FEEDBACK AMPLIFIERS

AMPLIFIERS, FEEDBACK

Feedback is the process of combining a portion of the output of a system with the system input to achieve modified performance characteristics. Found in a multitude of engineering applications, the process has become the foundation of several disciplines, such as feedback control systems, feedback receivers, feedback oscillators, and feedback amplifiers. It has become particularly pervasive in amplifier design since the advent of transistors that can provide high gain cheaply but cannot provide stable gain.

Negative feedback was originally applied by Harold S. Black in 1927 at Bell Labs to valve (vacuum tube) amplifiers to reduce distortion. In 1957 Black's work was described by Mervin Kelly, president of Bell Labs, as one of the two inventions of broadest scope and significance in electronics and communications is the first half of the century. Negative feedback is the process of mixing an inverted portion of the output of a system with the input to alter system performance characteristics. The process of negative feedback has become especially important in linear amplifier design and is characterized by several significant benefits:

- The gain of the amplifier is stabilized against variation in the characteristic parameters of the active devices due to voltage or current supply changes, temperature changes, or device degradation with age. Similarly, amplifier gain is stabilized within a group of amplifiers that have active devices with somewhat different characteristic parameters.
- Nonlinear signal distortion is reduced.
- The frequency range over which there is constant linear amplification (the midband region) is increased.
- The input and output impedance of the amplifier can be selectively increased or decreased.

It is a rare occurrence when benefits come without a price. In the case of negative feedback, the aforementioned benefits are accompanied by two primary drawbacks:

- The gain of the circuit is reduced. To regain the losses due to feedback, additional amplification stages often must be included in the system design. Complexity, size, weight, and cost may be added to the final amplifier design.
- There is a possibility for oscillation. Should oscillation occur, the basic gain properties of the amplifier are destroyed.

This article considers the benefits of negative feedback for amplifier design. Basic definitions are followed by a general discussion of the properties of a feedback system. Amplifiers are divided into four categories of feedback topology, and the specific properties of each topological type are derived. While the discussions must focus on circuit analysis techniques, a clear understanding of feedback in general, and effects of circuit topology in particular, is a necessity for good feedback amplifier design.

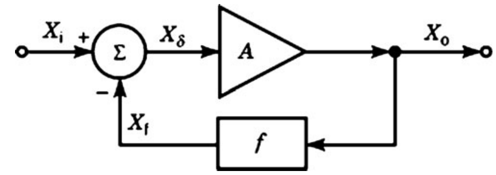

Figure 1. Basic negative feedback topology.

BASIC FEEDBACK CONCEPTS

Electronic amplifiers are fundamentally characterized by four properties:

- The gain of the amplifier. Gain is defined as the ratio of the output signal to the input signal. Each signal may be either a voltage signal or current signal.
- The range of frequencies over which the gain is essentially constant: This range of frequencies is identified as the midband region. It is bounded by the frequencies at which the output power is halved: the high and low 3 dB frequencies.
- The midband input impedance. Defined as the ratio of input voltage to input current.
- The midband output impedance. Defined as the Thévenin impedance seen at the amplifier output terminals.

The application of feedback to an amplifier alters each of these fundamental properties.

The basic topology of a feedback amplifier is shown in Fig. 1. An input signal, X_i, enters a summing (or *mixing*) junction, symbolized by a circular symbol. At the summing junction, the feedback signal, X_f, is subtracted from the input signal and the resultant signal, X_δ, is passed on to a linear amplifier, shown as a triangular symbol, of gain, A. The output of the linear amplifier, X_o, forms the output of the feedback system. The rectangular symbol indicates a feedback network that samples the output signal, scales it by a feedback factor, f, and passes it forward to the input of the system. Each signal can take the form of either a voltage or a current and ideally travels only in the direction of the indicated arrows. Subtraction of the two inputs at the summing junction is the key factor in negative feedback systems.

Feedback systems can be mathematically modeled with a set of simple relationships. The output, X_o, of the amplifier is related to the input signal, X_δ, by a linear amplification factor (gain), A, often called the *forward* or *open-loop gain:*

$$X_o = A(X_\delta) \tag{1}$$

Since the quantities X_o and X_δ can be either voltage or current signals, the forward gain, A, can be a voltage gain, a current gain, a transconductance, or a transresistance. Voltage gain is the ratio of output voltage to input voltage: Similarly, current gain relates output and input currents. Transresistance implies the ratio of an output voltage to

J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*.

an input current: Transconductance is the ratio of an output current to an input voltage. The feedback signal, X_f (a fraction of the output signal, X_o), is then subtracted from the input signal, X_i, to form the difference signal, X_δ.

$$X_\delta = (X_i - X_f) = (X_i - fX_o) \tag{2}$$

where f is the *feedback ratio* defining the relationship between X_f and X_o:

$$X_f = fX_o \tag{3}$$

As is the case with the amplifier gain, the feedback ratio, f, is either a ratio of voltages, a ratio of currents, a transconductance, or a transresistance. The product fA, called the *loop gain,* must be a positive, dimensionless quantity to have stable negative feedback. Thus it is necessary, for negative feedback, that the mathematical sign of f be the same as that of A within the midband region. The input–output relationship for the overall feedback system can be derived from Eqs. (1) and (2):

$$X_o = \frac{A}{1+fA}X_i = A_f X_i \tag{4}$$

The overall gain of the system, including the effects of feedback, is then written as

$$A_f = \frac{X_o}{X_i} = \frac{A}{1+fA} \tag{5}$$

The overall feedback amplifier gain, A_f, has the same dimensions as the forward gain, A. Equation (5) has special significance in the study of feedback systems and is typically identified as the *basic feedback equation.* The denominator of the basic feedback equation is identified as the *return difference, D*, but is also commonly referred to as the amount of feedback:

$$D = 1 + fA \tag{6}$$

The return difference, for negative feedback systems, has magnitude larger than unity (in the midband frequency region) and is often specified in decibels:

$$D_{dB} = 20\log_{10}|1+fA| \tag{7}$$

The return difference quantifies the reduction in gain due to the addition of feedback to the system. It also plays a significant role in quantifying changes in frequency bandwidth and input and output impedance. Specifically, the high and low 3 dB frequencies are increased and decreased, respectively, by approximately a factor of D; and the input and output impedances are increased or decreased by a factor of D depending on the sampling and mixing circuit topologies.

The derivation of the basic feedback equation is based on two basic assumptions:

- The reverse transmission through the amplifier is negligible (a signal at the output produces essentially no signal at the input) compared to the reverse transmission through the feedback network.
- The forward transmission (left to right in Fig. 1) through the feedback network is negligible compared to the forward transmission through the amplifier.

In most feedback amplifiers, the amplifier is an active device with significant forward gain and near-zero reverse gain: The feedback network is almost always a passive network. Thus, in the forward direction, the large active gain will exceed the passive attenuation significantly. Similarly, in the reverse direction, the gain of the feedback network, albeit typically small, is significantly greater than the near-zero reverse gain of the amplifier. In almost every electronic application, the aforementioned requirements for the use of the basic feedback equation are easily met by the typical feedback amplifier.

ANALYSIS OF FEEDBACK AMPLIFIER PROPERTIES

The analysis and design of electronic amplifiers is typically a multistep process. Complete, analytic characterization of an amplifier is a complex process whose results are in a form that often masks the individual amplifier properties. Amplifier designers therefore investigate the amplifier gain, frequency response, and impedance properties separately, carefully balancing system requirements to converge on a successful design. In addition to simplifying the process, separate investigation of the amplifier properties often leads to greater insight into design improvement. When a successful design is apparent, final fine-tuning is accomplished with the aid of a computerized circuit simulator [i.e., System Program with Integrated Circuit Emphasis (*SPICE*)] and a breadboard prototype.

Essentially all of the drawbacks and benefits of feedback systems can be investigated on a simple level by looking at the properties of the basic feedback equation [Eq. (5)]. Gain stabilization, the reduction in nonlinear signal distortion, the increase in the frequency range over which there is linear amplification, the reduction in gain, and the possibility of oscillation all can be investigated on a simple level. The change in the input and output impedances cannot be investigated at this level: It is necessary to specify the nature (voltage or current) of the input and output quantities and the circuit topology to investigate these impedance changes.

Amplifier Gain

In the section on basic feedback concepts, it was shown that feedback reduces amplifier gain by a factor of the return difference, D. While reduction of gain can be a significant drawback, the ancillary gains are significant. Primary among those benefits is the stabilization of the amplifier gain against variation in the characteristic parameters of the active devices. It is well known that the forward gain, A, of an electronic amplifier is highly dependent on the parameters of the active devices contained within that amplifier. These parameters are typically dependent on temperature, bias conditions, and manufacturing tolerances. To maintain consistent amplifier performance, it is desirable to design amplifiers that are reasonably insensitive to the variation of the device parameters.

