SUPERCONDUCTORS, STABILIZATION AGAINST FLUX JUMPS

INTRODUCTION

Superconducting conductors used for ac and dc magnets are usually composite conductors consisting of many "intrinsically" stable fine superconducting filaments in a normal metal matrix.

It is well known that a simple twisting of the wire as a whole cannot ensure an equal current distribution between each filament or each layer of filaments. It appears that during energization the current fills first the outer layers of filaments up to the critical current density, whereas hardly any current flows in the inner layers.

It has already been shown that this particular current distribution with the field totally shielded from the interior of the wire could result in instabilities and premature quenching of large multifilamentary wires as it does in a large monofilament. It has already been observed and explained how a highly conductive matrix can slow the process and eventually totally damp the flux jump. Several laboratories have also observed how highly resistive matrix wires could hardly be stable. This was not very surprising and could be explained in agreement with the theories of stability of wires made with many filaments in homogeneous matrix (copper or cupronickel).

This work was mainly devoted to superconducting wires made of filaments embedded in a resistive matrix (such as CuNi to reduce coupling current losses) with an outer shell of low resistivity (such as Cu) to provide enough stability. In order to help our understanding, this study was carried out in line with previous work on stability (1–6). It concerns the so-called "intrinsic stability." We have shown how most of the classical criteria (usually unrelated) can be integrated into one unique and general expression. In particular, in the case of superconducting composites with a highly resistive matrix, we investigated how the different physical parameters can be optimized for achieving stabilization.

Intrinsic stability of a superconducting composite refers, in this approach, to hampering the development of flux jumpings, the origins of which lie in the nonuniform current distribution inside the wire. This means that a wire is intrinsically stable if a "small" perturbation does not lead the conditions to enter in a diverging spiral, $\Delta T \rightarrow -\Delta J_c \rightarrow \Delta B \rightarrow E \rightarrow Q \rightarrow \Delta T$, caused by the decrease of critical current J_c with respect to temperature T.

The sources of perturbations are mainly of internal origin (change in current, mechanical hysteresis) or, by extension, any "small" change in temperature due to an external source (field change, for instance).

In the last section of this article, a more complete discussion is given about what can be considered to be a "small" perturbation with respect to the actual relationship between electric field and current density. Thus, intrinsic instability is directly ascribed to the nonuniform current distribution inside the wire (a direct consequence of superconductivity), which stores a source of magnetic energy, and the decrease in critical current density J_c with respect to temperature T.

Two ways of preventing this phenomenon are known: by absorbing heat generation itself (adiabatic stabilization) and by removing it by using the enthalpy of external materials or coolants (dynamic stabilization). In fact, the second process is different from the first only in that the flux jump has slowed enough to provide time for heat to be transferred to the coolant. This results only in obtaining more "available" enthalpy than the enthalpy of the wire itself, to absorb the magnetic energy stored in the system.

FUNDAMENTAL EQUATIONS AND CALCULATIONS

For a given transport current I_t less than the critical current I_c, the current density and field profile are shown in Fig. 1. (The transport current I_t and the background field B_a are kept constant.) The critical current density is a unique function of temperature. For simplicity, the dependence on temperature is taken to be linear:

$$J_c = J_{c0}\frac{T_c - T}{T_c - T_b} \qquad \text{for} \quad T \le T_c$$

$$J_c = 0 \qquad \text{for} \quad T > T_c$$

In the frame of a simplified critical model, Jc is a step function versus electric field:

$$J(E = 0) = 0$$

$$J(E \ne 0) = J_c$$

The effects of more realistic expressions of current density, such as $J_{c0} = J_{c0}(E/E_0)^{1/n}$, are discussed in the final section.

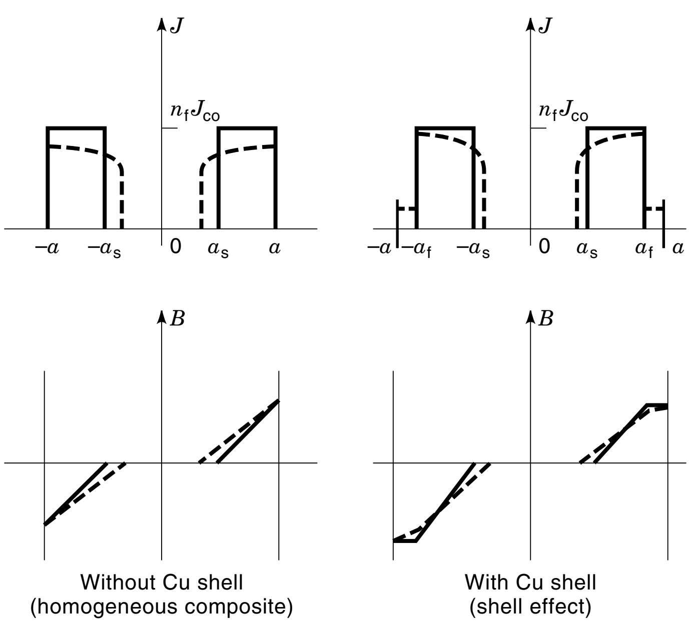

Figure 1. Schematic representation of current distribution and field profiles in a composite before and after a temperature perturbation.

In previous work (devoted to copper matrix composites), the resistivity of the normal metal matrix was taken into account by the expression (5)

$$E = \rho_m(J - \eta_s J_c)$$

where ρ_m is the effective or average resistivity of the wire in the critical state, J is the average current density in the wire, and η_s is the superconductor space factor.

In contrast to the previous work, we now consider the case of a multifilament wire made of two different regions: a central region consisting of the superconducting filaments in a highly resistive matrix (CuNi), for which a justified assumption is made in a later step in the calculations that the resistivity of this central zone is infinite, and an outer normal region with good electrical conductivity (Cu).

Calculations are derived for a slab model. Owing to the self-field effect, for a given transport current I_t, the field penetrates to radius a_s as shown in Fig. 1, with

$$i = \frac{I_t}{I_c} = 1 - \frac{a_s}{a_f}$$

where a_f is the limit of the filamentary region and a is the outer "dimension" of the wire ($a \approx R\sqrt{\pi}/2$ for equivalence with a round wire).

According to Fig. 1, there are three zones of study:

$$0 \le x \le a_s$$

$$a_s \le x \le a_f$$

$$a_f \le x \le a$$

A small temperature perturbation dT in the composite results in a magnetic flux motion together with an induced electric field. The deviations from the steady state are related by two electromagnetic equations and one thermodynamic equation in each region.

