J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*

HYSTERESIS IN CIRCUITS

Hysteresis, in simple terms, is the tendency of a system to resist change. This tendency may be created in a variety of ways, and so we may observe hysteresis in many diverse kinds of systems: electrical, mechanical, physical, biological, and others. Once a system with hysteresis is in one state, it requires a suitable amount of energy to overcome this tendency in order to move the system to another state. Hysteresis can also describe how a system "remembers" a past response. This "memory" may be considered a storage of energy, like a capacitor or inductor, but is really a form of dynamic tension.

Hysteresis is a nonlinear phenomenon. Thus, it cannot be described using simple linear operations (addition, subtraction, multiplication, differentiation, etc.). The mathematical description of hysteresis has been an intense area of research in recent years. However, hysteresis has been known and understood by electrical engineers for many years and in numerous forms: as a cause of energy loss and heating in the ferrite cores of transformers, as a fundamental building block in the design of some electronic oscillators, and so on.

The Hysteresis Function

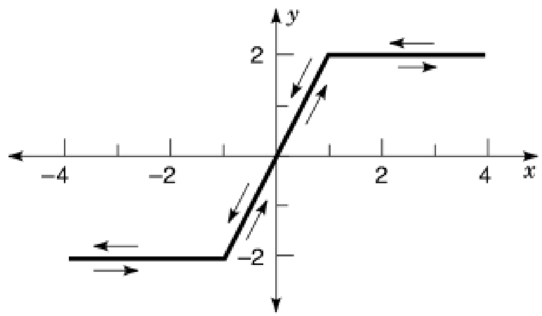
The hysteresis function can best be illustrated by comparison using the transfer functions shown in Fig. 1. The input to each function is given on the horizontal x axis, and the output is given on the vertical y axis. The curve in Fig. 1(a) is a simple piecewise linear transfer function with gain $A = y/x = 2$ limited above and below by ± 2. The three curves are identical, except that there exists a region in the domain of x for the curve in Fig. 1(b,c) for which there are two possible solutions in the range of y. In this example, the curve in Fig. 1(a) may be expressed as

$$y = \begin{cases} +2 & \text{for} \quad x \geq +1 \\ 2x & \text{for} \quad -1 < x < +1 \\ -2 & \text{for} \quad x \leq -1 \end{cases} \tag{1}$$

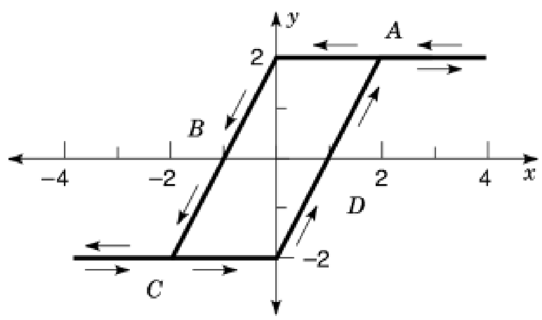
However, for the curve in Fig. 1(b), one must know something about the previous value of x in order to determine the corresponding y:

$$y = \begin{cases} +2 & \text{for} \quad x > +2 \text{ previously and } x \geq 0 \\ 2x+2 & \text{for} \quad x > +2 \text{ previously and } -2 < x < 0 \\ -2 & \text{for} \quad x < -2 \text{ previously and } x \leq 0 \\ 2x-2 & \text{for} \quad x < -2 \text{ previously and } 0 < x < +2 \end{cases} \tag{2}$$

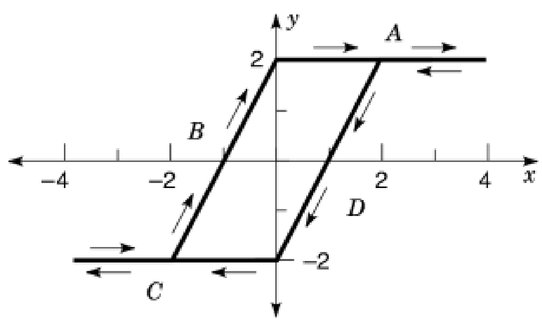
The determination of y based on previous values of x is illustrated by the arrows in Fig. 1(b). If the first value of x is larger than 2, then the output value of y begins at 2 and follows line segment A. If the following values of x remain larger than -2, the resulting values of y follow the arrows around the curve to the left, along



(a)



(b)



(c)

Fig. 1. Comparison of (a) simple nonlinear transfer function with (b) positive hysteresis and (c) negative hysteresis. Note that the directions of the arrows for (b) and (c) traverse the curve in opposite directions.

segments A and B. If, however, the value of x falls below -2, the resulting values of y will follow the arrows on line segments C and D. This tracing of the hysteresis curve is known as *positive* hysteresis. There is one other way to trace the curve, as shown in Fig. 1(c). This alternative path around the hysteresis curve is *negative* hysteresis.