The relationship between the differential change in gain due to device parameter variation with and without feedback is obtained by differentiating Eq. (5):

$$dA_f = \frac{1}{(1+fA)^2}\,dA \tag{8}$$

Stable negative feedback amplifiers require that the return difference have magnitude greater than unity:

$$D = 1 + fA > 1 \tag{9}$$

Thus the absolute variation in gain is reduced by a factor of the return ratio squared. Another measure of the change in gain variation can be found by regrouping terms.

$$\left|\frac{dA_f}{A_f}\right| = \left|\frac{1}{(1+fA)}\right|\left|\frac{dA}{A}\right| \tag{10}$$

The factors in this equation more realistically describe the benefits: Equation (10) demonstrates the change in the percentage of gain variation about the nominal value. It can be seen that the percentage variation of the overall amplifier gain, A_f, is reduced by a factor of the return ratio when compared to the percentage variation of the gain, A, of the forward amplifier.

For example, a feedback amplifier is constructed with an amplifier that is subject to a 3% variation in gain as its fundamental forward-gain element and it is desired that the feedback amplifier have no more than 0.1% variation in its overall gain due to the variation in this element. The necessary return difference to achieve this design goal can be obtained as follows: Equation (10) is the significant relationship in determining the gain variation:

$$\left|\frac{dA_f}{A_f}\right| = \left|\frac{1}{(1+fA)}\right|\left|\frac{dA}{A}\right|$$

The significant properties are

$$0.001 \geq \left|\frac{1}{(1+fA)}\right| 0.03 \Rightarrow D = 1 + Af \geq 30$$

The minimum necessary return ratio is 30, more often identified as its decibel equivalent,

$$D_{dB} = 20\log_{10} D \geq 29.54\,\text{dB}$$

Equation (10) is extremely useful for small changes in amplification due to parameter variation but is inaccurate for large changes. If the change in amplification is large, the mathematical process must involve differences rather than differentials:

$$\Delta A_f = A_{2f} - A_{1f} = \frac{A_2}{1+fA_2} - \frac{A_1}{1+fA_1} \tag{11}$$

To put this into the same format as Eq. (10), it is necessary to divide both sides of the equation by A_{1f}.

$$\left|\frac{\Delta A_f}{A_{1f}}\right| = \left|\frac{A_2}{1+fA_2}\left(\frac{1+fA_1}{A_1}\right) - 1\right| = \left|\frac{(A_2 - A_1)}{1+fA_2}\left(\frac{1}{A_1}\right)\right| \tag{12}$$

or

$$\left|\frac{\Delta A_f}{A_{1f}}\right| = \left|\frac{1}{1+fA_2}\right|\left|\frac{\Delta A}{A_1}\right| = \left|\frac{1}{1+f(A_1+\Delta A)}\right|\left|\frac{\Delta A}{A_1}\right| \tag{13}$$

The results are similarly a reduction in gain sensitivity by a factor of the form of the return difference.

The differential change in feedback amplifier gain due to variation in the feedback ratio, f, can be obtained by differentiating Eq. (5) with respect to the feedback ratio. Appropriate mathematical manipulation leads to the desired results:

$$\frac{dA_f}{A_f} = \frac{-fA}{1+fA}\frac{df}{f} \Rightarrow -\left|\frac{dA_f}{A_f}\right| = -fA_f\left|\frac{df}{f}\right|$$

It is easily seen that the percentage variation of the overall amplifier gain A_f is approximately the same magnitude (actually slightly smaller) than the percentage variation of the feedback ratio, f. Since electronic feedback amplifiers typically employ a feedback network constructed entirely with passive elements (usually resistors), variation in the feedback ratio can be kept relatively small through the utilization of precision elements in the feedback network. In good amplifier designs, the variation of amplifier gain due to variability in the feedback network is usually of lesser significance than that due to variability of the basic forward amplifier gain.

Nonlinear Signal Distortion

Stabilization of gain with parameter variation suggests that amplifier gain will be stabilized with respect to other gain-changing effects. One such effect is nonlinear distortion. Nonlinear distortion is a variation of the gain with respect to input signal amplitude. A simple example of nonlinear distortion is demonstrated in Fig. 2, in which the transfer characteristic of a simple amplifier is approximated by two regions, each of which is characterized by different amplification, A_1 and A_2. To this transfer characteristic, a small amount of feedback is applied so that $fA_1 = 1$, and the resultant feedback transfer characteristic is shown. As can be seen easily, the overall feedback transfer characteristic also consists of two regions with overall amplification A_{1f} and A_{2f}. In this demonstration, the amplification ratios are:

$$\frac{A_1}{A_2} = 3 \quad \text{and} \quad \frac{A_{1f}}{A_{2f}} = 1.5$$

Feedback has significantly improved the linearity of the system and consequently has reduced the nonlinear distortion. Larger amounts of feedback (increasing the feedback ratio, f) will continue to improve the linearity. For this example, increasing the feedback ratio by a factor of 5 will result in a ratio of overall gain in the two regions of 1.067 (as compared to 1.5 previously). The saturation level of an amplifier is not significantly altered by the introduction of negative feedback. Since the incremental gain in saturation is essentially zero, the incremental feedback difference is also zero. No significant change to the input occurs and the output remains saturated.

Another viewpoint on gain stabilization comes from a limiting form of the basic feedback equation:

$$A_f = \frac{A}{1+fA} = \frac{1}{f}\left(1 - \frac{1}{1+fA}\right) \approx \frac{1}{f} \tag{14}$$

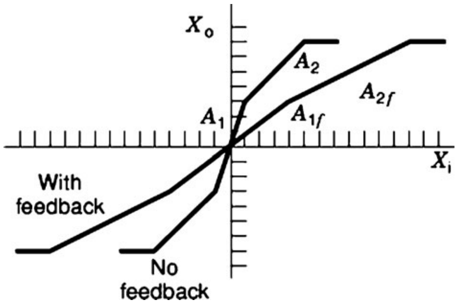

Figure 2. The effect on feedback on an amplifier transfer characteristic.

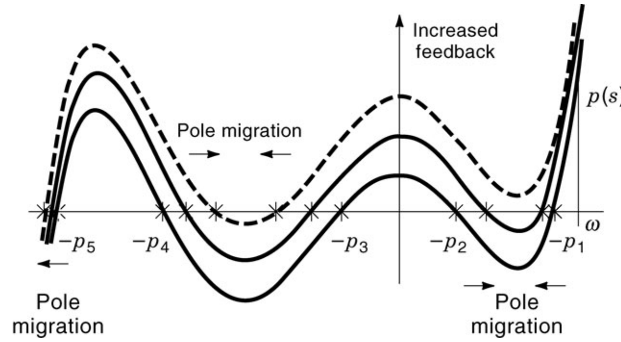

Figure 3. Pole migration due to feedback.

For large return difference ($D = 1 + Af$) the overall gain with feedback is dominated by the feedback ratio, f, and therefore virtually independent of the forward gain, A, and any variations in A.

Frequency Response

Typical linear amplifiers have a range of frequencies over which the gain is essentially constant: This frequency range is called the midband. As frequencies increase, the performance parameters of an amplifier degrade. Similarly, coupling and bypass capacitors internal to the amplifier, when present, will degrade low-frequency performance. Using feedback to broaden of the frequency range over which gain is relatively constant can be considered a special case of the stabilization of gain due to variation in amplifier performance characteristics. Feedback reduces the effects of these frequency-dependent degradations and thereby increases the frequency band over which the amplifier has stable gain.

A more exact description of the increase in the width of the midband region can be obtained through a frequency-domain analysis. It is common practice to use the frequencies at which the output power is reduced by 50% (the high and low 3 dB frequencies) as descriptors of the limits of the midband region. Discussion focuses on the change in these 3 dB frequencies.

It can be shown that the basic forward amplifier gain, A, is described as the midband gain, A_0, divided by a polynomial in frequency (written as s or $j\omega$):

$$A(s) = \frac{A_0}{P(s)} \tag{15}$$

The locations of the first few poles of $A(s)$ [or the zeroes of $P(s)$] closest to the midband region are the dominant predictors of amplifier frequency response: Specifically, their location controls the 3 dB frequencies.

