ANTENNA THEORY

FUNDAMENTALS

Maxwell's Equations

Antenna properties are analyzed with basic laws of physics. These laws have been collected into a set of equations commonly referred to as *Maxwell's equations.* (The presentation in this section follows the textbook by Stutzman and Thiele (1) where a more detailed treat may be found.) In the time domain, these equations are

$$\nabla \times \mathscr{E} = -\frac{\partial \mathscr{B}}{\partial t} - \mathscr{M} \tag{1}$$

$$\nabla \times \mathscr{H} = \frac{\partial \mathscr{D}}{\partial t} + \mathscr{T} \tag{2}$$

$$\nabla \cdot \mathscr{D} = \rho(t) \tag{3}$$

and

$$\nabla \cdot \mathscr{B} = m \tag{4}$$

The cross and dot derivatives are referred to as the curl and divergence respectively. A supplementary equation that may be deduced from the second and third equations is

$$\nabla \cdot \mathscr{T} = -\frac{\partial \rho(t)}{\partial t} \tag{5}$$

and is denoted the *continuity equation* to explicitly describe the electric current density $\mathscr{T}$ in terms of the movement of volumetric electric charge, ρ. A similar relationship holds for the magnetic current density $\mathscr{M}$ and volumetric magnetic charge, m. These latter two quantities have not been identified to date as actual physical quantities, but are found to be extremely useful in analysis. In fact, the concept of magnetic current is identical to the concept of ideal voltage sources in electrical networks. The remaining quantities, $\mathscr{E}$, $\mathscr{H}$, $\mathscr{D}$, and $\mathscr{B}$, describe the physical terms of electric and magnetic field intensities and the electric and magnetic field densities respectively.

In most antenna applications, we analyze sinusoidally varying sources in a linear environment. For such time-harmonic fields with a radian frequency of ω, we use the phasor form of the fields which may be written in the form

$$\mathscr{E} = \Re(\boldsymbol{E}e^{j\omega t}) \tag{6}$$

to obtain the phasor form of Maxwell's equations and the continuity equation as

$$\nabla \times \boldsymbol{E} = -j\omega \boldsymbol{B} - \boldsymbol{M} \tag{7}$$

$$\nabla \times \boldsymbol{H} = j\omega \boldsymbol{D} + \boldsymbol{J} \tag{8}$$

$$\nabla \cdot \boldsymbol{D} = \rho \tag{9}$$

$$\nabla \cdot \boldsymbol{B} = m \tag{10}$$

and

$$\nabla \cdot \boldsymbol{J} = -j\omega\rho \tag{11}$$

If multiple frequencies are present, the solution to the equations may be found for each frequency separately and the results combined for the total solution. The linearity restriction was only to ensure that the analysis would be properly restricted to a single frequency. For nonlinear media and some complex problems, it is advantageous to solve the time-domain equations and obtain the frequency-domain form through a Fourier (or Laplace) transform process. Computationally, the Fourier transform is usually obtained using a fast Fourier transform (FFT).

Maxwell's equations define relationships between the field quantities, but do not explicitly provide information about the media in which these fields exist. The material is characterized by three quantities: permittivity ϵ, permeability μ, and conductivity σ. Sometimes the material conductivity is given in inverse form as the resistivity $\rho = 1/\sigma$. These quantities relate the density and intensity quantities as well as the portion of the current due to conduction. Thus we have $\boldsymbol{D} = \epsilon\boldsymbol{E}$, $\boldsymbol{B} = \mu\boldsymbol{H}$, and $\boldsymbol{J} = \sigma\boldsymbol{E}$ that lead to

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H} - \boldsymbol{M}) \tag{12}$$

$$\nabla \times \boldsymbol{H} = (\sigma + j\omega\epsilon)\boldsymbol{E} + \boldsymbol{J}_i \tag{13}$$

$$\left(\epsilon - \frac{\sigma}{j\omega}\right)\nabla \cdot \boldsymbol{E} = \rho_i \tag{14}$$

$$\mu\nabla \cdot \boldsymbol{H} = (m) \tag{15}$$

and

$$\nabla \cdot \boldsymbol{J}_i = -j\omega\rho_i \tag{16}$$

where the i-subscript denotes the impressed sources in the system, equivalent to the independent sources of circuit theory. We find the "simple" media description limited in two ways in the last equations: (1) the medium is described by scalar quantities, implying isotropic media, and (2) the material parameters have been extracted from the derivatives, implying a constant, homogeneous media. These simplifications are valid for a large portion of antenna problems and the generalization is left for specific situations. It should be noted that Eqs. (14) and (15) can be obtained from Eqs. (13) and (12), respectively, with the appropriate continuity relations, such as Eq. (16).

Wave Equations

Along transmission lines and in the far-field of antennas, the solution of Maxwell's equations are solutions to the wave equation in source-free regions. The wave equation may be obtained by eliminating either $\boldsymbol{E}$ or $\boldsymbol{H}$ from Eqs. (12) through (15) with no impressed sources as

$$(k^2 + \nabla^2)\begin{Bmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{Bmatrix} = 0 \tag{17}$$

where $k = \omega\sqrt{\mu[\epsilon - (\sigma/j\omega)]}$. The quantity k is referred to as the propagation constant or wave number and may be written in terms of the phase and amplitude constants as $(\beta - j\alpha)$. In most antenna problems of interest, it is common to use β instead of k since the media is generally lossless. Similar steps may be taken for the transmission line to give a one-dimensional equation in either the voltage or current.

The solutions to Eq. (19) may be written in terms of either traveling or standing waves, with traveling waves being more common for antenna applications. The traveling wave solution to the electric field has a plane-wave solution form of

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_+ e^{-j\boldsymbol{k}\cdot\boldsymbol{r}} + \boldsymbol{E}_- e^{-j\boldsymbol{k}\cdot\boldsymbol{r}} \tag{18}$$

The corresponding magnetic field is given by

$$\boldsymbol{H}(\boldsymbol{r}) = \frac{1}{\eta}\boldsymbol{k} \times [\boldsymbol{E}_+ e^{-j\boldsymbol{k}\cdot\boldsymbol{r}} - \boldsymbol{E}_- e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}],\ \eta = \sqrt{\frac{\mu}{\left(\epsilon - \dfrac{\sigma}{j\omega}\right)}} \tag{19}$$

The form of Eq. (20) is called a generalized plane wave along $\pm\boldsymbol{k}$ with the restriction that $\boldsymbol{k}\cdot\boldsymbol{E} = 0$, since the divergence is zero. The more general solution requires additional work and is not presented here. The direct solution of the differential forms of Maxwell's equations may be obtained analytically in special cases and numerically in most other cases. Numerical procedures typically use finite differences (FD), the finite difference-time domain (FDTD) method, or finite-element (FE) techniques. The alternative is to transform the equations into integral forms for solution, where the solution structure is written in integal form and the integrals are used to solve for the field quantities.

Auxiliary Functions

Auxiliary functions are used to extend the solution of the wave equation beyond the simple traveling plane-wave form. If the magnetic sources are zero, then we may expand the magnetic-flux density in terms of the curl of an auxiliary function, the magnetic vector potential $\boldsymbol{A}$, or

$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \tag{20}$$

The corresponding electric-field intensity in simple media (using the Lorentz gauge for the potential) is given by

$$\boldsymbol{E} = \frac{1}{j\omega\mu\epsilon}[\beta^2\boldsymbol{A} + \nabla\nabla\cdot\boldsymbol{A}] \tag{21}$$

The use of a gauge condition completes the specification of the degrees of freedom for $\boldsymbol{A}$. The magnetic-vector potential must satisfy the Helmholtz equation given by

$$(\beta^2 + \nabla^2)\boldsymbol{A} = -\mu\boldsymbol{J} \tag{22}$$

having a solution in free space of

$$\boldsymbol{A}(\boldsymbol{r}) = \mu\int_V \boldsymbol{J}(\boldsymbol{r}')\frac{e^{-j\beta|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\,dv \tag{23}$$

for the geometry of Fig. 1. This general form can be specialized to the far-field case for an antenna located near the origin by expanding $R = |\boldsymbol{r} - \boldsymbol{r}'|$ in a binomial series as

$$\begin{aligned} R = |\boldsymbol{r}-\boldsymbol{r}'| &= \sqrt{r^2 - 2\boldsymbol{r}\cdot\boldsymbol{r}' + r'^2} \\ &= r - \frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r} + \frac{r'^2}{2r} - \frac{(\boldsymbol{r}\cdot\boldsymbol{r}')^2}{2r^3} + \cdots \end{aligned} \tag{24}$$

for r' sufficiently small. Only the first term in this expansion, r, needs to be retained for use in the denominator of Eq. (23). However, more accuracy is needed for R in the exponential to account for phase changes; so the second term of the expansion is used in the exponential:

$$R \approx r - \hat{\boldsymbol{r}}\cdot\boldsymbol{r}' \tag{25}$$

The complete far-field approximation is then

$$\boldsymbol{A}(\boldsymbol{r}) = \mu\frac{e^{-j\beta r}}{4\pi r}\int_V \boldsymbol{J}(\boldsymbol{r}')e^{j\beta\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}\,dv \tag{26}$$

which is a familiar Fourier transform representation.

In the far-field where Eq. (26) is applicable, we may approximate the corresponding electric and magnetic fields as

$$\boldsymbol{E} \approx \frac{1}{j\omega\mu\epsilon}[\beta^2\boldsymbol{A} - \boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{A})] \tag{27}$$

and

$$\boldsymbol{H} \approx \frac{1}{j\mu}[\boldsymbol{k}\times\boldsymbol{A}] \tag{28}$$

The second term in Eq. (27) simply removes the radial portion $\boldsymbol{A}$ from the electric field.

