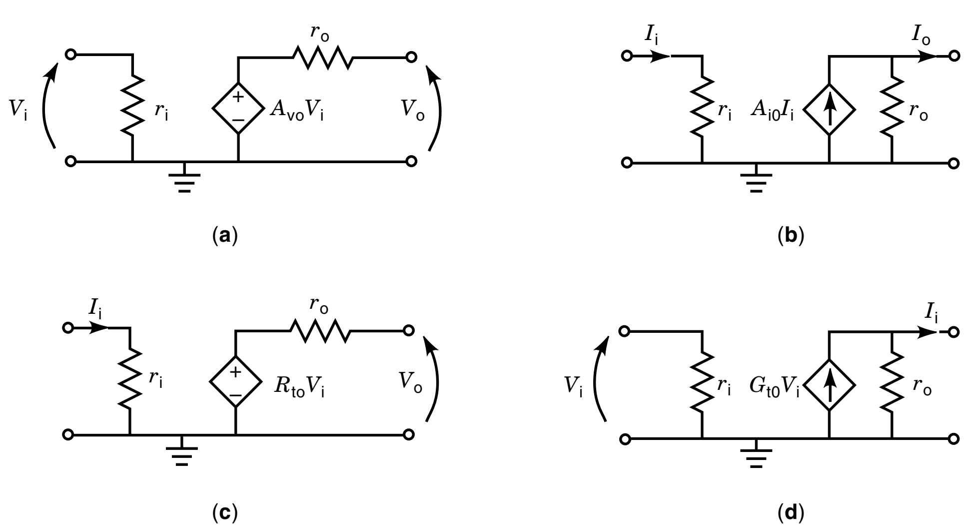


Figure 2. Circuit models for (a) voltage amplifier; (b) current amplifier; (c) transresistance amplifier; and (d) transconductance amplifier.

active device parameter spreads, power supply variations, and temperature changes. Feedback allows the input and output resistances of the circuit to be modified in any desired fashion. It improves the linearity of the amplifier, thus reducing the distortion produced in the output signal. Finally, it can lead to an increase in the bandwidth of the amplifier. However, all these features are paid for in terms of a proportional reduction in the gain. Moreover, negative feedback can cause the tendency of oscillation to occur in the circuit, and hence frequency compensation is usually mandatory (1–3).

The analysis of an ideal feedback system like that shown in Fig. 3 is straightforward, and leads to the transfer function

$$G_F = \frac{x_o}{x_s} = \frac{A}{1 + fA} \tag{3}$$

Unfortunately, for real cases where the blocks, A and f, are made up of active and passive components, the analysis is not so simple. Several techniques for the analysis of real feedback amplifiers have been reported in Refs. 1–7 and are critically discussed in Ref. 8 and Ref. 9. Each technique has its own benefits and drawbacks, but from a design point of view, two of them are the most interesting and powerful. The first was proposed in 1974 by Rosenstark (10), and was recently rediscovered and formalized using signal flow graphs (2); the second was proposed in 1990 by Choma and is based on signal flow analysis (11).

Rosenstark Method

The Rosenstark method is based on calculation of the *return ratio, T*, the *asymptotic term, G_∞*, and the *direct transmission term, G_0*. All these quantities, which are functions of the input source resistance, R_S, and output load resistance, R_L, must be calculated with respect to one and only one controlled source within the feedback amplifier. The exact transfer function between the input and output of the feedback amplifier (10) is thus given by

$$G_F(R_S, R_L) = \frac{G_\infty(R_S, R_L)T(R_S, R_L) + G_0(R_S, R_L)}{1 + T(R_S, R_L)} \tag{4}$$

More specifically, to evaluate the three terms, we have to relate a controlled source quantity, x_o, to the controlling quantity, x_c, by the parameter P (i.e., $x_o = Px_c$) and to follow the steps below:

1. Switch off the critical controlled source setting $P = 0$ and, to achieve the direct transmission term, G_0, compute the transfer function between the input and output.
2. Set the input source to zero. This means a short-circuit voltage source or an open-current source. Replace the critical controlled source by an independent one of value P. The return ratio, T, coincides with the resulting controlling quantity with the opposite sign, $-x_c$.
3. Set the critical parameter to infinity (i.e., $P \to \infty$). Since the controlled source is still finite, this is equivalent to having $x_c = 0$ with the related consequence. The Asymptotic term, G_∞, is the transfer function between the input and the output under these conditions.

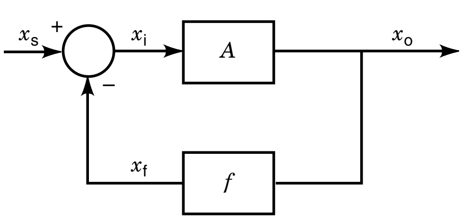


Figure 3. Block scheme of a feedback system.

Comparing Eq. (4) with Eq. (3), it is apparent that with a negligible direct transmission term, G_0, the return ratio, T, and the asymptotic gain, G_∞ are equal to the product between the amplifier gain, A, and the feedback factor, f, and to the inverse of the feedback factor, f, respectively (i.e., $T = fA$ and $G_\infty = 1/f$) (1–4). It is worth noting that the term G_∞ represents the ideal transfer function of the feedback network. Indeed, for well-designed feedback amplifiers which have a low G_0 and a high T, the transfer function of the feedback circuit is well approximated by G_∞.

Choma Method

The Choma method starts from the same assumptions made in the Rosenstark method. After choosing a controlled source P inside the feedback, we again have to calculate the return ratio, T, and the direct transmission term, G_0, as described in points 1 and 2 of the previous subsection. But now, instead of the asymptotic term, G_∞, we have to evaluate the *null return ratio, T_R*. Thus the desired exact transfer function between the input and output of the feedback amplifier (11) is given by

$$G_F(R_S, R_L) = G_0(R_S, R_L)\frac{1 + T_R(R_S, R_L)}{1 + T(R_S, R_L)} \tag{5}$$

More specifically, the null return ratio, T_R, can be evaluated by replacing the critical controlled source with an independent one of value P, as done in the point 2 of the previous subsection, but without nullifying the input source. It will coincide with the resulting controlling quantity with the opposite sign, $-x_c$, assuming the output voltage to be equal to zero.

The ratio between the return ratio and the null return ratio, $T(R_S, R_L)/T_R(R_S, R_L)$, quantifies the degree to which the local feedback approaches global feedback (11) (when it is ∞ the feedback is global), and hence gives interesting information regarding the kind of feedback. Of course, both methods presented give the same results, and combining Eq. (3) with Eq. (4) the degree to which the local feedback approaches global feedback versus the asymptotic gain is given by

$$\frac{T(R_S, R_L)}{T_R(R_S, R_L)} = \frac{G_0(R_S, R_L)}{G_\infty(R_S, R_L)} \tag{6}$$

Input and Output Resistances

The driving point input impedance and driving point output impedance of a feedback amplifier can be simply evaluated by using the Blackman theorem (12). The same relationships are obtained using signal flow analysis (11). The input and output resistance are given by

$$R_i = R_{iol}\frac{1+T(0,R_L)}{1+T(\infty,R_L)} \quad (7)$$

$$R_o = R_{ool}\frac{1+T(R_S,0)}{1+T(R_S,\infty)} \quad (8)$$

where R_{iol}, and R_{ool}, are the corresponding driving point input and output resistances with the critical parameter P equal to zero, and $T(0, R_L)$, $T(\infty, R_L)$, $T(R_S, 0)$, and $T(R_S, \infty)$ are the return ratios under the conditions specified for the source resistance, R_S, and load resistance, R_L.

FEEDBACK AMPLIFIER CONFIGURATIONS (13)

It follows from the previous discussion that the characteristics of the four amplifier types can be improved with the use of negative feedback. For each amplifier we have to sample the output signal by a suitable network and transmit a portion of this signal back to the input.

There are four basic types of single-loop feedback amplifiers analyzed below: (1) series-shunt, (2) shunt-series, (3) shunt-shunt, and (4) series-series. The four typical amplifiers are only implemented with bipolar transistors. However, since bipolar transistors are modeled with the equivalent-π circuit, the results can be extended to the MOS transistor quite simply by setting r_π to infinity.

Series-Shunt Feedback Amplifier

Figure 4(a) depicts the ac schematic diagram (a circuit diagram divorced of biasing details) of a series-shunt feedback amplifier. A portion of the output voltage, v_o, sampled by the feedback network R_E, R_F, is compared with the input voltage v_S. The small signal model of the amplifier in Fig. 4(a) is shown in Fig. 4(b).

A practical consideration regarding application of the Rosenstark approach is the choice of the critical controlled source. Although the approach is general, evaluation of the terms becomes simpler if one node of the controlled source is at ground potential. This, in other words, means that for multitransistor amplifiers we have to choose a controlled source associated to a common emitter transistor. Thus we choose the transconductance g_{m2}, as the controlled source P and follow the steps below:

1. Set $P = 0$ ($g_{m2} = 0$). This, unless there is a load effect on the collector of T1 due to $r_{\pi 2}$, means switching off transistor T2 and, hence, the ac schematic diagram is

