CIRCUIT TUNING

Circuit tuning refers to the process of adjusting the values of electronic components in a circuit to ensure that the fabricated or manufactured circuit performs to specifications. In digital circuits, where signals are switched functions in the time domain and correct operation depends largely on the active devices switching all the way between their ON and OFF states, tuning in the sense discussed in this chapter is rarely necessary. In analog continuous-time circuits, however, signals are continuous functions of time and frequency so that circuit performance depends critically on the component values. Consequently, in all but the most undemanding applications with wide tolerances, correct circuit operation almost always requires some form of tuning. Naturally, components could be manufactured with very tight tolerances, but the resulting fabrication costs would become prohibitive. In practice, therefore, electronic components used in circuit design are never or only rarely available as accurately as the nominal design requires, so we must assume that they are affected by fabrication and manufacturing tolerances. Furthermore, regardless of whether a circuit is assembled in discrete form with discrete components on a printed circuit board (as a hybrid circuit), or in integrated form on an integrated circuit chip, the circuit will be affected by parasitic components and changing operating conditions, all of which contribute to inaccurate circuit performance. Consider, for example, the requirement of implementing as a hybrid circuit a time of 1 s for a timer circuit via an RC time constant $\tau = RC$ with an accuracy of 0.1%. Assume that R and C are selected to have the nominal values $R = 100$ kΩ and $C = 10$ μF, that inexpensive chip capacitors with ±20% tolerances are used, and that the desired fabrication process of thin-film resistors results in components with ±10% tolerances. The fabricated time constant can therefore be expected to lie in the range

$$0.68\,\text{s} \le \tau = 100\,\text{k}\Omega(1 \pm 0.1)10\,\mu\text{F}(1 \pm 0.2) \le 1.32\,\text{s}$$

In other words, the τ-error must be expected to be ±32%, which is far above the specified 0.1%. Tuning is clearly necessary. Because capacitors are difficult to adjust and accurate capacitors are expensive, let us assume in this simple case that the capacitor was measured with 0.05% accuracy as $C = 11.125$ μF (i.e., the measured error was +11.25%). We can readily compute that the resistor should be adjusted (trimmed) to the nominal value $R = \tau/C = 1\,\text{s}/11.125\,\mu\text{F} = 89.888$ kΩ within a tolerance of ±45 Ω to yield the correctly

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implemented time constant of 1 s with ±0.1% tolerances. Observe that tuning generally allows the designer to construct a circuit with less expensive wide-tolerance parts because subsequent tuning of these or other components permits the errors to be corrected. Thus, C was fabricated with 20% tolerances but measured with a 0.05% error to permit the resistor with fabrication tolerances of 10% to be trimmed to a 0.05% accuracy. Note that implied in this process is the availability of measuring instruments with the necessary accuracy.

Tuning has two main purposes. Its most important function is to correct errors in circuit performance caused by such factors as fabrication tolerances such as in the preceding example. Second, it permits a circuit's function or parameters, such as the cut-off frequency of a given low-pass filter, to be changed to different values to make the circuit more useful or to be able to accommodate changing operating requirements. But even the best fabrication technology together with tuning will not normally result in a circuit operating with *zero* errors; rather, the aim of tuning is to trim the values of one or more, or in rare cases of all, components until the circuit's response is guaranteed to remain within a specified *tolerance range* when the circuit is put into operation. Figure 1 illustrates the idea for a low-pass filter. Examples are a gain error that is specified to remain within ±0.05 dB, the cut-off frequency f_c of a filter that must not deviate from the design value of, say, $f_c = 10$ kHz by more than 85 Hz, or the gain of an amplifier that must settle to, say, 1% of its final value within less than 1 μs. As these examples indicate, in general, a circuit's operation can be specified in the time domain, such as a transient response with a certain highest permissible overshoot or a maximal settling time, or in the frequency (s) domain through an input-output transfer function with magnitude, phase, or delay specifications and certain tolerances (see Fig. 1). This article focuses on the tuning of *filters,* that is, of frequency-selective networks. Such circuits are continuous functions of components, described by transfer functions in the s domain, where tuning of design parameters (e.g., cut-off frequency, bandwidth, quality factor, and gain), is particularly important in practice. The concepts discussed in connection with filters apply equally to other analog circuits. Obviously, in order to tune (adjust) a circuit, that circuit must be tunable. That is, its components must be capable of being varied in some manner (manually or electronically) by an amount sufficient to overcome the consequences of fabrication tolerances, parasitic effects, or other such factors.

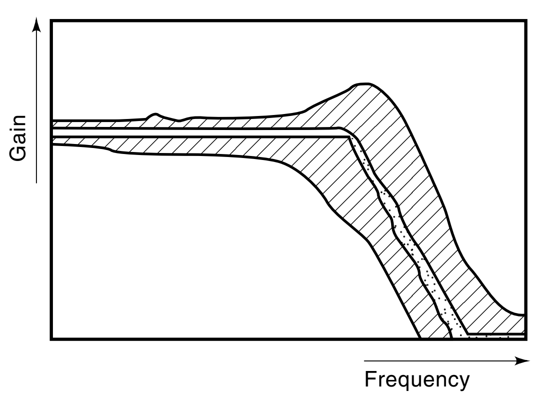

Figure 1. The shaded area in the gain vs. frequency plot shows the operating region for a low-pass filter that must be expected based on raw (untuned) fabrication tolerances; the dotted region is the acceptable tolerance range that must be maintained in operation after the filter is tuned.

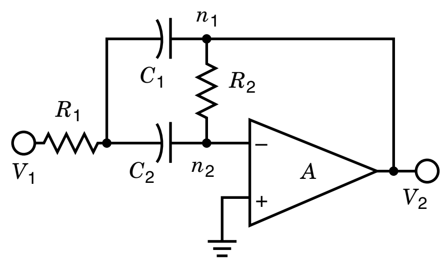

Figure 2. Active *RC* bandpass filter.

An example will help to illustrate the discussion and terminology. Consider the simple second-order active band-pass circuit in Fig. 2. Its voltage transfer function, under the assumption of ideal operational amplifiers, can be derived to be

$$T(s) = \frac{V_2}{V_1} = -\frac{b_1 s}{s^2 + a_1 s + a_0} = -\frac{\dfrac{1}{R_1C_1}s}{s^2 + \dfrac{1}{R_2}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}} \tag{1}$$

We see that $T(s)$ is a continuous function of the circuit components, as are all its coefficients that determine the circuit's behavior:

$$b_1 = \frac{1}{R_1C_1},\ a_1 = \frac{C_1 + C_2}{R_2C_1C_2},\ a_0 = \frac{1}{R_1R_2C_1C_2} \tag{2}$$

Just as in the earlier example of the *RC* time constant, the coefficients will not be implemented precisely if the component values have fabrication tolerances. If these component tolerances are "too large," generally the coefficient errors will become "too large" as well, and the circuit will not function correctly. In that case, the circuit must be tuned. Furthermore, circuits are generally affected by parasitic components. Parasitic components, or *parasitics,* are physical effects that often can be modeled as "real components" affecting the circuit's performance but that frequently are not specified with sufficient accuracy and are not included in the nominal design. For instance, in the filter of Fig. 2, a parasitic capacitor can be assumed to exist between any two nodes or between any individual node and ground; also, real "wires" are not ideal short circuit connections with zero resistance but are resistive and, at high frequencies, even inductive. In the filter of Fig. 2, a parasitic capacitor C_p between nodes n_1 and n_2 would let the resistor R_2 look like the frequency-dependent impedance $Z_2(s) = R_2/(1 + sC_pR_2)$. Similarly, real resistive wires would place small resistors r_w in series with C_1 and C_2 and would make these capacitors appear lossy. That is, the capacitors $C_i, i = 1, 2$, would present admittances of the form $Y_i(s) = sC_i/(1 + sC_ir_w)$. Substituting $Z_2(s)$ and $Y_i(s)$ for R_2 and C_i, respectively, into Eq. (1) shows that, depending on the frequency range of interest and the element values, the presence of these parasitics changes the coefficients of the transfer function, maybe even its type, and consequently the circuit's performance. Similarly, when changes occur in environmental operating conditions, such as bias voltages or temperature, the performance of electronic devices is altered, and as a result the fabricated circuit may not perform as specified.