Duality

Duality provides an extremely useful way to complete the development of the solution form as well as equating some forms of antennas. To complete the previous set of equations for the magnetic current and charge, we simply note that we may change the variable definitions to obtain an identical form of equations. Specifically, we replace

$$\boldsymbol{E} \to \boldsymbol{H} \tag{29a}$$

$$\boldsymbol{H} \to -\boldsymbol{E} \tag{29b}$$

$$\boldsymbol{J} \to \boldsymbol{M} \tag{29c}$$

$$\boldsymbol{A} \to \boldsymbol{F} \tag{29d}$$

$$\mu \to \epsilon,\ \epsilon \to \mu \tag{29e}$$

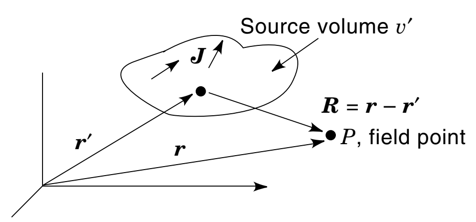

Figure 1. Coordinates and geometry for solving radiation problems.

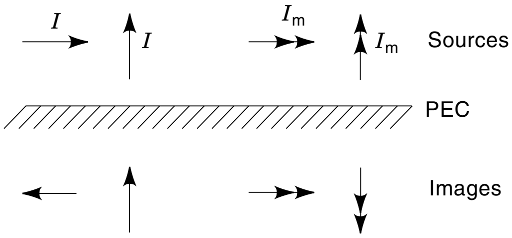

Figure 2. Images of elemental electric (I) and magnetic (I_m) currents over a perfect electric ground plane.

and

$$\beta \to \beta, \eta \to 1/\eta \tag{29f}$$

where η is the intrinsic impedance of the medium. The solution forms for $\boldsymbol{J}$ and $\boldsymbol{M}$ may be combined for the total solution as

$$\boldsymbol{E} = \frac{1}{j\omega\mu\epsilon}[\beta^2\boldsymbol{A} + \nabla\nabla\cdot\boldsymbol{A}] - \epsilon\nabla\times\boldsymbol{F} \tag{30a}$$

and

$$\boldsymbol{H} = \frac{1}{j\omega\mu\epsilon}[\beta^2\boldsymbol{F} + \nabla\nabla\cdot\boldsymbol{F}] + \mu\nabla\times\boldsymbol{A} \tag{30b}$$

The alternate use of duality is to equate similar dual problems numerically. A classic problem is the relationship between the input impedance of a slot dipole and strip dipole. The two structures are planar complements, each filling the void of the other, and have input impedances which satisfy

$$Z_{\text{slot}} Z_{\text{strip}} = \frac{\eta^2}{4} \tag{31}$$

This relationship incorporates several equivalencies, but most importantly the electric and magnetic quantities are scaled appropriately by η to preserve the proper units in the dual relationship. For a 72 Ω strip dipole, we find the complementary slot dipole has an input impedance of $Z_{\text{slot}} = 493.5$ Ω. Self-complementary planar structures such as spirals provide an input impedance of 188.5 Ω. A self-complementary structure is its own complement.

Images

Many antennas are constructed above a large metallic structure referred to as a ground plane. As long as the structure is greater than a half-wavelength in radius, the finite plane may be moldeled as an infinite structure for all but radiation behind the plane. The advantage of the infinite structure which is a perfect electric conductor (PEC) is that the planar sheet may be replaced by the images of the antenna elements in the plane. For the PEC, the images are constructed to provide a zero, tangential electric field at the plane. Figure 2 shows the equivalent current structure for the original and the image problems.

It is common to feed antennas at the ground plane through a coaxial cable. Then the equivalent voltage for the imaged problem is twice that of the source above the ground plane. The vertical electric current in Fig. 2 fed at the ground plane is called a monopole; it together with its image form a dipole and

$$Z_{\text{monopole}} = \frac{1}{2} Z_{\text{dipole}} \tag{32}$$

Since the corresponding field is radiated into only a half-space, the gain of the antenna defined as the peak power density in the far-field compared to the average power density over the radiation region of the antenna is double for the ground plane-fed antenna as

$$G_{\text{monopole}} = 2G_{\text{dipole}} \tag{33}$$

ANTENNA CHARACTERISTICS

There are a number of characteristics that describe an antenna as a device. Characteristics such as impedance and gain are common to any electrical device. On the other hand, a property such as radiation pattern is unique to the antenna. In this section we discuss patterns and impedance. Gain is discussed in the following section. We begin with a discussion of reciprocity.

Reciprocity

Circuit Form. Reciprocity plays an important role in antenna theory and can be used to great advantage in calculations and measurements. Fortunately, antennas usually behave as reciprocal devices. This permits characterization of the antenna as either a transmitting or receiving antenna. For example, radiation patterns are often measured with the test antenna operating in the receive mode. If the antenna is reciprocal, the measured pattern is identical when the antenna is in either a transmit or a receive mode. In fact, the following general statement applies: *If nonreciprocal materials are not present in an antenna, its transmitting and receiving properties are identical.* A case where reciprocity may not hold is when ferrite or active devices are included as a part of the antenna.

Reciprocity is also helpful when examining the terminal behavior of antennas. Consider two antennas, a and b shown in Fig. 3. Although connected through the intervening medium and not by a direct connection path, we can view this as a two-port network. Two port circuit analysis permits us to write the following:

$$V_a = Z_{aa} I_a + Z_{ab} I_b \tag{34a}$$

$$V_b = Z_{ba} I_a + Z_{bb} I_b \tag{34b}$$

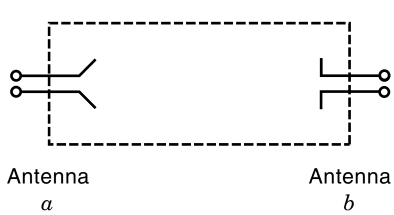

Figure 3. Two-port device representation for coupling between antennas.

where V_a and V_b are the terminal voltages and I_a and I_b are the currents of antennas a and b, respectively. Z_{aa} and Z_{bb} are self impedances and Z_{ab} and Z_{ba} are mutual impedances. To illustrate the use of these equations, suppose a generator of current I_a is placed on antenna a. The open circuit ($I_b = 0$) voltage at antenna b is then,

$$V_b = Z_{ba} I_a \tag{35}$$

Therefore, mutual impedance Z_{ba} provides the coupling between a transmitting antenna and a receiving antenna. Reversing the situation by using antenna b as the transmitter and antenna a as the receiver, leads to

$$V_a = Z_{ab} I_b \tag{36}$$

It can be seen from Eqs. (35) and (36) that if the applied currents are the same ($I_a = I_b = I$), then reciprocity is satisfied (i.e., $V_a = V_b$) if

$$Z_{ab} = Z_{ba} \quad \text{for reciprocal antennas} \tag{37}$$

If one antenna is rotated, the output voltage as a function of rotation angle becomes the radiation pattern. Since the coupling mechanism is via mutual impedances Z_{ab} and Z_{ba}, they must correspond to the radiation patterns. For example, if antenna b is rotated in the plane of Fig. 3, the pattern in that plane is proportional to the output of a receiver connected to antenna b due to a source of constant power attached to antenna a. For reciprocal antennas Eq. (37) implies the transmitting and receiving patterns for the rotated antenna are the same.

Another interesting result follows from Eq. (34). The input impedance of antenna a is

$$Z_a = \frac{V_a}{I_a} \quad \text{for reciprocal antennas} \tag{38}$$

If antennas a and b are far enough apart, such as in the far field, $Z_{ab} \ll Z_a$ and the input impedance of antenna a becomes

$$Z_a = (Z_{aa} I_a + 0)/I_a = Z_{aa} \tag{39}$$

That is, the input impedance equals the self impedance and antenna a acts as if it is in free space.

Reaction Theorem. Reciprocity may also be stated in integral form by cross multiplying Maxwell's equations by the opposite field for two separate problems, integrating and combining to obtain

$$\oint_S [\boldsymbol{E_a} \times \boldsymbol{H_b} - \boldsymbol{E_b} \times \boldsymbol{H_a}] \cdot d\boldsymbol{s} = \int_V [(\boldsymbol{J_a} \cdot \boldsymbol{E_b} - \boldsymbol{M_a} \cdot \boldsymbol{H_b}) - (\boldsymbol{J_b} \cdot \boldsymbol{E_a} - \boldsymbol{M_b} \cdot \boldsymbol{H_a})]\, dv \tag{40}$$

For antenna problems, the surface integral on the left of Eq. (40) is taken to an infinite radius and the integral becomes zero for finite antennas. This form is the typical field form of reciprocity. This form also suggests constructing a second problem that can be used as an auxiliary form to solve the original problem. For instance, if $\boldsymbol{M}_b = 0$ and $\boldsymbol{J}_b$ is a point dipole (or test antenna) of vector unit length $\hat{\boldsymbol{\ell}}_b$ located in the far field of the antenna a, then we have the radiated electric field of antenna a along the point dipole as

$$\hat{\boldsymbol{\ell}}_{\boldsymbol{b}} \cdot \boldsymbol{E_a} = \int_V [\boldsymbol{J_a} \cdot \boldsymbol{E_b} - \boldsymbol{M_a} \cdot \boldsymbol{H_b}]\, dv \tag{41}$$

If we go to the extreme of taking the test antenna to the surface of the problem antenna, then Eq. (41) becomes an equation that may be used for the solution of the currents on the antenna.

