CIRCUIT NOISE

Noise is present in all electronic circuits. It is generated by the random motion of electrons in a resistive material, by the random generation and recombination of holes and electrons in a semiconductor, and when holes and electrons diffuse through a potential barrier. This article covers the fundamentals of noise from a circuit viewpoint. The principal sources are described, circuit models are given, and methods for its measurement are discussed. Noise models for the bipolar junction transistor (BJT) and the field effect transistor (FET) are given. The conditions for minimum noise in each are derived.

The notation for voltages and currents corresponds to the following conventions: dc bias values are indicated by an uppercase letter with uppercase subscripts, e.g., V_{DS}, I_C. Instantaneous values of small-signal variables are indicated by a lowercase letter with lowercase subscripts, e.g. v_s, i_c. Mean squared values of small-signal variables are represented by a bar over the square of the variable, e.g., $\overline{v_s^2}$, $\overline{i_c^2}$, where the bar indicates an arithmetic average of an ensemble of functions. The root-mean-square (rms) value is the square root of the mean squared value. Phasors are indicated by an uppercase letter and lowercase subscripts (e.g. V_s, I_c). Circuit symbols for independent sources are circular, symbols for controlled sources have a diamond shape, and symbols for noise sources are square. Voltage sources have a $\pm$ sign within the symbol, and current sources have an arrow. In the numerical evaluation of noise equations, Boltzmann's constant is $k = 1.38 \times 10^{-23}$ J/K and the electronic charge is $q = 1.60 \times 10^{-19}$ C. The standard temperature is denoted by T_0 and is taken to be $T_0 = 290$ K. For this value, $4kT_0 = 1.60 \times 10^{-20}$ J and the thermal voltage is $V_T = kT_0/q = 25.0$ mV.

THERMAL NOISE

Thermal noise, also called Johnson noise, is generated by the random thermal motion of electrons in a resistive material. It is present in all circuit elements containing resistance and is independent of the composition of the resistance. It is modeled the same way in discrete-circuit resistors and in integrated circuit monolithic and thin film resistors. The phenomenon was discovered (or anticipated) by Schottky in 1928 and first measured and evaluated by Johnson in the same year. Shortly after its discovery, Nyquist used a thermodynamic argument to show that the mean squared open-circuit thermal noise voltage across a resistor is given by

$$\overline{v_t^2} = 4kTR\,\Delta f \tag{1}$$

where k is Boltzmann's constant, T is the absolute temperature, R is the resistance, and Δf is the bandwidth in hertz over which the noise is measured. The corresponding mean squared short-circuit thermal noise current is given by

$$\overline{i_t^2} = \frac{\overline{v_t^2}}{R^2} = \frac{4kT\,\Delta f}{R} \tag{2}$$

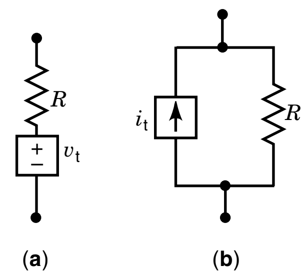

Figure 1. Thermal noise models of the resistor: (a) Thevenin model. (b) Norton model.

The Thevenin and Norton noise models of a resistor are given in Fig. 1. Because noise is random, the source polarities are arbitrary. In general, the polarities must be labeled when writing circuit equations that involve small-signal or phasor voltages and currents. The mean squared noise is independent of the assumed polarities.

The crest factor of thermal noise is defined as the level that is exceeded 0.01% of the time. To relate this to the rms value, a statistical model for the amplitude distribution is required. It is common to use a Gaussian, or normal, probability density function. For a Gaussian random variable, the probability that the instantaneous value exceeds four times the rms value is approximately 0.01%. Thus the crest factor of thermal noise is approximately 4.

The spectral density of a noise source is defined as the mean squared value of the source per unit bandwidth. It is equal to the average power per unit bandwidth delivered by the source to a normalized load resistance of 1 Ω. In general, the spectral density is a function of frequency. The voltage and current spectral densities, respectively, for the thermal noise generated by a resistor of value R are given by

$$S_{v_t}(f) = \frac{\overline{v_t^2}}{\Delta f} = 4kTR \tag{3}$$

$$S_{i_t}(f) = \frac{\overline{i_t^2}}{\Delta f} = \frac{4kT}{R} \tag{4}$$

Because these are independent of frequency, thermal noise is said to have a uniform or flat power distribution. It is sometimes called white noise by analogy to white light, which also has a flat spectral density in the optical band.

In the frequency band from f_1 to f_2, the mean squared open-circuit thermal noise voltage generated by any two-terminal network is given by

$$\overline{v_t^2} = 4kT\int_{f_1}^{f_2} \mathrm{Re}\,Z\,df \tag{5}$$

where Z is the complex impedance of the network, Re Z is the real part of Z, and f is the frequency in hertz. For $f_2 = f_1 +$

Δf and Δf small, the noise voltage divided by the square root of the bandwidth is given by

$$\sqrt{\frac{\overline{v_{\mathrm{t}}^2}}{\Delta f}} = \sqrt{4kT\,\mathrm{Re}\,Z} \tag{6}$$

This equation defines what is called the thermal spot noise voltage generated by the impedance. The units are read "volts per root hertz."

NOISE RESISTANCE AND CONDUCTANCE

A mean-square noise voltage can be represented in terms of an equivalent noise resistance R_n. Let $\overline{v_n^2}$ be the mean-square noise voltage in the band Δf. The noise resistance R_n is defined as the value of a resistor at the standard temperature $T_0 = 290$ K which generates the same noise voltage. It is given by

$$R_n = \frac{\overline{v_n^2}}{4kT_o\,\Delta f} \tag{7}$$

A mean-square noise current can be represented in terms of an equivalent noise conductance G_n. Let $\overline{i_n^2}$ be the mean-square noise current in the band Δf. The noise conductance G_n is defined as the value of a conductance at the standard temperature which generates the same noise current. It is given by

$$G_n = \frac{\overline{i_n^2}}{4kT_o\,\Delta f} \tag{8}$$

NOISE TEMPERATURE

The noise temperature T_{n} of a source is the temperature of a resistor having a value equal to the output resistance of the source that generates the same noise as the source. It is given by

$$T_{\mathrm{n}} = \frac{\overline{v_{\mathrm{ns}}^2}}{4kR_{\mathrm{S}}\,\Delta f} \tag{9}$$

where $\overline{v_{\mathrm{ns}}^2}$ is the mean squared open-circuit noise voltage generated by the source in the band Δf, and R_{S} is the real part of the output impedance of the source. If the source noise is expressed as a current, the noise temperature is given by

$$T_{\mathrm{n}} = \frac{\overline{i_{\mathrm{ns}}^2}}{4kG_{\mathrm{S}}\,\Delta f} \tag{10}$$

where $\overline{i_{\mathrm{ns}}^2}$ is the mean squared short-circuit noise current generated by the source in the band Δf, and G_{S} is the real part of the output admittance of the source.

SHOT NOISE

Shot noise is generated by the random emission of electrons or by the random passage of electrons and holes across a potential barrier. The shot noise generated in a device is modeled by a parallel noise current source. The mean squared shot noise current in the frequency band Δf is given by

$$\overline{i_{\mathrm{sh}}^2} = 2qI\,\Delta f \tag{11}$$

where q is the electronic charge and I is the dc current flowing through the device. This equation was derived by Shottky in 1928 and is known as the Shottky formula. The spectral density of shot noise is flat; thus shot noise is white noise. It is commonly assumed that the amplitude distribution can be modeled by a Gaussian distribution. Thus the relation between the crest factor and rms value for shot noise is the same as it is for thermal noise.

FLICKER NOISE

The imperfect contact between two conducting materials causes the conductivity to fluctuate in the presence of a dc current. This phenomenon generates flicker noise, also called contact noise. It occurs in any device where two conductors are joined together, for example the contacts of switches, potentiometers, or relays. Flicker noise in BJTs occurs in the base bias current. In FETs, it occurs in the drain bias current.

Flicker noise is modeled by a noise current source in parallel with the device. The mean squared flicker noise current in the frequency band Δf is given by

$$\overline{i_{\mathrm{f}}^2} = \frac{K_{\mathrm{f}} I^m\,\Delta f}{f^n} \tag{12}$$

where I is the dc current, $n \approx 1$, K_{f} is the flicker noise coefficient, and m is the flicker noise exponent. The spectral density of flicker noise is inversely proportional to frequency. For this reason, it is commonly referred to as "$1/f$ noise."

Flicker noise in BJTs can increase significantly if the base-to-emitter junction is subjected to reverse breakdown. This can be caused during power supply turn-on or by the application of too large an input voltage. A normally reverse-biased diode in parallel with the base-to-emitter junction is often used to prevent it.

EXCESS NOISE

Flicker noise in resistors is referred to as excess noise. It is caused by the variable contact between particles of the resistive material. Metal film resistors generate the least excess noise, carbon composition resistors generate the most, and carbon film resistors lie between the two. The mean squared short-circuit excess noise current generated by a resistor R is given by

$$\overline{i_{\mathrm{ex}}^2} = \frac{K_{\mathrm{f}} I_{\mathrm{DC}}^2\,\Delta f}{f} \tag{13}$$

where I_{DC} is the dc current through the resistor. The mean squared open-circuit excess noise voltage across the resistor is given by

$$\overline{v_{\mathrm{ex}}^2} = \frac{K_{\mathrm{f}} I_{\mathrm{DC}}^2 R_{\mathrm{S}}^2\,\Delta f}{f} = \frac{K_{\mathrm{f}} V_{\mathrm{DC}}^2\,\Delta f}{f} \tag{14}$$

where $V_{\mathrm{DC}} = I_{\mathrm{DC}}R$ is the dc voltage across the resistor.

The noise index of a resistor in decibels is the value of 20 log $(\sqrt{\overline{v_{ex}^2}}/V_{DC})$ for one decade of frequency, where $\sqrt{\overline{v_{ex}^2}}$ is expressed in microvolts. An alternative definition of the noise index is the value of 20 log $(\sqrt{\overline{i_{ex}^2}}/I_{DC})$ for one decade of frequency, where $\sqrt{\overline{i_{ex}^2}}$ is expressed in microamperes. Given the noise index NI, the value of $\sqrt{\overline{v_{ex}^2}}$ in microvolts and the value of $\sqrt{\overline{i_{ex}^2}}$ in microamperes in the range from f_1 to f_2 are given by

$$\sqrt{\overline{v_{ex}^2}} = 10^{\mathrm{NI}/20} \times V_{DC}\sqrt{\frac{\ln(f_2/f_1)}{\ln 10}} \tag{15}$$

$$\sqrt{\overline{i_{ex}^2}} = 10^{\mathrm{NI}/20} \times I_{DC}\sqrt{\frac{\ln(f_2/f_1)}{\ln 10}} \tag{16}$$

BURST NOISE

Burst noise is caused by a metallic impurity in a *pn* junction caused by a manufacturing defect. It is minimized by improved fabrication processes. When burst noise is amplified and reproduced by a loudspeaker, it sounds like corn popping. For this reason, it is also called popcorn noise. When viewed on an oscilloscope, burst noise appears as fixed-amplitude pulses of randomly varying width and repetition rate. The rate can vary from less than one pulse per minute to several hundred pulses per second. Typically, the amplitude of burst noise is 2 to 100 times that of the background thermal noise.

NOISE BANDWIDTH

In making noise measurements, it is common to precede the measuring voltmeter with a filter of known noise bandwidth. The noise bandwidth of a filter is defined as the bandwidth of an ideal filter that passes the same mean squared noise voltage, where the input signal is white noise. The filter and the ideal filter are assumed to have the same gain.

The noise bandwidth B in hertz of a filter is given by

$$B = \frac{1}{A_0^2}\int_0^\infty |A(f)|^2\,df \tag{17}$$

where $A(f)$ is the filter voltage-gain transfer function, A_0 is the maximum value of $|A(f)|$, and f is the frequency in hertz. Figure 2 graphically illustrates the concept of noise bandwidth for a low-pass filter and a band-pass filter. In each case, the actual filter response and the response of an ideal filter having the same noise bandwidth are shown. For equal noise bandwidths, the area under the actual filter curve must be equal to the area under the ideal filter curve. This makes the two indicated areas equal for the low-pass case. A similar interpretation holds for the band-pass case.

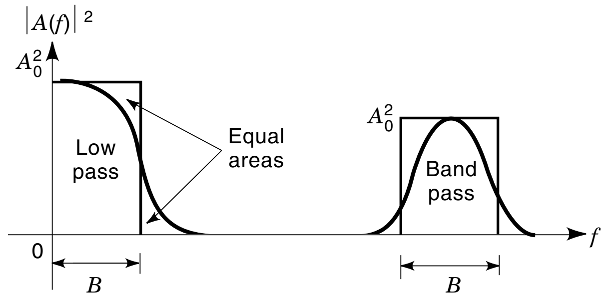

Figure 2. The bandwidth of an ideal filter is equal to the noise bandwidth of a physical filter if the two filters have the same area beneath their magnitude-squared response curves.

Table 1. Noise Bandwidth *B* of Low-Pass Filters

Number of poles	Slope (dB/decade)	*B* Real Pole	*B* Butterworth
1	20	$1.571f_0 = 1.571f_3$	$1.571f_3$
2	40	$0.785f_0 = 1.220f_3$	$1.111f_3$
3	60	$0.589f_0 = 1.155f_3$	$1.042f_3$
4	80	$0.491f_0 = 1.129f_3$	$1.026f_3$
5	100	$0.420f_0 = 1.114f_3$	$1.017f_3$

Two classes of low-pass filters are often used in measuring noise. One has n real poles, all with the same frequency. The other is a n-pole Butterworth filter. Table 1 gives the noise bandwidth for each filter as a function of the number of poles n for $1 \le n \le 5$. For the real-pole filter, the noise bandwidth is given as a function of both the pole frequency f_0 and the upper -3 dB cutoff frequency f_3. For the Butterworth filter, the noise bandwidth is given as a function of the upper -3 dB frequency. The table shows that the noise bandwidth approaches the -3 dB frequency as the number of poles is increased.

Band-pass filters are used in making spot noise measurements. The noise bandwidth of the filter must be small enough so that the spectral density of the input noise is approximately constant over the bandwidth. The spot noise voltage is obtained by dividing the rms noise output voltage from the filter by the square root of its noise bandwidth. Second-order bandpass filters are commonly used for these measurements. The noise bandwidth of a second-order band-pass filter having a resonance frequency f_0 and a quality factor Q is given by

$$B = \frac{\pi f_0}{2Q} \tag{18}$$

If f_a and f_b, respectively, are the lower and upper -3 dB frequencies of the filter, an alternative expression for the noise bandwidth is

$$B = \frac{\pi}{2}(f_b - f_a) \tag{19}$$

This expression is often used to approximate the noise bandwidth of unknown band-pass filters.

