TRANSMISSION USING CHAOTIC SYSTEMS

As surprising as it might seem, conveying information using chaotic carrier signals can be highly beneficial and desirable in many applications in which the information (1) has to be delivered to somebody with a certain level of secrecy and (2) has to be spread over a channel without interfering with other communications. The purpose of this article is to show why and how it is possible to design chaotic systems that encode information signals by mapping them onto chaotic signals. We will describe special modulation processes in which the information signal modulates a chaotic deterministic signal. Of course this kind of modulation is only meaningful if one is able to design a demodulation process that can extract the information from the modulated signal. Therefore the main problem we will address is the design of receiver systems that are able to detect the information signal by processing the modulated chaotic signal.

The information transmission systems we will deal with can be decomposed according to Fig. 1. An information signal $s(t)$ is injected into a chaotic dynamical system that produces a chaotic output signal $y(t)$. The transmission of $y(t)$ through some medium, called the *channel,* degrades it in such a way that a chaos like signal $\hat{y}(t)$ enters the receiver. The receiver, which will be explained further, extracts by a suitable procedure the information signal from $\hat{y}(t)$. This produces a signal $\hat{s}(t)$ that should be as accurate as possible a copy of the original information signal $s(t)$. Before going into details, we briefly list the advantages of chaos when properly used in communication systems.

1. Deterministic chaotic systems produce deterministic signals which "look like" noise and are wideband signals.
2. Two exact copies of a chaotic system, when started with infinitely small different initial conditions, produce decorrelated output signals after a short transient. In other words, it is said that one manifestation of chaos is the property of extreme sensitivity to initial conditions.
3. Very simple systems can be designed in such a way that they behave chaotically.
4. Certain classes of chaotic oscillators can be shown to be synchronizable.
5. Determinism in chaotic signals can be exploited to enhance these signals when they are corrupted by noise.

It is now easy to understand why chaotic signals and systems can be useful for transmission purposes. Property 1 suggests that chaotic oscillators could be used to spread (in frequency) information through a channel. Property 2 indicates that two identical oscillators started or modulated with two different initial conditions will produce two uncorrelated modulated signals. This decorrelation of modulated signals active in the same band suggests that a receiver properly tuned to one of the oscillators and started with the appropriate initial condition will barely notice the other transmissions if one uses a detection based on correlation techniques. Therefore chaotic modulation can be seen as an alternative spread-spectrum modulation in which other communications, although sharing the same channel, are transparent to each other as well as to other interfering signals. In other words, each transmitter–receiver pair possesses a transmission key that ensures that two different transmitter–receiver pairs will send two orthogonal signals through the channel. What would this key be? The key could be, for instance, a combination of parameter values of the oscillator and initial conditions in the transmitter. Figure 2 depicts an example. In this example, two communications have been sent through the same channel, which is modeled by a constant delay plus an additive constant noise. To highlight the good decorrelation properties of chaotic systems, the two information signals are modulated using two exact copies of discrete-time chaotic dynamical systems. The only difference in the modulation comes from two slighly different initial conditions in the oscillators. Note that this difference is very small, only 1×10^{-6}. As indicated by prop-

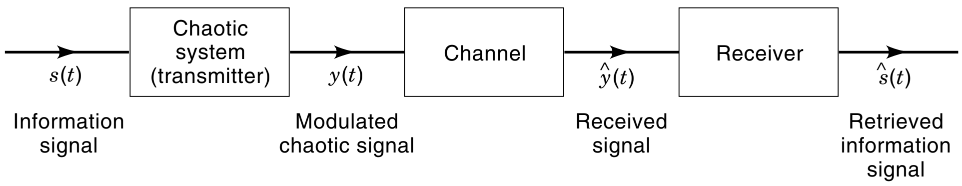

Figure 1. Transmission system.

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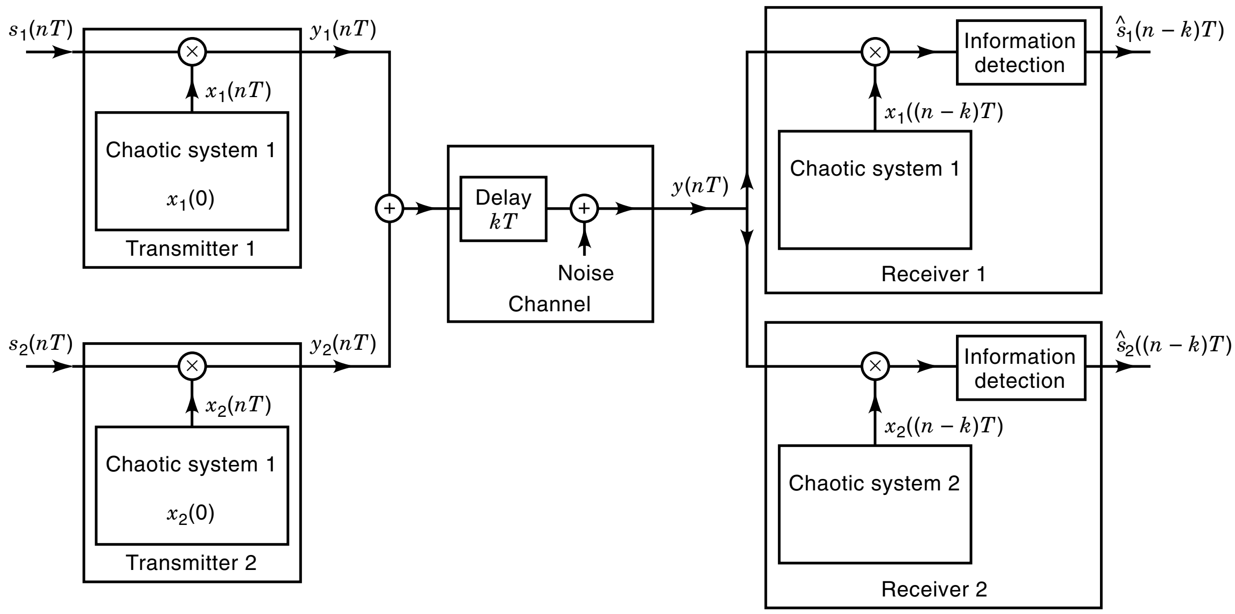

Figure 2. Chaotic modulation and demodulation of two information signals sent through the same channel.

erty 3, the chaotic system does not have to be very complicated. This is the case in the example, in which the oscillator is a skew tent map system that is described in Fig. 3. The parameter a of the skew tent map is chosen in such a way that the system operates in a chaotic regime ($0 < \alpha < 1$). For each bit of information to be sent to the channel a signal sequence of 500 iterations of the skew tent map system multiplies a constant signal whose value is -1 or $+1$ according to the value of the bit we want to transmit 0 or 1. This principle of modulation is shown in Fig. 4. The receiver oscillators are supposed to be synchronized by adjusting their delay to that of the channel for their corresponding signals of interest. Therefore in this (academic) example we have chosen to detect the information signal by measuring a filtered version of a sliding correlation function. Ideally the demodulation should provide a positive constant $+C$ when a bit 1 is transmitted while giving us $-C$ when a 0 is transmitted. The communication noise in the example has been adjusted in such a way that its power is equal to that of $y_1(n)$. Despite the consecutive very bad signal-to-noise ratio, the retrieved sequences after filtering provide the correct sequences of bits since any thresehold detector would produce the exact information sequences. This example illustrates some of the appealing aspects of chaos modulation; that is, we can spread information on a channel and make this information transparent. The user who wants to demodulate the information has to find the key. The key in our example is just an initial condition and a parameter value both the transmitter and the receiver have agreed to use in advance. The property of sensitivity tells us that there is a large number of keys available, and this number of keys can be enlarged if we decide to use more sophisticated maps or combinations of 1D skew tent maps. Therefore the chaotic modulation gives us some level of security since finding the key would suppose that a codebreaker would be able to identify both the parameter value and the initial con-

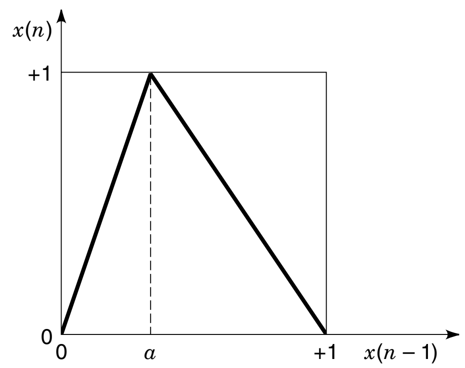

Figure 3. Skew tent map system.

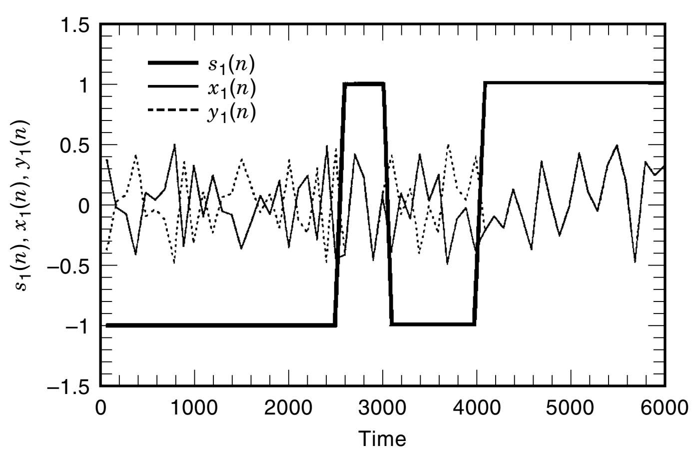

Figure 4. Principle of modulation of the chaotic sequence $x_1(n)$.

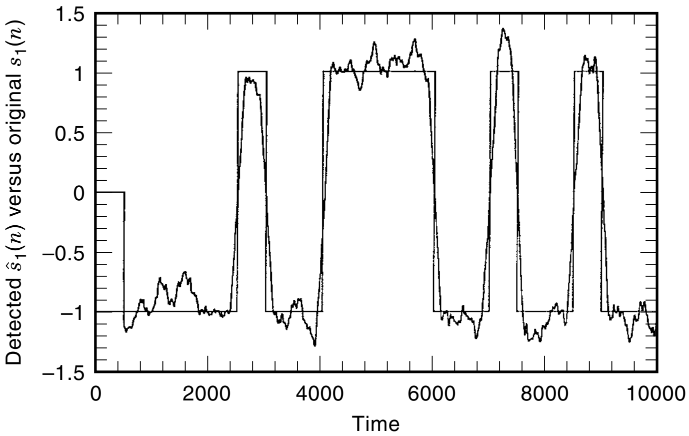

Figure 5. Detection of the $s_1(n)$ sequence using a correlation measure.

dition in oscillator state-space. Although certainly not impossible in theory, this operation of identification would require in general a great amount of computation time for high-dimensional chaotic systems.