The actual form of reaction is to suggest that the field b reaction with current a is equal to the field a reaction with current b, or

$$\int_V (\boldsymbol{J_a} \cdot \boldsymbol{E_b} - \boldsymbol{M_a} \cdot \boldsymbol{H_b})\, dv = \int_V (\boldsymbol{J_b} \cdot \boldsymbol{E_a} - \boldsymbol{M_b} \cdot \boldsymbol{H_a})\, dv \tag{42}$$

written symbolically as

$$\langle b, a \rangle = \langle a, b \rangle \tag{43}$$

Antenna Impedance

Reciprocity may be used to obtain the basic formula for the input impedance of an antenna. If we define the two problems for Eq. (42) as (a) the antenna current distribution in the presence of the antenna structure and (b) the same antenna current in free space, then we can apply Eq. (42) to obtain

$$\int_V (\boldsymbol{J} \cdot \boldsymbol{E_b})\, dv = \int_V (\boldsymbol{J} \cdot \boldsymbol{E_a})\, dv = -IV_a \tag{44}$$

Since $V_a = IZ$, we may write

$$Z = \frac{1}{I^2} \int_V (\boldsymbol{J} \cdot \boldsymbol{E_b})\, dv \tag{45}$$

Thus, if the current distribution on the antenna is known, or may be estimated, then Eq. (45) provides a means for computing the antenna impedance Z by integrating the near-field radiated by the antenna current in free space times the current distribution itself. A common approach to this computation results in the induced-EMF method (2).

Radiation Patterns

The radiation pattern is a description of the angular variation of radiation level around an antenna. This is perhaps the most important characteristic of an antenna. In this section we present definitions associated with patterns and develop the general procedures for calculating radiation patterns.

Radiation Pattern Basics. A *radiation pattern (antenna pattern)* is a graphical representation of the radiation (far-field) properties of an antenna. The radiation fields from a transmitting antenna vary inversely with distance, for example, $1/r$. The variation with observation angles (θ, ϕ), however, depends on the antenna and, in fact, forms the bulk of antenna investigations.

Radiation patterns can be understood by examining the ideal dipole. The fields radiated from an ideal dipole are shown in Fig. 4(a) over the surface of a sphere of radius r which is in the far field. The length and orientation of the

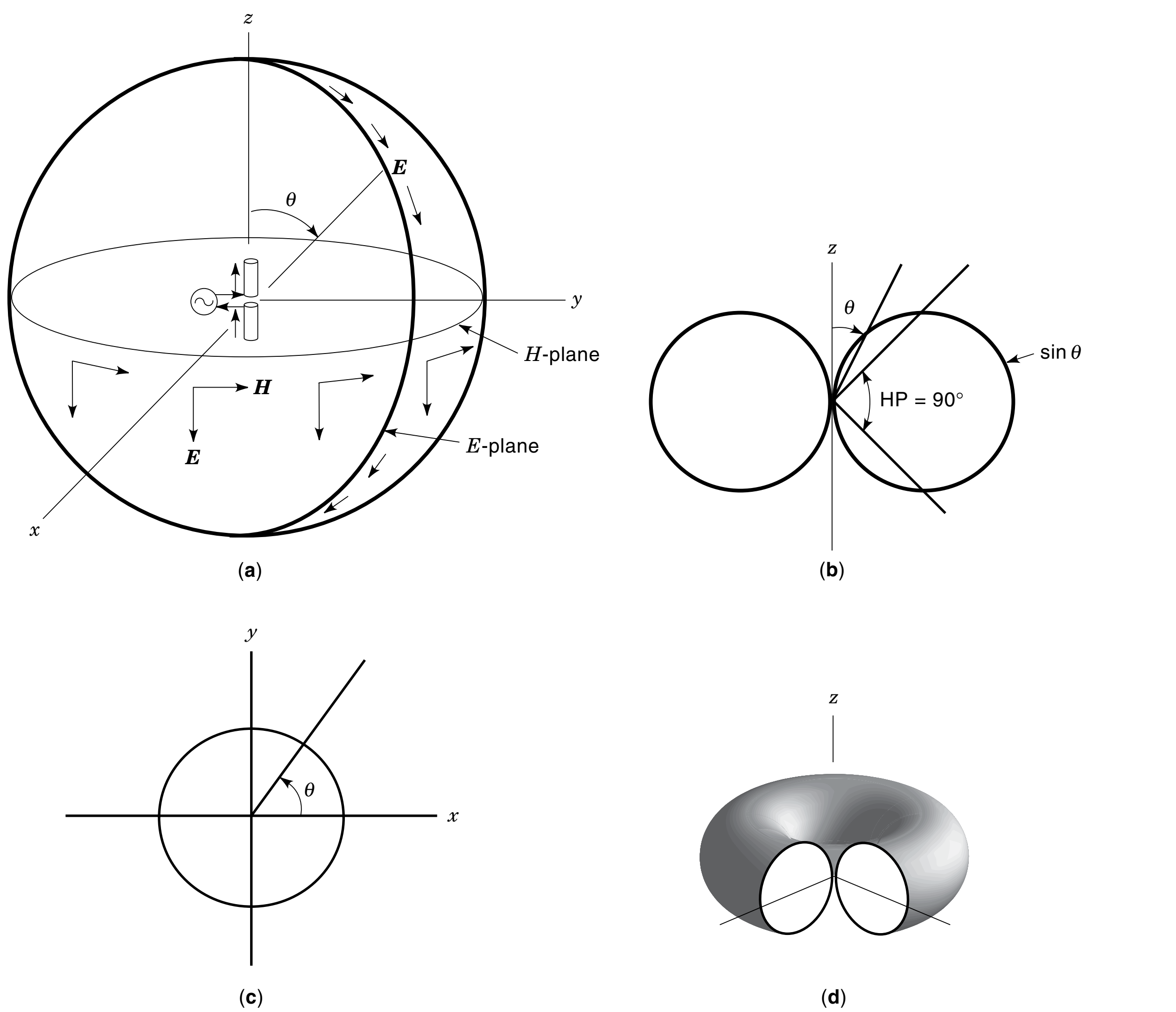

Figure 4. Radiation from an ideal dipole. (a) Field components. (b) E-plane radiation pattern polar plot. (c) H-plane radiation pattern polar plot. (d) Three-dimensional pattern plot.

field vectors follow from Eq. (30a); they are shown for an instant of time for which the fields are peak. The angular variation of E_θ and H_ϕ over the sphere is sin θ. An electric-field probe antenna moved over the sphere surface and oriented parallel to E_θ will have an output proportional to sin θ; see Fig. 4(b). Any plane containing the z-axis has the same radiation pattern since there is no ϕ variation in the fields. A pattern taken in one of these planes is called an *E-plane pattern* because it contains the electric vector. A pattern taken in a plane perpendicular to an E-plane and cutting through the test antenna (the xy-plane in this dipole case) is called an *H-plane pattern* because it contains the magnetic field H_ϕ. The E- and H-plane patterns, in general, are referred to as principal plane patterns. The E- and H-plane patterns for the ideal dipole are shown in Fig. 4(b) and (c). These are polar plots in which the distance from the origin to the curve is proportional to the field intensity; they are often called polar patterns or polar diagrams.

The complete pattern for the ideal dipole is shown in isometric view with a slice removed in Fig. 4(d). This solid polar radiation pattern resembles a doughnut with no hole. It is referred to as an *omni directional pattern* since it is uniform in the xy-plane. Omni directional antennas are very popular in ground-based applications with the omni directional plane horizontal. When encountering new antennas the reader should attempt to visualize the complete pattern in three dimensions.

Radiation patterns in general can be calculated in a manner similar to that used for the ideal dipole if the current distribution on the antenna is known. This calculation is done by first finding the vector potential given in Eq. (26). As a simple example consider a filament of current along the z-axis and located near the origin. Many antennas can be modeled by this line source; straight wire antennas are good examples. In this case the vector potential has only a z-component and the vector potential integral is one-dimensional

$$A_z = \mu \int I(z') \frac{e^{-j\beta R}}{4\pi R}\, dz' \tag{46}$$

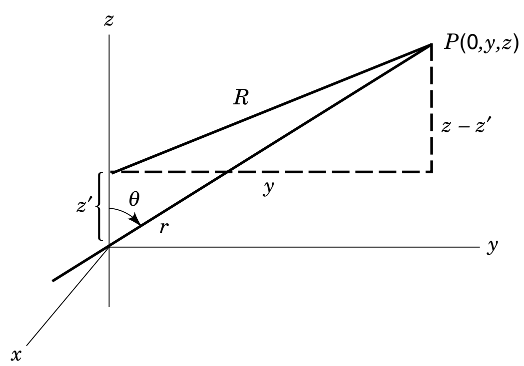

Figure 5. Geometry used for field calculations of a line source along the z-axis.

where β has been used for typical radiation media. Due to the symmetry of the source, we expect that the radiation fields will not vary with ϕ. This lack of variation is because as the observer moves around the source such that ρ and z are constant, the appearance of the source remains the same; thus, its radiation fields are also unchanged. Therefore, for simplicity we will confine the observation point to a fixed ϕ in the yz-plane ($\phi = 90°$) as shown in Fig. 5. Then from Fig. 5 we see that

$$y^2 = y^2 + z^2 \tag{47}$$

$$z = r\cos\theta \tag{48}$$

$$y = r\sin\theta \tag{49}$$

Applying the general geometry of Fig. 1 to this case, $\boldsymbol{r} = y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$ and $\boldsymbol{r}' = z\hat{\boldsymbol{z}}$ lead to $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' = y\hat{\boldsymbol{y}} + (z - z')\hat{\boldsymbol{z}}$ and then

$$R = \sqrt{y^2 + (z - z')^2} = \sqrt{y^2 + z^2 - 2zz' + (z')^2} \tag{50}$$

Substituting Eqs. (47) and (48) into Eq. (49), to put all field point coordinates into the spherical coordinate system, gives

$$R = \{r^2 + [-2rz'\cos\theta + (z')^2]\}^{1/2} \tag{51}$$

This result could also be obtained by using $J_z(\boldsymbol{r}') = I(z')\delta(x')\delta(y')$ in Eq. (23) where $dv' = dx'\ dy'\ dz'$. In order to develop approximate expressions for R, we expand Eq. (51) using the binomial theorem:

$$\begin{aligned} R &= r + \frac{1}{2r}[-2rz'\cos\theta + (z')^2] - \frac{1}{8r^3}[-2rz'\cos\theta + (z')^2]^2 + \cdots \\ &= r - z'\cos\theta + \frac{(z')^2\sin^2\theta}{2r} + \frac{(z')^3\sin^2\theta\cos\theta}{2r^2} + \cdots \end{aligned} \tag{52}$$