The application of feedback to an amplifier alters the gain expression through the basic feedback equation so that the total gain, A_f, is described by

$$A_f(s) = \frac{\dfrac{A_0}{P(s)}}{1 + f\dfrac{A_0}{P(s)}} = \frac{A_0}{P(s) + fA_0} \tag{16}$$

The application of feedback to the basic forward amplifier has the effect of vertically shifting the denominator polynomial by a constant, fA_0 (see Fig. 3). This shift upward causes movement in the zeroes of the denominator, thereby changing the location of the poles of the frequency response. Any movement of the poles nearest the region of constant gain (the midband region) equates into a change in the width of the midband region. Observation of the consequences of the result in graphical format is a great aid to understanding pole migration.

For example, when the high-frequency response is described by a single pole p_1, Eq. (21) takes the form

$$\begin{aligned} A_f(s) &= \frac{A_0}{P(s) + fA_0} = \frac{A_0}{\left(1 + \dfrac{s}{p_{H1}}\right) + fA_0} \\ &= \frac{1}{1 + fA_0} \frac{A_0}{\left(1 + \dfrac{s}{p_{H1}(1 + fA_0)}\right)} \end{aligned} \tag{17}$$

It can be seen easily that the gain has been reduced by a factor of the return difference, D, and the pole frequency has been increased by the same factor. Similarly, if the low-frequency response is described by a single pole, p_{L1}, the pole frequency will be reduced (that is, divided by) by a factor of D. Since, in single-pole systems, the 3 dB frequency coincides with the pole frequencies, the high and low 3 dB frequencies, ω_H and ω_L, are shifted by a factor of the return ratio:

$$\omega_{Hf} = (1 + fA_0)\omega_H = D\omega_H \quad \text{or} \quad \omega_{Lf} = \frac{\omega_L}{1 + fA_0} = \frac{\omega_L}{D} \tag{18}$$

As the number of poles increases, description of the bandwidth increases with the application of feedback increases in complexity. When the high or low frequency response can be described by two poles, the damping coefficient due to the application of feedback is function ratio of the initial pole locations, k, and the return difference. The damping coefficient can be calculated to be:

$$\zeta = \frac{1 + k}{2\sqrt{k(1 + f A_o)}} = \frac{1 + k}{2\sqrt{k D}} \tag{19}$$

where the k is defined as the ratio of the larger pole to the smaller pole:

$$k_H = \frac{\omega_{2H}}{\omega_{1H}} \quad \text{or} \quad k_L = \frac{\omega_{1L}}{\omega_{2L}} \tag{20}$$

This simple expression for the damping coefficient is a particularly important result in that it can tell the circuit designer the flatness of the frequency response in relation to the amount of feedback applied: a flat frequency response requires critically or overdamped pole pairs ($\zeta \geq 0.707$).

Once the damping coefficient is determined, the expression for the high or low 3 dB frequency shift with the application of feedback takes a form similar to the single pole case with an additional factor:

$$\omega_{\mathrm{H}\,f} = K(\zeta_{\mathrm{H}}, k_{\mathrm{H}}) \cdot D \cdot \omega_{1\mathrm{H}} \quad \text{or} \quad \omega_{\mathrm{L}\,f} = \frac{\omega_{1\mathrm{L}}}{K(\zeta_{\mathrm{L}}, k_{\mathrm{L}}) \cdot D}, \tag{21}$$

where k is the ratio of the initial pole spacing ($k \geq 1$), ζ is the pole-pair damping coefficient, $\omega_{1\mathrm{H}}$ and $\omega_{1\mathrm{L}}$ are the poles closest to the midband, and the factor, $K(\zeta, k)$, is given by:

$$K(\zeta, k) = \frac{2k\,\zeta}{k+1}\sqrt{1 - 2\zeta^2 + \sqrt{(1-2\zeta^2)^2 + 1}} \tag{22}$$

This relationship is shown in Fig. 4 for a variety of initial pole spacing ratios, k. In most amplifier applications, $0.9 < K(\zeta, k) < \sqrt{2}$: some designers use a $K(\zeta, k) \approx 1$ as a first-order approximation.

For amplifiers where the frequency response must be described by more than two poles, the description of the frequency shift is even more complicated. Fortunately, amplifiers with a high- or low-frequency response that is described by more than two poles are reasonably modeled by considering them to be two-pole systems (1). Equation (26) adequately approximates the change in bandwidth for these higher-order systems.

For example, an amplifier has a midband gain, A_0 = 1000 and has frequency response described by one low-frequency pole, f_{L} = 10, and two high-frequency poles, f_{H1} = 10 kHz and f_{H2} = 100 kHz, and feedback is applied so that the midband gain is reduced to $A_{0\mathrm{f}}$ = 140. The new low and high 3 dB frequencies can be determined as follows: The return difference is the ratio of the two gains:

$$D = \frac{A_0}{A_{0\mathrm{f}}} = \frac{1000}{140} = 7.14286$$

The low-frequency response is described by a single pole; thus the low 3 dB frequency is changed by a factor of D:

$$f_{\mathrm{Lf}} = \frac{f_{\mathrm{L}}}{D} = \frac{100}{7.14286} = 14\,\mathrm{Hz}$$

The high-frequency response is described by two poles with ratio, k.

$$k = \frac{\omega_{2\mathrm{H}}}{\omega_{1\mathrm{H}}} = \frac{f_{2\mathrm{H}}}{f_{1\mathrm{H}}} = \frac{10\,\mathrm{MHz}}{1\,\mathrm{MHz}} = 10.$$

The damping coefficient for the two poles are found to be:

$$\zeta_{\mathrm{H}} = \frac{1+k}{2\sqrt{k(1+\ f\,A_o)}} = \frac{1+10}{2\sqrt{10(7.14286)}} = 0.6508$$

Notice that the high poles of the feedback amplifier are slightly underdamped and that there will be a small "bump" ($\approx$ 0.1 dB) in the frequency response as a result. The high 3 dB frequency, f_{Hf}, is then found from $K(\zeta_{\mathrm{H}}, k)$, D, and f_1:

$$f_{\mathrm{Hf}} = K(\zeta_{\mathrm{H}}, k)D\ f_1 = (1.277)(7.4286)(1\,\mathrm{MHz}) = 9.12\,\mathrm{MHz}$$

The resultant frequency response plots are shown in Fig 5.

Input and Output Impedance

The input and output impedance of a feedback amplifier can be selectively increased or decreased through the application of feedback. As has been seen in the previous sections, general discussions provide great insight into many of the properties of feedback systems. To consider the design of electronic feedback amplifiers, it is necessary, however, to specify the details of the feedback sampling and mixing processes and the circuits necessary to accomplish these operations. The sampling and mixing processes have a profound effect on the input impedance, the output impedance, and the definition of the forward-gain quantity that undergoes *quantified* change due to the application of feedback. This subsection analyzes the various idealized feedback configurations. The following section looks at practical feedback configurations.

The mixing and the sampling processes for a feedback amplifier utilize either voltages or currents. Voltage mixing (subtraction) implies a series connection of voltages at the input of the amplifier: Current mixing implies a shunt connection. Voltage sampling implies a shunt connection of the sampling probes across the output voltage: Current sampling implies a series connection so that the output current flows into the sampling network. Either type of mixing can be combined with either type of sampling. Thus, a feedback amplifier may have one of four possible combinations of the mixing and sampling processes. These four combinations are commonly identified by a hyphenated term: (mixing topology)–(sampling topology). The four types are as follows:

- Shunt–shunt feedback (current mixing and voltage sampling)
- Shunt–series feedback (current mixing and current sampling)
- Series–shunt feedback (voltage mixing and voltage sampling)
- Series–series feedback (voltage mixing and current sampling)

The four basic feedback amplifier topologies are shown schematically in Fig. 6. A source and a load resistance have been attached to model complete operation. In each diagram the input, feedback, and output quantities are shown properly as voltages or currents. Forward gain, A, must be defined as the ratio of the output sampled quantity divided by the input quantity that undergoes mixing. As such it is a transresistance, current gain, voltage gain, or transconductance. The feedback network, as described by the feedback ratio (f), must sample the output quantity and present a quantity to the mixer that is of the same type (current or voltage) as the input quantity. As such it is a transconductance, current gain, voltage gain, or transresistance. Table 1 lists the appropriate quantities mixed at the input, the

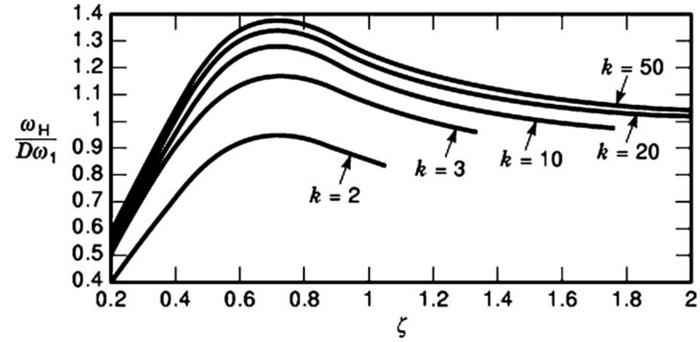

Figure 4. High 3 dB frequency as a function of ζ and nonfeedback pole spacing.