As discussed by Moschytz (1, Sec. 4.4, pp. 394–425), and Bowron and Stevenson (2, Sec. 9.5, pp. 247–251), the operation of tuning can be classified into *functional* and *deterministic* tuning. In functional tuning, the designed circuit is assembled, and its performance is measured. By analyzing the circuit, we can identify which component affects the performance parameter to be tuned. These predetermined components are then adjusted in situ (i.e., with the circuit in operation), until errors in performance parameters are reduced to acceptable tolerances. The process is complicated by the fact that tuning is most often interactive, meaning that adjusting a given component will vary several circuit parameters; thus iterative routines are normally called for. As an example, consider again the active *RC* filter in Fig. 2. If its bandpass transfer function, Eq. (1), is expressed in the measurable terms of center frequency ω_0, the pole quality factor $Q = \omega_0/\Delta\omega$, the parameter that determines the filter's bandwidth $\Delta\omega$, and midband (at $s = j\omega_0$) gain K as

$$T(s) = -\frac{\dfrac{1}{R_1C_1}s}{s^2 + \dfrac{1}{R_2}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}} = -\frac{K\dfrac{\omega_0}{Q}s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} \quad (3)$$

These parameters are expressed in terms of the circuit components and we arrive at the more meaningful and useful design equations

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}, Q = \sqrt{\frac{R_2}{R_1}}\frac{\sqrt{C_1C_2}}{C_1 + C_2}, K = \frac{R_2}{R_1}\frac{C_2}{C_1 + C_2} \quad (4)$$

instead of Eq. (2). It is clear that varying any of the passive components will change all three filter parameters, so that expensive and time-consuming iterative tuning is required. However, functional tuning has the advantage that the effects of all component and layout parasitics, losses, loading, and other hard-to-model or hard-to-predict factors are accounted for because the performance of the complete circuit is measured under actual operating conditions. In general, more accurate results are obtained by basing functional tuning on measurements of phase rather than of magnitude because phase tends to be more sensitive to component errors.

Deterministic tuning refers to calculating the needed value of a component from circuit equations and then adjusting the component to that value. We determined the resistor $R = \tau/C = 89.888$ kΩ to set a time constant of 1 s at the beginning of this article in this manner. Similarly, from Eq. (4) we can derive the three equations in the four unknowns R_1, R_2, C_1, and C_2

$$R_2 = \frac{Q}{\omega_0}\left(\frac{1}{C_1} + \frac{1}{C_2}\right), R_1 = \frac{1}{R_2\omega_0^2C_1C_2}, C_1 = \frac{1}{KR_1}\frac{Q}{\omega_0} \quad (5)$$

with C_2 a free parameter. That there are more circuit components than parameters is normal, so the additional "free" elements may be used at will, for example, to achieve practical element values or element-value *spreads* (i.e., the difference between the maximum and minimum of a component type, such as $R_{\max} - R_{\min}$). Technology or cost considerations may place further constraints on tuning by removing some components from the list of tunable ones. Thus, in hybrid circuits with thin- or thick-film technology as in the preceding example, the capacitors will likely be fixed; only the two resistors will be determined as in Eq. (5) from the prescribed circuit parameters ω_0 and Q and the selected and measured capacitor values. This choice leaves the midband gain fixed at the value $K = Q/(\omega_0C_1R_1)$. Precise deterministic tuning requires careful measurements and accurate models and design equations that, in contrast to the idealized expressions in Eq. (5), describe circuit behavior along with loss, parasitic, and environmental effects. As we saw in Eq. (5), the equations that must be solved are highly nonlinear and tend to be very complex, particularly if parasitic components also are involved. Computer tools are almost always used to find the solution. Typically, automatic laser trimming is employed to tune the resistors to the desired tolerances (e.g., 0.1%). A second tuning iteration using functional tuning may be required because the assembled circuit under power may still not meet the specifications as a result of further parasitic or loading effects that could not be accounted for in the initial deterministic tuning step.

SENSITIVITY

We mentioned earlier that a filter parameter P depends on the values k_i of the components used to manufacture a circuit, $P = P(k_i)$, and that real circuit components or parts can be realized only to within some tolerances $\pm\Delta k_i$. That is, the values of the parts used to assemble circuits are $k_i \pm \Delta k_i$. Clearly, the designer needs to know how much these tolerances will affect the circuit and whether the resulting errors can be corrected by adjusting (tuning) the circuit after fabrication. Obviously, the parameter to be tuned and must depend on the component to be varied. For example, Q in Eq. (4) is a function of the components R_1, R_2, C_1, and C_2, any one of which can be adjusted to correct fabrication errors in Q. In general, the questions of how large the adjustment of a component has to be, whether it should be increased or decreased, and what the best tuning sequence is are answered by considering the parameter's *sensitivity* to component tolerances. How sensitive P is to the component-value tolerances, that is how large the deviation ΔP of the parameter in question is, is computed for small changes via the derivative of $P(k_i)$ with respect to k_i, $\partial P/\partial k_i$, at the nominal value k_i:

$$\Delta P = \frac{\partial P(k_i)}{\partial k_i}\Delta k_i \quad (6)$$

Typically, designers are less interested in the absolute tolerances than in the relative ones, that is,

$$\frac{\Delta P}{P} = \frac{k_i}{P}\frac{\partial P}{\partial k_i}\frac{\Delta k_i}{k_i} = S_{k_i}^P\frac{\Delta k_i}{k_i} \quad (7)$$

$S^P_{k_i}$ is the sensitivity, defined as "the relative change of the parameter divided by the relative change of the component,"

$$S^P_{k_i} = \frac{\Delta P/P}{\Delta k_i/k_i} \tag{8}$$

A detailed discussion of sensitivity issues can be found in many text books [see Schaumann, Ghausi, and Laker (3), Chap. 3, pp. 124–196]. For example, the sensitivity of ω_0 in Eq. (4) to changes in R_1 is readily computed to be

$$\begin{aligned} S^{\omega_0}_{R_1} &= \frac{R_1}{\omega_0}\frac{\partial\omega_0}{\partial R_1} \\ &= \frac{R_1}{1/(R_1R_2C_1C_2)^{1/2}}\left(-\frac{1}{2}\right)\frac{R_2C_1C_2}{(R_1R_2C_1C_2)^{3/2}} = -\frac{1}{2} \end{aligned} \tag{9}$$

$S^{\omega_0}_{R_1} = -0.5$ means that the percentage error in the parameter ω_0 is one-half the size of the percentage error of R_1 and opposite in sign (i.e., if R_1 increases, ω_0 decreases). A large number of useful sensitivity relations that make sensitivity calculations easy can be derived [see, for example, Moschytz (1), Sec. 1.6, pp. 103–105, 1.5, pp. 71–102, and 4.3, pp. 371–393, or Schaumann, Ghausi, and Laker (3), Chap. 3, pp. 124–196]. Of particular use for our discussion of tuning are

$$\begin{aligned} &S^{P(k^n)}_k = nS^{P(k)}_k, S^{P(\alpha k)}_k = S^{P(k)}_k, \\ &S^{P(1/k)}_k = -S^{P(k)}_k, \text{ and } S^{P(\sqrt{k})}_k = \frac{1}{2}S^{P(k)}_k \end{aligned} \tag{10}$$

where α is a constant, independent of k. The last two of these equations are special cases of the first one for $n = -1$ and $n = 1/2$, respectively. The last equation generalizes the result obtained in Eq. (9). Equations (7) and (8) indicate that, for small differential changes, the parameter deviation caused by a component error and, conversely from the point of view of tuning, the change in component value necessary to achieve a desired change in parameter can be computed if the sensitivity is known.