We have assumed until now that the transmitter and the receiver are synchronized, and this synchronization can be attained in different ways ranging from absolute time measurement to periodic transmission of predefined synchronizing sequences. The illustrative example presupposes that both the transmitter and the receiver use two exact versions of a digital chaotic system and use the same rounded initial conditions in state-space; otherwise synchronization would fail. What happens if we are unable to build an accurate copy of the transmitter? For instance, if we decide to modulate a chaotic analog system, would we be able to retrieve the modulating signal? Since any analog implementation of an oscillator would mean that we are able to replicate parameter values and initial conditions in a range of 1% to 5%, it seems a priori impossible to retrieve the information. The purpose of this article is to show that in certain circumstances it is, however, possible to synchronize even imperfect copies of chaotic oscillators. This is given by property 4, which seems at first to contradict property 2. In addition, if the receiver knows in advance the equations of the dynamics of the chaotic signals sent by the transmitter, it can check to see if the received signals follow the a priori known trajectory constraints (determinism), and take advantage of the constraints to clean the noisy carrier (property 5) before extracting the information. This synchronization, the definition of which will be given later, means that the receiver is able to reconstruct the transmitter state-space independently of its initial conditions. Although this intriguing property seems to defy the intrinsic sensitivity property of chaotic systems, different methods of synchronization have been already developed. This article investigates two groups of chaotic-based transmission systems. One group is based on synchronization. The second is based on purely statistical properties of chaotic systems. In addition, we exploit property 5 and show how determinism can be used to improve the capabilities of the spread-spectrum systems.

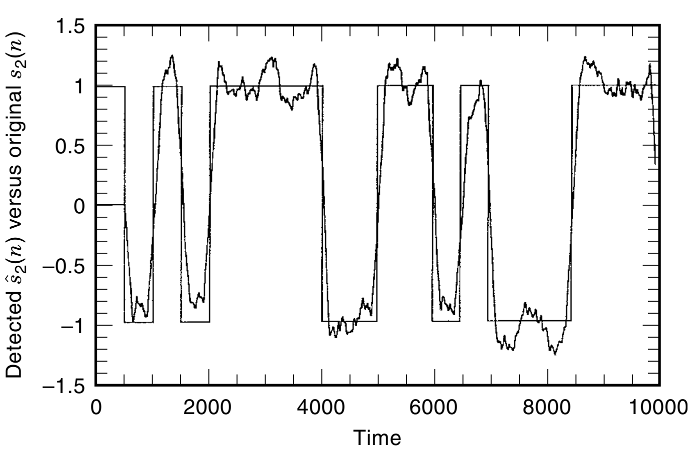

Figure 6. Detection of the $s_2(n)$ sequence using a correlation measure.

SYNCHRONIZATION OF CHAOTIC SYSTEMS

Listed here are three different methods used to synchronize chaotic systems:

1. Synchronization by decomposition into subsystems
2. Synchronization by linear feedback
3. Synchronization by inverse system design

The notion of synchronization is usually linked to periodic systems. Conventional communication systems generally use sinusoidal carriers that are either amplitude-modulated or frequency-modulated. In this context, phase-locked-loop systems (PLL) have been specifically designed to extract the information signal from the frequency-modulated carrier. The information extraction is facilitated by the carrier sinusoidal form, and the notion of synchronization refers to a notion of phase synchronization in which a feedback system, the PLL, is able to track the instantaneous phase of the carrier (with a constant shift of $\pi/2$). In the case of chaotic modulation the notion of phase synchronization is hardly exploitable and the notion of synchronization is generally considered as the asymptotical convergence of two signals when time tends towards infinity.

To understand this notion of synchronization, consider the master–slave relationship shown in Fig. 7. In this setup a master system (i.e., an autonomous dynamical system) sends its output signal to a slave system and controls this slave system in such a way that the slave produces a signal $\hat{y}(t)$. The signal $y(t)$ is often called the driving signal. The slave system

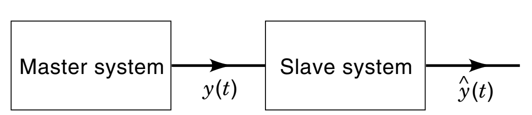

Figure 7. Master–slave setup.

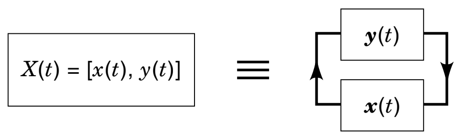

Figure 8. Decomposition of the system as an interaction between two subsystems.

synchronizes with the master system if

$$|\hat{y}(t) - y(t)| \to 0 \qquad \text{when } t \to \infty \tag{1}$$

independent of the initial conditions in both the master and the slave systems.

This definition can be enlarged to take into account both inaccurate system parameters and nonideal signal transmission. We will, however, stick to this definition in the scope of this article since the purpose is to arrive at an understanding of the basic principles of chaos synchronization.

It is possible to synchronize two systems with chaotic behavior. Such systems have sensitive dependence on initial conditions (i.e., any two solutions drift apart) even if their initial conditions are very close to each other. It is the driving signal that forces the slave system to follow the time evolution of the master system. The following sections present three methods to achieve this.

Synchronization by Decomposition into Subsystems

This idea was first proposed by Pecora and Carroll in 1990 (1) and is called *open-loop state estimation* in the control literature.

Suppose that a nonlinear dynamical system can be described by state equations of the form

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x}, y_1), \qquad \text{where } \boldsymbol{F} : \boldsymbol{R}^{n+1} \to \boldsymbol{R}^n \tag{2}$$

$$\frac{d\boldsymbol{y}}{dt} = \boldsymbol{F}(\boldsymbol{x}_1, y), \qquad \text{where } \boldsymbol{G} : \boldsymbol{R}^{m+1} \to \boldsymbol{R}^m \tag{3}$$

where $\boldsymbol{x} = (x_1, \ldots, x_n)$ and $\boldsymbol{y} = (y_1, \ldots, y_m)$ form the state space of the nonlinear dynamical system $\boldsymbol{X} = (\boldsymbol{x}, \boldsymbol{y})$. Therefore the system can be decomposed into two subsystems that interact through the signals x_1 and y_1 (see Fig. 8).

The synchronization by decomposition into subsystems consists of building a slave system which is an open-loop version of the master system. In this open-loop version the subsystem whose state is $\hat{\boldsymbol{x}}$ is forced with the signal y_1 (see Fig. 9) while the output $\hat{x}_1$ of this subsystem feeds the subsystem

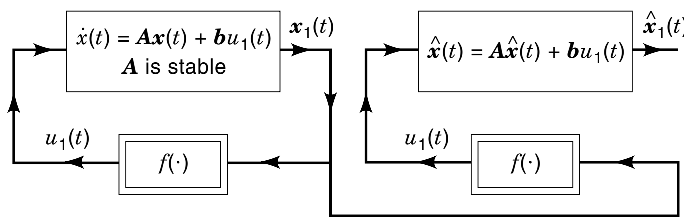

Figure 10. Synchronization by open-loop state estimation for Lure's systems.

whose state space is $\hat{\boldsymbol{y}}$. If both the systems of Fig. 9 were started with the same initial conditions $\boldsymbol{x}(0) = \hat{\boldsymbol{x}}(0)$ and $\boldsymbol{y}(0) = \hat{\boldsymbol{y}}(0)$, then the time evolution of the state variables in both systems would be identical; that is, the two systems would be perfectly synchronized. However, in practical situations either we have no control over the initial conditions and we are not able to design accurate subsystem copies. Therefore synchronization may or may not take place. Depending on the example, synchronization can be easy or it can be difficult, if not impossible, to prove. A typical example in which the proof is easy is the following coupling of two Lure's systems (Fig. 10). Lure's systems are such that a linear subsystem interacts with a nonlinear static nonlinearity. The master–slave setup of Fig. 10 can be described using the following equations:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u_1, \qquad \frac{d\hat{\boldsymbol{x}}}{dt} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{b}u_1 \tag{4}$$

$$u_1 = f(x_1), \qquad u_1 = f(x_1) \tag{5}$$

Thus, if all eigenvalues of $\boldsymbol{A}$ have negative real parts, the slave system synchronizes with the master system. A more complicated case is one in which the synchronization is mainly investigated using simulations; as an example we take Chua's circuit (see Fig. 11). The state space of this oscillator is a three-dimensional state space, where the state space variables are v_1, v_2, and i_L. Chua's circuit is described by the following state equations:

$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_2 - v_1) - g(v_1) \tag{6}$$

$$C_2 \frac{dv_2}{dt} = -\frac{1}{R}(v_2 - v_1) + i_L \tag{7}$$

$$L \frac{di_L}{dt} = -v_2 \tag{8}$$

in which the nonlinear resistor characteristic $g(v_1)$ is shown in Fig. 12.

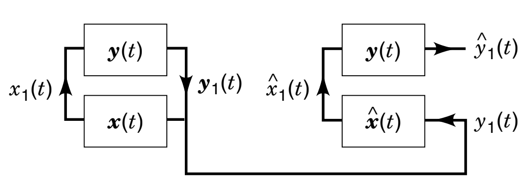

Figure 9. Synchronization by open loop state estimation.

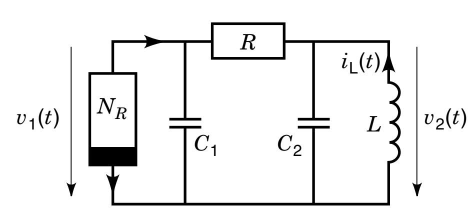

Figure 11. Chua's circuit.

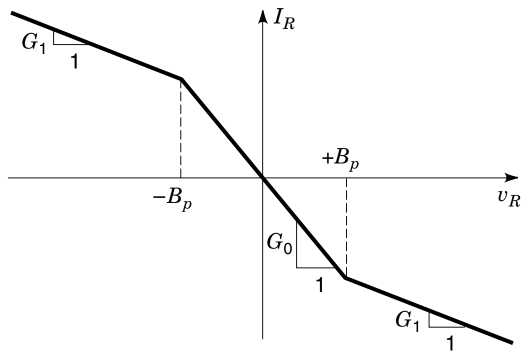

Figure 12. Nonlinear characteristic in Chua's circuit.

