RECEIVING ANTENNAS

An antenna can be used for both reception and transmission. This article discusses the properties of an antenna when it is used for the reception of an electromagnetic (EM) plane wave (1,2). Figure 1(a) shows an example of a receiving antenna, in which a half-wavelength dipole is used. A receiver, expressed as antenna load Z_L, is connected to the center terminals of the dipole. The arrows in this figure show the flow of the power density (Poynting power) of an incident EM plane wave, which propagates toward the dipole antenna from the right side. It is observed that the power of the incident EM plane wave moves toward the center terminals and is absorbed in the antenna load Z_L.

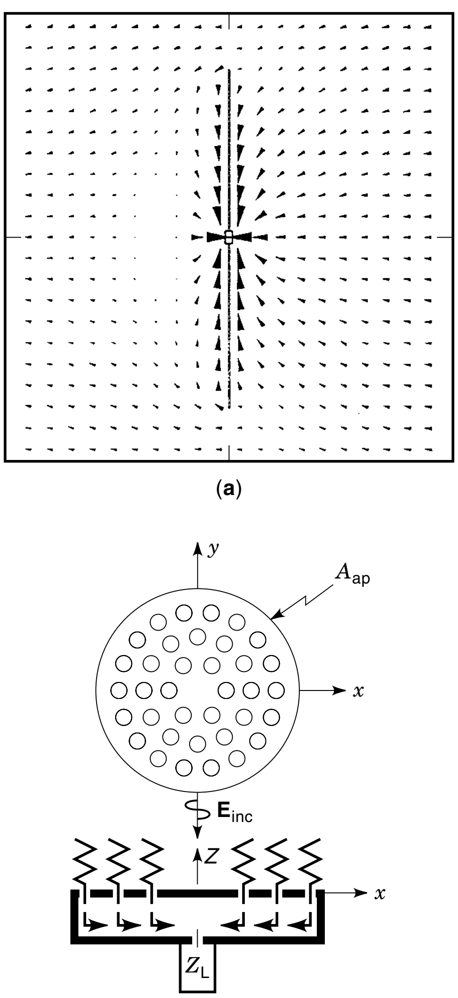

Figure 1. Reception of an electromagnetic plane wave. (a) Half-wavelength dipole antenna. (b) Array antenna on a circular cavity.

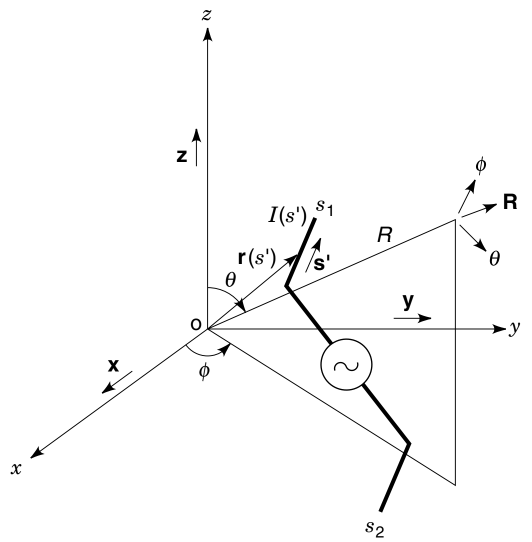

Figure 2. Coordinate system for radiation field.

The point of interest of a receiving antenna is the power W_L delivered to a receiver or antenna load Z_L, as shown in an example of Fig. 1(a). To calculate W_L, the induced current I_0 at the antenna terminals must be obtained. For this, an equivalent circuit for the receiving antenna is introduced. The maximum value of W_L is discussed on the basis of the vector effective height **h**.

