DIGITAL FILTER SYNTHESIS

OVERVIEW OF THE PROBLEM

Direct digital synthesis is applied to problems in signal enhancing, such as the following case study: Assume that a faulty tape recorder is used to record a critical message. Due to all the hissing noises created by the tape and recording systems, the message is unintelligible. In order to know the message, it is imperative to improve the quality of the audio signal. All the actions performed to attain that goal are said to be intended to *enhance the signal.*

By extension, any effect that tends to reduce the quality of data is called *noise,* and data are said to be *corrupted by noise.* Data signals may originate not only from voice or audio, but from many other sources such as radio and television, industrial and medical instrumentation, statistical sampling, and so on. In general, one strives to manipulate the corrupted signal and recreate the original data.

A significant feature of signal-enhancing problems is that neither the desired data nor the corrupting noise can be known exactly. In the example of the recorded message, one must enhance the signal to get the message; the noise, in general, is caused by random effects. At best, one may be able to identify the signals as members of certain classes.

The enhancing considered in this article is based on the concept of *frequency-selective response* and listening is an example of this type of processing. The human (animal) ears are *frequency-selective* in the following essential ways: (1) Sounds of different frequencies are recognized as being different (e.g., a sound wave vibrating at 440 Hz is the note A, and one vibrating at 494 Hz corresponds to the note B); (2) Ears are sensitive to certain frequencies and completely deaf to others; (3) Ears can discriminate between sounds regardless of their relative loudness; that is, they can differentiate between the A note played by a violin and the same note played by a trumpet, even if one is significantly louder than the other. The last characteristic is an instance of frequency-selective processing. Both instruments create a sound that is not a pure vibration at 440 Hz, but has components that vibrate at frequencies that are characteristic of each instrument. The vibration at 440 Hz is by far the strongest, allowing the ear to recognize the note while the vibrations at other frequencies create an aural perception unique to the instrument. The fact that loudness is not significant indicates that the *relative strength* of the vibrations is more important than the actual intensity.

Any sound can be viewed as a combination of basic sound waves, each of a unique frequency and strength. The relative strength becomes a signature that permits the identification of a sound regardless of its level. One can characterize *classes of sounds* by the form in which their strength is concentrated at the various frequencies; and more significantly, one can create or enhance sounds by adjusting the signal energy at the various frequencies.

Frequency-Selective Enhancement

Frequency-domain analysis is a generalization of the representation of sound waves as combinations of basic vibrations. The Fourier transform is the mathematical tool used to determine how the energy of the signal is distributed at different frequencies. Devices that have a frequency-selective behavior are mathematically described by their *frequency response.* The class of devices considered in this article are said to be linear and time-invariant (1, pp. 46–54). For these systems the frequency response is a complex-valued function which determines how the device responds to any linear combination of sinusoids.

The statement that a continuous time system, also referred to as a filter, has a frequency response, $H(2\pi f)$, conveys the following information: (a) The variable, f, corresponds to the frequency in cycles per second (Hz), which is related to the frequency measured in *radians per second* by the relationship $\Omega = 2\pi f$. (b) $H(2\pi f)$ is a complex number with magnitude $|H(2\pi f)|$ and argument $\phi(2\pi f) = \angle H$. (c) If one applies as input to this system the signal $u(t) = \cos 2\pi ft$, the output of the system will be the sinusoid, $y(t) = |H(2\pi f)| \cos[2\pi ft + \phi(2\pi f)]$. In an extreme case, if for some frequency, f_0, one has $|H(2\pi f_0)| = 0$, then that particular frequency is completely eliminated from the output of the system.

There are many practical signal-enhancing applications which use frequency-selective processing based on an *ideal band-pass* behavior. With the specified frequency range $f_l \le f \le f_h$, the ideal device is given by

$$H_{bp}(2\pi f) = \begin{cases} 1; & f_l \le |f| \le f_h \\ 0; & \text{elsewhere} \end{cases}$$

Standard variations of this ideal behavior are: ideal *low-pass filter* ($f_l = 0$), *high-pass filter* ($f_h = \infty$), and a *stop-band filter* ($1 - H_{bp}(2\pi f)$). All ideal filters are unrealizable since, in theory, they require complete knowledge of past, present, and future values of the input signal in order to operate. Creating realizable filters which "approximate" the behavior of ideal ones is still a very active research area.