Hysteresis in Ferromagnetism

Ferromagnetic materials include the elements iron (Fe), cobalt (Co), and nickel (Ni), and many of their alloys and composites, as discussed by Plonus (1). The use of ferromagnetic materials in electrical and electronic systems can be found in transformers, motors, generators, inductors, and the like (see Magnetic Materials and Induction Motor Drives).

Hysteresis in Electronic Systems

In addition to ferromagnetic systems, electronic systems may also exhibit hysteresis. In these instances, there are usually two major contributing features: upper and lower limits, such as power supply voltages, and a positive feedback loop. Hysteresis can be used effectively in electronic systems as a building block in oscillator design, as an input amplifier to help reject noise, or as a delay function.

Example: An Opamp Hysteresis Function. One simple hysteresis design using operational amplifiers (see Operational Amplifiers) is given in Fig. 2. Note that opamp 1 and resistors R_1 and R_2 constitute the actual hysteresis element. Resistor R_2 connects the output of opamp 1 to its positive input. This large amount of positive feedback will force the circuit voltage at v_h to be either the positive supply voltage V_D or the negative supply voltage V_S. If we sweep the input voltage v_i, the hysteresis voltage v_h will have the transfer function shown at the top of Fig. 3. To find the values of input voltage v_i for which the hysteresis voltage v_h will switch, we have

$$v_\mathrm{i} = V_\mathrm{R}\left(1+\frac{R_1}{R_2}\right) - v_\mathrm{h}\frac{R_1}{R_2} \qquad (3)$$

If we make $V_\mathrm{R} = 0$, the equation in Eq. (3) simplifies to

$$v_\mathrm{i} = -v_\mathrm{h}\frac{R_1}{R_2} \qquad (4)$$

This gives for the upper and lower trip voltages V_U and V_L respectively

$$V_\mathrm{U} = -V_\mathrm{S}\frac{R_1}{R_2}, \qquad V_L = -V_\mathrm{D}\frac{R_1}{R_2} \qquad (5)$$

and for the hysteresis width V_HW

$$V_\mathrm{HW} = V_\mathrm{U} - V_\mathrm{L} = (V_\mathrm{D} - V_\mathrm{S})\frac{R_1}{R_2} \qquad (6)$$

Thus, the trip voltages for the hysteresis may be designed using a ratio of resistors and the power supply voltages. However, it is usually undesirable to have a voltage in such a system that swings from rail to rail. To prevent this we may use another opamp amplifier such that the hysteresis voltage v_h is attenuated. This is shown using opamp 2 at the top of Fig. 2. By making R_3 much larger than R_4, the output voltage v_o may be designed so that its signal swing is well away from the power supply voltages, preventing the output of the hysteresis from saturating any following stages. This attenuation is shown at the bottom of Fig. 3. Also note that this second amplifier is in the inverting configuration. Thus the trip voltages for the hysteresis are

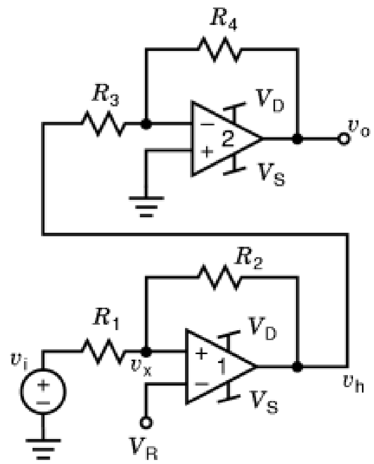


Fig. 2. An electronic hysteresis can be constructed using monolithic operational amplifiers. The opamp at the bottom of the figure has positive feedback. The amplifier at the top is in the inverting configuration. The center of the hysteresis can be adjusted using V_R.

determined by V_D, V_S, R_1, and R_2, and the output limits are determined by V_D, V_S, R_3, and R_4. We may construct an inverting hysteresis by exchanging V_i with V_R in Fig. 2.

