Elayn Martin-Gay

Basic College Mathematics

with Early Integers

Fourth Edition

Basic College Mathematics with Early Integers

Fourth Edition

Elayn Martin-Gay
University of New Orleans

Pearson

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Pearson

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This book is dedicated to students everywhere—
and we should all be students. After all, is there anyone among
us who truly knows too much? Take that hint and continue
to learn something new every day of your life.

Best wishes from a fellow student:
Elayn Martin-Gay

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Preface

***Basic College Mathematics with Early Integers*, Fourth Edition** was written to provide a solid foundation in the basics of college mathematics, including the topics of whole numbers, integers, fractions, decimals, ratio and proportion, percent, and algebra topics. Integers are introduced in Chapter 2 and integrated throughout the text. This allows students to gain confidence and mastery by working with integers throughout the course. Specific care was taken to make sure students have the most up-to-date relevant text preparation for their next mathematics course or for non-mathematical courses that require an understanding of basic mathematical concepts. I have tried to achieve this by writing a user-friendly text that is keyed to objectives and contains many worked-out examples. As suggested by AMATYC and the NCTM Standards (plus Addenda), real-life and real-data applications, data interpretation, conceptual understanding, problem solving, writing, cooperative learning, appropriate use of technology, number sense, estimation, critical thinking, and geometric concepts are emphasized and integrated throughout the book.

The many factors that contributed to the success of the previous edition have been retained. In preparing the Fourth Edition, I considered comments and suggestions of colleagues, students, and many users of the prior edition throughout the country.

What's New in the Fourth Edition?

- **The Martin-Gay Program** has been revised and enhanced with a new design in the text and MyLab Math to actively encourage students to use the text, video program, and Video Organizer as an integrated learning system.
- **New Getting Ready for the Test** can be found before each Chapter Test. These exercises can increase student success by helping students prepare for their Chapter Test. The purpose of these exercises is to check students' conceptual understanding of the topics in the chapter as well as common student errors. It is suggested that students complete and check these exercises before taking a practice Chapter Test. All Getting Ready for the Test exercises are either Multiple Choice or Matching, and all answers can be found in the answer section of this text.

  Video Solutions of all exercises can be found in MyLab Math. These video solutions contain brief explanations and reminders of material in the chapter. Where applicable, incorrect choices contain explanations.

  Getting Ready for the Test exercise numbers marked in blue indicate that the exercise is available in **Learning Catalytics**. LC

- **New Learning Catalytics** is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. Generate class discussion, guide your lecture, and promote peer-to-peer learning with real-time analytics. Accessible through MyLab Math, instructors can use Learning Catalytics to:
  - Pose a variety of open-ended questions that help your students develop critical thinking skills.
  - Monitor responses to find out where students are struggling.
  - Use real-time data to adjust your instructional strategy and try other ways of engaging students during class.
  - Manage student interactions by automatically grouping students for discussion, teamwork, and peer-to-peer learning.

  - Pearson-created questions for developmental math topics are available to allow you to take advantage of this exciting technology. Additionally, “Getting Ready for the Test” exercises (marked in blue) are available in Learning Catalytics. Search the question library for “MGBCMEI” and the chapter number, for example, MGBCMEI7 would be the questions from Chapter 7.

- **New Key Concept Activity Lab Workbook** includes Extension Exercises, Exploration Activities, Conceptual Exercises, and Group Activities. These activities are a great way to engage students in conceptual projects and exploration as well as group work. This workbook is available in MyLab Math, or can be packaged with a text or MyLab code.

- **Exercise Sets** have been carefully examined and revised. Special focus was placed on making sure that even- and odd-numbered exercises are carefully paired and that real-life applications are updated.

- **The Martin-Gay MyLab Math** course has been updated and revised to provide more exercise coverage, including assignable Video Check questions and an expanded video program. There are Lecture Videos for every section, which students can also access at the specific objective level; Student Success Tips videos; and an increased number of video clips at the exercise level to help students while doing homework in MyLab Math. Suggested homework assignments have been premade for assignment at the instructor’s discretion.

Key Pedagogical Features

The following key features have been retained and/or updated for the Fourth Edition of the text:

- **Problem-Solving Process** This is formally introduced in Chapter 1 with a four-step process that is integrated throughout the text. The four steps are **Understand, Translate, Solve,** and **Interpret.** The repeated use of these steps in a variety of examples shows their wide applicability. Reinforcing the steps can increase students’ comfort level and confidence in tackling problems.
- **Exercise Sets Revised and Updated** The exercise sets have been carefully examined and extensively revised. Special focus was placed on making sure that even- and odd-numbered exercises are paired and that real-life applications were updated.
- **Examples** Detailed, step-by-step examples were added, deleted, replaced, or updated as needed. Many examples reflect real life. Additional instructional support is provided in the annotated examples.
- **Practice Exercises** Throughout the text, each worked-out example has a parallel Practice exercise. These invite students to be actively involved in the learning process. Students should try each Practice Exercise after finishing the corresponding example. Learning by doing will help students grasp ideas before moving on to other concepts. Answers to the Practice Exercises are provided at the bottom of each page.
- **Helpful Hints** Helpful Hints contain practical advice on applying mathematical concepts. Strategically placed where students are most likely to need immediate reinforcement, Helpful Hints help students avoid common trouble areas and mistakes.
- **Concept Checks** This feature allows students to gauge their grasp of an idea as it is being presented in the text. Concept Checks stress conceptual understanding at the point-of-use and help suppress misconceived notions before they start. Answers appear at the bottom of the page. Exercises related to Concept Checks are included in the exercise sets.
- **Mixed Practice Exercises** In the section exercise sets, these exercises require students to determine the problem type and strategy needed to solve it just as they would need to do on a test.

- **Integrated Reviews** This unique mid-chapter exercise set helps students assimilate new skills and concepts that they have learned separately over several sections. These reviews provide yet another opportunity for students to work with "mixed" exercises as they master the topics.
- **Vocabulary Check** This feature provides an opportunity for students to become more familiar with the use of mathematical terms as they strengthen their verbal skills. These appear at the end of each chapter before the Chapter Highlights.
- **Vocabulary, Readiness & Video Check Questions** are assignable for each section of the text and in MyLab Math. **Vocabulary** exercises check student understanding of new terms. The **Readiness** exercises center on a student's understanding of a concept that is necessary in order to continue to the exercise set. The **Video Check questions** correlate to the videos in MyLab Math, and are a great way to assess whether students have viewed and understood the key concepts presented in the videos. Answers to all Video Check questions are available in an answer section at the back of the text.
- **Chapter Highlights** Found at the end of every chapter, these contain key definitions and concepts with examples to help students understand and retain what they have learned and help them organize their notes and study for tests.
- **Chapter Review** The end of every chapter contains a comprehensive review of topics introduced in the chapter. The Chapter Review offers exercises keyed to every section in the chapter, as well as Mixed Review exercises that are not keyed to sections.
- **Chapter Test and Chapter Test Prep Videos** The Chapter Test is structured to include those problems that involve common student errors. The **Chapter Test Prep Videos** gives students instant access to a step-by-step video solution of each exercise in the Chapter Test.
- **Cumulative Review** This review follows every chapter in the text (except Chapter 1). Each odd-numbered exercise contained in the Cumulative Review is an earlier worked example in the text that is referenced in the back of the book along with the answer.
- **Writing Exercises** ✎ These exercises occur in almost every exercise set and require students to provide a written response to explain concepts or justify their thinking.
- **Applications** Real-world and real-data applications have been thoroughly updated, and many new applications are included. These exercises occur in almost every exercise set and show the relevance of mathematics and help students gradually and continuously develop their problem-solving skills.
- **Review Exercises** These exercises occur in each exercise set (except in Chapter 1) and are keyed to earlier sections. They review concepts learned earlier in the text that will be needed in the next section or chapter.
- **Exercise Set Resource Icons** Located at the opening of each exercise set, these icons remind students of the resources available for extra practice and support:

MyLab Math ▶

See Student Resources descriptions on page xiii for details on the individual resources available.

- **Exercise Icons** These icons facilitate the assignment of specialized exercises and let students know what resources can support them.

  ▶ Video icon: exercise worked on the Interactive Lecture Series.

  △ Triangle icon: identifies exercises involving geometric concepts.

  ✎ Pencil icon: indicates a written response is needed.

  🖩 Calculator icon: optional exercises intended to be solved using a scientific or graphing calculator.

- **Group Activities** Found at the end of each chapter, these activities are for individual or group completion, and are usually hands-on or data-based activities that extend the concepts found in the chapter, allowing students to make decisions and interpretations and to think and write about algebra.

- **Optional: Calculator Exploration Boxes and Calculator Exercises** The optional Calculator Explorations provide keystrokes and exercises at appropriate points to give students an opportunity to become familiar with these tools. Section exercises that are best completed by using a calculator are identified by 🖩 for ease of assignment.

- **The Video Organizer** workbook is designed to help students take notes and work practice exercises while watching the Interactive Lecture Series videos in MyLab Math, making it easy for students to create a course notebook and build good study habits.
    - Covers all of the video examples in order.
    - Provides ample space for students to write down key definitions and properties.
    - Includes "Play" and "Pause" button icons to prompt students to follow along with the author for some exercises while they try others on their own.

The Video Organizer is available in a loose-leaf, notebook-ready format, or can be downloaded from the MyLab Math course.

- **Interactive Lecture Series,** featuring Elayn Martin-Gay, provides students with learning at their own pace. The videos offer the following resources and more:
    - **A complete lecture for each section of the text** highlights key examples and exercises from the text. "Pop-ups" reinforce key terms, definitions, and concepts.
    - **An interface with menu navigation features** allows students to quickly find and focus on the examples and exercises they need to review.
    - **Interactive Concept Check** exercises measure students' understanding of key concepts and common trouble spots.
    - **Student Success Tip Videos** are in short segments designed to be daily reminders to be organized and to study.
    - **The Chapter Test Prep Videos** help students during their most teachable moment—when they are preparing for a test. This innovation provides step-by-step solutions for the exercises found in each Chapter Test.
    - **The Practice Final Exam Videos** help students prepare for an end-of-course final. Students can watch full video solutions to each exercise in the Practice Final Exam at the end of this text.

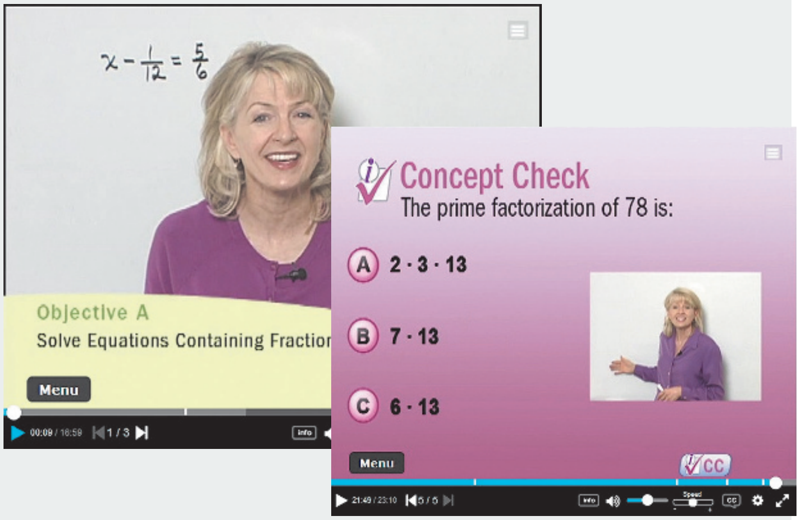
Resources for Success

Get the Most Out of MyLab Math for *Basic College Mathematics with Early Integers,* Fourth Edition by Elayn Martin-Gay

Elayn Martin-Gay believes that every student can succeed, and every MyLab course that accompanies her texts is infused with her student-centric approach. The seamless integration of Elayn's signature support with the #1 choice in digital learning for developmental math gives students a completely consistent experience from print to MyLab.

A Comprehensive and Dynamic Video Program

The **Martin-Gay video program** is 100% presented by Elayn Martin-Gay in her clear, approachable style. The video program includes full section lectures and smaller objective level videos. Within many section lecture videos, **Interactive Concept Checks** measure students' understanding of concepts and common trouble spots—students are asked to try a question within the video in order, after which correct and incorrect answers are explained.



$x - \frac{1}{12} = \frac{5}{6}$

Objective A

Solve Equations Containing Fractio

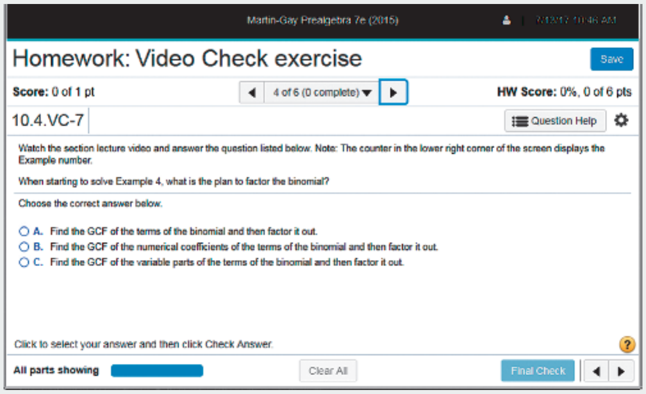
Concept Check

The prime factorization of 78 is:

Ⓐ $2 \cdot 3 \cdot 13$

Ⓑ $7 \cdot 13$

Ⓒ $6 \cdot 13$



Martin-Gay Prealgebra 7e (2015)

Homework: Video Check exercise

Score: 0 of 1 pt

4 of 6 (0 complete)

HW Score: 0%, 0 of 6 pts

10.4.VC-7

Watch the section lecture video and answer the question listed below. Note: The counter in the lower right corner of the screen displays the Example number.

When starting to solve Example 4, what is the plan to factor the binomial?

Choose the correct answer below.

○ A. Find the GCF of the terms of the binomial and then factor it out.
○ B. Find the GCF of the numerical coefficients of the terms of the binomial and then factor it out.
○ C. Find the GCF of the variable parts of the terms of the binomial and then factor it out.

Click to select your answer and then click Check Answer.

All parts showing

Assignable **Video Check questions** ensure that students have viewed and understood the key concepts from the section lecture videos.

Supporting College Success

Other hallmark Martin-Gay videos include **Student Success Tip videos**, which are short segments designed to be daily reminders to stay organized and to study. Additionally in keeping with Elayn's belief that every student can succeed, a new **Mindset module** is available in the course, with mindset-focused videos and exercises that encourage students to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as a learning opportunity.

pearson.com/mylab/math

Resources for Success

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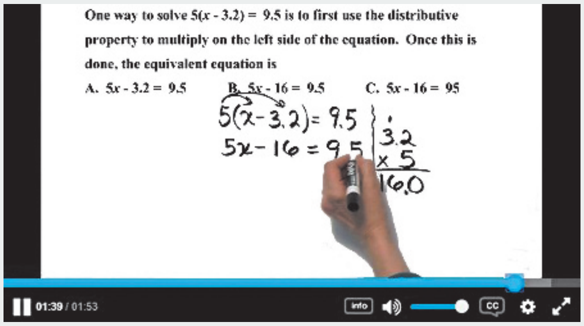
Resources for Review

New! Getting Ready for the Test video solutions cover every Getting Ready for the Test exercise. These appear at the end of each chapter and give students an opportunity to assess whether they understand the big picture concepts of the chapter, and help them focus on avoiding common errors. Students also have **Chapter Test Prep videos**, a Martin-Gay innovation, to help during their most teachable moment —when preparing for a test.



One way to solve 5(x - 3.2) = 9.5 is to first use the distributive property to multiply on the left side of the equation. Once this is done, the equivalent equation is

A. 5x - 3.2 = 9.5 B. 5x - 16 = 9.5 C. 5x - 16 = 95

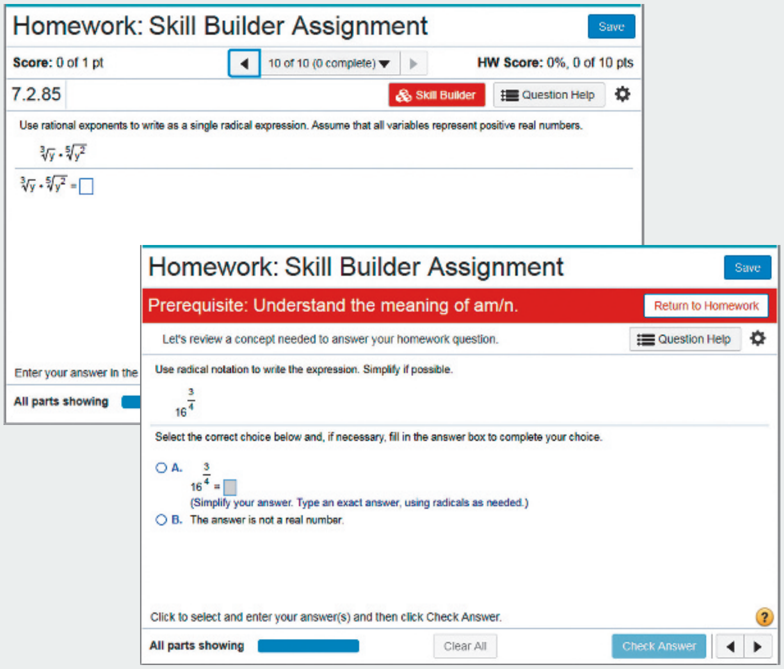
$5(x-3.2)=9.5$

$5x-16=9.5$

$\begin{array}{r} 3.2 \\ \times\ 5 \\ \hline 16.0 \end{array}$

Personalize Learning

New! Skill Builder exercises offer just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding.This new feature allows instructors to assign fewer questions for homework, allowing students to complete as many or as few questions as they need.



Homework: Skill Builder Assignment

Score: 0 of 1 pt

10 of 10 (0 complete)

HW Score: 0%, 0 of 10 pts

7.2.85

Use rational exponents to write as a single radical expression. Assume that all variables represent positive real numbers.

$\sqrt[3]{y}\cdot\sqrt[5]{y^2}$

$\sqrt[3]{y}\cdot\sqrt[5]{y^2}=\square$

Enter your answer in the

All parts showing

Homework: Skill Builder Assignment

Prerequisite: Understand the meaning of am/n.

Let's review a concept needed to answer your homework question.

Use radical notation to write the expression. Simplify if possible.

$16^{\frac{3}{4}}$

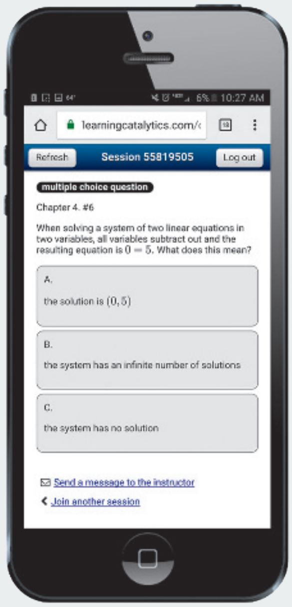
Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ A. $16^{\frac{3}{4}}=\square$
(Simplify your answer. Type an exact answer, using radicals as needed.)

○ B. The answer is not a real number.

Click to select and enter your answer(s) and then click Check Answer.

All parts showing



learningcatalytics.com/

Session 55819505

multiple choice question

Chapter 4. #6

When solving a system of two linear equations in two variables, all variables subtract out and the resulting equation is $0 = 5$. What does this mean?

A.
the solution is $(0, 5)$

B.
the system has an infinite number of solutions

C.
the system has no solution

Send a message to the instructor

Join another session

Get Students Engaged

New! Learning Catalytics Martin-Gay-specific questions are pre-built and available through MyLab Math. Learning Catalytics is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. **Getting Ready for the Test** exercises marked in blue in the text are pre-built in Learning Catalytics to use in class. These questions can be found in Learning Catalytics by searching for "MGBCMEI#" where # is the chapter number.

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Resources for Success

Instructor Resources

Annotated Instructor's Edition

Contains all the content found in the student edition, plus answers to even and odd exercises on the same text page, and Teaching Tips throughout the text placed at key points.

The resources below are available through Pearson's Instructor Resource Center, or from MyLab Math.

Instructor's Resource Manual with Tests and Mini-Lectures

Includes mini-lectures for each text section, additional practice worksheets for each section, several forms of tests per chapter—free response and multiple choice, and answers to all items.

Instructor's Solutions Manual

Contains detailed, worked-out solutions to even-numbered exercises in the text.

TestGen®

Enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.

Instructor-to-Instructor Videos

Provide instructors with suggestions for presenting course material as well as time-saving teaching tips.

PowerPoint Lecture Slides

Available for download only, these slides present key concepts and definitions from the text.

Student Resources

Video Organizer

Designed to help students take notes and work practice exercises while watching the Interactive Lecture Series videos.

- Covers all of the video examples in order.
- Provides prompts with ample space for students to write down key definitions and rules.
- Includes "Play" and "Pause" button icons to prompt students to follow along with the author for some exercises while they try others on their own.
- Includes Student Success Tips Outline and Questions.

Available printed in a loose-leaf, notebook-ready format and to download in MyLab Math. All answers are available in Instructor Resources in MyLab Math.

New! Key Concept Activity Lab Workbook

Includes Extension Exercises, Exploration Activities, Conceptual Exercises, and Group Activities. This workbook is available in MyLab Math, or can be packaged in printed form with a text or MyLab Math code. All answers available in Instructor Resources in MyLab Math.

Student Solutions Manual

Provides completely worked-out solutions to the odd-numbered section exercises; all exercises in the Integrated Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews.

pearson.com/mylab/math

Acknowledgments

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Elayn Martin-Gay

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About the Author

Elayn Martin-Gay has taught mathematics at the University of New Orleans for more than 25 years. Her numerous teaching awards include the local University Alumni Association's Award for Excellence in Teaching, and Outstanding Developmental Educator at University of New Orleans, presented by the Louisiana Association of Developmental Educators.

Prior to writing textbooks, Elayn Martin-Gay developed an acclaimed series of lecture videos to support developmental mathematics students in their quest for success. These highly successful videos originally served as the foundation material for her texts. Today, the videos are specific to each book in the Martin-Gay series. The author has also created Chapter Test Prep Videos to help students during their most "teachable moment"—as they prepare for a test—along with Instructor-to-Instructor videos that provide teaching tips, hints, and suggestions for each developmental mathematics course, including basic mathematics, prealgebra, beginning algebra, and intermediate algebra.

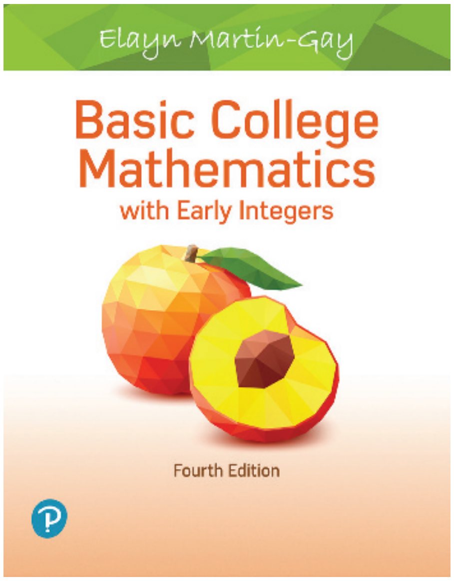
Elayn is the author of 13 published textbooks, and a new Interactive Assignment MyLab Math course, all specializing in developmental mathematics courses. She has also published series in Algebra 1, Algebra 2, and Geometry. She has participated as an author across the broadest range of educational materials: textbooks, videos, tutorial software, and courseware. This provides an opportunity of various combinations for an integrated teaching and learning package offering great consistency for the student.

Applications Index

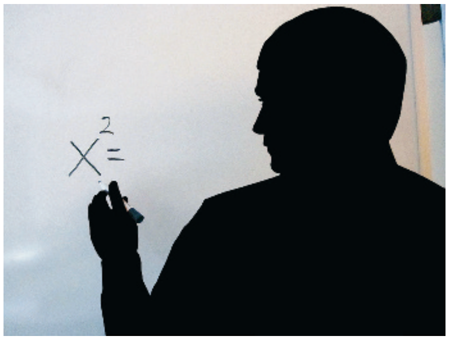
The Whole Numbers

1

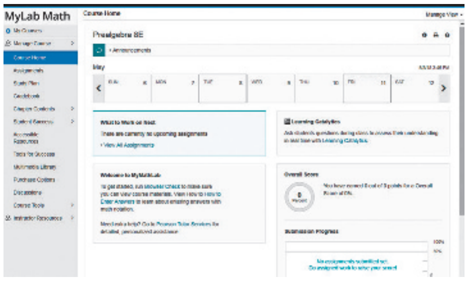
A Selection of Resources for Success in This Mathematics Course



Textbook



Instructor



MyLab Math and MathXL

Video Organizer

Interactive Lecture Series

For more information about the resources illustrated above, read Section 1.1.

Whole numbers are the basic building blocks of mathematics. The whole numbers answer the question "How many?"

This chapter covers basic operations on whole numbers. Knowledge of these operations provides a good foundation on which to build further mathematical skills.

Sections

Check Your Progress

1.1 Study Skill Tips for Success in Mathematics

Objectives

A Get Ready for This Course.

B Understand Some General Tips for Success.

C Know How to Use This Text.

D Know How to Use Text Resources.

E Get Help as Soon as You Need It.

F Learn How to Prepare for and Take an Exam.

G Develop Good Time Management.

Before reading this section, ask yourself a few questions.

1. Were you satisfied—really satisfied—with your performance in your last math course? In other words, do you feel that your outcome represented your best effort?
2. When you took your last math course, were your notes and materials from that course organized and easy to find, or were they disorganized and hard to find—if you saved them at all?

If the answer is "no" to these questions, then it is time to make a change. To begin, continue reading this section.

Objective A Let's Get Ready for This Course

1. *Start with a Positive Attitude.* 🙂

Now that you have decided to take this course, remember that a *positive attitude* will make all the difference in the world. Your belief that you can succeed is just as important as your commitment to this course. Make sure you are ready for this course by having the time and positive attitude that it takes to succeed.

2. *Understand How Your Course Material Is Presented—Lecture by Instructor, Online with Computer, or Both?*

Make sure that you are familiar with the way that this course is being taught. Is it a traditional course, in which you have a printed textbook and meet with an instructor? Is it taught totally online, and your textbook is electronic and you e-mail your instructor? Or is your course structured somewhere in between these two methods? (Not all of the tips that follow will apply to all forms of instruction.)

3. *Schedule Your Class So That It Does Not Interfere with Other Commitments.*

Make sure that you have scheduled your math course for a time that will give you the best chance for success. For example, if you are also working, you may want to check with your employer to make sure that your work hours will not conflict with your course schedule.

Objective B Here Are a Few General Tips for Success

Below are some general tips that will increase your chance for success in a mathematics class. Many of these tips will also help you in other courses you may be taking.

1. *Most Important! Organize Your Class Materials. Unless Told Otherwise, Use a 3-Ring Binder Solely for Your Mathematics Class.*

Helpful Hint

MyLab Math and MathXL When assignments are turned in online, keep a hard copy of your complete written work. You will need to refer to your written work to be able to ask questions and to study for tests later.

In the next couple pages, many ideas will be presented to help you organize your class materials—notes, any handouts, completed homework, previous tests, etc. In general, you MUST have these materials organized. All of them will be valuable references throughout your course and when studying for upcoming tests and the final exam. One way to make sure you can locate these materials when you need them is to use a three-ring binder. This binder should be used solely for your mathematics class and should be brought to each and every class or lab. This way, any material can be immediately inserted in a section of this binder and will be there when you need it.

2. *Choose to Attend All Class Periods.*

If possible, sit near the front of the classroom. This way, you will see and hear the presentation better. It may also be easier for you to participate in classroom activities.

3. *Complete Your Homework. This Means: Attempt All of It, Check All of It, Correct Any Mistakes, and Ask for Help If Needed.*

You've probably heard the phrase "practice makes perfect" in relation to music and sports. It also applies to mathematics. You will find that the more time you spend solving mathematics exercises, the easier the process becomes. Be sure to schedule enough time to complete your assignments before the due date assigned by your instructor.

Review the steps you took while working a problem. Learn to check your answers in the original exercises. You may also compare your answers with the "Answers to Selected Exercises" section in the back of the book. If you have made a mistake, try to figure out what went wrong. Then correct your mistake. If you can't find what went wrong, **don't** erase your work or throw it away. Show your work to your instructor, a tutor in a math lab, or a classmate. It is easier for someone to find where you had trouble if he or she looks at your original work.

It's all right to ask for help. In fact, it's a good idea to ask for help whenever there is something that you don't understand. Make sure you know when your instructor has office hours and how to find his or her office. Find out whether math tutoring services are available on your campus. Check on the hours, location, and requirements of the tutoring service.

Helpful Hint

MyLab Math and MathXL
If you are doing your homework online, you can work and rework those exercises that you struggle with until you master them. Try working through all the assigned exercises twice before the due date.

Helpful Hint

MyLab Math and MathXL
If you are completing your homework online, it's important to work each exercise on paper before submitting the answer. That way, you can check your work and follow your steps to find and correct any mistakes.

4. *Learn from Your Mistakes, and Be Patient with Yourself.*

Everyone, even your instructor, makes mistakes. (That definitely includes me—Elayn Martin-Gay.) Use your errors to learn and to become a better math student. The key is finding and understanding your errors.

Was your mistake a careless one, or did you make it because you can't read your own math writing? If so, try to work more slowly or write more neatly and make a conscious effort to carefully check your work.

Did you make a mistake because you don't understand a concept? Take the time to review the concept or ask questions to better understand it.

Did you skip too many steps? Skipping steps or trying to do too many steps mentally may lead to preventable mistakes.

5. *Turn In Assignments on Time.*

This way, you can be sure that you will not lose points for being late. Show every step of a problem and be neat and organized. Also be sure that you understand which problems are assigned for homework. If allowed, you can always double-check the assignment with another student in your class.

Helpful Hint

MyLab Math and MathXL
Be aware of assignments and due dates set by your instructor. Don't wait until the last minute to submit work online.

Objective C Knowing and Using Your Text or e-Text

Flip through the pages of this text or view the e-text pages on a computer screen. Start noticing examples, exercise sets, end-of-chapter material, and so on. Learn the way this text is organized by finding an example in your text of each type of resource listed below. Finding and using these resources throughout your course will increase your chance of success.

- ***Practice Exercises.*** Each example in every section has a parallel Practice exercise. Work each Practice exercise after you've finished the corresponding example. Answers are at the bottom of the page. This "learn-by-doing" approach will help you grasp ideas before you move on to other concepts.
- ***Objectives.*** Every section of this text is divided into objectives, such as A or B. They are listed at the beginning of the section and noted in that section. The main section of exercises in each exercise set is also referenced by an objective, such as A or B, and also an example(s). There is also often a section of exercises entitled "Mixed Practice," which is referenced by two or more objectives or sections. These are mixed exercises written to prepare you for your next exam. Use all of this referencing if you have trouble completing an assignment from the exercise set.

- ***Icons (Symbols).*** Make sure that you understand the meaning of the icons that are beside many exercises. ▶ tells you that the corresponding exercise may be viewed on the video Lecture Series that corresponds to that section. ✏ tells you that this exercise is a writing exercise in which you should answer in complete sentences. △ tells you that the exercise involves geometry.
- ***Integrated Reviews.*** Found in the middle of each chapter, these reviews offer you a chance to practice—in one place—the many concepts that you have learned separately over several sections.
- ***End-of-Chapter Opportunities.*** There are many opportunities at the end of each chapter to help you understand the concepts of the chapter.

  Vocabulary Checks contain key vocabulary terms introduced in the chapter.

  Chapter Highlights contain chapter summaries and examples.

  Chapter Reviews contain review problems. The first part is organized section by section and the second part contains a set of mixed exercises.

  Getting Ready for the Tests are multiple choice or matching exercises designed to check your knowledge of chapter concepts, before you attempt the chapter test. Video solutions are available for all these exercises.

  Chapter Tests are sample tests to help you prepare for an exam. The Chapter Test Prep Videos found in MyLab Math and YouTube provide the video solution to each question on each Chapter Test.

  Cumulative Reviews start at Chapter 2 and are reviews consisting of material from the beginning of the book to the end of that particular chapter.

- ***Student Resources in Your Textbook.*** You will find a **Student Resources** section at the back of this textbook. It contains the following to help you study and prepare for tests:

  Study Skill Builders contain study skills advice. To increase your chance for success in the course, read these study tips, and answer the questions.

  Bigger Picture—Study Guide Outline provides you with a study guide outline of the course, with examples.

  Practice Final provides you with a Practice Final Exam to help you prepare for a final.

- ***Resources to Check Your Work.*** The **Answers to Selected Exercises** section provides answers to all odd-numbered section exercises and to all integrated review, chapter review, getting ready for the test, chapter test, and cumulative review exercises. Use the **Solutions to Selected Exercises** to see the worked-out solution to every other odd-numbered exercise in the section exercises and chapter tests.

Objective D Knowing and Using Video and Notebook Organizer Resources ▶

Video Resources

Below is a list of video resources that are all made by me—the author of your text, Elayn Martin-Gay. By making these videos, I can be sure that the methods presented are consistent with those in the text. All video resources may be found in MyLab Math and some also on YouTube.

- ***Interactive Video Lecture Series.*** Exercises marked with a ▶ are fully worked out by the author. The lecture series provides approximately 20 minutes of instruction per section and is organized by objective.
- ***Getting Ready for the Test Videos.*** These videos provide solutions to all of the Getting Ready for the Test exercises.

Helpful Hint

MyLab Math

In MyLab Math, you have access to the following video resources:

- Lecture Videos for each section
- Getting Ready for the Test Videos
- Chapter Test Prep Videos
- Final Exam Videos

Use these videos provided by the author to prepare for class, review, and study for tests.

- ***Chapter Test Prep Videos.*** These videos provide solutions to all of the Chapter Test exercises worked out by the author. They can be found in MyLab Math and YouTube. This supplement is very helpful before a test or exam.
- ***Tips for Success in Mathematics.*** These video segments are about 3 minutes long and are daily reminders to help you continue practicing and maintaining good organizational and study habits.
- ***Final Exam Videos.*** These video segments provide solutions to each question.

Video Organizer

This organizer is in three-ring notebook ready form. It is to be inserted in a three-ring binder and completed. This organizer is numbered according to the sections in your text to which it refers.

It is closely tied to the Interactive (Video) Lecture Series. Each section should be completed while watching the lecture video on the same section. Once completed, you will have a set of notes to accompany the (Video) Lecture Series section by section.

Objective E Getting Help

If you have trouble completing assignments or understanding the mathematics, get help as soon as you need it! This tip is presented as an objective on its own because it is so important. In mathematics, usually the material presented in one section builds on your understanding of the previous section. This means that if you don't understand the concepts covered during a class period, there is a good chance that you will not understand the concepts covered during the next class period. If this happens to you, get help as soon as you can.

Where can you get help? Try your instructor, a tutoring center, or a math lab, or you may want to form a study group with fellow classmates. If you do decide to see your instructor or go to a tutoring center, make sure that you have a neat notebook and are ready with your questions.

Helpful Hint

MyLab Math and MathXL

- Use the **Help Me Solve This** button to get step-by-step help for the exercise you are working. You will need to work an additional exercise of the same type before you can get credit for having worked it correctly.
- Use the **Video** button to view a video clip of the author working a similar exercise.

Objective F Preparing for and Taking an Exam

Make sure that you allow yourself plenty of time to prepare for a test. If you think that you are a little "math anxious," it may be that you are not preparing for a test in a way that will ensure success. The way that you prepare for a test in mathematics is important. To prepare for a test:

1. Review your previous homework assignments.
2. Review any notes from class and section-level quizzes you have taken. (If this is a final exam, also review chapter tests you have taken.)
3. Review concepts and definitions by reading the Chapter Highlights at the end of each chapter.
4. Practice working out exercises by completing the Chapter Review found at the end of each chapter. (If this is a final exam, go through a Cumulative Review. There is one found at the end of each chapter except Chapter 1. Choose the review found at the end of the latest chapter that you have covered in your course.) *Don't stop here!*
5. Take the Chapter Getting Ready for the Test. All answers to these exercises are available to you as well as video solutions.
6. Take a sample test with no notes, etc, available for help. It is important that you place yourself in conditions similar to test conditions to find out how you

Helpful Hint

MyLab Math and MathXL Review your written work for previous assignments. Then, go back and re-work previous assignments. Open a previous assignment, and click **Similar Exercise** to generate new exercises. Re-work the exercises until you fully understand them and can work them without help features.

will perform. There is a Chapter Test available at the end of each chapter, or you can work selected problems from the Chapter Review. Your instructor may also provide you with a review sheet. Then check your sample test. If your sample test is the Chapter Test in the text, don't forget that the video solutions are in MyLab Math and YouTube.

7. On the day of the test, allow yourself plenty of time to arrive at where you will be taking your exam.

When taking your test:

1. Read the directions on the test carefully.
2. Read each problem carefully as you take the test. Make sure that you answer the question asked.
3. Watch your time and pace yourself so that you can attempt each problem on your test.
4. If you have time, check your work and answers.
5. Do not turn your test in early. If you have extra time, spend it double-checking your work.

Objective G Managing Your Time

As a college student, you know the demands that classes, homework, work, and family place on your time. Some days you probably wonder how you'll ever get everything done. One key to managing your time is developing a schedule. Here are some hints for making a schedule:

1. Make a list of all of your weekly commitments for the term. Include classes, work, regular meetings, extracurricular activities, etc. You may also find it helpful to list such things as laundry, regular workouts, grocery shopping, etc.
2. Next, estimate the time needed for each item on the list. Also make a note of how often you will need to do each item. Don't forget to include time estimates for the reading, studying, and homework you do outside of your classes. You may want to ask your instructor for help estimating the time needed.
3. In the exercise set that follows, you are asked to block out a typical week on the schedule grid given. Start with items with fixed time slots like classes and work.
4. Next, include the items on your list with flexible time slots. Think carefully about how best to schedule items such as study time.
5. Don't fill up every time slot on the schedule. Remember that you need to allow time for eating, sleeping, and relaxing! You should also allow a little extra time in case some items take longer than planned.
6. If you find that your weekly schedule is too full for you to handle, you may need to make some changes in your workload, classload, or other areas of your life. You may want to talk to your advisor, manager or supervisor at work, or someone in your college's academic counseling center for help with such decisions.

1.1 Exercise Set MyLab Math ▶

1. What is your instructor's name?

2. What are your instructor's office location and office hours?

3. What is the best way to contact your instructor?

4. Do you have the name and contact information of at least one other student in class?

5. Will your instructor allow you to use a calculator in this class?

6. Why is it important that you write step-by-step solutions to homework exercises and keep a hard copy of all work submitted?

7. Is there a tutoring service available on campus? If so, what are its hours? What services are available?

8. Have you attempted this course before? If so, write down ways that you might improve your chances of success during this attempt.

9. List some steps that you can take if you begin having trouble understanding the material or completing an assignment. If you are completing your homework in MyLab Math and MathXL, list the resources you can use for help.

10. How many hours of studying does your instructor advise for each hour of instruction?

11. What does the ✏ icon in this text mean?

12. What does the △ icon in this text mean?

13. What does the ▶ icon in this text mean?

14. Search the minor columns in your text. What are Practice exercises?

15. When might be the best time to work a Practice exercise?

16. Where are the answers to Practice exercises?

17. What answers are contained in this text and where are they?

18. What are Tips for Success in Mathematics and where are they located?

19. What and where are Integrated Reviews?

20. How many times is it suggested that you work through the homework exercises in MyLab Math or MathXL before the submission deadline?

21. How far in advance of the assigned due date is it suggested that homework be submitted online? Why?

22. Chapter Highlights are found at the end of each chapter. Find the Chapter 1 Highlights and explain how you might use it and how it might be helpful.

23. Chapter Reviews are found at the end of each chapter. Find the Chapter 1 Review and explain how you might use it and how it might be helpful.

24. Chapter Tests are found at the end of each chapter. Find the Chapter 1 Test and explain how you might use it and how it might be helpful when preparing for an exam on Chapter 1. Include how the Chapter Test Prep Videos may help. If you are working in MyLab Math and MathXL, how can you use previous homework assignments to study?

25. What is the Video Organizer? Explain the contents and how it might be used.

26. Read or reread objective **G** and fill out the schedule grid on the next page.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
4:00 a.m.							
5:00 a.m.							
6:00 a.m.							
7:00 a.m.							
8:00 a.m.							
9:00 a.m.							
10:00 a.m.							
11:00 a.m.							
12:00 p.m.							
1:00 p.m.							
2:00 p.m.							
3:00 p.m.							
4:00 p.m.							
5:00 p.m.							
6:00 p.m.							
7:00 p.m.							
8:00 p.m.							
9:00 p.m.							
10:00 p.m.							
11:00 p.m.							
Midnight							
1:00 a.m.							
2:00 a.m.							
3:00 a.m.							

1.2 Place Value, Names for Numbers, and Reading Tables

Objectives

A Find the Place Value of a Digit in a Whole Number.

B Write a Whole Number in Words and in Standard Form.

C Write a Whole Number in Expanded Form.

D Read Tables.

The **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 can be used to write numbers. For example, the **whole numbers** are

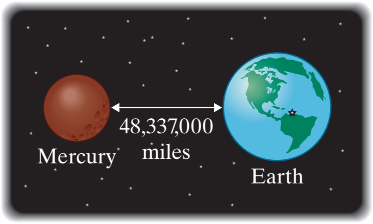
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .

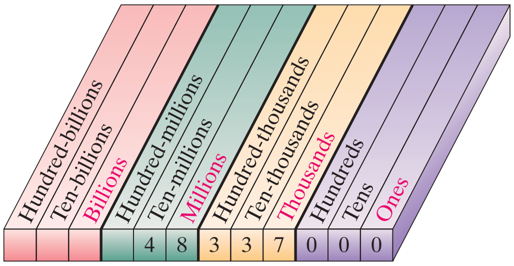
and the **natural numbers** are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .

The three dots (. . .) after each 11 means that these lists continue indefinitely. That is, there is no largest whole number. The smallest whole number is 0. Also, there is no largest natural number. The smallest natural number is 1.

Objective A Finding the Place Value of a Digit in a Whole Number

The position of each digit in a number determines its **place value.** For example, the distance (in miles) between the planet Mercury and the planet Earth can be represented by the whole number 48,337,000. Next is a place-value chart for this whole number.





The two 3s in 48,337,000 represent different amounts because of their different placements. The place value of the 3 on the left is hundred-thousands. The place value of the 3 on the right is ten-thousands.

Examples Find the place value of the digit 3 in each whole number.

1. 396,418
↑
hundred-thousands

2. 93,192
↑
thousands

3. 534,275,866
↑
ten-millions

Work Practice 1–3

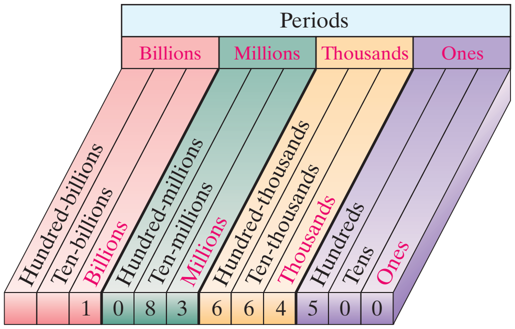
Practice 1–3

Find the place value of the digit 8 in each whole number.

1. 38,760,005
2. 67,890
3. 481,922

Objective B Writing a Whole Number in Words and in Standard Form

A whole number such as 1,083,664,500 is written in **standard form.** Notice that commas separate the digits into groups of three, starting from the right. Each group of three digits is called a **period.** The names of the first four periods are shown in red.



Writing a Whole Number in Words

To write a whole number in words, write the number in each period followed by the name of the period. (The ones period is usually not written.) This same procedure can be used to read a whole number.

For example, we write 1,083,664,500 as

one **billion,**
eighty-three **million,**
six hundred sixty-four **thousand,**
five hundred

Helpful Hint Notice the commas after the name of each period.

Answers
1. millions **2.** hundreds
3. ten-thousands

Helpful Hint

The name of the ones period is not used when reading and writing whole numbers. For example,

9,265

is read as

"nine **thousand,** two hundred sixty-five."

Practice 4–6

Write each number in words.

4. 67

5. 395

6. 12,804

Examples Write each number in words.

4.	85	eighty-five
5.	126	one hundred twenty-six
6.	27,034	twenty-seven thousand, thirty-four

Work Practice 4–6

Helpful Hint The word "and" is *not* used when reading and writing whole numbers. It is used when reading and writing mixed numbers and some decimal values, as shown later in this text.

Practice 7

Write 321,670,200 in words.

Example 7 Write 106,052,447 in words.

Solution: 106,052,447 is written as

one hundred six **million**, fifty-two **thousand**, four hundred forty-seven

Work Practice 7

✓Concept Check True or false? When writing a check for $2600, the word name we write for the dollar amount of the check is "two thousand sixty." Explain your answer.

Practice 8–11

Write each number in standard form.

8. twenty-nine

9. seven hundred ten

10. twenty-six thousand, seventy-one

11. six million, five hundred seven

Writing a Whole Number in Standard Form

To write a whole number in standard form, write the number in each period, followed by a comma.

Examples Write each number in standard form.

8. sixty-one 61 **9.** eight hundred five 805

10. nine thousand, three hundred eighty-six

9,386 or 9386

11. two million, five hundred sixty-four thousand, three hundred fifty

2,564,350

Work Practice 8–11

Answers

4. sixty-seven **5.** three hundred ninety-five **6.** twelve thousand, eight hundred four **7.** three hundred twenty-one million, six hundred seventy thousand, two hundred **8.** 29 **9.** 710 **10.** 26,071 **11.** 6,000,507

✓Concept Check Answer

false

Helpful Hint

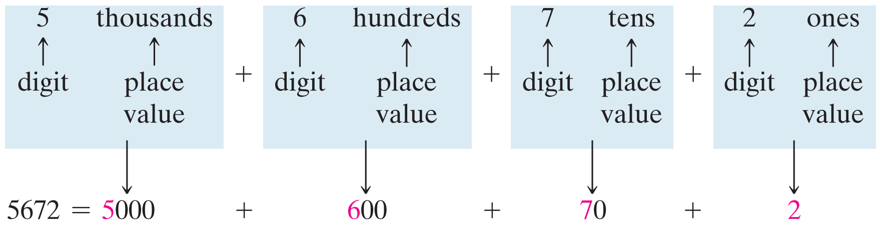
A comma may or may not be inserted in a four-digit number. For example, both

9,386 and 9386

are acceptable ways of writing nine thousand, three hundred eighty-six.

Objective C Writing a Whole Number in Expanded Form

The place value of a digit can be used to write a number in expanded form. The **expanded form** of a number shows each digit of the number with its place value. For example, 5672 is written in expanded form as



Example 12 Write 2,706,449 in expanded form.

Solution: $2{,}000{,}000 + 700{,}000 + 6000 + 400 + 40 + 9$

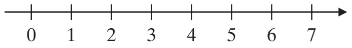
Work Practice 12

Practice 12

Write 1,047,608 in expanded form.

We can visualize whole numbers by points on a line. The line below is called a **number line.** This number line has equally spaced marks for each whole number. The arrow to the right simply means that the whole numbers continue indefinitely. In other words, there is no largest whole number.

Number Line



We will study number lines further in Section 1.5.

Objective D Reading Tables

Now that we know about place value and names for whole numbers, we introduce one way that whole numbers may be presented. **Tables** are often used to organize and display facts that involve numbers. The table on the next page shows the ten countries with the most Nobel Prize winners since the inception of the Nobel Prize in 1901, and the categories of the prizes. The numbers for the Economics prize reflect the winners since 1969, when this category was established. (The numbers may seem large because the annual Nobel Prize is often awarded to more than one individual.)

Answer

12. $1{,}000{,}000 + 40{,}000 + 7000 + 600 + 8$

Most Nobel Prize Winners by Country of Birth, 1901–2016

Country	Chemistry	Economics	Literature	Peace	Physics	Physiology & Medicine	Total
United States	52	43	9	19	66	70	259
United Kingdom	24	8	7	11	23	26	99
Germany	24	1	7	5	23	17	77
France	9	4	11	10	9	12	55
Sweden	4	2	7	5	4	7	29
Russia (USSR)	3	2	5	2	11	2	25
Japan	6	0	2	1	11	4	24
Canada	4	3	2	1	4	4	18
Netherlands	4	2	0	1	9	2	18
Italy	1	1	5	0	5	5	17

Source: Nobelprize.org

For example, by reading from left to right along the row marked “United States,” we find that the United States is the birthplace of 52 Chemistry, 43 Economics, 9 Literature, 19 Peace, 66 Physics, and 70 Physiology and Medicine Nobel Prize winners.

Example 13 Use the Nobel Prize Winner table to answer each question.

a. How many total Nobel Prize winners were born in Sweden?

b. Which countries shown have fewer Nobel Prize winners than Russia?

Solution:

a. Find “Sweden” in the left column. Then read from left to right until the “Total” column is reached. We find that 29 Nobel Prize winners were born in Sweden.

b. There are 25 Russian-born Nobel Prize winners. Japan has 24, Canada has 18, Netherlands has 18, and Italy has 17, so they have fewer Nobel Prize winners.

Work Practice 13

Practice 13

Use the Nobel Prize Winner table to answer the following questions:

a. How many Nobel Prize winners in Literature were born in France?

b. Which countries shown have more than 60 Nobel Prize winners?

Answers

13. a. 11 **b.** United States, United Kingdom, and Germany

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

standard form	period	whole
expanded form	place value	words

1. The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . . are called ____________ numbers.

2. The number 1,286 is written in ____________.

3. The number "twenty-one" is written in ____________.

4. The number 900 + 60 + 5 is written in ____________.

5. In a whole number, each group of three digits is called a(n) ____________.

6. The ____________ of the digit 4 in the whole number 264 is ones.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.

See Video 1.2

Objective A **7.** In Example 1, what is the place value of the digit 6?

Objective B **8.** Complete this statement based on Example 3: To read (or write) a number, read from _____ to _____.

Objective C **9.** In Example 5, what is the expanded-form value of the digit 8?

Objective D **10.** Use the table given in Example 6 to determine which mountain in the table is the shortest.

1.2 Exercise Set MyLab Math

Objective A *Determine the place value of the digit 5 in each whole number. See Examples 1 through 3.*

1. 657

2. 905

3. 5423

4. 6527

5. 43,526,000

6. 79,050,000

7. 5,408,092

8. 51,682,700

Objective B *Write each whole number in words. See Examples 4 through 7.*

9. 354

10. 316

11. 8279

12. 5445

13. 26,990

14. 42,009

15. 2,388,000

16. 3,204,000

17. 24,350,185

18. 47,033,107

Write each number in the sentence in words. See Examples 4 through 7.

19. As of March 2017, the population of Iceland was 322,653. (*Source:* livepopulation.com)

20. The land area of Belize is 22,806 square kilometers. (*Source:* CIA World Factbook)

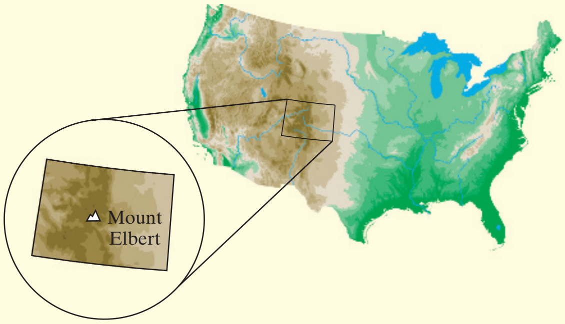
21. The Burj Khalifa, in Dubai, United Arab Emirates, a hotel and office building, is the world's tallest building at a height of 2717 feet. (*Source:* Council on Tall Buildings and Urban Habitat)

22. As of March 2017, there were 118,049 patients in the United States waiting for an organ transplant. (*Source:* Organ Procurement and Transplantation Network)

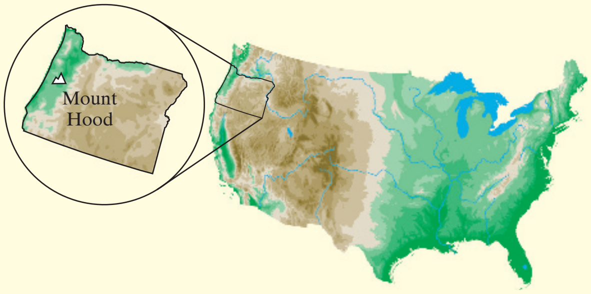
23. In 2016, UPS received an average of 101,500,000 online tracking requests per day. (*Source:* UPS)

24. In 2024, there are projected to be 11,378,000 undergraduate students enrolled in four-year colleges and universities in the United States. (*Source:* Digest of Education Statistics)

25. The highest point in Colorado is Mount Elbert, at an elevation of 14,433 feet. (*Source:* U.S. Geological Survey)



26. The highest point in Oregon is Mount Hood, at an elevation of 11,239 feet. (*Source:* U.S. Geological Survey)



27. In 2016, the Great Internet Mersenne Prime Search, a cooperative computing project, helped find a prime number that has 22,338,618 digits. (*Source:* Mersenne Research, Inc.)

28. The Goodyear blimp *Eagle* holds 202,700 cubic feet of helium. (*Source:* The Goodyear Tire & Rubber Company)

Write each whole number in standard form. See Examples 8 through 11.

29. Six thousand, five hundred eighty-seven

30. Four thousand, four hundred sixty-eight

31. Fifty-nine thousand, eight hundred

32. Seventy-three thousand, two

33. Thirteen million, six hundred one thousand, eleven

34. Sixteen million, four hundred five thousand, sixteen

35. Seven million, seventeen

36. Two million, twelve

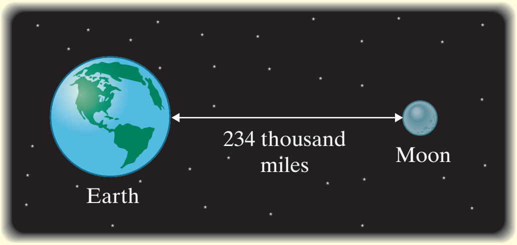
37. Two hundred sixty thousand, nine hundred ninety-seven

38. Six hundred forty thousand, eight hundred eighty-one

Write the whole number in each sentence in standard form. See Examples 8 through 11.

39. After an orbit correction in October 2013, the International Space Station orbited Earth at an average altitude of about four hundred eighteen kilometers. (*Source:* Heavens Above)

40. The average distance between the surfaces of Earth and the Moon is about two hundred thirty-four thousand miles.



41. La Rinconada, Peru, is the highest town in the world. It is located sixteen thousand, seven hundred thirty-two feet above sea level. (*Source:* Russell Ash: *Top 10 of Everything*)

42. The world's tallest freestanding tower is the Tokyo Sky Tree in Japan. Its height is two thousand eighty feet tall. (*Source:* Council on Tall Buildings and Urban Habitat)

43. The Buena Vista film *Star Wars VII: The Force Awakens* holds the record for U.S./Canada opening day box office gross when it took in approximately one hundred nineteen million, one hundred nineteen thousand dollars on its opening day in 2015. (*Source:* Box Office Mojo)

44. The Buena Vista film *Beauty and the Beast* set the U.S./Canada record for highest opening weekend box office gross for a PG film when it took in approximately one hundred seventy-four million, seven hundred fifty-one thousand dollars on its opening day in 2017. (*Source:* Box Office Mojo)

45. In 2017, the UPS delivery fleet consisted of more than one hundred eight thousand vehicles. (*Source:* UPS)

46. Morten Andersen, who played football for New Orleans, Atlanta, N.Y. Giants, Kansas City, and Minnesota between 1982 and 2007, holds the record for the most points scored in a career. Over his 25-year career he scored two thousand, five hundred forty-four points. (*Source:* NFL.com)

Objective C *Write each whole number in expanded form. See Example 12.*

47. 406

48. 789

49. 3470

50. 6040

▶ **51.** 80,774

52. 20,215

53. 66,049

54. 99,032

55. 39,680,000

56. 47,703,029

Objectives B C D Mixed Practice *The table shows the six tallest mountains in New England and their elevations. Use this table to answer Exercises 57 through 62. See Example 13.*

Mountain (State)	Elevation (in feet)
Boott Spur (NH)	5492
Mt. Adams (NH)	5793
Mt. Clay (NH)	5532
Mt. Jefferson (NH)	5712
Mt. Sam Adams (NH)	5584
Mt. Washington (NH)	6288

Source: U.S. Geological Survey

▶ **57.** Write the elevation of Mt. Clay in standard form and then in words.

58. Write the elevation of Mt. Washington in standard form and then in words.

▶ **59.** Write the height of Boott Spur in expanded form.

60. Write the height of Mt. Jefferson in expanded form.

▶ **61.** Which mountain is the tallest in New England?

62. Which mountain is the second tallest in New England?

The table shows the top ten museums in the world in 2015. Use this table to answer Exercises 63 through 68. See Example 13.

Top 10 Museums Worldwide in 2015

Museum	**Location**	**Visitors**
Louvre	Paris, France	8,700,000
National Museum of China	Beijing, China	7,290,000
National Museum of Natural History	Washington, DC, United States	6,900,000
National Air and Space Museum	Washington, DC, United States	6,900,000
British Museum	London, England	6,821,000
The Metropolitan Museum of Art	New York, NY, United States	6,300,000
Vatican Museums	Vatican, Vatican City	6,002,000
Shanghai Science and Technology Museum	Shanghai, China	5,948,000
National Gallery	London, England	5,908,000
National Palace Museum (Taiwan)	Taipei, Taiwan	5,288,000

(*Source:* Themed Entertainment Association)

63. Which museum had fewer visitors, the National Gallery in London or the National Air and Space Museum in Washington, DC?

64. Which museum had more visitors, the British Museum in London or the Shanghai Science and Technology Museum in Shanghai?

65. How many people visited the Vatican Museums? Write the number of visitors in words.

66. How many people visited the Louvre? Write the number of visitors in words.

67. How many of 2015's top ten museums in the world were located in the United States?

68. How many of 2015's top ten museums in the world were visited by fewer than 6,000,000 people?

Concept Extensions

69. Write the largest four-digit number that can be made from the digits 1, 9, 8, and 6 if each digit must be used once. _____ _____ _____ _____

70. Write the largest five-digit number that can be made using the digits 5, 3, and 7 if each digit must be used at least once. _____ _____ _____ _____ _____

Check to see whether each number written in standard form matches the number written in words. If not, correct the number in words. See the Concept Check in this section.

71.

60–8124/7233
1000613331
1401

DATE

PAY TO THE ORDER OF $ 105.00

One Hundred Fifty and $\frac{00}{100}$ DOLLARS

FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422

MEMO

⑆621497260⑆ 1000613331⑈ 1401

72.

60–8124/7233
1000613331
1402

DATE

PAY TO THE ORDER OF $ 7030.00

Seven Thousand Thirty and $\frac{00}{100}$ DOLLARS

FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422

MEMO

⑆621497260⑆ 1000613331⑈ 1402

73. If a number is given in words, describe the process used to write this number in standard form.

74. If a number is written in standard form, describe the process used to write this number in expanded form.

75. As of June 2017, the Chinese supercomputer Sunway TaihuLight was still ranked as the world's fastest computer. Its speed was clocked at 93 petaflops, or more than 93 quadrillion arithmetic operations per second. Look up "quadrillion" (in the American system) and use the definition to write this number in standard form. (*Source:* top500.org)

76. As of March 2017, the national debt of France was approximately $5 trillion. Look up "trillion" (in the American system) and use the definition to write 5 trillion in standard form. (*Source:* CIA World Factbook)

77. The Pro Football Hall of Fame was established on September 7, 1963, in this town. Use the information and the diagram to the right to find the name of the town.

- Alliance is east of Massillon.
- Dover is between Canton and New Philadelphia.
- Massillon is not next to Alliance.
- Canton is north of Dover.

1.3 Adding Whole Numbers and Perimeter

Objective A Adding Whole Numbers

According to Gizmodo, the iPod nano (currently in its seventh generation) is still the best overall MP3 player.

Suppose that an electronics store received a shipment of two boxes of iPod nanos one day and an additional four boxes of iPod nanos the next day. The **total** shipment in the two days can be found by adding 2 and 4.

2 boxes of iPod nanos + 4 boxes of iPod nanos = 6 boxes of iPod nanos

The **sum** (or total) is 6 boxes of iPod nanos. Each of the numbers 2 and 4 is called an **addend,** and the process of finding the sum is called **addition.**

$$\underset{\text{addend}}{\underset{\uparrow}{2}} + \underset{\text{addend}}{\underset{\uparrow}{4}} = \underset{\text{sum}}{\underset{\uparrow}{6}}$$

To add whole numbers, we add the digits in the ones place, then the tens place, then the hundreds place, and so on. For example, let's add 2236 + 160.

$$\begin{array}{r} 2\,2\,3\,6 \\ +\ \ 1\,6\,0 \\ \hline 2\,3\,9\,6 \end{array}$$

Line up numbers vertically so that the place values correspond. Then add digits in corresponding place values, starting with the ones place.

sum of ones
sum of tens
sum of hundreds
sum of thousands

Example 1 Add: 23 + 136

Solution:

$$\begin{array}{r} 23 \\ +136 \\ \hline 159 \end{array}$$

Work Practice 1

When the sum of digits in corresponding place values is more than 9, **carrying** is necessary. For example, to add 365 + 89, add the ones-place digits first.

Carrying

$$\begin{array}{r} {\scriptstyle 1}\ \ \\ 3\,6\,5 \\ +\ \ 8\,9 \\ \hline 4 \end{array}$$

5 ones + 9 ones = **14 ones** or **1 ten** + **4 ones**

Write the 4 ones in the ones place and carry the 1 ten to the tens place.

Next, add the tens-place digits.

$$\begin{array}{r} {\scriptstyle 1\ 1}\ \ \\ 3\,6\,5 \\ +\ \ 8\,9 \\ \hline 5\,4 \end{array}$$

1 ten + 6 tens + 8 tens = **15 tens** or **1 hundred** + **5 tens**

Write the 5 tens in the tens place and carry the 1 hundred to the hundreds place.

Next, add the hundreds-place digits.

$$\begin{array}{r} {\scriptstyle 1\ 1}\ \ \\ 3\,6\,5 \\ +\ \ 8\,9 \\ \hline 4\,5\,4 \end{array}$$

1 hundred + 3 hundreds = 4 hundreds

Write the 4 hundreds in the hundreds place.

Objectives

A Add Whole Numbers.

B Find the Perimeter of a Polygon.

C Solve Problems by Adding Whole Numbers.

Practice 1

Add: 7235 + 542

Answer

1. 7777

Practice 2

Add: 27,364 + 92,977

Example 2 Add: 34,285 + 149,761

Solution:

$$\begin{array}{r} \scriptstyle 1\,1\;\;1\;\;\;\;\;\; \\ 34{,}285 \\ +\ 149{,}761 \\ \hline 184{,}046 \end{array}$$

Work Practice 2

✓Concept Check What is wrong with the following computation?

$$\begin{array}{r} 394 \\ +\ 283 \\ \hline 577 \end{array}$$

Before we continue adding whole numbers, let's review some properties of addition that you may have already discovered. The first property that we will review is the **addition property of 0.** This property reminds us that the sum of 0 and any number is that same number.

Addition Property of 0

The sum of 0 and any number is that number. For example,

$7 + 0 = 7$
$0 + 7 = 7$

Next, notice that we can add any two whole numbers in any order and the sum is the same. For example,

$4 + 5 = 9$ and $5 + 4 = 9$

We call this special property of addition the **commutative property of addition.**

Commutative Property of Addition

Changing the **order** of two addends does not change their sum. For example,

$2 + 3 = 5$ and $3 + 2 = 5$

Another property that can help us when adding numbers is the **associative property of addition.** This property states that when adding numbers, the grouping of the numbers can be changed without changing the sum. We use parentheses to group numbers. They indicate which numbers to add first. For example, let's use two different groupings to find the sum of 2 + 1 + 5.

$$\underbrace{(2 + 1)} + 5 = 3 + 5 = 8$$

Also,

$$2 + \underbrace{(1 + 5)} = 2 + 6 = 8$$

Both groupings give a sum of 8.

Answer

2. 120,341

✓Concept Check Answer

forgot to carry 1 hundred to the hundreds place

Associative Property of Addition

Changing the **grouping** of addends does not change their sum. For example,

$$3 + \underbrace{(5 + 7)} = 3 + 12 = 15 \quad \text{and} \quad \underbrace{(3 + 5)} + 7 = 8 + 7 = 15$$

The commutative and associative properties tell us that we can add whole numbers using any order and grouping that we want.

When adding several numbers, it is often helpful to look for two or three numbers whose sum is 10, 20, and so on. Why? Adding multiples of 10 such as 10 and 20 is easier.

Example 3 Add: $13 + 2 + 7 + 8 + 9$

Solution: $13 + 2 + 7 + 8 + 9 = 39$

$$20 + 10 + 9$$

$$39$$

Work Practice 3

Practice 3

Add: $11 + 7 + 8 + 9 + 13$

Feel free to use the process of Example 3 anytime when adding.

Example 4 Add: $1647 + 246 + 32 + 85$

Solution:

$$\begin{array}{r} {}^{1\,2\,2}\\ 1647 \\ 246 \\ 32 \\ +\quad 85 \\ \hline 2010 \end{array}$$

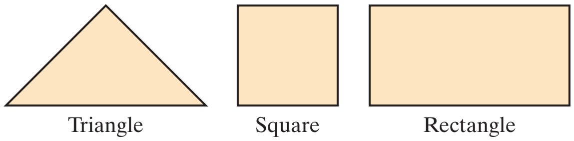
Work Practice 4

Practice 4

Add: $19 + 5042 + 638 + 526$

Objective B Finding the Perimeter of a Polygon

In geometry, addition is used to find the perimeter of a polygon. A **polygon** can be described as a flat figure formed by line segments connected at their ends. (For more review, see Appendix A.3.) Geometric figures such as triangles, squares, and rectangles are called polygons.



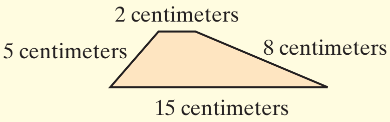
The **perimeter** of a polygon is the *distance around* the polygon. This means that the perimeter of a polygon is the sum of the lengths of its sides.

Answers

3. 48 **4.** 6225

Practice 5

Find the perimeter of the polygon shown. (A centimeter is a unit of length in the metric system.)



Practice 6

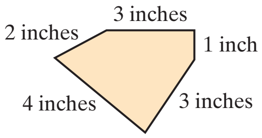
A park is in the shape of a triangle. Each of the park's three sides is 647 feet. Find the perimeter of the park.

Answers

5. 30 cm **6.** 1941 ft

Example 5

Find the perimeter of the polygon shown.



Solution: To find the perimeter (distance around), we add the lengths of the sides.

$$2 \text{ in.} + 3 \text{ in.} + 1 \text{ in.} + 3 \text{ in.} + 4 \text{ in.} = 13 \text{ in.}$$

The perimeter is 13 inches.

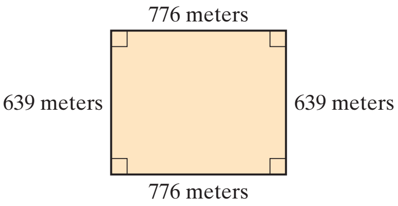
Work Practice 5

To make the addition appear simpler, we will often not include units with the addends. If you do this, make sure units are included in the final answer.

Example 6 Calculating the Perimeter of a Building

The world's largest commercial building under one roof is the flower auction building of the cooperative VBA in Aalsmeer, Netherlands. The floor plan is a rectangle that measures 776 meters by 639 meters. Find the perimeter of this building. (A meter is a unit of length in the metric system.) (*Source: The Handy Science Answer Book,* Visible Ink Press)

Solution: Recall that opposite sides of a rectangle have the same length. To find the perimeter of this building, we add the lengths of the sides. The sum of the lengths of its sides is



$$\begin{array}{r} 639 \\ 639 \\ 776 \\ +\ 776 \\ \hline 2830 \end{array}$$

The perimeter of the building is 2830 meters.

Work Practice 6

Objective C Solving Problems by Adding

Often, real-life problems occur that can be solved by adding. The first step in solving any word problem is to *understand* the problem by reading it carefully.

Descriptions of problems solved through addition *may* include any of these key words or phrases:

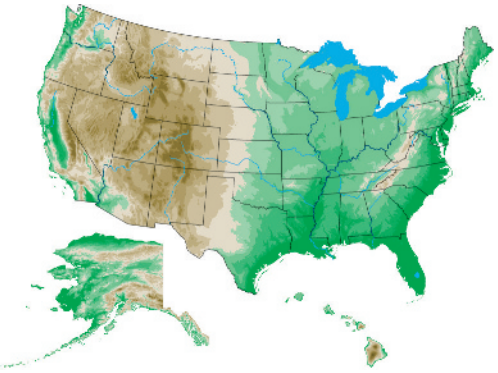
Addition

Key Words or Phrases	Examples	Symbols
added to	5 added to 7	7 + 5
plus	0 plus 78	0 + 78
increased by	12 increased by 6	12 + 6
more than	11 more than 25	25 + 11
total	the total of 8 and 1	8 + 1
sum	the sum of 4 and 133	4 + 133

To solve a word problem that involves addition, we first use the facts given to write an addition statement. Then we write the corresponding solution of the real-life problem. It is sometimes helpful to write the statement in words (brief phrases) and then translate to numbers.

Example 7 Finding the Number of Trucks Sold in the United States

In 2015, a total of 9,879,465 trucks were sold in the United States. In 2016, total truck sales in the United States had increased by 712,397 vehicles. Find the total number of trucks sold in the United States in 2016. (*Source:* Alliance of Automobile Manufacturers)



Solution: The key phrase here is "had increased by," which suggests that we add. To find the number of trucks sold in 2016, we add the increase 712,397 to the number of trucks sold in 2015.

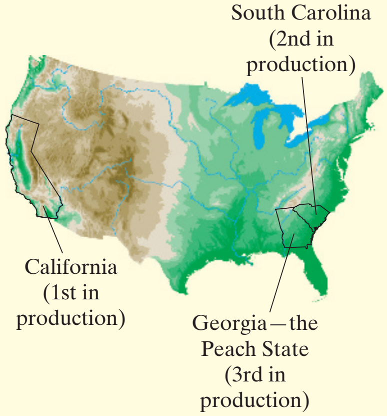
In Words	**Translate to Numbers**
Number sold in 2015	9,879,465
+ increase	+ 712,397
Number sold in 2016	10,591,862

The number of passenger vehicles sold in the United States in 2016 was 10,591,862.

Work Practice 7

Practice 7

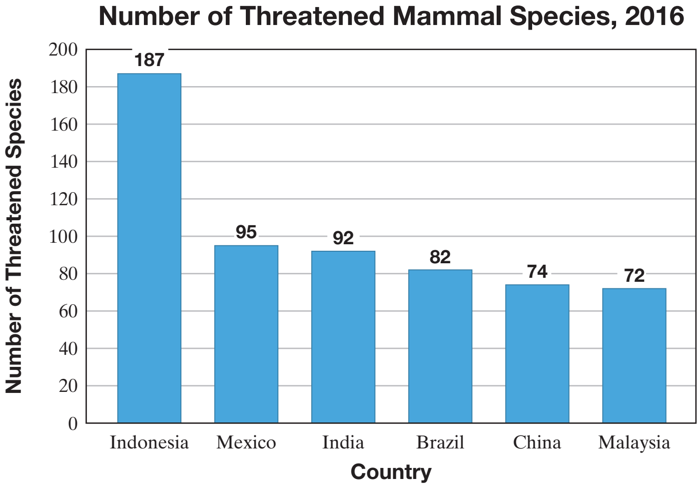
Georgia produces 70 million pounds of freestone peaches per year. The second largest U.S. producer of peaches, South Carolina, produces 50 million pounds more freestone peaches than Georgia. How much does South Carolina produce? (*Source:* farms.com)



Graphs can be used to visualize data. The graph shown next is called a **bar graph.** For this bar graph, the height of each bar is labeled above the bar. To check this height, follow the top of each bar to the vertical line to the left. For example, the first bar is labeled 187. Follow the top of that bar to the left until the vertical line is reached, between 180 and 200, but closer to 180, or 187.

Example 8 Reading a Bar Graph

In the following graph, each bar represents a country and the height of each bar represents the number of threatened mammal species identified in that country.



Source: International Union for Conservation of Nature

(*Continued on next page*)

Practice 8

Use the graph in Example 8 to answer the following:

a. Which country shown has the fewest threatened mammal species?

b. Find the total number of threatened mammal species for Brazil, India, and Mexico.

Answers

7. 120 million lb

8. a. Malaysia **b.** 269

a. Which country shown has the greatest number of threatened mammal species?

b. Find the total number of threatened mammal species for Malaysia, China, and Indonesia.

Solution:

a. The country with the greatest number of threatened mammal species corresponds to the tallest bar, which is Indonesia.

b. The key word here is "total." To find the total number of threatened mammal species for Malaysia, China, and Indonesia, we add.

In Words		Translate to Numbers	
Malaysia	→		72
China	→		74
Indonesia	→		+ 187
		Total	333

The total number of threatened mammal species for Malaysia, China, and Indonesia is 333.

Work Practice 8

Calculator Explorations Adding Numbers

To add numbers on a calculator, find the keys marked $\boxed{+}$ and $\boxed{=}$ or $\boxed{\text{ENTER}}$.

For example, to add 5 and 7 on a calculator, press the keys $\boxed{5}$ $\boxed{+}$ $\boxed{7}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

The display will read $\boxed{\quad 12}$.

Thus, $5 + 7 = 12$.

To add 687, 981, and 49 on a calculator, press the keys $\boxed{687}$ $\boxed{+}$ $\boxed{981}$ $\boxed{+}$ $\boxed{49}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

The display will read $\boxed{\quad 1717}$.

Thus, $687 + 981 + 49 = 1717$. (Although entering 687, for example, requires pressing more than one key, here numbers are grouped together for easier reading.)

Use a calculator to add.

1. $89 + 45$

2. $76 + 97$

3. $285 + 55$

4. $8773 + 652$

5.
$$\begin{array}{r} 985 \\ 1210 \\ 562 \\ \underline{+\quad 77} \end{array}$$

6.
$$\begin{array}{r} 465 \\ 9888 \\ 620 \\ \underline{+\,1550} \end{array}$$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

sum	order	addend	associative
perimeter	number	grouping	commutative

1. The sum of 0 and any number is the same ___________.

2. The sum of any number and 0 is the same ___________.

3. In $35 + 20 = 55$, the number 55 is called the ___________ and 35 and 20 are each called a(n) ___________.

4. The distance around a polygon is called its ___________.

5. Since $(3 + 1) + 20 = 3 + (1 + 20)$, we say that changing the __________ in addition does not change the sum. This property is called the __________ property of addition.

6. Since $7 + 10 = 10 + 7$, we say that changing the __________ in addition does not change the sum. This property is called the __________ property of addition.

Martin-Gay Interactive Videos

See Video 1.3

Watch the section lecture video and answer the following questions.

Objective A **7.** Complete this statement based on the lecture before Example 1: To add whole numbers, we line up __________ values and add from __________ to __________.

Objective B **8.** In Example 4, the perimeter of what type of polygon is found? How many addends are in the resulting addition problem?

Objective C **9.** In Example 6, what key word or phrase indicates addition?

1.3 Exercise Set MyLab Math

Objective A *Add. See Examples 1 through 4.*

1. $\begin{array}{r} 14 \\ +\,22 \\ \hline \end{array}$

2. $\begin{array}{r} 27 \\ +\,31 \\ \hline \end{array}$

3. $\begin{array}{r} 62 \\ +\,230 \\ \hline \end{array}$

4. $\begin{array}{r} 37 \\ +\,542 \\ \hline \end{array}$

5. $\begin{array}{r} 12 \\ 13 \\ +\,24 \\ \hline \end{array}$

6. $\begin{array}{r} 23 \\ 45 \\ +\,30 \\ \hline \end{array}$

7. $\begin{array}{r} 5267 \\ +\;\;132 \\ \hline \end{array}$

8. $\begin{array}{r} 236 \\ +\,6243 \\ \hline \end{array}$

9. $53 + 64$

10. $41 + 74$

11. $22 + 490$

12. $35 + 470$

13. $22{,}781 + 186{,}297$

14. $17{,}427 + 821{,}059$

15. $\begin{array}{r} 8 \\ 9 \\ 2 \\ 5 \\ +\,1 \\ \hline \end{array}$

16. $\begin{array}{r} 3 \\ 5 \\ 8 \\ 5 \\ +\,7 \\ \hline \end{array}$

17. $\begin{array}{r} 6 \\ 21 \\ 14 \\ 9 \\ +\,12 \\ \hline \end{array}$

18. $\begin{array}{r} 12 \\ 4 \\ 8 \\ 26 \\ +\,10 \\ \hline \end{array}$

19. $\begin{array}{r} 81 \\ 17 \\ 23 \\ 79 \\ +\,12 \\ \hline \end{array}$

20. $\begin{array}{r} 64 \\ 28 \\ 56 \\ 25 \\ +\,32 \\ \hline \end{array}$

21. $62 + 18 + 14$

22. $23 + 49 + 18$

23. $40 + 800 + 70$

24. $90 + 900 + 20$

25. $7542 + 49 + 682$

26. $1624 + 32 + 976$

27. $24 + 9006 + 489 + 2407$

28. $16 + 1056 + 748 + 7770$

29. $$\begin{array}{r} 627 \\ 628 \\ +629 \\ \hline \end{array}$$

30. $$\begin{array}{r} 427 \\ 383 \\ +229 \\ \hline \end{array}$$

31. $$\begin{array}{r} 6820 \\ 4271 \\ +5626 \\ \hline \end{array}$$

32. $$\begin{array}{r} 6789 \\ 4321 \\ +5555 \\ \hline \end{array}$$

33. $$\begin{array}{r} 507 \\ 593 \\ +\ \ 10 \\ \hline \end{array}$$

34. $$\begin{array}{r} 864 \\ 33 \\ +356 \\ \hline \end{array}$$

35. $$\begin{array}{r} 4200 \\ 2107 \\ +2692 \\ \hline \end{array}$$

36. $$\begin{array}{r} 5000 \\ 1400 \\ +3021 \\ \hline \end{array}$$

37. $$\begin{array}{r} 49 \\ 628 \\ 5\,762 \\ +29{,}462 \\ \hline \end{array}$$

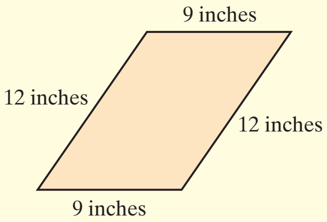
38. $$\begin{array}{r} 26 \\ 582 \\ 4\,763 \\ +62{,}511 \\ \hline \end{array}$$

39. $$\begin{array}{r} 121{,}742 \\ 57{,}279 \\ 26{,}586 \\ +426{,}782 \\ \hline \end{array}$$

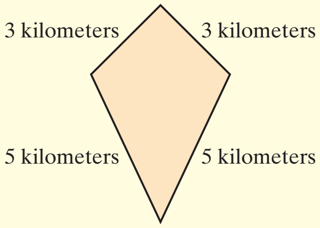
40. $$\begin{array}{r} 504{,}218 \\ 321{,}920 \\ 38{,}507 \\ +594{,}687 \\ \hline \end{array}$$

Objective B *Find the perimeter of each figure. See Examples 5 and 6.*

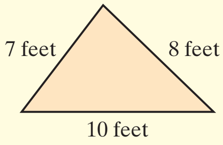
△ **41.**



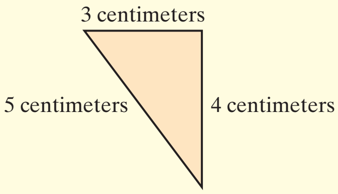
△ **42.**



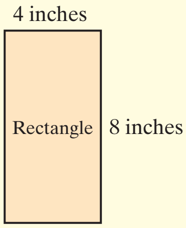
△ **43.**



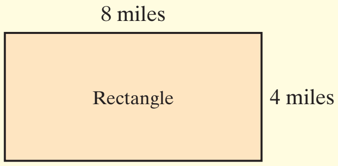
△ **44.**



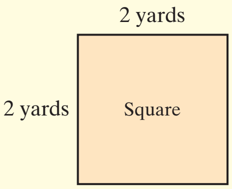
△ **45.**



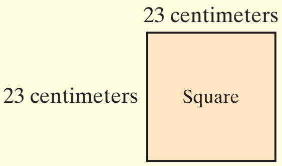
△ **46.**



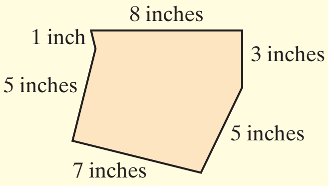
△ **47.**



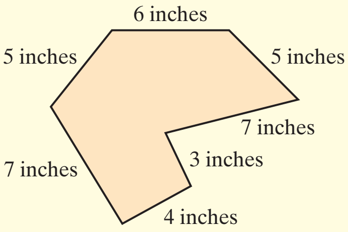
△ **48.**



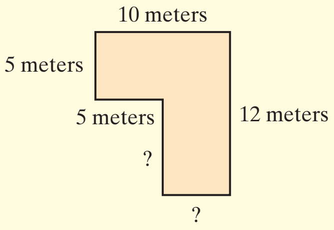
△ **49.**



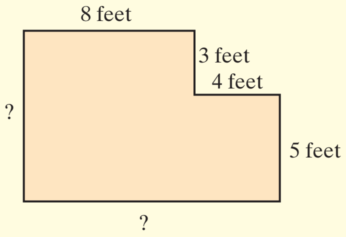
△ **50.**



△ **51.**



△ **52.**



Objectives A B C Mixed Practice–Translating *Solve. See Examples 1 through 8.*

▶ **53.** Find the sum of 297 and 1796.

54. Find the sum of 802 and 6487.

55. Find the total of 76, 39, 8, 17, and 126.

56. Find the total of 89, 45, 2, 19, and 341.

57. What is 452 increased by 92?

58. What is 712 increased by 38?

59. What is 2686 plus 686 plus 80?

60. What is 3565 plus 565 plus 70?

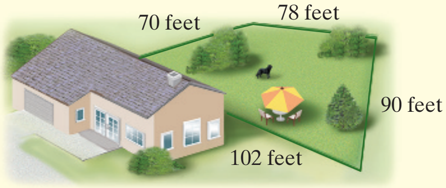
61. The estimated population of Florida was 20,148 thousand in 2016. If it is projected to increase by 2286 thousand by 2025, what is Florida's projected population in 2025? (*Source:* University of Florida, Bureau of Economic and Business research)

62. The estimated population of California was 39,250 thousand in 2016. It is projected in increase by 4850 thousand by 2030. What is California's projected population in 2030? (*Source:* Public Policy Institute of California)

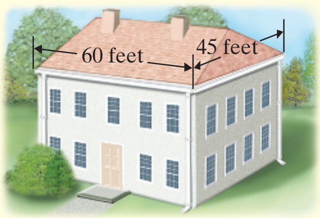
▶ **63.** The highest point in South Carolina is Sassafras Mountain at 3560 feet above sea level. The highest point in North Carolina is Mt. Mitchell, whose peak is 3124 feet increased by the height of Sassafras Mountain. Find the height of Mt. Mitchell. (*Source:* U.S. Geological Survey)

64. The distance from Kansas City, Kansas, to Hays, Kansas, is 285 miles. Colby, Kansas, is 98 miles farther from Kansas City than Hays. Find the total distance from Kansas City to Colby.

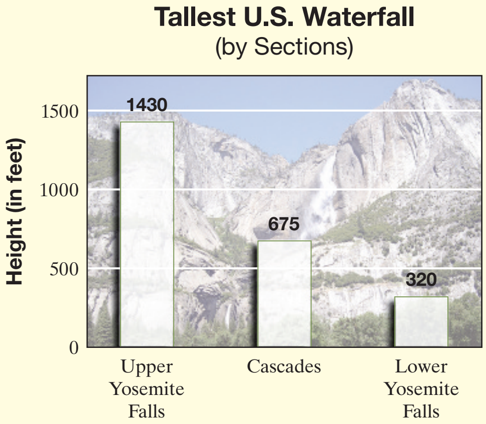
△ **65.** Leo Callier is installing an invisible fence in his backyard. How many feet of wiring are needed to enclose the yard below?



△ **66.** A homeowner is considering installing gutters around her home. Find the perimeter of her rectangular home.



67. The tallest waterfall in the United States is Yosemite Falls in Yosemite National Park in California. Yosemite Falls is made up of three sections, as shown in the graph. What is the total height of Yosemite Falls? (*Source:* U.S. Department of the Interior)



68. Jordan White, a nurse at Mercy Hospital, is recording fluid intake on a patient's medical chart. During his shift, the patient had the following types and amounts of intake measured in cubic centimeters (cc). What amount should Jordan record as the total fluid intake for this patient?

Oral	Intravenous	Blood
240	500	500
100	200	
355		

69. In 2016, Harley-Davidson sold 161,839 motorcycles domestically. In addition, 100,352 Harley-Davidson motorcycles were sold internationally. What was the total number of Harley-Davidson motorcycles sold in 2016? (*Source:* Harley-Davidson, Inc.)

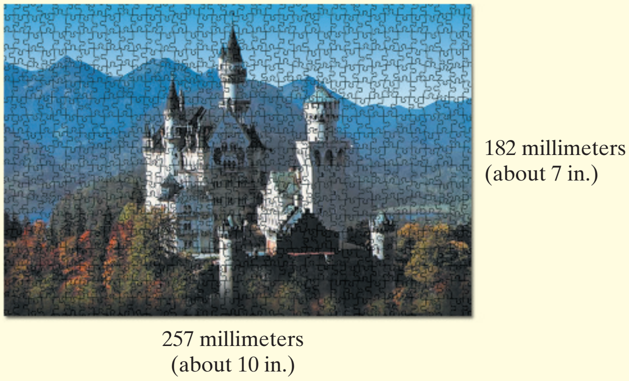
70. Hank Aaron holds Major League Baseball's record for the most runs batted in over his career. He batted in 1305 runs from 1954 to 1965. He batted in another 992 runs from 1966 until he retired in 1976. How many total runs did Hank Aaron bat in during his career in professional baseball?

71. During one month is 2016, the two top-selling vehicles in the United States were the Ford F-Series and the Chevrolet Silverado, both trucks. There were 65,542 F-Series trucks and 49,768 Silverados sold that month. What was the total number of these trucks sold in that month? (*Source:* www.goodcarbadcar.com)

72. In 2016, the country of New Zealand had 22,867,835 more sheep than people. If the human population of New Zealand in 2016 was 4,573,567, what was the sheep population? (*Source:* www.stats.govt.nz)

73. The largest permanent Monopoly board is made of granite and located in San Jose, California. Find the perimeter of the square playing board.

74. The smallest commercially available jigsaw puzzle (with a minimum of 1000 pieces) is manufactured in Hong Kong, China. Find the exact perimeter of this rectangular-shaped puzzle in millimeters. (*Source: Guinness World Records*)



75. In 2016, there were 2669 Gap Inc. (Gap, Banana Republic, Athleta, Old Navy, and Intermix North America) stores located in the United States and 606 located outside the United States. How many Gap Inc. stores were located worldwide? (*Source:* Gap Inc.)

76. Wilma Rudolph, who won three gold medals in track and field events in the 1960 Summer Olympics, was born in 1940. Allyson Felix, who also won three gold medals in track and field events but in the 2012 Summer Olympics, was born 45 years later. In what year was Allyson Felix born?

The table shows the number of CVS Pharmacies in ten states. Use this table to answer Exercises 77 through 82.

Top Ten States for CVS Pharmacies in 2016

State	Number of Pharmacies
Massachusetts	356
California	867
Florida	756
Georgia	313
Indiana	301
New York	486
North Carolina	313
Ohio	309
Pennsylvania	408
Texas	659

(*Source:* CVS Pharmacy Inc.)

77. Which state had the most CVS pharmacies?

78. Which of the states listed in the table had the fewest CVS pharmacies?

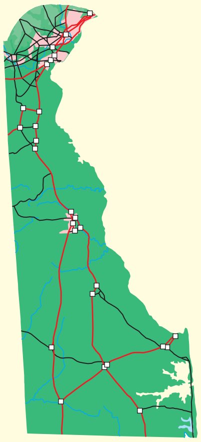
79. What was the total number of CVS pharmacies located in the three states with the most CVS pharmacies?

80. How many CVS pharmacies were located in the ten states listed in the table?

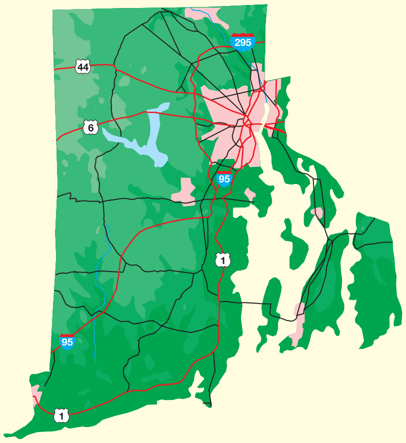
81. Which pair of neighboring states had more CVS pharmacies combined, Pennsylvania and New York or Florida and Georgia?

82. There were 3048 CVS pharmacies located in the states not listed in the table. How many CVS pharmacies were there in the 50 states?

83. The state of Delaware has 2997 miles of urban highways and 3361 miles of rural highways. Find the total highway mileage in Delaware. (*Source:* U.S. Federal Highway Administration)



84. The state of Rhode Island has 5260 miles of urban highways and 1225 miles of rural highways. Find the total highway mileage in Rhode Island. (*Source:* U.S. Federal Highway Administration)



Concept Extensions

85. In your own words, explain the commutative property of addition.

86. In your own words, explain the associative property of addition.

87. Give any three whole numbers whose sum is 100.

88. Give any four whole numbers whose sum is 25.

89. Add: 56,468,980 + 1,236,785 + 986,768,000

90. Add: 78,962 + 129,968,350 + 36,462,880

Check each addition below. If it is incorrect, find the correct answer. See the Concept Check in this section.