A second-order band-pass filter is often realized by a first-order high-pass filter in cascade with a first-order low-pass filter. The noise bandwidth is given by

$$B = \frac{\pi}{2}(f_1 + f_2) \tag{20}$$

where f_1 is the pole frequency of the high-pass filter and f_2 is the pole frequency of the low-pass filter. These frequencies are not the -3 dB frequencies of the bandpass filter.

The noise bandwidth of any filter can be measured if a white noise source and another filter with a known noise bandwidth are available. With both filters driven simultaneously from the white noise source, the ratio of the noise bandwidths is equal to the square of the ratio of the output voltages.

MEASURING NOISE

Noise is usually measured at an amplifier output, where the voltage is the largest and easiest to measure. The output noise is referred to the input by dividing by the gain. A filter with a known noise bandwidth should precede the voltmeter to limit the bandwidth. The measuring voltmeter should have a bandwidth that is at least 10 times the filter bandwidth. The voltmeter crest factor is the ratio of the peak input voltage to the full scale rms meter reading at which the internal meter circuits overload. For a sine wave, the minimum voltmeter crest factor is $\sqrt{2}$. For noise measurements, a higher crest factor is required. For Gaussian noise, a crest factor of 3 gives an error less than 1.5%. A crest factor of 4 gives an error less than 0.5%. To minimize errors caused by overload on noise peaks, measurements should be made on the lower one-third to one-half of the voltmeter scale.

A true rms voltmeter is preferred over one that responds to the average rectified value of the input voltage but has a scale calibrated to read rms. When the latter type of voltmeter is used to measure noise, the reading will be low. For Gaussian noise, the reading can be corrected by multiplying the measured voltage by 1.13.

ADDITION OF NOISE SIGNALS

Real Signals

Let $v_a(t)$ and $v_b(t)$ be two noise voltages having the mean squared values $\overline{v_a^2}$ and $\overline{v_b^2}$. Define the sum voltage $v_{sum}(t) = v_a(t) + v_b(t)$. The mean squared value of the sum is given by

$$\begin{aligned}\overline{v_{\mathrm{sum}}^2} &= \overline{[v_{\mathrm{a}}(t)+v_{\mathrm{b}}(t)]^2} \\ &= \overline{v_{\mathrm{a}}^2(t)} + 2\overline{v_{\mathrm{a}}(t)v_{\mathrm{b}}(t)} + \overline{v_{\mathrm{b}}^2(t)} \\ &= \overline{v_{\mathrm{a}}^2} + 2\rho\sqrt{\overline{v_{\mathrm{a}}^2}}\sqrt{\overline{v_{\mathrm{b}}^2}} + \overline{v_{\mathrm{b}}^2}\end{aligned} \tag{21}$$

where ρ is the correlation coefficient defined by

$$\rho = \frac{\overline{v_{\mathrm{a}}(t)v_{\mathrm{b}}(t)}}{\sqrt{\overline{v_{\mathrm{a}}^2}}\sqrt{\overline{v_{\mathrm{b}}^2}}} \tag{22}$$

The correlation coefficient can take on values in the range $-1 \le \rho \le +1$. For the case $\rho = 0$, the voltages $v_a(t)$ and $v_b(t)$ are said to be statistically independent or uncorrelated.

Phasor Signals

It is often necessary to use phasor representations of noise voltages and currents in writing equations for circuits containing capacitors and/or inductors. The mathematical basis for this is involved and is omitted here. When a phasor representation is used, the ensemble average of the squared magnitude of the phasor represents the mean squared value of the noise voltage or current in the band Δf centered on the frequency of analysis. Thus the magnitude of the phasor is taken here to represent the rms value rather than the peak value of the variable at that frequency. To illustrate the addition of noise phasors, let V_a and V_b be the phasor representations of two noise voltages at a particular frequency. The sum is given by $V_{sum} = V_a + V_b$. The mean squared sum is calculated as follows:

$$\begin{aligned}\overline{v_{\mathrm{sum}}^2} &= \overline{(V_{\mathrm{a}}+V_{\mathrm{b}})(V_{\mathrm{a}}^*+V_{\mathrm{b}}^*)} \\ &= \overline{|V_{\mathrm{a}}|^2} + 2\,\mathrm{Re}(\overline{V_{\mathrm{a}}V_{\mathrm{b}}^*}) + \overline{|V_{\mathrm{b}}|^2} \\ &= \overline{v_{\mathrm{a}}^2} + 2(\mathrm{Re}\,\gamma)\sqrt{\overline{v_{\mathrm{a}}^2}}\sqrt{\overline{v_{\mathrm{b}}^2}} + \sqrt{\overline{v_{\mathrm{b}}^2}}\end{aligned} \tag{23}$$

where γ is the complex correlation coefficient defined by

$$\gamma = \gamma_{\mathrm{r}} + j\gamma_{\mathrm{i}} = \frac{\overline{V_{\mathrm{a}}V_{\mathrm{b}}^*}}{\sqrt{\overline{v_{\mathrm{a}}^2}}\sqrt{\overline{v_{\mathrm{b}}^2}}} \tag{24}$$

Equation (23) seems to imply that only the real part of γ needs to be known. In general, it is necessary to know both the real and imaginary parts. To illustrate this, consider the sum $V_{sum} = V_n + I_n Z$, where Z is a complex impedance. The mean squared sum is given by

$$\begin{aligned}\overline{v_{\mathrm{sum}}^2} &= \overline{|V_{\mathrm{n}}|^2} + 2\,\mathrm{Re}(\overline{V_{\mathrm{n}}I_{\mathrm{n}}^*Z^*}) + \overline{|I_{\mathrm{n}}Z|^2} \\ &= \overline{v_{\mathrm{n}}^2} + 2(\mathrm{Re}\,\gamma Z^*)\sqrt{\overline{v_{\mathrm{n}}^2}}\sqrt{\overline{i_{\mathrm{n}}^2}} + \overline{i_{\mathrm{n}}^2}|Z|^2\end{aligned} \tag{25}$$

where the correlation coefficient γ is given by

$$\gamma = \gamma_{\mathrm{r}} + j\gamma_{\mathrm{i}} = \frac{\overline{V_{\mathrm{n}}I_{\mathrm{n}}^*}}{\sqrt{\overline{v_{\mathrm{n}}^2}}\sqrt{\overline{i_{\mathrm{n}}^2}}} \tag{26}$$

For arbitrary Z, both γ_r and γ_i must be known to evaluate the factor Re γZ^*.

Noise formulas derived by a phasor analysis of circuits containing complex impedances can be converted into formulas for circuits containing real impedances, i.e. resistors, by setting to zero in the formulas the reactive, or imaginary, part of all impedances and setting $\gamma_i = 0$ and $\gamma_r = \rho$, where ρ is real. However, the procedure cannot be done in reverse. For this reason, noise formulas derived by a phasor analysis are the more general form of the formulas.

Correlation Impedance and Admittance

The concepts of a correlation impedance Z_γ and a correlation admittance Y_γ between a noise voltage V_n and a noise current I_n are often used in the noise literature. These are defined by

$$Z_\gamma = R_\gamma + jX_\gamma = \frac{\overline{V_{\mathrm{n}}I_{\mathrm{n}}^*}}{\overline{i_{\mathrm{n}}^2}} = \gamma\frac{\sqrt{\overline{v_{\mathrm{n}}^2}}}{\sqrt{\overline{i_{\mathrm{n}}^2}}} \tag{27}$$

$$Y_\gamma = G_\gamma + jB_\gamma = \frac{\overline{V_{\mathrm{n}}^*I_{\mathrm{n}}}}{\overline{v_{\mathrm{n}}^2}} = \gamma^*\frac{\sqrt{\overline{i_{\mathrm{n}}^2}}}{\sqrt{\overline{v_{\mathrm{n}}^2}}} \tag{28}$$

It follows from these definitions that $Z_\gamma Y_\gamma = |\gamma|^2$.

v_n–i_n AMPLIFIER NOISE MODEL

A general noise model of an amplifier can be obtained by reflecting all internal noise sources to the input. In order for

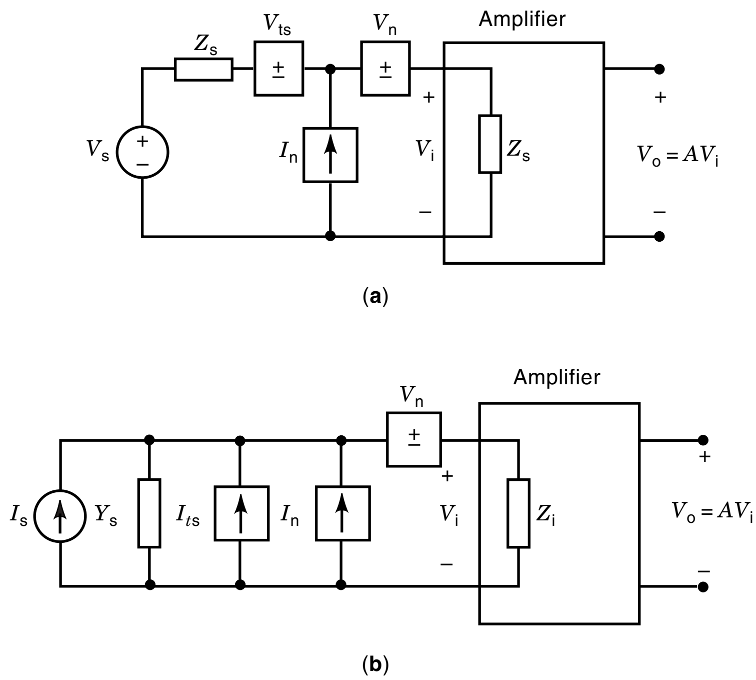

Figure 3. The v_n–i_n noise model amplifier: (a) With Thevenin source. (b) With Norton source.

the reflected sources to be independent of the source impedance, two noise sources are required—a series voltage source and a shunt current source. In general, these sources are correlated. The amplifier noise model is described in this section. The equivalent noise input voltage is derived for the case where the source is represented by a Thevenin equivalent. The equivalent noise input current is derived for the case where the source is represented by a Norton equivalent. A more general phasor analysis is used.

Thevenin Source

Figure 3(a) shows the amplifier model with a Thevenin input source, where V_s is the source voltage, $Z_s = R_S + jX_S$ is the source impedance, and V_{ts} is the thermal noise voltage generated by the source. The output voltage is given by

$$V_o = \frac{AZ_i}{Z_s + Z_i}[V_s + (V_{ts} + V_n + I_n Z_s)] \tag{29}$$

where A is the loaded voltage gain and Z_i is the input impedance. The equivalent noise input voltage V_{ni} is defined as the voltage in series with the amplifier input that generates the same noise voltage at the output as all noise sources in the circuit. Its value is given by the sum of the noise terms in the parentheses in Eq. (29) and is independent of the amplifier input impedance.

The mean squared value of V_{ni} is solved for as follows:

$$\begin{aligned}\overline{v_{ni}^2} &= \overline{(V_{ts} + V_n + I_n Z_s)(V_{ts}^* + V_n^* + I_n^* Z_s^*)} \\ &= \overline{V_{ts}V_{ts}^*} + \overline{V_n V_n^*} + 2\,\mathrm{Re}(\overline{V_n I_n^* Z_s^*}) + \overline{(I_n Z_s)(I_n^* Z_s^*)} \\ &= 4kTR_S\,\Delta f + \overline{v_n^2} + 2(\mathrm{Re}\,\gamma Z_s^*)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}|Z_s|^2 \\ &= 4kTR_S\,\Delta f + \overline{v_n^2} + 2(\gamma_r R_S + \gamma_i X_S)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}(R_S^2 + X_S^2)\end{aligned} \tag{30}$$

where $\gamma = \gamma_r + j\gamma_i$ is the correlation coefficient between V_n and I_n and it is assumed that V_{ts} is independent of both V_n and I_n. For $|Z_s|$ very small, $\overline{v_{ni}^2} \approx \overline{v_n^2}$ and the correlation coefficient is not important. Similarly, for $|Z_s|$ very large, $\overline{v_{ni}^2} \approx \overline{i_n^2}\,|Z_s|^2$ and the correlation coefficient is again not important. Unless it can be assumed that $\gamma = 0$, the v_n–i_n amplifier noise model can be cumbersome for making noise calculations. For $\gamma \neq 0$, it is often simpler to use the original circuit with its internal noise sources.

With $V_s = 0$ in Eq. (29), the mean squared noise voltage at the amplifier output is given by

$$\overline{v_{no}^2} = \left|\frac{AZ_i}{Z_s + Z_i}\right|^2 \Big[4kTR_S\,\Delta f + \overline{v_n^2} + 2(\mathrm{Re}\,\gamma Z_s^*)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}|Z_s|^2\Big] \tag{31}$$

If $Z_s = 0$, this equation can be solved for $\overline{v_n^2}$ to obtain

$$\overline{v_n^2} = \frac{\overline{v_{no}^2}}{|A|^2} \qquad \text{for} \quad Z_s = 0 \tag{32}$$

If $|Z_s|$ is very large, $\overline{i_n^2}$ can be solved for to obtain

$$\overline{i_n^2} = \left|\frac{1}{Z_s} + \frac{1}{Z_i}\right|^2 \frac{\overline{v_{no}^2}}{|A|^2} \qquad \text{for} \quad |Z_s|\ \text{large} \tag{33}$$

These equations suggest methods for experimental determination of $\overline{v_n^2}$ and $\overline{i_n^2}$. In measuring $\overline{v_{no}^2}$, it is common to use a filter with a known noise bandwidth preceding the voltmeter.

Norton Source

Figure 3(b) shows the amplifier model with a Norton input source, where I_s is the source current, $Y_s = G_S + jB_S$ is the source admittance, and I_{ts} is the thermal noise current generated by the source. The output voltage is given by

$$V_o = \frac{A}{Y_s + Y_i}[I_s + I_{ts} + V_n Y_s + I_n] \tag{34}$$

where A is the loaded voltage gain and Y_i is the input admittance. The equivalent noise input current I_{ni} is defined as the current in parallel with the amplifier input that generates the same noise voltage at the output as all noise sources in the circuit. Its value is given by the sum of the terms in the brackets in Eq. (34) with the exception of the I_s.

