J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*

RADAR TRACKING

Radar tracking is the ability to determine the position and velocity vector of a target at any particular instant in time, to predict its position in the future, and to distinguish the desired target from other targets and clutter. For a typical radar, the direction from the radar antenna (or antennas) to the target is generally determined in the polar coordinates of range (distance), azimuth (horizontal) angle, and possibly vertical angle. For a sophisticated coherent radar, tracking targets in Doppler frequency space may also be required. Thus radar tracking can be one dimensional (range, angle, or Doppler), two dimensional (range and azimuth angle), three dimensional (range, azimuth angle, and elevation angle), or four dimensional (range, azimuth angle, elevation angle, and Doppler). For some systems, radar information is converted to Cartesian coordinates, and the tracking functions are performed in coordinates such as latitude, longitude, and height.

Target tracking is necessary for a number of reasons. In order to direct a weapon such as a missile or a projectile to a target, the range, future range, and angles from the radar to the target must be determined by the radar. By knowing the position of the target relative to that of the missile, the guidance computer can direct the missile to the target. Aircraft controllers must know an aircraft's location relative to other aircraft in the vicinity, and by tracking the positions of all the aircraft in their assigned sectors, they can control the spacing of the aircraft to ensure flight safety.

Examples of Radar Trackers

A police radar can determine the speed of the vehicle in the field of view of the radar by measuring the Doppler frequency of the return (echo) signal from the vehicle because the Doppler frequency is directly proportional to the vehicle's velocity. Most police radars must track the Doppler frequency over a given period of time to ensure measurement. A missile guidance radar must continually track the target's range, azimuth angle, and elevation angle in order to predict the future target position; thus, it is an example of a three-dimensional tracker. An airborne radar such as the APG-70 in the F15-E aircraft utilizes Doppler processing for clutter rejection, as well as range, azimuth angle, and elevation for target-tracking purposes, and is thus an example of a four-dimensional tracking radar. A phased-array radar must be capable of maintaining track simultaneously on multiple targets, while still scanning its field of regard for new targets.

History of Radar Tracking

In the early days of radar, range and angle-tracking functions were performed manually. Using a device such as a track ball, the operator could keep the cross-hairs positioned on the range and azimuth angle of a detected target viewed on a display such as a plan position indicator (*PPI*) display. The PPI display, such as that shown in Fig. 1, provides a two-dimensional display of range and azimuth angle for a radar with an azimuth-scanning antenna. Targets result in blips on the display where the brightness (and size) of the blips are related to

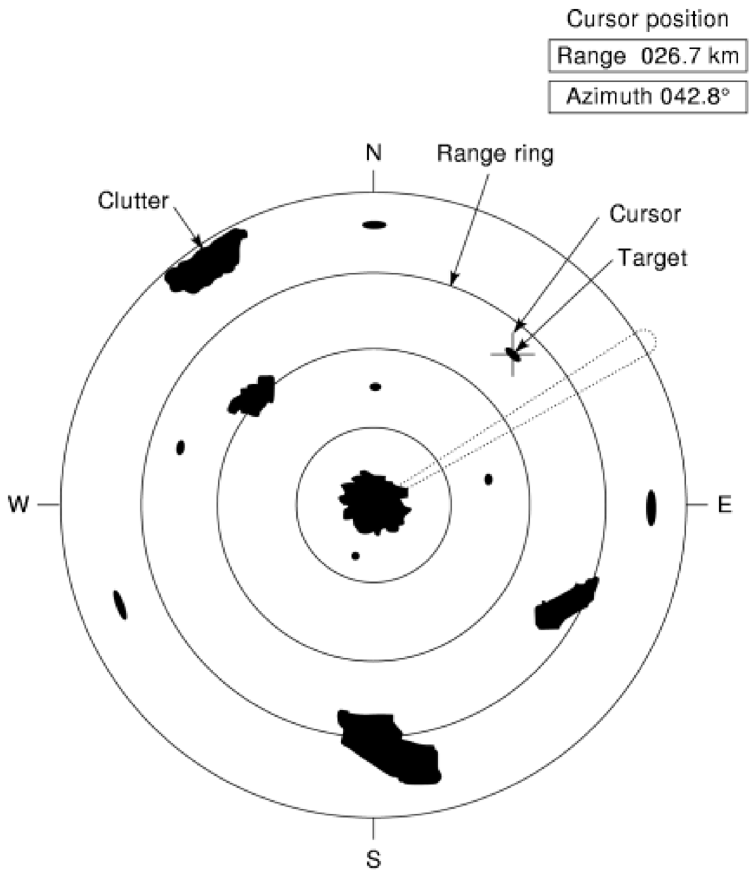

Fig. 1. PPI radar display showing targets and clutter.

the amplitude of the target echoes at the receiver. The output of the track ball can provide readout of the target range and azimuth angle or provide the required range and angle information to weapons systems for targeting purposes. Although this was a satisfactory technique for tracking slow-moving targets such as ships, it is certainly a tedious process.

To aid in the tracking of ships and aircraft, a rate-aided device was added to some systems. With rate-aided tracking, the operator needed to make only fine adjustments to account for changes of the target range and angle rates with respect to the radar. With this configuration, the radar operators were better able to track faster-moving objects such as aircraft. Still, this tracking function required the constant attention of the radar operator.

Automated target tracking evolved as a necessary tool to allow the radar operator to perform the tracking function efficiently. After range and angle trackers are locked onto the target, the tracker then senses any error between the current target position and that predicted by the tracker and automatically and continuously adjusts the tracker functions either on a pulse-to-pulse or scan-to-scan basis. As a result, automatic radar tracking can maintain target track more accurately than a human operator and can better follow fast maneuvering targets.

Tracking Basics

For automatic target tracking, a sequential procedure must be used to acquire the target and initiate track. The three steps are target detection, target acquisition, and target track.

Target Detection. In order for the received echo signal from the target to be detected by the radar, the receive signal strength in that particular range cell must be stronger than the residual noise in the radar and other interfering signals in that range cell. For a target separated from clutter, the primary interfering source is receiver noise. Although it is desired to declare a target's presence with high probability, it is also necessary to keep the probability of false alarm (declaring a target detection when no target is present) as low as possible. The two values are tied closely together: for a given signal-to-noise (*SNR*), lowering the detection threshold to increase the probability of detection threshold also increases the probability of false alarm. Depending upon the target detection criteria, a SNR of 8 to 15 dB is generally required to keep the probability of detection reasonably high, while keeping the probability of false alarms at or below 10^{-6}. Probability of detection vs. false alarm curves are available in Blake (1) and a number of other sources.

In many cases, the single-pulse SNR may be below the threshold, but the SNR can be improved by integrating a number of pulses. For coherent operation, the SNR improvement is directly proportional to n, the number of pulses coherently integrated. For noncoherent operation, the SNR improvement for small n, is usually near $n^{0.8}$ in practical radar systems where $n < 20$. Most real targets are composed of complex reflecting surfaces; the scattering contributions of these separate reflecting surfaces tend to add and subtract vectorially to the overall radar cross section (*RCS*) of the target. The fluctuations in RCS caused by these surfaces will affect the probability of detection and false alarm. Swerling (2) has derived the probability of detection and false alarm curves for both slowly varying and rapidly varying target RCS fluctuations. For these cases, the required SNR required can be obtained from this set of curves.

Target Acquisition. Target acquisition for tracking can be done either manually or automatically. For manual target acquisition, the operator needs to point the radar antenna (or an angle cursor) on the azimuth angle to the target and designate the desired target range. Alternately, the operator could use a light pen if available to designate the target azimuth angle and range to the tracker. When the particular target is within the acquisition limits of the tracker, the acquisition process can be initiated to lock the tracker up on the target range and azimuth.

For automatic target acquisition, the tracker must have either a designated philosophy for selecting the target for track acquisition, or the tracker must have sufficient capability of tracking all the targets satisfying the track initiation criteria. For example, for a radar altimeter, the track would be initiated on the closest radar returns to the radar. For a scanning surveillance radar, the tracker would need to have sufficient capability to track all the targets satisfying the track criteria.