91.
$$\begin{array}{r} 566 \\ 932 \\ +\ 871 \\ \hline 2369 \end{array}$$

92.
$$\begin{array}{r} 773 \\ 659 \\ +\ 481 \\ \hline 1913 \end{array}$$

93.
$$\begin{array}{r} 14 \\ 173 \\ 86 \\ +257 \\ \hline 520 \end{array}$$

94.
$$\begin{array}{r} 19 \\ 214 \\ 49 \\ +651 \\ \hline 923 \end{array}$$

1.4 Subtracting Whole Numbers

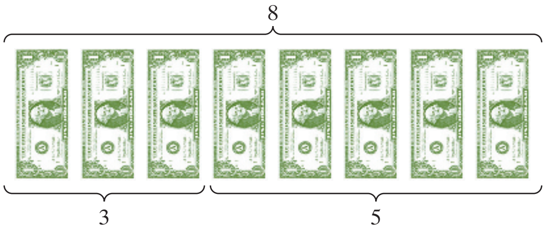
Objectives

A Subtract Whole Numbers.

B Solve Problems by Subtracting Whole Numbers.

Objective A Subtracting Whole Numbers

If you have \$5 and someone gives you \$3, you have a total of \$8, since 5 + 3 = 8. Similarly, if you have \$8 and then someone borrows \$3, you have \$5 left. **Subtraction** is finding the **difference** of two numbers.



$$\underset{\text{minuend}}{\underset{\uparrow}{8}} \quad - \quad \underset{\text{subtrahend}}{\underset{\uparrow}{3}} \quad = \quad \underset{\text{difference}}{\underset{\uparrow}{5}}$$

In this example, 8 is the **minuend,** and 3 is the **subtrahend.** The **difference** between these two numbers, 8 and 3, is 5.

Notice that addition and subtraction are very closely related. In fact, subtraction is defined in terms of addition.

$8 - 3 = 5$ because $5 + 3 = 8$

This means that subtraction can be *checked* by addition, and we say that addition and subtraction are reverse operations.

Example 1 Subtract. Check each answer by adding.

a. $12 - 9$ **b.** $22 - 7$ **c.** $35 - 35$ **d.** $70 - 0$

Solution:

a. $12 - 9 = 3$ because $3 + 9 = 12$
b. $22 - 7 = 15$ because $15 + 7 = 22$
c. $35 - 35 = 0$ because $0 + 35 = 35$
d. $70 - 0 = 70$ because $70 + 0 = 70$

Work Practice 1

Practice 1

Subtract. Check each answer by adding.

a. $14 - 6$
b. $20 - 8$
c. $93 - 93$
d. $42 - 0$

Look again at Examples 1(c) and 1(d).

$$1(c)\ \ \underset{\text{same number}}{35 - 35} = \underset{\text{difference is 0}}{\underset{\uparrow}{0}} \qquad 1(d)\ \ \underbrace{70 - 0}_{\substack{\uparrow \\ \text{a number minus 0}}} = \underset{\text{difference is the same number}}{\underset{\uparrow}{70}}$$

These two examples illustrate the subtraction properties of 0.

Subtraction Properties of 0

The difference of any number and that same number is 0. For example,

$11 - 11 = 0$

The difference of any number and 0 is that same number. For example,

$45 - 0 = 45$

To subtract whole numbers we subtract the digits in the ones place, then the tens place, then the hundreds place, and so on. When subtraction involves numbers

Answers
1. a. 8 **b.** 12 **c.** 0 **d.** 42

of two or more digits, it is more convenient to subtract vertically. For example, to subtract 893 − 52,

$$\begin{array}{rl} 8\,9\,3 & \leftarrow \text{minuend} \\ -\quad 5\,2 & \leftarrow \text{subtrahend} \\ \hline 8\,4\,1 & \leftarrow \text{difference} \end{array}$$

$3 - 2$

$9 - 5$

$8 - 0$

Line up the numbers vertically so that the minuend is on top and the place values correspond. Subtract in corresponding place values, starting with the ones place.

To check, add.

$$\begin{array}{r} \text{difference} \\ +\ \text{subtrahend} \\ \hline \text{minuend} \end{array} \quad \text{or} \quad \begin{array}{r} 841 \\ +\ \ 52 \\ \hline 893 \end{array} \leftarrow \text{Since this is the original minuend, the problem checks.}$$

Practice 2

Subtract. Check by adding.

a. 9143 − 122

b. 978 − 851

Example 2 Subtract: 7826 − 505. Check by adding.

Solution:

$$\begin{array}{r} 7826 \\ -\ \ 505 \\ \hline 7321 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 7321 \\ +\ \ 505 \\ \hline 7826 \end{array}$$

Work Practice 2

Subtracting by Borrowing

When subtracting vertically, if a digit in the second number (subtrahend) is larger than the corresponding digit in the first number (minuend), **borrowing** is necessary. For example, consider

$$\begin{array}{r} 8\boxed{1} \\ -\,6\boxed{3} \\ \hline \end{array}$$

Since the 3 in the ones place of 63 is larger than the 1 in the ones place of 81, borrowing is necessary. We borrow 1 ten from the tens place and add it to the ones place.

Borrowing

$$\underset{\text{tens}}{8} - \underset{\text{ten}}{1} = \underset{\text{tens}}{7} \rightarrow \begin{array}{r} \overset{7}{\cancel{8}}\ \overset{11}{\cancel{1}} \\ -\,6\ \ 3 \\ \hline \end{array} \leftarrow 1 \text{ ten} + 1 \text{ one} = 11 \text{ ones}$$

Now we subtract the ones-place digits and then the tens-place digits.

$$\begin{array}{rl} \overset{7}{\cancel{8}}\,\overset{11}{\cancel{1}} & \\ -\,6\,3 & \\ \hline 1\,8 & \leftarrow 11 - 3 = 8 \\ & \ \ 7 - 6 = 1 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 18 \\ +63 \\ \hline 81 \end{array} \quad \text{The original minuend.}$$

Practice 3

Subtract. Check by adding.

$$\textbf{a.}\ \begin{array}{r} 697 \\ -\ \ 49 \\ \hline \end{array} \qquad \textbf{b.}\ \begin{array}{r} 326 \\ -245 \\ \hline \end{array} \qquad \textbf{c.}\ \begin{array}{r} 1234 \\ -\ \ 822 \\ \hline \end{array}$$

Example 3 Subtract: 543 − 29. Check by adding.

Solution:

$$\begin{array}{r} 5\,\overset{3}{\cancel{4}}\,\overset{13}{\cancel{3}} \\ -\ \ 2\,9 \\ \hline 5\,1\,4 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 514 \\ +\ \ 29 \\ \hline 543 \end{array}$$

Work Practice 3

Answers

2. a. 9021 **b.** 127

3. a. 648 **b.** 81 **c.** 412

Sometimes we may have to borrow from more than one place. For example, to subtract 7631 − 152, we first borrow from the tens place.

$$\begin{array}{r} 7\,6\,\overset{2}{\cancel{3}}\,\overset{11}{\cancel{1}} \\ -\;1\,5\,2 \\ \hline 9 \end{array} \leftarrow 11 - 2 = 9$$

In the tens place, 5 is greater than 2, so we borrow again. This time we borrow from the hundreds place.

6 hundreds − **1 hundred** = 5 hundreds

$$\begin{array}{r} 7\,\overset{5}{\cancel{6}}\,\overset{\overset{12}{\cancel{2}}}{\cancel{3}}\,\overset{11}{\cancel{1}} \\ -\;1\,5\,2 \\ \hline 7\,4\,7\,9 \end{array}$$

1 hundred + 2 tens

or

10 tens + 2 tens = 12 tens

Check:

$$\begin{array}{r} 7479 \\ +\;152 \\ \hline 7631 \end{array}$$ The original minuend.

Example 4 Subtract: 900 − 174. Check by adding.

Solution: In the ones place, 4 is larger than 0, so we borrow from the tens place. But the tens place of 900 is 0, so to borrow from the tens place we must first borrow from the hundreds place.

$$\begin{array}{r} \overset{8}{\cancel{9}}\;\overset{10}{\cancel{0}}\;0 \\ -\;1\;7\;4 \\ \hline \end{array}$$

Now borrow from the tens place.

$$\begin{array}{r} \overset{8}{\cancel{9}}\;\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\;\overset{10}{\cancel{0}} \\ -\;1\;7\;4 \\ \hline 7\;2\;6 \end{array}$$

Check:

$$\begin{array}{r} 726 \\ +174 \\ \hline 900 \end{array}$$

Work Practice 4

Practice 4

Subtract. Check by adding.

a. $\begin{array}{r} 400 \\ -164 \\ \hline \end{array}$

b. $\begin{array}{r} 1000 \\ -\;762 \\ \hline \end{array}$

Objective B Solving Problems by Subtracting

Often, real-life problems occur that can be solved by subtracting. The first step in solving any word problem is to *understand* the problem by reading it carefully.

Descriptions of problems solved through subtraction *may* include any of these key words or phrases:

Helpful Hint Be careful when solving applications that suggest subtraction. Although order *does not* matter when adding, order *does* matter when subtracting. For example, 20 − 15 and 15 − 20 do not simplify to the same number.

Subtraction

Key Words or Phrases	Examples	Symbols
subtract	subtract 5 from 8	8 − 5
difference	the difference of 10 and 2	10 − 2
less	17 less 3	17 − 3
less than	2 less than 20	20 − 2
take away	14 take away 9	14 − 9
decreased by	7 decreased by 5	7 − 5
subtracted from	9 subtracted from 12	12 − 9

Answers
4. a. 236 **b.** 238

✓Concept Check In each of the following problems, identify which number is the minuend and which number is the subtrahend.

a. What is the result when 6 is subtracted from 40?

b. What is the difference of 15 and 8?

c. Find a number that is 15 fewer than 23.

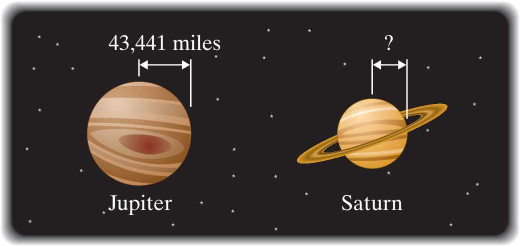
To solve a word problem that involves subtraction, we first use the facts given to write a subtraction statement. Then we write the corresponding solution of the real-life problem. It is sometimes helpful to write the statement in words (brief phrases) and then translate to numbers.

Practice 5

The radius of Uranus is 15,759 miles. The radius of Neptune is 458 miles less than the radius of Uranus. What is the radius of Neptune? (*Source:* National Space Science Data Center)

Example 5 Finding the Radius of a Planet

The radius of Jupiter is 43,441 miles. The radius of Saturn is 7257 miles less than the radius of Jupiter. Find the radius of Saturn. (Source: National Space Science Data Center)



Solution: **In Words** **Translate to Numbers**

$$\begin{array}{rcr} \text{radius of Jupiter} & \longrightarrow & \overset{3}{\cancel{4}}\overset{13}{\cancel{3}},\overset{3}{\cancel{4}}\overset{\overset{13}{\cancel{3}}}{\cancel{4}}\overset{11}{\cancel{1}} \\ -\qquad 7257 & \longrightarrow & -\ 7\,257 \\ \hline \text{radius of Saturn} & \longrightarrow & 36,184 \end{array}$$

The radius of Saturn is 36,184 miles.

Helpful Hint Since subtraction and addition are reverse operations, don't forget that a subtraction problem can be checked by adding.

Work Practice 5

Practice 6

During a sale, the price of a new suit is decreased by $47. If the original price was $92, find the sale price of the suit.

Example 6 Calculating Miles per Gallon

A subcompact car gets 42 miles per gallon of gas. A full-size car gets 17 miles per gallon of gas. Find the difference between the subcompact car miles per gallon and the full-size car miles per gallon.

Solution: **In Words** **Translate to Numbers**

$$\begin{array}{rcr} \text{subcompact miles per gallon} & \longrightarrow & \overset{3}{\cancel{4}}\overset{12}{\cancel{2}} \\ -\quad \text{full-size miles per gallon} & \longrightarrow & -17 \\ \hline \text{difference in miles per gallon} & & 25 \end{array}$$

The difference in the subcompact car miles per gallon and the full-size car miles per gallon is 25 miles per gallon.

Work Practice 6

Answers

5. 15,301 miles **6.** $45

✓Concept Check Answers

a. minuend: 40; subtrahend: 6

b. minuend: 15; subtrahend: 8

c. minuend: 23; subtrahend: 15

Helpful Hint

Once again, because subtraction and addition are reverse operations, don't forget that a subtraction problem can be checked by adding.

Calculator Explorations **Subtracting Numbers**

To subtract numbers on a calculator, find the keys marked $\boxed{-}$ and $\boxed{=}$ or $\boxed{\text{ENTER}}$.

For example, to find 83 − 49 on a calculator, press the keys $\boxed{83}$ $\boxed{-}$ $\boxed{49}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

The display will read $\boxed{\quad 34}$.

Thus, 83 − 49 = 34.

Use a calculator to subtract.

1. 865 − 95

2. 76 − 27

3. 147 − 38

4. 366 − 87

5. 9625 − 647

6. 10,711 − 8925

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

0 minuend difference
number subtrahend

1. The difference of any number and that same number is __________.

2. The difference of any number and 0 is the same __________.

3. In 37 − 19 = 18, the number 37 is the __________, and the number 19 is the __________.

4. In 37 − 19 = 18, the number 18 is called the __________.

Find each difference.

5. 6 − 6

6. 93 − 93

7. 600 − 0

8. 5 − 0

Martin-Gay Interactive Videos

See Video 1.4

Watch the section lecture video and answer the following questions.

Objective A **9.** In Example 2, explain how we end up subtracting 7 from 12 in the ones place.

Objective B **10.** Complete this statement based on Example 4: Order does not matter when ____________, but order does matter when ____________.

1.4 Exercise Set MyLab Math

Objective A *Subtract. Check by adding. See Examples 1 and 2.*

1. $\begin{array}{r} 67 \\ -23 \\ \hline \end{array}$

2. $\begin{array}{r} 72 \\ -41 \\ \hline \end{array}$

3. $\begin{array}{r} 389 \\ -124 \\ \hline \end{array}$

4. $\begin{array}{r} 572 \\ -321 \\ \hline \end{array}$

5. $\begin{array}{r} 167 \\ -\ 32 \\ \hline \end{array}$

6. $\begin{array}{r} 286 \\ -\ 45 \\ \hline \end{array}$

7. 2677 − 423

8. 5766 − 324

9. 6998 − 1453

10. 4912 − 2610

11. $\begin{array}{r} 749 \\ -149 \\ \hline \end{array}$

12. $\begin{array}{r} 257 \\ -257 \\ \hline \end{array}$

Subtract. Check by adding. See Examples 1 through 4.

13. $\begin{array}{r} 62 \\ -37 \\ \hline \end{array}$

14. $\begin{array}{r} 55 \\ -29 \\ \hline \end{array}$

15. $\begin{array}{r} 70 \\ -25 \\ \hline \end{array}$

16. $\begin{array}{r} 80 \\ -37 \\ \hline \end{array}$

17. $\begin{array}{r} 938 \\ -792 \\ \hline \end{array}$

18. $\begin{array}{r} 436 \\ -275 \\ \hline \end{array}$

19. $\begin{array}{r} 922 \\ -634 \\ \hline \end{array}$

20. $\begin{array}{r} 674 \\ -299 \\ \hline \end{array}$

21. $\begin{array}{r} 600 \\ -432 \\ \hline \end{array}$

22. $\begin{array}{r} 300 \\ -149 \\ \hline \end{array}$

23. $\begin{array}{r} 142 \\ -\ 36 \\ \hline \end{array}$

24. $\begin{array}{r} 773 \\ -\ 29 \\ \hline \end{array}$

25. $\begin{array}{r} 923 \\ -476 \\ \hline \end{array}$

26. $\begin{array}{r} 813 \\ -227 \\ \hline \end{array}$

27. $\begin{array}{r} 6283 \\ -\ 560 \\ \hline \end{array}$

28. $\begin{array}{r} 5349 \\ -\ 720 \\ \hline \end{array}$

29. $\begin{array}{r} 533 \\ -\ 29 \\ \hline \end{array}$

30. $\begin{array}{r} 724 \\ -\ 16 \\ \hline \end{array}$

31. $\begin{array}{r} 200 \\ -111 \\ \hline \end{array}$

32. $\begin{array}{r} 300 \\ -211 \\ \hline \end{array}$

33. $\begin{array}{r} 1983 \\ -1904 \\ \hline \end{array}$

34. $\begin{array}{r} 1983 \\ -1914 \\ \hline \end{array}$

35. $\begin{array}{r} 56{,}422 \\ -16{,}508 \\ \hline \end{array}$

36. $\begin{array}{r} 76{,}652 \\ -29{,}498 \\ \hline \end{array}$

37. 50,000 − 17,289

38. 40,000 − 23,582

39. 7020 − 1979

40. 6050 − 1878

41. 51,111 − 19,898

42. 62,222 − 39,898

Objective B *Solve. See Examples 5 and 6.*

43. Subtract 5 from 9.

44. Subtract 9 from 21.

45. Find the difference of 41 and 21.

46. Find the difference of 16 and 5.

47. Subtract 56 from 63.

48. Subtract 41 from 59.

49. Find 108 less 36.

50. Find 25 less 12.

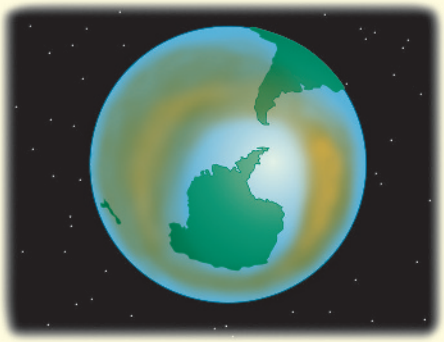
51. Find 12 subtracted from 100.

52. Find 86 subtracted from 90.

53. Professor Graham is reading a 503-page book. If she has just finished reading page 239, how many more pages must she read to finish the book?

54. When a couple began a trip, the odometer read 55,492. When the trip was over, the odometer read 59,320. How many miles did they drive on their trip?

55. In 2012, the hole in the Earth's ozone layer over Antarctica was about 18 million square kilometers in size. By mid 2016, the hole peaked at about 23 million square kilometers. By how much did the hole grown from 2012 to 2016? (*Source:* NASA Ozone Watch)



56. Bamboo can grow to 98 feet while Pacific giant kelp (a type of seaweed) can grow to 197 feet. How much taller is the kelp than the bamboo?

Bamboo

Kelp

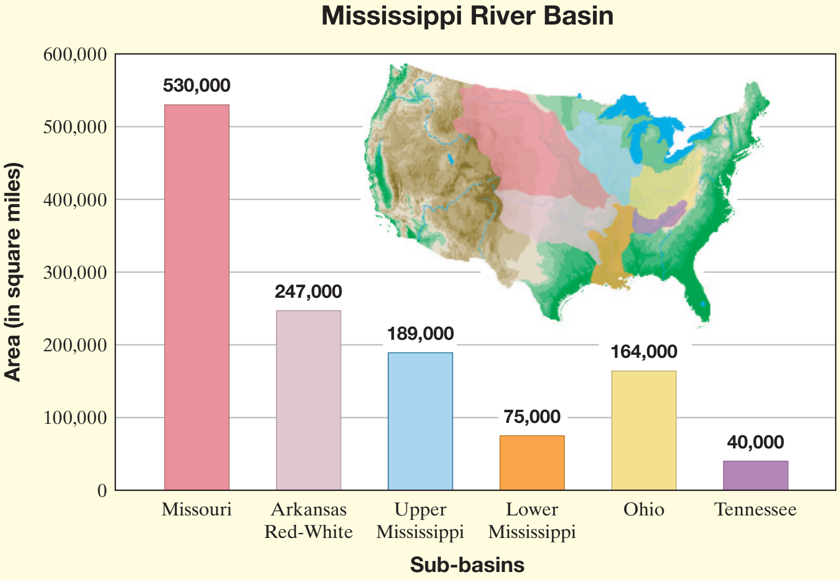
A river basin is the geographic area drained by a river and its tributaries. The Mississippi River Basin is the third largest in the world and is divided into six sub-basins, whose areas are shown in the following bar graph. Use this graph for Exercises 57 through 60.

57. Find the total U.S. land area drained by the Upper Mississippi and Lower Mississippi sub-basins.

58. Find the total U.S. land area drained by the Ohio and Tennessee sub-basins.

59. How much more land is drained by the Missouri sub-basin than the Arkansas Red-White sub-basin?

60. How much more land is drained by the Upper Mississippi sub-basin than the Lower Mississippi sub-basin?



61. The peak of Denali in Alaska is 20,320 feet above sea level. The peak of Long's Peak in Colorado is 14,255 feet above sea level. How much higher is the peak of Denali than Long's Peak? (*Source:* U.S. Geological Survey)

Denali, Alaska

Long's Peak, Colorado

62. On January 12, 1916, the city of Indianapolis, Indiana, had the greatest temperature change in a day. It dropped 58 degrees. If the high temperature was 68° Fahrenheit, what was the low temperature?

63. The Oroville Dam, on the Feather River, is the tallest dam in the United States at 754 feet. The Hoover Dam, on the Colorado River, is 726 feet high. How much taller is the Oroville Dam than the Hoover Dam? (*Source:* U.S. Bureau of Reclamation)

64. A new iPhone 7 with 32 GB costs $649. Jocelyn Robinson has $845 in her savings account. How much will she have left in her savings account after she buys the iPhone? (*Source:* Apple, Inc.)

65. The distance from Kansas City to Denver is 645 miles. Hays, Kansas, lies on the road between the two and is 287 miles from Kansas City. What is the distance between Hays and Denver?

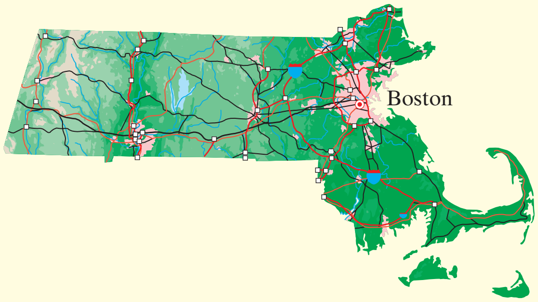
66. Pat Salanki's blood cholesterol level is 243. The doctor tells him it should be decreased to 185. How much of a decrease is this?

67. A new 4D Blu-ray player with streaming and Wi-Fi costs $295. A college student has $914 in her savings account. How much will she have left in her savings account after she buys the Blu-ray player?

68. A stereo that regularly sells for $547 is discounted by $99 in a sale. What is the sale price?

69. The population of Arizona is projected to grow from 6927 thousand in 2017 to 8536 thousand in 2030. What is Arizona's projected population increase over that time? (*Source:* Arizona Office of Economic Opportunity)

70. In 1996, the centennial of the Boston Marathon, the official number of participants was 38,708. In 2017, there were 6208 fewer participants. How many official participants were there for the 2017 Boston Marathon? (*Source:* Boston Athletic Association)



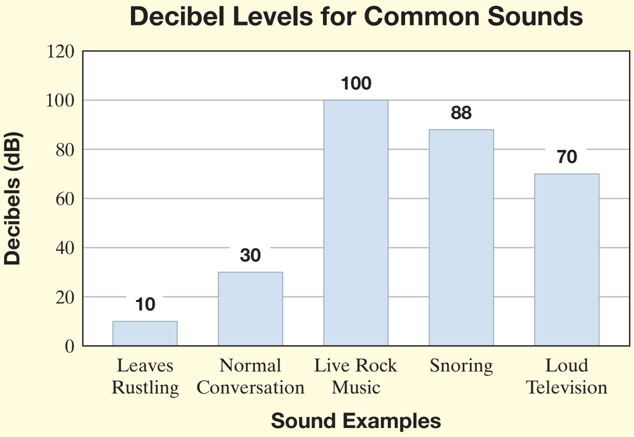
The decibel (dB) is a unit of measurement for sound. Every increase of 10 dB is a tenfold increase in sound intensity. The bar graph below shows the decibel levels for some common sounds. Use this graph for Exercises 71 through 74.

71. What is the dB rating for live rock music?

72. Which is the quietest of all the sounds shown in the graph?

73. How much louder is the sound of snoring than normal conversation?

74. What is the difference in sound intensity between live rock music and loud television?



75. As of this writing, there have been 45 U.S. presidents. Of these 15, were freemasons. How many U.S. presidents were not freemasons?

76. In 2016, the population of Springfield, Illinois, was 117,006, and the population of Champaign, Illinois, was 83,424. How much larger was Springfield than Champaign? (*Source:* U.S. Census Bureau)

77. Until recently, the world's largest permanent maze was located in Ruurlo, Netherlands. This maze of beech hedges covers 94,080 square feet. A new hedge maze using hibiscus bushes at the Dole Plantation in Wahiawa, Hawaii, covers 100,000 square feet. How much larger is the Dole Plantation maze than the Ruurlo maze? (*Source: The Guinness Book of Records*)

78. There were only 27 California condors in the entire world in 1987. To date, the number has increased to an estimated 276 living in the wild. How much of an increase is this? (*Source:* California Department of Fish and Wildlife)

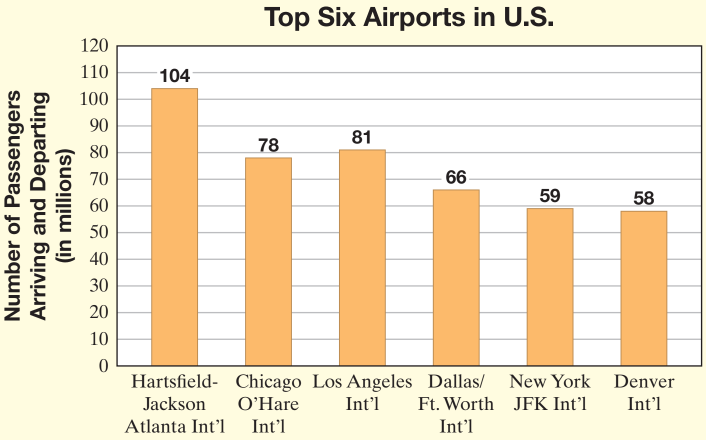
The bar graph shows the top six U.S. airports according to number of passengers arriving and departing in 2016. Use this graph to answer Exercises 79 through 82.

79. Which airport was the busiest?

80. Which airports had 60 million passengers or fewer per year?

81. How many more passengers per year did the Chicago O'Hare International Airport have than the Dallas/Ft. Worth International Airport?

82. How many more passengers per year did the Hartsfield-Jackson Atlanta International Airport have than the Los Angeles International Airport?



Source: Airports Council International

Solve.

83. Two seniors, Jo Keen and Trudy Waterbury, were candidates for student government president. Who won the election if the votes were cast as follows? By how many votes did the winner win?

	Candidate	
Class	**Jo**	**Trudy**
Freshman	276	295
Sophomore	362	122
Junior	201	312
Senior	179	18

84. Two students submitted advertising budgets for a student government fund-raiser.

	Student A	**Student B**
Radio ads	$600	$300
Newspaper ads	$200	$400
Posters	$150	$240
Handbills	$120	$170

If $1200 is available for advertising, how much excess would each budget have?

Mixed Practice *(Sections 1.3 and 1.4)* *Add or subtract as indicated.*

85. $\begin{array}{r} 986 \\ +\ 48 \\ \hline \end{array}$

86. $\begin{array}{r} 986 \\ -\ 48 \\ \hline \end{array}$

87. $76 - 67$

88. $80 + 93 + 17 + 9 + 2$

89. $\begin{array}{r} 9000 \\ -\ 482 \\ \hline \end{array}$

90. $\begin{array}{r} 10{,}000 \\ -\ 1786 \\ \hline \end{array}$

91. $\begin{array}{r} 10{,}962 \\ 4851 \\ +\ 7063 \\ \hline \end{array}$

92. $\begin{array}{r} 12{,}468 \\ 3211 \\ +\ 1988 \\ \hline \end{array}$

Concept Extensions

For each exercise, identify which number is the minuend and which number is the subtrahend. See the Concept Check in this section.

93. $\begin{array}{r} 48 \\ -\ 1 \\ \hline \end{array}$

94. $\begin{array}{r} 2863 \\ -1904 \\ \hline \end{array}$

95. Subtract 7 from 70.

96. Find 86 decreased by 25.

Identify each answer as correct or incorrect. Use addition to check. If the answer is incorrect, then write the correct answer.

97.
$$\begin{array}{r} 741 \\ -\ 56 \\ \hline 675 \end{array}$$

98.
$$\begin{array}{r} 478 \\ -\ 89 \\ \hline 389 \end{array}$$

99.
$$\begin{array}{r} 1029 \\ -\ 888 \\ \hline 141 \end{array}$$

100.
$$\begin{array}{r} 7615 \\ -\ 547 \\ \hline 7168 \end{array}$$

Fill in the missing digits in each problem.

101.
$$\begin{array}{r} 526\_ \\ -2\_85 \\ \hline 28\_4 \end{array}$$

102.
$$\begin{array}{r} 10,\_4\_ \\ -\ 85\_4 \\ \hline \_710 \end{array}$$

103. Is there a commutative property of subtraction? In other words, does order matter when subtracting? Why or why not?

104. Explain why the phrase "Subtract 7 from 10" translates to "10 − 7."

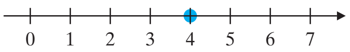
105. The local college library is having a Million Pages of Reading promotion. The freshmen have read a total of 289,462 pages; the sophomores have read a total of 369,477 pages; the juniors have read a total of 218,287 pages; and the seniors have read a total of 121,685 pages. Have they reached a goal of one million pages? If not, how many more pages need to be read?

1.5 Rounding and Estimating

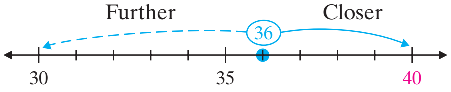
Objective A Rounding Whole Numbers

Rounding a whole number means approximating it. A rounded whole number is often easier to use, understand, and remember than the precise whole number. For example, instead of trying to remember the Missouri state population as 6,063,589, it is much easier to remember it rounded to the nearest million: 6,000,000, or 6 million people. (*Source:* U.S. Census)

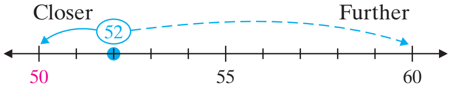
Recall from Section 1.2 that the line below is called a number line. To **graph** a whole number on this number line, we darken the point representing the location of the whole number. For example, the number 4 is graphed below.



On a number line, the whole number 36 is closer to 40 than 30, so 36 rounded to the nearest ten is 40.



The whole number 52 is closer to 50 than 60, so 52 rounded to the nearest ten is 50.

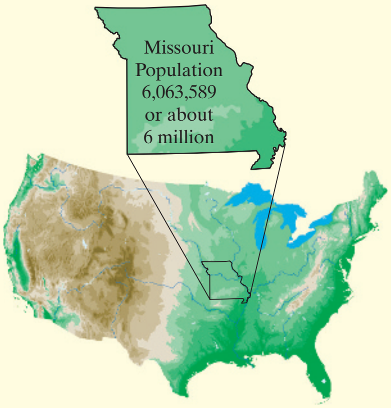


Objectives

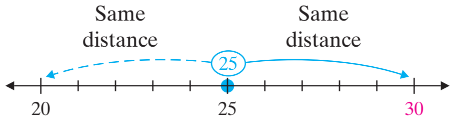
A Round Whole Numbers.

B Use Rounding to Estimate Sums and Differences.

C Solve Problems by Estimating.



In trying to round 25 to the nearest ten, we see that 25 is halfway between 20 and 30. It is not closer to either number. In such a case, we round to the larger ten, that is, to 30.



Here, we round "up."

To round a whole number without using a number line, follow these steps:

Rounding Whole Numbers to a Given Place Value

Step 1: Locate the digit to the right of the given place value.

Step 2: If this digit is 5 or greater, add 1 to the digit in the given place value and replace each digit to its right by 0.

Step 3: If this digit is less than 5, replace it and each digit to its right by 0.

Practice 1

Round to the nearest ten.

a. 57
b. 641
c. 325

Example 1

Round 568 to the nearest ten.

Solution: 5 6⑧ The digit to the right of the tens place is the ones place, which is circled.

↑
tens place

5 6⑧ Since the circled digit is 5 or greater, add 1 to the 6 in the tens place and replace the digit to the right by 0.

↑ Add 1. ↖ Replace with 0.

We find that 568 rounded to the nearest ten is 570.

Work Practice 1

Practice 2

Round to the nearest thousand.

a. 72,304
b. 9222
c. 671,800

Example 2

Round 278,362 to the nearest thousand.

Solution:

Thousands place ↓
↓ 3 is less than 5.

278,③62

↑ Do not add 1.
↑ Replace with zeros.

The number 278,362 rounded to the nearest thousand is 278,000.

Work Practice 2

Answers

1. a. 60 **b.** 640 **c.** 330
2. a. 72,000 **b.** 9000 **c.** 672,000

Example 3 Round 248,982 to the nearest hundred.

Solution:

Hundreds place

8 is greater than or equal to 5.

$248{,}9\,⑧\,2$

Add 1.

9 + 1 = 10, so replace the digit 9 by 0 and carry 1 to the place value to the left.

$2\ \ 4\ \ \overset{8+1}{\cancel{8}},\ \ \overset{0}{\cancel{9}}\ \ \underbrace{8\ \ 2}$

Add 1. Replace with zeros.

The number 248,982 rounded to the nearest hundred is 249,000.

Work Practice 3

Practice 3

Round to the nearest hundred.

- **a.** 3474
- **b.** 76,243
- **c.** 978,965

✓Concept Check Round each of the following numbers to the nearest *hundred*. Explain your reasoning.

a. 59 **b.** 29

Objective B Estimating Sums and Differences

By rounding addends, minuends, and subtrahends, we can estimate sums and differences. An estimated sum or difference is appropriate when the exact number is not necessary. Also, an estimated sum or difference can help us determine if we made a mistake in calculating an exact amount. To estimate the sum below, round each number to the nearest hundred and then add.

$$\begin{array}{rcr} 768 & \text{rounds to} & 800 \\ 1952 & \text{rounds to} & 2000 \\ 225 & \text{rounds to} & 200 \\ \underline{+\ 149} & \text{rounds to} & \underline{+\ 100} \\ & & 3100 \end{array}$$

The estimated sum is 3100, which is close to the **exact** sum of 3094.

Example 4 Round each number to the nearest hundred to find an estimated sum.

$$\begin{array}{r} 294 \\ 625 \\ 1071 \\ \underline{+\ 349} \end{array}$$

Solution:

$$\begin{array}{rcr} \textbf{Exact:} & & \textbf{Estimate:} \\ 294 & \text{rounds to} & 300 \\ 625 & \text{rounds to} & 600 \\ 1071 & \text{rounds to} & 1100 \\ \underline{+\ 349} & \text{rounds to} & \underline{+\ 300} \\ & & 2300 \end{array}$$

The estimated sum is 2300. (The exact sum is 2339.)

Work Practice 4

Practice 4

Round each number to the nearest ten to find an estimated sum.

$$\begin{array}{r} 49 \\ 25 \\ 32 \\ 51 \\ \underline{+\ 98} \end{array}$$

Answers

3. a. 3500 **b.** 76,200 **c.** 979,000
4. 260

✓Concept Check Answers

a. 100 **b.** 0

Practice 5

Round each number to the nearest thousand to find an estimated difference.

$$\begin{array}{r} 3785 \\ -\ 2479 \\ \hline \end{array}$$

Practice 6

Tasha Kilbey is trying to estimate how far it is from Gove, Kansas, to Hays, Kansas. Round each given distance on the map to the nearest ten to estimate the total distance.

Answers

5. 2000 **6.** 80 mi

Example 5

Round each number to the nearest hundred to find an estimated difference.

$$\begin{array}{r} 4725 \\ -\ 2879 \\ \hline \end{array}$$

Solution:

Exact:		**Estimate:**
4725	rounds to	4700
−2879	rounds to	−2900
		1800

The estimated difference is 1800. (The exact difference is 1846.)

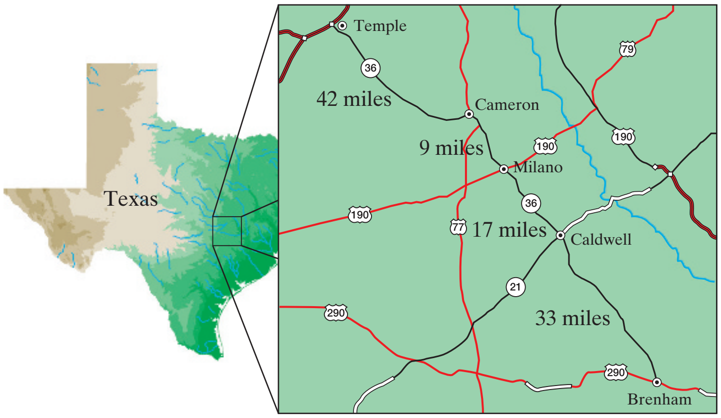
Work Practice 5

Objective C Solving Problems by Estimating

Making estimates is often the quickest way to solve real-life problems when solutions do not need to be exact.

Example 6 Estimating Distances

A driver is trying to quickly estimate the distance from Temple, Texas, to Brenham, Texas. Round each distance given on the map to the nearest ten to estimate the total distance.



Solution:

Exact Distance:		**Estimate:**
42	rounds to	40
9	rounds to	10
17	rounds to	20
+33	rounds to	+30
		100

It is approximately 100 miles from Temple to Brenham. (The exact distance is 101 miles.)

Work Practice 6

Example 7 Estimating Data

In three months in 2016, the numbers of tons of mail that went through Hartsfield-Jackson Atlanta International Airport were 1993, 2538, and 3033. Round each number to the nearest hundred to estimate the tons of mail that passed through this airport.

Solution:

Exact Tons of Mail:		Estimate:
1993	rounds to	2000
2538	rounds to	2500
+3033	rounds to	+3000
		7500

The approximate tonnage of mail that moved through Atlanta's airport over this period was 7500 tons. (The exact tonnage was 7564 tons.)

Work Practice 7

Practice 7

In 2015, there were 2930 reported cases of mumps, 18,166 reported cases of pertussis (whooping cough), and 189 reported cases of measles. Round each number to the nearest thousand to estimate the total number of cases reported for these preventable diseases. (*Source:* Centers for Disease Control)

Answer

7. 21,000 total cases

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

60
70
rounding
estimate
exact
graph

1. To __________ a number on a number line, darken the point representing the location of the number.

2. Another word for approximating a whole number is __________.

3. The number 65 rounded to the nearest ten is __________, but the number 61 rounded to the nearest ten is __________.

4. A(n) __________ number of products is 1265, but a(n) __________ is 1000.

Martin-Gay Interactive Videos

See Video 1.5

Watch the section lecture video and answer the following questions.

Objective A **5.** In Example 1, when rounding the number to the nearest ten, why do we replace the digit 3 with a 4?

Objective B **6.** As discussed in Example 3, explain how a number line can help us understand how to round 22 to the nearest ten.

Objective C **7.** What is the significance of the circled digit in each height value in Example 5?

1.5 Exercise Set MyLab Math

Objective A *Round each whole number to the given place. See Examples 1 through 3.*

1. 423 to the nearest ten

2. 273 to the nearest ten

3. 635 to the nearest ten

4. 846 to the nearest ten

5. 2791 to the nearest hundred

6. 8494 to the nearest hundred

7. 495 to the nearest ten

8. 898 to the nearest ten

9. 21,094 to the nearest thousand

10. 82,198 to the nearest thousand

11. 33,762 to the nearest thousand

12. 42,682 to the nearest ten-thousand

13. 328,495 to the nearest hundred

14. 179,406 to the nearest hundred

15. 36,499 to the nearest thousand

16. 96,501 to the nearest thousand

17. 39,994 to the nearest ten

18. 99,995 to the nearest ten

19. 29,834,235 to the nearest ten-million

20. 39,523,698 to the nearest million

Complete the table by estimating the given number to the given place value.

		Tens	Hundreds	Thousands
21.	5281			
22.	7619			
23.	9444			
24.	7777			
25.	14,876			
26.	85,049			

Round each number to the indicated place.

27. The University of California, Los Angeles, had a total undergraduate enrollment of 27,214 students in fall 2016. Round this number to the nearest thousand. (*Source:* UCLA)

28. In 2016, there were 15,667 Burger King restaurants worldwide. Round this number to the nearest thousand. (*Source:* Burger King Worldwide, Inc.)

29. Kareem Abdul-Jabbar holds the NBA record for points scored, a total of 38,387 over his NBA career. Round this number to the nearest thousand. (*Source:* National Basketball Association)

30. It takes 60,149 days for Neptune to make a complete orbit around the Sun. Round this number to the nearest hundred. (*Source:* National Space Science Data Center)

31. In 2016, the most valuable brand in the world was Apple, Inc. The estimated brand value of Apple was $154,118,000,000. Round this to the nearest ten billion. (*Source: Forbes*)

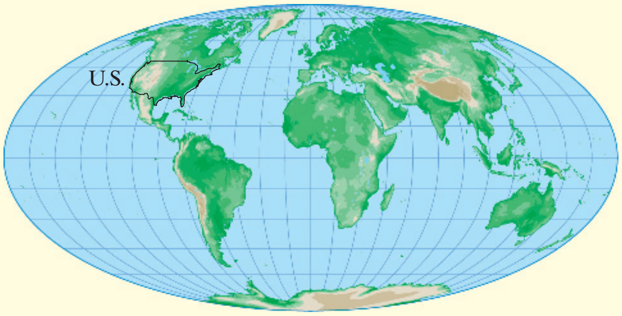
32. According to the U.S. Population Clock, the population of the United States was 324,758,293 in March 2017. Round this population figure to the nearest million. (*Source:* U.S. Census population clock)

33. The average salary for a professional baseball player in 2016 was $4,155,907. Round this average salary to the nearest hundred thousand. (*Source: ESPN*)

34. The average salary for a professional football player in 2016 was $2,110,000. Round this average salary to the nearest million. (*Source: ESPN*)

35. The United States currently has 219,600,000 smart phone users. Round this number to the nearest million. (*Source:* Pew Internet Research)

36. U.S. farms produced 15,226,000,000 bushels of corn in 2016. Round the corn production figure to the nearest ten million. (*Source:* U.S. Department of Agriculture)



Objective B *Estimate the sum or difference by rounding each number to the nearest ten. See Examples 4 and 5.*

▶ **37.**
$$\begin{array}{r} 39\\ 45\\ 22\\ +17\\ \hline \end{array}$$

38.
$$\begin{array}{r} 52\\ 33\\ 15\\ +29\\ \hline \end{array}$$

39.
$$\begin{array}{r} 449\\ -373\\ \hline \end{array}$$

40.
$$\begin{array}{r} 555\\ -235\\ \hline \end{array}$$

Estimate the sum or difference by rounding each number to the nearest hundred. See Examples 4 and 5.

41.
$$\begin{array}{r} 1913\\ 1886\\ +1925\\ \hline \end{array}$$

42.
$$\begin{array}{r} 4050\\ 3133\\ +1220\\ \hline \end{array}$$

▶ **43.**
$$\begin{array}{r} 1774\\ -1492\\ \hline \end{array}$$

44.
$$\begin{array}{r} 1989\\ -1870\\ \hline \end{array}$$

45.
$$\begin{array}{r} 3995\\ 2549\\ +4944\\ \hline \end{array}$$

46.
$$\begin{array}{r} 799\\ 1655\\ +271\\ \hline \end{array}$$

Three of the given calculator answers below are incorrect. Find them by estimating each sum.

47. 463 + 219 602

48. 522 + 785 1307

49. 229 + 443 + 606 1278

50. 542 + 789 + 198 2139

51. 7806 + 5150 12,956

52. 5233 + 4988 9011

Helpful Hint

Estimation is useful to check for incorrect answers when using a calculator. For example, pressing a key too hard may result in a double digit, while pressing a key too softly may result in the digit not appearing in the display.

Objective C *Solve each problem by estimating. See Examples 6 and 7.*

53. An appliance store advertises three refrigerators on sale at $899, $1499, and $999. Round each cost to the nearest hundred to estimate the total cost.

54. Suppose you scored 89, 97, 100, 79, 75, and 82 on your biology tests. Round each score to the nearest ten to estimate your total score.

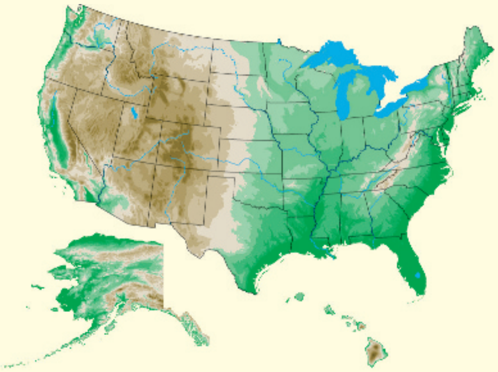
55. The distance from Kansas City to Boston is 1429 miles and from Kansas City to Chicago is 530 miles. Round each distance to the nearest hundred to estimate how much farther Boston is from Kansas City than Chicago is.

56. The Gonzales family took a trip and traveled 588, 689, 277, 143, 59, and 802 miles on six consecutive days. Round each distance to the nearest hundred to estimate the distance they traveled.

57. The peak of Denali, in Alaska, is 20,320 feet above sea level. The top of Mt. Rainier, in Washington, is 14,410 feet above sea level. Round each height to the nearest thousand to estimate the difference in elevation of these two peaks.(*Source:* U.S. Geological Survey)

58. A student is pricing new car stereo systems. One system sells for $1895 and another system sells for $1524. Round each price to the nearest hundred dollars to estimate the difference in price of these systems.

59. In 2014, the United States Postal Service delivered 155,410,000,000 pieces of mail. In 2016, it delivered 154,239,000,000 pieces of mail. Round each number to the nearest billion to estimate how much the mail volume decreased from 2014 to 2016. (*Source:* United States Postal Service)



60. Round each distance given on the map to the nearest ten to estimate the total distance from North Platte, Nebraska, to Lincoln, Nebraska.

61. Head Start is a national program that provides developmental and social services for America's low-income preschool children ages three to five. Enrollment figures in Head Start programs showed a decrease from 1,128,030 in 2012 to 946,357 in 2016. Round each number of children to the nearest thousand to estimate this decrease. (*Source:* U.S. Department of Health and Human Services)

62. Enrollment figures at a local community college showed an increase from 49,713 credit hours in 2015 to 51,746 credit hours in 2016. Round each number to the nearest thousand to estimate the increase.

Mixed Practice ***(Sections 1.2 and 1.5)*** *The following table shows the top five countries that spent the most in mobile Internet advertising in 2015 and the amount of money spent that year on advertising. Complete this table. The first line is completed for you.* (*Source:* eMarketer)

	Country	Amount Spent on Mobile Internet Advertising in 2015 (in millions of dollars)	Amount Written in Standard Form	Standard Form Rounded to Nearest Hundred-Million	Standard Form Rounded to Nearest Billion
	United States	$28,240	$28,240,000,000	$28,200,000,000	$28,000,000,000
63.	China	$12,140			
64.	Japan	$3370			
65.	Germany	$2110			
66.	United Kingdom	$4670			

Concept Extensions

67. Find one number that when rounded to the nearest hundred is 5700.

68. Find one number that when rounded to the nearest ten is 5700.

Round each number to the nearest hundred. See the Concept Check in this section.

69. 999

70. 950

71. 38

72. 48

A number rounded to the nearest hundred is 8600. Use this for Exercise 73 and 74.

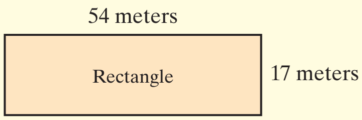
73. Determine the smallest possible number.

74. Determine the largest possible number.

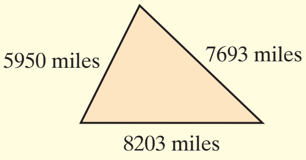
✎ **75.** In your own words, explain how to round a number to the nearest thousand.

✎ **76.** In your own words, explain how to round 9660 to the nearest thousand.

△ **77.** Estimate the perimeter of the rectangle by first rounding the length of each side to the nearest ten.



△ **78.** Estimate the perimeter of the triangle by first rounding the length of each side to the nearest hundred.

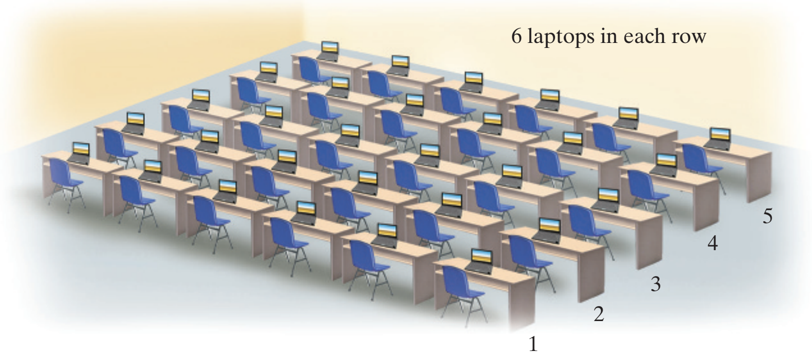


1.6 Multiplying Whole Numbers and Area

Objectives

- **A** Use the Properties of Multiplication.
- **B** Multiply Whole Numbers.
- **C** Multiply by Whole Numbers Ending in Zero(s).
- **D** Find the Area of a Rectangle.
- **E** Solve Problems by Multiplying Whole Numbers.

Multiplication Shown as Repeated Addition Suppose that we wish to count the number of laptops provided in a computer class. The laptops are arranged in 5 rows, and each row has 6 laptops.



Adding 5 sixes gives the total number of laptops. We can write this as $6 + 6 + 6 + 6 + 6 = 30$ laptops. When each addend is the same, we refer to this as **repeated addition.**

Multiplication is repeated addition but with different notation.

$$\begin{array}{ccccccc} 6 + 6 + 6 + 6 + 6 & = & 5 & \times & 6 & = & 30 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{5 addends; each addend is 6} & & \text{(number of addends) factor} & & \text{(each addend) factor} & & \text{product} \end{array}$$

The × is called a **multiplication sign.** The numbers 5 and 6 are called **factors.** The number 30 is called the **product.** The notation 5×6 is read as "five times six." The symbols · and ( ) can also be used to indicate multiplication.

$$5 \times 6 = 30, \quad 5 \cdot 6 = 30, \quad (5)(6) = 30, \quad \text{and} \quad 5(6) = 30$$

✓ Concept Check

a. Rewrite $5 + 5 + 5 + 5 + 5 + 5 + 5$ using multiplication.

b. Rewrite 3×16 as repeated addition. Is there more than one way to do this? If so, show all ways.

Objective A Using the Properties of Multiplication

As with addition, we memorize products of one-digit whole numbers and then use certain properties of multiplication to multiply larger numbers. (If necessary, review the multiplication of one-digit numbers in Appendix A.2.)

Notice that when any number is multiplied by 0, the result is always 0. This is called the **multiplication property of 0.**

✓ Concept Check Answers

a. $7 \times 5 = 35$

b. $16 + 16 + 16 = 48$; yes, $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 48$

Multiplication Property of 0

The product of 0 and any number is 0. For example,

$5 \cdot 0 = 0$ and $0 \cdot 8 = 0$

Also notice in Appendix A.2 that when any number is multiplied by 1, the result is always the original number. We call this result the **multiplication property of 1.**

Multiplication Property of 1

The product of 1 and any number is that same number. For example,

$1 \cdot 9 = 9$ and $6 \cdot 1 = 6$

Example 1 Multiply.

a. 6×1 **b.** $0(18)$ **c.** $1 \cdot 45$ **d.** $(75)(0)$

Solution:

a. $6 \times 1 = 6$ **b.** $0(18) = 0$

c. $1 \cdot 45 = 45$ **d.** $(75)(0) = 0$

Work Practice 1

Practice 1

Multiply.

a. 3×0

b. $4(1)$

c. $(0)(34)$

d. $1 \cdot 76$

Like addition, multiplication is commutative and associative. Notice that when multiplying two numbers, the order of these numbers can be changed without changing the product. For example,

$3 \cdot 5 = 15$ and $5 \cdot 3 = 15$

This property is the **commutative property of multiplication.**

Commutative Property of Multiplication

Changing the **order** of two factors does not change their product. For example,

$9 \cdot 2 = 18$ and $2 \cdot 9 = 18$

Another property that can help us when multiplying is the **associative property of multiplication.** This property states that when multiplying numbers, the grouping of the numbers can be changed without changing the product. For example,

$$\underbrace{(2 \cdot 3)} \cdot 4 = 6 \cdot 4 = 24$$

Also,

$$2 \cdot \underbrace{(3 \cdot 4)} = 2 \cdot 12 = 24$$

Both groupings give a product of 24.

Answers

1. a. 0 **b.** 4 **c.** 0 **d.** 76

Associative Property of Multiplication

Changing the **grouping** of factors does not change their product. From the previous page, we know that for example,

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$$

With these properties, along with the **distributive property,** we can find the product of any whole numbers. The distributive property says that multiplication **distributes** over addition. For example, notice that $3(2 + 5)$ simplifies to the same number as $3 \cdot 2 + 3 \cdot 5$.

$$3\underbrace{(2 + 5)} = 3(7) = 21$$

$$\underbrace{3 \cdot 2} + \overbrace{3 \cdot 5} = 6 + 15 = 21$$

Since $3(2 + 5)$ and $3 \cdot 2 + 3 \cdot 5$ both simplify to 21, then

$$3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$$

Notice in $3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$ that each number inside the parentheses is multiplied by 3.

Distributive Property

Multiplication distributes over addition. For example,

$$2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$$

Practice 2

Rewrite each using the distributive property.

a. $5(2 + 3)$
b. $9(8 + 7)$
c. $3(6 + 1)$

Example 2 Rewrite each using the distributive property.

a. $3(4 + 5)$ **b.** $10(6 + 8)$ **c.** $2(7 + 3)$

Solution: Using the distributive property, we have

a. $3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$
b. $10(6 + 8) = 10 \cdot 6 + 10 \cdot 8$
c. $2(7 + 3) = 2 \cdot 7 + 2 \cdot 3$

Work Practice 2

Objective B Multiplying Whole Numbers

Let's use the distributive property to multiply $7(48)$. To do so, we begin by writing the expanded form of 48 (see Section 1.2) and then applying the distributive property.

$$\begin{aligned} 7(48) &= 7(40 + 8) && \text{Write 48 in expanded form.} \\ &= 7 \cdot 40 + 7 \cdot 8 && \text{Apply the distributive property.} \\ &= 280 + 56 && \text{Multiply.} \\ &= 336 && \text{Add.} \end{aligned}$$

Answers

2. a. $5(2 + 3) = 5 \cdot 2 + 5 \cdot 3$
b. $9(8 + 7) = 9 \cdot 8 + 9 \cdot 7$
c. $3(6 + 1) = 3 \cdot 6 + 3 \cdot 1$

This is how we multiply whole numbers. When multiplying whole numbers, we will use the following notation.

First:

$$\begin{array}{r} \overset{5}{4}8 \\ \times\ \ 7 \\ \hline 6 \end{array} \longleftarrow 7 \cdot 8 = 56$$

Write 6 in the ones place and carry 5 to the tens place.

Next:

$$\begin{array}{r} \overset{5}{4}8 \\ \times\ \ 7 \\ \hline 336 \end{array} \quad 7 \cdot 4 + 5 = 28 + 5 = 33$$

The product of 48 and 7 is 336.

Example 3 Multiply:

a. $\begin{array}{r} 25 \\ \times\ \ 8 \\ \hline \end{array}$ **b.** $\begin{array}{r} 246 \\ \times\ \ \ 5 \\ \hline \end{array}$

Solution:

a. $\begin{array}{r} \overset{4}{2}5 \\ \times\ \ 8 \\ \hline 200 \end{array}$ **b.** $\begin{array}{r} \overset{2}{2}\overset{3}{4}6 \\ \times\ \ \ 5 \\ \hline 1230 \end{array}$

Work Practice 3

To multiply larger whole numbers, use the following similar notation. Multiply 89 × 52.

Step 1

$$\begin{array}{r} \overset{1}{8}9 \\ \times\ 52 \\ \hline 178 \end{array} \longleftarrow \text{Multiply } 89 \times 2.$$

Step 2

$$\begin{array}{r} \overset{4}{8}9 \\ \times\ 52 \\ \hline 178 \\ 4450 \\ \hline \end{array} \longleftarrow \text{Multiply } 89 \times 50.$$

Step 3

$$\begin{array}{r} 89 \\ \times\ 52 \\ \hline 178 \\ 4450 \\ \hline 4628 \end{array} \longleftarrow \text{Add.}$$

The numbers 178 and 4450 are called **partial products.** The sum of the partial products, 4628, is the product of 89 and 52.

Example 4 Multiply: 236 × 86

Solution:

$$\begin{array}{rl} 236 & \\ \times\ \ \ 86 & \\ \hline 1416 & \longleftarrow 6(236) \\ 18880 & \longleftarrow 80(236) \\ \hline 20{,}296 & \text{Add.} \end{array}$$

Work Practice 4

Example 5 Multiply: 631 × 125

Solution:

$$\begin{array}{rl} 631 & \\ \times\ \ 125 & \\ \hline 3155 & \longleftarrow 5(631) \\ 12620 & \longleftarrow 20(631) \\ 63100 & \longleftarrow 100(631) \\ \hline 78{,}875 & \text{Add.} \end{array}$$

Work Practice 5

Practice 3

Multiply.

a. $\begin{array}{r} 36 \\ \times\ \ 4 \\ \hline \end{array}$ **b.** $\begin{array}{r} 132 \\ \times\ \ \ 9 \\ \hline \end{array}$

Practice 4

Multiply.

a. $\begin{array}{r} 594 \\ \times\ 72 \\ \hline \end{array}$ **b.** $\begin{array}{r} 306 \\ \times\ 81 \\ \hline \end{array}$

Practice 5

Multiply.

a. $\begin{array}{r} 726 \\ \times 142 \\ \hline \end{array}$ **b.** $\begin{array}{r} 288 \\ \times\ \ \ 4 \\ \hline \end{array}$

Answers

3. a. 144 **b.** 1188
4. a. 42,768 **b.** 24,786
5. a. 103,092 **b.** 1152

✓ **Concept Check** Find and explain the error in the following multiplication problem.

$$\begin{array}{r} 102 \\ \times\ \ 33 \\ \hline 306 \\ 306 \\ \hline 612 \end{array}$$

Objective C Multiplying by Whole Numbers Ending in Zero(s)

Interesting patterns occur when we multiply by a number that ends in zeros. To see these patterns, let's multiply a number, say 34, by 10, then 100, then 1000.

$$34 \cdot 1\overset{\overset{\text{1 zero}}{\downarrow}}{0} = 340 \qquad \text{1 zero attached to 34.}$$

$$34 \cdot 1\overbrace{00}^{\text{2 zeros}} = 3400 \qquad \text{2 zeros attached to 34.}$$

$$34 \cdot 1\overbrace{000}^{\text{3 zeros}} = 34{,}000 \qquad \text{3 zeros attached to 34.}$$

These patterns help us develop a shortcut for multiplying by whole numbers ending in zeros.

To multiply by 10, 100, 1000, and so on,

Form the product by attaching the number of zeros in that number to the other factor.

For example, $41 \cdot 1\underbrace{00}_{\text{2 zeros}} = 41\overset{\uparrow}{00}$.

Examples Multiply.

6. $176 \cdot 1000 = 176{,}\underbrace{000}$ Attach 3 zeros.

7. $2041 \cdot 100 = 204{,}1\underbrace{00}$ Attach 2 zeros.

Work Practice 6–7

Practice 6–7

Multiply.

6. $75 \cdot 100$

7. $808 \cdot 1000$

We can use a similar format to multiply by any whole number ending in zeros. For example, since

$$15 \cdot 500 = 15 \cdot 5 \cdot 100,$$

we find the product by multiplying 15 and 5, then attaching two zeros to the product.

$$\begin{array}{r} \overset{2}{1}5 \\ \times\ 5 \\ \hline 75 \end{array} \qquad 15 \cdot 500 = 7500$$

Answers

6. 7500 **7.** 808,000

✓ **Concept Check Answer**

$$\begin{array}{r} 102 \\ \times\ \ 33 \\ \hline 306 \\ 3060 \\ \hline 3366 \end{array}$$

Examples Multiply.

8. $25 \cdot 9000 = 225{,}000$ $\qquad \begin{array}{r} \overset{4}{2}5 \\ \times\ 9 \\ \hline 225 \end{array}$ Attach 3 zeros.

9. $20 \cdot 7000 = \underset{2 \cdot 7}{14}0{,}000$ Attach 4 zeros.

Work Practice 8–9

Practice 8–9

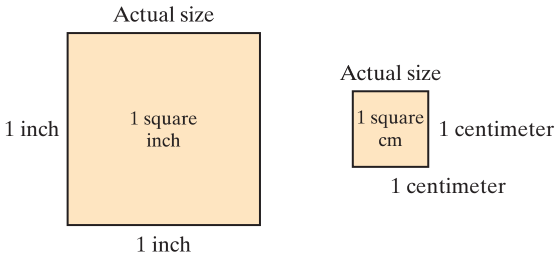
Multiply.

8. $35 \cdot 3000$

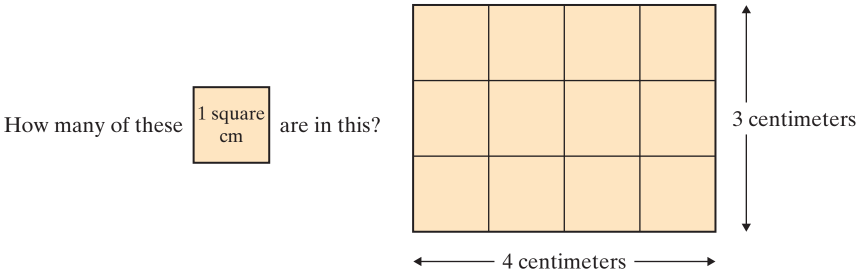
9. $600 \cdot 600$

Objective D Finding the Area of a Rectangle

A special application of multiplication is finding the **area** of a region. Area measures the amount of surface of a region. For example, we measure a plot of land or the living space of a home by its area. The figures below show two examples of units of area measure. (A centimeter is a unit of length in the metric system.)



For example, to measure the area of a geometric figure such as the rectangle below, count the number of square units that cover the region.



This rectangular region contains 12 square units, each 1 square centimeter. Thus, the area is 12 square centimeters. This total number of squares can be found by counting or by multiplying $\mathbf{4 \cdot 3}$ (length · width).

$$\begin{aligned} \text{Area of a rectangle} &= \text{length} \cdot \text{width} \\ &= (4 \text{ centimeters})(3 \text{ centimeters}) \\ &= 12 \text{ square centimeters} \end{aligned}$$

In this section, we find the areas of rectangles only. In later sections, we will find the areas of other geometric regions.

Helpful Hint

Notice that area is measured in **square** units while perimeter is measured in units.

Answers

8. 105,000 **9.** 360,000

Practice 10

The state of Wyoming is in the shape of a rectangle whose length is 360 miles and whose width is 280 miles. Find its area.

Example 10 Finding the Area of a State

The state of Colorado is in the shape of a rectangle whose length is 380 miles and whose width is 280 miles. Find its area.

Solution: The area of a rectangle is the product of its length and its width.

$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= (380 \text{ miles})(280 \text{ miles}) \\ &= 106{,}400 \text{ square miles}\end{aligned}$$

The area of Colorado is 106,400 square miles.

Work Practice 10

Objective E Solving Problems by Multiplying

There are several words or phrases that indicate the operation of multiplication. Some of these are as follows:

Multiplication

Key Words or Phrases	Examples	Symbols
multiply	multiply 5 by 7	$5 \cdot 7$
product	the product of 3 and 2	$3 \cdot 2$
times	10 times 13	$10 \cdot 13$

Many key words or phrases describing real-life problems that suggest addition might be better solved by multiplication instead. For example, to find the **total** cost of 8 shirts, each selling for $27, we can either add

$$27 + 27 + 27 + 27 + 27 + 27 + 27 + 27$$

or we can multiply $8(27)$.

Practice 11

A particular computer printer can print 16 pages per minute in color. How many pages can it print in 45 minutes?

Example 11 Finding DVD Space

A digital video disc (DVD) can hold about 4800 megabytes (MB) of information. How many megabytes can 12 DVDs hold?

Solution: Twelve DVDs will hold 12 × 4800 megabytes.

In Words		Translate to Numbers
megabytes per disc	→	4800
× number of DVDs	→	× 12
		9600
		48000
total megabytes		57,600

Twelve DVDs will hold 57,600 megabytes.

Work Practice 11

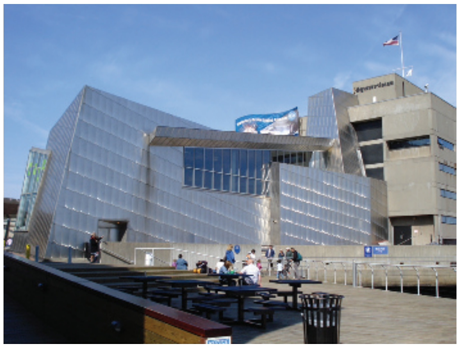
Answers
10. 100,800 sq mi **11.** 720 pages

Example 12 Budgeting Money

Suzanne Scarpulla and a friend plan to take their children to the New England Aquarium in Boston. The peak hour ticket price for each child is \$19 and for each adult \$27. If five children and two adults plan to go, how much money is needed for admission? (*Source:* New England Aquarium)

Solution: If the price of one child's ticket is \$19, the cost for 5 children is $5 \times 19 = \$95$. The price of one adult ticket is \$27, so the cost for two adults is $2 \times 27 = \$54$. The total cost is:

In Words		Translate to Numbers
cost for 5 children	→	95
+ cost for 2 adults	→	+ 54
total cost		149



The total cost is \$149.

Work Practice 12

Practice 12

Ken Shimura purchased DVDs and CDs through a club. Each DVD was priced at \$11, and each CD cost \$9. Ken bought eight DVDs and five CDs. Find the total cost of the order.

Example 13 Estimating Word Count

The average page of a book contains 259 words. Estimate, rounding each number to the nearest hundred, the total number of words contained on 212 pages.

Solution: The exact number of words is 259×212. Estimate this product by rounding each factor to the nearest hundred.

259	rounds to	300
× 212	rounds to	× 200,

$$300 \times 200 = 60{,}000$$

$$3 \cdot 2 = 6$$

There are approximately 60,000 words contained on 212 pages.

Work Practice 13

Practice 13

If an average page in a book contains 163 words, estimate, rounding each number to the nearest hundred, the total number of words contained on 391 pages.

Answers

12. \$133 **13.** 80,000 words

Calculator Explorations Multiplying Numbers

To multiply numbers on a calculator, find the keys marked $\boxed{\times}$ and $\boxed{=}$ or $\boxed{\text{ENTER}}$. For example, to find $31 \cdot 66$ on a calculator, press the keys $\boxed{31}$ $\boxed{\times}$ $\boxed{66}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$. The display will read $\boxed{\quad 2046}$. Thus, $31 \cdot 66 = 2046$.

Use a calculator to multiply.

1. 72×48

2. 81×92

3. $163 \cdot 94$

4. $285 \cdot 144$

5. 983(277)

6. 1562(843)

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

area
factor
grouping
order
commutative
associative
1
0
product
distributive
length
number

1. The product of 0 and any number is ________.
2. The product of 1 and any number is the ________.
3. In $8 \cdot 12 = 96$, the 96 is called the ________ and 8 and 12 are each called a(n) ________.
4. Since $9 \cdot 10 = 10 \cdot 9$, we say that changing the ________ in multiplication does not change the product. This property is called the ________ property of multiplication.
5. Since $(3 \cdot 4) \cdot 6 = 3 \cdot (4 \cdot 6)$, we say that changing the ________ in multiplication does not change the product. This property is called the ________ property of multiplication.
6. ________ measures the amount of surface of a region.
7. Area of a rectangle = ________ · width.
8. We know $9(10 + 8) = 9 \cdot 10 + 9 \cdot 8$ by the ________ property.

Martin-Gay Interactive Videos

See Video 1.6

Watch the section lecture video and answer the following questions.

Objective A **9.** The expression in Example 3 is rewritten using what property?

Objective B **10.** During the multiplication process for Example 5, why is a single zero placed at the end of the second partial product?

Objective C **11.** Explain two different approaches to solving the multiplication problem $50 \cdot 900$ in Example 7.

Objective D **12.** Why are the units to the answer to Example 8 not just meters? What are the correct units?

Objective E **13.** In Example 9, why can “total” imply multiplication as well as addition?

1.6 Exercise Set MyLab Math

Objective A *Multiply. See Example 1.*

1. $1 \cdot 24$
2. $55 \cdot 1$
3. $0 \cdot 19$
4. $27 \cdot 0$
5. $8 \cdot 0 \cdot 9$
6. $7 \cdot 6 \cdot 0$
7. $87 \cdot 1$
8. $1 \cdot 41$

Use the distributive property to rewrite each expression. See Example 2.

9. $6(3 + 8)$
10. $5(8 + 2)$
11. $4(3 + 9)$
12. $6(1 + 4)$
13. $20(14 + 6)$
14. $12(12 + 3)$

Objective B *Multiply. See Example 3.*

15. $\begin{array}{r} 64 \\ \times\ 8 \\ \hline \end{array}$

16. $\begin{array}{r} 79 \\ \times\ 3 \\ \hline \end{array}$

17. $\begin{array}{r} 613 \\ \times\ \ 6 \\ \hline \end{array}$

18. $\begin{array}{r} 638 \\ \times\ \ 5 \\ \hline \end{array}$

19. 277×6

20. 882×2

21. 1074×6

22. 9021×3

Objectives A B Mixed Practice *Multiply. See Examples 1 through 5.*

23. $\begin{array}{r} 89 \\ \times 13 \\ \hline \end{array}$

24. $\begin{array}{r} 91 \\ \times 72 \\ \hline \end{array}$

25. $\begin{array}{r} 421 \\ \times\ 58 \\ \hline \end{array}$

26. $\begin{array}{r} 526 \\ \times\ 23 \\ \hline \end{array}$

27. $\begin{array}{r} 306 \\ \times\ 81 \\ \hline \end{array}$

28. $\begin{array}{r} 708 \\ \times\ 21 \\ \hline \end{array}$

29. $(780)(20)$

30. $(720)(80)$

31. $(495)(13)(0)$

32. $(593)(47)(0)$

33. $(640)(1)(10)$

34. $(240)(1)(20)$

35. 1234×39

36. 1357×79

37. 609×234

38. 807×127

39. $\begin{array}{r} 8649 \\ \times\ \ 274 \\ \hline \end{array}$

40. $\begin{array}{r} 1234 \\ \times\ \ 567 \\ \hline \end{array}$

41. $\begin{array}{r} 589 \\ \times 110 \\ \hline \end{array}$

42. $\begin{array}{r} 426 \\ \times 110 \\ \hline \end{array}$

43. $\begin{array}{r} 1941 \\ \times 2035 \\ \hline \end{array}$

44. $\begin{array}{r} 1876 \\ \times 1407 \\ \hline \end{array}$

Objective C *Multiply. See Examples 6 through 9.*

45. 8×100

46. 6×100

47. 11×1000

48. 26×1000

49. $7406 \cdot 10$

50. $9054 \cdot 10$

51. $6 \cdot 4000$

52. $3 \cdot 9000$

53. $50 \cdot 900$

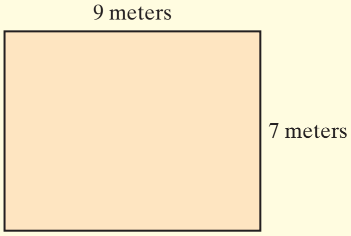
54. $70 \cdot 300$

55. $41 \cdot 80{,}000$

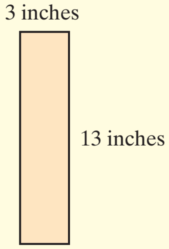
56. $27 \cdot 50{,}000$

Objective D Mixed Practice *(Sections 1.3 and 1.6)* *Find the area and the perimeter of each rectangle. See Example 10.*

57.

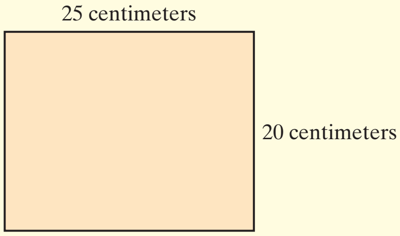


58.



59.

60.



Objective E Mixed Practice *(Section 1.5)*

Estimate the products by rounding each factor to the nearest hundred. See Example 13.

61. 576 × 354

62. 982 × 650

63. 604 × 451

64. 111 × 999

Without actually calculating, mentally round, multiply, and choose the best estimate.

65. 38 × 42 =
- **a.** 16
- **b.** 160
- **c.** 1600
- **d.** 16,000

66. 2872 × 12 =
- **a.** 2872
- **b.** 28,720
- **c.** 287,200
- **d.** 2,872,000

67. 612 × 29 =
- **a.** 180
- **b.** 1800
- **c.** 18,000
- **d.** 180,000

68. 706 × 409 =
- **a.** 280
- **b.** 2800
- **c.** 28,000
- **d.** 280,000

Objectives D E Mixed Practice–Translating

Solve. See Examples 10 through 13.

69. Multiply 80 by 11.

70. Multiply 70 by 12.

71. Find the product of 6 and 700.

72. Find the product of 9 and 900.

73. Find 2 times 2240.

74. Find 3 times 3310.

75. One tablespoon of olive oil contains 125 calories. How many calories are in 3 tablespoons of olive oil? (*Source: Home and Garden Bulletin No. 72,* U.S. Department of Agriculture)

76. One ounce of hulled sunflower seeds contains 14 grams of fat. How many grams of fat are in 8 ounces of hulled sunflower seeds? (*Source: Home and Garden Bulletin No. 72,* U.S. Department of Agriculture)

77. The textbook for a course in biology costs $94. There are 35 students in the class. Find the total cost of the biology books for the class.

78. The seats in a lecture hall are arranged in 14 rows with 34 seats in each row. Find how many seats are in this room.

79. Cabot Creamery is packing a pallet of 20-lb boxes of cheddar cheese to send to a local restaurant. There are five layers of boxes on the pallet, and each layer is four boxes wide by five boxes deep.
- **a.** How many boxes are in one layer?
- **b.** How many boxes are on the pallet?
- **c.** What is the weight of the cheese on the pallet?

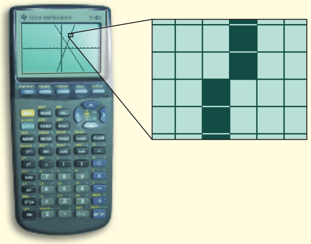
80. An apartment building has *three floors*. Each floor has five rows of apartments with four apartments in each row.
- **a.** How many apartments are on 1 floor?
- **b.** How many apartments are in the building?

△ **81.** A plot of land measures 80 feet by 110 feet. Find its area.

△ **82.** A house measures 45 feet by 60 feet. Find the floor area of the house.

△ **83.** The largest hotel lobby can be found at the Hyatt Regency in San Francisco, CA. It is in the shape of a rectangle that measures 350 feet by 160 feet. Find its area.

85. A pixel is a rectangular dot on a graphing calculator screen. If a graphing calculator screen contains 62 pixels in a row and 94 pixels in a column, find the total number of pixels on a screen.



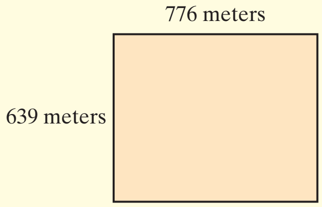
87. A line of print on a computer contains 60 characters (letters, spaces, punctuation marks). Find how many characters there are in 35 lines.

89. One ounce of Planters® Dry Roasted Peanuts has 170 calories. How many calories are in 8 ounces? (*Source:* Kraft Foods)

91. The Thespian club at a local community college is ordering T-shirts. T-shirts size S, M, or L cost $10 each and T-shirts size XL or XXL cost $12 each. Use the table below to find the total cost. (The first row is filled in for you.)

T-Shirt Size	Number of Shirts Ordered	Cost per Shirt	Cost per Size Ordered
S	4	$10	$40
M	6		
L	20		
XL	3		
XXL	3		

△ **84.** Recall from an earlier section that the world's largest commercial building under one roof is the flower auction building of the cooperative VBA in Aalsmeer, Netherlands. The floor plan is a rectangle that measures 776 meters by 639 meters. Find the area of this building. (*Source: The Handy Science Answer Book,* Visible Ink Press)



86. A certain compact disc (CD) can hold 700 megabytes (MB) of information. How many MB can 17 discs hold?

88. An average cow eats 3 pounds of grain per day. Find how much grain a cow eats in a year. (Assume 365 days in 1 year.)

90. One ounce of Planters® Dry Roasted Peanuts has 14 grams of fat. How many grams of fat are in 16 ounces? (*Source:* Kraft Foods)

92. The student activities group at North Shore Community College is planning a trip to see the local minor league baseball team. Tickets cost $5 for students, $7 for nonstudents, and $2 for children under 12. Use the following table to find the total cost.

Person	Number of Persons	Cost per Person	Cost per Category
Student	24	$5	$120
Nonstudent	4		
Children under 12	5		

93. Celestial Seasonings of Boulder, Colorado, is a tea company that specializes in herbal teas, accounting for over $100,000,000 in herbal tea blend sales in the United States annually. Their plant in Boulder has bagging machines capable of bagging over 1000 bags of tea per minute. If the plant runs 24 hours a day, how many tea bags are produced in one day? (*Source:* Celestial Seasonings)

94. The number of "older" Americans (ages 65 and older) has increased sixteenfold since 1900. If there were 3 million "older" Americans in 1900, how many were there in 2016? (*Source:* U.S. Census Bureau)

Mixed Practice (*Sections 1.3, 1.4, and 1.6*) *Perform each indicated operation.*

95. $$\begin{array}{r} 128 \\ \underline{+\quad 7} \end{array}$$

96. $$\begin{array}{r} 126 \\ \underline{-\quad 8} \end{array}$$

97. $$\begin{array}{r} 134 \\ \underline{\times\ \ 16} \end{array}$$

98. $47 + 26 + 10 + 231 + 50$

99. Find the sum of 19 and 4.

100. Find the product of 19 and 4.

101. Find the difference of 19 and 4.

102. Find the total of 19 and 4.

Concept Extensions

Solve. See the first Concept Check in this section.

103. Rewrite $7 + 7 + 7 + 7$ using multiplication.

104. Rewrite $11 + 11 + 11 + 11 + 11 + 11$ using multiplication.

105. **a.** Rewrite $3 \cdot 5$ as repeated addition.
b. Explain why there is more than one way to do this.

106. **a.** Rewrite $4 \cdot 5$ as repeated addition.
b. Explain why there is more than one way to do this.

Find and explain the error in each multiplication problem. See the second Concept Check in this section.

107. $$\begin{array}{r} 203 \\ \underline{\times\ 14} \\ 812 \\ \underline{203} \\ 1015 \end{array}$$

108. $$\begin{array}{r} 31 \\ \underline{\times\ 50} \\ 155 \end{array}$$

Fill in the missing digits in each problem.

109. $$\begin{array}{r} 4\underline{\ \ } \\ \underline{\times\quad \underline{\ \ }3} \\ 126 \\ \underline{3780} \\ 3906 \end{array}$$

110. $$\begin{array}{r} \underline{\ \ }7 \\ \underline{\times\ \ 6\underline{\ \ }} \\ 171 \\ \underline{3420} \\ 3591 \end{array}$$

111. Explain how to multiply two 2-digit numbers using partial products.

112. In your own words, explain the meaning of the area of a rectangle and how this area is measured.

113. A window washer in New York City is bidding for a contract to wash the windows of a 23-story building. To write a bid, the number of windows in the building is needed. If there are 7 windows in each row of windows on 2 sides of the building and 4 windows per row on the other 2 sides of the building, find the total number of windows.

114. During the 2015–2016 NBA regular season, Stephen Curry of the Golden State Warriors was named the Most Valuable Player. He scored 402 three-point field goals, 403 two-point field goals, and 363 free throws (worth one point each). How many points did Stephen Curry score during the 2015–2016 regular season? (*Source:* National Basketball Association)

1.7 Dividing Whole Numbers

Objectives

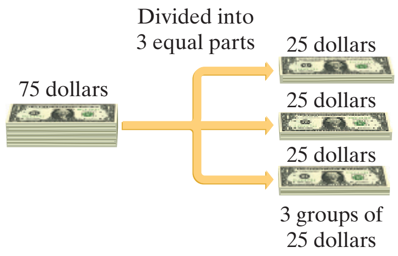
A Divide Whole Numbers.

B Perform Long Division.

C Solve Problems That Require Dividing by Whole Numbers.

D Find the Average of a List of Numbers.

Suppose three people pooled their money and bought a raffle ticket at a local fundraiser. Their ticket was the winner, and they won a $75 cash prize. They then divided the prize into three equal parts so that each person received $25.



Objective A Dividing Whole Numbers

The process of separating a quantity into equal parts is called **division.** The division above can be symbolized by several notations.

$$\begin{array}{l} \text{quotient} \\ \quad\llcorner\!\rightarrow 25 \\ \quad 3\overline{)75} \longleftarrow \text{dividend} \\ \quad\uparrow \\ \quad\text{divisor} \end{array} \qquad \frac{\overset{\substack{\text{dividend}\\ \downarrow}}{75}}{\underset{\substack{\uparrow\\ \text{divisor}}}{3}} = 25 \longleftarrow \text{quotient}$$

$$\underset{\substack{\uparrow\\ \text{dividend}}}{75} \div \underset{\substack{\uparrow\\ \text{divisor}}}{3} = \overset{\substack{\text{quotient}\\ \downarrow}}{25} \qquad \overset{\substack{\text{dividend}\\ \downarrow}}{75}/\underset{\substack{\uparrow\\ \text{divisor}}}{3} = \overset{\substack{\text{quotient}\\ \downarrow}}{25}$$

(In the notation $\frac{75}{3}$, the bar separating 75 and 3 is called a **fraction bar.**) Just as subtraction is the reverse of addition, division is the reverse of multiplication. This means that division can be checked by multiplication.

$$\overset{25}{3\overline{)75}} \quad \text{because} \quad 25 \cdot 3 = 75$$

$$\boxed{\text{Quotient}} \cdot \boxed{\text{Divisor}} = \boxed{\text{Dividend}}$$

Since multiplication and division are related in this way, you can use your knowledge of multiplication facts (or study Appendix A.2) to review quotients of one-digit divisors if necessary.

Practice 1

Find each quotient. Check by multiplying.

a. $9\overline{)72}$

b. $40 \div 5$

c. $\frac{24}{6}$

Example 1

Find each quotient. Check by multiplying.

a. $42 \div 7$ **b.** $\frac{64}{8}$ **c.** $3\overline{)21}$

Solution:

a. $42 \div 7 = 6$ because $6 \cdot 7 = 42$

b. $\frac{64}{8} = 8$ because $8 \cdot 8 = 64$

c. $\begin{array}{r}7\\3\overline{)21}\end{array}$ because $7 \cdot 3 = 21$

Work Practice 1

Practice 2

Find each quotient. Check by multiplying.

a. $\frac{7}{7}$ **b.** $5 \div 1$

c. $1\overline{)11}$ **d.** $4 \div 1$

e. $\frac{10}{1}$ **f.** $21 \div 21$

Example 2

Find each quotient. Check by multiplying.

a. $1\overline{)7}$ **b.** $12 \div 1$ **c.** $\frac{6}{6}$ **d.** $9 \div 9$ **e.** $\frac{20}{1}$ **f.** $19\overline{)19}$

Solution:

a. $\begin{array}{r}7\\1\overline{)7}\end{array}$ because $7 \cdot 1 = 7$

b. $12 \div 1 = 12$ because $12 \cdot 1 = 12$

c. $\frac{6}{6} = 1$ because $1 \cdot 6 = 6$

d. $9 \div 9 = 1$ because $1 \cdot 9 = 9$

e. $\frac{20}{1} = 20$ because $20 \cdot 1 = 20$

f. $\begin{array}{r}1\\19\overline{)19}\end{array}$ because $1 \cdot 19 = 19$

Work Practice 2

Example 2 illustrates the important properties of division described next:

Division Properties of 1

The quotient of any number (except 0) and that same number is 1. For example,

$$8 \div 8 = 1 \qquad \frac{5}{5} = 1 \qquad \begin{array}{r}1\\4\overline{)4}\end{array}$$

The quotient of any number and 1 is that same number. For example,

$$9 \div 1 = 9 \qquad \frac{6}{1} = 6 \qquad \begin{array}{r}3\\1\overline{)3}\end{array} \qquad \frac{0}{1} = 0$$

Practice 3

Find each quotient. Check by multiplying.

a. $\frac{0}{7}$ **b.** $8\overline{)0}$

c. $5 \div 0$ **d.** $0 \div 14$

Answers

1. a. 8 **b.** 8 **c.** 4 **2. a.** 1 **b.** 5 **c.** 11 **d.** 4 **e.** 10 **f.** 1 **3. a.** 0 **b.** 0 **c.** undefined **d.** 0

Example 3

Find each quotient. Check by multiplying.

a. $9\overline{)0}$ **b.** $0 \div 12$ **c.** $\frac{0}{5}$ **d.** $\frac{3}{0}$

Solution:

a. $\begin{array}{r}0\\9\overline{)0}\end{array}$ because $0 \cdot 9 = 0$

b. $0 \div 12 = 0$ because $0 \cdot 12 = 0$

c. $\frac{0}{5} = 0$ because $0 \cdot 5 = 0$

d. If $\frac{3}{0}$ = a *number,* then the *number* times $0 = 3$. Recall from Section 1.6 that any number multiplied by 0 is 0 and not 3. We say, then, that $\frac{3}{0}$ is **undefined.**

Work Practice 3

Example 3 illustrates important division properties of 0.

Division Properties of 0

The quotient of 0 and any number (except 0) is 0. For example,

$$0 \div 9 = 0 \qquad \frac{0}{5} = 0 \qquad \begin{array}{r} 0 \\ 14\overline{)0} \end{array}$$

The quotient of any number and 0 is not a number. We say that

$$\frac{3}{0}, \quad 0\overline{)3}, \quad \text{and} \quad 3 \div 0$$

are **undefined.**

Objective B Performing Long Division

When dividends are larger, the quotient can be found by a process called **long division.** For example, let's divide 2541 by 3.

$$\text{divisor} \longrightarrow 3\overline{)\underset{\underset{\text{dividend}}{\uparrow}}{2541}}$$

We can't divide 3 into 2, so we try dividing 3 into the first two digits.

$$\begin{array}{rl} 8 & 25 \div 3 = 8 \text{ with 1 left, so our best estimate is 8. We place 8 over} \\ 3\overline{)2541} & \text{the 5 in 25.} \end{array}$$

Next, multiply 8 and 3 and subtract this product from 25. Make sure that this difference is less than the divisor.

$$\begin{array}{rl} 8 & \\ 3\overline{)2541} & \\ -24 & 8(3) = 24 \\ \hline 1 & 25 - 24 = 1, \text{ and 1 is less than the divisor 3.} \end{array}$$

Bring down the next digit and go through the process again.

$$\begin{array}{rl} 84 & 14 \div 3 = 4 \text{ with 2 left} \\ 3\overline{)2541} & \\ -24\downarrow & \\ \hline 14 & \\ -12 & 4(3) = 12 \\ \hline 2 & 14 - 12 = 2 \end{array}$$

Once more, bring down the next digit and go through the process.

$$\begin{array}{rl} 847 & 21 \div 3 = 7 \\ 3\overline{)2541} & \\ -24 & \\ \hline 14 & \\ -12\downarrow & \\ \hline 21 & \\ -21 & 7(3) = 21 \\ \hline 0 & 21 - 21 = 0 \end{array}$$

The quotient is 847. To check, see that $847 \times 3 = 2541$.

Practice 4

Divide. Check by multiplying.

a. $4908 \div 6$
b. $2212 \div 4$
c. $753 \div 3$

Example 4

Divide: $3705 \div 5$. Check by multiplying.

Solution:

$$\begin{array}{rl}
7 & 37 \div 5 = 7 \text{ with 2 left. Place this estimate, 7, over the 7 in 37.} \\
5\overline{)3705} & \\
\underline{-35\downarrow} & 7(5) = 35 \\
20 & 37 - 35 = 2\text{, and 2 is less than the divisor 5.} \\
 & \text{Bring down the 0.}
\end{array}$$

$$\begin{array}{rl}
74 & 20 \div 5 = 4 \\
5\overline{)3705} & \\
\underline{-35} & \\
20 & \\
\underline{-20\downarrow} & 4(5) = 20 \\
05 & 20 - 20 = 0\text{. and 0 is less than the divisor 5.} \\
 & \text{Bring down the 5.}
\end{array}$$

$$\begin{array}{rl}
741 & 5 \div 5 = 1 \\
5\overline{)3705} & \\
\underline{-35} & \\
20 & \\
\underline{-20\downarrow} & \\
5 & \\
\underline{-5} & 1(5) = 5 \\
0 & 5 - 5 = 0
\end{array}$$

Helpful Hint Since division and multiplication are reverse operations, don't forget that a division problem can be checked by multiplying.

Check:

$$\begin{array}{r}
741 \\
\underline{\times \quad 5} \\
3705
\end{array}$$

Work Practice 4

Practice 5

Divide and check by multiplying.

a. $7\overline{)2128}$
b. $9\overline{)45{,}900}$

Example 5

Divide and check: $1872 \div 9$

Solution:

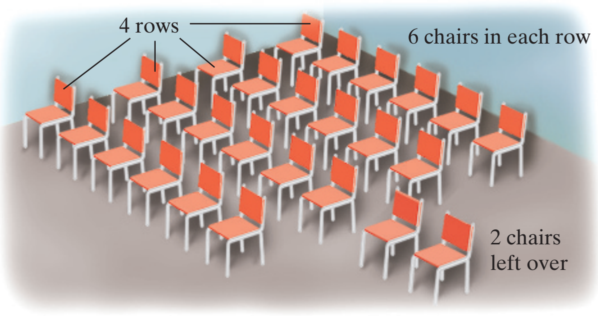
$$\begin{array}{rl}
208 & \\
9\overline{)1872} & \\
\underline{-18\downarrow} & 2(9) = 18 \\
07 & 18 - 18 = 0\text{; bring down the 7.} \\
\underline{-0\downarrow} & 0(9) = 0 \\
72 & 7 - 0 = 7\text{; bring down the 2.} \\
\underline{-72} & 8(9) = 72 \\
0 & 72 - 72 = 0
\end{array}$$

Check: $208 \cdot 9 = 1872$

Work Practice 5

Answers
4. a. 818 **b.** 553 **c.** 251
5. a. 304 **b.** 5100

Naturally, quotients don't always "come out even." Making 4 rows out of 26 chairs, for example, isn't possible if each row is supposed to have exactly the same number of chairs. Each of 4 rows can have 6 chairs, but 2 chairs are still left over.