In the central region (filamentary zone with no current)

$$\frac{\partial H}{\partial t} = 0$$

$$J = 0$$

$$C_f\frac{\partial T}{\partial t} = \operatorname{div}(K \operatorname{grad} T)$$

In the central region carrying the current,

$$\operatorname{rot} E = -\mu_0\frac{\partial H}{\partial t}$$

$$\operatorname{rot}\frac{\partial H}{\partial t} = \eta_f\frac{\partial J_c}{\partial t}$$

$$C_f\frac{\partial T}{\partial t} = \eta_f J_c E + \operatorname{div}(K \operatorname{grad} T)$$

In the outer copper shell,

$$\operatorname{rot} E = -\mu_0\frac{\partial H}{\partial t}$$

$$\operatorname{rot} H = J_n$$

$$C_n\frac{\partial T}{\partial t} = \rho_n J_n^2 + \operatorname{div}(K_n \operatorname{grad} T)$$

For simplicity, the physical properties of materials are assumed to be independent of temperature. Equivalent and average characteristics are derived for each zone after a homogenization that takes into account the local structure of the composite strand.

In each zone, according to the differential equations, the solution for the temperature can be developed in the form of a sum of terms with time and space, for separate variables:

$$T(x,t) = \sum X_i(x) \exp(\lambda_i t/\tau) \tag{1}$$

where

$$\tau = \frac{C_f a_f^2}{K_f} \tag{2}$$

A dimensionless differential equation with respect to space can be derived in the filamentary region:

$$X^{(4)} - \lambda(1-\nu)X^{(2)} + \lambda(\lambda\nu - \beta)X = 0 \tag{3}$$

with the fundamental parameters

$$\beta = \frac{\mu_0 \eta_s^2 J_{c0}^2 a^2}{C_f(T_c - T_b)} \tag{4}$$

$$\nu = \frac{K_f \mu_0}{C_f \rho_f} \tag{5}$$

Note that $\eta_s a = \eta_f a_f$ and that

$$\eta = \frac{D_{\theta f}}{D_{mf}}$$

is the ratio of the thermal diffusivity over the magnetic diffusivity.

In composites with a highly resistive matrix, ν is much less than 1, which allows a further simplification of the differential equation

$$X^{(4)} - \lambda X^{(2)} - \lambda\beta X = 0 \tag{6}$$

In the outer copper shell, if the Joule heating is neglected, the heat equation becomes

$$X'' - \frac{\lambda}{\gamma} X = 0 \tag{7}$$

with

$$\gamma = \frac{K_n}{C_n}\frac{C_f}{K_f} \tag{8}$$

For a given constant transport current, there are eight boundary conditions among the three zones concerning the electric field, magnetic field, temperature, flux, and heat transfer to the outer coolant. The general solutions of the differential equations are combinations of eight hyperbolic functions, the coefficients of which can be derived from the set of eight linear equations.

In order to ensure self-consistency and to obtain a nonunique trivial solution for λ, the determinant of the system must be equal to zero. From Eq. (1), it can be inferred that any positive λ value leads to an irrevocable increase of temperature with time. The first positive value for λ determines the limit between stability and instability (see also Refs. 2 and 3).

Our calculations were carried out in the same way as in previous papers (2–7). One of the goals of this work was to find an *analytical* expression for the stability conditions that included the parameter β and other physical characteristics.

As stability is violated for the very first positive λ value, a power-series expansion of the hyperbolic functions can be made. Let us set the dimensionless parameters

$$\gamma = \frac{K_n C_f}{K_f C_n}, \qquad \alpha = \frac{K_n}{K_f}, \qquad \delta = \frac{\rho_n C_f}{\mu_0 K_f}, \qquad \epsilon = \frac{e_n}{a_f}, \qquad h = \frac{h_t a_f}{K_f}$$

When λ tends to zero, it can be written in the form

$$\lambda = \frac{4\beta h \alpha}{4\beta(1+h\epsilon)\left(\frac{\beta i^3}{3} - 1\right) - \alpha\left[\frac{4\beta\epsilon}{\gamma}\left(1 + \frac{h\epsilon}{2}\right) + h\left(1 + \frac{2\beta\epsilon^2}{\delta} + 2\beta + \frac{4\beta\epsilon}{\delta} i - \frac{\beta^2}{2} i^4\right)\right]} \tag{9}$$

GENERAL STABILITY CRITERION

From our final expression Eq. (9), we can find a condition for β when an instability occurs ($\lambda \geq 0$). Conversely, a general stability criterion can be written in the form

$$\beta < \frac{A + (A^2 + 4hB)^{1/2}}{2B} \tag{10}$$

with

$$A = 4\left(1 + \frac{h\epsilon}{\alpha}\right) + 2\frac{h\epsilon}{\delta}(2i + \epsilon) + 2h + 4\epsilon\frac{\gamma}{\alpha}\left(1 + \frac{h\epsilon}{2\alpha}\right) \tag{11}$$

$$B = \frac{4}{3}\left(1 + \frac{h\epsilon}{\alpha}\right) i^3 + \frac{h}{2} i^4 \tag{12}$$

For greater clarity, it is convenient to present criterion Eq. (10) in terms of an energy balance. The magnetic energy released by the flux jump should be less than the available enthalpy in the system (composite and exchange to the surrounding helium layer)

$$\frac{\mu_0(\eta_s J_{co} a)^2}{3} i^3 < C_f(T_c - T_b)[1 + f(h_t, K, C, \rho, \ldots)]$$

where f is a function containing all the extra terms issuing from Eq. (10). To confirm the general character of this criterion for composites with a highly resistive matrix, we can show how some usual criteria can be found, at the cost of a few approximations.

We must bear in mind in any case that the first assumption that has been made in our calculations is $\nu \to 0$ in the filamentary zone ($\rho_f \to \infty$).

Let us recall a few typical orders of magnitude for physical properties (Table 1). It can be seen that thermal and electrical diffusivities are exactly permutable for the inner region and the outer shell.

Table 1. Physical Properties of the Materials of Superconducting Composites

Zone	K (W·mK^{-1})	C (J·m^{-3}·K^{-1})	ρ (Ω·m)	D_θ (m^2·s^{-1})	D_m (m^2·s^{-1})
NbTi, CuNi	1.0	1500	3×10^{-7}	6×10^{-4}	0.25
Cu	300	1000	3×10^{-10}	0.3	2.5×10^{-4}

It takes 0.5 μs to diffuse heat over 0.5 mm in the copper or magnetic flux in the CuNi matrix, whereas it takes a much longer time (0.3 ms) to diffuse magnetic flux in the copper or heat in the CuNi matrix. This means that the process in the filamentary region is almost locally adiabatic. There is hardly any current generated in the inner core due to the fact that at the initiation of the flux jump the electric field in the inner core is zero when it is at a maximum at the interface between the filamentary region and the outer copper shell. The self-field effect tends to expel the excess current to the periphery.