Range Track

When the target has been acquired by the tracker, the tracker must determine not only the range and angular positions but also the velocity vector of the target, and it must determine the velocity components in range and angle in order to maintain track on the target. This is especially important in order to maintain track during conditions of track fade or during momentary passage of other targets or clutter returns. Target trackers differ in complexity and include: (1) dedicated single target trackers, (2) track-while-scan target trackers, and (3) multiple target trackers. For scan-to-scan and multiple target trackers, association algorithms are required to keep track of the targets, especially during crossing target events.

Dedicated Range Trackers. Dedicated target trackers generally use radars with antennas that spotlight the desired target with the antenna beam and keep the antenna beam spotlighted on the target during the entire tracking process. This type of tracker is generally used with weapon systems that require continuously

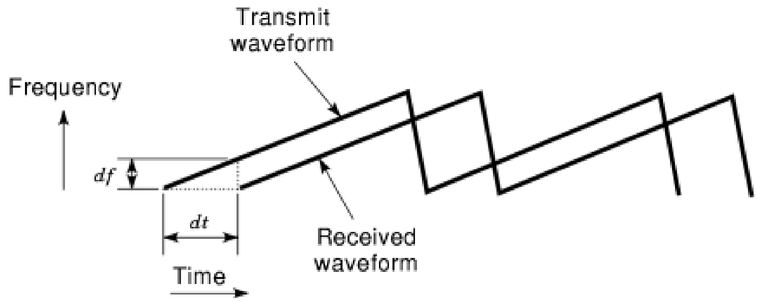

Fig. 2. FMCW transmit and receive waveforms.

updated position information on the target. This is a relatively simple type of tracker and will be used to explain the principles of tracking. The tracking process will be described as composed of the following process: range tracking, angle tracking, and Doppler tracking.

For a radar system, the range from the radar to a target is precisely determined from the time delay between the transmission of the radar signal and the receipt of the radar echo from the target arriving back at the radar's receive antenna. The range (R) from the radar antenna to the target is then given by

$$R = \frac{c\tau}{2}$$

where

c = speed of light (2.997×10^8 m/s),
τ = delay time between transmit and receive target echo.

Because radar signals travel at the speed of light, the range to the target is approximately 150 m for each microsecond of time delay between the time the radar signal is transmitted and when the return echo signal reflected from the target arrives at the receiver.

Range Tracking with an FMCW Radar. The simplest type of radar used for range tracking is that of frequency modulated carrier wave (commonly referred to as an *FMCW*) radar. One of the prime advantages of using an FMCW radar is that, for a given signal to noise ratio, the average transmit power is much less than the peak power required for a pulse-type radar. Transmit signal frequency is generally swept linearly over a period of time, such as shown in Fig. 2. This signal is transmitted toward the target and returns with a time delay (τ). By comparing the frequency of the received signal with that currently being generated by the transmitter, the time delay (and hence the range) can be determined from the equation

$$R = \frac{c\tau}{2} = \frac{c\Delta f}{2}\left(\frac{dt}{df}\right)$$

where

Δf = difference frequency between the received signal and the transmit signal,
df/dt = rate of change in frequency versus time for the transmit signal.

The circuitry for a FMCW ranging system is rather simple, as shown in Fig. 3. The transmitter is coupled to the antenna through a circulator, for example to isolate the transmit signal from the receiver input. Transmit signal is reflected by the target, received by the antenna, and then mixed with the current transmit signal. The mixed signal is then amplified, filtered to remove the radio-frequency (*RF*) transmitter and receive signal

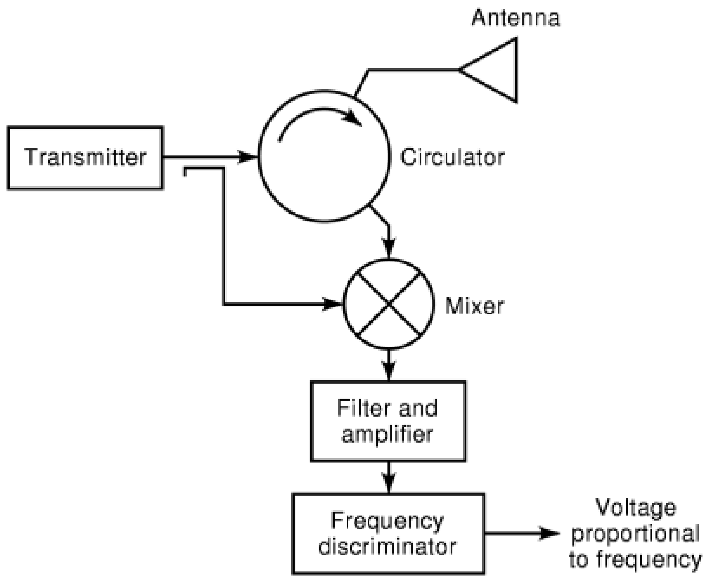

Fig. 3. Typical FMCW circuit.

components, and coupled to a frequency discriminator circuit. The frequency discriminator provides an output voltage that is proportional to the input frequency. Thus the output signal is proportional to the range to the target.

For a moving target, the frequency of the returns from the target are not only affected by the range to the target but also by the target velocity with respect to the radar. In order to separate frequency change effects resulting from range from those resulting from target velocity, an up/down ramp waveform, such as that shown in Fig. 4 can be used. The frequency change caused by velocity essentially moves the entire receive frequency up or down, and by averaging the frequency difference between the up frequency and down frequency portions of the waveform, both the range and the target velocity can be determined from the following equations. The range is determined from the average of the frequency differences during the positive frequency ramp and the negative frequency ramp, thus

$$R = \frac{c\left(\frac{\tau_{\mathrm{p}} + \tau_{\mathrm{n}}}{2}\right)}{2} = \frac{c(\Delta f_{\mathrm{p}} + \Delta f_{\mathrm{n}})}{4}\left(\frac{dt}{df}\right)$$

The velocity of the target relative to that of the radar is a function of the frequency difference between the positive ramp portion and the negative ramp portion. The velocity of the target in the direction toward the radar (positive Doppler frequency) is then

$$v = \frac{\lambda f}{2} = \frac{\lambda(\Delta f_{\mathrm{n}} - \Delta f_{\mathrm{p}})}{4}$$

where λ is the wavelength of the transmit frequency.

Range tracking using an FMCW radar can be accomplished simply by averaging the output voltage from the frequency discriminator, which is proportional to the time delay between the transmit and receive signals,

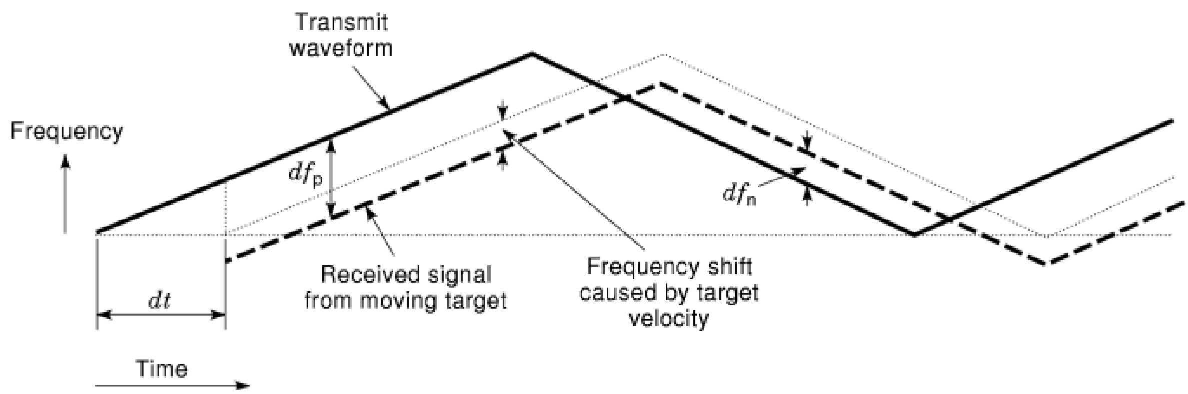

Fig. 4. FMCW waveform for resolving target range from velocity.