The mean squared value of I_{ni} is solved for as follows:

$$\begin{aligned}\overline{i_{ni}^2} &= \overline{(I_{ts} + V_n Y_s + I_n)(I_{ts}^* + V_n^* Y_s^* + I_n^*)} \\ &= \overline{I_{ts} I_{ts}^*} + \overline{(V_n Y_s)(V_n^* Y_s^*)} + 2\,\mathrm{Re}\,(\overline{V_n Y_s I_n^*}) + \overline{I_n I_n^*} \\ &= 4kTG_S\,\Delta f + \overline{v_n^2}|Y_s|^2 + 2(\mathrm{Re}\,\gamma Y_s)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2} \\ &= 4kTG_S\,\Delta f + \overline{v_n^2}(G_S^2 + B_S^2) + 2(\gamma_r G_S - \gamma_i B_S)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}\end{aligned} \tag{35}$$

where $\gamma = \gamma_r + j\gamma_i$ is given by Eq. (26).

SIGNAL-TO-NOISE RATIO

Thevenin Source

When the source is modeled by a Thevenin equivalent circuit as in Fig. 3(a), the signal-to-noise ratio is given by

$$\mathrm{SNR} = \frac{\overline{v_s^2}}{\overline{v_{ni}^2}} \tag{36}$$

where $\overline{v_{ni}^2}$ is given by Eq. (30). The SNR is often expressed in decibels as $10\log(\overline{v_s^2}/\overline{v_{ni}^2})$. The SNR is maximized by minimizing $\overline{v_{ni}^2}$. The source impedance that minimizes $\overline{v_{ni}^2}$ can be obtained by setting $d\overline{v_{ni}^2}/dR_S = 0$ and $d\overline{v_{ni}^2}/dX_S = 0$ and solving for R_S and X_S. The solution for R_S is negative. Because this is not realizable, $R_S = 0$ is the realizable solution for the least noise. The source impedance which minimizes $\overline{v_{ni}^2}$ is given by

$$Z_{sm} = R_{sm} + jX_{sm} = 0 - j\gamma_i\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \tag{37}$$

Because minimum noise occurs for $R_S = 0$, it can be concluded that a resistor should never be connected in series with a source at an amplifier input if noise performance is a design criterion. Although the output impedance of a source is usually fixed, the SNR can be improved by adding a reactance in series with the source that makes the total series reactance equal to the imaginary part of Z_{sm}. When this is the case, $\overline{v_{ni}^2}$ is given by

$$\begin{aligned}\overline{v_{ni}^2} &= 4kTR_S\,\Delta f + \overline{v_n^2}(1-\gamma_i^2) + 2\gamma_r R_S\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}R_S^2 \\ &= 4kTR_S\,\Delta f + \overline{v_n^2} + 2\gamma_r R_S\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}(R_S^2 - X_S^2)\end{aligned} \tag{38}$$

Norton Source

When the source is modeled by a Norton equivalent circuit as in Fig. 3(b), the signal-to-noise ratio is given by

$$\mathrm{SNR} = \frac{\overline{i_s^2}}{\overline{i_{ni}^2}} \tag{39}$$

where $\overline{i_{ni}^2}$ is given by Eq. (35). The SNR is expressed in decibels as $10\log(\overline{i_s^2}/\overline{i_{ni}^2})$. The source admittance that minimizes $\overline{i_{ni}^2}$ can be obtained by setting $d\overline{i_{ni}^2}/dG_S = 0$ and $d\overline{i_{ni}^2}/dB_S = 0$ and solving for G_S and B_S. The solution for G_S is negative. Because this is not realizable, $G_S = 0$ is the realizable solution for the least noise. The source admittance that minimizes $\overline{i_{ni}^2}$ is given by

$$Y_{sm} = G_{sm} + jB_{sm} = 0 + j\gamma_i\sqrt{\frac{\overline{i_n^2}}{\overline{v_n^2}}} \tag{40}$$

Because minimum noise occurs for $G_S = 0$, it can be concluded that a resistor should never be connected in parallel with a source at an amplifier input if noise performance is a design criterion. Although the output admittance of a source is usually fixed, the SNR can be improved by adding a susceptance in parallel with the source that makes the total parallel susceptance equal to the imaginary part of Y_{sm}. When this is the case, $\overline{i_{ni}^2}$ is given by

$$\begin{aligned}\overline{i_{ni}^2} &= 4kTG_S\,\Delta f + \overline{v_n^2}G_S^2 + 2\gamma_r G_S\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}(1-\gamma_i^2) \\ &= 4kTG_S\,\Delta f + \overline{v_n^2}(G_S^2 - B_{sm}^2) + 2\gamma_r G_S\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}\end{aligned} \tag{41}$$

NOISE FACTOR AND NOISE FIGURE

Thevenin Source

The noise factor F of an amplifier is defined as the ratio of its actual SNR to the SNR if the amplifier were noiseless. When it is expressed in decibels, it is called the noise figure and is given by $\mathrm{NF} = 10\log F$. It follows from Eq. (30) that the noise factor for the amplifier model of Fig. 3(a) in which the source is modeled by a Thevenin equivalent circuit is given by

$$F = \frac{\overline{v_{ni}^2}}{\overline{v_{ts}^2}} = 1 + \frac{\overline{v_n^2} + 2(\gamma_r R_S + \gamma_i X_S)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \sqrt{\overline{i_n^2}}(R_S^2 + X_S^2)}{4kTR_S\,\Delta f} \tag{42}$$

A noiseless amplifier has the noise factor $F = 1$.

The value of Z_s that minimizes F is called the optimum source impedance and is denoted by Z_{so}. It is obtained by setting $dF/dR_S = 0$ and $dF/dX_S = 0$ and solving for R_S and X_S. The impedance is given by

$$Z_{so} = R_{so} + jX_{so} = \left[\sqrt{1-\gamma_i^2} - j\gamma_i\right]\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \tag{43}$$

Note that the imaginary part of Z_{so} is equal to the imaginary part of Z_{sm} that maximizes the signal-to-noise ratio. The corresponding minimum value of the noise factor is given by

$$F_o = 1 + \frac{\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}}{2kT\,\Delta f}\left(\gamma_r + \sqrt{1-\gamma_i^2}\right) \tag{44}$$

With the relations $\sqrt{1-\gamma_i^2} = R_{so}\sqrt{\overline{i_n^2}}/\sqrt{\overline{v_n^2}}$ and $\gamma_r = R_\gamma\sqrt{\overline{i_n^2}}/\sqrt{\overline{v_n^2}}$, the minimum noise factor can be written

$$F_o = 1 + \frac{\overline{i_n^2}}{2kT\,\Delta f}[R_\gamma + R_{so}] \tag{45}$$

where R_γ is the real part of the correlation impedance Z_γ defined by Eq. (27). Let $\overline{i_n^2}$ be expressed in terms of the noise conductance G_n, so that $\overline{i_n^2} = 4kT_0G_n\Delta f$. It follows that F_o can be written in the alternate form

$$F_o = 1 + 2G_n\frac{T_0}{T}[R_\gamma + R_{so}] \tag{46}$$

It is straightforward to show that the noise factor in Eq. (42) can be expressed as a function of F_o as follows:

$$F = F_o + \frac{i_n^2}{4kTR_S\Delta f}[(R_S - R_{so})^2 + (X_S - X_{so})^2] \tag{47}$$

In this expression, let $\overline{i_n^2}$ be expressed in terms of the noise conductance G_n. Let the source noise be expressed in terms of its noise resistance R_{ns}, so that $TR_S = T_0R_{ns}$. It follows that F can be written

$$F = F_o + \frac{G_n}{R_{ns}}[(R_S - R_{so})^2 + (X_S - X_{so})^2] \tag{48}$$

Norton Source

For the amplifier model of Fig. 3(b) in which the source is modeled by a Norton equivalent circuit, F is given by

$$F = \frac{\overline{i_{ni}^2}}{\overline{i_{ts}^2}} = 1 + \frac{\overline{v_n^2}(G_S^2 + B_S^2) + 2(\gamma_r G_S - \gamma_i B_S)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2}}{4kTG_S\,\Delta f} \tag{49}$$

The optimum source admittance Y_{so} that minimizes F is obtained by setting $dF/dG_S = 0$ and $dF/dB_S = 0$ and solving for G_S and B_S. The admittance that is obtained is equal to the reciprocal of the optimum source impedance and is given by

$$Y_{so} = \frac{1}{Z_{so}} = G_{so} + jB_{so} = \left[\sqrt{1-\gamma_i^2} + j\gamma_i\right]\sqrt{\frac{\overline{i_n^2}}{\overline{v_n^2}}} \tag{50}$$

When $Y_s = Y_{so}$, F is given by Eq. (44). Note that the imaginary part of Y_{so} is equal to the imaginary part of Y_{sm} that maximizes the signal-to-noise ratio.

With the relations $\sqrt{1-\gamma_i^2} = G_{so}\sqrt{\overline{v_n^2}}/\sqrt{\overline{i_n^2}}$ and $\gamma_r = G_\gamma\sqrt{\overline{v_n^2}}/\sqrt{\overline{i_n^2}}$, Eq. (44) can be written

$$F_o = 1 + \frac{\overline{e_n^2}}{2kT\,\Delta f}(G_\gamma + G_{so}) \tag{51}$$

where G_γ is the real part of the correlation admittance Y_γ defined by Eq. (28). Let $\overline{v_n^2}$ be expressed in terms of the noise resistance R_n, so that $\overline{v_n^2} = 4kT_0R_n\Delta f$. It follows that F_o can be written in the alternative form

$$F_o = 1 + 2R_n\frac{T_0}{T}(G_\gamma + G_{so}) \tag{52}$$

It is straightforward to show that the noise factor in Eq. (49) can be expressed as a function of F_o as follows:

$$F = F_o + \frac{\overline{v_n^2}}{4kTG_S\,\Delta f}[(G_S - G_{so})^2 + (B_S - B_{so})^2] \tag{53}$$

In this expression, let $\overline{v_n^2}$ be expressed in terms of the noise resistance R_n. Let the source noise be expressed in terms of its noise conductance G_{ns}, so that $TG_S = T_oG_{ns}$. It follows that F can be written

$$F = F_o + \frac{R_n}{G_{ns}}[(G_S - G_{so})^2 + (B_S - B_{so})^2] \tag{54}$$

The noise factor can be a misleading specification. If an attempt is made to minimize F by adding resistors either in series or in parallel with the source at the input of an amplifier, the SNR is always decreased. This is referred to as the noise factor fallacy or the noise figure fallacy. Confusion can be avoided if low-noise amplifiers are designed to maximize the SNR. This is accomplished by minimizing the equivalent noise input voltage referred to the source.

NOISE IN MULTISTAGE AMPLIFIERS

Figure 4 shows the first two stages of a multistage amplifier having N stages. The input impedance to each stage is modeled by a resistor. Each output circuit is modeled by a Norton equivalent circuit consisting of a parallel current source and resistor. The equivalent noise input voltage for each stage is shown as a series voltage source preceding that stage. For the jth stage, it is given by $v_{nij} = v_{nj} + i_{nj}R_{o(j-1)}$.

The short-circuit output current from the jth stage can be written $i_{oj} = G_{mj}v_{ij(oc)}$, where $v_{ij(oc)}$ is the open-circuit input voltage and G_{mj} is the transconductance gain from the open-circuit input voltage to the short-circuit output current. This transconductance is given by $G_{mj} = g_{mj}R_{ij}/(R_{o(j-1)} + R_{ij})$, where g_m is the ratio of the short-circuit output current to the actual or loaded input voltage. The open-circuit voltage gain of the jth stage is given by $G_{mj}R_{oj}$. For the overall circuit, the voltage gain is given by $K = G_{m1}R_{o1}G_{m2}R_{o2} \cdots G_{mN}R_{oN} \parallel R_L$.

It is straightforward to show that the output voltage is given by

$$v_o = K\left[v_s + v_{n1} + i_{n1}R_S + \frac{v_{n2} + i_{n2}R_{o1}}{G_{m1}R_{o1}} + \frac{v_{n3} + i_{n3}R_{o2}}{G_{m1}R_{o1}G_{m2}R_{o2}} + \cdots + \frac{v_{nN} + i_{nN}R_{o(N-1)}}{G_{m1}R_{o1}G_{m2}R_{o2}\cdots G_{m(N-1)}R_{o(N-1)}}\right] \tag{55}$$

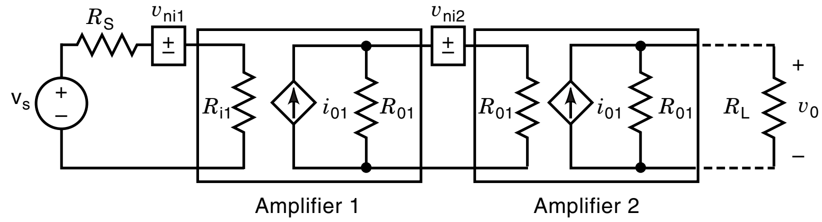

Figure 4. Model used to calculate the equivalent input noise voltage of a multistage amplifier.

The equivalent noise input voltage v_{ni} is given by the sum of all terms in the brackets in this equation except the v_s term. It is given by

$$v_{ni} = v_{n1} + \frac{v_{n2}}{G_{m1}R_{o1}} + \frac{v_{n3}}{G_{m1}R_{o1}G_{m2}R_{o2}} + \cdots + i_{n1}R_S + \frac{i_{n2}}{G_{m1}} + \frac{i_{n3}}{G_{m1}R_{o1}G_{m2}} + \cdots \tag{56}$$

It can be seen that the v_n noise of any stage following the first stage is divided by the open-circuit voltage gain of the first stage. If this gain is sufficiently high, the v_n noise of all stages after the first stage can be neglected. This also minimizes the i_n noise of all stages after the second stage. The i_n noise of the second stage is divided by G_{m1}. Unless G_{m1} is large, the only way to minimize the i_{n2} term is to use a second stage that exhibits a low i_n noise. For a single bipolar transistor, $G_m \le g_m = I_C/V_T$, where I_C is the collector bias current and V_T is the thermal voltage. For I_C = 1 mA and V_T = 25 mV, $g_m = 0.04$. For field effect devices, the g_m is usually lower. Therefore, minimization of the i_{n2} noise can be difficult to achieve by maximizing G_{m1}.

NOISE REDUCTION WITH PARALLEL DEVICES

A method that can be used to reduce the noise generated in an amplifier input stage is to realize that stage with several active devices in parallel, e.g. parallel BJTs or parallel FETs. Figure 5 shows the diagram of an amplifier input stage having N identical devices in parallel. For simplicity, only the first two are shown. The noise source V_{ts} models the thermal noise generated by the source resistance R_S = Re Z_s. Each amplifier stage is modeled by the v_n–i_n amplifier noise model having an input impedance Z_i. The output circuit is modeled by a Norton equivalent circuit consisting of a parallel current source and impedance. The short-circuit output current from the jth stage can be written $I_{oj} = g_m V_{ij}$, where g_m is the transconductance and V_{ij} is the input voltage for that stage.