The terms in this series decrease as the power of z' increases if z' is small compared to r. This expression for R is used in the radiation integral Eq. (46) to different degrees of approximation. In the denominator of Eq. (46) (which affects only the amplitude) we let

$$R \approx r \tag{53}$$

We can do this because in the far field r is very large compared to the antenna size, so $r \gg z' \geq z'\cos\theta$. In the phase term $-\beta R$, we must be more accurate when computing the distance from points along the line source to the observation point. The integral Eq. (46) sums the contributions from all the points along the line source. Although the amplitude of waves due to each source point is essentially the same, the phase can be different if the path length differences are a sizable fraction of a wavelength. We, therefore, include the first two terms of the series in Eq. (52) for the R in the numerator of Eq. (46) giving

$$R \approx r - z'\cos\theta \tag{54}$$

Using the far-field approximations Eqs. (53) and (54) in Eq. (46) yields

$$A_z = \mu \int I(z') \frac{e^{-j\beta(r - z'\cos\theta)}}{4\pi r} dz' = \mu \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z'\cos\theta}\, dz' \tag{55}$$

where the integral is over the extent of the line source.

The electric field is found from Eq. (27), which is

$$\boldsymbol{E} = -j\omega\mu\boldsymbol{A} - \frac{j}{\omega\epsilon}\boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{A}) \tag{56}$$

This far-field result for a z-directed current, as in Eq. (46), reduces to

$$\boldsymbol{E} \approx -j\omega A_\theta\hat{\boldsymbol{\theta}} = j\omega\sin\theta A_z\hat{\boldsymbol{\theta}} \tag{57}$$

Note that this result is the portion of the first term of Eq. (56) which is transverse to $\hat{\boldsymbol{r}}$ because $-j\omega\boldsymbol{A} = -j\omega(-A_z \sin\theta\, \hat{\boldsymbol{\theta}} + A_z \cos\theta\, \hat{\boldsymbol{r}})$. This form is an important general result for z-directed sources that is not restricted to line sources.

The radiation fields from a z-directed line source (any z-directed current source in general) are H_ϕ and E_θ, and are found from Eqs. (27) and (28). The only remaining problem is to calculate A_z, which is given by Eq. (26) in general and by Eq. (55) for z-directed line sources. Calculation of A_z is the focal point of linear antenna analysis. We shall return to this topic after pausing to further examine the characteristics of the far-field region.

The radiation field components given by Eqs. (27) and (28) yield

$$E_\theta = \frac{\omega\mu}{\beta}H_\phi = \eta H_\phi \tag{58}$$

where $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance of the medium. An interesting conclusion can be made at this point. The radiation fields are perpendicular to each other and to the direction of propagation $\hat{\boldsymbol{r}}$ and their magnitudes are related by Eq. (58).

These are the familiar properties of a plane wave. They also hold for the general form of a *transverse electromagnetic* (TEM) *wave* which has both the electric and magnetic fields transverse to the direction of propagation. Radiation from a finite antenna is a special case of a TEM wave, called a *spherical wave* which propagates radially outward from the antenna and the radiation fields have no radial components. Spherical wave behavior is also characterized by the $e^{-j\beta r}/4\pi r$

factor in the field expressions; see Eq. (55). The $e^{-j\beta r}$ phase factor indicates a traveling-wave propagating radially outward from the origin and the $1/r$ magnitude dependence leads to constant power flow just as with the infinitesimal dipole. In fact, the radiation fields of all antennas of finite extent display this dependence with distance from the antenna.

Another way to view radiation field behavior is to note that spherical waves appear to an observer in the far field to be a plane wave. This local plane wave behavior occurs because the radius of curvature of the spherical wave is so large that the phase front is nearly planar over a local region.

If parallel lines (or rays) are drawn from each point on a line current as shown in Fig. 6, the distance R to the far field is geometrically related to r by Eq. (54), which was derived by neglecting high order terms in the expression for R in Eq. (52). The parallel ray assumption is exact only when the observation point is at infinity, but it is a good approximation in the far field. Radiation calculations often start by assuming parallel rays and then determining R for the phase by geometrical techniques. From the general source shown in Fig. 6, we see that

$$R = r - r' \cos\alpha \tag{59}$$

Using the definition of dot product, we have

$$R = r - \hat{\boldsymbol{r}} \cdot \boldsymbol{r}' \tag{60}$$

This form is a general approximation to R for the phase factor in the radiation integral. Notice that if $\boldsymbol{r}' = z'\hat{\boldsymbol{z}}$, as for line sources along the z-axis, Eq. (60) reduces to Eq. (54).

The definition of the distance from the source where the far field begins is where errors due to the parallel ray approximation become insignificant. The distance where the far field begins, r_{ff}, is taken to be that value of r for which the path length deviation due to neglecting the third term of Eq. (52) is a sixteenth of a wavelength. This corresponds to a phase error (by neglecting the third term) of $2\pi/\lambda \times \lambda/16 = \pi/8$ rad $= 22.5°$.

If D is the length of the line source, r_{ff} is found by equating the maximum value of the third term of Eq. (52) to a sixteenth of a wavelength; that is, for $z' = D/2$ and $\theta = 90°$, the third term of Eq. (52) is

$$\frac{(D/2)^2}{2r_{ff}} = \frac{\lambda}{16} \tag{61}$$

Solving for r_{ff} gives

$$r_{ff} = \frac{2D^2}{\lambda} \tag{62}$$

The far-field region is $r \geq r_{ff}$ and r_{ff} is called the far-field distance, or Rayleigh distance. The far-field conditions are summarized as follows:

$$r > \frac{2D^2}{\lambda} \tag{63a}$$

$$r \gg D \tag{63b}$$

$$r \gg \lambda \tag{63c}$$

The condition $r \gg D$ was mentioned in association with the approximation $R \approx r$ of Eq. (53) for use in the magnitude dependence. The condition $r \gg \lambda$ follows from $\beta r = (2\pi r/\lambda) \gg 1$ which was used to reduce Eq. (46) to Eq. (55). Usually the far field is taken to begin at a distance given by Eq. (62) where D is the maximum dimension of the antenna. This is usually a sufficient condition for antennas operating in the ultra high frequency (UHF) region and above. At lower frequencies, where the antenna can be small compared to the wavelength, the far-field distance may have to be greater than $2D^2/\lambda$ in order that all conditions in Eq. (63) are satisfied.

The concept of field regions was introduced in an earlier section and illustrated with the fields of an ideal dipole. We can now generalize that discussion to any finite antenna of maximum extent D. The distance to the far field is $2D^2/\lambda$. This zone was historically called the Fraunhofer region if the antenna is focused at infinity; that is, if the rays at large distances from the antenna when transmitting are parallel. In the far-field region the radiation pattern is independent of distance. For example, the sin θ pattern of an ideal dipole is valid anywhere in its far field. The zone interior to this distance from the center of the antenna, called the near field, is divided into two subregions. The reactive near-field region is closest to the antenna and is that region for which the reactive field dominates over the radiative fields. This region extends to a distance $0.62\sqrt{D^3/\lambda}$ from the antenna, as long as $D \gg \lambda$. For an ideal dipole, for which $D = \Delta z \ll \lambda$, this distance is $\lambda/2\pi$. Between the reactive near-field and far-field regions is the radiating near-field region in which the radiation fields dominate and where the angular field distribution depends on distance from the antenna. For an antenna focused at infinity the region is sometimes referred to as the Fresnel region. We can summarize the field region distances for cases where $D \gg \lambda$ as follows:

Region	Distance from antenna (r)	
Reactive near field	0 to $0.62\sqrt{D^3/\lambda}$	(64a)
Radiating near field	$0.62\sqrt{D^3/\lambda}$ to $2D^2/\lambda$	(64b)
Far field	$2D^2/\lambda$ to ∞	(64c)

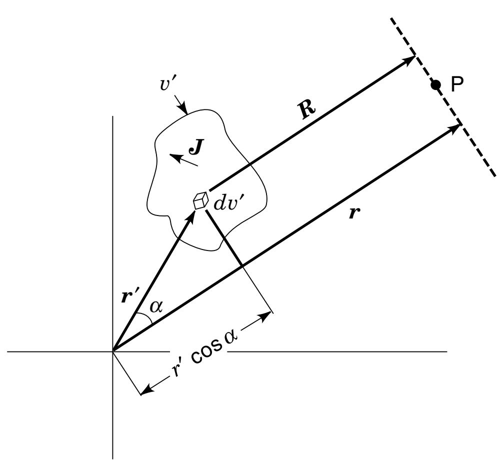

Figure 6. Parallel ray approximation for far-field calculations of radiation from a general source.