In Eqs. (6) and (7) we purposely used partial derivatives, $\partial P/\partial k_i$, to indicate that circuit parameters normally depend on more than one component [see Eq. (4)], all of which affect the accuracy of the parameter. To get a more complete picture of the combined effect of the tolerances and to gain insight into the operation of tuning involving several parameters, total derivatives need to be computed. Assuming P depends on n components, we find [see Schaumann, Ghausi, and Laker (3), Chap. 3, pp. 124–196]

$$\Delta P = \frac{\partial P}{\partial k_1}\Delta k_1 + \frac{\partial P}{\partial k_2}\Delta k_2 + \cdots + \frac{\partial P}{\partial k_n}\Delta k_n$$

that is

$$\begin{aligned} \frac{\Delta P}{P} &= \frac{k_1}{P}\frac{\partial P}{\partial k_1}\frac{\Delta k_1}{k_1} + \cdots + \frac{k_n}{P}\frac{\partial P}{\partial k_n}\frac{\Delta k_n}{k_n} \\ &= S^P_{k_1}\frac{\Delta k_1}{k_1} + \cdots + S^P_{k_n}\frac{\Delta k_n}{k_n} = \sum_{i=1}^{n} S^P_{k_i}\frac{\Delta k_i}{k_i} \end{aligned} \tag{11}$$

indicating that the sum of all relative component tolerances, weighted by their sensitivities, contributes to the parameter error. To illustrate the calculations, let us apply Eq. (11) to ω_0 in Eq. (4). Using Eqs. (9) and (10), the result is

$$\begin{aligned} \frac{\Delta\omega_0}{\omega_0} &= \frac{R_1}{\omega_0}\frac{\partial\omega_0}{\partial R_1}\frac{\Delta R_1}{R_1} + \frac{R_2}{\omega_0}\frac{\partial\omega_0}{\partial R_2}\frac{\Delta R_2}{R_2} + \frac{C_1}{\omega_0}\frac{\partial\omega_0}{\partial C_1}\frac{\Delta C_1}{C_1} + \frac{C_2}{\omega_0}\frac{\partial\omega_0}{\partial C_2}\frac{\Delta C_2}{C_2} \\ &= S^{\omega_0}_{R_1}\frac{\Delta R_1}{R_1} + S^{\omega_0}_{R_2}\frac{\Delta R_2}{R_2} + S^{\omega_p}_{C_1}\frac{\Delta C_1}{C_1} + S^{\omega_0}_{C_2}\frac{\Delta C_2}{C_2} \\ &= -\frac{1}{2}\frac{\Delta R_1}{R_1} - \frac{1}{2}\frac{\Delta R_2}{R_2} - \frac{1}{2}\frac{\Delta C_1}{C_1} - \frac{1}{2}\frac{\Delta C_2}{C_2} \\ &= -\frac{1}{2}\left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta C_1}{C_1} + \frac{\Delta C_2}{C_2}\right) \end{aligned} \tag{12}$$

The last expression gives insight into whether and how ω_0 can be tuned. Because the effects of the errors are additive, tuning just one component, say R_1, will suffice for given tolerances of R_2, C_1, and C_2 if ΔR_1 can be large enough. If we have measured the R_2 errors at -12%, and those of C_1 and C_2 at $+15\%$ and $+10\%$, respectively, Eq. (12) results in

$$\begin{aligned} \frac{\Delta\omega_0}{\omega_0} &= -\frac{1}{2}\left(\frac{\Delta R_1}{R_1} - 0.12 + 0.15 + 0.10\right) \\ &= -0.5\left(\frac{\Delta R_1}{R_1} + 0.13\right) \end{aligned} \tag{13}$$

indicating that R_1 must be *decreased* by 13% to yield, within the linearized approximations made, $\Delta\omega_0 \approx 0$. Inserting components with these tolerances into Eq. (4) for ω_0 confirms the result obtained.

To expand these results and gain further insight into the effects of tolerances, as well as beneficial tuning strategies and their constraints, we remember that a transfer function generally depends on more than one parameter. Returning to the example of Fig. 2 described by the function $T(s)$ in Eq. (3) with the three parameters ω_0, Q, and K given in Eq. (4) and applying Eq. (11) leads to

$$\frac{\Delta\omega_0}{\omega_0} = S^{\omega_0}_{R_1}\frac{\Delta R_1}{R_1} + S^{\omega_0}_{R_2}\frac{\Delta R_2}{R_2} + S^{\omega_0}_{C_1}\frac{\Delta C_1}{C_1} + S^{\omega_0}_{C_2}\frac{\Delta C_2}{C_2} \tag{14a}$$

$$\frac{\Delta Q}{Q} = S^{Q}_{R_1}\frac{\Delta R_1}{R_1} + S^{Q}_{R_2}\frac{\Delta R_2}{R_2} + S^{Q}_{C_1}\frac{\Delta C_1}{C_1} + S^{Q}_{C_2}\frac{\Delta C_2}{C_2} \tag{14b}$$

$$\frac{\Delta K}{K} = S^{K}_{R_1}\frac{\Delta R_1}{R_1} + S^{K}_{R_2}\frac{\Delta R_2}{R_2} + S^{K}_{C_1}\frac{\Delta C_1}{C_1} + S^{K}_{C_2}\frac{\Delta C_2}{C_2} \tag{14c}$$

These equations can be expressed in matrix form as follows:

$$\begin{pmatrix} \dfrac{\Delta\omega_0}{\omega_0} \\ \dfrac{\Delta Q}{Q} \\ \dfrac{\Delta K}{K} \end{pmatrix} = \begin{pmatrix} S^{\omega_0}_{R_1} & S^{\omega_0}_{R_2} & S^{\omega_0}_{C_1} & S^{\omega_0}_{C_2} \\ S^{Q}_{R_1} & S^{Q}_{R_2} & S^{Q}_{C_1} & S^{Q}_{C_2} \\ S^{K}_{R_1} & S^{K}_{R_2} & S^{K}_{C_1} & S^{K}_{C_2} \end{pmatrix} \begin{pmatrix} \dfrac{\Delta R_1}{R_1} \\ \dfrac{\Delta R_2}{R_2} \\ \dfrac{\Delta C_1}{C_1} \\ \dfrac{\Delta C_2}{C_2} \end{pmatrix} \tag{15}$$

The *sensitivity matrix* in Eq. (15) [see Moschytz (1), Sec. 4.3, pp. 376–393, or Schaumann, Ghausi, and Laker (3), Sec. 3.3, pp. 161–188], a 3 × 4 matrix in this case, shows how the tolerances of all the filter parameters depend on the component tolerances. We see that adjusting any one of the circuit components will vary all filter parameters as long as *all* the sensitivities are nonzero, which is indeed the case for the circuit in Fig. 2. Thus, noninteractive tuning is not possible. To illustrate the form of the sensitivity matrix, we calculate for the circuit in Fig. 2

$$\begin{pmatrix} \frac{\Delta\omega_0}{\omega_0} \\ \frac{\Delta Q}{Q} \\ \frac{\Delta K}{K} \end{pmatrix} = \begin{pmatrix} -0.5 & -0.5 & -0.5 & -0.5 \\ -0.5 & 0.5 & -\frac{1}{2}\frac{C_1 - C_2}{C_1 + C_2} & \frac{1}{2}\frac{C_1 - C_2}{C_1 + C_2} \\ -1 & 1 & -\frac{C_1}{C_1 + C_2} & \frac{C_1}{C_1 + C_2} \end{pmatrix} \begin{pmatrix} \frac{\Delta R_1}{R_1} \\ \frac{\Delta R_2}{R_2} \\ \frac{\Delta C_1}{C_1} \\ \frac{\Delta C_2}{C_2} \end{pmatrix} \tag{16}$$

Note that the first line of Eq. (16) is equal to the last part of Eq. (12).