In the early days of radio, telephony, and industrial instrumentation, all signals were converted into voltages or currents. All filtering was performed using analog components such as resistors, inductors, and capacitors creating devices known as *passive filters.* With the advent of electronics—in particular, solid-state devices—it is now possible to emulate and improve on the traditional passive filters. The new devices require an external power supply and are referred to as *active filters.*

There is a significant amount of literature dealing with the design of the basic filters. The reader is referred to Ref. 2 (pp. 666–692) for designs based on analog filters, including Butterworth, Tchebychev, elliptic, and Bessel filters. There are also well-established computer-aided tools that only require from the user the specification of the frequency range and the type of the filter desired, based on which, they deliver a realizable filter which approximates the desired behavior (3).

DIRECT DIGITAL DESIGN

Nowadays, it is possible to use computers to perform the recording and enhancing of signals. Figure 1 establishes basic notation by showing a block diagram for filtering a signal using two techniques. In the figure, the continuous time signal, $x(t)$, is to be enhanced (e.g., the voltage from the audio amplifier going to the speakers). The block **Analog Filter** represents the conventional processing producing an enhanced signal, $y(t)$. The block **Sampling** represents the physical process of collecting the samples of the signal, $x(t)$; the actual device is an *analog-to-digital converter* (ADC). The sequence of values is modeled as a discrete time signal, $x_d[k]$. If the sampling process is ideal and the sampling period is T, then

$$x_{\rm d}[k] = x(kT)$$

The block *Digital Filter* corresponds to the numerical process that will be applied to the samples of the input signal to produce samples of the enhanced signal, $y_d[k]$. It is said to be a realization of a discrete time system. The block *Reconstruct* represents the process used by the computer to create an analog signal from a sequence of numerical values, $y_d[k]$. The device is called a *digital-to-analog converter* (DAC). A sound card (in a computer) or a CD player performs such a reconstruction operation to produce sounds from the digits stored in the computer or in the compact disc. The result is the *reconstructed enhanced signal,* $y_r(t)$ (4, pp. 91–99). An ideal, and hence unrealizable, reconstruction uses the formula

$$y_{\rm r}(t) = \sum_{k=-\infty}^{\infty} y_{\rm d}[k] \frac{\sin(\pi(t-nT)/T)}{\pi(t-nT)/T}$$

In practice, there are several efficient computational techniques to create reconstructed signals that satisfactorily approximate this ideal behavior (see also Ref. 5, pp. 100–110; Ref. 2, pp. 763–774).

The goal of direct digital synthesis is to define the numerical processing that must be done to the samples so that the complete sequence *sampling–digital filtering–signal reconstruction* creates desired enhancing effects on the signal.

Numerical algorithms designed to process the samples are considered *discrete time systems* (the independent variable is integer-valued). Conceptually, one can apply as input a discrete time sinusoid and characterize the frequency-selective behavior using a *discrete frequency response.* In this case, the frequency response is a 2π periodic function of the discrete frequency, ω. The notation $H_d(e^{j\omega})$ is used to emphasize the fact that the function is periodic. Knowledge of the discrete frequency response permits, ideally, the complete determination of the numerical algorithm that is required. Using the *exponential Fourier series expansion* (see Ref. 4, pp. 39–51) one can write

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_n e^{jn\omega}$$
$$h_n = \int_{-\pi}^{\pi} H(e^{j\omega}) \frac{d\omega}{2\pi}$$

The coefficients of the expansion, h_n, define the *impulse response* of the discrete time system. The enhanced digital signal values are computed either by using directly the *convolution formula*

$$y_{\rm d}[k] = \sum_{n=-\infty}^{\infty} h_n x_{\rm d}[k-n] \tag{1}$$

or by using an equivalent efficient numerical algorithm. Notice that when

$$h_n = 0; \qquad \forall n < 0$$

the value of $y_d[k]$ depends only on input samples, $x_d[m]$, where $m \leq k$. The algorithm is said to be causal and is suitable for on-the-fly processing where the input samples arrive in real time.