We signify "leftovers" or **remainders** in this way:

$$\begin{array}{r} 6 \text{ R } 2 \\ 4\overline{)26} \end{array}$$

The **whole number part of the quotient** is 6; the **remainder part of the quotient** is 2. Checking by multiplying,

$$\begin{array}{ccccccc} \text{whole number part} & \cdot & \text{divisor} & + & \text{remainder part} & = & \text{dividend} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 6 & \cdot & 4 & + & 2 & & \\ & 24 & & + & 2 & = & 26 \end{array}$$

Example 6 Divide and check: $2557 \div 7$

Solution:

$$\begin{array}{r} 365 \text{ R } 2 \\ 7\overline{)2557} \\ -21\downarrow\downarrow \\ 45 \\ -42\downarrow \\ 37 \\ -35 \\ 2 \end{array}$$

$3(7) = 21$

$25 - 21 = 4$; bring down the 5.

$6(7) = 42$

$45 - 42 = 3$; bring down the 7.

$5(7) = 35$

$37 - 35 = 2$; the remainder is 2.

Check:

$$\begin{array}{ccccccc} 365 & \cdot & 7 & + & 2 & = & 2557 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{whole number part} & \cdot & \text{divisor} & + & \text{remainder part} & = & \text{dividend} \end{array}$$

Work Practice 6

Practice 6

Divide and check.

a. $4\overline{)939}$

b. $5\overline{)3287}$

Answers

6. a. 234 R 3 **b.** 657 R 2

Practice 7

Divide and check.

a. $9\overline{)81,605}$

b. $4\overline{)23,310}$

Example 7

Divide and check: $56,717 \div 8$

Solution:

7089 R 5	
$8\overline{)56717}$	
$-56\downarrow$	$7(8) = 56$
07	Subtract and bring down the 7.
$-0\downarrow$	$0(8) = 0$
71	Subtract and bring down the 1.
$-64\downarrow$	$8(8) = 64$
77	Subtract and bring down the 7.
-72	$9(8) = 72$
5	Subtract. The remainder is 5.

Check:

Check:	7089	·	8	+	5	=	56,717
	↓		↓		↓		↓
	whole number part	·	divisor	+	remainder part	=	dividend

Work Practice 7

When the divisor has more than one digit, the same pattern applies. For example, let's find $1358 \div 23$.

5	
$23\overline{)1358}$	$135 \div 23 = 5$ with 20 left over. Our estimate is 5.
$-115\downarrow$	$5(23) = 115$
208	$135 - 115 = 20$. Bring down the 8.

Now we continue estimating.

59 R 1	$208 \div 23 = 9$ with 1 left over.
$23\overline{)1358}$	
-115	
208	
-207	$9(23) = 207$
1	$208 - 207 = 1$. The remainder is 1.

To check, see that $59 \cdot 23 + 1 = 1358$.

Practice 8

Divide: $8920 \div 17$

Example 8

Divide: $6819 \div 17$

Solution:

401 R 2	
$17\overline{)6819}$	
$-68\downarrow$	$4(17) = 68$
01	Subtract and bring down the 1.
$-0\downarrow$	$0(17) = 0$
19	Subtract and bring down the 9.
-17	$1(17) = 17$
2	Subtract. The remainder is 2.

To check, see that $401 \cdot 17 + 2 = 6819$.

Work Practice 8

Answers

7. a. 9067 R 2 **b.** 5827 R 2

8. 524 R 12

Example 9

Divide: 51,600 ÷ 403

Solution:

$$\begin{array}{rl}
128 \quad \text{R } 16 & \\
403\overline{)51600} & \\
-403\downarrow & 1(403) = 403 \\
\hline
1130 & \text{Subtract and bring down the 0.} \\
-806\downarrow & 2(403) = 806 \\
\hline
3240 & \text{Subtract and bring down the 0.} \\
-3224 & 8(403) = 3224 \\
\hline
16 & \text{Subtract. The remainder is 16.}
\end{array}$$

To check, see that $128 \cdot 403 + 16 = 51{,}600$.

Work Practice 9

Practice 9

Divide: 33,282 ÷ 678

Division Shown as Repeated Subtraction To further understand division, recall from Section 1.6 that addition and multiplication are related in the following manner:

$$\underbrace{3 + 3 + 3 + 3}_{\text{4 addends; each addend is 3}} = 4 \times 3 = 12$$

In other words, multiplication is repeated addition. Likewise, division is repeated subtraction.

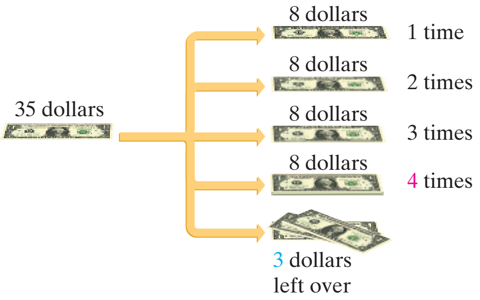
For example, let's find

$$35 \div 8$$

by repeated subtraction. Keep track of the number of times 8 is subtracted from 35. We are through when we can subtract no more because the difference is less than 8.

35 ÷ 8: Repeated Subtraction

$$\begin{array}{rl}
\left.\begin{array}{r} 35 \\ -8 \\ \hline \end{array}\right\} & \text{1 time} \\
\left.\begin{array}{r} 27 \\ -8 \\ \hline \end{array}\right\} & \text{2 times} \\
\left.\begin{array}{r} 19 \\ -8 \\ \hline \end{array}\right\} & \text{3 times} \\
\left.\begin{array}{r} 11 \\ -8 \\ \hline \end{array}\right\} & \text{4 times} \\
3 & \longleftarrow \text{Remainder (We cannot subtract 8 again.)}
\end{array}$$



Thus, 35 ÷ 8 = 4 R 3.

To check, perform the same multiplication as usual and finish by adding in the remainder.

$$\begin{array}{ccccccc}
\boxed{\text{whole number part of quotient}} & \cdot & \boxed{\text{divisor}} & + & \boxed{\text{remainder}} & = & \boxed{\text{dividend}} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
4 & \cdot & 8 & + & 3 & = & 35
\end{array}$$

Answer

9. 49 R 60

Objective C Solving Problems by Dividing

Below are some key words and phrases that may indicate the operation of division:

Division

Key Words or Phrases	**Examples**	**Symbols**
divide	divide 10 by 5	$10 \div 5$ or $\frac{10}{5}$
quotient	the quotient of 64 and 4	$64 \div 4$ or $\frac{64}{4}$
divided by	9 divided by 3	$9 \div 3$ or $\frac{9}{3}$
divided or shared equally among	\$100 divided equally among five people	$100 \div 5$ or $\frac{100}{5}$
per	100 miles per 2 hours	$\frac{100 \text{ miles}}{2 \text{ hours}}$

✓**Concept Check** Determine whether each of the following is the correct way to represent "the quotient of 60 and 12." Explain your answer.

a. $12 \div 60$
b. $60 \div 12$

Practice 10

Three students bought 171 blank CDs to share equally. How many CDs did each person get?

Example 10 Finding Shared Earnings

Three college students share a paper route to earn money for expenses. The total in their fund after expenses was \$2895. How much is each person's equal share?

Solution:

In words: Each person's share = total money ÷ number of persons

Translate: Each person's share = 2895 ÷ 3

Then

$$\begin{array}{r} 965 \\ 3\overline{)2895} \\ \underline{-27} \\ 19 \\ \underline{-18} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

Each person's share is \$965.

Work Practice 10

Answer
10. 57 CDs

✓**Concept Check Answers**
a. incorrect **b.** correct

Example 11 Dividing Number of Downloads

As part of a promotion, an executive receives 238 cards, each good for one free song download. If she wants to share them evenly with 19 friends, how many download cards will each friend receive? How many will be left over?

Solution:

$$\begin{array}{lccccc} \text{In words:} & \text{Number of cards for each person} & = & \text{number of cards} & \div & \text{number of friends}\\ & \downarrow & & \downarrow & & \downarrow \\ \text{Translate:} & \text{Number of cards for each person} & = & 238 & \div & 19 \end{array}$$

$$\begin{array}{r} 12\ \text{R}\ 10\\ 19\overline{)238}\\ -19\\ \hline 48\\ -38\\ \hline 10 \end{array}$$

Each friend will receive 12 download cards. The cards cannot be divided equally among her friends since there is a nonzero remainder. There will be 10 download cards left over.

Work Practice 11

Objective D Finding Averages

A special application of division (and addition) is finding the average of a list of numbers. The **average** of a list of numbers is the sum of the numbers divided by the *number* of numbers.

$$\text{average} = \frac{\text{sum of numbers}}{\textit{number}\text{ of numbers}}$$

Example 12 Averaging Scores

A mathematics instructor is checking a simple program she wrote for averaging the scores of her students. To do so, she averages a student's scores of 75, 96, 81, and 88 by hand. Find this average score.

Solution: To find the average score, we find the sum of the student's scores and divide by 4, the number of scores.

$$\begin{array}{r} 75\\ 96\\ 81\\ +88\\ \hline 340 \end{array}\ \text{sum} \qquad \text{average} = \frac{340}{4} = 85 \qquad \begin{array}{r} 85\\ 4\overline{)340}\\ -32\\ \hline 20\\ -20\\ \hline 0 \end{array}$$

The average score is 85.

Work Practice 12

Practice 11

Printers can be packed 12 to a box. If 532 printers are to be packed but only full boxes are shipped, how many full boxes will be shipped? How many printers are left over and not shipped?

Practice 12

To compute a safe time to wait for reactions to occur after allergy shots are administered, a lab technician is given a list of elapsed times between administered shots and reactions. Find the average of the times 4 minutes, 7 minutes, 35 minutes, 16 minutes, 9 minutes, 3 minutes, and 52 minutes.

Answers

11. 44 full boxes; 4 printers left over

12. 18 minutes

Calculator Explorations Dividing Numbers

To divide numbers on a calculator, find the keys marked $\boxed{\div}$ and $\boxed{=}$ or $\boxed{\text{ENTER}}$. For example, to find 435 ÷ 5 on a calculator, press the keys $\boxed{435}$ $\boxed{\div}$ $\boxed{5}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$. The display will read $\boxed{87}$. Thus, 435 ÷ 5 = 87.

Use a calculator to divide.

1. 848 ÷ 16

2. 564 ÷ 12

3. $95\overline{)5890}$

4. $27\overline{)1053}$

5. $\frac{32,886}{126}$

6. $\frac{143,088}{264}$

7. 0 ÷ 315

8. 315 ÷ 0

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

1
0
number
undefined
divisor
average
dividend
quotient

1. In 90 ÷ 2 = 45, the answer 45 is called the ____________, 90 is called the ____________, and 2 is called the ____________.

2. The quotient of any number and 1 is the same ____________.

3. The quotient of any number (except 0) and the same number is ____________.

4. The quotient of 0 and any number (except 0) is ____________.

5. The quotient of any number and 0 is ____________.

6. The ____________ of a list of numbers is the sum of the numbers divided by the ____________ of numbers.

Martin-Gay Interactive Videos

See Video 1.7

Watch the section lecture video and answer the following questions.

Objective A **7.** Look at Examples 6–8. What number can never be the divisor in division?

Objective B **8.** In Example 10, how many 102s are in 21? How does this result affect the quotient?

9. What calculation would you use to check the answer in Example 10?

Objective C **10.** In Example 11, what is the importance of knowing that the distance to each hole is the same?

Objective D **11.** As shown in Example 12, what two operations are used when finding an average?

1.7 Exercise Set MyLab Math

Objective A *Find each quotient. See Examples 1 through 3.*

1. $54 \div 9$

2. $72 \div 9$

▶ **3.** $36 \div 3$

4. $24 \div 3$

5. $0 \div 8$

6. $0 \div 4$

▶ **7.** $31 \div 1$

8. $38 \div 1$

▶ **9.** $\frac{18}{18}$

10. $\frac{49}{49}$

11. $\frac{24}{3}$

12. $\frac{45}{9}$

▶ **13.** $26 \div 0$

14. $\frac{12}{0}$

15. $26 \div 26$

16. $6 \div 6$

▶ **17.** $0 \div 14$

18. $7 \div 0$

19. $18 \div 2$

20. $18 \div 3$

Objectives A B Mixed Practice *Divide and then check by multiplying. See Examples 1 through 5.*

21. $3\overline{)87}$

22. $5\overline{)85}$

23. $3\overline{)222}$

24. $8\overline{)640}$

25. $3\overline{)1014}$

26. $4\overline{)2104}$

27. $\frac{30}{0}$

28. $\frac{0}{30}$

29. $63 \div 7$

30. $56 \div 8$

31. $150 \div 6$

32. $121 \div 11$

Divide and then check by multiplying. See Examples 6 and 7.

33. $7\overline{)479}$

34. $7\overline{)426}$

35. $6\overline{)1421}$

36. $3\overline{)1240}$

37. $305 \div 8$

38. $167 \div 3$

39. $2286 \div 7$

40. $3333 \div 4$

Divide and then check by multiplying. See Examples 8 and 9.

▶ **41.** $55\overline{)715}$

42. $23\overline{)736}$

43. $23\overline{)1127}$

44. $42\overline{)2016}$

45. $97\overline{)9417}$

46. $44\overline{)1938}$

47. $3146 \div 15$

48. $7354 \div 12$

49. $6578 \div 13$

50. $5670 \div 14$

51. $9299 \div 46$

52. $2505 \div 64$

53. $\frac{12{,}744}{236}$

54. $\frac{5781}{123}$

55. $\frac{10{,}297}{103}$

56. $\frac{23{,}092}{240}$

▶ **57.** $20{,}619 \div 102$

58. $40{,}853 \div 203$

59. $244{,}989 \div 423$

60. $164{,}592 \div 543$

Divide. See Examples 1 through 9.

61. $7\overline{)119}$

62. $8\overline{)104}$

63. $7\overline{)3580}$

64. $5\overline{)3017}$

65. $40\overline{)85,312}$

66. $50\overline{)85,747}$

67. $142\overline{)863,360}$

68. $214\overline{)650,560}$

Objective C Translating *Solve. See Examples 10 and 11.*

69. Find the quotient of 117 and 5.

70. Find the quotient of 94 and 7.

71. Find 200 divided by 35.

72. Find 116 divided by 32.

73. Find the quotient of 62 and 3.

74. Find the quotient of 78 and 5.

75. Martin Thieme teaches American Sign Language classes for $65 per student for a 7-week session. He collects $2145 from the group of students. Find how many students are in the group.

76. Kathy Gomez teaches Spanish lessons for $85 per student for a 5-week session. From one group of students, she collects $4930. Find how many students are in the group.

77. The gravity of Jupiter is 318 times as strong as the gravity of Earth, so objects on Jupiter weigh 318 times as much as they weigh on Earth. If a person would weigh 52,470 pounds on Jupiter, find how much the person weighs on Earth.

78. Twenty-one people pooled their money and bought lottery tickets. One ticket won a prize of $5,292,000. Find how many dollars each person received.

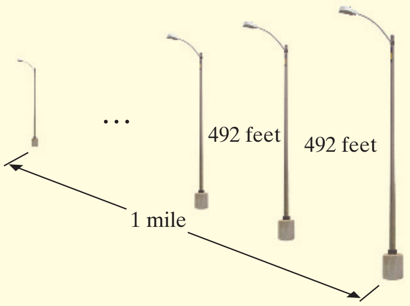
79. An 18-hole golf course is 5580 yards long. If the distance to each hole is the same, find the distance between holes.

80. A truck hauls wheat to a storage granary. It carries a total of 5768 bushels of wheat in 14 trips. How much does the truck haul each trip if each trip it hauls the same amount?

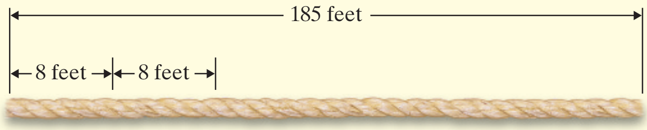
81. There is a bridge over highway I-35 every three miles. The first bridge is at the beginning of a 265-mile stretch of highway. Find how many bridges there are over 265 miles of I-35.

82. The white stripes dividing the lanes on a highway are 25 feet long, and the spaces between them are 25 feet long. Let's call a "lane divider" a stripe followed by a space. Find how many whole "lane dividers" there are in 1 mile of highway. (A mile is 5280 feet.)

83. Ari Trainor is in the requisitions department of Central Electric Lighting Company. Light poles along a highway are placed 492 feet apart. The first light pole is at the beginning of a 1-mile strip. Find how many poles he should order for the 1-mile strip of highway. (A mile is 5280 feet.)



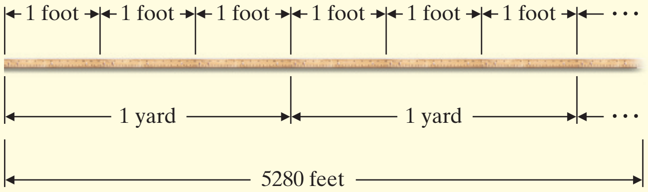
84. Professor Lopez has a piece of rope 185 feet long that she wants to cut into pieces for an experiment in her physics class. Each piece of rope is to be 8 feet long. Determine whether she has enough rope for her 22-student class. Determine the amount extra or the amount short.



85. Broad Peak in Pakistan is the twelfth-tallest mountain in the world. Its elevation is 26,400 feet. A mile is 5280 feet. How many miles tall is Broad Peak? (*Source:* National Geographic Society)

86. David Johnson of the Arizona Cardinals led the NFL in touchdowns during the 2016 regular football season, scoring a total of 120 points from touchdowns. If a touchdown is worth 6 points, how many touchdowns did Johnson make during 2016? (*Source:* National Football League)

87. Find how many yards are in 1 mile. (A mile is 5280 feet; a yard is 3 feet.)



88. Find how many whole feet are in 1 rod. (A mile is 5280 feet; 1 mile is 320 rods.)

Objective D *Find the average of each list of numbers. See Example 12.*

89. 10, 24, 35, 22, 17, 12

90. 37, 26, 15, 29, 51, 22

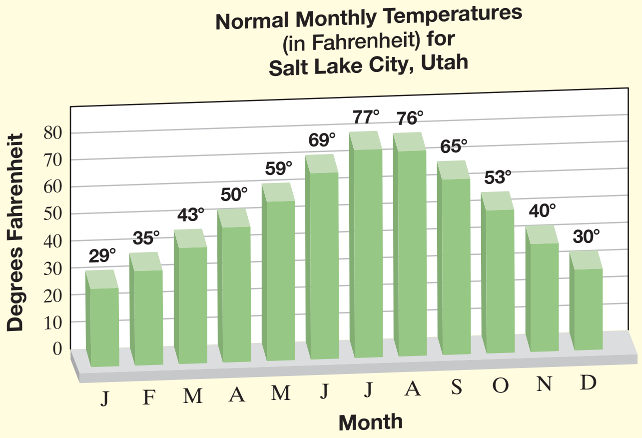
91. 205, 972, 210, 161

92. 121, 200, 185, 176, 163

93. 86, 79, 81, 69, 80

94. 92, 96, 90, 85, 92, 79

The normal monthly temperatures in degrees Fahrenheit for Salt Lake City, Utah, are given in the graph. Use this graph to answer Exercises 95 and 96. (Source: National Climatic Data Center)



95. Find the average temperature for June, July, and August.

96. Find the average temperature for October, November, and December.

Mixed Practice (Sections 1.3, 1.4, 1.6, and 1.7) *Perform each indicated operation. Watch the operation symbol.*

97. 82 + 463 + 29 + 8704

98. 23 + 407 + 92 + 7011

99. $\begin{array}{r} 546 \\ \times\ 28 \\ \hline \end{array}$

100. $\begin{array}{r} 712 \\ \times\ 54 \\ \hline \end{array}$

101. $\begin{array}{r} 722 \\ -\ 43 \\ \hline \end{array}$

102. $\begin{array}{r} 712 \\ -\ 54 \\ \hline \end{array}$

103. $\frac{45}{0}$

104. $\frac{0}{23}$

105. 228 ÷ 24

106. 304 ÷ 31

Concept Extensions

Match each word phrase to the correct translation. (Not all letter choices will be used.) See the Concept Check in this section.

107. The quotient of 40 and 8

108. The quotient of 200 and 20

109. 200 divided by 20

110. 40 divided by 8

a. 20 ÷ 200
b. 200 ÷ 20
c. 40 ÷ 8
d. 8 ÷ 40

The following table shows the top seven countries with the most Nobel Prize winners by country of birth. Use this table to answer Exercises 111 and 112. (Source: Nobel Prize Organization)

111. Find the average number of Nobel Prize winners for United Kingdom, Germany, and France.

112. Find the average number of Nobel Prize winners for Sweden, Russia, and Japan.

Most Nobel Prize Winners by Country of Birth, 1901–2016

Country	Chemistry	Economics	Literature	Peace	Physics	Physiology & Medicine	Total
United States	52	43	9	19	66	70	259
United Kingdom	24	8	7	11	23	26	99
Germany	24	1	7	5	23	17	77
France	9	4	11	10	9	12	55
Sweden	4	2	7	5	4	7	29
Russia (USSR)	3	2	5	2	11	2	25
Japan	6	0	2	1	11	4	24

In Example 12 in this section, we found that the average of 75, 96, 81, and 88 is 85. Use this information to answer Exercises 113 and 114.

113. If the number 75 is removed from the list of numbers, does the average increase or decrease? Explain why.

114. If the number 96 is removed from the list of numbers, does the average increase or decrease? Explain why.

115. Without computing it, tell whether the average of 126, 135, 198, 113 is 86. Explain why it is possible or why it is not.

116. Without computing it, tell whether the average of 38, 27, 58, and 43 is 17. Explain why it is possible or why it is not.

117. If the area of a rectangle is 60 square feet and its width is 5 feet, what is its length?

118. If the area of a rectangle is 84 square inches and its length is 21 inches, what is its width?

119. Write down any two numbers whose quotient is 25.

120. Write down any two numbers whose quotient is 1.

121. Find 26 ÷ 5 using the process of repeated subtraction.

122. Find 86 ÷ 10 using the process of repeated subtraction.

Sections 1.2–1.7

Integrated Review

Operations on Whole Numbers

1. $\begin{array}{r} 23 \\ 46 \\ +79 \\ \hline \end{array}$

2. $\begin{array}{r} 7006 \\ -\ 451 \\ \hline \end{array}$

3. $\begin{array}{r} 36 \\ \times 45 \\ \hline \end{array}$

4. $8\overline{)4496}$

5. $1 \cdot 79$

6. $\frac{36}{0}$

7. $9 \div 1$

8. $9 \div 9$

9. $0 \cdot 13$

10. $7 \cdot 0 \cdot 8$

11. $0 \div 2$

12. $12 \div 4$

13. $4219 - 1786$

14. $1861 + 7965$

15. $5\overline{)1068}$

16. $\begin{array}{r} 1259 \\ \times \quad 63 \\ \hline \end{array}$

17. $3 \cdot 9$

18. $45 \div 5$

19. $\begin{array}{r} 207 \\ -\ \ 69 \\ \hline \end{array}$

20. $\begin{array}{r} 207 \\ +\ \ 69 \\ \hline \end{array}$

21. $7\overline{)7695}$

22. $9\overline{)1000}$

23. $32\overline{)21{,}222}$

24. $65\overline{)70{,}000}$

25. $4000 - 2976$

26. $10{,}000 - 101$

27. $\begin{array}{r} 303 \\ \times 101 \\ \hline \end{array}$

28. $(475)(100)$

29. Find the total of 57 and 8.

30. Find the product of 57 and 8.

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______
25. ______
26. ______
27. ______
28. ______
29. ______
30. ______

31. Find the quotient of 62 and 9.

32. Find the difference of 62 and 9.

33. Subtract 17 from 200.

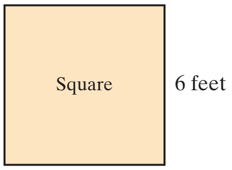
34. Find the difference of 432 and 201.

Complete the table by rounding the given number to the given place value.

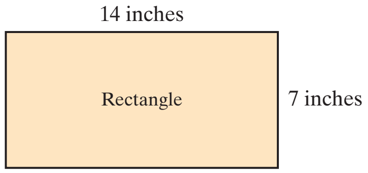
		Tens	Hundreds	Thousands
35.	9735			
36.	1429			
37.	20,801			
38.	432,198			

Find the perimeter and area of each figure.

△ **39.**

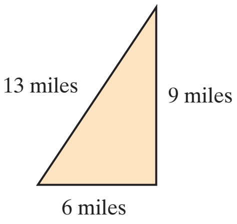


△ **40.**

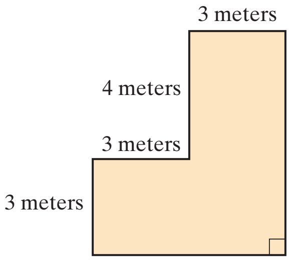


Find the perimeter of each figure.

△ **41.**



△ **42.**

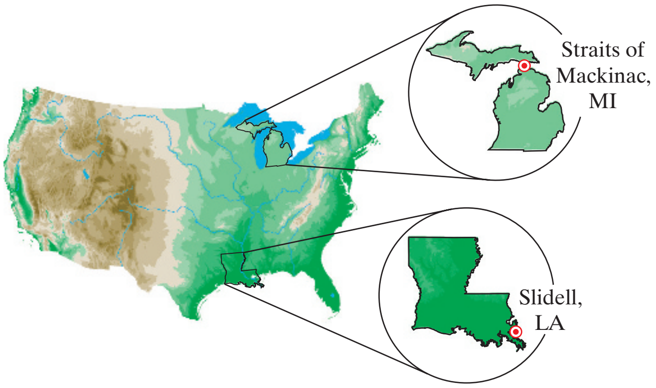


Find the average of each list of numbers.

43. 19, 15, 25, 37, 24

44. 108, 131, 98, 159

45. The Mackinac Bridge is a suspension bridge that connects the lower and upper peninsulas of Michigan across the Straits of Mackinac. Its total length is 26,372 feet. The Lake Pontchartrain Bridge is a twin concrete trestle bridge in Slidell, Louisiana. Its total length is 28,547 feet. Which bridge is longer and by how much? (*Sources:* Mackinac Bridge Authority and Federal Highway Administration, Bridge Division)



46. The average teenage male American consumes 2 quarts of carbonated soft drinks per day. On average, how many quarts of carbonated soft drinks would be consumed in a year? (Use 365 for the number of days.) (*Source:* American Beverage Association)

31. ____________________

32. ____________________

33. ____________________

34. ____________________

35. ____________________

36. ____________________

37. ____________________

38. ____________________

39. ____________________

40. ____________________

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

1.8 An Introduction to Problem Solving

Objective A Solving Problems Involving Addition, Subtraction, Multiplication, or Division

In this section, we decide which operation to perform in order to solve a problem. Don't forget the key words and phrases that help indicate which operation to use. Some of these are listed below and were introduced earlier in the chapter. Also included are several words and phrases that translate to the symbol " = ".

Addition (+)	**Subtraction** (−)	**Multiplication** (·)	**Division** (÷)	**Equality** (=)
sum	difference	product	quotient	equals
plus	minus	times	divide	is equal to
added to	subtract	multiply	shared equally among	is/was
more than	less than	multiply by	per	yields
increased by	decreased by	of	divided by	
total	less	double/triple	divided into	

The following problem-solving steps may be helpful to you:

Problem-Solving Steps

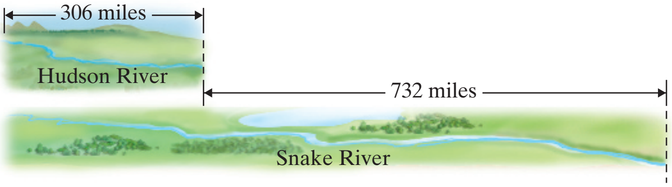
1. UNDERSTAND the problem. Some ways of doing this are to read and reread the problem, construct a drawing, and look for key words to identify an operation.
2. TRANSLATE the problem. That is, write the problem in short form using words, and then translate to numbers and symbols.
3. SOLVE the problem. It is helpful to estimate the solution by rounding. Then carry out the indicated operation from step 2.
4. INTERPRET the results. *Check* the proposed solution in the stated problem and *state* your conclusions. Write your results with the correct units attached.

Example 1 Calculating the Length of a River

The Hudson River in New York State is 306 miles long. The Snake River in the northwestern United States is 732 miles longer than the Hudson River. How long is the Snake River? (*Source:* U.S. Department of the Interior)

Solution:

1. UNDERSTAND. Read and reread the problem, and then draw a picture. Notice that we are told that Snake River is 732 miles longer than the Hudson River. The phrase "longer than" means that we add.



(*Continued on next page*)

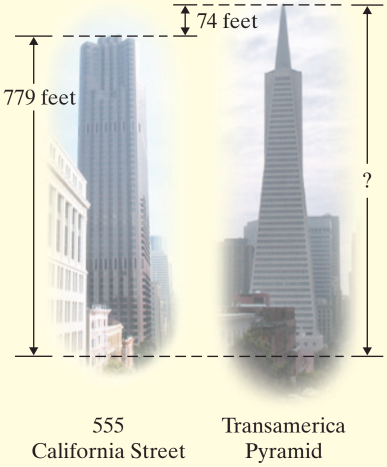
Objectives

A Solve Problems by Adding, Subtracting, Multiplying, or Dividing Whole Numbers.

B Solve Problems That Require More Than One Operation.

Practice 1

The building called 555 California Street is the third-tallest building in San Francisco, California, at 779 feet. The second-tallest building in San Francisco is the Transamerica Pyramid, which is 74 feet taller than 555 California Street. How tall is the Transamerica Pyramid? (*Source: The World Almanac*)



Answer

1. 853 ft

2. TRANSLATE.

In words:	Snake River	is	732 miles	longer than	the Hudson River
	↓	↓	↓	↓	↓
Translate:	Snake River	=	732	+	306

3. SOLVE: Let's see if our answer is reasonable by also estimating. We will estimate each addend to the nearest hundred.

732	rounds to	700	
+306	rounds to	+300	
1038	exact	1000	estimate

4. INTERPRET. *Check* your work. The answer is reasonable since 1038 is close to our estimated answer of 1000. *State* your conclusion: The Snake River is 1038 miles long.

■ Work Practice 1

Practice 2

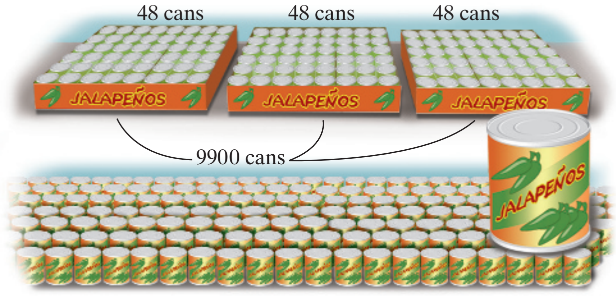
Four friends bought a lottery ticket and won $65,000. If each person is to receive the same amount of money, how much does each person receive?

Example 2 Filling a Shipping Order

How many cases can be filled with 9900 cans of jalapeños if each case holds 48 cans? How many cans will be left over? Will there be enough cases to fill an order for 200 cases?

Solution:

1. UNDERSTAND. Read and reread the problem. Draw a picture to help visualize the situation.



Since each case holds 48 cans, we want to know how many 48s there are in 9900. We find this by dividing.

2. TRANSLATE.

In words:	Number of cases	is	9900	divided by	48
	↓	↓	↓	↓	↓
Translate:	Number of cases	=	9900	÷	48

3. SOLVE: Let's estimate a reasonable solution before we actually divide. Since 9900 rounded to the nearest thousand is 10,000 and 48 rounded to the nearest ten is 50,10,000 ÷ 50 = 200. Now find the exact quotient.

$$\begin{array}{r} 206 \ \text{R}12 \\ 48\overline{)9900} \\ -96 \\ \hline 300 \\ -288 \\ \hline 12 \end{array}$$

Answer

2. $16,250

4. INTERPRET. *Check* your work. The answer is reasonable since 206 R 12 is close to our estimate of 200. *State* your conclusion: 206 cases will be filled, with 12 cans left over. There will be enough cases to fill an order for 200 cases.

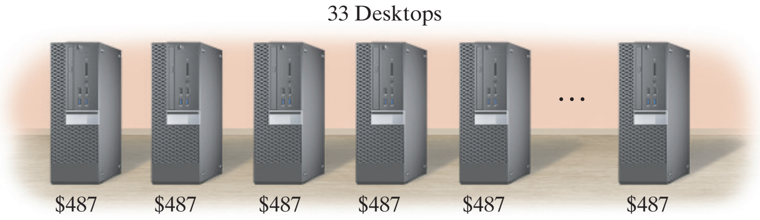
Work Practice 2

Example 3 Calculating Budget Costs

The director of a computer lab at a local state college is working on next year's budget. Thirty-three new desktop computers are needed at a cost of $487 each. What is the total cost of these desktops?

Solution:

1. UNDERSTAND. Read and reread the problem, and then draw a diagram.



From the phrase "total cost," we might decide to solve this problem by adding. This would work, but repeated addition, or multiplication, would save time.

2. TRANSLATE.

In words:	Total cost	is	number of desktops	times	cost of a desktop
	↓	↓	↓	↓	↓
Translate:	Total cost	=	33	×	$487

3. SOLVE: Once again, let's estimate a reasonable solution.

$$\begin{array}{rlrl} 487 & \text{rounds to} & 500 & \\ \times\ 33 & \text{rounds to} & \times\ 30 & \\ \hline 1461 & & 15{,}000 & \text{estimate} \\ 14610 & & & \\ \hline 16{,}071 & \text{exact} & & \end{array}$$

4. INTERPRET. *Check* your work. *State* your conclusion: The total cost of the desktops is $16,071.

Work Practice 3

Example 4 Calculating a Public School Teacher's Salary

In 2017, the average salary for a public school teacher in California was $69,320. For the same year, the average salary for a public school teacher in Iowa was $20,370 less than this. What was the average public school teacher's salary in Iowa? (*Source:* National Education Association)

Solution:

1. UNDERSTAND. Read and reread the problem. Notice that we are told that the Iowa salary is $20,370 less than the California salary. The phrase "less than" indicates subtraction.

(*Continued on next page*)

Practice 3

The director of the learning lab also needs to include in the budget a line for 425 flash drives at a cost of $4 each. What is this total cost for the flash drives?

Practice 4

In 2017, the average salary for a public school teacher in New York was $69,118. For the same year, the average salary for a public school teacher in Illinois was $7774 less than this. What was the average public school teacher's salary in Illinois? (*Source:* National Education Association)

Answers

3. $1700 **4.** $61,344

2. TRANSLATE. Remember that order matters when subtracting, so be careful when translating.

In words:	Iowa salary	is	California salary	minus	\$20,370
	↓		↓	↓	↓
Translate:	Iowa salary	=	69,320	−	20,370

3. SOLVE. This time, instead of estimating, let's check by adding.

$$\begin{array}{r} 69{,}320 \\ -20{,}370 \\ \hline 48{,}950 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 48{,}950 \\ +20{,}370 \\ \hline 69{,}320 \end{array}$$

4. INTERPRET. *Check* your work. The check is above. *State* your conclusion: The average Iowa teacher's salary in 2017 was \$48,950.

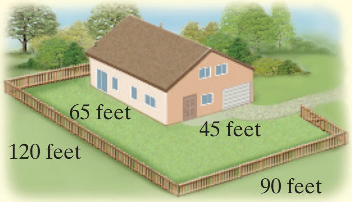
Work Practice 4

Objective B Solving Problems That Require More Than One Operation

We must sometimes use more than one operation to solve a problem.

Practice 5

A gardener is trying to decide how much fertilizer to buy for his yard. He knows that his lot is in the shape of a rectangle that measures 90 feet by 120 feet. He also knows that the floor of his house is in the shape of a rectangle that measures 45 feet by 65 feet. How much area of the lot is not covered by the house?

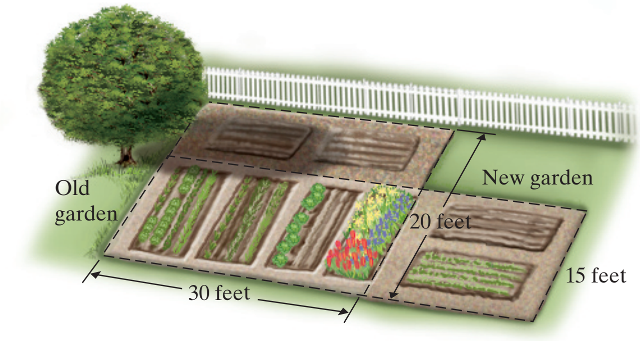


Example 5 Planting a New Garden

A gardener bought enough plants to fill a rectangular garden with length 30 feet and width 20 feet. Because of shading problems from a nearby tree, the gardener changed the width of the garden to 15 feet. If the area is to remain the same, what is the new length of the garden?

Solution:

1. UNDERSTAND. Read and reread the problem. Then draw a picture to help visualize the problem.



2. TRANSLATE. Since the area of the new garden is to be the same as the area of the old garden, let's find the area of the old garden.

$$\text{Area} = \text{length} \times \text{width} = 30 \text{ feet} \times 20 \text{ feet} = 600 \text{ square feet}$$

Answer

5. 7875 sq ft

Since the area of the new garden is to be 600 square feet also, we need to see how many 15s there are in 600. This means division. In other words,

In words:	New length	=	Area of garden	÷	New width
	↓		↓		↓
Translate:	New length	=	600	÷	15

3. SOLVE.

$$\begin{array}{r} 40 \\ 15\overline{)600} \\ \underline{-60} \\ 00 \end{array}$$

4. INTERPRET. *Check* your work. *State* your conclusion: The length of the new garden is 40 feet.

Work Practice 5

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 1.8

Watch the section lecture video and answer the following questions.

Objective A **1.** The answer to the calculations in Example 3 is 3500. What is the final interpreted solution, written as a sentence?

Objective B **2.** What two operations are used to solve Example 4?

1.8 Exercise Set MyLab Math

Objective A *Solve. Exercises 1, 2, 11, and 12 have been started for you. See Examples 1 through 4.*

1. 41 increased by 8 is what number?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blanks below.)

41	increased by	8	is	what number
↓	↓	↓	↓	↓
41	___	8	___	what number

Finish with:

3. SOLVE

4. INTERPRET

2. What is 12 multiplied by 9?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blanks below.)

what	is	12	multiplied by	9
↓	↓	↓	↓	↓
what number	___	12	___	9

Finish with:

3. SOLVE

4. INTERPRET

▶ **3.** What is the quotient of 1185 and 5?

4. 78 decreased by 12 is what number?

▶ **5.** What is the total of 35 and 7?

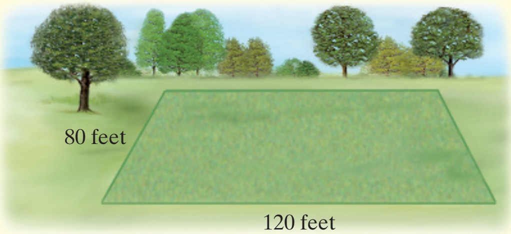
6. What is the difference of 48 and 8?

7. 60 times 10 is what number?

8. 60 divided by 10 is what number?

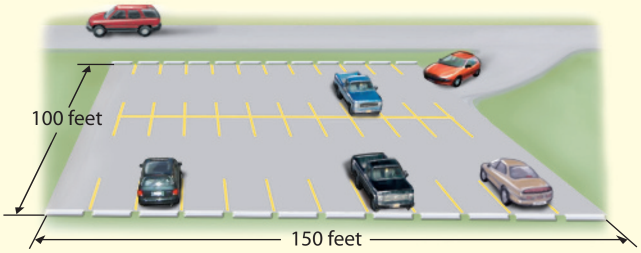
△ **9.** A vacant lot in the shape of a rectangle measures 120 feet by 80 feet.

- **a.** What is the perimeter of the lot?
- **b.** What is the area of the lot?



△ **10.** A parking lot in the shape of a rectangle measures 100 feet by 150 feet.

- **a.** What is the perimeter of the lot?
- **b.** What is the area of the parking lot?



11. A family bought a house for $185,700 and later sold the house for $201,200. How much money did they make by selling the house?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks below.)

money made	is	selling price	minus	purchase price
↓	↓	↓	↓	↓
money made	=	____	−	____

Finish with:

3. SOLVE

4. INTERPRET

12. Three people dream of equally sharing a $147 million lottery. How much would each person receive if they have the winning ticket?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks below.)

each person's share	is	lottery amount	divided by	number by persons
↓	↓	↓	↓	↓
each person's share	=	____	÷	____

Finish with:

3. SOLVE

4. INTERPRET

13. There are 24 hours in a day. How many hours are in a week?

14. There are 60 minutes in an hour. How many minutes are in a day?

15. The Verrazano Narrows Bridge is the longest bridge in New York, measuring 4260 feet. The George Washington Bridge, also in New York, is 760 feet shorter than the Verrazano Narrows Bridge. Find the length of the George Washington Bridge.

16. In 2013, the Goodyear Tire & Rubber Company began replacing its fleet of nonrigid GZ-20 blimps with new Goodyear NT semi-rigid airships. The new Goodyear NT airship can hold 297,527 cubic feet of helium. Its GZ-20 predecessor held 94,827 fewer cubic feet of helium. How much helium did a GZ-20 blimp hold? (*Source:* Goodyear Tire & Rubber Company)

17. Yellowstone National Park in Wyoming was the first national park in the United States. It was created in 1872. One of the more recent additions to the National Park System is First State National Monument. It was established in 2013. How much older is Yellowstone than First State? (*Source:* National Park Service)

18. Razor scooters were introduced in 2000. Radio Flyer Wagons were first introduced 83 years earlier. In what year were Radio Flyer Wagons introduced? (*Source:* Toy Industry Association, Inc.)

19. Since their introduction, the number of LEGO building bricks that have been sold is equivalent to the world's current population of approximately 6 billion people owning 62 LEGO bricks each. About how many LEGO bricks have been sold since their introduction? (*Source:* LEGO Company)

20. In 2016, the average weekly pay for a home health aide in the United States was about $425. At this rate, how much will a home health aide earn working a 52-week year? (*Source:* Bureau of Labor Statistics)

21. The three most common city names in the United States are Fairview, Midway, and Riverside. There are 287 towns named Fairview, 252 named Midway, and 180 named Riverside. Find the total number of towns named Fairview, Midway, or Riverside.

22. In the game of Monopoly, a player must own all properties in a color group before building houses. The yellow color-group properties are Atlantic Avenue, Ventnor Avenue, and Marvin Gardens. These cost $260, $260, and $280, respectively, when purchased from the bank. What total amount must a player pay to the bank before houses can be built on the yellow properties? (*Source:* Hasbro, Inc.)

23. In 2016, the average weekly pay for a fire inspector was $1040. If such an inspector works 40 hours in one week, what is his or her hourly pay? (*Source:* Bureau of Labor Statistics)

25. Three ounces of canned tuna in oil has 165 calories. How many calories does 1 ounce have? (*Source: Home and Garden Bulletin No. 72,* U.S. Department of Agriculture)

27. The average estimated 2016 U.S. population was 324,000,000. Between Memorial Day and Labor Day, 7 billion hot dogs are consumed. Approximately how many hot dogs were consumed per person between Memorial Day and Labor Day in 2016? Divide, but do not give the remainder part of the quotient. (*Source:* U.S. Census Bureau, National Hot Dog and Sausage Council)

29. The Museum of Modern Art in New York City had approximately 268,300 visitors on average each month in a recent year. Use the fact that there are 12 months in a year to find the total number of visitors to this museum in one year.

31. In 2016, Typhoon Lagoon at Walt Disney World in Orlando, Florida, hosted 2,277,000 visitors. Aquatica, also in Orlando, Florida, received 1,536,000. How many more people visited Typhoon Lagoon than Aquatica? (*Source:* Themed Entertainment Association)

33. The length of the southern boundary of the conterminous United States is 1933 miles. The length of the northern boundary of the conterminous United States is 2054 miles longer than this. What is the length of the northern boundary? (*Source:* U.S. Geological Survey)

24. In 2016, the average weekly pay for a paralegal was $960. If the paralegal works 40 hours in one week, what is his or her hourly pay? (*Source:* Bureau of Labor Statistics)

26. A whole cheesecake has 3360 calories. If the cheesecake is cut into 12 equal pieces, how many calories will each piece have? (*Source: Home and Garden Bulletin No. 72,* U.S. Department of Agriculture)

28. In 2016, PetSmart employed approximately 55,000 associates and operated roughly 1500 stores. What is the average number of associates employed at each of its stores? Divide, but do not give the remainder part of the quotient. (*Source:* PetSmart)

30. The National Air and Space Museum in Washington, D.C. had approximately 625,100 visitors on average each month in 2016. Use the fact that there are 12 months in a year to find the total number of visitors to this museum in 2016.

32. In 2016, Target Corporation operated 1806 stores in the United States. Of these, 193 were in California. How many Target Stores were located in states other than California? (*Source:* Target Corporation)

34. In humans, 14 muscles are required to smile. It takes 29 more muscles to frown. How many muscles does it take to frown?

35. An instructor at the University of New Orleans receives a paycheck every four weeks. Find how many paychecks he receives in a year. (A year has 52 weeks.)

36. A loan of $6240 is to be paid in 48 equal payments. How much is each payment?

Objective B *Solve. See Example 5.*

37. Find the total cost of 3 sweaters at $38 each and 5 shirts at $25 each.

38. Find the total cost of 10 computers at $2100 each and 7 boxes of diskettes at $12 each.

39. A college student has $950 in an account. She spends $205 from the account on books and then deposits $300 in the account. How much money is now in the account?

40. The temperature outside was 57°F (degrees Fahrenheit). During the next few hours, it decreased by 18 degrees and then increased by 23 degrees. Find the new temperature.

The table shows the menu from a concession stand at the county fair. Use this menu to answer Exercises 41 and 42.

41. A hungry college student is debating between the following two orders:

a. a hamburger, an order of onion rings, a candy bar, and a soda.

b. a hot dog, an apple, an order of french fries, and a soda.

Which order will be cheaper? By how much?

42. A family of four is debating between the following two orders:

a. 6 hot dogs, 4 orders of onion rings, and 4 sodas.

b. 4 hamburgers, 4 orders of french fries, 2 apples, and 4 sodas.

Will the family save any money by ordering **(b)** instead of **(a)**? If so, how much?

Corky's Concession Stand Menu

Item	Price
Hot dog	$3
Hamburger	$4
Soda	$1
Onion rings	$3
French fries	$2
Apple	$1
Candy bar	$2

Objectives A B Mixed Practice *Use the bar graph to answer Exercises 43 through 50. (Source: Internet World Stats)*

43. Which region of the world listed had the greatest number of Internet users in 2017?

44. Which region of the world listed had the least number of Internet users in 2017?

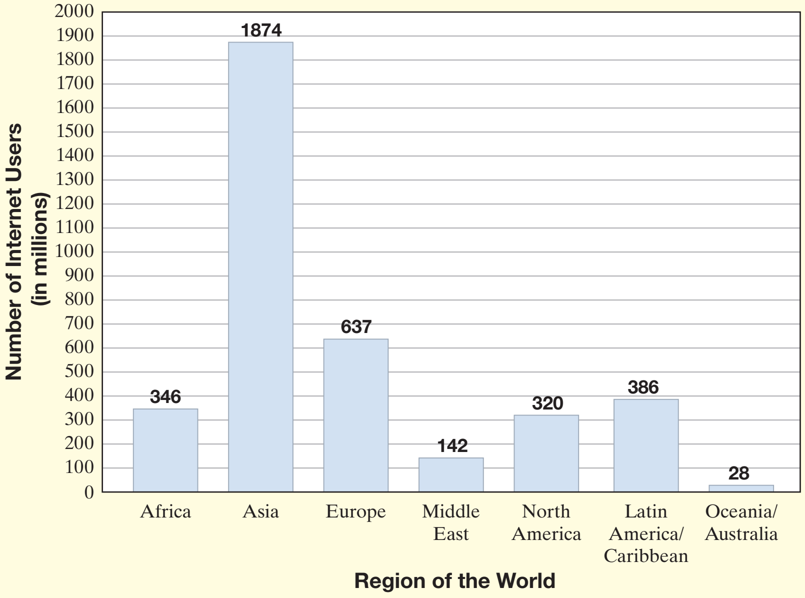
45. How many more Internet users (in millions) did the world region with the most Internet users have than the world region with the fewest Internet users?

46. How many more Internet users did Africa have than the Middle East in 2017?

47. How many more Internet users did Latin America/Caribbean have than North America?

48. Which region of the world had more Internet users, Europe or North America? How many more Internet users did it have?

World Internet Usage in 2017



Find the average number of Internet users for the world regions listed in the graph.

49. The world region with the greatest number of Internet users and the world region with the least number of Internet users.

50. The four world regions with the least number of Internet users.

Solve.

51. The learning lab at a local university is receiving new equipment. Twenty-two computers are purchased for $615 each and three printers for $408 each. Find the total cost for this equipment.

52. The washateria near the local community college is receiving new equipment. Thirty-six washers are purchased for $585 each and ten dryers are purchased for $388 each. Find the total cost for this equipment.

53. The American Heart Association recommends consuming no more than 2400 milligrams of salt per day. (This is about the amount in 1 teaspoon of salt.) How many milligrams of sodium is this in a week?

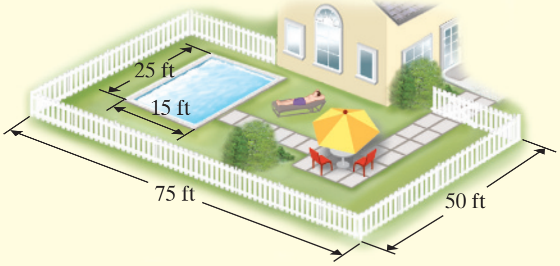
54. This semester a particular student pays $1750 for room and board, $709 for a meal ticket plan, and $2168 for tuition. What is her total bill?

△**55.** The Meishs' yard is in the shape of a rectangle and measures 50 feet by 75 feet. In their yard, they have a rectangular swimming pool that measures 15 feet by 25 feet.

a. Find the area of the entire yard.

b. Find the area of the swimming pool.

c. Find the area of the yard that is not part of the swimming pool.

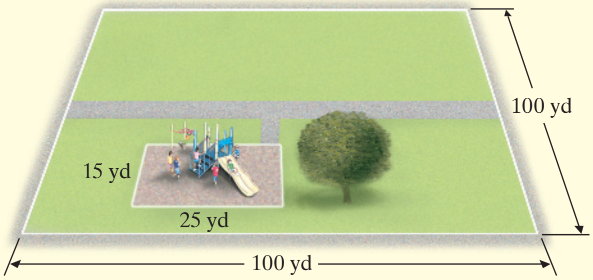


56. The community is planning to construct a rectangular-shaped playground within the local park. The park is in the shape of a square and measures 100 yards on each side. The playground is to measure 15 yards by 25 yards.

a. Find the area of the entire park.

b. Find the area of the playground.

c. Find the area of the park that is not part of the playground.



Concept Extensions

57. In 2016, United Parcel Service delivered about 4,893,000,000 packages worldwide, which generated revenue of approximately $49,906,000,000. Round the revenue and number of packages to the nearest billion to estimate the average revenue generated by each package. (*Source:* UPS)

58. In 2015, the United States Post Office received about 919,500,000 customer visits at its retail outlets. The total retail revenue for that year was approximately $19,790,000,000. Round the retail revenue and customer visits to the nearest hundred million to estimate the average revenue generated by each customer. (*Source:* United States Postal Service)

59. Write an application of your own that uses the term "bank account" and the numbers 1036 and 524.

1.9 Exponents, Square Roots, and Order of Operations

Objectives

- **A** Write Repeated Factors Using Exponential Notation.
- **B** Evaluate Expressions Containing Exponents.
- **C** Evaluate the Square Root of a Perfect Square.
- **D** Use the Order of Operations.
- **E** Find the Area of a Square.

Objective A Using Exponential Notation

In the product $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, notice that 3 is a factor several times. When this happens, we can use a shorthand notation, called an **exponent,** to write the repeated multiplication.

$\underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{3 is a factor 5 times}}$ can be written as

exponent

3^5 Read as "three to the fifth power."

base

This is called **exponential notation.** The **exponent,** 5, indicates how many times the **base,** 3, is a factor.

The table below shows examples of reading exponential notation in words.

Expression	In Words
5^2	"five to the second power" or "five squared"
5^3	"five to the third power" or "five cubed"
5^4	"five to the fourth power"

Usually, an exponent of 1 is not written, so when no exponent appears, we assume that the exponent is 1. For example, $2 = 2^1$ and $7 = 7^1$.

Examples Write using exponential notation.

1. $7 \cdot 7 \cdot 7 = 7^3$
2. $3 \cdot 3 = 3^2$
3. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$
4. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 17 \cdot 17 \cdot 17 = 3^4 \cdot 17^3$

Work Practice 1–4

Practice 1–4

Write using exponential notation.

1. $8 \cdot 8 \cdot 8 \cdot 8$
2. $3 \cdot 3 \cdot 3$
3. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
4. $5 \cdot 5 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Objective B Evaluating Exponential Expressions

To **evaluate** an exponential expression, we write the expression as a product and then find the value of the product.

Examples Evaluate.

5. $9^2 = 9 \cdot 9 = 81$
6. $6^1 = 6$
7. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
8. $5 \cdot 6^2 = 5 \cdot 6 \cdot 6 = 180$

Work Practice 5–8

Practice 5–8

Evaluate.

5. 4^2 **6.** 7^3

7. 11^1 **8.** $2 \cdot 3^2$

Answers

1. 8^4 **2.** 3^3 **3.** 10^5 **4.** $5^2 \cdot 4^6$

5. 16 **6.** 343 **7.** 11 **8.** 18

Example 8 illustrates an important property: An exponent applies only to its base. The exponent 2, in $5 \cdot 6^2$, applies only to its base, 6.

Helpful Hint

An exponent applies only to its base. For example, $4 \cdot 2^3$ means $4 \cdot 2 \cdot 2 \cdot 2$.

Helpful Hint

Don't forget that 2^4, for example, is *not* $2 \cdot 4$. The expression 2^4 means repeated multiplication of the same factor.

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16, \quad \text{whereas } 2 \cdot 4 = 8$$

✓ **Concept Check** Which of the following statements is correct?

a. 3^6 is the same as $6 \cdot 6 \cdot 6$.
b. "Eight to the fourth power" is the same as 8^4.
c. "Ten squared" is the same as 10^3.
d. 11^2 is the same as $11 \cdot 2$.

Objective C Evaluating Square Roots

A **square root** of a number is one of two identical factors of the number. For example,

$7 \cdot 7 = 49$, so a square root of 49 is 7.

We use this symbol $\sqrt{}$ (called a radical sign) for finding square roots. Since

$7 \cdot 7 = 49$, then $\sqrt{49} = 7.$

Practice 9–11

Find each square root.

9. $\sqrt{100}$
10. $\sqrt{4}$
11. $\sqrt{1}$

Examples Find each square root.

9. $\sqrt{25} = 5$ because $5 \cdot 5 = 25$
10. $\sqrt{81} = 9$ because $9 \cdot 9 = 81$
11. $\sqrt{0} = 0$ because $0 \cdot 0 = 0$

Work Practice 9–11

Helpful Hint

Make sure you understand the difference between squaring a number and finding the square root of a number.

$9^2 = 9 \cdot 9 = 81 \quad \sqrt{9} = 3$ because $3 \cdot 3 = 9$

Not every square root simplifies to a whole number. We will study this more in a later chapter. In this section, we will find square roots of perfect squares only.

Answers

9. 10 **10.** 2 **11.** 1

✓ **Concept Check Answer**

b

Objective D Using the Order of Operations

Suppose that you are in charge of taking inventory at a local cell phone store. An employee has given you the number of a certain cell phone in stock as the expression

$$6 + 2 \cdot 30$$

To calculate the value of this expression, do you add first or multiply first? If you add first, the answer is 240. If you multiply first, the answer is 66.

Mathematical symbols wouldn't be very useful if two values were possible for one expression. Thus, mathematicians have agreed that, given a choice, we multiply first.

$$\begin{aligned} 6 + 2 \cdot 30 &= 6 + 60 && \text{Multiply.} \\ &= 66 && \text{Add.} \end{aligned}$$

This agreement is one of several **order of operations** agreements.

Order of Operations

1. Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars or square roots, starting with the innermost set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Below we practice using order of operations to simplify expressions.

Example 12 Simplify: $2 \cdot 4 - 3 \div 3$

Solution: There are no parentheses and no exponents, so we start by multiplying and dividing, from left to right.

$$\begin{aligned} 2 \cdot 4 - 3 \div 3 &= 8 - 3 \div 3 && \text{Multiply.} \\ &= 8 - 1 && \text{Divide.} \\ &= 7 && \text{Subtract.} \end{aligned}$$

Work Practice 12

Practice 12

Simplify: $9 \cdot 3 - 8 \div 4$

Example 13 Simplify: $4^2 \div 2 \cdot 4$

Solution: We start by evaluating 4^2.

$$4^2 \div 2 \cdot 4 = 16 \div 2 \cdot 4 \quad \text{Write } 4^2 \text{ as 16.}$$

Next we multiply or divide *in order* from left to right. Since division appears before multiplication from left to right, we divide first, then multiply.

$$\begin{aligned} 16 \div 2 \cdot 4 &= 8 \cdot 4 && \text{Divide.} \\ &= 32 && \text{Multiply.} \end{aligned}$$

Work Practice 13

Practice 13

Simplify: $48 \div 3 \cdot 2^2$

Answers

12. 25 **13.** 64

Practice 14

Simplify: $(10 - 7)^4 + 2 \cdot 3^2$

Example 14 Simplify: $(8 - 6)^2 + 2^3 \cdot 3$

Solution:

$$\begin{aligned} (8 - 6)^2 + 2^3 \cdot 3 &= 2^2 + 2^3 \cdot 3 && \text{Simplify inside parentheses.} \\ &= 4 + 8 \cdot 3 && \text{Write } 2^2 \text{ as 4 and } 2^3 \text{ as 8.} \\ &= 4 + 24 && \text{Multiply.} \\ &= 28 && \text{Add.} \end{aligned}$$

Work Practice 14

Practice 15

Simplify:

$36 \div [20 - (4 \cdot 2)] + 4^3 - 6$

Example 15 Simplify: $4^3 + [3^2 - (10 \div 2)] - 7 \cdot 3$

Solution: Here we begin with the innermost set of parentheses.

$$\begin{aligned} 4^3 + [3^2 - \overbrace{(10 \div 2)}] - 7 \cdot 3 &= 4^3 + [3^2 - 5] - 7 \cdot 3 && \text{Simplify inside parentheses.} \\ &= 4^3 + \underbrace{[9 - 5]} - 7 \cdot 3 && \text{Write } 3^2 \text{ as 9.} \\ &= 4^3 + 4 - 7 \cdot 3 && \text{Simplify inside brackets.} \\ &= 64 + 4 - \underbrace{7 \cdot 3} && \text{Write } 4^3 \text{ as 64.} \\ &= 64 + 4 - 21 && \text{Multiply.} \\ &= 47 && \text{Add and subtract from left to right.} \end{aligned}$$

Work Practice 15

Practice 16

Simplify: $\dfrac{25 + 8 \cdot 2 - 3^3}{2(3 - 2)}$

Example 16 Simplify: $\dfrac{7 - 2 \cdot 3 + 3^2}{5(2 - 1)}$

Solution: Here, the fraction bar is like a grouping symbol. We simplify above and below the fraction bar separately.

$$\begin{aligned} \frac{7 - 2 \cdot 3 + 3^2}{5(2 - 1)} &= \frac{7 - 2 \cdot 3 + 9}{5(1)} && \text{Evaluate } 3^2 \text{ and } (2 - 1). \\ &= \frac{7 - 6 + 9}{5} && \text{Multiply } 2 \cdot 3 \text{ in the numerator and multiply 5 and 1 in the denominator.} \\ &= \frac{10}{5} && \text{Add and subtract from left to right.} \\ &= 2 && \text{Divide.} \end{aligned}$$

Work Practice 16

Practice 17

Simplify: $81 \div \sqrt{81} \cdot 5 + 7$

Example 17 Simplify: $64 \div \sqrt{64} \cdot 2 + 4$

Solution:

$$\begin{aligned} 64 \div \sqrt{64} \cdot 2 + 4 &= \underbrace{64 \div 8} \cdot 2 + 4 && \text{Find the square root.} \\ &= \underbrace{8 \cdot 2} + 4 && \text{Divide.} \\ &= 16 + 4 && \text{Multiply.} \\ &= 20 && \text{Add.} \end{aligned}$$

Work Practice 17

Answers

14. 99 **15.** 61 **16.** 7 **17.** 52

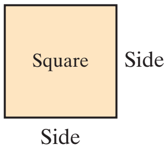
Objective E Finding the Area of a Square

Since a square is a special rectangle, we can find its area by finding the product of its length and its width.

Area of a rectangle = length · width

By recalling that each side of a square has the same measurement, we can use the following procedure to find its area:

$$\text{Area of a square} = \text{length} \cdot \text{width}$$
$$= \text{side} \cdot \text{side}$$
$$= (\text{side})^2$$



Helpful Hint

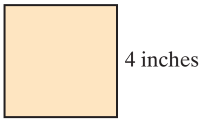
Recall from Section 1.6 that area is measured in **square** units while perimeter is measured in units.

Example 18 Find the area of a square whose side measures 4 inches.

Solution: Area of a square = (side)^2

= (4 inches)^2

= 16 square inches



The area of the square is 16 square inches.

Work Practice 18

Practice 18

Find the area of a square whose side measures 12 centimeters.

Answer

18. 144 sq cm

Calculator Explorations Exponents

To evaluate an exponent such as 4^7 on a calculator, find the keys marked $\boxed{y^x}$ or $\boxed{\wedge}$ and $\boxed{=}$ or $\boxed{\text{ENTER}}$. To evaluate 4^7, press the keys $\boxed{4}$ $\boxed{y^x}$ (or $\boxed{\wedge}$) $\boxed{7}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$. The display will read $\boxed{16384}$. Thus, $4^7 = 16{,}384$.

Use a calculator to evaluate.

1. 4^6

2. 5^6

3. 5^5

4. 7^6

5. 2^{11}

6. 6^8

Order of Operations

To see whether your calculator has the order of operations built in, evaluate $5 + 2 \cdot 3$ by pressing the keys $\boxed{5}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\times}$ $\boxed{3}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$. If the display reads $\boxed{11}$, your calculator does have the order of operations built in. This means that most of the time, you can key in a problem exactly as it is written and the calculator will perform operations in the proper order. When evaluating an expression containing parentheses, key in the parentheses. (If an expression contains brackets, key in parentheses.) For example, to evaluate $2[25 - (8 + 4)] - 11$, press the keys $\boxed{2}$ $\boxed{\times}$ $\boxed{(}$ $\boxed{25}$ $\boxed{-}$ $\boxed{(}$ $\boxed{8}$ $\boxed{+}$ $\boxed{4}$ $\boxed{)}$ $\boxed{)}$ $\boxed{-}$ $\boxed{11}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

The display will read $\boxed{15}$.

Use a calculator to evaluate.

7. $7^4 + 5^3$

8. $12^4 - 8^4$

9. $63 \cdot 75 - 43 \cdot 10$

10. $8 \cdot 22 + 7 \cdot 16$

11. $4(15 \div 3 + 2) - 10 \cdot 2$

12. $155 - 2(17 + 3) + 185$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

addition multiplication exponent base
subtraction division square root

1. In $2^5 = 32$, the 2 is called the ____________ and the 5 is called the ____________.
2. To simplify $8 + 2 \cdot 6$, which operation should be performed first? ____________
3. To simplify $(8 + 2) \cdot 6$, which operation should be performed first? ____________
4. To simplify $9(3 - 2) \div 3 + 6$, which operation should be performed first? ____________
5. To simplify $8 \div 2 \cdot 6$, which operation should be performed first? ____________
6. The ____________ of a whole number is one of two identical factors of the number.

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 1.9

Objective A **7.** In the Example 1 expression, what is the 3 called and what is the 12 called?

Objective B **8.** As mentioned in Example 4, what "understood exponent" does any number we've worked with before have?

Objective C **9.** From Example 7, how do we know that $\sqrt{64} = 8$?

Objective D **10.** List the three operations needed to evaluate Example 9 in the order they should be performed.

Objective E **11.** As explained in the lecture before Example 12, why does the area of a square involve an exponent whereas the area of a rectangle usually does not?

1.9 Exercise Set MyLab Math

Objective A *Write using exponential notation. See Examples 1 through 4.*

1. $4 \cdot 4 \cdot 4$
2. $5 \cdot 5 \cdot 5 \cdot 5$
3. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$
4. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$
5. $12 \cdot 12 \cdot 12$
6. $10 \cdot 10 \cdot 10$
7. $6 \cdot 6 \cdot 5 \cdot 5 \cdot 5$
8. $4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$
9. $9 \cdot 8 \cdot 8$
10. $7 \cdot 4 \cdot 4 \cdot 4$
11. $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
12. $4 \cdot 6 \cdot 6 \cdot 6 \cdot 6$
13. $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$
14. $6 \cdot 6 \cdot 2 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Objective B *Evaluate. See Examples 5 through 8.*

15. 8^2
16. 6^2
▶ **17.** 5^3
18. 6^3
19. 2^5
20. 3^5
21. 1^{10}
22. 1^{12}
▶ **23.** 7^1
24. 8^1
25. 2^7
26. 5^4
27. 2^8
28. 3^3
29. 4^4
30. 4^3
31. 9^3
32. 8^3
33. 12^2
34. 11^2
▶ **35.** 10^2
36. 10^3
37. 20^1
38. 14^1
39. 3^6
40. 4^5
41. $3 \cdot 2^6$
42. $5 \cdot 3^2$
43. $2 \cdot 3^4$
44. $2 \cdot 7^2$

Objective C *Find each square root. See Examples 9 through 11.*

▶ **45.** $\sqrt{9}$
46. $\sqrt{36}$
▶ **47.** $\sqrt{64}$
48. $\sqrt{121}$
49. $\sqrt{144}$
50. $\sqrt{0}$
51. $\sqrt{16}$
52. $\sqrt{169}$

Objective D *Simplify. See Examples 12 through 16. (This section does not contain square roots.)*

▶ **53.** $15 + 3 \cdot 2$
54. $24 + 6 \cdot 3$
▶ **55.** $14 \div 7 \cdot 2 + 3$
56. $100 \div 10 \cdot 5 + 4$
57. $32 \div 4 - 3$
58. $42 \div 7 - 6$
59. $13 + \frac{24}{8}$
60. $32 + \frac{8}{2}$
61. $6 \cdot 5 + 8 \cdot 2$
62. $3 \cdot 4 + 9 \cdot 1$
63. $\frac{5 + 12 \div 4}{1^7}$
64. $\frac{6 + 9 \div 3}{3^2}$
65. $(7 + 5^2) \div 4 \cdot 2^3$
66. $6^2 \cdot (10 - 8)$
67. $5^2 \cdot (10 - 8) + 2^3 + 5^2$
68. $5^3 \div (10 + 15) + 9^2 + 3^3$
69. $\frac{18 + 6}{2^4 - 2^2}$
70. $\frac{40 + 8}{5^2 - 3^2}$
71. $(3 + 5) \cdot (9 - 3)$
72. $(9 - 7) \cdot (12 + 18)$
▶ **73.** $\frac{7(9 - 6) + 3}{3^2 - 3}$

74. $\dfrac{5(12-7)-4}{5^2-18}$

75. $8 \div 0 + 37$

76. $18 - 7 \div 0$

77. $2^4 \cdot 4 - (25 \div 5)$

78. $2^3 \cdot 3 - (100 \div 10)$

79. $3^4 - [35 - (12 - 6)]$

80. $[40 - (8 - 2)] - 2^5$

▶ **81.** $(7 \cdot 5) + [9 \div (3 \div 3)]$

82. $(18 \div 6) + [(3 + 5) \cdot 2]$

83. $8 \cdot \left[2^2 + (6 - 1) \cdot 2\right] - 50 \cdot 2$

84. $35 \div \left[3^2 + (9 - 7) - 2^2\right] + 10 \cdot 3$

85. $\dfrac{9^2 + 2^2 - 1^2}{8 \div 2 \cdot 3 \cdot 1 \div 3}$

86. $\dfrac{5^2 - 2^3 + 1^4}{10 \div 5 \cdot 4 \cdot 1 \div 4}$

Simplify. See Examples 12 through 17. (This section does contain square roots.)

87. $6 \cdot \sqrt{9} + 3 \cdot \sqrt{4}$

88. $3 \cdot \sqrt{25} + 2 \cdot \sqrt{81}$

89. $4 \cdot \sqrt{49} - 0 \div \sqrt{100}$

90. $7 \cdot \sqrt{36} - 0 \div \sqrt{64}$

91. $\dfrac{\sqrt{4} + 4^2}{5(20 - 16) - 3^2 - 5}$

92. $\dfrac{\sqrt{9} + 9^2}{3(10 - 6) - 2^2 - 1}$

93. $\sqrt{81} \div \sqrt{9} + 4^2 \cdot 2 - 10$

94. $\sqrt{100} \div \sqrt{4} + 3^3 \cdot 2 - 20$

95. $\left[\sqrt{225} \div (11 - 6) + 2^2\right] + \left(\sqrt{25} - \sqrt{1}\right)^2$

96. $\left[\sqrt{169} \div (20 - 7) + 2^5\right] - \left(\sqrt{4} + \sqrt{9}\right)^2$

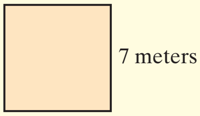
97. $7^2 - \left\{18 - \left[40 \div (4 \cdot 2) + \sqrt{4}\right] + 5^2\right\}$

98. $29 - \{5 + 3[8 \cdot (10 - \sqrt{64})] - 50\}$

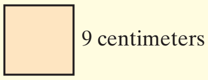
Objective E Mixed Practice *(Sections 1.3 and 1.6)*

Find the area and perimeter of each square. See Example 18.

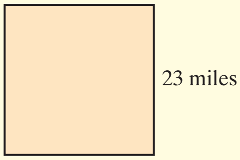
▶ △ **99.**



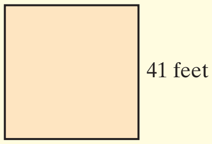
△ **100.**



△ **101.**



△ **102.**



Concept Extensions

Answer the following true or false. See the Concept Check in this section.

103. "Six to the fifth power" is the same as 6^5.

104. "Seven squared" is the same as 7^2.

105. 2^5 is the same as $5 \cdot 5$.

106. 4^9 is the same as $4 \cdot 9$.

Insert grouping symbols (parentheses) so that each given expression evaluates to the given number.

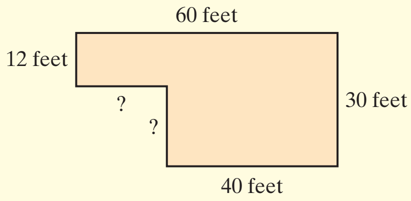
107. $2 + 3 \cdot 6 - 2$; evaluates to 28

108. $2 + 3 \cdot 6 - 2$; evaluates to 20

109. $24 \div 3 \cdot 2 + 2 \cdot 5$; evaluates to 14

110. $24 \div 3 \cdot 2 + 2 \cdot 5$; evaluates to 15

△ **111.** A building contractor is bidding on a contract to install gutters on seven homes in a retirement community, all in the shape shown. To estimate the cost of materials, she needs to know the total perimeter of all seven homes. Find the total perimeter.



112. The building contractor from Exercise **111** plans to charge $4 per foot for installing vinyl gutters. Find the total charge for the seven homes given the total perimeter answer to Exercise **111**.

Simplify.

🖩 **113.** $(7 + 2^4)^5 - (3^5 - 2^4)^2$

🖩 **114.** $25^3 \cdot (45 - 7 \cdot 5) \cdot 5$

✎ **115.** Write an expression that simplifies to 5. Use multiplication, division, addition, subtraction, and at least one set of parentheses. Explain the process you would use to simplify the expression.

✎ **116.** Explain why $2 \cdot 3^2$ is not the same as $(2 \cdot 3)^2$.

Chapter 1 Group Activity

Modeling Subtraction of Whole Numbers

A mathematical concept can be represented or modeled in many different ways. For instance, subtraction can be represented by the following symbolic model:

$$11 - 4$$

The following verbal models can also represent subtraction of these same quantities:

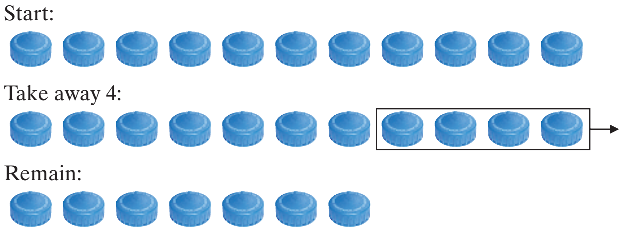
"Four subtracted from eleven" or
"Eleven take away four"

Physical models can also represent mathematical concepts. In these models, a number is represented by that many objects. For example, the number 5 can be represented by five pennies, squares, paper clips, tiles, or bottle caps.

A physical representation of the number 5

Take-Away Model for Subtraction: 11 – 4

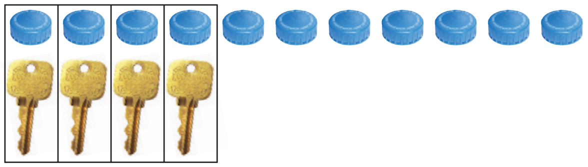
- Start with 11 objects.
- Take 4 objects away.
- How many objects remain?



Comparison Model for Subtraction: 11 – 4

- Start with a set of 11 of one type of object and a set of 4 of another type of object.

- Make as many pairs that include one object of each type as possible.



- How many more objects left are in the larger set?

Missing Addend Model for Subtraction: 11 – 4

- Start with 4 objects.
- Continue adding objects until a total of 11 is reached.
- How many more objects were needed to give a total of 11?

Group Activity

Use an appropriate physical model for subtraction to solve each of the following problems. Explain your reasoning for choosing each model.

1. Sneha has assembled 12 computer components so far this shift. If her quota is 20 components, how many more components must she assemble to reach her quota?
2. Yuko has 14 daffodil bulbs to plant in her yard. She planted 5 bulbs in the front yard. How many bulbs does she have left for planting in the backyard?
3. Todd is 19 years old and his sister Tanya is 13 years old. How much older is Todd than Tanya?

Chapter 1 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

difference
place value
sum
area
factor
whole numbers
square root
quotient
perimeter
addend
subtrahend
dividend
divisor
exponent
average
minuend
digits
product

1. The ____________ are 0, 1, 2, 3, . . .

2. The ____________ of a polygon is its distance around or the sum of the lengths of its sides.

3. The position of each digit in a number determines its ____________.

4. A(n) ____________ is a shorthand notation for repeated multiplication of the same factor.

5. To find the ____________ of a rectangle, multiply length times width.

6. A(n) ____________ of a number is one of two identical factors of the number.

7. The ____________ used to write numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

8. The ____________ of a list of numbers is their sum divided by the number of numbers.

Use the facts below for Exercises 9 through 18.

$$2 \cdot 3 = 6 \quad 4 + 17 = 21 \quad 20 - 9 = 11 \quad \begin{array}{r}7\\5\overline{)35}\end{array}$$

9. The 5 above is called the ____________.

10. The 35 above is called the ____________.

11. The 7 above is called the ____________.

12. The 3 above is called a(n) ____________.

13. The 6 above is called the ____________.

14. The 20 above is called the ____________.

15. The 9 above is called the ____________.

16. The 11 above is called the ____________.

17. The 4 above is called a(n) ____________.

18. The 21 above is called the ____________.

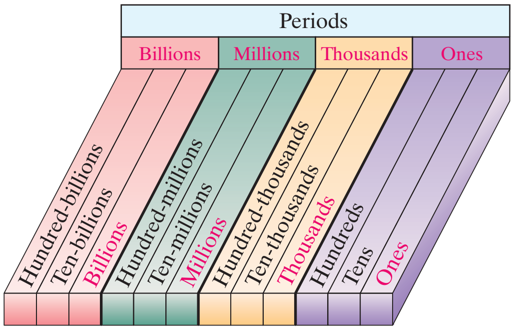
Helpful Hint

Are you preparing for your test? To help, don't forget to take these:

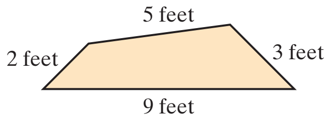
- Chapter 1 Getting Ready for the Test on page 108
- Chapter 1 Test on page 109

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

1 Chapter Highlights

Definitions and Concepts	Examples
Section 1.2 Place Value, Names for Numbers, and Reading Tables	
The **whole numbers** are 0, 1, 2, 3, 4, 5, ... The **natural numbers** are 1, 2, 3, 4, 5, ... The position of each digit in a number determines its **place value.** A place-value chart is shown next with the names of the periods given.  	0, 14, 968, 5,268,619 are whole numbers. Zero is a whole number but not a natural number.

(continued)

Definitions and Concepts	Examples
Section 1.2 Place Value, Names for Numbers, and Reading Tables (*continued*)	
To write a whole number in words, write the number in each period followed by the name of the period. (The name of the ones period is not included.)	9,078,651,002 is written as nine billion, seventy-eight million, six hundred fifty-one thousand, two.
To write a whole number in standard form, write the number in each period, followed by a comma.	Four million, seven hundred six thousand, twenty-eight is written as 4,706,028.
Section 1.3 Adding Whole Numbers and Perimeter	
To add whole numbers, add the digits in the ones place, then the tens place, then the hundreds place, and so on, carrying when necessary.	Find the sum: $\begin{array}{rl} \overset{2\,1\,1}{2689} & \leftarrow \text{addend} \\ 1735 & \leftarrow \text{addend} \\ +\ 662 & \leftarrow \text{addend} \\ \hline 5086 & \leftarrow \text{sum} \end{array}$
The **perimeter** of a polygon is its distance around or the sum of the lengths of its sides.	△ Find the perimeter of the polygon shown.  The perimeter is 5 feet + 3 feet + 9 feet + 2 feet = 19 feet.
Section 1.4 Subtracting Whole Numbers	
To subtract whole numbers, subtract the digits in the ones place, then the tens place, then the hundreds place, and so on, borrowing when necessary.	Subtract: $\begin{array}{rl} 7\overset{8}{\cancel{9}}\overset{15}{\cancel{5}}4 & \leftarrow \text{minuend} \\ -5673 & \leftarrow \text{subtrahend} \\ \hline 2281 & \leftarrow \text{difference} \end{array}$
Section 1.5 Rounding and Estimating	
Rounding Whole Numbers to a Given Place Value **Step 1:** Locate the digit to the right of the given place value. **Step 2:** If this digit is 5 or greater, add 1 to the digit in the given place value and replace each digit to its right with 0. **Step 3:** If this digit is less than 5, replace it and each digit to its right with 0.	Round 15,721 to the nearest thousand. $\underset{\text{Add 1}\uparrow}{15},\underbrace{\textcircled{7}21}_{\text{Replace with zeros.}}$ Since the circled digit is 5 or greater, add 1 to the given place value and replace digits to its right with zeros. 15,721 rounded to the nearest thousand is 16,000.

Definitions and Concepts | **Examples**

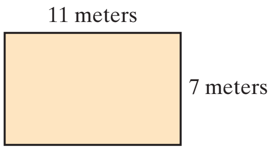
Section 1.6 Multiplying Whole Numbers and Area

To multiply 73 and 58, for example, multiply 73 and 8, then 73 and 50. The sum of these partial products is the product of 73 and 58. Use the notation to the right.

$$\begin{array}{r l}
73 & \leftarrow \text{factor}\\
\times\ 58 & \leftarrow \text{factor}\\
\hline
584 & \leftarrow 73 \times 8\\
3650 & \leftarrow 73 \times 50\\
\hline
4234 & \leftarrow \text{product}
\end{array}$$

To find the **area** of a rectangle, multiply length times width.

△ Find the area of the rectangle shown.



$$\begin{aligned}
\text{area of rectangle} &= \text{length} \cdot \text{width}\\
&= (11 \text{ meters})(7 \text{ meters})\\
&= 77 \text{ square meters}
\end{aligned}$$

Section 1.7 Dividing Whole Numbers

Division Properties of 0

The quotient of 0 and any number (except 0) is 0.

$\frac{0}{5} = 0$

The quotient of any number and 0 is not a number. We say that this quotient is undefined.

$\frac{7}{0}$ is undefined

To divide larger whole numbers, use the process called **long division** as shown to the right.

$$\begin{array}{r l}
507 \text{ R } 2 & \leftarrow \text{quotient and remainder}\\
\text{divisor} \rightarrow 14\overline{)7100} & \leftarrow \text{dividend}\\
-70\downarrow & 5(14) = 70\\
\hline
10 & \text{Subtract and bring down the 0.}\\
-0\downarrow & 0(14) = 0\\
\hline
100 & \text{Subtract and bring down the 0.}\\
-98 & 7(14) = 98\\
\hline
2 & \text{Subtract. The remainder is 2.}
\end{array}$$

To check, see that $507 \cdot 14 + 2 = 7100$.

The **average** of a list of numbers is

$$\text{average} = \frac{\text{sum of numbers}}{\textit{number} \text{ of numbers}}$$

Find the average of 23, 35, and 38.

$$\text{average} = \frac{23 + 35 + 38}{3} = \frac{96}{3} = 32$$

Definitions and Concepts	Examples
Section 1.8 An Introduction to Problem Solving	
Problem-Solving Steps	Suppose that 225 tickets are sold for each performance of a play. How many tickets are sold for 5 performances?
1. UNDERSTAND the problem.	**1.** UNDERSTAND. Read and reread the problem. Since we want the number of tickets for 5 performances, we multiply.
2. TRANSLATE the problem.	**2.** TRANSLATE. $\begin{array}{ccccc} \text{number of tickets} & \text{is} & \text{number of performances} & \text{times} & \text{tickets per performance} \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Number of tickets} & = & 5 & \cdot & 225 \end{array}$
3. SOLVE the problem.	**3.** SOLVE: See if the answer is reasonable by also estimating. $\begin{array}{r} \overset{1\,2}{22}5 \\ \times\ \ 5 \\ \hline 1125 \end{array}$ exact rounds to $\begin{array}{r} 200 \\ \times\ \ 5 \\ \hline 1000 \end{array}$ estimate
4. INTERPRET the results.	**4.** INTERPRET. **Check** your work. The product is reasonable since 1125 is close to our estimated answer of 1000, and **state** your conclusion: There are 1125 tickets sold for 5 performances.
Section 1.9 Exponents, Square Roots, and Order of Operations	
An **exponent** is a shorthand notation for repeated multiplication of the same factor.	$\underset{\text{base}}{3}^{\overset{\text{exponent}}{4}} = \underbrace{3\cdot3\cdot3\cdot3}_{\text{4 factors of 3}} = 81$
A **square root** of a number is one of two identical factors of the number.	$\sqrt{36} = 6$ because $6\cdot6 = 36$ $\sqrt{121} = 11$ because $11\cdot11 = 121$ $\sqrt{0} = 0$ because $0\cdot0 = 0$
Order of Operations **1.** Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as square roots or fraction bars, starting with the innermost set. **2.** Evaluate any expressions with exponents. **3.** Multiply or divide in order from left to right. **4.** Add or subtract in order from left to right.	Simplify: $\frac{5 + 3^2}{2(7 - 6)}$ Simplify above and below the fraction bar separately. $\frac{5 + 3^2}{2(7 - 6)} = \frac{5 + 9}{2(1)}$ Evaluate 3^2 above the fraction bar. Subtract: $7 - 6$ below the fraction bar. $= \frac{14}{2}$ Add. Multiply. $= 7$ Divide.
The **area of a square** is $(\text{side})^2$.	Find the area of a square with side length 9 inches. Area of the square $= (\text{side})^2$ $= (9\,\text{inches})^2$ $= 81\,\text{square inches}$

Chapter 1 Review

(1.2) *Determine the place value of the digit 4 in each whole number.*

1. 7640

2. 46,200,120

Write each whole number in words.

3. 7640

4. 46,200,120

Write each whole number in expanded form.

5. 3158

6. 403,225,000

Write each whole number in standard form.

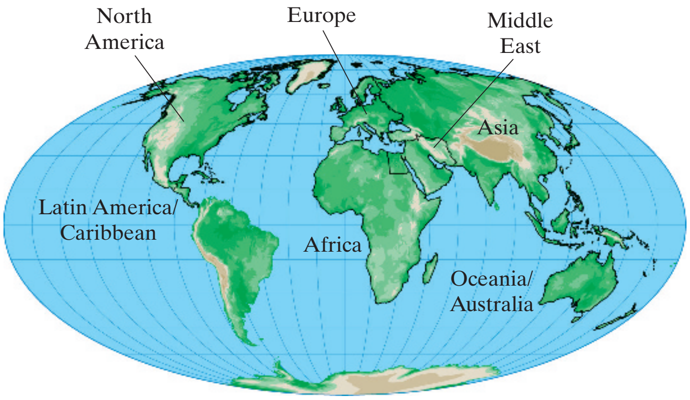
7. Eighty-one thousand, nine hundred

8. Six billion, three hundred four million

The following table shows the Internet and Facebook use of world regions as of June 2016. Use this table to answer Exercises 9 through 12.

World Region	Internet Users	Facebook Users
Africa	345,436,500	146,637,000
Asia	1,873,522,600	559,003,000
Europe	636,831,820	328,273,740
Middle East	141,711,760	76,000,000
North America	320,021,200	223,081,200
Latin America/ Caribbean	385,842,300	326,975,340
Oceania/Australia	27,506,000	19,463,250

(*Source:* Internet World Stats)



9. Find the number of Internet users in Europe.

10. Find the number of Facebook users in Latin America/Caribbean.

11. Which world region had the largest number of Facebook users?

12. Which world region had the smallest number of Internet users?

(1.3) *Add.*

13. 17 + 46

14. 28 + 39

15. 25 + 8 + 15

16. 27 + 9 + 41

17. 932 + 24

18. 819 + 21

19. 567 + 7383

20. 463 + 6787

21. 91 + 3623 + 497

22. 82 + 1647 + 238

Solve.

23. Find the sum of 86, 331, and 909.

24. Find the sum of 49, 529, and 308.

25. What is 26,481 increased by 865?

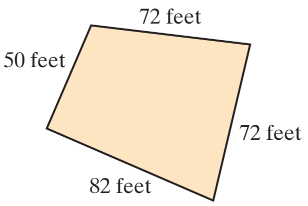
26. What is 38,556 increased by 744?

27. The distance from Chicago to New York City is 714 miles. The distance from New York City to New Delhi, India, is 7318 miles. Find the total distance from Chicago to New Delhi if traveling by air through New York City.

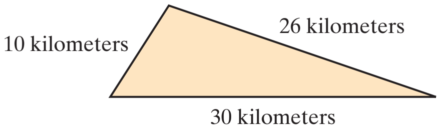
28. Susan Summerline earned salaries of $62,589, $65,340, and $69,770 during the years 2014, 2015, and 2016, respectively. Find her total earnings during those three years.

Find the perimeter of each figure.

△ **29.**



△ **30.**



(1.4) *Subtract and then check.*

31. 93 − 79

32. 61 − 27

33. 462 − 397

34. 583 − 279

35. 4000 − 86

36. 8000 − 92

Solve.

37. Subtract 7965 from 25,862.

38. Subtract 4349 from 39,007.

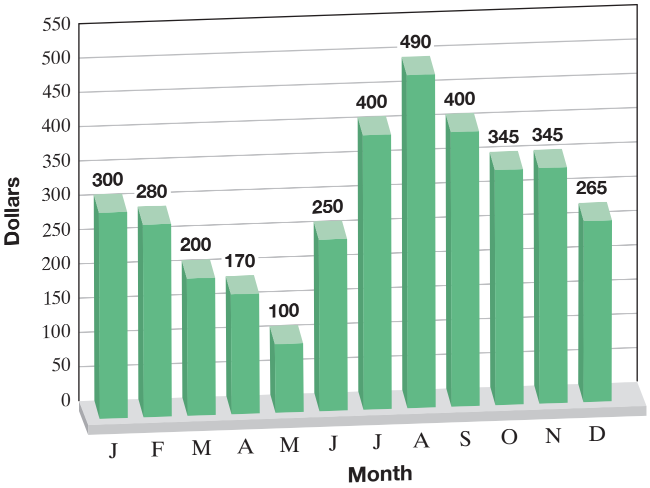
39. Find the increase in population for San Antonio, Texas, from 2010 (population: 1,327,551) to 2016 (population: 1,469,485). (*Source:* U.S. Census Bureau)

40. Find the decrease in population for Detroit, Michigan, from 2010 (population: 713,862) to 2016 (population: 677,116). (*Source:* U.S. Census Bureau)

41. Bob Roma is proofreading the Yellow Pages for his county. If he has finished 315 pages of the total 712 pages, how many pages does he have left to proofread?

42. Shelly Winters bought a new car listed at $28,425. She received a discount of $1599 and a factory rebate of $1200. Find how much she paid for the car.

The following bar graph shows the monthly savings account balance for a freshman attending a local community college. Use this graph to answer Exercises 43 through 46.



43. During what month was the balance the least?

44. During what month was the balance the greatest?

45. By how much did his balance decrease from February to April?

46. By how much did his balance increase from June to August?

(1.5) *Round to the given place.*

47. 93 to the nearest ten

48. 45 to the nearest ten

49. 467 to the nearest ten

50. 493 to the nearest hundred

51. 4832 to the nearest hundred

52. 57,534 to the nearest thousand

53. 49,683,712 to the nearest million

54. 768,542 to the nearest hundred-thousand

55. In 2017, there were 17,546,645 new vehicles sold in the United States. Round this number to the nearest million. (*Source:* U.S. Department of Transportation)

56. In a recent year, there were 98,454 public elementary and secondary schools in the United States. Round this number to the nearest thousand. (*Source:* National Center for Education Statistics)

Estimate the sum or difference by rounding each number to the nearest hundred.

57. $4892 + 647 + 1876$

58. $5925 - 1787$

59. A group of students took a week-long driving trip and traveled 628, 290, 172, 58, 508, 445, and 383 miles on seven consecutive days. Round each distance to the nearest hundred to estimate the distance they traveled.