The receiving antenna is recognized as an electrical net for collecting an EM plane wave. For example, the power of the EM plane wave in Fig. 1(b) is collected by many elements on a circular cavity of area A_{ap} (aperture) and transferred to the center port to which a receiver (Z_L) is connected. Generally, the collected power W_L at the center port is less than the power W_{ap} given by A_{ap} times the power density at the receiving antenna aperture. In other words, 100% of A_{ap} is not used for the reception of the EM plane wave. In the final section, the aperture efficiency η_{ap} as a measure of receiving antenna performance is defined after the discussion of the receiving cross section A_r. (Note that some fundamental relationships used in the discussion of receiving antennas are summarized in the last part of this article.)

VECTOR EFFECTIVE HEIGHT

Consider an antenna isolated in free space specified by permittivity ϵ_0 and permeability μ_0, as shown in Fig. 2, where spherical coordinates (R, θ, ϕ) are used with unit vectors ($\mathbf{R}$, $\boldsymbol{\theta}$, $\boldsymbol{\phi}$). The antenna is driven by a voltage source of frequency f. The current $I(s')$ flows along the antenna conductor of length $L = s_2 - s_1$, radiating the electric field **E** expressed as

$$\mathbf{E} = -j\,30k\frac{e^{-jkR}}{R}I_0\mathbf{h} \tag{1}$$

where **h**, called the *vector effective height,* is defined as

$$\mathbf{h} = \frac{1}{I_0}\{(\mathbf{S}\cdot\Theta)\Theta + (\mathbf{S}\cdot\boldsymbol{\phi})\boldsymbol{\phi}\} \tag{2}$$

with

$$S = \int_{s_1}^{s_2} I(s')\mathbf{s}' e^{jk\mathbf{r}(s')\cdot\mathbf{R}}\, ds' \tag{3}$$

The notations in Eqs. (1) to (3) are as follows: $k(= \omega\sqrt{\mu_0\epsilon_0} = 2\pi/\lambda$ with $\omega = 2\pi f$, where λ is wavelength) is the phase constant, I_0 is the input-terminal current [i.e., $I_0 = I(0)$], s' is the distance from the driving point to a source point along the antenna conductor, $\mathbf{s}'$ is the unit vector tangential to the antenna conductor at the source point, and $\mathbf{r}(s')$ is the position vector directed toward the source point from the coordinate origin.

Example. Let us consider an infinitesimal dipole antenna ($k\mathbf{r} \approx 0$) on the z axis, assuming that the current has constant amplitude and phase over the antenna length l. Equation (3) is calculated to be $\mathbf{S} = I_0 l\mathbf{s}'$. From Eq. (2), $\mathbf{h} = l(\mathbf{s}' \cdot \boldsymbol{\theta})\boldsymbol{\theta} = -l \sin\theta\boldsymbol{\theta}$. In the direction normal to the dipole axis ($\theta = 90°$), $\mathbf{h} = -l\boldsymbol{\theta} = l\mathbf{z} \equiv \mathbf{h}_d$.

OPEN-CIRCUIT VOLTAGE

It should be noted that the vector effective height $\mathbf{h}$ is defined for the situation where the antenna is used for transmission. Let us consider how $\mathbf{h}$ is related to receiving antenna operation.

Figure 3 shows two antenna systems in which antenna 1 with vector effective height $\mathbf{h}$ is used as a transmitted antenna in (a) and as a receiving antenna in (b). I_{01} is the terminal current and V_{01} is the open terminal voltage (*open-circuit voltage*).