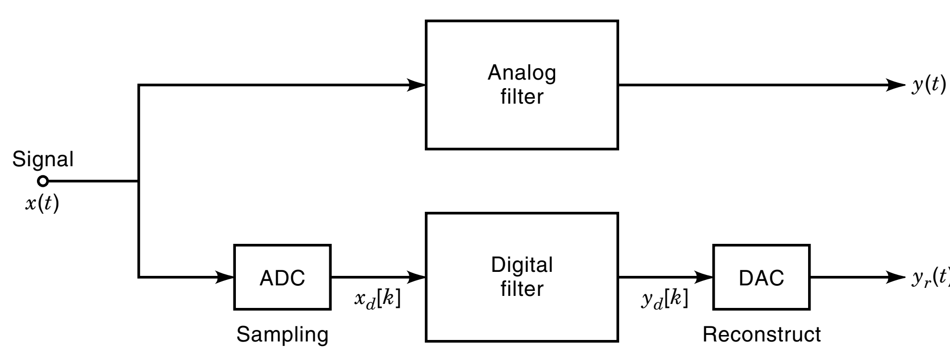

Figure 1. Analog and digital signal filtering.

If the number of nonzero coefficients is finite, one has a *finite impulse response* (FIR); otherwise it is *infinite impulse response* (IIR). For this latter class of filters, the formula in Eq. (1) is not an efficient algorithm and one must find an alternative representation. A standard form suitable for real time operation is the recursive expression

$$y_\mathrm{d}[k] + a_1 y_\mathrm{d}[k-1] + \cdots + a_m y_\mathrm{d}[k-m] = b_0 x_\mathrm{d}[k] + \cdots + b_m x_\mathrm{d}[k-m] \quad (2)$$

However, it is known that some impulse responses, $\{h_n\}$, do not allow such a recursive representation, and there are no general techniques to establish such a representation for a given impulse response. The direct determination of a recursive implementation with a desired frequency response is still an open research problem. References 6 and 7 provide partial results for the direct design of IIR filters.

Relating Continuous and Discrete Frequency Responses

It is possible to relate the continuous time frequency, f, to the discrete time frequency, ω. The procedure requires the development of a continuous time model for discrete time signals and leads to the equation

$$2\pi f T = \omega, \qquad -\pi \le \omega \le \pi$$

This equation highlights two important issues in digital signal processing: (1) Under the ideal reconstruction scheme, the analog signal created from the samples cannot contain any frequencies above the frequency $f_N = 1/2T$; that is, it is *bandlimited.* (2) If the original analog signal, $x(t)$, contains frequencies above the value $1/2T$, it will not be possible to recreate the analog signal from the samples; even the ideal reconstruction will be different. This phenomenon is known as *aliasing,* and the frequency, f_N, is called the Nyquist frequency in honor of *H*. Nyquist (8), who first stated the result in his Sampling Theorem (2, pp. 21–33).

The relationship has also been used to define discrete time frequency responses when the desired continuous time response is known. In one approach (see Ref. 4, pp. 97–99), one period of the discrete frequency response is defined as follows:

$$H_d(e^{j\omega}) = H(2\pi f), \qquad f = \frac{\omega}{2\pi T}\ |\omega| \le \pi$$

This approach actually relates the discrete impulse response to samples of the continuous time frequency response. The reader is referred to Ref. 4 (pp. 406–438) for techniques based on *discretization of the transfer function.*

In a simpler approach, the relationship between discrete- and continuous-time frequency is used to define basic frequency-selective responses for discrete-time systems. Different techniques are used to determine a numerical processing algorithm which gives (approximately) the desired frequency response.

Example 1. In theory, the human ear can perceive frequencies up to 20 kHz. For simple voice applications, one can preserve intelligibility with a much smaller bandwidth. The smaller the bandwidth, the larger the amount of noise that is eliminated. For most applications this increases signal-to-noise ratio; but if the bandwidth is made too narrow, then the message itself will be destroyed. If understanding the message is the goal, one may recognize sentences even if all frequencies above 2 kHz are eliminated. Bandlimiting is thus a viable technique to increase signal-to-noise ratio and would be one of the techniques considered for recovering the message in the opening case study. As an example, assume that the voice signal needs to be enhanced by eliminating all frequency components below $f_\mathrm{l} = 60$ Hz and above $f_\mathrm{h} = 12{,}000$ Hz. The signal is sampled at a frequency of $f_\mathrm{s} = 44.1$ kHz, or with a sampling period, $T = 1/f_\mathrm{s}$, which is the normal sampling rate for storing music in compact discs.