60. The estimated 2016 population of Houston, Texas, was 2,239,558, and for San Diego, California, it was 1,376,410. Round each number to be nearest hundred-thousand and estimate how much larger Houston is than San Diego. (*Source:* U.S. Census Bureau)

(1.6) *Multiply.*

61. $\begin{array}{r} 273 \\ \times \quad 7 \\ \hline \end{array}$

62. $\begin{array}{r} 349 \\ \times \quad 4 \\ \hline \end{array}$

63. $\begin{array}{r} 47 \\ \times\, 30 \\ \hline \end{array}$

64. $\begin{array}{r} 69 \\ \times\, 42 \\ \hline \end{array}$

65. 20(8)(5)

66. 25(9)(4)

67. $\begin{array}{r} 48 \\ \times\, 77 \\ \hline \end{array}$

68. $\begin{array}{r} 77 \\ \times\, 22 \\ \hline \end{array}$

69. $49 \cdot 49 \cdot 0$

70. $62 \cdot 88 \cdot 0$

71. $\begin{array}{r} 586 \\ \times \;\, 29 \\ \hline \end{array}$

72. $\begin{array}{r} 242 \\ \times \;\, 37 \\ \hline \end{array}$

73. $\begin{array}{r} 642 \\ \times\, 177 \\ \hline \end{array}$

74. $\begin{array}{r} 347 \\ \times \;\, 129 \\ \hline \end{array}$

75. $\begin{array}{r} 1026 \\ \times \;\, 401 \\ \hline \end{array}$

76. $\begin{array}{r} 2107 \\ \times \;\, 302 \\ \hline \end{array}$

77. $375 \cdot 1000$

78. $108 \cdot 1000$

79. $30 \cdot 400$

80. $50 \cdot 700$

81. $1700 \cdot 3000$

82. $1900 \cdot 4000$

Solve.

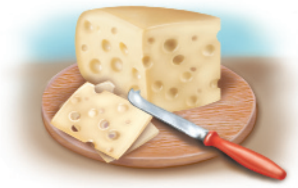
83. Find the product of 5 and 230.

84. Find the product of 6 and 820.

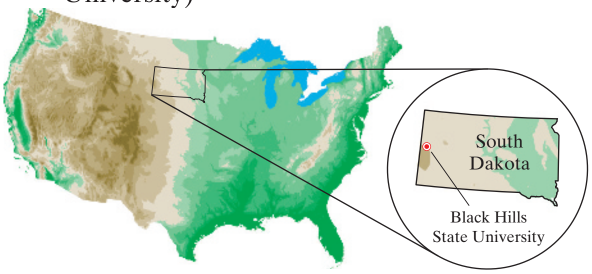
85. Multiply 9 and 12.

86. Multiply 8 and 14.

87. One ounce of Swiss cheese contains 8 grams of fat. How many grams of fat are in 3 ounces of Swiss cheese? (*Source: Home and Garden Bulletin No. 72*, U.S. Department of Agriculture)

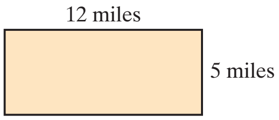


88. The cost for a South Dakota resident to attend Black Hills State University full-time is $7949 per semester. Determine the cost for 20 students to attend full-time. (*Source:* Black Hills State University)

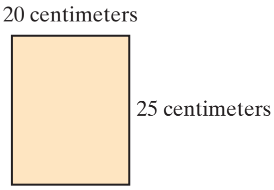


Find the area of each rectangle.

△ **89.**



△ **90.**



(1.7) *Divide and then check.*

91. $\frac{18}{6}$ **92.** $\frac{36}{9}$ **93.** $42 \div 7$ **94.** $35 \div 5$ **95.** $27 \div 5$

96. $18 \div 4$ **97.** $16 \div 0$ **98.** $0 \div 8$ **99.** $9 \div 9$ **100.** $10 \div 1$

101. $0 \div 668$ **102.** $918 \div 0$ **103.** $5\overline{)167}$ **104.** $8\overline{)159}$ **105.** $26\overline{)626}$

106. $19\overline{)680}$ **107.** $47\overline{)23{,}792}$ **108.** $53\overline{)48{,}111}$ **109.** $207\overline{)578{,}291}$ **110.** $306\overline{)615{,}732}$

Solve.

111. Find the quotient of 92 and 5.

112. Find the quotient of 86 and 4.

113. One foot is 12 inches. Find how many feet there are in 5496 inches.

114. One mile is 1760 yards. Find how many miles there are in 22,880 yards.

115. Find the average of the numbers 76, 49, 32, and 47.

116. Find the average of the numbers 23, 85, 62, and 66.

(1.8) *Solve.*

117. A box can hold 24 cans of corn. How many boxes can be filled with 648 cans of corn?

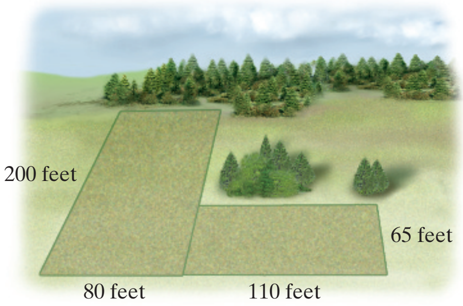
118. If a ticket to a movie costs $6, how much do 32 tickets cost?

119. In a recent year, General Motors spent $3,500,000,000 on television advertising, while Toyota spent only $1,800,000,000. How much more did General Motors spend on television advertising than Toyota? (*Source:* Automotive News)

120. The cost to banks when a person uses an automatic teller machine (ATM) is 27¢. The cost to banks when a person deposits a check with a teller is 48¢ more. How much is this cost?

121. A golf pro orders shirts for the company sponsoring a local charity golfing event. Shirts size large cost $32 while shirts size extra-large cost $38. If 15 large shirts and 11 extra-large shirts are ordered, find the cost.

122. Two rectangular pieces of land are purchased: one that measures 65 feet by 110 feet and one that measures 80 feet by 200 feet. Find the total area of land purchased. (*Hint:* Find the area of each rectangle, then add.)



(1.9) *Simplify.*

123. 7^2

124. 5^3

125. $5 \cdot 3^2$

126. $4 \cdot 10^2$

127. $18 \div 3 + 7$

128. $12 - 8 \div 4$

129. $\dfrac{5(6^2 - 3)}{3^2 + 2}$

130. $\dfrac{7(16 - 8)}{2^3}$

131. $48 \div 8 \cdot 2$

132. $27 \div 9 \cdot 3$

133. $2 + 3\left[1^5 + (20 - 17) \cdot 3\right] + 5 \cdot 2$

134. $21 - [2^4 - (7 - 5) - 10] + 8 \cdot 2$

Simplify. (These exercises contain square roots.)

135. $\sqrt{81}$

136. $\sqrt{4}$

137. $\sqrt{1}$

138. $\sqrt{0}$

139. $4 \cdot \sqrt{25} - 2 \cdot 7$

140. $8 \cdot \sqrt{49} - 3 \cdot 9$

141. $(\sqrt{36} - \sqrt{16})^3 \cdot [10^2 \div (3 + 17)]$

142. $(\sqrt{49} - \sqrt{25})^3 \cdot [9^2 \div (2 + 7)]$

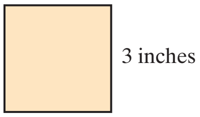
143. $\dfrac{5 \cdot 7 - 3 \cdot \sqrt{25}}{2(\sqrt{121} - 3^2)}$

144. $\dfrac{4 \cdot 8 - 1 \cdot \sqrt{121}}{3(\sqrt{81} - 2^3)}$

Find the area of each square.

△ **145.** A square with side length of 7 meters.

△ **146.**



Mixed Review

Perform the indicated operations.

147. $375 - 68$

148. $729 - 47$

149. 723×3

150. 629×4

151. $264 + 39 + 598$

152. $593 + 52 + 766$

153. $13\overline{)5962}$

154. $18\overline{)4267}$

155. 1968×36

156. 5324×18

157. $2000 - 356$

158. $9000 - 519$

Round to the given place.

159. 736 to the nearest ten

160. 258,371 to the nearest thousand

161. 1999 to the nearest hundred

162. 44,499 to the nearest ten thousand

Write each whole number in words.

163. 36,911

164. 154,863

Write each whole number in standard form.

165. Seventy thousand, nine hundred forty-three

166. Forty-three thousand, four hundred one

Simplify.

167. 4^3

168. 5^3

169. $\sqrt{144}$

170. $\sqrt{100}$

171. $24 \div 4 \cdot 2$

172. $\sqrt{256} - 3 \cdot 5$

173. $\dfrac{8(7 - 4) - 10}{4^2 - 3^2}$

174. $\dfrac{(15 + \sqrt{9}) \cdot (8 - 5)}{2^3 + 1}$

Solve.

175. 36 divided by 9 is what number?

176. What is the product of 2 and 12?

177. 16 increased by 8 is what number?

178. 7 subtracted from 21 is what number?

The following table shows the 2015 and 2016 average Major League Baseball salaries (rounded to the nearest thousand) for the five teams with the largest payrolls for 2016. Use this table to answer Exercises 179 and 180. (Source: CBSSports.com, Associated Press)

Team	2016 Average Salary	2015 Average Salary
Los Angeles Dodgers	$7,445,000	$9,092,000
New York Yankees	$7,361,000	$7,309,000
Boston Red Sox	$6,072,000	$6,247,000
Detroit Tigers	$6,891,000	$5,794,000
San Francisco Giants	$5,946,000	$5,756,000

179. How much more was the average salary for a San Francisco Giants player in 2016 than in 2015?

180. How much less was the average Boston Red Sox salary in 2016 than the average New York Yankee salary in 2016?

181. A manufacturer of drinking glasses ships his delicate stock in special boxes that can hold 32 glasses. If 1714 glasses are manufactured, how many full boxes are filled? Are there any glasses left over?

182. A teacher orders 2 small white boards for $27 each and 8 boxes of dry erase pens for $4 each. What is her total bill before taxes?

Chapter 1 Getting Ready for the Test LC

MATCHING Exercises 1 through 16 are **Matching** exercises. Choices may be used more than once or not at all.

For Exercises 1 through 4, match the digit from the number 189,570,264 in the left column with the correct place value listed in the right columns.

1. 9
2. 7
3. 0
4. 8

A. hundreds place
B. thousands place
C. ten-thousands place
D. millions place
E. ten-millions place
F. billions place

The number 5,726,953 is rounded to different place values. For Exercises 5 through 8, match each rounding of the number in the left column to the place it is rounded to in the right column. One exercise has 2 correct answers.

5. 6,000,000
6. 5,726,950
7. 5,727,000
8. 5,730,000

A. the nearest ten
B. the nearest hundred
C. the nearest million
D. the nearest ten-thousand
E. the nearest thousand

The number 27,600 is multiplied and divided by different powers of 10. For Exercises 9 through 12, match each number in the left column with the correct operation described in the right column.

9. 276,000
10. 276
11. 2760
12. 2,760,000

A. 27,600 divided by 10
B. 27,600 multiplied by 10
C. 27,600 divided by 100
D. 27,600 multiplied by 100

For Exercises 13 through 16, match each exercise in the left column with its correct answer in the right column.

13. 9^2
14. $\sqrt{9}$
15. 9^1
16. $16 - 4 \div 2^2$

A. 3
B. 81
C. 15
D. 0
E. 9

MULTIPLE CHOICE *For Exercises 17 through 20, do not calculate, but use rounding to choose the best estimated answer.*

17. 49×52
A. 25 **B.** 250 **C.** 2500 **D.** 25,000

18. 3275×11
A. 32,750 **B.** 3275 **C.** 327,500 **D.** 3,275,000

19. $87 + 86 + 91$
A. 27,000 **B.** 270 **C.** 2700 **D.** 27

20. $1000 - 62$
A. 940 **B.** 400 **C.** 40 **D.** 9400

For additional practice go to your study plan in MyLab Math.

MyLab Math

Test Chapter 1

Simplify.

1. Write 82,426 in words.

2. Write "four hundred two thousand, five hundred fifty" in standard form.

3. $59 + 82$

4. $600 - 487$

5.
$$\begin{array}{r} 496 \\ \times\ 30 \\ \hline \end{array}$$

6. $52{,}896 \div 69$

7. $2^3 \cdot 5^2$

8. $\sqrt{4} \cdot \sqrt{25}$

9. $0 \div 49$

10. $62 \div 0$

11. $(2^4 - 5) \cdot 3$

12. $16 + 9 \div 3 \cdot 4 - 7$

13. $\dfrac{64 \div 8 \cdot 2}{(\sqrt{9} - \sqrt{4})^2 + 1}$

14. $2[(6 - 4)^2 + (22 - 19)^2] + 10$

15. $5698 \cdot 1000$

16. $8000 \cdot 1400$

17. Round 52,369 to the nearest thousand.

Estimate each sum or difference by rounding each number to the nearest hundred.

18. $6289 + 5403 + 1957$

19. $4267 - 2738$

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ____________________

18. ____________________

19. ____________________

20.

21.

22.

23.

24.

25.

26.

27.

28.

29.

Solve.

20. Subtract 15 from 107.

21. Find the sum of 15 and 107.

22. Find the product of 15 and 107.

23. Find the quotient of 107 and 15.

24. Twenty-nine cans of Sherwin-Williams paint cost $493. How much was each can?

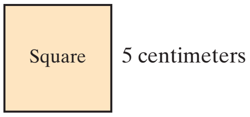
25. Jo McElory is looking at two new refrigerators for her apartment. One costs $599 and the other costs $725. How much more expensive is the higher-priced one?

26. One tablespoon of white granulated sugar contains 45 calories. How many calories are in 8 tablespoons of white granulated sugar? (*Source: Home and Garden Bulletin No. 72,* U.S. Department of Agriculture)

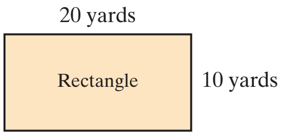
27. A small business owner recently ordered 16 digital cameras that cost $430 each and 5 printers that cost $205 each. Find the total cost for these items.

Find the perimeter and the area of each figure.

28.



29.



Integers and Introduction to Variables

2

The system in Boston, built in 1897, is the oldest in the U.S.

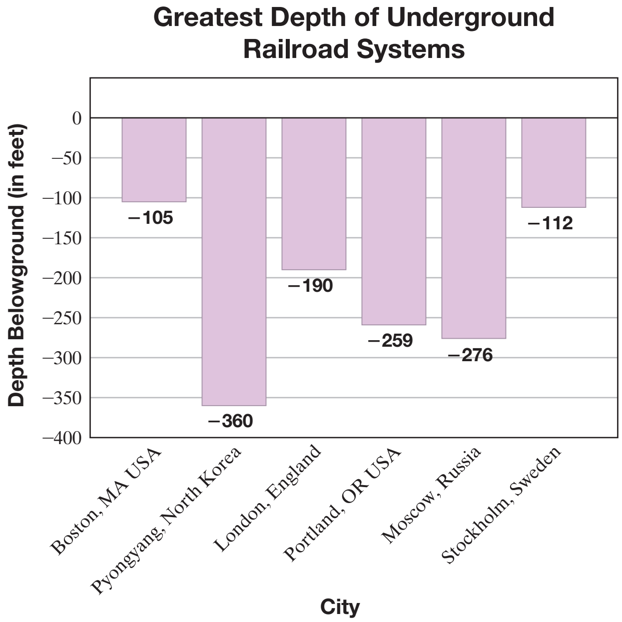
The bedrock through which the system in Stockholm is built has been made an architectural feature, and there are murals and quantities of art located everywhere throughout the system.

What Is "the Tube"?

Many major cities in the world boast a subterranean (belowground) rail system, known by various names such as subway, metro, underground, or even tube. These transportation systems vary greatly by length, depth, age, and style. The bar graph below shows the deepest point (distance below the surface) of the listed subway systems. London, England, boasts the oldest system in the world, which is often referred to as the Tube, built in 1863. The first subway in the United States was built in 1897 in Boston, while Portland, Oregon, boasts the deepest subway system in the United States. The world's subway station with the greatest depth can be found in Pyongyang, North Korea.

While Moscow subways may not compete with Stockholm in total beauty, the stations designed by Stalin on the circle line are considered masterpieces in themselves, and are a must-see on any trip to Moscow.

Exercises throughout this chapter deal with such negative numbers.



Greatest Depth of Underground Railroad Systems

Thus far, we have studied whole numbers, but these numbers are not sufficient for representing many situations in real life. For example, to express 5 degrees below zero or $100 in debt, numbers less than 0 are needed. This chapter is devoted to integers, which include numbers less than 0, and operations on these numbers.

Sections

Check Your Progress

2.1 Introduction to Variables and Algebraic Expressions

Objectives

A Evaluate Algebraic Expressions Given Replacement Values.

B Translate Phrases into Variable Expressions.

Objective A Evaluating Algebraic Expressions

Perhaps the most important quality of mathematics is that it is a science of patterns. Communicating about patterns is often made easier by using a letter to represent all the numbers fitting a pattern. We call such a letter a **variable.** For example, in Section 1.3 we presented the addition property of 0, which states that the sum of 0 and any number is that number. We might write

$$\begin{aligned} 0 + 1 &= 1 \\ 0 + 2 &= 2 \\ 0 + 3 &= 3 \\ 0 + 4 &= 4 \\ 0 + 5 &= 5 \\ 0 + 6 &= 6 \\ &\vdots \end{aligned}$$

continuing indefinitely. This is a pattern, and all whole numbers fit the pattern. We can communicate this pattern for all whole numbers by letting a letter, such as a, represent all whole numbers. We can then write

$$0 + a = a$$

Using variable notation is a primary goal of learning **algebra.** We now take some important first steps in beginning to use variable notation.

A combination of operations on letters (variables) and numbers is called an **algebraic expression** or simply an **expression.**

Algebraic Expressions

$$3 + x \qquad 5 \cdot y \qquad 2 \cdot z - 1 + x$$

If two variables or a number and a variable are next to each other, with no operation sign between them, the operation is multiplication. For example,

$$2x \quad \text{means} \quad 2 \cdot x$$

and

$$xy \text{ or } x(y) \quad \text{means} \quad x \cdot y$$

Also, the meaning of an exponent remains the same when the base is a variable. For example,

$$x^2 = \underbrace{x \cdot x}_{\text{2 factors of } x} \quad \text{and} \quad y^5 = \underbrace{y \cdot y \cdot y \cdot y \cdot y}_{\text{5 factors of } y}$$

Algebraic expressions such as $3x$ have different values depending on replacement values for the variables—in this case, the variable x. For example, if x is 2, then $3x$ becomes

$$\begin{aligned} 3x &= 3 \cdot 2 \\ &= 6 \end{aligned}$$

If x is 7, then $3x$ becomes

$$\begin{aligned} 3x &= 3 \cdot 7 \\ &= 21 \end{aligned}$$

Replacing a variable in an expression by a number and then finding the value of the expression is called **evaluating the expression** for the variable. When finding the value of an expression, remember to follow the order of operations given in Section 1.9.

Example 1 Evaluate $x + 7$ if x is 8.

Solution: Replace x with 8 in the expression $x + 7$.

$$\begin{aligned} x + 7 &= 8 + 7 && \text{Replace } x \text{ with 8.} \\ &= 15 && \text{Add.} \end{aligned}$$

Work Practice 1

When we write a statement such as "x is 5," we can use an equality symbol to represent "is" so that

x is 5 can be written as $x = 5$.

Example 2 Evaluate $2(x - y)$ for $x = 8$ and $y = 4$.

Solution:

$$\begin{aligned} 2(x - y) &= 2(8 - 4) && \text{Replace } x \text{ with 8 and } y \text{ with 4.} \\ &= 2(4) && \text{Subtract.} \\ &= 8 && \text{Multiply.} \end{aligned}$$

Work Practice 2

Example 3 Evaluate $\frac{x - 5y}{y}$ for $x = 21$ and $y = 3$.

Solution:

$$\begin{aligned} \frac{x - 5y}{y} &= \frac{21 - 5(3)}{3} && \text{Replace } x \text{ with 21 and } y \text{ with 3.} \\ &= \frac{21 - 15}{3} && \text{Multiply.} \\ &= \frac{6}{3} && \text{Subtract.} \\ &= 2 && \text{Divide.} \end{aligned}$$

Work Practice 3

Example 4 Evaluate $x^2 + z - 3$ for $x = 5$ and $z = 4$.

Solution:

$$\begin{aligned} x^2 + z - 3 &= 5^2 + 4 - 3 && \text{Replace } x \text{ with 5 and } z \text{ with 4.} \\ &= 25 + 4 - 3 && \text{Evaluate } 5^2. \\ &= 26 && \text{Add and subtract from left to right.} \end{aligned}$$

Work Practice 4

Practice 1

Evaluate $x - 2$ if x is 5.

Practice 2

Evaluate $y(x - 3)$ for $x = 3$ and $y = 7$.

Practice 3

Evaluate $\frac{y + 6}{x}$ for $x = 2$ and $y = 8$.

Practice 4

Evaluate $25 - z^3 + x$ for $z = 2$ and $x = 1$.

Answers

1. 3 **2.** 0 **3.** 7 **4.** 18

Helpful Hint

If you are having difficulty replacing variables with numbers, first replace each variable with a set of parentheses, then insert the replacement number within the parentheses.

Using this method in Example 4, we have:

$$\begin{aligned} x^2 + z - 3 &= (\ \ )^2 + (\ \ ) - 3 \\ &= (5)^2 + (4) - 3 \\ &= 25 + 4 - 3 \\ &= 26 \end{aligned}$$

✓ **Concept Check** What's wrong with the solution to the following problem? Evaluate $3x + 2y$ for $x = 2$ and $y = 3$.

Solution:

$$\xcancel{\begin{aligned} 3x + 2y &= 3(3) + 2(2) \\ &= 9 + 4 \\ &= 13 \end{aligned}}$$

Practice 5

Evaluate $\frac{5(F - 32)}{9}$ for $F = 41$.

Example 5 The expression $\frac{5(F - 32)}{9}$ can be used to write degrees Fahrenheit F as degrees Celsius C. Find the value of this expression for $F = 86$.

Solution:

$$\begin{aligned} \frac{5(F - 32)}{9} &= \frac{5(86 - 32)}{9} \\ &= \frac{5(54)}{9} \\ &= \frac{270}{9} \\ &= 30 \end{aligned}$$

Thus 86°F is the same temperature as 30°C.

Work Practice 5

Objective B Translating Phrases into Variable Expressions

To aid us in solving problems later, we practice translating verbal phrases into algebraic expressions. Recall from Section 1.8 that certain key words and phrases suggest addition, subtraction, multiplication, or division. These are reviewed next.

Addition (+)	Subtraction (−)	Multiplication (·)	Division (÷)
sum	difference	product	quotient
plus	minus	times	divide
added to	subtract	multiply	shared equally among
more than	less than	multiply by	per
increased by	decreased by	of	divided by
total	less	double/triple	divided into

Answer

5. 5

✓ **Concept Check Answer**

The replacement values were switched. If done correctly, we have:

$$\begin{aligned} 3x + 2y &= 3(2) + 2(3) \\ &= 6 + 6 \\ &= 12 \end{aligned}$$

Example 6

Write each phrase as an algebraic expression. Use x to represent "a number."

a. 7 increased by a number
b. 15 decreased by a number
c. The product of 2 and a number
d. The quotient of a number and 5
e. 2 subtracted from a number

Solution:

a.

In words:	7	increased by	a number
Translate:	7	$+$	x

b.

In words:	15	decreased by	a number
Translate:	15	$-$	x

c.

In words:		The product of		
	2	and	a number	
Translate:	2	$\cdot$	x	or $2x$

d.

In words:		The quotient of		
	a number	and	5	
Translate:	x	$\div$	5	or $\frac{x}{5}$

e.

In words:	2	subtracted from	a number
Translate:	x	$-$	2

Work Practice 6

Helpful Hint

Remember that order is important when subtracting. Study the order of numbers and variables below.

Phrase	**Translation**
a number *decreased by* 5	$x - 5$
a number *subtracted from* 5	$5 - x$

Practice 6

Write each phrase as an algebraic expression. Use x to represent "a number."

a. Twice a number
b. 8 increased by a number
c. 10 minus a number
d. 10 subtracted from a number
e. The quotient of a number and 16

Answers

6. a. $2x$ **b.** $8 + x$ **c.** $10 - x$ **d.** $x - 10$ **e.** $x \div 16$ or $\frac{x}{16}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

evaluating the expression   variable(s)   expression

1. A combination of operations on letters (variables) and numbers is a(n) ____________________.

2. A letter that represents a number is a(n) ____________________.

3. $3x - 2y$ is called a(n) ____________________ and the letters x and y are ____________________.

4. Replacing a variable in an expression by a number and then finding the value of the expression is called ____________________.

Martin-Gay Interactive Videos

See Video 2.1

Watch the section lecture video and answer the following questions.

Objective A **5.** Complete this statement based on the lecture before Example 1: When a letter and a variable are next to each other, the operation is an understood ______________.

Objective B **6.** In Example 4, what phrase translates to subtraction?

2.1 Exercise Set MyLab Math

Objective A *Complete the table. The first row has been done for you. See Examples 1 through 5.*

	a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
	45	9	54	36	405	5
1.	21	7				
2.	24	6				
3.	152	0				
4.	298	0				
5.	56	1				
6.	82	1				

Evaluate each expression for $x = 2$, $y = 5$, and $z = 3$. See Examples 1 through 5.

7. $3 + 2z$

8. $7 + 3z$

9. $6xz - 5x$

10. $4yz + 2x$

11. $z - x + y$

12. $x + 5y - z$

13. $3x - z$

14. $2y + 5z$

15. $y^3 - 4x$

16. $y^3 - z$

17. $2xy^2 - 6$

18. $3yz^2 + 1$

19. $8 - (y - x)$

20. $5 + (2x - 1)$

21. $y^4 + (z - x)$

22. $x^4 - (y - z)$

23. $\dfrac{6xy}{z}$

24. $\dfrac{8yz}{15}$

25. $\dfrac{2y - 2}{x}$

26. $\dfrac{6 + 3x}{z}$

27. $\frac{x + 2y}{z}$

28. $\frac{2z + 6}{3}$

29. $\frac{5x}{y} - \frac{10}{y}$

30. $\frac{70}{2y} - \frac{15}{z}$

31. $2y^2 - 4y + 3$

32. $3z^2 - z + 10$

33. $(3y - 2x)^2$

34. $(4y + 3z)^2$

35. $(xy + 1)^2$

36. $(xz - 5)^4$

37. $2y(4z - x)$

38. $3x(y + z)$

39. $xy(5 + z - x)$

40. $xz(2y + x - z)$

41. $\frac{7x + 2y}{3x}$

42. $\frac{6z + 2y}{4}$

43. The expression $16t^2$ gives the distance in feet that an object falls after t seconds. Complete the table by evaluating $16t^2$ for each given value of t.

t	1	2	3	4
$16t^2$				

44. The expression $\frac{5(F - 32)}{9}$ gives the equivalent degrees Celsius for F degrees Fahrenheit. Complete the table by evaluating this expression for each given value of F.

F	50	59	68	77
$\frac{5(F - 32)}{9}$				

Objective B

Write each phrase as a variable expression. Use x to represent "a number." See Example 6.

45. The sum of a number and five

46. Ten plus a number

47. The total of a number and eight

48. The difference of a number and five hundred

49. Twenty decreased by a number

50. A number less thirty

51. The product of 5 and a number

52. A number times twenty

53. A number divided by 2

54. The quotient of seven and a number

55. Seventeen added to the product of five and a number

56. The difference of twice a number, and four

57. The quotient of a number and five, decreased by 12

58. The quotient of twenty and a number, decreased by three

59. A number subtracted from 11

60. Twelve subtracted from a number

61. A number less 5

62. The product of a number and 7

63. 6 divided by a number

64. The sum of a number and 7

65. Fifty decreased by eight times a number

66. Twenty decreased by twice a number

Review

Determine the place value of the digit 7 in each whole number. See Section 1.2.

67. 720

68. 2307

69. 67,522

70. 179

Concept Extensions

Solve. See the Concept Check in this section. Determine whether each expression is correctly evaluated for $x = 2$, $y = 0$, and $z = 7$. If incorrect, then correctly evaluate.

71.
$$\begin{aligned} 2y + 3z &\stackrel{?}{=} 2(2) + 3(7) \\ &\stackrel{?}{=} 4 + 21 \\ &\stackrel{?}{=} 25 \end{aligned}$$

72.
$$\begin{aligned} 2z - 4x &\stackrel{?}{=} 2(7) - 4(2) \\ &\stackrel{?}{=} 14 - 8 \\ &\stackrel{?}{=} 6 \end{aligned}$$

73.
$$\begin{aligned} \frac{4z}{2x} &\stackrel{?}{=} \frac{4(7)}{2(2)} \\ &\stackrel{?}{=} \frac{28}{4} \\ &\stackrel{?}{=} 7 \end{aligned}$$

74.
$$\begin{aligned} \frac{2xy}{z} &\stackrel{?}{=} \frac{2(2)(0)}{7} \\ &\stackrel{?}{=} \frac{0}{7} \end{aligned}$$

is undefined

Use a calculator to evaluate each expression for $x = 23$ and $y = 72$.

75. $x^4 - y^2$

76. $2(x + y)^2$

77. $x^2 + 5y - 112$

78. $16y - 20x + x^3$

79. If x is a whole number, which expression is the largest: $2x$, $5x$, or $\frac{x}{3}$? Explain your answer.

80. If x is a whole number, which expression is the smallest: $2x$, $5x$, or $\frac{x}{3}$? Explain your answer.

81. In Exercise **43**, what do you notice about the value of $16t^2$ as t gets larger?

82. In Exercise **44**, what do you notice about the value of $\frac{5(F - 32)}{9}$ as F gets larger?

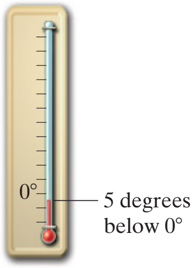
2.2 Introduction to Integers

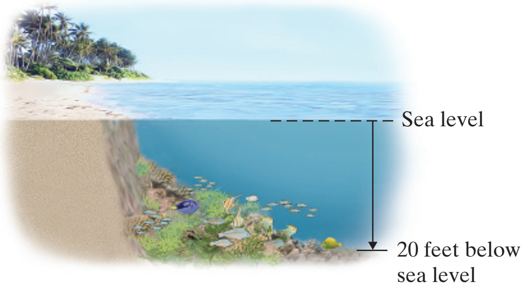
Objectives

- **A** Represent Real-Life Situations with Integers.
- **B** Graph Integers on a Number Line.
- **C** Compare Integers.
- **D** Find the Absolute Value of a Number.
- **E** Find the Opposite of a Number.
- **F** Read Bar Graphs Containing Integers.

Objective A Representing Real-Life Situations

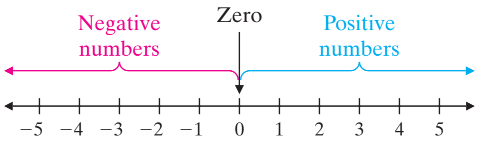
Thus far in this text, all numbers have been 0 or greater than 0. Numbers greater than 0 are called **positive numbers.** However, sometimes situations exist that cannot be represented by a number greater than 0. For example,





To represent these situations, we need numbers less than 0.

Extending the number line to the left of 0 allows us to picture **negative numbers,** which are numbers that are less than 0.



When a single + sign or no sign is in front of a number, the number is a positive number. When a single − sign is in front of a number, the number is a negative number. Together, we call positive numbers, negative numbers, and zero the **signed numbers.**

−5 indicates "negative five."

5 and +5 both indicate "positive five."

The number 0 is neither positive nor negative.

Helpful Hint Notice that 0 is neither positive nor negative.

Some signed numbers are integers. The **integers** consist of the numbers labeled on the number line above. The integers are

. . . , −3, −2, −1, 0, 1, 2, 3, . . .

Now we have numbers to represent the situations previously mentioned.

5 degrees below 0 −5°

20 feet below sea level −20 feet

Helpful Hint

A − sign, such as the one in −1, tells us that the number is to the left of 0 on a number line. −1 is read "negative one."

A + sign or no sign tells us that a number lies to the right of 0 on a number line. For example, 3 and +3 both mean "positive three."

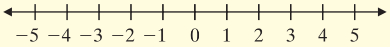
Practice 1

a. The world's deepest bat colony spends each winter in a New York zinc mine at a depth of 3805 feet. Represent this position with an integer. (*Source: Guinness Book of World Records*)

b. The deepest point of the underground railway system in Kiev, Russia, is 346 feet below ground. Represent this with an integer.

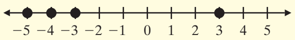
Practice 2

Graph −5, −4, 3, and −3 on the number line.



Answers

1. a. −3805 **b.** −346

2.


✓ **Concept Check Answer**

No

Example 1 Representing Depth with an Integer

The world's deepest cave is Krubera (or Voronja), in the country of Georgia, located by the Black Sea in Asia. It has been explored to a depth of 7188 feet below the surface of Earth. Represent this position using an integer. (*Source:* MessagetoEagle.com and Wikipedia)

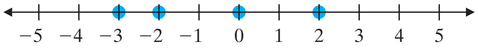
Solution: If 0 represents the surface of Earth, then 7188 feet below the surface can be represented by −7188.

Work Practice 1

Objective B Graphing Integers

Example 2 Graph 0, −3, 2, and −2 on the number line.

Solution:

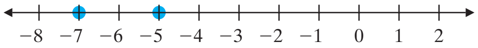


Work Practice 2

Objective C Comparing Integers

We can compare integers by using a number line. For any two numbers graphed on a number line, the number to the **right** is the **greater number** and the number to the **left** is the **smaller number.** Also, the symbols $<$ and $>$ are called **inequality symbols.**

The inequality symbol $>$ means "is greater than" and
the inequality symbol $<$ means "is less than."
For example, both −5 and −7 are graphed on the number line below.



On the graph, −7 is **to the left of** −5, so −7 **is less than** −5, written as

$$-7 < -5$$

We can also write

$$-5 > -7$$

since −5 is **to the right of** −7, so −5 **is greater than** −7.

✓ **Concept Check** Is there a largest positive number? Is there a smallest negative number? Explain.

Example 3

Insert < or > between each pair of numbers to make a true statement.

a. $-7 \quad 7$ **b.** $0 \quad -4$ **c.** $-9 \quad -11$

Solution:

a. -7 is to the left of 7 on a number line, so $-7 < 7$.

b. 0 is to the right of -4 on a number line, so $0 > -4$.

c. -9 is to the right of -11 on a number line, so $-9 > -11$.

Work Practice 3

Practice 3

Insert < or > between each pair of numbers to make a true statement.

a. $0 \quad -3$

b. $-5 \quad 5$

c. $-8 \quad -12$

Helpful Hint

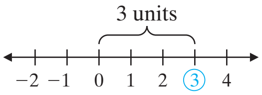
If you think of < and > as arrowheads, notice that in a true statement, the arrow always points to the smaller number.

$$\underset{\substack{\uparrow \\ \text{smaller number}}}{5 > -4} \qquad \underset{\substack{\uparrow \\ \text{smaller number}}}{-3 < -1}$$

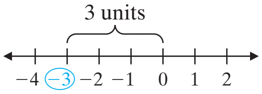
Objective D Finding the Absolute Value of a Number

The **absolute value** of a number is the number's distance from 0 on a number line. The symbol for absolute value is | |. For example, $|3|$ is read as "the absolute value of 3."

$|3| = 3$ because 3 is 3 units from 0.



$|-3| = 3$ because -3 is 3 units from 0.



Example 4

Simplify.

a. $|-2|$ **b.** $|8|$ **c.** $|0|$

Solution:

a. $|-2| = 2$ because -2 is 2 units from 0.

b. $|8| = 8$ because 8 is 8 units from 0.

c. $|0| = 0$ because 0 is 0 units from 0.

Work Practice 4

Practice 4

Simplify.

a. $|-4|$ **b.** $|2|$

c. $|-8|$

Helpful Hint

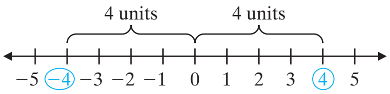
Since the absolute value of a number is that number's *distance* from 0, the absolute value of a number is always 0 or positive. It is never negative.

$$\underset{\substack{\uparrow \\ \text{zero}}}{|0| = 0} \qquad \underset{\substack{\uparrow \\ \text{a positive number}}}{|-6| = 6}$$

Objective E Finding Opposites

Two numbers that are the same distance from 0 on a number line but are on opposite sides of 0 are called **opposites.**

4 and -4 are opposites.



When two numbers are opposites, we say that each is the opposite of the other. Thus **4 is the opposite of -4** and **-4 is the opposite of 4.**

Answers

3. a. > **b.** < **c.** >

4. a. 4 **b.** 2 **c.** 8

The phrase "the opposite of" is written in symbols as "−". For example,

$$\begin{array}{cccc} \text{The opposite of} & 5 & \text{is} & -5 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ - & (5) & = & -5 \end{array}$$

$$\begin{array}{ccccc} \text{The opposite of} & -3 & \text{is} & 3 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \\ - & (-3) & = & 3 & \text{or} \end{array}$$

$$-(-3) = 3$$

In general, we have the following:

Opposites

If a is a number, then $-(-a) = a$.

Notice that because "the opposite of" is written as "−", to find the opposite of a number, we place a "−" sign in front of the number.

Practice 5

Find the opposite of each number.

a. 7 **b.** −17

Example 5 Find the opposite of each number.

a. 11 **b.** −2 **c.** 0

Solution:

a. The opposite of 11 is −11.

b. The opposite of −2 is $-(-2)$ or 2.

c. The opposite of 0 is 0.

Helpful Hint Remember that 0 is neither positive nor negative.

Work Practice 5

✓ **Concept Check** True or false? The number 0 is the only number that is its own opposite.

Practice 6

Simplify.

a. $-|-2|$

b. $-|5|$

c. $-(-11)$

Example 6 Simplify.

a. $-(-4)$ **b.** $-|-5|$ **c.** $-|6|$

Solution:

a. $-(-4) = 4$ The opposite of negative 4 is 4.

b. $-\underbrace{|-5|} = -5$ The opposite of the absolute value of −5 is the opposite of 5, or −5.

c. $-\underbrace{|6|} = -6$ The opposite of the absolute value of 6 is the opposite of 6, or −6.

Work Practice 6

Practice 7

Evaluate $-|x|$ if $x = -9$.

Example 7 Evaluate $-|-x|$ if $x = -2$.

Solution: Carefully replace x with −2; then simplify.

$-|-x| = -|-(-2)|$ Replace x with −2.

Then $-|-(-2)| = -\underbrace{|2|} = -2$.

Work Practice 7

Answers

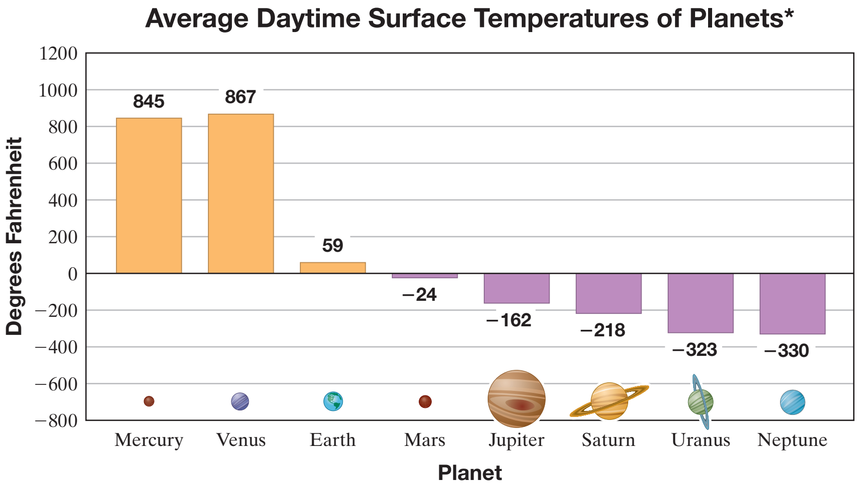
5. a. −7 **b.** 17 **6. a.** −2 **b.** −5 **c.** 11 **7.** −9

✓ **Concept Check Answer**

True

Objective F Reading Bar Graphs Containing Integers

The bar graph below shows the average daytime surface temperature (in degrees Fahrenheit) of the eight planets, excluding the newly classified "dwarf planet," Pluto. Notice that a negative temperature is illustrated by a bar below the horizontal line representing 0 °F, and a positive temperature is illustrated by a bar above the horizontal line representing 0 °F.



Source: The World Almanac
*For some planets, the temperature given is the temperature where the atmospheric pressure equals 1 Earth atmosphere.

Example 8 Which planet has the lowest average daytime surface temperature?

Solution: The planet with the lowest average daytime surface temperature is the one that corresponds to the bar that extends the farthest in the negative direction (downward). Neptune has the lowest average daytime surface temperature, −330°F.

Work Practice 8

Practice 8

Which planet has the highest average daytime surface temperature? What is this temperature?

Answer

8. Venus; 867 °F

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

opposites
inequality symbols
signed
absolute value
negative
integers
right
positive
is greater than
is less than
left

1. The numbers . . . , −3, −2, −1, 0, 1, 2, 3, . . . are called ________________.

2. Positive numbers, negative numbers, and zero, together, are called ________________ numbers.

3. The symbols "<" and ">" are called ________________.

4. Numbers greater than 0 are called ________________ numbers, while numbers less than 0 are called ________________ numbers.

5. The sign "<" means ________________ and ">" means ________________.

6. On a number line, the greater number is to the ________________ of the lesser number.

7. A number's distance from 0 on a number line is the number's ________________.

8. The numbers −5 and 5 are called ________________.

Martin-Gay Interactive Videos

See Video 2.2

Watch the section lecture video and answer the following questions.

Objective A **9.** In Example 1, what application is used to represent a negative number?

Objective B **10.** In Example 2, the tick marks are labeled with what kind of numbers on the number line?

Objective C **11.** From Example 3 and your knowledge of a number line, complete this statement: 0 will always be greater than any of the ____________ integers.

Objective D **12.** What is the answer to Example 5? The absolute value of what other integer has this same answer?

Objective E **13.** Complete this statement based on Example 10: A negative sign can be translated to the phrase "____________."

Objective F **14.** In Examples 13 and 14, what other lake has a negative integer elevation?

2.2 Exercise Set MyLab Math

Objective A *Represent each quantity by an integer. See Example 1.*

1. A worker in a silver mine in Nevada works 1445 feet underground.

2. A scuba diver is swimming 35 feet below the surface of the water in the Gulf of Mexico.

3. The peak of Mount Elbert in Colorado is 14,433 feet above sea level. (*Source:* U.S. Geological Survey)

4. The lowest elevation in the United States is found at Death Valley, California, at an elevation of 282 feet below sea level. (*Source:* U.S. Geological Survey)

5. The record high temperature in Arkansas is 120 degrees above zero Fahrenheit. (*Source:* National Climatic Data Center)

6. The record high temperature in California is 134 degrees above zero Fahrenheit. (*Source:* National Climatic Data Center)

7. The average depth of the Atlantic Ocean is 11,810 feet below its surface. (*Source: The World Almanac*)

8. The average depth of the Pacific Ocean is 14,040 feet below its surface. (*Source: The World Almanac*)

9. Occidental Petroleum reported a long-term debt of \$9800 million in 2017. (*Source:* Occidental Petroleum)

10. Uber reported a net loss of \$1460 million in the third quater of 2017. (*Source:* Bloomberg)

11. Two divers are exploring the wreck of the *Andrea Doria,* south of Nantucket Island, Massachusetts. Guillermo is 160 feet below the surface of the ocean and Luigi is 147 feet below the surface. Represent each quantity by an integer and determine who is deeper.

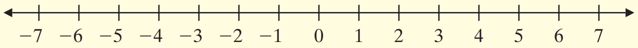
12. The temperature on one January day in Minneapolis was 10° below 0° Celsius. Represent this quantity by an integer and tell whether this temperature is cooler or warmer than 5° below 0° Celsius.

13. During December 2017, the average gasoline price across the country dropped by 10¢ per gallon. (*Source:* American Automobile Association)

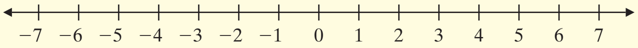
14. The Challenger Deep in the Mariana Trench is the lowest known point in the Earth's seabed hydrosphere. The bottom is estimated to be 35,814 feet below sea level.

Objective B *Graph each integer in the list on the same number line. See Example 2.*

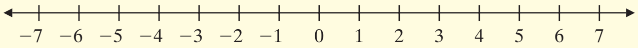
15. 1, 2, 4, 6



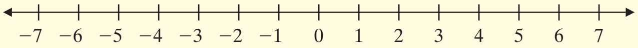
16. 3, 5, 2, 0



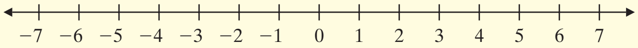
17. 1, −1, 2, −2, −4



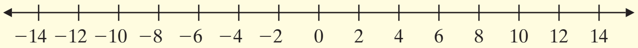
18. 3, −3, 5, −5, 6



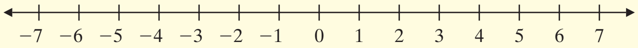
19. 0, 2, 5, 7



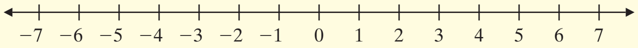
20. 0, 3, 6, 10



21. 0, −2, −7, −5



22. 0, −7, 3, −6



Objective C *Insert $<$ or $>$ between each pair of integers to make a true statement. See Example 3.*

23. 0 −7

24. −8 0

25. −7 −5

26. −12 −10

27. −30 −35

28. −27 −29

29. −26 26

30. 13 −13

Objective D *Simplify. See Example 4.*

31. $|5|$

32. $|7|$

33. $|-8|$

34. $|-19|$

35. $|0|$

36. $|100|$

37. $|-5|$

38. $|-10|$

Objective E *Find the opposite of each integer. See Example 5.*

39. 5 **40.** 8 **41.** -4 **42.** -6

43. 23 **44.** 123 **45.** -85 **46.** -13

Objectives C D E Mixed Practice *Simplify. See Example 6.*

47. $|-7|$ **48.** $|-11|$ **49.** $-|20|$ **50.** $-|43|$

51. $-|-3|$ **52.** $-|-18|$ **53.** $-(-43)$ **54.** $-(-27)$

55. $-|-15|$ **56.** $-|-29|$ **57.** $-(-33)$ **58.** $-(-14)$

Evaluate. See Example 7.

59. $|-x|$ if $x = -6$ **60.** $-|x|$ if $x = -8$ **61.** $-|-x|$ if $x = 2$ **62.** $-|-x|$ if $x = 10$

63. $|x|$ if $x = -32$ **64.** $|x|$ if $x = 32$ **65.** $-|x|$ if $x = 7$ **66.** $|-x|$ if $x = 1$

Insert <, >, or = between each pair of numbers to make a true statement. See Examples 3 through 6.

67. $-12 \quad -6$ **68.** $-4 \quad -17$ **69.** $|-8| \quad |-11|$ **70.** $|-8| \quad |-4|$

71. $|-47| \quad -(-47)$ **72.** $-|17| \quad -(-17)$ **73.** $-|-12| \quad -(-12)$ **74.** $|-24| \quad -(-24)$

75. $0 \quad -9$ **76.** $-45 \quad 0$ **77.** $|0| \quad |-9|$ **78.** $|-45| \quad |0|$

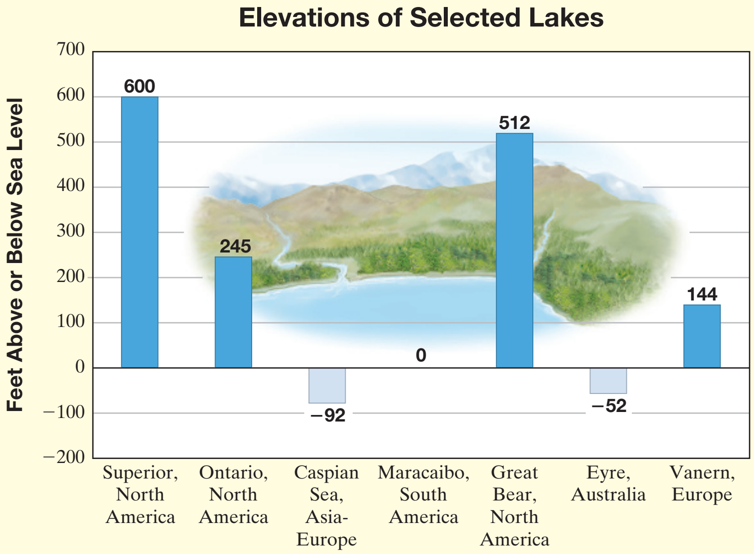
79. $-|-2| \quad -|-10|$ **80.** $-|-8| \quad -|-4|$ **81.** $-(-12) \quad -(-18)$ **82.** $-22 \quad -(-38)$

Objectives D E Mixed Practice *Fill in the chart. See Examples 4 through 7.*

	Number	Absolute Value of Number	Opposite of Number
83.	31		
85.			−28

	Number	Absolute Value of Number	Opposite of Number
84.	−13		
86.			90

Objective F *The bar graph shows the elevations of selected lakes. Use this graph for Exercises 87 through 90. (Source: U.S. Geological Survey) See Example 8.*



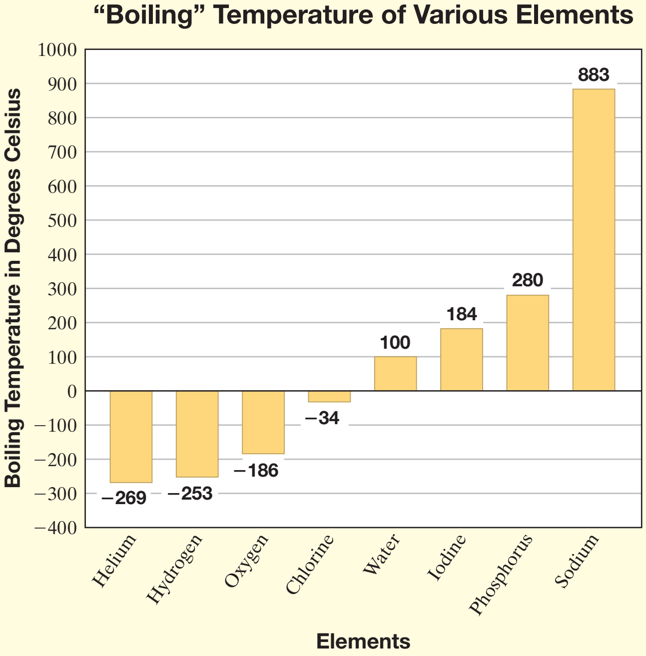
87. Which lake shown has the lowest elevation?

88. Which lake shown has an elevation at sea level?

89. Which lake shown has the highest elevation?

90. Which lake shown has the second-lowest elevation?

The following bar graph represents the boiling temperature, the temperature at which a substance changes from liquid to gas at standard atmospheric pressure. Use this graph to answer Exercises 91 through 94.



91. Which element has a positive boiling temperature closest to that of water?

92. Which element has the lowest boiling temperature?

93. Which element has a boiling temperature closest to −200°C?

94. Which element has an average boiling temperature closest to +300°C?

Review

Add. See Section 1.3.

95. $0 + 13$

96. $9 + 0$

97. $15 + 20$

98. $20 + 15$

99. $47 + 236 + 77$

100. $362 + 37 + 90$

Concept Extensions

Write the given numbers in order from least to greatest.

101. $2^2, -|3|, -(-5), -|-8|$

102. $|10|, 2^3, -|-5|, -(-4)$

103. $|-1|, -|-6|, -(-6), -|1|$

104. $1^4, -(-3), -|7|, |-20|$

105. $-(-2), 5^2, -10, -|-9|, |-12|$

106. $3^3, -|-11|, -(-10), -4, -|2|$

Choose all numbers for x from each given list that make each statement true.

107. $|x| > 8$
a. 0 **b.** −5 **c.** 8 **d.** −12

108. $|x| > 4$
a. 0 **b.** 4 **c.** −1 **d.** −100

109. Evaluate: $-(-|-5|)$

110. Evaluate: $-(-|-(-7)|)$

Answer true or false for Exercises 111 through 115.

111. If $a > b$, then a must be a positive number.

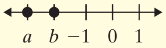
112. The absolute value of a number is *always* a positive number.

113. A positive number is always greater than a negative number.

114. Zero is always less than a positive number.

115. The number $-a$ is always a negative number. (*Hint:* Read "−" as "the opposite of.")

116. Given the number line, is it true that $b < a$?



117. Write in your own words how to find the absolute value of a signed number.

118. Explain how to determine which of two signed numbers is larger.

For Exercises 119 and 120, see the Concept Check in this section.

119. Is there a largest negative number? If so, what is it?

120. Is there a smallest positive number? If so, what is it?

2.3 Adding Integers

Objectives

A Add Integers.

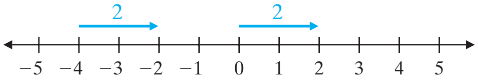
B Evaluate an Algebraic Expression by Adding.

C Solve Problems by Adding Integers.

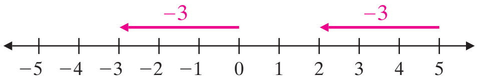
Objective A Adding Integers

Adding integers can be visualized using a number line. A positive number can be represented on the number line by an arrow of appropriate length pointing to the right, and a negative number by an arrow of appropriate length pointing to the left.

Both arrows represent 2 or +2. They both point to the right and they are both 2 units long.

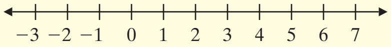


Both arrows represent −3. They both point to the left and they are both 3 units long.



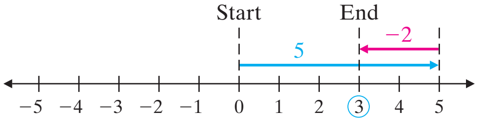
Practice 1

Add using a number line: $5 + (-1)$



Example 1 Add using a number line: $5 + (-2)$

Solution: To add integers on a number line, such as $5 + (-2)$, we start at 0 on the number line and draw an arrow representing 5. From the tip of this arrow, we draw another arrow representing −2. The tip of the second arrow ends at their sum, 3.

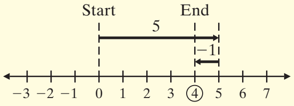


$$5 + (-2) = 3$$

Work Practice 1

Answer

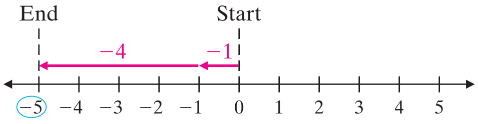
1.



$5 + (-1) = 4$

Example 2 Add using a number line: $-1 + (-4)$

Solution: Start at 0 and draw an arrow representing -1. From the tip of this arrow, we draw another arrow representing -4. The tip of the second arrow ends at their sum, -5.

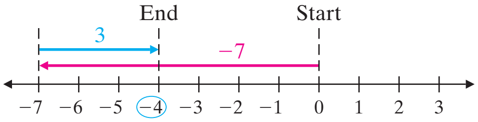


$$-1 + (-4) = -5$$

Work Practice 2

Example 3 Add using a number line: $-7 + 3$

Solution:



Work Practice 3

Using a number line each time we add two numbers can be time consuming. Instead, we can notice patterns in the previous examples and write rules for adding signed numbers.

Rules for adding signed numbers depend on whether we are adding numbers with the same sign or different signs. When adding two numbers with the same sign, as in Example 2, notice that the sign of the sum is the same as the sign of the addends.

Adding Two Numbers with the Same Sign

Step 1: Add their absolute values.

Step 2: Use their common sign as the sign of the sum.

Example 4 Add: $-2 + (-21)$

Solution:

Step 1: $|-2| = 2$, $|-21| = 21$, and $2 + 21 = 23$.

Step 2: Their common sign is negative, so the sum is negative:

$$-2 + (-21) = -23$$

Work Practice 4

Examples Add.

5. $-5 + (-10) = -15$

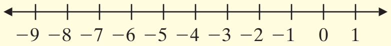
6. $2 + 6 = 8$

Work Practice 5–6

When adding two numbers with different signs, as in Examples 1 and 3, the sign of the result may be positive or negative, or the result may be 0.

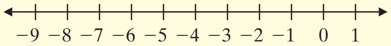
Practice 2

Add using a number line:
$-6 + (-2)$



Practice 3

Add using a number line:
$-8 + 3$



Practice 4

Add: $(-3) + (-9)$

Practice 5–6

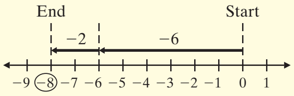
Add.

5. $-12 + (-3)$

6. $9 + 5$

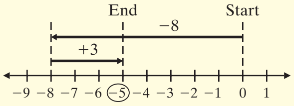
Answers

2.



$-6 + (-2) = -8$

3.



$-8 + 3 = -5$

4. -12 **5.** -15 **6.** 14

Adding Two Numbers with Different Signs

Step 1: Find the larger absolute value minus the smaller absolute value.

Step 2: Use the sign of the number with the larger absolute value as the sign of the sum.

Practice 7

Add: $-3 + 59$

Example 7

Add: $-14 + 35$

Solution:

Step 1: $|-14| = 14$, $|35| = 35$, and $35 - 14 = 21$.

Step 2: 35 has the larger absolute value and its sign is an understood +:

$-14 + 35 = +21$ or 21

Work Practice 7

Practice 8

Add: $22 + (-28)$

Example 8

Add: $13 + (-17)$

Solution:

Step 1: $|13| = 13$, $|-17| = 17$, and $17 - 13 = 4$.

Step 2: -17 has the larger absolute value and its sign is $-$:

$13 + (-17) = -4$

Work Practice 8

Practice 9–11

Add.

9. $-46 + 20$

10. $8 + (-6)$

11. $-2 + 0$

Examples

Add.

9. $-18 + 10 = -8$

10. $12 + (-8) = 4$

11. $0 + (-5) = -5$ The sum of 0 and any number is the number.

Work Practice 9–11

Recall that numbers such as 7 and −7 are called opposites. In general, the sum of a number and its opposite is always 0.

$$\underbrace{7 + (-7)}_{\text{opposites}} = 0 \qquad \underbrace{-26 + 26}_{\text{opposites}} = 0 \qquad \underbrace{1008 + (-1008)}_{\text{opposites}} = 0$$

If a is a number, then

$-a$ is its opposite. Also,

$$\left.\begin{aligned} a + (-a) &= 0 \\ -a + a &= 0 \end{aligned}\right\} \text{ The sum of a number and its opposite is 0.}$$

Practice 12–13

Add.

12. $15 + (-15)$

13. $-80 + 80$

Examples

Add.

12. $-21 + 21 = 0$

13. $36 + (-36) = 0$

Work Practice 12–13

✓Concept Check What is wrong with the following calculation?

$$\xcancel{6 + (-22) = 16}$$

Answers

7. 56 **8.** −6 **9.** −26 **10.** 2
11. −2 **12.** 0 **13.** 0

✓Concept Check Answer

$6 + (-22) = -16$

In the following examples, we add three or more integers. Remember that by the associative and commutative properties for addition, we may add numbers in any order that we wish. In Examples 14 and 15, let's add the numbers from left to right.

Example 14 Add: $(-3) + 4 + (-11)$

Solution: $(-3) + 4 + (-11) = 1 + (-11)$

$= -10$

Work Practice 14

Practice 14

Add: $8 + (-3) + (-13)$

Example 15 Add: $1 + (-10) + (-8) + 9$

Solution:

$$\begin{aligned} 1 + (-10) + (-8) + 9 &= -9 + (-8) + 9 \\ &= -17 + 9 \\ &= -8 \end{aligned}$$

The sum will be the same if we add the numbers in any order. To see this, let's first add the positive numbers together and then the negative numbers together.

$1 + 9 = 10$ Add the positive numbers.

$(-10) + (-8) = -18$ Add the negative numbers.

$10 + (-18) = -8$ Add these results.

The sum is -8.

Work Practice 15

Practice 15

Add: $5 + (-3) + 12 + (-14)$

Helpful Hint

Don't forget that addition is commutative and associative. In other words, numbers may be added in any order and in any grouping.

Objective B Evaluating Algebraic Expressions

We can continue our work with algebraic expressions by evaluating expressions given integer replacement values.

Example 16 Evaluate $2x + y$ for $x = 3$ and $y = -5$.

Solution: Replace x with 3 and y with -5 in $2x + y$.

$$\begin{aligned} 2x + y &= 2 \cdot 3 + (-5) \\ &= 6 + (-5) \\ &= 1 \end{aligned}$$

Work Practice 16

Practice 16

Evaluate $x + 3y$ for $x = -4$ and $y = 1$.

Example 17 Evaluate $x + y$ for $x = -2$ and $y = -10$.

Solution: $x + y = (-2) + (-10)$ Replace x with -2 and y with -10.

$= -12$

Work Practice 17

Practice 17

Evaluate $x + y$ for $x = -11$ and $y = -6$.

Answers

14. -8 **15.** 0 **16.** -1 **17.** -17

Practice 18

If the temperature was −8° Fahrenheit at 6 a.m., and it rose 4 degrees by 7 a.m. and then rose another 7 degrees in the hour from 7 a.m. to 8 a.m., what was the temperature at 8 a.m.?

Answer

18. 3 °F

Objective C Solving Problems by Adding Integers

Next, we practice solving problems that require adding integers.

Example 18 Calculating Temperature

In Philadelphia, Pennsylvania, the record extreme high temperature is 104 °F. Add to this temperature the opposite of 111 degrees, and the result is the record extreme low temperature. Find this temperature. (*Source:* National Climatic Data Center)

Solution:

In words:	extreme low temperature	=	extreme high temperature	add to this	opposite of 111°
	↓		↓		↓
Translate:	extreme low temperature	=	104	+	(−111)
		= −7			

The record extreme low temperature in Philadelphia, Pennsylvania, is −7°F.

Work Practice 18

Calculator Explorations Entering Negative Numbers

To enter a negative number on a calculator, find the key marked $\boxed{+/-}$. (Some calculators have a key marked $\boxed{\text{CHS}}$ and some calculators have a special key $\boxed{(-)}$ for entering a negative sign.) To enter the number −2, for example, press the keys $\boxed{2}$ $\boxed{+/-}$. The display will read $\boxed{\quad -2}$.

To find −32 + (−131), press the keys

$\boxed{32}$ $\boxed{+/-}$ $\boxed{+}$ $\boxed{131}$ $\boxed{+/-}$ $\boxed{=}$ or

$\boxed{(-)}$ $\boxed{32}$ $\boxed{+}$ $\boxed{(-)}$ $\boxed{131}$ $\boxed{\text{ENTER}}$

The display will read $\boxed{\quad -163}$. Thus −32 + (−131) = −163.

Use a calculator to perform each indicated operation.

1. −256 + 97

2. 811 + (−1058)

3. 6(15) + (−46)

4. −129 + 10(48)

5. −108,650 + (−786,205)

6. −196,662 + (−129,856)

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used. (Review Section 1.3 if needed.)

−*a* *a* 0 commutative associative

1. If n is a number, then $-n + n =$ __________.

2. Since $x + n = n + x$, we say that addition is __________.

3. If a is a number, then $-(-a) =$ __________.

4. Since $n + (x + a) = (n + x) + a$, we say that addition is __________.

Add.

5. 5 + 0

6. 0 + 3

7. 0 + (−35)

8. (−2) + 0

9. −12 + 12

10. −9 + 9

11. 28 + (−28)

12. 48 + (−48)

Martin-Gay Interactive Videos

See Video 2.3

Watch the section lecture video and answer the following questions.

Objective A **13.** What is the sign of the sum in Example 6 and why?

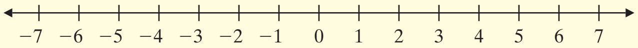
Objective B **14.** What is the sign of the sum in Example 8 and why?

Objective C **15.** What does the answer to Example 10, -231, mean in the context of the application?

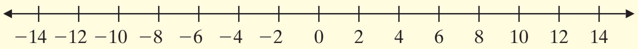
2.3 Exercise Set MyLab Math

Objective A *Add using a number line. See Examples 1 through 3.*

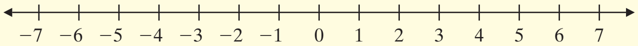
1. $-1 + (-6)$



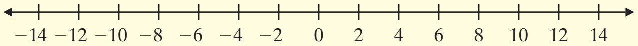
2. $9 + (-4)$



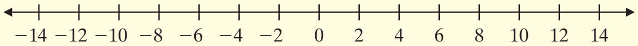
3. $-4 + 7$



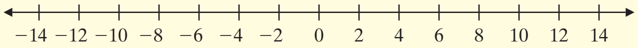
4. $10 + (-3)$



5. $-13 + 7$



6. $-6 + (-5)$



Add. See Examples 4 through 13.

7. $23 + 12$
8. $15 + 42$
9. $-6 + (-2)$
10. $-5 + (-4)$
11. $-43 + 43$
12. $-62 + 62$
13. $6 + (-2)$
14. $8 + (-3)$
15. $-6 + 8$
16. $-8 + 12$
17. $3 + (-5)$
18. $5 + (-9)$
19. $-2 + (-7)$
20. $-6 + (-1)$
21. $-12 + (-12)$
22. $-23 + (-23)$
23. $-25 + (-32)$
24. $-45 + (-90)$
25. $-123 + (-100)$
26. $-500 + (-230)$
27. $-7 + 7$
28. $-10 + 10$
29. $12 + (-5)$
30. $24 + (-10)$
31. $-6 + 3$
32. $-8 + 2$
33. $-12 + 3$
34. $-15 + 5$
35. $56 + (-26)$
36. $89 + (-37)$
37. $-37 + 57$
38. $-25 + 65$

39. $-42 + 93$

40. $-64 + 164$

41. $34 + (-67)$

42. $42 + (-83)$

43. $124 + (-144)$

44. $325 + (-375)$

▶ **45.** $-82 + (-43)$

46. $-56 + (-33)$

Add. See Examples 14 and 15.

▶ **47.** $-4 + 2 + (-5)$

48. $-1 + 5 + (-8)$

49. $-52 + (-77) + (-117)$

50. $-103 + (-32) + (-27)$

51. $12 + (-4) + (-4) + 12$

52. $18 + (-9) + 5 + (-2)$

53. $-10 + 14 + 25 + (-16)$

54. $34 + (-12) + (-11) + 213$

Objective A Mixed Practice *Add. See Examples 1 through 15.*

55. $-8 + (-14) + (-11)$

56. $-10 + (-6) + (-1)$

57. $-26 + 5$

58. $-35 + (-12)$

59. $5 + (-1) + 17$

60. $3 + (-23) + 6$

61. $-14 + (-31)$

62. $-100 + 70$

63. $13 + 14 + (-18)$

64. $(-45) + 22 + 20$

65. $-87 + 87$

66. $-87 + 0$

67. $-3 + (-8) + 12 + (-1)$

68. $-16 + 6 + (-14) + (-20)$

69. $0 + (-103)$

70. $94 + (-94)$

Objective B *Evaluate $x + y$ for the given replacement values. See Examples 16 and 17.*

▶ **71.** $x = -20$ and $y = -50$

72. $x = -1$ and $y = -29$

Evaluate $3x + y$ for the given replacement values. See Examples 16 and 17.

▶ **73.** $x = 2$ and $y = -3$

74. $x = 7$ and $y = -11$

75. $x = 3$ and $y = -30$

76. $x = 13$ and $y = -17$

Objective C Translating *Translate each phrase; then simplify. See Example 18.*

77. Find the sum of -8 and 25.

78. Find the sum of -30 and 10.

79. Find the sum of -31, -9, and 30.

80. Find the sum of -49, -2, and 40.

Solve. See Example 18.

81. Suppose a deep-sea diver dives from the surface to 215 feet below the surface. He then dives down 16 more feet. Use integers to represent this situation. Then find the diver's present depth.

82. Suppose a diver dives from the surface to 248 meters below the surface and then swims up 6 meters, down 17 meters, down another 24 meters, and then up 23 meters. Use integers to represent this situation. Then find the diver's depth after these movements.

In golf, it is possible to have positive, negative, and zero scores. The following table shows the results of the eighteen-hole Round 4 for Rickie Fowler and Tiger Woods at the 2017 PGA Tour Hero World Challenge at New Providence, Bahamas. Use the table to answer Exercises 83 and 84.

Player/Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Fowler	−1	−1	−1	−1	−1	−1	−1	0	−1	0	−1	0	−1	0	−1	0	0	0
Woods	0	0	−1	0	−1	0	−2	0	−1	2	−1	0	0	−1	−1	0	1	1

83. Find the total score for each of the athletes in the round.

84. In golf, the lower score is the winner. Use the result of Exercise **83** to determine who won Round 4.

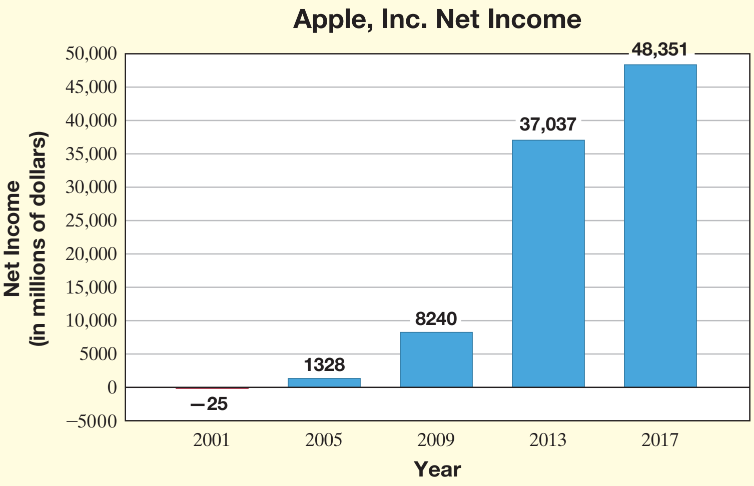
The following bar graph shows the yearly net income for Apple, Inc. Net income is one indication of a company's health. It measures revenue (money taken in) minus cost (money spent). Use this graph to answer Exercises 85 through 88. (Source: Apple, Inc.)

85. What was the net income (in dollars) for Apple, Inc. in 2017?

86. What was the net income (in dollars) for Apple, Inc. in 2001?

87. What was the increase in net income (in dollars) for Apple, Inc. from 2013 to 2017?

88. Find the total net income (in dollars) for the years shown on the graph.



89. The temperature at 4 p.m. on February 2 was −10° Celsius. By 11 p.m. the temperature had risen 12 degrees. Find the temperature at 11 p.m.

90. In some card games, it is possible to have both positive and negative scores. After four rounds of play, Michelle had scores of 14, −5, −8, and 7. What was her total score for the game?

A small company reports the following net incomes. Use this table for Exercises 91 and 92.

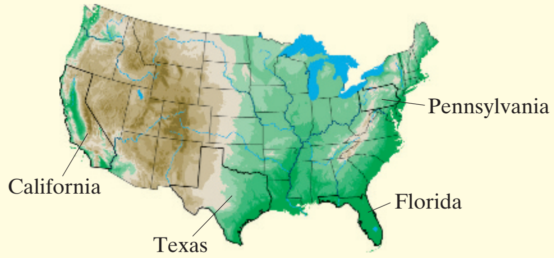
91. Find the sum of the net incomes for 2015 and 2016.

92. Find the sum of the net incomes for all four years shown.

Year	**Net Income (in Dollars)**
2014	$75,083
2015	−$10,412
2016	−$1786
2017	$96,398

93. The all-time record low temperature for Texas is −23°F. Florida's all-time record low temperature is 21°F higher than Texas' record low. What is Florida's record low temperature? (*Source:* National Climatic Data Center)

94. The all-time record low temperature for California is −45°F. In Pennsylvania, the lowest temperature ever recorded is 3°F higher than California's all-time low temperature. What is the all-time record low temperature for Pennsylvania? (*Source:* National Climatic Data Center)



95. The deepest spot in the Pacific Ocean is the Mariana Trench, which has an elevation of 10,924 meters below sea level. The bottom of the Pacific's Aleutian Trench has an elevation 3245 meters higher than that of the Mariana Trench. Use a negative number to represent the depth of the Aleutian Trench. (*Source:* Defense Mapping Agency)

96. The deepest spot in the Atlantic Ocean is the Puerto Rico Trench, which has an elevation of 8605 meters below sea level. The bottom of the Atlantic's Cayman Trench has an elevation 1070 meters above the level of the Puerto Rico Trench. Use a negative number to represent the depth of the Cayman Trench. (*Source:* Defense Mapping Agency)

Review

Subtract. See Section 1.4.

97. $44 - 0$

98. $91 - 0$

99. $200 - 59$

100. $400 - 18$

Concept Extensions

101. Name 2 numbers whose sum is −17.

102. Name 2 numbers whose sum is −30.

Each calculation below is incorrect. Find the error and correct it. See the Concept Check in this section.

103. $\xcancel{7 + (-10) \overset{?}{=} 17}$

104. $\xcancel{-4 + 14 \overset{?}{=} -18}$

105. $\xcancel{-10 + (-12) \overset{?}{=} -120}$

106. $\xcancel{-15 + (-17) \overset{?}{=} 32}$

For Exercises 107 through 110, determine whether each statement is true or false.

107. The sum of two negative numbers is always a negative number.

108. The sum of two positive numbers is always a positive number.

109. The sum of a positive number and a negative number is always a negative number.

110. The sum of zero and a negative number is always a negative number.

111. In your own words, explain how to add two negative numbers.

112. In your own words, explain how to add a positive number and a negative number.

2.4 Subtracting Integers

In Section 2.2, we discussed the opposite of an integer.

The opposite of 3 is -3.
The opposite of -6 is 6.

In this section, we use opposites to subtract integers.

Objectives

A Subtract Integers.

B Add and Subtract Integers.

C Evaluate an Algebraic Expression by Subtracting.

D Solve Problems by Subtracting Integers.

Objective A Subtracting Integers

To subtract integers, we will write the subtraction problem as an addition problem. To see how to do this, study the examples below.

$$10 - 4 = 6$$
$$10 + (-4) = 6$$

Since both expressions simplify to 6, this means that

$$10 - 4 = 10 + (-4) = 6$$

Also,

$$3 - 2 = 3 + (-2) = 1$$
$$15 - 1 = 15 + (-1) = 14$$

Thus, to subtract two numbers, we add the first number to the opposite of the second number. (The opposite of a number is also known as its **additive inverse**.)

Subtracting Two Numbers

If a and b are numbers, then $a - b = a + (-b)$.

Examples Subtract.

	Subtraction	=	first number	+	opposite of the second number		
	↓		↓	↓	↓		
1.	$8 - 5$	=	8	+	(-5)	=	3
2.	$-4 - 10$	=	-4	+	(-10)	=	-14
3.	$6 - (-5)$	=	6	+	5	=	11
4.	$-11 - (-7)$	=	-11	+	7	=	-4

Work Practice 1–4

Practice 1–4

Subtract.

1. $12 - 7$

2. $-6 - 4$

3. $11 - (-14)$

4. $-9 - (-1)$

Examples Subtract.

5. $-10 - 5 = -10 + (-5) = -15$

6. $8 - 15 = 8 + (-15) = -7$

7. $-4 - (-5) = -4 + 5 = 1$

Work Practice 5–7

Practice 5–7

Subtract.

5. $5 - 9$

6. $-12 - 4$

7. $-2 - (-7)$

Answers

1. 5 **2.** -10 **3.** 25 **4.** -8
5. -4 **6.** -16 **7.** 5

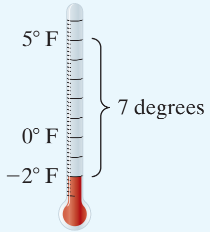
Helpful Hint

To visualize subtraction, try the following:

The difference between 5°F and −2°F can be found by subtracting. That is,

$$5 - (-2) = 5 + 2 = 7$$

Can you visually see from the thermometer on the right that there are actually 7 degrees between 5°F and −2°F?



Concept Check What is wrong with the following calculation?

$$\xcancel{-9 - (-6)} = -15$$

Practice 8

Subtract 5 from −10.

Example 8 Subtract 7 from −3.

Solution: To subtract 7 *from* −3, we find

$$-3 - 7 = -3 + (-7) = -10$$

Work Practice 8

Objective B Adding and Subtracting Integers

If a problem involves adding or subtracting more than two integers, we rewrite differences as sums and add. Recall that by associative and commutative properties, we may add numbers in any order. In Examples 9 and 10, we will add from left to right.

Practice 9

Simplify: $-4 - 3 - 7 - (-5)$

Example 9 Simplify: $7 - 8 - (-5) - 1$

Solution:

$$\begin{aligned} 7 - 8 - (-5) - 1 &= \underbrace{7 + (-8)} + 5 + (-1) \\ &= \underbrace{-1 \quad + 5} + (-1) \\ &= \underbrace{4 \quad + (-1)} \\ &= 3 \end{aligned}$$

Work Practice 9

Practice 10

Simplify:
$3 + (-5) - 6 - (-4)$

Example 10 Simplify: $7 + (-12) - 3 - (-8)$

Solution:

$$\begin{aligned} 7 + (-12) - 3 - (-8) &= \underbrace{7 + (-12)} + (-3) + 8 \\ &= \underbrace{-5 \quad + (-3)} + 8 \\ &= \underbrace{-8 \quad + 8} \\ &= 0 \end{aligned}$$

Work Practice 10

Objective C Evaluating Expressions

Now let's practice evaluating expressions when the replacement values are integers.

Answers

8. −15 **9.** −9 **10.** −4

Concept Check Answer

$-9 - (-6) = -9 + 6 = -3$

Example 11 Evaluate $x - y$ for $x = -3$ and $y = 9$.

Solution: Replace x with -3 and y with 9 in $x - y$.

$$\begin{array}{rccc} & x & - & y \\ & \downarrow & \downarrow & \downarrow \\ = & (-3) & - & 9 \\ = & (-3) & + & (-9) \\ = & -12 & & \end{array}$$

Work Practice 11

Practice 11

Evaluate $x - y$ for $x = -2$ and $y = 14$.

Example 12 Evaluate $2a - b$ for $a = 8$ and $b = -6$.

Solution: Watch your signs carefully!

$$\begin{array}{rcccl} & 2a & - & b & \\ & \downarrow & \downarrow & \downarrow & \\ = & \overbrace{2 \cdot 8} & - & (-6) & \text{Replace } a \text{ with 8 and } b \text{ with } -6. \\ = & 16 & + & 6 & \text{Multiply.} \\ = & 22 & & & \text{Add.} \end{array}$$

Helpful Hint Watch carefully when replacing variables in the expression $2a - b$. Make sure that all symbols are inserted and accounted for.

Work Practice 12

Practice 12

Evaluate $3y - z$ for $y = 9$ and $z = -4$.

Objective D Solving Problems by Subtracting Integers

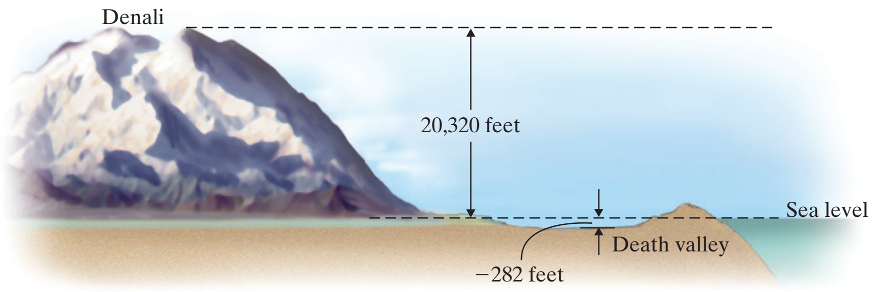
Solving problems often requires subtraction of integers.

Example 13 Finding a Change in Elevation

The highest point in the United States is the top of Denali, at a height of 20,320 feet above sea level. The lowest point is Death Valley, California, which is 282 feet below sea level. How much higher is Denali than Death Valley? (*Source:* U.S. Geological Survey)

Solution:

1. UNDERSTAND. Read and reread the problem. To find "how much higher," we subtract. Don't forget that since Death Valley is 282 feet *below* sea level, we represent its height by -282. Draw a diagram to help visualize the problem.



2. TRANSLATE.

In words:	how much higher is Denali	=	height of Denali	minus	height of Death Valley
	↓	↓	↓	↓	↓
Translate:	how much higher is Denali	=	20,320	−	(−282)

(Continued on next page)

Practice 13

The highest point in Asia is the top of Mount Everest, at a height of 29,028 feet above sea level. The lowest point is the Dead Sea, which is 1312 feet below sea level. How much higher is Mount Everest than the Dead Sea? (*Source:* National Geographic Society)

Answers

11. −16 **12.** 31 **13.** 30,340 ft

3. SOLVE:

$$20{,}320 - (-282) = 20{,}320 + 282 = 20{,}602$$

4. INTERPRET. Check and state your conclusion: Denali is 20,602 feet higher than Death Valley.

Work Practice 13

Vocabulary, Readiness & Video Check

Multiple choice: Select the correct lettered response following each exercise.

1. It is true that $a - b =$ ________.

a. $b - a$ **b.** $a + (-b)$ **c.** $a + b$

2. The opposite of n is ____________.

a. $-n$ **b.** $-(-n)$ **c.** n

3. To evaluate $x - y$ for $x = -10$ and $y = -14$, we replace x with -10 and y with -14 and evaluate ____________.

a. $10 - 14$ **b.** $-10 - 14$ **c.** $-14 - 10$ **d.** $-10 - (-14)$

4. The expression $-5 - 10$ equals __________.

a. $5 - 10$ **b.** $5 + 10$ **c.** $-5 + (-10)$ **d.** $10 - 5$

Subtract.

5. $5 - 5$ **6.** $7 - 7$ **7.** $8642 - 8642$ **8.** $9012 - 9012$

Martin-Gay Interactive Videos

See Video 2.4

Watch the section lecture video and answer the following questions.

Objective A **9.** In the lecture before Example 1, what can the "opposite" of a number also be called?

Objective B **10.** In Example 7, how is the example rewritten in the first step of simplifying and why?

Objective C **11.** In Example 8, why do we multiply first?

Objective D **12.** What does the answer to Example 9, 263, mean in the context of the application?

2.4 Exercise Set MyLab Math

Objective A *Subtract. See Examples 1 through 7.*

1. $-5 - (-5)$ **2.** $-6 - (-6)$ **3.** $8 - 3$ **4.** $5 - 2$

5. $3 - 8$ **6.** $2 - 5$ **7.** $7 - (-7)$ **8.** $12 - (-12)$

9. $-5 - (-8)$ **10.** $-25 - (-25)$ **11.** $-14 - 4$ **12.** $-2 - 42$

13. $2 - 16$
14. $8 - 9$
15. $22 - 55$
16. $17 - 63$

17. $362 - (-40)$
18. $844 - (-20)$
19. $-4 - 10$
20. $-5 - 8$

21. $-7 - (-3)$
22. $-12 - (-5)$
23. $16 - 23$
24. $16 - 45$

Translating *Translate each phrase; then simplify. See Example 8.*

25. Subtract 17 from -25.
26. Subtract 10 from -22.
27. Find the difference of -20 and -3.

28. Find the difference of -8 and -13.
29. Subtract -11 from 2.
30. Subtract -50 from -50.

Mixed Practice (*Sections 2.3 and 2.4*) *Add or subtract as indicated.*

31. $-21 + (-17)$
32. $-35 + (-11)$
33. $9 - 20$
34. $7 - 30$

35. $-49 - 78$
36. $-105 - 68$
37. $48 - 59$
38. $86 - 98$

Objective B *Simplify. See Examples 9 and 10.*

39. $7 - 3 - 2$
40. $8 - 4 - 1$
41. $12 - 5 - 7$

42. $30 - 7 - 12$
43. $-5 - 8 - (-12)$
44. $-10 - 6 - (-9)$

45. $-10 + (-5) - 12$
46. $-15 + (-8) - 4$
47. $12 - (-34) + (-6)$

48. $23 - (-17) + (-9)$
49. $-(-6) - 12 + (-16)$
50. $-(-9) - 7 + (-23)$

51. $-9 - (-12) + (-7) - 4$
52. $-6 - (-8) + (-12) - 7$
53. $-3 + 4 - (-23) - 10$

54. $5 + (-18) - (-21) - 2$

Objective C *Evaluate $x - y$ for the given replacement values. See Examples 11 and 12.*

55. $x = -4$ and $y = 7$
56. $x = -7$ and $y = 1$

57. $x = 8$ and $y = -23$
58. $x = 9$ and $y = -2$

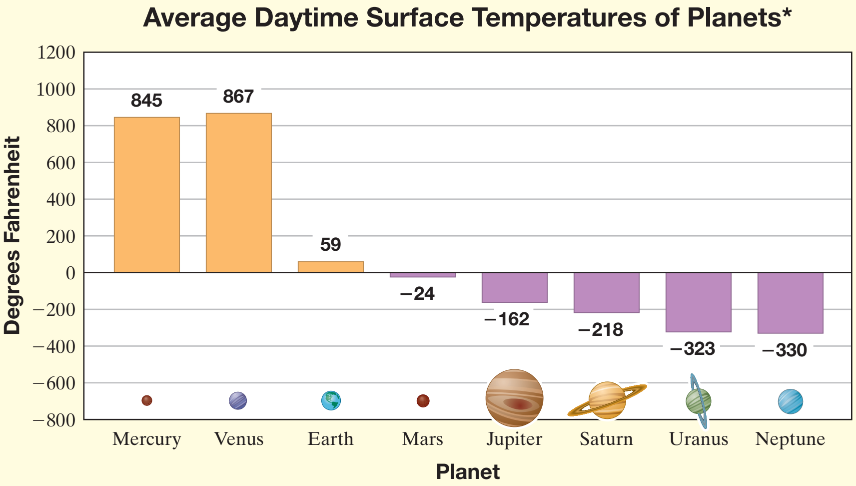
Evaluate $2x - y$ for the given replacement values. See Examples 11 and 12.

59. $x = 4$ and $y = -4$
60. $x = 8$ and $y = -10$

61. $x = 1$ and $y = -18$
62. $x = 14$ and $y = -12$

Objective D *Solve. See Example 13.*

The bar graph from Section 2.2 showing the average daytime surface temperature in degrees Fahrenheit of known planets is reprinted below. Notice that a negative temperature is illustrated by a bar below the horizontal line representing 0°F*, and a positive temperature is illustrated by a bar above the horizontal line representing* 0°F*.*



Source: The World Almanac

*For some planets, the temperature given is the temperature where the atmospheric pressure equals 1 Earth atmosphere.

63. Find the difference in temperature between Earth and Neptune.

64. Find the difference in temperature between Venus and Mars.

65. Find the difference in temperature between the two planets with the lowest temperatures.

66. Find the difference in temperature between Jupiter and Saturn.

67. The coldest temperature ever recorded on Earth was −129°F in Antarctica. The warmest temperature ever recorded was 134°F in Death Valley, California. How many degrees warmer is 134°F than −129°F? (*Source: The World Almanac*)

68. The coldest temperature ever recorded in the United States was −80°F in Alaska. The warmest temperature ever recorded was 134°F in California. How many degrees warmer is 134°F than −80°F? (*Source: The World Almanac*)

Solve.

69. Aaron Aiken has $125 in his checking account. He writes a check for $117, makes a deposit of $45, and then writes another check for $69. Find the balance in his account. (Write the amount as an integer.)

70. A woman received a statement of her charge account at Old Navy. She spent $93 on purchases last month. She returned an $18 top because she didn't like the color. She also returned a $26 nightshirt because it was damaged. What does she actually owe on her account?

71. The temperature on a February morning is −6° Celsius at 6 a.m. If the temperature drops 3 degrees by 7 a.m., rises 4 degrees between 7 a.m. and 8 a.m., and then drops 7 degrees between 8 a.m. and 9 a.m., find the temperature at 9 a.m.

72. Mauna Kea in Hawaii has an elevation of 13,796 feet above sea level. The Mid-America Trench in the Pacific Ocean has an elevation of 21,857 feet below sea level. Find the difference in elevation between those two points. (*Source: National Geographic Society and Defense Mapping Agency*)

Some places on Earth lie below sea level, which is the average level of the surface of the oceans. Use this diagram to answer Exercises 73 through 76. (Source: Fantastic Book of Comparisons, *Russell Ash)*

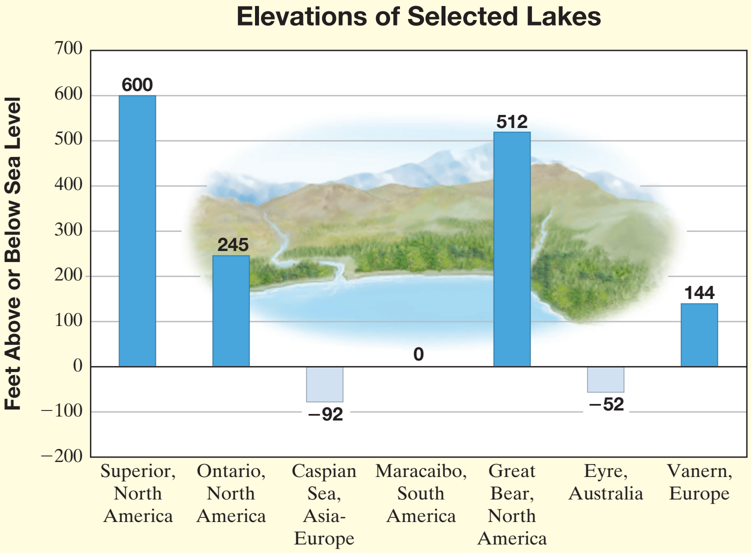
73. Find the difference in elevation between Death Valley and Quattâra Depression.

74. Find the difference in elevation between Danakil and Turfan Depressions.

75. Find the difference in elevation between the two lowest elevations shown.

76. Find the difference in elevation between the highest elevation shown and the lowest elevation shown.

The bar graph from Section 2.2 shows heights of selected lakes. For Exercises 77 through 80, find the difference in elevation for the lakes listed. (Source: U.S. Geological Survey)



77. Lake Superior and Lake Eyre

78. Great Bear Lake and Caspian Sea

79. Lake Maracaibo and Lake Vanern

80. Lake Eyre and Caspian Sea

Solve.

81. The difference between a country's exports and imports is called the country's *trade balance*. In October 2017, the United States had $196 billion in exports and $245 billion in imports. What was the U.S. trade balance in October 2017? (*Source:* U.S. Department of Commerce)

82. In 2016, the United States exported an average of 5261 thousand barrels of petroleum products per day and imported an average of 10,055 thousand barrels of petroleum products per day. What was the average U.S. trade balance for petroleum products per day in 2016? (*Source:* U.S. Energy Information Administration)

Mixed Practice—Translating (*Sections 2.3 and 2.4*)

Translate each phrase to an algebraic expression. Use "x" to represent "a number."