APPLICATIONS TO A FEW SIMPLIFIED OR USUAL CRITERIA

Adiabatic Criterion

If one assumes $h_t \to 0$ (no heat exchange with the helium bath), Eq. (10) becomes

$$\beta < \frac{3}{i^3}\left(1 + \frac{e_n C_n}{a_f C_f}\right) \tag{13}$$

and using Eq. (11) we obtain

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} i^3 < C_f(T_c - T_b)\left(1 + \frac{e_n C_n}{a_f C_f}\right) \tag{14}$$

This criterion indicates the role of the enthalpy of the outer normal metal shell. With $i = 1$ and $e_n = 0$ (no shell), we obtain the usual so-called adiabatic criterion (1–4). The "stable" parameter β varies with i as $1/i^3$. A direct application of this expression is the evaluation of the maximum average current density J that can be carried in a wire of half-dimension a. Letting

$$J = \eta_s J_c i \qquad \text{and} \qquad C = C_f\left(1 + \frac{e_n C_n}{a_f C_f}\right)$$

yields

$$J < \left(\frac{C(T_c - T_b)}{\mu_0}\right)^{1/3} (\eta_s J_c)^{1/3} a^{-2/3} \tag{15}$$

Whereas using a wire with the highest possible critical current density seems important, the gain in stable density is not so significant, however. On the other hand, the stability is not as dependent on the size a as is often considered. Although it is true that the stability parameter β varies as a^2, the maximum stable average current density varies as $a^{-2/3}$. For instance, doubling the thickness a results in a stable average current density multiplied by 0.63.

Dynamic Criterion

Several workers have proposed analytical expressions for stability criteria of homogeneous composites (6–8). To our knowledge, they have always been presented for $i = 1$ (critical current) and for nonzero values for ν in the case of exact solutions obtained along a similar approach.

Let us consider a typical composite wire as a guide to justify some approximations, with $a_f = 0.7 \times 10^{-3}$ m, $e_n = 0.3 \times 10^{-3}$ m, $\eta_s = 0.2$, $\eta_{Cu} = 0.5$, $\eta_{CuNi} = 0.3$, and $h_t = 10^3$ W·m^{-2}·K^{-1}. The basic assumption ($\nu = 0$) is satisfied by $\rho_f/\mu_0 \gg K_f/C_f$. To derive a dynamic criterion, it is necessary to assume that the heat conductivity in the inner region is not negligible (ρ_f is kept infinite). The two terms $h_t a_f/K_f$ and $h_t e_n/K_n$ are considered to be much less than 1.

The general criterion can then be written as follows:

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} i^3 < C_f(T_c - T_b)\left(1 + \frac{e_n C_n}{a_f C_f}\right)\left(1 + A_1 \frac{h_t a_f}{K_f} + A_2 \frac{h_t e_n}{K_n}\right) \tag{16}$$

with

$$A_1 = \frac{2 + 3\left(\frac{e_n C_n}{a_f C_f}\right) + \frac{i^3}{3}}{4\left(1 + \frac{e_n C_n}{a_f C_f}\right)^2} - \tfrac{3}{8} i \tag{17}$$

$$A_2 = \frac{\left(1 + \frac{e_n C_n}{a_f C_f}\right)\left[2 + \frac{\mu_0 K_n}{\rho_n C_f}\left(\frac{e_f}{a_f} + 2i\right) + \frac{e_n C_n}{a_f C_f}\right] + 2 + 3\left(\frac{e_n C_n}{a_f C_f}\right) + \left(\frac{e_n C_n}{a_f C_f}\right)^2}{4\left(1 + \frac{e_n C_n}{a_f C_f}\right)^2} + \frac{\frac{\mu_0 K_n}{\rho_n C_f}\left(\frac{e_n}{a_f} + 2i\right)\left(2 + \frac{e_n C_n}{a_f C_f}\right)}{4\left(1 + \frac{e_n C_n}{a_f C_f}\right)^2} - 1 \tag{18}$$

Equation (16) emphasizes the respective influences of the inner region and of the outer shell. From Eq. (16), two particular cases can be derived as follows.

1. Letting $e_n = 0$ (no outer normal shell) and $i = 1$, we find

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} < C_f(T_c - T_b)\left(1 + \frac{5}{24}\frac{h_t a_f}{K_f}\right) \tag{19}$$

This expression is similar to the expression obtained previously (6) in the case of a homogeneous multifilamentary composite for low values of the ratio ν:

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} < C_m(T_c - T_b)\left(1 + \frac{7}{20}\frac{h_t a}{K_m}\right) \tag{20}$$

The small discrepancy between the two expressions can be mainly ascribed to the fact that the calculations performed previously (6) were only correct for $\nu \neq 0$, whereas the present calculations were carried out with $\nu = 0$.

2. If we keep $e_n \neq 0$ and assume a very low resistivity for the outer normal metal shell (made of copper, for instance), Eq. (16) becomes

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} i^3 < C_f(T_c - T_b)\left(1 + \frac{e_n C_n}{a_f C_f}\right)\left(1 + A_2 \frac{h_t e_n}{K_n}\right) \quad (21)$$

with

$$A_2 = \frac{\mu_0 K_n}{\rho_n C_f}\left(\frac{e_n}{a_f} + 2i\right)\frac{3 + 2\left(\dfrac{e_n C_n}{a_f C_f}\right)}{4\left(1 + \dfrac{e_n C_n}{a_f C_f}\right)^2} \quad (22)$$

Again we find a simplified criterion that emphasizes the beneficial influence of a low resistivity ρ_n as in the case of homogeneous composites. This is already visible in Eqs. (21) and (22). It can be pointed out more clearly for $e_n/a_f \ll 1$, which permits another simplification step. It follows that

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} i^3 < C_f(T_c - T_b)\left[1 + \frac{e_n C_n}{a_f C_f} + \frac{3}{4}\frac{\mu_0}{\rho_n}\frac{h_t e_n}{C_f}\left(\frac{e_n}{a_f} + 2i\right)\right] \quad (23)$$

We can see that the simplified Eq. (23) of our general criterion Eq. (10) can also be put in a form similar to more conventional criteria derived for homogeneous composites. For instance, for $i = 1$ (and in the frame of the particular assumptions), we find stability for

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} < (T_c - T_b)\left(C_f + C_n \frac{e_n}{a_f} + \frac{3}{2}\frac{\mu_0 h_t e_n}{\rho_n}\right) \quad (24)$$

when it was determined for homogeneous composites in previous work

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} < (T_c - T_b)\left(C_m + \frac{3}{10}\frac{\mu_0 h_t a}{\rho_m}\right)$$

(exact solution in Ref. 6) and

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} < (T_c - T_b)\left(C_m + \frac{4}{\pi^2}\frac{\mu_0 h_t a}{\rho_m}\right)$$

(approximate solution in Ref. 8). Both expressions were given with no outer copper shell.