An ideal analog filter to perform this operation is the filter

$$H(2\pi f) = \begin{cases} 1; & f_\mathrm{l} < |f| < f_\mathrm{h} \\ 0; & \text{elsewhere} \end{cases}$$

To each of the frequencies, f_l and f_h, one associates the discrete frequencies, $\omega_\mathrm{l} = 2\pi f_\mathrm{l} T$, $\omega_\mathrm{h} = 2\pi f_\mathrm{h} T$. One period of the ideal discrete frequency response will be

$$H_\mathrm{d}(e^{j\omega}) = \begin{cases} 1; & \omega_\mathrm{l} \le |\omega| \le \omega_\mathrm{h} \\ 0; & \text{elsewhere in } [-\pi\ \pi] \end{cases}$$

Once the desired ideal frequency response is defined, one can use the Fourier series expansion to determine the coefficients, h_n, of the impulse response. The result in this case is

$$h_n = \begin{cases} \dfrac{1}{\pi n}(\sin n\omega_h - \sin n\omega_\mathrm{l}); & n \ne 0 \\ 0; & n = 0 \end{cases}$$

It is clear from this expression that there are infinitely many nonzero values of h_n and the convolution representation in Eq. (1) is not efficient. Moreover, there are nonzero values of h_n for negative values of n, showing that the system is noncausal and cannot be implemented in real time. It is also apparent that the values of h_n get smaller and smaller as n increases. From a practical point of view, one could consider that after a certain value N, they are zero. This effectively implies a truncation of the impulse response—that is, using an FIR approximation. Once the response is truncated to a finite number of terms, the problem of $h_n \ne 0$ for $n < 0$, can be solved by introducing a time delay in the computation of the response (see Ref. 4, pp. 250–254).

There are many situations where a simple truncation of the impulse response introduces a significant deterioration of performance (see Ref. 4, pp. 444–462). A simple and effective technique to overcome this is to perform the truncation by means of a smooth window function. The new coefficients of the impulse response are given by

$$\hat{h}_n = w(n) h_n$$

An example of a window function is the *generalized Hamming* window:

$$w_\mathrm{H}(n) = \begin{cases} \alpha + (1-\alpha)\cos\left(\dfrac{2\pi}{N}\right); & -\left(\dfrac{N-1}{2}\right) \le n \le \dfrac{N-1}{2},\ 0 \le \alpha \le 1 \\ 0; & \text{elsewhere} \end{cases}$$

If $\alpha = 0.54$, the window is called a Hamming window; and if $\alpha = 0.5$, it is called a Hanning window (see Ref. 9, pp. 92–93).

The computation of the impulse response coefficients can be simplified to a great extent if one resorts to the discrete Fourier transform (DFT), $H(k)$, given by

$$H(k) = \sum_{n=0}^{N-1} h_n e^{-j(2\pi nk/N)}; \qquad k = 0, 1, \ldots, N-1 \tag{3}$$

The values, $H(k)$, correspond to samples of the Fourier transform for an N-periodic signal at the frequency points, $\omega_k = 2\pi k/N$, where $k = 0, 1, \ldots, N - 1$. For direct digital filter design, one would proceed as follows:

1. Define the discrete frequency selective behavior that is required.
2. Select the number, N, that determines the number of coefficients, h_n, to be used. This value is application dependent and can be as low as 4 and as high as 256 or more.
3. Determine the values of the desired discrete frequency-selective response, $H(k)$, at the frequencies, $\omega_k = 2\pi k/N$, where $k = 0, 1, \ldots, N - 1$.
4. Use the definition of the DFT as a system of algebraic equations and solve for the values, h_n. This operation is the *computation of the inverse DFT.* The computation is performed with high numerical efficiency using a variety of fast Fourier transform (FFT) algorithms (see Ref. 10, pp. 114–152).