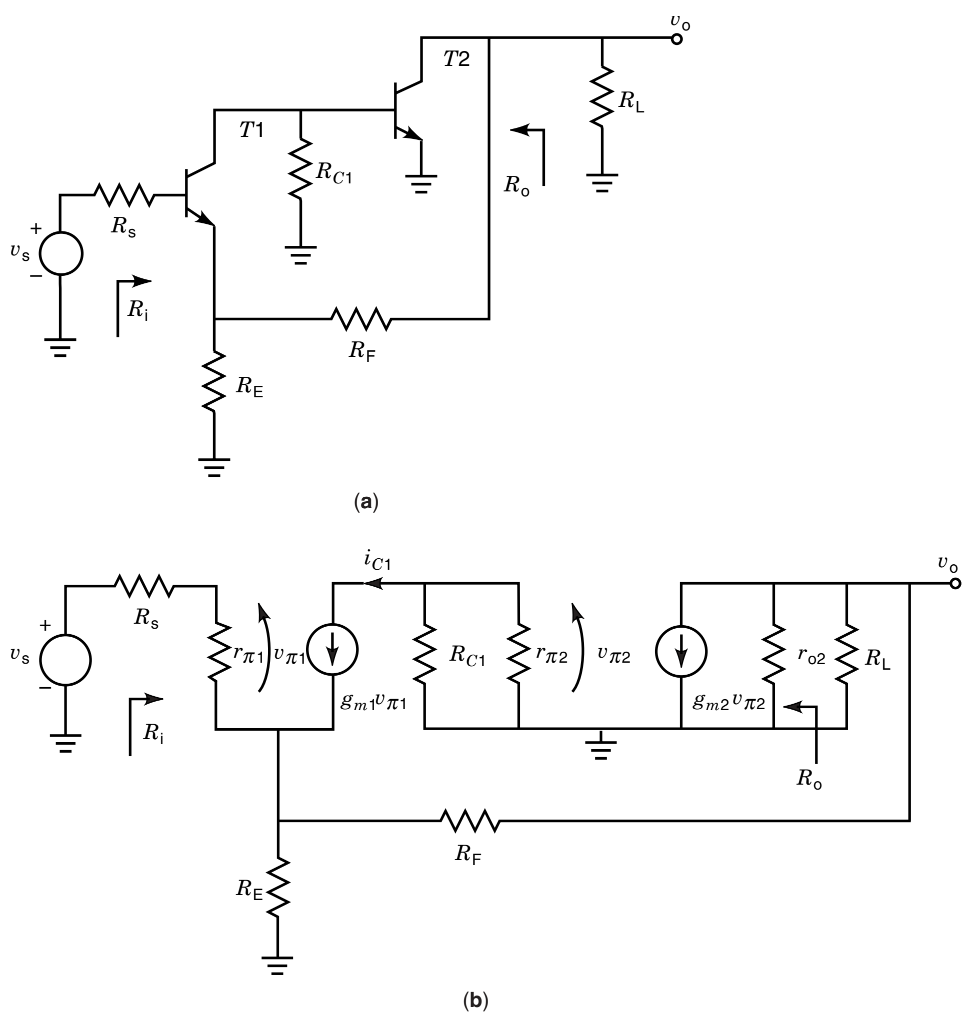


Figure 4. (a) Ac schematic of series-shunt feedback amplifier; (b) small signal equivalent circuit of the series-shunt feedback amplifier in (a), obtained by replacing each transistor with its small-signal model.

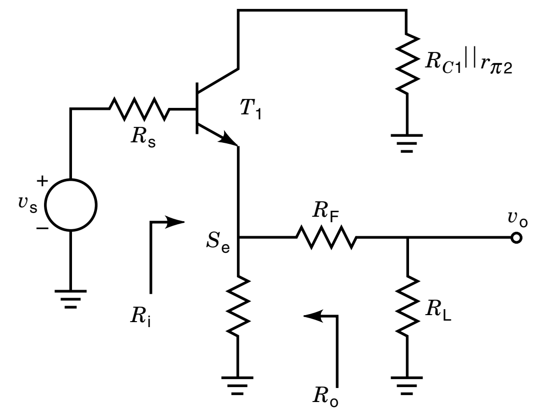


Figure 5. Ac schematic for the evaluation of the direct transmission term for the circuit in Fig. 4(a). On having nullified the transconductance of T2 in Fig. 4(a), the circuit becomes a simple emitter follower.

the one in Fig. 5, which is a voltage follower whose transfer function, assuming the transistor output resistance, r_{o1}, to be much greater than $R_{C1}\|r_{\pi 2}$, is given by

$$\frac{v_e}{v_S} = \frac{(g_{m1}r_{\pi 1}+1)[R_E\|(R_L+R_F)]}{(g_{m1}r_{\pi 1}+1)[R_E\|(R_L+R_F)]+r_{\pi 1}+R_S} \approx 1 \qquad (9)$$

where v_e is the voltage on the emitter of T1. Thus, including the term $R_L/(R_L + R_F)$, which takes into account the voltage partition at the output of the voltage buffer, we get the gain, G_0, under the special condition of zero feedback

$$G_0 = \left.\frac{v_o}{v_S}\right|_{g_{m2}=0} \approx \frac{R_L}{R_L+R_F}\frac{v_e}{v_S} \approx \frac{R_L}{R_L+R_F} \qquad (10)$$

It is apparent that this contribution is always lower than one. Since closed-loop resistances are evaluated with $P = 0$, we can compute the corresponding driving point input and output resistances, R_{iol}, and R_{ool}, given by

$$R_{iol} = r_{\pi 1} + (g_{m1}r_{\pi 1}+1)[R_E\|(R_L+R_F)] \qquad (11)$$

$$R_{ool} = R_F + \frac{r_{\pi 1}+R_S}{g_{m1}r_{\pi 1}+1}\|R_E \approx R_F \qquad (12)$$

2. Set v_s to zero and replace the original controlled current generator, $g_{m2}v_{\pi 2}$, with an independent current source, i, of value P. Again, transistor T2 can be considered to be switched off while transistor T1 is in a common base configuration [Fig. 6(a)]. Then, by introducing a Norton equivalent generator at the input, as shown in Fig. 6(b), where the current i' is given by

$$i' = \frac{R_L}{R_L+R_F}i \qquad (13)$$

and neglecting the resistance r_{o1}, we get

$$\frac{i_{c1}}{i} = \frac{R_L}{R_L+R_F}\frac{g_{m1}r_{\pi 1}}{g_{m1}r_{\pi 1}+1}\frac{(R_F+R_L)\|R_E}{(R_F+R_L)\|R_E+\dfrac{r_{\pi 1}+R_S}{g_{m1}r_{\pi 1}+1}} \approx \frac{R_L}{R_L+R_F} \qquad (14)$$

Hence, the return ratio, T, with respect to the critical parameter g_{m2} is

$$T = -g_{m2}\frac{v_{\pi 2}}{i} = (R_{C1}\|r_{\pi 2})g_{m2}\frac{i_{c1}}{i} = \frac{R_L}{R_L+R_F}(R_{C1}\|r_{\pi 2})g_{m2} \qquad (15)$$

3. Now evaluate the *closed loop asymptotic gain, G_∞*, by setting the parameter g_{m2} infinitely large. By inspection of Fig. 4(b), to be the current of generator $g_{m2}v_{\pi 2}$ finite, the voltage $v_{\pi 2}$ must be zero and this holds only if the current generator i_{c1} is zero which, in turn, means $v_{\pi 1} = 0$. Therefore, $v_{b1} = v_S$ (where $b1$ is the base of $T1$), and all the input voltage is found across the resistance R_E

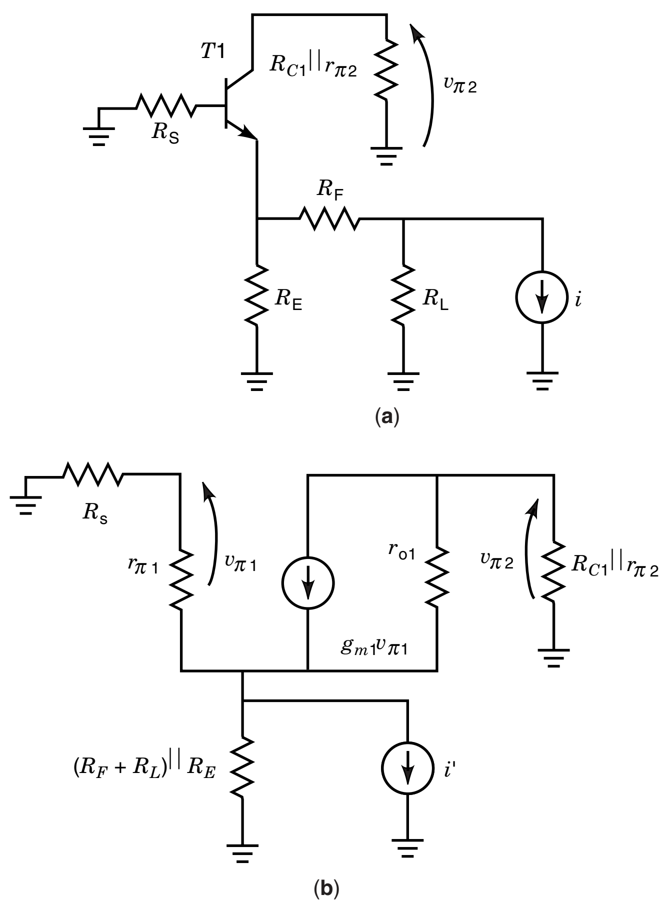


Figure 6. (a) Ac schematic for the evaluation of the return ratio for the circuit in Fig. 4(a). On nullifying the input signal and replacing the controlled generator of T2 with an independent current source i, T1 becomes a common-base transistor. (b) Small-signal circuit of (a).

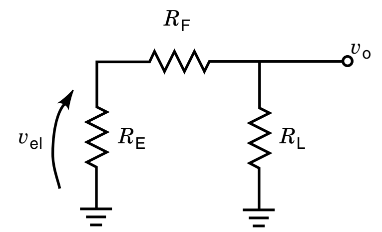


Figure 7. Equivalent circuit for the evaluation of the asymptotic gain for the circuit in Fig. 4(a). The transconductance of T2 in Fig. 4(a) has been made infinitely large.

(i.e., the typical virtual short-circuit condition). According to Fig. 7, which follows from these considerations,

$$G_\infty = \left.\frac{v_o}{v_S}\right|_{g_{m2}\to\infty} = 1 + \frac{R_F}{R_E} \tag{16}$$

The final closed-loop gain is obtained by substituting G_0, T, and G_∞, into Eq. (14). In order to calculate the input and output resistance, according to Eqs. (7) and (8), since the relationship of the return ratio in Eq. (15) is independent of the source resistance, it is necessary to introduce the exact expression of Eq. (14) into Eq. (15), before evaluating the terms $T(0, R_L)$ and $T(\infty, R_L)$. The four return ratios needed are

$$T(0, R_L) = \frac{R_L}{R_L + R_F}(R_{C1}\|r_{\pi 2})g_{m2} \tag{17a}$$

$$T(\infty, R_L) = T(R_S, 0) = 0 \tag{17b}$$

$$T(R_S, \infty) = (R_{C1}\|r_{\pi 2})g_{m2} \tag{17c}$$

And, hence, the input and output resistances are

$$R_i = R_{iol}\left[1 + \frac{R_L}{R_L + R_F}(R_{C1}\|r_{\pi 2})g_{m2}\right] \tag{18}$$

$$R_o = \frac{R_{ool}}{1 + (R_{C1}\|r_{\pi 2})g_{m2}} \tag{19}$$

To follow the Choma method, one must replace point 3 with the following step, in order to evaluate the null return ratio T_R.

First substitute the original controlled current generator, $g_{m2}v_{\pi 2}$, with an independent current source, i. By inspection of Figure 4(b), to have an output voltage equal to zero means that the critical current, i, is forced to flow through the resistance R_F. Hence, the equivalent circuits in Fig. 8 can be used. Under the assumption that the voltage on the emitter of transistor T1 follows the input source voltage (i.e., $v_e \approx v_S$), the voltage on the collector of T1 and the critical current i, are, respectively,

$$v_{\pi 2} = \frac{g_{m1}r_{\pi 1}}{1 + g_{m1}r_{\pi 1}}\frac{R_{C1}\|r_{\pi 2}}{R_E\|R_F}v_S \approx \frac{R_{C1}\|r_{\pi 2}}{R_E\|R_F}v_S \tag{20}$$

$$i = -\frac{v_S}{R_F} \tag{21}$$

Thus, the null return ratio is given by

$$T_R = -\frac{g_{m2}v_{\pi 2}}{i} = g_{m2}\frac{R_E + R_F}{R_E}(R_{C1}\|r_{\pi 2}) \tag{22}$$

It is apparent that the relationship displayed in Eq. (6) is verified.