The tuning situation is simpler if the matrix elements above the main diagonal are zero as was assumed for an arbitrary different circuit in Eq. (17a):

$$\begin{aligned} \begin{pmatrix} \frac{\Delta\omega_0}{\omega_0} \\ \frac{\Delta Q}{Q} \\ \frac{\Delta K}{K} \end{pmatrix} &= \begin{pmatrix} S^{\omega_0}_{R_1} & 0 & 0 & S^{\omega_0}_{C_2} \\ S^{Q}_{R_1} & S^{Q}_{R_2} & 0 & S^{Q}_{C_2} \\ S^{K}_{R_1} & S^{K}_{R_2} & S^{K}_{C_1} & S^{K}_{C_2} \end{pmatrix} \begin{pmatrix} \frac{\Delta R_1}{R_1} \\ \frac{\Delta R_2}{R_2} \\ \frac{\Delta C_1}{C_1} \\ \frac{\Delta C_2}{C_2} \end{pmatrix} \\ &= \begin{pmatrix} S^{\omega_0}_{R_1} & 0 & 0 \\ S^{Q}_{R_1} & S^{Q}_{R_2} & 0 \\ S^{K}_{R_1} & S^{K}_{R_2} & S^{K}_{C_1} \end{pmatrix} \begin{pmatrix} \frac{\Delta R_1}{R_1} \\ \frac{\Delta R_2}{R_2} \\ \frac{\Delta C_1}{C_1} \end{pmatrix} + \begin{pmatrix} S^{\omega_0}_{C_2} \\ S^{Q}_{C_2} \\ S^{K}_{C_2} \end{pmatrix} \frac{\Delta C_2}{C_2} \end{aligned} \tag{17a}$$

Here the sensitivities to C_2 are irrelevant because C_2 is a free parameter and is assumed fixed so that the effects of C_2 tolerances can be corrected by varying the remaining elements. We see then that first ω_0 can be tuned by R_1, next Q is tuned by R_2 without disturbing ω_0 because $S^{\omega_0}_{R_2}$ is zero, and finally K is tuned by C_1 without disturbing the previous two adjustments. Thus a sensitivity matrix of the structure indicated in Eq. (17a) with elements above the main diagonal equal to zero permits sequential "noninteractive" tuning *if the tuning order is chosen correctly.* Completely noninteractive tuning without regard to the tuning order requires all elements in the sensitivity matrix off the main diagnonal to be zero as indicated for another circuit in Eq. (17b):

$$\begin{pmatrix} \frac{\Delta\omega_0}{\omega_0} \\ \frac{\Delta Q}{Q} \\ \frac{\Delta K}{K} \end{pmatrix} - \begin{pmatrix} S^{\omega_0}_{C_2} \\ S^{Q}_{C_2} \\ S^{K}_{C_2} \end{pmatrix} \frac{\Delta C_2}{C_2} = \begin{pmatrix} S^{\omega_0}_{R_1} & 0 & 0 \\ 0 & S^{Q}_{R_2} & 0 \\ 0 & 0 & S^{K}_{C_1} \end{pmatrix} \begin{pmatrix} \frac{\Delta R_1}{R_1} \\ \frac{\Delta R_2}{R_2} \\ \frac{\Delta C_1}{C_1} \end{pmatrix} \tag{17b}$$

As can be verified readily, each component affects only one circuit parameter. Again, sensitivities to C_2 are irrelevant because C_2 is fixed, and the effects of its tolerances can be corrected by the remaining components.

An important observation on the effects of tolerances on circuit parameters and the resultant need for tuning can be made from Eq. (16). We see that the sensitivities of the *dimensionless* parameters (parameters with no physical unit) Q and K to the two resistors and similarly to the two capacitors are equal in magnitude but opposite in sign. Because dimensionless parameters are determined by *ratios* of *like components* [see Eq. (4)], we obtain from Eq. (4) with Eq. (10)

$$\begin{aligned} S^{Q}_{R_1} &= -S^{Q}_{R_2} = S^{Q}_{R} = -0.5 \text{ and} \\ S^{Q}_{C_1} &= -S^{Q}_{C_2} = S^{Q}_{C} = -\frac{1}{2}\frac{C_1 - C_2}{C_1 + C_2} \end{aligned} \tag{18}$$

Thus, the tolerances of Q are

$$\frac{\Delta Q}{Q} = S^{Q}_{R}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right) + S^{Q}_{C}\left(\frac{\Delta C_1}{C_1} - \frac{\Delta C_2}{C_2}\right) \tag{19}$$

with analogous expressions obtained for the gain K [see the last line of Eq. (16)]. Thus, if the technology chosen to implement the filter permits *ratios* of resistors and capacitors to be realized accurately (i.e., if all resistors have equal tolerances, as do all capacitors), tuning of dimensionless parameters will generally not be necessary. A prime example is integrated circuit technology, where absolute value tolerances of resistors and capacitors may reach 20 to 50%, but ratios, depending mainly on processing mask dimensions, are readily implemented with tolerances of a fraction of 1%. As an example, assume that the circuit in Fig. 2 was designed, as is often the case, with two identical capacitors $C_1 = C_2 = C$ with tolerances of 20% and that R_1 and R_2 have tolerances of 10% each, that is,

$$\begin{aligned} &C_1 = C_2 = C_n + \Delta C = C_n(1 + 0.2), \\ &R_1 = R_{1n} + \Delta R_1 = R_{1n}(1 + 0.1), \text{ and} \\ &R_2 = R_{2n}(1 + 0.1) \end{aligned} \tag{20}$$

where the subscript n stands for the *nominal* values. From Eq. (19), we find

$$\Delta Q = [S^{Q}_{R}(0.1 - 0.1) + S^{Q}_{C}(0.2 - 0.2)]Q = 0$$

That is, the quality factor Q, depending only on *ratios* of like components, is basically unaffected because all like components have equal fabrication tolerances. This result can be confirmed directly from Eq. (4) where, for equal capacitors,

$$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} = \frac{1}{2}\sqrt{\frac{R_{2n}(1 + 0.1)}{R_{1n}(1 + 0.1)}} \approx Q_n \tag{21}$$

Naturally, if R_1 and R_2 are selected from different manufacturing lots, or if R_1 and R_2 are from physically different fabrication processes (such as a carbon and a metal-film resistor), tolerances cannot be assumed to be equal, Q errors are not zero, and tuning will be required.

The situation is quite different for any *dimensioned* circuit parameter, that is, a parameter with a physical unit (e.g., a frequency or time constant, or a voltage or a current). Such parameters are determined by *absolute* values of components, as seen for ω_0 in Eq. (4). Absolute values, depending on physical process parameters e.g., resistivity, permittivity, or diffusion depth, are very difficult to control and will usually suffer from large process variations. Thus, for the component tolerances in Eq. (20), sensitivity calculations predict from Eqs. (10) and (12) the realized center frequency error

$$\begin{aligned}\Delta\omega_0 &\approx -\frac{1}{2}\left(\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}+\frac{\Delta C_1}{C_1}+\frac{\Delta C_2}{C_2}\right)\omega_0 \\ &= -\frac{1}{2}(0.1+0.1+0.2+0.2) = -0.3\omega_0\end{aligned} \tag{22a}$$

that is, all individual component tolerances add to a −30% frequency error. Again, the validity of this sensitivity result can be confirmed directly from Eq. (4):

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{R_1R_2C_1C_2}} = \frac{1}{C\sqrt{R_1R_2}} \\ &= \frac{1}{C_n(1+0.2)\sqrt{R_{1n}R_{2n}(1+0.1+0.1+0.01)}} \\ &\approx \frac{\omega_{0n}}{(1+.02)\sqrt{1+0.2}} \approx \frac{\omega_{0n}}{(1+0.2)(1+0.1)} \\ &= \frac{\omega_{0n}}{(1+0.32)} \approx \omega_{0n}(1-0.24)\end{aligned} \tag{22b}$$

The difference between the exact result in Eq. (22b) and the one obtained via the sensitivity approach in Eq. (22a) arises because the latter assumes *incremental* component changes whereas the former assumed the relatively large changes of 10 and 20%. The center frequency ω_0 is approximately 25–30% smaller than specified and must be corrected by tuning. This can be accomplished, for example, by trimming the two resistors to be 27% smaller than their *fabricated* values, that is,

$$\begin{aligned}R_1 &= R_{1n}(1+0.1)(1-0.27) \approx R_{1n}(1-0.2), \\ R_2 &\approx R_{2n}(1-0.2)\end{aligned}$$

so that sensitivity calculations yield

$$\Delta\omega_0 \approx -0.5(-0.2-0.2+0.2+0.2) = 0$$

More exact deterministic tuning requires the resistors to be trimmed to 24.2% smaller than the fabricated value as shown in Eq. (23):

$$\omega_0 \approx \frac{1}{C_n(1+0.2)\sqrt{R_{1n}R_{2n}}(1+0.1)(1-\delta)} = \frac{\omega_{0n}}{1.32(1-\delta)} \Rightarrow \omega_{0n} \tag{23}$$

where δ is the trimming change to be applied to the resistors as fabricated. Equation (23) results in δ = 0.242. Of course, ω_0 tuning could have been accomplished by adjusting only one of the resistors by a larger amount; we trimmed both resistors by equal amounts to maintain *the value of their ratio* that determines Q according to Eq. (21), thereby avoiding the need to retune Q.