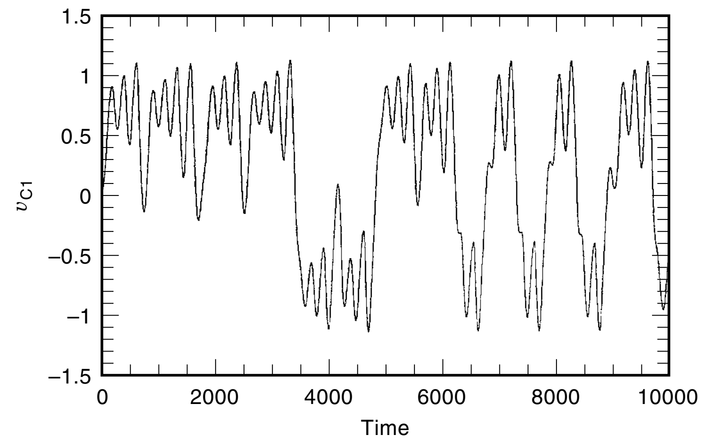

Figure 14. Behavior of $v_1(t)$ for the Chua's circuit operating in chaotic mode.

By choosing the parameters to be $R = 1730\ \Omega$, $L = 18\ m$H, $C_1 = 10\ n$F, $C_2 = 100\ n$F, $B_p = 1\ V$, $G_0 = -0.44/R$ and $G_1 = -0.23/R$, Chua's circuit is set in a chaotic mode. A projection of the state-space attractor of the oscillator is shown in the plane $v_1 - i_L$ (see Fig. 13), while Fig. 14 gives a sample of the irregular behavior of $v_1(t)$.

Suppose that we decide to send $v_1(t)$ and want to design a slave system so that this slave system will produce a signal $\hat{v}_1(t)$ that converges asymptotically to $v_1(t)$. By decomposing Chua's circuit in two subsystems, the role of $\boldsymbol{x}$ and $\boldsymbol{y}$ in Fig. 9 are played by $\boldsymbol{x} = (\boldsymbol{v}_2, \boldsymbol{i}_L)$ and $\boldsymbol{y} = \boldsymbol{v}_1$. The first subsystem driven by $v_1(t)$ aims to reproduce the state space variables v_2 and i_L. This first subcircuit is shown in Fig. 15. This subsystem is linear and its elements have positive values, and therefore it is an asymptotically stable circuit. This implies that the state variables $\hat{v}_2$ and $\hat{\imath}_L$ will converge toward the state variables of the master circuit as t tends toward infinity. It is now clear that for accurate values of C_2, L, and R the first subsystem of the slave will accurately reproduce two of the state variables of the master system. The second subsystem shown in Fig. 16 will be driven by the signal $\hat{v}_2(t)$, which is the output of the first slave subsystem. The aim of this second subsystem is to reproduce the state-space variable $v_1(t)$. Depending on the circuit parameters, synchronization may occur or not (2). The exact conditions for synchronization can, however, be computed in this case. Derivation of these conditions are beyond the scope of this article and can be found in Ref. 3.

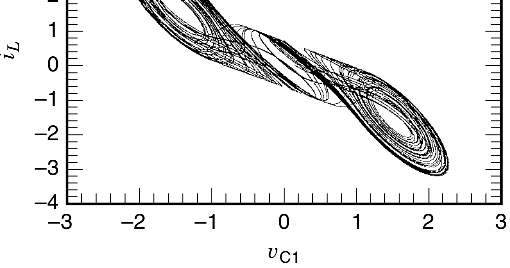

Figure 13. Chua's circuit attractor projection in the plane v_1–i_L.

Synchronization by Linear Feedback

This approach is a typical automatic control approach in which the master and the slave compare their output to form a synchronization error signal. The synchronization error signal is fed back as a control input of the slave system, which is a copy of the master system (see Fig. 17). This approach has been introduced in Ref. 4 under the topic of control of chaos. Usually the synchronization is linearly fed back to the state variables, and therefore the state equations of the whole system are such that

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}) \tag{9}$$

$$y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) \tag{10}$$

$$\frac{d\hat{\boldsymbol{x}}}{dt} = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{k}e(t) \tag{11}$$

$$\hat{y}(t) = \boldsymbol{c}^{\mathrm{T}}(\hat{\boldsymbol{x}}) \tag{12}$$

$$e(t) = y(t) - \hat{y}(t) \tag{13}$$

If both the master and the slave had agreed to start from the same initial conditions, then at all times we have $\hat{\boldsymbol{x}}(t) = \boldsymbol{x}(t)$, $\hat{y}(t) = y(t)$, and $e(t) = 0$. With no constraints on initial conditions, the slave system may or may not synchronize and the synchronization has to be studied case by case. In some instances, conditions on the coupling matrix $\boldsymbol{k}$ that ensures synchronization can be derived by using Lyapounov functions (4).

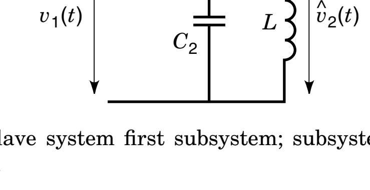

Figure 15. Slave system first subsystem; subsystem driven by the master circuit.

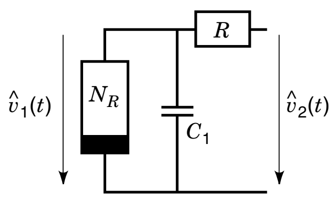

Figure 16. Slave system second subsystem.

Synchronization of the Inverse System

In this method we consider the general setup of Fig. 1. The goal is to control a chaotic system with an information signal. The output of the transmitter, a chaotic broadband signal where the information is hidden, becomes, after transmission, the input of the receiver that must retrieve the information signal independent of the state space and the initial conditions of the receiver. In order to do this, the receiver must perform an input–output relationship that is inverse to that of the transmitter. Therefore this kind of synchronization has been referred to as *inverse system synchronization.* Consider the following example borrowed from Ref. 5 (see Fig. 18). Similar examples can be also found in Ref. 6. In Fig. 18 it is supposed that the function $f(\boldsymbol{x}, s)$ is invertible with respect to s. Independent of the state-space initial conditions in the receiver, it can be seen that after N iterations of the transmitter

$$\hat{\boldsymbol{x}}(k) = \boldsymbol{x}(k) \qquad \text{if } k \geq N \tag{14}$$

Therefore as $f(\boldsymbol{x}, s)$ is invertible with respect to s, we get $\hat{s} = s$ for $k \geq N$.

For continuous-time systems the synchronization by the inverse system approach can be explained as follows. Consider the nonlinear dynamical 1-port of Fig. 19 excited by an independent current source with current $i(t)$. If we would take the voltage across the current source and drive an exact copy of the same 1-port by a voltage-controlled source, we would find exactly the same current $i(t)$ flowing through the voltage source, provided that the initial currents in the inductors and the initial voltages across the capacitors in the two 1-ports are identical (see Fig. 19). This method of synchronization has been sucessfully applied to various circuits. Consider the example borrowed from Ref. 7 in which two Chua's circuits have been synchronized using the inverse system approach (see Fig. 20). In this case it is possible to prove exactly the synchronization because imposing the voltage across the nonlinear resistor forces the current in the receiver nonlinear resistor to follow exactly the current in the transmitter nonlinear

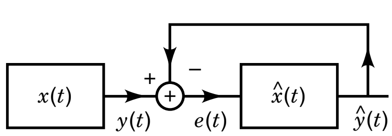

Figure 17. Master–slave setup for synchronization by linear feedback.

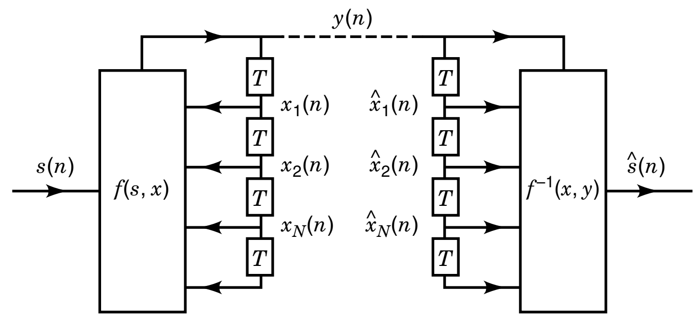

Figure 18. Discrete-time system and its inverse.

resistor. Therefore $\hat{i}_1(t) = i_1(t)$ for all times, and consecutively $\hat{i}_2(t)$ converges asymptotically to $i_2(t)$ since i_2 is the port current of a linear passive 1-port. An analysis of the inverse system approach can be found in Ref. 5. Despite its clear foundation the inverse system approach suffers from different drawbacks that limit its applicability. It has been reported by many researchers that the inverse system approach is very sensitive to channel noise. Furthermore, the transmitter should be designed in such a way that it remains chaotic with all admissible signals; unfortunately there is no general method known for this.

EXPLOITING CHAOS SYNCHRONIZATION FOR TRANSMISSION OF INFORMATION

So far we have explained three methods to ensure synchronization between a master system and a slave system. Now we will present different methods for modulating chaotic carriers and retrieving this information from the observation of the chaotic carrier:

1. Chaotic masking
2. Chaotic shift keying
3. Direct chaotic modulation.

Chaotic Masking

In this method (8) an analog information carrying signal $s(t)$ is added to the output $y(t)$ of the chaotic system in the transmitter. On the receiver side an identical chaotic system tries to synchronize with $y(t)$. From this point of view, the information signal $s(t)$ is a perturbation, and synchronization will

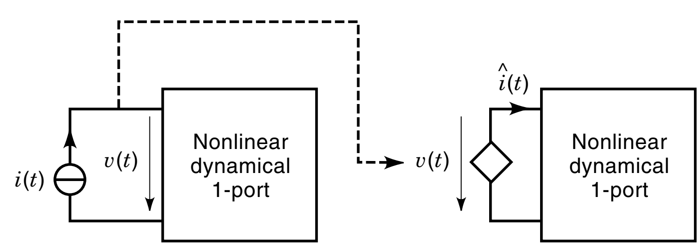

Figure 19. Realization of the inverse system for analog circuits.