Remark 1. Using the DFT algorithm to determine the impulse response corresponding to a discrete frequency response, $H(e^{j\omega})$, has the implicit assumption that the impulse response is a periodic function. Therefore, there is an aliasing effect in this approach. If $h_p(n)$ is the nth coefficient computed using the DFT technique, its relationship to the exact coefficients, determined from the series expansion, is given by

$$h_p(n) = \sum_m h(n - mN)$$

The values $h(k)$ are the exact values of the impulse response computed using the Fourier series expansion. In practice, by using a suitably large value of N, the aliasing effect can usually be neglected.

Remark 2. In principle, one can specify any frequency-selective behavior which may lead to coefficients, h_n, that are complex numbers. By placing constraints on the frequency response, $H_d(e^{j\omega})$, one can guarantee that the coefficients will be real numbers. Also, it is known that if the argument of the discrete frequency response is a linear function of the discrete frequency over the interval $[-\pi\ \pi]$—linear-phase digital filters—one can obtain performances closer to the ideal using fewer terms than nonlinear-phase filters.

OPTIMAL DIGITAL FILTER DESIGN

The techniques described in the previous section are aimed at determining a numerical algorithm that approximates the behavior of an ideal frequency-selective processing algorithm. It is significant that they cannot ensure that the *best approximation* is being used. Moreover, the techniques do not permit the design of special, or customized, frequency-selective characteristics. This section presents an overview of the extensive literature on optimal digital filter design, which can be used to define any type of filter. In this approach, the design is transformed into a nonlinear optimization problem over a set of parameters, or filter coefficients. The user must (a) specify a merit index or cost function, which measures the quality of a given solution and (b) specify possible constraints in the optimization process. The various methods differ in the choice of cost function and constraints.

If the desired frequency behavior is IIR, the problem cannot be solved or implemented unless the filter can be put in the form of a recursive equation such as Eq. (2). Even in this case, the frequency response is a highly nonlinear function of the coefficients, (a_l, b_l), and a general solution to even a simple optimization problem is not known. Several special cases have been considered in the literature, and the readers are referred to Refs. 6, 7, and 9 (pp. 75–293). The FIR case has received much more attention because the frequency response is a linear function of the coefficients. Moreover, FIR filters have good numerical properties and can be easily implemented, even if the number of coefficients (taps) is large.

A causal FIR filter with N taps has a frequency response

$$H(e^{j\omega}) = \sum_{k=0}^{N-1} h_k e^{-jk\omega}$$

Let $\boldsymbol{h} = \mathrm{col}\{h_0, h_1, \ldots, h_{N-1}\}$ be the vector of real-valued variables over which the optimization is performed. The desired specification is given as an *ideal,* or desired, frequency response, $H_d(e^{j\omega})$. A commonly used merit index is the L_p, $1 \le p < \infty$, one which has the cost function of the form

$$J(\boldsymbol{h}) = \int_{-\pi}^{\pi} W(\omega)|H_d(e^{j\omega}) - H(e^{j\omega})|^p \frac{d\omega}{2\pi} \tag{4}$$

The function $W(\omega)$ is a *weighting function* used to assign different weights, corresponding to the relative importance of different frequency ranges (e.g., zero on a frequency band of no importance to the designer). The value of the exponent, p, has a definite effect on the solution. If $p = 1$, all errors have the same importance while $p > 1$ assigns increasingly more significance to larger errors. The most common constraint is the requirement of *linear phase,* which can be easily incorporated as a symmetry condition in the coefficients, leading to standard *canonical filter types* (4, pp. 250–270).

Least-Squares Solution

The special case, $p = 2$, is called the *least-squares method* and has a particularly simple analytical solution. Define

$$\Omega_N = \mathrm{col}\{e^{-jk\omega};\ k = 0, 1, \ldots, N-1\}$$
$$Q = 2Re\left\{\int_{-\pi}^{\pi} W(\omega)\Omega_N \overline{\Omega_N}^T \frac{d\omega}{2\pi}\right\} \tag{5}$$

$$u = Re\left\{\int_{-\pi}^{\pi} W(\omega)\Omega_N \frac{d\omega}{2\pi}\right\} \tag{6}$$

where Re$\{\cdot\}$ indicates *the real part of* $\{\cdot\}$, and Ω_N^T denotes the transposed (row) vector. The optimal solution is known to exist and is unique whenever the matrix, Q in Eq. (5) is nonsingular. The expression for the solution is

$$\boldsymbol{h} = Q^{-1}u \tag{7}$$

The matrix, Q, is nonsingular for most practical cases of the weighting function. For example, if $W(\omega)$ is piecewise constant, it is sufficient to require that it be nonzero over at least one interval (i.e., nontrivial). The trivial case, $W \equiv 1$, leads to $Q = I$ and $\boldsymbol{h} = u$.