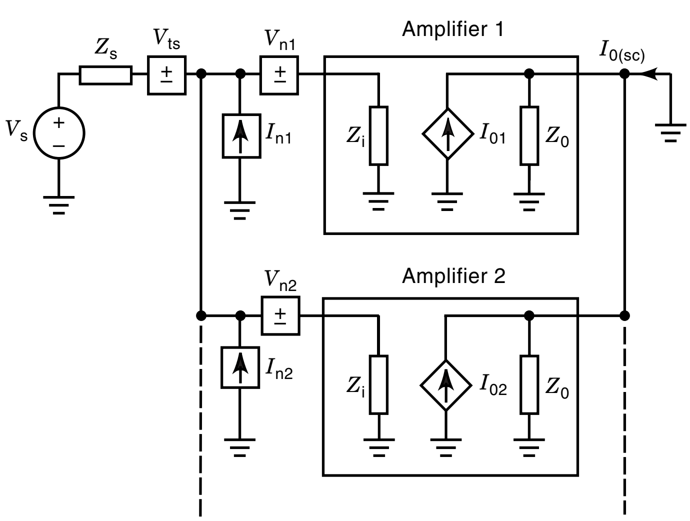

Figure 5. Model used to calculate the equivalent input noise voltage of paralleled amplifier stages.

The short-circuit output current from the circuit can be written

$$I_{o(sc)} = g_m \left[N \left(Z_s \parallel \frac{Z_i}{N} \right) \left(\frac{V_s + V_{ts}}{Z_s} + \sum_{j=1}^{N} I_{nj} \right) + \sum_{j=1}^{N} \left(\frac{Z_i}{Z_i + Z_s \parallel \left(\frac{Z_i}{N-1} \right)} V_{nj} - \frac{Z_s \parallel \left(\frac{Z_i}{N-1} \right)}{Z_i + Z_s \parallel \left(\frac{Z_i}{N-1} \right)} \sum_{\substack{k=1 \\ k \neq j}}^{N} V_{nk} \right) \right] \tag{57}$$

To define the equivalent noise input voltage, the expression multiplying V_s must be factored from the outer brackets in this equation. All remaining terms with the exception of the V_s term then represent V_{ni}. When this is done and the expression for V_{ni} is converted into a mean-square sum, a significant simplification occurs. The final expression for $\overline{v_{ni}^2}$ is

$$\overline{v_{ni}^2} = 4kTR_S\,\Delta f + \frac{\overline{v_n^2}}{N} + 2(\mathrm{Re}\,\gamma Z_s^*)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + N\overline{i_n^2}|Z_s|^2 \tag{58}$$

where γ is the correlation coefficient between V_n and I_n for any one of the N identical stages.

If Z_s = 0, Eq. (58) reduces to $\overline{v_{ni}^2} = \overline{v_n^2}/N$. In this case, the noise can theoretically be reduced to any desired level if N is made large enough. For $Z_s \neq 0$, Eq. (58) predicts that $\overline{v_{ni}^2} \to \infty$ for $N \to 0$ or $N \to \infty$. Thus there is a value of N that minimizes the noise. It is solved for by setting $d\overline{v_{ni}^2}/dN$ = 0 and solving for N. It is given by

$$N = \frac{1}{|Z_s|}\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \tag{59}$$

This expression shows that N decreases as $|Z_s|$ increases. It follows that the noise cannot be reduced by paralleling input devices if the source impedance is too large.

NOISE REDUCTION WITH AN INPUT TRANSFORMER

A transformer at the input of an amplifier may improve its noise performance. Figure 6(a) shows a signal source connected to an amplifier through a transformer with a turns ratio 1 : n. Resistors R_1 and R_2, respectively, represent the

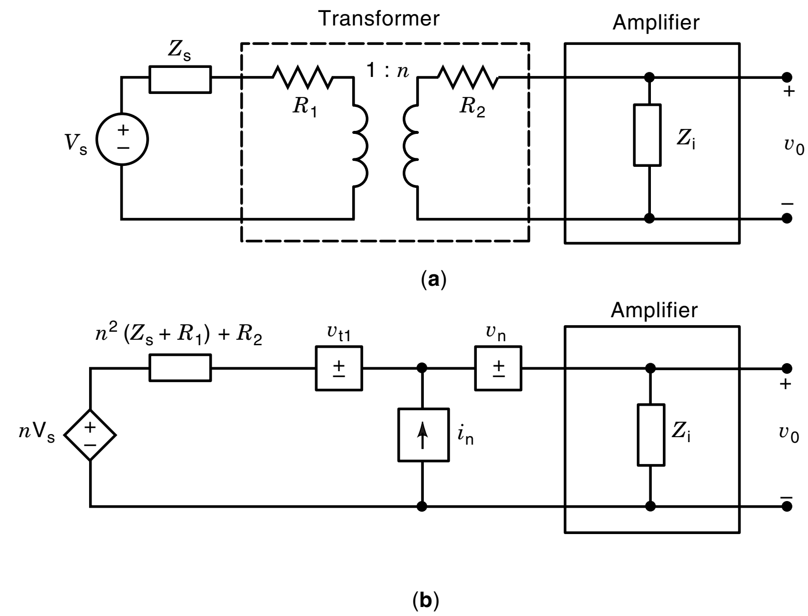

Figure 6. (a) Model used to calculate the equivalent input noise voltage of a transformer coupled amplifier. (b) Equivalent circuit.

primary and the secondary winding resistances. Figure 6(b) shows the equivalent circuit seen by the amplifier input with all noise sources shown. The source V_{t1} represents the thermal noise generated by the effective source resistance $n^2 (R_S + R_1) + R_2$, where $R_S = \mathrm{Re}\, Z_s$. By analogy to Eq. (29), the amplifier output voltage is given by

$$V_o = \frac{AZ_i}{n^2(Z_s + R_1) + R_2 + Z_i} \times \{nV_s + V_{t1} + V_n + I_n[n^2(Z_s + R_1) + R_2]\} \tag{60}$$

The equivalent noise input voltage referred to the source is obtained by factoring the turns ratio n from the braces in Eq. (60) and retaining all terms except the V_s term. The expression obtained can be converted into a mean-square sum to obtain

$$\begin{aligned}\overline{v_{nis}^2} &= 4kT\left(R_S + R_1 + \frac{R_2}{n^2}\right)\Delta f + \frac{\overline{v_n^2}}{n^2} \\ &\quad + 2\,\mathrm{Re}\left[\gamma\left(Z_s^* + R_1 + \frac{R_2}{n^2}\right)\right]\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} \\ &\quad + n^2\overline{i_n^2}\left|Z_s + R_1 + \frac{R_2}{n^2}\right|^2\end{aligned} \tag{61}$$

where γ is the correlation coefficient between V_n and I_n.

Because the series resistance of a transformer winding is proportional to the number of turns in the winding, it follows that $R_2/R_1 \propto n$. This makes it difficult to determine the value of n which minimizes $\overline{v_{nis}^2}$. In the case that $|Z_s| \gg R_1 + R_2/n^2$, the expression for $\overline{v_{nis}^2}$ is given approximately by

$$\overline{v_{nis}^2} \approx 4kTR_S\,\Delta f + \frac{\overline{v_n^2}}{n^2} + 2(\mathrm{Re}\,\gamma Z_s^*)\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + n^2\overline{i_n^2}|Z_s|^2 \tag{62}$$

This is minimized when n^2 is given by

$$n^2 = \frac{1}{|Z_s|}\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \tag{63}$$

In this case, the magnitude of the effective source impedance seen by the amplifier is $n^2\,|Z_s| = \sqrt{\overline{v_n^2}}/\sqrt{\overline{i_n^2}}$. The transformer also minimizes the noise factor, but it is not equal to the optimum noise factor F_o unless γ and Z_s are real.

If the source resistance is small, the transformer winding resistance can be a significant contributor to the thermal noise at the amplifier input. For this reason, a transformer can result in a decreased SNR compared to the case without the transformer.

JUNCTION DIODE NOISE MODEL

The current in a *pn* junction diode consists of two components—the forward diffusion current I_F and the reverse saturation current I_S. The total current is given by $I = I_F - I_S$. The forward diffusion current is a function of the diode voltage V and is given by $I_F = I_S \exp(V/\eta V_T)$, where η is the emission coefficient and V_T is the thermal voltage. (For discrete silicon diodes $\eta \approx 2$, whereas for integrated circuit diodes $\eta \approx 1$.) Both I_F and I_S generate uncorrelated shot noise. The total mean squared noise is given by

$$\overline{i_n^2} = 2q(I_F + I_S)\,\Delta f = 2q(I + 2I_S)\,\Delta f \approx 2qI\,\Delta f \tag{64}$$

where the approximation holds for a foward biased diode for which $I \gg I_S$. Figure 7(a) shows the diode noise model. In Fig.

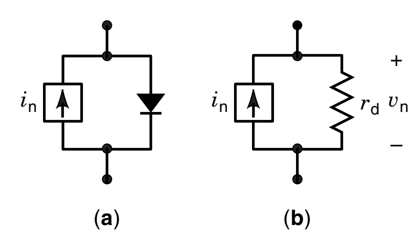

Figure 7. (a) The noise model of a diode. (b) Small-signal model with the diode replaced with its small-signal resistance.

7(b), the diode is replaced by its small-signal resistance $r_d = \eta V_T/(I + I_S) \approx \eta V_T/I$.

At low frequencies, the diode exhibits flicker noise. When this is included, the total mean squared noise current is given by

$$\overline{i_n^2} = 2qI\,\Delta f + \frac{K_f I\,\Delta f}{f} \tag{65}$$

where it is assumed that $I \gg I_S$. A plot of $\sqrt{\overline{i_n^2}}$ versus f for a constant Δf exhibits a slope of -10 dB/decade for very low frequencies and a slope of zero for higher frequencies. The two terms in Eq. (65) are equal at the frequency where the noise current is up 3 dB from its high-frequency limit. This frequency is called the flicker noise corner frequency.

Diodes are often used as noise sources in circuits. Specially processed zener diodes are fabricated as solid-state noise diodes. The noise mechanism in these is called avalanche noise, and it is associated with the diode reverse breakdown current. For a given breakdown current, avalanche noise is much greater than the shot noise in the same current.

NOISE IN BIPOLAR JUNCTION TRANSISTORS

BJT Noise Model

The principal noise sources in a BJT are thermal noise in the base spreading resistance, shot noise and flicker noise in the base bias current, and shot noise in the collector bias current. Figure 8(a) shows the small-signal T model of the BJT with the collector node grounded and all noise sources shown. The short-circuit collector output current is labeled $i_{c(sc)}$. The circuit contains two signal sources: one connects to the base (v_1 and R_1) and the other to the emitter (v_2 and R_2). With $v_2 = 0$, the circuit models a common–emitter (CE) amplifier. With $v_1 = 0$, it models a common–base (CB) amplifier.

In the figure, r_x is the base spreading resistance, α is the emitter-to-collector current gain, r_e is the intrinsic emitter resistance, and r_o is the collector-to-emitter resistance. The latter two are given by

$$r_e = \frac{V_T}{I_E} \tag{66}$$

$$r_o = \frac{V_{CB} + V_A}{I_C} \tag{67}$$

where $V_T = kT/q$ is the thermal voltage, I_E is the emitter bias current V_{CB} is the collector-to-base bias voltage, and V_A is the Early voltage. The collector, emitter, and base bias currents are related by

$$I_C = \alpha I_E = \beta I_B \tag{68}$$

where

$$\alpha = \frac{\beta}{1+\beta} \tag{69}$$

The noise sources v_{t1}, v_{tx}, and v_{t2}, respectively, model the thermal noise in R_1, r_x, and R_2. The shot noise and flicker noise, respectively, in I_B are modeled by i_{shb} and by i_{fb}. The shot noise

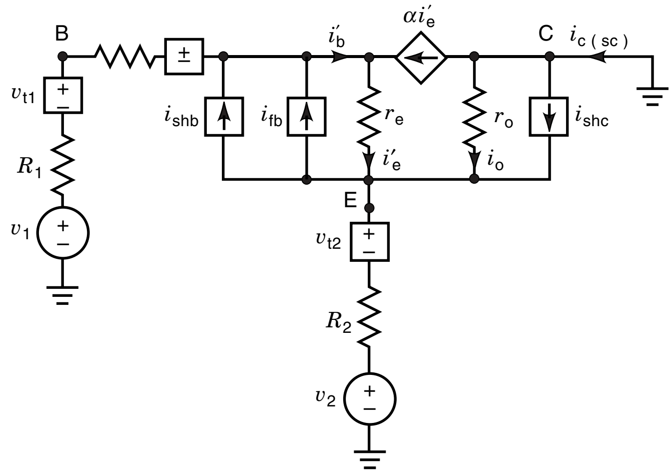

Figure 8. Small-signal T-model of the BJT with all noise sources.

in I_C is modeled by i_{shc}. In the band Δf, these have the mean squared values

$$\overline{v_{t1}^2} = 4kTR_1\Delta f, \qquad \overline{v_{tx}^2} = 4kTr_x\Delta f, \qquad \overline{v_{t2}^2} = 4kTR_2\Delta f \tag{70}$$

$$\overline{i_{shb}^2} = 2qI_B\Delta f, \qquad \overline{i_{fb}^2} = \frac{K_f I_B \Delta f}{f}, \qquad \overline{i_{shc}^2} = 2qI_C\Delta f \tag{71}$$

Equivalent Noise Input Voltage

Looking to the left in Fig. 8 into the branch where the current i'_b is labeled, the Thevenin equivalent circuit consists of the voltage $v_1 + v_{nb}$ in series with the resistance $R_1 + r_x$, where v_{nb} is given by

$$v_{nb} = v_{t1} + v_{tx} + (i_{shb} + i_{fb})(R_1 + r_x) \tag{72}$$

Looking down into the branch where the current i'_e is labeled, the Thevenin equivalent circuit consists of the voltage $v_2 + v_{ne}$ in series with the resistor R_2, where u_{ne} is given by

$$v_{ne} = v_{t2} + (i_0 + i_{shc} - i_{shb} - i_{fb})R_2 \tag{73}$$

The current i'_e can be solved for from the loop equation

$$(v_1 + v_{nb}) - (v_2 + v_{ne}) = \frac{i'_e}{1+\beta}(R_1 + r_x) + i'_e(r_e + R_2) \tag{74}$$

to obtain

$$i'_e = \frac{(v_1 + v_{nb}) - (v_2 + v_{ne})}{(R_1 + r_x)/(1+\beta) + r_e + R_2} \tag{75}$$