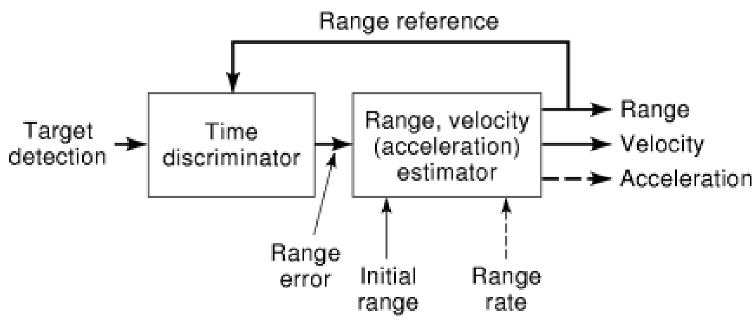

Fig. 5. Basic pulsed radar range tracker.

and hence is proportional to the target range. Extreme linearity and slope calibration of the frequency sweep is required for accurate range determination. For example, a 1% error in the linearity of the linearity or slope, can have an equivalent error in the range determination.

Range Tracking with Pulsed Radar. For pulsed radar, the target range is measured from the time delay (τ) between transmit pulse and received echo from the target. Figure 5 shows the basic configuration of a range tracker used with pulsed radar. The "heart" of a range tracker is the time discriminator that enables the tracker to determine the time difference between the range reference (estimated delay time) and the actual range of the target return. The range error (ϵ_r) is normally bipolar and proportional to the range (or time) difference between the estimated range and measured range. The range error is then input to a range and velocity estimator (and possibly acceleration estimator) circuit. The function of the range error output is to drive estimated range to the measured range. In most cases, an initial range (and possibly range rate) in the general vicinity of the target range must be input to the range, velocity estimator circuit in order to enable it to acquire the target.

There are three basic classes of range trackers, which will be designated as analog, digital, and computer tracker range trackers. The most common analog-type tracker circuit uses early and early–late gates, such as those shown in Fig. 6. The detected target video is input to both early and late gates. During the early gate time, the portion of the video signal existing during that time period is fed through to an integrator circuit, which integrates the signal energy during that time period. The late gate likewise feeds the video signal during the late gate time period to a second integrator. The outputs of the two integrators are compared in a difference circuit. If there is more video energy in one of the integrators, an error signal proportional to the difference is

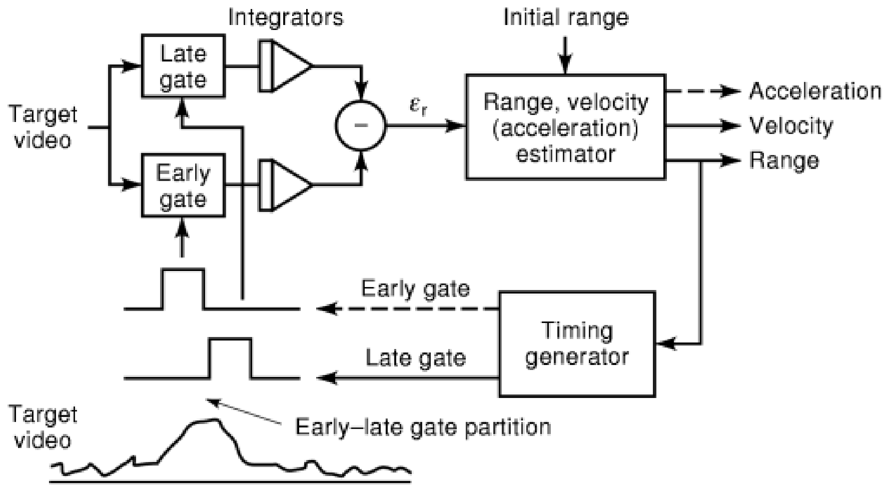

Fig. 6. Analog early–late gate range tracker.

generated. The polarity of the error signal depends upon which integrator output is greater. The error voltage then is provided to the range servo loop circuit, which generates voltages proportional to the estimated range, velocity, and possibly acceleration. The range voltage (estimated range) drives the timing generator, which generates the early and late gate times dependent upon the range voltage. If more video energy is in the late gate time, the error voltage causes the range voltage to increase so that the partition between the early and late gates moves out in range and becomes aligned on the centroid of the video pulse. In order for the range tracker to initially acquire track, the early–late gates must be positioned so that a significant portion of the target video energy appears in the early or late gate times. An operator can accomplish this by observing a radar display and setting the initial range into the track circuit.

The range tracking accuracy of the range tracker is dependent upon the signal to noise power ratio (SNR) of the signal compared to the noise in the early and late gate time periods. According to Barton (3), the standard deviation of the range error (σ_{r1}) on a single pulse basis is given by

$$(\sigma_{r1}) = \frac{1}{2B\sqrt{SNR}} \qquad \text{for } (B\tau_o \geq 1)$$

where

B = receiver frequency bandwidth

τ_0 = pulse width.

Normally, the servo loop integrates a number of pulses to provide smoothing of the range voltage, which reduces the effects of noise jitter upon the range determination. For noncoherent operation, the range error is effectively reduced by $1/n$, the number of pulses integrated. The resulting range error is then given by

$$\sigma_r = \frac{1}{2B\sqrt{f_r t_o (SNR)}}$$

where

f_r = pulse repetition frequency,

t_0 = observation time.

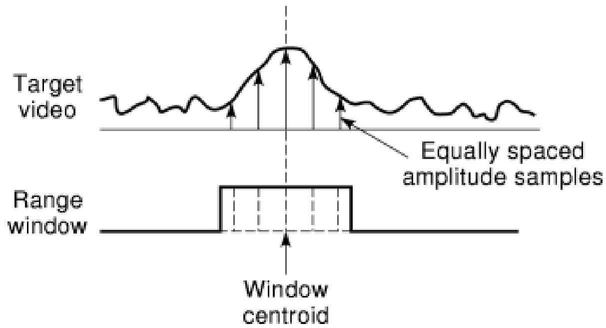

Fig. 7. Digital range track sampler.

Digital range trackers can be implemented using a number of techniques. In most cases, the range information (estimated range) is stored in a digital counter and is updated (up or down counted) depending upon the actual range compared to the estimated range. An early–late gate discriminator, such as that shown for the analog range tracker can be used, and the error voltage then drives the up/down count. A simpler method for accomplishing the discrimination function is shown in Fig. 7. For this case, a range window is positioned about the radar video, and the video voltage is sampled at equal increments across the pulse. It should be noted that the digital discriminator of Fig. 7 requires that the signal be passed through an approximately matched filter prior to sampling, if the *SNR* is to be optimized. The split-gate tracker performs the matched filter function by averaging over the gates, and hence can be preceded by an *IF* amplifier with wider bandwidth. The digital circuit then drives the center of the range window to the centroid of the target video signal by equalizing the voltages in the early and late sample times. Three samples are required, as a minimum, for this type of discriminator: an early sample, a late sample, and an on-target sample. When the tracker is centered on the target, the on-target sample voltage is maximized, thus indicating that a true target is being tracked, rather than noise. Again, the range window must initially be set to the approximate target range or caused to slew automatically until a target is detected.

The analog and digital trackers described earlier are primarily intended for range tracking a single target, and in most cases the radar antenna is boresighted on the target either manually or by using an angle tracker. Tracking circuits, either analog or digital, can be designed to track targets using continuously scanning antennas. For this case, the target returns are received only by the radar during the time when the antenna beam a scans by the target, and the tracker must use prediction algorithms to estimate the position of the target on the next scan. If multiple targets are to be tracked, then individual analog or digital tracking circuits must be used for each target tracked. In most cases where multiple targets are to be tracked, especially in scanning-type radars, the tracking functions are performed in a computer using specialized tracking algorithms. Because track-while-scan tracking normally involves angle tracking as well as range tracking, the discussion on multiple target computer tracking will be deferred.