TUNING DISCRETE CIRCUITS

Whether implemented on a printed circuit board, with chip and thin- or thick-film components in hybrid form, by use of wire-wrapping, or in any other technology, an advantage of discrete circuits for the purpose of tuning is that circuit elements are accessible individually before or after assembly for deterministic or functional adjusting. Thus, after a circuit is assembled and found not to meet the design specifications, the circuit components (most commonly the resistors or inductors), can be varied until the performance is as required. All the previous general discussion applies to the rest of the article so we shall present only those special techniques and considerations that have been found particularly useful or important for passive and active filters.

Passive Filters

Discrete passive filters are almost always implemented as lossless ladder circuits, that is, the components are inductors L and capacitors C as is illustrated in the typical circuit in Fig. 3. These LC filters are designed such that the maximum signal power is transmitted from a resistive source to a resistive load in the frequency range of interest; a brief treatment can be found in Schaumann, Ghausi, and Laker (3), Chap. 2, pp. 71–123. As pointed out in our earlier discussion, accurate filter behavior depends on precise element values so that it is normally necessary to trim components. This tuning is almost always accomplished via variable inductors whose values are changed by screwing a ferrite slug (the "trimmer") into or out of the magnetic core of the inductive windings. Variable discrete capacitors are hard to construct, expensive, and rarely used.

LC filters have the advantage of very low sensitivities to all their elements [see Schaumann, Ghausi, and Laker (3), Chaps. 2 and 3, pp. 71–196], which makes it possible to assemble the filter using less expensive wide-tolerance components. This property is further enhanced by the fact that lossless ladders are very easy to tune so that large tolerances of one component can be compensated by accurately tuning another. For example, the resonant frequency $f_0 = 1/\sqrt{LC}$ of an LC resonance circuit has ±15% tolerances if both L and C have ±15% tolerances; if then L is trimmed to ±0.5% of its correct value for the existing capacitor (with ±15% tolerances), f_0 is accurate to within 0.25% without requiring any narrower manufacturing tolerances. Without tuning, a 0.25% f_0 error would require the same narrow 0.25% tolerance in

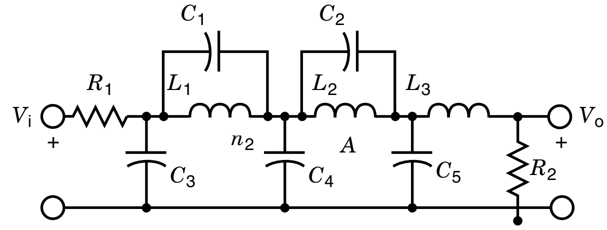

Figure 3. Sixth-order LC low-pass filter. The filter is to realize a maximally flat passband with a 2 dB bandwidth of f_c = 6 kHz, minimum stopband attenuation α_s = 57.5 dB with transmission zeros at 12 and 24 kHz. The nominal components are listed in Table 1. Note that at dc the filter has $-20\log[R_2/(R_1+R_2)]$ = 6.02 dB attenuation.

Table 1. *LC* Low-pass Filter (elements in mH, nF, and kΩ)

Components	L_1	C_1	L_2	C_2	L_3	C_3	C_4	C_5	R_1	R_2
Nominal values	27.00	6.490	46.65	0.943	12.67	6.977	45.55	33.90	1.00	1.00
Performance	f_c = 6.0 kHz @ α_p = 8.03 dB; f_{z1} = 12.0 kHz, f_{z2} = 24.0 kHz, α_s = 57.5 dB									
15% tolerance values	31	7.5	52	1.1	14	8	51	38	1.05	1.05
Performance untuned	f_c = 5.36 kHz @ α_p = 8.01 dB; f_{z1} = 10.3 kHz, f_{z2} = 20.7 kHz, α_s = 56.7 dB									
Tuned values	**23.5**	7.5	**40**	1.1	14	8	51	38	1.05	1.05
Performance tuned	f_c = 6.07 kHz @ α_p = 8.03 dB; f_{z1} = 12.0 kHz, f_{z2} = 24.0 kHz, α_s = 57.8 dB									

both components, which is likely more expensive than a simple tuning step.

It is well known that lossless ladders can be tuned quite accurately simply by adjusting the components to realize the prescribed transmission zeros [see Heinlein and Holmes (4), Sec. 12.3, pp. 591–604, and Christian (5), Chap. 8, pp. 167–176]. Transmission zeros, frequencies where the attenuation is infinite, usually depend on only two elements, a capacitor and an inductor in a parallel resonant ciruit (see Fig. 3) with the parallel tank circuits L_1, C_1 and L_2, C_2 in the series branches of the filter, or alternatively with series *LC* resonance circuits in the shunt branches. The resonant frequencies $f_{zi} = 1/\sqrt{L_iC_i}, i = 1, 2$, of the *LC* tank circuits are not affected by other elements in the filter, so that tuning is largely noninteractive. As mentioned, the effect of the tolerances of one component, say *C*, are corrected by tuning *L*. It is performed by adjusting the inductors for maximum attenuation at the readily identified frequencies of zero transmission while observing the response of the complete manufactured filter on a network analyzer. Tuning accurancies of the transmission zeros of 0.05% or less should be aimed at. Such tuning of the transmission zeros is almost always sufficient even if the circuit elements have fairly large tolerances [see Heinlein and Holmes (4), Sec. 12.3, pp. 594–604]. If even better accuracy is needed, adjustments of those inductors that do not cause finite transmission zeros, such as L_3 in Fig. 3, may need to be performed [see Christian (5), Chap. 8, pp. 167–176]. For instance, consider the filter in Fig. 3 realized with unreasonably large tolerances of ±15%, using the components shown in Table 1. This places the two resonant frequencies at 10.3 and 20.7 kHz, with the minimum stopband attenuation equal to only 56.7 dB; the 2 dB passband corner is reduced to 5.36 kHz. If we next tune the transmission zero frequencies to 12 and 24 kHz by adjusting *only* the inductors L_1 and L_2 to 23.5 and 40 mH, respectively, the minimum stopband attenuation is increased to 57.8 dB, and the 2 dB bandwidth of the passband is measured as f_c = 6.07 kHz (refer to Table 1).

We still note that when functional tuning is performed, the filter must be operated with the correct terminations for which it was designed [see Christian (5), Sec. 8.2, pp. 168–173]. Large performance errors, not just at dc or low frequencies, will result if the nominal terminations are severely altered. For example, an *LC* filter designed for 600 Ω terminations cannot be correctly tuned by connecting it directly without terminations to a high-frequency network analyzer whose input and source impedances are 50 Ω. Also, if maintaining an accurate narrow passband ripple is important, the tolerances of the untuned capacitors must not be too large. Finally, we observe that the tuning properties of passive *LC* ladders translate directly to active simulations of these filters via transconductance-*C* and gyrator-*C* circuits, which are widely used in high-frequency integrated circuits for communications (see the following discussion).

Active Filters

Several differences must be kept in mind when tuning active filters as compared to passive lossless filters, particularly to ladders:

1. Active filters are almost always more sensitive to component tolerances than *LC* ladders. Consequently, tuning is always required in practice.
2. Tuning in active filters is almost always interactive; that is, a filter parameter depends on many or all circuit components as discussed in connection with the circuit in Fig. 2 and the sensitivity discussion related to Eqs. (15) and (16). Consequently, tuning active filters usually requires computer aids to solve the complicated nonlinear tuning equations [see, for example, the relatively simple case in Eq. (4)].
3. The performance of the active devices, such as operational amplifiers (op amps), and their often large tolerances almost always strongly affects the filter performance and must be accounted for in design and in tuning. Because active-device behavior is often hard to model or account for, functional tuning of the fabricated circuit is normally the only method to ensure accurate circuit performance.

In discrete active filters constructed with resistors, capacitors, and operational amplifiers on a circuit board or in thin- or thick-film form, tuning is almost always performed by varying the resistors. Variable resistors, potentiometers, are available in many forms, technologies, and sizes required to make the necessary adjustments.