It follows that the short-circuit collector output current is given by

$$\begin{aligned} i_{c(sc)} &= i_0 + i_{shc} + \alpha i'_e \\ &= i_0 + i_{shc} + G_m(v_1 + v_{nb} - v_2 - v_{ne}) \end{aligned} \tag{76}$$

where G_m is the effective transconductance given by

$$G_m = \frac{\alpha}{r_{ie} + R_2} \tag{77}$$

and r_{ie} is the resistance given by

$$r_{\mathrm{ie}} = \frac{R_1 + r_{\mathrm{x}}}{1+\beta} + r_{\mathrm{e}} \tag{78}$$

With these definitions, Eq. (76) can be written

$$i_{\mathrm{c(sc)}} = G_{\mathrm{m}}\left[v_1 - v_2 + \left(v_{\mathrm{nb}} - v_{\mathrm{ne}} + \frac{i_0 + i_{\mathrm{shc}}}{G_{\mathrm{m}}}\right)\right] \tag{79}$$

The collector output resistance is given by

$$r_{\mathrm{ic}} = \frac{r_0 + r_{\mathrm{ie}} \parallel R_2}{1 - G_{\mathrm{m}}R_2} \tag{80}$$

The terms in the parentheses in Eq. (79) represent the equivalent noise input voltage v_{ni}. It will be assumed that the collector-to-emitter resistance r_o is large enough so that the current i_o can be neglected. It follows that v_{ni} is given by

$$\begin{aligned} v_{\mathrm{ni}} &= v_{\mathrm{t1}} + v_{\mathrm{tx}} - v_{\mathrm{t2}} + (i_{\mathrm{shb}} + i_{\mathrm{fb}})(R_1 + r_{\mathrm{x}} + R_2) \\ &\quad + i_{\mathrm{shc}}\left(\frac{R_1 + r_{\mathrm{x}} + R_2}{\beta} + \frac{V_{\mathrm{T}}}{I_{\mathrm{C}}}\right) \end{aligned} \tag{81}$$

This has the mean squared value

$$\begin{aligned} \overline{v_{\mathrm{ni}}^2} &= 4kT(R_1 + r_{\mathrm{x}} + R_2)\,\Delta f \\ &\quad + \left(2qI_{\mathrm{B}}\,\Delta f + \frac{K_{\mathrm{f}}I_{\mathrm{B}}\,\Delta f}{f}\right)(R_1 + r_{\mathrm{x}} + R_2)^2 \\ &\quad + 2qI_{\mathrm{C}}\,\Delta f\left(\frac{R_1 + r_{\mathrm{x}} + R_2}{\beta} + \frac{V_{\mathrm{T}}}{I_{\mathrm{C}}}\right)^2 \end{aligned} \tag{82}$$

This expression gives the mean squared equivalent noise input voltage for both the CE and the CB amplifier. The SNR for either amplifier is given by SNR $= \overline{v_i^2}/\overline{v_{ni}^2}$, where $\overline{v_i^2}$ is the mean squared value of v_1 for the CE amplifier and the mean squared value of v_2 for the CB amplifier.

Optimum Bias Current

Except at low frequencies, the flicker noise term in Eq. (82) can be neglected. When this is done, $\overline{v_{ni}^2}$ can be written

$$\begin{aligned} \overline{v_{\mathrm{ni}}^2} &= 4kT(R_1 + r_{\mathrm{x}} + R_2)\,\Delta f \\ &\quad + 2q\frac{I_{\mathrm{C}}}{\beta}\,\Delta f\,(R_1 + r_{\mathrm{x}} + R_2)^2 \\ &\quad + 2qI_{\mathrm{C}}\,\Delta f\left(\frac{R_1 + r_{\mathrm{x}} + R_2}{\beta} + \frac{V_{\mathrm{T}}}{I_{\mathrm{C}}}\right)^2 \end{aligned} \tag{83}$$

It can be seen that $\overline{v_{ni}^2} \to \infty$ if $I_C \to 0$ or if $I_C \to \infty$. It follows that there is a value of I_C that minimizes $\overline{v_{ni}^2}$. This current is called the optimum collector bias current, and it is denoted by $I_{C(opt)}$. It is obtained by setting $d\overline{v_{ni}^2}/dI_C = 0$ and solving for I_C. It is given by

$$I_{\mathrm{C(opt)}} = \frac{V_{\mathrm{T}}}{R_1 + r_{\mathrm{x}} + R_2} \times \frac{\beta}{\sqrt{1+\beta}} \tag{84}$$

When the BJT is biased at $I_{C(opt)}$, let the equivalent noise input voltage be denoted by $\overline{v_{ni(min)}^2}$. Its given by

$$\overline{v_{\mathrm{ni(min)}}^2} = 4kT(R_1 + r_x + R_2)\,\Delta f \times \frac{\sqrt{1+\beta}}{\sqrt{1+\beta} - 1} \tag{85}$$

For minimum noise, this equation shows that the series resistance in the external base and emitter circuits should be minimized and that the BJT should have a small r_x and a high β.

Although $\overline{v_{ni(min)}^2}$ decreases as β increases, the sensitivity is not great for the range of β for most BJTs. For example, as β increases from 100 to 1000, $\overline{v_{ni(min)}^2}$ decreases by 0.32 dB. Superbeta transistors have a β in the range $1000 \le \beta \le 10{,}000$. As β increases from 1000 to 10,000, $\overline{v_{ni(min)}^2}$ decreases by only 0.096 dB. It can be concluded that only a slight improvement in noise performance can be expected by using higher-β BJTs when the device is biased at $I_{C(opt)}$.

If $I_C \neq I_{C(opt)}$, then $\overline{v_{ni}^2}$ can be written

$$\overline{v_{\mathrm{ni}}^2} = \overline{v_{\mathrm{ni(min)}}^2}\left[1 + \frac{0.5(I_{\mathrm{C}}/I_{\mathrm{C(opt)}} + I_{\mathrm{C(opt)}}/I_{\mathrm{C}}) - 1}{1 + \sqrt{1+\beta}}\right] \tag{86}$$

This equation shows that a plot of $\overline{v_{ni}^2}$ versus $\log(I_C/I_{C(opt)})$ would exhibit even symmetry about the value $I_C/I_{C(opt)} = 1$. This means, for example, that $\overline{v_{ni}^2}$ is the same for $I_C = I_{C(opt)}/2$ as for $I_C = 2I_{C(opt)}$. In addition, the sensitivity of $\overline{v_{ni}^2}$ to changes in I_C decreases as β increases. For example, at $I_C = I_{C(opt)}/2$ and $I_C = 2I_{C(opt)}$, $\overline{v_{ni}^2}$ is greater than $\overline{v_{ni(min)}^2}$ by 0.097 dB for $\beta = 100$, by 0.033 dB for $\beta = 1000$, and by 0.010 dB for $\beta = 10{,}000$.

Comparison of CE and CB Stages

The above analysis shows that the noise performance of the CE amplifier is the same as that of the CB amplifier. This assumes that the noise generated by the stages driven from the collector can be neglected. Let the second stage be modeled by a v_n–i_n amplifier noise model having the noise sources v_{n2} and i_{n2} and the correlation coefficient ρ_2. Let $\overline{v_{ni}^2}'$ be the new noise equivalent input voltage. Following Eq. (56), this is given by

$$\overline{v_{\mathrm{ni}}^2}' = \overline{v_{\mathrm{ni}}^2} + \frac{\overline{v_{\mathrm{n2}}^2}}{G_{\mathrm{m}}^2 r_{\mathrm{ic}}^2} + 2\rho_2\frac{\sqrt{\overline{v_{\mathrm{n2}}^2}}\sqrt{\overline{i_{\mathrm{n2}}^2}}}{G_{\mathrm{m}}^2 r_{\mathrm{ic}}} + \frac{\overline{i_{\mathrm{n2}}^2}}{G_{\mathrm{m}}^2} \tag{87}$$

where r_{ic} is the collector output resistance of the first stage. It follows from this equation that the noise contributed by the second stage is the lowest for the first-stage configuration that exhibits the largest G_m.

For a CE first stage, let $R_1 = R_S$ and $R_2 = 0$, where R_S is the source resistance. For a CB first stage, let $R_1 = 0$ and $R_2 = R_S$. The ratio of the G_m's for these two configurations is

$$\frac{G_{\mathrm{m(CE)}}}{G_{\mathrm{m(CB)}}} = \frac{r_{\mathrm{x}}/(1+\beta) + r_{\mathrm{e}} + R_{\mathrm{S}}}{(R_{\mathrm{S}} + r_{\mathrm{x}})/(1+\beta) + r_{\mathrm{e}}} \tag{88}$$

$R_S = 0$, the ratio is unity. In this case, the noise performance of the two amplifiers is the same. For R_S large, the ratio approaches $1 + \beta$, so that the effect of the second-stage noise on the CB amplifier is greater than for the CE amplifier. There-

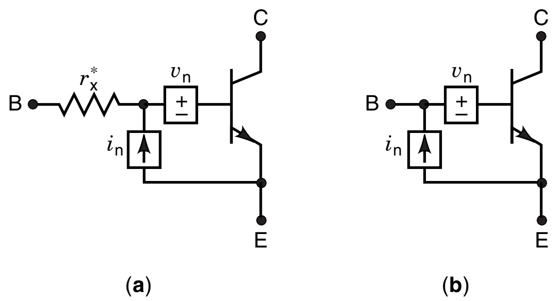

Figure 9. The v_n–i_n noise models of the BJT. (a) First model. (b) Second model.

fore, the CE amplifier is the preferred topology for low-noise applications when the source resistance is not small.

Two BJT v_n–i_n Noise Models

First Model. There are two formulations for $\overline{v_n^2}$ and $\overline{i_n^2}$ for the BJT, which differ by the placement of r_x in the model. For the first, Eq. (81) can be written

$$v_{ni} = v_{t1} - v_{t2} + v_n + i_n(R_1 + r_x + R_2) \tag{89}$$

where v_n and i_n are given by

$$v_n = v_{tx} + i_{shc}\frac{V_T}{I_C} \tag{90}$$

$$i_n = i_{shb} + i_{fb} + \frac{i_{shc}}{\beta} \tag{91}$$

These expressions can be converted into mean squared sums to obtain

$$\overline{v_n^2} = 4kTr_x\Delta f + 2kT\frac{V_T}{I_C}\Delta f \tag{92}$$

$$\overline{i_n^2} = 2qI_B\Delta f + \frac{K_f I_B}{f}\Delta f + \frac{2qI_C}{\beta^2}\Delta f \tag{93}$$

Because i_{shc} appears in the expressions for both v_n and i_n, the correlation coefficient is not zero. It is given by

$$\rho = \frac{2kT\Delta f}{\beta\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}} \tag{94}$$

The first form of the v_n–i_n BJT noise model is shown in Fig. 9(a). The asterisk indicates that the base spreading resistance r_x^* is considered to be a noiseless resistor. Its noise is included in the expression for $\overline{v_n^2}$.

Second Model. For the second formulation, Eq. (81) can be written

$$v_{ni} = v_{t1} - v_{t2} + v_n + i_n(R_1 + R_2) \tag{95}$$

where v_n and i_n are given by

$$v_n = v_{tx} + \left(i_{shb} + i_{fb} + \frac{i_{shc}}{\beta}\right)r_x + i_{shc}\frac{V_T}{I_C} \tag{96}$$

$$i_n = i_{shb} + i_{fb} + \frac{i_{shc}}{\beta} \tag{97}$$

These expressions can be converted into mean squared sums to obtain

$$\begin{aligned}\overline{v_n^2} &= 4kTr_x\Delta f + \left(2qI_B\Delta f + \frac{K_f I_B\Delta f}{f}\right)r_x^2 \\ &\quad + 2qI_C\Delta f\left(\frac{r_x}{\beta} + \frac{V_T}{I_C}\right)^2\end{aligned} \tag{98}$$

$$\overline{i_n^2} = 2qI_B\Delta f + \frac{K_f I_B}{f}\Delta f + \frac{2qI_C}{\beta^2}\Delta f \tag{99}$$

In this case, i_{shb}, i_{fb}, and i_{shc} appear in the expressions for both v_n and i_n. The correlation coefficient is given by

$$\begin{aligned}\rho = \frac{1}{\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}}\Bigg[&\left(2qI_B\Delta f + \frac{K_f I_B\Delta f}{f}\right)r_x \\ &+ 2q\frac{I_C}{\beta}\Delta f\left(\frac{r_x}{\beta} + \frac{V_T}{I_C}\right)\Bigg]\end{aligned} \tag{100}$$

The second form of the v_n–i_n BJT noise model is shown in Fig. 9(b). The first form has the simpler equations.

Flicker Noise Corner Frequency. The expression for $\overline{i_n^2}$ in each form of the BJT v_n–i_n noise model is the same. The equations predict that a plot of $\sqrt{\overline{i_n^2}}$ versus frequency would exhibit a slope of -10 dB/decade at low frequencies and a slope of zero at higher frequencies. The flicker noise corner frequency is the frequency at which $\sqrt{\overline{i_n^2}}$ is up 3 dB from its higher-frequency value. This is the frequency for which the middle term in Eqs. (93) and (99) is equal to the sum of the first and last terms.

