LOW-PASS FILTERS

A low-pass filter suppresses the high-frequency components of a signal, leaving intact the low-frequency ones. A low-pass

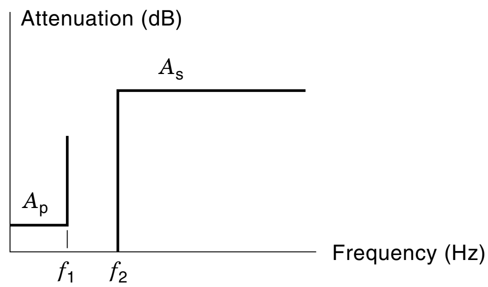

Figure 1. Low-pass filter requirements.

filter specification can be expressed as shown in Fig. 1. In the stopband (above f_2 Hz), the attenuation is at least A_s dB. In the passband (below f_1 Hz), the attenuation is at most A_p dB. The band from f_1 to f_2 is called the transition band.

A first-order low-pass filter is shown in Fig. 2. The transfer function is

$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

The pole-zero diagram is shown in Fig. 3.

An active second-order low-pass filter is shown in Fig. 4. The circuit is known as the Sallen and Key low-pass circuit. The transfer function is

$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{\frac{\alpha}{R_1R_2C_1C_2}}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-\alpha}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}} \qquad (1)$$

where $\alpha = 1 + r_2/r_1$. The pole-zero diagram is shown in Fig. 5.

An approach to design a circuit (a low-pass filter) whose frequency response satisfies the low-pass requirements shown in Fig. 1 consists of two steps: approximation of the requirements by a transfer function and synthesis of the transfer function.

There are several approximation methods, for instance, Butterworth, Chebyshev, inverse Chebyshev, and Cauer approximations. A transfer function obtained by one method is different from those obtained by the others, and has different properties. However, the frequency response of each of the transfer functions satisfies the low-pass requirements. The Butterworth approximation method is described below.

Compute the scaling factor k given by the equation below:

$$k = \sqrt{10^{0.1A_p} - 1}$$

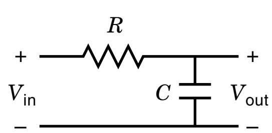

Figure 2. First-order low-pass filter.

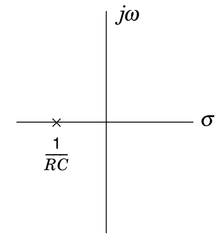

Figure 3. Pole-zero diagram of the first-order low-pass filter.

Next, compute the quantity

$$\frac{\log_{10}\left(\frac{10^{0.1A_s} - 1}{k^2}\right)}{2\log_{10}\left(\frac{f_2}{f_1}\right)}$$

Choose the order, n, of the filter to be the smallest integer not smaller than the above quantity. Solve for all the left-half-plane roots, Z_is, of the equation

$$(-1)^n S^{2n} + 1 = 0$$

The Butterworth low-pass transfer function is formed as follows:

$$T_{\text{LP}}(s) = \left.\frac{1}{\prod_{i=1}^{n}(S - Z_i)}\right|_{s=k^{1/n}s/2\pi f_1}$$

Example. Find the Butterworth transfer function for a low-pass filter with $A_s = 15$ dB, $A_p = 0.5$ dB, $f_1 = 1$ kHz, and $f_2 = 5$ kHz.

$$k = \sqrt{10^{0.1(0.5)} - 1} = 0.35$$

$$\frac{\log_{10}\left(\frac{10^{0.1(15)} - 1}{0.35^2}\right)}{2\log_{10}\left(\frac{5\text{k}}{1\text{k}}\right)} = 1.72$$

Choose $n = 2$. The left-half-plane roots of the equation

$$(-1)^2 s^{2(2)} + 1 = 0$$

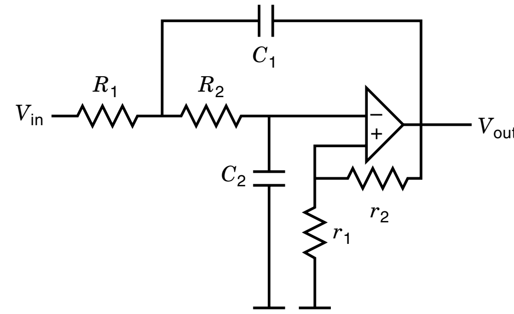

Figure 4. Sallen and Key low-pass circuit.

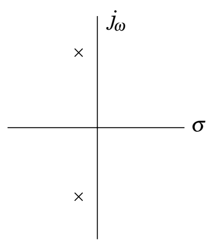

Figure 5. Pole-zero diagram of the Sallen and Key low-pass filter.

are $1/\sqrt{2}(-1 \pm j)$. Therefore, the Butterworth transfer function is

$$T_{\mathrm{LP}}(s) = \left|\frac{1}{s^2 + \sqrt{2}s + 1}\right|_{s=\sqrt{0.35}s/2\pi(1000)}$$

which simplifies to

$$T_{\mathrm{LP}}(s) = \frac{1.128 \times 10^8}{s^2 + 1.502 \times 10^4 s + 1.128 \times 10^8}$$

The above calculations have all been performed before, and the results are available in tabular and computer program forms.

The Sallen and Key low-pass circuit can be used to realized a second-order low-pass transfer function of the form

$$T(s) = \frac{K}{s^2 + as + b}$$

Compare $T(s)$ with Eq. (1)

$$K = \frac{\alpha}{R_1R_2C_1C_2} \quad a = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-\alpha}{R_2C_2} \quad b = \frac{1}{R_1R_2C_1C_2}$$

Since there are more unknowns than equations, one can assign values to certain unknowns and then solve for the remaining unknowns. As an example, choose $C_1 = 1$, $C_2 = 1$, $\alpha = 2$. Then

$$K = 2b \qquad R_1 = \frac{1}{a} \qquad R_2 = \frac{a}{b}$$

The impedance-scaling method can be used to scale the values of R's and C's into the practical ranges.

In general, a higher-order low-pass transfer function obtained by Butterworth and Chebyshev approximations can be factorized into a product of biquadratic functions and possibly one first-order expression. Each of the biquadratic functions can be synthesized by using the Sallen and Key or other low-pass circuits. The first-order transfer function can be realized by an active or passive circuit. By cascading all the biquad and first-order circuits together, the low-pass filter is realized.

Reading List

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