83. The sum of -5 and a number.

84. The difference of -3 and a number.

85. Subtract a number from -20.

86. Add a number and -36.

Review

Multiply or divide as indicated. See Sections 1.6 and 1.7.

87. $\frac{100}{20}$

88. $\frac{96}{3}$

89. $\begin{array}{r} 23 \\ \times\, 46 \\ \hline \end{array}$

90. $\begin{array}{r} 51 \\ \times\, 89 \\ \hline \end{array}$

Concept Extensions

91. Name two numbers whose difference is -3.

92. Name two numbers whose difference is -10.

Each calculation below is ***incorrect.*** *Find the error and correctly calculate. See the Concept Check in this section.*

93. $\xcancel{9 - (-7) \overset{?}{=} 2}$

94. $\xcancel{-4 - 8 \overset{?}{=} 4}$

95. $\xcancel{10 - 30 \overset{?}{=} 20}$

96. $\xcancel{-3 - (-10) \overset{?}{=} -13}$

Simplify. (Hint: Find the absolute values first.)

97. $|-3| - |-7|$

98. $|-12| - |-5|$

99. $|-6| - |6|$

100. $|-9| - |9|$

101. $|-17| - |-29|$

102. $|-23| - |-42|$

For Exercises 103 and 104, determine whether each statement is true or false.

103. $|-8 - 3| \overset{?}{=} 8 - 3$

104. $|-2 - (-6)| \overset{?}{=} |-2| - |-6|$

105. In your own words, explain how to subtract one signed number from another.

106. A student explains to you that the first step to simplify $8 + 12 \cdot 5 - 100$ is to add 8 and 12. Is the student correct? Explain why or why not.

Sections 2.1–2.4

Integrated Review

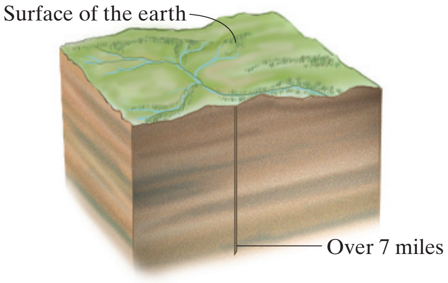
Integers

Represent each quantity by an integer.

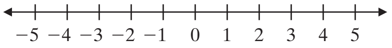
1. The peak of Mount Everest in Asia is 29,028 feet above sea level. (*Source:* U.S. Geological Survey)

2. The Mariana Trench in the Pacific Ocean is 35,840 feet below sea level. (*Source: The World Almanac*)

3. The deepest hole ever drilled in the Earth's crust is in Russia and its depth is over 7 miles below sea level. (*Source: Fantastic Book of Comparisons*)



4. Graph the signed numbers on the given number line. $-4, 0, -1, 3$



Insert $<$ or $>$ *between each pair of numbers to make a true statement.*

5. $0 \quad -3$

6. $-15 \quad -5$

7. $-1 \quad 1$

8. $-2 \quad -7$

Simplify.

9. $|-1|$

10. $-|-4|$

11. $|-8|$

12. $-(-5)$

Find the opposite of each number.

13. 6

14. -3

15. 89

16. 0

Add or subtract as indicated.

17. $-7 + 12$

18. $-9 + (-11)$

19. $25 + (-35)$

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. ______

18. ______

19. ______

20. ____________
21. ____________
22. ____________
23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________

20. $1 - 3$

21. $26 - (-26)$

22. $-2 - 1$

23. $-18 - (-102)$

24. $-8 + (-6) + 20$

25. $-11 - 7 - (-19)$

26. $-4 + (-8) - 16 - (-9)$

27. Subtract 14 from 26.

28. Subtract -8 from -12.

29. Find the sum of -17 and -27.

Choose all numbers for x from each given list that make each statement true.

30. $|x| > 0$
a. 0 **b.** 18 **c.** -3 **d.** -21

31. $|x| > -5$
a. 0 **b.** 3 **c.** -1 **d.** -1000

Evaluate the expressions below for $x = -1$ and $y = 11$.

32. $x + y$

33. $x - y$

34. $y - x$

35. $y + x$

36. $5y - x$

37. $x - 3y$

2.5 Multiplying and Dividing Integers

Multiplying and dividing integers is similar to multiplying and dividing whole numbers. One difference is that we need to determine whether the result is a positive number or a negative number.

Objectives

A Multiply Integers.

B Divide Integers.

C Evaluate an Algebraic Expression by Multiplying or Dividing.

D Solve Problems by Multiplying and Dividing Integers.

Objective A Multiplying Integers

Consider the following pattern of products.

First factor decreases by 1 each time.

$$\begin{aligned} 3 \cdot 2 &= 6 \\ 2 \cdot 2 &= 4 \\ 1 \cdot 2 &= 2 \\ 0 \cdot 2 &= 0 \downarrow \end{aligned}$$

Product decreases by 2 each time.

This pattern can be continued, as follows.

$$\begin{aligned} -1 \cdot 2 &= -2 \\ -2 \cdot 2 &= -4 \\ -3 \cdot 2 &= -6 \end{aligned}$$

This suggests that the product of a negative number and a positive number is a negative number.

What is the sign of the product of two negative numbers? To find out, we form another pattern of products. Again, we decrease the first factor by 1 each time, but this time the second factor is negative.

$$\begin{aligned} 2 \cdot (-3) &= -6 \\ 1 \cdot (-3) &= -3 \\ 0 \cdot (-3) &= 0 \downarrow \end{aligned}$$

Product increases by 3 each time.

This pattern continues as:

$$\begin{aligned} -1 \cdot (-3) &= 3 \\ -2 \cdot (-3) &= 6 \\ -3 \cdot (-3) &= 9 \end{aligned}$$

This suggests that the product of two negative numbers is a positive number. Thus we can determine the sign of a product when we know the signs of the factors.

Multiplying Numbers

The product of two numbers having the same sign is a positive number.

The product of two numbers having different signs is a negative number.

Product of Like Signs	Product of Different Signs
$(+)(+) = +$	$(-)(+) = -$
$(-)(-) = +$	$(+)(-) = -$

Examples Multiply.

1. $-7 \cdot 3 = -21$

2. $-3(-5) = 15$

3. $0 \cdot (-4) = 0$

4. $10(-8) = -80$

■ Work Practice 1–4

Practice 1–4

Multiply.

1. $-3 \cdot 8$

2. $-5(-2)$

3. $0 \cdot (-20)$

4. $10(-5)$

Answers

1. -24 **2.** 10 **3.** 0 **4.** -50

Recall that by the associative and commutative properties for multiplication, we may multiply numbers in any order that we wish. In Example 5, we multiply from left to right.

Practice 5–7

Multiply.

5. $8(-6)(-2)$
6. $(-9)(-2)(-1)$
7. $(-3)(-4)(-5)(-1)$

Examples

Multiply.

5. $\overbrace{7(-6)}(-2) = -42(-2)$
$= 84$

6. $\overbrace{(-2)(-3)}(-4) = 6(-4)$
$= -24$

7. $(-1)(-2)(-3)(-4) = -1(-24)$ We have -24 from Example 6.
$= 24$

Work Practice 5–7

✓ **Concept Check** What is the sign of the product of five negative numbers? Explain.

Recall from our study of exponents that $2^3 = 2 \cdot 2 \cdot 2 = 8$. We can now work with bases that are negative numbers. For example,

$$(-2)^3 = (-2)(-2)(-2) = -8$$

Practice 8

Evaluate $(-2)^4$.

Example 8

Evaluate: $(-5)^2$

Solution: Remember that $(-5)^2$ means 2 factors of -5.

$$(-5)^2 = (-5)(-5) = 25$$

Work Practice 8

Helpful Hint

Have you noticed a pattern when multiplying signed numbers?

If we let $(-)$ represent a negative number and $(+)$ represent a positive number, then

The product of an even number of negative numbers is a positive result.

$(-)(-) = (+)$

$(-)(-)(-) = (-)$

$(-)(-)(-)(-) = (+)$

$(-)(-)(-)(-)(-) = (-)$

The product of an odd number of negative numbers is a negative result.

Notice in Example 8 the parentheses around -5 in $(-5)^2$. With these parentheses, -5 is the base that is squared. Without parentheses, such as -5^2, only the 5 is squared. In other words, $-5^2 = -(5 \cdot 5) = -25$.

Practice 9

Evaluate: -8^2

Example 9

Evaluate: -7^2

Solution: Remember that without parentheses, only the 7 is squared.

$$-7^2 = -(7 \cdot 7) = -49$$

Work Practice 9

Answers
5. 96 **6.** -18 **7.** 60 **8.** 16 **9.** -64

✓ **Concept Check Answer**
negative; answers may vary

Helpful Hint

Make sure you understand the difference between Examples 8 and 9.

$$\overbrace{(-5)}^{\text{parentheses, so } -5 \text{ is squared}}{}^2 = (-5)(-5) = 25$$

$$\overbrace{-7^2}^{\text{no parentheses, so only the 7 is squared}} = -(7 \cdot 7) = -49$$

Objective B Dividing Integers

Division of integers is related to multiplication of integers. The sign rules for division can be discovered by writing a related multiplication problem. For example,

$$\frac{6}{2} = 3 \quad \text{because } 3 \cdot 2 = 6$$

$$\frac{-6}{2} = -3 \quad \text{because } -3 \cdot 2 = -6$$

$$\frac{6}{-2} = -3 \quad \text{because } -3 \cdot (-2) = 6$$

$$\frac{-6}{-2} = 3 \quad \text{because } 3 \cdot (-2) = -6$$

Helpful Hint

Just as for whole numbers, division can be checked by multiplication.

Dividing Numbers

The quotient of two numbers having the same sign is a positive number.

The quotient of two numbers having different signs is a negative number.

Quotient of Like Signs

$$\frac{(+)}{(+)} = + \qquad \frac{(-)}{(-)} = +$$

Quotient of Different Signs

$$\frac{(+)}{(-)} = - \qquad \frac{(-)}{(+)} = -$$

Examples Divide.

10. $\frac{-12}{6} = -2$

11. $-20 \div (-4) = 5$

12. $\frac{48}{-3} = -16$

Work Practice 10–12

✓ **Concept Check** What is wrong with the following calculation?

$$\xcancel{\frac{-36}{-9} = -4}$$

Practice 10–12

Divide.

10. $\frac{42}{-7}$

11. $-16 \div (-2)$

12. $\frac{-80}{10}$

Answers

10. -6 **11.** 8 **12.** -8

✓ **Concept Check Answer**

$\frac{-36}{-9} = 4$

Practice 13–14

Divide, if possible.

13. $\frac{-6}{0}$ **14.** $\frac{0}{-7}$

Examples Divide, if possible.

13. $\frac{0}{-5} = 0$ because $0 \cdot -5 = 0$

14. $\frac{-7}{0}$ is undefined because there is no number that gives a product of -7 when multiplied by 0.

Work Practice 13–14

Objective C Evaluating Expressions

Next, we practice evaluating expressions given integer replacement values.

Practice 15

Evaluate xy for $x = 5$ and $y = -8$.

Example 15 Evaluate xy for $x = -2$ and $y = 7$.

Solution: Recall that xy means $x \cdot y$.

Replace x with -2 and y with 7.

$$\begin{aligned} xy &= -2 \cdot 7 \\ &= -14 \end{aligned}$$

Work Practice 15

Practice 16

Evaluate $\frac{x}{y}$ for $x = -12$ and $y = -3$.

Example 16 Evaluate $\frac{x}{y}$ for $x = -24$ and $y = 6$.

Solution: $\frac{x}{y} = \frac{-24}{6}$ Replace x with -24 and y with 6.

$= -4$

Work Practice 16

Objective D Solving Problems by Multiplying and Dividing Integers

Many real-life problems involve multiplication and division of signed numbers.

Practice 17

A card player had a score of -13 for each of four games. Find her total score.

Example 17 Calculating a Total Golf Score

A professional golfer finished seven strokes under par (-7) for each of three days of a tournament. What was his total score for the tournament?

Solution:

1. UNDERSTAND. Read and reread the problem. Although the key word is "total," since this is repeated addition of the same number, we multiply.

2. TRANSLATE.

In words:	golfer's total score	=	number of days	·	score each day
	↓	↓	↓	↓	↓
Translate:	golfer's total	=	3	·	(−7)

3. SOLVE: $3 \cdot (-7) = -21$

4. INTERPRET. Check and state your conclusion: The golfer's total score was -21, or 21 strokes under par.

Work Practice 17

Answers

13. undefined **14.** 0 **15.** −40
16. 4 **17.** −52

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Each choice may be used more than once.

negative	0
positive	undefined

1. The product of a negative number and a positive number is a(n) ________ number.
2. The product of two negative numbers is a(n) ________ number.
3. The quotient of two negative numbers is a(n) ________ number.
4. The quotient of a negative number and a positive number is a(n) ________ number.
5. The product of a negative number and zero is ________.
6. The quotient of 0 and a negative number is ________.
7. The quotient of a negative number and 0 is ________.

Martin-Gay Interactive Videos

See Video 2.5

Watch the section lecture video and answer the following questions.

Objective A **8.** Explain the role of parentheses when comparing Examples 3 and 4.

Objective B **9.** Complete this statement based on the lecture before Example 6: We can find out about sign rules for division because we know sign rules for __________.

Objective C **10.** In Example 10, what are you asked to remember about the algebraic expression *ab*?

Objective D **11.** In Example 12, how do we know the example will involve a negative number?

2.5 Exercise Set MyLab Math

Objective A *Multiply. See Examples 1 through 4.*

1. $-6(-2)$
2. $-7(-2)$
3. $-4(9)$
4. $-9(8)$
5. $9(-9)$
6. $5(-3)$
7. $0(-11)$
8. $-6(0)$

Multiply. See Examples 5 through 7.

9. $6(-2)(-4)$
10. $-2(3)(-7)$
11. $-1(-3)(-4)$
12. $-8(-3)(-3)$
13. $-4(4)(-5)$
14. $2(-5)(-4)$
15. $10(-5)(0)(-7)$
16. $3(0)(-4)(-8)$
17. $-5(3)(-1)(-1)$
18. $-2(-1)(3)(-2)$

Evaluate. See Examples 8 and 9.

▶ **19.** -3^2

20. -2^4

▶ **21.** $(-3)^2$

22. $(-1)^4$

23. -6^2

24. -4^3

25. $(-4)^3$

26. $(-3)^3$

Objective B *Find each quotient. See Examples 10 through 14.*

27. $-24 \div 3$

28. $90 \div (-9)$

▶ **29.** $\frac{-30}{6}$

30. $\frac{56}{-8}$

31. $\frac{-77}{-11}$

32. $\frac{-32}{4}$

33. $\frac{0}{-21}$

34. $\frac{-13}{0}$

▶ **35.** $\frac{-10}{0}$

▶ **36.** $\frac{0}{-15}$

37. $\frac{56}{-4}$

38. $\frac{-24}{-12}$

Objectives A B Mixed Practice *Multiply or divide as indicated. See Examples 1 through 14.*

39. $-14(0)$

40. $0(-100)$

41. $-5(3)$

42. $-6 \cdot 2$

43. $-9 \cdot 7$

44. $-12(13)$

▶ **45.** $-7(-6)$

46. $-9(-5)$

▶ **47.** $-3(-4)(-2)$

48. $-7(-5)(-3)$

49. $(-7)^2$

50. $(-6)^2$

51. $-\frac{25}{5}$

52. $-\frac{30}{5}$

53. $-\frac{72}{8}$

54. $-\frac{49}{7}$

55. $-18 \div 3$

56. $-15 \div 3$

57. $4(-10)(-3)$

58. $6(-5)(-2)$

59. $-30(6)(-2)(-3)$

60. $-20 \cdot 5 \cdot (-5) \cdot (-3)$

61. $\frac{-25}{0}$

62. $\frac{0}{-14}$

63. $\frac{120}{-20}$

64. $\frac{63}{-9}$

65. $280 \div (-40)$

66. $480 \div (-8)$

▶ **67.** $\frac{-12}{-4}$

68. $\frac{-36}{-3}$

69. -1^4

70. -2^3

71. $(-2)^5$

72. $(-11)^2$

73. $-2(3)(5)(-6)$

74. $-1(2)(7)(-3)$

75. $(-1)^{32}$

76. $(-1)^{33}$

77. $-2(-3)(-5)$

78. $-2(-2)(-3)(-2)$

79. $-48 \cdot 23$

80. $-56 \cdot 43$

81. $35 \cdot (-82)$

82. $70 \cdot (-23)$

Objective C *Evaluate ab for the given replacement values. See Example 15.*

83. $a = -8$ and $b = 7$

84. $a = 5$ and $b = -1$

85. $a = 9$ and $b = -2$

86. $a = -8$ and $b = 8$

87. $a = -7$ and $b = -5$

88. $a = -9$ and $b = -6$

Evaluate $\frac{x}{y}$ *for the given replacement values. See Example 16.*

89. $x = 5$ and $y = -5$

90. $x = 9$ and $y = -3$

91. $x = -15$ and $y = 0$

92. $x = 0$ and $y = -5$

93. $x = -36$ and $y = -6$

94. $x = -10$ and $y = -10$

Evaluate xy and also $\frac{x}{y}$ *for the given replacement values. See Examples 15 and 16.*

95. $x = -8$ and $y = -2$

96. $x = 20$ and $y = -5$

97. $x = 0$ and $y = -8$

98. $x = -3$ and $y = 0$

Objective D **Translating** *Translate each phrase; then simplify. See Example 17.*

99. Find the quotient of −54 and 9.

100. Find the quotient of −63 and −3.

101. Find the product of −42 and −6.

102. Find the product of −49 and 5.

Translating *Translate each phrase to an expression. Use x to represent "a number." See Example 17.*

103. The product of −71 and a number

104. The quotient of −8 and a number

105. Subtract a number from −16.

106. The sum of a number and −12

107. −29 increased by a number

108. The difference of a number and −10

109. Divide a number by −33.

110. Multiply a number by −17.

Solve. See Example 17.

111. A football team lost four yards on each of three consecutive plays. Represent the total loss as a product of signed numbers and find the total loss.

112. An investor lost $400 on each of seven consecutive days in the stock market. Represent his total loss as a product of signed numbers and find his total loss.

113. A deep-sea diver must move up or down in the water in short steps in order to keep from getting a physical condition called the "bends." Suppose a diver moves down from the surface in five steps of 20 feet each. Represent his total movement as a product of signed numbers and find the product.

114. A weather forecaster predicts that the temperature will drop five degrees each hour for the next six hours. Represent this drop as a product of signed numbers and find the total drop in temperature.

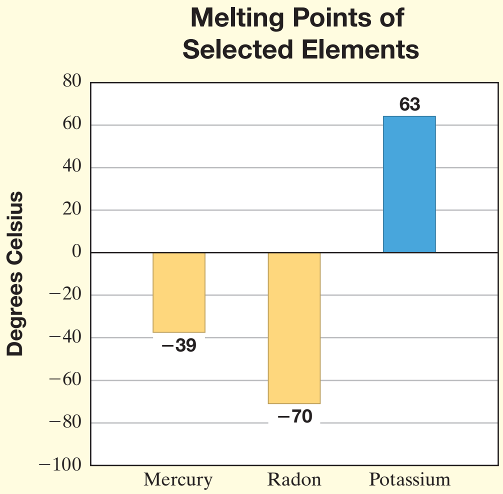
The graph shows melting points in degrees Celsius of selected elements. Use this graph to answer Exercises 115 through 118.

115. The melting point of nitrogen is 3 times the melting point of radon. Find the melting point of nitrogen.

116. The melting point of rubidium is −1 times the melting point of mercury. Find the melting point of rubidium.

117. The melting point of argon is −3 times the melting point of potassium. Find the melting point of argon.

118. The melting point of strontium is −11 times the melting point of radon. Find the melting point of strontium.



Solve. See Example 17.

119. Bees play an essential role in ecosystems. Nearly one-third of all food we eat depends on their pollination. However, from 1947 to 2017, the estimated number of colonies of bees in North America declined by approximately 3150 thousand colonies. If the loss of bee colonies were evenly spaced over these years, by how many bee colonies would you expect the population to drop each year? (*Source:* National Agricultural Statistics Service)

120. For the third quarter of 2017, Sears Holding Corporation posted a loss of $558 million due to a decline in sales. If this trend were consistent for each month of the quarter, how much would you expect this loss to have been for each month? (*Source:* Sears Holdings Corporation)

121. In 2010, there were approximately 24,810 analog (nondigital) U.S. movie screens. In 2016, this number of screens dropped to about 870. (*Source:* Motion Picture Association of America)

a. Find the change in the number of U.S. analog movie screens from 2010 to 2016.

b. Find the average change per year in the number of analog movie screens over this period.

122. In 1987, the California Condor was all but extinct in the wild, with about 30 condors in the world. The condors in the wild were captured by the U.S. Fish and Wildlife Service in an aggressive move to rebuild the population by breeding them in captivity and releasing the chicks into the wild. The condor population increased to approximately 436 birds in 2016. (*Source:* Arizona Game and Fish Department)

a. Find the change in the number of California Condors from 1987 to 2016.

b. Find the average change per year in the California Condor population over the period in part **a.**

Review

Perform each indicated operation. See Section 1.9.

123. $90 + 12^2 - 5^3$

124. $3 \cdot (7 - 4) + 2 \cdot 5^2$

125. $12 \div 4 - 2 + 7$

126. $12 \div (4 - 2) + 7$

Concept Extensions

Mixed Practice (*Sections 2.3, 2.4, and 2.5*) *Perform the indicated operations.*

127. $-57 \div 3$

128. $-9(-11)$

129. $-8 - 20$

130. $-4 + (-3) + 21$

131. $-4 - 15 - (-11)$

132. $-16 - (-2)$

Solve. For Exercises 133 and 134, see the first Concept Check in this section.

133. What is the sign of the product of seven negative numbers?

134. What is the sign of the product of ten negative numbers?

Without actually finding the product, write the list of numbers in Exercises 135 and 136 in order from least to greatest. For help, see a Helpful Hint box in this section.

135. $(-2)^{12}, (-2)^{17}, (-5)^{12}, (-5)^{17}$

136. $(-1)^{50}, (-1)^{55}, 0^{15}, (-7)^{20}, (-7)^{23}$

137. In your own words, explain how to divide two integers.

138. In your own words, explain how to multiply two integers.

2.6 Order of Operations

Objectives

A Simplify Expressions by Using the Order of Operations.

B Evaluate an Algebraic Expression.

C Find the Average of a List of Numbers.

Objective A Simplifying Expressions

We first discussed the order of operations in Chapter 1. In this section, you are given an opportunity to practice using the order of operations when expressions contain signed numbers. The rules for the order of operations from Section 1.9 are repeated here.

Order of Operations

1. Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars or square roots, starting with the innermost set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Before simplifying other expressions, make sure you are confident simplifying Examples 1 through 3.

Practice 1–3

Find the value of each expression.

1. $(-2)^4$
2. -2^4
3. $3 \cdot 6^2$

Examples Find the value of each expression.

1. $(-3)^2 = (-3)(-3) = 9$ The base of the exponent is −3.

2. $-3^2 = -(3)(3) = -9$ The base of the exponent is 3.

3. $2 \cdot 5^2 = 2 \cdot (5 \cdot 5) = 2 \cdot 25 = 50$ The base of the exponent is 5.

Work Practice 1–3

Helpful Hint

When simplifying expressions with exponents, remember that parentheses make an important difference.

> $(-3)^2$ and -3^2 **do not** mean the same thing.
> $(-3)^2$ means $(-3)(-3) = 9$.
> -3^2 means the opposite of $3 \cdot 3$, or -9.

Only with parentheses around it is the −3 squared.

Practice 4

Simplify: $\dfrac{-25}{5(-1)}$

Example 4 Simplify: $\dfrac{-6(2)}{-3}$

Solution: First we multiply −6 and 2. Then we divide.

$$\frac{-6(2)}{-3} = \frac{-12}{-3}$$
$$= 4$$

Work Practice 4

Practice 5

Simplify: $\dfrac{-18 + 6}{-3 - 1}$

Example 5 Simplify: $\dfrac{12 - 16}{-1 + 3}$

Solution: We simplify above and below the fraction bar separately. Then we divide.

$$\frac{12 - 16}{-1 + 3} = \frac{-4}{2}$$
$$= -2$$

Work Practice 5

Practice 6

Simplify: $30 + 50 + (-4)^3$

Example 6 Simplify: $60 + 30 + (-2)^3$

Solution: $60 + 30 + (-2)^3 = 60 + 30 + (-8)$ Write $(-2)^3$ as −8.
$= 90 + (-8)$ Add from left to right.
$= 82$

Work Practice 6

Answers
1. 16 **2.** −16 **3.** 108 **4.** 5
5. 3 **6.** 16

Example 7

Simplify: $-4^2 + (-3)^2 - 1^3$

Solution:

$$\begin{aligned} -4^2 + (-3)^2 - 1^3 &= -16 + 9 - 1 && \text{Simplify expressions with exponents.}\\ &= -7 - 1 && \text{Add or subtract from left to right.}\\ &= -8 \end{aligned}$$

Work Practice 7

Example 8

Simplify: $3(4 - 7) + (-2) - \sqrt{25}$

Solution:

$$\begin{aligned} 3(4 - 7) + (-2) - \sqrt{25} &= 3(-3) + (-2) - 5 && \text{Simplify inside parentheses and replace } \sqrt{25} \text{ with 5.}\\ &= -9 + (-2) - 5 && \text{Multiply.}\\ &= -11 - 5 && \text{Add or subtract from left to right.}\\ &= -16 \end{aligned}$$

Work Practice 8

Example 9

Simplify: $(-3) \cdot |-5| - (-2) + 4^2$

Solution:

$$\begin{aligned} (-3) \cdot |-5| - (-2) + 4^2 &= (-3) \cdot 5 - (-2) + 4^2 && \text{Write } |-5| \text{ as 5.}\\ &= (-3) \cdot 5 - (-2) + 16 && \text{Write } 4^2 \text{ as 16.}\\ &= -15 - (-2) + 16 && \text{Multiply.}\\ &= -13 + 16 && \text{Add or subtract from left to right.}\\ &= 3 \end{aligned}$$

Work Practice 9

Example 10

Simplify: $-2[-3 + 2(-1 + 6)] - 5$

Solution: Here we begin with the innermost set of parentheses.

$$\begin{aligned} -2[-3 + 2(-1 + 6)] - 5 &= -2[-3 + 2(5)] - 5 && \text{Write } -1 + 6 \text{ as 5.}\\ &= -2[-3 + 10] - 5 && \text{Multiply.}\\ &= -2(7) - 5 && \text{Add.}\\ &= -14 - 5 && \text{Multiply.}\\ &= -19 && \text{Subtract.} \end{aligned}$$

Work Practice 10

✓ *Concept Check* True or false? Explain your answer. The result of

$$-4(3 - 7) - 8(9 - 6)$$

is positive because there are four negative signs.

Practice 7

Simplify: $-2^3 + (-4)^2 + 1^5$

Practice 8

Simplify:

$2(2 - 9) + (-12) - \sqrt{9}$

Practice 9

Simplify:

$(-5) \cdot |-8| + (-3) + 2^3$

Practice 10

Simplify:

$-4[-6 + 5(-3 + 5)] - 7$

Answers

7. 9 **8.** −29 **9.** −35 **10.** −23

✓ Concept Check Answer

false; $-4(3 - 7) - 8(9 - 6) = -8$

Objective B Evaluating Expressions

Now we practice evaluating expressions.

Practice 11

Evaluate x^2 and $-x^2$ for $x = -15$.

Example 11 Evaluate x^2 and $-x^2$ for $x = -11$.

Solution:

$$x^2 = (-11)^2 = (-11)(-11) = 121$$

$$-x^2 = -(-11)^2 = -(-11)(-11) = -121$$

Work Practice 11

Practice 12

Evaluate $5y^2$ for $y = 4$ and $y = -4$.

Example 12 Evaluate $6z^2$ for $z = 2$ and $z = -2$.

Solution:

$$6z^2 = 6(2)^2 = 6(4) = 24$$

$$6z^2 = 6(-2)^2 = 6(4) = 24$$

Work Practice 12

Practice 13

Evaluate $x - y + 3z$ for $x = -6$, $y = -3$, and $z = 12$.

Example 13 Evaluate $x + 2y - z$ for $x = 3$, $y = -5$, and $z = -4$.

Solution: Replace x with 3, y with -5, and z with -4, and simplify.

Helpful Hint Remember to rewrite the subtraction sign.

$$\begin{aligned} x + 2y - z &= 3 + 2(-5) - (-4) && \text{Let } x = 3, y = -5, \text{ and } z = -4. \\ &= 3 + (-10) + 4 && \text{Replace } 2(-5) \text{ with its product, } -10. \\ &= -3 && \text{Add.} \end{aligned}$$

Work Practice 13

Practice 14

Evaluate $4 - x^2$ for $x = -8$.

Example 14 Evaluate $7 - x^2$ for $x = -4$.

Solution: Replace x with -4 and simplify carefully!

$$\begin{aligned} 7 - x^2 &= 7 - (-4)^2 \\ &= 7 - 16 && (-4)^2 = (-4)(-4) = 16 \\ &= -9 && \text{Subtract.} \end{aligned}$$

Work Practice 14

Objective C Finding Averages

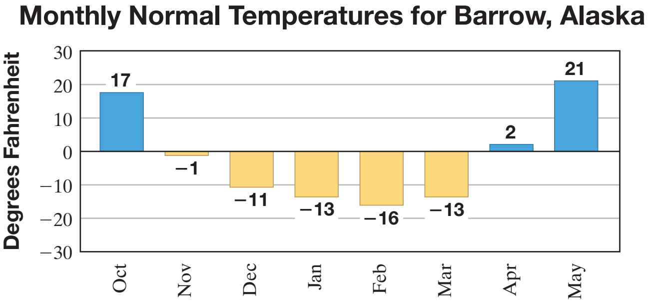
Recall from Chapter 1 that the average of a list of numbers is

$$\text{average} = \frac{\text{sum of numbers}}{\textit{number} \text{ of numbers}}$$

Answers

11. 225; −225 **12.** 80; 80 **13.** 33
14. −60

Example 15 The graph shows some monthly normal temperatures for Barrow, Alaska. Use this graph to find the average of the temperatures for the months January through April.



Solution: By reading the graph, we have

$$\text{average} = \frac{-13 + (-16) + (-13) + 2}{4} \quad \text{There are 4 months from January through April.}$$

$$= \frac{-40}{4}$$

$$= -10$$

The average of the temperatures is −10°F.

Work Practice 15

Practice 15

Find the average of the temperatures for the months October through April.

Answer

15. −5°F

Calculator Explorations Simplifying an Expression Containing a Fraction Bar

Recall that even though most calculators follow the order of operations, parentheses must sometimes be inserted. For example, to simplify $\frac{-8 + 6}{-2}$ on a calculator, enter parentheses around the expression above the fraction bar so that it is simplified separately.

To simplify $\frac{-8 + 6}{-2}$, press the keys

$\boxed{(}\ \boxed{8}\ \boxed{+/-}\ \boxed{+}\ \boxed{6}\ \boxed{)}\ \ \boxed{\div}\ \ \boxed{2}\ \boxed{+/-}\ \boxed{=}$ or

$\boxed{(}\ \boxed{(-)}\ \boxed{8}\ \boxed{+}\ \boxed{6}\ \boxed{)}\ \ \boxed{\div}\ \ \boxed{(-)}\ \boxed{2}\ \boxed{\text{ENTER}}$

The display will read $\boxed{\quad 1 \quad}$.

Thus, $\frac{-8 + 6}{-2} = 1$.

Use a calculator to simplify.

1. $\frac{-120 - 360}{-10}$

2. $\frac{4750}{-2 + (-17)}$

3. $\frac{-316 + (-458)}{28 + (-25)}$

4. $\frac{-234 + 86}{-18 + 16}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

average subtraction division $-7 - 3(1)$

addition multiplication $-7 - 3(-1)$

1. To simplify $-2 \div 2 \cdot (3)$, which operation should be performed first? ______________

2. To simplify $-9 - 3 \cdot 4$, which operation should be performed first? ______________

3. The ______________ of a list of numbers is $\frac{\text{sum of numbers}}{\textit{number} \text{ of numbers}}$.

4. To simplify $5[-9 + (-3)] \div 4$, which operation should be performed first? ______________

5. To simplify $-2 + 3(10 - 12) \cdot (-8)$, which operation should be performed first? ______________

6. To evaluate $x - 3y$ for $x = -7$ and $y = -1$, replace x with −7 and y with −1 and evaluate ______________.

Identify the bases and exponents of each expression. Do not simplify.

7. -3^2

8. $(-3)^2$

9. $4 \cdot 2^3$

10. $9 \cdot 5^6$

11. $(-7)^5$

12. -9^4

13. $5^7 \cdot 10$

14. $2^8 \cdot 11$

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 2.6

Objective A **15.** In Example 1, what two things about the fraction bar are we reminded of?

Objective B **16.** In Example 5, why is it important to place the replacement value for x within parentheses?

Objective C **17.** From the lecture before Example 6, explain why finding the average is a good example of an application for this section.

2.6 Exercise Set MyLab Math

Objective A *Simplify. See Examples 1 through 10.*

1. $(-4)^3$

2. -2^4

3. -4^3

4. $(-2)^4$

5. $6 \cdot 2^2$

6. $5 \cdot 2^3$

7. $\sqrt{9} + (-8) \div 2$

8. $\sqrt{49} + (-18) \div 2$

9. $9 - 12 - 4$

10. $10 - 23 - 12$

11. $4 + 3(-6)$

12. $-8 + 4(3)$

13. $5(-9) + 2$

14. $7(-6) + 3$

15. $(-10) + 4 \div 2$

16. $(-12) + 6 \div 3$

17. $6 + 7 \cdot 3 - 40$

18. $5 + 9 \cdot 4 - 52$

19. $\dfrac{16 - 13}{-3}$

20. $\dfrac{20 - 15}{-1}$

21. $\dfrac{24}{10 + (-4)}$

22. $\dfrac{88}{-8 - 3}$

23. $5(-3) - (-12)$

24. $7(-4) - (-6)$

25. $(-19) - 12(3)$

26. $(-24) - 14(2)$

27. $-8 + 4^2$

28. $-12 + 3^3$

29. $[8 + (-4)]^2$

30. $[9 + (-2)]^3$

31. $8 \cdot 6 - 3 \cdot 5 + (-20)$

32. $7 \cdot 6 - 6 \cdot 5 + (-10)$

33. $4 - (-3)^4$

34. $20 - (-5)^2$

35. $|5 + 3| \cdot 2^3$

36. $|-3 + 7| \cdot 7^2$

37. $7 \cdot 8^2 + 4$

38. $10 \cdot 5^3 + 7$

39. $5^3 - (4 - 2)^3$

40. $8^2 - (5 - 2)^4$

41. $|3 - 12| \div 3$

42. $|12 - 19| \div 7$

43. $-(-2)^2$

44. $-(-2)^3$

45. $(5-9)^2 \div (4-2)^2$

46. $(2-7)^2 \div (4-3)^4$

▶ **47.** $|8-24| \cdot (-2) \div (-2)$

48. $|3-15| \cdot (-4) \div (-16)$

49. $(-12-20) \div 16 - 25$

50. $(-20-5) \div 5 - 15$

51. $\sqrt{25}(5-2) + (-5)^2 - 6$

52. $\sqrt{9}(8-3) + (-4)^2 - 10$

53. $(2-7) \cdot (6-19)$

54. $(4-12) \cdot (8-17)$

55. $2 - 7 \cdot 6 - 19$

56. $4 - 12 \cdot 8 - 17$

57. $(-36 \div 6) - (4 \div 4)$

58. $(-4 \div 4) - (8 \div 8)$

59. $-5^2 - 6^2$

60. $-4^4 - 5^4$

61. $(-5)^2 - 6^2$

62. $(-4)^4 - 5^4$

63. $(\sqrt{100} - 4^2)^2$

64. $(\sqrt{121} - 3^2)^3$

65. $2(8-10)^2 - 5(1-6)^2$

66. $-3(4-8)^2 + 5(14-16)^3$

67. $3(-10) \div [5(-3) - 7(-2)]$

68. $12 - [7 - (3-6)] + (2-3)^3$

69. $\dfrac{(-7)(-3) - (4)(3)}{3[7 \div (3-10)]}$

70. $\dfrac{10(-1) - (-2)(-3)}{2[-8 \div (-2-2)]}$

71. $-5[4 + 5(-3+5)] + 11$

72. $-2[1 + 3(7-12)] - 35$

▶ **73.** $-3[5 + 2(-4+9)] + 15$

74. $-2[6 + 4(2-8)] - 25$

Objective B *Evaluate each expression for* $x = -2$, $y = 4$, *and* $z = -1$. *See Examples 11 through 14.*

75. $x + y + z$

76. $x - y - z$

77. $2x - 3y - 4z$

78. $5x - y + 4z$

▶ **79.** $x^2 - y$

80. $x^2 + z$

81. $\dfrac{5y}{z}$

82. $\dfrac{4x}{y}$

Evaluate each expression for $x = -3$ *and* $z = -4$. *See Examples 11 through 14.*

83. x^2

84. z^2

85. $-z^2$

86. $-x^2$

87. $10 - x^2$

88. $3 - z^2$

89. $2x^3 - z$

90. $3z^2 - x$

Objective C *Find the average of each list of numbers. See Example 15.*

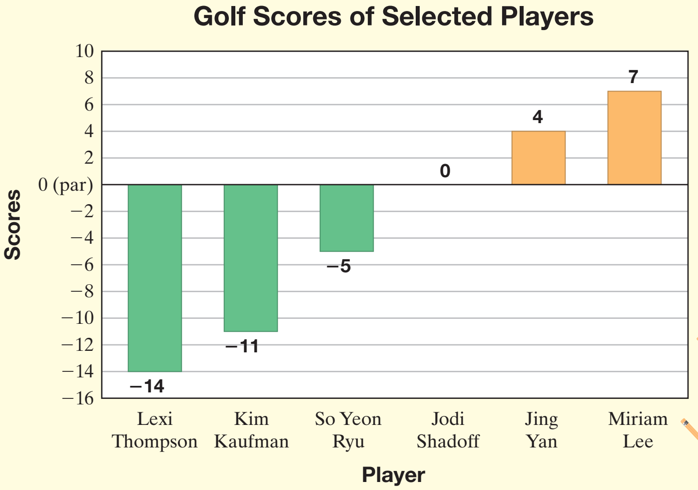
▶ **91.** −10, 8, −4, 2, 7, −5, −12

92. −18, −8, −1, −1, 0, 4

93. −17, −26, −20, −13

94. −40, −20, −10, −15, −5

Scores in golf can be 0 (also called par), a positive integer (also called above par), or a negative integer (also called below par). The bar graph shows final scores of selected golfers from a 2017 tournament. Use this graph for Exercises 95 through 100. (Source: LPGA)



95. Find the difference between the lowest score shown and the highest score shown.

96. Find the difference between the two lowest scores.

97. Find the average of the scores for Kaufman, Ryu, Shadoff, and Yan. (*Hint:* Here the average is the sum of the scores divided by the number of players.)

98. Find the average of the scores for Kaufman, Yan, and Lee.

99. Can the average for the scores in Exercise **98** be greater than the highest score, 7? Explain why or why not.

100. Can the average of the scores in Exercise **98** be less than the lowest score, −14? Explain why or why not.

Review

Perform each indicated operation. See Sections 1.3, 1.4, 1.6, and 1.7.

101. $45 \cdot 90$

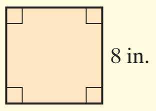
102. $90 \div 45$

103. $90 - 45$

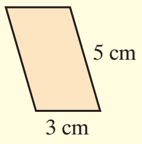
104. $45 + 90$

Find the perimeter of each figure. See Section 1.3.

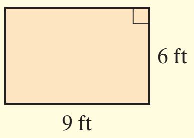
△ **105.** Square



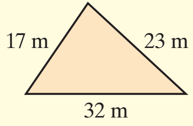
△ **106.** Parallelogram



△ **107.** Rectangle



△ **108.** Triangle



Concept Extensions

Insert parentheses where needed so that each expression evaluates to the given number.

109. $2 \cdot 7 - 5 \cdot 3$; evaluates to 12

110. $7 \cdot 3 - 4 \cdot 2$; evaluates to 34

111. $-6 \cdot 10 - 4$; evaluates to −36

112. $2 \cdot 8 \div 4 - 20$; evaluates to −36

113. Are parentheses necessary in the expression $3 + (4 \cdot 5)$? Explain your answer.

114. Are parentheses necessary in the expression $(3 + 4) \cdot 5$? Explain your answer.

115. Discuss the effect parentheses have in an exponential expression. For example, what is the difference between $(-6)^2$ and -6^2?

116. Discuss the effect parentheses have in an exponential expression. For example, what is the difference between $(2 \cdot 4)^2$ and $2 \cdot 4^2$?

Evaluate.

117. $(-12)^4$

118. $(-17)^6$

119. $x^3 - y^2$ for $x = 21$ and $y = -19$

120. $3x^2 + 2x - y$ for $x = -18$ and $y = 2868$

121. $(xy + z)^x$ for $x = 2$, $y = -5$, and $z = 7$

122. $5(ab + 3)^b$ for $a = -2$, $b = 3$

Chapter 2 Group Activity

Magic Squares

Sections 2.2–2.4

A magic square is a set of numbers arranged in a square table so that the sum of the numbers in each column, row, and diagonal is the same. For instance, in the magic square below, the sum of each column, row, and diagonal is 15. Notice that no number is used more than once in the magic square.

2	9	4
7	5	3
6	1	8

The properties of magic squares have been known for a very long time and once were thought to be good luck charms. The ancient Egyptians and Greeks understood their patterns, and a magic square even made it into a famous work of art. The engraving titled *Melencolia I,* created by German artist Albrecht Dürer in 1514, features the following four-by-four magic square on the building behind the central figure.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Exercises

1. Verify that what is shown in the Dürer engraving is, in fact, a magic square. What is the common sum of the columns, rows, and diagonals?

2. Negative numbers can also be used in magic squares. Complete the following magic square:

		–2
	–1	
0		–4

3. Use the numbers $-16, -12, -8, -4, 0, 4, 8, 12,$ and 16 to form a magic square:

Chapter 2 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

signed positive opposites negative absolute value variable integers

1. Two numbers that are the same distance from 0 on a number line but are on opposite sides of 0 are called ______________.

2. Together, positive numbers, negative numbers, and 0 are called ____________ numbers.

3. The ______________ of a number is that number's distance from 0 on a number line.

4. The ______________ are . . . , −3, −2, −1, 0, 1, 2, 3, ...

5. A letter used to represent a number is called a(n) ____________.

6. The ______________ numbers are numbers less than zero.

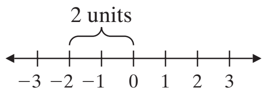
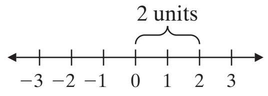
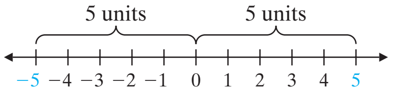
7. The ______________ numbers are numbers greater than zero.

Helpful Hint Are you preparing for your test? To help, don't forget to take these:

- Chapter 2 Getting Ready for the Test on page 171
- Chapter 2 Test on page 172

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

2 Chapter Highlights

Definitions and Concepts	Examples
Section 2.1 Introduction to Variables and Algebraic Expressions	
A letter used to represent a number is called a **variable.**	Variables: $x, \quad y, \quad z, \quad a, \quad b$
A combination of operations on variables and numbers is called an **algebraic expression.**	Algebraic expressions: $3 + x, \quad 7y, \quad x^3 + y - 10$
Replacing a variable in an expression by a number, and then finding the value of the expression, is called **evaluating the expression** for the variable.	Evaluate $2x + y$ for $x = 22$ and $y = 4$. $2x + y = 2 \cdot 22 + 4$ Replace x with 22 and y with 4. $= 44 + 4$ Multiply. $= 48$ Add.
Section 2.2 Introduction to Integers	
Together, positive numbers, negative numbers, and 0 are called **signed numbers.**	−432, −10, 0, 15
The **integers** are . . . , −3, −2, −1, 0, 1, 2, 3, ...	
The **absolute value** of a number is that number's distance from 0 on a number line. The symbol for absolute value is | |.	$\|-2\| = 2$  $\|2\| = 2$ 
Two numbers that are the same distance from 0 on a number line but are on opposite sides of 0 are called **opposites.**	5 and −5 are opposites. 
If a is a number, then $-(-a) = a$.	$-(-11) = 11$. Do not confuse with $-\|-3\| = -3$.

Definitions and Concepts	Examples
Section 2.3 Adding Integers	
Adding Two Numbers with the Same Sign **Step 1:** Add their absolute values. **Step 2:** Use their common sign as the sign of the sum.	Add: $-3 + (-2) = -5$ $-7 + (-15) = -22$
Adding Two Numbers with Different Signs **Step 1:** Find the larger absolute value minus the smaller absolute value. **Step 2:** Use the sign of the number with the larger absolute value as the sign of the sum.	$-6 + 4 = -2$ $17 + (-12) = 5$ $-32 + (-2) + 14 = -34 + 14$ $= -20$
Section 2.4 Subtracting Integers	
Subtracting Two Numbers If a and b are numbers, then $a - b = a + (-b)$.	Subtract: $-35 - 4 = -35 + (-4) = -39$ $3 - 8 = 3 + (-8) = -5$ $-10 - (-12) = -10 + 12 = 2$ $7 - 20 - 18 - (-3) = 7 + (-20) + (-18) + (+3)$ $= -13 + (-18) + 3$ $= -31 + 3$ $= -28$
Section 2.5 Multiplying and Dividing Integers	
Multiplying Numbers The product of two numbers having the same sign is a positive number. The product of two numbers having different signs is a negative number.	Multiply: $(-7)(-6) = 42$ $9(-4) = -36$ Evaluate: $(-3)^2 = (-3)(-3) = 9$
Dividing Numbers The quotient of two numbers having the same sign is a positive number. The quotient of two numbers having different signs is a negative number.	Divide: $-100 \div (-10) = 10$ $\frac{14}{-2} = -7, \quad \frac{0}{-3} = 0, \quad \frac{22}{0}$ is undefined.
Section 2.6 Order of Operations	
Order of Operations **1.** Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars or square roots, starting with the innermost set. **2.** Evaluate any expressions with exponents. **3.** Multiply or divide in order from left to right. **4.** Add or subtract in order from left to right.	Simplify: $3 + 2 \cdot (-5) = 3 + (-10)$ $= -7$ Simplify: $\frac{-2(5 - 7)}{-7 + \lvert -3 \rvert} = \frac{-2(-2)}{-7 + 3}$ $= \frac{4}{-4}$ $= -1$

Chapter 2 Review

(2.1) *Evaluate each expression for $x = 5$, $y = 0$, and $z = 2$.*

1. $\frac{2x}{z}$

2. $4x - 3$

3. $\frac{x + 7}{y}$

4. $\frac{y}{5x}$

5. $x^3 - 2z$

6. $\frac{7 + x}{3z}$

7. $(y + z)^2$

8. $\frac{100}{x} + \frac{y}{3}$

Translate each phrase into a variable expression. Use x to represent "a number."

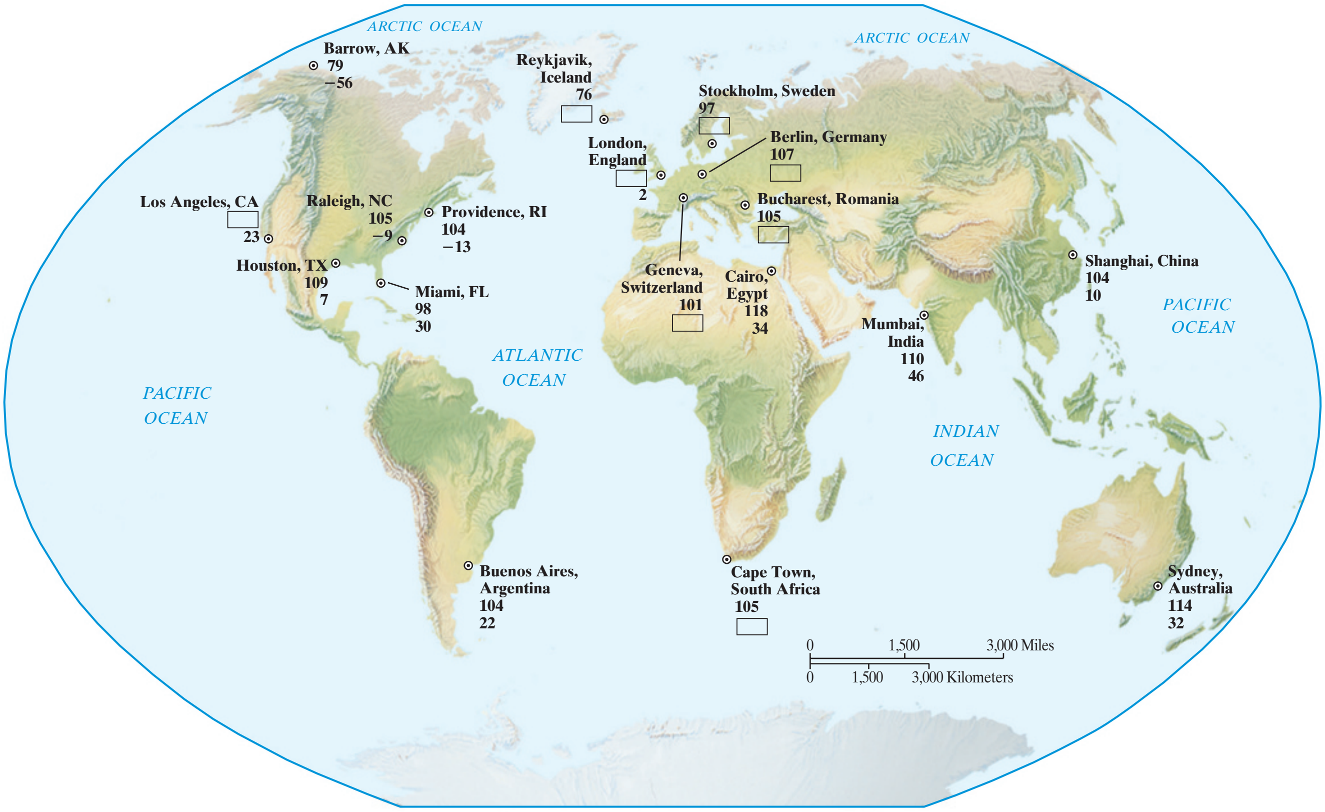
9. Five subtracted from a number

10. Seven more than a number

11. Ten divided by a number

12. The product of 5 and a number

The map below shows selected cities and their record high and low temperatures in degrees Fahrenheit. Use this map as indicated throughout the rest of this Chapter Review to fill in each missing temperature in the map. The table on the next page may help to insert missing temperatures. Exercise **49** *is the first exercise to use this map.*



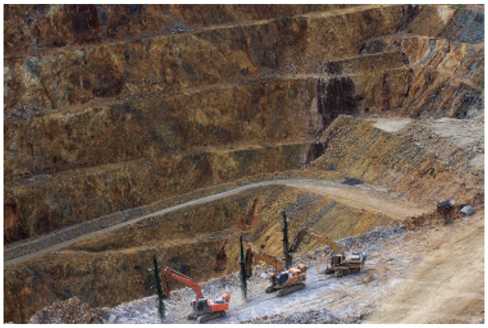
Source: World Almanac, 2014

Record High and Low Temperatures for Selected Locations (in degrees Fahrenheit)

	Max	Min		Max	Min
Berlin, Germany	107		Barrow, AK	79	−56
Raleigh, NC	105	−9	London, England		2
Houston, TX	109	7	Cairo, Egypt	118	34
Miami, FL	98	30	Sydney, Australia	114	32
Los Angeles, CA		23	Shanghai, China	104	10
Bucharest, Romania	105		Reykjavik, Iceland	76	
Geneva, Switzerland	101		Cape Town, South Africa	105	
Providence, RI	104	−13	Buenos Aires, Argentina	104	22
Stockholm, Sweden	97		Mumbai, India	110	46

(2.2) *Represent each quantity by an integer.*

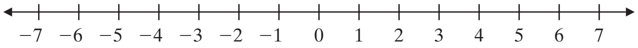
13. A gold miner is working 1435 feet down in a mine.



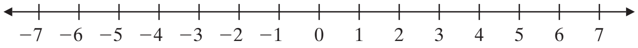
14. Mount Hood, in Oregon, has an elevation of 11,239 feet.

Graph each integer in the list on the same number line.

15. $-2, -5, 0, 5$



16. $-7, -1, 0, 7$



Simplify.

17. $|-12|$

18. $|0|$

19. $-|6|$

20. $-(-9)$

21. $-|-9|$

22. $-(-2)$

Insert < or > between each pair of integers to make a true statement.

23. $-18 \quad -20$

24. $-5 \quad 5$

25. $|-123| \quad -|-198|$

26. $8 - |-12| \quad -|-16|$

Find the opposite of each integer.

27. -12

28. $-(-3)$

Answer true or false for each statement.

29. If $a < b$, then a must be a negative number.

30. The absolute value of an integer is always 0 or a positive number.

31. A negative number is always less than a positive number.

32. If a is a negative number, then $-a$ is a positive number.

(2.3) *Add.*

33. $5 + (-3)$

34. $18 + (-4)$

35. $-12 + 16$

36. $-23 + 40$

37. $-8 + (-15)$

38. $-5 + (-17)$

39. $-24 + 3$

40. $-89 + 19$

41. $15 + (-15)$

42. $-24 + 24$

43. $-43 + (-108)$

44. $-100 + (-506)$

45. The temperature at 5 a.m. on a day in January was −15° Celsius. By 6 a.m. the temperature had fallen 5 degrees. Use a signed number to represent the temperature at 6 a.m.

46. A diver starts out at 127 feet below the surface and then swims downward another 23 feet. Use a signed number to represent the diver's current depth.

47. During the PGA 2017 CJ Cup at Nine Bridges tournament in South Korea, the winner, Justin Thomas, had scores of −9, 2, −2, and 0. What was his total score for the tournament? (*Source:* Professional Golfers Association)

48. The Ryder Cup, a biennial (every two years) pro men golfers' tournament between an American team and a European team, scores holes won. During the 2016 Ryder Cup, the winners, the American team, had a score of 17. The losing team, the Europeans, had a score 6 less than the Americans' score. What was the European team's score? (*Source*: Ryder Cup 2016)

For Exercises 49 and 50, use the map near the beginning of this Chapter Review.

49. The high temperature for London, England, is 155 degrees greater than the low temperature for Barrow, Alaska. Find the high temperature for London.

50. The high temperature for Los Angeles, California, is 123 degrees greater than the low temperature for Providence, Rhode Island. Find the high temperature for Los Angeles.

(2.4) *Subtract.*

51. $12 - 4$

52. $-12 - 4$

53. $8 - 19$

54. $-8 - 19$

55. $7 - (-13)$

56. $-6 - (-14)$

57. $16 - 16$

58. $-16 - 16$

59. $-12 - (-12)$

60. $|-5| - |-12|$

61. $-(-5) - 12 + (-3)$

62. $-8 + |-12| - 10 - |-3|$

Solve.

63. Josh Weidner has $142 in his checking account. He writes a check for $125, makes a deposit of $43, and then writes another check for $85. Represent the final dollar amount in his account by an integer.

64. If the elevation of Lake Superior is 600 feet above sea level and the elevation of the Caspian Sea is 92 feet below sea level, find the difference in the elevations.

For Exercises 65 and 66, use the map near the beginning of this Chapter Review.

65. The low temperature for Reykjavik is 35 degrees less than the low temperature for Sydney, Australia. Find the low temperature for Reykjavik.

66. The low temperature for Berlin, Germany, is 14 degrees less than the low temperature for Shanghai, China. Find the low temperature for Berlin.

Answer true or false for each statement.

67. $|-5| - |-6| = 5 - 6$

68. $|-5 - (-6)| = 5 + 6$

69. If $b > a$, then $b - a$ is a positive number.

70. If $b < a$, then $b - a$ is a negative number.

(2.5) *Multiply.*

71. $-3(-7)$

72. $-6(3)$

73. $-4(16)$

74. $-5(-12)$

75. $(-5)^2$

76. $(-1)^5$

77. $12(-3)(0)$

78. $-1(6)(2)(-2)$

Divide.

79. $-15 \div 3$

80. $\frac{-24}{-8}$

81. $\frac{0}{-3}$

82. $\frac{-46}{0}$

83. $\frac{100}{-5}$

84. $\frac{-72}{8}$

85. $\frac{-38}{-1}$

86. $\frac{45}{-9}$

87. A football team lost 5 yards on each of two consecutive plays. Represent the total loss by a product of integers, and find the product.

88. A race horse bettor lost \$50 on each of four consecutive races. Represent the total loss by a product of integers, and find the product.

For Exercises 89 through 92, use the map near the beginning of this Chapter Review.

89. The low temperature for Bucharest, Romania, is 2 times the low temperature for Raleigh, North Carolina. Find the low temperature for Bucharest.

90. The low temperature for Geneva, Switzerland, is the same as the low temperature for Miami, Florida, divided by -10. Find the low temperature for Geneva.

91. The low temperature for Cape Town, South Africa, is the same as the low temperature for Barrow, Alaska, divided by -2. Find the low temperature for Cape Town.

92. The low temperature for Stockholm, Sweden, is 2 times the low temperature for Providence, Rhode Island. Find the low temperature for Stockholm.

(2.6) *Simplify.*

93. $(-7)^2$

94. -7^2

95. -2^5

96. $(-2)^5$

97. $-7 + 5 + (-8) - 11$

98. $-3 + 12 + (-7) - 10$

99. $-10 + 3 \cdot (-2)$

100. $5 - 10 \cdot (-3)$

101. $16 \cdot (-2) + 4$

102. $3 \cdot (-12) - 8$

103. $5 + 6 \div (-3)$

104. $-6 + (-10) \div (-2)$

105. $16 + (-3) \cdot 12 \div 4$

106. $-12 + 25 \cdot 1 \div (-5)$

107. $4^3 - (8 - 3)^2$

108. $4^3 - 90$

109. $-(-4)\cdot|-3|-5$ **110.** $-(-8)\cdot|-7|-10$ **111.** $\dfrac{(-4)(-3)-(-2)(-1)}{-10+5}$ **112.** $\dfrac{4(12-18)}{-10\div(-2-3)}$

Find the average of each list of numbers.

113. $-18, 25, -30, 7, 0, -2$ **114.** $-45, -40, -30, -25$

Evaluate each expression for $x=-2$ and $y=1$.

115. $2x-y$ **116.** y^2+x^2 **117.** $\dfrac{3x}{6}$ **118.** $\dfrac{5y-x}{-y}$

119. x^2 **120.** $-x^2$ **121.** $7-x^2$ **122.** $100-x^3$

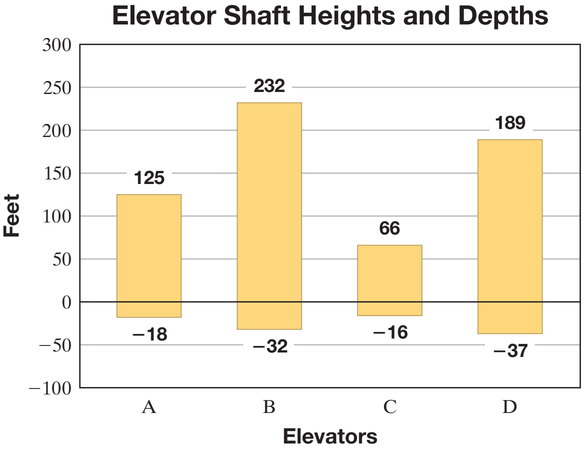
Mixed Review

Perform the indicated operations.

123. $(-4)^2$ **124.** -4^2 **125.** $-6+(-9)$ **126.** $-16-3$

127. $-4(-12)$ **128.** $\dfrac{84}{-4}$ **129.** $-76-(-97)$ **130.** $-9+4$

Elevator shafts in some buildings extend not only aboveground but in many cases belowground to accommodate basements, underground parking, and so on. The bar graph shows four such elevators and their shaft distance above- and belowground. Use the bar graph to answer Exercises 131 and 132.



131. Which elevator shaft extends the farthest belowground?

132. Which elevator shaft extends the highest aboveground?

133. The top of a mountain has an altitude of 12,923 feet. The bottom of a valley is 195 feet below sea level. Find the difference between these two elevations.

134. Wednesday's lowest temperature was −18°C. The cold weather continued and by Friday it had dropped another 9°C. What was the temperature on Friday?

Simplify.

135. $(3-7)^2\div(6-4)^3$ **136.** $(4+6)^2\div(2-7)^2$ **137.** $3(4+2)+(-6)-3^2$

138. $4(5-3)-(-2)+3^3$ **139.** $2-4\cdot3+\sqrt{25}$ **140.** $4-6\cdot5+\sqrt{1}$

141. $\dfrac{-|-14|-6}{7+2(-3)}$ **142.** $5(7-6)^3-4(2-3)^2+2^4$

Getting Ready for the Test LC

MULTIPLE CHOICE All the exercises below are **Multiple Choice**. Choose the correct letter. Not all letters may be used, and some may be used more than once.

For Exercises 1 through 4, identify each answer as

A. 2 **B.** -2 **C.** 0

1. -2; choose the opposite

2. -2; choose the absolute value

3. 2; choose the absolute value

4. 2; choose the opposite

For Exercises 5 through 10, identify each answer as

A. addition **B.** subtraction **C.** multiplication **D.** division

5. For 5(6), the operation is ______.

6. For $-12(+3)$, the operation is ______.

7. For $-12 + 3$, the operation is ______.

8. For $\frac{-12}{+3}$, the operation is ______.

9. For $4 + 6 \cdot 2$, which operation is performed first?

10. For $4 + 6 \div 2$, which operation is performed first?

For Exercises 11 through 24, fill in the blank with the correct choice.

A. 0 **B.** a positive number **C.** a negative number **D.** cannot be determined

11. (a negative number) · (a negative number) = _____.

12. (a negative number) + (a negative number) = _____.

13. (a negative number) ÷ (a negative number) = _____.

14. (a negative number) − (a negative number) = _____.

15. 0 · (a negative number) = _____.

16. 0 + (a negative number) = _____.

17. 0 − (a negative number) = _____.

18. 0 ÷ (a negative number) = _____.

19. (a positive number) · (a negative number) = _____.

20. (a negative number) · (a positive number) = _____.

21. (a positive number) + (a negative number) = _____.

22. (a positive number) − (a negative number) = _____.

23. (a negative number) − (a positive number) = _____.

24. (a positive number) − (a positive number) = _____.

Chapter 2 Test MyLab Math

For additional practice go to your study plan in MyLab Math.

Answers

1. ____

2. ____

3. ____

4. ____

5. ____

6. ____

7. ____

8. ____

9. ____

10. ____

11. ____

12. ____

13. ____

14. ____

15. ____

16. ____

17. ____

18. ____

Simplify each expression.

1. $-5 + 8$

2. $18 - 24$

3. $5 \cdot (-20)$

4. $(-16) \div (-4)$

5. $(-18) + (-12)$

6. $-7 - (-19)$

7. $(-5) \cdot (-13)$

8. $\dfrac{-25}{-5}$

9. $|-25| + (-13)$

10. $14 - |-20|$

11. $|5| \cdot |-10|$

12. $\dfrac{|-10|}{-|-5|}$

13. $(-8) + 9 \div (-3)$

14. $-7 + (-32) - 12 + 5$

15. $(-5)^3 - 24 \div (-3)$

16. $(5 - 9)^2 \cdot (8 - 2)^3$

17. $-(-7)^2 \div 7 \cdot (-4)$

18. $3 - (8 - 2)^3$

19. $-6 + (-15) \div (-3)$

20. $\frac{4}{2} - \frac{8^2}{16}$

21. $\frac{-3(-2) + 12}{-1(-4 - 5)}$

22. $\frac{|25 - 30|^2}{2(-6) + 7}$

23. $5(-8) - [6 - (2 - 4)] + (12 - 16)^2$

24. $-2^3 - 2^2$

Evaluate each expression for $x = 0$, $y = -3$, *and* $z = 2$.

25. $3x + y$

26. $|y| + |x| + |z|$

27. $\frac{3z}{2y}$

28. $2y^3$

29. $10 - y^2$

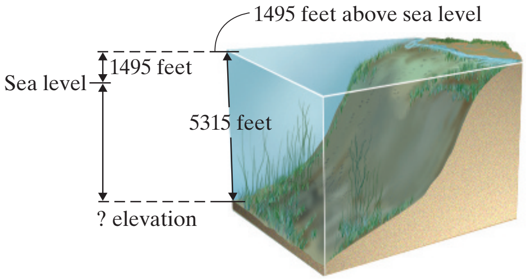
30. $7x + 3y - 4z$

31. Mary Dunstan, a diver, starts at sea level and then makes 4 successive descents of 22 feet. After the descents, what is her elevation?

32. Aaron Hawn has $129 in his checking account. He writes a check for $79, withdraws $40 from an ATM, and then deposits $35. Represent the new balance in his account by an integer.

33. Mt. Washington in New Hampshire has an elevation of 6288 feet above sea level. The Romanche Gap in the Atlantic Ocean has an elevation of 25,354 feet below sea level. Represent the difference in elevation between these two points by an integer. (*Source:* National Geographic Society and Defense Mapping Agency)

34. Lake Baykal in Siberian Russia is the deepest lake in the world, with a maximum depth of 5315 feet. The elevation of the lake's surface is 1495 feet above sea level. What is the elevation (with respect to sea level) of the deepest point in the lake? (*Source:* U.S. Geological Survey)



35. Find the average of −12, −13, 0, 9.

36. Translate the following phrases into mathematical expressions. Use *x* to represent "a number."

a. The product of a number and 17

b. Twice a number subtracted from 20

19. ____________

20. ____________

21. ____________

22. ____________

23. ____________

24. ____________

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. a. ____________

b. ____________

Chapters 1–2 Cumulative Review

Answers

1.
2.
3.
4.
5.
6.
7. a.
b.
c.
8. a.
b.
c.
9.
10.
11.
12.
13.
14.
15.
16.
17.
18.
19. a.
b.
c.
20. a.
b.
c.
21.
22.
23. a.
b.
c.
24. a.
b.
c.

Find the place value of the digit 3 in each whole number.

1. 396,418

2. 4308

3. 93,192

4. 693,298

5. 534,275,866

6. 267,301,818

7. Insert $<$ or $>$ to make a true statement.
- **a.** $-7 \quad 7$
- **b.** $0 \quad -4$
- **c.** $-9 \quad -11$

8. Insert $<$ or $>$ to make a true statement.
- **a.** $12 \quad 4$
- **b.** $-13 \quad 31$
- **c.** $-79 \quad -82$

9. Add: $13 + 2 + 7 + 8 + 9$

10. Add: $11 + 3 + 9 + 16$

11. Subtract: $7826 - 505$ Check by adding.

12. Subtract: $3285 - 272$ Check by adding.

13. The radius of Jupiter is 43,441 miles. The radius of Saturn is 7257 miles less than the radius of Jupiter. Find the radius of Saturn. (*Source:* National Space Science Data Center)

14. C. J. Dufour wants to buy a digital camera. She has \$762 in her savings account. If the camera costs \$237, how much money will she have in her account after buying the camera?

15. Round 568 to the nearest ten.

16. Round 568 to the nearest hundred.

17. Round each number to the nearest hundred to find an estimated difference.

$$\begin{array}{r} 4725 \\ -2879 \\ \hline \end{array}$$

18. Round each number to the nearest thousand to find an estimated difference.

$$\begin{array}{r} 8394 \\ -2913 \\ \hline \end{array}$$

19. Rewrite each using the distributive property.
- **a.** $3(4 + 5)$
- **b.** $10(6 + 8)$
- **c.** $2(7 + 3)$

20. Rewrite each using the distributive property.
- **a.** $5(2 + 12)$
- **b.** $9(3 + 6)$
- **c.** $4(8 + 1)$

21. Multiply: 631×125

22. Multiply: 299×104

23. Find each quotient. Check by multiplying.
- **a.** $42 \div 7$
- **b.** $\frac{64}{8}$
- **c.** $3\overline{)21}$

24. Find each quotient. Check by multiplying.
- **a.** $\frac{35}{5}$
- **b.** $64 \div 8$
- **c.** $4\overline{)48}$

25. Divide: $3705 \div 5$
Check by multiplying.

26. Divide: $3648 \div 8$
Check by multiplying.

27. As part of a promotion, an executive receives 238 cards, each good for one free song download. If she wants to share them evenly with 19 friends, how many download cards will each friend receive? How many will be left over?

28. Mrs. Mallory's first-grade class is going to the zoo. She pays a total of \$324 for 36 admission tickets. How much does each ticket cost?

Evaluate.

29. 9^2

30. 5^3

31. 6^1

32. 4^1

33. $5 \cdot 6^2$

34. $2^3 \cdot 7$

35. Simplify: $\dfrac{7 - 2 \cdot 3 + 3^2}{5(2 - 1)}$

36. Simplify: $\dfrac{6^2 + 4 \cdot 4 + 2^3}{37 - 5^2}$

37. Evaluate $x + 7$ if x is 8.

38. Evaluate $5 + x$ if x is 9.

39. Simplify:
a. $|-2|$
b. $|8|$
c. $|0|$

40. Simplify:
a. $|4|$
b. $|-7|$

41. Add: $-14 + 35$

42. Add: $8 + (-3)$

43. Evaluate $2a - b$ for $a = 8$ and $b = -6$.

44. Evaluate $x - y$ for $x = -2$ and $y = -7$.

45. Multiply: $-7 \cdot 3$

46. Multiply: $5(-2)$

47. Multiply: $0 \cdot (-4)$

48. Multiply: $-6 \cdot 9$

49. Simplify: $3(4 - 7) + (-2) - \sqrt{25}$

50. Simplify: $4 - 8(7 - 3) - (-1)$

25. ______
26. ______
27. ______
28. ______
29. ______
30. ______
31. ______
32. ______
33. ______
34. ______
35. ______
36. ______
37. ______
38. ______
39. a. ______
b. ______
c. ______
40. a. ______
b. ______
41. ______
42. ______
43. ______
44. ______
45. ______
46. ______
47. ______
48. ______
49. ______
50. ______

3

Fractions and Mixed Numbers

Fractions are numbers, and like whole numbers and integers, they can be added, subtracted, multiplied, and divided. Fractions are very useful and appear frequently in everyday language, in common phrases such as “half an hour,” “quarter of a pound,” and “third of a cup.” This chapter reviews the concepts of fractions and mixed numbers and demonstrates how to add, subtract, multiply, and divide these numbers.

Sections

Check Your Progress

Pinnacles National Park in California was signed into law on January 10, 2013, as the United States’ 59th national park. (The former Pinnacles National Monument was established in 1908 by U.S. President Theodore Roosevelt.)

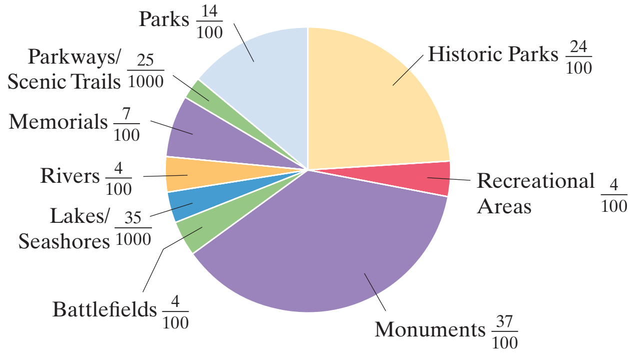
The National Park Service Turned 100

The National Park Service is well known for its stewardship of national parks and protecting natural resources for future generations. Many of us recognize the National Park ranger by the “Smokey Bear” hat, but the National Park Service also employs archeologists, architects, curators, historians, and other professionals to help manage and preserve the vastly different properties under its oversight.

The following graph is called a circle graph or pie chart. Each sector (shaped like a piece of a pie) shows the fraction of the areas managed by the National Park System by type of national designation.

In Sections 3.2, Exercises 99–102, and Section 3.5, Exercises 87–90, we show this same circle graph but in 3-D design. We simplify some of the fractions in it and also study sector size versus fraction value.

Areas Maintained by the National Park Service



Source: National Park Service

3.1 Introduction to Fractions and Mixed Numbers

Objectives

A Identify the Numerator and the Denominator of a Fraction.

B Write a Fraction to Represent Parts of Figures or Real-Life Data.

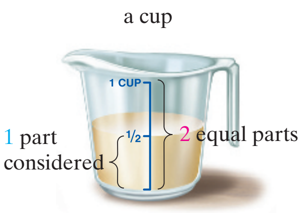
C Identify Proper Fractions, Improper Fractions, and Mixed Numbers.

D Graph Fractions on a Number Line.

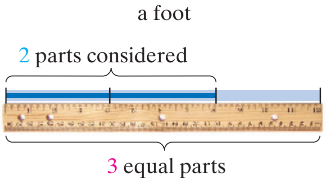
E Review Division Properties of 0 and 1.

Objective A Identifying Numerators and Denominators

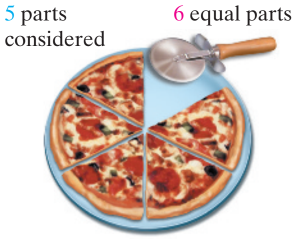
Whole numbers are used to count whole things or units, such as cars, horses, dollars, and people. To refer to a part of a whole, fractions can be used. Here are some examples of **fractions.** Study these examples for a moment.



$\frac{1}{2}$ of a cup



$\frac{2}{3}$ of a foot



$\frac{5}{6}$ of a pizza

In a fraction, the top number is called the **numerator** and the bottom number is called the **denominator.** The bar between the numbers is called the **fraction bar.**

Names	Fraction	Meaning
numerator ⟶	5	⟵ number of parts being considered
denominator ⟶	6	⟵ number of equal parts in the whole

Examples Identify the numerator and the denominator of each fraction.

1. $\dfrac{3}{7}\begin{array}{l}\leftarrow \text{numerator}\\ \leftarrow \text{denominator}\end{array}$

2. $\dfrac{13}{5}\begin{array}{l}\leftarrow \text{numerator}\\ \leftarrow \text{denominator}\end{array}$

■ Work Practice 1–2

Practice 1–2

Identify the numerator and the denominator of each fraction.

1. $\dfrac{11}{2}$

2. $\dfrac{10}{17}$

Helpful Hint

$\dfrac{3}{7}$ ← Remember that the bar in a fraction means division. Since division by 0 is undefined, a fraction with a denominator of 0 is undefined. For example, $\dfrac{3}{0}$ is undefined.

Objective B Writing Fractions to Represent Parts of Figures or Real-Life Data

One way to become familiar with the concept of fractions is to visualize fractions with shaded figures. We can then write a fraction to represent the shaded area of the figure (or diagram).

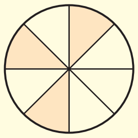
Answers

1. numerator $= 11$, denominator $= 2$
2. numerator $= 10$, denominator $= 17$

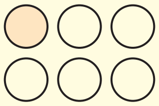
Practice 3–4

Write a fraction to represent the shaded part of each figure.

3.

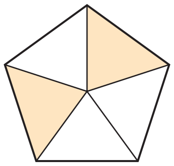


4.



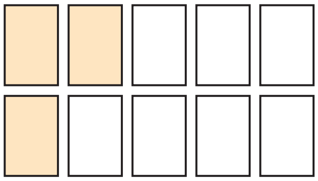
Examples Write a fraction to represent the shaded part of each figure.

3. In this figure, 2 of the 5 equal parts are shaded. Thus, the fraction is $\frac{2}{5}$.



$\begin{array}{l} 2 \leftarrow \text{number of parts shaded} \\ \overline{5} \leftarrow \text{number of equal parts} \end{array}$

4. In this figure, 3 of the 10 rectangles are shaded. Thus, the fraction is $\frac{3}{10}$.



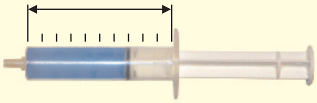
$\begin{array}{l} 3 \leftarrow \text{number of parts shaded} \\ \overline{10} \leftarrow \text{number of equal parts} \end{array}$

Work Practice 3–4

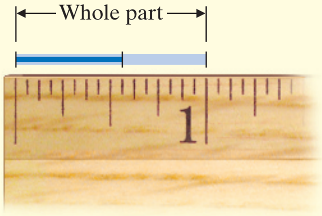
Practice 5–6

Write a fraction to represent the part of the whole shown.

5. Just consider this part of the syringe



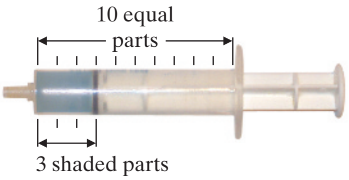
6.



Examples Write a fraction to represent the shaded part of the diagram.

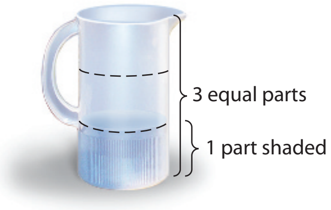
5.

The fraction is $\frac{3}{10}$.



6.

The fraction is $\frac{1}{3}$.



Work Practice 5–6

Practice 7

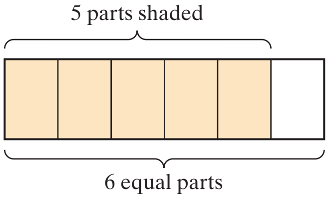
Draw and shade a part of a figure to represent the fraction.

$\frac{2}{3}$ of a figure

Examples Draw a figure and then shade a part of it to represent each fraction.

7. $\frac{5}{6}$ of a figure

We will use a geometric figure such as a rectangle. Since the denominator is 6, we divide it into 6 equal parts. Then we shade 5 of the equal parts.



$\frac{5}{6}$ of the rectangle is shaded

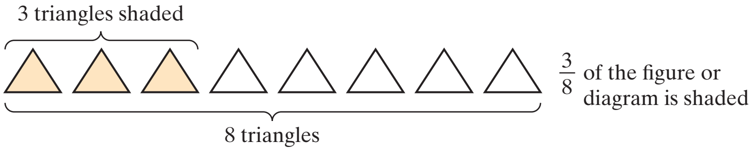
Answers

3. $\frac{3}{8}$ **4.** $\frac{1}{6}$ **5.** $\frac{7}{10}$ **6.** $\frac{9}{16}$

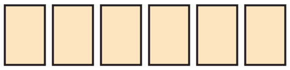
7. answers may vary; for example,

8. $\frac{3}{8}$ of a figure

If you'd like, our figure can consist of 8 triangles of the same size. We will shade 3 of the triangles.

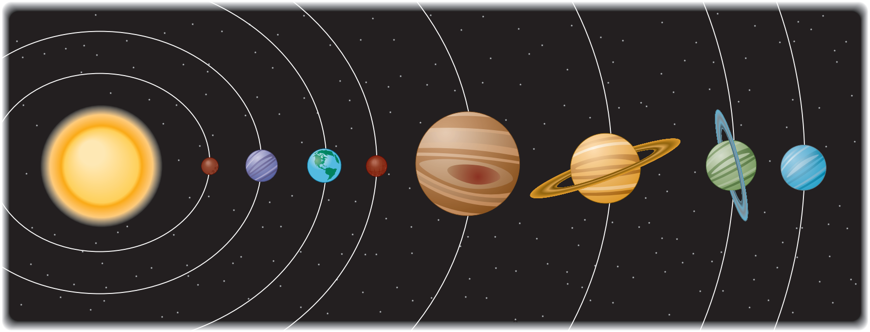


Work Practice 7–8

✓Concept Check If  represents $\frac{6}{7}$ of a whole diagram, sketch the whole diagram.

Example 9 Writing Fractions from Real-Life Data

Of the eight planets in our solar system (Pluto is now a dwarf planet), three are closer to the sun than Mars is. What fraction of the planets are closer to the sun than Mars is?



Solution: The fraction of planets closer to the sun than Mars is:

$\frac{3}{8}$ ← number of planets closer
← number of planets in our solar system

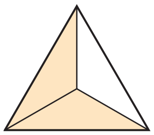
Thus, $\frac{3}{8}$ of the planets in our solar system are closer to the sun than Mars is.

Work Practice 9

Objective C Identifying Proper Fractions, Improper Fractions, and Mixed Numbers

The definitions and statements below apply to positive fractions.

A **proper fraction** is a fraction whose numerator is less than its denominator. Proper fractions are less than 1. For example, the shaded portion of the triangle is represented by $\frac{2}{3}$.



$\frac{2}{3}$

An **improper fraction** is a fraction whose numerator is greater than or equal to its denominator. Improper fractions are greater than or equal to 1.

Practice 8

Draw and shade a part of a figure to represent the fraction.

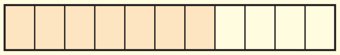
$\frac{7}{11}$ of a figure

Practice 9

Of the eight planets in our solar system, five are farther from the sun than Earth is. What fraction of the planets are farther from the sun than Earth is?

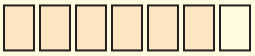
Answers

8. answers may vary; for example,

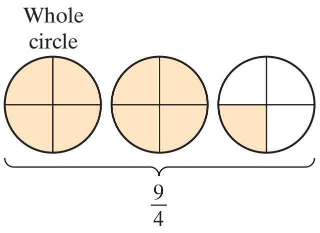


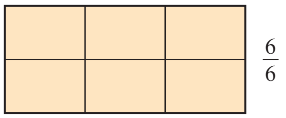
9. $\frac{5}{8}$

✓Concept Check Answer

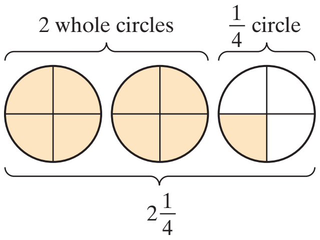


The shaded part of the group of circles below is $\frac{9}{4}$. The shaded part of the rectangle is $\frac{6}{6}$. Recall that $\frac{6}{6}$ simplifies to 1 and notice that the entire rectangle (1 whole figure) is shaded below.





A **mixed number** contains a whole number and a fraction. Mixed numbers are greater than 1. Above, we wrote the shaded part of the group of circles as the improper fraction $\frac{9}{4}$. Now let's write the shaded part of the same group of circles as a mixed number (see below). The shaded part of the group of circles' area is $2\frac{1}{4}$. Read this as "two and one-fourth."



Example 10 Identify each number as a proper fraction, improper fraction, or mixed number.

a. $\frac{6}{7}$ is a proper fraction.

b. $\frac{13}{12}$ is an improper fraction.

c. $\frac{2}{2}$ is an improper fraction.

d. $\frac{99}{101}$ is a proper fraction.

e. $1\frac{7}{8}$ is a mixed number.

f. $\frac{93}{74}$ is an improper fraction.

Work Practice 10

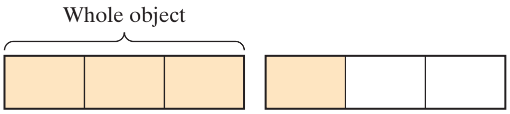
Helpful Hint

The mixed number $2\frac{1}{4}$ represents $2 + \frac{1}{4}$.

The mixed number $-3\frac{1}{5}$ represents $-\left(3 + \frac{1}{5}\right)$ or $-3 - \frac{1}{5}$. We review this later in this chapter.

Examples Represent the shaded part of each figure group as both an improper fraction and a mixed number.

11.



improper fraction: $\frac{4}{3}$

mixed number: $1\frac{1}{3}$

Practice 10

Identify each number as a proper fraction, improper fraction, or mixed number.

a. $\frac{5}{8}$

b. $\frac{7}{7}$

c. $\frac{14}{13}$

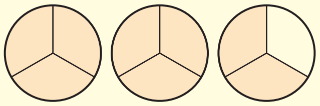
d. $\frac{13}{14}$

e. $5\frac{1}{4}$

f. $\frac{100}{49}$

Practice 11

Represent the shaded part of the figure group as both an improper fraction and a mixed number.

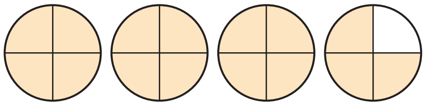


Answers

10. a. proper fraction **b.** improper fraction **c.** improper fraction **d.** proper fraction **e.** mixed number **f.** improper fraction

11. $\frac{8}{3}, 2\frac{2}{3}$

12.



improper fraction: $\frac{15}{4}$

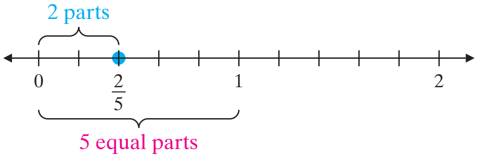
mixed number: $3\frac{3}{4}$

Work Practice 11–12

✓Concept Check If you were to round $3\frac{3}{4}$, shown in Example 12 above, to the nearest whole number, would you choose 3 or 4? Why?

Objective D Graphing Fractions on a Number Line

Another way to visualize fractions is to graph them on a number line. To do this, think of 1 unit on the number line as a whole. To graph $\frac{2}{5}$, for example, divide the distance from 0 to 1 into 5 equal parts. Then start at 0 and count 2 parts to the right.



Notice that the graph of $\frac{2}{5}$ lies between 0 and 1. This means

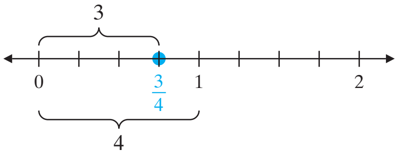
$$0 < \frac{2}{5}\left(\text{or } \frac{2}{5} > 0\right) \text{ and also } \frac{2}{5} < 1$$

Example 13 Graph each fraction on a number line.

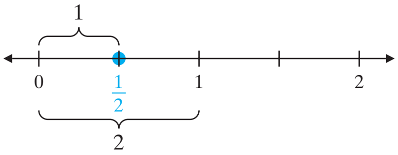
a. $\frac{3}{4}$ **b.** $\frac{1}{2}$ **c.** $\frac{3}{6}$ **d.** $\frac{9}{5}$ **e.** $\frac{6}{6}$

Solution:

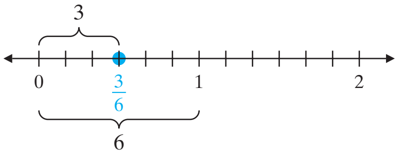
a. To graph $\frac{3}{4}$, divide the distance from 0 to 1 into 4 parts. Then start at 0 and count over 3 parts.



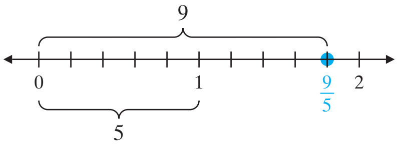
b.



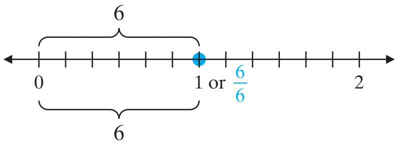
c.



d.



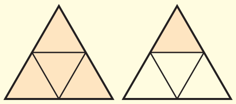
e.



Work Practice 13

Practice 12

Represent the shaded part of the figure group as both an improper fraction and a mixed number.



Practice 13

Graph each fraction on a number line.

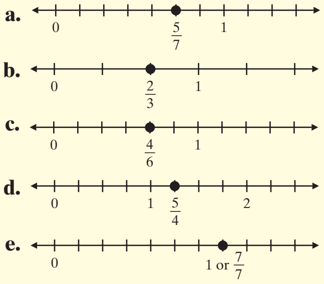
a. $\frac{5}{7}$ **b.** $\frac{2}{3}$ **c.** $\frac{4}{6}$

d. $\frac{5}{4}$ **e.** $\frac{7}{7}$

Answers

12. $\frac{5}{4}, 1\frac{1}{4}$

13.



✓Concept Check Answer

4; answers may vary

The statements that follow apply to positive fractions.

The fractions in Example 13, parts a, b, and c, are proper fractions. Notice that the value of each is less than 1. This is always true for proper fractions since the numerator of a proper fraction is less than the denominator.

The fractions in Example 13, parts d and e, are improper fractions. Notice that improper fractions are greater than or equal to 1. This is always true since the numerator of an improper fraction is greater than or equal to the denominator. *Note:* We will graph mixed numbers at the end of this chapter.

Objective E Reviewing Division Properties of 0 and 1

Before we continue further, don't forget from Section 1.7 that the fraction bar indicates division. Let's review some division properties of 1 and 0.

$\frac{9}{9} = 1$ because $1 \cdot 9 = 9$ $\qquad \frac{-11}{1} = -11$ because $-11 \cdot 1 = -11$

$\frac{0}{6} = 0$ because $0 \cdot 6 = 0$ $\qquad \frac{6}{0}$ *is undefined* because there is no number that when multiplied by 0 gives 6.

In general, we can say the following.

Helpful Hint Thus, any integer can be written as a fraction. For example,

$$5 = \frac{5}{1} \text{ and } -3 = \frac{-3}{1}$$

Let n be any integer except 0.

$\frac{n}{n} = 1 \qquad \frac{0}{n} = 0$

$\frac{n}{1} = n \qquad \frac{n}{0}$ is undefined.

Examples Simplify.

14. $\frac{15}{15} = 1$ **15.** $\frac{-2}{-2} = 1$ **16.** $\frac{0}{-5} = 0$

17. $\frac{-9}{1} = -9$ **18.** $\frac{41}{1} = 41$ **19.** $\frac{19}{0}$ is undefined

Work Practice 14–19

Practice 14–19

Simplify.

14. $\frac{9}{9}$ **15.** $\frac{-6}{-6}$

16. $\frac{0}{-1}$ **17.** $\frac{4}{1}$

18. $\frac{-13}{0}$ **19.** $\frac{-13}{1}$

Notice from Example 17 that we can have negative fractions. In fact,

$$\frac{-5}{1} = -5, \quad \frac{5}{-1} = -5, \quad \text{and } -\frac{5}{1} = -5$$

Because all of the fractions equal -5, we have

$$\frac{-5}{1} = \frac{5}{-1} = -\frac{5}{1}$$

This means that the negative sign in a fraction can be written in the numerator, in the denominator, or in front of the fraction. Remember this as we work with negative fractions.