Circuit Model. In some cases it is desirable to have a model of the hysteresis function for use in a hierarchical design instead of a complete circuit, as shown by Varrientos and Sanchez-Sinencio (2). The hysteresis function may then be modeled as shown in Fig. 4.

A circuit symbol for the hysteresis function is shown in Fig. 4(a). The underlying model is given in Fig. 4(b). The resistors r_a and r_b are used as a voltage divider, and resistor r_d and capacitor c_d constitute a high-pass filter, such that the difference voltage $v_a - v_b$ is an all-pass filtering of the differential input voltage $v_p - v_n$. This difference voltage is then converted to a current using the transconductance i_o. The transconductance equation is given as

$$i_o = 2G[(v_a - v_b) + \tfrac{1}{2}(v_o - v_a)] \tag{7}$$

From this equation, one may note that this transconductance current i_o depends somewhat on the output voltage v_o across it, and that this dependence is positive. Therefore, the term $v_o - v_a$ is positive feedback. Also, note that the term $v_a - v_b$ is a phase delay term, making the hysteresis element a function of the speed and/or frequency of the input signal $v_p - v_n$. Generally, this is the case for practical implementations.

The resistor r_o and capacitor c_o at the output of the hysteresis element help determine the rise and fall times, or slew rate, of the output voltage. The diodes d_n and d_p and voltages v_n and v_p define the limits of the output voltage. For this model, the voltages v_n and v_p also give the trip voltages, like those illustrated in Fig. 3. Finally, this circuit model can be described using the circuit simulator SPICE. An example appears in Fig. 5. In this example, the diode model used gives a near-ideal diode, clamping the limit voltages at 0.5 V and the trip voltages at twice the limit voltages, or ±1 V.

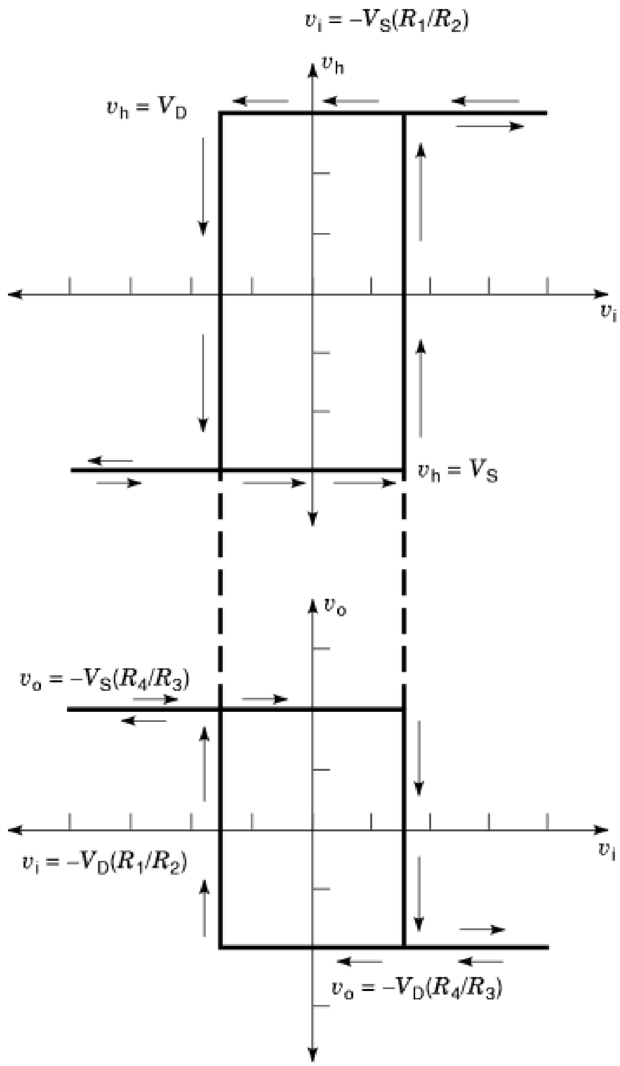


Fig. 3. The transfer functions for the opamp hysteresis in Fig. 2. The output of the first opamp will have the transfer function shown at the top. Note that it has limits at the power supply voltages. After attentuation using the second opamp, the transfer function will look like the trace at the bottom of the figure. The trip voltages are given by the first opamp, and the output limits are given by the second opamp.