Steps in the Evaluation of Radiation Fields. The derivation for the fields radiated by a line source can be generalized for application to any antenna. The analysis of the line source, and its generalizations, can be reduced to the following three step procedure:

1. Find $\boldsymbol{A}$. Select a coordinate system most compatible with the geometry of the antenna, using the notation of Fig. 1. In general, use Eq. (23) with $R \approx r$ in the magnitude factor and the parallel ray approximation of Eq. (60) for determining phase differences over the antenna. These yield

$$\boldsymbol{A} = \mu \frac{e^{-j\beta r}}{4\pi r} \int_V \boldsymbol{J} e^{j\beta \hat{\boldsymbol{r}}\cdot \boldsymbol{r}'} dv' \tag{65}$$

For z-directed sources

$$\boldsymbol{A} = \hat{\boldsymbol{z}} \mu \frac{e^{-j\beta r}}{4\pi r} \int_V J_z e^{j\beta \hat{\boldsymbol{r}}\cdot \boldsymbol{r}'} dv' \tag{66}$$

For z-directed line sources on the z-axis

$$\boldsymbol{A} = \hat{\boldsymbol{z}} \mu \frac{e^{-j\beta r}}{4\pi r} \int_z I(z') e^{j\beta z' \cos\theta} dz' \tag{67}$$

which is Eq. (55).

2. Find $\boldsymbol{E}$. In general, use the component of

$$\boldsymbol{E} = -j\omega \boldsymbol{A} \tag{68}$$

which is transverse to the direction of propagation, $\hat{\boldsymbol{r}}$. This result is expressed formally as

$$\boldsymbol{E} = -j\omega \boldsymbol{A} + j\omega(\hat{\boldsymbol{r}}\cdot \boldsymbol{A})\hat{\boldsymbol{r}} = -j\omega(A_\theta \hat{\boldsymbol{\theta}} + A_\phi \hat{\boldsymbol{\phi}}) \tag{69}$$

which arises from the component of $\boldsymbol{A}$ tangent to the far-field sphere. For z-directed sources this form becomes

$$\boldsymbol{E} = j\omega A_z \sin\theta \hat{\boldsymbol{\theta}} \tag{70}$$

which is Eq. (57).

3. Find $\boldsymbol{H}$. In general, use the plane-wave relation

$$\boldsymbol{H} = \frac{1}{\eta} \hat{\boldsymbol{r}} \times \boldsymbol{E} \tag{71}$$

This equation expresses the fact that in the far field the directions of $\boldsymbol{E}$ and $\boldsymbol{H}$ are perpendicular to each other and to the direction of propagation, and also that their magnitudes are related by η. For z-directed sources

$$H_\phi = \frac{E_\theta}{\eta} \tag{72}$$

which is Eq. (58). The most difficult step is the first, calculating the radiation integral. To develop an appreciation for the process, we present an example. This uniform line source example will also serve to provide a specific setting for introducing general radiation pattern concepts and definitions.

Example: The Uniform Line Source. The uniform line source is a line source for which the current is constant along its extent. If we use a z-directed uniform line source centered on the origin and along the z-axis, the current is

$$I(z') = \begin{cases} I_o & x' = 0, y' = 0, |z'| \le \dfrac{L}{2} \\ 0 & \text{elsewhere} \end{cases} \tag{73}$$

where L is the length of the line source; see Fig. 5. We first find A_z from Eq. (67) as follows:

$$A_z = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{-L/2}^{L/2} I_o e^{j\beta z' \cos\theta} dz' = \mu \frac{e^{-j\beta r}}{4\pi r} I_o L \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta} \tag{74}$$

The electric field from Eq. (69) is then

$$\boldsymbol{E} = j\omega A_z \sin\theta \hat{\boldsymbol{\theta}} = j\omega\mu I_o L \frac{e^{-j\beta r}}{4\pi r} \sin\theta \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta} \hat{\boldsymbol{\theta}} \tag{75}$$

The magnetic field is simply found from this form using $H_\phi = E_\theta/\eta$.

Radiation Pattern Definitions. Since the radiation pattern is the variation over a sphere centered on the antenna, r is constant and we have only θ and ϕ variation of the field. It is convenient to normalize the field expression such that its maximum value is unity. This is accomplished as follows for a z-directed source which has only a θ-component of $\boldsymbol{E}$

$$F(\theta, \phi) = \frac{E_\theta}{E_\theta(\max)} \tag{76}$$

where $F(\theta, \phi)$ is the normalized field pattern and $E_\theta(\max)$ is the maximum value of E_θ over a sphere of radius r.

In general E_θ can be complex-valued and, therefore, so can $F(\theta, \phi)$. In this case the phase is usually set to zero at the same point the magnitude is normalized to unity. This is appropriate since we are only interested in relative phase behavior. This variation is, of course, independent of r. An element of current on the z-axis has a normalized field pattern of

$$F(\theta) = \sin\theta \tag{77}$$

and there is no ϕ variation. The normalized field pattern for the uniform line source is from Eq. (75) in Eq. (76)

$$F(\theta) = \sin\theta \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta} \tag{78}$$

and again there is no ϕ variation. The second factor of this expression is the function $\sin(u)/u$. It has a maximum value of unity at $u = 0$; this corresponds to $\theta = 90°$ where $u = (\beta L/2)\cos\theta$. Substituting $\theta = 90°$ in Eq. (78) gives unity and we see that $F(\theta)$ is properly normalized.

In general, a normalized field pattern can be written as the product

$$F(\theta, \phi) = g(\theta, \phi) f(\theta, \phi) \tag{79}$$

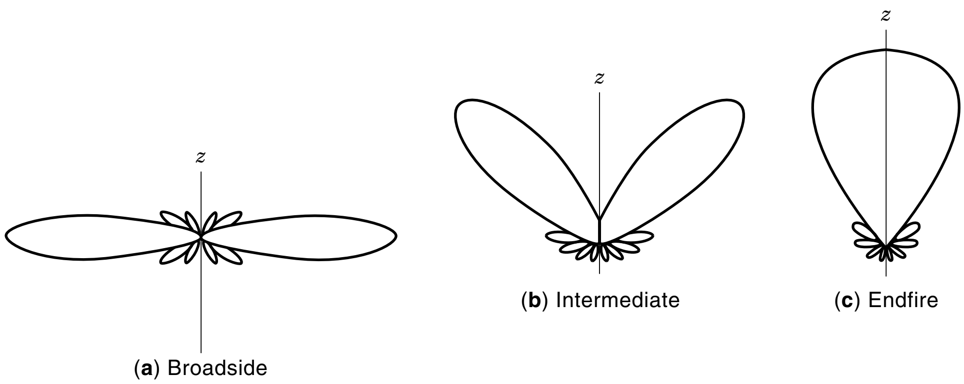

Figure 7. Polar plots of uniform line source patterns. (a) Broadside. (b) Intermediate. (c) Endfire.

where $g(\theta, \phi)$ is the element factor and $f(\theta, \phi)$ is the pattern factor. The pattern factor comes from the integral over the current and is strictly due to the distribution of current in space. The element factor is the pattern of an infinitesimal current element in the current distribution. For example, for a z-directed current element the total pattern is given by the element factor:

$$F(\theta) = g(\theta) = \sin\theta \tag{80}$$

for a z-directed current element. Actually this factor originates from Eq. (57) and can be interpreted as the projection of the current element in the θ-direction. In other words, at $\theta = 90°$ we see the maximum length of the current, whereas at $\theta = 0°$ or 180° we see the end view of an infinitesimal current which yields no radiation. The sin θ factor expresses the fraction of the size of the current as seen from the observation angle θ. On the other hand, the pattern factor $f(\theta, \phi)$ represents the integrated effect of radiation contributions from the current distribution, which can be treated as being made up of many current elements. The pattern value in a specific direction is then found by summing the parallel rays from each current element to the far field with the magnitude and phase of each included. The radiation integral of Eq. (65) sums the far-field contributions from the current elements and when normalized yields the pattern factor.

Antenna analysis is usually easier to understand by considering the antenna to be transmitting as we have here. However, most antennas are reciprocal and thus their radiation properties are identical when used for reception; as discussed in the section on reciprocity.

For the z-directed uniform line source pattern Eq. (78) we can identify the factors as

$$g(\theta) = \sin\theta \tag{81}$$

and

$$f(\theta) = \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta} \tag{82}$$

For long line sources ($L \gg \lambda$) the pattern factor of Eq. (82) is much sharper than the element factor sin θ, and the total pattern is approximately that of Eq. (82), that is, $F(\theta) \approx f(\theta)$. Hence, in many cases we need only work with $f(\theta)$, which is obtained from Eq. (67). If we allow the beam to be scanned as in Fig. 7, the element factor becomes important as the pattern maximum approaches the z-axis.