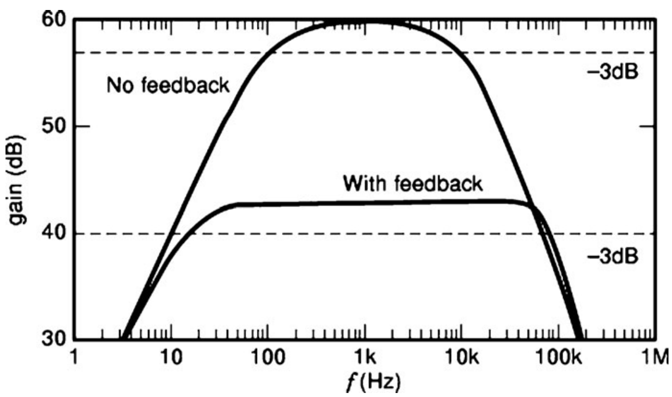

Figure 5. An example of the effect of feedback on frequency response.

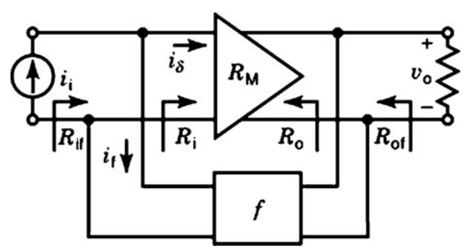

Figure 7. Input and output resistance for shunt–shunt feedback.

output sampled quantity, the forward gain, and the feedback ratio for each of the four feedback amplifier topologies. It is important to remember that the product, fA, must be dimensionless and, in the midband region of operation, positive.

In the previous section, all benefits of feedback were discussed except the modification of input and output impedance. The specific definitions of the four feedback amplifier topologies allow for that discussion to begin here. The mixing process alters the input impedance of a negative feedback amplifier. Heuristically, one can see that subtraction of a feedback quantity at the mixing junction increases the input quantity necessary for similar performance. Thus, subtracting current (shunt mixing) requires an increase in overall input current and decreases the input impedance. Similarly, subtracting voltage (series mixing) requires an increase in overall input voltage and increases input impedance.

Shunt Mixing Decreases the Input Resistance. For the *shunt–shunt* feedback amplifier (Fig. 7), the voltage across its input terminals (arbitrarily identified as v) and the input current, i_i, are related by the feedback amplifier input resistance, R_{if}:

$$v = i_i R_{if} \tag{23}$$

Similarly, the forward-gain amplifier has input quantities related by its input impedance, R_i:

$$v = i_\delta R_i \tag{24}$$

The two input currents, i_i and i_δ, are related through the forward gain and the feedback ratio:

$$i_\delta = i_i - i_f = i_i - i_\delta (fR_M) \Rightarrow i_i = i_\delta (1 + fR_M) \tag{25}$$

Therefore, combining Eqs. (32) and (33) yields

$$R_{if} = \frac{v}{i_i} = \frac{v}{i_\delta (1 + fR_M)} = \frac{R_i}{1 + fR_M} \tag{26}$$

The input resistance to feedback amplifier is the input resistance of the forward-gain amplifier reduced by a factor of the return difference. *Shunt–series* feedback amplifier input resistance is similarly derived (replacing R_M by A_I). The same basic reduction in input resistance occurs:

$$R_{if} = \frac{R_i}{1 + fA_I} \tag{27}$$

Series Mixing Increases Input Resistance. For the *series–series* feedback amplifier of Fig. 8, the voltage across its input terminals, v_i, and the input current (arbitrarily identified as i) are related by the feedback amplifier input resistance, R_{if}:

$$v_i = iR_{if} \tag{28}$$

Table 1. Feedback Amplifier Topology Parameters

	Shunt–shunt	Shunt–series	Series–shunt	Series–series
Input quantity, X_i	Current, i_s	Current, i_s	Voltage, v_s	Voltage, v_s
Output quantity, X_o	Voltage, v_o	Current, i_o	Voltage, v_o	Current, i_o
Forward gain, A	Transresistance, R_M	Current gain, A_I	Voltage gain, A_V	Transconductance, G_M
Feedback ratio, f	i_f/v_o	i_f/i_o	v_f/v_o	v_f/i_o

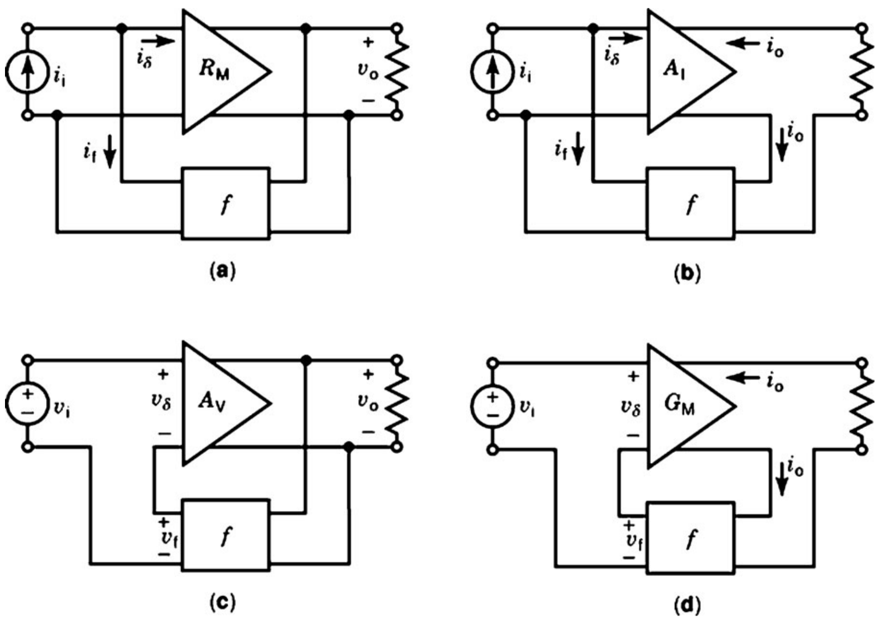

Figure 6. Feedback amplifier topologies. (a) Shunt–shunt feedback. (b) Shunt–series feedback. (c) Series–shunt feedback. (d) Series–series feedback.

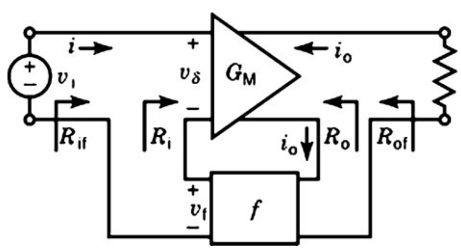

Figure 8. Input and output resistance for series–series feedback.

Similarly, the forward-gain amplifier has input quantities related by its input impedance, R_i:

$$v_\delta = iR_i \tag{29}$$

The two input voltages, v_i and v_δ, are related through the forward gain and the feedback ratio:

$$v_\delta = v_i - v_f = v_i - v_\delta(fG_M) \Rightarrow v_i = v_\delta(1 + fG_M) \tag{30}$$

Therefore, combining Eqs. (37) and (38) yields

$$R_{if} = \frac{v_i}{i} = \frac{v_\delta(1 + fG_M)}{i} = R_i(1 + fG_M) \tag{31}$$

The input resistance to feedback amplifier is the input resistance of the forward-gain amplifier increased by a factor of the return difference. *Series–shunt* feedback amplifier input resistance is similarly derived (replacing G_M by A_V). The same basic reduction in input resistance occurs:

$$R_{if} = R_i(1 + fA_V) \tag{32}$$

Resistors shunting the input, such as biasing resistors, often do not fit within the topological standards of series mixing. Thus, they must be considered separate from the feedback amplifier to model feedback amplifier characteristics properly using the techniques outlined in this and other articles. Examples of such resistors are found in the next section of this article.

The sampling process alters the output impedance of the feedback amplifier. As was the case for shunt mixing, shunt sampling decreases the output resistance: Series sampling increases the output resistance.

Shunt Sampling Decreases the Output Resistance. For the *shunt–shunt* feedback amplifier of Fig. 7, the output resistance is measured by applying a voltage source of value, v, to the output terminals with the input, i_i, set to zero value. A simplified schematic representation of that measurement is shown in Fig. 9. In this figure, the forward-gain amplifier has been shown with its appropriate gain parameter, R_M, and output resistance, R_o.