Shunt-Series Feedback Amplifier

While the series-shunt feedback circuit functions as a voltage amplifier, the shunt-series configuration, whose ac schematic diagram is depicted in Fig. 9(a), is best suited as a current amplifier. In the subject circuit, the current through the emitter of transistor T2, which is approximately equal to the output signal current, i_o, is sampled by the feedback network formed on the resistance, R_E and R_F. The sampled current is fed back as a current to a current-driven input port. Thus the resulting driving point output resistance is large, and the driving point input resistance is small. These characteristics allow for a closed-loop current gain, $G_i(R_S, R_L) = i_o/i_s$, which is relatively independent of the source and load resistances and insensitive to transistor parameters.

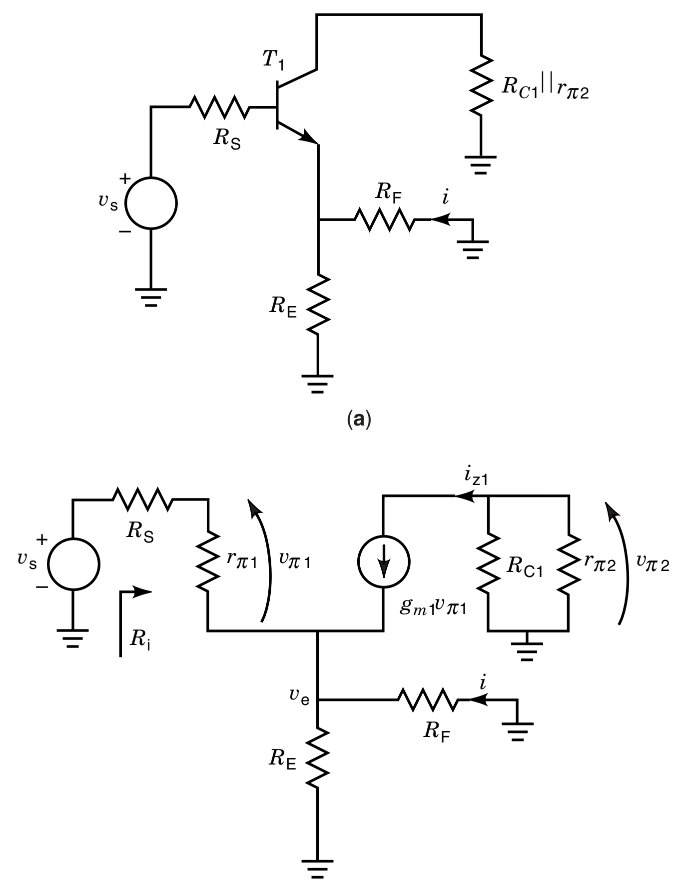
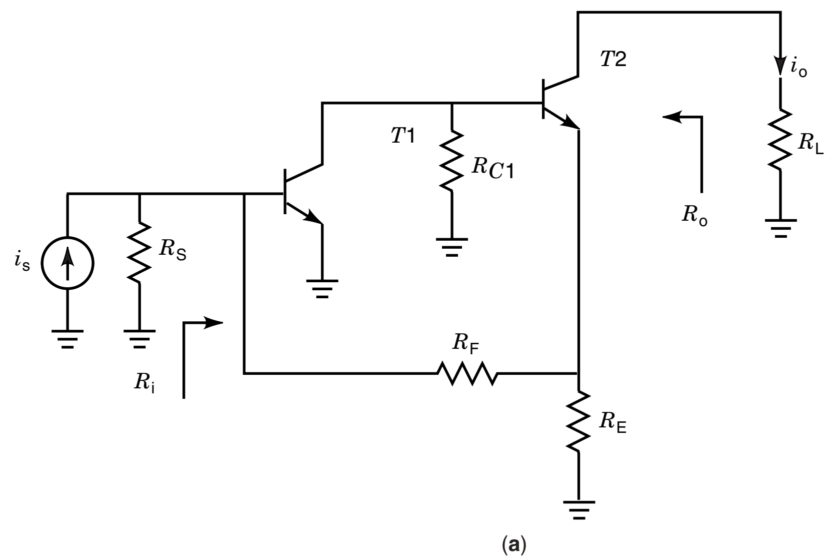


Figure 8. (a) Ac schematic for the evaluation of the null return ratio for the circuit in Fig. 4(a). The controlled generator of T2 is replaced by an independent current source i. (b) Small-signal circuit of Fig. 8(a).



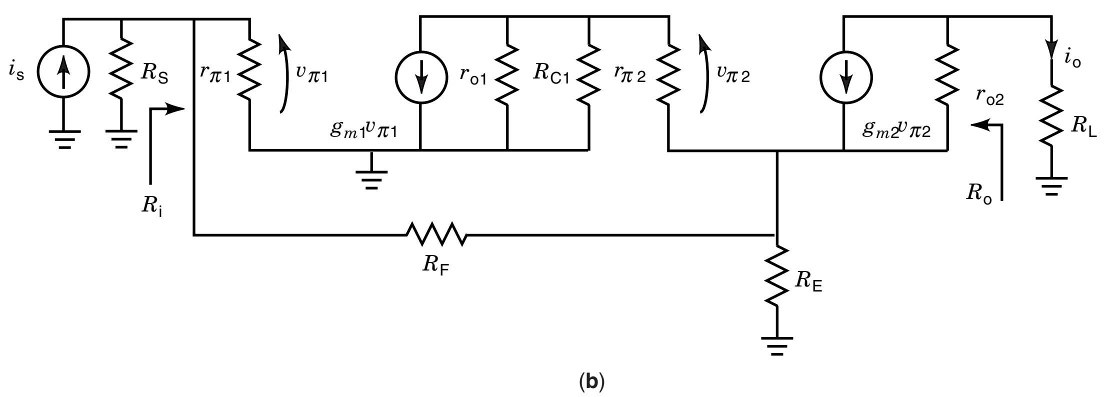


Figure 9. (a) Ac schematic of shunt-series feedback amplifier; (b) small-signal equivalent circuit of the shunt-series-shunt feedback amplifier in (a), obtained by replacing each transistor with its small-signal model.

To analyze the circuit in Fig. 9(a), consider its small signal model shown in Fig. 9(b), and assume the transconductance g_{m1} as the critical parameter P.

1. Set $P = 0$ ($g_{m1} = 0$), which means switch off transistor T1; then, taking into account the input resistance of T1 which still exists, the circuit has the ac schematic diagram depicted in Fig. 10(a), and the small signal model shown in Fig. 10(b) where the resistance R'_S and the current i'_s are given by

$$R'_S = (R_F + R_S \| r_{\pi 1}) \| R_E \tag{23}$$

$$i'_s = \frac{R_S \| r_{\pi 1}}{R_F + R_S \| r_{\pi 1}} i_s \tag{24}$$

The circuit in Fig. 10 represents a common base configuration; from it one obtains

$$G_o = \left.\frac{i_o}{i_S}\right|_{g_{m1}=0} = \frac{R'_S}{R'_S + \dfrac{R_{C1} + r_{\pi 2}}{g_{m2} r_{\pi 2} + 1}} \frac{g_{m2} r_{\pi 2}}{g_{m2} r_{\pi 2} + 1} \frac{R_S \| r_{\pi 1}}{R_F + R_S \| r_{\pi 1}}$$
$$\approx \frac{R_S \| r_{\pi 1}}{R_F + R_S \| r_{\pi 1}} \tag{25}$$

which is always lower than one. The corresponding input and output resistance, R_{iol}, and R_{ool}, are given by

$$R_{iol} = \frac{r_{\pi 2} + R_{C1}}{g_{m2} r_{\pi 2} + 1} \| R'_S \tag{26}$$

$$R_{ool} = r_{o2} + (1 + g_{m2} r_{o2})[R'_S \| (R_{C1} + r_{\pi 2})] \tag{27}$$

2. Set i_s to zero and replace the original controlled current generator, $g_{m1} v_{\pi 1}$, with an independent current source, i. Now, as shown from the equivalent ac circuit shown in Fig. 11, transistor T2 works as a voltage follower, and the voltage $v_{\pi 1}$ is a portion of the emitter voltage of T2. Therefore, since r_{o2} is usually much higher than R_L and R_E, and assuming R_{C1} to be lower than the equivalent resistance at the base terminal of T2, the loop gain is

$$T = \frac{g_{m2} r_{\pi 2}}{g_{m2} r_{\pi 2} + 1 + \dfrac{r_{\pi 2}}{R_E \| (R_F + R_S \| r_{\pi 1})}} \frac{R_S \| r_{\pi 1}}{R_S \| r_{\pi 1} + R_F} g_{m1} R_{C1}$$
$$\approx \frac{R_S \| r_{\pi 1}}{R_S \| r_{\pi 1} + R_F} g_{m1} R_{C1} \tag{28}$$

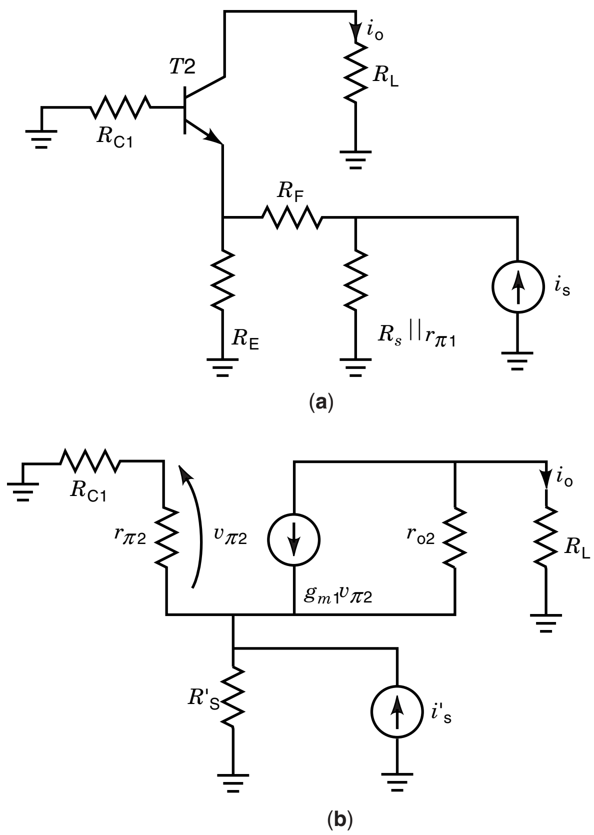


Figure 10. (a) Ac schematic for the evaluation of the direct transmission term for the circuit in Fig. 9(a). On having nullified the input signal and the transconductance of T1, the circuit acquires a common-base configuration. (b) Small-signal equivalent circuit of the circuit in (a). The current generator i'_S and resistor R'_S represent the Norton equivalent seen by the emitter of T2.