Frequently the directional properties of the radiation from an antenna are described by another form of radiation pattern, the power pattern. The *power pattern* gives angular dependence of the power density and is found from the θ, ϕ variation of the r-component of the Poynting vector. For z-directed sources $H_\phi = E_\theta/\eta$ so the r-component of the Poynting vector is $\frac{1}{2}E_\theta H_\phi = |E_\theta|^2/(2\eta)$ and the normalized power pattern is simply the square of its field pattern magnitude $P(\theta) = |F(\theta)|^2$. The general normalized power pattern is

$$P(\theta, \phi) = |F(\theta, \phi)|^2 \tag{83}$$

The normalized power pattern for a z-directed current element is

$$P(\theta, \phi) = \sin^2\theta \tag{84}$$

and for a z-directed uniform line source is

$$P(\theta) = \left\{\sin\theta \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta}\right\}^2 \tag{85}$$

Frequently patterns are plotted in decibels. It is important to recognize that the field (magnitude) pattern and power pattern are the same in decibels. This follows directly from the definitions. For field intensity in decibels

$$|F(\theta, \phi)|_{\text{dB}} = 20\log|F(\theta, \phi)| \tag{86}$$

and for power in decibels

$$P(\theta, \phi)_{\text{dB}} = 10\log P(\theta, \phi) = 10\log|F(\theta, \phi)|^2 = 20\log|F(\theta, \phi)| \tag{87}$$

and we see that

$$P(\theta, \phi)_{\text{dB}} = |F(\theta, \phi)|_{\text{dB}} \tag{88}$$

Radiation Pattern Parameters. A typical antenna power pattern is shown in Fig. 8 as a polar plot in linear units (rather than decibels). It consists of several lobes. The *main lobe* (or *main beam* or *major lobe*) is the lobe containing the direction of maximum radiation. There is also usually a series of lobes smaller than the main lobe. Any lobe other than the main lobe is called a *minor lobe.* Minor lobes are composed of side lobes and back lobes. *Back lobes* are directly opposite the main lobe, or sometimes they are taken to be the lobes in

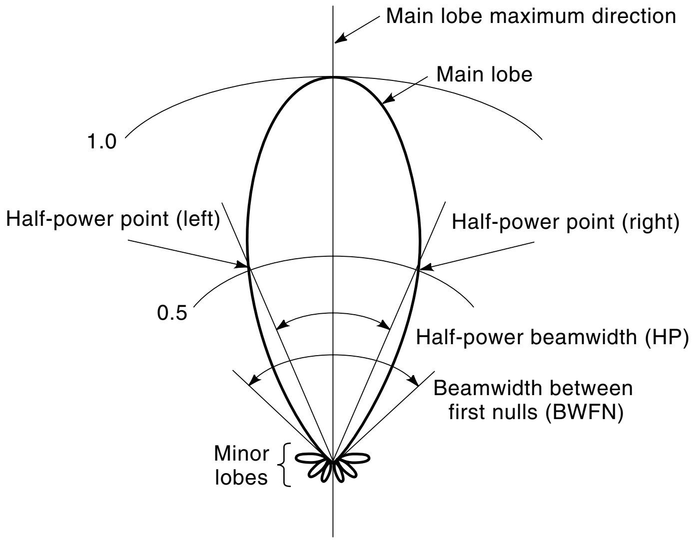

Figure 8. A typical power pattern polar plot.

the half-space opposite the main lobe. The term *side lobe* is sometimes reserved for those minor lobes near the main lobe, but is most often taken to be synonymous with minor lobe; we will use the latter convention.

The radiation from an antenna is represented mathematically through the radiation pattern function, $F(\theta, \phi)$ for field and $P(\theta, \phi)$ for power. This angular distribution of radiation is visualized through various graphical representations of the pattern, which we discuss in this section. Graphical representations also are used to introduce definitions of pattern parameters that are commonly used to quantify radiation pattern characteristics.

A three-dimensional plot as in Fig. 4(d) gives a good overall impression of the entire radiation pattern, but cannot convey accurate quantitative information. Cuts through this pattern in various planes are the most popular pattern plots. They usually include the E- and H-plane patterns; see Figs. 4(b) and (c). Pattern cuts are often given various fixed ϕ values, leaving the pattern a function of θ alone; we will assume that is the case here. Typically the side lobes are alternately positive and negative valued. In fact, a pattern in its most general form may be complex-valued. Then we use the magnitude of the field pattern $|F(\theta)|$ or the power pattern $P(\theta)$.

A measure of how well the power is concentrated into the main lobe is the (relative) *side lobe level,* which is the ratio of the pattern value of a side lobe peak to the pattern value of the main lobe. The largest side lobe level for the whole pattern is the maximum (relative) side lobe level, frequently abbreviated as SLL. In decibels it is given by

$$SLL = 20 \log \left| \frac{F(SLL)}{F(\max)} \right| \tag{89}$$

where $|F(\max)|$ is the maximum value of the pattern magnitude and $|F(SLL)|$ is the pattern value of the maximum of the highest side lobe magnitude. For a normalized pattern $F(\max) = 1$.

The width of the main beam is quantified through half-power beamwidth, HP, which is the angular separation of the points where the main beam of the power pattern equals one-half the maximum value:

$$HP = |\theta_{\text{HP left}} - \theta_{\text{HP right}}| \tag{90}$$

where $\theta_{\text{HP left}}$ and $\theta_{\text{HP right}}$ are points to the left and right of the main beam maximum for which the normalized power pattern has a value of one-half (see Fig. 8). On the field pattern $|F(\theta)|$ these points correspond to the value $1/\sqrt{2}$. For example, the $\sin\theta$ pattern of an ideal dipole has a value of $1/\sqrt{2}$ for θ values of $\theta_{\text{H-P left}} = 135°$ and $\theta_{\text{H-P right}} = 45°$. Then HP = $|135° - 45°| = 90°$. This is shown in Fig. 4(b). Note that the definition of HP is the magnitude of the difference of the half-power points and the assignment of left and right can be interchanged without changing HP. In three dimensions the radiation pattern major lobe becomes a solid object and the half-power contour is a continuous curve. If this curve is essentially elliptical, the pattern cuts that contain the major and minor axes of the ellipse determine what the Institute of Electrical and Electronics Engineers (IEEE) defines as the *principal half-power beamwidths.*

Antennas are often referred to by the type of pattern they produce. An *isotropic antenna,* which is hypothetical, radiates equally in all directions giving a constant radiation pattern. An *omnidirectional antenna* produces a pattern which is constant in one plane; the ideal dipole of Fig. 4 is an example. The pattern shape resembles a doughnut. We often refer to antennas as being broadside or endfire. A *broadside antenna* is one for which the main beam maximum is in a direction normal to the plane containing the antenna. An *endfire antenna* is one for which the main beam is in the plane containing the antenna. For a linear current on the z-axis, the broadside direction is $\theta = 90°$ and the endfire directions are 0° and 180°. For example, an ideal dipole is a broadside antenna. For z-directed line sources several patterns are possible. Figure 7 illustrates a few $|f(\theta)|$ patterns. The entire pattern (in three dimensions) is imagined by rotating the pattern about the z-axis. The full pattern can then be generated from the E-plane patterns shown. The broadside pattern of Fig. 7(a) is called *fan beam.* The full three dimensional endfire pattern for Fig. 7(c) has a single lobe in the endfire direction. This single lobe is referred to as a *pencil beam.* Note that the $\sin\theta$ element factor, which must multiply these patterns to obtain the total pattern, will have a significant effect on the endfire pattern. Intermediate scan angles are also possible, as shown in Fig. 7(b).

ANTENNA PERFORMANCE MEASURES

Antennas are devices that are used in systems for communications or sensing. There are many parameters used to quantify the performance of the antenna as a device, which in turn impacts on system performance. In this section we consider the most important of these parameters when they are employed in their primary application area of communication links, such as the simple communication link as shown in Fig. 9. We first discuss the basic properties of a receiving antenna.

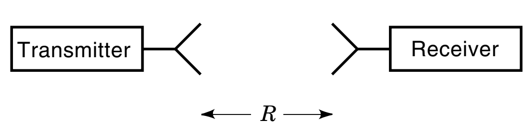

Figure 9. A communication link.

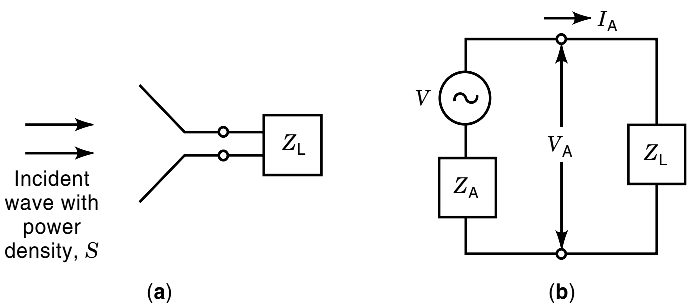

Figure 10. Equivalent circuit for a receiving antenna. (a) Receive antenna connected to a receiver with load impedance Z_L. (b) Equivalent circuit.

The receiving antenna with impedance Z_A and terminated in load impedance Z_L is modeled as shown in Fig. 10. The total power incident on the receiving antenna is found by summing up the incident power density over the area of the receive antenna, called effective aperture. How an antenna converts this incident power into available power at its terminals depends on the type of antenna used, its pointing direction, and polarization. In this section we discuss the basic relationships for power calculations and illustrate their use in communication links.

Directivity and Gain. For system calculations it is usually easier to work with directivity rather than its equivalent, maximum effective aperture. The relation can be established by examining an infinitestimal dipole and generalizing. The maximum effective aperture of an ideal, lossless dipole of length Δz is found by orienting the dipole for maximum response, which is parallel to the incoming linearly polarized electric field E^i. Then the open circuit voltage is found from

$$V_A = E^i \Delta z \quad \text{Ideal dipole receiving antenna} \tag{91}$$

The power available from the antenna is realized when the antenna impedance is matched by a load impedance of $Z_L = R_r - jX_A$ assuming $R_{\text{ohmic}} = 0$. R_r is the radiation resistance. The maximum available power is then

$$P_{\text{Am}} = \frac{1}{8}\frac{|V_A|^2}{R_r} = \frac{1}{8}\frac{|E^i|^2}{R_r}(\Delta z)^2 \tag{92}$$

where Eq. (91) was used. The available power can also be calculated by examining the incident wave. The power density (Poynting vector magnitude) in the incoming wave is

$$S = \frac{1}{2}|\boldsymbol{E} \times \boldsymbol{H}^*| = \frac{1}{2}\frac{|E^i|^2}{\eta} \tag{93}$$

The available power is found using the *maximum effective aperture* A_{em}, which is the collecting area of the antenna. The receiving antenna collects power from the incident wave in proportion to its maximum effective aperture

$$P_{\text{Am}} = SA_{\text{em}} \tag{94}$$

The maximum available power P_{Am} will be realized if the antenna is directed for maximum response, is polarization matched to the wave, and is impedance matched to its load. The maximum refers to the assumption that there are no ohmic losses on the antenna.