The output resistance of the feedback system is the ratio,

$$R_{of} = \frac{v}{i} \tag{33}$$

The current, i, is calculated from Ohm's law at the output of the amplifier:

$$i = \frac{v - R_M i_\delta}{R_o} \tag{34}$$

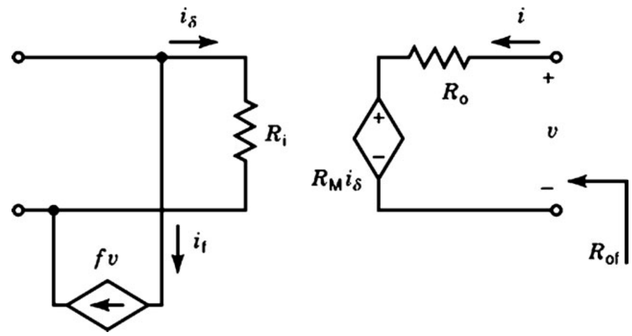

Figure 9. Schematic representation of shunt–shunt feedback for output resistance calculations.

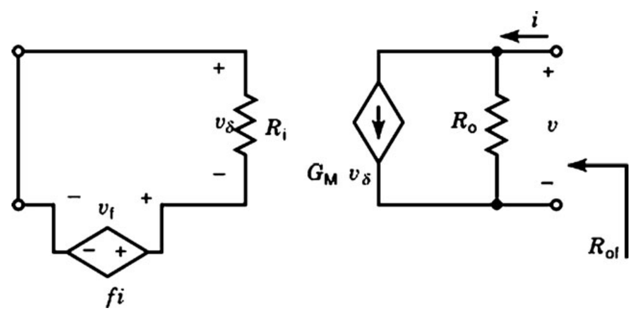

Figure 10. Schematic representation of series–series feedback for output resistance calculations.

In the case where the input current has been set to zero,

$$i_\delta = -i_f = -fv \tag{35}$$

Combining Eqs. (43) and (44) yields

$$i = \frac{v - R_M(-fv)}{R_o} \Rightarrow R_{of} = \frac{v}{i} = \frac{R_o}{1 + fR_M} \tag{36}$$

The output resistance of the feedback amplifier is the output resistance of the forward-gain amplifier decreased by a factor of the return difference. *Series–shunt* feedback amplifier output resistance is similarly derived (replacing R_M by A_V). The same basic reduction in input resistance occurs:

$$R_{of} = \frac{R_o}{1 + fA_V} \tag{37}$$

Resistors that shunt the output terminals, such as a load resistor, are considered as part of the feedback amplifier. The forward-gain parameter (R_M or A_V) must be calculated in a consistent fashion with the consideration of these elements.

Series Sampling Increases the Output Resistance. For the *series–series* feedback amplifier of Fig. 8, the output resistance is measured by applying a current source of value, i, to the output terminals with the input, v_i, set to zero value. A simplified schematic representation of that measurement is shown in Fig. 10. In this figure, the forward-gain amplifier has been shown with its appropriate gain parameter, A_V, and output resistance, R_o.

The output resistance of the feedback system is the ratio

$$R_{of} = \frac{v}{i} \tag{38}$$

The voltage, v, is given by

$$v = (i - G_M v_\delta)R_o \tag{39}$$

Since the input voltage, v_i, has been set to zero value,

$$v_\delta = -v_f = -fi \tag{40}$$

Combining Eqs. (48) and (49) yields

$$v = (i - G_M\{-fi\}R_o = i(1 + fG_M)R_o \tag{41}$$

The output resistance is then given by

$$R_{of} = \frac{v}{i} = (1 + fG_M)R_o \tag{42}$$

The output resistance of the feedback amplifier is the output resistance of the forward-gain amplifier increased by a factor of the return difference. *Shunt–series* feedback amplifier output resistance is similarly derived (replacing G_M by A_I). The same basic increase in input resistance occurs:

$$R_{of} = (1 + fA_I)R_o \tag{43}$$

Resistances shunting the output, such as load resistances, do not fit within the topological standards of series sampling. Thus, they must be considered separate from the feedback amplifier to model feedback amplifier characteristics properly using the techniques outlined in this and other articles. The forward-gain parameters, A_I and G_M, must be calculated excluding these resistances.

PRACTICAL FEEDBACK CONFIGURATIONS

Previous discussions of feedback and feedback configurations have been limited to idealized systems and amplifiers. The four idealized feedback schematic diagrams of Fig. 6 identify the forward-gain amplifier and the feedback network as two-port networks with a very specific property: Each is a device with one-way gain. Realistic electronic feedback amplifiers can only approximate that idealized behavior. In addition, in practical feedback amplifiers there is always some interaction between the forward-gain amplifier and the feedback network. This interaction most often takes the form of input and output resistive loading of the forward-gain amplifier. The division of the practical feedback amplifier into its forward-gain amplifier and feedback network is also not always obvious. These apparent obstacles to using idealized feedback analysis can be resolved through the use of two-port network relationships in the derivation of practical feedback amplifier properties. Once amplifier gain and impedance relationships have been derived, the utility of the two-port representations becomes minimal and is typically discarded.

Identification of the Feedback Topology

Feedback topology is determined through careful observation of the interconnection of the feedback network and forward-gain amplifier. Shunt mixing occurs at the input terminal of the amplifier. Thus, shunt mixing is identified by a connection of feedback network and the forward-gain amplifier at the input terminal of first active device within the amplifier; that is,

- At the base of a BJT for a common-emitter or common-collector first stage
- At the emitter of a BJT for a common-base first stage
- At the gate of a FET for a common-source or common-drain first stage, or
- At the source of a FET for a common-gate first stage

Series mixing occurs in a loop that contains the input terminal of the forward-gain amplifier and the controlling port of the first active device. The controlling port of a BJT in the forward-active region is the base-emitter junction: A FET in the saturation region is controlled by the voltage across the gate-source input port. Series mixing is characterized by a circuit element or network that is both connected to the output and in series with the input voltage and the input port of the first active device.

Identification of the sampling is derived from direct observation of the connection of the output of the basic forward amplifier and the feedback network. Shunt sampling is typically characterized by a direct connection of the feedback network to the output node: Series sampling implies a series connection of the amplifier output, the feedback network, and the load. Two tests performed at the feedback amplifier output can aid in the determination of sampling topology:

- If the feedback quantity vanishes for a short-circuit load, the output voltage must be the sampled quantity. Thus, zero feedback for a short-circuit load implies shunt sampling.
- If the feedback quantity vanishes for an open-circuit load, the output current must be the sampled quantity. Thus, zero feedback for an open-circuit load implies series sampling.

After the topological type has been identified, each amplifier must be transformed into a form that allows for the use of the idealized feedback formulations. This transformation includes modeling the amplifier and the feedback network with a particular two-port representation that facilitates combination of elements. Once the transformations are accomplished, the amplifier performance parameters are easily obtained using the methods previously outlined. The particular operations necessary to transform each of the four feedback amplifier topological types require separate discussion. Only the shunt–shunt topology is discussed in detail: The other three topologies use similar techniques that lead to the results shown in Fig. 13 and described in Table 2.

Shunt–Shunt Feedback: a Detailed Derivation. Figure 11 is a small-signal model representation of a typical shunt–shunt feedback amplifier. In this representation, the forward-gain amplifier and the feedback network have been replaced by their equivalent y-parameter two-port network representations so that parallel parameters can be easily combined. A resistive load has been applied to the output port; and, since shunt–shunt feedback amplifiers are transresistance amplifiers, a Norton equivalent source has been shown as the input. The forward-gain parameter of each two-port, y_{21}, is the transadmittance.

The basic feedback equation for a transresistance amplifier takes the form:

$$R_{\mathrm{Mf}} = \frac{R_{\mathrm{M}}}{1 + fR_{\mathrm{M}}} \tag{44}$$

The application of the basic feedback equation to this circuit in its current form is not immediately clear. It is necessary to transform the feedback amplifier circuit into a form that allows for easy application of the basic feedback equation, Eq. (53). Such a transformation must meet the previously stated feedback requirements:

- The forward-gain amplifier is to be a forward transmission system only—its reverse transmission must be negligible.
- The feedback network is to be a reverse transmission system that presents a feedback current, dependent on the output voltage, to the amplifier input port.

While a mathematically rigorous derivation of the transformation is possible, greater insight to the process comes with a heuristic approach.