Angle Tracking

Angle tracking can differ depending upon the application. For dedicated target-tracking radars, the antenna is kept boresighted on the target by the angle-tracking circuits and the antenna servo. With a continuously scanning antenna, the centroid of the target returns is measured each time the radar scans by the target, and uses an estimator to predict the position of the target on the next scan. For multifunction or phase array radars, the target track is updated each time the antenna is scanned to the target location. Because track-while-scan

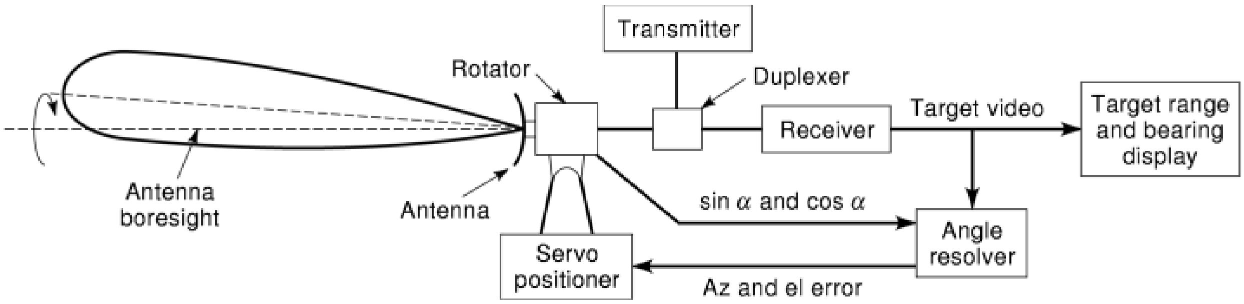

Fig. 8. Conical scan radar.

and multitarget trackers normally require range (and possible Doppler tracking), the angle tracking described in this section is limited to a single target, boresighted angle tracking systems. The most common types of on-boresight trackers use conical scan, sequential lobe, or monopulse-angle-sensing techniques.

Conical Scan Angle Trackers. Conical scan is the simplest angle-sensing technique in that only a single receiver channel is required. As shown in Fig. 8, the antenna beam is squinted off the antenna rotational axis. The squinted antenna beam is rotated about the antenna boresight by either rotating the antenna or nutating an offset feed. If the target is located on the antenna boresight, the target video signal maintains a constant amplitude as the antenna rotates. However, if the target moves off boresight, the target video signal will have a sinusoidal amplitude variation given by

$$E_{\mathrm{t}} = E_0[1 + K_{\mathrm{s}}\epsilon\cos(\omega_{\mathrm{s}}t + \phi)]$$

where

E_0 = average magnitude of received signal,
ϵ = angular distance of target from boresight,
K_{s} = antenna error slope,
ω_{s} = antenna rotator scan frequency,
ϕ = phase angle of the return modulation relative to the scan rotation.

In order to determine the transverse (azimuth) and elevation angle error components, the equation can be rewritten in the form

$$E_{\mathrm{t}} = E_0[1 + K_{\mathrm{s}}\epsilon_{\mathrm{t}}\cos\omega_{\mathrm{s}}t + K_{\mathrm{s}}\epsilon_{\mathrm{e}}\cos\omega_{\mathrm{s}}t]$$

where

ϵ_{t} = transverse (azimuth) angle error component,
ϵ_{e} = elevation angle error component.

By using the preceding equation, the angle resolver can determine the azimuth and elevation error components of the target direction from boresight. These angle error components are then coupled into the azimuth and elevation inputs of the antenna servo positioner, which then drives the antenna boresight onto the target direction. Although this is conceivably the easiest angle-sensing technique, it is susceptible to tracking errors produced by amplitude fluctuations of the target. Also, for military applications, because the conical scan modulation can be detected, modulation jammers can drive the antenna off the target.

Sequential Lobe. Sequential lobe angle sensing is similar to that of conical scan, except that the beam is switched electronically between beam positions. For dual-axis (azimuth and elevation) angle sensing, generally four beam positions are used (up, down, left, right). By comparing the amplitude of the received signals in the upper and lower beams, and knowing the shape of the antenna beams, the angular elevation angle of the target from the antenna boresight can be determined. A similar technique can be used for azimuth angle sensing. The technique can either use a single receiver channel or separate receivers for azimuth and elevation angle sensing. The advantage of the technique is that the switching of the beams can be accomplished on a pulse-to-pulse basis, thus making it less vulnerable to target radar cross-sectioned fluctuations. It, however, still has a vulnerability to modulation-type jammers.

Lobe on receive only (*LORO*) is a variation of sequential lobe sensing. With this technique, the transmitter either uses a separate transmit horn on boresight or transmits simultaneously through all four horns. The sequential lobing is accomplished only on receive, through sequential sampling of the signals in the four horns. The advantage of LORO is that modulation jammers cannot detect the sequential modulation pattern of the receivers.

Monopulse. Monopulse sensing provides the ability to determine the angle of arrival in a single pulse by simultaneously processing the signals in multiple receive beams. Figure 9 shows an example of a four-horn monopulse configuration for dual-plane (elevation and azimuth) angle sensing. The four-horn configuration shown in Fig. 9 is useful for a description of the basic process, but practical radars built since the 1960s have used more complex feed systems to optimize the sum gain, difference error slopes, and sidelobes of all channels. Amplitude-type monopulse uses simultaneous antenna beams squinted at angles off the elevation and azimuth boresights. The relative amplitude of receive signals determines the angular distance of the target off the boresight. Another type of monopulse, referred to as phase-sensing monopulse, uses separate receive apertures spaced a short distance apart, but with the beams pointed parallel with the antenna boresight. For this type of monopulse sensing, the phase difference between the receive signals determines the angular distance of the target from boresight. The monopulse feed, such as shown in Fig. 9, is normally used either to illuminate a parabolodial reflector directly or to illuminate a subreflector for a Cassegrain-type antenna. The monopulse feed is normally attached directly to a Σ and Δ comparator. The Σ and Δ comparator combines the received signals in the four beams to form a Σ signal, a Δ_{AZ} signal, and a Δ_{EL} signal. According to Rhodes (4), amplitude sensing and phase sensing are equivalent and can be converted to Σ and Δ sensing. Within the 3 dB beamwidth of the Σ pattern of the monopulse antenna, the function Δ/Σ is approximately linear. The target azimuth angle θ off the azimuth boresight and the elevation angle β off the elevation boresight can be determined from

$$\theta \approx K\frac{|\Delta_{AZ}|}{|\Sigma|}\cos\phi_{AZ}$$

$$\beta \approx K\frac{|\Delta_{EL}|}{|\Sigma|}\cos\phi_{EL}$$

where

K = antenna slope (a function of the squint angle of the beams),
ϕ_{AZ} = phase angle of the Δ_{AZ} signal relative to the Σ signal,
ϕ_{EL} = phase angle of the Δ_{EL} signal relative to the Σ signal.

For a point like target (target extent ≪ less than the antenna beamwidth), the phase angle between the Σ and Δ signals is normally either 0° or 180°, depending upon which side of the boresight the target is located.

A typical configuration for a three-channel monopulse receiver is shown in Fig. 10. The Σ and Δ signals, after down-conversion to IF are amplified in gain-controlled amplifiers. The Δ IF outputs from the gain-controlled amplifiers are input to amplitude-sensitive phase detectors, along with the Σ IF outputs. The

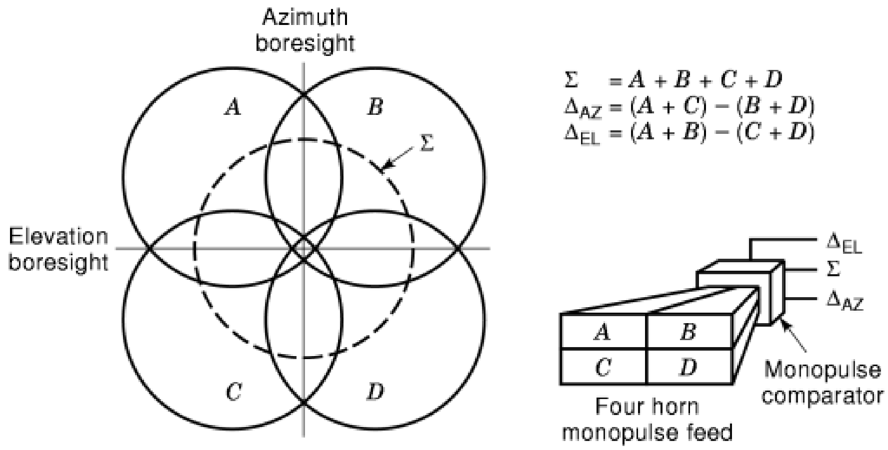

Fig. 9. Four-horn monopulse antenna beam patterns.