Common–Collector Stage. Figure 10 shows the circuit diagram of a common–collector (CC) amplifer with its output connected to the input of a second stage that is modeled with the v_n–v_n amplifer noise model. For simplicity, the bias

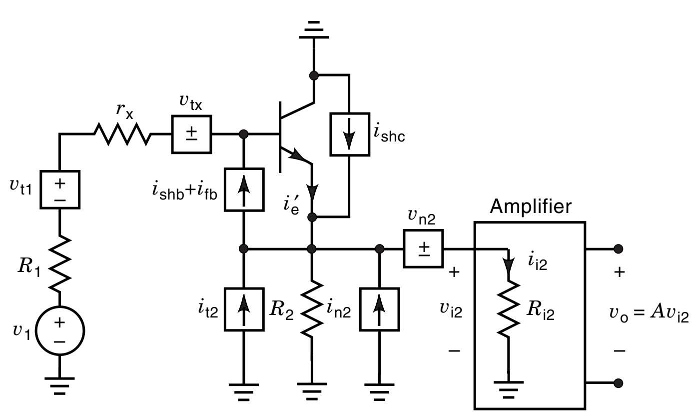

Figure 10. Circuit for calculating the equivalent noise input voltage of an amplifier preceded by a BJT common–collector stage.

sources are not shown. The resistor r_x and all BJT noise sources are shown external to the BJT. The source i_{t2} models the thermal noise current in R_2. The voltage across R_{i2} is proportional to the short-circuit current through R_{i2}, that is, the current i_{i2} evaluated with $R_{i2} = 0$. It is given by

$$\begin{aligned} i_{i2(sc)} &= i'_e - (i_{shb} + i_{fb}) + i_{shc} + i_{t2} + \frac{v_{n2}}{R_2} + i_{n2} \\ &= \frac{G_m}{\alpha}[v_1 + v_{t1} + v_{tx} + (i_{shb} + i_{fb})(R_1 + r_x) + v_{n2}] \\ &\quad - (i_{shb} + i_{fb}) + i_{shc} + i_{t2} + \frac{v_{n2}}{R_2} + i_{n2} \end{aligned} \tag{101}$$

where G_m is given by Eq. (77) with $R_2 = 0$. It follows that the equivalent noise input voltage is given by

$$\begin{aligned} v_{ni} &= v_{t1} + v_{tx} + v_{n2}\left(1 + \frac{\alpha}{G_m R_2}\right) \\ &\quad + (i_{shb} + i_{fb})\left(R_1 + r_x - \frac{\alpha}{G_m}\right) + \frac{\alpha}{G_m}(i_{shc} + i_{t2} + i_{n2}) \end{aligned} \tag{102}$$

This can be converted into a mean squared sum to obtain

$$\begin{aligned} \overline{v_{ni}^2} &= 4kT(R_1 + r_x)\Delta f + \overline{v_{n2}^2}\left(1 + \frac{R_1 + r_x}{R_2}\right)^2 \\ &\quad + 2\rho_2\sqrt{\overline{v_{n2}^2}}\sqrt{\overline{i_{n2}^2}}\left(\frac{R_1 + r_x}{1+\beta} + r_e\right)\left(1 + \frac{R_1 + r_x}{R_2}\right) \\ &\quad + \left(2qI_B\Delta f + \frac{K_f I_B \Delta f}{f}\right)[\alpha(R_1 + r_x) - r_e]^2 \\ &\quad + \left(2qI_C\Delta f + \frac{4kT\Delta f}{R_2} + \overline{i_{n2}^2}\right)\left(\frac{R_1 + r_x}{1+\beta} + r_e\right)^2 \end{aligned} \tag{103}$$

where ρ_2 is the correlation coefficient between v_{n2} and i_{n2}.

It can be seen from Eq. (103) that the v_{n2} noise is increased by the CC stage. The i_{n2} noise is decreased if $R_1 > r_x/\beta + r_e/\alpha$. The noise voltage generated by the base shot and flicker noise currents can be canceled if $r_e = \alpha(R_1 + r_x)$. For $R_2 \gg 2V_T/I_C$, the thermal noise generated by R_2 can be neglected compared to the shot noise in I_C.

BJT Differential Amplifier

Figure 11 shows the circuit diagram of a BJT differential amplifier. For simplicity, the bias sources are not shown. It is assumed that the BJTs are matched and biased at equal currents. The emitter resistors labeled R_2 are included for completeness. For lowest noise, these should be omitted. The source i_{nt} models the noise current generated by the tail current source, and the resistor r_t models its output resistance.

For minimum noise output from the differential amplifier, the output signal must be proportional to the differential short-circuit output current $i_{od(sc)} = i_{c1(sc)} - i_{c2(sc)}$. The subtraction cancels the common-mode output noise generated by the tail current i_{nt}. Although a current-mirror active load can be used to realize the subtraction, the lowest-noise performance is obtained with a resistive load. With a resistive load on each collector, a second differential amplifier is required to subtract the output signals. The analysis presented here assumes that the circuit output is taken differentially. In addition, it is assumed that r_t is large enough so that it can be approximated by an open circuit. This is equivalent to the assumption of a high common-mode rejection ratio.

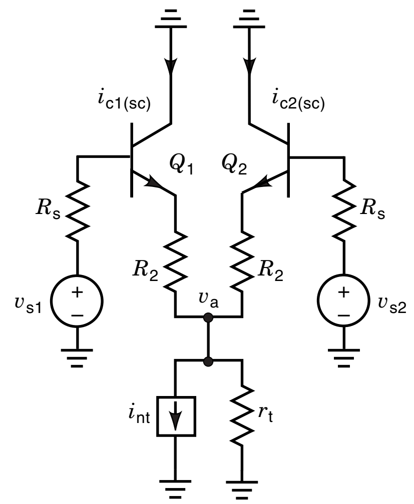

Figure 11. Circuit for calculating the equivalent noise input voltage of a BJT differential amplifier.

The simplest method to calculate the equivalent noise input voltage of the differential amplifier is to exploit symmetry by resolving all sources into their differential and common-mode components. The common-mode components are all canceled when the output is taken differentially. Therefore, only the differential components are required. When the sources are replaced by their differential components, the node labeled v_a in Fig. 11 can be grounded. This decouples the differential amplifier into two CE stages.

Consider the effect of the base shot noise currents. For Q_1, i_{shb1} is replaced with $i'_{shb1} = (i_{shb1} - i_{shb2})/2$. For Q_2, i_{shb2} is replaced by $i'_{shb2} = (i_{shb2} - i_{shb1})/2$. The differential short-circuit collector output current $i_{od(sc)} = i_{c1(sc)} - i_{c2(sc)}$ is proportional to $i'_{shb1} - i'_{shb2} = i_{shb1} - i_{shb2}$. If i_{shb1} and i_{shb2} are not correlated, it follows that $\overline{i_{od(sc)}^2}$ contains a term that is proportional to $\overline{i_{shb1}^2} + \overline{i_{shb2}^2}$. Because $\overline{i_{shb1}^2} = \overline{i_{shb2}^2}$, the base current shot noise is increased by 3 dB over that in a CE amplifier. Likewise, the thermal noise of the base spreading resistance, the thermal noise of R_1 and R_2, the base current flicker noise, and the collector current shot noise are increased by 3 dB over those of a CE amplifier. The mean squared noise input voltage of the differential amplifier is given by $2\overline{v_{ni}^2}$, where $\overline{v_{ni}^2}$ is given by Eq. (82). Above the flicker noise frequency band, the noise is minimized when each BJT is biased at a collector current given by Eq. (84).

BJT at High Frequencies

Figure 12 shows the high-frequency T model of the BJT with the emitter grounded and the base driven by a voltage source having the output impedance $Z_s = R_S + jX_S$. The base-to-emitter capacitance and the collector-to-base capacitance are given by

$$c_\pi = \frac{\tau_F I_C}{V_T} \tag{104}$$

$$c_\mu = \frac{c_{jc0}}{(1 + V_{CB}/\Phi_C)^{m_c}} \tag{105}$$

where τ_F is the forward transit time of the base-to-emitter junction, c_{jco} is the zero-bias junction capacitance of the base-

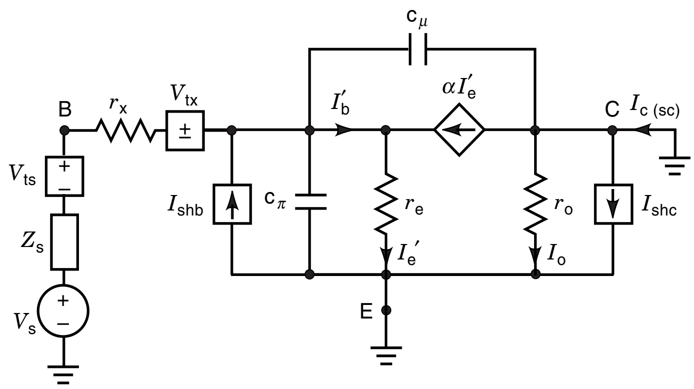

Figure 12. Small-signal T-model of the common–emitter amplifier used to calculate the equivalent noise input voltage at high frequencies.

to-collector junction, Φ_C is the built-in potential, and m_c is the junction exponential factor. All noise sources are shown in the circuit except the base flicker noise current, which can be neglected at high frequencies.

If the current I_0 through r_0 is neglected, it is straightforward to show that the short-circuit collector output current is given by

$$I_{c(sc)} = G_m \left[V_s + V_{ts} + V_{tx} + I_{shb}(Z_s + r_x) + \frac{I_{shc}}{G_m} \right] \tag{106}$$

where G_m is given by

$$G_m = \frac{\alpha + j\omega c_\mu r_e}{\left(\dfrac{1}{1+\beta} + j\omega r_e c_T \right)(Z_s + r_x) + r_e} \tag{107}$$

and $c_T = c_\pi + c_\mu$. The equivalent noise input voltage is given by the sum of the terms in the brackets in Eq. (106) with the V_s term omitted. It has a mean squared value given by

$$\begin{aligned} \overline{v_{ni}^2} &= 4kT(R_S + r_x)\Delta f + 2qI_B \Delta f[(R_S + r_x)^2 + X_S^2] \\ &\quad + \frac{2qI_C\Delta f}{\alpha^2 + \omega^2 c_\mu^2 r_e^2} \left[\left(\frac{R_S + r_x}{1+\beta} + r_e(1 - \omega X_S c_T) \right)^2 \right. \\ &\quad \left. + \left(\frac{X_S}{1+\beta} + \omega r_e c_T (R_S + r_x) \right)^2 \right] \end{aligned} \tag{108}$$

The noise factor is given by

$$F = \frac{\overline{v_{ni}^2}}{4kTR_S\Delta f} \tag{109}$$

If it is assumed that $c_\mu \ll c_\pi$ and that $\omega^2 c_\mu^2 r_e^2 \ll \alpha^2$, the values of X_S and I_C can be solved for to minimize $\overline{v_{ni}^2}$. These are obtained by setting $d\overline{v_{ni}^2}/dX_S = 0$ and $d\overline{v_{ni}^2}/dI_C = 0$ and solving the equations simultaneously. It is straightforward to show that X_S and I_C are given by

$$X_S = \omega\tau_F(R_S + r_x)\frac{\beta}{\sqrt{1+\beta}} \tag{110}$$

$$I_C = \frac{V_T}{(R_S + r_x)(1 + \alpha\beta\omega^2\tau_F^2)} \times \frac{\beta}{\sqrt{1+\beta}} \tag{111}$$

The corresponding expressions for $\overline{v_{ni(min)}^2}$ and F are

$$\overline{v_{ni(min)}^2} = 4kT(R_S + r_x)\Delta f \frac{\sqrt{1+\beta}}{\sqrt{1+\beta} - 1} \tag{112}$$

$$F = \left(1 + \frac{r_x}{R_S}\right)\frac{\sqrt{1+\beta}}{\sqrt{1+\beta} - 1} \tag{113}$$

NOISE IN FEEDBACK AMPLIFIERS

Series–Shunt Amplifier

Figure 13(a) shows the simplified diagram of a series–shunt feedback amplifier with a BJT input stage. The bias sources and networks are omitted for simplicity. If the loop gain is sufficiently high, the voltage gain is approximately the reciprocal of the feedback ratio and is given by $v_o/v_s \approx 1 + R_F/R_E$.

The circuit in Fig. 13(b) can be used to solve for the equivalent noise input voltage. The figure shows the BJT with its collector connected to signal ground and the circuit seen looking out of the emitter replaced by a Thevenin equivalent circuit with respect to v_o. The equivalent noise input voltage is modeled by the source v_{ni}. If flicker noise is neglected, the

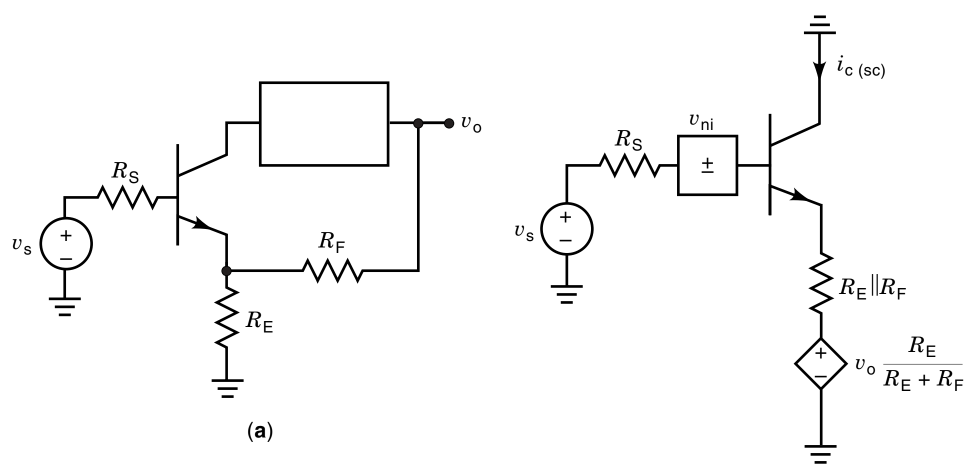

Figure 13. (a) Series–shunt feedback amplifier with a BJT input stage. (b) Equivalent circuit of the input stage used to calculate the equivalent noise input voltage.

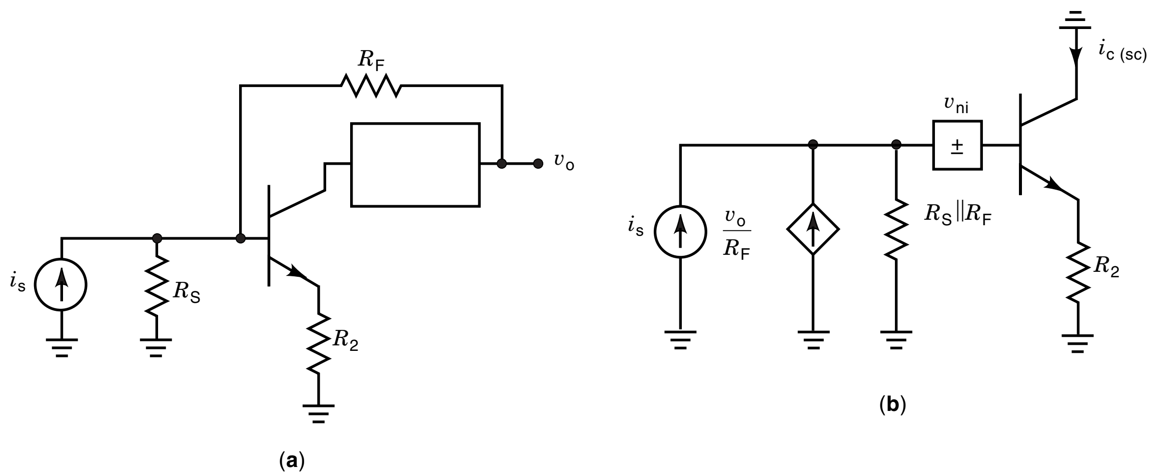

Figure 14. (a) Shunt–shunt feedback amplifier with a BJT input stage. (b) Equivalent circuit of the input stage used to calculate the equivalent noise input voltage.

value of $\overline{v_{ni}^2}$ is obtained from Eq. (83) with R_2 replaced with $R_E \parallel R_F$. It is given by

$$\begin{aligned}\overline{v_{ni}^2} &= 4kT(R_1 + r_x + R_E \| R_F)\,\Delta f \\ &\quad + 2q\frac{I_C}{\beta}\,\Delta f\,(R_1 + r_x + R_E \| R_F)^2 \\ &\quad + 2qI_C\,\Delta f \left(\frac{R_1 + r_x + R_E \| R_F}{\beta} + \frac{V_T}{I_C}\right)^2\end{aligned} \tag{114}$$

For minimum noise, $R_E \| R_F$ should be small compared to $R_1 + r_x$ and the BJT should be biased at $I_{C(opt)}$. The resistance $R_E \| R_F$ cannot be zero, because the amplifier gain is set by the ratio of R_F to R_E.