We can see that the criterion given in Eqs. (23) and (24) indicates the enhancement of stability by the heat removal to the coolant due to the "shell effect." In addition to the enthalpy of the composite, the third term in Eq. (24) represents the enthalpy transferred to the helium during the diffusion time of the current in the copper shell.

The enhancement of stability with increase in the Cu shell thickness is clear. Evidently a good heat transfer is of no help when no copper can damp or slow the development of the flux jump. In contrast, with a sufficient copper thickness, time is provided to take advantage of the heat transfer to the coolant and therefore for accounting for the enthalpy absorbed by helium in the heat energy balance.

Similarity with the Cryogenic Criterion

Equation (23) contains two terms: the enthalpy of the composite and the enthalpy absorbed by the helium. If the resistivity of the shell is small enough or the heat transfer large enough, the enthalpy of the composite becomes negligible, and stability is completely ensured by the transfer to helium.

The criterion becomes

$$\frac{\mu_0(\eta_s J_{c0} a)^2}{3} i^3 < \frac{3\mu_0 h_t e_n}{4\rho_n}\left(\frac{e_n}{a_f} + 2i\right)(T_c - T_b) \quad (25)$$

Under these conditions, the criterion can be written, to a first approximation, in the form of generated and exchanged heat fluxes. For a thin outer copper shell

$$\frac{4\rho_n(\eta_s J_{c0} i)^2 a^2}{9e_n} < h_t(T_c - T_b) \quad (26)$$

Effect of Thermal Conductivity

In the general expression Eq. (10), assuming now a very high heat exchange to the helium ($h_t \to \infty$) and good thermal and electrical properties of the shell (high K_n and low ρ_n), we obtain

$$A \approx \frac{4\mu_0 h_t e_n}{\rho_n C_f}$$

$$B \approx \frac{h_t a_f}{2K_f} i^4$$

hence

$$A^2 \gg 4\frac{h_t a_f}{K_f} B$$

In particular, when $i = 1$, it can be found that the condition for stability becomes

$$\frac{\rho_n(\eta_s J_{c0} a)^2}{8K_f(T_c - T_b)}\frac{a_f}{e_n} < 1 \quad (27)$$

This expression can be compared with the expression that was regarded as being the dynamic stability criterion:

$$\frac{\rho_m(\eta_s J_{c0} a)^2}{3K_m(T_c - T_b)} < 1$$

This criterion, developed by several groups, quoted in Ref. 8, assumes an infinite heat transfer coefficient to the coolant. As a result, it is too optimistic in most practical cases. However, it is correct in that it determines the limit for the existence of stationary solutions for temperature-dependent critical cur-

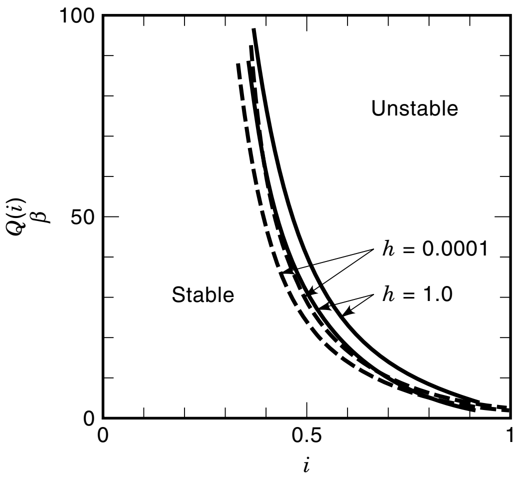

Figure 2. Comparison of stability curves for composites with no outer shell with usual curves given for homogeneous composites. Dashed lines, this work ($\nu = 0$) ; solid lines, Ref. 5 ($\nu = 0.1$).

rent profiles inside the filamentary region. It points out clearly, in this case, the effect of poor thermal conductivity.

Simplified Analytical Expression for Homogeneous Composites

In the general criterion Eq. (10) written in a dimensionless form, stability is ensured when $\beta < Q(i)$, where

$$Q(i) = \frac{A + (A^2 + 4hB)^{1/2}}{2B}$$

The coefficients A and B include all the physical parameters, the heat transfer, and the reduced transport current. For a given set of physical parameters, it is possible to plot the expression $Q(i)$ as a function of the current i in the form of a boundary line between stability and instability for any given value of the pair (β, i).

In order to compare with theoretical and numerical calculations performed previously (6), we give here an analytical expression for the particular case of a highly resistive matrix superconducting composite without a stabilizing shell ($e_n = 0$).

In the general criterion Eq. (10), let us set $e_n = 0$ and define $h = (h_t a_f)/K_f$. We find

$$\beta = \frac{3}{i^3} \frac{2(h+2) + [2h^2(2+i^4) + 16h(1+i^3/3) + 16]^{1/2}}{8 + 3hi} \qquad (28)$$

This expression can be compared with the expression obtained for the homogeneous composite studied previously (6) in the case of a low ratio of thermal diffusivity to magnetic diffusivity (ν):

$$\nu = \frac{D_\theta}{D_m}$$

Figure 2 shows the stability function $Q(i)$ for two values of reduced heat transfer h. In the present approach $\nu = 0$ because ρ_f is infinite. In the previous approach (6), the limiting case $\nu = 0.1$ was considered. It can be seen that the results are similar if it is remembered that the theory of the homogeneous composite was developed for a round wire (a correction coefficient of $\pi/4$ was applied to the curves plotted previously (6) to obtain the curves plotted in Fig. 2.

Developing Eq. (28) for $i = 1$ and for small values of the reduced heat transfer h leads to

$$\beta_s = 3\left(1 + \frac{5h}{24}\right)$$

[another form of Eq. (19)].