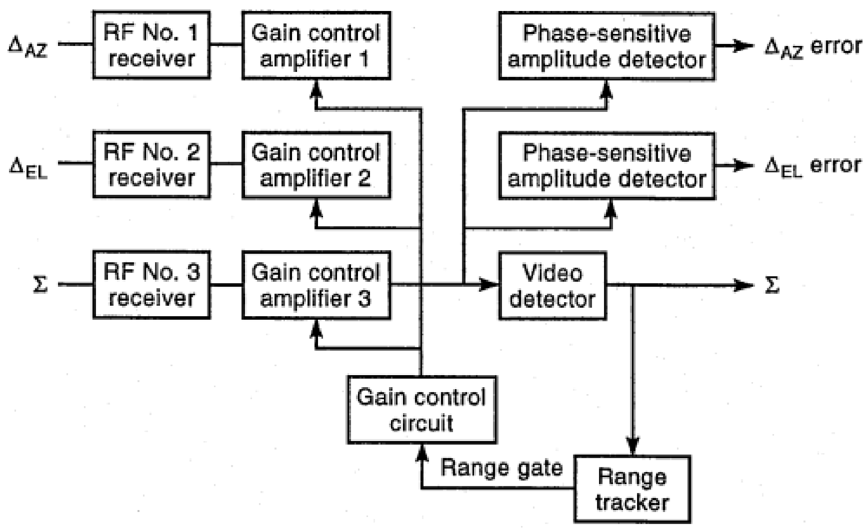

Fig. 10. Three-channel monopulse circuit.

phase-sensitive amplitude detector provides a video output signal proportional to the amplitude of the Σ signal and the cosine of the phase angle between the Δ signal and the Σ signal. In order to maintain a constant number of volts per degree for the output phase amplitude phase detectors, the gain of the receivers must be maintained to provide a constant output signal level at the range of the target. To do this, the sum signal is detected to provide a Σ *video* signal to a range tracker circuit, which then locks the range onto the target. The output video signal is then sampled at the range of the target, and this is then used to form the gain control voltage in the three receiver channels. This Σ signal normalization maintains the desired number of output volts per degree from the Δ channel receivers. Close tolerances on gain and phase track of the three gain control amplifiers are required to provide the integrity of the angle error calibration.

Figure 11 shows an example of a two-channel monopulse receiver. The Σ, Δ_{AZ}, and Δ_{EL} microwave signals out of the monopulse comparator are switched in a RF commutator so that on receive pulse 1, $\Sigma + \Delta_{AZ}$, and

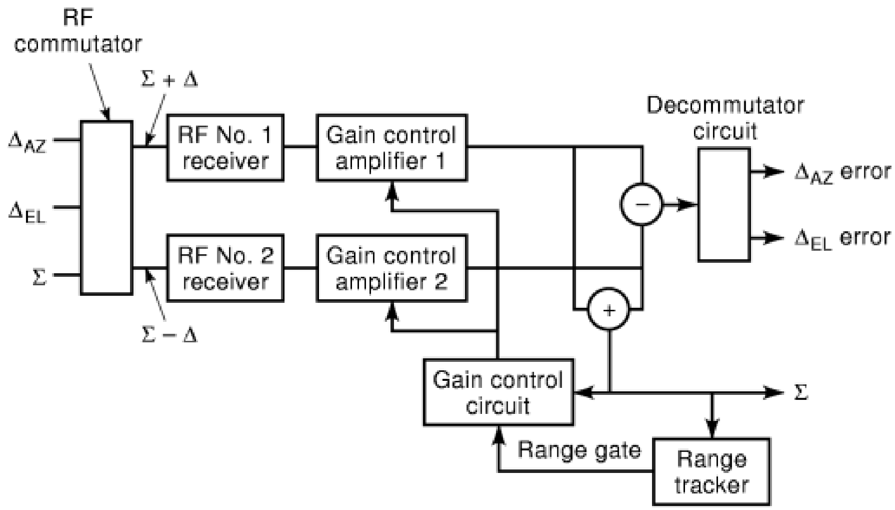

Fig. 11. Two-channel monopulse circuit.

$\Sigma - \Delta_{AZ}$ signals are in receiver channels 1 and 2, respectively. On the second receive pulse, $\Sigma + \Delta_{EL}$, and $\Sigma - \Delta_{EL}$ are coupled in to receive channels 1 and 2. On the third receive pulses, the Δ polarities are switched, so that the $\Sigma - \Delta_{AZ}$, and $\Sigma + \Delta_{AZ}$ signals are input to channels 1 and 2 on the third pulse, and the $\Sigma - \Delta_{EL}$, and $\Sigma + \Delta_{EL}$ signals are in channels 1 and 2 on the fourth pulse. The receive signals in channels 1 and 2 are down-converted from RF to intermediate frequency (*IF*), amplified in gain-controlled amplifiers and subsequently converted to video. The decommutator circuit then uses the difference in the video outputs to form the Δ_{AZ} *error* and the Δ_{EL} *error* signals. The error signals are then coupled into the antenna servo to maintain the antenna boresight on the tracked target.

In order for the angle circuit to maintain a constant number of volts per degree for the angle error output, the gain of the receivers must be maintained to provide a constant (on the average) output signal level at the target range. In order to accomplish this, the sum of the video receive signals is provided to a range tracker circuit, which then determines the range to the tracked target. The output video signal is then sampled at the range of the target, and this is then used to form the gain control voltage to both the receiver channels, in order to maintain relatively constant target output levels in the receivers.

The advantage of the two-channel receiver is that it eliminates the need for a third receiver channel, and that it eliminates any zero drift in the phase-sensitive amplitude detectors. This is at the expense of 3 dB less efficiency (as compared to the three-channel configuration), and potential sensitivity to target amplitude fluctuation if the two channel gains are not identical. The disadvantage is that the angle error is only determined on alternate pulses, and any noise or differential losses in the switching process will tend to degrade the accuracy and precision of track. Thus, some sacrifice in tracking precision will be suffered in comparison to a full three-channel monopulse angle tracker.

Angle Error Sources. The accurate determination of the angle to the target is influenced by a number of factors including radar-dependent errors, target-dependent errors, and propagation effects. Radar-dependent errors include the effects of thermal noise, antenna misalignment and cross coupling errors, and radar instrumentation error sources. The angular errors resulting from thermal noise can be quantified and are primarily dependent upon the signal-to-noise ratio. For a conical scan radar, the variance in the angle determination is given by

$$\sigma_t = \frac{1.4\theta_c}{K_s\sqrt{B\tau(SNR)(f_r/\beta_n)}}, \text{ for } SNR > 4$$

where

K_s = conical scan angle error slope,
θ_c = antenna 3-dB bandwidth,
SNR = signal-to-noise power ratio,
f_r = pulse repetition frequency,
β_n = servo bandwidth,
B = receiver bandwidth,
τ = pulse width.

For a monopulse angle tracker, the variance is given by

$$\sigma_t = \frac{\theta_m}{K_m\sqrt{B\tau(SNR)(f_r/\beta_n)}}, \text{ for } B\tau > 1$$

where

K_m = monopulse error slope,
θ_m = antenna 3 dB Σ beamwidth.

Glint is one of the most significant target dependent, angle error sources for complex targets such as aircraft and ships. Complex targets consist of multiple scatterers separated in angle and range. Rather small variations in target aspect angle can change the phase relationship of the separate scatterers, resulting in large variations of the amplitude and indicated angle to target. Depending upon the extent of the scatterers and their phase relationships, the indicated angle to the target can actually be outside the physical dimensions of the target. In order to understand the phenomena, the slope of the phase front resulting from two isolated point targets is given by Dunn and Howard (5) as

$$\delta' = \frac{L\cos\psi}{2}\left[\frac{1-a^2}{1+2^2+2a\cos\left(\phi+\dfrac{2\pi L}{\lambda}\sin\psi\right)}\right]$$

where

a = relative amplitude of the one scatterer to the stronger scatterer,
L = lateral distance separating the two scatterers,
ϕ = relative phase of the two scatterers,
ψ = angle between the perpendicular bisector of the scatterers and direction to the radar.