Maximum effective aperture for the ideal dipole is found using Eqs. (92) and (93) with Eq. (94) to give

$$A_{\text{em}} = \frac{P_{\text{Am}}}{S} = \frac{\frac{1}{8}\frac{|V_A|^2}{R_r}}{\frac{1}{2}\frac{|E^i|^2}{\eta}} = \frac{1}{4}\frac{\eta}{R_r}(\Delta z)^2 = \frac{3}{8\pi}\lambda^2 \tag{95}$$

where the ideal dipole radiation resistance value of $[2\pi/3\,\eta(\Delta z/\lambda)^2]$ was used. The maximum effective aperture of an ideal dipole is independent of its length Δz (as long as $\Delta z \ll \lambda$). However, it is important to note that R_r is proportional to $(\Delta z/\lambda)^2$ so that even though A_{em} remains constant as the dipole is shortened, its radiation resistance decreases rapidly and it is more difficult to realize this maximum effective aperture because of the required conjugate impedance match of the receiver to the antenna.

The directivity of the ideal dipole can be written in the following manner:

$$D = \frac{3}{2} = \frac{4\pi}{\lambda^2}\frac{3}{8\pi}\lambda^2 \qquad \text{Ideal dipole} \tag{96}$$

Grouping factors this way permits identification of A_{em} from Eq. (95). Thus

$$D = \frac{4\pi}{\lambda^2}A_{\text{em}} \tag{97}$$

Although we derived this for an ideal dipole, this relationship is true for any antenna. For an isotropic antenna, the directivity by definition is unity; so from Eq. (97) with $D = 1$

$$A_{\text{em}} = \frac{\lambda^2}{4\pi} \qquad \text{Isotropic antenna} \tag{98}$$

Comparing this to the definition of directivity in (100) below we see that

$$\lambda^2 = A_{\text{em}}\Omega_A \tag{99}$$

which is also a general relationship. We can extract some interesting concepts from this relation. For a fixed wavelength A_{em} and Ω_A, are inversely proportional; that is, as the maximum effective aperture increases (as a result of increasing its physical size), the beam solid angle decreases, which means power is more concentrated in angular space (i.e., directivity goes up). For a fixed maximum effective aperture (i.e., antenna size), as wavelength decreases (frequency increases) the beam solid angle also decreases, leading to increased directivity.

Directivity is more directly related to its definition through this inverse dependence on beam solid angle as

$$D = \frac{4\pi}{\Omega_A} \tag{100}$$

where

$$\Omega_A = \int\!\!\int |F(\theta,\phi)|^2\, d\Omega \tag{101}$$

This directivity definition has a simple interpretation. Directivity is a measure of how much greater the power density at a fixed distance is in a given direction than if all power were radiated isotropically. This view is illustrated in Fig. 11. For an isotropic antenna, as in Fig. 11(a), the beam solid angle is 4π, and thus Eq. (100) gives a directivity of unity.

In practice antennas are not completely lossless. Earlier we saw that power available at the terminals of a transmitting antenna was not all transformed into radiated power. The power received by a receiving antenna is reduced to the fraction e_r (radiation efficiency) from what it would be if the antenna were lossless. This is represented by defining *effective aperture*

$$A_e = e_r A_{em} \qquad (102)$$

and the available power with antenna losses included, analogous to Eq. (94), is

$$P_A = SA_e \qquad (103)$$

This simple equation is very intuitive and indicates that a receiving antenna acts to convert incident power (flux) density in W/m^2 to power delivered to the load in watts. Losses associated with mismatch between the polarization of the incident wave and receiving antenna as well as impedance mismatch between the antenna and load are not included in A_e. These losses are not inherent to the antenna, but depend on how it is used in the system. The concept of gain is introduced to account for losses on an antenna, that is, $G = e_r D$. We can form a gain expression from the directivity expression by multiplying both sides of Eq. (97) by e_r and using Eq. (102):

$$G = e_r D = \frac{4\pi}{\lambda^2} e_r A_{em} = \frac{4\pi}{\lambda^2} A_e \qquad (104)$$

For electrically large antennas effective aperture is equal to or less than the physical aperture area of the antenna A_p, which is expressed using *aperture efficiency* ϵ_{ap}:

$$A_e = \epsilon_{ap} A_p \qquad (105)$$

It is important to note that although we developed the general relationships of Eqs. (97), (99), and (104) for receiving antennas, they apply to transmitting antennas as well. The relationships are essential for communication system computations that we consider next.

Communication Links. We are now ready to completely describe the power transfer in the communication link of Fig. 9.

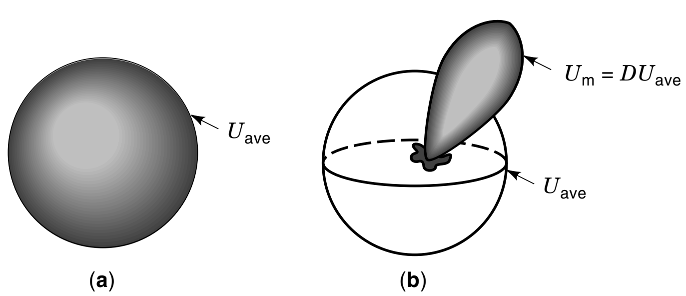

Figure 11. Illustration of directivity. (a) Radiation intensity distributed isotropically. (b) Radiation intensity from an actual antenna.

If the transmitting antenna were isotropic, it would have power density at distance R of

$$S = \frac{U_{ave}}{R^2} = \frac{P_t}{4\pi R^2} \qquad (106)$$

where P_t is the time-averaging input power (P_{in}) accepted by the transmitting antenna. The quantity U_{ave} denotes the time average radiation intensity given in the units of power per solid angle see Fig. 11. For a transmitting antenna that is not isotropic but has gain G_t and is pointed for maximum power density in the direction of the receiver, we have for the power density incident on the receiving antenna,

$$S = \frac{G_t U_{ave}}{R^2} = \frac{G_t P_t}{4\pi R^2} \qquad (107)$$

Using this in Eq. (103) gives the available received power as

$$P_r = SA_{er} = \frac{G_t P_t A_{er}}{4\pi R^2} \qquad (108)$$

where A_{er} is the effective aperture of the receiving antenna and we assume it to be pointed and polarized for maximum response. Now from Eq. (104) $A_{er} = G_r\lambda^2/4\pi$, so Eq. (108) becomes

$$P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi R)^2} \qquad (109)$$

which gives the available power in terms of the transmitted power, antenna gains, and wavelength. Or, we could use $G_t = 4\pi A_{et}/\lambda^2$ in Eq. (108) giving

$$P_r = P_t \frac{A_{et} A_{er}}{R^2 \lambda^2} \qquad (110)$$

which is called the *Friis transmission formula* (2).

The power transmission formula Eq. (109) is very useful for calculating signal power levels in communication links. It assumes that the transmitting and receiving antennas are matched in impedance to their connecting transmission lines, have identical polarizations, and are aligned for polarization match. It also assumes the antennas are pointed toward each other for maximum gain. If any of these conditions are not met, it is a simple matter to correct for the loss introduced by polarization mismatch, impedance mismatch, or antenna misalignment.

The antenna misalignment effect is easily included by using the power gain value in the appropriate direction. The effect and evaluation of polarization and impedance mismatch are additional considerations. Figure 10 shows the network model for a receiving antenna with input antenna impedance Z_A and an attached load impedance Z_L, which can be a transmission line connected to a distant receiver. The power delivered to the terminating impedance is

$$P_D = pqP_r \qquad (111)$$

where

P_D = power delivered from the antenna
P_r = power available from the receiving antenna
p = polarization efficiency (or polarization mismatch factor), $0 \le p \le 1$
q = impedance mismatch factor, $0 \le q \le 1$

An overall efficiency, or total efficiency ϵ_{total}, can be defined to include the effects of polarization and impedance mismatch:

$$\epsilon_{total} = pq\epsilon_{ap} \tag{112}$$

Then $P_D = \epsilon_{total}P_r$. It is convenient to express Eq. (111) in dB form:

$$P_D(\text{dBm}) = 10\log p + 10\log q + P_r(\text{dBm}) \tag{113}$$

where the unit dBm is power in decibels above a milliwatt; for example, 30 dBm is 1 W. Both powers could also be expressed in units of decibels above a watt, dBW. The power transmission formula Eq. (109) can also be expressed in dB form as

$$\begin{aligned} P_r(\text{dBm}) = {} & P_t(\text{dBm}) + G_t(\text{dB}) + G_r(\text{dB}) \\ & - 20\log R(\text{km}) - 20\log f(\text{MHz}) - 32.44 \end{aligned} \tag{114}$$

where G_t(dB) and G_r(dB) are the transmit and receive antenna gains in decibels, R(km) is the distance between the transmitter and receiver in kilometers, and f(MHz) is the frequency in megahertz.

Effective Isotropically Radiated Power. A frequently used concept in communication systems is that of effective (or equivalent) isotropically radiated power, EIRP. It is formally defined as the power gain of a transmitting antenna in a given direction multiplied by the net power accepted by the antenna from the connected transmitter. Sometimes it is denoted as ERP, but this term, effective radiated power, is usually reserved for EIRP with antenna gain relative to that of a half-wave dipole instead of gain relative to an isotropic antenna. As an example of EIRP, suppose an observer is located in the direction of maximum radiation from a transmitting antenna with input power P_t. Then the EIRP may be expressed as

$$EIRP = P_t G_t \tag{115}$$

For a radiation intensity U_m, as illustrated in Fig. 11(b), and $G_t = 4\pi U_m/P_t$, we obtain

$$EIRP = P_t\frac{4\pi U_m}{P_t} = 4\pi U_m \tag{116}$$

The same radiation intensity could be obtained from a lossless isotropic antenna (with power gain $G_i = 1$) if it had an input power P_{in} equal to P_tG_t. In other words, to obtain the same radiation intensity produced by the directional antenna in its pattern maximum direction, an isotropic antenna would have to have an input power G_t times greater. Effective isotropically radiated power is a frequently used parameter. For example, FM radio stations often mention their effective radiated power when they sign off at night.