Helpful Hint Remember, for example, that

$$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$$

Answers

14. 1 **15.** 1 **16.** 0 **17.** 4
18. undefined **19.** -13

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

improper	fraction	proper	is undefined	mixed number	$= 0$
≥ 1	denominator	$= 1$	< 1	numerator	

1. The number $\frac{17}{31}$ is called a(n) __________. The number 31 is called its __________ and 17 is called its __________.

2. If we simplify each fraction, $\frac{-9}{-9}$ __________, $\frac{0}{-4}$ __________, and we say $\frac{-4}{0}$ __________.

3. The fraction $\frac{8}{3}$ is called a(n) __________ fraction, the fraction $\frac{3}{8}$ is called a(n) __________ fraction, and $10\frac{3}{8}$ is called a(n) __________.

4. The value of an improper fraction is always __________, and the value of a proper fraction is always __________.

Martin-Gay Interactive Videos

See Video 3.1

Watch the section lecture video and answer the following questions.

Objective A **5.** Complete this statement based on Example 1: When the numerator is greater than or __________ to the denominator, you have a(n) __________ fraction.

Objective B **6.** In Example 4, there are two shapes in the diagram, so why do the representative fractions have a denominator of 3?

Objective C **7.** Based on the lecture during Example 4, fill in the blanks: A mixed number has a(n) __________ part and a(n) __________ part.

Objective D **8.** From Examples 6 and 7, when graphing a positive fraction on a number line, how does the denominator help? That is, what does the denominator tell you?

Objective E **9.** From Example 10, what can you conclude about any fraction where the numerator and denominator are the same nonzero number?

3.1 Exercise Set MyLab Math

Objectives A C *Identify the numerator and the denominator of each fraction and identify each fraction as proper or improper. See Examples 1, 2, and 10.*

1. $\frac{1}{2}$

2. $\frac{1}{4}$

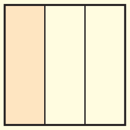
3. $\frac{10}{3}$

4. $\frac{53}{21}$

5. $\frac{15}{15}$

6. $\frac{26}{26}$

Objectives B C *Write a proper or improper fraction to represent the shaded part of each diagram. If an improper fraction is appropriate, write the shaded part of the diagram as (a) an improper fraction and (b) a mixed number. (Note to students: If you are familiar with simplifying fractions, note that none of the fractions in this section are simplified.) See Examples 3 through 6, 11, and 12.*

7.


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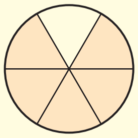

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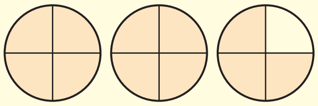

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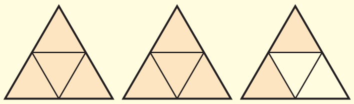

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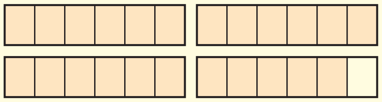

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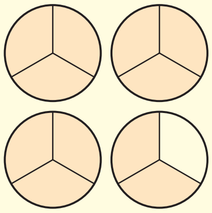
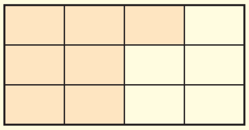


13.


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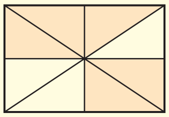

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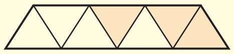

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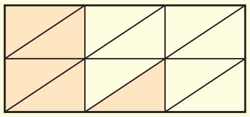

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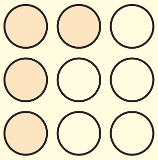

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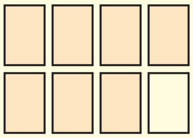

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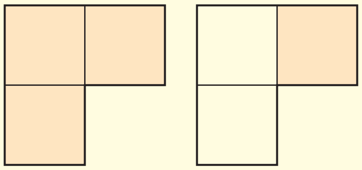

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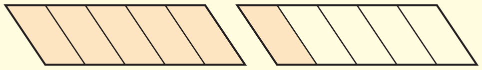

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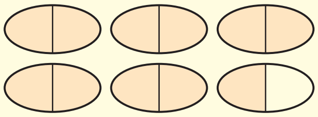

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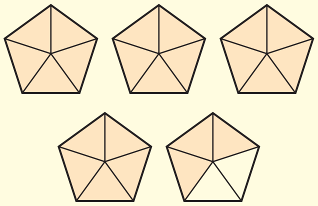

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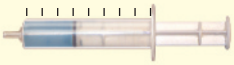

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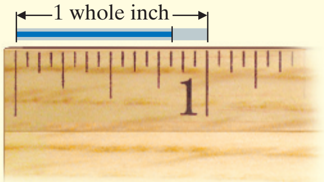


Objective B *Draw and shade a part of a diagram to represent each fraction. See Examples 7 and 8.*

27. $\frac{1}{5}$ of a diagram

28. $\frac{1}{16}$ of a diagram

29. $\frac{6}{7}$ of a diagram

30. $\frac{7}{9}$ of a diagram

31. $\frac{4}{4}$ of a diagram

32. $\frac{6}{6}$ of a diagram

Write each fraction. (Note to students: In case you are familiar with simplifying fractions, note that none of the fractions in this section are simplified.) See Example 9.

33. Of the 131 students at a small private school, 42 are freshmen. What fraction of the students are freshmen?

34. Of the 63 employees at a new biomedical engineering firm, 22 are men. What fraction of the employees are men?

35. Use Exercise 33 to answer **a** and **b.**

a. How many students are *not* freshmen?
b. What fraction of the students are *not* freshmen?

36. Use Exercise 34 to answer **a** and **b.**

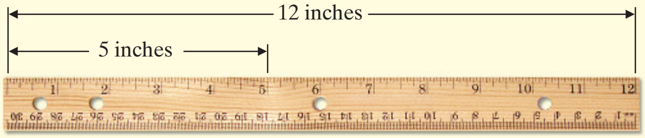
a. How many of the employees are women?
b. What fraction of the employees are women?

37. As of 2018, the United States has had 44 different presidents. A total of seven U.S. presidents were born in the state of Ohio, second only to the state of Virginia in producing U.S. presidents. What fraction of U.S. presidents were born in Ohio? (*Source: World Almanac,* 2014)

38. Of the eight planets in our solar system, four have days that are longer than the 24-hour Earth day. What fraction of the planets have longer days than Earth has? (*Source:* National Space Science Data Center)

39. There were ten certified Atlantic hurricanes in 2017. Three of these hurricanes—Harvey, Maria, and Irma—were category 4 or greater. What fraction of the 2017 hurricanes were category 4 or greater?

40. There are 12 inches in a foot. What fractional part of a foot does 5 inches represent?



41. There are 31 days in the month of March. What fraction of the month does 11 days represent?

Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

42. There are 60 minutes in an hour. What fraction of an hour does 37 minutes represent?

43. In a prealgebra class containing 31 students, there are 18 freshmen, 10 sophomores, and 3 juniors. What fraction of the class is sophomores?

44. In a sports team with 20 children, there are 9 boys and 11 girls. What fraction of the team is boys?

45. Thirty-three out of the fifty states in the United States contain federal Indian reservations.

- **a.** What fraction of the states contain federal Indian reservations?
- **b.** How many states do not contain federal Indian reservations?
- **c.** What fraction of the states do not contain federal Indian reservations? (*Source:* Tiller Research, Inc., Albuquerque, NM)

46. Consumer fireworks are legal in 46 out of the 50 states in the United States.

- **a.** In what fraction of the states are consumer fireworks legal?
- **b.** In how many states are consumer fireworks illegal?
- **c.** In what fraction of the states are consumer fireworks illegal? (*Source:* United States Fireworks Safety Council)

47. A bag contains 50 red or blue marbles. If 21 marbles are blue,

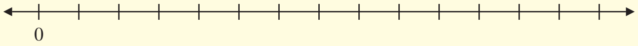
- **a.** What fraction of the marbles are blue?
- **b.** How many marbles are red?
- **c.** What fraction of the marbles are red?

48. An art dealer is taking inventory. His shop contains a total of 37 pieces, which are all sculptures, watercolor paintings, or oil paintings. If there are 15 watercolor paintings and 17 oil paintings, answer each question.

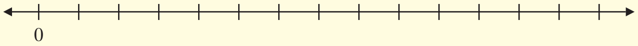
- **a.** What fraction of the inventory is watercolor paintings?
- **b.** What fraction of the inventory is oil paintings?
- **c.** How many sculptures are there?
- **d.** What fraction of the inventory is sculptures?

Objective D *Graph each fraction on a number line. See Example 13.*

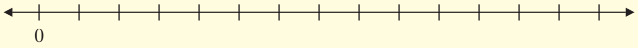
49. $\frac{1}{4}$



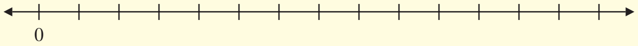
50. $\frac{1}{3}$



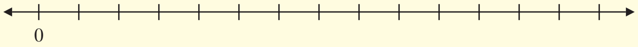
51. $\frac{4}{7}$



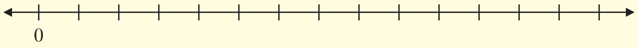
52. $\frac{5}{6}$



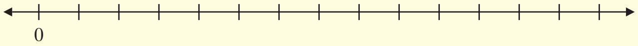
53. $\frac{8}{5}$



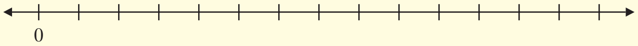
54. $\frac{9}{8}$



55. $\frac{7}{3}$



56. $\frac{13}{7}$



Objective **E** *Simplify by dividing. See Examples 14 through 19.*

57. $\frac{12}{12}$

58. $\frac{-3}{-3}$

59. $\frac{-5}{1}$

60. $\frac{-20}{1}$

61. $\frac{0}{-2}$

62. $\frac{0}{-8}$

63. $\frac{-8}{-8}$

64. $\frac{-14}{-14}$

65. $\frac{-9}{0}$

66. $\frac{-7}{0}$

67. $\frac{3}{1}$

68. $\frac{5}{1}$

Review

Simplify. See Section 1.9.

69. 3^2

70. 4^3

71. 5^3

72. 3^4

Write each using exponents.

73. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

74. $5 \cdot 5 \cdot 5 \cdot 5$

75. $2 \cdot 2 \cdot 2 \cdot 3$

76. $4 \cdot 4 \cdot 10 \cdot 10 \cdot 10$

Concept Extensions

For Exercises 77 through 80, write each fraction in two other equivalent ways by inserting the negative sign in different places.

77. $-\frac{11}{2} = \qquad =$

78. $-\frac{21}{4} = \qquad =$

79. $\frac{-13}{15} = \qquad =$

80. $\frac{45}{-57} = \qquad =$

81. In your own words, explain why $\frac{0}{10} = 0$ and $\frac{10}{0}$ is undefined.

82. In your own words, explain why $\frac{14}{-2} = \frac{-14}{2} = -\frac{14}{2}$.

Solve. See the Concept Checks in this section.

83. If ◯◯◯◯ represents $\frac{4}{9}$ of a whole diagram, sketch the whole diagram.

84. If △△ represents $\frac{1}{3}$ of a whole diagram, sketch the whole diagram.

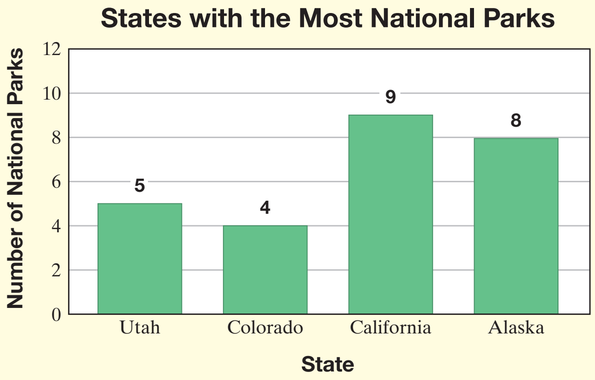
85. Round the mixed number $7\frac{1}{8}$ to the nearest whole number.

86. Round the mixed number $5\frac{11}{12}$ to the nearest whole number.

87. IKEA Group operates 355 stores in five different regions worldwide, as shown on the bar graph. What fraction of IKEA stores are located in the North American region?

Source: IKEA Group

88. There are currently 60 national parks in the United States. The bar graph below shows the four states with the greatest number of national parks. What fraction of the national parks found in these four states are located in California? (*Source:* National Park Service)



89. Habitat for Humanity is a nonprofit organization that helps provide affordable housing to families in need. Habitat for Humanity does its work of building and renovating houses through 1500 local affiliates in the United States and 80 international affiliates. What fraction of the total Habitat for Humanity affiliates are located in the United States? (*Hint:* First find the total number of affiliates.) (*Source:* Habitat for Humanity International)

90. The United States Marine Corps (USMC) has five principal training centers in California, three in North Carolina, two in South Carolina, one in Arizona, one in Hawaii, and one in Virginia. What fraction of the total USMC principal training centers are located in California? (*Hint:* First find the total number of USMC training centers.) (*Source:* U.S. Department of Defense)

3.2 Factors and Simplest Form

Objectives

A Write a Number as a Product of Prime Numbers.

B Write a Fraction in Simplest Form.

C Determine Whether Two Fractions Are Equivalent.

D Solve Problems by Writing Fractions in Simplest Form.

Objective A Writing a Number as a Product of Prime Numbers

To perform operations on fractions, it is necessary to be able to factor a number. Factoring a number means writing a number as a product. We first practice writing a number as a product of prime numbers.

Recall from Section 1.6 that since $12 = 2 \cdot 6$, the numbers 2 and 6 are called *factors* of 12. A **factor** is any number that divides a number evenly (with a remainder of 0).

Of all the ways to factor a number, one special way is called the **prime factorization.** To help us write prime factorizations, we first review prime and composite numbers.

Prime Numbers

A **prime number** is a natural number that has exactly two different factors, 1 and itself.

The first several prime numbers are

2, 3, 5, 7, 11, 13, 17

It would be helpful to memorize these.

If a natural number other than 1 is not a prime number, it is called a **composite number.**

Composite Numbers

A **composite number** is any natural number, other than 1, that is not prime.

Helpful Hint

The natural number 1 is neither prime nor composite.

Now we are ready to define, and then find, **prime factorizations** of numbers.

Prime Factorization

The **prime factorization** of a number is the factorization in which all the factors are prime numbers.

Earlier, we wrote $12 = 2 \cdot 6$. Although 2 and 6 are factors of 12, the product $2 \cdot 6$ is *not* the prime factorization of 12 because 6 is *not* a prime number.

The prime factorization of 12 is $2 \cdot 2 \cdot 3$ because

$$12 = \underbrace{2 \cdot 2 \cdot 3}_{\substack{\text{This product is 12 and each} \\ \text{number is a prime number.}}}$$

Helpful Hint

Don't forget that multiplication is commutative, so the order of the factors is not important. We can write the factorization $2 \cdot 2 \cdot 3$ as $2 \cdot 3 \cdot 2$ or $3 \cdot 2 \cdot 2$. Any of these is called the *prime factorization* of 12.

Every whole number greater than 1 has exactly one prime factorization.

One method for finding the prime factorization of a number is by using a factor tree, as shown in the next example.

Practice 1

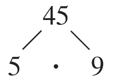
Use a factor tree to find the prime factorization of each number.

a. 30 **b.** 56 **c.** 72

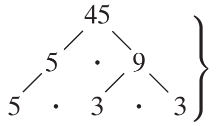
Example 1

Write the prime factorization of 45.

Solution: We can begin by writing 45 as the product of two numbers, say, 5 and 9.



The number 5 is prime but 9 is not, so we write 9 as $3 \cdot 3$.



A factor tree

Each factor is now a prime number, so the prime factorization of 45 is $3 \cdot 3 \cdot 5$ or $3^2 \cdot 5$.

Work Practice 1

✓Concept Check True or false? Two different numbers can have exactly the same prime factorization. Explain your answer.

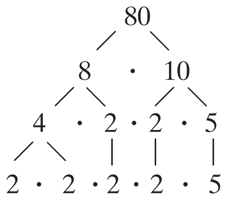
Practice 2

Write the prime factorization of 60.

Example 2

Write the prime factorization of 80.

Solution: Write 80 as a product of two numbers. Continue this process until all factors are prime.



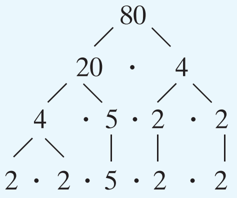
All factors are now prime, so the prime factorization of 80 is

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \quad \text{or} \quad 2^4 \cdot 5.$$

Work Practice 2

Helpful Hint

It makes no difference which factors you start with. The prime factorization of a number will be the same.



Same factors as in Example 2

There are a few quick **divisibility tests** to determine whether a number is divisible by the primes 2, 3, or 5. (A number is divisible by 2, for example, if 2 divides it evenly so that the remainder is 0.)

Answers

1. a. $2 \cdot 3 \cdot 5$ b. $2^3 \cdot 7$ c. $2^3 \cdot 3^2$
2. $2^2 \cdot 3 \cdot 5$

✓Concept Check Answer

false; answers may vary

Divisibility Tests

A whole number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.

  ↓

  132 is divisible by 2 since the last digit is a 2.
- 3 if the sum of the digits is divisible by 3.

  144 is divisible by 3 since $1 + 4 + 4 = 9$, which is divisible by 3.
- 5 if the last digit is 0 or 5.

  ↓

  1115 is divisible by 5 since the last digit is a 5.

Helpful Hint

Here are a few other divisibility tests you may want to use. A whole number is divisible by:

- 4 if its last two digits are divisible by 4.

  1712 is divisible by 4.
- 6 if it's divisible by 2 and 3.

  9858 is divisible by 6.
- 9 if the sum of its digits is divisible by 9.

  5238 is divisible by 9 since $5 + 2 + 3 + 8 = 18$, which is divisible by 9.

When finding the prime factorization of larger numbers, you may want to use the procedure shown in Example 3.

Example 3 Write the prime factorization of 252.

Solution: For this method, we divide prime numbers into the given number. Since the ones digit of 252 is 2, we know that 252 is divisible by 2.

$$\begin{array}{r} 126 \\ 2\overline{)252} \end{array}$$

126 is divisible by 2 also.

$$\begin{array}{r} 63 \\ 2\overline{)126} \\ 2\overline{)252} \end{array}$$

63 is not divisible by 2 but is divisible by 3. Divide 63 by 3 and continue in this same manner until the quotient is a prime number.

$$\begin{array}{r} 7 \\ 3\overline{)\ 21} \\ 3\overline{)\ 63} \\ 2\overline{)126} \\ 2\overline{)252} \end{array}$$

Helpful Hint

The order of choosing prime numbers does not matter. For consistency, we use the order 2, 3, 5, 7, ...

The prime factorization of 252 is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$ or $2^2 \cdot 3^2 \cdot 7$.

Work Practice 3

Practice 3

Write the prime factorization of 297.

Answer

3. $3^3 \cdot 11$

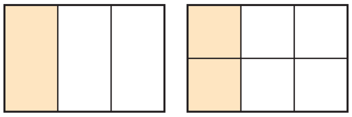
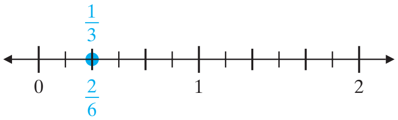
In this text, we will write the factorization of a number from the smallest factor to the largest factor.

✓Concept Check True or false? The prime factorization of 117 is $9 \cdot 13$. Explain your reasoning.

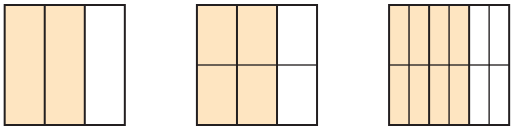
Objective B Writing Fractions in Simplest Form

Fractions that represent the same portion of a whole or the same point on a number line are called **equivalent fractions.** Study the table below to see two ways to visualize equivalent fractions.

Equivalent Fractions

Figures	Number Line
When we shade $\frac{1}{3}$ and $\frac{2}{6}$ on the same-sized figures,  $\frac{1}{3}$ $\frac{2}{6}$ notice that both $\frac{1}{3}$ and $\frac{2}{6}$ represent the same portion of a whole. These fractions are called **equivalent fractions,** and we write $\frac{1}{3} = \frac{2}{6}$.	When we graph $\frac{1}{3}$ and $\frac{2}{6}$ on a number line,  notice that both $\frac{1}{3}$ and $\frac{2}{6}$ correspond to the same point. These fractions are called **equivalent fractions,** and we write $\frac{1}{3} = \frac{2}{6}$.

Thus, $\frac{1}{3} = \frac{2}{6}$ and $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent.



$\frac{2}{3}$ $\frac{4}{6}$ $\frac{8}{12}$

For example, $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ all represent the same shaded portion of the rectangle's area, so they are equivalent fractions. To show that these fractions are equivalent, we place an equal sign between them. In other words,

$$\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$$

There are many equivalent forms of a fraction. A special equivalent form of a fraction is called **simplest form.**

Simplest Form of a Fraction

A fraction is written in **simplest form** or **lowest terms** when the numerator and the denominator have no common factors other than 1.

For example, the fraction $\frac{2}{3}$ *is* in simplest form because 2 and 3 have no common factor other than 1. The fraction $\frac{4}{6}$ *is not* in simplest form because 4 and 6 both

✓Concept Check Answer

false; 9 is not prime

have a factor of 2. That is, 2 is a common factor of 4 and 6. The process of writing a fraction in simplest form is called **simplifying** the fraction.

To simplify $\frac{4}{6}$ and write it as $\frac{2}{3}$, let's first study a few properties. Recall from Section 3.1 that any nonzero whole number n divided by itself is 1.

> Any nonzero number n divided by itself is 1.
>
> $\frac{5}{5} = 1$, $\frac{17}{17} = 1$, $\frac{24}{24} = 1$, or, in general, $\frac{n}{n} = 1$

Also, in general, if $\frac{a}{b}$ and $\frac{c}{d}$ are fractions (with b and d not 0), the following is true.

> $$\frac{a \cdot c}{b \cdot d} = \frac{a}{b} \cdot \frac{c}{d}^{*}$$

These properties allow us to do the following:

$$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \underbrace{\frac{2}{2}}_{\text{This is 1}} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$$

When 1 is multiplied by a number, the result is the same number.

Example 4

Write in simplest form: $\frac{12}{20}$

Solution: Notice that 12 and 20 have a common factor of 4.

$$\frac{12}{20} = \frac{4 \cdot 3}{4 \cdot 5} = \frac{4}{4} \cdot \frac{3}{5} = 1 \cdot \frac{3}{5} = \frac{3}{5}$$

Since 3 and 5 have no common factors (other than 1), $\frac{3}{5}$ is in simplest form.

Work Practice 4

If you have trouble finding common factors, write the prime factorization of the numerator and the denominator.

Example 5

Write in simplest form: $\frac{42}{66}$

Solution: Let's write the prime factorizations of 42 and 66.

$$\frac{42}{66} = \frac{2 \cdot 3 \cdot 7}{2 \cdot 3 \cdot 11} = \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{7}{11} = 1 \cdot 1 \cdot \frac{7}{11} = \frac{7}{11}$$

Work Practice 5

In the example above, you may have saved time by noticing that 42 and 66 have a common factor of 6.

$$\frac{42}{66} = \frac{6 \cdot 7}{6 \cdot 11} = \frac{6}{6} \cdot \frac{7}{11} = 1 \cdot \frac{7}{11} = \frac{7}{11}$$

Helpful Hint

Writing the prime factorizations of the numerator and the denominator is helpful in finding any common factors.

Helpful Hint

These two properties together are called the Fundamental Property of Fractions.

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b}$$

Practice 4

Write in simplest form: $\frac{30}{45}$

Practice 5

Write in simplest form: $\frac{39}{51}$

Answers

4. $\frac{2}{3}$ **5.** $\frac{13}{17}$

---

*Note: We will study this concept further in the next section.

The method for simplifying negative fractions is the same as for positive fractions.

Practice 6

Write in simplest form: $-\frac{9}{50}$

Example 6 Write in simplest form: $-\frac{10}{27}$

Solution:

$$-\frac{10}{27} = -\frac{2 \cdot 5}{3 \cdot 3 \cdot 3} \quad \text{Prime factorizations of 10 and 27}$$

Since 10 and 27 have no common factors, $-\frac{10}{27}$ is already in simplest form.

Work Practice 6

Practice 7

Write in simplest form: $\frac{49}{112}$

Example 7 Write in simplest form: $\frac{30}{108}$

Solution:

$$\frac{30}{108} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{5}{2 \cdot 3 \cdot 3} = 1 \cdot 1 \cdot \frac{5}{18} = \frac{5}{18}$$

Work Practice 7

We can use a shortcut procedure with common factors when simplifying.

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3} = \frac{1 \cdot 2}{1 \cdot 3} = \frac{2}{3} \quad \text{Divide out the common factor of 2 in the numerator and denominator.}$$

This procedure is possible because dividing out a common factor in the numerator and denominator is the same as removing a factor of 1 in the product.

Writing a Fraction in Simplest Form

To write a fraction in simplest form, write the prime factorization of the numerator and the denominator and then divide both by all common factors.

Practice 8

Write in simplest form: $-\frac{64}{20}$

Example 8 Write in simplest form: $-\frac{72}{26}$

Solution:

$$-\frac{72}{26} = -\frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 3 \cdot 3}{\underset{1}{\cancel{2}} \cdot 13} = -\frac{1 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{1 \cdot 13} = -\frac{36}{13}$$

Work Practice 8

✓ **Concept Check** Which is the correct way to simplify the fraction $\frac{15}{25}$? Or are both correct? Explain.

a. $\frac{15}{25} = \frac{3 \cdot \overset{1}{\cancel{5}}}{5 \cdot \underset{1}{\cancel{5}}} = \frac{3}{5}$

b. $\frac{1\overset{1}{\cancel{5}}}{2\underset{1}{\cancel{5}}} = \frac{11}{21}$

Answers

6. $-\frac{9}{50}$ **7.** $\frac{7}{16}$ **8.** $-\frac{16}{5}$

✓ **Concept Check Answers**

a. correct **b.** incorrect

Example 9 Write in simplest form: $\frac{6}{60}$

Solution:

$$\frac{6}{60} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 5} = \frac{1 \cdot 1}{1 \cdot 2 \cdot 1 \cdot 5} = \frac{1}{10}$$

Work Practice 9

Practice 9

Write in simplest form: $\frac{8}{56}$

Helpful Hint

Be careful when all factors of the numerator or denominator are divided out. In Example 9, the numerator was $1 \cdot 1 = 1$, so the final result was $\frac{1}{10}$.

In the fraction of Example 9, $\frac{6}{60}$, you may have immediately noticed that the largest common factor of 6 and 60 is 6. If so, you may simply divide out that common factor.

$$\frac{6}{60} = \frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \cdot 10} = \frac{1}{1 \cdot 10} = \frac{1}{10} \quad \text{Divide out the common factor of 6.}$$

Notice that the result, $\frac{1}{10}$, is in simplest form. If it were not, we would repeat the same procedure until the result is in simplest form.

Objective C Determining Whether Two Fractions Are Equivalent

Recall from Objective B that two fractions are equivalent if they represent the same part of a whole. One way to determine whether two fractions are equivalent is to see whether they simplify to the same fraction.

Example 10 Determine whether $\frac{16}{40}$ and $\frac{10}{25}$ are equivalent.

Solution: Simplify each fraction.

$$\frac{16}{40} = \frac{\overset{1}{\cancel{8}} \cdot 2}{\underset{1}{\cancel{8}} \cdot 5} = \frac{1 \cdot 2}{1 \cdot 5} = \frac{2}{5}$$

$$\frac{10}{25} = \frac{2 \cdot \overset{1}{\cancel{5}}}{5 \cdot \underset{1}{\cancel{5}}} = \frac{2 \cdot 1}{5 \cdot 1} = \frac{2}{5}$$

Since these fractions are the same, $\frac{16}{40} = \frac{10}{25}$.

Work Practice 10

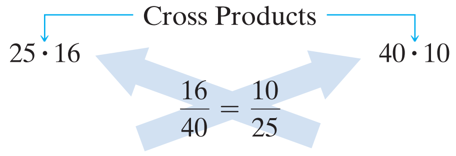
Practice 10

Determine whether $\frac{7}{9}$ and $\frac{21}{27}$ are equivalent.

There is a shortcut method you may use to check or test whether two fractions are equivalent. In the example above, we learned that the fractions are equivalent, or

$$\frac{16}{40} = \frac{10}{25}$$

In this example above, we call $25 \cdot 16$ and $40 \cdot 10$ **cross products** because they are the products one obtains by multiplying diagonally across the equal sign, as shown below.



Answers

9. $\frac{1}{7}$ **10.** equivalent

Notice that these cross products are equal:

$25 \cdot 16 = 400, \quad 40 \cdot 10 = 400$

In general, this is true for equivalent fractions.

Equality of Fractions

$$\begin{array}{ccc} 8 \cdot 6 & & 24 \cdot 2 \\ & \dfrac{6}{24} \stackrel{?}{=} \dfrac{2}{8} & \end{array}$$

Since the cross products ($8 \cdot 6 = 48$ and $24 \cdot 2 = 48$) are equal, the fractions are equal.

Note: If the cross products are not equal, the fractions are not equal.

Practice 11

Determine whether $\frac{4}{13}$ and $\frac{5}{18}$ are equivalent.

Example 11 Determine whether $\frac{8}{11}$ and $\frac{19}{26}$ are equivalent.

Solution: Let's check cross products.

$$\begin{array}{ccc} 26 \cdot 8 & & 11 \cdot 19 \\ = 208 & \dfrac{8}{11} \stackrel{?}{=} \dfrac{19}{26} & = 209 \end{array}$$

Since $208 \neq 209$, then $\frac{8}{11} \neq \frac{19}{26}$.

Helpful Hint

"Not equal to" symbol

Work Practice 11

Objective D Solving Problems by Writing Fractions in Simplest Form

Many real-life problems can be solved by writing fractions. To make the answers clearer, these fractions should be written in simplest form.

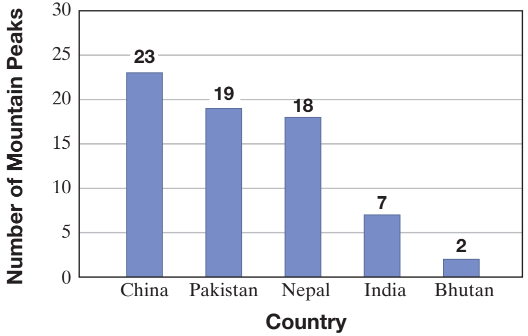
Practice 12

There are only 14 mountains over 8000 meters in the world, all located in Asia. If 7 of these mountains are located in Nepal, determine what fraction of the mountains over 8000 meters are located in Nepal. Write the fraction in simplest form.

Example 12 Calculating Fraction of High Mountain Peaks

There are 49 mountain peaks over 7500 meters in the world, all located in Asia. Many of them form the borders between two countries and are thus counted twice on the bar graph below. Use the graph below to determine what fraction of the 49 mountain peaks over 7500 meters are located in India. Write the fraction in simplest form.

Location of Mountain Peaks Over 7500 Meters in Height



Source: Internet Research

Answers

11. not equivalent **12.** $\frac{1}{2}$

Solution: First we determine the fraction of highest mountain peaks found in India.

$$\frac{7}{49} \begin{array}{l} \leftarrow \text{mountain peaks over 7500 meters in India} \\ \leftarrow \text{total mountain peaks over 7500 meters} \end{array}$$

Next, we simplify the fraction.

$$\frac{7}{49} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}} \cdot 7} = \frac{1}{1 \cdot 7} = \frac{1}{7}$$

Thus, $\frac{1}{7}$ of the world's mountain peaks over 7500 meters in height are located in India.

Work Practice 12

Calculator Explorations Simplifying Fractions

Scientific Calculator

Many calculators have a fraction key, such as $\boxed{a^b/_c}$, that allows you to simplify a fraction on the calculator. For example, to simplify $\frac{324}{612}$, enter

$$\boxed{324} \quad \boxed{a^b/_c} \quad \boxed{612} \quad \boxed{=}$$

The display will read

$$\boxed{\quad 9 \,\lrcorner\, 17}$$

which represents $\frac{9}{17}$, the original fraction simplified.

Graphing Calculator

Graphing calculators also allow you to simplify fractions. The fraction option on a graphing calculator may be found under the $\boxed{\text{MATH}}$ menu.

To simplify $\frac{324}{612}$, enter

$$\boxed{324} \quad \boxed{\div} \quad \boxed{612} \quad \boxed{\text{MATH}} \quad \boxed{\text{ENTER}} \quad \boxed{\text{ENTER}}$$

The display will read

$$\boxed{324/612 \blacktriangleright \text{Frac } 9/17}$$

Helpful Hint

The Calculator Explorations boxes in this chapter provide only an introduction to fraction keys on calculators. Any time you use a calculator, there are both advantages and limitations to its use. Never rely solely on your calculator. It is very important that you understand how to perform all operations on fractions by hand in order to progress through later topics. For further information, talk to your instructor.

Use your calculator to simplify each fraction.

1. $\frac{128}{224}$ **2.** $\frac{231}{396}$ **3.** $\frac{340}{459}$

4. $\frac{999}{1350}$ **5.** $\frac{432}{810}$ **6.** $\frac{225}{315}$

7. $\frac{54}{243}$ **8.** $\frac{455}{689}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

cross products
simplest form
equivalent
prime factorization
composite
prime

1. The number 40 equals $2 \cdot 2 \cdot 2 \cdot 5$. Since each factor is prime, we call $2 \cdot 2 \cdot 2 \cdot 5$ the ______________ of 40.

2. A natural number, other than 1, that is not prime is called a(n) ______________ number.

3. A natural number that has exactly two different factors, 1 and itself, is called a(n) ______________ number.

4. In $\frac{11}{48}$, since 11 and 48 have no common factors other than 1, $\frac{11}{48}$ is in ______________.

5. Fractions that represent the same portion of a whole are called ______________ fractions.

6. In the statement $\frac{5}{12} = \frac{15}{36}$, $5 \cdot 36$ and $12 \cdot 15$ are called ______________.

Without dividing, answer the questions in Exercises 7 and 8.

7. Is 2430 divisible by 2? By 3? By 5?

8. Is 1155 divisible by 2? By 3? By 5?

Write the prime factorization of each number.

9. 15

10. 10

11. 6

12. 21

13. 4

14. 9

Martin-Gay Interactive Videos

See Video 3.2

Watch the section lecture video and answer the following questions.

Objective A **15.** From Example 1, what two things should you check to make sure your prime factorization of a number is correct?

Objective B **16.** From the lecture before Example 3, when you have a common factor in the numerator and denominator of a fraction, essentially you have what?

Objective C **17.** Describe another way to solve Example 8 besides using cross products.

Objective D **18.** Why isn't $\frac{10}{24}$ the final answer to Example 9? What is the final answer?

3.2 Exercise Set MyLab Math

Objective A Write the prime factorization of each number. See Examples 1 through 3.

1. 20
2. 12
3. 48
4. 75
5. 50
6. 64
7. 162
8. 128
9. 110
10. 130
11. 85
12. 93
13. 240
14. 836
15. 828
16. 504

Objective B Write each fraction in simplest form. See Examples 4 through 9.

17. $\frac{3}{12}$
18. $\frac{5}{30}$
19. $\frac{4}{42}$
20. $\frac{9}{48}$
21. $\frac{14}{16}$
22. $\frac{22}{34}$
23. $\frac{20}{30}$
24. $\frac{70}{80}$
25. $\frac{35}{50}$
26. $\frac{25}{55}$
27. $-\frac{63}{81}$
28. $-\frac{21}{49}$
29. $\frac{24}{40}$
30. $\frac{36}{54}$
31. $\frac{27}{64}$
32. $\frac{32}{63}$
33. $\frac{25}{40}$
34. $\frac{36}{42}$
35. $-\frac{40}{64}$
36. $-\frac{28}{60}$
37. $\frac{36}{24}$
38. $\frac{60}{36}$
39. $\frac{90}{120}$
40. $\frac{60}{150}$
41. $\frac{70}{196}$
42. $\frac{98}{126}$
43. $\frac{66}{308}$
44. $\frac{65}{234}$
45. $-\frac{55}{85}$
46. $-\frac{78}{90}$
47. $\frac{189}{216}$
48. $\frac{144}{162}$
49. $\frac{224}{16}$
50. $\frac{270}{15}$

Objective C Determine whether each pair of fractions is equivalent. See Examples 10 and 11.

51. $\frac{3}{6}$ and $\frac{4}{8}$
52. $\frac{3}{9}$ and $\frac{2}{6}$
53. $\frac{7}{11}$ and $\frac{5}{8}$
54. $\frac{2}{5}$ and $\frac{4}{11}$
55. $\frac{10}{15}$ and $\frac{6}{9}$
56. $\frac{4}{10}$ and $\frac{6}{15}$
57. $\frac{3}{9}$ and $\frac{6}{18}$
58. $\frac{2}{8}$ and $\frac{7}{28}$
59. $\frac{10}{13}$ and $\frac{12}{15}$
60. $\frac{16}{20}$ and $\frac{9}{12}$
61. $\frac{8}{18}$ and $\frac{12}{24}$
62. $\frac{6}{21}$ and $\frac{14}{35}$

Objective D *Solve. Write each fraction in simplest form. See Example 12.*

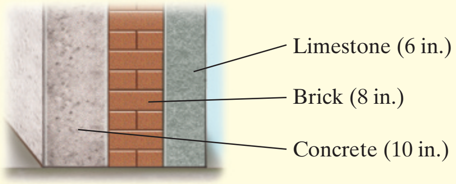
63. A work shift for an employee at Starbucks consists of 8 hours. What fraction of the employee's work shift is represented by 2 hours?

65. There are 5280 feet in a mile. What fraction of a mile is represented by 2640 feet?

67. There have been a total of 27 gold medals won in men's eight plus coxswain rowing competition in the Olympic Summer Games. An American team has earned 12 of them.

- **a.** What fraction of these gold medals has been won by an American team?
- **b.** How many of these gold medals have been won by non-American teams?
- **c.** Write the fraction of gold medals won in men's eight plus coxswain rowing competition by teams other than American.

69. The outer wall of the Pentagon is 24 inches thick. Ten inches is concrete, 8 inches is brick, and 6 inches is limestone. What fraction of the wall is concrete?



71. The Internet is the path of choice for many income tax filers. Currently, 37 states out of the total 50 states allow residents to file their federal and state income taxes electronically at the same time. (*Source:* TheBalance.com)

- **a.** How many states do not allow filing both state and federal income taxes at the same time online?
- **b.** What fraction of the states do not allow this option of tax filing?

73. Of the 20 most popular films released in 2017, five had a movie rating of PG. What fraction of 2017's most popular movies were PG-rated? (*Source:* IMDB)

64. Two thousand baseball caps were sold one year at the U.S. Open Golf Tournament. What fractional part of this total does 200 caps represent?

66. There are 100 centimeters in 1 meter. What fraction of a meter is 20 centimeters?

68. Katy Biagini just bought a brand-new 2018 Toyota Camry hybrid for $28,000. Her old car was traded in for $12,000.

- **a.** How much of her purchase price was not covered by her trade-in?
- **b.** What fraction of the purchase price was not covered by the trade-in?

70. There are 35 students in a biology class. If 10 students made an A on the first test, what fraction of the students made an A?

72. Albertsons Companies merged with Safeway, Inc. and operates grocery stores under multiple banners in 33 states in the United States. (*Source:* Safeway, Inc.)

- **a.** How many states do not have one of Albertsons Companies stores?
- **b.** What fraction of states do not have an Albertsons Companies store?

74. Worldwide, Hallmark employs nearly 9600 full-time employees. About 2700 employees work at the Hallmark headquarters in Kansas City, Missouri. What fraction of Hallmark employees work in Kansas City? (*Source:* Hallmark Cards, Inc.)

Review

Multiply. See Section 1.6.

75. $\begin{array}{r} 91 \\ \times\ \ 4 \\ \hline \end{array}$ **76.** $\begin{array}{r} 73 \\ \times\ \ 8 \\ \hline \end{array}$ **77.** $\begin{array}{r} 387 \\ \times\ \ 6 \\ \hline \end{array}$ **78.** $\begin{array}{r} 562 \\ \times\ \ 9 \\ \hline \end{array}$ **79.** $\begin{array}{r} 72 \\ \times\ 35 \\ \hline \end{array}$ **80.** $\begin{array}{r} 238 \\ \times\ 26 \\ \hline \end{array}$

Concept Extensions

81. In your own words, define equivalent fractions.

82. Given a fraction, say, $\frac{3}{8}$, how many fractions are there that are equivalent to it? Explain your answer.

Write each fraction in simplest form.

83. $\frac{3975}{6625}$

84. $\frac{9506}{12{,}222}$

There are generally considered to be eight basic blood types. The table shows the number of people with the various blood types in a typical group of 100 blood donors. Use the table to answer Exercises 85 through 88. Write each answer in simplest form.

Distribution of Blood Types in Blood Donors

Blood Type	Number of People
O Rh-positive	37
O Rh-negative	7
A Rh-positive	36
A Rh-negative	6
B Rh-positive	9
B Rh-negative	1
AB Rh-positive	3
AB Rh-negative	1

(*Source:* American Red Cross Biomedical Services)

85. What fraction of blood donors have blood type A Rh-positive?

86. What fraction of blood donors have an O blood type?

87. What fraction of blood donors have an AB blood type?

88. What fraction of blood donors have a B blood type?

Find the prime factorization of each number.

89. 34,020

90. 131,625

Solve.

91. In your own words, define a prime number.

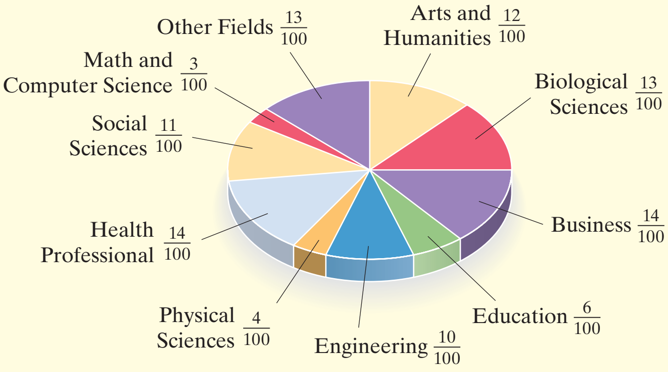
92. The number 2 is a prime number. All other even natural numbers are composite numbers. Explain why.

93. Two students have different prime factorizations for the same number. Is this possible? Explain.

94. Two students work to prime-factor 120. One student starts by writing 120 as 12 × 10. The other student writes 120 as 24 × 5. Finish each prime factorization. Are they the same? Why or why not?

The following graph is called a ***circle graph*** *or* ***pie chart****. Each sector (shaped like a piece of pie) shows the fraction of entering college freshmen who choose to major in each discipline shown. The whole circle represents the entire class of college freshmen. Use this graph to answer Exercises 95 through 98. Write each fraction answer in simplest form.*

College Freshmen Majors



Source: The Higher Education Research Institute

95. What fraction of entering college freshmen plan to major in education?

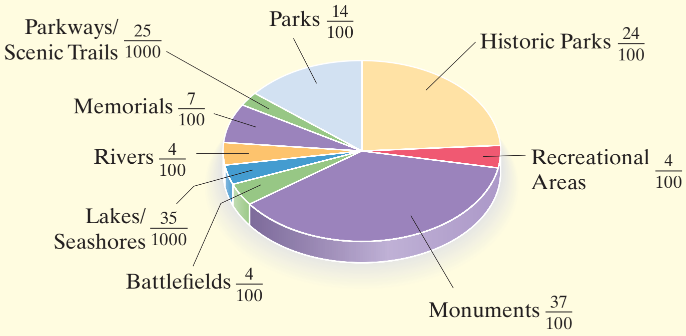
96. What fraction of entering college freshmen plan to major in engineering?

97. Why is the Business sector the same size as the Health Professional sector?

98. Why is the Physical Sciences sector smaller than the Business sector?

Use this circle graph to answer Exercises 99 through 102.

Areas Maintained by the National Park Service



Source: National Park Service

99. What fraction of National Park Service areas are rivers?

100. What fraction of National Park Service areas are parks (not including Historic Parks)?

101. Why is the rivers sector smaller than the monuments sector?

102. Why is the recreation area sector the same size as the battle fields sector?

Use the following numbers for Exercises 103 through 106.

8691 786 1235 2235 85 105 22 222 900 1470

103. List the numbers divisible by both 2 and 3.

104. List the numbers that are divisible by both 3 and 5.

105. The answers to Exercise **103** are also divisible by what number? Tell why.

106. The answers to Exercise **104** are also divisible by what number? Tell why.

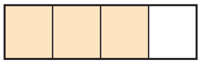
3.3 Multiplying and Dividing Fractions

Objectives

- **A** Multiply Fractions.
- **B** Evaluate Exponential Expressions with Fractional Bases.
- **C** Divide Fractions.
- **D** Multiply or Divide Given Fractional Replacement Values.
- **E** Solve Applications That Require Multiplication of Fractions.

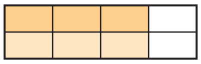
Objective A Multiplying Fractions

Let's use a diagram to discover how fractions are multiplied. For example, to multiply $\frac{1}{2}$ and $\frac{3}{4}$, we find $\frac{1}{2}$ of $\frac{3}{4}$. To do this, we begin with a diagram showing $\frac{3}{4}$ of a rectangle's area shaded.



$\frac{3}{4}$ of the rectangle's area is shaded.

To find $\frac{1}{2}$ of $\frac{3}{4}$, we heavily shade $\frac{1}{2}$ of the part that is already shaded.



By counting smaller rectangles, we see that $\frac{3}{8}$ of the larger rectangle is now heavily shaded, so that

$$\frac{1}{2} \text{ of } \frac{3}{4} \text{ is } \frac{3}{8}, \text{ or } \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8} \qquad \text{Notice that } \frac{1}{2}\cdot\frac{3}{4} = \frac{1\cdot 3}{2\cdot 4} = \frac{3}{8}.$$

Multiplying Fractions

To multiply two fractions, multiply the numerators and multiply the denominators. If a, b, c, and d represent numbers, and b and d are not 0, we have

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d}$$

Examples Multiply.

1. $\frac{2}{3}\cdot\frac{5}{11} = \frac{2\cdot 5}{3\cdot 11} = \frac{10}{33}$ Multiply numerators. Multiply denominators.

This fraction is in simplest form since 10 and 33 have no common factors other than 1.

2. $\frac{1}{4}\cdot\frac{1}{2} = \frac{1\cdot 1}{4\cdot 2} = \frac{1}{8}$ This fraction is in simplest form.

Work Practice 1–2

Practice 1–2

Multiply.

1. $\frac{3}{7}\cdot\frac{5}{11}$ **2.** $\frac{1}{3}\cdot\frac{1}{9}$

Example 3 Multiply and simplify: $\frac{6}{7}\cdot\frac{14}{27}$

Solution:

$$\frac{6}{7}\cdot\frac{14}{27} = \frac{6\cdot 14}{7\cdot 27}$$

We can simplify by finding the prime factorizations and using our shortcut procedure of dividing out common factors in the numerator and denominator.

$$\frac{6\cdot 14}{7\cdot 27} = \frac{2\cdot\overset{1}{\cancel{3}}\cdot 2\cdot\overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}}\cdot\underset{1}{\cancel{3}}\cdot 3\cdot 3} = \frac{2\cdot 2}{3\cdot 3} = \frac{4}{9}$$

Work Practice 3

Practice 3

Multiply and simplify: $\frac{6}{77}\cdot\frac{7}{8}$

Answers

1. $\frac{15}{77}$ **2.** $\frac{1}{27}$ **3.** $\frac{3}{44}$

Helpful Hint

Remember that the shortcut procedure in Example 3 is the same as removing factors of 1 in the product.

$$\frac{6\cdot 14}{7\cdot 27}=\frac{2\cdot 3\cdot 2\cdot 7}{7\cdot 3\cdot 3\cdot 3}=\frac{7}{7}\cdot\frac{3}{3}\cdot\frac{2\cdot 2}{3\cdot 3}=1\cdot 1\cdot\frac{4}{9}=\frac{4}{9}$$

Practice 4

Multiply and simplify: $\frac{4}{27}\cdot\frac{3}{8}$

Helpful Hint Don't forget that we may identify common factors that are not prime numbers.

Example 4

Multiply and simplify: $\frac{23}{32}\cdot\frac{4}{7}$

Solution: Notice that 4 and 32 have a common factor of 4.

$$\frac{23}{32}\cdot\frac{4}{7}=\frac{23\cdot 4}{32\cdot 7}=\frac{23\cdot\overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}}\cdot 8\cdot 7}=\frac{23}{8\cdot 7}=\frac{23}{56}$$

Work Practice 4

After multiplying two fractions, always check to see whether the product can be simplified.

Practice 5

Multiply.

$$\frac{1}{2}\cdot\left(-\frac{11}{28}\right)$$

Example 5

Multiply: $-\frac{1}{4}\cdot\frac{1}{2}$

Solution: Recall that the product of a negative number and a positive number is a negative number.

$$-\frac{1}{4}\cdot\frac{1}{2}=-\frac{1\cdot 1}{4\cdot 2}=-\frac{1}{8}$$

Work Practice 5

Practice 6–7

Multiply.

6. $\left(-\frac{4}{11}\right)\left(-\frac{33}{16}\right)$

7. $\frac{1}{6}\cdot\frac{3}{10}\cdot\frac{25}{16}$

Examples

Multiply.

6. $\left(-\frac{6}{13}\right)\left(-\frac{26}{30}\right)=\frac{6\cdot 26}{13\cdot 30}=\frac{\overset{1}{\cancel{6}}\cdot\overset{1}{\cancel{13}}\cdot 2}{\underset{1}{\cancel{13}}\cdot\underset{1}{\cancel{6}}\cdot 5}=\frac{2}{5}$ The product of two negative numbers is a positive number.

7. $\frac{1}{3}\cdot\frac{2}{5}\cdot\frac{9}{16}=\frac{1\cdot 2\cdot 9}{3\cdot 5\cdot 16}=\frac{1\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{3}}\cdot 5\cdot\underset{1}{\cancel{2}}\cdot 8}=\frac{3}{40}$

Work Practice 6–7

Objective B Evaluating Exponential Expressions with Fractional Bases

The base of an exponential expression can also be a fraction.

$$\left(\frac{1}{3}\right)^4=\underbrace{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}_{\frac{1}{3}\text{ is a factor }4\text{ times.}}=\frac{1\cdot 1\cdot 1\cdot 1}{3\cdot 3\cdot 3\cdot 3}=\frac{1}{81}$$

Answers

4. $\frac{1}{18}$ **5.** $-\frac{11}{56}$ **6.** $\frac{3}{4}$ **7.** $\frac{5}{64}$

Example 8 Evaluate.

a. $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{16}{625}$

b. $\left(-\frac{1}{4}\right)^2 = \left(-\frac{1}{4}\right) \cdot \left(-\frac{1}{4}\right) = \frac{1 \cdot 1}{4 \cdot 4} = \frac{1}{16}$ The product of two negative numbers is a positive number.

Work Practice 8

Practice 8

Evaluate.

a. $\left(\frac{3}{4}\right)^3$ **b.** $\left(-\frac{4}{5}\right)^2$

Objective C Dividing Fractions

Before we can divide fractions, we need to know how to find the **reciprocal** of a fraction.

Reciprocal of a Fraction

Two numbers are **reciprocals** of each other if their product is 1. The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ because $\frac{a}{b} \cdot \frac{b}{a} = \frac{a \cdot b}{b \cdot a} = 1$.

For example,

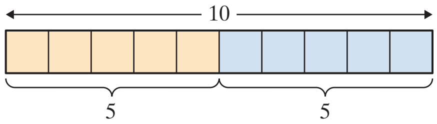
- The reciprocal of $\frac{7}{2}$ is $\frac{2}{7}$ because $\frac{7}{2} \cdot \frac{2}{7} = \frac{14}{14} = 1$.
- The reciprocal of 3 is $\frac{1}{3}$ because $3 \cdot \frac{1}{3} = \frac{3}{1} \cdot \frac{1}{3} = \frac{3}{3} = 1$.
- The reciprocal of $-\frac{5}{6}$ is $-\frac{6}{5}$ because $-\frac{5}{6} \cdot -\frac{6}{5} = \frac{30}{30} = 1$.

Helpful Hint

Every number has a reciprocal except 0. The number 0 has no reciprocal because there is no number such that $0 \cdot a = 1$.

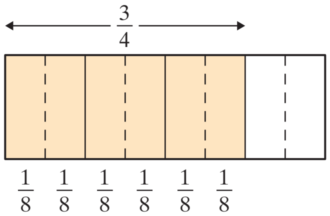
Division of fractions has the same meaning as division of whole numbers. For example,

$10 \div 5$ means: How many 5s are there in 10?



There are two 5s in 10, so $10 \div 5 = 2$.

$\frac{3}{4} \div \frac{1}{8}$ means: How many $\frac{1}{8}$s are there in $\frac{3}{4}$?



There are six $\frac{1}{8}$s in $\frac{3}{4}$, so $\frac{3}{4} \div \frac{1}{8} = 6$.

We use reciprocals to divide fractions.

Answers

8. a. $\frac{27}{64}$ **b.** $\frac{16}{25}$

Dividing Fractions

To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

If a, b, c, and d represent numbers, and b, c, and d are not 0, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

reciprocal

For example,

multiply by reciprocal

$$\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \cdot \frac{8}{1} = \frac{3 \cdot 8}{4 \cdot 1} = \frac{3 \cdot 2 \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \cdot 1} = \frac{6}{1} \quad \text{or} \quad 6$$

Just as when you are multiplying fractions, *always* check to see whether the result can be simplified when you divide fractions.

Practice 9–11

Divide and simplify.

9. $\frac{3}{2} \div \frac{14}{5}$ **10.** $\frac{8}{7} \div \frac{2}{9}$

11. $\frac{4}{9} \div \frac{1}{2}$

Examples

Divide and simplify.

9. $\frac{7}{8} \div \frac{2}{9} = \frac{7}{8} \cdot \frac{9}{2} = \frac{7 \cdot 9}{8 \cdot 2} = \frac{63}{16}$

10. $\frac{5}{16} \div \frac{3}{4} = \frac{5}{16} \cdot \frac{4}{3} = \frac{5 \cdot 4}{16 \cdot 3} = \frac{5 \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \cdot 4 \cdot 3} = \frac{5}{12}$

11. $\frac{2}{5} \div \frac{1}{2} = \frac{2}{5} \cdot \frac{2}{1} = \frac{2 \cdot 2}{5 \cdot 1} = \frac{4}{5}$

Work Practice 9–11

Helpful Hint

When dividing by a fraction, do not look for common factors to divide out until you rewrite the division as multiplication.

Do not try to divide out these two 2s.

$$\frac{1}{\mathbf{2}} \div \frac{\mathbf{2}}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$

Practice 12

Divide: $-\frac{10}{21} \div \frac{2}{9}$

Example 12

Divide: $-\frac{7}{12} \div -\frac{5}{6}$

Solution: Recall that the quotient (or product) of two negative numbers is a positive number.

$$-\frac{7}{12} \div -\frac{5}{6} = -\frac{7}{12} \cdot -\frac{6}{5} = \frac{7 \cdot \overset{1}{\cancel{6}}}{2 \cdot \underset{1}{\cancel{6}} \cdot 5} = \frac{7}{10}$$

Work Practice 12

Answers

9. $\frac{15}{28}$ **10.** $\frac{36}{7}$ **11.** $\frac{8}{9}$ **12.** $-\frac{15}{7}$

✓**Concept Check** Which is the correct way to divide $\frac{3}{5}$ by $\frac{5}{12}$? Or are both correct? Explain.

a. $\frac{3}{5} \div \frac{5}{12} = \frac{3}{5} \cdot \frac{12}{5}$

b. $\frac{3}{5} \div \frac{5}{12} = \frac{5}{3} \cdot \frac{5}{12}$

Objective D Multiplying or Dividing with Fractional Replacement Values

Example 13 If $x = \frac{7}{8}$ and $y = -\frac{1}{3}$, evaluate **(a)** xy and **(b)** $x \div y$.

Solution: Replace x with $\frac{7}{8}$ and y with $-\frac{1}{3}$.

a.
$$xy = \frac{7}{8} \cdot -\frac{1}{3} = -\frac{7 \cdot 1}{8 \cdot 3} = -\frac{7}{24}$$

b.
$$x \div y = \frac{7}{8} \div -\frac{1}{3} = \frac{7}{8} \cdot -\frac{3}{1} = -\frac{7 \cdot 3}{8 \cdot 1} = -\frac{21}{8}$$

Work Practice 13

Objective E Solving Problems by Multiplying Fractions

To solve real-life problems that involve multiplying fractions, we use our four problem-solving steps from Chapter 1. In Example 14, a key word that implies multiplication is used. That key word is "**of.**"

Example 14 Finding the Number of Roller Coasters in an Amusement Park

Cedar Point is an amusement park located in Sandusky, Ohio. Its collection of 72 rides is the largest in the world. Of the rides, $\frac{2}{9}$ are roller coasters. How many roller coasters are in Cedar Point's collection of rides? (*Source:* Wikipedia)

Solution:

1. UNDERSTAND the problem. To do so, read and reread the problem. We are told that $\frac{2}{9}$ of Cedar Point's rides are roller coasters. The word "of" here means multiplication.

(*Continued on next page*)

Practice 13

If $x = -\frac{3}{4}$ and $y = \frac{9}{2}$, evaluate (a) xy and (b) $x \div y$.

Helpful Hint "of" usually translates to multiplication.

Practice 14

Kings Dominion is an amusement park in Doswell, Virginia. Of its 48 rides, $\frac{5}{16}$ of them are roller coasters. How many roller coasters are in Kings Dominion? (*Source:* Cedar Fair Parks)

Answers

13 a. $-\frac{27}{8}$ **b.** $-\frac{1}{6}$

14. 15 roller coasters

✓**Concept Check Answers**

a. correct **b.** incorrect

2. TRANSLATE.

In words:	number of roller coasters	is	$\frac{2}{9}$	of	total rides at Cedar Point
	↓	↓	↓	↓	↓
Translate:	number of roller coasters	=	$\frac{2}{9}$	$\cdot$	72

3. SOLVE: Before we solve, let's estimate a reasonable answer. The fraction $\frac{2}{9}$ is less than $\frac{1}{4}$ (draw a diagram, if needed), and $\frac{1}{4}$ of 72 rides is 18 rides, so the number of roller coasters should be less than 18.

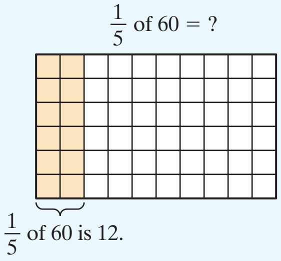
$$\frac{2}{9}\cdot 72 = \frac{2}{9}\cdot\frac{72}{1} = \frac{2\cdot 72}{9\cdot 1} = \frac{2\cdot \overset{1}{\cancel{9}}\cdot 8}{\underset{1}{\cancel{9}}\cdot 1} = \frac{16}{1} \quad \text{or} \quad 16$$

4. INTERPRET. *Check* your work. From our estimate, our answer is reasonable. *State* your conclusion: The number of roller coasters at Cedar Point is 16.

Work Practice 14

Helpful Hint

To help visualize a fractional part of a whole number, look at the diagram below.



Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

multiplication $\quad \frac{a\cdot d}{b\cdot c} \quad \frac{a\cdot c}{b\cdot d} \quad \frac{2\cdot 2\cdot 2}{7} \quad \frac{2}{7}\cdot\frac{2}{7}\cdot\frac{2}{7}$

division $\quad 0 \quad$ reciprocals

1. To multiply two fractions, we write $\frac{a}{b}\cdot\frac{c}{d} =$ ____________.

2. Two numbers are ____________ of each other if their product is 1.

3. The expression $\frac{2^3}{7} =$ ____________ whereas $\left(\frac{2}{7}\right)^3 =$ ____________.

4. Every number has a reciprocal except ____________.

5. To divide two fractions, we write $\frac{a}{b} \div \frac{c}{d} =$ ____________.

6. The word "of" indicates____________.

Find each product.

7. $\frac{1}{3}\cdot\frac{2}{5}$

8. $\frac{2}{3}\cdot\frac{4}{7}$

9. $\frac{6}{5}\cdot\frac{1}{7}$

10. $\frac{7}{3}\cdot\frac{2}{3}$

11. $\frac{3}{1}\cdot\frac{3}{8}$

12. $\frac{2}{1}\cdot\frac{7}{11}$

Martin-Gay Interactive Videos

See Video 3.3

Watch the section lecture video and answer the following questions.

Objective A 13. In Example 1, how do we know that the answer is positive?

Objective B 14. In Example 4, does the exponent apply to the negative sign? Why or why not?

Objective C 15. Complete this statement based on Example 5: When writing the reciprocal of a fraction, the denominator becomes the ______________, and the numerator becomes the ______________ .

Objective D 16. In Example 9a, why don't we write out the prime factorizations of 4 and 9 in the numerator?

Objective E 17. What formula is used to solve Example 10?

3.3 Exercise Set MyLab Math

Objective A *Multiply. Write the product in simplest form. See Examples 1 through 7.*

1. $\frac{2}{7}\cdot\frac{6}{11}$

2. $\frac{5}{9}\cdot\frac{7}{4}$

3. $-\frac{1}{5}\cdot\frac{9}{10}$

4. $\frac{5}{8}\cdot-\frac{1}{3}$

5. $\left(-\frac{1}{2}\right)\left(-\frac{2}{15}\right)$

6. $\left(-\frac{1}{5}\right)\left(-\frac{5}{12}\right)$

7. $\frac{6}{5}\cdot\frac{1}{7}$

8. $\frac{7}{3}\cdot\frac{1}{4}$

9. $\frac{2}{7}\cdot\frac{5}{8}$

10. $\frac{7}{8}\cdot\frac{2}{3}$

11. $\frac{5}{28}\cdot\frac{2}{25}$

12. $\frac{4}{35}\cdot\frac{5}{24}$

13. $0\cdot\frac{8}{9}$

14. $\frac{11}{12}\cdot 0$

15. $\frac{18}{20}\cdot\frac{36}{99}$

16. $\frac{5}{32}\cdot\frac{64}{100}$

17. $\frac{11}{20}\cdot\frac{1}{7}\cdot\frac{5}{22}$

18. $\frac{27}{32}\cdot\frac{10}{13}\cdot\frac{16}{30}$

Objective B *Evaluate. See Example 8.*

19. $\left(\frac{1}{5}\right)^3$

20. $\left(-\frac{1}{2}\right)^4$

▶ **21.** $\left(-\frac{2}{3}\right)^2$

22. $\left(\frac{8}{9}\right)^2$

23. $\left(-\frac{2}{3}\right)^3 \cdot \frac{1}{2}$

24. $\left(-\frac{3}{4}\right)^3 \cdot \frac{1}{3}$

Objective C *Divide. Write all quotients in simplest form. See Examples 9 through 12.*

▶ **25.** $\frac{2}{3} \div \frac{5}{6}$

26. $\frac{5}{8} \div \frac{2}{3}$

27. $-\frac{6}{15} \div \frac{12}{5}$

28. $-\frac{4}{15} \div -\frac{8}{3}$

29. $\frac{8}{9} \div -\frac{1}{2}$

30. $\frac{10}{11} \div -\frac{4}{5}$

31. $-\frac{2}{3} \div 4$

▶ **32.** $-\frac{5}{6} \div 10$

33. $\frac{1}{10} \div \frac{10}{1}$

34. $\frac{3}{13} \div \frac{13}{3}$

35. $\frac{7}{45} \div \frac{4}{25}$

36. $\frac{14}{52} \div \frac{1}{13}$

37. $\frac{3}{25} \div \frac{27}{40}$

38. $\frac{6}{15} \div \frac{7}{10}$

▶ **39.** $\frac{8}{13} \div 0$

40. $0 \div \frac{4}{11}$

▶ **41.** $0 \div \frac{7}{8}$

42. $\frac{2}{3} \div 0$

Objectives A B C Mixed Practice *Perform each indicated operation. See Examples 1 through 12.*

43. $\frac{2}{3} \cdot \frac{5}{9}$

44. $\frac{8}{15} \cdot \frac{5}{32}$

45. $-\frac{5}{28} \cdot \frac{35}{25}$

46. $\frac{24}{45} \cdot -\frac{5}{8}$

47. $-\frac{3}{5} \div -\frac{4}{5}$

48. $-\frac{11}{16} \div -\frac{13}{16}$

49. $\left(-\frac{3}{4}\right)^2$

50. $\left(-\frac{1}{2}\right)^5$

51. $7 \div \frac{2}{11}$

52. $-100 \div \frac{1}{2}$

53. $\frac{4}{25} \div \frac{3}{16}$

54. $\frac{9}{2} \div \frac{16}{15}$

55. $-\frac{1}{8} \cdot \frac{3}{7} \cdot \frac{14}{27}$

56. $-\frac{1}{10} \cdot \frac{7}{11} \cdot \frac{22}{35}$

Objective D *Given the following replacement values, evaluate (a) xy and (b) x ÷ y. See Example 13.*

57. $x = \frac{2}{5}$ and $y = \frac{5}{6}$

58. $x = \frac{8}{9}$ and $y = \frac{1}{4}$

▶ **59.** $x = -\frac{4}{5}$ and $y = \frac{9}{11}$

60. $x = \frac{7}{6}$ and $y = -\frac{1}{2}$

Objective E Translating

Solve. Write each answer in simplest form. For Exercises 61 through 64, recall that "of" translates to multiplication. See Example 14.

61. Find $\frac{1}{4}$ of 200.

62. Find $\frac{1}{5}$ of 200.

63. Find $\frac{5}{6}$ of 24.

64. Find $\frac{5}{8}$ of 24.

Solve. For Exercises 65 and 66, the solutions have been started for you. See Example 14.

65. In the United States, $\frac{7}{50}$ of college freshmen major in business. A community college in Pennsylvania has a freshmen enrollment of approximately 800 students. How many of these freshmen might we project are majoring in business?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blank below.)

freshmen majoring in business	is	$\frac{7}{50}$	of	community college freshmen enrollment
↓	↓	↓	↓	↓
freshmen majoring in business	=	$\frac{7}{50}$	·	______

Finish with:

3. SOLVE

4. INTERPRET

66. A patient was told that, at most, $\frac{1}{5}$ of his calories should come from fat. If his diet consists of 3000 calories a day, find the maximum number of calories that can come from fat.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blank below.)

patient's fat calories	is	$\frac{1}{5}$	of	his daily calories
↓	↓	↓	↓	↓
patient's fat calories	=	$\frac{1}{5}$	·	______

Finish with:

3. SOLVE

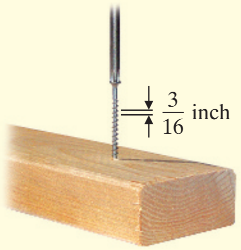
4. INTERPRET

67. In 2016, there were approximately 250 million moviegoers in the United States and Canada. Of these, about $\frac{12}{25}$ were male. Find the approximate number of males who attended the movies in that year. (*Source:* Motion Picture Association of America)

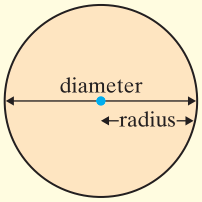
68. In 2016, cinemas in the United States and Canada sold about 1325 million movie tickets. Frequent moviegoers who go to the cinema once or more per month purchased about $\frac{12}{25}$ of these tickets. Find the number of tickets purchased by frequent moviegoers in 2016. (*Source:* Motion Picture Association of America)

69. The Oregon National Historic Trail is 2170 miles long. It begins in Independence, Missouri, and ends in Oregon City, Oregon. Manfred Coulon has hiked $\frac{2}{5}$ of the trail before. How many miles has he hiked? (*Source:* National Park Service)

70. Each turn of a screw sinks it $\frac{3}{16}$ of an inch deeper into a piece of wood. Find how deep the screw is after 8 turns.



▶ △ **71.** The radius of a circle is one-half of its diameter, as shown. If the diameter of a circle is $\frac{3}{8}$ of an inch, what is its radius?



△ **72.** The diameter of a circle is twice its radius, as shown in the Exercise **71** illustration. If the radius of a circle is $\frac{7}{20}$ of a foot, what is its diameter?

73. A special on a cruise to the Bahamas is advertised to be $\frac{2}{3}$ of the regular price. If the regular price is \$2757, what is the sale price?

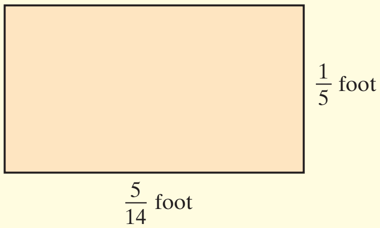
74. A family recently sold their house for \$102,000, but $\frac{3}{50}$ of this amount goes to the real estate companies that helped them sell their house. How much money does the family pay to the real estate companies?

75. The state of Mississippi houses $\frac{1}{184}$ of the total U.S. libraries. If there are about 9200 libraries in the United States, how many libraries are in Mississippi?

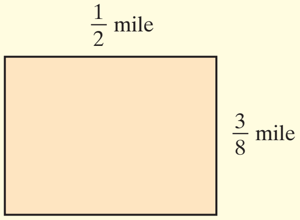
76. There have been about 516 different contestants on the reality television show *Survivor* over 35 seasons. Some of these contestants appeared in two or more different series and/or seasons. If the number of repeat contestants was $\frac{5}{86}$ of the total number of participants in the first 35 seasons, how many contestants participated more than once? (*Source:* Survivor.com)

Find the area of each rectangle. Recall that area = length · width.

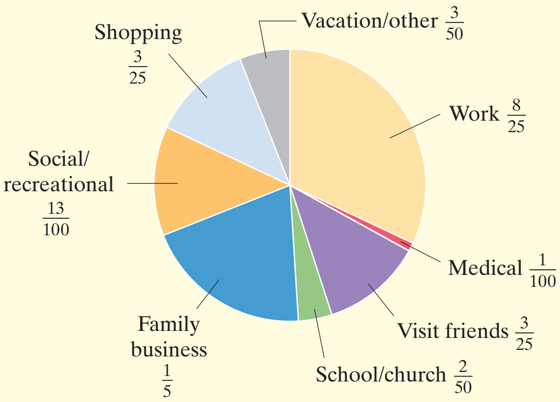
△ **77.**



△ **78.**



Recall from Section 3.2 that the following graph is called a ***circle graph*** *or* ***pie chart.*** *Each sector (shaped like a piece of pie) shows the fractional part of a car's total mileage that falls into a particular category. The whole circle represents a car's total mileage.*



Source: The American Automobile Manufacturers Association and The National Automobile Dealers Association

In one year, a family drove 12,000 miles in the family car. Use the circle graph to determine how many of these miles might be expected to have fallen in the categories shown in Exercises 79 through 82.

79. Work

80. Shopping

81. Family business

82. Medical

Review

Perform each indicated operation. See Sections 1.3 and 1.4.

83. $$\begin{array}{r} 27 \\ 76 \\ +\ 98 \\ \hline \end{array}$$

84. $$\begin{array}{r} 811 \\ 42 \\ +\ 69 \\ \hline \end{array}$$

85. $$\begin{array}{r} 968 \\ -\ 772 \\ \hline \end{array}$$

86. $$\begin{array}{r} 882 \\ -\ 773 \\ \hline \end{array}$$

Concept Extensions

87. In your own words, describe how to divide fractions.

88. In your own words, explain how to multiply fractions.

Simplify.

89. $\left(\frac{1}{2} \cdot \frac{2}{3}\right) \div \frac{5}{6}$

90. $\left(\frac{3}{4} \cdot \frac{8}{9}\right) \div \frac{2}{5}$

91. $\frac{42}{25} \cdot \frac{125}{36} \div \frac{7}{6}$

92. $\left(\frac{8}{13} \cdot \frac{39}{16} \cdot \frac{8}{9}\right)^2 \div \frac{1}{2}$

Solve.

93. Approximately $\frac{1}{8}$ of the U.S. population lives in the state of California. If the U.S. population is approximately 325,748,000, find the approximate population of California. (*Source:* U.S. Census Bureau)

94. The estimated population of New Zealand was 4,693,000 in 2017. About $\frac{3}{20}$ of New Zealand's population is of Māori descent. How many Māori lived in New Zealand in 2017? (*Source:* Statistics New Zealand)

95. In 2017, a survey found that about $\frac{19}{25}$ of all Americans in the United States owned a smartphone. There were approximately 325,748,000 Americans at that time. How many Americans owned a smartphone in 2017? (*Sources:* Pew Research Center, U.S. Census Bureau)

96. If $\frac{3}{4}$ of 36 students on a first bus are girls and $\frac{2}{3}$ of the 30 students on a second bus are *boys*, how many students on the two buses are girls?

3.4 Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions

Objectives

- **A** Add or Subtract Like Fractions.
- **B** Add or Subtract Given Fractional Replacement Values.
- **C** Solve Problems by Adding or Subtracting Like Fractions.
- **D** Find the Least Common Denominator of a List of Fractions.
- **E** Write Equivalent Fractions.

Fractions with the same denominator are called **like fractions.** Fractions that have different denominators are called **unlike fractions.**

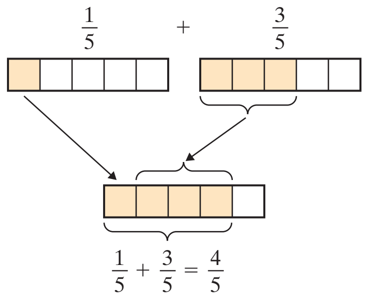
Like Fractions	Unlike Fractions
$\frac{2}{5}$ and $\frac{3}{5}$ same denominator	$\frac{2}{5}$ and $\frac{3}{4}$ different denominators
$\frac{5}{21}, \frac{16}{21},$ and $\frac{7}{21}$ same denominator	$\frac{5}{7}$ and $\frac{5}{9}$ different denominators

Objective A Adding or Subtracting Like Fractions

To see how we add like fractions (fractions with the same denominator), study one or both illustrations below.

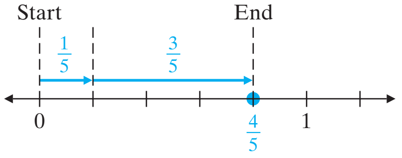
Add: $\frac{1}{5} + \frac{3}{5}$

Figures



Number Line

To add $\frac{1}{5} + \frac{3}{5}$, start at 0 and draw an arrow $\frac{1}{5}$ of a unit long pointing to the right. From the tip of this arrow, draw an arrow $\frac{3}{5}$ of a unit long, also pointing to the right. The tip of the second arrow ends at their sum, $\frac{4}{5}$.



$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

Thus, $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$**.**

Notice that the numerator of the sum is the sum of the numerators. Also, the denominator of the sum is the **common denominator.** This is how we add fractions. Similar illustrations can be shown for subtracting fractions.

Adding or Subtracting Like Fractions (Fractions with the Same Denominator)

If a, b, and c are numbers and b is not 0, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and also} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

In other words, to add or subtract fractions with the same denominator, add or subtract their numerators and write the sum or difference over the **common** denominator.

For example,

$$\frac{1}{4}+\frac{2}{4}=\frac{1+2}{4}=\frac{3}{4}$$ Add the numerators.
Keep the common denominator.

$$\frac{4}{5}-\frac{2}{5}=\frac{4-2}{5}=\frac{2}{5}$$ Subtract the numerators.
Keep the common denominator.

Helpful Hint

As usual, don't forget to write all answers in simplest form.

Examples Add and simplify.

1. $\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\frac{5}{7}$ ← Add the numerators.
← Keep the common denominator.

2. $\frac{3}{16}+\frac{7}{16}=\frac{3+7}{16}=\frac{10}{16}=\frac{\overset{1}{\cancel{2}}\cdot 5}{\underset{1}{\cancel{2}}\cdot 8}=\frac{5}{8}$

3. $\frac{7}{8}+\frac{6}{8}+\frac{3}{8}=\frac{7+6+3}{8}=\frac{16}{8}$ or 2

Work Practice 1–3

✓Concept Check Find and correct the error in the following:

$$\xcancel{\frac{1}{5}+\frac{1}{5}=\frac{2}{10}}$$

Examples Subtract and simplify.

4. $\frac{8}{9}-\frac{1}{9}=\frac{8-1}{9}=\frac{7}{9}$ ← Subtract the numerators.
← Keep the common denominator.

5. $\frac{7}{8}-\frac{5}{8}=\frac{7-5}{8}=\frac{2}{8}=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}\cdot 4}=\frac{1}{4}$

Work Practice 4–5

From our earlier work, we know that

$$\frac{-12}{6}=\frac{12}{-6}=-\frac{12}{6}$$ since these all simplify to −2.

In general, the following is true:

$$\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$$ as long as b is not 0.

Example 6 Add: $-\frac{11}{8}+\frac{6}{8}$

Solution: $-\frac{11}{8}+\frac{6}{8}=\frac{-11+6}{8}$

$=\frac{-5}{8}$ or $-\frac{5}{8}$

Work Practice 6

Practice 1–3

Add and simplify.

1. $\frac{6}{13}+\frac{2}{13}$

2. $\frac{5}{8}+\frac{1}{8}$

3. $\frac{20}{11}+\frac{6}{11}+\frac{7}{11}$

Practice 4–5

Subtract and simplify.

4. $\frac{11}{12}-\frac{6}{12}$

5. $\frac{7}{15}-\frac{2}{15}$

Practice 6

Add: $-\frac{8}{17}+\frac{4}{17}$

Answers

1. $\frac{8}{13}$ **2.** $\frac{3}{4}$ **3.** 3 **4.** $\frac{5}{12}$
5. $\frac{1}{3}$ **6.** $-\frac{4}{17}$

✓Concept Check Answer

We don't add denominators together;
correction: $\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$.

Practice 7

Subtract: $\frac{2}{5} - \frac{7}{5}$

Example 7 Subtract: $\frac{3}{4} - \frac{7}{4}$

Solution: $\frac{3}{4} - \frac{7}{4} = \frac{3 - 7}{4} = \frac{3 + (-7)}{4} = \frac{-4}{4} = -1$

Work Practice 7

Practice 8

Subtract: $\frac{4}{11} - \frac{6}{11} - \frac{3}{11}$

Example 8 Subtract: $\frac{3}{7} - \frac{6}{7} - \frac{3}{7}$

Solution: $\frac{3}{7} - \frac{6}{7} - \frac{3}{7} = \frac{3 - 6 - 3}{7} = \frac{-6}{7}$ or $-\frac{6}{7}$

Work Practice 8

Helpful Hint

Recall that $\frac{-6}{7} = -\frac{6}{7}$ $\left(\text{Also, } \frac{6}{-7} = -\frac{6}{7}, \text{ if needed.}\right)$

Objective B Adding or Subtracting Given Fractional Replacement Values

Practice 9

Evaluate $x + y$ if $x = -\frac{10}{12}$ and $y = \frac{7}{12}$.

Example 9 Evaluate $y - x$ if $x = -\frac{3}{10}$ and $y = -\frac{8}{10}$.

Solution: Be very careful when replacing x and y with replacement values.

$$y - x = -\frac{8}{10} - \left(-\frac{3}{10}\right) \quad \text{Replace } x \text{ with } -\frac{3}{10} \text{ and } y \text{ with } -\frac{8}{10}.$$

$$= \frac{-8 - (-3)}{10}$$

$$= \frac{-5}{10} = \frac{-1 \cdot 5}{2 \cdot 5} = \frac{-1}{2} \text{ or } -\frac{1}{2}$$

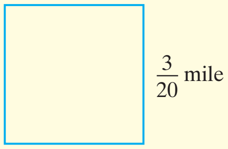
Work Practice 9

Objective C Solving Problems by Adding or Subtracting Like Fractions

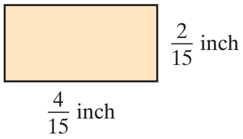
Many real-life problems involve finding the perimeters of square- or rectangular-shaped figures such as pastures, swimming pools, and so on. We can use our knowledge of adding fractions to find perimeters.

Practice 10

Find the perimeter of the square.



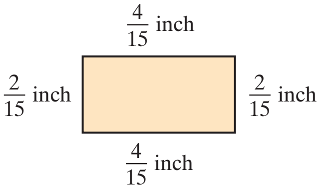
Example 10 Find the perimeter of the rectangle.



Answers

7. -1 8. $-\frac{5}{11}$ 9. $-\frac{1}{4}$ 10. $\frac{3}{5}$ mi

Solution: Recall that perimeter means distance around and that opposite sides of a rectangle are the same length.



$$\text{Perimeter} = \frac{2}{15} + \frac{4}{15} + \frac{2}{15} + \frac{4}{15} = \frac{2+4+2+4}{15}$$

$$= \frac{12}{15} = \frac{\overset{1}{\cancel{3}} \cdot 4}{\underset{1}{\cancel{3}} \cdot 5} = \frac{4}{5}$$

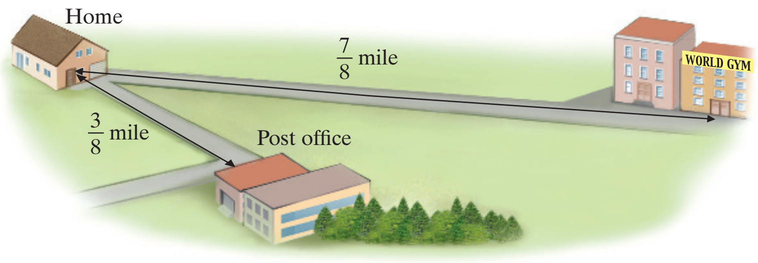
The perimeter of the rectangle is $\frac{4}{5}$ inch.

Work Practice 10

We can combine our skills in adding and subtracting fractions with our four problem-solving steps from Chapter 1 to solve many kinds of real-life problems.

Example 11 Calculating Distance

The distance from home to the World Gym is $\frac{7}{8}$ of a mile and from home to the post office is $\frac{3}{8}$ of a mile. How much farther is it from home to the World Gym than from home to the post office?



Solution:

1. UNDERSTAND. Read and reread the problem. The phrase "How much farther" tells us to subtract distances.
2. TRANSLATE.

In words:	distance farther	is	home to World Gym distance	minus	home to post office distance
	↓	↓	↓	↓	↓
Translate:	distance farther	=	$\frac{7}{8}$	−	$\frac{3}{8}$

3. SOLVE: $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{\overset{1}{\cancel{4}}}{2 \cdot \underset{1}{\cancel{4}}} = \frac{1}{2}$
4. INTERPRET. *Check* your work. *State* your conclusion: The distance from home to the World Gym is $\frac{1}{2}$ mile farther than from home to the post office.

Work Practice 11

Practice 11

A jogger ran $\frac{13}{4}$ miles on Monday and $\frac{11}{4}$ miles on Wednesday. How much farther did he run on Monday than on Wednesday?

Answer

11. $\frac{1}{2}$ mi

Objective D Finding the Least Common Denominator

In the next section, we will add and subtract fractions that have different, or unlike, denominators. To do so, we first write them as equivalent fractions with a common denominator.

Although any common denominator can be used to add or subtract unlike fractions, we will use the **least common denominator (LCD).** The LCD of a list of fractions is the same as the **least common multiple (LCM)** of the denominators. Why do we use this number as the common denominator? Since the LCD is the *smallest* of all common denominators, operations are usually less tedious with this number.

The **least common denominator (LCD)** of a list of fractions is the smallest positive number divisible by all the denominators in the list. (The least common denominator is also the **least common multiple (LCM) of the denominators**).

For example, the LCD of $\frac{1}{4}$ and $\frac{3}{10}$ is 20 because 20 is the smallest positive number divisible by both 4 and 10.

Finding the LCD: Method 1

One way to find the LCD is to see whether the larger denominator is divisible by the smaller denominator. If so, the larger number is the LCD. If not, then check consecutive multiples of the larger denominator until the LCD is found.

Method 1: Finding the LCD of a List of Fractions Using Multiples of the Largest Number

Step 1: Write the multiples of the largest denominator (starting with the number itself) until a multiple common to all denominators in the list is found.

Step 2: The multiple found in Step 1 is the LCD.

Practice 12

Find the LCD of $\frac{7}{8}$ and $\frac{11}{16}$.

Example 12 Find the LCD of $\frac{3}{7}$ and $\frac{5}{14}$.

Solution: The denominators are 7 and 14. We write the multiples of 14 until we find one that is also a multiple of 7.

$14 \cdot 1 = 14$ A multiple of 7

The LCD is 14.

Work Practice 12

Practice 13

Find the LCD of $\frac{23}{25}$ and $\frac{1}{30}$.

Example 13 Find the LCD of $\frac{11}{12}$ and $\frac{7}{20}$.

Solution: We write the multiples of the larger denominator, 20, until we find one that is also a multiple of 12.

$20 \cdot 1 = 20$ Not a multiple of 12

$20 \cdot 2 = 40$ Not a multiple of 12

$20 \cdot 3 = 60$ A multiple of 12

The LCD is 60.

Work Practice 13

Answers

12. 16 **13.** 150

Method 1 for finding multiples works fine for smaller numbers, but may get tedious for larger numbers. For this reason, let's study a second method, which uses prime factorization.

Finding the LCD: Method 2

For example, to find the LCD of $\frac{11}{12}$ and $\frac{7}{20}$, such as in Example 13, let's look at the prime factorization of each denominator.

$$\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 \\ 20 &= 2 \cdot 2 \cdot 5 \end{aligned}$$

Recall that the LCD must be a multiple of both 12 and 20. Thus, to build the LCD, we will circle the greatest number of factors for each different prime number. The LCD is the product of the circled factors.

Prime Number Factors

$$\begin{aligned} 12 &= \textcircled{2 \cdot 2} \cdot \textcircled{3} \\ 20 &= 2 \cdot 2 \cdot \qquad \textcircled{5} \\ \text{LCD} &= 2 \cdot 2 \cdot 3 \cdot 5 = 60 \end{aligned}$$

Circle either pair of 2s, but not both.

The number 60 is the smallest number that both 12 and 20 divide into evenly. This method is summarized below:

Method 2: Finding the LCD of a List of Denominators Using Prime Factorization

Step 1: Write the prime factorization of each denominator.

Step 2: For each different prime factor in Step 1, circle the *greatest* number of times that factor occurs in any one factorization.

Step 3: The LCD is the product of the circled factors.

Example 14 Find the LCD of $-\frac{23}{72}$ and $\frac{17}{60}$.

Solution: First we write the prime factorization of each denominator.

$$\begin{aligned} 72 &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \\ 60 &= 2 \cdot 2 \cdot 3 \cdot 5 \end{aligned}$$

For the prime factors shown, we circle the greatest number of factors found in either factorization.

$$\begin{aligned} 72 &= \textcircled{2 \cdot 2 \cdot 2} \cdot \textcircled{3 \cdot 3} \\ 60 &= 2 \cdot 2 \cdot 3 \cdot \textcircled{5} \end{aligned}$$

The LCD is the product of the circled factors.

$$\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 360$$

The LCD is 360.

Work Practice 14

Practice 14

Find the LCD of $-\frac{3}{40}$ and $\frac{11}{108}$.

Helpful Hint

If you prefer working with exponents, circle the factor with the greatest exponent.

Example 14:

$$\begin{aligned} 72 &= \textcircled{2^3} \cdot \textcircled{3^2} \\ 60 &= 2^2 \cdot 3 \cdot \textcircled{5} \\ \text{LCD} &= 2^3 \cdot 3^2 \cdot 5 = 360 \end{aligned}$$

Answer

14. 1080

Helpful Hint

If the number of factors of a prime number is equal, circle either one, but not both. For example,

$$12 = \boxed{2 \cdot 2} \cdot \boxed{3}$$
$$15 = 3 \cdot \boxed{5}$$

Circle either 3 but not both.

The LCD is $2 \cdot 2 \cdot 3 \cdot 5 = 60$.

Practice 15

Find the LCD of $\frac{7}{20}, \frac{1}{24}$, and $\frac{13}{60}$.

Example 15 Find the LCD of $\frac{1}{15}, \frac{5}{18}$, and $\frac{53}{54}$.

Solution:

$$15 = 3 \cdot \boxed{5}$$
$$18 = \boxed{2} \cdot 3 \cdot 3$$
$$54 = 2 \cdot \boxed{3 \cdot 3 \cdot 3}$$

The LCD is $2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$ or 270.

Work Practice 15

✓Concept Check True or false? The LCD of the fractions $\frac{1}{6}$ and $\frac{1}{8}$ is 48.

Objective E Writing Equivalent Fractions

To add or subtract unlike fractions in the next section, we first write equivalent fractions with the LCD as the denominator.

To write $\frac{1}{3}$ as an equivalent fraction with a denominator of 6, we multiply by 1 in the form of $\frac{2}{2}$. Why? Because $3 \cdot 2 = 6$, so the new denominator will become 6, as shown below.

$$\frac{1}{3} = \frac{1}{3} \cdot 1 = \frac{1}{3} \cdot \frac{2}{2} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$$

$$\frac{2}{2} = 1$$

So $\frac{1}{3} = \frac{2}{6}$.

Helpful Hint

Recall from the Helpful Hint on p. 193 that this is also called the Fundamental Property of Fractions.

$$\frac{a}{b} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a \cdot c}{b \cdot c}$$

To write an equivalent fraction,

$$\frac{a}{b} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a \cdot c}{b \cdot c}$$

where a, b, and c are nonzero numbers.

✓Concept Check Which of the following is *not* equivalent to $\frac{3}{4}$?

a. $\frac{6}{8}$ **b.** $\frac{18}{24}$ **c.** $\frac{9}{14}$ **d.** $\frac{30}{40}$

Answer

15. 120

✓Concept Check Answers

false; it is 24

c

Example 16 Write $\frac{3}{4}$ as an equivalent fraction with a denominator of 20.

$$\frac{3}{4} = \frac{}{20}$$

Solution: In the denominators, since $4 \cdot 5 = 20$, we will multiply by 1 in the form of $\frac{5}{5}$.

$$\frac{3}{4} = \frac{3}{4} \cdot \frac{5}{5} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$$

Thus, $\frac{3}{4} = \frac{15}{20}$.

Work Practice 16

Helpful Hint

To check Example 16, write $\frac{15}{20}$ in simplest form.

$$\frac{15}{20} = \frac{3 \cdot \overset{1}{\cancel{5}}}{4 \cdot \underset{1}{\cancel{5}}} = \frac{3}{4}, \text{ the original fraction.}$$

If the original fraction is in lowest terms, we can check our work by writing the new, equivalent fraction in simplest form. This form should be the original fraction.