Shunt–Shunt Amplifier

Figure 14(a) shows the simplified diagram of a shunt–shunt feedback amplifier with a BJT input stage. The bias sources and networks are omitted for simplicity. The signal source is represented by the current source i_s in parallel with the resistor R_S. If the loop gain is sufficiently high, the transresistance gain is given by $v_o/i_s \approx -R_F$.

The circuit in Fig. 14(b) can be used to evaluate the input stage noise. The figure shows the BJT with its collector connected to signal around and the circuit seen looking out of the base replaced by a Norton equivalent circuit with respect to i_s and v_o. The equivalent noise input voltage is modeled by the source v_{ni}. The short-circuit collector current $i_{c(sc)}$. is given by

$$i_{c(sc)} = G_m\left(i_s(R_s \| R_F) + v_0\frac{R_S \| R_F}{R_F} + v_{ni}\right) \tag{115}$$

where G_m is given by Eq. (77) with R_1 replaced with $R_S \| R_F$.

Because the signal source is a current as opposed to a voltage, the noise-equivalent input current i_{ni} in parallel with i_s must be calculated. This is obtained by factoring the coefficient of i_s from Eq. (115) and retaining only the term involving v_{ni}. It follows that i_{ni} is given by $i_{ni} = v_{ni}/(R_S \| R_F)$. When flicker noise is neglected, $\overline{v_{ni}^2}$ is given by Eq. (83) with R_1 replaced with $R_S \| R_F$). It follows that $\overline{i_{ni}^2}$ is given by

$$\begin{aligned}\overline{i_{ni}^2} &= \frac{4kT\,\Delta f}{R_S \| R_F}\left(1 + \frac{r_x + R_2}{R_S \| R_F}\right) \\ &\quad + 2qI_B\,\Delta f\left(1 + \frac{r_x + R_2}{R_S \| R_F}\right)^2 \\ &\quad + 2qI_C\,\Delta f\left[\frac{1}{\beta} + \frac{1}{R_S \| R_F}\left(\frac{r_x + R_2}{\beta} + \frac{V_T}{I_C}\right)\right]^2\end{aligned} \tag{116}$$

The noise is minimized by making R_2 small and by making R_F large compared to R_S. In addition, the BJT should be biased at $I_{C(opt)}$.

NOISE IN FIELD EFFECT TRANSISTORS

General Noise Model

The principal noise sources in the FET are thermal noise and flicker noise generated in the channel. If the gate bias current in the junction FET (JFET) is not negligible, the shot noise generated by it must also be included. Flicker noise in a MOSFET is usually larger than in a JFET because the MOSFET is a surface device in which the fluctuating occupancy of traps in the oxide modulate the conducting surface channel all along the channel. The relations between the flicker noise in a MOSFET and its geometry and bias conditions depend on the fabrication process. In most cases, the flicker noise, when referred to the input, is independent of the bias voltage and current and is inversely proportional to the product of the active gate area and the gate oxide capacitance per unit area. Because the JFET has less flicker noise, it is usually preferred over the MOSFET in low-noise applications at low frequencies.

Figure 15 shows the MOSFET small-signal T model with the drain node grounded and all noise sources shown. The bulk lead is shown connected to signal ground. A simple modification to the noise equations for this connection can be made to obtain the equations for the case where the bulk is connected to the source. The equations so obtained also apply to

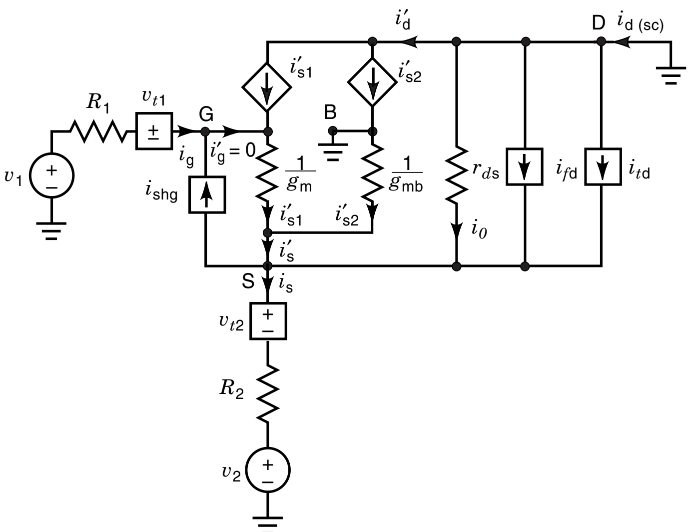

Figure 15. Small-signal T-model of the FET with all noise sources.

the JFET. The small-signal transconductances and drain-to-source resistance are given by

$$g_m = 2\sqrt{K(1+\lambda V_{DS})I_D} \approx 2\sqrt{KI_D} \tag{117}$$

$$g_{mb} = \chi g_m \tag{118}$$

$$r_{ds} = \frac{V_{DS} + 1/\lambda}{I_D} \tag{119}$$

where K is the transconductance parameter, λ is the channel length modulation factor, V_{DS} is the drain-to-source bias voltage, and I_D is the drain bias current. For the JFET, the transconductance parameter is given by $K = I_{DSS}/V_P^2$, where I_{DSS} is the drain-to-source saturation current and V_P is the pinchoff voltage. For the MOSFET, K is given by $K = \mu_0 C_{ox} W/2L$, where μ_0 is the average carrier mobility in the channel, C_{ox} is the gate oxide capacitance per unit area, W is the effective channel width, and L is the effective channel length.

The parameter χ is referred to here as the transconductance ratio. It is the ratio of the bulk transconductance g_{mb} to the transconductance g_m and is given by

$$\chi = \frac{\gamma}{2\sqrt{\Phi + V_{SB}}} \tag{120}$$

where γ is the body threshold parameter, Φ is the surface potential, and V_{SB} is the source-to-body bias voltage. Any equation derived from the circuit of Fig. 15 can be converted into a corresponding equation for the case where the body is connected to the source simply by setting $\chi = 0$ in the equation. The T model for the JFET is the same as the T model for the MOSFET with $\chi = 0$.

In Fig. 15, the short-circuit drain output current is labeled $i_{d(sc)}$. There are two signal sources in the circuit: one connects to the gate (v_1 and R_1) and the other to the source (v_2 and R_2). With $v_2 = 0$, the circuit models a common–source (CS) amplifier. With $v_1 = 0$, it models a common–gate (CG) amplifier. The sources v_{t1} and v_{t2}, respectively, represent the thermal noise generated by R_1 and R_2. The sources i_{td} and i_{shg}, respectively, represent the thermal noise generated in the channel and the shot noise generated in the gate bias current. The latter is zero for the MOSFET. The mean squared values of these sources are

$$\overline{v_{t1}^2} = 4kTR_1\Delta f, \qquad \overline{v_{t2}^2} = 4kTR_2\Delta f \tag{121}$$

$$\overline{i_{td}^2} = 4kT\frac{g_m}{1.5}\Delta f, \qquad \overline{i_{shg}^2} = 2qI_G\Delta f \tag{122}$$

The source i_{fd} represents flicker noise generated in the drain. Its mean squared values are

$$\overline{i_{fd}^2} = \frac{K_f I_D \Delta f}{f L^2 C_{ox}} \quad \text{for the MOSFET} \tag{123}$$

$$\overline{i_{fd}^2} = \frac{K_f I_D \Delta f}{f} \quad \text{for the JFET} \tag{124}$$

It follows from the equation for $\overline{i_{td}^2}$ that the mean squared thermal noise current generated in the channel is the same as the thermal noise current generated by a resistor of value $1.5/g_m$.

Looking to the left in Fig. 15 into the branch where the current i_g' is labeled, the Thevenin equivalent circuit consists of the voltage $v_1 + v_{ng}$ in series with the resistance R_1, where v_{ng} is given by

$$v_{ng} = v_{t1} + i_{shg}R_1 \tag{125}$$

Looking up into the branch where the current i_s' is labeled, the Thevenin equivalent circuit has the open-circuit voltage and output resistance

$$v_{is} = \frac{v_1 + v_{ng}}{1+\chi} \tag{126}$$

$$r_{is} = \frac{1}{(1+\chi)g_m} \tag{127}$$

Looking down into the branch where the current i_s' is labeled, the Thevenin equivalent circuit consists of the voltage $v_2 + v_{ns}$ in series with the resistor R_2, where v_{ns} is given by

$$v_{ns} = v_{t2} + (i_{td} + i_{fd} + i_0 - i_{shg})R_2 \tag{128}$$

It follows that

$$i_d' = i_s' = \frac{v_{is} - (v_2 + v_{ns})}{r_{is} + R_2} \tag{129}$$

The short-circuit drain current is given by

$$\begin{aligned} i_{d(sc)} &= i_{td} + i_{fd} + i_0 + i_d' \\ &= i_{td} + i_{fd} + i_0 + \frac{v_{is} - (v_2 + v_{ns})}{r_{is} + R_2} \\ &= i_{td} + i_{fd} + i_0 + G_m\left(\frac{v_1 + v_{ng}}{1+\chi} - v_2 - v_{ns}\right) \end{aligned} \tag{130}$$

where G_m is the effective transconductance given by

$$G_m = \frac{1}{r_{is} + R_2} \tag{131}$$

The drain output resistance is given by

$$r_{\mathrm{id}} = \frac{r_{\mathrm{ds}} + r_{\mathrm{is}} \| R_2}{1 - G_{\mathrm{m}} R_2} \tag{132}$$

Equivalent Noise Input Voltage in a Metal–Oxide–Semiconductor Field Effect Transistor

Unless $\chi = 0$, the equivalent noise input voltage for the MOSFET is not the same if it is reflected to the gate as it is if it is reflected to the source. If it is reflected to the gate, it is obtained from Eq. (130) by factoring out $G_{\mathrm{m}}/(1 + \chi)$, setting $i_{\mathrm{shg}} = 0$, and retaining all remaining terms except $v_1 - v_2$. It will be assumed that the drain-to-source resistance r_{ds} is large enough so that the current i_0 can be neglected. It follows that v_{ni} is given by

$$v_{\mathrm{ni}} = v_{\mathrm{t1}} - v_{\mathrm{t2}}(1+\chi) + \frac{i_{\mathrm{td}} + i_{\mathrm{fd}}}{g_{\mathrm{m}}} \tag{133}$$

This has the mean squared value

$$\overline{v_{\mathrm{ni}}^2} = 4kT[R_1 + R_2(1+\chi)^2]\,\Delta f + \frac{4kT}{3\sqrt{KI_{\mathrm{D}}}}\,\Delta f + \frac{K_{\mathrm{f}}}{4KfL^2C_{\mathrm{ox}}}\,\Delta f \tag{134}$$

where $\chi = 0$ for the case where the bulk is connected to the source.

For minimum noise in the MOSFET, it can be concluded from Eq. (134) that the series resistance in the external gate and source circuits should be minimized and the MOSFET should have a high transconductance parameter K and a low flicker noise coefficient K_{f}. The component of $\overline{v_{\mathrm{ni}}^2}$ due to the channel thermal noise is proportional to $1/\sqrt{I_{\mathrm{D}}}$. This decreases by 1.5 dB each time I_{D} is doubled.

v_{n}–i_{n} Noise Model for a Metal–Oxide–Semiconductor Field Effect Transistor

For the MOSFET v_{n}–i_{n} noise model, the mean squared values of v_{n} and i_{n} are given by

$$\overline{v_{\mathrm{n}}^2} = \frac{4kT}{3\sqrt{KI_{\mathrm{D}}}}\,\Delta f + \frac{K_f}{4KfL^2C_{\mathrm{ox}}}\,\Delta f \tag{135}$$

$$\overline{i_{\mathrm{n}}^2} = 0 \tag{136}$$

Figure 16(a) shows the MOSFET model.

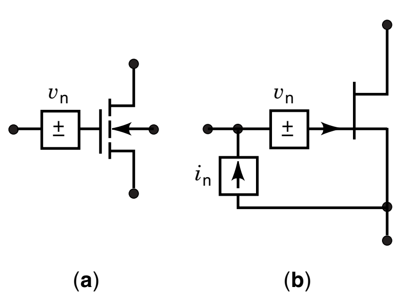

Figure 16. The v_n–i_n noise models of the FET: (a) MOSFET model. (b) JFET model.

Equivalent Noise Input Voltage in a Junction Field Effect Transistor

The equivalent noise input voltage for the JFET is obtained from Eq. (130), setting $\chi = 0$, factoring out G_{m}, and retaining all terms except $v_1 - v_2$. It will be assumed that the drain-to-source resistance r_{ds} is large enough so that the current i_0 can be neglected. It follows that $\overline{v_{\mathrm{ni}}^2}$ is given by

$$\overline{v_{\mathrm{ni}}^2} = 4kT(R_1 + R_2)\,\Delta f + 2qI_{\mathrm{G}}\,\Delta f + \frac{4kT}{3\sqrt{KI_{\mathrm{D}}}}\,\Delta f + \frac{K_{\mathrm{f}}}{4Kf}\,\Delta f \tag{137}$$

where I_{G} is the gate bias current. This current is commonly assumed to be zero when the gate-to-channel junction is reverse biased. For a high source impedance, the effect of the gate current on the noise might not be negligible. In particular, attention must be paid to the variation of the gate current with drain-to-gate voltage. In general, the gate current increases with drain-to-gate voltage. Some devices exhibit a threshold effect such that the gate current increases rapidly when the drain-to-gate voltage exceeds some value. The drain-to-gate voltage at which this occurs is called the I_{G} breakpoint. It is typically in the range of 8 V to 40 V. The JFET noise is minimized in the same way that the MOSFET noise is reduced.

v_{n}–i_{n} Noise Model for a Junction Field Effect Transistor

For the JFET v_{n}–i_{n} noise model, the mean squared values of v_{n} and i_{n} are given by

$$\overline{v_{\mathrm{n}}^2} = \frac{4kT}{3\sqrt{KI_{\mathrm{D}}}}\,\Delta f + \frac{K_{\mathrm{f}}}{4Kf}\,\Delta f \tag{138}$$

$$\overline{i_{\mathrm{n}}^2} = 2qI_{\mathrm{G}}\,\Delta f \tag{139}$$

It is common to assume that i_{shg} is independent of both i_{td} and i_{fd}. Thus the correlation coefficient between v_{n} and i_{n} is zero. Figure 16(b) shows the v_{n}–i_{n} JFET model.