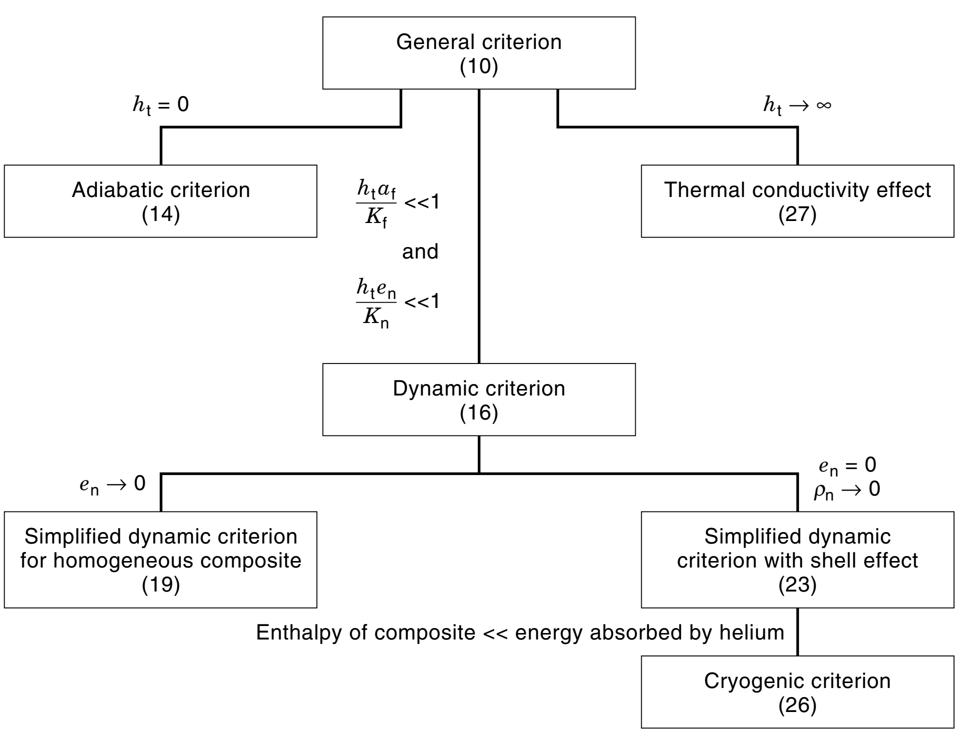

Figure 3. Summary of conditions to deduce usual criteria from the general stability criterion

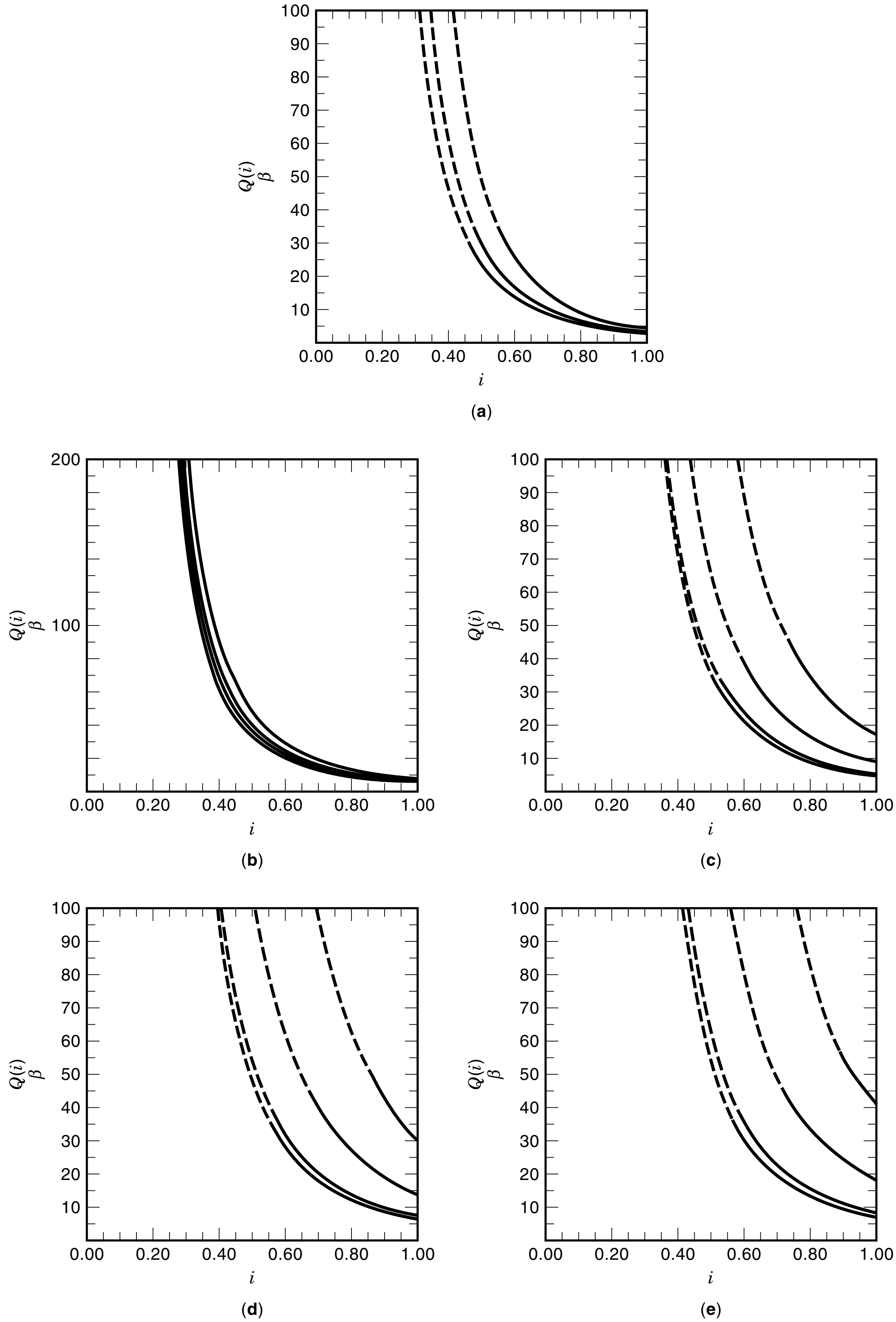

Figure 4. Stability is improved when both copper shell thickness and the heat transfer coefficient attain significant values for copper shell thickness $\epsilon = e_n/a_f$: (a) 0; (b) 0.3; (c) 0.6; (d) 1.0; (e) 1.3. Lines from top to bottom: h = 10, 1, 0.1, 0.01.