✓**Concept Check** True or false? When the fraction $\frac{2}{9}$ is rewritten as an equivalent fraction with 27 as the denominator, the result is $\frac{2}{27}$.

Example 17 Write an equivalent fraction with the given denominator.

$$\frac{2}{5} = \frac{}{15}$$

Solution: Since $5 \cdot 3 = 15$, we multiply by 1 in the form of $\frac{3}{3}$.

$$\frac{2}{5} = \frac{2}{5} \cdot \frac{3}{3} = \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15}$$

Then $\frac{2}{5}$ is equivalent to $\frac{6}{15}$. They both represent the same part of a whole.

Work Practice 17

Example 18 Write an equivalent fraction with the given denominator.

$$3 = \frac{}{7}$$

Solution: Recall that $3 = \frac{3}{1}$. Since $1 \cdot 7 = 7$, multiply by 1 in the form $\frac{7}{7}$.

$$\frac{3}{1} = \frac{3}{1} \cdot \frac{7}{7} = \frac{3 \cdot 7}{1 \cdot 7} = \frac{21}{7}$$

Work Practice 18

✓**Concept Check** What is the first step in writing $\frac{3}{10}$ as an equivalent fraction whose denominator is 100?

Practice 16

Write $\frac{7}{8}$ as an equivalent fraction with a denominator of 56.

$$\frac{7}{8} = \frac{}{56}$$

Practice 17

Write an equivalent fraction with the given denominator.

$$\frac{1}{4} = \frac{}{44}$$

Practice 18

Write an equivalent fraction with the given denominator.

$$4 = \frac{}{6}$$

Answers

16. $\frac{49}{56}$ **17.** $\frac{11}{44}$ **18.** $\frac{24}{6}$

✓**Concept Check Answers**

false; the correct result is $\frac{6}{27}$

answers may vary

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

least common denominator (LCD) like $-\frac{a}{b}$ $\frac{a-c}{b}$ $\frac{a+c}{b}$ $-\frac{a}{-b}$

perimeter unlike equivalent

1. The fractions $\frac{9}{11}$ and $\frac{13}{11}$ are called ________________ fractions while $\frac{3}{4}$ and $\frac{1}{3}$ are called ________________ fractions.

2. $\frac{a}{b} + \frac{c}{b} =$ ________________ and $\frac{a}{b} - \frac{c}{b} =$ ________________.

3. As long as b is not 0, $\frac{-a}{b} = \frac{a}{-b} =$ ________________.

4. The distance around a figure is called its ________________.

5. The smallest positive number divisible by all the denominators of a list of fractions is called the ________________.

6. Fractions that represent the same portion of a whole are called ________________ fractions.

State whether the fractions in each list are like or unlike fractions.

7. $\frac{7}{8}, \frac{7}{10}$

8. $\frac{2}{3}, \frac{4}{9}$

9. $\frac{9}{10}, \frac{1}{10}$

10. $\frac{8}{11}, \frac{2}{11}$

11. $\frac{2}{31}, \frac{30}{31}, \frac{19}{31}$

12. $\frac{3}{10}, \frac{3}{11}, \frac{3}{13}$

13. $\frac{5}{12}, \frac{7}{12}, \frac{12}{11}$

14. $\frac{1}{5}, \frac{2}{5}, \frac{4}{5}$

Martin-Gay Interactive Videos

See Video 3.4

Watch the section lecture video and answer the following questions.

Objective A **15.** Complete this statement based on the lecture before Example 1: To add like fractions, we add the ____________ and keep the same ____________.

Objective B **16.** In Example 8, why are we told to be careful when substituting the replacement value for y?

Objective C **17.** What is the perimeter equation used to solve Example 9? What is the final answer?

Objective D **18.** In Example 10, the LCD is found to be 45. What does this mean in terms of the specific fractions in the problem?

Objective E **19.** From Example 12, why can we multiply a fraction by a form of 1 to get an equivalent fraction?

3.4 Exercise Set MyLab Math

Objective A *Add and simplify. See Examples 1 through 3 and 6.*

1. $\frac{1}{7} + \frac{2}{7}$

2. $\frac{9}{17} + \frac{2}{17}$

3. $\frac{1}{10} + \frac{1}{10}$

4. $\frac{1}{4} + \frac{1}{4}$

5. $\frac{2}{9} + \frac{4}{9}$

6. $\frac{3}{10} + \frac{2}{10}$

7. $-\frac{6}{20} + \frac{1}{20}$

8. $-\frac{1}{8} + \frac{3}{8}$

9. $-\frac{3}{14} + \left(-\frac{4}{14}\right)$

10. $-\frac{5}{24} + \left(-\frac{7}{24}\right)$

11. $\frac{10}{11} + \frac{3}{11}$

12. $\frac{13}{17} + \frac{9}{17}$

13. $\frac{4}{13} + \frac{2}{13} + \frac{1}{13}$

14. $\frac{5}{11} + \frac{1}{11} + \frac{2}{11}$

15. $-\frac{7}{18} + \frac{3}{18} + \frac{2}{18}$

16. $-\frac{7}{15} + \frac{3}{15} + \frac{1}{15}$

Subtract and simplify. See Examples 4, 5, and 7.

17. $\frac{10}{11} - \frac{4}{11}$

18. $\frac{9}{13} - \frac{5}{13}$

19. $\frac{4}{5} - \frac{1}{5}$

20. $\frac{7}{8} - \frac{4}{8}$

21. $\frac{7}{4} - \frac{3}{4}$

22. $\frac{18}{5} - \frac{3}{5}$

23. $\frac{7}{8} - \frac{1}{8}$

24. $\frac{5}{6} - \frac{1}{6}$

25. $-\frac{25}{12} - \frac{15}{12}$

26. $-\frac{30}{20} - \frac{15}{20}$

27. $\frac{11}{10} - \frac{3}{10}$

28. $\frac{14}{15} - \frac{4}{15}$

29. $-\frac{27}{33} - \left(-\frac{8}{33}\right)$

30. $-\frac{37}{45} - \left(-\frac{18}{45}\right)$

Mixed Practice *Perform the indicated operation. See Examples 1 through 8.*

31. $\frac{8}{21} + \frac{5}{21}$

32. $\frac{7}{37} + \frac{9}{37}$

33. $\frac{99}{100} - \frac{9}{100}$

34. $\frac{85}{200} - \frac{15}{200}$

35. $-\frac{13}{28} - \frac{13}{28}$

36. $-\frac{15}{26} - \frac{15}{26}$

37. $-\frac{3}{16} + \left(-\frac{7}{16}\right) + \left(-\frac{2}{16}\right)$

38. $-\frac{5}{18} + \left(-\frac{1}{18}\right) + \left(-\frac{6}{18}\right)$

Objective B *Evaluate each expression for the given replacement values. See Example 9.*

39. $x + y; x = \frac{3}{4}, y = \frac{2}{4}$

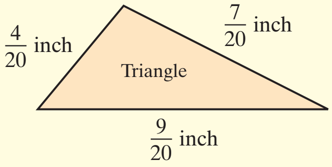
40. $x - y; x = \frac{7}{8}, y = \frac{9}{8}$

41. $x - y; x = -\frac{1}{5}, y = -\frac{3}{5}$

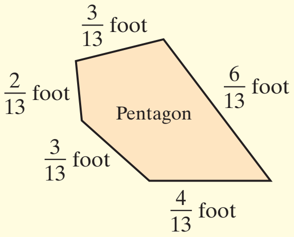
42. $x + y; x = -\frac{1}{6}, y = \frac{5}{6}$

Objective C *Find the perimeter of each figure. (Hint: Recall that perimeter means distance around.) See Example 10.*

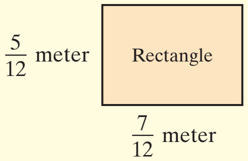
△ **43.**



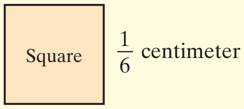
△ **44.**



▶ △ **45.**



△ **46.**



Solve. For Exercises 47 and 48, the solutions have been started for you. Write each answer in simplest form. See Example 11.

47. A railroad inspector must inspect $\frac{19}{20}$ of a mile of railroad track. If she has already inspected $\frac{5}{20}$ of a mile, how much more does she need to inspect?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blanks.)

distance left to inspect	is	distance needed to inspect	minus	distance already inspected
↓	↓	↓	↓	↓
distance left to inspect	=	______	−	______

Finish with:

3. SOLVE.

4. INTERPRET.

48. Scott Davis has run $\frac{11}{8}$ miles already and plans to complete $\frac{16}{8}$ miles. To do this, how much farther must he run?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blanks.)

distance left to run	is	distance planned to run	minus	distance already run
↓	↓	↓	↓	↓
distance left to run	=	______	−	______

Finish with:

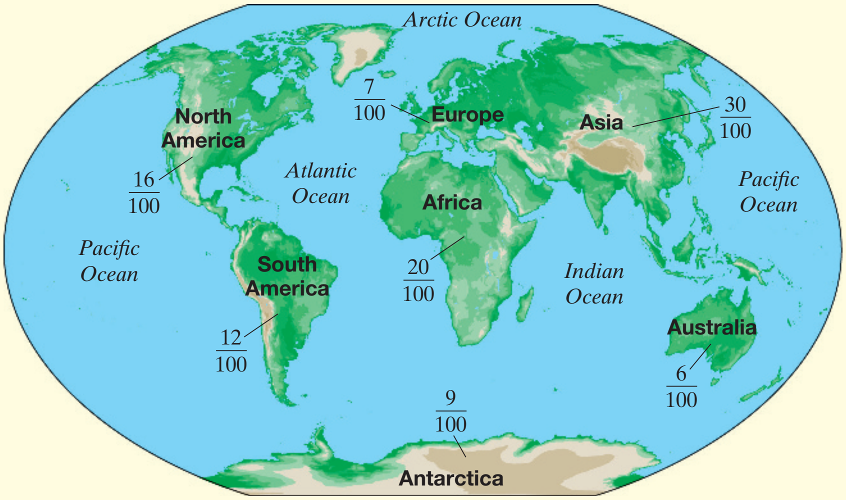
3. SOLVE.

4. INTERPRET.

49. As of 2017, the fraction of states in the United States with maximum interstate highway speed limits up to and including 70 mph was $\frac{33}{50}$. The fraction of states with 70 mph speed limits was $\frac{20}{50}$. What fraction of states had speed limits that were less than 70 mph? (*Source:* Insurance Institute for Highway Safety)

50. When people take aspirin, $\frac{31}{50}$ of the time it is used to treat some type of pain. Approximately $\frac{7}{50}$ of all aspirin use is for treating headaches. What fraction of aspirin use is for treating pain other than headaches? (*Source:* Bayer Market Research)

The map of the world below shows the fraction of the world's surface land area taken up by each continent. In other words, the continent of Africa, for example, makes up $\frac{20}{100}$ *of the land in the world. Use this map to solve Exercises 51 through 54. Write answers in simplest form.*



51. Find the fractional part of the world's land area within the continents of North America and South America.

52. Find the fractional part of the world's land area within the continents of Asia and Africa.

53. How much greater is the fractional part of the continent of Antarctica than the fractional part of the continent of Europe?

54. How much greater is the fractional part of the continent of Asia than the fractional part of the continent of Australia?

Objective D *Find the LCD of each list of fractions. See Examples 12 through 15.*

55. $\frac{2}{9}, \frac{6}{15}$

56. $\frac{7}{8}, \frac{3}{20}$

57. $-\frac{1}{36}, \frac{1}{24}$

58. $-\frac{1}{15}, \frac{1}{90}$

59. $\frac{2}{25}, \frac{3}{15}, \frac{5}{6}$

60. $\frac{3}{4}, \frac{1}{14}, \frac{13}{20}$

61. $-\frac{7}{24}, -\frac{5}{7}$

62. $-\frac{11}{3}, -\frac{13}{64}$

63. $\frac{23}{18}, \frac{1}{21}$

64. $\frac{45}{24}, \frac{2}{45}$

65. $\frac{4}{3}, \frac{8}{21}, \frac{3}{56}$

66. $\frac{12}{11}, \frac{20}{33}, \frac{12}{121}$

Objective E *Write each fraction as an equivalent fraction with the given denominator. See Examples 16 through 18.*

67. $\frac{2}{3} = \frac{\quad}{21}$

68. $\frac{5}{6} = \frac{\quad}{24}$

69. $\frac{4}{7} = \frac{\quad}{35}$

70. $\frac{3}{5} = \frac{\quad}{100}$

71. $\frac{1}{2} = \frac{\quad}{50}$

72. $\frac{1}{5} = \frac{\quad}{50}$

73. $\frac{14}{17} = \frac{\quad}{68}$

74. $\frac{19}{21} = \frac{\quad}{126}$

75. $\frac{2}{3} = \frac{\quad}{12}$

76. $\frac{3}{2} = \frac{\quad}{12}$

77. $\frac{5}{9} = \frac{\quad}{36}$

78. $\frac{7}{6} = \frac{\quad}{36}$

A nonstore retailer is a mail-order business that sells goods via catalogs, toll-free telephone numbers, or online media. The table shows the fraction of nonstore retailers' goods that were sold online in a recent year by types of goods. Use this table to answer Exercises 79 through 82. (Hint for Exercises 80, 81, and 82: To compare fractions with the same denominator, simply compare their numerators.)

Types of Goods Sold by Nonstore Retailers	Fraction of Goods That Were Sold Online	Equivalent Fraction with a Denominator of 100
Books and magazines	$\frac{43}{50}$	
Clothing and accessories	$\frac{4}{5}$	
Computer hardware	$\frac{29}{50}$	
Computer software	$\frac{17}{25}$	
Drugs, health and beauty aids	$\frac{3}{25}$	
Electronics and appliances	$\frac{21}{25}$	
Food, beer, and wine	$\frac{17}{25}$	
Home furnishings	$\frac{81}{100}$	
Music and videos	$\frac{9}{10}$	
Office equipment and supplies	$\frac{79}{100}$	
Sporting goods	$\frac{37}{50}$	
Toys, hobbies, and games	$\frac{39}{50}$	

(*Source:* U.S. Census Bureau)

79. Complete the table by writing each fraction as an equivalent fraction with a denominator of 100.

80. Which of these types of goods had the largest fraction sold online?

81. Which of these types of goods had the smallest fraction sold online?

82. Which of the types of goods had **more than** $\frac{4}{5}$ of the goods sold online? (*Hint:* Write $\frac{4}{5}$ as an equivalent fraction with a denominator of 100.)

Review

Simplify. See Section 1.9.

83. 3^2

84. 4^3

85. 3^4

86. 7^2

87. $2^3 \cdot 3$

88. $4^2 \cdot 5$

Concept Extensions

Find and correct the error. See the first Concept Check in this section.

89. $\xcancel{\frac{2}{7} + \frac{9}{7} = \frac{11}{14}}$

90. $\xcancel{\frac{3}{4} - \frac{1}{4} = \frac{2}{8} = \frac{1}{4}}$

Solve.

91. In your own words, explain how to add like fractions.

92. In your own words, explain how to subtract like fractions.

93. Use the map of the world for Exercises **51** through **54** and find the sum of all the continents' fractions. Explain your answer.

94. Mike Cannon jogged $\frac{3}{8}$ of a mile from home and then rested. Then he continued jogging farther from home for another $\frac{3}{8}$ of a mile until he discovered his watch had fallen off. He walked back along the same path for $\frac{4}{8}$ of a mile until he found his watch. Find how far he was from his home.

Write each fraction as an equivalent fraction with the indicated denominator.

95. $\frac{37}{165} = \frac{}{3630}$

96. $\frac{108}{215} = \frac{}{4085}$

97. In your own words, explain how to find the LCD of two fractions.

98. In your own words, explain how to write a fraction as an equivalent fraction with a given denominator.

Solve. See the second through fourth Concept Checks in this section.

99. Which of the following are equivalent to $\frac{2}{3}$?

a. $\frac{10}{15}$ **b.** $\frac{40}{60}$

c. $\frac{16}{20}$ **d.** $\frac{200}{300}$

100. True or false? When the fraction $\frac{7}{12}$ is rewritten with a denominator of 48, the result is $\frac{11}{48}$. If false, give the correct fraction.

Answers

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21.

22.

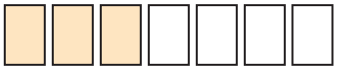
23. a. b. c.

24. a. b. c.

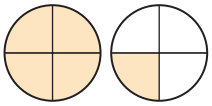
Summary on Fractions and Operations on Fractions

Use a fraction to represent the shaded area of each figure. If the fraction is improper, also write the fraction as a mixed number.

1.



2.



Solve.

3. In a survey, 73 people out of 85 get less than 8 hours of sleep each night. What fraction of people in the survey get less than 8 hours of sleep?

4. Sketch a diagram to represent $\frac{9}{13}$.

Simplify.

5. $\frac{11}{-11}$ **6.** $\frac{17}{1}$ **7.** $\frac{0}{-3}$ **8.** $\frac{7}{0}$

Write the prime factorization of each composite number. Write any repeated factors using exponents.

9. 65 **10.** 70 **11.** 315 **12.** 441

Write each fraction in simplest form.

13. $\frac{2}{14}$ **14.** $\frac{24}{20}$ **15.** $-\frac{56}{60}$ **16.** $-\frac{72}{80}$

17. $\frac{54}{135}$ **18.** $\frac{90}{240}$ **19.** $\frac{165}{210}$ **20.** $\frac{245}{385}$

Determine whether each pair of fractions is equivalent.

21. $\frac{7}{8}$ and $\frac{9}{10}$ **22.** $\frac{10}{12}$ and $\frac{15}{18}$

23. Of the 50 states, 2 states are not adjacent to any other states.

a. What fraction of the states are not adjacent to other states?
b. How many states are adjacent to other states?
c. What fraction of the states are adjacent to other states?

24. In 2017, 363 new films were released and rated. Of these, 129 were rated PG-13. (*Source:* Nash Information, LLC)

a. What fraction were rated PG-13?
b. How many films were rated other than PG-13?
c. What fraction of films were rated other than PG-13?

Find the LCD of each list of fractions.

25. $\frac{3}{5}, \frac{1}{6}$

26. $\frac{1}{2}, \frac{11}{14}$

27. $\frac{5}{6}, \frac{13}{18}, \frac{17}{30}$

Write each fraction as an equivalent fraction with the indicated denominator.

28. $\frac{7}{9} = \frac{\quad}{36}$

29. $\frac{11}{15} = \frac{\quad}{75}$

30. $\frac{5}{6} = \frac{\quad}{48}$

The following summary will help you with the following review of operations on fractions.

Operations on Fractions

Let a, b, c, and d be integers.

Addition: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$(b \neq 0)$ $\uparrow \quad \uparrow \quad \uparrow$

common denominator

Subtraction: $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

$(b \neq 0)$ $\uparrow \quad \uparrow \quad \uparrow$

common denominator

Multiplication: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$(b \neq 0, d \neq 0)$

Division: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

$(b \neq 0, d \neq 0, c \neq 0)$

Perform each indicated operation.

31. $\frac{9}{10} + \frac{3}{10}$

32. $\frac{9}{10} - \frac{3}{10}$

33. $\frac{9}{10} \cdot \frac{2}{3}$

34. $\frac{9}{10} \div \frac{2}{3}$

35. $\frac{21}{70} - \frac{3}{70}$

36. $\frac{21}{70} + \frac{3}{70}$

37. $\frac{21}{25} \div \frac{3}{70}$

38. $\frac{21}{25} \cdot \frac{3}{70}$

39. $-\frac{7}{9} \cdot \frac{4}{5}$

40. $-\frac{3}{11} \div \left(-\frac{3}{10}\right)$

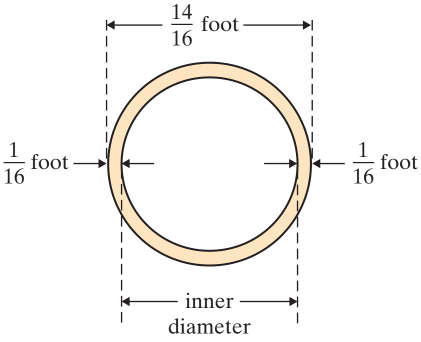
41. $-\frac{14}{27} - \frac{4}{27}$

42. $-\frac{8}{45} + \frac{6}{45}$

Solve.

43. A contractor is using 18 acres of his land to sell $\frac{3}{4}$-acre lots. How many lots can he sell?

44. Suppose that the cross-section of a piece of pipe looks like the diagram shown. What is the inner diameter?



25.

26.

27.

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42.

43.

44.

3.5 Adding and Subtracting Unlike Fractions

Objectives

A Add or Subtract Unlike Fractions.

B Write Fractions in Order.

C Evaluate Expressions Given Fractional Replacement Values.

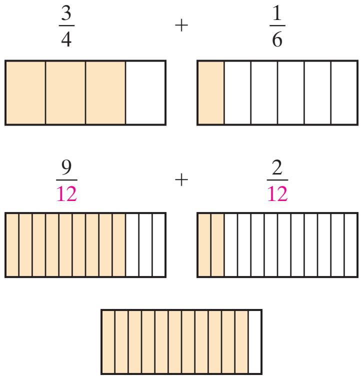
D Solve Problems by Adding or Subtracting Unlike Fractions.

Objective A Adding or Subtracting Unlike Fractions

In this section we add and subtract fractions with unlike denominators. To add or subtract these unlike fractions, we first write the fractions as equivalent fractions with a common denominator and then add or subtract the like fractions. Recall from the previous section that the common denominator we use is called the **least common denominator (LCD).**

To begin, let's add the unlike fractions $\frac{3}{4} + \frac{1}{6}$.

The LCD of these fractions is 12. So we write each fraction as an equivalent fraction with a denominator of 12, and then add as usual. This addition process is shown next and is also illustrated by figures.

Add: $\frac{3}{4} + \frac{1}{6}$	**The LCD is 12.**
Figures	**Algebra**
 $\frac{9}{12} + \frac{2}{12} = \frac{11}{12}$	$\frac{3}{4} = \frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}$ and $\frac{1}{6} = \frac{1}{6} \cdot \frac{2}{2} = \frac{2}{12}$ Remember, $\frac{3}{3} = 1$ and $\frac{2}{2} = 1$. Now we can add just as we did in Section 3.4. $\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$
Thus, the sum is $\frac{11}{12}$.	

Adding or Subtracting Unlike Fractions

Step 1: Find the least common denominator (LCD) of the fractions.

Step 2: Write each fraction as an equivalent fraction whose denominator is the LCD.

Step 3: Add or subtract the like fractions.

Step 4: Write the sum or difference in simplest form.

Example 1 Add: $\frac{2}{5} + \frac{4}{15}$

Solution:

Step 1: The LCD of the fractions is 15. In later examples, we will simply say, for example, that the LCD of the denominators 5 and 15 is 15.

Step 2: $\frac{2}{5} = \frac{2}{5} \cdot \underset{\uparrow}{\frac{3}{3}} = \frac{6}{15} \qquad \frac{4}{15} = \frac{4}{15}$ ← This fraction already has a denominator of 15.

Multiply by 1 in the form $\frac{3}{3}$.

Step 3: $\frac{2}{5} + \frac{4}{15} = \frac{6}{15} + \frac{4}{15} = \frac{10}{15}$

Step 4: Write in simplest form.

$$\frac{10}{15} = \frac{2 \cdot \overset{1}{\cancel{5}}}{3 \cdot \underset{1}{\cancel{5}}} = \frac{2}{3}$$

Work Practice 1

Practice 1

Add: $\frac{2}{7} + \frac{8}{21}$

Example 2 Add: $\frac{2}{15} + \frac{3}{10}$

Solution:

Step 1: The LCD of the denominators 15 and 10 is 30.

Step 2: $\frac{2}{15} = \frac{2}{15} \cdot \frac{2}{2} = \frac{4}{30} \qquad \frac{3}{10} = \frac{3}{10} \cdot \frac{3}{3} = \frac{9}{30}$

Step 3: $\frac{2}{15} + \frac{3}{10} = \frac{4}{30} + \frac{9}{30} = \frac{13}{30}$

Step 4: $\frac{13}{30}$ is in simplest form.

Work Practice 2

Practice 2

Add: $\frac{5}{6} + \frac{2}{9}$

Example 3 Add: $-\frac{1}{6} + \frac{1}{2}$

Solution: The LCD of the denominators 6 and 2 is 6.

$$-\frac{1}{6} + \frac{1}{2} = \frac{-1}{6} + \frac{1 \cdot 3}{2 \cdot 3}$$
$$= \frac{-1}{6} + \frac{3}{6}$$
$$= \frac{2}{6}$$

Helpful Hint Recall that

$$-\frac{1}{6} = \frac{-1}{6} = \frac{1}{-6}$$

Next, simplify $\frac{2}{6}$.

$$\frac{2}{6} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot 3} = \frac{1}{3}$$

Work Practice 3

Practice 3

Add: $-\frac{1}{5} + \frac{9}{20}$

Answers

1. $\frac{2}{3}$ **2.** $\frac{19}{18}$ **3.** $\frac{1}{4}$

✓Concept Check Find and correct the error in the following:

$$\xcancel{\frac{2}{9}+\frac{4}{11}=\frac{6}{20}=\frac{3}{10}}$$

Practice 4

Subtract: $\frac{7}{12}-\frac{5}{24}$

Example 4

Subtract: $\frac{2}{5}-\frac{3}{20}$

Solution:

Step 1: The LCD of the denominators 5 and 20 is 20.

Step 2: $\frac{2}{5}=\frac{2}{5}\cdot\frac{4}{4}=\frac{8}{20}\qquad\frac{3}{20}=\frac{3}{20}$ ← This fraction already has a denominator of 20.

Step 3: $\frac{2}{5}-\frac{3}{20}=\frac{8}{20}-\frac{3}{20}=\frac{5}{20}$

Step 4: Write in simplest form.

$$\frac{5}{20}=\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\cdot 4}=\frac{1}{4}$$

Work Practice 4

Practice 5

Subtract: $\frac{5}{7}-\frac{9}{10}$

Example 5

Subtract: $\frac{2}{3}-\frac{10}{11}$

Solution:

Step 1: The LCD of the denominators 3 and 11 is 33.

Step 2: $\frac{2}{3}=\frac{2}{3}\cdot\frac{11}{11}=\frac{22}{33}\qquad\frac{10}{11}=\frac{10}{11}\cdot\frac{3}{3}=\frac{30}{33}$

Step 3: $\frac{2}{3}-\frac{10}{11}=\frac{22}{33}-\frac{30}{33}=\frac{-8}{33}$ or $-\frac{8}{33}$

Step 4: $-\frac{8}{33}$ is in simplest form.

Work Practice 5

Practice 6

Find: $\frac{5}{8}-\frac{1}{3}-\frac{1}{12}$

Example 6

Find: $-\frac{3}{4}-\frac{1}{14}+\frac{6}{7}$

Solution: The LCD of the denominators 4, 14, and 7 is 28.

$$\begin{aligned}-\frac{3}{4}-\frac{1}{14}+\frac{6}{7}&=-\frac{3\cdot 7}{4\cdot 7}-\frac{1\cdot 2}{14\cdot 2}+\frac{6\cdot 4}{7\cdot 4}\\&=-\frac{21}{28}-\frac{2}{28}+\frac{24}{28}\\&=\frac{1}{28}\end{aligned}$$

Work Practice 6

✓Concept Check Find and correct the error in the following:

$$\xcancel{\frac{7}{12}-\frac{3}{4}=\frac{4}{8}=\frac{1}{2}}$$

Answers

4. $\frac{3}{8}$ **5.** $-\frac{13}{70}$ **6.** $\frac{5}{24}$

✓Concept Check Answers

When adding or subtracting fractions, we don't add or subtract the denominators. Correct solutions:

$\frac{2}{9}+\frac{4}{11}=\frac{22}{99}+\frac{36}{99}=\frac{58}{99}$

$\frac{7}{12}-\frac{3}{4}=\frac{7}{12}-\frac{9}{12}=-\frac{2}{12}=-\frac{1}{6}$

Objective B Writing Fractions in Order

One important application of the least common denominator is using the LCD to help order or compare fractions.

Example 7 Insert < or > to form a true sentence.

$$\frac{3}{4} \qquad \frac{9}{11}$$

Solution: The LCD for these fractions is 44. Let's write each fraction as an equivalent fraction with a denominator of 44.

$$\frac{3}{4} = \frac{3 \cdot 11}{4 \cdot 11} = \frac{33}{44} \qquad \frac{9}{11} = \frac{9 \cdot 4}{11 \cdot 4} = \frac{36}{44}$$

Since 33 $<$ 36, then

$$\frac{33}{44} < \frac{36}{44} \text{ or}$$

$$\frac{3}{4} < \frac{9}{11}$$

Work Practice 7

Practice 7

Insert < or > to form a true sentence.

$$\frac{5}{8} \qquad \frac{11}{20}$$

Example 8 Insert < or > to form a true sentence.

$$-\frac{2}{7} \qquad -\frac{1}{3}$$

Solution: The LCD of the denominators 7 and 3 is 21.

$$-\frac{2}{7} = -\frac{2 \cdot 3}{7 \cdot 3} = -\frac{6}{21} \text{ or } \frac{-6}{21} \qquad -\frac{1}{3} = -\frac{1 \cdot 7}{3 \cdot 7} = -\frac{7}{21} \text{ or } \frac{-7}{21}$$

Since $-6 > -7$, then

$$-\frac{6}{21} > -\frac{7}{21} \text{ or}$$

$$-\frac{2}{7} > -\frac{1}{3}$$

Work Practice 8

Practice 8

Insert < or > to form a true sentence.

$$-\frac{17}{20} \qquad -\frac{4}{5}$$

Objective C Evaluating Expressions Given Fractional Replacement Values

Example 9 Evaluate $x - y$ if $x = \frac{7}{18}$ and $y = \frac{2}{9}$.

Solution: Replace x with $\frac{7}{18}$ and y with $\frac{2}{9}$ in the expression $x - y$.

$$x - y = \frac{7}{18} - \frac{2}{9}$$

(*Continued on next page*)

Practice 9

Evaluate $x - y$ if $x = \frac{10}{11}$ and $y = \frac{9}{22}$.

Answers

7. $>$ **8.** $<$ **9.** $\frac{1}{2}$

The LCD of the denominators 18 and 9 is 18. Then

$$\frac{7}{18} - \frac{2}{9} = \frac{7}{18} - \frac{2 \cdot 2}{9 \cdot 2}$$
$$= \frac{7}{18} - \frac{4}{18}$$
$$= \frac{3}{18} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot 6} = \frac{1}{6} \quad \text{Simplified}$$

Work Practice 9

Objective D Solving Problems by Adding or Subtracting Unlike Fractions

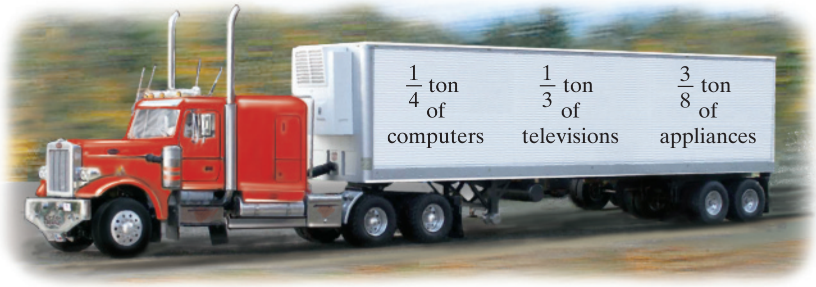
Very often, real-world problems involve adding or subtracting unlike fractions.

Practice 10

To repair her sidewalk, a homeowner must pour cement in three different locations. She needs $\frac{3}{5}$ of a cubic yard, $\frac{3}{10}$ of a cubic yard, and $\frac{1}{15}$ of a cubic yard for these locations. Find the total amount of cement the homeowner needs.

Example 10 Finding Total Weight

A freight truck has $\frac{1}{4}$ ton of computers, $\frac{1}{3}$ ton of televisions, and $\frac{3}{8}$ ton of small appliances. Find the total weight of its load.



Solution:

1. UNDERSTAND. Read and reread the problem. The phrase "total weight" tells us to add.
2. TRANSLATE.

In words:	total weight	is	weight of computers	plus	weight of televisions	plus	weight of appliances
	↓	↓	↓	↓	↓	↓	↓
Translate:	total weight	=	$\frac{1}{4}$	+	$\frac{1}{3}$	+	$\frac{3}{8}$

3. SOLVE: The LCD is 24.

$$\frac{1}{4} + \frac{1}{3} + \frac{3}{8} = \frac{1}{4} \cdot \frac{6}{6} + \frac{1}{3} \cdot \frac{8}{8} + \frac{3}{8} \cdot \frac{3}{3}$$
$$= \frac{6}{24} + \frac{8}{24} + \frac{9}{24}$$
$$= \frac{23}{24}$$

4. INTERPRET. *Check* the solution. *State* your conclusion: The total weight of the truck's load is $\frac{23}{24}$ ton.

Work Practice 10

Answer

10. $\frac{29}{30}$ cu yd

Example 11 Calculating Flight Time

A flight from Tucson to Phoenix, Arizona, requires $\frac{5}{12}$ of an hour. If the plane has been flying $\frac{1}{4}$ of an hour, find how much time remains before landing.

Solution:

1. UNDERSTAND. Read and reread the problem. The phrase "how much time remains" tells us to subtract.

2. TRANSLATE.

In words:	time remaining	is	flight time from Tucson to Phoenix	minus	flight time already passed
	↓	↓	↓	↓	↓
Translate:	time remaining	=	$\frac{5}{12}$	−	$\frac{1}{4}$

3. SOLVE: The LCD is 12.

$$\frac{5}{12} - \frac{1}{4} = \frac{5}{12} - \frac{1}{4} \cdot \frac{3}{3}$$
$$= \frac{5}{12} - \frac{3}{12}$$
$$= \frac{2}{12} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot 6} = \frac{1}{6}$$

4. INTERPRET. *Check* the solution. *State* your conclusion: The remaining flight time is $\frac{1}{6}$ of an hour.

Work Practice 11

Practice 11

Find the difference in length of two boards if one board is $\frac{3}{4}$ of a foot long and the other is $\frac{2}{3}$ of a foot long.

Answer

11. $\frac{1}{12}$ ft

Calculator Explorations Performing Operations on Fractions

Scientific Calculator

Many calculators have a fraction key, such as $\boxed{a^b/_c}$, that allows you to enter fractions and perform operations on fractions and will give the result as a fraction. If your calculator has a fraction key, use it to calculate

$$\frac{3}{5} + \frac{4}{7}$$

Enter the keystrokes

$\boxed{3}\ \boxed{a^b/_c}\ \boxed{5}\ \boxed{+}\ \boxed{4}\ \boxed{a^b/_c}\ \boxed{7}\ \boxed{=}$

The display should read $\boxed{1\_6 \mid 35}$

which represents the mixed number $1\frac{6}{35}$. Let's write the result as a fraction. To convert from mixed number notation to fractional notation, press

$\boxed{2^{\text{nd}}}\ \boxed{d/c}$

The display now reads $\boxed{41 \mid 35}$

which represents $\frac{41}{35}$, the sum in fractional notation.

Graphing Calculator

Graphing calculators also allow you to perform operations on fractions and will give exact fractional results. The fraction option on a graphing calculator may be found under the $\boxed{\text{MATH}}$ menu. To perform the addition in the left column, try the keystrokes

$\boxed{3}\ \boxed{\div}\ \boxed{5}\ \boxed{+}\ \boxed{4}\ \boxed{\div}\ \boxed{7}\ \boxed{\text{MATH}}\ \boxed{\text{ENTER}}$
$\boxed{\text{ENTER}}$

The display should read

$\boxed{3/5 + 4/7 ▶ \text{Frac } 41/35}$

Use a calculator to add the following fractions. Give each sum as a fraction.

1. $\frac{1}{16} + \frac{2}{5}$

2. $\frac{3}{20} + \frac{2}{25}$

3. $\frac{4}{9} + \frac{7}{8}$

4. $\frac{9}{11} + \frac{5}{12}$

5. $\frac{10}{17} + \frac{12}{19}$

6. $\frac{14}{31} + \frac{15}{21}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used. Any numerical answers are not listed.

expression	least common denominator	$>$
equivalent	$<$	

1. To add or subtract unlike fractions, we first write the fractions as ______________ fractions with a common denominator. The common denominator we use is called the ______________.

2. The LCD for $\frac{1}{6}$ and $\frac{5}{8}$ is ______________.

3. $\frac{1}{6}+\frac{5}{8}=\frac{1}{6}\cdot\frac{4}{4}+\frac{5}{8}\cdot\frac{3}{3}=\underline{\quad}+\underline{\quad}=\underline{\quad}$.

4. $\frac{1}{6}-\frac{5}{8}=\frac{1}{6}\cdot\frac{4}{4}-\frac{5}{8}\cdot\frac{3}{3}=\underline{\quad}-\underline{\quad}=\underline{\quad\ }$.

5. $x - y$ is a(n) ______________ .

6. Since $-10 < -1$, we know that $-\frac{10}{13}$ ______________ $-\frac{1}{13}$.

Write the LCD of each pair of fractions.

7. $\frac{1}{2}, \frac{2}{3}$

8. $\frac{1}{2}, \frac{3}{7}$

9. $\frac{1}{6}, \frac{5}{12}$

10. $\frac{2}{5}, \frac{7}{10}$

11. $\frac{4}{7}, \frac{1}{8}$

12. $\frac{2}{3}, \frac{3}{11}$

13. $\frac{11}{12}, \frac{3}{4}$

14. $\frac{23}{24}, \frac{1}{3}$

Martin-Gay Interactive Videos

See Video 3.5

Watch the section lecture video and answer the following questions.

Objective A 15. In Example 4, why can't we add the two terms in the numerators?

Objective B 16. In Example 6, when comparing two fractions, how does writing each fraction with the same denominator help?

Objective C 17. In Example 7, if we had chosen to simplify the first fraction before adding, what would our addition problem have become and what would our LCD have been?

Objective D 18. Is the answer to Example 8 a proper or an improper fraction?

3.5 Exercise Set MyLab Math

Objective A *Add or subtract as indicated. See Examples 1 through 6.*

1. $\frac{2}{3}+\frac{1}{6}$

2. $\frac{5}{6}+\frac{1}{12}$

3. $\frac{1}{2}-\frac{1}{3}$

4. $\frac{2}{3}-\frac{1}{4}$

▶ **5.** $-\frac{2}{11}+\frac{2}{33}$

6. $-\frac{5}{9}+\frac{1}{3}$

7. $\frac{3}{14}-\frac{3}{7}$

8. $\frac{2}{15}-\frac{2}{5}$

9. $\frac{11}{35} + \frac{2}{7}$

10. $\frac{2}{5} + \frac{3}{25}$

11. $2 - \frac{5}{12}$

12. $5 - \frac{3}{20}$

13. $\frac{5}{12} - \frac{1}{9}$

14. $\frac{7}{12} - \frac{5}{18}$

15. $\frac{5}{7} + 1$

16. $-10 + \frac{7}{10}$

17. $\frac{5}{11} + \frac{4}{9}$

18. $\frac{7}{18} + \frac{2}{9}$

19. $\frac{2}{3} - \frac{1}{6}$

20. $\frac{5}{6} - \frac{1}{12}$

21. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27}$

22. $\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$

23. $-\frac{2}{11} - \frac{2}{33}$

24. $-\frac{5}{9} - \frac{1}{3}$

25. $\frac{9}{14} - \frac{3}{7}$

26. $\frac{4}{5} - \frac{2}{15}$

27. $\frac{11}{35} - \frac{2}{7}$

28. $\frac{2}{5} - \frac{3}{25}$

29. $\frac{1}{9} - \frac{5}{12}$

30. $\frac{5}{18} - \frac{7}{12}$

31. $\frac{7}{15} - \frac{5}{12}$

32. $\frac{5}{8} - \frac{3}{20}$

33. $\frac{5}{7} - \frac{1}{8}$

34. $\frac{10}{13} - \frac{7}{10}$

35. $\frac{7}{8} + \frac{3}{16}$

36. $-\frac{7}{18} - \frac{2}{9}$

37. $\frac{5}{9} + \frac{3}{9}$

38. $\frac{4}{13} - \frac{1}{13}$

39. $-\frac{2}{5} + \frac{1}{3} - \frac{3}{10}$

40. $-\frac{1}{3} - \frac{1}{4} + \frac{2}{5}$

41. $-\frac{5}{6} - \frac{3}{7}$

42. $\frac{1}{2} - \frac{3}{29}$

43. $\frac{7}{9} - \frac{1}{6}$

44. $\frac{9}{16} - \frac{3}{8}$

45. $\frac{5}{11} + \frac{3}{13}$

46. $\frac{3}{7} + \frac{9}{17}$

47. $\frac{7}{30} - \frac{5}{12}$

48. $\frac{7}{30} - \frac{3}{20}$

49. $\frac{6}{5} - \frac{3}{4} + \frac{1}{2}$

50. $\frac{6}{5} + \frac{3}{4} - \frac{1}{2}$

51. $\frac{4}{5} + \frac{4}{9}$

52. $\frac{11}{12} - \frac{7}{24}$

53. $-\frac{9}{12} + \frac{17}{24} - \frac{1}{6}$

54. $-\frac{5}{14} + \frac{3}{7} - \frac{1}{2}$

55. $\frac{1}{1000} - \frac{1}{100}$

56. $\frac{1}{500} - \frac{1}{50}$

Objective B *Insert $<$ or $>$ to form a true sentence. See Examples 7 and 8.*

57. $\frac{2}{7} \quad \frac{3}{10}$

58. $\frac{5}{9} \quad \frac{6}{11}$

59. $\frac{5}{6} \quad -\frac{13}{15}$

60. $-\frac{7}{8} \quad \frac{5}{6}$

61. $-\frac{3}{4} \quad -\frac{11}{14}$

62. $-\frac{2}{9} \quad -\frac{3}{13}$

Objective C and Section 3.3 Mixed Practice *Evaluate each expression if $x = \frac{1}{3}$ and $y = \frac{3}{4}$. See Example 9.*

63. $x + y$

64. $x - y$

65. xy

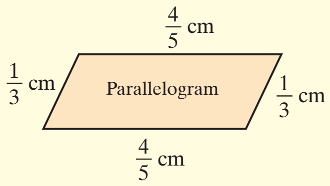
66. $x \div y$

67. $2y + x$

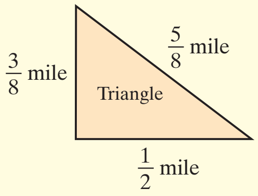
68. $2x + y$

Objective D *Find the perimeter of each geometric figure. (Hint: Recall that perimeter means distance around.)*

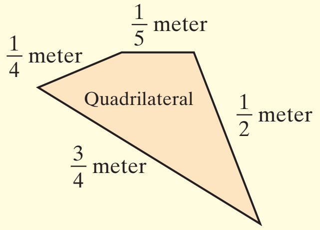
△ **69.**



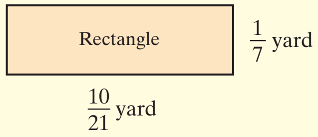
△ **70.**



△ **71.**



△ **72.**



Translating *Translate each phrase to an algebraic expression. Use "x" to represent "a number." See Examples 10 and 11.*

73. The sum of a number and $\frac{1}{2}$

74. A number increased by $-\frac{2}{5}$

75. A number subtracted from $-\frac{3}{8}$

76. The difference of a number and $\frac{7}{20}$

Solve. For Exercises 77 and 78, the solutions have been started for you. See Examples 10 and 11.

77. The slowest mammal is the three-toed sloth from South America. The sloth has an average ground speed of $\frac{1}{10}$ mph. In the trees, it can accelerate to $\frac{17}{100}$ mph. How much faster can a sloth travel in the trees? (*Source: The Guinness Book of World Records*)

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blanks.)

how much faster sloth travels in trees	is	sloth speed in trees	minus	sloth speed on ground
↓	↓	↓	↓	↓
how much faster sloth travels in trees	=	____	−	____

Finish with:

3. SOLVE.

4. INTERPRET.

78. Killer bees have been known to chase people for up to $\frac{1}{4}$ of a mile, while domestic European honeybees will normally chase a person for no more than 100 feet, or $\frac{5}{264}$ of a mile. How much farther will a killer bee chase a person than a domestic honeybee? (*Source:* Coachella Valley Mosquito & Vector Control District)

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

2. TRANSLATE into an equation. (Fill in the blanks.)

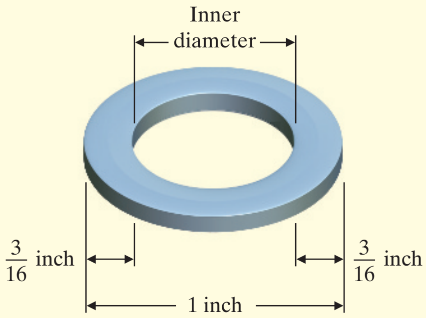
how much farther killer bee will chase than honeybee	is	distance killer bee chases	minus	distance honeybee chases
↓	↓	↓	↓	↓
how much farther killer bee will chase than honeybee	=	____	−	____

Finish with:

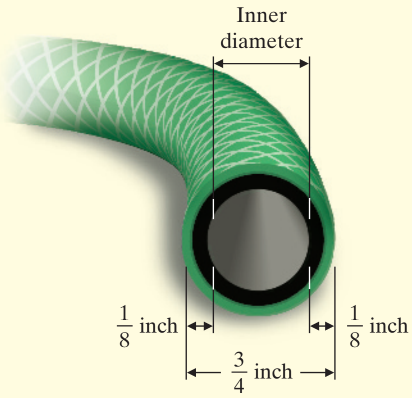
3. SOLVE.

4. INTERPRET.

79. Find the inner diameter of the washer.



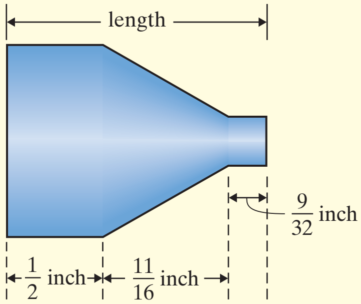
80. Find the inner diameter of the tubing.



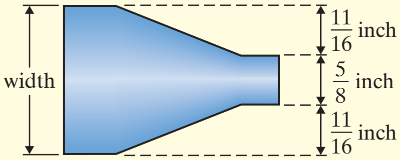
81. About $\frac{13}{20}$ of American students ages 10 to 17 name math, science, or art as their favorite subject in school. Art is the favorite subject for about $\frac{4}{25}$ of the American students ages 10 to 17. For what fraction of students this age is math or science their favorite subject? (*Source:* Peter D. Hart Research Associates for the National Science Foundation)

82. In the 2017 Formula One world-racing season, the Mercedes team won $\frac{3}{5}$ of the races. If Mercedes driver Lewis Hamilton won $\frac{9}{20}$ of the races, what fraction did the other Mercedes driver, Valtteri Bottas, win? (*Source:* Formula1.com)

83. Given the following diagram, find its total length.



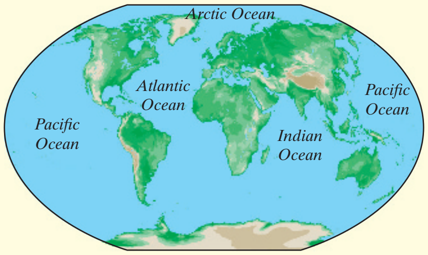
84. Given the following diagram, find its total width.



The table below shows the fraction of the Earth's water area taken up by each ocean. Use this table for Exercises 85 and 86.

Fraction of Earth's Water Area per Ocean

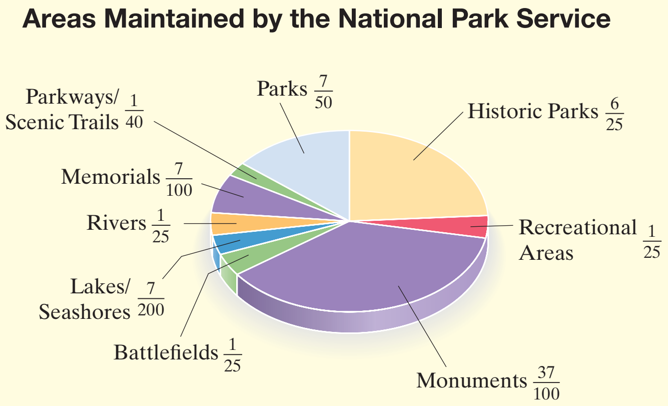
Ocean	Fraction
Arctic	$\frac{1}{25}$
Atlantic	$\frac{13}{50}$
Pacific	$\frac{1}{2}$
Indian	$\frac{1}{5}$



85. What fraction of the world's water surface area is accounted for by the Pacific and Atlantic Oceans?

86. What fraction of the world's water surface area is accounted for by the Arctic and Indian Oceans?

Use this circle graph to answer Exercises 87 through 90.



Source: National Park Service

87. What fraction of areas maintained by the National Park Service are designated as rivers or lakes/seashores?

88. What fraction of areas maintained by the National Park Service are designated as recreation areas or memorials?

89. What fraction of areas maintained by the National Park Service are NOT national monuments?

90. What fraction of areas maintained by the National Park Service are NOT national parkways or scenic trails?

Review

Use the order of operations to simplify. See Section 1.9.

91. $50 \div 5 \cdot 2$

92. $8 - 6 \cdot 4 - 7$

93. $(8 - 6) \cdot (4 - 7)$

94. $50 \div (5 \cdot 2)$

Concept Extensions

For Exercises 95 and 96 below, do the following:

a. *Draw three rectangles of the same size and represent each fraction in the sum or difference, one fraction per rectangle, by shading.*

b. *Using these rectangles as estimates, determine whether there is an error in the sum or difference.*

c. *If there is an error, correctly calculate the sum or difference.*

See the Concept Checks in this section.

95. $\frac{3}{5} + \frac{4}{5} \stackrel{?}{=} \frac{7}{10}$

96. $\frac{5}{8} - \frac{3}{4} \stackrel{?}{=} \frac{2}{4}$

Subtract from left to right.

97. $\frac{2}{3} - \frac{1}{4} - \frac{2}{540}$

98. $\frac{9}{10} - \frac{7}{200} - \frac{1}{3}$

Perform each indicated operation.

99. $\frac{30}{55} + \frac{1000}{1760}$

100. $\frac{19}{26} - \frac{968}{1352}$

101. In your own words, describe how to add or subtract two fractions with different denominators.

102. Find the sum of the fractions in the circle graph above. Did the sum surprise you? Why or why not?

103. In 2016, about $\frac{2}{5}$ of the total number of pieces of mail delivered by the United States Postal Service was first-class mail. That same year, about $\frac{13}{25}$ of the total number of pieces of mail delivered by the United States Postal Service was marketing mail. Which of these two categories accounts for a greater portion of the mail delivered? (*Source:* United States Postal Service)

3.6 Complex Fractions, Order of Operations, and Mixed Numbers

Objectives

- **A** Simplify Complex Fractions.
- **B** Review the Order of Operations.
- **C** Evaluate Expressions Given Replacement Values.
- **D** Write Mixed Numbers as Improper Fractions.
- **E** Write Improper Fractions as Mixed Numbers or Whole Numbers.

Objective A Simplifying Complex Fractions

Thus far, we have studied operations on fractions. We now practice simplifying fractions whose numerators or denominators themselves contain fractions. These fractions are called **complex fractions.**

Complex Fraction

A fraction whose numerator or denominator or both numerator and denominator contain fractions is called a **complex fraction.**

Examples of complex fractions are

$$\frac{\frac{1}{4}}{\frac{3}{2}} \qquad \frac{\frac{1}{2}+\frac{3}{8}}{\frac{3}{4}-\frac{1}{6}} \qquad \frac{\frac{4}{5}-2}{\frac{3}{10}}$$

Method 1 for Simplifying Complex Fractions

Two methods are presented to simplify complex fractions. The first method makes use of the fact that a fraction bar means division.

Example 1 Simplify: $\dfrac{\frac{1}{4}}{\frac{3}{2}}$

Solution: Since a fraction bar means division, the complex fraction $\dfrac{\frac{1}{4}}{\frac{3}{2}}$ can be written as $\frac{1}{4} \div \frac{3}{2}$. Then divide as usual to simplify.

$$\frac{1}{4} \div \frac{3}{2} = \frac{1}{4}\cdot\frac{2}{3} \qquad \text{Multiply by the reciprocal.}$$

$$= \frac{1\cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}\cdot 2\cdot 3}$$

$$= \frac{1}{6}$$

Work Practice 1

Practice 1

Simplify: $\dfrac{\frac{7}{10}}{\frac{1}{5}}$

Example 2 Simplify: $\dfrac{\frac{1}{2}+\frac{3}{8}}{\frac{3}{4}-\frac{1}{6}}$

Solution: Recall the order of operations. Since the fraction bar is considered a grouping symbol, we simplify the numerator and the denominator of the complex fraction separately. Then we divide.

$$\frac{\frac{1}{2}+\frac{3}{8}}{\frac{3}{4}-\frac{1}{6}} = \frac{\frac{1\cdot 4}{2\cdot 4}+\frac{3}{8}}{\frac{3\cdot 3}{4\cdot 3}-\frac{1\cdot 2}{6\cdot 2}} = \frac{\frac{4}{8}+\frac{3}{8}}{\frac{9}{12}-\frac{2}{12}} = \frac{\frac{7}{8}}{\frac{7}{12}}$$

(Continued on next page)

Practice 2

Simplify: $\dfrac{\frac{1}{2}+\frac{1}{6}}{\frac{3}{4}-\frac{2}{3}}$

Answers

1. $\frac{7}{2}$ **2.** $\frac{8}{1}$ or 8

Thus,

$$\frac{\frac{1}{2}+\frac{3}{8}}{\frac{3}{4}-\frac{1}{6}} = \frac{\frac{7}{8}}{\frac{7}{12}}$$

$$= \frac{7}{8} \div \frac{7}{12} \qquad \text{Rewrite the quotient using the } \div \text{ sign.}$$

$$= \frac{7}{8} \cdot \frac{12}{7} \qquad \text{Multiply by the reciprocal.}$$

$$= \frac{\overset{1}{\cancel{7}} \cdot 3 \cdot \overset{1}{\cancel{4}}}{2 \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{7}}} \qquad \text{Multiply.}$$

$$= \frac{3}{2} \qquad \text{Simplify.}$$

Work Practice 2

Method 2 for Simplifying Complex Fractions

The second method for simplifying complex fractions is to multiply the numerator and the denominator of the complex fraction by the LCD of all the fractions in its numerator and its denominator. This has the effect of leaving sums and differences of integers in the numerator and the denominator, as we shall see in the example below.

Let's use this second method to simplify the complex fraction in Example 2 again.

Practice 3

Use Method 2 to simplify:

$$\frac{\frac{1}{2}+\frac{1}{6}}{\frac{3}{4}-\frac{2}{3}}$$

Example 3 Simplify: $\dfrac{\frac{1}{2}+\frac{3}{8}}{\frac{3}{4}-\frac{1}{6}}$

Solution: The complex fraction contains fractions with denominators 2, 8, 4, and 6. The LCD is 24. By the fundamental property of fractions, recall that we can multiply the numerator and the denominator of the complex fraction by 24. Notice below that by the distributive property, this means that we multiply each term in the numerator and denominator by 24.

$$\frac{\frac{1}{2}+\frac{3}{8}}{\frac{3}{4}-\frac{1}{6}} = \frac{24\left(\frac{1}{2}+\frac{3}{8}\right)}{24\left(\frac{3}{4}-\frac{1}{6}\right)}$$

$$= \frac{\left(\overset{12}{\cancel{24}} \cdot \frac{1}{\underset{1}{\cancel{2}}}\right) + \left(\overset{3}{\cancel{24}} \cdot \frac{3}{\underset{1}{\cancel{8}}}\right)}{\left(\overset{6}{\cancel{24}} \cdot \frac{3}{\underset{1}{\cancel{4}}}\right) - \left(\overset{4}{\cancel{24}} \cdot \frac{1}{\underset{1}{\cancel{6}}}\right)} \qquad \text{Apply the distributive property. Then divide out common factors to aid in multiplying.}$$

$$= \frac{12+9}{18-4} \qquad \text{Multiply.}$$

$$= \frac{21}{14}$$

$$= \frac{\overset{1}{\cancel{7}} \cdot 3}{\underset{1}{\cancel{7}} \cdot 2} = \frac{3}{2} \qquad \text{Simplify.}$$

Work Practice 3

The simplified result is the same, of course, no matter which method is used.

Answer

3. $\frac{8}{1}$ or 8

Example 4 Simplify: $\dfrac{\frac{4}{5}-2}{\frac{3}{10}}$

Solution: Use the second method and multiply the numerator and the denominator of the complex fraction by the LCD of all fractions. Recall that $2 = \frac{2}{1}$. The LCD of the denominators 5, 1, and 10 is 10.

$$\frac{\frac{4}{5}-\frac{2}{1}}{\frac{3}{10}} = \frac{10\left(\frac{4}{5}-\frac{2}{1}\right)}{10\left(\frac{3}{10}\right)}$$ Multiply the numerator and denominator by 10.

$$= \frac{\left(\overset{2}{\cancel{10}}\cdot\frac{4}{\underset{1}{\cancel{5}}}\right)-\left(10\cdot\frac{2}{1}\right)}{\overset{1}{\cancel{10}}\cdot\frac{3}{\underset{1}{\cancel{10}}}}$$ Apply the distributive property. Then divide out common factors to aid in multiplying.

$$= \frac{8-20}{3}$$ Multiply.

$$= \frac{-12}{3} = -4$$ Simplify.

Work Practice 4

Practice 4

Simplify: $\dfrac{\frac{3}{4}}{\frac{3}{5}-1}$

Helpful Hint

Don't forget to multiply the numerator and the denominator of the complex fraction by the same number—the LCD.

Objective B Reviewing the Order of Operations

At this time, it is probably a good idea to review the order of operations on expressions containing fractions. Before we do so, let's review how we perform operations on fractions.

Review of Operations on Fractions

Operation	Procedure	Example
Multiply	Multiply the numerators and multiply the denominators.	$\frac{5}{9}\cdot\frac{1}{2} = \frac{5\cdot 1}{9\cdot 2} = \frac{5}{18}$
Divide	Multiply the first fraction by the reciprocal of the second fraction.	$\frac{2}{3}\div\frac{11}{13} = \frac{2}{3}\cdot\frac{13}{11} = \frac{2\cdot 13}{3\cdot 11} = \frac{26}{33}$
Add or Subtract	**1.** Write each fraction as an equivalent fraction whose denominator is the LCD. **2.** Add or subtract numerators and write the result over the common denominator.	$\frac{3}{4}+\frac{1}{8} = \frac{3}{4}\cdot\frac{2}{2}+\frac{1}{8} = \frac{6}{8}+\frac{1}{8} = \frac{7}{8}$

Now let's review the order of operations.

Order of Operations

1. Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars or square roots, starting with the innermost set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Example 5 Simplify: $\left(\frac{4}{5}\right)^2 - 1$

Solution: According to the order of operations, first evaluate $\left(\frac{4}{5}\right)^2$.

$$\left(\frac{4}{5}\right)^2 - 1 = \frac{16}{25} - 1$$ Write $\left(\frac{4}{5}\right)^2$ as $\frac{16}{25}$.

(Continued on next page)

Practice 5

Simplify: $\left(\frac{2}{3}\right)^3 - 2$

Answers

4. $-\frac{15}{8}$ **5.** $-\frac{46}{27}$

Next, combine the fractions. The LCD of 25 and 1 is 25.

$$\frac{16}{25} - 1 = \frac{16}{25} - \frac{25}{25} \qquad \text{Write 1 as } \frac{25}{25}.$$

$$= \frac{-9}{25} \text{ or } -\frac{9}{25} \qquad \text{Subtract.}$$

Work Practice 5

Practice 6

Simplify: $\left(-\frac{1}{2}+\frac{1}{5}\right)\left(\frac{7}{8}+\frac{1}{8}\right)$

Example 6 Simplify: $\left(\frac{1}{4}+\frac{2}{3}\right)\left(\frac{11}{12}+\frac{1}{4}\right)$

Solution: First perform operations inside parentheses. Then multiply.

$$\left(\frac{1}{4}+\frac{2}{3}\right)\left(\frac{11}{12}+\frac{1}{4}\right) = \left(\frac{1\cdot 3}{4\cdot 3}+\frac{2\cdot 4}{3\cdot 4}\right)\left(\frac{11}{12}+\frac{1\cdot 3}{4\cdot 3}\right) \qquad \text{Each LCD is 12.}$$

$$= \left(\frac{3}{12}+\frac{8}{12}\right)\left(\frac{11}{12}+\frac{3}{12}\right)$$

$$= \left(\frac{11}{12}\right)\left(\frac{14}{12}\right) \qquad \text{Add.}$$

$$= \frac{11\cdot \overset{1}{\cancel{2}}\cdot 7}{\underset{1}{\cancel{2}}\cdot 6\cdot 12} \qquad \text{Multiply.}$$

$$= \frac{77}{72} \qquad \text{Simplify.}$$

Work Practice 6

Concept Check What should be done first to simplify the expression $\frac{1}{5}\cdot\frac{5}{2} - \left(\frac{2}{3}+\frac{4}{5}\right)^2$?

Helpful Hint If you find it difficult replacing a variable with a number, try the following. First, replace the variable with a set of parentheses, and then place the replacement number between the parentheses.

If $x = \frac{4}{5}$, find $2x + x^2$.

$$2x + x^2 = 2(\ \ ) + (\ \ )^2$$

$$= 2\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 \ldots$$

and then continue simplifying.

Objective C Evaluating Algebraic Expressions

Practice 7

Evaluate $-\frac{3}{5} - xy$ if $x = \frac{3}{10}$ and $y = \frac{2}{3}$.

Example 7 Evaluate $2x + y^2$ if $x = -\frac{1}{2}$ and $y = \frac{1}{3}$.

Solution: Replace x and y with the given values and simplify.

$$2x + y^2 = 2\left(-\frac{1}{2}\right) + \left(\frac{1}{3}\right)^2 \qquad \text{Replace } x \text{ with } -\frac{1}{2} \text{ and } y \text{ with } \frac{1}{3}.$$

$$= 2\left(-\frac{1}{2}\right) + \frac{1}{9} \qquad \text{Write } \left(\frac{1}{3}\right)^2 \text{ as } \frac{1}{9}.$$

$$= -1 + \frac{1}{9} \qquad \text{Multiply.}$$

$$= -\frac{9}{9} + \frac{1}{9} \qquad \text{The LCD is 9.}$$

$$= -\frac{8}{9} \qquad \text{Add.}$$

Work Practice 7

Answers

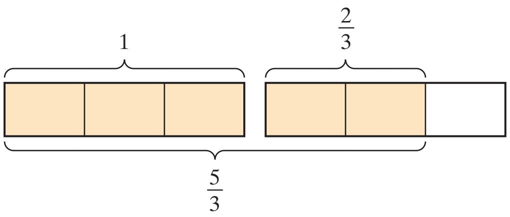
6. $-\frac{3}{10}$ **7.** $-\frac{4}{5}$

Concept Check Answer

Add inside parentheses.

Objective D Writing Mixed Numbers as Improper Fractions

In Section 3.1, mixed numbers and improper fractions were both used to represent the shaded part of figure groups. For example,



$1\frac{2}{3}$ or $\frac{5}{3}$ Thus, $1\frac{2}{3} = \frac{5}{3}$.

The following steps may be used to write a mixed number as an improper fraction:

Writing a Mixed Number as an Improper Fraction

To write a mixed number as an improper fraction:

Step 1: Multiply the denominator of the fraction by the whole number.

Step 2: Add the numerator of the fraction to the product from Step 1.

Step 3: Write the sum from Step 2 as the numerator of the improper fraction over the original denominator.

For example,

$$1\frac{2}{3} = \frac{\overbrace{3\cdot 1}^{\text{Step 1}} + \overbrace{2}^{\text{Step 2}}}{\underset{\text{Step 3}}{3}} = \frac{3+2}{3} = \frac{5}{3} \quad\text{or}\quad 1\frac{2}{3} = \frac{5}{3}\text{, as stated above.}$$

Example 8 Write each as an improper fraction.

a. $4\frac{2}{9} = \frac{9\cdot 4 + 2}{9} = \frac{36 + 2}{9} = \frac{38}{9}$

b. $1\frac{8}{11} = \frac{11\cdot 1 + 8}{11} = \frac{11 + 8}{11} = \frac{19}{11}$

Work Practice 8

Objective E Writing Improper Fractions as Mixed Numbers or Whole Numbers

Just as there are times when an improper fraction is preferred, sometimes a mixed or a whole number better suits a situation. To write improper fractions as mixed or whole numbers, we use division. Recall once again from Section 1.7 that the fraction bar means division. This means that the fraction

$$\frac{5\ \text{numerator}}{3\ \text{denominator}} \qquad \text{means}\ \ \underset{\uparrow\ \text{denominator}}{3}\overset{}{)}\underset{\uparrow\ \text{numerator}}{5}$$

Practice 8

Write each as an improper fraction.

a. $5\frac{2}{7}$ **b.** $6\frac{2}{3}$ **c.** $10\frac{9}{10}$ **d.** $4\frac{1}{5}$

Answers

8. a. $\frac{37}{7}$ **b.** $\frac{20}{3}$ **c.** $\frac{109}{10}$ **d.** $\frac{21}{5}$

Writing an Improper Fraction as a Mixed Number or a Whole Number

To write an improper fraction as a mixed number or a whole number:

Step 1: Divide the denominator into the numerator.

Step 2: The whole number part of the mixed number is the quotient. The fraction part of the mixed number is the remainder over the original denominator.

$$\text{quotient}\,\frac{\text{remainder}}{\text{original denominator}}$$

For example,

$$\frac{5}{3}:\quad \begin{array}{r} \overbrace{1}^{\text{Step 1}} \\ 3\overline{)5} \\ -3 \\ \hline 2 \end{array} \qquad \frac{5}{3} = \underset{\substack{\uparrow \\ \text{quotient}}}{1}\overbrace{\frac{2}{3}}^{\text{Step 2}} \begin{array}{l} \leftarrow \text{remainder} \\ \leftarrow \text{original denominator} \end{array}$$

Example 9

Write each as a mixed number or a whole number.

a. $\frac{30}{7}$ **b.** $\frac{16}{15}$ **c.** $\frac{84}{6}$

Solution:

a. $$\frac{30}{7}:\quad \begin{array}{r} 4 \\ 7\overline{)30} \\ -28 \\ \hline 2 \end{array} \qquad \frac{30}{7} = 4\frac{2}{7}$$

b. $$\frac{16}{15}:\quad \begin{array}{r} 1 \\ 15\overline{)16} \\ -15 \\ \hline 1 \end{array} \qquad \frac{16}{15} = 1\frac{1}{15}$$

c. $$\frac{84}{6}:\quad \begin{array}{r} 14 \\ 6\overline{)84} \\ -6 \\ \hline 24 \\ -24 \\ \hline 0 \end{array} \qquad \frac{84}{6} = 14$$

Since the remainder is 0, the result is the whole number 14.

Work Practice 9

Helpful Hint

When the remainder is 0, the improper fraction is a whole number. For example, $\frac{92}{4} = 23$.

$$\begin{array}{r} 23 \\ 4\overline{)92} \\ -8 \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

Practice 9

Write each as a mixed number or a whole number.

a. $\frac{9}{5}$ **b.** $\frac{23}{9}$ **c.** $\frac{48}{4}$

d. $\frac{62}{13}$ **e.** $\frac{51}{7}$ **f.** $\frac{21}{20}$

Answers

9. a. $1\frac{4}{5}$ **b.** $2\frac{5}{9}$ **c.** 12 **d.** $4\frac{10}{13}$ **e.** $7\frac{2}{7}$ **f.** $1\frac{1}{20}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

addition multiplication evaluate the exponential expression

subtraction division complex

1. A fraction whose numerator or denominator or both numerator and denominator contain fractions is called a(n) ________________________ fraction.

2. To simplify $-\frac{1}{2} + \frac{2}{3}\cdot\frac{7}{8}$, which operation do we perform first? ________________________

3. To simplify $-\frac{1}{2} \div \frac{2}{3} \cdot \frac{7}{8}$, which operation do we perform first? ______________________________

4. To simplify $\frac{7}{8} \cdot \left(\frac{1}{2} - \frac{2}{3}\right)$, which operation do we perform first? ______________________________

5. To simplify $\frac{1}{3} \div \frac{1}{4} \cdot \left(\frac{9}{11} + \frac{3}{8}\right)^3$, which operation do we perform first? ______________________________

6. To simplify $9 - \left(-\frac{3}{4}\right)^2$, which operation do we perform first? ______________________________

Martin-Gay Interactive Videos

See Video 3.6

Watch the section lecture video and answer the following questions.

Objective A **7.** In Example 3, what property is used to simplify the denominator of the complex fraction?

Objective B **8.** In Example 4, why can we add the fractions in the first set of parentheses right away?

Objective C **9.** In Example 5, why do we use parentheses when substituting the replacement value for x? What would happen if we didn't use parentheses?

Objective D **10.** Complete this statement based on the lecture before Example 6: The operation of ______ is understood in a mixed number notation; for example, $1\frac{1}{3}$ means 1 ______ $\frac{1}{3}$.

Objective E **11.** From the lecture before Example 9, what operation is used to write an improper fraction as a mixed number?

3.6 Exercise Set MyLab Math

Objective A *Simplify each complex fraction. See Examples 1 through 4.*

1. $\dfrac{\frac{1}{8}}{\frac{3}{4}}$

2. $\dfrac{\frac{5}{12}}{\frac{10}{8}}$

3. $\dfrac{\frac{2}{3}}{\frac{2}{7}}$

4. $\dfrac{\frac{9}{25}}{\frac{6}{25}}$

5. $\dfrac{\frac{2}{27}}{\frac{4}{9}}$

6. $\dfrac{\frac{3}{11}}{\frac{1}{2}}$

7. $\dfrac{\frac{3}{4} + \frac{2}{5}}{\frac{1}{2} + \frac{3}{5}}$

8. $\dfrac{\frac{7}{6} + \frac{2}{3}}{\frac{3}{2} - \frac{8}{9}}$

9. $\dfrac{\frac{3}{4}}{5 - \frac{1}{8}}$

10. $\dfrac{\frac{3}{10} + 2}{\frac{2}{5}}$

Objective B *Use the order of operations to simplify each expression. See Examples 5 and 6.*

11. $\frac{1}{5} + \frac{1}{3} \cdot \frac{1}{4}$

12. $\frac{1}{2} + \frac{1}{5} \cdot \frac{1}{3}$

13. $\frac{5}{6} \div \frac{1}{3} \cdot \frac{1}{4}$

14. $\frac{7}{8} \div \frac{1}{4} \cdot \frac{1}{7}$

15. $2^2 - \left(\frac{1}{3}\right)^2$

16. $3^2 - \left(\frac{1}{2}\right)^2$

17. $\left(\frac{2}{9} + \frac{4}{9}\right)\left(\frac{1}{3} - \frac{9}{10}\right)$

18. $\left(\frac{1}{5} - \frac{1}{10}\right)\left(\frac{1}{5} + \frac{1}{10}\right)$

19. $\left(\frac{7}{8} - \frac{1}{2}\right) \div \frac{3}{11}$

20. $\left(-\frac{2}{3} - \frac{7}{3}\right) \div \frac{4}{9}$

21. $2 \cdot \left(\frac{1}{4} + \frac{1}{5}\right) + 2$

22. $\frac{2}{5} \cdot \left(5 - \frac{1}{2}\right) - 1$

23. $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{4} - \frac{1}{12}\right)$

24. $\left(\frac{8}{9}\right)^2 \div \left(2 - \frac{2}{3}\right)$

25. $\left(\frac{2}{5} - \frac{3}{10}\right)^2$

26. $\left(\frac{3}{2} - \frac{4}{3}\right)^3$

27. $\left(\frac{3}{4} + \frac{1}{8}\right)^2 - \left(\frac{1}{2} + \frac{1}{8}\right)$

28. $\left(\frac{1}{6} + \frac{1}{3}\right)^3 + \left(\frac{2}{5} + \frac{1}{4}\right)$

Objective C *Evaluate each expression if* $x = -\frac{1}{3}$, $y = \frac{2}{5}$, *and* $z = \frac{5}{6}$. *See Example 7.*

29. $5y - z$

30. $2z - x$

31. $\frac{x}{z}$

32. $\frac{y + x}{z}$

33. $x^2 - yz$

34. $x^2 - z^2$

35. $(1 + x)(1 + z)$

36. $(1 - x)(1 - z)$

Objectives A B Mixed Practice *Simplify the following. See Examples 1 through 6.*

37. $\frac{\frac{5}{24}}{\frac{1}{12}}$

38. $\frac{\frac{7}{10}}{\frac{14}{25}}$

39. $\left(\frac{4}{9} \div \frac{3}{2}\right) + \left(\frac{1}{9} \cdot \frac{2}{3}\right)$

40. $\left(\frac{5}{21} \div \frac{1}{2}\right) + \left(\frac{1}{7} \cdot \frac{1}{3}\right)$

41. $\left(-\frac{1}{2}\right)^2 + \frac{1}{5}$

42. $\left(-\frac{3}{4}\right)^2 + \frac{3}{8}$

43. $\frac{2 + \frac{1}{6}}{1 - \frac{4}{3}}$

44. $\frac{3 - \frac{1}{2}}{4 + \frac{1}{5}}$

45. $\left(1 - \frac{2}{5}\right)^2$

46. $\left(\frac{1}{2} - \frac{3}{4}\right)^2$

47. $\left(\frac{3}{4} - 1\right)\left(\frac{1}{8} + \frac{1}{2}\right)$

48. $\left(\frac{1}{10} + \frac{3}{20}\right)\left(\frac{1}{5} - 1\right)$

49. $\left(-\frac{2}{9}-\frac{7}{9}\right)^4$

50. $\left(\frac{5}{9}-\frac{2}{3}\right)^2$

51. $\dfrac{\frac{1}{3}-\frac{5}{6}}{\frac{3}{4}+\frac{1}{2}}$

52. $\dfrac{\frac{7}{10}+\frac{1}{2}}{\frac{4}{5}+\frac{3}{4}}$

53. $\left(\frac{3}{4}\div\frac{6}{5}\right)-\left(\frac{3}{4}\cdot\frac{6}{5}\right)$

54. $\left(\frac{1}{2}\cdot\frac{2}{7}\right)-\left(\frac{1}{2}\div\frac{2}{7}\right)$

Objective D *Write each mixed number as an improper fraction. See Example 8.*

▶ **55.** $2\frac{1}{3}$

56. $1\frac{13}{17}$

▶ **57.** $3\frac{3}{5}$

58. $2\frac{5}{9}$

59. $6\frac{5}{8}$

60. $7\frac{3}{8}$

61. $11\frac{6}{7}$

62. $12\frac{2}{5}$

▶ **63.** $9\frac{7}{20}$

64. $10\frac{14}{27}$

65. $166\frac{2}{3}$

66. $114\frac{2}{7}$

Objective E *Write each improper fraction as a mixed number or a whole number. See Example 9.*

▶ **67.** $\frac{17}{5}$

68. $\frac{13}{7}$

▶ **69.** $\frac{37}{8}$

70. $\frac{64}{9}$

71. $\frac{47}{15}$

72. $\frac{65}{12}$

73. $\frac{225}{15}$

74. $\frac{196}{14}$

75. $\frac{182}{175}$

76. $\frac{149}{143}$

77. $\frac{737}{112}$

78. $\frac{901}{123}$

Review

Perform each indicated operation. If the result is an improper fraction, also write the improper fraction as a mixed number. See Sections 3.5 and 3.6.

79. $3+\frac{1}{2}$

80. $2+\frac{2}{3}$

81. $9-\frac{5}{6}$

82. $4-\frac{1}{5}$

Concept Extensions

✎ **83.** In your own words, explain how to write an improper fraction as a mixed number.

✎ **84.** In your own words, explain how to write a mixed number as an improper fraction.

✎ **85.** Calculate $\frac{2^3}{3}$ and $\left(\frac{2}{3}\right)^3$. Do both of these expressions simplify to the same number? Explain why or why not.

✎ **86.** Calculate $\left(\frac{1}{2}\right)^2\cdot\left(\frac{3}{4}\right)^2$ and $\left(\frac{1}{2}\cdot\frac{3}{4}\right)^2$. Do both of these expressions simplify to the same number? Explain why or why not.

Recall that to find the average of two numbers, we find their sum and divide by 2. For example, the average of $\frac{1}{2}$ *and* $\frac{3}{4}$ *is* $\frac{\frac{1}{2}+\frac{3}{4}}{2}$*. Find the average of each pair of numbers.*

87. $\frac{1}{2}, \frac{3}{4}$

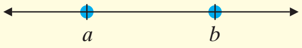
88. $\frac{3}{5}, \frac{9}{10}$

89. $\frac{1}{4}, \frac{2}{14}$

90. $\frac{5}{6}, \frac{7}{9}$

Solve.

91. Two positive numbers, a and b, are graphed below. Where should the graph of their average lie?



92. Study Exercise **91**. Without calculating, can $\frac{1}{3}$ be the average of $\frac{1}{2}$ and $\frac{8}{9}$? Explain why or why not.

For Exercises 93 through 98, answer true or false for each statement.

93. It is possible for the average of two numbers to be greater than both numbers.

94. It is possible for the average of two numbers to be less than both numbers.

95. The sum of two negative fractions is always a negative number.

96. The sum of a negative fraction and a positive fraction is always a positive number.

97. It is possible for the sum of two fractions to be a whole number.

98. It is possible for the difference of two fractions to be a whole number.

99. What operation should be performed first to simplify

$$\frac{1}{5}\cdot\frac{5}{2} - \left(\frac{2}{3} + \frac{4}{5}\right)^2?$$

Explain your answer.

100. A student is to evaluate $x - y$ when $x = \frac{1}{5}$ and $y = -\frac{1}{7}$. This student is asking you if he should evaluate $\frac{1}{5} - \frac{1}{7}$. What do you tell this student and why?

Each expression contains one addition, one subtraction, one multiplication, and one division. Write the operations in the order that they should be performed. Do not actually simplify. See the Concept Check in this section.

101. $[9 + 3(4 - 2)] \div \frac{10}{21}$

102. $[30 - 4(3 + 2)] \div \frac{5}{2}$

103. $\frac{1}{3} \div \left(\frac{2}{3}\right)\left(\frac{4}{5}\right) - \frac{1}{4} + \frac{1}{2}$

104. $\left(\frac{5}{6} - \frac{1}{3}\right)\cdot\frac{1}{3} + \frac{1}{2} \div \frac{9}{8}$

Evaluate each expression if $x = \frac{3}{4}$ *and* $y = -\frac{4}{7}$.

105. $\frac{2 + x}{y}$

106. $4x + y$

107. $x^2 + 7y$

108. $\frac{\frac{9}{14}}{x + y}$

3.7 Operations on Mixed Numbers

Objective A Graphing Fractions and Mixed Numbers

Let's review graphing fractions and practice graphing mixed numbers on a number line. This will help us visualize rounding and estimating operations with mixed numbers.

Recall that $5\frac{2}{3}$ means $5 + \frac{2}{3}$ and that

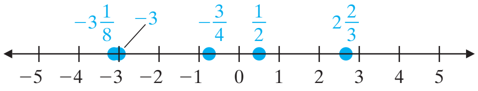
$$-4\frac{1}{6} \text{ means } -\left(4 + \frac{1}{6}\right) = -4 - \frac{1}{6} \quad \text{or} \quad -4 + \left(-\frac{1}{6}\right)$$

Example 1 Graph the numbers on a number line:

$$\frac{1}{2},\ -\frac{3}{4},\ 2\frac{2}{3},\ -3,\ -3\frac{1}{8}$$

Solution: Remember that $2\frac{2}{3}$ means $2 + \frac{2}{3}$.

Also, $-3\frac{1}{8}$ means $-\left(3 + \frac{1}{8}\right) = -3 - \frac{1}{8}$, so $-3\frac{1}{8}$ lies to the left of −3.



Work Practice 1

✓Concept Check

Which of the following are equivalent to 9?

a. $7\frac{6}{3}$ **b.** $8\frac{4}{4}$ **c.** $8\frac{9}{9}$ **d.** $\frac{18}{2}$ **e.** all of these

Objective B Multiplying or Dividing with Mixed Numbers or Whole Numbers

When multiplying or dividing a fraction and a mixed or a whole number, remember that mixed and whole numbers can be written as improper fractions.

Multiplying or Dividing Fractions and Mixed Numbers or Whole Numbers

To multiply or divide with mixed numbers or whole numbers, first write any mixed or whole numbers as improper fractions and then multiply or divide as usual.

(*Note:* If an exercise contains a mixed number, we will write the answer as a mixed number, if possible.)

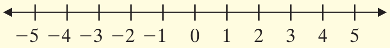
Objectives

- **A** Graph Positive and Negative Fractions and Mixed Numbers.
- **B** Multiply or Divide Mixed or Whole Numbers.
- **C** Add or Subtract Mixed Numbers.
- **D** Solve Problems Containing Mixed Numbers.
- **E** Perform Operations on Negative Mixed Numbers.

Practice 1

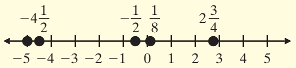
Graph the numbers on a number line.

$$-5,\ -4\frac{1}{2},\ 2\frac{3}{4},\ \frac{1}{8},\ -\frac{1}{2}$$



Answer

1.



✓Concept Check Answer

e

Practice 2

Multiply and simplify: $1\frac{2}{3}\cdot\frac{11}{15}$

Example 2 Multiply: $3\frac{1}{3}\cdot\frac{7}{8}$

Solution: Recall from Section 3.6 that the mixed number $3\frac{1}{3}$ can be written as the fraction $\frac{10}{3}$. Then

$$3\frac{1}{3}\cdot\frac{7}{8} = \frac{10}{3}\cdot\frac{7}{8} = \frac{\overset{1}{\cancel{2}}\cdot 5\cdot 7}{3\cdot\underset{1}{\cancel{2}}\cdot 4} = \frac{35}{12} \quad\text{or}\quad 2\frac{11}{12}$$

Work Practice 2

Helpful Hint Recall that any integer or whole number can be written as a fraction.

Don't forget that a whole number can be written as a fraction by writing the whole number over 1. For example,

$$20 = \frac{20}{1} \quad\text{and}\quad 7 = \frac{7}{1}$$

Practice 3

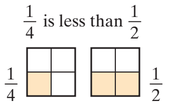
Multiply: $\frac{5}{6}\cdot 18$

Example 3 Multiply: $\frac{3}{4}\cdot 20$

Solution: $\frac{3}{4}\cdot 20 = \frac{3}{4}\cdot\frac{20}{1} = \frac{3\cdot 20}{4\cdot 1} = \frac{3\cdot\overset{1}{\cancel{4}}\cdot 5}{\underset{1}{\cancel{4}}\cdot 1} = \frac{15}{1}$ or 15

Work Practice 3

When both numbers to be multiplied are mixed or whole numbers, it is a good idea to estimate the product to see if your answer is reasonable. To do this, we first practice rounding mixed numbers to the nearest whole. If the fraction part of the mixed number is $\frac{1}{2}$ or greater, we round the whole number part up. If the fraction part of the mixed number is less than $\frac{1}{2}$, then we do not round the whole number part up. Study the table below for examples.

Mixed Number	Rounding
$5\frac{1}{4}$  $\frac{1}{4}$ is less than $\frac{1}{2}$	Thus, $5\frac{1}{4}$ rounds to 5.
$3\frac{9}{16}$ ← 9 is greater than 8 → Half of 16 is 8.	Thus, $3\frac{9}{16}$ rounds to 4.
$1\frac{3}{7}$ ← 3 is less than $3\frac{1}{2}$. → Half of 7 is $3\frac{1}{2}$.	Thus, $1\frac{3}{7}$ rounds to 1.

Practice 4

Multiply. Check by estimating.

$3\frac{1}{5}\cdot 2\frac{3}{4}$

Example 4 Multiply $1\frac{2}{3}\cdot 2\frac{1}{4}$. Check by estimating.

Solution: $1\frac{2}{3}\cdot 2\frac{1}{4} = \frac{5}{3}\cdot\frac{9}{4} = \frac{5\cdot 9}{3\cdot 4} = \frac{5\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{3}}\cdot 4} = \frac{15}{4}$ or $3\frac{3}{4}$ Exact

Let's check by estimating.

$1\frac{2}{3}$ rounds to 2, $2\frac{1}{4}$ rounds to 2, and $2\cdot 2 = 4$. Estimate

The estimate is close to the exact value, so our answer is reasonable.

Work Practice 4

Answers

2. $1\frac{2}{9}$ **3.** 15 **4.** $8\frac{4}{5}$

Example 5 Multiply: $7 \cdot 2\frac{11}{14}$. Check by estimating.

Solution: $7 \cdot 2\frac{11}{14} = \frac{7}{1} \cdot \frac{39}{14} = \frac{7 \cdot 39}{1 \cdot 14} = \frac{\overset{1}{\cancel{7}} \cdot 39}{1 \cdot 2 \cdot \underset{1}{\cancel{7}}} = \frac{39}{2}$ or $19\frac{1}{2}$ Exact

To estimate,

$2\frac{11}{14}$ rounds to 3 and $7 \cdot 3 = 21$. Estimate

The estimate is close to the exact value, so our answer is reasonable.

Work Practice 5

✓Concept Check Find the error.

$$\cancel{2\frac{1}{4} \cdot \frac{1}{2} = 2\frac{1 \cdot 1}{4 \cdot 2} = 2\frac{1}{8}}$$

Examples Divide.

6. $\frac{3}{4} \div 5 = \frac{3}{4} \div \frac{5}{1} = \frac{3}{4} \cdot \frac{1}{5} = \frac{3 \cdot 1}{4 \cdot 5} = \frac{3}{20}$

7. $\frac{11}{18} \div 2\frac{5}{6} = \frac{11}{18} \div \frac{17}{6} = \frac{11}{18} \cdot \frac{6}{17} = \frac{11 \cdot 6}{18 \cdot 17} = \frac{11 \cdot \overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \cdot 3 \cdot 17} = \frac{11}{51}$

8. $5\frac{2}{3} \div 2\frac{5}{9} = \frac{17}{3} \div \frac{23}{9} = \frac{17}{3} \cdot \frac{9}{23} = \frac{17 \cdot 9}{3 \cdot 23} = \frac{17 \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{3}} \cdot 23} = \frac{51}{23}$ or $2\frac{5}{23}$

Work Practice 6–8

Objective C Adding or Subtracting Mixed Numbers

We can add or subtract mixed numbers, too, by first writing each mixed number as an improper fraction. But it is often easier to add or subtract the whole-number parts and add or subtract the proper-fraction parts vertically.

Adding or Subtracting Mixed Numbers

To add or subtract mixed numbers, add or subtract the fraction parts and then add or subtract the whole number parts separately.

Example 9 Add: $2\frac{1}{3} + 5\frac{3}{8}$. Check by estimating.

Solution: The LCD of the denominators 3 and 8 is 24.

$$\begin{array}{rcl} 2\frac{1 \cdot 8}{3 \cdot 8} & = & 2\frac{8}{24} \\ +\,5\frac{3 \cdot 3}{8 \cdot 3} & = & 5\frac{9}{24} \\ \hline & & 7\frac{17}{24} \end{array}$$

← Add the fractions.

Add the whole numbers.

To check by estimating, we round as usual. The fraction $2\frac{1}{3}$ rounds to 2, $5\frac{3}{8}$ rounds to 5, and $2 + 5 = 7$, our estimate.

Our exact answer is close to 7, so our answer is reasonable.

Work Practice 9

Practice 5

Multiply. Check by estimating.

$3 \cdot 6\frac{7}{15}$

Practice 6–8

Divide.

6. $\frac{4}{9} \div 7$ **7.** $\frac{8}{15} \div 3\frac{4}{5}$

8. $3\frac{2}{7} \div 2\frac{3}{14}$

Practice 9

Add: $2\frac{1}{6} + 4\frac{2}{5}$.

Check by estimating.

Answers

5. $19\frac{2}{5}$ **6.** $\frac{4}{63}$ **7.** $\frac{8}{57}$ **8.** $1\frac{15}{31}$

9. $6\frac{17}{30}$

✓Concept Check Answer

forgot to change mixed number to improper fraction

Practice 10

Add: $3\frac{5}{14} + 2\frac{6}{7}$

Practice 11

Add: $12 + 3\frac{6}{7} + 2\frac{1}{5}$

Practice 12

Subtract: $32\frac{7}{9} - 16\frac{5}{18}$. Check by estimating.

Answers

10. $6\frac{3}{14}$ **11.** $18\frac{2}{35}$ **12.** $16\frac{1}{2}$

✓Concept Check Answer

Round each mixed number to the nearest whole number and add. $5\frac{1}{9}$ rounds to 5 and $14\frac{10}{11}$ rounds to 15, and the estimated sum is $5 + 15 = 20$.

Helpful Hint

When adding or subtracting mixed numbers and whole numbers, it is a good idea to estimate to see if your answer is reasonable.

For the rest of this section, we leave most of the checking by estimating to you.

Example 10

Add: $3\frac{4}{5} + 1\frac{4}{15}$

Solution: The LCD of the denominators 5 and 15 is 15.

$$\begin{array}{rcl} 3\frac{4}{5} & = & 3\frac{12}{15} \\ +1\frac{4}{15} & = & 1\frac{4}{15} \\ \hline & & 4\frac{16}{15} \end{array}$$

Add the fractions; add the whole numbers.

Notice that the fraction part is improper.

Since $\frac{16}{15}$ is $1\frac{1}{15}$, we can write the sum as

$$4\frac{16}{15} = 4 + 1\frac{1}{15} = 5\frac{1}{15}$$

Work Practice 10

✓Concept Check Explain how you could estimate the following sum: $5\frac{1}{9} + 14\frac{10}{11}$.

Example 11

Add: $2\frac{4}{5} + 5 + 1\frac{1}{2}$

Solution: The LCD of the denominators 5 and 2 is 10.

$$\begin{array}{rcl} 2\frac{4}{5} & = & 2\frac{8}{10} \\ 5 & = & 5 \\ +\ 1\frac{1}{2} & = & 1\frac{5}{10} \\ \hline & & 8\frac{13}{10} = 8 + 1\frac{3}{10} = 9\frac{3}{10} \end{array}$$

Work Practice 11

Example 12

Subtract: $8\frac{3}{7} - 5\frac{2}{21}$. Check by estimating.

