FRIIS FREE-SPACE TRANSMISSION FORMULA

STATEMENT OF THE FORMULA

For a free-space transmission path, the available power P_r at the receiving antenna terminals is related to the power input P_t at the transmitting antenna terminals by the ratio

$$\frac{P_r}{P_t} = \frac{A_t A_r}{\lambda^2 R^2} \tag{1}$$

In this expression A_r denotes the effective area of the receiving antenna in the direction of the transmitter, A_t is the effective area of the transmitting antenna in the direction of the receiver, λ is the wavelength, and R is the distance separating the transmitting and receiving antenna, all measured in compatible units (e.g., both powers in watts, both areas in square meters, λ and R in meters). An alternative form is

$$\frac{P_r}{P_t} = \frac{G_t G_r \lambda^2}{(4\pi)^2 R^2} \tag{2}$$

where G_t and G_r denote, respectively, the gain of the transmitting antenna in the direction of the receiver and the gain of the receiving antenna in the direction of the transmitter, and λ and R must be given in the same units. For convenience, a decibel formulation is sometimes used. Such formulations are obtained by setting $\lambda = c/f$, where c is the free-space velocity of light and f the frequency, converting quantities to the desired units (e.g., R from meters to kilometers) taking the common logarithm of both sides of the resulting equation, and multiplying by 10. Algebraic and arithmetic manipulation then yield expressions such as

$$\begin{aligned} P_{r,dbW} = {} & P_{t,dbW} + G_{t,dB} + G_{r,dB} - 20\log_{10} R_{km} \\ & - 20\log_{10} f_{MHz} - 32.4 \end{aligned} \tag{3}$$

In this example of the decibel form of the formula, the power at the transmitting antenna terminals and the power available at the receiving antenna terminals are expressed in dBW, (i.e., the power relative to 1 W expressed in decibels), the distance is given in kilometers, and the frequency in megahertz. When other units are used, the form of the equation remains the same, but the value of the constant term may differ. In the decibel form of the formula, the antenna gains are always specified in decibels.

ASSUMPTIONS

Free Space

The term "free-space" implies that environmental effects, such as the effects of the ground and of the atmosphere, are negligible. This is sometimes a very good approximation. For example, in the case of a transmission from the earth surface to a satellite appearing not too near the horizon in the frequency range 500 MHz to 10 GHz, the antenna will usually be sufficiently directive to prevent substantial power from impinging on the ground. Also, in this frequency range atmospheric effects are small. Thus the Friis formulation is directly applicable. In other situations the free-space equation may be the starting formulation to which corrections for other effects (e.g., atmospheric attenuation, attenuation due to the ground) can be added. In fact, attenuation over a path is often specified as the attenuation relative to that for the same distance in free space; the latter would be calculated by the Friis formula.

Far Field

The antennas must be in the far field (Fraunhofer region) with respect to one another. This requires a sufficient distance so that the transmitted field at the receiving antenna is a spherical wave which may be approximated as a plane wave over the receiving antenna aperture. A frequently used criterion is that the longest and shortest paths between the two antenna apertures should not differ by more than 1/16 wavelength. Also the distance must be greater than five times the sum of the largest transmitting antenna dimension and the largest receiving antenna dimension, and it must exceed 1.6 wavelengths.

Polarization

In this formulation it has been assumed that the polarization of the wave at the receiving antenna is optimal for that antenna; otherwise a polarization-mismatch factor (in the decibel formula, a polarization-mismatch term) must be included.

DERIVATION

Equations (1) and (2) can be derived easily from basic physical principles. For a hypothetical lossless, isotropic transmitting antenna the power applied to the input terminals would be spread uniformly over a sphere at radius R, giving a flux density

$$S_i = \frac{P_t}{4\pi R^2} \tag{4}$$

For an actual antenna, by the definition of antenna gain, the field in the receiver direction is obtained by

$$S = S_i G_t = \frac{P_t G_t}{4\pi R^2} \tag{5}$$

By definition of the effective area, the power available at the receiving antenna terminals for optimum polarization is given by

$$P_r = S A_r = \frac{P_t G_t A_r}{4\pi R^2} \tag{6}$$

Use of the relationship between gain and effective area

$$G = \frac{4\pi A}{\lambda^2} \tag{7}$$

then leads to either Eq. (1) or Eq. (2).

HISTORICAL NOTE

The formulas in Eqs. (1) and (2) are named after Harald Trap Friis, who published Eq. (1) in 1946 while he was a research engineer at Bell Telephone Laboratories, Inc. (1). Mr. Friis later became Director of Research in High Frequency and Electronics at Bell Telephone Laboratories and was honored with numerous medals and awards for his technical work and his leadership.

BIBLIOGRAPHY

1. H. T. Friis, A note on a simple transmission formula, *Proc. IRE* (subsequently *Proc. IEEE*) **34**: 254–256, 1946.

Reading List

An interesting summary of derivations of the Friis formula from various perspectives can be found in D. C. Hogg, Fun with the Friis free-space transmission formula, *IEEE Antennas Propag., Mag.* **35** (4): 33–35, 1993.

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FUEL CELL.

See HYDROGEN ENERGY SYSTEMS.