Flicker Noise Corner Frequency

It can be seen from Eq. (135) for the MOSFET and Eq. (138) for the JFET that a plot of $\sqrt{\overline{v_{\mathrm{n}}^2}}$ versus frequency would exhibit a slope of -10 dB/decade at low frequencies and a slope of zero at higher frequencies. The flicker noise corner frequency f_{f} is defined as the frequency at which $\sqrt{\overline{v_{\mathrm{n}}^2}}$ is up to 3 dB from its higher-frequency value. For the MOSFET, this is the frequency for which the two terms in Eq. (135) are equal. For the JFET, it is the frequency for which the two terms in Eq. (138) are equal.

Field-Effect-Transistor Differential Amplifier

It has been shown in a preceding section that the BJT noise is 3 dB higher in the differential amplifier than in the CE amplifier. The same comparison holds between the FET differential amplifier and the CS amplifier. This assumes that the signal output from the differential amplifier is taken differentially. Otherwise, the common-mode noise generated by the tail current bias supply is not canceled.

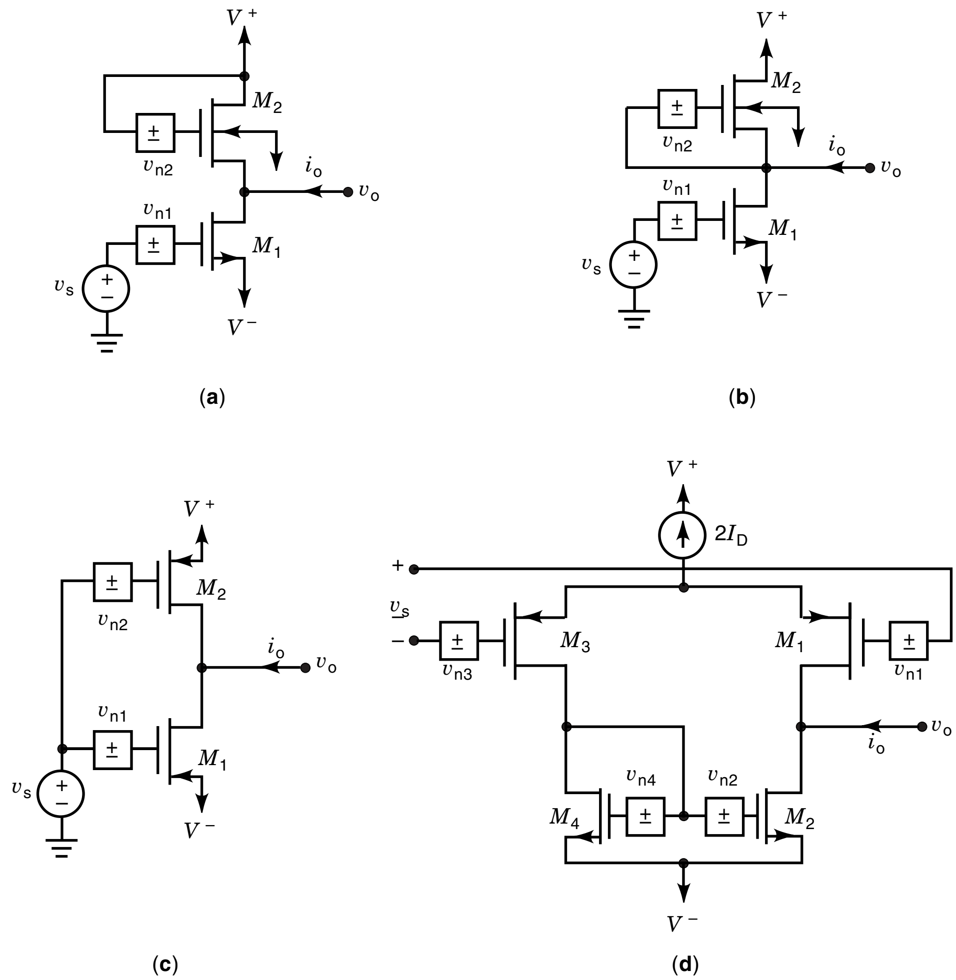

Figure 17. MOSFET circuits for example noise calculations.

Examples of Low-Frequency Noise in Metal–Oxide–Semiconductor Field Effect Transistors

The equivalent noise input voltage at low frequencies is determined in this subsection for four example MOSFET circuits. It is assumed that the frequency is low enough so that the dominant component of the noise is flicker noise. It is straightforward to modify the results for the higher-frequency case where the dominant component of the noise is thermal noise or for the more general case where both thermal noise and flicker noise are included. The circuits are shown in Fig. 17. The analysis assumes that each transistor is operated in the saturation region and that the noise sources are uncorrelated. Because the MOSFET exhibits no current noise, the output resistance of the signal source is omitted with no loss in generality.

Common–Source Amplifier with Enhancement-Mode Load. Figure 17(a) shows a single-channel NMOS enhancement-mode common–source (CS) amplifier with an active NMOS enhancement-mode load. It is assumed that the two MOSFETs have matched model parameters and are biased at the same current. With $v_0 = 0$, the short-circuit output current can be written

$$i_{o(sc)} = g_{m1}(v_s + v_{n1}) - g_{m2}v_{n2} \tag{140}$$

The equivalent noise input voltage is obtained by factoring g_{m1} from the expression and retaining all terms except the v_s term. It has the mean squared value

$$\overline{v_{ni}^2} = \overline{v_{n1}^2} + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \overline{v_{n2}^2} \tag{141}$$

When the low-frequency approximation is used for $\overline{v_n^2}$ in Eq. (135), the expression for $\overline{v_{ni}^2}$ reduces to

$$\overline{v_{ni}^2} = \frac{K_f \Delta f}{2\mu_n C_{ox}^2 W_1 L_1 f}\left[1 + \left(\frac{L_1}{L_2}\right)^2\right] \tag{142}$$

The value of L_1 that minimizes this is $L_1 = L_2$. The noise can be reduced further by increasing W_1 and L_2. The noise is independent of W_2.

Common–Source Amplifier with Depletion-Mode Load. Figure 17(b) shows a single-channel NMOS enhancement-mode CS amplifier with an active NMOS depletion-mode load. It is assumed that the two MOSFETs are biased at the same current. With $v_o = 0$, the expression for $i_{o(sc)}$ is the same as for the circuit of Fig. 17(a). Therefore, the expression for v_{ni} is the same. However, the two MOSFETs cannot be assumed to have the same flicker noise coefficient. The low-frequency expression for $\overline{v_{ni}^2}$ is

$$\overline{v_{\rm ni}^2} = \frac{K_{\rm f1}\,\Delta f}{2\mu_{\rm n}C_{\rm ox}^2 W_1 L_1 f}\left[1 + \frac{K_{\rm f2}}{K_{\rm f1}}\left(\frac{L_1}{L_2}\right)^2\right] \tag{143}$$

The value of L_1 that minimizes this is $L_1 = L_2\sqrt{K_{f1}/K_{f2}}$. The noise can be reduced further by increasing W_1 and L_2. The noise is independent of W_2.

Complementary Metal–Oxide–Semiconductor Amplifier. Figure 17(c) shows a push–pull complementary MOSFET (CMOS) amplifier. It is assumed that the two MOSFETs are biased at the same current. With $v_o = 0$, the short-circuit output current is given by

$$i_{\rm o(sc)} = g_{\rm m1}(v_{\rm s} + v_{\rm n1}) + g_{\rm m2}(v_{\rm s} + v_{\rm n2}) \tag{144}$$

The equivalent noise input voltage is obtained by factoring $g_{m1} + g_{m2}$ from the equation and retaining all terms except the v_s term. It has the mean squared value

$$\overline{v_{\rm ni}^2} = \frac{g_{\rm m1}^2\overline{v_{\rm n1}^2} + g_{\rm m2}^2\overline{v_{\rm n2}^2}}{(g_{\rm m1} + g_{\rm m2})^2} \tag{145}$$

In order for the quiescent output voltage to be midway between the rail voltages, the circuit is commonly designed with $g_{m1} = g_{m2}$. When this is true and the low-frequency approximation is used for $\overline{v_n^2}$ in Eq. (135), $\overline{v_{ni}^2}$ can be written

$$\overline{v_{\rm ni}^2} = \frac{1}{2}\left(\frac{K_{\rm f1}\,\Delta f}{2\mu_{\rm n}C_{\rm ox}^2 W_1 L_1 f} + \frac{K_{\rm f2}\,\Delta f}{2\mu_{\rm p}C_{\rm ox}^2 W_2 L_2 f}\right) \tag{146}$$

The noise can be decreased by increasing the size of both transistors. For $g_{m1} \neq g_{m2}$, a technique for further reducing $\overline{v_{ni}^2}$ is to increase L for the MOSFET for which K_f/μ_o is the largest.

Differential Amplifier with Active Load

Figure 17(d) shows a differential amplifier with a current-mirror active load. It is assumed that M_1 and M_3 have matched model parameters and similarly for M_2 and M_4. In addition, it is assumed that all four transistors are biased at the same current so that $g_{m1} = g_{m3}$ and $g_{m2} = g_{m4}$. Because the noise generated by the tail source is a common-mode signal, it is canceled at the output by the current-mirror load and is not modeled in the circuit. The differential input voltage is given by $v_{id} = v_s + v_{n1} - v_{n3}$. The component of the short-circuit output current due to v_{id} is $i_{o(sc)} = g_{m1}v_{id}$. To solve for the component of $i_{o(sc)}$ due to v_{n2} and v_{n4}, the sources v_s, v_{n1}, and v_{n3} are set to zero. This forces M_4 to have zero drain signal current. Thus the gate of M_4 is a signal ground and $i_{o(sc)} = -g_{m2}(v_{n2} - v_{n4})$. The total short-circuit output current is given by

$$i_{\rm o(sc)} = g_{\rm m1}(v_{\rm s} + v_{\rm n1} - v_{\rm n3}) + g_{\rm m2}(v_{\rm n2} - v_{\rm n4}) \tag{147}$$

The equivalent noise input voltage is obtained by factoring g_{m1} from this equation and retaining all terms except the v_s term. The mean squared value is

$$\overline{v_{\rm ni}^2} = \overline{v_{\rm n1}^2} + \overline{v_{\rm n3}^2} + \left(\frac{g_{\rm m2}}{g_{\rm m1}}\right)^2 (\overline{v_{\rm n2}^2} + \overline{v_{\rm n4}^2}) \tag{148}$$

When the low-frequency approximation is used for $\overline{v_n^2}$ in Eq. (135), $\overline{v_{ni}^2}$ can be written

$$\overline{v_{\rm ni}^2} = \frac{K_{\rm f1}\,\Delta f}{\mu_{\rm p}C_{\rm ox}^2 W_1 L_1 f}\left[1 + \frac{K_{\rm f2}}{K_{\rm f1}}\left(\frac{L_1}{L_2}\right)^2\right] \tag{149}$$

The noise can be reduced by increasing W_1 and L_2 and by making $L_1 = L_2\sqrt{K_{f1}/K_{f2}}$. The expression is independent of W_2.

COMPARISON OF THE BIPOLAR JUNCTION TRANSISTOR AND THE FIELD EFFECT TRANSISTOR

An exact comparison of the BJT and the FET is impossible, in general, because the noise performance of each is so dependent on device parameters and bias currents. The FET exhibits only v_n noise, whereas the BJT exhibits both v_n and i_n noise. For a low source resistance, the BJT's v_n noise is its dominant noise. In this case, the BJT usually has lower noise than the FET. For a high source resistance, the BJT's i_n noise can cause it to exhibit more noise than the FET. This assumes that the BJT bias current remains fixed as the source resistance is increased. If the BJT is biased for minimum noise, the collector bias current must be decreased as the source resistance is increased. In this case, the FET may not be the better choice device for the lowest noise. However, a very small bias current is a disadvantage when the amplifier slew rate, for example an *op amp,* is a design consideration. For this reason, the FET may be preferable when the source resistance is high and a low bias current is a disadvantage.

The above considerations neglect flicker noise effects. Flicker noise is so device-dependent that it is difficult to reach general conclusions. However, the FET usually exhibits more flicker noise at low frequencies than the BJT. In a JFET not selected for low flicker noise, the flicker noise corner frequency can be as high as several kilohertz. In MOSFETs, it can be even higher.

OPERATIONAL AMPLIFIER NOISE

Op amp noise models are variations of the v_n–i_n amplifier noise model. The general noise model puts a noise voltage source and a noise current source at each input to the op amp. Thus four noise sources are required. In general, the sources are correlated. However, the correlation between the two sources on one input and the two sources on the other input would be expected to be weak and might be neglected. The general noise model is given in Fig. 18(a).

Because the op amp responds only to the differential input voltage, the two noise voltage sources in the general model can be replaced by a single noise voltage source in either op amp input lead. Figure 18(b) shows the modified model, where the source is input in the noninverting input lead and $v_n = v_{n1} - v_{n2}$. In general, v_n in this circuit is correlated to both i_{n1} and i_{n2}.

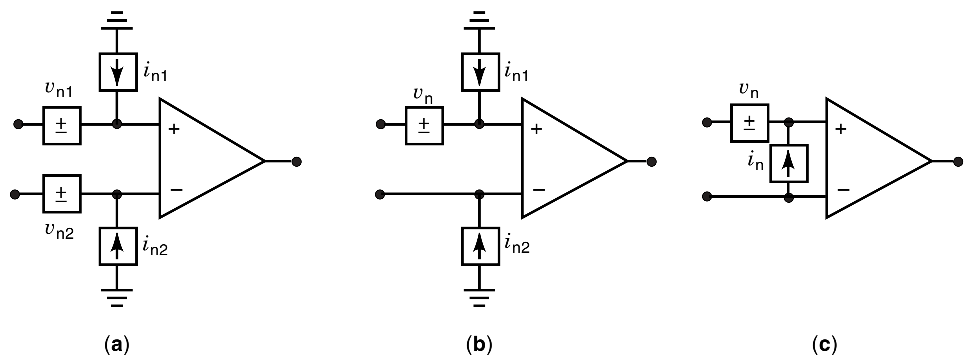

Figure 18. Noise models of the op amp.

An even simpler noise model can be obtained if the two noise current sources are replaced by a single differential noise current source as shown in Fig. 18(c). This model is not as accurate as the other two. In making calculations that use specified noise data on op amps, it is important to use the noise model for which the data apply.

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