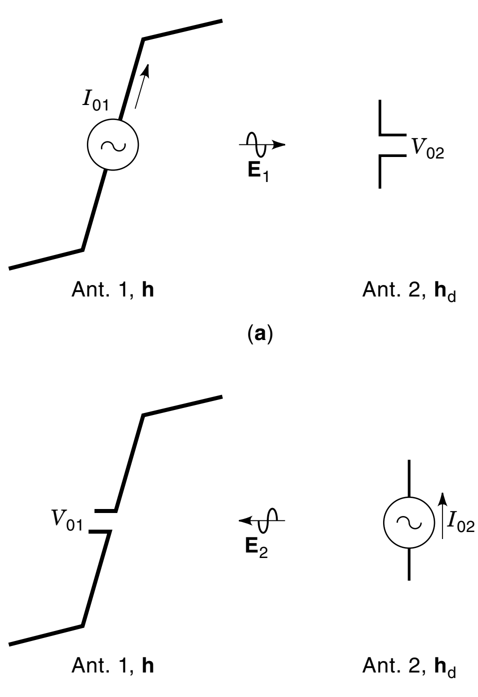

Figure 3. Determination of open-circuit voltage at antenna 1. (a) Antenna 1 for transmission. Antenna 2 is an infinitesimal dipole for reception. (b) Antenna 1 for reception. Antenna 2 is an infinitesimal dipole antenna for transmission.

Antenna 2 is an infinitesimal dipole antenna with vector effective height $\mathbf{h}_d = l\mathbf{z}$, which is used as receiving and transmitting antennas in (a) and (b), respectively.

The radiation field $\mathbf{E}_1$ from antenna 1 induces an open-circuit voltage at antenna 2

$$\begin{aligned} V_{02} &= \mathbf{E}_1 \cdot \ell\mathbf{z} \\ &= \left(-j\,30\,k\frac{e^{-jkR}}{R}I_{01}\mathbf{h}\right)\cdot\mathbf{h}_d \end{aligned} \tag{4}$$

Using Eq. (1), the radiation field from antenna 2 with terminal current I_{02} is written as

$$\mathbf{E}_2 = -j\,30\,k\frac{e^{-jkR}}{R}I_{02}\mathbf{h}_d \tag{5}$$

The open-circuit voltage V_{01} at antenna 1 induced by the radiation field from antenna 2 satisfies the relationship according to the [*reciprocity theorem* (1)]

$$V_{01}I_{01} = V_{02}I_{02} \tag{6}$$

Substituting Eq. (4) into Eq. (6) yields

$$V_{01} = -j\,30\,k\frac{e^{-jkR}}{R}\mathbf{h}\cdot I_{02}\mathbf{h}_d \tag{7}$$

Using Eq. (5) and replacing V_{01} and $\mathbf{E}_2$ with V_0 and $\mathbf{E}_{inc}$, respectively, Eq. (7) is written as

$$V_0 = \mathbf{E}_{inc}\cdot\mathbf{h} \tag{8}$$

Equation (8) means that the open-circuit voltage is given by the inner product of the incident wave field and the antenna vector effective height.

Example. Let us obtain the maximum open-circuit voltage for a *half-wavelength dipole* antenna ($L = \lambda/2$) located on the z axis. The maximum open circuit voltage is obtained when the polarization of an incident wave $\mathbf{E}_{inc}$ is parallel to the dipole; that is, the incident wave illuminates the dipole from the $\theta = 90°$ direction. When the half-wavelength dipole has a current distribution of $I(z') = I_0 \cos kz'$ over the antenna conductor from $z' = -\lambda/4$ to $z' = +\lambda/4$, Eq. (3) yields $\mathbf{S} = I_0 \lambda/\pi\mathbf{z}$ using $\mathbf{r}\cdot\mathbf{R} = 0$. Hence, $\mathbf{h} = \lambda/\pi\mathbf{z}$. From Eq. (8), the maximum open-circuit voltage is $|V_0| = |\mathbf{E}_{inc}|\lambda/\pi$.