If ψ is set equal to zero, then

$$\delta' = \frac{L}{2}\left[\frac{1-a^2}{1+a^2+2a\cos\phi}\right]$$

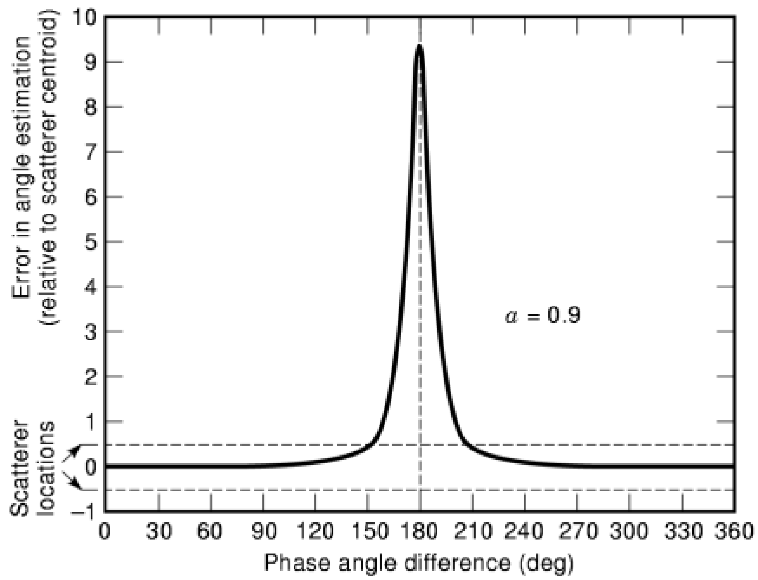

Fig. 12. Phase front warpage caused by two scatterers.

The preceding equation has been plotted in Fig. 12 for $a = 0.9$. As can be seen from the plot, when the relative phase angle between the two scatterers approaches 180°, the indicated angular position is outside the directions to the two scatterers.

Propagation effects such as multipath and ducting can also affect the angular indication, especially for elevation angle sensing. Multipath is a severe problem for low angle tracking of targets, where the multipath return from the terrain is in the main beam (or possibly even the sidelobes) of the antenna. Multipath contributions can be both from specular and diffuse reflections from the terrain, and their contributions are a function of the surface roughness. For specular reflections, the return signals can be expressed as

$$E = A_t f(\theta_t) + A_r \rho_o f(-\theta_r) e^{-j\alpha}$$

where

A_t = free-space target amplitude at the antenna,
A_r = free-space multipath (target image) amplitude at the antenna,
$f(\theta_t)$ = antenna voltage gain in the direction of the target,
$f(-\theta_r)$ = antenna voltage gain in the direction of the multipath return,
ρ_o = magnitude of the reflection coefficient,
α = relative phase angle between the direct and the multipath return.

In a sense, the angular errors caused multipath effects are similar to those associated with the two-point scatterer situation.

Doppler Tracking

Tracking targets in a clutter background is one of the major problems for radar trackers. Fortunately, terrain clutter generally has a narrow specular extent. If the target is moving, the Doppler frequency of its return is normally outside that of the terrain clutter. Doppler filtering can then be used to reject clutter, while keeping the target returns. The simplest type of Doppler filtering is obtained by using moving target indication (*MTI*) processing. More advanced Doppler processing enables the determination of the Doppler frequency (and hence the radial velocity) of the target. MTI or Doppler filtering must be applied to both sum and difference channels of a monopulse tracker, and in conical scan or lobing radar must be able to cancel the modulation induced on the clutter by scanning.

MTI Processing. For a ground-based radar system, MTI processing provides the capability to reject clutter by filtering out the returns whose spectral content is close to the pulse repetition frequency (*PRF*) of the radar. This is accomplished by comparing the phase and amplitude of the target returns on successive pulse intervals. Coherent radar operation is normally used for MTI processing, however, coherent on-receive MTI processing can be used with noncoherent radars to provide most of the benefits achieved with coherent radar processing. In MTI processing, if the phase and amplitude of the returns stay constant over two, three, or more pulse intervals, then the returns are assumed to be associated with clutter and are rejected. The phase (and possibly the amplitude) of moving target returns will change on a pulse-to-pulse basis and are not rejected by the MTI filtering. MTI-filtered target returns can then be tracked by range and angle tracking circuits.

Doppler Filtering. Full Doppler tracking requires coherent radar operation and can improve the tracking ability of the radar using narrow filter bandwidths, thus increasing the sensitivity of the radar. The Doppler filtering can also enable the determination of the actual Doppler frequency of the radar returns, thus providing an exact determination of the target radial velocity. In addition, for airborne radars, the Doppler frequencies of the clutter returns are a function of the aircraft velocity and the aspect angles to the clutter patch. Thus MTI processing cannot be used for clutter rejection with airborne radars.

Continuous wave (*CW*) radar provides the ability to track a moving target while rejecting clutter. Normally, separate transmit and receive antennas are used for CW tracking radars. An example of a Doppler phase lock loop, a simplified version of that shown by Morris (6), is given in Fig. 13. The Σ signal input is mixed down to the center frequency (f_2) of the narrow band-pass filter. The input to the Σ signal mixer is derived from a combination of the output of the phase locked oscillator (*PLO*) which is then mixed with the IF local oscillator (*IF LO*) frequency to provide the IF signal necessary to mix the Σ signal down to frequency f_2. Any increase or decrease in the Doppler frequency (f_D) will cause the PLO output frequency to change in order to maintain the input to the band-pass filter at frequency f_2. The Δ_{AZ} and Δ_{EL} signals are also mixed down to frequency f_2. The Δ_{AZ} and Δ_{EL} signals are narrowband filtered, and used to derive the Δ_{AZ} *error* and Δ_{EL} *error* signals.

For a high PRF pulsed Doppler radar, a narrow pass-band filter is normally used to limit the receive spectrum to $f_o \pm$ PRF/2. This has the effect of converting the pulsed signal to CW, at which time the CW Doppler tracking configuration described earlier can be used for the Doppler and angle tracking. If range tracking of the signal is also required, the signal must first be sampled at the range of the target prior to narrow band filtering. A minimum of two adjacent range cell samplers, each followed by narrow band Doppler filter, are required to accomplish range tracking. In this case, the range samplers act as early and late gate samplers, and by comparing the output Doppler amplitudes, the range tracker can keep the received pulses centered between the two range samplers. Acquisition with a pulsed Doppler tracker can be a complicated process. In order for the Doppler tracker to acquire the target, both the range and Doppler frequency must be established in order to provide the Doppler output signals required for tracking. Thus, unless the range and the Doppler frequency is known (and normally they are not), a search process both in range and Doppler frequency must be initiated to find the target and initiate track. Other configurations exist for Doppler filter trackers. Barton (3) describes a technique using narrow-band filters offset above and below from a center frequency. By

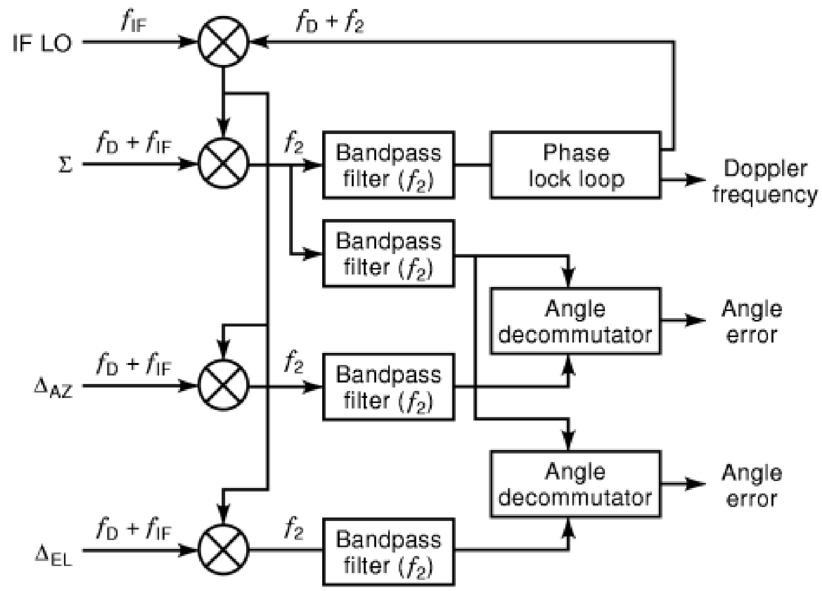

Fig. 13. Doppler phase lock loop.

comparing the amplitudes out of the high- and low-frequency narrowband filters, an estimate of the Doppler frequency can be obtained on a single pulse basis.