Higher-Order Hysteresis

As was mentioned in the introduction, a hysteresis element may be thought of as a single energy storage unit. The amount of storage is determined by the width and height of the hysteresis. It is then possible to increase the *order* of the system in which the hysteresis appears, for example, by increasing the number of hysteresis storage elements. To do this, we may add other hysteresis elements in a given system, or we may increase the number of storage elements in a given hysteresis element. This latter method of increasing the order of a hysteretic element gives *higher-order hysteresis.* Hysteresis of this type has uses in complex nonlinear systems such as chaotic oscillators (3) (see Chaos, Bifurcation, and their control).

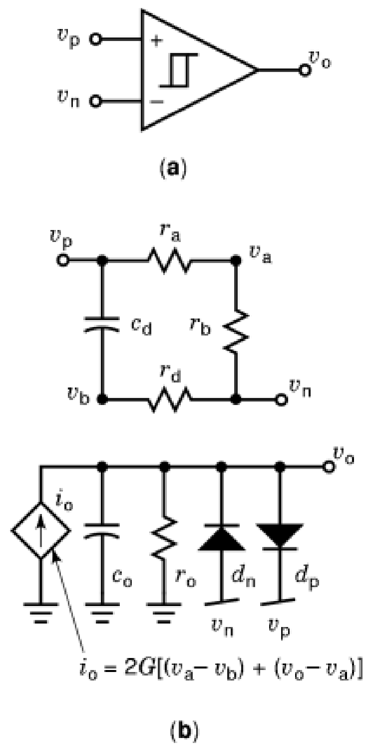


Fig. 4. The hysteresis symbol (a) can be modeled using the simple nonlinear circuit (b) shown at the bottom of the figure. All the components are linear except for the diodes d_p and d_n. The trip voltages and output limits are a function of v_p and v_n.

Linear Combination. By combing several hysteresis elements linearly using inverting and/or summing amplifiers, higher-order hysteresis functions can be derived (2). A block diagram of a general design approach for higher-order hysteresis is given in Fig. 6. The approach begins with the design of trip values for each hysteresis by determining a reference value to shift the hysteresis either left or right and then amplifying (or attenuating) the hysteresis output of each element. Continuing, we may then combine the outputs of several such elements to construct a composite hysteresis with many storage regions. By example, we may combine two hysteresis elements shown in Fig. 2 to construct a composite hysteresis with two nonoverlapping storage regions. This result is shown in Fig. 7. The value of ε gives the hysteresis width, δ is the distance of the center of each width from the origin, and D is the magnitude of the limit.

Differential Hysteresis. In addition to linear combinations, complex hysteresis functions can also be constructed using cross-coupled feedback and nonlinear combinations as shown by Varrientos and Sanchez-Sinencio (2). One example appears as Fig. 8. This hysteresis element is comprised of two hysteresis elements and two comparators referenced to a common ground. Because of the finite response of each amplifier, this hysteresis composite will have two hysteresis regions. The result is similar to the transfer function given for linear combinations in Fig. 7. Note that by increasing the limit voltages on the comparator elements, we also vary the reference voltage for each hysteresis. The result is that each storage region reduces in width ε and increases in distance δ from common ground.

```
.subckt hysteresis in+ in- out
+ cap=10.0p res=1.0k
vp   v2      0    0.5
vn    0      v3   0.5
dp  out     v2  diode
dn   v3   out  diode
ra   in+    va   0.5t
rb   va     in-  0.5t
rp   in-    vb   res
cp   vb     in+  cap
go    0     out  poly(2) va vb out
+ va 0  20.0m 10.0m
ro  out      0    1meg
co  out      0    10n
.ends hysteresis
.model diode d(is=1.0f n=0.001)
```

Fig. 5. This SPICE input file can be used to model the hysteresis shown in Fig. 4. The parameters given for the diode model will make the diodes almost ideal.

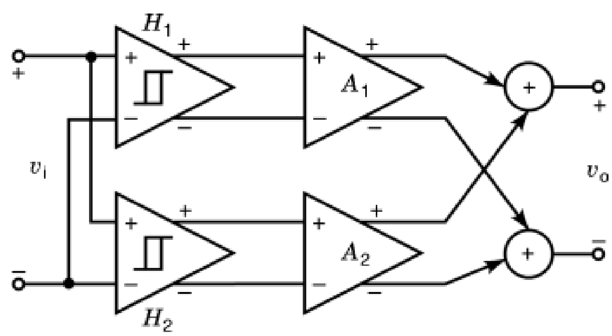


Fig. 6. Higher-order hysteresis can be constructed using linear combinations of simpler hysteresis elements. Here the elements are combined using amplifiers (multiplication) and summing nodes.