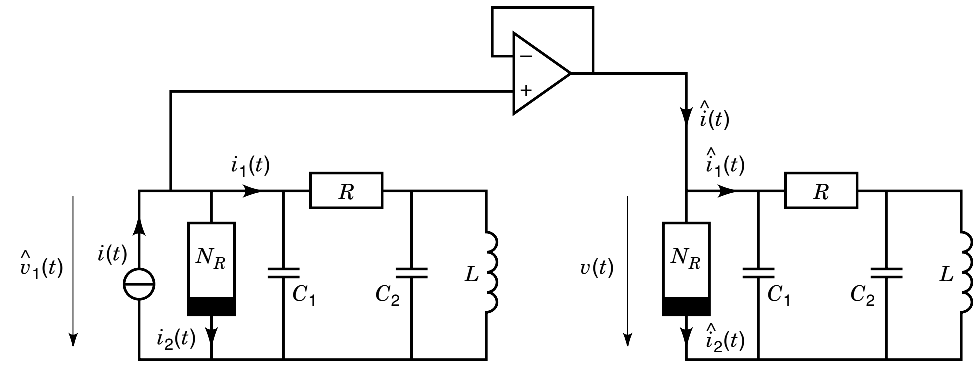

Figure 20. Synchronization of Chua's circuit by the inverse system.

take place only approximately. However, if the synchronization error is small with respect to $s(t)$, the latter can be approximately retrieved by subtraction (see Fig. 21). This is the case if the signal $s(t)$ is small with respect to $y(t)$ and/or if the spectra of the two signals do not overlap too much. Both of these requirements can apparently be relaxed (9). However, if the purpose of using a chaotic signal for transmission is to hide the information, $s(t)$ should not be large. Therefore, it can be expected that the method is sensitive to channel noise. Indeed, additive noise cannot be distinguished from $s(t)$ by the setup of Fig. 21, and it has to be eliminated at a later stage. This is a difficult, if not impossible, task if the amplitude of $s(t)$ is not large with respect to the noise level.

Transmission Using Chaotic Shift Keying (CSK)

In this method (2,10) the information signal is supposed to be binary. This binary information modulates a chaotic system by controlling a switch whose action is is to change the parameter values of a chaotic system (see Fig. 22). Thus according to the binary value of $s(t)$ at any given time, the chaotic system can use either the parameter vector $\boldsymbol{p}$ or the parameter $\boldsymbol{p}'$. The receiver consists of two sets of receivers: One uses the parameter vector $\boldsymbol{p}$, while the other uses the parameter $\boldsymbol{p}'$. At any given time, the receiver which is able to synchronize or which synchronizes best tells us what set of parameters has been used in the transmitter (see Fig. 22). When the transmitter switch is on position $\boldsymbol{p}$, the receiver with parameter $\boldsymbol{p}$ will synchronize, whereas the receiver with parameter $\boldsymbol{p}'$ will desynchronize. Therefore when the parameter $\boldsymbol{p}$ is used in the transmitter the error signal $e(t)$ converges toward 0 while the error signal $e'(t)$ has an irregular waveform with nonzero amplitude. Information is retrieved by detecting synchronization and desynchronization of errors signals. In such a method the switching speed is inversely proportional to the time of synchronization. We give below a detailed example in which this synchronization speed is assessed.

Example of Chaos Shift Keying Transmission. We take the example already developed in Ref. 2. The transmitter is based on a Chua's circuit (see Fig. 23) whose chaotic behavior has been widely studied (11–14). It consists of a single nonlinear resistor (see Fig. 24) and four linear circuit elements: two capacitors, an inductor, and a resistor. Details of the synthesis of the nonlinear resistor can be found in Ref. 15. The modulation device runs as follows. A binary data stream (the signal to be transmitted) "modulates" the chaotic carrier $v_{C_1}(t)$. If an input bit $+1$ has to be transmitted, the switch of Fig. 23 is kept open for a time interval T. If the next bit to be transmitted is -1, the switch is closed, connecting in parallel the resistor r with the nonlinear negative resistor. During the transmission of the -1 bit, the device can be seen as a Chua's circuit with a three-segment piecewise-linear resistor having a slope $G_0' = G_0 + 1/r$ in the central region and a slope $G_1' = G_1 + 1/r$ in the outer region. The breakpoints B_p remain unchanged. The following three equations describe the dynamics of the modulation system:

$$C_1 \frac{dv_{C_1}(t)}{dt} = \frac{1}{R}(v_{C_2}(t) - v_{C_1}(t)) - h_{\pm}(v_{C_1}(t)) \tag{15}$$

$$C_2 \frac{dv_{C_2}(t)}{dt} = -\frac{1}{R}(v_{C_2}(t) - v_{C_1}(t)) + i_{L_1}(t) \tag{16}$$

$$L \frac{di_{L_1}(t)}{dt} = -v_{C_2}(t) \tag{17}$$

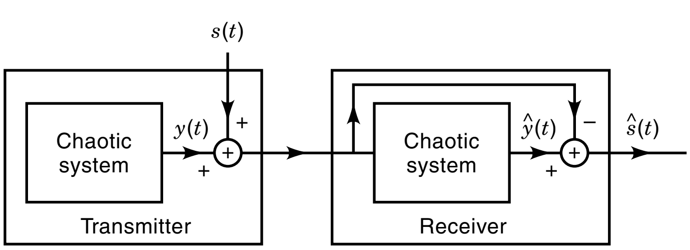

Figure 21. Transmission using chaotic masking.

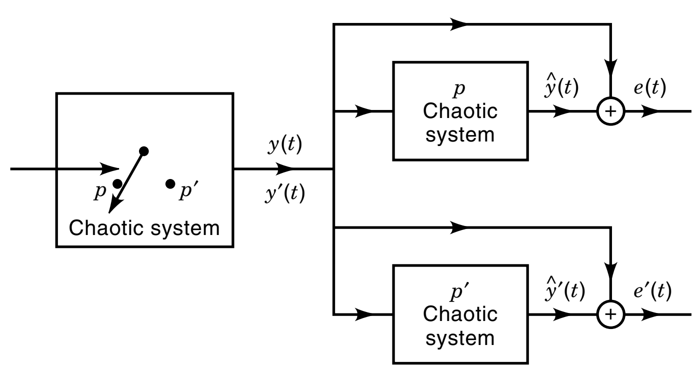

Figure 22. Transmission using chaotic shift keying.

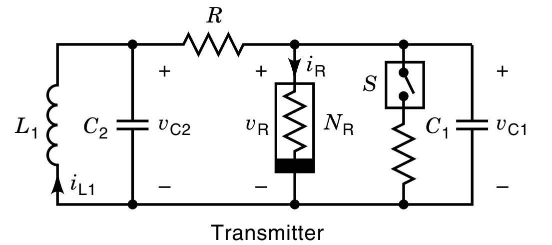

Figure 23. Transmitter for binary chaotic shift keying using Chua's circuit. Chaotic signal v_{C_1} is transmitted.

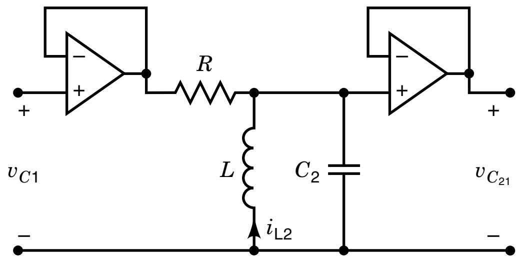

Figure 25. First receiver subsystem.

The function $h_\pm$ in Eq. (15) has the following meaning: During a bit 1 transmission we have $h_\pm = h_+$, where h_+ is the three-segment piecewise-linear function with slopes G_0, G_1 and breakpoints $-B_p$ and $+B_p$; during a bit -1 transmission we have $h_\pm = h_-$, where h_- is the three-segment piecewise-linear function with slopes G'_0, G'_1 and breakpoints $-B_p$ and $+B_p$. We suppose that the signal $v_{C_1}(t)$ is transmitted to the receiver without any alteration.

The receiver is made of three subsystems (see Figs. 25 to 27). The goal of the first subsystem is to create as close as possible a copy of the signal $v_{C_2}(t)$; this signal will be referred to as $v_{C_{21}}(t)$. The first subsystem is governed by the two following equations:

$$C_2 \frac{dv_{C_{21}}(t)}{dt} = \frac{1}{R}(v_{C_1}(t) - v_{C_{21}}(t)) + i_{L_2}(t) \tag{18}$$

$$L\frac{di_{L_2}(t)}{dt} = -v_{C_{21}}(t) \tag{19}$$

The second and the third subsystems are designed to produce the signals $v_{C_{12}}(t)$ and $v'_{C_{12}}(t)$. As will be shown in the theoretical part of this article, $v_{C_{12}}(t)$ converges to $v_{C_1}(t)$ during the transmission of $+1$ bit while $v'_{C_{12}}(t)$ converges to $v_{C_1}(t)$ during the transmission of -1 bit. Equation (20) governs the second subsystem, while Eq. (21) governs the third subsystem:

$$C_1 \frac{dv_{C_{12}}(t)}{dt} = \frac{1}{R}(v_{C_{21}}(t) - v_{C_{12}}(t)) - h_+(v_{C_{12}}(t)) \tag{20}$$

$$C_1 \frac{dv'_{C_{12}}(t)}{dt} = \frac{1}{R}(v_{C_{21}}(t) - v'_{C_{12}}) - h_-(v'_{C_{12}}(t)) \tag{21}$$

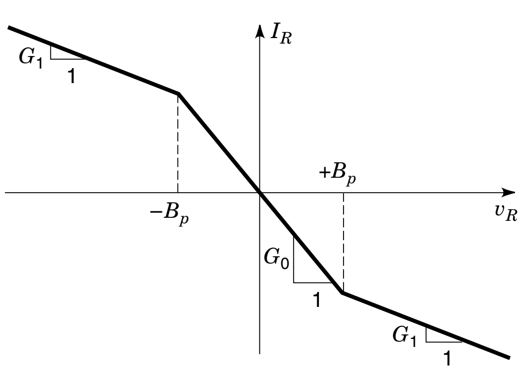

Figure 24. Three-segment piecewise-linear function. The inner region has slope G_0; the outer regions have slopes G_1.

Additional elements for the circuits in Figs. 25 to 27 remain to be designed for the synchronization detectors. The interested reader can find in Ref. 2 a method explaining how it is possible to compute a bound for the synchronization time. This method is based on the computation of the relative duration time of the driving signal in the different areas of the piecewise characteristic of the nonlinear resistor of the transmitter.