Our expression especially developed for the case of an outer good conducting shell is in perfect agreement with previous models devoted to homogeneous composites.

SUMMARY

At the cost of some particular assumptions, it has been shown that the general criterion Eq. (10) encompasses the usual criteria that have been already developed in various particular cases. This can be represented schematically as in Fig. 3.

INFLUENCE OF THE OUTER SHELL THICKNESS AND OF THE HEAT TRANSFER COEFFICIENTS

A set of curves have been plotted in Fig. 4 using the more general criterion Eq. (10) directly. The particular case under investigation corresponds to typical orders of magnitude of $\gamma = 200$, $\alpha = 200$, and $\delta = 1$.

The thickness of the shell and the heat transfer coefficients are given in the form of the dimensionless parameters $\epsilon = e_n/a_f$ and $h = h_t a_f/K_f$.

It is clearly seen in Fig. 4(a), for $\epsilon = 0$, that because of the low thermal conductivity of the filamentary region, little is expected in improving the stability from a good heat transfer to the coolant. Figures 4(a)–4(e) show the significant role of the heat transfer coefficient when ϵ is not negligible (for low values of h, improvement is only provided by the increase in the equivalent heat capacity of the composite). A noticeable improvement in stability is provided when both ϵ and h simultaneously attain significant values.

The copper fraction in the conductor is given by $\eta_{Cu} = \epsilon/(1 + \epsilon)$. This expression means that the amount of copper is totally concentrated in the outer shell. It is, for instance, 23% for $\epsilon = 0.3$ and 56% for $\epsilon = 1.3$. The reduced heat transfer is about $h = 1$ for $h_t = 10^3\ \mathrm{W^3 \cdot m^{-2} \cdot K^{-1}}$ and $a_f = 0.6 \times 10^{-3}$ m.

The influence of the shell thickness is summarized in one set of curves for $h = 1$ in Fig. 5.

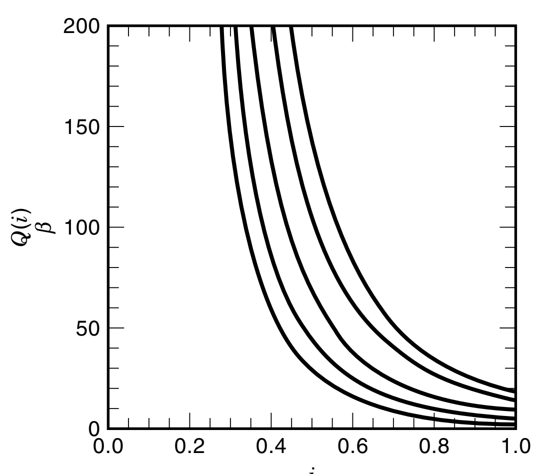

Figure 5. Influence of the copper shell thickness $\epsilon = e_n/a_f$ on stability curves for $h = 1$. Lines from left to right: $\epsilon = 0, 0.3, 0.6, 1.0, 1.3$.

DETERMINATION OF THE STABLE CURRENT I_s IN A PARTICULAR EXAMPLE

For any given composite there is no special difficulty in plotting the general curve $Q(i)$ of criterion Eq. (10). Then, the evaluation of the parameter

$$\beta = \frac{\mu_0(\eta_s J_{c0} a)^2}{C_f(T_c - T_b)}$$

leads immediately to the determination of the maximum stable current i_s.

As a practical example, let us consider a multifilamentary composite with a cupronickel matrix surrounded by a copper shell with the two fixed parameters $2R = 1.35$ mm (diameter) and $\eta_s = 0.20$ (superconductor volume fraction).

In order to point out again the influence of the outer copper, let us present the curves in terms of copper volume fraction (the CuNi is entirely located in the filamentary region)

$$R_f^2 = (1 - \eta_{Cu})R^2 \quad \text{and} \quad \eta_{CuNi} = 1 - \eta_s - \eta_{Cu}$$

The equivalent dimensions used in the slab model are given by

$$a = \frac{\sqrt{\pi}}{2} R \quad \text{and} \quad a_f = \frac{\sqrt{\pi}}{2} R_f$$

respectively. All the parameters intervening in the expressions for $Q(i)$ can be derived as equivalent physical properties.

The expressions $Q(i)$ are plotted in Figs. 6(a)–6(e) with the copper fraction as a parameter for given values of the actual heat transfer coefficient h_t. Up to a heat transfer of $10^2\ \mathrm{W \cdot m^{-2} \cdot K^{-1}}$, the composite can be considered to behave as in adiabatic conditions.

In order to determine the stable operating current, we can proceed as follows. Calculate the parameter

$$\beta = \frac{\mu_0(\eta_s J_{c0} a)^2}{C_f(T_c - T_b)}$$

and then, for a given copper fraction and a given heat transfer coefficient, determine the maximum operating current. For instance, Fig. 6(d) shows the evaluation of the stable current for two amounts of copper for a particular case. In a field of 11 T and if one assumes the strand to be cooled in superfluid helium to 1.8 K ($h_t = 5 \times 10^3\ \mathrm{W \cdot m^{-2}}$), $\beta \approx 90$, which means that the transport current can be increased safely up to 60% of the critical current for 60% of copper, whereas it is 77% of the critical current for 70% of copper.

SMALL PERTURBATION AND THE CRITICAL STATE MODEL

The theoretical calculations that have been discussed in the previous sections assume an ideal critical state model, that is, the electric field and the current density in the superconductor are related simply by $E \neq 0 \Rightarrow J = J_{co}$. Actually, in composites made up with a large number of fine filaments, a continuous dependence of E on J, caused mainly by inhomogeneities, can be observed.