Antenna Noise Temperature and Radiometry

Receiving systems are vulnerable to noise and a major contribution is the receiving antenna, which collects noise from its surrounding environment. In most situations a receiving antenna is surrounded by a complex environment as shown in

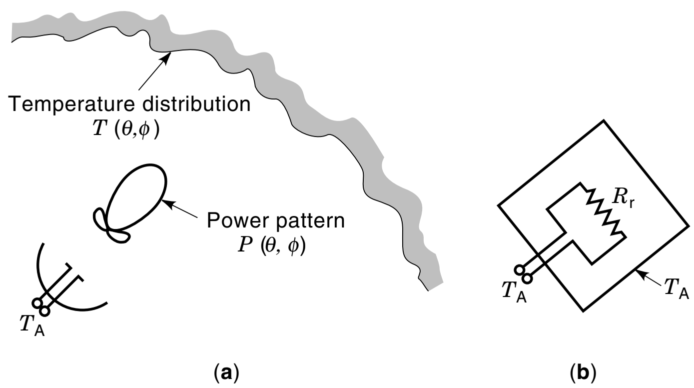

Figure 12. Antenna temperature. (a) An antenna receiving noise from directions (θ, ϕ) producing antenna temperature T_A. (b) Equivalent model.

Fig. 12(a). Any object (except a perfect reflector) that is above absolute zero temperature will radiate electromagnetic waves. An antenna picks up this radiation through its antenna pattern and produces noise power at its output. The equivalent terminal behavior is modeled in Fig. 12(b) by considering the radiation resistance of the antenna to be a noisy resistor at a temperature T_A such that the same output noise power from the antenna in the actual environment is produced. The antenna temperature T_A is not the actual physical temperature of the antenna, but is an equivalent temperature that produces the same noise power, P_{NA}, as the antenna operating in its surroundings. This equivalence is established by assuming the model of Fig. 12(b); the noise power available from the noise resistor in bandwidth Δf at temperature T_A is

$$P_{NA} = kT_A\Delta f \tag{117}$$

where

P_{NA} = available power due to antenna noise [W]
k = Boltzmann's constant = 1.38×10^{-23} JK^{-1}
T_A = antenna temperature [K]
Δf = receiver bandwidth [Hz]

Such noise is often referred to as Nyquist or Johnson noise for system calculations. The system noise power P_N is calculated using the total system noise temperature T_{sys} in place of T_A in Eq. (117) with $T_{sys} = T_A + T_r$, where T_r is the receiver noise temperature.

Antenna noise is important in several system applications including communications and radiometry. Communication systems are evaluated through carrier-to-noise ratio, which is determined from the signal power and the system noise power as

$$CNR = \frac{P_D}{P_N} \tag{118}$$

where $P_N = kT_{sys}\,\Delta f$ is the system noise power. This noise power equals the sum of P_{NA} and noise power generated in the receiver connected to the antenna.

Noise power is found by first evaluating antenna temperature. As seen in Fig. 12(a), T_A is found from the collection of noise through the scene temperature distribution $T(\theta, \phi)$ weighted by the response function of the antenna, the normalized power pattern $P(\theta, \phi)$. This is expressed mathematically by integrating over the temperature distribution:

$$T_A = \frac{1}{\Omega_A}\int_0^{\pi}\int_0^{2\pi} T(\theta,\phi)P(\theta,\phi)\,d\Omega \tag{119}$$

If the scene is of constant temperature T_o over all angles, T_o comes out of the integral and then

$$T_A = \frac{T_o}{\Omega_A}\int_0^{\pi}\int_0^{2\pi} P(\theta,\phi)\,d\Omega = \frac{T_o}{\Omega_A}\Omega_A = T_o \tag{120}$$

using Eq. (101) for Ω_A. The antenna is completely surrounded by noise of temperature T_o and its output antenna temperature equals T_o independent of the antenna pattern shape.

In general, antenna noise power P_{NA} is found from Eq. (117) using T_A from Eq. (119) once the temperature distribution $T(\theta, \phi)$ is determined. Of course, this depends on the scene, but in general $T(\theta, \phi)$ consists of two components: sky noise and ground noise. Ground noise temperature in most situations is well approximated for soils by the value of 290 K, but is much less for surfaces that are highly reflective due to reflection of low temperature sky noise. Also, smooth surfaces have high reflection for near grazing incidence angles.

Unlike ground noise, sky noise is a strong function of frequency. Sky noise is made up of atmospheric, cosmic, and manmade noise. Atmospheric noise increases with decreasing frequency below 1 GHz and is primarily due to lightning, which propagates over large distances via ionospheric reflection below several MHz. Atmospheric noise increases with frequency above 10 GHz due to water vapor and hydrometeor absorption; these depend on time, season, and location. It also increases with decreasing elevation angle. Atmospheric gases have strong, broad spectral lines, such as water vapor and oxygen lines at 22 and 60 GHz, respectively.

Cosmic noise originates from discrete sources such as the sun, moon, and radio stars as well as our galaxy, which has strong emissions for directions toward the galactic center. Galactic noise increases with decreasing frequency below 1 GHz. Manmade noise is produced by power lines, electric motors, and other sources and usually can be ignored except in urban areas at low frequencies. Sky noise is very low for frequencies between 1 and 10 GHz, and can be as low as a few K for high elevation angles.

Of course, the antenna pattern strongly influences antenna temperature; see Eq. (119). The ground noise temperature contribution to antenna noise can be very low for high-gain antennas having low side lobes in the direction of the earth. Broad beam antennas, on the other hand, pick up a significant amount of ground noise as well as sky noise. Losses on the antenna structure also contribute to antenna noise. A figure of merit used with satellite earth terminals is G/T_{sys}, which is the antenna gain divided by system noise temperature usually expressed in dB/K. It is desired to have high values of G to increase signal and to have low values of T_{sys} to decrease noise, giving high values of G/T_{sys}.

Example: Direct Broadcast Satellite Reception. Reception of high quality television channels at home in the 1990s, with inexpensive, small terminals, is the result of three decades of technology development, including new antenna designs. DirecTv (trademark of Hughes Network Systems) transmits from 12.2 to 12.7 GHz with 120 W of power and an EIRP of about 55 dBW in each 24 MHz transponder that handles several compressed digital video channels. The receiving system uses a 0.46 m (18 in) diameter offset fed reflector antenna. In this example we perform the system calculations using the following link parameter values:

$$\begin{aligned}
f &= 12.45\,\text{GHz (midband)}\\
P_t(\text{dBW}) &= 20.8\,\text{dBW (120 W)}\\
G_t(\text{dB}) &= \text{EIRP(dBW)} - P_t(\text{dBW}) = 55 - 20.8 = 34.2\,\text{dB}\\
R &= 38,000\,\text{km (typical slant path length)}\\
G_r &= \frac{4\pi}{\lambda^2}\epsilon_{ap}A_p = \frac{4\pi}{(0.024)^2}0.7\pi\left(\frac{0.46}{2}\right)^2 = 2538\\
&= 34\,\text{dB (70\% aperture efficiency)}
\end{aligned}$$

The received power from Eq. (114) is

$$\begin{aligned}
P_r(\text{dBm}) &= 20.8 + 34.2 + 34 - 20\log(38,000) - 20\log(12450)\\
&\quad - 32.44 = -116.9\,\text{dBW}
\end{aligned} \tag{121}$$

This is 2 × 10^{-12} W! Without the high gains of the antennas (68 dB combined) this signal would be hopelessly lost in noise.

The receiver uses a 67 K noise temperature low noise block downconverter. This is the dominant receiver contribution, and when combined with antenna temperature leads to a system noise temperature of T_{sys} = 125 K. The noise power in the effective signal bandwidth Δf = 20 MHz is

$$\begin{aligned}
P_N &= kT_{sys}\Delta f\\
&= 1.38\times10^{-23}\cdot125\cdot20\times10^6 = 3.45\times10^{-14}\\
&= -134.6\,\text{dBW}
\end{aligned} \tag{122}$$

Thus the carrier to noise ratio from Eqs. (118) and (121) is

$$\begin{aligned}
\text{CNR}(dB) &= P_D(dBW) - P_N(dBW)\\
&= -116.9 - (-134.6) = 17.7\,\text{dB}
\end{aligned} \tag{123}$$

Antenna Bandwidth

Bandwidth is a measure of the range of operating frequencies over which antenna performance is acceptable. Bandwidth is computed in one of two ways. Let f_U and f_L be the upper and lower frequencies of operation for which satisfactory performance is obtained. The center (or sometimes the design frequency) is denoted as f_C. Then bandwidth as a percent of the center frequency, B_p, is

$$B_p = \frac{f_U - f_L}{f_C}\times100 \tag{124}$$

Bandwidth is also defined as a ratio, B_r, by

$$B_r = \frac{f_U}{f_L} \tag{125}$$

The bandwidth of narrow band antennas is usually expressed as a percent whereas wide band antennas are quoted as a ratio. Resonant antennas have small bandwidths. For example, half-wave dipoles have bandwidths of up to 16%, (f_U and f_L determined by the VSWR = 2.0). On the other hand, antennas that have traveling waves on them rather than standing waves (as in resonant antennas), operate over wider frequency ranges.

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ANTIFERROMAGNETISM. See MAGNETIC MATERIALS.

APD. See AVALANCHE DIODES.