Digital Doppler Processing. With the advent of high-speed digital processors, the Doppler frequencies can be computed directly. For this type of processing, the receive signals are normally converted to I and Q digital format using high-speed analog to digital converters. The range-sampled I and Q signals can be stored for a selected number of pulse repetition intervals (*PRI*), and input to fast Fourier transform (*FFT*) computational routines. The FFT computes the detected amplitude versus Doppler frequency for each sampled range. Tracking algorithms can then use the detected targets out of the FFT processor to establish the range track, and subsequently angle and Doppler track.

Radar Ambiguities

Generally radars are classified as low, medium, or high PRF radars. For low PRF radars, all the target (and clutter) returns are received prior to transmission of the next radar pulse. With high PRF radars, the Doppler frequencies of all the target (and clutter) signals are less than that of the radar PRF. Low PRF radars, which are unambiguous in range, are generally ambiguous in Doppler, whereas high PRF radars are normally highly ambiguous in range. Medium PRF radars can be ambiguous in both range and Doppler.

Range Ambiguities and Eclipsing. Figure 14 shows receive signals over several PRI. The returns from target 1 occur within the same PRI as the transmit pulse that initiated the target returns, and so target 1 range is unambiguous. The returns from target 2 occur at the same times when other transmit pulses are being generated. Because receiver returns are normally disabled during the transmit pulse times to prevent receiver saturation (and possibly burn-out), target 2 returns are eclipsed, and not detected in the receiver. The returns from target 3 are from a range exceeding the unambiguous range, so that the returns in the current PRI are associated with pulses transmitted several pulses earlier. Thus, from the radar display, the returns from target 3 appear to be from a much closer range.

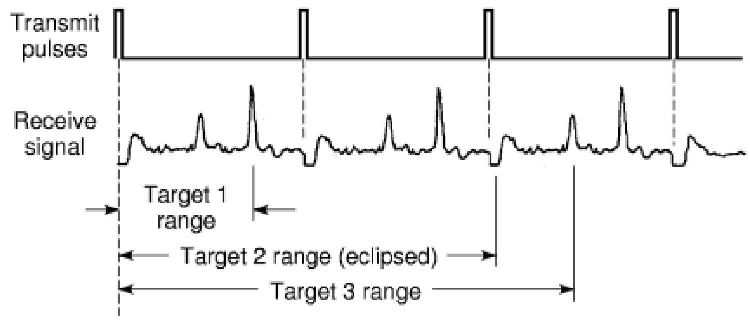

Fig. 14. Range ambiguities and eclipsing.

Range eclipsing occurs quite frequently in high PRF radars because of the relatively high transmit time duty factors. Even on medium PRF radars eclipsing must be avoided for reliable target detection. Eclipsing can be avoided by changing the PRF when the radar determines that the tracked target range is approaching an eclipse situation. An alternate solution is to switch between two or more PRI, so that the target will be visible in the PRIs in which it is not eclipsed.

Clutter returns with delay times exceeding the PRI (second-time around returns) can cause serious problems to an MTI radar. This is because many MTI radars employ pulse-to-pulse stagger to avoid blind ranges. With PRF-staggered MTI radar, second time around clutter returns are not cancelled because the apparent range changes from pulse to pulse. In general, range ambiguities need be resolved, especially for medium and high PRF radars. Even for relatively low PRF radar, such as the AN/MPS-36 instrumentation tracking radar with a 320 Hz PRF (unambiguous range of 253 nm), the radar when its return is augmented by a transponder can track missiles many thousands of miles. A number of methods are available for resolving range ambiguities. One method is to use a form of PRF stagger in which the transmission time is varied on a pulse-to-pulse basis. The only receive pulses that align on a pulse-to-pulse basis are those corresponding to the destagger associated with that specific number of PRI. Another method is to apply intrapulse coding on the transmit pulse in which the coding is changed on a pulse-to-pulse basis. On receipt, receive signals can then be associated with the particular transmit pulse responsible for the target returns.

Doppler Ambiguities and Blind Speeds. Figure 15 illustrates receive signals (in frequency space) for a pulsed coherent radar. The spectral content of the clutter returns are centered about the PRF frequency lines denoted by $f_o \pm n_{\text{PRF}}$. Target 1 has a Doppler frequency that is less than the PRF, and so its Doppler can be determined unambiguously. Target 2 Doppler frequency is at a multiple of the PRF, and because the clutter returns are normally much higher than those of the target, it is highly unlikely that the target will be detected. In fact, most coherent radars intentionally reject frequencies around the $f_o \pm n$PRF frequencies, specifically to reject clutter. Target speeds associated with Doppler frequencies of $f_o \pm n$PRF are referred to as blind speeds. Target 3 Doppler frequency exceeds that of the PRF so that the actual Doppler frequency cannot be determined from the receive spectrum.

Blind speeds can be avoided by several methods. Many coherent MTI radars avoid blind speeds by varying the PRF on a pulse-to-pulse basis. By appropriately selecting a number of different PRFs, and switching PRFs on a pulse-to-pulse basis, blind speeds can be avoided over a large range of target velocities. However, as noted previously, second-time-around clutter returns will pose a problem to this type of processing. Most Doppler radars require a constant PRF during the coherent processing interval (*CPI*). Doppler radars can avoid blind speeds by switching to a different PRF when it notes that it is approaching a blind speed. Alternately, the radar could transmit groups of pulses at different PRFs so that at most only one group would be at the blind speed.

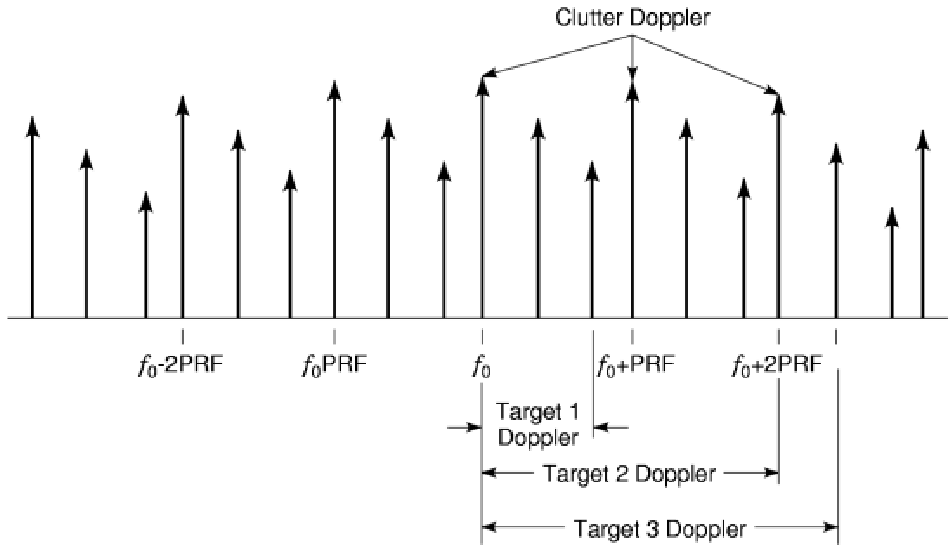

Fig. 15. Doppler ambiguity and blind speeds.

Resolving Doppler ambiguities can be accomplished by several techniques. If the radar is tracking the target range, the range rate determination is generally accurate enough to enable determination of which PRF multiple the target Doppler is located. If two or more groups of Doppler PRFs are used, the ambiguity can often be resolved from the measured Doppler frequencies resulting from the multiple PRFs.

Multiple Target Tracking. In many cases there is a need to track multiple targets simultaneously. Continuously scanning surveillance radars, such as the FAA's ASR-9 airport surveillance radars, must track all the targets (airplanes) within their coverage regime. This tracking must be performed on a scan-to-scan basis, and thus this type of tracking is commonly referred to as track-while-scan processing. Phased array radars are also multiple target trackers, because they normally interleave switched beam locations to track a number of targets, with scanning for new targets, as well as performing other possible functions. With these radars, the targets (or aircraft) are only viewed for a number of pulses on a scan-to-scan (or look-to-look) basis.