EQUIVALENT CIRCUIT OF A RECEIVING ANTENNA

The original receiving antenna problem shown in Fig. 4(a) can be handled by superimposing two cases, shown in Fig. 4(b) and (c). In Fig. 4(b) an EM plane wave illuminating the antenna induces an open-circuit voltage of $V_0 = \mathbf{E}_{inc}\cdot\mathbf{h}$. In Fig. 4(c) the antenna acts as a transmitting antenna with terminal current I. Superimposing these two cases leads to a relationship of

$$V_{rec} = V_0 - V_{trans} \tag{9}$$

where $V_{rec} = Z_L I$ and $V_{trans} = Z_A I$, with Z_A defined as the antenna input impedance. From Eq. (9), the terminal current I

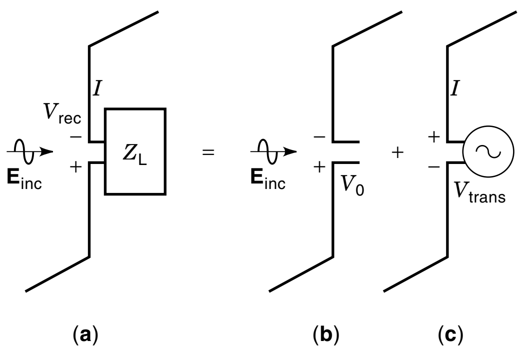

Figure 4. A receiving antenna. (a) Original receiving antenna problem. (b) Antenna with open terminals. (c) Transmitting antenna.

is given as

$$I = \frac{V_0}{Z_A + Z_L} \tag{10}$$

Therefore, the equivalent circuit for Eq. (10) is as shown in Fig. 5, where the open-circuit voltage V_0 is used as a voltage generator with an internal impedance Z_A.

Let us express the load impedance Z_L and the antenna input impedance Z_A as

$$Z_L = R_L + jX_L \tag{11}$$

$$Z_A = (R_A + r_A) + jX_A \tag{12}$$

where R_A and r_A are the radiation resistance and the loss resistance (not contributing to the radiation), respectively. Then, the power delivered to the antenna load is given as

$$W_L = \frac{|V_0|^2 R_L}{|Z_A + Z_L|^2} \tag{13}$$

and the power consumed in the generator is given as

$$W_A = \frac{|V_0|^2 (R_A + r_A)}{|Z_A + Z_L|^2} \tag{14}$$

When an impedance matching condition is satisfied [i.e., when Z_L is complex conjugate of $Z_A (R_L = R_A + r_A$ and $X_L = -X_A)$], Eqs. (13) and (14) become

$$W_L = \frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2}{4\,R_L} \equiv W_{L\,max} \tag{15}$$

$$W_R = \frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2}{4(R_A + r_A)} = \frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2}{4\,R_L} \tag{16}$$

It follows that half of the power provided by the generator is delivered to the antenna load. Note that $\mathbf{W}_{L\,max}$ in Eq. (15) is the maximum power that can be delivered to the load Z_L, because the antenna is perfectly matched to the load. Also, note that W_R is recognized as the scattered (reradiated) power from the receiving antenna (1,4,5).

Example. The relationship $W_R = W_L$ obtained when the impedance is matched can be checked by a numerical technique called the method of moments (3), where the incident power W_{inc} and the received power W_L are calculated. The scattered power is obtained from $W_{inc} - W_L$. For a center-load dipole antenna (4), the relationship $W_R = W_L$ is obtained when the antenna length L is less than 0.8λ. Limits on the validity of the equivalent circuit shown in Fig. 5 are discussed elsewhere (1,4,5).

RECEIVING CROSS-SECTION AND APERTURE EFFICIENCY

The *receiving cross-section* A_r of an antenna is defined as the ratio of the power W_L [Eq. (13)] received by the antenna to the Poynting power (power density of the incident wave, $P_{inc} = |\mathbf{E}_{inc}|^2/Z_0$, where $Z_0 = 120\pi\Omega$):

$$A_r = \frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2 R_L}{|Z_A + Z_L|^2} \frac{Z_0}{|\mathbf{E}_{inc}|^2} \tag{17}$$