In spite of the analytical solution, one normally uses numerical algorithms to compute the solution. A popular algorithm uses the *conjugate gradient technique* (11), which has the advantage of being relatively simple to implement and has a guaranteed convergence to the solution in a finite number of iterations.

A variation of the least-squares approach is presented in Ref. 12. Here, instead of defining an ideal response, the authors constrain the filter at specific frequencies and convert the minimization into an eigenvalue problem.

The existence of a formula for the best filter makes the least-squares method very popular. However, it has recognized limitations: Because of the squaring operation, small errors are given less importance and may exist over larger frequency ranges than with other methods; and the merit index will accept large errors over small frequency ranges. As a result, the approximations that are obtained may display large peaks (overshoots) and less desirable soft transitions, as opposed to the sharp transitions of ideal filters. The choice of weighting function will have a significant effect on the final solution obtained; but there are no established criteria for its definition. Lawson (13) provided a significant link between this method and the equiripple approach presented next. He proved that the equiripple solution can be obtained as a least-squares solution for some weighting function, and he developed an iterative algorithm to modify the weighting function. The algorithm can be used to make the optimization less sensitive to the choice of weighting function, or as a way to improve on a given least squares solution.

Equiripple Solution

The *worst-case design* avoids the problems in the least-squares method by minimizing the largest error. The cost function in this case is

$$J(\boldsymbol{h}) = \max_{-\pi \leq \omega \leq \pi} W(\omega)|H_{\mathrm{d}}(e^{j\omega}) - H(e^{j\omega})| \tag{8}$$

This case can be shown to be equivalent to the L_p case for $p = \infty$. Only special cases of this cost function have known solutions (14,15), the most important being the case where the function $W(\omega)$ is piecewise constant and the FIR filter is linear-phase. Using Tchebychev polynomials, it is possible to give necessary conditions to characterize the optimal solution: It alternates from maximum to minimum a known number of times (which depends on the number of parameters); all maxima have the same value and so do all the various minima. This characteristics give the solution the property called *equiripple.* The case does not have an analytical closed-form solution, but there are excellent numerical algorithms such as the Parks–McClellan technique (see Refs. 16 and 17). Kootsookos et al. (18), considered the case, $W(\omega) \equiv 1$, and developed an algorithm based on the solution of a number of related extension (Nehari) problems. Their algorithm compares favorably with the exchange algorithm implemented by Parks and McClellan, with the added advantages that linear phase is not required and the algorithm is applicable to multivariable filters (filters which process several inputs at the same time).

Extensions and Further Readings

The optimal design of IIR filters is an active area of research with only partial results available. Least-squares techniques have been applied to special forms, such as all pole systems (see Ref. 4, pp. 701–725). Zhang and Ikawara (6) use the worst-case criterion and reformulate the design problem as a generalized eigenvalue problem.

Perhaps the strongest extension of the direct design of digital filters has been motivated by problem in computer vision and image processing. For such application, the signals (images) are modeled as functions of two independent *time* variables and are referred to as multidimensional signals. Least-squares design of FIR filters has been formulated and solved (e.g., in Ref. 19). Theoretical results characterizing multidimensional equiripple solutions are not complete, but some practical results have been attained by using a modification of Lawson's algorithm (20).

The technical literature on digital filtering is vast. The reader can find good lists of references in textbooks such as Refs. 2 and 4 for conventional signals and Ref. 21 for multidimensional signals.

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DIGITAL HALFTONING. See HALFTONING.

DIGITAL IMAGE PROCESSING. See IMAGE PROCESSING.

DIGITAL LOGIC. See BOOLEAN FUNCTIONS; NAND CIRCUITS; NOR CIRCUITS.

DIGITAL LOGIC BASED ON MAGNETIC ELEMENTS. See MAGNETIC LOGIC.

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