The two-port y-parameter representation, in conjunction with the shunt-shunt connection, is used to describe the two main elements of this feedback amplifier so that all the input port elements of both two-port networks are in parallel. Similarly, all output port elements are in parallel. It is well known that circuit elements in parallel may be rearranged and, as long as they remain in parallel, the circuit continues to function in an identical fashion. Hence, it is possible, for analysis purposes only, to move elements conceptually from one section of the circuit into another (from the feedback circuit to the amplifier circuit or the reverse). The necessary conceptual changes made for the transformation are as follows:

- The source resistance, the load resistance, and all input and output admittances, y_{11} and y_{22}, are placed in the modified amplifier circuit. While inclusion of the source and load resistance in the amplifier seems, at first, counterproductive, it is necessary to include these resistances so that the use of the feedback properties produces correct results for input and output resistance (after appropriate transformations).
- All forward transadmittances, y_{21} (represented by current sources dependent on the input voltage, v_1), are placed in the modified amplifier circuit.
- All reverse transadmittances, y_{12} (represented by current sources dependent on the output voltage, v_o), are placed in the modified feedback circuit.

Table 2. Feedback Amplifier Analysis

Characteristic	Topology: Shunt–Shunt	Shunt–Series	Series–Shunt	Series–Series
Input, X_i	Current, i_s	Current, i_s	Voltage, v_s	Voltage, v_s
Output, X_o	Voltage, v_o	Current, i_o	Voltage, v_o	Current, i_o
Signal source	Norton	Norton	Thévenin	Thévenin
Input circuit	Include shunting resistances; set v_o 0	Include shunting resistances; set i_o 0	Exclude all shunt resistances; set v_o 0	Exclude all shunt resistances; set i_o 0
Output circuit	Include shunting resistances; set v_i 0	Exclude all shunt resistances; set v_i 0	Include shunting resistances; set i_i 0	Exclude all shunt resistances; set i_i 0
Feedback ratio, f	i_f/v_o	i_f/i_o	v_f/v_o	v_f/v_o
Forward gain, A	Transresistance, R_M	Current gain, A_I	Voltage gain, A_V	Transconductance, G_M
Input resistance	R_{if} $\frac{R_i}{1 \cdot fR_M}$	R_{if} $\frac{R_i}{1 \cdot fA_I}$	R_{if} $R_i(1 \cdot fA_V)$	R_{if} $R_i(1 \cdot fG_M)$
Output resistance	R_{of} $\frac{R_o}{1 \cdot fR_M}$	R_{of} $R_o(1 \cdot fA_I)$	R_{of} $\frac{R_o}{1 \cdot fA_V}$	R_{of} $R_o(1 \cdot fG_M)$

Notes
Input/Output Circuit: These procedures give the basic forward amplifier without feedback but including the effects of loading due to the feedback network.
Resistance: The resistance modified at shunted ports will include all shunting resistances that were included in the basic forward amplifier. The resistance modified at series ports will only include the resistances included in the basic forward amplifier. The true amplifier input and output impedance must be modified to reflect the point of measurement desired.

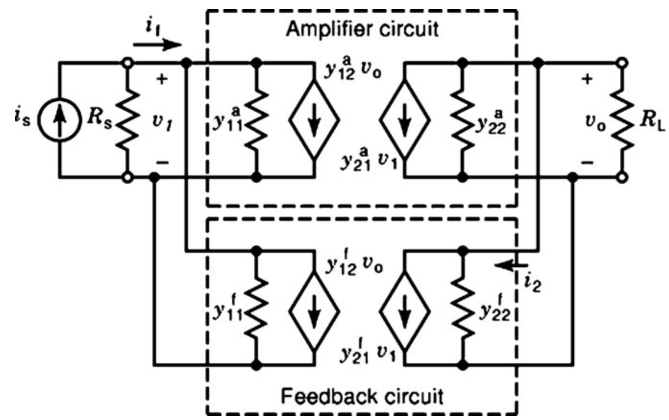

Figure 11. Two-port realization of a shunt–shunt feedback amplifier.

The dependent current source can be easily combined:

$$y^t_{12} = y^a_{12} + y^f_{12} \tag{45}$$

and

$$y^t_{21} = y^a_{21} + y^f_{21} \tag{46}$$

In virtually every practical feedback amplifier, the reverse transadmittance of the forward-gain amplifier is much smaller than that of the feedback network ($y_{12}{}^a < y_{12}{}^f$) and the forward transadmittance of the feedback network is much smaller than that of the forward-gain amplifier ($y_{21}{}^f < y_{21}{}^a$). Thus approximate simplifications of the amplifier representation can be made:

$$y^t_{12} = y^a_{12} + y^f_{12} \approx y^f_{12} \tag{47}$$

and

$$y^t_{21} = y^f_{21} + y^a_{21} \approx y^a_{21} \tag{48}$$

The shunt–shunt feedback amplifier circuit of Fig. 11 is, with these changes and approximations, thereby transformed into the circuit shown in Fig. 12.

This transformed circuit is composed of two simple elements:

- The original amplifier, with its input shunted by the source resistance; the feedback network short-circuit input admittance, $y_{11}{}^f$, and its output shunted by the load resistance; the feedback network short-circuit output admittance, $y_{22}{}^f$
- A feedback network composed solely of the feedback network reverse transadmittance, $y_{12}{}^f$

It is also important to notice that the input resistance, R_{if}, of this circuit includes the source resistance, R_s. As such, it is not the same as the input resistance of the true amplifier, R_{in}. The input resistance of the true amplifier can be obtained as

$$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} \tag{49}$$

Similarly, the output resistance, R_{of}, of this circuit includes the load resistance, R_L: Similar operations may be necessary to obtain the true output resistance of the amplifier.

The y-parameters of the feedback network can be obtained:

$$y^f_{11} = \left.\frac{i_f}{v_1}\right|_{v_o=0}, \quad y^f_{22} = \left.\frac{i_2}{v_o}\right|_{v_1=0}, \quad \text{and} \quad y^f_{12} = \left.\frac{i_f}{v_o}\right|_{v_1=0} \tag{50}$$

where i_2 is the current entering the output port of the feedback network (see Fig. 10). With the determination of these two-port parameters, the circuit has been transformed into a form that is compatible with all previous discussions. The forward-gain parameter (in this case, G_M) of the loaded basic amplifier must be calculated, while the feedback ratio has been determined from the two-port analysis of the feedback network:

$$f = y^f_{12} \tag{51}$$

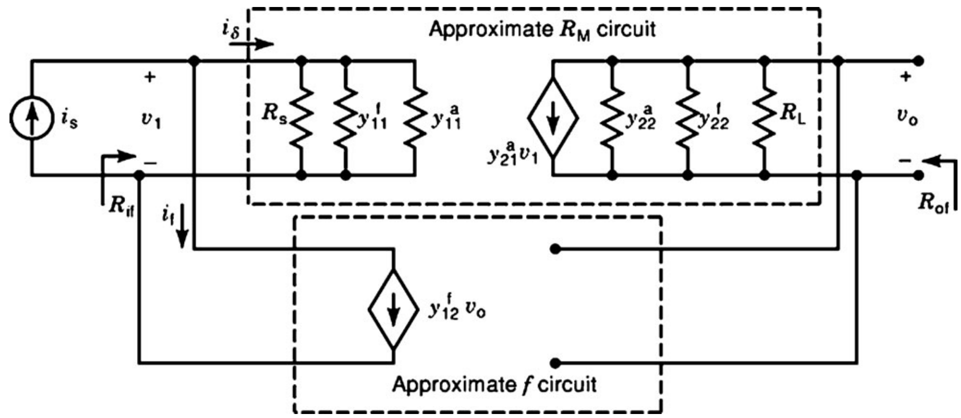

Figure 12. Redistributed shunt–shunt realization.

In the case of totally resistive feedback networks, the shunting resistances can be found in a simple fashion:

- $r_{in} = (y_{11}^f)^{-1}$ is found by setting the output voltage to zero value, $v_o = 0$, and determining the resistance from the input port of the feedback network to ground.
- $r_{out} = (y_{22}^f)^{-1}$ is found by setting the input voltage to zero value, $v_i = 0$, and determining the resistance from the output port of the feedback network to ground.

The feedback ratio, f, is simply the ratio of the feedback current, i_f, to the output voltage when the input port of the feedback network, v_i, is set to zero value. All idealized feedback methods can be applied to this transformed amplifier, and all previously derived feedback results are valid.

The other three feedback amplifier topologies can be similarly analyzed using various two-port parameters for analysis:

- Shunt–series—g parameters
- Series–shunt—h parameters
- Series–series—z parameters

Such analysis leads to a characterization of the loading of the basic forward amplifier as is described in Fig. 13. As is the case with the shunt–shunt topology, individual elements within the feedback network may appear more than once in the loaded basic forward amplifier equivalent circuit. Table 2 summarizes the analysis of feedback amplifier properties.

STABILITY IN FEEDBACK AMPLIFIERS

Under certain conditions, feedback amplifiers have the possibility of being unstable. This instability stems from the frequency-dependent nature of the forward gain of the basic amplifier, A, and the feedback factor, f. The frequency dependency is exhibited in the changes in magnitude and phase of the product, fA, as a function of frequency.