Using an impedance mismatch factor M_{imp} defined as $W_L = W_{L\,max} M_{imp}$, Eq. (17) is rewritten as

$$\begin{aligned} A_r &= \left(\frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2}{4(R_A + r_A)} M_{imp} \right) \frac{Z_0}{|\mathbf{E}_{inc}|^2} \\ &= \left(\eta \frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2}{4R_A} M_{imp} \right) \frac{Z_0}{|\mathbf{E}_{inc}|^2} \end{aligned} \tag{18}$$

where $R_L = R_A + r_A = R_A/\eta$ (Appendix Eq. (a1)) is used. Using the absolute gain G_a [see Eq. (a5)], Eq. (18) is written as

$$A_r = \frac{\lambda^2}{4\pi} G_a \frac{|\mathbf{E}_{inc} \cdot \mathbf{h}|^2}{|\mathbf{E}_{inc}|^2 |\mathbf{h}|^2} M_{imp} \tag{19}$$

Because the received power is proportional to the square of the open-circuit voltage, that is, $|\mathbf{E}_{inc} \cdot \mathbf{h}|^2$, the third factor $|\mathbf{E}_{inc} \cdot \mathbf{h}|^2/|\mathbf{E}_{inc}|^2|\mathbf{h}|^2$ in Eq. (19), is the reduction factor of the received power from polarization mismatch. Let the third factor be denoted as M_{pol}, which has a maximum value of 1 for the case in which $\mathbf{h}$ is a real constant multiplied by the complex conjugate of $\mathbf{E}_{inc}$. Eq. (19) now can be written as

$$A_r = \frac{\lambda^2}{4\pi} G_a M_{pol} M_{imp} \tag{20}$$

The maximum receiving cross-section, which is called the *effective area* A_{eff}, is obtained when $M_{pol} = M_{imp} = 1$.

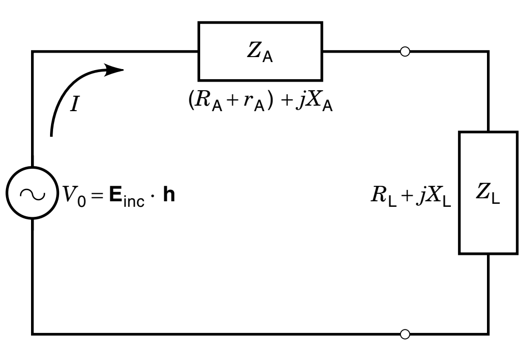

Figure 5. Equivalent circuit of a receiving antenna.

$$A_{\text{eff}} = \frac{\lambda^2}{4\pi} G_{\text{a}} \tag{21}$$

Let the receiving antenna gain be defined as the ratio of the effective area of the antenna, A_{eff}, to the effective area of an isotropic antenna, $A_{\text{eff, iso}} = \lambda^2/4\pi$. Then, from Eq. (21), the receiving antenna gain is equal to the absolute gain when the same antenna is used as a transmitting antenna.

Example. An isotropic antenna has a gain of $G_{\text{a}} = 1$ (=0 dB) by definition. Using Eq. (21), the effective area is calculated to be $A_{\text{eff}} = 0.0796\lambda^2$. An infinitesimal dipole, whose vector effective height is $|\mathbf{h}| = l|\sin\theta|$, has a gain of $G_{\text{a}} = 1.5$ (=1.76 dB) at $\theta = 90°$ and an effective area of $A_{\text{eff}} = 0.119\lambda^2$. A half-wavelength dipole antenna with $|\mathbf{h}| = \lambda/\pi$ yields an effective area of $0.130\lambda^2$, because $G_{\text{a}} = 1.64$ (=2.15 dB). Note that all G_{a}s are calculated using Appendix (a6) and (a7) with $\eta = 1$.