Mathematical Descriptions

Often, hysteresis is found in systems with steady-state sinusoidal solutions where the input to the hysteresis is sinusoidal. The hysteresis, whether desirable or not, contributes to the solution, and as engineers we would like to know the extent of the interaction between the linear and nonlinear portions of the system.

Describing Functions. One way of doing this is with the use of describing functions as shown by Gelb and Vander Velde (4). A describing function is a nonlinear function in the steady-state *s* domain. Thus, we may, using the describing function for a given nonlinearity, solve for the solution of a system using linear algebra.

Differential Model for Hysteresis. Often, however, we would like a time-varying, or transient, solution for our system with hysteresis, where the *s*-domain description is not sufficient. In recent years, mathematicians have been investigating the underlying fundamental dynamics of hysteresis as shown by Visintin (5). It remains an active area of research because of the difficulty mathematicians and engineers have had in constructing a sufficiently accurate differential expression for many types of common hysteresis, including the hysteresis found in ferromagnetism.

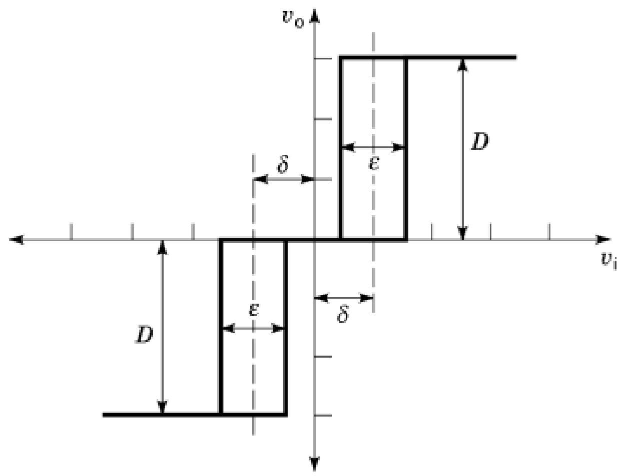


Fig. 7. The transfer function of a second-order hysteresis function. Note that there are two hysteresis regions, one for each simple hysteresis element. The value of D is the upper and lower limit, ε is the hysteresis width, and δ is the distance from the center of each hysteresis width of the origin.

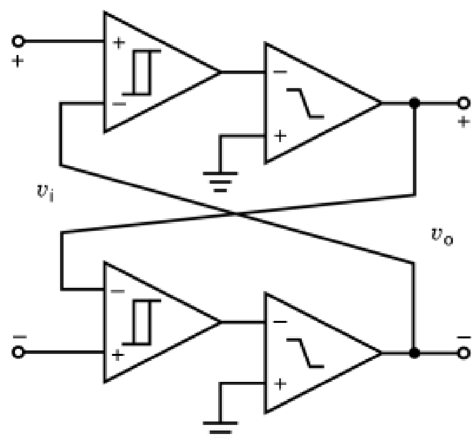


Fig. 8. Higher-order hysteresis can also be constructed using cross-coupling and nonlinear feedback. The output limits of the comparators shown determine the width of the hysteresis and their offset from the origin.

However, several differential expressions exist. One of the most common is a variation on the differential equation given by Rössler (6):

$$e\dot{y}(t) = [1 - y^2(t)][Sx(t) - D + y(t)] - \delta y(t) \qquad (8)$$

where $x(t)$ is the time-varying input and $y(t)$ is the output. The values of S and D are derived from the upper and lower trip values; generally, $S = 2.0/(V_U - V_L)$ and $D = SV_L + 1.0$. Loss in the hysteresis is due to a variety of sources in practical systems and is modeled with the damping factor δ. The value of ε is small so that the hysteresis response reaches its saturation quickly.