Instability can be visualized by studying the basic feedback equation as a function of frequency:

$$A_f(j\omega) = \frac{A(j\omega)}{1 + f(j\omega)A(j\omega)} \tag{52}$$

It is important that an amplifier be designed so that stability is present at all frequencies, not only those in the midband region. If the product $-f(j\omega)A(j\omega)$ approaches unity at any frequency, the denominator of Eq. (61) approaches zero value: The total gain of the amplifier approaches infinity. This condition represents an output that is truncated only by power supply limitations regardless of input magnitude and is an unstable condition that is intolerable in amplifiers. To avoid this instability, it is necessary to avoid a simultaneous approach of $|f(j\omega)A(j\omega)| = 1$ and $\angle f(j\omega)A(j\omega) = \pm 180^\circ$. Since each pole can only provide a phase shift of between 0 and -90°, the second condition is only possible for amplifiers that have high- or low-frequency responses described by three or more poles. Simultaneously satisfying both conditions can be avoided if the magnitude of fA is always less than unity when the phase angle of fA is $\pm 180^\circ$. Designers of feedback amplifiers typically verify that this is the case through the use of amplifier frequency-response plots.

Gain Margin and Phase Margin

A frequency-response plot of the loop gain, fA, for a typical amplifier is shown in Fig 14. The frequency at which $|f(j\omega)A(j\omega)| = 1$ is identified as ω_m and the frequency at which $\angle f(j\omega)A(j\omega) = -180^\circ$ is identified as ω_p. Since $\omega_m \neq \omega_p$, it is apparent that this is a stable amplifier—that is, the two instability conditions are not simultaneously met. It is, however, important to ensure that the two conditions are not met simultaneously with a margin of safety. The margin of safety is defined by the gain margin and the phase margin of the feedback amplifier.

Gain margin is defined as the difference in the loop gain magnitude (in decibels) between 0 dB (unity gain) and the loop gain magnitude at frequency ω_p:

$$\text{Gain margin} = -20\log|f(j\omega_p)A(j\omega_p)| \tag{53}$$

Phase margin is the difference between the loop gain phase angle at frequency ω_m and -180°:

$$\text{Phase margin} = \angle f(j\omega_m)A(j\omega_m) + 180^\circ \tag{54}$$

Each safety margin is shown in Fig. 14. It is common to specify the design of feedback amplifiers with gain and phase margins greater than 10 dB and 50°, respectively. These margins ensure stable amplifier operation

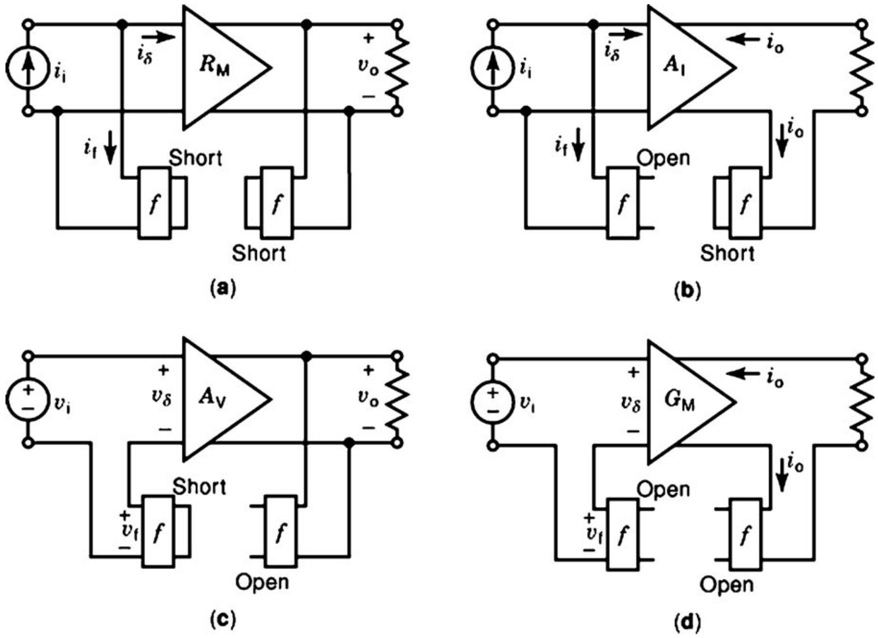

Figure 13. Feedback network loading of basic forward amplifier. (a) Shunt–shunt feedback. (b) Shunt–series feedback. (c) Series–shunt feedback. (d) Series–series feedback.

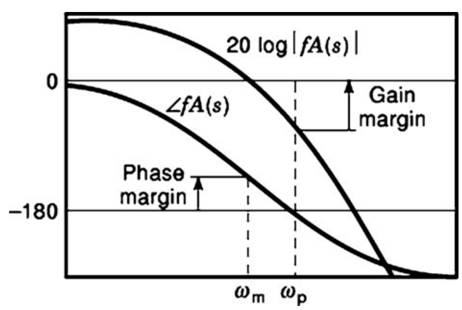

Figure 14. Gain margin and phase margin.

over component parameter variation, temperature change, and other variations found in typical amplifiers.

Compensation

Two fundamental techniques are available for ensuring amplifier stability:

- Reducing the midband loop gain, fA, of the amplifier
- Adding a compensation network to the amplifier to shape the loop gain frequency response so that the phase and gain margins are positive and in an acceptable range

Careful design is required in each of these cases to ensure stable amplifier operation over typical performance conditions.

In many cases, decreasing the loop gain to achieve stability is not an acceptable design possibility. Additionally, as is often the case in operational amplifier circuits, the feedback ratio, f, may be determined by the user rather than the amplifier designer and can range widely. In such cases, compensation networks are added within the feedback loop of the amplifier to increase the gain and phase margins. Such compensation networks add poles or a combination of poles and zeros to the loop gain characteristic. The most commonly used compensation techniques are as follows:

- Dominant pole compensation
- Lag–lead (pole–zero) compensation
- Lead compensation

Each technique modifies the gain and phase profiles of the basic forward amplifier through pole and zero manipulation.

In dominant pole compensation, the amplifier is modified by adding a dominant pole that is much smaller in magnitude than all other poles in the amplifier gain function: Typically it is chosen so that the loop gain, fA, reaches 0 dB at the frequency of the next pole (the first pole of the uncompensated amplifier). Consequently, the modified loop gain falls below 0 dB before the nondominant poles shift the total phase shift near 180° and the circuit is inherently stable. Dominant pole compensation will typically result in a phase margin of approximately 45°.

The location of the new compensation pole can be determined by modeling the loop gain response with a single pole and setting its value to 0 dB at the first pole of the uncompensated amplifier:

$$20\log|fA_o| - 20\log\left|1+\frac{j\omega_c}{\omega_{p1}}\right| = 0\ \text{dB} \tag{55}$$

Solving Eq. (64) for the compensation pole frequency, ω_c, results in:

$$\omega_c \approx \frac{\omega_{p1}}{fA_o} \tag{56}$$

If the design goals of the feedback amplifier includes a range of feedback ratios, the frequency of the compensation pole is determined by the maximum value of the feedback ratio. That is, ω_c is chosen to be the smallest value predicted by Eq. (65).

An example of dominant pole compensation is shown in Fig. 15 in the frequency domain. For clarity, the gain plots are represented by straight-line Bode approximations, while the exact phase plots are retained. The example amplifier is described by a midband gain of 60 dB with poles at 1 MHz, 5 MHz, and 50 MHz: the feedback ratio is $f = 0.1$.

The possibility of feedback amplifier instability is focused at the frequency where $|fA| = 1$ or equivalently where $|A|_{dB} = -20\log(f)$. For this particular three-pole example, instability may occur at $\omega_m \approx 21$ MHz. Here the phase margin is very small and negative ($\approx -7°$). After compensation, the focus is again centered where the gain plot (now compensated) intersects the negated feedback ratio plot. The addition of the compensation pole, ω_c, shifts this intersection to the frequency of the first uncompensated pole, ω_{p1}. For this example, the compensation pole is placed at 10 kHz and yields a phase margin of $\approx 43°$ and a gain margin of ≈ 14 dB.

Dominant pole compensation reduces the open-loop bandwidth drastically. Still, it is common in many circuits with inherently large gain: Operational amplifiers commonly utilize dominant pole compensation.

Lead–lag (or pole–zero) compensation is similar to dominant pole compensation with one major exception. In addition to a dominant pole, a higher-frequency zero is added. This zero is used to cancel the first pole of the uncompensated amplifier. The added dominant pole can then be chosen so that the gain reaches 0 dB at the frequency of the next pole (the second pole of the uncompensated amplifier). Lead–lag compensation has a distinct bandwidth advantage over dominant pole compensation.