When a receiving antenna has an aperture A_{ap} which is much larger than the wavelength λ, the performance of the receiving antenna is evaluated by how efficiently the aperture is utilized for reception. Since the receiving antenna of A_{ap} has the potential to collect an EM wave power of $W_{\text{ap}} = P_{\text{inc}}A_{\text{ap}}$, the ratio of the received power $P_{\text{inc}}A_{\text{eff}}$ to W_{ap} is defined as the *aperture efficiency* η_{ap}, where

$$\eta_{\text{ap}} = \frac{A_{\text{eff}}}{A_{\text{ap}}} \leq 1 \tag{22}$$

Example. A parabolic reflector antenna has an aperture efficiency of less than 70% (6–8). Flat antennas, such as curl and helical element array antennas (9,10), have aperture efficiencies ranging from 70 to 90%, where the elements are arrayed using a circular cavity, as shown in Fig. 1(b).

APPENDIX

The input power $W_{\text{in}}[= |I(0)|^2(R_{\text{A}} + r_{\text{A}})]$ to a test transmitting antenna is transformed to the radiation power $W_{\text{rad}}(= R_{\text{A}}|I(0)|^2)$ with a relationship of $W_{\text{rad}} = \eta W_{\text{in}}$, where η is called the *radiation efficiency:*

$$\eta = \frac{W_{\text{rad}}}{W_{\text{in}}} = \frac{R_{\text{A}}}{R_{\text{A}} + r_{\text{A}}} \tag{a1}$$

The radiation power W_{rad} can be calculated by integrating the Poynting power $P = |\mathbf{E}(R, \theta, \phi)|^2/Z_0$ over the surface of a sphere of radius R

$$R_{\text{A}}|I(0)|^2 = R^2 \int \frac{|\mathbf{E}(R,\theta,\phi)|^2}{Z_0}\, d\Omega \tag{a2}$$

where $d\Omega$ is the solid angle subtended by the area $dA = R^2 \sin\theta\, d\theta\, d\phi$.

Using Eq. (1), the radiation resistance R_{A} is given as

$$R_{\text{A}} = \left(\frac{k}{4\pi}\right)^2 Z_0 \int |\mathbf{h}|^2\, d\Omega \tag{a3}$$

The *gain* $G(\theta, \phi)$ is defined as the ratio of the radiation power density from a test antenna in a direction of (θ, ϕ) to the radiation power density from a reference antenna that has the same input power as the test antenna:

$$\mathrm{G}(\theta,\phi) = \frac{|\mathbf{E}(\mathrm{R},\theta,\phi)|^2/\mathrm{W}_{\text{in}}}{|\mathbf{E}_0|^2/\mathrm{W}_{\text{in},0}} \tag{a4}$$

where $\mathbf{E}_0$ is the far-field radiated from the reference antenna, and $W_{\text{in},0}$ is the power input to the reference antenna. Note that the maximum value of the gain is conventionally used for the antenna gain if the coordinates (θ, ϕ) for the direction of interest are not specified.

When an isotropic antenna (hypothetical antenna radiating with uniform radiation power density in all directions) is chosen as the reference antenna, the gain is called the *absolute gain* and denoted as G_{a}. Equation (a4) becomes

$$\begin{aligned} G_{\text{a}} &= \frac{4\pi R^2}{Z_0}\frac{|\mathbf{E}(R,\theta,\phi)|^2}{W_{\text{in}}} \\ &= \eta\, \frac{4\pi R^2}{Z_0}\frac{|\mathbf{E}(R,\theta,\phi)|^2}{W_{\text{rad}}} \end{aligned} \tag{a5}$$

Using Eq. (1) and Eq. (a2),

$$G_{\text{a}} = \eta D \tag{a6}$$

where

$$D = \frac{4\pi|\mathbf{h}(\theta,\phi)|^2}{\int |\mathbf{h}(\theta,\phi)|^2\, d\Omega} \tag{a7}$$

D is called the *directivity*.

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RECEPTION, DIVERSITY. See Diversity reception.

RECLOSERS. See Circuit breakers.

RECOGNITION, FACE. See Face recognition.

RECOGNITION OF HANDWRITING. See Online handwriting recognition.

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