3. Now evaluate the *closed-loop asymptotic gain,* G_∞. By inspection of Fig. 9(b), setting the parameter g_{m1} infinitely large leads to $v_{\pi1}$, equal to zero which, in turn, means that all the input current, i_s, enters the feedback resistance, R_F. Moreover, since a finite value for the current $g_{m1}v_{\pi1}$ determines a $v_{\pi1}$ other than zero, the term $g_{m1}v_{\pi1}$ itself must be equal to zero. Under these hypotheses, one can model the circuit with the one shown in Fig. 12, and by inspection we find that the current entering into the emitter of transistor T2 is equal to

$$i_{e2} = \left(1 + \frac{R_F}{R_E}\right) i_S \tag{29}$$

Hence, neglecting the resistance r_{o2}, one obtains

$$G_\infty = \left.\frac{i_o}{i_s}\right|_{g_m \to \infty} = \frac{g_{m2}r_{\pi2}}{1 + g_{m2}r_{\pi2}}\left(1 + \frac{R_F}{R_E}\right) \approx 1 + \frac{R_F}{R_E} \tag{30}$$

Therefore, combining Eqs. (25), (28), and (30) the exact expression of the closed-loop gain of a shunt-series feedback amplifier can be found quite simply. For common values, the loop gain is much greater than one and the closed-loop gain is equal to the asymptotic one.

Finally, the resulting input and output resistances are given by Eqs. (7) and (8), where the return ratios are

$$T(0, R_L) = 0 \tag{31}$$

$$T(\infty, R_L) = \frac{r_{\pi1}}{r_{\pi1} + R_F} g_{m1} R_{C1} \tag{32}$$

$$T(R_S, 0) = \frac{R_S \| r_{\pi1}}{R_S \| r_{\pi1} + R_F} g_{m1} R_{C1} \tag{33}$$

$$T(R_S, \infty) = \frac{R_S \| r_{\pi1}}{R_S \| r_{\pi1} + R_F} g_{m1} \{R_{C1} \| [r_{\pi2} + R_E \| (R_F + R_S \| r_{\pi1})]\} \tag{34}$$

Shunt-Shunt Feedback Amplifier

The ac schematic diagram of the third type of the single-loop feedback amplifier, the shunt-shunt triple, is drawn in Fig. 13(a). A cascade interconnection of three transistors, T1, T2, and T3, forms the open loop, while the feedback subcircuit is a single resistance, R_F. This resistance samples the output voltage, v_o, and feeds it back as current to the input port. Therefore, both the driving point input and output resistance are very small. Accordingly, the circuit operates best as a transresistance amplifier, in that its closed-loop transresis-

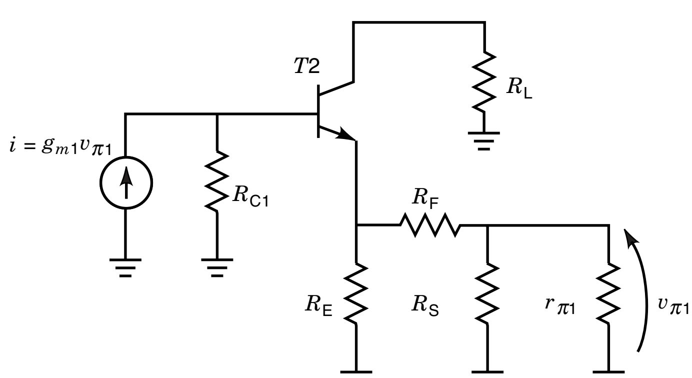


Figure 11. Ac schematic for the evaluation of the return ratio for the circuit in Fig. 9. On replacing the controlled generator of T1 with an independent current source i, T1 becomes an emitter follower.

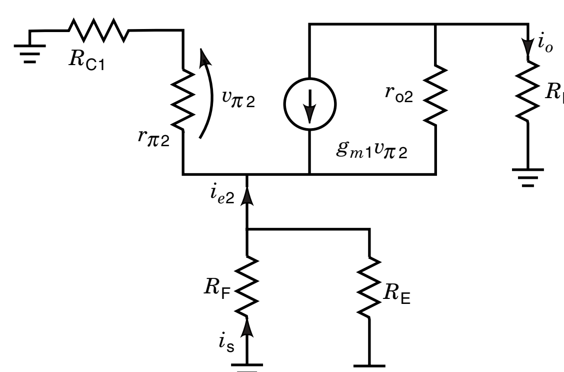


Figure 12. Equivalent circuit for the evaluation of the asymptotic gain for the circuit in Fig. 9. The transconductance of T1 has been made infinitely large.

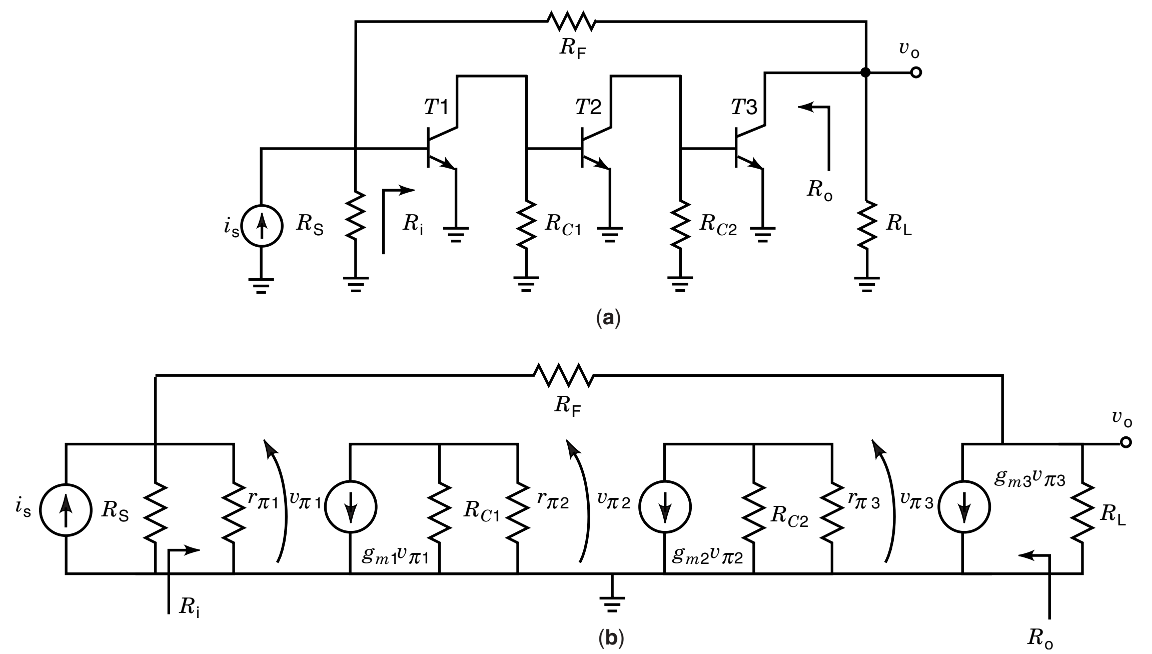


Figure 13. (a) Ac schematic of shunt-shunt feedback amplifier; (b) small-signal equivalent circuit of the shunt-shunt feedback amplifier, obtained by replacing each transistor in (a) with its small-signal model.

tance, $R_M(R_S, R_L) = v_o/i_s$, is nominally invariant with source resistance, load resistance, and transistor parameters.

Considering the equivalent small signal model of the shunt-shunt circuit shown in Fig. 13(b), we can arbitrarily choose the transconductance g_{m1} as the parameter P (other choices do not lead to any substantial differences). Setting $g_{m1} = 0$, the feedforward transresistance and the corresponding input and output resistances are

$$R_{fo} = \left.\frac{v_o}{i_s}\right|_{g_{m1}=0} = \frac{R_S \| r_{\pi 1}}{R_S \| r_{\pi 1} + R_F + R_L} R_L \tag{35}$$

$$R_{iol} = (R_F + R_L) \| r_{\pi 1} \tag{36}$$

$$R_{ool} = R_F + R_S \| r_{\pi 1} \tag{37}$$

The return ratio is

$$T = g_{m1}(R_{C1} \| r_{\pi 2}) g_{m2} (R_{C2} \| r_{\pi 3}) g_{m3} \frac{R_L}{R_L + R_F + R_S \| r_{\pi 1}} R_S \| r_{\pi 1} \tag{38}$$

and the asymptotic transresistance is

$$R_{f\infty} = \left.\frac{v_o}{i_s}\right|_{g_{m1}\to\infty} = -R_F \tag{39}$$

Hence, substituting R_{fo}, T and $R_{f\infty}$ into Eq. (4), we get the closed-loop transresistance R_{cl}.

Finally, the input and output resistance can be simply obtained by properly evaluating the particular return ratio by using Eq. (38).

Series-Series Feedback Amplifier

Figure 14 reports the ac schematic diagram of the series-series feedback amplifier. Three transistors, T1, T2, and T3, are embedded in the open-loop amplifier. Although it is possible to achieve series-series feedback via emitter or source degeneration of a single-stage amplifier, the series-series triple offers substantially more loop gain and thus a better desensitization of the forward gain with respect to both transistor parameters and source and load termination. Of course, these benefits are paid for in terms of frequency response.