Most modern scan-to-scan (or look-to-look) radars use computers for multiple target tracking. Because the aircraft positions typically change on a look-to-look basis, tracking algorithms must be derived to predict the estimated target positions on the next scan, based upon on previous scans. The accuracy of these predicted positions is limited by the maneuver capabilities of the targets being tracked, so that the predicted positions are only estimates of their actual positions. Association algorithms must then be used to determine (1) if detected target is associated with an established track, (2) which established target track the target should be associated with, and (3) to determine if a new track should be established, if no track association is made.

Figure 16 shows a typical flow diagram for a multiple target tracker. The raw target position information, such as range and azimuth angle (and possibly elevation angle or height), is derived in the radar. Most multiple target tracker associative algorithms prefer to track in rectilinear coordinates (R_{N}, R_{E}, R_{V}) rather than polar coordinates so that the conversion must be made from polar coordinates. If North is assumed to be at zero degrees, then

$$R_{\mathrm{N}} = R \cos\theta \cos\beta$$
$$R_{\mathrm{E}} = R \sin\theta \cos\beta$$
$$R_{\mathrm{V}} = R \sin\beta$$

where

R = range,
θ = azimuth angle,
β = elevation angle.

The current target position (R_{C}) is then

$$R_{\mathrm{C}} = \sqrt{(R_{\mathrm{N}})^2 + (R_{\mathrm{E}})^2 + (R_{\mathrm{V}})^2}$$

After the coordinate transformations are performed on the incoming radar target data, present target detections are then compared in association algorithms to determine if the target data are associated with established target tracks. For the FAA airport surveillance radars, the radar target detections are also associated with the secondary (beacon) radar target reports. The beacon returns also include aircraft identification and reported aircraft height. The combined associations are then used to update the target tracks and predict the aircraft locations on the next scan using the track prediction and smoothing algorithms. If the target detection is not associated with any of the present target tracks, then the position information is considered for the establishment of a new target track. In order to establish a new target track, generally the target must be associated with m on n of the previous scans in order to establish a new track. After the m out of n association is made, a new track is established, and the past position information on those target detections is used to predict the target location on the next scan. Target tracks are generally dropped after a certain number of successive target track associations are missed. The information on established target tracks is then routed to radar displays and possibly to weapons systems.

Smoothing and Prediction Algorithms. Radar measurements of target positions and velocities are often imprecise as a result of a number of factors, such as signal-to-noise ratio, target RCS fluctuations, multipath, and clutter contamination. Various algorithms can be used for the track smoothing and prediction to mitigate the effects of scan-to-scan position and velocity measurement errors, and thus to improve the accuracy of tracking. Kalman filters (7) are probably the best -known smoothing and prediction algorithms. Alpha, beta (α, β or α, β, γ) trackers are a subset of the Kalman filters and are the simplestbecause they use precomputed fixed gains. The α, β, γ equations applied to position and velocity smoothing are

$$\hat{R}_{\mathrm{C}} = \hat{R}_{\mathrm{PC}} + \alpha(R_{\mathrm{C}} - \hat{R}_{\mathrm{PC}})$$
$$\hat{\dot{R}}_{\mathrm{C}} = \hat{\dot{R}}_{\mathrm{C}} + \beta \frac{(R_{\mathrm{C}} - \hat{R}_{\mathrm{PC}})}{T}$$
$$\hat{\ddot{R}}_{\mathrm{C}} = \hat{\ddot{R}}_{\mathrm{PC}} + \gamma \frac{\left(\hat{\dot{R}}_{\mathrm{C}} - \hat{\dot{R}}_{\mathrm{PC}}\right)}{T}$$

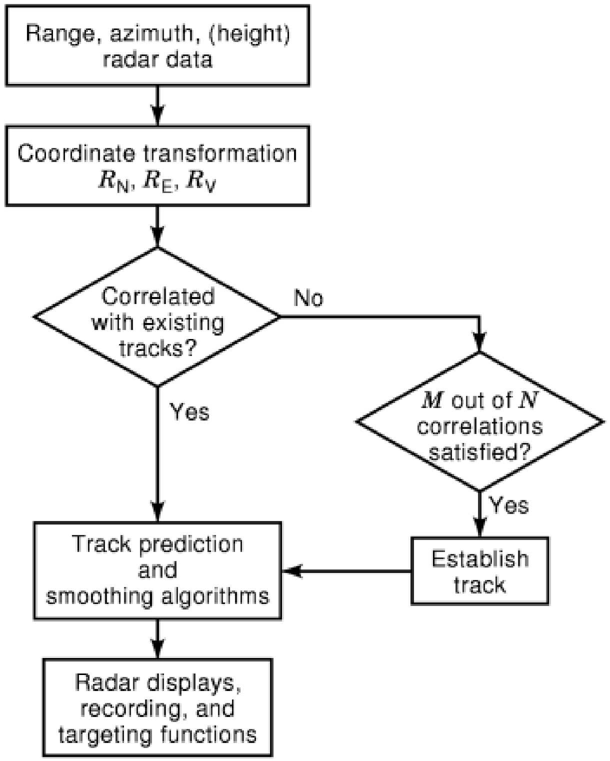

Fig. 16. Multiple target tracking.

and for prediction are

$$\hat{R}_{P(C+1)} = \hat{R}_C + \hat{\dot{R}}_C T + (T^2/2)\hat{\ddot{R}}_C$$
$$\hat{\dot{R}}_{P(C+1)} = \hat{\dot{R}}_C + \hat{\ddot{R}}_C T$$
$$\hat{\ddot{R}}_{P(C+1)} = \hat{\ddot{R}}_C$$

where

T = sampling period,
R_C = measured position,
$\hat{R}_C$ = smoothed estimate of current position,
$\hat{R}_{PC}$ = predicted position at the time of the measurement,
$\hat{R}_{P(C+1)}$ = predicted position T s later,
$\hat{R}C$ = Smoothed estimate of current velocity,
$\hat{R}_{PC}$ = predicted velocity at the time of the measurement,
$\hat{R}_{P(C+1)}$ = predicted velocity T s later,
$\hat{R}_C$ = smoothed current acceleration,
$\hat{R}_{P(C+1)}$ = predicted acceleration T s later,
T = time between measurements.

The precomputed fixed gains α, β, γ can vary between zero and one, with values toward one giving the greatest emphasis toward the current measurements, whereas values toward zero provide the greatest smoothing. Benedict and Bordner (8) analyzed the gains for an α, β for track-while scan application and determined the optimal selection for this application as

$$\beta = \alpha^2/(2-\alpha)$$

The performance of the α, β, γ trackers are limited by the selection of the fixed gains, which may not be optimal for all situations. Bar-Shalom and Li (9) discusses the use of Bayesian data association techniques, as well as multiple model estimators for providing superior performance for multitarget tracking.

BIBLIOGRAPHY

1. L. V. Blake, Prediction of radar range, in M. I. Skolnik, (ed.), *Radar Handbook*, New York: McGraw-Hill, 1990, chap. 2.
2. P. Swerling, Probability of detection for fluctuating targets, *IRE Trans.*, **IT-6**: 269–308, 1960.
3. D. K. Barton, *Radar System Analysis*, Englewood Cliffs, NJ: Prentice-Hall, 1964.
4. D. R. Rhodes, *Introduction to Monopulse*, New York: McGraw-Hill, 1959, p. 41; reprint, Norwood, MA: Artech House, 1980.
5. J. H. Dunn, D. D. Howard, Radar target amplitude, angle, and Doppler scintillation from analysis of the echo signal propagating through space, *IEEE Trans. Microw. Theory Tech.*, **MTT-16**: 715–728, 1968.
6. G. V. Morris, Doppler frequency tracking, in J. L. Eaves and E. K. Reedy (eds.), *Principles of Modern Radar*, New York: Van Nostrand Reinhold, 1987, Chap. 19.
7. R. E. Kalman, New results in linear filtering and prediction theory, *ASME Trans.*, **83D**: 95–108, 1961.
8. T. R. Benedict, G. W. Bordner, Synthesis of an optimal set of track-while-scan smoothing equations, *IRE Trans. Autom. Control*, **AC-7**: 27–32, 1962.
9. Y. Bar-Shalom, Xiao-Rong Li, *Multitarget-Multisensor Tracking: Principles and Techniques*, Storrs, CT: Yaakov Bar-Shalom, 1995.

JOSEPH A. BRUDER
Air Force Research Laboratory