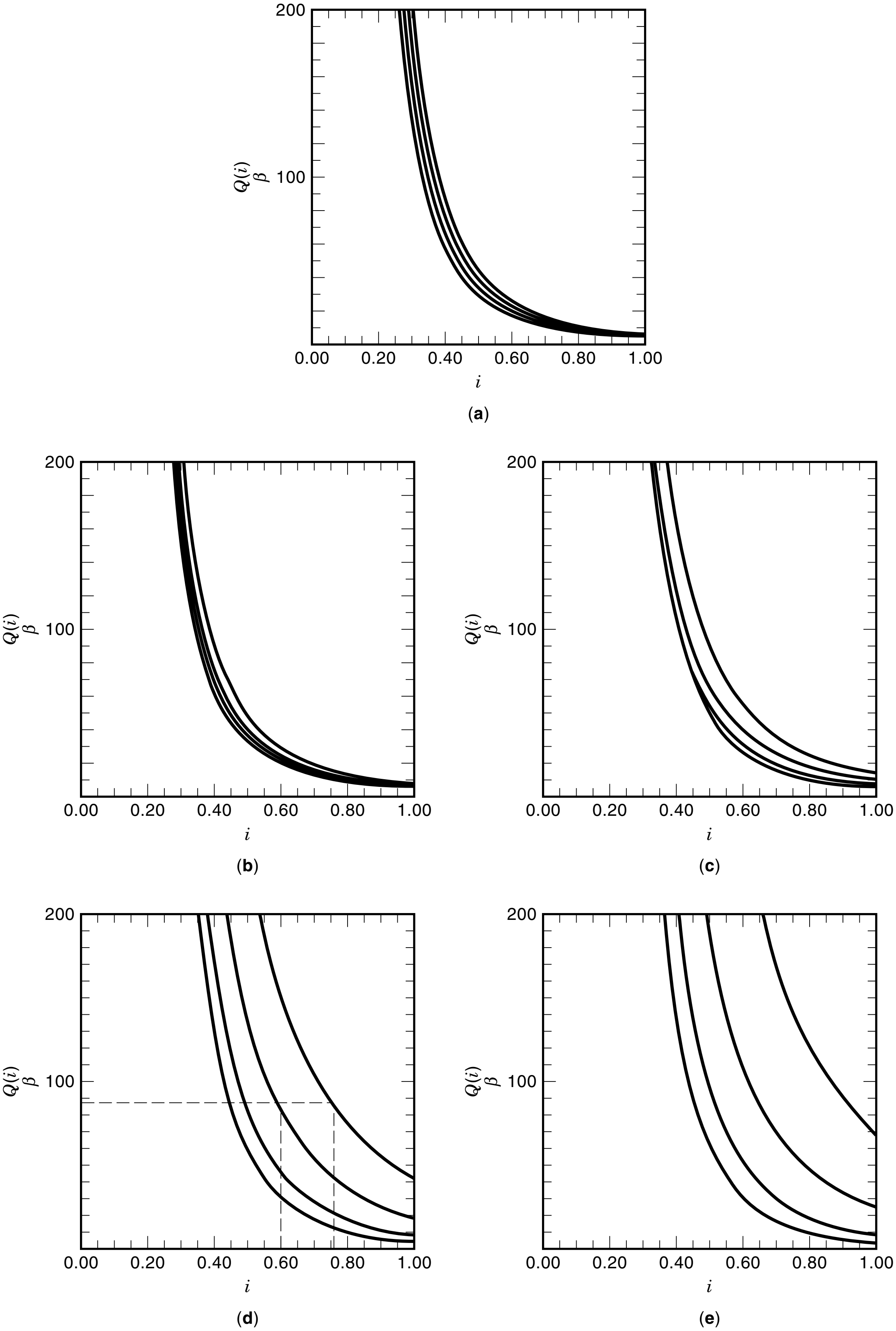

Figure 6. Stability curves for a 1.35 mm diameter wire with 20% of superconductor. h_t = (a) 0; (b) 10^2; (c) 10^3; (d) 5×10^3; (e) 10^4 $W \cdot m^{-2} \cdot K^{-1}$. Lines from left to right: η_{Cu} = 0.3, 0.5, 0.6, 0.7.

One of the expressions that is most often accepted is

$$E = E_0 \left(\frac{J_c}{J_{c0}}\right)^n$$

where n is of the order of 10–100 depending on the field level and on the quality of manufacture. Under these conditions, the "critical current" is defined for a given electric field. For instance, the conditions $E_0 = 10^{-5}\ \mathrm{V \cdot m^{-1}}$ and $J_{c0} = 2 \times 10^9\ \mathrm{A \cdot m^{-2}}$ result in another definition of critical current for $E = 10^{-4}\ \mathrm{V \cdot m^{-1}}$: $J_c = 2.16 \times 10^9\ \mathrm{A \cdot m^{-2}}$ for $n = 30$ and $J_c = 2.046 \times 10^9\ \mathrm{A \cdot m^{-2}}$ for $n = 100$.

One of the consequences is that the concept of a small perturbation is more difficult to define, since there is no step function in the $E(J)$ relationship. Another way of pointing out this effect is to compare the apparent resistivity in this resistive "critical state" with the resistivity ρ_{Cu} of the stabilizing copper.

The differential resistivity is

$$\frac{\partial E}{\partial J} = n\,\frac{E_0}{J_{c0}} \left(\frac{J}{J_{c0}}\right)^{n-1}$$

The "effective critical" current J_c^* for which this value equals the copper resistivity can be estimated. For $\partial E/\partial J = \rho_{Cu}$,

$$\frac{J_c^*}{J_{c0}} = \left(\rho_{Cu}\,\frac{J_{c0}}{nE_0}\right)^{1/n-1}$$

Assuming $\rho_{Cu} \approx 5 \times 10^{-10}\ \Omega \cdot \mathrm{m}$ and $E_0 = 10^{-5}\ \mathrm{V \cdot m^{-1}}$, for $n = 30$

$$J_c^* = 1.32 J_{c0}$$

and for $n = 100$

$$J_c^* = 1.07 J_{c0}$$

This shows that the critical current density J_{c0} defined for E_0 has to be noticeably exceeded in order to obtain a stabilizing effect by the copper matrix.

In order to establish the full relevance to our model, the perturbation has to generate enough electric field so that J_c^* is locally exceeded. This is hardly to be expected from the ac losses produced by an external changing field or by the self-field of the increasing current itself, except in case of very high field or current change rate (in less than 10 ms).

A way to cope with this problem is to evaluate the stability through β and i, not in using J_{c0} to define β but with J_c^* taken as 10 to 30% more than J_{c0} depending on the n value (Fig. 7). This will result in a larger value for β (20 to 60%) and a smaller value for i. However, the stable transport current I_s^* will be larger

$$I_s^* = i_s I_c^*$$

As a result, one can see that a low n value could suggest that the conductor is more stable, when actually it is only the "true" critical current that has been underestimated.

Some other factors can give rise to a more stable current than given in the calculations. They arise from the existence of a steady-state current distribution (for $E \approx 0$) that is different from the distribution given by the ideal critical state model. The spiral aspect of the filament enables some current to flow in the unsaturated region (5); after a current increase, the current decays in the outer layers of filaments because of the resistive effect (n value), and in a conductor with limited length, the current distribution is mainly imposed by the transfer resistances from layer to layer.