One can conveniently choose the transconductance g_{m2} as the parameter P. Hence, assuming ideal behavior for the transistor working as a current or voltage follower, the fundamental relationships are given by

$$G_0 = \left.\frac{i_o}{v_S}\right|_{g_{m2}=0} \approx -\frac{1}{R_F} \tag{40}$$

$$R_{iol} = r_{\pi 1} + (g_{m1} r_{\pi 1} + 1)\left[R_{E1} \| \left(R_F + R_{E2} \| \frac{R_{C2} + r_{\pi 3}}{g_{m3} r_{\pi 3} + 1}\right)\right] \tag{41}$$

$$R_{ool} = r_{o3} + (g_{m3} r_{o3} + 1)\left[(r_{\pi 3} + R_{C2}) \| R_{E2} \| \left(R_F + R_{E2} \left\| \frac{R_S + r_{\pi 3}}{g_{m3} r_{\pi 3} + 1}\right.\right)\right] \tag{42}$$

$$T = R_{C1} g_{m2} \frac{R_{C2}}{R_F} \tag{43}$$

$$G_\infty = \left.\frac{i_o}{v_S}\right|_{g_{m2}\to\infty} \approx -\left(\frac{1}{R_{E1}} + \frac{1}{R_{E2}} + \frac{R_F}{R_{E1} R_{E2}}\right) \tag{44}$$

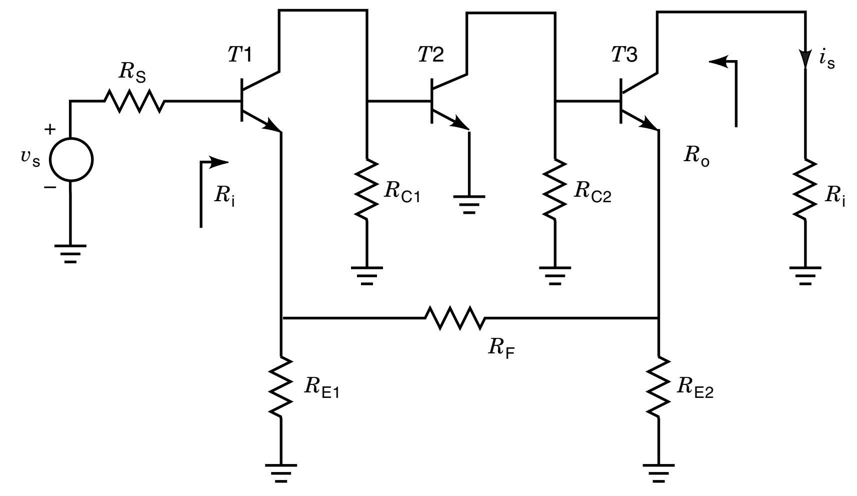


Figure 14. Ac schematic of series-series feedback amplifier.

Since the loop gain generally has a very high value, the closed-loop transconductance is almost equal to the asymptotic one, G_∞. Moreover, the particular return ratios needed to calculate the input and output resistance, assuming r_{o1} to be a very high resistance, are

$$T(0, R_L) = T(R_S, 0) = R_{C1} g_{m2} \frac{g_{C2}}{R_F} \tag{45}$$

$$T(\infty, R_L) \approx 0 \tag{46}$$

$$T_S(R_S, \infty) \approx (R_{C1} \| r_{\pi 2}) g_{m2} \frac{R_{C2}}{R_{C2} + r_{\pi 3} + R_{E2} \| R_F} \frac{R_{E2}}{R_{E2} + R_F} \tag{47}$$

STABILITY

In order show the increase in bandwidth of a feedback amplifier, consider an amplifier having the following single-pole transfer function

$$A(s) = \frac{A_0}{1 + \dfrac{s}{p_1}} \tag{48}$$

Assuming a pure resistive feedback network, the closed-loop transfer function is

$$G_F(s) = \frac{G_{F0}}{1 + \dfrac{s}{(1 + fA_0)p_1}} \approx \frac{G_{F0}}{1 + \dfrac{s}{T_0 p_1}} \tag{49}$$

where G_{F0} is the dc closed-loop gain equal to

$$G_{F0} = \frac{A_0}{1 + fA_0} \approx G_\infty \tag{50}$$

Hence, the resulting pole is shifted to a higher frequency by a factor equal to the dc gain of the return ratio, T_0. It is worth noting that, when a feedback amplifier can be modeled with the scheme in Fig. 3, the gain-bandwidth product of the open-loop amplifier is equal to that of the closed-loop amplifier. Thus the gain-bandwidth product is an invariant amplifier parameter which is independent of the degree of feedback applied. Moreover, it is equal to the maximum bandwidth achieved with a unitary feedback factor, $f = 1$ (i.e., with the amplifier in a unity gain feedback configuration).

Two-Pole Amplifier

Real amplifiers have transfer functions with more than one pole and instability problems arise. Consider now an amplifier with a two-pole transfer function

$$A(s) = \frac{A_0}{\left(1 + \dfrac{s}{p_1}\right)\left(1 + \dfrac{s}{p_2}\right)} \tag{51}$$

Its closed-loop transfer function is

$$G_F(s) = \frac{G_{F0}}{1 + 2\dfrac{\xi}{\omega_0} s + \dfrac{s^2}{\omega_0^2}} \tag{52}$$

where ω_o is the *pole frequency* and ξ is the *damping factor,*

$$\omega_0 = \sqrt{p_1 p_2 (1 + fA_0)} \tag{53}$$

$$\xi = \frac{p_1 + p_2}{2\omega_0} \approx \frac{1}{2\sqrt{T_0}} \left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}} \right) \tag{54}$$

Normalizing the closed-loop transfer function to ω_0, the frequency and step responses for different values of ξ are those plotted in Fig. 15(a) and Fig. 15(b), respectively. The behavior is overdamped, critically damped, or underdamped if the ξ value is greater than, equal to, or lower than 1, respectively. The underdamped condition (i.e., with two complex poles) is critical since overshoot occurs in both the frequency and the time domain and, to keep the peak in both the frequency and step responses below the desired value, the parameter ξ must be properly set.

According to Eq. (54), to avoid overshoot one needs an amplifier, A, with widely spaced poles. More specifically, in order to avoid an excess of underdamping, open-loop amplifiers are designed with a dominant-pole behavior and a second pole at a frequency higher than the gain-bandwidth product, ω_{GBW}, of the return ratio transfer function (i.e., $p_2 > T_0 p_1$). Thus it is

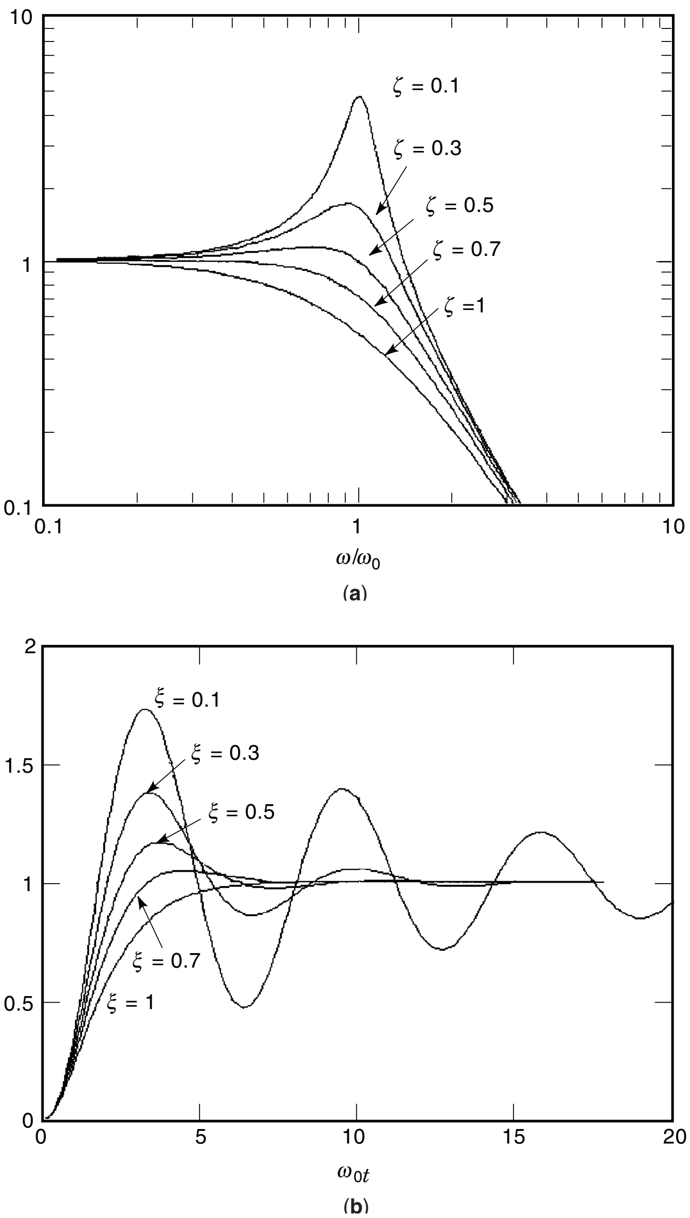


Figure 15. (a) Frequency-response module for a two-pole feedback amplifier in the traditional representation. Overshoot arises for ξ lower than $1/\sqrt{2}$. The overshoot is around the pole frequency ω_0. (b) Step response for a two-pole feedback amplifier in the traditional representation.

useful to define the separation factor, K, between the second pole and the gain-bandwidth product of the return ratio T (observe that the return ratio transfer function has the same pole as the amplifier transfer function) (14)

$$K = \frac{p_2}{\omega_{\mathrm{GBW}}} = \frac{p_2}{T_0 p_1} \tag{55}$$

A well-known parameter which gives the degree of stability of a feedback system is the phase margin, Φ, defined as 180° plus the phase of the return ratio evaluated at the transition frequency ω_T. For a two-pole system it is

$$\Phi = 180^\circ - \mathrm{arctg}\frac{\omega_T}{\omega_1} - \mathrm{arctg}\frac{\omega_T}{\omega_2} = \mathrm{arctg}\frac{\omega_1}{\omega_T} + \mathrm{arctg}\frac{\omega_2}{\omega_T} \tag{56}$$

Since for a dominant-pole amplifier the gain–bandwidth product, ω_{GBW}, is about equal to the transition frequency, ω_{T}, and arctg $(\omega_1/\omega_{\mathrm{T}}) \approx 0$, from Eqs. (55) and (56) one obtains

$$K \approx \tan\phi \tag{57}$$

Hence, for a required phase margin one obtains the value of the separation factor needed during the compensation design step. Of course, there is the well-known rule that the phase margin must be greater than 45° to avoid excessive underdamped behavior. Moreover, the underdamped natural frequency and the damping factor can be represented as

$$\omega_0 = p_1\sqrt{KT_0(1+T_0)} \approx \omega_{\mathrm{GBW}}\sqrt{K} \tag{58}$$

$$\xi = \frac{1}{2}\frac{1+KT_0}{\sqrt{K}T_0} \approx \frac{\sqrt{K}}{2} \tag{59}$$

and hence the closed-loop transfer function is

$$G_{\mathrm{F}}(s') = \frac{G_{\mathrm{F0}}}{1 + s' + \dfrac{s'^2}{K}} \tag{60}$$

where the complex frequency s' is the complex frequency s normalized to ω_{GBW}. This is a useful representation of a closed-loop amplifier, since it is simple and depends on K (or the phase margin) and ω_{GBW}, which are two fundamental parameters in amplifier design. The frequency and step responses for different values of K are those plotted in Fig. 16(a) and Fig. 16(b), respectively. The overshoot in the frequency domain of the transfer function in Eq. (60) occurs at a frequency ω_{cp} given by

$$\omega_{\mathrm{cp}} = \omega_{\mathrm{GBW}}\sqrt{K - \frac{K^2}{2}} \tag{61}$$

It is apparent that for values of K greater than 2 peaking is avoided in the frequency domain. In order to optimize the closed-loop amplifier time response (15), useful information for the designer is the time, t_{pcl}, at which the first peak occurs and its overshoot, D, given by

$$t_{\mathrm{p}} = \frac{2\pi}{\omega_{\mathrm{GBW}}\sqrt{4K - K^2}} \tag{62}$$

$$D = e^{-\pi\sqrt{\frac{K}{4-K}}} \tag{63}$$

For example, having the minimum settling time at 0.1%, Eq. (63) gives a K equal to 2.75 (i.e., a phase margin of 70°); then from Eq. (62) the amplifier gain-bandwidth product needed to achieve the required settling time can be found.