We present here simulations using the subsystems described in Figs. 25 to 27. The value of r was chosen in order that G'_0 and G'_1 exhibit variations of 1% with respect to G_0 and G_1. The values of circuit elements were fixed as follows: $R = 1680\ \Omega$, $L = 18\ m$H, $C_1 = 10\ n$F, $C_2 = 100\ nF$, $G_0 = -753\ \mu$s, $G_1 = -396\ \mu$s, and $B_p = 1\ V$. The value of T was 4.65 ms.

Figure 28 shows the chaotic message from 10 ms to 40 ms and the corresponding binary message.

Figure 29 shows the double-scroll attractor in the phase-plane $(v_{C_1}(t), v_{C_2}(t))$ during the transmission of the binary message presented in Fig. 28.

Figure 30 shows the relationship between $v_{C_{12}}(t)$ and $v_{C_1}(t)$ during the transmission of 120 bits which were alternatively $+1$, -1 while Fig. 31 displays the relationship between $v'_{C_{12}}(t)$ and $v_{C_1}(t)$ during the same transmission.

Obviously the receiver subsystems are not synchronized during the whole transmission (it is hoped that the receiver not matched to receive the right bit exhibits a desynchronized behavior). Finally, Fig. 32 shows the relationship between $v_{C_{12}}(t)$ and $v_{C_1}(t)$ during the transmission of 60 nonconsecutive $+1$ bits. The signals were sampled during the last half duration of each bit in order to avoid transients. Figure 33 is the alter ego figure of Fig. 32; it shows the relationship between $v'_{C_{12}}(t)$ and $v_{C_1}(t)$ during the transmission of 60 nonconsecutive -1 bits. The interested reader can find in Ref. 2 an implementation of the chaotic shift keying method which confirms the above-presented simulations.

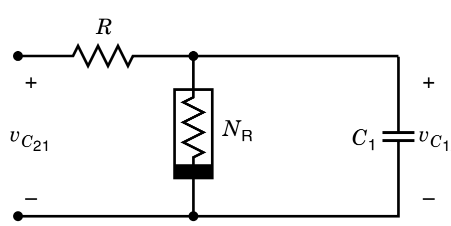

Figure 26. Second receiver subsystem.

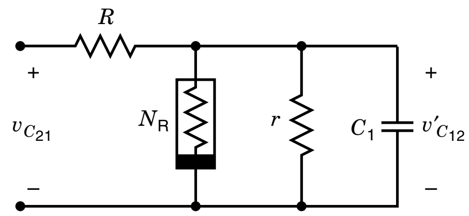

Figure 27. Third receiver subsystem.

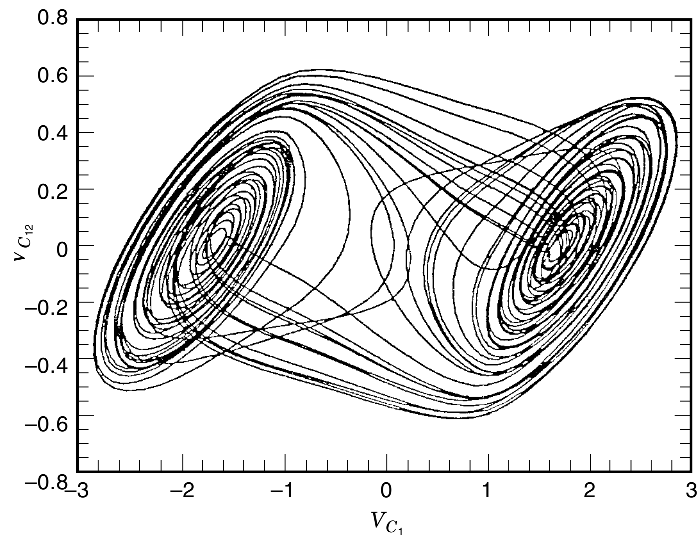

Figure 29. Double-scroll attractor in the phase plane $v_{C_1}(t)$, $v_{C_2}(t)$ during modulation.

Direct Chaotic Modulation

In this method the inverse system approach is used directly in a straightforward manner. Thus, no additional circuitry must be used, the chaotic system is the transmitter, and the inverse system is the receiver. If we look at a circuit realization (e.g., in Fig. 18), we can see that $s(n)$ drives the chaotic circuit and thus modulates the chaotic signal in some way. The information can be injected directly in analog form, as proposed in Ref. 7, or $s(t)$ can be itself an analog signal modulated by binary information, as proposed in Ref. 16, with the obvious advantages and drawbacks. The transmission of a digital signal modulated onto $s(t)$ can be expected to reach higher bitrates than with chaotic switching. In chaotic switching, whenever the signal changes its value, one has to wait for synchronization since the initial conditions in the transmitter and the receiver subsystem that have to synchronize are different. In direct chaotic modulation, the receiver continuously tracks the transmitter and thus the states of the two chaotic systems are never very different. This can be seen in Fig. 34, where a phase-modulated signal is transmitted on a chaotic carrier, using Saito's circuit. At the beginning, the receiver needs some time to synchronize, but afterwards the receiver tracks the 180° phase shifts perfectly. In Fig. 35, the transmitted signal is represented for the same experiment. Both figures were created with computer simulation.

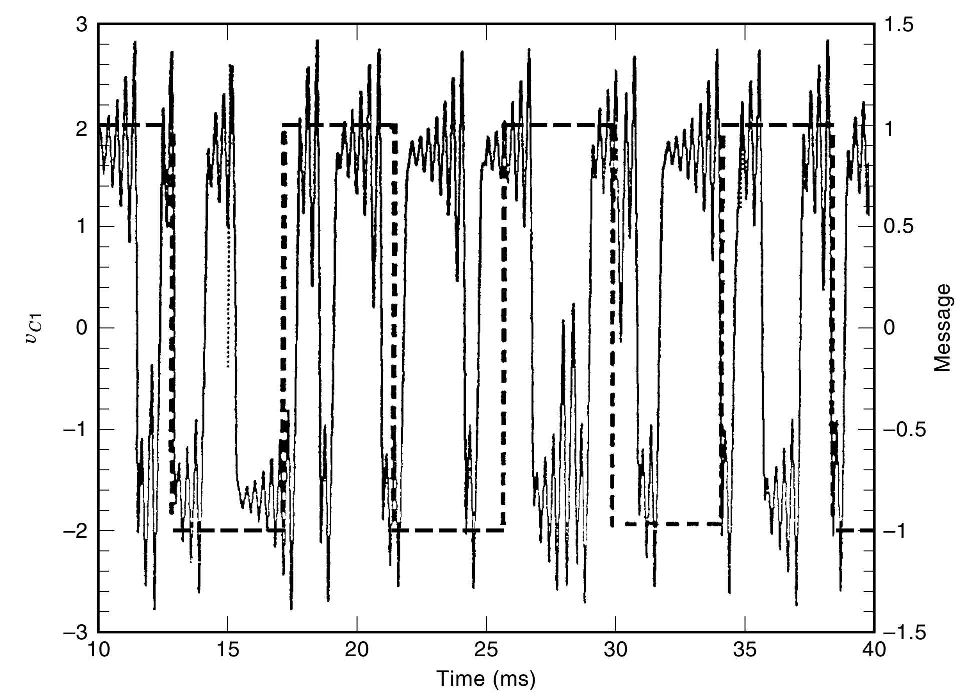

Figure 28. Behavior of the chaotic signal $v_{C_1}(t)$ (———) and of the binary message (– – –).

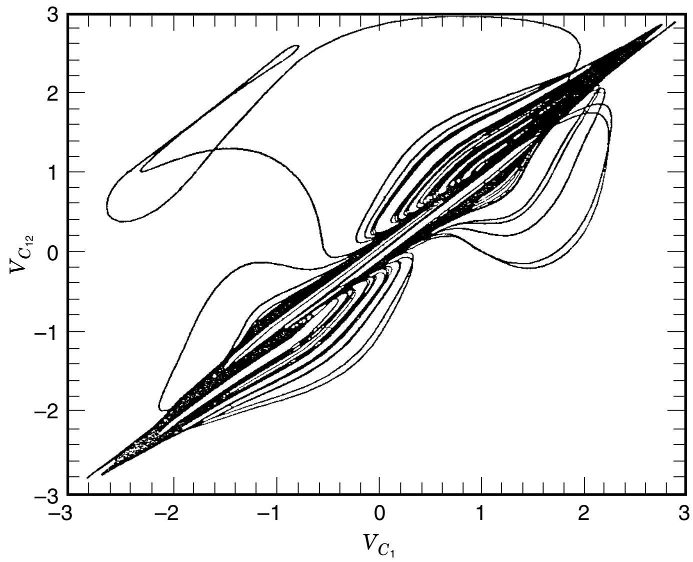

Figure 30. $v_{C_{12}}(t)$ as a function of $v_{C_1}(t)$ during the transmission of several bits $+1$, -1.

EXPLOITING CHAOS FOR INFORMATION TRANSMISSION BY MEANS OF STATISTICAL DECISION

Until now we have presented examples in which synchronization has been used in order to detect the information signal. In digital communication systems, this approach would belong to the coherent detection approach in which the signal sent to the channel is made of a combination of basis functions. At the receiver the basis functions are usually regenerated using synchronization and the symbols which were associated with the combination of the basis functions are therefore detected. This is in fact the scheme which has been used for the CSK technique in which a bit ± 1 is associated with a segment of chaotic waveform belonging to one of two different attractors (i.e., produced by two slighly different systems). At the receiver, two systems are also used, and the system which best synchronizes with the incoming signal allows the detection of the right bit of modulation.

We now present some basic ideas that are based on the noncoherent detection approach; in contrast to the coherent detection approach, we now rely on the estimation of one or more characteristics of the basis functions, and will not assume that the basis functions are regenerated at the receiver.