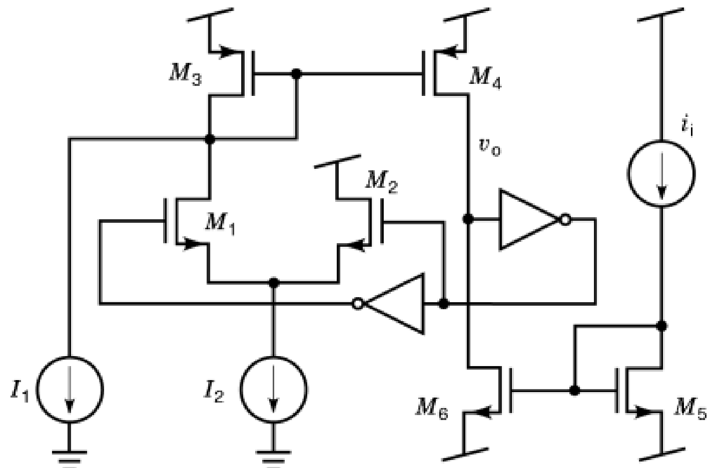


Fig. 9. In CMOS design, is it possible to design hysteresis using currents as parameters? Here the upper and lower trip input currents are determined by I_1 and I_2.

Another popular hysteresis equation is the Bouc–Wen differential equation used by Wong, Ni, and Lau (7):

$$\dot{y}(t) = \kappa \dot{x}(t) - \beta |\dot{x}(t)| \cdot |y(t)|^{n-1} y(t) - \gamma \dot{x}(t) |y(t)|^n \qquad (9)$$

where the quantities κ, β, γ, and n are system parameters that determine the shape and magnitude of the hysteresis this equation produces. The parameters κ, β, and n are positive real numbers, and γ may be a positive or negative real number. Typical values are $\kappa = 1.0$, $\beta = 0.1$, $\gamma = 0.9$, and $n = 1.0$.

Both of the equations above give hysteretic responses that are like those given for ferromagnetic materials (see Magnetic Materials) and for the monolithic responses that follow. The difficulty in using such equations is that, unlike the circuit model given above, they are sensitive to the speed of the input signal and will have responses that vary widely for a given set of inputs. Generally, these equations work best with systems that have well-behaved limit cycles as solutions, sinusoidal oscillations being one example .

Monolithic Implementations

In integrated circuit design, simple implementations of hysteresis elements can be constructed. One such hysteresis element, proposed by Wang and Guggenbühl (8), is shown in Fig. 9. This hysteresis is performed in the *current mode,* since the signals and the trip values are determined by currents rather than voltages. In this circuit, the input current i_i is mirrored through the current mirror M_5, M_6 into the high impedance node v_o. If the input current i_i is smaller than I_1, the voltage at v_o will be near V_D, turning on M_1 and turning off M_2. The total current flowing into v_o from the current mirror M_3, M_4 will now be $I_1 + I_2$. Now, in order to lower the voltage at v_o, the input current i_i must exceed $I_1 + I_2$. Once this happens, the voltage at v_o will be near V_S, turning off M_1 and turning on M_2, so that the current entering the node v_o from the current mirror M_3, M_4 will again be I_1. The upper and lower trip current values will be

$$I_H = I_1 + I_2, \qquad I_L = I_1 \qquad (10)$$

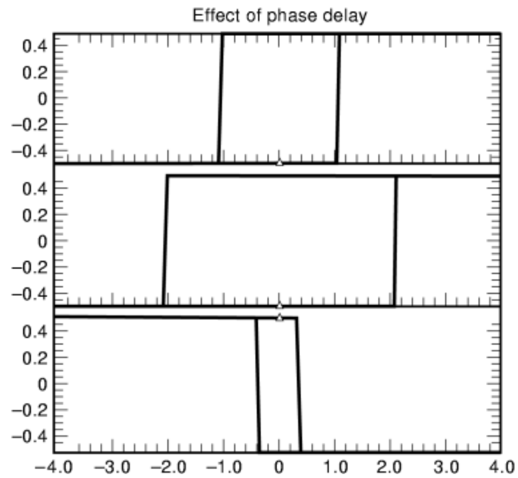


Fig. 10. An input signal to a hysteresis element that is fast in comparison with the switching speed of the hysteresis will cause the width of the hysteresis to increase. If the input signal is too fast, the phase of the hysteresis transfer function will be inverted.