Three-Pole Amplifier

For amplifiers with more than two poles, more accurate relationships have to be used during compensation. Of course, a

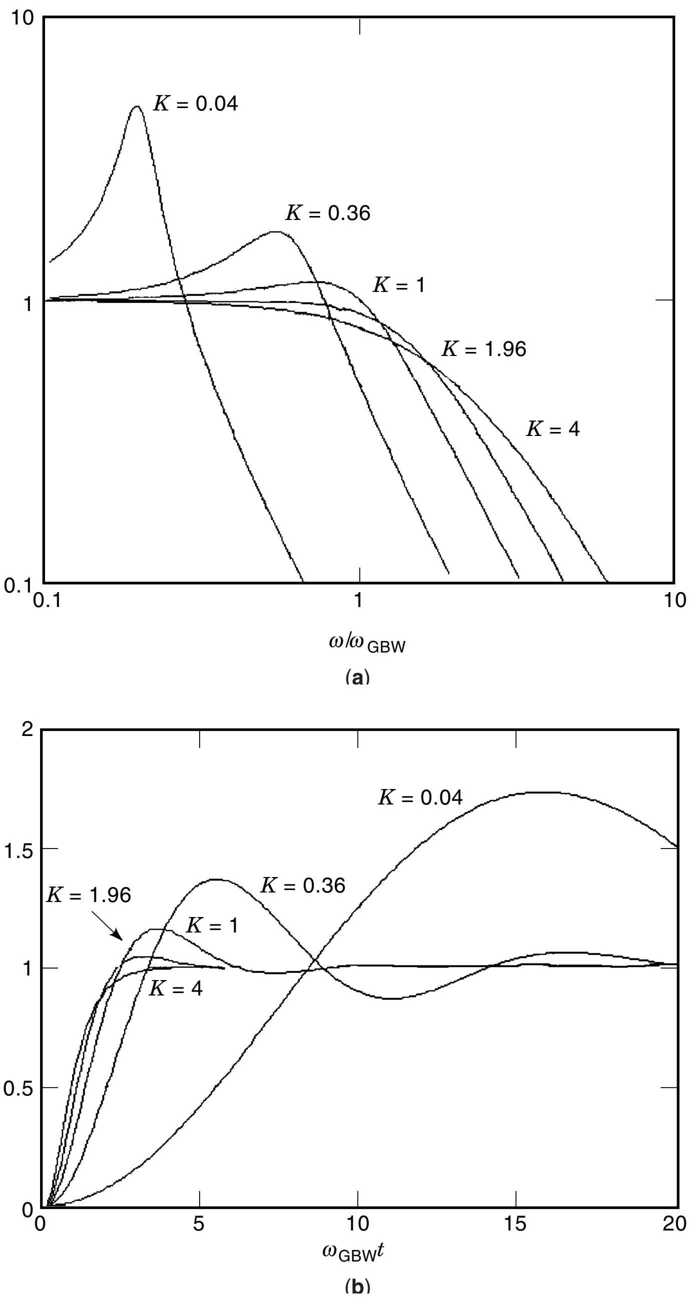


Figure 16. (a) Frequency-response module for a two-pole feedback amplifier in the proposed representation. Overshoot arises for values of K lower than 2. (b) Step response for a two-pole feedback amplifier in the proposed representation. Rise time and settling time increase for values of K greater than 2.

dominant-pole behavior is mandatory to achieve stability. Consider an amplifier with three separate poles:

$$A(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p2}\right)\left(1+\frac{s}{p_3}\right)} \tag{64}$$

If ω_1 is the dominant pole, the phase margin is equal to

$$\phi \approx \text{arctg}\frac{\omega_2}{\omega_{\text{GBW}}} + \text{arctg}\frac{\omega_3}{\omega_{\text{GBW}}} - 90^\circ \tag{65}$$

Hence, remembering that $\tan(a + b) = \tan(a) + \tan(b)/1 - \tan(a)\tan(b)$ and $\tan(a + 90°) = -1/\tan(a)$, one obtains

$$\frac{1}{\omega_{\text{GBW}}^2} - \tan(\phi)\left(\frac{1}{\omega_2} + \frac{1}{\omega_3}\right)\frac{1}{\omega_{\text{GBW}}} - \frac{1}{\omega_2\omega_3} = 0 \tag{66}$$

By solving Eq. (66) the required gain-bandwidth product for a fixed phase margin is obtained:

$$\frac{1}{\omega_{\text{GBW}}} = \frac{\tan(\phi)}{2}\left(\frac{1}{\omega_2} + \frac{1}{\omega_3}\right)\left[1 + \sqrt{1 + \frac{4}{\tan^2(\phi)}\frac{\omega_2\omega_3}{(\omega_2+\omega_2)^2}}\right] \approx \tan(\phi)\left(\frac{1}{\omega_2} + \frac{1}{\omega_3}\right) \tag{67}$$

It is worth noting that compensation of a three-stage amplifier can be performed like that of a two-pole amplifier, where the equivalent time constant of the second pole is equal to the sum of the second and third pole time constants of the three-pole amplifier.

POLE SPLITTING COMPENSATION

Generally, the return ratio of amplifiers used in negative feedback is not characterized by a dominant-pole frequency response. Therefore, compensation is needed to achieve the required phase margin. Compensation can be simply performed by increasing the capacitance at the node which determines the lower pole. However, except in the case of a one-stage amplifier, such as a cascade amplifier, a more efficient approach based on pole splitting compensation can be used.

Open-Loop Amplifier

The return ratio of a two-stage feedback amplifier such as the series-shunt and the shunt-series feedback amplifiers in Fig. 4(a) and Fig. 9(a), can be evaluated with the simplified scheme plotted in Fig. 17, which is composed of two equivalent transconductances and two equivalent resistances with the associated parasitic capacitances. More specifically, C_i is the equivalent capacitance at the interstage node, the capacitance C_o is the equivalent one at the output node, and C_r is the equivalent capacitance across the two stages (C_C is the capacitor used to achieve compensation). Moreover, for the se-

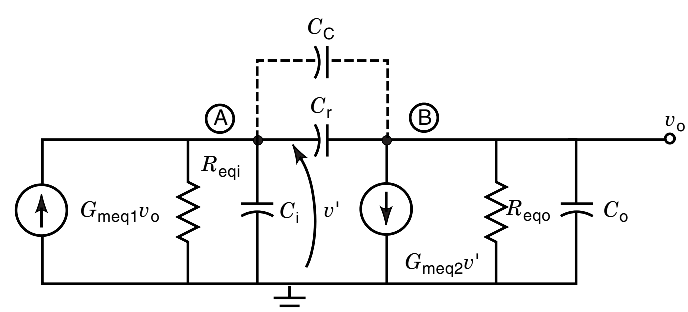


Figure 17. Small-signal equivalent circuit for the evaluation of the generalized loop gain frequency behavior of two-stage amplifiers. The two stages are represented by G_{meq1}, R_{eqi} and by G_{meq2}, R_{eqo}. Since we assume the amplifier in unity-gain configuration, the output voltage v_o drives the input stage.

ries-shunt amplifier G_{meq2} is equal to g_{m2} and G_{meq1}; R_{eqi} and R_{eqo} are about equal to $1/R_F$, $R_{C1}//r_{\pi 2}$, and $R_{LT}//R_F$, respectively. For the shunt-series amplifier G_{meq2} is equal to g_{m1} and G_{meq1}, R_{eqi} and R_{eqo} are about equal to $1/(R_S//r_{\pi 1} + R_F)$, $R_S//r_{\pi 1}$ and R_{C1}, respectively.

Referring to Fig. 17, the return ratio, $T(s)$, can be written in the form

$$T(s) = T_0 \frac{1 - \dfrac{s}{z_r}}{1 + \left(\dfrac{1}{p_1} + \dfrac{1}{p_2}\right) s + \dfrac{s^2}{p_1 p_2}} \tag{68}$$

where

$$T_0 = G_{meq1} R_{eqi} G_{meq2} R_{eqo} \tag{69}$$

The dashed branch containing the capacitor C_C, which will be addressed later, is the pole splitting compensation element. The frequency, z_r, of the right half-plane zero due to the forward path through the feedback capacitance, C_r, to the output is given by

$$z_r = \frac{G_{meq2}}{C_r} \tag{70}$$

and the lower pole frequency, p_1, and the higher pole frequency, p_2, derive implicitly from

$$\frac{1}{p_1} + \frac{1}{p_2} = R_{eqo}(C_o + C_r) + R_{eqi}[C_i + (1 + G_{meq2} R_{eqo}) C_r] \tag{71}$$

and

$$\frac{1}{p_1 p_2} = R_{eqi} R_{eqo} C_o \left[C_i + \left(1 + \frac{C_i}{C_o}\right) C_r\right] \tag{72}$$

Pole Splitting Analysis

Under the assumption that the amplifier has a dominant-pole behavior (fundamental to use the amplifier in feedback), one can neglect the term $1/p_2$ in Eq. (71) with respect to the term $1/p_1$. Consequently, the following pole expressions are obtained:

$$p_1 \approx \frac{1}{R_{eqo}(C_o + C_r) + R_{eqi}[C_i + (1 + G_{meq2} R_{eqo}) C_r]} \approx \frac{1}{R_{eqi}[C_i + (1 + G_{meq2} R_{eqo}) C_r]} \tag{73a}$$

$$p_2 \approx \frac{R_{eqo}(C_o + C_r) + R_{eqi}[C_i + (1 + G_{meq2} R_{eqo}) C_r]}{R_{eqi} R_{eqo} C_o \left[C_i + \left(1 + \dfrac{C_i}{C_o}\right) C_r\right]} \approx \frac{C_i + (1 + G_{meq2} R_{eqo}) C_r}{R_{eqo} C_o \left[C_i + \left(1 + \dfrac{C_i}{C_o}\right) C_r\right]} \tag{73b}$$

It is worth noting that the above approximations hold since, in practical cases, the Miller effect represented by the term $(1 + G_{meq2} R_{eqo}) C_r$ leads to an input dominant pole. From Eqs. (73) the pole splitting due to Miller effect it is apparent. An increase in the internal feedback capacitance, C_r, moves the dominant pole and the second pole to lower and higher frequencies, respectively. Thus, in order to improve the separation of the two poles it is efficient to increase C_r since its contribution is magnified by the gain factor $(1 + G_{meq2} R_{eqo})$. Actually, this is the technique followed to perform compensation (i.e., to obtain a phase margin greater than 45° or $K > 1$), which allows the amplifier to be connected in a closed loop without an excess of underdamped behavior. In this case one adds to the internal feedback capacitance C_r a compensation capacitance, C_c and, since the Miller effect becomes the dominant capacitive contribution, Eqs. (73) can be further simplified:

$$P_{1c} \approx \frac{1}{R_{oeqi} G_{meq2} R_{eqo} C_p} \tag{74a}$$

$$p_{2c} \approx \frac{G_{meq2}}{C_o + C_i} \tag{74b}$$

where the capacitance C_p, which is the sum of C_r and C_c, has been assumed to be greater than C_i or C_o. After compensation the value of the second pole given by Eq. (74b) finds an intuitive justification. At the frequency at which it occurs, the capacitance C_p can be considered short-circuited, and Eq. (74b) can be simply obtained by inspection of the circuit in Fig. 17.