Coherent detection-based receivers are known to have advantages over noncoherent receivers in terms of noise performance and bandwidth efficiency. However, these advantages are lost if synchronization cannot be maintained—for example, under poor propagation conditions. In these circumstances, communication without synchronization may be preferable. In the following sections we present two classes of systems which exploit the noncoherent detection approach. In that kind of approach, it is mainly the macroscopic features of deterministic chaos which will be used—that is, the good

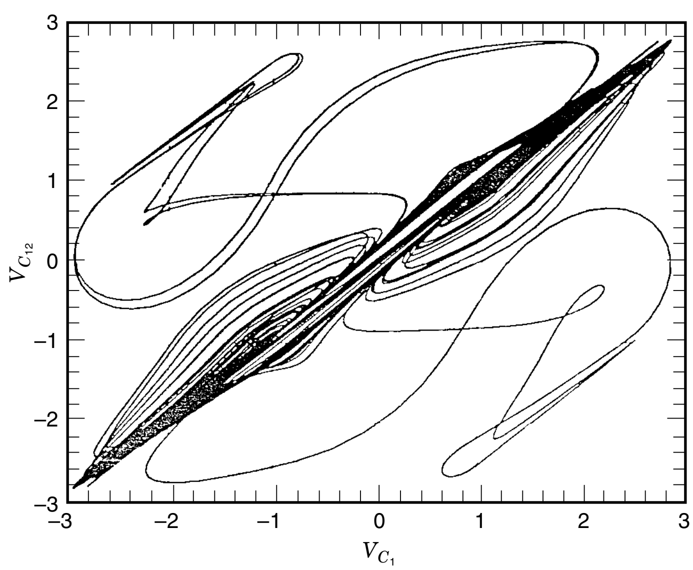

Figure 31. $v'_{C_{12}}(t)$ as a function of $v_{C_1}(t)$ during the transmission of several bits $+1$, -1.

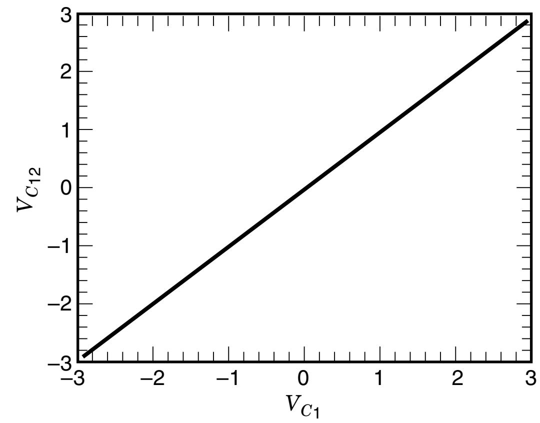

Figure 32. $v_{C_{12}}(t)$ as a function of $v_{C_1}(t)$ during the transmission of several bits $+1$.

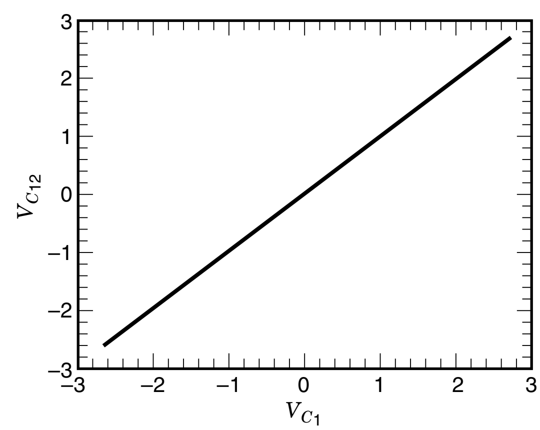

Figure 33. $v_{C_{12}}(t)$ as a function of $v_{C_1}(t)$ during the transmission of several bits -1.

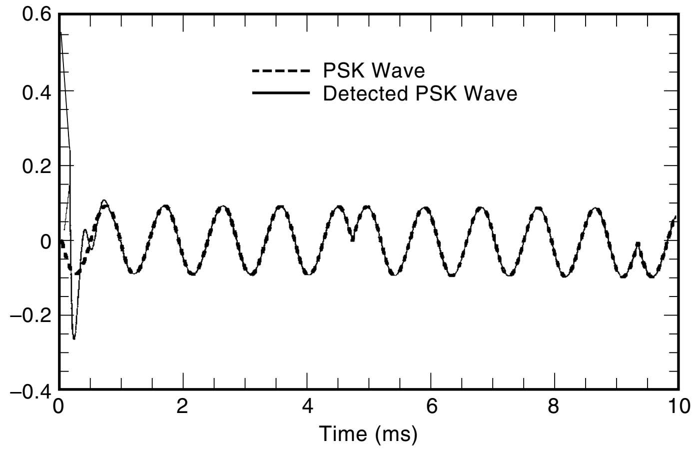

Figure 34. Original and retrieved information signal, for direct modulation with Saito's circuit (– – – PSK wave; ——— detected PSK wave) (From Ref. 16.)

decorrelation properties between two different segments of chaotic trajectories. This is in contrast with the methods presented in the section entitled "Synchronization of Chaotic Systems," in which we mainly used the microscopic nature of chaos (i.e., the determinism), since we implicitly used (in the synchronization approach) the dynamical constraints between consecutive points of a chaotic trajectory.

Example 1: Chaotic Direct-Sequence Spread-Spectrum Systems

Spread-spectrum systems are systems that are designed to resist to external interference, to operate with a low-energy spectral density, to provide multiple-access capability without external control, or to make it difficult to unauthorized receivers to observe the message. Among different classes of methods, we briefly describe the direct-sequence spread-spectrum approach which uses as a carrier signal a pseudonoise (PN) which consists usually of a *binary* PN sequence; our objective will be to show that a better alternative would be to use a *chaotic* sequence.

Basic Principle for Standard Direct-Sequence Spread-Spectrum Systems. Let $m(t)$ be a binary message ± 1 and let $c(t) = \pm 1$

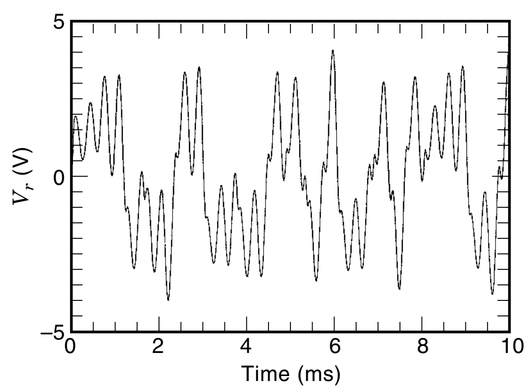

Figure 35. Transmitted signal, for direct modulation with Saito's circuit. (From Ref. 16.)

be a PN signal—that is, a signal which is formed by linearly modulating the output sequence $\{c_n\}$ of a pseudorandom number generator onto a train of pulses, each pulse having a duration T_c called the chip time:

$$c(t) = \sum_{n=-\infty}^{n=+\infty} c_n p(t - nT_c) \tag{22}$$

where $p(t)$ is the basic pulse shape which is assumed to be of rectangular form.

The bit duration T_b of the information signal $m(t)$ is such that

$$T_b \gg T_c \tag{23}$$

A standard direct-sequence spread-spectrum (DS–SS) system operates with a double modulation. At the emitter the message $m(t)$ is "spread" by multiplying $m(t)$ by $c(t)$. This first modulation is followed by a second standard modulation which centers the spectrum of the transmitted signal around a frequency carrier ω_0. At the receiver the same PN sequence is used to unspread the signal. For the sake of simplicity we will ignore in the following the second modulation. The complete simplified modulation–demodulation scheme is shown in Fig. 36. As shown in Fig. 36 the product of the received signal $v(t)$ by a delayed version of the spread spectrum is integrated. Let us suppose that the integration time is equal to the message bit duration T_b. Furthermore, let us suppose that in an ideal manner the output of the integrator is reset to zero at the beginning of each message bit while this output is precisely observed at the end of each message bit. In our simplified scheme of transmission the received signal is of the form

$$v(t) = m(t - \tau)c(t - \tau) + \eta(t) \tag{24}$$

where $\eta(t)$ is the observation noise. If we suppose that $\eta(t)$ and $c(t - \tau)$ are uncorrelated, we get at the output of the integrator

$$\hat{m}(kT_b) = \int_{(k-1)T_b}^{kT_b} v(t)c(t - \tau')\,dt = m(kT_b)R_c(\tau - \tau') \tag{25}$$

where $R_c(\tau)$ is the cross-correlation function of $c(t)$. The result will therefore be maximum if $\tau - \tau'$ equals zero—that is, if the emitter and receiver sequences are synchronized.

Limiting Properties of PN Sequences. In order to spread bandwidth, PN sequences have been used extensively in spread-spectrum communication systems. The maximal-length linear code sequence (m-sequence) has very desirable autocorrelation functions. However, in the case of multipath environments, large spikes can be found in their cross-correlation functions. Another limitation is that they are very small in number. In order to overcome these limitations, Heidari–Bateni and McGillem (17) have been among the first to propose the use of chaotic sequences as spreading sequences.

Advantages Associated with Chaotic Sequences. Chaotic sequences present the following advantages:

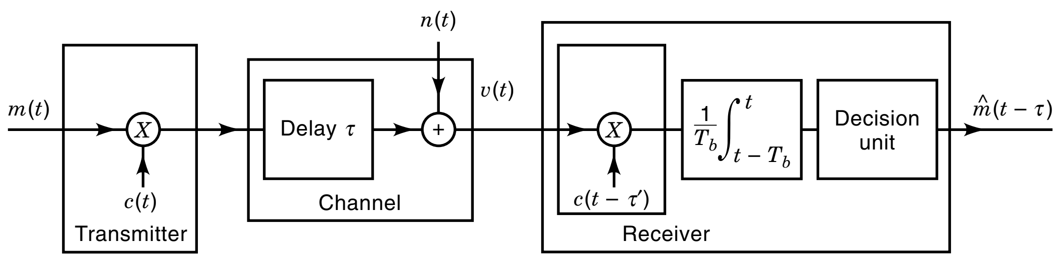

Figure 36. Simplified modulation–demodulation scheme for a DS–SS system.

- It is easy to generate a great number of distinct sequences.
- The transmission security is increased.
- Correlation properties which are very similar to those of random binary sequences. In some cases, superior properties have been reported (18,19).

We should add to this list one major difference: these sequences are nonbinary sequences. This property and the fact that these sequences are produced by deterministic systems can be used to clean the chaotic sequences—that is, to get rid of the noise by checking the deterministic constraints between consecutive points of the sequence. This property will be exploited in the section entitled "Improving Chaos Transmission by Exploiting the Deterministic Feature of Chaos."

