NONLINEAR FILTERS

Numerous linear and nonlinear digital filters have been developed for a wide range of applications. Linear filters enjoy the benefits of having a well-established and rich theoretical framework. Furthermore, real-time implementation of linear filters is relatively easy since they employ only standard operations (multiply and add) and can also be implemented using fast Fourier transforms. In many cases, however, the restriction of linearity can lead to highly suboptimal results. In such

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cases, it may be desirable to employ a nonlinear filter (1,2). Furthermore, as digital signal processing hardware becomes ever more sophisticated and capable, complex nonlinear operations can be realized in real time. For these reasons, the field of nonlinear filters has grown and continues to grow rapidly.

While many applications benefit from the use of nonlinear methods, there exist broad classes of problems that are fundamentally suited to nonlinear methods and which have motivated the development of many nonlinear algorithms. Included in these classes of problems are the following:

1. *Suppression of Heavy-Tailed Noise Processes.* Simplifying approximations and the Central Limit Theorem often lead to the assumption that corrupting noise processes are Gaussian. However, many noise processes are decidedly heavy-tailed, or impulsive, in nature (i.e., have probability density functions with relatively high valued tails). Linear filters often do a poor job suppressing such noise, necessitating the use of robust nonlinear methods.
2. *Processing of Nonstationary Signals.* Linear filters tend to be sensitive to nonstationarities (changes in local signal statistics), which are common in images and biomedical signals, for example. In images and video sequences, nonstationarities in the form of edges and scene changes are abundant. Linear processing of such data for restoration or enhancement may produce blurred edges and/or ringing artifacts, which can seriously degrade visually important features.
3. *Super-resolution Frequency Extension.* Frequency analysis shows that linear methods can be designed to either amplify or attenuate signal power at selected frequencies. However, linear filters are incapable of restoring frequency content to a signal from which it has been completely eliminated. Such frequency content extension requires nonlinear methods and is important in applications such as the restoration of high-resolution broad-band images from low-resolution narrowband realizations.
4. *Modeling and Inversion of Nonlinear Physical Systems.* Signals are generally acquired through physical systems (such as transducers or optics) that are inherently nonlinear. Both the accurate modeling of such systems and the inversion of their effects on acquired signals necessitate the use of nonlinear methods.

Here we describe a variety of nonlinear filters and identify some current areas of research on nonlinear methods. An extensive treatment of nonlinear filters can be found in the books by Astola and Kuosmanen (1) and by Pitas and Venetsanopoulos (2). The fact that nonlinear methods lack a unifying framework makes presenting a general overview difficult. However, we organize the presented filters into two general methodologies:

1. *Weighted Sum Filters.* The output of a linear filter is formed as a linear combination, or weighted sum, of observed samples. Nonlinearities can be introduced into this general filtering methodology by transforming the observation samples, through reordering or nonlinear warping for instance, prior to the weighting and summing operations. We refer to all filters that form an estimate through linear combinations in this way as weighted sum filters.
2. *Selection Filters.* Combining samples to form an estimate, especially in a weighted sum fashion, can lead to cases where corrupted samples have a disproportionate influence on the estimate. Moreover, the output of a weighted sum estimate is generally an intermediary sample that is not equal to any of the observed samples, which is undesirable in some applications. These issues are addressed by a broad class of nonlinear filters (referred to as selection filters) that restrict their output to be one of input samples.

Weighted sum filters combine the benefits of linear filters with some strategically designed nonlinearity to provide the desired result. Selection filters tend to offer robustness from outliers, provided that a proper selection rule is implemented. That is, as long as an outlier is not selected to be the output, the outlier is effectively removed and generally has little impact on the filter output. Furthermore, selection filters generally do not blur edges in signals since the output is forced to be one of the input samples and no intermediate transition samples are created by the filter. In the following analysis, we show that many useful nonlinear filters can be placed into these two broad categories. Signal and image processing examples are included at the end of this article to illustrate the performance of selected filtering methods. Also, numerous references are provided to allow the reader to pursue the study of the filters described in greater detail.

The organization of the remainder of this article is as follows. In the section entitled “The Filtering Problem,” the filtering problem is described and much of the notation is defined. The general class of nonlinear weighted sum filters is described in the section entitled “Nonlinear Weighted Sum Filters.” The family of selection filters is described in the section entitled “Selection Filters.” Illustrative filtering examples are provided in the section entitled “Filtering Examples,” where selected filters are applied to the restoration of an image embedded in Gaussian noise, to the restoration of an image contaminated by impulsive noise, and, finally, to edge enhancement. Some conclusions are provided in the section entitled “Conclusions.”

THE FILTERING PROBLEM

The goal in many filtering applications is to transform an observed signal into an approximation of a desired signal, where the transformation is designed to optimize some fidelity criterion. This scenario is illustrated in Fig. 1, where $\{x(\boldsymbol{n})\}$ and $\{y(\boldsymbol{n})\}$ represent the observation (input) and approximation (output) sequences, respectively. In this representation,

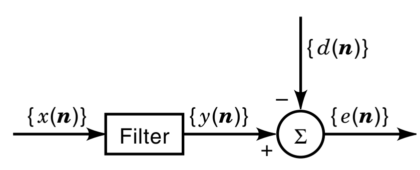

Figure 1. The filtering problem where an observed signal is transformed by the filtering operation to approximate a desired signal.