Second-Order Filters. The main building blocks of active filters are second-order sections, such as the bandpass circuit in Fig. 2. Many of the tuning strategies and concepts were presented earlier in connection with that circuit and the discussion of sensitivity. An important consideration when tuning an active filter is its dependence on the active devices as mentioned previously in point 3. To illustrate the problem, consider again the bandpass filter in Fig. 2. The transfer function $T(s)$ in Eq. (1) is independent of the frequency-dependent gain $A(s)$ of the op amp only because the analysis assumed that the amplifier is ideal, that is, it has constant and very large (ideally infinite) gain, $A = \infty$. In practice, $T(s)$ is also a function of $A(s)$ as a more careful analysis shows:

$$T(s) = \frac{V_2}{V_1} = -\frac{\frac{1}{R_1C_1}s\frac{A(s)}{1+A(s)}}{s^2+\frac{1}{R_2}\left\{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{R_1C_1[1+A(s)]}\right\}s+\frac{1}{R_1R_2C_1C_2}} \tag{24}$$

Evidently, for $A = \infty$, Eq. (24) reduces to Eq. (1), but finite and frequency-dependent gain can cause severe changes in $T(s)$ in all but the lowest-frequency applications. Consider the often-used integrator model for the operational amplifier, $A(s) \approx \omega_t/s$, where ω_t is the unity gain frequency (or the *gain-bandwidth product*) of the op amp with the typical value $\omega_t = 2\pi \times f_t = 2\pi \times 1.5$ MHz. Using this simple model, which is valid for frequencies up to about 10 to 20% of f_t, and assuming $\omega_t \gg \omega$, the transfer function becomes

$$T(s) = \frac{V_2}{V_1} \approx -\frac{\frac{1}{C_1R_1}s}{s^2\left(1+\frac{1}{\omega_tC_1R_1}\right)+\frac{1}{R_2}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)s+\frac{1}{R_1R_2C_1C_2}} \tag{25}$$

To get an estimate of the resulting error, let the circuit be designed with $C_1 = C_2 = C = 10$ nF, $R_1 = 66.32\ \Omega$ and $R_2 = 9.55$ kΩ to realize the nominal parameters $f_0 = 20$ kHz, $Q = 6$, and $K = 72$. Simulation (or measurement with a very fast op amp) shows that the resulting circuit performance is as desired. However, if the filter is implemented with a 741-type op amp with $f_t = 1.5$ MHz, the measured performance indicates $f_0 = 18.5$ kHz, $Q = 6.85$, and $K = 76.75$. Because of the complicated expressions involving a real op amp, it is appropriate to use functional tuning with the help of a network analyzer. Keeping C constant, the resulting resistor values, $R_1 = 68.5\ \Omega$ and $R_2 = 8.00$ kΩ, lead to $f_0 = 20$ kHz and $Q = 6.06$. The midband gain for these element values equals $K = 62.4$ (remember from the earlier discussion that K for the circuit in Fig. 2 cannot be separately adjusted if the capacitors are predetermined).

High-Order Filters. The two main methods for realizing active filters of order greater than two are active simulations of lossless ladders and cascading second-order sections. We mentioned in connection with the earlier discussion of LC ladders that tuning of active ladder simulations is completely analogous to that of the passive LC ladder: the electronic circuits that simulate the inductors are adjusted until the transmission zeros are implemented correctly. It remains to discuss tuning for the most frequently used method of realizing high-order filters, the cascading of first- and second-order sections. Apart from good sensitivity properties, relatively easy tuning is a main advantage of cascade implementations because each section performs in isolation from the others so that it can be tuned without interactions from the rest of the circuit. Remember, though, that each section by itself may require interactive tuning. Figure 4 shows the circuit structure where each of the blocks is a second-order section such as the ones in Figs. 2 and 5. If the total filter order is odd, one of the sections is, of course, of first order.

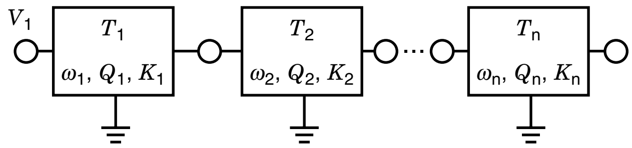

Figure 4. Cascade realization of 2nth-order filter. The n second-order sections do not interact with each other and can be tuned independently, that is, each section T_i can be tuned to its nominal values ω_i, Q_i, and H_i, $i = 1, 2, \ldots, n$, without being affected by the other sections.

To illustrate this point, assume a fourth-order Chebyshev low-pass filter is to be realized with a 1 dB ripple passband in $0 \le f \le 28$ kHz with passband gain equal to $H = 20$ dB. The transfer function is found to be

$$T(s) = T_1(s) \times T_2(s) = \frac{1.66\omega_0^2}{s^2+0.279\omega_0 s+0.987\omega_0^2}\,\frac{1.66\omega_0^2}{s^2+0.674\omega_0 s+0.279\omega_0^2} \tag{26}$$

with $\omega_0 = 2\pi \times 28{,}000\ \text{s}^{-1} = 175.93 \times 10^3\ \text{s}^{-1}$ [see Schaumann, Ghausi, and Laker (3), Sec. 1.6, pp. 36–64]. Let the function be realized by two sections of the form shown in Figure 5. Assuming that the op amps are ideal, the transfer function of the low-pass section is readily derived as

$$\frac{V_2}{V_1} = \frac{K\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2} = \frac{\alpha_1\alpha_2\frac{1}{C_1R_1C_2R_2}}{s^2+s\left(\frac{1}{C_1R_1}+\frac{1-\alpha_1\alpha_2}{C_2R_2}\right)+\frac{1}{C_1R_1C_2R_2}} \tag{27}$$

If the op amp gain is modeled as $A(s) = \omega_t/s$, α_i is to be replaced by

$$\alpha_i \Rightarrow \frac{\alpha_i}{1+\alpha_i/A(s)} \approx \frac{\alpha_i}{1+s\alpha_i/\omega_t} \tag{28}$$

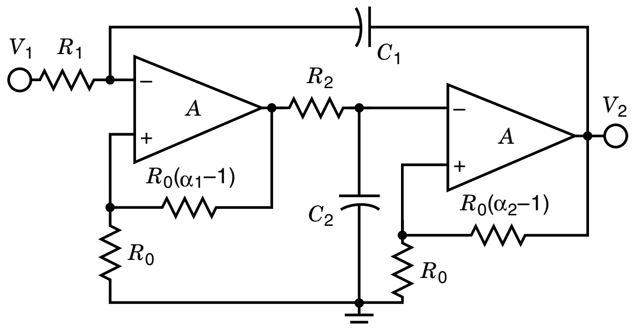

Figure 5. Two-amplifier active low-pass filter.

We observe again that the circuit parameters ω_0, Q, and gain K are functions of all the circuit elements so that design and tuning of each section will require iterative procedures, although section 1 is independent of section 2 as just discussed. Because there are six "components" (R_1, R_2, C_1, C_2, α_1, and α_2) and only three parameters, some simplifying design choices can be made. Choosing $C_1 = C_2 = C$, $R_1 = R$, and $R_2 = k^2R$ (and assuming ideal op amps), Eq. (27) leads to in the expressions

$$\omega_0 = \frac{1}{kRC}, Q = \frac{1}{k + \dfrac{1}{k}(1-K)}, \text{ and } K = \alpha_1\alpha_2 \tag{29}$$

The circuit is designed by first computing k from the given values Q and K; next we choose a suitable capacitor value C and calculate $R = 1/(k\omega_0 C)$. Finally, we determine the feedback resistors on the two op amps. Because only the product $\alpha_1\alpha_2$ is relevant, we choose $\alpha_1\alpha_2 = \alpha^2 = K$ (i.e., $\alpha = \sqrt{K} = \sqrt{1.66} = 1.288$). Working through the design equations and choosing all capacitors equal to $C = 150$ pF (standard 5% values) and $R_0 = 10$ kΩ, results in $(\alpha - 1)R_0 = 2.87$ kΩ for both sections: $k = 0.965, R_1 = 40.2$ kΩ, $R_2 = 36.5$ kΩ for section 1 and $k = 1.671$, $R_1 = 42.2$ kΩ, $R_2 = 120.1$ kΩ for section 2. All resistors have standard 1% tolerance values. Building the circuit with 741-type op amps with $f_t = 1.5$ MHz results in a ripple width of almost 3 dB, the reduced cut-off frequency of 27.2 kHz, and noticeable peaking at the band-edge. Thus, tuning is required. The errors can be attributed largely to the 5% capacitor errors and the transfer function changes as a result of the finite f_t in Eq. (28).