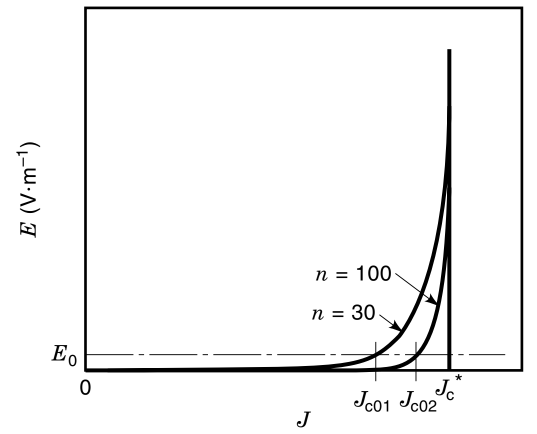

Figure 7. How the determination of J_c is affected by the "n" value.

To conclude with these considerations, we can describe a typical perturbation that can obviously give rise to the required electric field for transferring enough current in the copper (Fig. 8). If we consider that the current-sharing temperature is given by

$$T_{cs} - T_b = (T_c - T_b)(1 - i)$$

and if we consider that a shift in critical current of typically 10% is at least necessary, because of the shape of the $E(J)$ curve, a small perturbation $\Delta T \approx 0.1(T_c - T_b)$ is typical for initiating a flux jump in a wire whose operating parameters (β, h, i, ϵ) just lie on the theoretical curves between stable and unstable behavior [in other words, $J_{c0}(E_0, T_b)$ can also be considered as J_c^* at the temperature $T_b + \Delta T$; see Fig. 7].

CONCLUSION

The general analytical criterion that has been presented concerns the intrinsic stability of superconducting composites

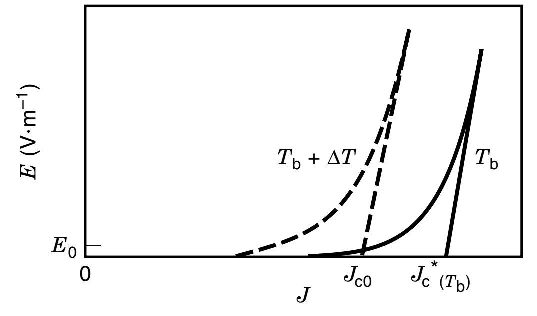

Figure 8. Effect of a small temperature perturbation on the $E(J)$ curve.

with a highly resistive matrix. It has shown consistency and continuity with previous theoretical results.

Apart from the general character of this criterion, the influence of an outer copper shell on intrinsic stability conditions has been clearly emphasized with the help of some analytical expressions.

The "critical" stability curve $Q(i)$ for a given composite under precise operating conditions acts as a useful basis to predict its behavior and to determine a "safe" current for intrinsic stability.

Conversely, our criterion can be used in a straightforward manner for a composite design. It becomes possible to consider how a modification of physical characteristics (ratios, relative positions of the materials, etc.) can improve the stability conditions. The general criterion Eq. (10), given in an analytical form, can be of great help in the optimization of the structure of low-loss composites.

We have also pointed out the great advantage of locating the copper at the periphery of the composite in order to take full advantage of the appearance of a maximum electric field to help current flow in the copper. On the other hand, copper around the axis is of almost no help, as the electric field is always negligible in the unsaturated region. However, the debate is not totally settled. In fact, the self-field effect is reduced and the conditions for a flux jump are less likely when the filaments are distributed in a concentric layer with no superconducting region close to the axis (9). The theory has to be developed for this case. Also undetermined is whether a good compromise exists with some copper both in the center and at the periphery.

NOMENCLATURE

a Composite external radius (slab model) (m)
a_f Multifilamentary zone radius (m)
a_s Saturated zone radius (m)
B_a Background magnetic field
C Specific heat of the composite ($J \cdot m^{-3} \cdot K^{-1}$)
C_f Average specific heat in the filamentary zone ($J \cdot m^{-3} \cdot K^{-1}$)
C_m Average specific heat in homogeneous composites ($J \cdot m^{-3} \cdot K^{-1}$)
C_n Average specific heat in the normal metal shell ($J \cdot m^{-3} \cdot K^{-1}$)
D_m Magnetic diffusivity ($m^2 \cdot s^{-1}$)
D_θ Thermal diffusivity ($m^2 \cdot s^{-1}$)
E Electric field ($V \cdot m^{-1}$)
e_n Normal metal shell thickness (m)
h $h_t a_f / K_f$, reduced heat transfer coefficient
h_t Heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
i I_t/I_c, reduced transport current
I_c Critical current (A)
I_s Stable transport current (A)
I_t Transport current (A)
J Current density in the composite ($A \cdot m^{-2}$)
J_c Critical current density at T for the given B ($A \cdot m^{-2}$)
J_{c0} Critical current density at T_b and B ($A \cdot m^{-2}$)
J_n Current density in a normal metal region
K Thermal conductivity in the composite ($W \cdot m^{-1} \cdot K^{-1}$
K_f Average thermal conductivity in filamentary zone ($W \cdot m^{-1} \cdot K^{-1}$)
K_m Average thermal conductivity in homogeneous composites ($W \cdot m^{-1} \cdot K^{-1}$)
K_n Average thermal conductivity in normal metal shell ($W \cdot m^{-1} \cdot K^{-1}$)
T_b Helium bath temperature (K)
T_c Critical temperature for a given field (K)

Greek Letters

α K_n/K_f, dimensionless ratio
β Dimensionless stability parameter
β_s Critical value of β bounding stable and unstable domains
γ $(K_n/K_f)(C_f/C_n)$, dimensionless ratio
δ $(\rho_n/\mu_0)(C_f/K_f)$, dimensionless ratio
ϵ e_n/a_f, reduced thickness of normal metal shell
η_f Superconductor ratio in the filamentary zone
η_s Superconductor ratio in the composite
η_{Cu} Copper ratio in the composite
η_{CuNi} Cupronickel ratio in the composite
λ Eigenvalue of the system
μ_0 Magnetic permeability of vacuum
ν $(K_f/C_f)(\mu_0/\rho_f)$, dimensionless ratio of the diffusivities
ρ_f Matrix resistivity ($\Omega \cdot m$)
ρ_m Average composite normal resistivity ($\Omega \cdot m$)
ρ_n Normal metal shell resistivity ($\Omega \cdot m$)

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