Example. Here is a system proposed in Ref. 17 in which both the emitter and the receiver have agreed to use two chaotic systems. A first digital chaotic system has a clock rate $1/T_b$ while the second has a clock rate $1/T_c$. The first system implements a one-dimensional chaotic map $x_{n+1} = C_1(x_n, r_1)$, where r_1 is a bifurcation parameter; the second one implements also a one-dimensional chaotic map $y_{n+1} = C_2(y_n, r_2)$ with bifurcation parameter r_2. The chaotic maps and their bifurcation parameters may or may not be the same, and their uniqueness among the different pairs of transmitters and receivers is not necessary.

The two chaotic systems are associated in such a way that the first chaotic system forces periodically the second one by imposing the initial condition of the second system at the starting time of each bit. Let N be the number of chips per bit, that is, $N = T_b/T_c$.

$$y_{n+1} = C_2(y_n, r_2) \qquad \text{if } n \text{ modulo } N \neq 0 \tag{26}$$

$$y_{n+1} = C_1(y_{n-N+1}, r_1) \qquad \text{if } n \text{ modulo } N = 0 \text{ and } N \neq 0 \tag{27}$$

The process is pictured in Fig. 37. If both the receiver and emitter have agreed on y_0, C_1, C_2, r_1, r_2, the sequence can be regenerated at the receiver in exactly the same manner as the emitter does. Note that as there is a change in initial conditions at each new bit transmission, the decorrelation between adjacent $y(t)$ corresponding to adjacent bits will be almost zero. Every receiver will be assigned distinct y_0, C_1, C_2, r_1, r_2, and therefore the resulting spreading sequences for each receiver in a multiple-access communication system will be completely different and almost decorrelated. Since there is a large number of initial conditions, bifurcation parameters, and chaotic maps to choose from, there are no limitations on the number of users that could be accommodated by these spreading sequences. The modulation and detection of the data sequence is otherwise the same as the conventional DS–SS systems. The detection will be based on correlating the received signal with the chaotic sequence of the receiver. Transmitter–receiver synchronization is supposedly attained either from absolute time measurement or from periodic transmission of predefined synchronizing sequences.

Example 2: Differential Chaos Shift Keying (DCSK)

Kolumban proposed in Ref. 23 to develop a noncoherent receiver based on an idea which combines differential encoding with chaos shift keying (CSK) modulation. Every incoming symbol is mapped to two sample chaotic signals, one of which acts as a reference while the other carries the information according to some very simple transformation of the first one. At the receiver the information is retrieved by correlating the two samples and finding out the relationship which links two adjacent signals. For example, let us denote as $x(t)$ the output of the chaos generator. For the sake of simplicity, let us consider the binary case. The symbol $+1$ will be represented by a positive correlation between two adjacent signals which are built according to

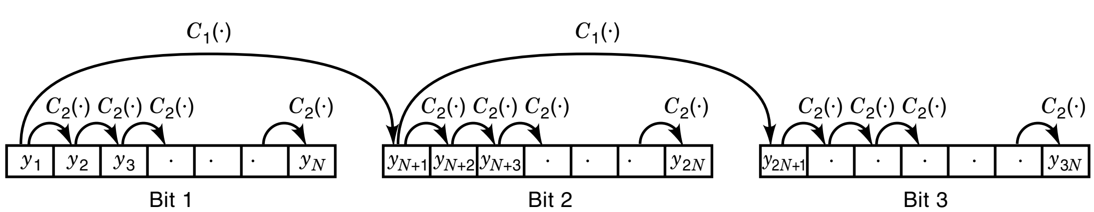

Figure 37. Illustration of the proposed method of generating the chaotic spreading sequences.

$$s_1(t) = \begin{cases} x(t) & (2k)\frac{T_b}{2} \le t < (2k+1)\frac{T_b}{2} \\ x\left(t - \frac{T_b}{2}\right) & (2k+1)\frac{T_b}{2} \le t < (2k+2)\frac{T_b}{2} \end{cases} \quad (28)$$

For the symbol 0 the magnitude of the correlation remains the same, but its sign becomes negative.

$$s_0(t) = \begin{cases} x(t) & (2k)\frac{T_b}{2} \le t < (2k+1)\frac{T_b}{2} \\ -x\left(t - \frac{T_b}{2}\right) & (2k+1)\frac{T_b}{2} \le t < (2k+2)\frac{T_b}{2} \end{cases} \quad (29)$$

The reference part of the transmitted signal is the inherently nonperiodic output signal of the chaotic generator. A block diagram of a differential chaos shift keying (DCSK) demodulator is shown in Fig. 38. The received noisy signal is delayed with the half symbol duration $T = T_b/2$, and the cross-correlation between the received signal and the delayed copy of itself is determined. The cross-correlation of the reference and information-bearing sample signals is estimated from signals of finite duration and therefore this estimation has a variance, even in the noise-free case. The variance can be reduced by increasing the estimation time (i.e., the symbol duration), but of course a larger estimation time results in a lower data rate. Reference 20 gives an example in which an analog chaotic phase lock loop is used in a DCSK framework. The authors give some indication about how the optimum value of the estimation time could be determined experimentally. They show from simulations that the noise performance of a DSCK communication system in terms of bit error ratio (BER) versus E_b/N_0 (E_b is the energy per bit and N_0 is the power spectral density of the noise introduced in the channel) outperforms the BER of a standard CSK system. The DCSK technique offers different advantages:

- Because synchronization is not required, a DCSK receiver can be implemented using very simple circuitry. Demodulation is very robust and as in noncoherent receivers, problems such as loss of synchronization and imperfect recovery of basis functions do not arise.
- DCSK is not as sensitive to channel distorsion as coherent methods since both the reference and the information-bearing signal pass through the same channel.

The main disadvantage of DCSK results from differential coding: E_b is doubled and the symbol rate is halved.

IMPROVING CHAOS TRANSMISSION BY EXPLOITING THE DETERMINISTIC FEATURE OF CHAOS

We have until now seen some principles of communication which exploited the features of deterministic chaotic systems. Indeed when we presented in the sections entitled "Synchronization of Chaotic Systems" and "Exploiting Chaos Synchronization for Transmission of Information" some synchronization principles we took advantage of the deterministic aspects of the chaotic systems since we built the receiver system by using a perfect knowledge of the emitter system. Therefore in these sections we implicitly used through the synchronization scheme the "microscopic" nature of chaos—that is, the fact that successive points in the state space of the emitter system belongs to some deterministic trajectory. In the section entitled "Exploiting Chaos for Information Transmission by Means of Statistical Decision," we used the "macroscopic" nature (i.e., the statistical feature of chaos) since we considered chaotic signals as noise carriers which mainly ensured information spreading and no correlation between successive signals bearing information. It may be argued that we could still improve our transmission features introduced in the aforementioned section if we were able to take into account the "microscopic" nature of deterministic chaos—that is, the deterministic constraints which link successive points of the state space trajectory. How to use both the statistical properties and the dynamical constraints when dealing with deterministic chaotic signals is the subject of this final section.

Mixing the Deterministic and Statistical Aspects

In the section entitled "Exploiting Chaos for Information Transmission by means of Statistical Decision" we were faced with the problem of the measurement of the correlation between a received chaotic signal and a chaotic signal which is generated by a local chaotic system which is assumed to be identical and synchronized with that of the emitter (see section entitled "Example 1: Chaotic Direct-Sequence Spread-Spectrum Systems"). In the section entitled "Example 2: Differential Chaos Shift Keying," we were faced with the problem of correlating two consecutive pieces of signal (DCSK framework). In both cases we had to compute a correlation function from a finite number of samples. According to the

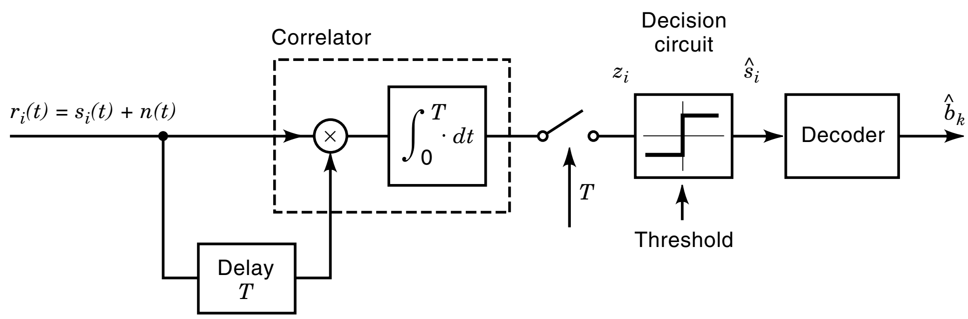

Figure 38. Block diagram of a DCSK receiver. (From Ref. 23.)

central limit theorem, the variance of the estimate of this correlation function varies as the inverse of the number of samples used. In order to decrease this variance we could use the dynamical constraints that are imposed by the emitter system, the dynamics of which are assumed to be known by the receiver. The basic problem we have to deal with is a problem of noise reduction in which a deterministic signal (the chaotic one) is corrupted by a noise time series which is considered to be a realization of a stochastic process. There are different methods which have been developed to solve the problem of the decontamination of chaotic signals. We will give details in the following, a simple approach which is inspired from the work of Ref. 21 in taking from a more sophisticated method developed by Farmer and Sidorowich (22).

We are given an M-dimensional system described by the difference equation

$$\boldsymbol{x}_{n+1} = f(\boldsymbol{x}_n) \tag{30}$$

Let $\boldsymbol{x}$ be a system orbit made of N consecutive M-dimensional points of the system (21), that is,

$$\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N]^T \tag{31}$$

We can observe a contaminated version of Ref. 22 such that each coordinate of $\boldsymbol{x}_n$, $n = 1, \ldots N$, is corrupted with an additive Gaussian noise. The corrupted version of the orbit is denoted $\boldsymbol{y}$, that is,

$$\boldsymbol{y}_n = \boldsymbol{x}_n + \boldsymbol{w}_n, \qquad n = 1 \ldots N \tag{32}$$

Our goal is to find an estimate $\hat{\boldsymbol{x}}$ of the noise-free orbit $\boldsymbol{x}$ given $\boldsymbol{y}$.

In order to achieve such a goal, the estimation problem can be seen as an optimization problem in which a cost function has to be minimized with respect to $\hat{\boldsymbol{x}}$. In order to exploit the deterministic nature of the system, we can design a cost function which is made of the sum of two terms:

1. A first term ensures that the global shape of $\hat{\boldsymbol{x}}$ is close to $\boldsymbol{y}$ according to a Euclidean distance or a correlation distance.
2. A second term involves the dynamical nature of the system using the deterministic relationship existing between consecutive points of the orbits.