The location of the new compensation pole can be determined in a similar fashion to the method utilized under dominant pole compensation with the exception that the loop gain (without the first original pole) is to reach 0 dB at the second pole of the uncompensated amplifier. Solving Eq. (64) for the compensation pole frequency, ω_c, results in:

$$\omega_c \approx \frac{\omega_{p2}}{fA_o} \tag{57}$$

As with dominant pole compensation, design goals including a range of feedback ratios lead to the determination of frequency of the compensation pole by the maximum value of the feedback ratio. That is, ω_c is chosen to be the smallest value predicted by Eq. (66).

The previously described, uncompensated amplifier is compensated with a lag–lead pole–zero pair and the frequency domain results are displayed in Fig. 16. The possibility of feedback amplifier instability is again focused at the intersection of the gain and the negated feedback ratio plots. After compensation, the focus is centered where the gain plot (now compensated) intersects the negated feedback ratio plot. The addition of the compensation pole, ω_c, and a zero at the first uncompensated pole, ω_{p1}, shifts this intersection to the frequency of the second uncompensated pole, ω_{p2}. The previously identified amplifier parameters lead to a compensation pole at 50 kHz, a positive phase margin of $\approx 48°$, and a gain margin of ≈ 20 dB.

Lead compensation can lead to the largest bandwidth of the three most common compensation networks. Here, as in lead–lag compensation, a pole and a zero are added. The zero is used to cancel the second pole of the uncompensated amplifier, and the added pole is positioned at a frequency higher than the zero. The objective is to reduce the phase shift of the uncompensated amplifier at the frequency where the loop gain reaches 0 dB (ω_m). Lead compensation can be extremely effective in feedback amplifiers where there are two or three dominant poles in the uncompensated amplifier.

The previously described, uncompensated amplifier is compensated with a lead pole–zero pair and the frequency domain results are displayed in Fig. 17. After compensation, the focus is again centered where the gain plot (now compensated) intersects the negated feedback ratio plot. The addition a zero at the second uncompensated pole, ω_{p2}, and a high-frequency compensation pole, ω_c, shifts this intersection a frequency beyond the second uncompensated pole, ω_{p2}. For this example, the high-frequency pole was chosen at 500 MHz. This design choice leads to a positive phase margin of $\approx 35°$ and a gain margin of ≈ 15 dB. Notice that with lead compensation there is no significant reduction in the frequency response of the feedback amplifier.

A passive component circuit implementation of each of the three compensation techniques is schematically shown in Fig. 18. For the circuit of Fig. 18(a), the compensation network component values are chosen so that

$$(R_p + R_o)C_p = \frac{fA_o}{\omega_{p1}} \tag{58}$$

where A_o, R_o, and ω_{p1} are the midband gain, the output resistance, and the first pole frequency, of the basic forward amplifier, respectively. For the circuit of Fig. 18(b), the compensation network component values are chosen so that

$$R_b C_c = \frac{1}{\omega_{p1}} \tag{59}$$

and

$$(R_a + R_b + R_o)C_c = \frac{fA_o}{\omega_{p2}} \tag{60}$$

For the circuit of Fig. 18(c), the compensation network component values are chosen so that

$$R_a C_c = \frac{1}{\omega_{p2}} \tag{61}$$

and

$$(R_a \| R_b)C_c \ll \frac{1}{\omega_{p1}} \tag{62}$$

While the placement of a compensation network at the output of the basic forward amplifier is an effective technique for feedback topologies with shunt sampling, other placement may be necessary. In particular, connections at the output of a feedback amplifier with series sampling are not within the feedback loop and are therefore invalid. In such cases, alternate placement of the compensation is

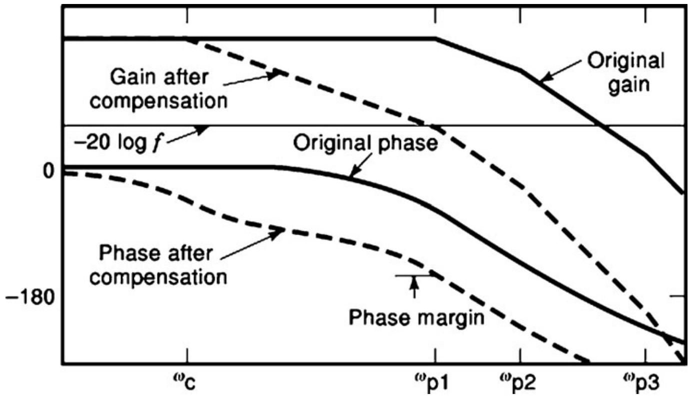

Figure 15. Bode diagram for a dominant pole compensated amplifier.

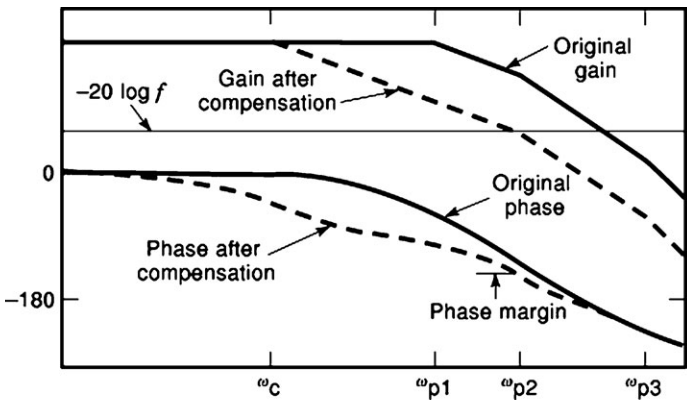

Figure 16. Bode diagram for a lag–lead compensated amplifier.

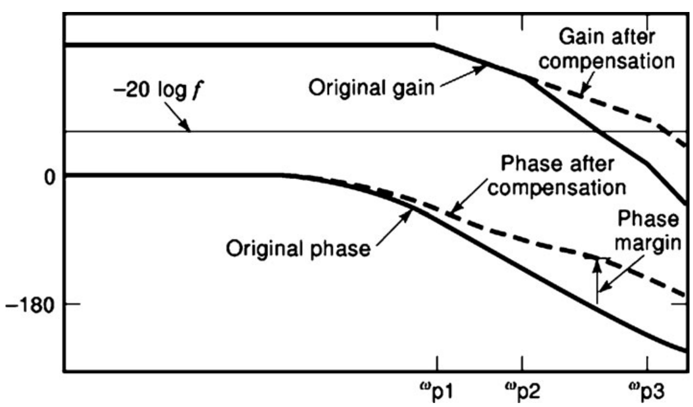

Figure 17. Bode diagram for a lead compensated amplifier.

necessary. One common placement intersperses the compensation network between individual gain stages of the amplifier. Similarly, it is possible to compensate a feedback amplifier within the feedback network rather than the basic forward amplifier. Unfortunately, analysis of compensation within the feedback network is often extremely complex due to the loading of the basic forward amplifier by feedback network components.

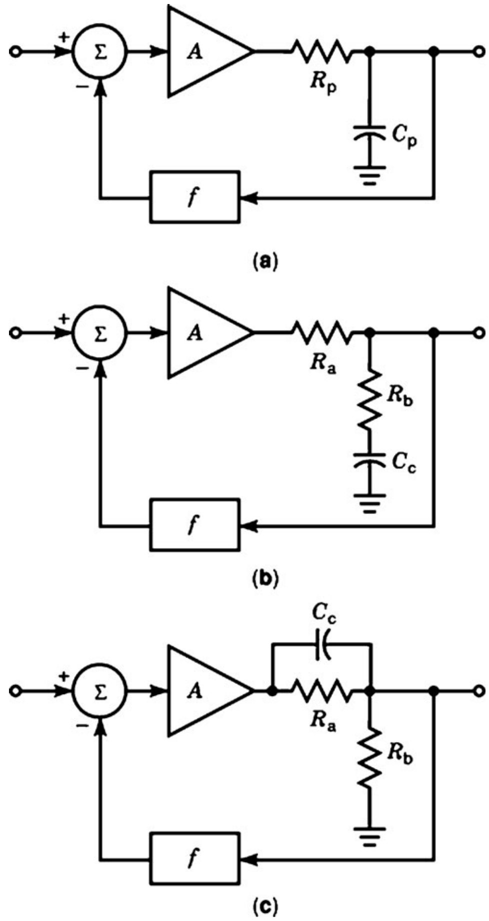

Figure 18. Compensation networks. (a) Dominant pole compensation. (b) Lag–lead (pole–zero) compensation. (c) Lead compensation.

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