To accomplish tuning in this case, deterministic tuning may be employed if careful modeling of the op amp behavior, using Eq. (28), and of parasitic effects is used and if the untuned components (the capacitors) are measured carefully and accurately. Because of the many interacting effects in the second-order sections, using a computer program to solve the coupled nonlinear equations is unavoidable, and the resistors are trimmed to their computed values. Functional tuning in this case may be more convenient, as well as more reliable in practice. For this purpose, the circuit is analyzed, and sensitivities are computed to help understand which components affect the circuit parameters most strongly. Because the sections do not interact, the high-order circuit is separated into its sections, and each section's functional performance is measured and adjusted on a network analyzer. After the performance of all second-order blocks is found to lie within the specified tolerances, the sections are reconnected in cascade.

TUNING INTEGRATED CIRCUITS

With the increasing demand for fully integrated microelectronic systems, naturally, analog circuits will have to be placed on an integrated circuit (IC) along with digital ones. Of considerable interest are communication circuits where bandwidths may reach many megahertz. Numerous applications call for on-chip high-frequency analog filters. Their frequency parameters, which in discrete active filters are set by RC time constants, are in integrated filters most often designed with voltage-to-current converters (transconductors), $I_o = g_m V_i$, and capacitors (i.e., as $\omega = 1/\tau = g_m/C$). As discussed earlier, filter performance must be tuned regardless of the implementation method because fabrication tolerances and parasitic effects are generally too large for filters to work correctly without adjustment. Understandably, tuning in the traditional sense is impossible when the complete circuit is integrated on an IC because individual components are not accessible and cannot be varied. To handle this problem, several techniques have been developed. They permit tuning the circuits electronically by varying the bias voltages V_B or bias currents I_B of the active electronic components (transconductors or amplifiers). In the usual approach, the performance of the fabricated circuit is compared to a suitably chosen accurate reference, such as an external precision resistor R_e to set the value of an electronic on-chip transconductance to $g_m = 1/R_e$, or to a reference frequency ω_r to set the time constant to $C/g_m = 1/\omega_r$. This approach is indeed used in practice, where often the external parameters, R_e or ω_e, are adjusted manually to the required tolerances. Tuning can be handled by connecting the circuit to be tuned into an on-chip control loop, which automatically adjusts bias voltages or currents until the errors are reduced to zero or an acceptable level [see Schaumann, Ghausi, and Laker (3), Sec. 7.3, pp. 418–446, and Johns and Martin (6), Sec. 15.7, pp. 626–635]. [A particularly useful reference is Tsividis and Voorman (7); it contains papers on all aspects of integrated filters, including tuning.] Naturally, this process requires that the circuit is designed to be tunable, that is, that the components are variable over a range sufficiently wide to permit errors caused by fabrication tolerances or temperature drifts to be recovered. We also must pay attention to keeping the tuning circuitry relatively simple because chip area and power consumption are at a premium. Although digital tuning schemes are conceptually attractive, analog methods are often preferred. The reason is the need to minimize or eliminate generating digital (switching) noise, which can enter the sensitive analog signal path through parasitic capacitive coupling or through the substrate, causing the dynamic range or the signal-to-noise ratio to deteriorate.

Automatic Tuning

Let us illustrate the concepts and techniques with a simple second-order example. Higher-order filters are treated in an entirely analogous fashion; the principles do not change. Consider the g_m–C filter in Fig. 6, which realizes the transfer function

$$T(s) = \frac{V_0}{V_1} = \frac{\alpha s^2 + s\left(\alpha\dfrac{g_{m1}}{C_1} - \beta\dfrac{g_{m2}}{C_2}\right) + \dfrac{g_{m0}g_{m2}}{C_1C_2}}{s^2 + s\dfrac{g_{m1}}{C_1} + \dfrac{g_{m1}g_{m2}}{C_1C_2}} \tag{30}$$

with pole frequency and pole Q equal to

$$\omega_0 = \sqrt{\frac{g_{m1}g_{m2}}{C_1C_2}}, Q = \frac{\omega_0 C_1}{g_{m1}} = \sqrt{\frac{C_1/C_2}{g_{m1}/g_{m2}}} \tag{31}$$

Comparing Eq. (31) to Eq. (2) indicates that the filter parameters for this technology are determined in fundamentally the same way as for discrete active circuits: the frequency is determined by time constants (C_i/g_{mi}) and the quality factor, by ratios of like components. Analogous statements are true for the numerator coefficients of $T(s)$. We can conclude then that, in principle, tuning can proceed in a manner quite similar to the one discussed in the beginning of this article if we can just develop a procedure for varying the on-chip components. To gain an understanding of what needs to be tuned in an integrated filter, let us introduce a more convenient notation that uses the ratios of the components to some suitably chosen unit values g_m and C,

$$g_{mi} = g_i g_m, C_i = c_i C, i = 1, 2, \text{ and } \omega_u = \frac{g_m}{C} \tag{32}$$

where ω_u is a unit frequency parameter and g_i and c_i are the dimensionless component ratios. With this notation, Eq. (30) becomes

$$T(s) = \frac{V_0}{V_1} = \frac{\alpha s^2 + s\left(\alpha\frac{g_1}{c_1} - \beta\frac{g_2}{c_2}\right)\omega_u + \frac{g_0 g_2}{c_1 c_2}\omega_u^2}{s^2 + s\frac{g_1}{c_1}\omega_u + \frac{g_1 g_2}{c_1 c_2}\omega_u^2} \tag{33}$$

Casting the transfer function in the form shown in Eq. (33) makes clear that the coefficient of s^i is proportional to ω_u^{n-i}, where n is the order of the filter, $n = 2$ in Eq. (33); the constants of proportionality are determined by ratios of like components, which are very accurately designable with IC technology. The same is true for filters of arbitrary order. For example, the pole frequency for the circuit in Fig. 6 is determined as ω_u times a designable quantity, $\omega_0 = \omega_u\sqrt{g_1 g_2/(c_1 c_2)}$. We may conclude therefore that it is only necessary to tune $\omega_u = g_m/C$, which, as stated earlier, as a ratio of two electrically dissimilar components will have large fabrication tolerances. In addition, the electronic circuit that implements the transconductance g_m depends on temperature, bias, and other conditions, so that ω_u can be expected to drift during operation. It can be seen from Eq. (33) that ω_u simply scales the frequency, that is, the only effect of varying ω_u is a shift of the filter's transfer function along the frequency axis.

We stated earlier that tuning a time constant, or, in the present case, the frequency parameter ω_u, is accomplished by equating it via a control loop to an external reference, in this case a reference frequency ω_R such as a clock frequency. Conceptually, the block diagram in Fig. 7 shows the method (8). The control loop equates the inaccurate unit frequency $\omega_u = g_m/C$ to the accurate reference frequency ω_R in the following way: ω_R is chosen in the vicinity of the most critical frequency parameters of the filter (the band-edge for a low-pass, mid-band for a bandpass filter), where sensitivities are highest. The transconductance g_m to be tuned is assumed to be proportional to the bias voltage V_B, such that $g_m = kV_B$ where k is a constant of proportionality with units of A/V^2. g_m generates an output current $I = g_m V_R$, which results in the capacitor voltage $V_C = g_m V_R/(j\omega_R C)$. The two matched peak detectors PD convert the two signals V_R and V_C to their dc peak values, so that any phase differences do not matter when comparing the signals at the input of g_{mc}. The dc output current $I_c = g_{mc}V_R\{1 - [g_m/(j\omega_R C)]\}$ of the control-transconductance g_{mc} charges the storage capacitor C_c to the required bias voltage V_B for the transconductance g_m. The values g_{mc} and C_c determine the loop gain; they influence the speed of conversion but are otherwise not critical. If the value of g_m gets too large because of fabrication tolerances, temperature, or other effects, I_c becomes negative, C_c discharges, and V_B, that is $g_m = kV_B$, is reduced. Conversely, if g_m is too small, I_c becomes positive and charges C_c, and the feedback loop acts to increase V_B and g_m. The loop stabilizes when V_C and V_R are equal, that is, when $g_m(V_B)/C$ is equal to the accurate reference frequency ω_R. The g_{mc}–C_c combination is, of course, an integrator with ideally infinite dc gain to amplify the shrinking error signal at the input of g_{mc}. In practice, the open loop dc gain of a transconductance of 35 to 50 dB is more than adequate. Note that the loop sets the value of ω_u to ω_R regardless of the causes of any errors: fabrication tolerances, parasitic effects, temperature drifts, aging, or changes in dc bias.