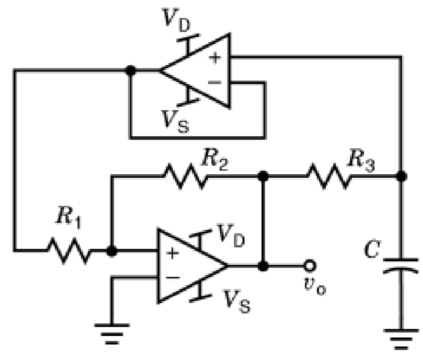
The hysteresis width I_{HW} will be $I_H - I_L = I_2$. Thus the hysteresis is easily adjustable using only two currents, and is independent of the power supply voltages. However, matching between the transistors of each current mirror and the slew rate of the output node v_o may be critical for fast-moving input signals.

Speed Tradeoffs. As was mentioned earlier, the width of the storage region of the hysteresis element will be affected by the speed and/or frequency of the input signal. This is because every practical implementation has practical limits to how fast a signal or node can move. These limits are a function of the voltage swing, load and parasitic capacitances, and bias currents. These effects are modeled in the hysteresis circuit model given in Fig. 4. A transient response of this hysteresis for 1 kHz sinusoidal signal is given in the top trace of Fig. 10.

If we keep the frequency of the input signal constant, but increase the value of the capacitance c_d in Fig. 4, we will increase the response time (phase delay) of the hysteresis. The effect will be to slow the hysteresis down with respect to its input signal. The effect of increasing the response time is shown in the second trace of Fig. 10. Even though we have not changed the value of the trip or limit voltages v_p and v_n, we have, nonetheless, increased the width of the hysteresis. Parasitic capacitances will have the same effect if we try to make the hysteresis go faster than its fundamental limits will allow it to go. However, the effect can be even more drastic if we increase the amount of phase delay even further. This is shown in the third trace in Fig. 10. Note that a further increase in phase delay has caused the hysteresis to actually become inverted. Also note that the width of the hysteresis is smaller than its design value. Here the input signal is so much faster than the hysteresis circuit than the magnitude of the phase delay has changed the sign of the hysteresis element.

Another effect of fast signals is in the speed at which the output voltage can change from a low value to a high value. This effect is shown in Fig. 12. We may note that as we increase the capacitance c_o in Fig. 4, the slope of the output voltages begins to reduce in magnitude, decreasing the slew rate of the output voltage swing. This effect is caused by additional capacitive loading at the output of the hysteresis. Compare this with Fig. 10 and note that there the voltage transitions are very steep in comparison. Also note that the two phenomena have a similar effect on the width of the hysteresis; for both, an increase in capacitance increases the hysteresis width.

Example: A Relaxation Oscillator. As an example of hysteresis design, consider the relaxation oscillator in Fig. 11 (see Relaxation Oscillator). This oscillator works by charging and discharging the capacitor C through R_3 at the limit voltages at the output of opamp 1. Recognize that opamp 1 and resistors R_1 and



$$T = (2R_3C)\ln V_D \left(\frac{1 + (R_1/R_2)}{1 - (R_1/R_2)} \right)$$

$$V_D = -V_S$$

Fig. 11. A hysteresis-based relaxation oscillator. The first opamp is in the positive feedback configuration, and the second opamp is a voltage buffer. Oscillations are caused by the charging and discharging of the capacitor.

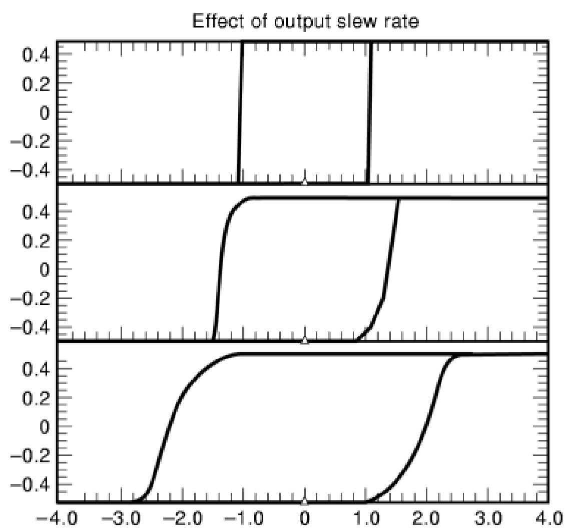


Fig. 12. A large capacitance on the output of the hysteresis may cause it to slew. Large load capacitances will make the width of the hysteresis increase.

R_2 constitute a hysteresis element, that opamp 2 is in the voltage follower configuration, and that R_3 and C constitute an RC time constant.

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