For instance, as described in Ref. 21 the first cost function can be a simple Euclidean distance

$$C_1(\hat{\boldsymbol{x}}, \boldsymbol{y}) = \sum_{n=1}^{N} \|\hat{\boldsymbol{x}}_n - \boldsymbol{y}_n\|^2 \tag{33}$$

or can be associated with the correlation coefficient between the enhanced orbit $\hat{\boldsymbol{x}}$ and the noisy orbit $\boldsymbol{y}$:

$$C_1(\hat{\boldsymbol{x}}, \boldsymbol{y}) = 1 - \frac{\sum_{n=1}^{N} \hat{\boldsymbol{x}}_n^T \boldsymbol{y}_n}{\sqrt{\sum_{n=1}^{N} \|\hat{\boldsymbol{x}}_n\|^2} \sqrt{\sum_{n=1}^{N} \|\boldsymbol{y}_n\|^2}} \tag{34}$$

The second cost function measures the compatibility of the enhanced points with the dynamics of the system, for instance,

$$C_2(\hat{\boldsymbol{x}}) = \sum_{n=1}^{N} \|f(\hat{\boldsymbol{x}}_n) - \hat{\boldsymbol{x}}_{n+1}\|^2 \tag{35}$$

The global cost function appears as a linear combination of $C_1(\hat{\boldsymbol{x}}, \boldsymbol{y})$ and $C_2(\hat{\boldsymbol{x}})$ such as

$$c(\hat{\boldsymbol{x}}, \boldsymbol{y}) = C_1(\hat{\boldsymbol{x}}, \boldsymbol{y}) + \Gamma C_2(\hat{\boldsymbol{x}}) \tag{36}$$

where Γ is some positive scalar weight.

The problem addressed amounts to finding an iterative method in $\hat{\boldsymbol{x}}^{(i)}$ which converges toward a local minimum of Eq. (36). Such a method can be easily implemented by using a simple gradient method in which the update of the current estimate is in the direction opposite to that of the gradient of the cost function, that is,

$$\hat{\boldsymbol{x}}^{(i)} = \hat{\boldsymbol{x}}^{(i-1)} - \mu \left[\frac{\partial c(\hat{\boldsymbol{x}}, \boldsymbol{y})}{\partial \hat{\boldsymbol{x}}} \right]_{\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}}^{(i-1)}} \tag{37}$$

Provided that μ is chosen small enough, the gradient method will converge to a local minima of the cost function [Eq. (37)].

Applying the method for the cost function given in Eq. (37) with $\Gamma = 1$, we obtain for the gradient method

$$\begin{aligned}
\hat{\boldsymbol{x}}_1^{(i)} &= \hat{\boldsymbol{x}}_1^{(i-1)} + \mu \left[\frac{\sqrt{m_x m_y}\,\boldsymbol{y}_1 - m_{xy} \dfrac{\sqrt{m_y}}{\sqrt{m_x}} \hat{\boldsymbol{x}}_1^{(i-1)}}{m_x m_y} \right] \\
&\quad + \mu[-2\boldsymbol{D}_f(\hat{\boldsymbol{x}}_1^{(i-1)})(f(\hat{\boldsymbol{x}}_1^{(i-1)}) - \hat{\boldsymbol{x}}_2^{(i-1)})] \\
\hat{\boldsymbol{x}}_n^{(i)} &= \hat{\boldsymbol{x}}_n^{(i-1)} + \mu \left[\frac{\sqrt{m_x m_y}\,\boldsymbol{y}_n - m_{xy} \dfrac{\sqrt{m_y}}{\sqrt{m_x}} \hat{\boldsymbol{x}}_n^{(i-1)}}{m_x m_y} \right] \\
&\quad + 2\mu[(f(\hat{\boldsymbol{x}}_{n-1}^{(i-1)}) - \hat{\boldsymbol{x}}_n^{(i-1)}) - \boldsymbol{D}_f(\hat{\boldsymbol{x}}_n^{(i-1)})(f(\hat{\boldsymbol{x}}_n^{(i-1)}) \\
&\quad - \hat{\boldsymbol{x}}_{n+1}^{(i-1)})], \qquad n = 2 \ldots N-1 \\
\hat{\boldsymbol{x}}_N^{(i)} &= \hat{\boldsymbol{x}}_N^{(i-1)} + \mu \left[\frac{\sqrt{m_x m_y}\,\boldsymbol{y}_N - m_{xy} \dfrac{\sqrt{m_y}}{\sqrt{m_x}} \hat{\boldsymbol{x}}_N^{(i-1)}}{m_x m_y} \right] \\
&\quad + \mu[2(f(\hat{\boldsymbol{x}}_{N-1}^{(i-1)}) - \hat{\boldsymbol{x}}_N^{(i-1)})]
\end{aligned} \tag{38}$$

The gradient method is a first-order optimization method that can converge slowly. Second-order methods are known to converge quickly in terms of the number of iterations; however the computational load per iteration is considerably greater compared to that of the gradient method since a Hessian matrix has to be inverted at each iteration. Alternative de-noising methods have been developed in Ref. 22, in which the minimization of the cost function $C_1(\hat{\boldsymbol{x}}, \boldsymbol{y})$ has been considered under equality constraints (constrained optimization). When de-noising N trajectory points, $N - 1$ equality constraints are given by the $N - 1$ trajectory constraints $f(\hat{\boldsymbol{x}}_n) - \hat{\boldsymbol{x}}_{n+1} = 0$, $n = 1 \ldots N - 1$. The constrained optimization amounts to introduce a cost function which mixes $C_1(\hat{\boldsymbol{x}}, \boldsymbol{y})$ and a linear combination of the equality constraints. This linear combination involves the Lagrange multipliers. The augmented cost function is the co-called Lagrangian and the fundamental problem is to find an extremum of the Lagrangian with respect to $\hat{x}$ and to the $N - 1$ Lagrange multipliers ($2N - 1$ unknowns). Writing the derivatives of the Lagrangian with respect to $\hat{x}$ and to the Lagrange multipliers one has to solve $2N - 1$ nonlinear equations. A large variety of search algorithms can be used; a Newton's method can be implemented to linearize the nonlinear equations under their current solution. Ref. 22 contains the details of such a constrained optimization method. Please note that the method is computationally demanding since a $2N - 1 \times 2N - 1$ dimensional matrix has to be inverted at each iteration. Other denoising methods have been developed recently. Of particular interest is the probabilistic approach introduced by Marteau and Abarbanel (24). The de-noising method is referred to as probabilistic because it relies on the p of the attractor generated by the dynamical signal we observe. These probabilistic properties are expressed in terms of the invariant distributions of data points on the attractor. Although the term probability is not fully adequate, it is used in the sense that these invariant distributions act like probability distributions (24). After a quantization of the phase space of the chaotic trajectories the probabilistic approach relies on the computation or measure of the probability of passing from one quanta to any other quanta. At the receiver end the signal can be seen as a first order Markov chain process, the states of which are hidden by the channel noise. Given the Markov chain transition probabilities which are known in advance at the receiver and the noisy trajectory, the problem amounts to finding the best Markov chain which maximizes a maximum likelihood criterion. The determinist constraints in the chaotic trajectory are taken into account through the Markov chain transition probabilities. These probabilities can be evaluated using two techniques; for some simple piecewise linear systems they can be computed analytically, and for most systems they can be computed by producing a very long reference clean orbit. The maximum likelihood problem can be solved elegantly by using powerful dynamic programming algorithms such as the Viterbi algorithm.

Example

Let us consider the Hénon map, the dynamics of which are given by

$$x_{1,n+1} = 1 - 1.4x_{1,n}^2 + x_{2,n} \tag{39}$$

$$x_{2,n+1} = 0.3x_{1,n} \tag{40}$$

Figure 39 shows the performances of the noise reduction algorithm when applied to the the state of the map. The number of points was set to $N = 200$.

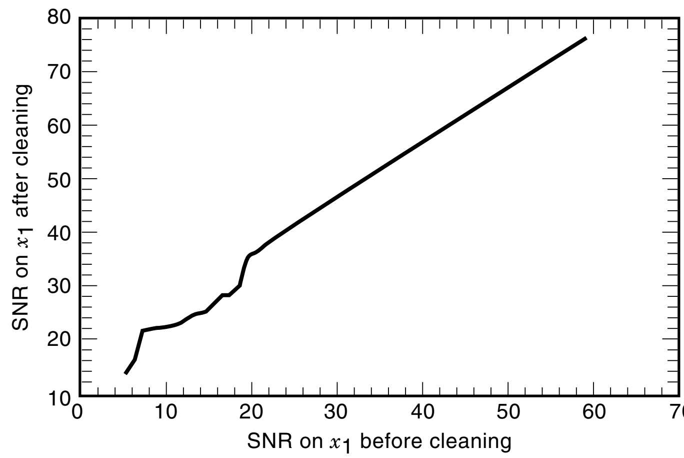

Figure 39. Mean-squared error according to various signal-to-noise ratios (SNRs).

SUMMARY

We have given an overview of the different methods which allow the transmission of information with chaotic carriers. We have higlighted that chaotic carriers could be of great interest in communication applications in which one needs to spread the information and/or desires some level of secrecy. The demodulation of the information can be achieved in different ways. Synchronization can be one of these ways if one wants to develop coherent receivers. Until now, synchronization of chaos seems to be a solution which is not suitable for imperfect communication channels since it is difficult to maintain synchronization when noise is added in the channel. These aspects have been addressed in the sections entitled "Synchronization of Chaotic Systems" and "Exploiting Chaos Synchronization for Transmission of Information." The statistical properties of the chaotic signals can be exploited in different manners, by replacing the conventional PN sequences of standard spread-spectrum systems by a chaotic oscillator (see section entitled "Example 1: Chaotic Direct-Sequence Spread-Spectrum Systems") or by using differential coding techniques (see section entitled "Example 2: Differential Chaos Shift Keying") if one wants to develop noncoherent receivers. The statistical aspects as well as the deterministic aspect of chaos can be exploited together to improve the statistical approach. This view which has been developed in the section entitled "Improving Chaos Transmission by Exploiting the Deterministic Feature of Chaos" offers new avenues in communicating with chaos and will be certainly exploited in a next generation of spread-spectrum systems.

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