From Eqs. (69) and (74a), the gain-bandwidth product is

$$\omega_{GBW} = \frac{B_{meq1}}{C_p} \tag{75}$$

Sometimes the large transconductance, G_{meq2}, allows the zero which is now given by Eq. (70) substituting C_p for C_r to be neglected. Otherwise, the right half-plane zero determines a negative contribution on the phase margin, and it must be compensated as discussed in the next subsection to achieve the required phase margin.

Considering the return ratio of a two-stage amplifier compensated by using the Miller effect, where the zero has also been compensated, it is apparent that the bandwidth performance of the amplifier is only set by the frequency of second pole given in Eq. (74b), and the separation factor, K, is

$$K = \frac{G_{meq2}}{G_{meq1}} \frac{C_p}{C_o + C_i} \tag{76}$$

Hence, one has to choose the compensation capacitance, C_c, to provide the value of K (always greater than 1) which gives the required frequency- or time-domain behavior.

The compensation technique is extensively used in the design of two-stage amplifiers. Moreover, for off-the-shelf bipolar amplifiers including a voltage buffer output stage, compensation is still achieved by means of the Miller effect by using Eq. (67). It is worth noting that when using three-stage gain amplifiers such as the shunt-shunt feedback amplifier in Fig. 13, the series-series feedback amplifier in Fig. 14, or CMOS power amplifiers, low-voltage signal amplifiers, and so forth, *nested* Miller, employing two or more compensation capacitors, is mandatory (16,17).

Zero Compensation Techniques

Various techniques for compensation of the right half-plane zero have been proposed for two-stage MOS opamps. They are based on the concept of breaking the forward path through the compensation capacitor by using active or passive components. The original of these was first applied in an NMOS opamp (18) and then in a CMOS opamp (19). It breaks the forward path by introducing a voltage buffer in the compensation branch. Then a compensation technique was proposed which uses a nulling resistor in series with the compensation capacitor (20). Another solution works like the former but uses a current buffer to break the forward path (21). Finally, both current and voltage buffers can be used for compensation of the right half-plane zero (22).

The most popular compensation technique is that based on the nulling resistor, since it can be implemented using only a MOS transistor biased in the triode region (which approximates a linear resistor) and does not reduce the input or output dynamic range of the original amplifier. It is achieved by introducing the resistance R_c in series with the compensation capacitor, as shown in Fig. 18(a). Neglecting the capacitance C_r (usually much lower than C_C), the zero is now at a frequency of

$$z_r = \frac{1}{\left(\dfrac{1}{G_{meq2}} - R_c\right) C_c} \tag{77}$$

and is moved to infinite frequency by setting R_c equal to $1/G_{meq2}$. Thus from Eqs. (55), (74), and (75) assuming $C_p \approx C_c$, one gets

$$\omega_{GBW} \approx \frac{G_{meq1}}{K(C_o + C_i)} \tag{78}$$

Hence the gain-bandwidth product is inversely proportional to the sum of the output and interstage capacitance.

The resistance R_c can also be set to compensate the second pole giving a new second pole $1/(R_c C_c)$, as proposed in (23), but this approach has a worse ω_{GBW} than the other optimized compensation strategies described below.

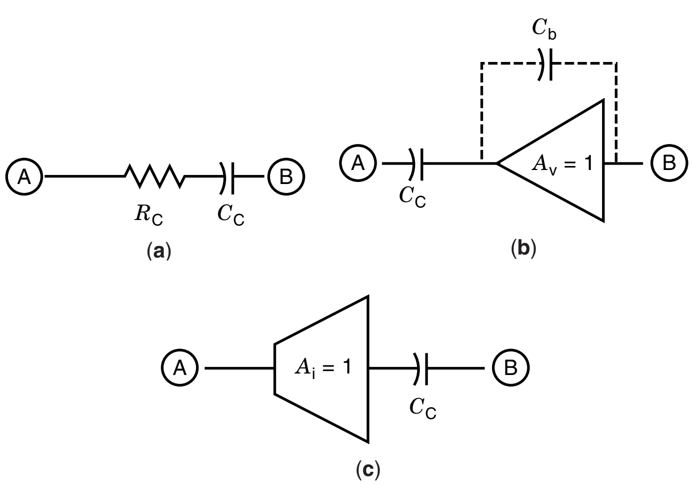


Figure 18. (a) Compensation network with nulling resistor. The technique allows one to modify the zero z_r according to Eq. (77). (b) Compensation network with voltage buffer that breaks the feedforward path. (c) Compensation network with current buffer: another way to break the feedforward path.

Figure 18(b) shows the compensation branch with a voltage buffer. Use of an ideal voltage buffer (i.e., with zero output resistance) to compensate the right half-plane zero gives the same second pole as Eq. (74b) without the dependence on the interstage capacitance, C_i and, hence, about the same ω_{GBW}. On the other hand, the finite output resistance of a real voltage buffer leads to a left half-plane zero, which can be efficiently exploited to perform a pole-zero compensation and to increase the amplifier gain-bandwidth (24). Following this last compensation strategy the second pole is given by

$$p_2 = \frac{G_{meq2}}{C_o} \frac{C_C - C_b}{C_i + C_b} \tag{79}$$

where C_b is the feedforward capacitance of the voltage buffer. After solving C_c by substituting Eq. (79) into Eq. (55), the gain-bandwidth product is

$$\omega_{GBW} \approx \frac{G_{meq1}}{\dfrac{C_b}{2} + \sqrt{\dfrac{G_{meq1}}{G_{meq2}} K(C_i + C_b) C_o}} \tag{80}$$

The resulting ω_{GBW} has a higher value than that given by Eq. (78), and, apart from the small contribution of C_b, is inversely dependent on the geometric mean of $C_i + C_b$ and C_o.

Compensation based on a current buffer, as shown in Fig. 18(c), is very efficient both for the gain-bandwidth (25,26) and the PSRR performance (21,27,28). Moreover, unlike the voltage buffer, it does not have the drawback of reducing the amplifier output swing.

Considering an ideal current buffer in the compensation branch, the second pole is given by

$$p_2 = \frac{G_{meq2}}{C_i \left(1 + \dfrac{C_o}{C_C}\right)} \tag{81}$$

which leads to the gain-bandwidth product

$$\omega_{GBW} \approx \frac{G_{meq1}}{\dfrac{G_{meq1}}{2G_{meq2}} K C_i + \sqrt{\dfrac{G_{meq1}}{G_{meq2}} K C_i C_o}} \tag{82}$$

Since usually $C_b \le C_i < C_o$ and $G_{meq1} < G_{meq2}$, the first term of the denominator of Eq. (82) is negligible and, hence, the performance obtainable with an ideal current buffer is slightly better than that obtained using an optimized design based on a voltage buffer. However, compensation with a real current buffer (i.e., with finite input resistance) is not as straightforward as other compensation approaches. As shown in Ref. 29, in order to achieve compensation, one needs to guarantee that the input resistance of the current buffer, R_b, must be equal to or lower than half $1/G_{meq1}$. Moreover, the condition

$$R_b = \frac{1}{2G_{meq1}} \tag{83}$$

allows compensation based on a current buffer to be optimized. Under the condition of Eq. (83) one obtains the following gain-bandwidth product:

$$\omega_{\rm GBW} \approx \frac{G_{meq1}}{\dfrac{G_{meq1}}{2G_{meq2}}\dfrac{2K-1}{2+K}C_{\rm i} + \sqrt{\dfrac{G_{meq1}}{G_{meq2}}\left(\dfrac{2K-1}{2+K}+\dfrac{1}{2}\right)C_{\rm o}C_{\rm i}}} \tag{84}$$

Thus, $\omega_{\rm GBW}$ is at least 20% higher than that obtained with an ideal current buffer (29). On the other hand, for the same gain-bandwidth product this kind of compensation needs more area and/or power than that based on a voltage buffer.

DISTORTION IN CLOSED-LOOP AMPLIFIERS

To characterize the effects of nonlinearity in circuits and systems used as linear blocks, harmonic distortion terms are often used. More specifically, consider the open-loop amplifier to be nonlinear with its transfer function, $A(x_{\rm i})$, well represented by the first three terms of a power series

$$x_{\rm o} = A(x_{\rm i}) \approx a_1 x_{\rm i} + a_2 x_{\rm i}^2 + a_3 x_{\rm i}^3 \tag{85}$$

Assuming that the incremental input voltage is a pure sinusoidal tone $x_{\rm i} = X_{\rm i}\cos(\omega_1 t)$, one obtains the following output:

$$x_{\rm o} = b_0 + b_1\cos(\omega_1 t) + b_2\cos(2\omega_1 t) + b_3\cos(3\omega_1 t) \tag{86}$$

where terms $b_{\rm i}$ up to the third order are

$$b_0 = \frac{a_2}{2}X_{\rm i}^2 \tag{87a}$$

$$b_1 = a_1 X_{\rm i} + \frac{3}{4}a_3 X_{\rm i}^3 \tag{87b}$$

$$b_2 = \frac{a_2}{2}X_{\rm i}^2 \tag{87c}$$

$$b_3 = \frac{a_3}{4}X_{\rm i}^3 \tag{87d}$$

and hence the second and third harmonic distortion factors are given by

$$HD_{2o} = \frac{|b_2|}{|b_1|} \approx \frac{1}{2}\frac{a_2}{a_1}X_{\rm i} = \frac{1}{2}\frac{a_2}{a_1^2}X_{\rm o} \tag{88a}$$

$$HD_{3o} = \frac{|b_3|}{|b_1|} \approx \frac{1}{4}\frac{a_3}{a_1}X_{\rm i}^2 = \frac{1}{4}\frac{a_3}{a_1^3}X_{\rm o}^2 \tag{88b}$$

in which the gain compression, which arises in term b_1 and is due to term a_3 (30), has been neglected. In order to allow a simple comparison with the closed-loop case, the harmonic factors refer to the output voltage magnitude.