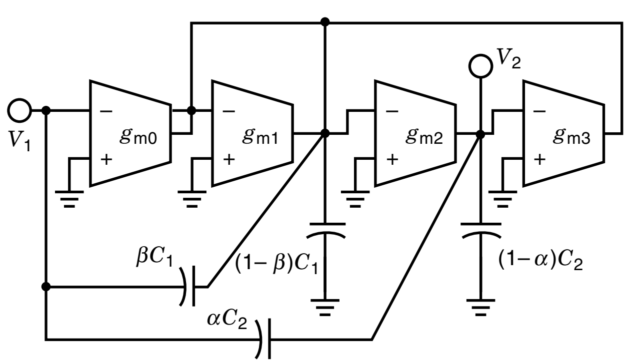

Figure 6. A general second-order transconductance-C filter. The circuit realizes arbitrary zeros by feeding the input signal into portions βC_1 and αC_2 of the capacitors C_1 and C_2.

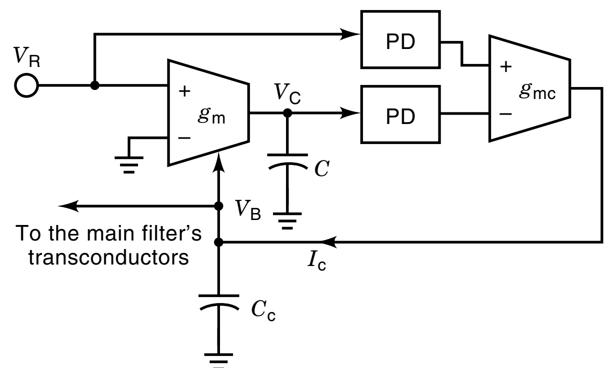

Figure 7. Automatic control loop to set $\omega_u = g_m/C$ via an applied reference signal V_R with frequency ω_R. The capacitor voltage equals $V_C = V_R(g_m/j\omega_R C)$, which makes the control current $I_c = g_{mc}V_R(1 - g_m/j\omega_R C)$. The operation is explained in the text.

We point out that although the scheme just discussed only varies g_m, it actually controls the time constant C/g_m, that is, errors in both g_m and C are accounted for. If one wishes to control only g_m, the capacitor C in Fig. 7 is replaced by an accurate resistor R_e, and the feedback loop will converge to $g_m = 1/R_e$.

Notice that the feedback loop in Fig. 7 controls directly only the transconductance g_m (as does the frequency control circuit in Fig. 8) such that the unit frequency parameter ω_u *within the control circuit* is realized correctly. The actual filter is *not* tuned. However, good matching and tracking can be

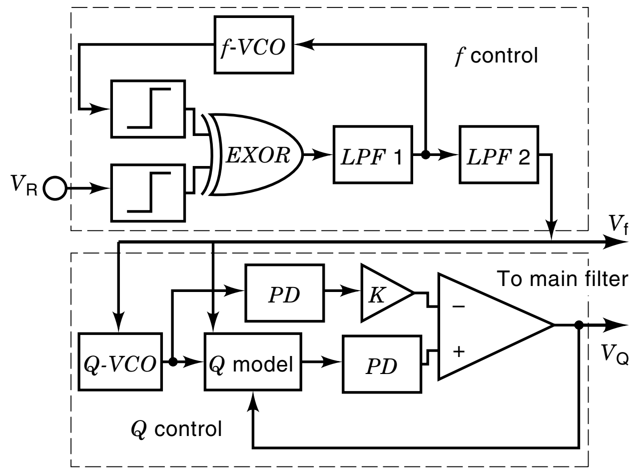

Figure 8. Dual-control loop-tuning system for tuning frequency parameters and quality factors of an integrated filter. Note that the frequency loop converges always, but for the Q loop to converge on the correct Q value, the frequency must be correct. Details of the operation are explained in the text.

assumed across the IC because all g_m cells are on the same chip and subject to the same error-causing effects. This assumes that the ratios g_i defined in Eq. (32) are not so large that matching problems will arise and that care is taken to account for (model the effect of) filter parasitics in the control circuit. The same is true for the unit capacitor C in the control loop and the filter capacitors (again, if the ratios c_i are not too large). Consequently, the control bias current I_B can be sent to all the main filter's transconductance cells as indicated in Fig. 7 and thereby tune the filter. Clearly, this scheme depends on good matching properties across the IC chip. Accurate tuning cannot be performed if matching and tracking cannot be relied upon or, in other words, if the g_m–C circuit in the control loop is not a good representative model of the filter cells.

An alternative method for frequency tuning [see Schaumann, Ghausi, and Laker (3), Sec. 7.3, pp. 418–446, and Johns and Martin (6), Sec. 15.7, pp. 626–635] relies on phase-locked loops [see Johns and Martin (6), Chap. 16, pp. 648–695]. The top half of Fig. 8 shows the principle. A sinusoidal reference signal V_R at $\omega = \omega_R$ and the output of a voltage-controlled oscillator (f-VCO) at ω_{vco} are converted to square waves by two matched limiters. Their outputs enter an EXOR gate acting as a phase detector whose output contains a dc component proportional to the frequency difference $\Delta\omega = \omega_{vco} - \omega_R$ of the two input signals. The low-pass filter LPF 1 eliminates second- and higher-order harmonics of the EXOR output and sends the dc component to the oscillator f-VCO, locking its frequency to ω_R. Just as the g_m–C circuit in Fig. 7, the oscillator is designed with transconductances and capacitors to represent (model) any frequency parameter errors of the filter to be tuned so that, relying on matching, the filter is tuned correctly by applying the tuning signal also to its g_m cells. The low-pass filter LPF 2 is used to clean the tuning signal V_f further before applying it to the filter.

We saw in Eq. (33) that all filter parameters depend, apart from ω_u, only on ratios of like components and are, therefore, accurately manufacturable and should require no tuning. This is indeed correct for moderate frequencies and filters with relatively low Q. However, Q is extremely sensitive [see Schaumann, Ghausi, and Laker (3), Chap. 7, pp. 410–486] to small parasitic phase errors in the feedback loops of active filters, so that Q errors may call for tuning as well, especially as operating frequencies increase. The problem is handled in much the same way as frequency tuning. One devises a model (the Q-*model* in Fig. 8) that represents the Q errors to be expected in the filter and encloses this model circuit in a control loop where feedback acts to reduce the error to zero. Figure 8 illustrates the principle. In the Q control loop, a Q-VCO (tuned correctly by the applied frequency control signal V_f) sends a test signal to the Q model that is designed to represent correctly the Q errors to be expected in the filter to be tuned, and through a peak detector PD to an amplifier of gain K. K is the gain of an accurately designable dc amplifier. Note that the positions of PD and K could be interchanged in principle, but a switch would require that K is the less well-controlled gain of a high-frequency amplifier. The output of the Q model goes through a second (matched) peak detector. Rather than measuring Q directly, which is very difficult in practice, because it would require accurate measurements of two amplitudes and two frequencies, the operation relies on the fact that Q errors are usually proportional to magnitude errors. The diagram in Fig. 8 assumes that for correct Q the output of the Q model is K times as large as its input so that for correct Q the inputs of the comparator are equal. The dc error signal V_Q resulting from the comparison is fed back to the Q model circuit to adjust the bias voltages appropriately, as well as to the filter. In these two interacting control loops, the frequency loop will converge independently of the Q control loop, but to converge on the correct value of Q, the frequency must be accurate. Hence, the two loops must operate together. The correct operation and convergence of the frequency and Q control scheme in Fig. 8 has been verified by experiments [see Schaumann, Ghausi, and Laker (3), Chapter 7, pp. 410–486] but because of the increased noise, power consumption, and chip area needed for the control circuitry, the method has not found its way into commercial applications.

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CIRCULAR BIREFRINGENCE. See CHIRALITY.
CIRCULAR DICHROISM, MAGNETIC. See MAGNETIC STRUCTURE.
CIRCULATORS, NUMERICAL MODELING. See NUMERICAL MODELING OF CIRCULATORS.
CLEAR WRITING. See DOCUMENT AND INFORMATION DESIGN.