Linear Feedback

If we close the amplifier in a loop with a linear feedback, f (which means a return ratio $T_0 = fa_1$), the harmonic distortion terms given by Eq. (88) must be reduced by the factors $(1 + T_0)^2$ and $(1 + T_0)^3$, respectively. This implies a reduction, approximately equal to the return ratio T_0, in the harmonic distortion terms referring to the output signal magnitude.

As reported in Ref. 31, a more accurate analysis shows that the harmonic distortion terms for a closed-loop amplifier are given by

$$HD_{2fl} = \frac{1}{2}\frac{a_2}{a_1}\frac{1}{(1+T_0)^2}X_{\rm s} = \frac{1}{2}\frac{a_2}{a_1^2}\frac{1}{1+T_0}X_{\rm o} \tag{89a}$$

$$HD_{3fl} = \frac{1}{4}\frac{a_3}{a_1}\frac{1-\dfrac{2fa_2^2}{a_3(1+T_0)}}{(1+T_0)^3}X_s^2 = \frac{1}{4}\frac{a_3}{a_1^3}\frac{1-\dfrac{2fa_2^2}{a_3(1+T_0)}}{1+T_0}X_{\rm o}^2 \tag{89b}$$

The third harmonic distortion can be further minimized by canceling its numerator, according to the following relation:

$$\frac{f}{1+T_0} = \frac{a_3}{2a_2^2} \tag{90a}$$

which with high return ratios, T_0, simplifies to

$$a_1 = \frac{2a_2^2}{a_3} \tag{90b}$$

Moreover, according to Eq. (89b) for amplifiers where the term a_3 is negligible, the third harmonic is still determined by the term a_2.

Nonlinear Feedback

Considering a feedback amplifier where the feedback path is also nonlinear and is represented by the following relation:

$$x_{\rm f} = F(x_{\rm o}) = f_1 x_{\rm o} + f_2 x_{\rm o}^2 + f_3 x_{\rm o}^3 + \cdots \tag{91}$$

it is demonstrated in (32) that, assuming the return ratio, $T_{\rm o}$, to be much greater than 1, the second and third harmonic distortion coefficients are respectively given by

$$HD_{2f} = \frac{1}{2}\left(\frac{1}{T_{\rm o}a_1}a_{2N} - f_{2N}\right)X_{\rm o} \tag{92a}$$

$$HD_{3f} = \frac{1}{4}\left[\frac{1}{T_{\rm o}a_1^2}(a_{3N} - 2a_{2N}^2) - (f_{3N} - 2f_{2N}^2) - 4\frac{1}{T_{\rm o}a_1}a_{2N}f_{2N}\right]X_{\rm o}^2 \tag{92b}$$

where a_{2N} and a_{3N} represent the amplifier terms normalized to the amplifier gain a_1, and f_{2N} and f_{3N} represent the feedback terms normalized to the feedback gain f_1. It is apparent that feedback does not reduce the nonlinearity of the feedback network. Thus one cannot obtain an amplifier having a nonlinearity lower than that of the feedback network, and even small nonlinearity terms of feedback networks cannot be neglected, but must be taken into account during harmonic distortion evaluation. It is also worth noting that for negative feedback, distortion due to the feedback network has an opposite sign to that due to the amplifier. A more compact and clear representation of the harmonic distortion in a nonlinear amplifier with nonlinear feedback is

$$HD_{2f} = HD_{2fl} + HD_{2fn} \tag{93a}$$

$$HD_{3f} = HD_{3l} + HD_{3fn} + 4HD_{2fl}HD_{2fn} \tag{93b}$$

where HD_{2fn}, and HD_{3fn}, are the harmonic distortion terms of the feedback amplifier assuming nonlinear feedback but a linear amplifier, which are given by

$$HD_{2fn} = -\frac{1}{2}\frac{f_2 a_1^2}{(1+T_0)^2} X_S = -\frac{1}{2}\frac{f_2 a_1}{(1+T_0)} X_o \quad (94a)$$

$$HD_{3fn} = -\frac{1}{4}\frac{f_3 a_1^3 - \dfrac{2f_2^2 a_1^4}{(1+T_0)}}{(1+T_0)^3} X_s^2 = -\frac{1}{4}\frac{f_3 a_1 - \dfrac{2f_2^2 a_1^2}{(1+T_0)}}{1+T_0} X_0^2 \quad (94b)$$

Hence, the second and third harmonic distortion terms can be compactly represented by Eqs. (93), which are only a simple function of the second and third harmonic distortion of the whole feedback network evaluated in two particular cases:

1. A nonlinear amplifier with a linearized feedback network.
2. A linearized amplifier with a nonlinear feedback network.

BIBLIOGRAPHY

1. M. Ghausi, *Electronic Devices and Circuits: Discrete and Integrated,* Philadelphia: Saunders, 1985.
2. J. Millman and A. Grabel, *Microelectronics,* 2nd ed., Singapore: McGraw-Hill, 1987.
3. A. Sedra and K. Smith, *Microelectronic Circuits,* 3rd ed., Philadelphia: Saunders, 1991.
4. S. Ben-Yaakov, A unified approach to teaching feedback in electronic circuits courses, *IEEE Trans. Educ.,* **34**: 310–316, 1991.
5. P. Gray and R. Meyer, *Analysis and Design of Analog Integrated Circuits,* 3rd ed., New York: Wiley, 1993.
6. K. Laker and W. Sansen, *Design of Analog Integrated Circuits and Systems,* New York: McGraw-Hill, 1994.
7. A. Arbel, Negative feedback revisited, *Analog Integ. Circuits Signal Process.,* **10** (3): 157–178, 1996.
8. P. Hurst, A comparison of two approaches to feedback circuit analysis, *IEEE Trans. Educ.,* **35**: 253–261, 1992.
9. G. Palumbo and J. Choma Jr., An overview of analog feedback, Part I: Basic theory, *Analog Integ. Circuits Signal Process.,* in press.
10. S. Rosenstark, A simplified method of feedback amplifier analysis, *IEEE Trans. Educ.,* **E-17**: 192–198, 1974.
11. J. Choma, Jr., Signal flow analysis of feedback networks, *IEEE Trans. Circ. Syst.,* **37**: 455–463, 1990.
12. R. Blackman, Effect of feedback on impedance, *Bell Syst. Tech. J.,* **22**: 269–277, 1943.
13. G. Palumbo and J. Choma, Jr., An overview of analog feedback, Part II: Amplifier configurations in generic device technologies, in print *Analog Integ. Circuits Signal Process.*
14. G. Palmisano and G. Palumbo, A novel representation for two-pole feedback amplifiers, *IEEE Trans. Educ.,* **41**: 216–218, 1998.
15. H. Yang and D. Allstot, Considerations for fast settling operational amplifiers, *IEEE Trans. Circuits Syst.,* **37**: 326–334, 1990.
16. E. Cherry, A new result in negative feedback theory, and its application to audio power amplifiers, *Int. J. Circuit Theory,* **6**: 265–288, 1978.
17. R. Eschauzier and J. Huijsing, *Frequency Compensation Techniques for Low-Power Operational Amplifiers,* Norwell, MA: Kluwer, 1995.
18. Y. Tsividis and P. Gray, An integrated NMOS operational amplifier with internal compensation, *IEEE J. Solid-State Circuits,* **SC-11**: 748–754, 1976.
19. G. Smaradoiu et al., CMOS pulse-code-modulation voice codec, *IEEE J. Solid-State Circuits,* **SC-13**: 504–510, 1978.
20. D. Senderowicz, P. Gray, and D. Hodges, High-performance NMOS operational amplifier, *IEEE J. Solid-State Circuits,* **SC-13**: 760–766, 1978.
21. B. Ahuja, An improved frequency compensation technique for CMOS operational amplifiers, *IEEE J. Solid-State Circuits,* **SC-18**: 629–633, 1983.
22. C. Makris and C. Toumazou, Current-mode active compensation techniques, *Electron. Lett.,* **26**: 1792–1794, 1990.
23. W. Black, Jr., D. Allstot, and R. Reed, A high performance low power CMOS channel filter, *IEEE J. Solid-State Circuits,* **SC-15**: 929–938, 1980.
24. G. Palmisano and G. Palumbo, An optimized compensation strategy for two-stage CMOS Op Amps, *IEEE Trans. Circuits Syst.,* **42**: 178–182, 1995.
25. R. Castello, CMOS Buffer Amplifier, in J. Huijsing, R. van der Plassche, and W. Sansen (eds.), *Analog Circuit Design,* Norwell, MA: Kluwer, 1993, pp. 113–138.
26. R. Reay and G. Kovacs, An unconditionally stable two-stage CMOS amplifier, *IEEE J. Solid-State Circuits,* **30**: 591–594, 1995.
27. D. Ribner and M. Copeland, Design techniques for cascoded CMOS op amps with improved PSRR and common-mode input range, *IEEE J. Solid-State Circuits,* **SC-19**: 919–925, 1984.
28. M. Steyaert and W. Sansen, Power supply rejection ratio in operational transconductance amplifier, *IEEE J. Solid-State Circuits,* **37**: 1077–1084, 1990.
29. G. Palmisano and G. Palumbo, Two-stage CMOS OPAMPS based on current buffer, *IEEE Trans. Circuits Syst. I,* **44**: 257–262, 1997.
30. R. Meyer and A. Wong, Blocking and desensitization in RF amplifier, *IEEE J. Solid-State Circuits,* **30**: 944–946, 1995.
31. D. Pederson and K. Mayaram, *Analog Integrated Circuits for Communication: Principle, Simulation and Design,* Norwell, MA: Kluwer, 1991.
32. G. Palumbo and S. Pennisi, Harmonic distortion in nonlinear amplifier with nonlinear feedback, in print, *Int. J. Circuit Theory Appl.*, **26**: 293–299, 1998.

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SIGNAL COMPRESSION. See LOGARITHMIC AMPLIFIERS.
SIGNAL DELAY. See INTEGRATED CIRCUIT SIGNAL DELAY.