Solution: The LCD of the denominators 7 and 21 is 21.

$$\begin{array}{rcl} 8\frac{3}{7} & = & 8\frac{9}{21} \\ -5\frac{2}{21} & = & -5\frac{2}{21} \\ \hline & & 3\frac{7}{21} \end{array}$$

← The LCD of 7 and 21 is 21.

← Subtract the fractions.

↑ Subtract the whole numbers.

Then $3\frac{7}{21}$ simplifies to $3\frac{1}{3}$. The difference is $3\frac{1}{3}$.

To check, $8\frac{3}{7}$ rounds to 8, $5\frac{2}{21}$ rounds to 5, and $8 - 5 = 3$, our estimate.

Our exact answer is close to 3, so our answer is reasonable.

Work Practice 12

When subtracting mixed numbers, borrowing may be needed, as shown in the next example.

Example 13 Subtract: $7\frac{3}{14} - 3\frac{6}{7}$

Solution: The LCD of the denominators 7 and 14 is 14.

$$\begin{array}{rcr} 7\frac{3}{14} & = & 7\frac{3}{14} \\ \underline{-3\frac{6}{7}} & = & \underline{-3\frac{12}{14}} \end{array}$$

Notice that we cannot subtract $\frac{12}{14}$ from $\frac{3}{14}$, so we borrow from the whole number, 7.

borrow 1 from 7

$$7\frac{3}{14} = 6 + 1\frac{3}{14} = 6 + \frac{17}{14} \text{ or } 6\frac{17}{14}$$

Now subtract.

$$\begin{array}{rcrcr} 7\frac{3}{14} & = & 7\frac{3}{14} & = & 6\frac{17}{14} \\ \underline{-3\frac{6}{7}} & = & \underline{-3\frac{12}{14}} & = & \underline{-3\frac{12}{14}} \\ & & & & 3\frac{5}{14} \end{array}$$

← Subtract the fractions.

↑ Subtract the whole numbers.

Work Practice 13

✓ **Concept Check** In the subtraction problem $5\frac{1}{4} - 3\frac{3}{4}$, $5\frac{1}{4}$ must be rewritten because $\frac{3}{4}$ cannot be subtracted from $\frac{1}{4}$. Why is it incorrect to rewrite $5\frac{1}{4}$ as $5\frac{5}{4}$?

Example 14 Subtract: $14 - 8\frac{3}{7}$

Solution:

$$\begin{array}{rcr} 14 & = & 13\frac{7}{7} \\ \underline{-8\frac{3}{7}} & = & \underline{-8\frac{3}{7}} \\ & & 5\frac{4}{7} \end{array}$$

Borrow 1 from 14 and write it as $\frac{7}{7}$.

← Subtract the fractions.

↑ Subtract the whole numbers.

Work Practice 14

Practice 13

Subtract: $9\frac{7}{15} - 4\frac{3}{5}$

Practice 14

Subtract: $25 - 10\frac{2}{9}$

Answers

13. $4\frac{13}{15}$ **14.** $14\frac{7}{9}$

✓ **Concept Check Answer**

Rewrite $5\frac{1}{4}$ as $4\frac{5}{4}$ by borrowing from the 5.

Objective D Solving Problems Containing Mixed Numbers

Now that we know how to perform operations on mixed numbers, we can solve real-life problems.

Practice 15

The measurement around the trunk of a tree just below shoulder height is called its girth. The largest known American beech tree in the United States has a girth of $23\frac{1}{4}$ feet. The largest known sugar maple tree in the United States has a girth of $19\frac{5}{12}$ feet. How much larger is the girth of the largest known American beech tree than the girth of the largest known sugar maple tree? (*Source: American Forests*)

Answer

15. $3\frac{5}{6}$ ft

Example 15 Finding Legal Lobster Size

Lobster fishermen must measure the upper body shells of the lobsters they catch. Lobsters that are too small are thrown back into the ocean. Each state has its own size standard for lobsters to help control the breeding stock. Massachusetts divides its waters into four Lobster Conservation Management Areas, with a different minimum lobster size permitted in each area. In area three, the legal lobster size increased from $3\frac{13}{32}$ inches to $3\frac{1}{2}$ inches. How much of an increase was this? (*Source:* Massachusetts Division of Marine Fisheries)

Solution:

1. UNDERSTAND. Read and reread the problem carefully. The word "increase" found in the problem might make you think that we add to solve the problem. But the phrase "how much of an increase" tells us to subtract to find the increase.
2. TRANSLATE.

In words:	increase	is	new lobster size	minus	old lobster size
	↓	↓	↓	↓	↓
Translate:	increase	$=$	$3\frac{1}{2}$	$-$	$3\frac{13}{32}$

3. SOLVE: Before we solve, let's estimate by rounding to the nearest wholes. The fraction $3\frac{1}{2}$ can be rounded up to 4, $3\frac{13}{32}$ rounds to 3, and $4 - 3 = 1$. The increase is not 1 but will be smaller since we rounded $3\frac{1}{2}$ up and rounded $3\frac{13}{32}$ down.

$$\begin{array}{rcr} 3\frac{1}{2} & = & 3\frac{16}{32} \\ -3\frac{13}{32} & = & -3\frac{13}{32} \\ \hline & & \frac{3}{32} \end{array}$$

4. INTERPRET. *Check* your work. Our estimate tells us that the exact increase of $\frac{3}{32}$ is reasonable. *State* your conclusion: The increase in lobster size was $\frac{3}{32}$ of an inch.

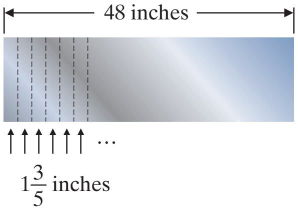
Work Practice 15

Example 16 Calculating Manufacturing Materials Needed

In a manufacturing process, a metal-cutting machine cuts strips $1\frac{3}{5}$ inches long from a piece of metal stock. How many such strips can be cut from a 48-inch piece of stock?

Solution:

1. UNDERSTAND the problem. To do so, read and reread the problem. Then draw a diagram:



We want to know how many $1\frac{3}{5}$s there are in 48.

2. TRANSLATE.

In words:	number of strips	is	48	divided by	$1\frac{3}{5}$
	↓	↓	↓	↓	↓
Translate:	number of strips	=	48	÷	$1\frac{3}{5}$

3. SOLVE: Let's estimate a reasonable answer. The mixed number $1\frac{3}{5}$ rounds to 2 and $48 \div 2 = 24$.

$$48 \div 1\frac{3}{5} = 48 \div \frac{8}{5} = \frac{48}{1} \cdot \frac{5}{8} = \frac{48 \cdot 5}{1 \cdot 8} = \frac{\overset{1}{\cancel{8}} \cdot 6 \cdot 5}{1 \cdot \underset{1}{\cancel{8}}} = \frac{30}{1} \quad \text{or} \quad 30$$

4. INTERPRET. *Check* your work. Since the exact answer of 30 is close to our estimate of 24, our answer is reasonable. *State* your conclusion: Thirty strips can be cut from the 48-inch piece of stock.

Work Practice 16

Practice 16

A designer of women's clothing designs a woman's dress that requires $3\frac{1}{7}$ yards of material. How many dresses can be made from a 44-yard bolt of material?

Objective E Operating on Negative Mixed Numbers

To perform operations on negative mixed numbers, let's first practice writing these numbers as negative fractions and negative fractions as negative mixed numbers.

To understand negative mixed numbers, recall our work on p. 251. For example,

$$-3\frac{2}{5} \text{ means } -\left(3\frac{2}{5}\right)$$

Thus, to write a negative mixed number as a fraction, we do the following.

$$-3\frac{2}{5} = -\left(3\frac{2}{5}\right) = -\left(\frac{5 \cdot 3 + 2}{5}\right) = -\left(\frac{17}{5}\right) \quad \text{or} \quad -\frac{17}{5}$$

Examples

Write each as a fraction.

17. $-\underbrace{1\frac{7}{8}} = -\underbrace{\frac{8 \cdot 1 + 7}{8}} = -\frac{15}{8}$ Write $1\frac{7}{8}$ as an improper fraction and keep the negative sign.

18. $-\underbrace{23\frac{1}{2}} = -\underbrace{\frac{2 \cdot 23 + 1}{2}} = -\frac{47}{2}$ Write $23\frac{1}{2}$ as an improper fraction and keep the negative sign.

Work Practice 17–18

Practice 17–18

Write each as a fraction.

17. $-9\frac{3}{7}$ **18.** $-5\frac{10}{11}$

Answers

16. 14 dresses **17.** $-\frac{66}{7}$ **18.** $-\frac{65}{11}$

To write a negative fraction as a negative mixed number, we use a similar procedure. We simply disregard the negative sign, convert the improper fraction to a mixed number, and then reinsert the negative sign.

Practice 19–20

Write each as a mixed number.

19. $-\frac{37}{8}$ **20.** $-\frac{46}{5}$

Examples Write each as a mixed number.

19. $\underbrace{-\frac{22}{5}} = \underbrace{-4\frac{2}{5}}$

$$\begin{array}{r} 4 \\ 5\overline{)22} \\ -20 \\ \hline 2 \end{array} \qquad \frac{22}{5} = 4\frac{2}{5}$$

20. $\underbrace{-\frac{9}{4}} = \underbrace{-2\frac{1}{4}}$

$$\begin{array}{r} 2 \\ 4\overline{)9} \\ -8 \\ \hline 1 \end{array} \qquad \frac{9}{4} = 2\frac{1}{4}$$

Work Practice 19–20

We multiply or divide with negative mixed numbers the same way that we multiply or divide with positive mixed numbers. We first write each mixed number as a fraction.

Practice 21–22

21. $2\frac{3}{4}\cdot\left(-3\frac{3}{5}\right)$

22. $-4\frac{2}{7} \div 1\frac{1}{4}$

Examples Perform the indicated operations.

21. $-4\frac{2}{5}\cdot 1\frac{3}{11} = -\frac{22}{5}\cdot\frac{14}{11} = -\frac{22\cdot 14}{5\cdot 11} = -\frac{2\cdot \overset{1}{\cancel{11}}\cdot 14}{5\cdot \underset{1}{\cancel{11}}} = -\frac{28}{5}$ or $-5\frac{3}{5}$

22. $-2\frac{1}{3} \div \left(-2\frac{1}{2}\right) = -\frac{7}{3} \div \left(-\frac{5}{2}\right) = -\frac{7}{3}\cdot\left(-\frac{2}{5}\right) = \frac{7\cdot 2}{3\cdot 5} = \frac{14}{15}$

Work Practice 21–22

Helpful Hint Recall that $(-)\cdot(-) = +$

To add or subtract with negative mixed numbers, we must be very careful! Recall that

$$-3\frac{2}{5} \text{ means } -\left(3\frac{2}{5}\right)$$

This means that

$$-3\frac{2}{5} = -\left(3\frac{2}{5}\right) = -\left(3+\frac{2}{5}\right) = -3-\frac{2}{5}$$ This can sometimes be easily overlooked.

To avoid problems, we will add or subtract negative mixed numbers by rewriting them as addition and recalling how to add signed numbers.

Answers

19. $-4\frac{5}{8}$ **20.** $-9\frac{1}{5}$ **21.** $-9\frac{9}{10}$

22. $-3\frac{3}{7}$

Example 23

Add: $6\frac{3}{5} + \left(-9\frac{7}{10}\right)$

Solution: Here we are adding two numbers with different signs. Recall that we then subtract the absolute values and keep the sign of the larger absolute value.

Since $-9\frac{7}{10}$ has the larger absolute value, the answer is negative.

First, subtract absolute values:

$$\begin{array}{rcr} 9\frac{7}{10} & = & 9\frac{7}{10} \\ -6\frac{3\cdot 2}{5\cdot 2} & = & -6\frac{6}{10} \\ \hline & & 3\frac{1}{10} \end{array}$$

Thus,

$$6\frac{3}{5} + \left(-9\frac{7}{10}\right) = -3\frac{1}{10}$$ The result is negative since $-9\frac{7}{10}$ has the larger absolute value.

Work Practice 23

Practice 23

Add: $6\frac{2}{3} + \left(-12\frac{3}{4}\right)$

Example 24

Subtract: $-11\frac{5}{6} - 20\frac{4}{9}$

Solution: Let's write as an equivalent addition: $-11\frac{5}{6} + \left(-20\frac{4}{9}\right)$. Here, we are adding two numbers with like signs. Recall that we add their absolute values and keep the common negative sign.

First, add absolute values:

$$\begin{array}{rcr} 11\frac{5\cdot 3}{6\cdot 3} & = & 11\frac{15}{18} \\ +\,20\frac{4\cdot 2}{9\cdot 2} & = & +20\frac{8}{18} \\ \hline & & 31\frac{23}{18} \text{ or } 32\frac{5}{18} \end{array}$$

Since $\frac{23}{18} = 1\frac{5}{18}$

Thus,

$$-11\frac{5}{6} - 20\frac{4}{9} = -32\frac{5}{18}$$

Keep the common sign.

Work Practice 24

Practice 24

Subtract: $-9\frac{2}{7} - 30\frac{11}{14}$

Answers

23. $-6\frac{1}{12}$ **24.** $-40\frac{1}{14}$

Calculator Explorations Converting Between Mixed Number and Fraction Notation

If your calculator has a fraction key, such as $\boxed{a^b/c}$, you can use it to convert between mixed number notation and fraction notation.

To w rite $13\frac{7}{16}$ as an improper fraction, press

$\boxed{13}$ $\boxed{a^b/c}$ $\boxed{7}$ $\boxed{a^b/c}$ $\boxed{16}$ $\boxed{\text{2nd}}$ $\boxed{d/c}$

The display will read

$\boxed{215\,|\,16}$

which represents $\frac{215}{16}$. Thus $13\frac{7}{16} = \frac{215}{16}$.

To convert $\frac{190}{13}$ to a mixed number, press

$\boxed{190}$ $\boxed{a^b/c}$ $\boxed{13}$ $\boxed{=}$

The display will read

$\boxed{14\_8/13}$

which represents $14\frac{8}{13}$. Thus $\frac{190}{13} = 14\frac{8}{13}$.

Write each mixed number as a fraction and each fraction as a mixed number.

1. $25\frac{5}{11}$
2. $67\frac{14}{15}$
3. $107\frac{31}{35}$
4. $186\frac{17}{21}$
5. $\frac{365}{14}$
6. $\frac{290}{13}$
7. $\frac{2769}{30}$
8. $\frac{3941}{17}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

round
improper
fraction
mixed number
whole number

1. The number $5\frac{3}{4}$ is called a(n) ______________.

2. For $5\frac{3}{4}$, the 5 is called the ______________ part and $\frac{3}{4}$ is called the ______________ part.

3. To estimate operations on mixed numbers, we ______________ mixed numbers to the nearest whole number.

4. The mixed number $2\frac{5}{8}$ written as a(n) ______________ fraction is $\frac{21}{8}$.

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 3.7

Objective A **5.** In Example 1, why is the unit distance between −4 and −3 on the number line split into 5 equal parts?

Objective B **6.** Why do we need to know how to multiply fractions to solve Example 2?

Objective C **7.** In Example 4, why is the first form of the answer not an appropriate form?

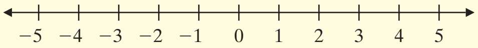
Objective D **8.** Why do we need to know how to subtract fractions to solve Example 5?

Objective E **9.** In Example 6, how is it determined whether the answer is positive or negative?

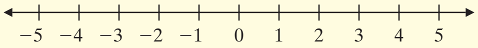
3.7 Exercise Set MyLab Math

Objective A *Graph each list of numbers on the given number line. See Example 1.*

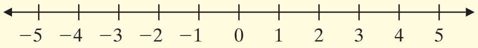
1. $-2, -2\frac{2}{3}, 0, \frac{7}{8}, -\frac{1}{3}$



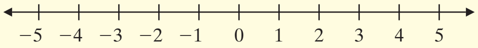
2. $-1, -1\frac{1}{4}, -\frac{1}{4}, 3\frac{1}{4}, 3$



3. $4, \frac{1}{3}, -3, -3\frac{4}{5}, 1\frac{1}{3}$



4. $3, \frac{3}{8}, -4, -4\frac{1}{3}, -\frac{9}{10}$



Objective B *Choose the best estimate for each product or quotient. See Examples 4 and 5.*

5. $2\frac{11}{12} \cdot 1\frac{1}{4}$

a. 2 **b.** 3 **c.** 1 **d.** 12

6. $5\frac{1}{6} \cdot 3\frac{5}{7}$

a. 9 **b.** 15 **c.** 8 **d.** 20

7. $12\frac{2}{11} \div 3\frac{9}{10}$

a. 3 **b.** 4 **c.** 36 **d.** 9

8. $20\frac{3}{14} \div 4\frac{8}{11}$

a. 5 **b.** 80 **c.** 4 **d.** 16

Multiply or divide. For Exercises 13 through 16, find the exact answer and an estimated answer. See Examples 2 through 8.

9. $2\frac{2}{3} \cdot \frac{1}{7}$

10. $\frac{5}{9} \cdot 4\frac{1}{5}$

11. $7 \div 1\frac{3}{5}$

12. $9 \div 1\frac{2}{3}$

13. $2\frac{1}{5} \cdot 3\frac{1}{2}$

Exact:

Estimate:

14. $2\frac{1}{4} \cdot 7\frac{1}{8}$

Exact:

Estimate:

15. $3\frac{4}{5} \cdot 6\frac{2}{7}$

Exact:

Estimate:

16. $5\frac{5}{6} \cdot 7\frac{3}{5}$

Exact:

Estimate:

17. $5 \cdot 2\frac{1}{2}$

18. $6 \cdot 3\frac{1}{4}$

19. $3\frac{2}{3} \cdot 1\frac{1}{2}$

20. $2\frac{4}{5} \cdot 2\frac{5}{8}$

21. $2\frac{2}{3} \div \frac{1}{7}$

22. $\frac{5}{9} \div 4\frac{1}{5}$

Objective C *Choose the best estimate for each sum or difference. See Examples 9 and 12.*

23. $3\frac{7}{8} + 2\frac{1}{5}$

a. 6 **b.** 5 **c.** 1 **d.** 2

24. $3\frac{7}{8} - 2\frac{1}{5}$

a. 6 **b.** 5 **c.** 1 **d.** 2

25. $8\frac{1}{3} + 1\frac{1}{2}$

a. 4 **b.** 10 **c.** 6 **d.** 16

26. $8\frac{1}{3} - 1\frac{1}{2}$

a. 4 **b.** 10 **c.** 6 **d.** 16

Add. For Exercises 27 through 30, find the exact sum and an estimated sum. See Examples 9 through 11.

27.
$$\begin{array}{r} 4\frac{7}{12} \\ +2\frac{1}{12} \\ \hline \end{array}$$
Exact:

Estimate:

28.
$$\begin{array}{r} 7\frac{4}{11} \\ +3\frac{2}{11} \\ \hline \end{array}$$
Exact:

Estimate:

29.
$$\begin{array}{r} 10\frac{3}{14} \\ +\ 3\frac{4}{7} \\ \hline \end{array}$$
Exact:

Estimate:

30.
$$\begin{array}{r} 12\frac{5}{12} \\ +\ 4\frac{1}{6} \\ \hline \end{array}$$
Exact:

Estimate:

31.
$$\begin{array}{r} 9\frac{1}{5} \\ +8\frac{2}{25} \\ \hline \end{array}$$

32.
$$\begin{array}{r} 6\frac{2}{13} \\ +8\frac{7}{26} \\ \hline \end{array}$$

33.
$$\begin{array}{r} 12\frac{3}{14} \\ 10 \\ +25\frac{5}{12} \\ \hline \end{array}$$

34.
$$\begin{array}{r} 8\frac{2}{9} \\ 32 \\ +\ 9\frac{10}{21} \\ \hline \end{array}$$

35.
$$\begin{array}{r} 15\frac{4}{7} \\ +9\frac{11}{14} \\ \hline \end{array}$$

36.
$$\begin{array}{r} 23\frac{3}{5} \\ +8\frac{8}{15} \\ \hline \end{array}$$

37.
$$\begin{array}{r} 3\frac{5}{8} \\ 2\frac{1}{6} \\ +7\frac{3}{4} \\ \hline \end{array}$$

38.
$$\begin{array}{r} 4\frac{2}{3} \\ 9\frac{2}{5} \\ +3\frac{1}{6} \\ \hline \end{array}$$

Subtract. For Exercises 39 through 42, find the exact difference and an estimated difference. See Examples 12 through 14.

39.
$$\begin{array}{r} 4\frac{7}{10} \\ -2\frac{1}{10} \\ \hline \end{array}$$
Exact:

Estimate:

40.
$$\begin{array}{r} 7\frac{4}{9} \\ -3\frac{2}{9} \\ \hline \end{array}$$
Exact:

Estimate:

41.
$$\begin{array}{r} 10\frac{13}{14} \\ -\ 3\frac{4}{7} \\ \hline \end{array}$$
Exact:

Estimate:

42.
$$\begin{array}{r} 12\frac{5}{12} \\ -\ 4\frac{1}{6} \\ \hline \end{array}$$
Exact:

Estimate:

43. $$\begin{array}{r} 9\frac{1}{5} \\ -\ 8\frac{6}{25} \\ \hline \end{array}$$

44. $$\begin{array}{r} 5\frac{2}{13} \\ -\ 4\frac{7}{26} \\ \hline \end{array}$$

45. $$\begin{array}{r} 6 \\ -\ 2\frac{4}{9} \\ \hline \end{array}$$

46. $$\begin{array}{r} 8 \\ -\ 1\frac{7}{10} \\ \hline \end{array}$$

47. $$\begin{array}{r} 63\frac{1}{6} \\ -\ 47\frac{5}{12} \\ \hline \end{array}$$

48. $$\begin{array}{r} 86\frac{2}{15} \\ -27\frac{3}{10} \\ \hline \end{array}$$

Objectives B C Mixed Practice *Perform each indicated operation. See Examples 2 through 14.*

49. $$\begin{array}{r} 2\frac{3}{4} \\ +\ 1\frac{1}{4} \\ \hline \end{array}$$

50. $$\begin{array}{r} 5\frac{5}{8} \\ +\ 2\frac{3}{8} \\ \hline \end{array}$$

51. $$\begin{array}{r} 15\frac{4}{7} \\ -\ 9\frac{11}{14} \\ \hline \end{array}$$

52. $$\begin{array}{r} 23\frac{8}{15} \\ -\ 8\frac{3}{5} \\ \hline \end{array}$$

53. $3\frac{1}{9} \cdot 2$

54. $4\frac{1}{2} \cdot 3$

55. $1\frac{2}{3} \div 2\frac{1}{5}$

56. $5\frac{1}{5} \div 3\frac{1}{4}$

57. $22\frac{4}{9} + 13\frac{5}{18}$

58. $15\frac{3}{25} + 5\frac{2}{5}$

59. $5\frac{2}{3} - 3\frac{1}{6}$

60. $5\frac{3}{8} - 2\frac{3}{16}$

61. $$\begin{array}{r} 15\frac{4}{5} \\ 20\frac{3}{10} \\ +\ 37\frac{2}{15} \\ \hline \end{array}$$

62. $$\begin{array}{r} 3\frac{7}{16} \\ 6\frac{1}{2} \\ +9\frac{3}{8} \\ \hline \end{array}$$

63. $6\frac{4}{7} - 5\frac{11}{14}$

64. $47\frac{5}{12} - 23\frac{19}{24}$

65. $4\frac{2}{7} \cdot 1\frac{3}{10}$

66. $6\frac{2}{3} \cdot 2\frac{3}{4}$

67. $$\begin{array}{r} 6\frac{2}{11} \\ 3 \\ +\ 4\frac{10}{33} \\ \hline \end{array}$$

68. $$\begin{array}{r} 7\frac{3}{7} \\ 15 \\ +\ 20\frac{1}{2} \\ \hline \end{array}$$

Objective D Translating *Translate each phrase to an algebraic expression. Use x to represent "a number." See Examples 15 and 16.*

69. $-5\frac{2}{7}$ decreased by a number

70. The sum of $8\frac{3}{4}$ and a number

71. Multiply $1\frac{9}{10}$ by a number.

72. Divide a number by $-6\frac{1}{11}$.

Solve. For Exercises 73 and 74, the solutions have been started for you. Write each answer in simplest form. See Examples 15 and 16.

73. A heart attack patient in rehabilitation walked on a treadmill $12\frac{3}{4}$ miles over 4 days. How many miles is this per day?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

miles per day	is	total miles	divided by	number of days
↓	↓	↓	↓	↓
miles per day	=	______	÷	______

Finish with:
3. SOLVE. and **4.** INTERPRET.

74. A local restaurant is selling hamburgers from a booth on Memorial Day. A total of $27\frac{3}{4}$ pounds of hamburger have been ordered. How many quarter-pound hamburgers can this make?

Start the solution:

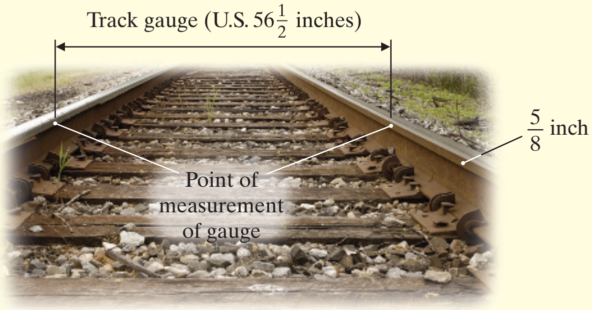
1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

how many quarter-pound hamburgers	is	total pounds of hamburger	divided by	a quarter-pound
↓	↓	↓	↓	↓
how many quarter-pound hamburgers	=	______	÷	______

Finish with:
3. SOLVE. and **4.** INTERPRET.

75. The Gauge Act of 1846 set the standard gauge for U.S. railroads at $56\frac{1}{2}$ inches. (See figure.) If the standard gauge in Spain is $65\frac{9}{10}$ inches, how much wider is Spain's standard gauge than the U.S. standard gauge? (*Source:* San Diego Railroad Museum)

76. The standard railroad track gauge (see figure) in Spain is $65\frac{9}{10}$ inches, while in neighboring Portugal it is $65\frac{11}{20}$ inches. Which gauge is wider and by how much? (*Source:* San Diego Railroad Museum)

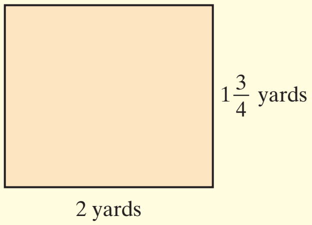


77. If Tucson's average rainfall is $11\frac{1}{4}$ inches and Yuma's is $3\frac{3}{5}$ inches, how much more rain, on the average, does Tucson get than Yuma?

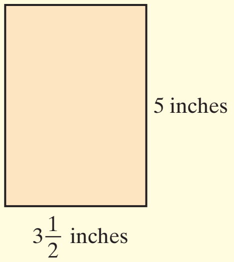
78. A pair of crutches needs adjustment. One crutch is 43 inches and the other is $41\frac{5}{8}$ inches. Find how much the shorter crutch should be lengthened to make both crutches the same length.

For Exercises 79 and 80, find the area of each figure.

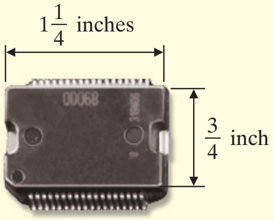
△ **79.**



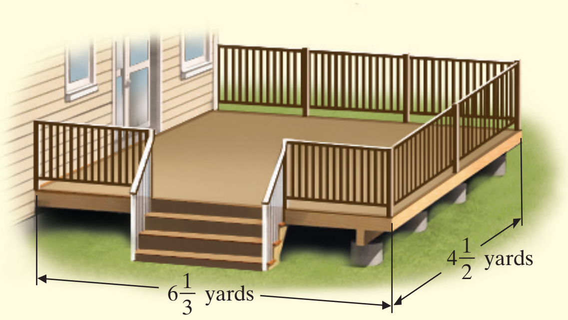
△ **80.**



△ **81.** A model for a proposed computer chip measures $\frac{3}{4}$ inch by $1\frac{1}{4}$ inches. Find its area.

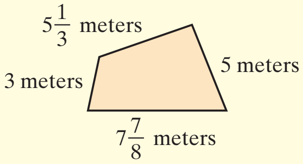


△ **82.** The Saltalamachios are planning to build a deck that measures $4\frac{1}{2}$ yards by $6\frac{1}{3}$ yards. Find the area of their proposed deck.

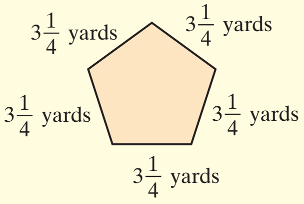


For Exercises 83 and 84, find the perimeter of each figure.

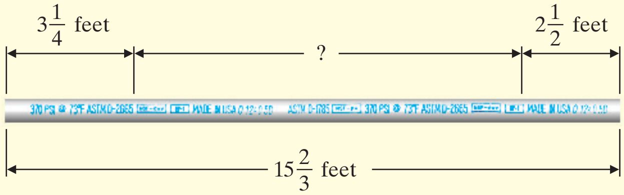
△ **83.**



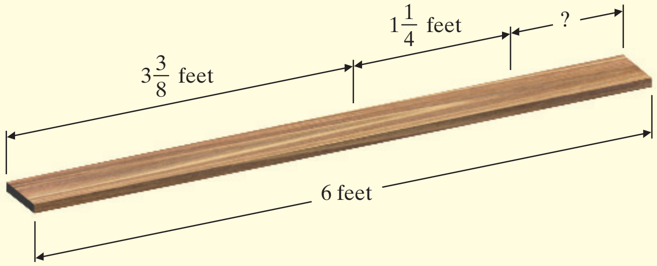
△ **84.**



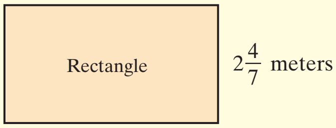
85. A homeowner has $15\frac{2}{3}$ feet of plastic pipe. She cuts off a $2\frac{1}{2}$-foot length and then a $3\frac{1}{4}$-foot length. If she now needs a 10-foot piece of pipe, will the remaining piece do? If not, by how much will the piece be short?



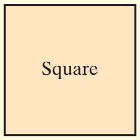
86. A trim carpenter has a 6-foot piece of board. He cuts off a $3\frac{3}{8}$-foot length and then a $1\frac{1}{4}$-foot length. How long is the remaining piece?



△ **87.** The area of the rectangle below is 12 square meters. If its width is $2\frac{4}{7}$ meters, find its length.



△ **88.** The perimeter of the square below is $23\frac{1}{2}$ feet. Find the length of each side.



The following table lists three upcoming total eclipses of the sun that will be visible from North America. The duration of each eclipse is listed in the table. Use the table to answer Exercises 89 through 92.

Total Solar Eclipses Visible from North America

Date of Eclipse	Duration (in Minutes)
April 8, 2024	$4\frac{7}{15}$
August 12, 2026	$2\frac{3}{10}$
March 30, 2033	$2\frac{37}{60}$

(*Source:* NASA/Goddard Space Flight Center)

89. What is the total duration for the three eclipses?

90. What is the total duration for the two eclipses occurring in even-numbered years?

91. How much longer will the April 8, 2024, eclipse be than the August 12, 2026, eclipse?

92. How much longer will the March 30, 2033, eclipse be than the August 12, 2026, eclipse?

Objective E *Perform the indicated operations. See Examples 17 through 24.*

93. $-4\frac{2}{5} \cdot 2\frac{3}{10}$

94. $-3\frac{5}{6} \div \left(-3\frac{2}{3}\right)$

95. $-5\frac{1}{8} - 19\frac{3}{4}$

96. $17\frac{5}{9} + \left(-14\frac{2}{3}\right)$

▶ **97.** $-31\frac{2}{15} + 17\frac{3}{20}$

98. $-31\frac{7}{8} - \left(-26\frac{5}{12}\right)$

99. $-1\frac{5}{7} \cdot \left(-2\frac{1}{2}\right)$

100. $1\frac{3}{4} \div \left(-3\frac{1}{2}\right)$

101. $11\frac{7}{8} - 13\frac{5}{6}$

102. $-20\frac{2}{5} + \left(-30\frac{3}{10}\right)$

103. $-7\frac{3}{10} \div (-100)$

104. $-4\frac{1}{4} \div 2\frac{3}{8}$

Review

Evaluate each expression. See Section 1.9.

105. $20 \div 10 \cdot 2$

106. $36 - 5 \cdot 6 + 10$

107. $2 + 3(8 \cdot 7 - 1)$

108. $2(10 - 2 \cdot 5) + 13$

Concept Extensions

Solve. See the first Concept Check in this section.

109. Which of the following are equivalent to 10?

a. $9\frac{5}{5}$ **b.** $9\frac{100}{100}$ **c.** $6\frac{44}{11}$ **d.** $8\frac{13}{13}$

110. Which of the following are equivalent to $7\frac{3}{4}$?

a. $6\frac{7}{4}$ **b.** $5\frac{11}{4}$ **c.** $7\frac{12}{16}$ **d.** all of them

Solve. See the second Concept Check in this section.

111. A student asked you to check her work below. Is it correct? If not, where is the error?

$$20\frac{2}{3} \div 10\frac{1}{2} \stackrel{?}{=} 2\frac{1}{3}$$

112. A student asked you to check his work below. Is it correct? If not, where is the error?

$$3\frac{2}{3} \cdot 1\frac{1}{7} \stackrel{?}{=} 3\frac{2}{21}$$

113. In your own words, describe how to divide mixed numbers.

114. In your own words, explain how to multiply

a. fractions

b. mixed numbers

Solve. See the third Concept Check in this section.

115. In your own words, explain how to round a mixed number to the nearest whole number.

116. Use rounding to estimate the best sum for $11\frac{19}{20} + 9\frac{1}{10}$.

a. 2 **b.** 3 **c.** 20 **d.** 21

Solve.

117. Explain in your own words why $9\frac{13}{9}$ is equal to $10\frac{4}{9}$.

118. In your own words, explain

a. when to borrow when subtracting mixed numbers, and

b. how to borrow when subtracting mixed numbers.

Chapter 3 Group Activity

Lobster Classification

Sections 3.1, 3.6, 3.7

This activity may be completed by working in groups or individually.

Lobsters are normally classified by weight. Use the weight classification table to answer the questions in this activity.

Classification of Lobsters

Class	Weight (in Pounds)
Chicken	1 to $1\frac{1}{8}$
Quarter	$1\frac{1}{4}$
Half	$1\frac{1}{2}$ to $1\frac{3}{4}$
Select	$1\frac{3}{4}$ to $2\frac{1}{2}$
Large select	$2\frac{1}{2}$ to $3\frac{1}{2}$
Jumbo	Over $3\frac{1}{2}$

(*Source:* The Maine Lobster Promotion Council)

1. A lobster fisher has kept four lobsters from a lobster trap. Classify each lobster if they have the following weights:

a. $1\frac{7}{8}$ pounds

b. $1\frac{9}{16}$ pounds

c. $2\frac{3}{4}$ pounds

d. $2\frac{3}{8}$ pounds

2. A recipe requires 5 pounds of lobster. Using the minimum weight for each class, decide whether a chicken, half, and select lobster will be enough for the recipe, and explain your reasoning. If not, suggest a better choice of lobsters to meet the recipe requirements.

3. A lobster market customer has selected two chickens, a select, and a large select. What is the most that these four lobsters could weigh? What is the least that these four lobsters could weigh?

4. A lobster market customer wishes to buy three quarters. If lobsters sell for $7 per pound, how much will the customer owe for her purchase?

5. Why do you think there is no classification for lobsters weighing under 1 pound?

Chapter 3 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

mixed number
composite number
prime number
reciprocals
complex fraction
equivalent
improper fraction
proper fraction
like
cross products
simplest form
numerator
least common denominator
undefined
prime factorization
denominator
0

1. Two numbers are ______________________ of each other if their product is 1.
2. A(n) ______________________ is a natural number greater than 1 that is not prime.
3. Fractions that represent the same portion of a whole are called ______________________ fractions.

4. A(n) ____________________ is a fraction whose numerator is greater than or equal to its denominator.

5. A(n) ____________________ is a natural number greater than 1 whose only factors are 1 and itself.

6. A fraction is in ____________________ when the numerator and the denominator have no factors in common other than 1.

7. A(n) ____________________ is one whose numerator is less than its denominator.

8. A(n) ____________________ contains a whole number part and a fraction part.

9. In the fraction $\frac{7}{9}$, the 7 is called the ____________________ and the 9 is called the ____________________.

10. The ____________________ of a number is the factorization in which all the factors are prime numbers.

11. The fraction $\frac{3}{0}$ is ____________________.

12. The fraction $\frac{0}{5} =$ ____________________.

13. Fractions that have the same denominator are called ____________________ fractions.

14. The LCM of the denominators in a list of fractions is called the ____________________.

15. A fraction whose numerator or denominator or both numerator and denominator contain fractions is called a(n) ____________________.

16. In $\frac{a}{b} = \frac{c}{d}$, $a \cdot d$ and $b \cdot c$ are called ____________________.

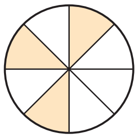
Helpful Hint

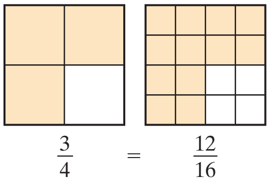
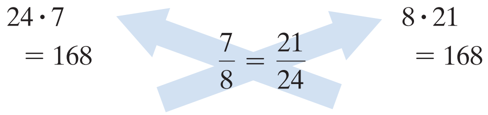
Are you preparing for your test? To help, don't forget to take these:

- Chapter 3 Getting Ready for the Test on page 279
- Chapter 3 Test on page 280

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyMathLab.

3 Chapter Highlights

Definitions and Concepts	Examples
Section 3.1 Introduction to Fractions and Mixed Numbers	
A **fraction** is of the form $\frac{\text{numerator}}{\text{denominator}}$ ← number of parts being considered ← number of equal parts in the whole	Write a fraction to represent the shaded part of the figure.  3 ← number of parts shaded 8 ← number of equal parts
A fraction is called a **proper fraction** if its numerator is less than its denominator.	Proper Fractions: $\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{100}{101}$
A fraction is called an **improper fraction** if its numerator is greater than or equal to its denominator.	Improper Fractions: $\frac{5}{4}, \frac{2}{2}, \frac{9}{7}, \frac{101}{100}$
A **mixed number** contains a whole number and a fraction.	Mixed Numbers: $1\frac{1}{2}, 5\frac{7}{8}, 25\frac{9}{10}$

Definitions and Concepts	Examples
Section 3.2 Factors and Simplest Form	
A **prime number** is a natural number that has exactly two different factors, 1 and itself.	2, 3, 5, 7, 11, 13, 17, . . .
A **composite number** is any natural number other than 1 that is not prime.	4, 6, 8, 9, 10, 12, 14, 15, 16, . . .
The **prime factorization** of a number is the factorization in which all the factors are prime numbers.	Write the prime factorization of 60. $60 = 6 \cdot 10$ $= 2\cdot3\cdot2\cdot5 \quad \text{or} \quad 2^2\cdot3\cdot5$
Fractions that represent the same portion of a whole are called **equivalent fractions.**	
A fraction is in **simplest form** or **lowest terms** when the numerator and the denominator have no common factors other than 1.	The fraction $\frac{2}{3}$ is in simplest form.
To write a fraction in simplest form, write the prime factorizations of the numerator and the denominator and then divide both by all common factors.	Write in simplest form: $\frac{30}{36}$ $\frac{30}{36} = \frac{2\cdot3\cdot5}{2\cdot2\cdot3\cdot3} = \frac{2}{2}\cdot\frac{3}{3}\cdot\frac{5}{2\cdot3} = 1\cdot1\cdot\frac{5}{6} = \frac{5}{6}$ or $\frac{30}{36} = \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot5}{\underset{1}{\cancel{2}}\cdot2\cdot\underset{1}{\cancel{3}}\cdot3} = \frac{5}{6}$
Two fractions are equivalent if **Method 1:** They simplify to the same fraction. **Method 2:** Their cross products are equal. $24\cdot7 = 168$ $\quad \frac{7}{8} = \frac{21}{24} \quad$ $8\cdot21 = 168$ Since $168 = 168$, $\frac{7}{8} = \frac{21}{24}$.	Determine whether $\frac{7}{8}$ and $\frac{21}{24}$ are equivalent. $\frac{7}{8}$ is in simplest form. $\frac{21}{24} = \frac{\overset{1}{\cancel{3}}\cdot7}{\underset{1}{\cancel{3}}\cdot8} = \frac{1\cdot7}{1\cdot8} = \frac{7}{8}$ Since both simplify to $\frac{7}{8}$, then $\frac{7}{8} = \frac{21}{24}$.
Section 3.3 Multiplying and Dividing Fractions	
To multiply two fractions, multiply the numerators and multiply the denominators.	Multiply. $\frac{2}{3}\cdot\frac{5}{7} = \frac{2\cdot5}{3\cdot7} = \frac{10}{21}$ $\frac{3}{4}\cdot\frac{1}{6} = \frac{3\cdot1}{4\cdot6} = \frac{\overset{1}{\cancel{3}}\cdot1}{4\cdot\underset{1}{\cancel{3}}\cdot2} = \frac{1}{8}$
To find the **reciprocal** of a fraction, interchange its numerator and denominator.	The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.	Divide. $-\frac{3}{10} \div \frac{7}{9} = -\frac{3}{10}\cdot\frac{9}{7} = -\frac{3\cdot9}{10\cdot7} = -\frac{27}{70}$

Definitions and Concepts	Examples
Section 3.4 Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions	
Fractions that have the same denominator are called **like fractions**.	$-\frac{1}{3}$ and $\frac{2}{3}$; $\frac{5}{7}$ and $\frac{6}{7}$
To add or subtract like fractions, combine the numerators and place the sum or difference over the common denominator.	$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$ ← Add the numerators. ← Keep the common denominator. $\frac{7}{8}-\frac{4}{8}=\frac{3}{8}$ ← Subtract the numerators. ← Keep the common denominator.
The **least common denominator (LCD)** of a list of fractions is the smallest positive number divisible by all the denominators in the list.	The LCD of $\frac{1}{2}$ and $\frac{5}{6}$ is 6 because 6 is the smallest positive number that is divisible by both 2 and 6.
Method 1 for Finding the LCD of a List of Fractions Using Multiples **Step 1:** Write the multiples of the largest denominator (starting with the number itself) until a multiple common to all denominators in the list is found. **Step 2:** The multiple found in Step 1 is the LCD.	Find the LCD of $\frac{1}{4}$ and $\frac{5}{6}$ using Method 1. $6\cdot 1 = 6$ Not a multiple of 4 $6\cdot 2 = 12$ A multiple of 4 The LCD is 12.
Method 2 for Finding the LCD of a List of a Fractions Using Prime Factorization **Step 1:** Write the prime factorization of each denominator. **Step 2:** For each different prime factor in Step 1, circle the greatest number of times that factor occurs in any one factorization. **Step 3:** The LCD is the product of the circled factors.	Find the LCD of $\frac{5}{6}$ and $\frac{11}{20}$ using Method 2. $6 = 2\cdot\enclose{circle}{3}$ $20 = \enclose{circle}{2\cdot 2}\cdot\enclose{circle}{5}$ The LCD is $2\cdot 2\cdot 3\cdot 5 = 60$
Equivalent fractions represent the same portion of a whole.	Write an equivalent fraction with the indicated denominator. $\frac{2}{8}=\frac{}{16}$ $\frac{2\cdot 2}{8\cdot 2}=\frac{4}{16}$
Section 3.5 Adding and Subtracting Unlike Fractions	
To Add or Subtract Fractions with Unlike Denominators	Add: $\frac{3}{20}+\frac{2}{5}$
Step 1: Find the LCD.	**Step 1:** The LCD of the denominators 20 and 5 is 20.
Step 2: Write each fraction as an equivalent fraction whose denominator is the LCD.	**Step 2:** $\frac{3}{20}=\frac{3}{20}$; $\frac{2}{5}=\frac{2}{5}\cdot\frac{4}{4}=\frac{8}{20}$
Step 3: Add or subtract the like fractions.	**Step 3:** $\frac{3}{20}+\frac{2}{5}=\frac{3}{20}+\frac{8}{20}=\frac{11}{20}$
Step 4: Write the sum or difference in simplest form.	**Step 4:** $\frac{11}{20}$ is in simplest form.

Definitions and Concepts	Examples
Section 3.6 Complex Fractions, Order of Operations, and Mixed Numbers	
A fraction whose numerator or denominator or both contain fractions is called a **complex fraction.**	Complex Fractions: $\dfrac{\frac{11}{4}}{\frac{7}{10}}, \quad \dfrac{\frac{1}{6} - 11}{\frac{4}{3}}$
One method for simplifying complex fractions is to multiply the numerator and the denominator of the complex fraction by the LCD of all fractions in its numerator and its denominator.	$\dfrac{\frac{1}{6} - 11}{\frac{4}{3}} = \dfrac{6\left(\frac{1}{6} - 11\right)}{6\left(\frac{4}{3}\right)} = \dfrac{6\left(\frac{1}{6}\right) - 6(11)}{6\left(\frac{4}{3}\right)}$ $= \dfrac{1 - 66}{8} = \dfrac{-65}{8} = -\dfrac{65}{8}$
To Write a Mixed Number as an Improper Fraction **1.** Multiply the denominator of the fraction by the whole number. **2.** Add the numerator of the fraction to the product from Step 1. **3.** Write the sum from Step 2 as the numerator of the improper fraction over the original denominator.	$5\dfrac{2}{7} = \dfrac{7 \cdot 5 + 2}{7} = \dfrac{35 + 2}{7} = \dfrac{37}{7}$
To Write an Improper Fraction as a Mixed Number or a Whole Number **1.** Divide the denominator into the numerator. **2.** The whole number part of the mixed number is the quotient. The fraction is the remainder over the original denominator. $\text{quotient}\,\dfrac{\text{remainder}}{\text{original denominator}}$	$\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{-15} \\ 2 \end{array} \qquad \dfrac{17}{3} = 5\dfrac{2}{3}$
Section 3.7 Operations on Mixed Numbers	
To multiply with mixed numbers or whole numbers, first write any mixed or whole numbers as improper fractions and then multiply as usual.	$2\dfrac{1}{3} \cdot \dfrac{1}{9} = \dfrac{7}{3} \cdot \dfrac{1}{9} = \dfrac{7 \cdot 1}{3 \cdot 9} = \dfrac{7}{27}$
To divide with mixed numbers or whole numbers, first write any mixed or whole numbers as fractions and then divide as usual.	$2\dfrac{5}{8} \div 3\dfrac{7}{16} = \dfrac{21}{8} \div \dfrac{55}{16} = \dfrac{21}{8} \cdot \dfrac{16}{55} = \dfrac{21 \cdot 16}{8 \cdot 55}$ $= \dfrac{21 \cdot 2 \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot 55} = \dfrac{42}{55}$
To add or subtract with mixed numbers, add or subtract the fractions and add or subtract the whole numbers.	Add: $2\dfrac{1}{2} + 5\dfrac{7}{8}$ $2\dfrac{1}{2} = 2\dfrac{4}{8}$ $\underline{+5\dfrac{7}{8} = 5\dfrac{7}{8}}$ $7\dfrac{11}{8} = 7 + 1\dfrac{3}{8} = 8\dfrac{3}{8}$

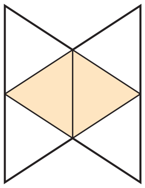
Chapter 3 Review

(3.1) *Determine whether each number is an improper fraction, a proper fraction, or a mixed number.*

1. $\frac{11}{23}$ **2.** $\frac{9}{8}$ **3.** $\frac{1}{2}$ **4.** $2\frac{1}{4}$

Write a fraction to represent the shaded area. If the fraction is improper, write the shaded area as a mixed number as well. (For Exercises 5 through 8, do not simplify answers.)

5.



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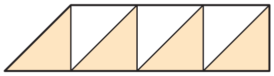

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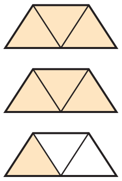

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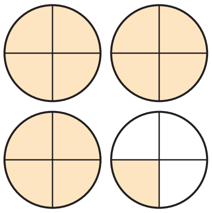


9. A basketball player made 11 free throws out of 12 attempts during a game. What fraction of free throws did the player make?

10. A new car lot contains 23 blue cars out of a total of 131 cars.

a. How many cars on the lot are not blue?

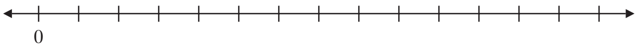
b. What fraction of cars on the lot are not blue?

Simplify by dividing.

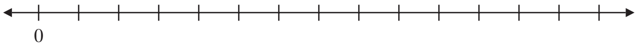
11. $\frac{3}{-3}$ **12.** $\frac{-20}{-20}$ **13.** $\frac{0}{-1}$ **14.** $\frac{4}{0}$

Graph each fraction on a number line.

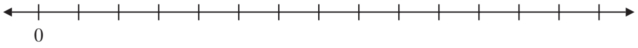
15. $\frac{7}{9}$



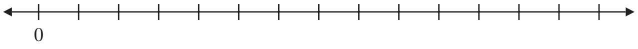
16. $\frac{4}{7}$



17. $\frac{5}{4}$



18. $\frac{7}{5}$



(3.2) *Find the prime factorization of each number.*

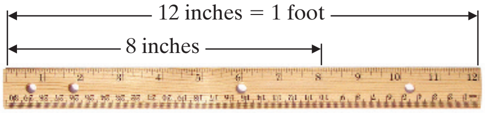
19. 68 **20.** 90 **21.** 825 **22.** 255

Write each fraction in simplest form.

23. $\frac{12}{28}$ **24.** $\frac{15}{27}$ **25.** $-\frac{25}{75}$ **26.** $-\frac{36}{72}$

27. $\frac{29}{32}$ **28.** $\frac{18}{23}$ **29.** $\frac{48}{6}$ **30.** $\frac{54}{9}$

31. There are 12 inches in a foot. What fractional part of a foot does 8 inches represent?



32. Six out of 15 cars are white. What fraction of the cars are *not* white?

Determine whether each set of two fractions is equivalent.

33. $\frac{10}{34}$ and $\frac{4}{14}$

34. $\frac{30}{50}$ and $\frac{9}{15}$

(3.3) *Multiply. Write each answer in simplest form.*

35. $-\frac{3}{5} \cdot \frac{1}{2}$ **36.** $\frac{6}{7} \cdot \frac{5}{12}$ **37.** $-\frac{24}{5} \cdot \left(-\frac{15}{8}\right)$ **38.** $\frac{39}{3} \cdot \frac{7}{13} \cdot \frac{5}{21}$

39. $\left(-\frac{1}{3}\right)^3$ **40.** $\left(-\frac{5}{12}\right)^2$

41. Evaluate xy if $x = \frac{2}{3}$ and $y = \frac{1}{5}$.

42. Evaluate ab if $a = -7$ and $b = \frac{9}{10}$.

Find the reciprocal of each number.

43. 7

44. $\frac{14}{23}$

Divide. Write each answer in simplest form.

45. $-\frac{3}{4} \div \frac{3}{8}$ **46.** $\frac{21}{4} \div \frac{7}{5}$ **47.** $\frac{5}{3} \div (-2)$

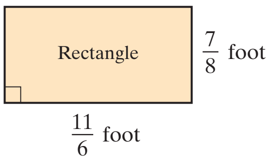
48. $-\frac{9}{2} \div -\frac{1}{3}$

49. Evaluate $x \div y$ if $x = \frac{9}{7}$ and $y = \frac{3}{4}$.

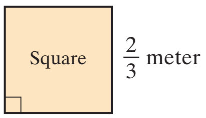
50. Evaluate $a \div b$ if $a = -5$ and $b = \frac{2}{3}$.

Find the area of each figure.

△ **51.**



△ **52.**



(3.4) *Add or subtract as indicated.*

53. $\frac{7}{11} + \frac{3}{11}$

54. $\frac{4}{9} + \frac{2}{9}$

55. $\frac{1}{12} - \frac{5}{12}$

56. $\frac{11}{15} + \frac{1}{15}$

57. $\frac{4}{21} - \frac{3}{21}$

58. $\frac{4}{15} - \frac{3}{15} - \frac{2}{15}$

Find the LCD of each list of fractions.

59. $\frac{2}{3}, \frac{5}{7}$

60. $\frac{3}{4}, \frac{3}{8}, \frac{7}{12}$

Write each fraction as an equivalent fraction with the given denominator.

61. $\frac{2}{3} = \frac{?}{30}$

62. $\frac{5}{8} = \frac{?}{56}$

63. $\frac{7}{6} = \frac{?}{42}$

64. $\frac{9}{4} = \frac{?}{20}$

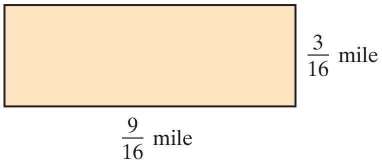
65. $\frac{4}{5} = \frac{?}{50}$

66. $\frac{5}{9} = \frac{?}{18}$

Solve.

67. One evening Mark Alorenzo did $\frac{3}{8}$ of his homework before supper, another $\frac{2}{8}$ of it while his children did their homework, and $\frac{1}{8}$ after his children went to bed. What part of his homework did he do that evening?

△ **68.** The Simpsons will be fencing in their land, which is in the shape of a rectangle. In order to do this, they need to find its perimeter. Find the perimeter of their land.



(3.5) *Add or subtract as indicated.*

69. $\frac{7}{18} + \frac{2}{9}$

70. $\frac{4}{13} - \frac{1}{26}$

71. $-\frac{1}{3} + \frac{1}{4}$

72. $-\frac{2}{3} + \frac{1}{4}$

73. $-\frac{9}{14} - \frac{3}{7}$

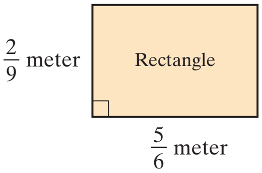
74. $\frac{5}{12} - \frac{2}{9}$

75. $\frac{4}{25} + \frac{23}{75} + \frac{7}{50}$

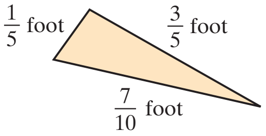
76. $\frac{2}{3} - \frac{2}{9} - \frac{1}{6}$

Find the perimeter of each figure.

△ **77.**

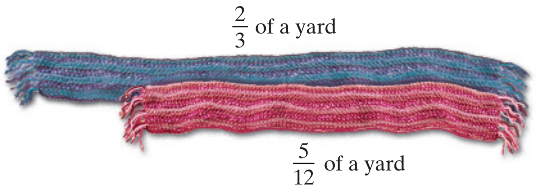


△ **78.**



79. In a group of 100 blood donors, typically $\frac{9}{25}$ have type A Rh-positive blood and $\frac{3}{50}$ have type A Rh-negative blood. What fraction have type A blood?

80. Find the difference in length of two scarves if one scarf is $\frac{5}{12}$ of a yard long and the other is $\frac{2}{3}$ of a yard long.



Insert < or > to form a true statement.

81. $\frac{5}{11}$ $\quad$ $\frac{6}{11}$

82. $\frac{4}{35}$ $\quad$ $\frac{3}{35}$

83. $-\frac{5}{14}$ $\quad$ $-\frac{16}{42}$

84. $-\frac{6}{35}$ $\quad$ $-\frac{17}{105}$

(3.6) *Simplify each complex fraction.*

85. $\dfrac{\frac{2}{5}}{\frac{7}{10}}$

86. $\dfrac{\frac{3}{7}}{\frac{11}{7}}$

87. $\dfrac{\frac{2}{5}-\frac{1}{2}}{\frac{3}{4}-\frac{7}{10}}$

88. $\dfrac{\frac{5}{6}-\frac{1}{4}}{-\frac{1}{12}}$

Evaluate each expression if $x = \frac{1}{2}$, $y = -\frac{2}{3}$, *and* $z = \frac{4}{5}$.

89. y^2

90. $x - z$

Write each improper fraction as a mixed number or a whole number.

91. $\frac{15}{4}$

92. $\frac{39}{13}$

93. $\frac{7}{7}$

94. $\frac{125}{4}$

Write each mixed number as an improper fraction.

95. $2\frac{1}{5}$

96. $3\frac{8}{9}$

Evaluate each expression. Use the order of operations to simplify.

97. $\frac{5}{13} \div \frac{1}{2} \cdot \frac{4}{5}$

98. $\frac{2}{27} - \left(\frac{1}{3}\right)^2$

99. $\frac{9}{10} \cdot \frac{1}{3} - \frac{2}{5} \cdot \frac{1}{11}$

100. $-\frac{2}{7} \cdot \left(\frac{1}{5} + \frac{3}{10}\right)$

(3.7) *Perform operations as indicated. Simplify your answers. Estimate where indicated.*

101. $31\frac{2}{7} + 14\frac{10}{21}$

102. $\begin{array}{r} 7\frac{3}{8} \\ 9\frac{5}{6} \\ +\,3\frac{1}{12} \\ \hline \end{array}$

103. $\begin{array}{r} 9\frac{1}{7} \\ -\,4\frac{3}{5} \\ \hline \end{array}$

104. $\begin{array}{r} 8\frac{1}{5} \\ -\,5\frac{3}{11} \\ \hline \end{array}$

105. $1\frac{5}{8} \cdot 3\frac{1}{5}$

Exact:

Estimate:

106. $3\frac{6}{11} \cdot 1\frac{7}{13}$

Exact:

Estimate:

107. $6\frac{3}{4} \div 1\frac{2}{7}$

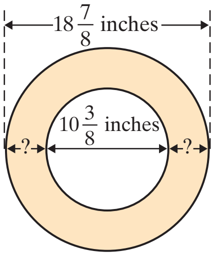
108. $5\frac{1}{2} \div 2\frac{1}{11}$

109. A truck traveled 341 miles on $15\frac{1}{2}$ gallons of gas. How many miles might we expect the truck to travel on 1 gallon of gas?

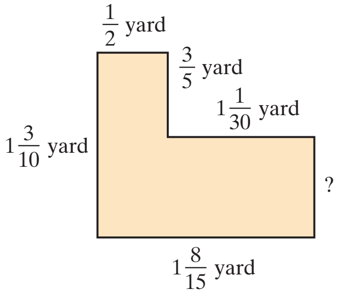
110. There are $7\frac{1}{3}$ grams of fat in each ounce of hamburger. How many grams of fat are in a 5-ounce hamburger patty?

Find the unknown measurements.

111.



112.



Perform the indicated operations.

113. $-12\frac{1}{7} + \left(-15\frac{3}{14}\right)$

114. $23\frac{7}{8} - 24\frac{7}{10}$

$-\frac{33}{40}$

115. $-3\frac{1}{5} \div \left(-2\frac{7}{10}\right)$

116. $-2\frac{1}{4} \cdot 1\frac{3}{4}$

Mixed Review

Perform indicated operations. Write each answer in simplest form. Estimate where indicated.

117. $\frac{7}{8} \cdot \frac{2}{3}$

118. $\frac{6}{15} \cdot \frac{5}{8}$

119. $\frac{18}{5} \div \frac{2}{5}$

120. $\frac{9}{2} \div \frac{1}{3}$

121. $\frac{5}{12} - \frac{3}{12}$

122. $\frac{3}{10} - \frac{1}{10}$

123. $\frac{2}{3} + \frac{1}{4}$

124. $\frac{5}{11} + \frac{2}{55}$

125. $4\frac{1}{6} \cdot 2\frac{2}{5}$

Exact:

Estimate:

126. $5\frac{2}{3} \cdot 2\frac{1}{4}$

Exact:

Estimate:

127. $\frac{7}{2} \div 1\frac{1}{2}$

128. $1\frac{3}{5} \div \frac{1}{4}$

129. $\begin{array}{r} 7\frac{3}{4} \\ +5\frac{2}{3} \\ \hline \end{array}$

130. $\begin{array}{r} 2\frac{7}{8} \\ +9\frac{1}{2} \\ \hline \end{array}$

131. $\begin{array}{r} 12\frac{3}{5} \\ -9\frac{1}{7} \\ \hline \end{array}$

132. $\begin{array}{r} 32\frac{10}{21} \\ -24\frac{3}{7} \\ \hline \end{array}$

Evaluate each expression. Use the order of operations to simplify.

133. $\frac{2}{5} + \left(\frac{2}{5}\right)^2 - \frac{3}{25}$

134. $\left(\frac{5}{6} - \frac{3}{4}\right)^2$

135. $-\frac{3}{8} \cdot \left(\frac{2}{3} - \frac{4}{9}\right)$

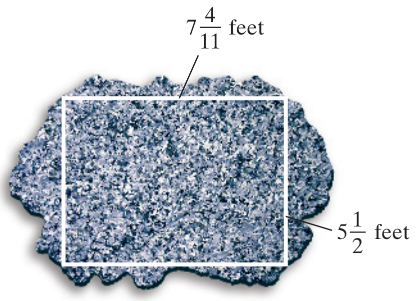
Solve.

136. Two packages to be mailed weigh $3\frac{3}{4}$ pounds and $2\frac{3}{5}$ pounds. Find their combined weight.

137. An area of Mississippi received $23\frac{1}{2}$ inches of rain in $30\frac{1}{2}$ hours. How many inches per 1 hour is this?

138. A ribbon $5\frac{1}{2}$ yards long is cut from a reel of ribbon with 50 yards on it. Find the length of the piece remaining on the reel.

△ **139.** A slab of natural granite is purchased and a rectangle with length $7\frac{4}{11}$ feet and width $5\frac{1}{2}$ feet is cut from it. Find the area of the rectangle.



Chapter 3 Getting Ready for the Test LC

MULTIPLE CHOICE Exercises 1 through 10 are **Multiple Choice**. Choose the correct letter.

For Exercises 1 through 4, choose whether the expression simplifies to

A. 1 **B.** −1 **C.** 0 **D.** undefined

1. $\frac{-2}{-2}$ **2.** $\frac{-2}{2}$ **3.** $\frac{2}{0}$ **4.** $\frac{0}{-2}$

5. The mixed number $4\frac{3}{5}$ written as a fraction is:

A. $\frac{12}{5}$ **B.** $\frac{5}{23}$ **C.** $\frac{23}{5}$ **D.** $\frac{23}{3}$

6. The improper fraction $\frac{23}{8}$ written as a mixed number is:

A. 2.3 **B.** $2\frac{7}{8}$ **C.** $8\frac{2}{3}$ **D.** $2\frac{8}{7}$

For Exercises 7 through 10, the exercises statement and correct answer are given. Choose whether the correct operation in the box should be:

A. + (addition) **B.** − (subtraction) **C.** · (multiplication) **D.** ÷ (division)

7. $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{16}{121}$ **8.** $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{8}{2}$ or 4

9. $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{6}{11}$ **10.** $\frac{8}{11} \square \frac{2}{11}$; Answer: $\frac{10}{11}$

MATCHING *For Exercises 11 through 14, **match** each operation of fractions in the first column with the correct answer in the second or third column.*

11. $\frac{5}{7} + \frac{1}{7}$

12. $\frac{5}{7} \cdot \frac{1}{7}$

13. $\frac{5}{7} \div \frac{1}{7}$

14. $\frac{5}{7} - \frac{1}{7}$

A. $\frac{5}{7}$

B. $\frac{5}{1}$ or 5

C. $\frac{6}{14}$ or $\frac{3}{7}$

D. $\frac{4}{7}$

E. undefined

F. $\frac{6}{7}$

G. $\frac{6}{49}$

H. $\frac{5}{49}$

MULTIPLE CHOICE Exercises 15 through 20 are **Multiple Choice**. Choose the correct answer.

For each expression in Exercises 15 through 18, use order of operations and decide which operation below should be performed first to simplify.

A. Addition **B.** Subtraction **C.** Multiplication **D.** Division

15. $\frac{1}{2} + \frac{1}{5} \cdot \frac{1}{4}$ **16.** $\left(\frac{1}{2} + \frac{1}{5}\right) \cdot \frac{1}{4}$ **17.** $\frac{1}{2} \div \frac{1}{5} \cdot \frac{1}{4}$ **18.** $\frac{1}{2} \div \left(\frac{1}{5} \cdot \frac{1}{4}\right)$

19. The expression $\left(\frac{1}{8}\right)^2$ evaluates to:

A. $\frac{1}{16}$ **B.** $\frac{2}{16} = \frac{1}{8}$ **C.** $\frac{1}{64}$

20. Choose the mixed number that is closest to $18 - 8\frac{9}{10}$.

A. 10 **B.** 9 **C.** 26

Chapter 3 Test MyLab Math

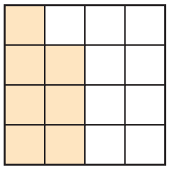
For additional practice go to your study plan in MyLab Math.

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______

Write a fraction to represent the shaded area.

1.



Write the mixed number as an improper fraction.

2. $7\frac{2}{3}$

Write the improper fraction as a mixed number.

3. $\frac{75}{4}$

Write each fraction in simplest form.

4. $\frac{24}{210}$

5. $-\frac{42}{70}$

Determine whether these fractions are equivalent.

6. $\frac{5}{7}$ and $\frac{8}{11}$

7. $\frac{6}{27}$ and $\frac{14}{63}$

Find the prime factorization of each number.

8. 84

9. 495

10. Find the LCM of 8, 9, and 12.

Perform each indicated operation. Simplify your answers.

11. $\frac{7}{9}+\frac{1}{9}$

12. $-\frac{8}{15}-\frac{2}{15}$

13. $\frac{4}{4}\div\frac{3}{4}$

14. $-\frac{4}{3}\cdot\frac{4}{4}$

15. $\frac{1}{6}+\frac{3}{14}$

16. $\frac{7}{8}-\frac{1}{3}$

17. $-\frac{2}{3}\cdot-\frac{8}{15}$

18. $8\div\frac{1}{2}$

19. $\frac{6}{21}-\frac{1}{7}$

20. $\frac{16}{25}-\frac{1}{2}$

21. $\frac{3}{8}\cdot\frac{16}{6}\cdot\frac{4}{11}$

22. $5\frac{1}{4}\div\frac{7}{12}$

23. $\frac{11}{12} - \frac{3}{8} + \frac{5}{24}$

24. $\begin{array}{r} 3\frac{7}{8} \\ 7\frac{2}{5} \\ +2\frac{3}{4} \\ \hline \end{array}$

25. $\begin{array}{r} 19 \\ -\ 2\frac{3}{11} \\ \hline \end{array}$

26. $-\frac{16}{3} \div -\frac{3}{12}$

27. $3\frac{1}{3} \cdot 6\frac{3}{4}$

28. $-\frac{2}{7} \cdot \left(6 - \frac{1}{6}\right)$

29. $\frac{1}{2} \div \frac{2}{3} \cdot \frac{3}{4}$

30. $\left(-\frac{3}{4}\right)^2 \div \left(\frac{2}{3} + \frac{5}{6}\right)$

31. Find the average of $\frac{5}{6}$, $\frac{4}{3}$, and $\frac{7}{12}$.

Simplify the complex fraction.

32. $\dfrac{5 + \frac{3}{7}}{2 - \frac{1}{2}}$

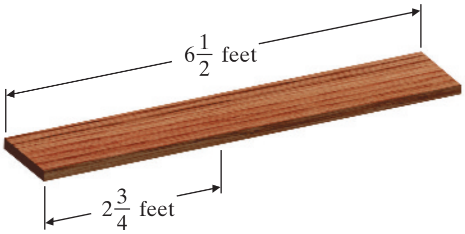
Evaluate the expression for the given replacement value.

33. $-5x$; $x = -\frac{1}{2}$

34. $x \div y$; $x = \frac{1}{2}$, $y = 3\frac{7}{8}$

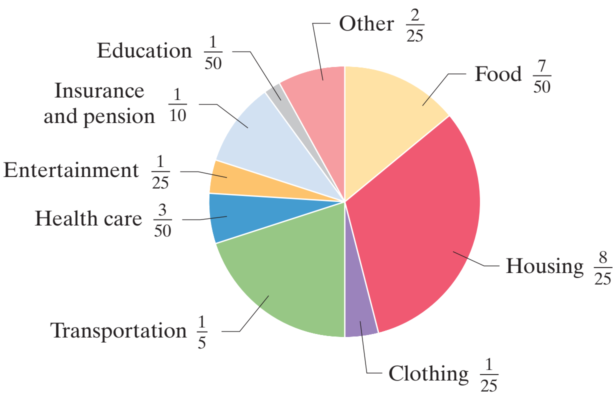
Solve.

35. A carpenter cuts a piece $2\frac{3}{4}$ feet long from a cedar plank that is $6\frac{1}{2}$ feet long. How long is the remaining piece?



The circle graph below shows how the average consumer spends money. For example, $\frac{7}{50}$ of your spending goes for food. Use this information for Exercises 36 through 38.

Consumer Spending



Source: U.S. Bureau of Labor Statistics; based on survey

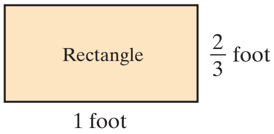
36. What fraction of spending goes for housing and food combined?

37. What fraction of spending goes for education, transportation, and clothing?

38. Suppose your family spent $47,000 on the items in the graph. How much might we expect was spent on health care?

Find the perimeter and area of the figure.

39.



40. During a 258-mile trip, a car used $10\frac{3}{4}$ gallons of gas. How many miles would we expect the car to travel on 1 gallon of gas?

23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________
38. ____________
39. ____________
40. ____________

Chapters 1–3 Cumulative Review

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______

Write each number in words.

1. 126

2. 115

3. 27,034

4. 6573

5. Add: $23 + 136$

6. Add: $587 + 44$

7. Subtract: $543 - 29$. Check by adding.

8. Subtract: $995 - 62$. Check by adding.

9. Round 278,362 to the nearest thousand.

10. Round 1436 to the nearest ten.

11. A digital video disc (DVD) can hold about 4800 megabytes (MB) of information. How many megabytes can 12 DVDs hold?

12. On a trip across the country, Daniel Daunis travels 435 miles per day. How many total miles does he travel in 3 days?

13. Divide and check: $56{,}717 \div 8$

14. Divide and check: $4558 \div 12$

Write using exponential notation.

15. $7 \cdot 7 \cdot 7$

16. $7 \cdot 7$

17. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 17 \cdot 17 \cdot 17$

18. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 5 \cdot 5$

19. Evaluate $2(x - y)$ for $x = 8$ and $y = 4$.

20. Evaluate $8a + 3(b - 5)$ for $a = 5$ and $b = 9$.

21. The world's deepest cave is Krubera (or Voronja), in the country of Georgia, located by the Black Sea in Asia. It has been explored to a depth of 7188 feet below the surface of Earth. Represent this position using an integer.

22. The temperature on a cold day in Minneapolis, MN, is 21°F below zero. Represent this temperature using an integer.

23. Add using a number line: $-7 + 3$

24. Add using a number line: $-3 + 8$

25. Simplify: $7 - 8 - (-5) - 1$

26. Simplify: $6 + (-8) - (-9) + 3$

27. Evaluate: $(-5)^2$

28. Evaluate: -2^4

29. Simplify: $\dfrac{12 - 16}{-1 + 3}$

30. Simplify: $(20 - 5^2)^2$

31. Write the prime factorization of 45.

32. Write the prime factorization of 92.

Multiply.

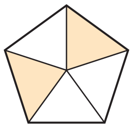
33. $\dfrac{2}{3} \cdot \dfrac{5}{11}$

34. $\dfrac{1}{7} \cdot \dfrac{2}{5}$

35. $\dfrac{1}{4} \cdot \dfrac{1}{2}$

36. $\dfrac{3}{5} \cdot \dfrac{1}{5}$

37. Write a fraction to represent the shaded part of the figure.



38. Write the prime factorization of 156.

39. Write each as an improper fraction.

a. $4\dfrac{2}{9}$ **b.** $1\dfrac{8}{11}$

40. Write $7\dfrac{4}{5}$ as an improper fraction.

41. Write in simplest form: $\dfrac{42}{66}$

42. Write in simplest form: $\dfrac{70}{105}$

43. Multiply: $3\dfrac{1}{3} \cdot \dfrac{7}{8}$

44. Multiply: $\dfrac{2}{3} \cdot 4$

45. Divide and simplify: $\dfrac{5}{16} \div \dfrac{3}{4}$

46. Divide: $1\dfrac{1}{10} \div 5\dfrac{3}{5}$

23. ______

24. ______

25. ______

26. ______

27. ______

28. ______

29. ______

30. ______

31. ______

32. ______

33. ______

34. ______

35. ______

36. ______

37. ______

38. ______

39. a. ______

b. ______

40. ______

41. ______

42. ______

43. ______

44. ______

45. ______

46. ______

4

Decimals

Decimal numbers represent parts of a whole, just like fractions. For example, one penny is 0.01 or $\frac{1}{100}$ of a dollar. In this chapter, we learn to perform arithmetic operations on decimals and to analyze the relationship between fractions and decimals. We also learn how decimals are used in the real world.

Sections

Check Your Progress

Places Where Cacao Trees Grow

Cacao Tree

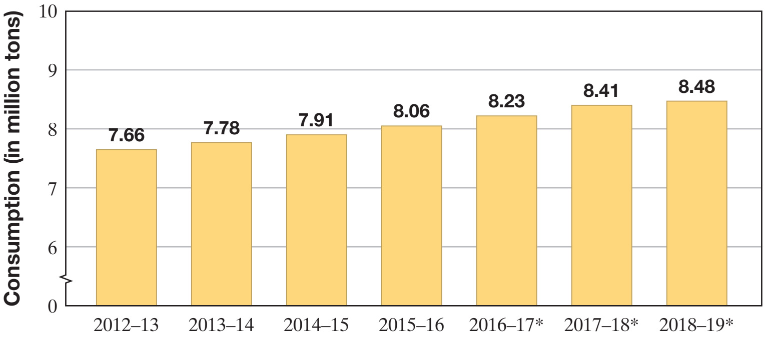
Cacao Pod

How Is Chocolate Made, and How Popular Is It?

Chocolate history begins in Latin America, where cacao trees grew wild. The first people to use chocolate extensively were the Maya. They not only created a beverage from the cacao beans but also used the beans themselves as a currency. From them, the secret of chocolate passed onto the Aztecs, who conquered the Maya. Through interactions between the Aztecs and the Spanish explorers, chocolate made it to Spain. Eventually it made its way to the rest of Europe. In fact, chocolate was the first caffeine to reach Europe, predating the introduction of coffee and tea by a few years. Solid chocolate did not appear until the mid 1800s, when an Englishman invented a way to create a chocolate bar.

In Section 4.2, Exercises 87–92, we will explore the top chocolate-consuming nations in the world.

Retail Consumption of Chocolate Confectionery Worldwide
(in million tons)



Note: ⌇ means that numbers are missing

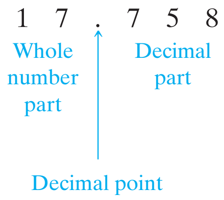
* predicted years

Source: Statista

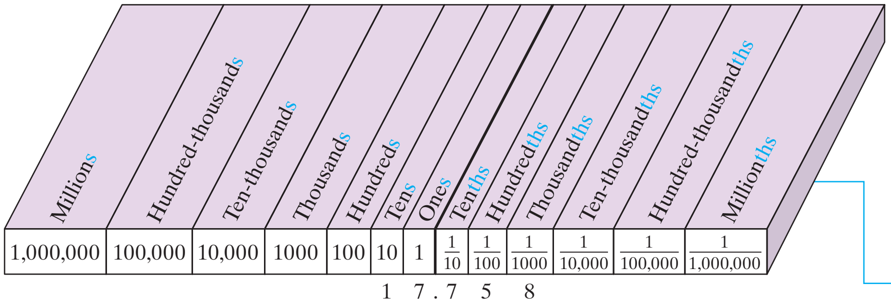
4.1 Introduction to Decimals

Objective A Decimal Notation and Writing Decimals in Words

Like fractional notation, decimal notation is used to denote a part of a whole. Numbers written in decimal notation are called **decimal numbers,** or simply **decimals.** The decimal 17.758 has three parts.



In Section 1.2, we introduced place value for whole numbers. Place names and place values for the whole number part of a decimal number are exactly the same. Place names and place values for the decimal part are shown below.



Notice that the value of each place is $\frac{1}{10}$ of the value of the place to its left. For example,

$$\underset{\text{ones}}{1}\cdot\frac{1}{10}=\underset{\text{tenths}}{\frac{1}{10}} \quad \text{and} \quad \underset{\text{tenths}}{\frac{1}{10}}\cdot\frac{1}{10}=\underset{\text{hundredths}}{\frac{1}{100}}$$

The decimal number 17.758 means

1 ten + 7 ones + 7 tenths + 5 hundredths + 8 thousandths

or $1\cdot 10 + 7\cdot 1 + 7\cdot\frac{1}{10} + 5\cdot\frac{1}{100} + 8\cdot\frac{1}{1000}$

or $10 + 7 + \frac{7}{10} + \frac{5}{100} + \frac{8}{1000}$

Writing (or Reading) a Decimal in Words

Step 1: Write the whole number part in words.

Step 2: Write "and" for the decimal point.

Step 3: Write the decimal part in words as though it were a whole number, followed by the place value of the last digit.

Objectives

A Know the Meaning of Place Value for a Decimal Number and Write Decimals in Words.

B Write Decimals in Standard Form.

C Write Decimals as Fractions.

D Compare Decimals.

E Round Decimals to Given Place Values.

Helpful Hint Notice that place values to the left of the decimal point end in "s." Place values to the right of the decimal point end in "ths."

Practice 1

Write each decimal in words.

a. 0.08 **b.** −500.025

c. 0.0329

Example 1 Write each decimal in words.

a. 0.3 **b.** −50.82 **c.** 21.093

Solution:

a. Three tenths

b. Negative fifty and eighty-two hundredths

c. Twenty-one and ninety-three thousandths

Work Practice 1

Practice 2

Write the decimal 97.28 in words.

Example 2 Write the decimal in the following sentence in words: The Golden Jubilee Diamond is a 545.67-carat cut diamond. (*Source: The Guinness Book of Records*)

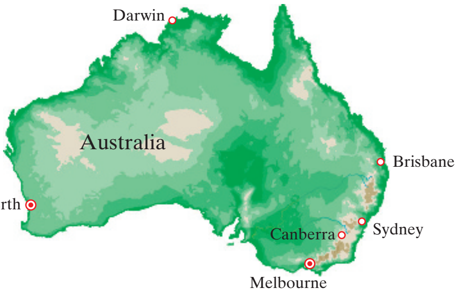
Solution: five hundred forty-five and sixty-seven hundredths

Work Practice 2

Practice 3

Write the decimal 72.1085 in words.

Example 3 Write the decimal in the following sentence in words: The oldest known fragments of the Earth's crust are Zircon crystals; they were discovered in Australia and are thought to be 4.276 billion years old. (*Source: The Guinness Book of Records*)

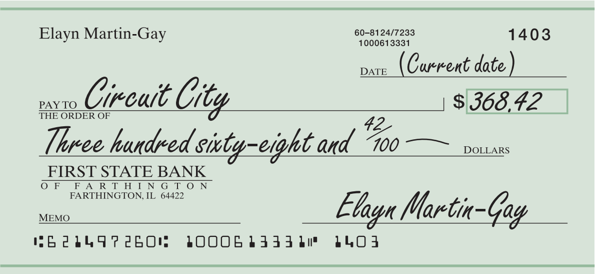


Solution: four and two hundred seventy-six thousandths

Work Practice 3

Suppose that you are paying for a purchase of $368.42 at Circuit City by writing a check. Checks are usually written using the following format.

Helpful Hint Although the number of checks written in the United States is decreasing, there are still about 21 million checks written each day. (*Source:* Federal Reserve System)



Elayn Martin-Gay
60–8124/7233
1000613331
1403
DATE (Current date)
PAY TO THE ORDER OF Circuit City $ 368.42
Three hundred sixty-eight and 42/100 —— DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO
Elayn Martin-Gay
⑆621497260⑆ 1000613331⑈ 1403

Answers

1. a. eight hundredths **b.** negative five hundred and twenty-five thousandths **c.** three hundred twenty-nine ten-thousandths **2.** ninety-seven and twenty-eight hundredths **3.** seventy-two and one thousand eighty-five ten-thousandths

Example 4 Fill in the check to Camelot Music to pay for your purchase of $92.98.

Solution:

Your Preprinted Name
Your Preprinted Address
60-8124/7233
1000613331
1404
DATE (Current date)
PAY TO THE ORDER OF Camelot Music $ 92.98
Ninety-two and 98/100 DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO (Your signature)
⑆621497260⑆ 1000613331⑈ 1404

Work Practice 4

Practice 4

Fill in the check to CLECO (Central Louisiana Electric Company) to pay for your monthly electric bill of $207.40.

Your Preprinted Name
Your Preprinted Address
60-8124/7233
1000613331
1406
DATE
PAY TO THE ORDER OF $
DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO
⑆621497260⑆ 1000613331⑈ 1406

Objective B Writing Decimals in Standard Form

A decimal written in words can be written in standard form by reversing the procedure in Objective A.

Examples Write each decimal in standard form.

5. Forty-eight and twenty-six hundredths is

48.26

hundredths place

6. Six and ninety-five thousandths is

6.095

thousandths place

Work Practice 5–6

Practice 5–6

Write each decimal in standard form.

5. Three hundred and ninety-six hundredths

6. Thirty-nine and forty-two thousandths

Helpful Hint

When converting a decimal from words to decimal notation, make sure the last digit is in the correct place by inserting 0s if necessary. For example,

Two and thirty-eight thousandths is 2.038

thousandths place

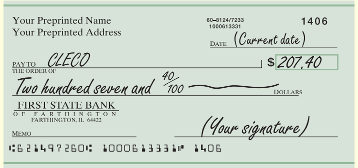
Objective C Writing Decimals as Fractions

Once you master reading and writing decimals, writing a decimal as a fraction follows naturally.

Decimal	In Words	Fraction
0.7	seven tenths	$\frac{7}{10}$
0.51	fifty-one hundredths	$\frac{51}{100}$
0.009	nine thousandths	$\frac{9}{1000}$
0.05	five hundredths	$\frac{5}{100} = \frac{1}{20}$

Answers

4.



Your Preprinted Name
Your Preprinted Address
60-8124/7233
1000613331
1406
DATE (Current date)
PAY TO THE ORDER OF CLECO $ 207.40
Two hundred seven and 40/100 DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO (Your signature)
⑆621497260⑆ 1000613331⑈ 1406

5. 300.96 **6.** 39.042

Notice that the number of decimal places in a decimal number is the same as the number of zeros in the denominator of the equivalent fraction. We can use this fact to write decimals as fractions.

$$0.\underbrace{31}_{\substack{\uparrow\\ \text{2 decimal}\\ \text{places}}} = \frac{31}{1\underbrace{00}_{\substack{\uparrow\\ \text{2 zeros}}}} \qquad 0.\underbrace{007}_{\substack{\uparrow\\ \text{3 decimal}\\ \text{places}}} = \frac{7}{1\underbrace{000}_{\substack{\uparrow\\ \text{3 zeros}}}}$$

Practice 7

Write 0.037 as a fraction.

Example 7

Write 0.43 as a fraction.

Solution:

$$0.\underset{\substack{\uparrow\\ 2\\ \text{decimal places}}}{43} = \frac{43}{1\underset{\substack{\uparrow\\ 2\\ \text{zeros}}}{00}}$$

Work Practice 7

Practice 8

Write 14.97 as a mixed number.

Example 8

Write 5.7 as a mixed number.

Solution:

$$5.\underset{\substack{\uparrow\\ 1\\ \text{decimal place}}}{7} = 5\frac{7}{1\underset{\substack{\uparrow\\ 1\\ \text{zero}}}{0}}$$

Work Practice 8

Practice 9–11

Write each decimal as a fraction or mixed number. Write your answer in simplest form.

9. 0.12

10. 57.8

11. −209.086

Examples

Write each decimal as a fraction or a mixed number. Write your answer in simplest form.

$$\textbf{9.}\ 0.125 = \frac{125}{1000} = \frac{\overset{1}{\cancel{125}}}{8\cdot\underset{1}{\cancel{125}}} = \frac{1}{8}$$

$$\textbf{10.}\ 23.5 = 23\frac{5}{10} = 23\frac{\overset{1}{\cancel{5}}}{2\cdot\underset{1}{\cancel{5}}} = 23\frac{1}{2\cdot 1} = 23\frac{1}{2}$$

$$\textbf{11.}\ -105.083 = -105\frac{83}{1000}$$

Work Practice 9–11

Later in the chapter, we write fractions as decimals. If you study Examples **7–11**, you already know how to write fractions with denominators of 10, 100, 1000, and so on, as decimals.

Objective D Comparing Decimals

One way to compare positive decimals is by comparing digits in corresponding places. To see why this works, let's compare 0.5 or $\frac{5}{10}$ and 0.8 or $\frac{8}{10}$. We know

$$\underset{\substack{\downarrow\\ 0.5}}{\frac{5}{10}} \ \underset{\substack{\\ <}}{<}\ \underset{\substack{\downarrow\\ 0.8}}{\frac{8}{10}} \quad \text{since } 5 < 8, \text{ so}$$

$$0.5 < 0.8 \text{ since } 5 < 8$$

This leads to the following.

Answers

7. $\frac{37}{1000}$ **8.** $14\frac{97}{100}$ **9.** $\frac{3}{25}$

10. $57\frac{4}{5}$ **11.** $-209\frac{43}{500}$

Comparing Two Positive Decimals

Compare digits in the same places from left to right. When two digits are not equal, the number with the larger digit is the larger decimal. If necessary, insert 0s after the last digit to the right of the decimal point to continue comparing.

Compare hundredths place digits

$$\begin{array}{lcl} 28.253 & & 28.263 \\ \uparrow & & \uparrow \\ 5 & < & 6 \\ \text{so } 28.253 & < & 28.263 \end{array}$$

Helpful Hint

For any decimal, writing 0s after the last digit to the right of the decimal point does not change the value of the number.

$7.6 = 7.60 = 7.600$, and so on

When a whole number is written as a decimal, the decimal point is placed to the right of the ones digit.

$25 = 25.0 = 25.00$, and so on

Example 12 Insert $<$, $>$, or $=$ to form a true statement.

0.378   0.368

Solution:

0.3 78   0.3 68   The tenths places are the same.

0.3 7 8   0.3 6 8   The hundredths places are different.

Since $7 > 6$, then $0.378 > 0.368$.

Work Practice 12

Practice 12

Insert $<$, $>$, or $=$ to form a true statement.

13.208   13.28

Example 13 Insert $<$, $>$, or $=$ to form a true statement.

0.052   0.236

Solution: $0.\underset{\uparrow}{\boxed{0}}52 < 0.\underset{\uparrow}{\boxed{2}}36$ 0 is smaller than 2 in the tenths place.

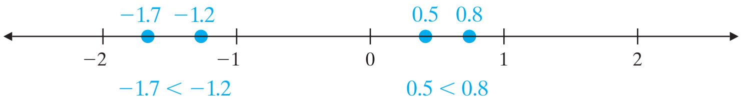
Work Practice 13

Practice 13

Insert $<$, $>$, or $=$ to form a true statement.

0.12   0.086

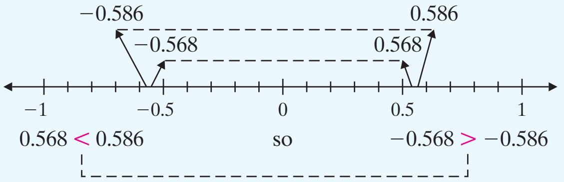
We can also use a number line to compare decimals. This is especially helpful when comparing negative decimals. Remember, the number whose graph is to the left is smaller, and the number whose graph is to the right is larger.



Answers
12. < **13.** >

Helpful Hint

If you have trouble comparing two negative decimals, try the following: Compare their absolute values. Then to correctly compare the negative decimals, reverse the direction of the inequality symbol.



Practice 14

Insert $<$, $>$, or $=$ to form a true statement.

$-0.029 \qquad -0.0209$

Example 14

Insert $<$, $>$, or $=$ to form a true statement.

$-0.0101 \qquad -0.00109$

Solution: Since $0.0101 > 0.00109$, then $-0.0101 < -0.00109$.

Work Practice 14

Objective E Rounding Decimals

We **round the decimal part** of a decimal number in nearly the same way as we round whole numbers. The only difference is that we drop digits to the right of the rounding place, instead of replacing these digits with 0s. For example,

$$36.95\underset{\uparrow}{4} \quad \text{rounded to the nearest hundredth is} \quad 36.95$$

Rounding Decimals to a Place Value to the Right of the Decimal Point

Step 1: Locate the digit to the right of the given place value.

Step 2: If this digit is 5 or greater, add 1 to the digit in the given place value and drop all digits to its right. If this digit is less than 5, drop all digits to the right of the given place.

Practice 15

Round 123.7815 to the nearest thousandth.

Example 15

Round 736.2359 to the nearest tenth.

Solution:

Step 1: We locate the digit to the right of the tenths place.

$$736.\overset{\text{tenths place}}{2}\,\underset{\text{digit to the right}}{\boxed{3}}\,59$$

Step 2: Since the digit to the right is less than 5, we drop it and all digits to its right.

Thus, 736.2359 rounded to the nearest tenth is 736.2.

Work Practice 15

The same steps for rounding can be used when the decimal is negative.

Answers

14. $<$ **15.** 123.782

Example 16

Round −0.027 to the nearest hundredth.

Solution:

Step 1: Locate the digit to the right of the hundredths place.

hundredths place

$-0.02\boxed{7}$

digit to the right

Step 2: Since the digit to the right is 5 or greater, we add 1 to the hundredths digit and drop all digits to its right.

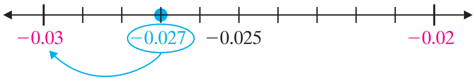
Thus, −0.027 is −0.03 rounded to the nearest hundredth.

Work Practice 16

Practice 16

Round −0.072 to the nearest hundredth.

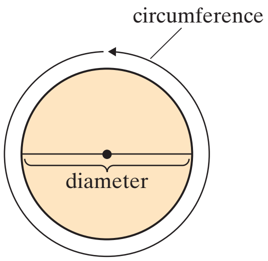
The following number line illustrates the rounding of negative decimals.



In Section 4.3, we will introduce a formula for the distance around a circle. The distance around a circle is given the special name **circumference.**

The symbol π is the Greek letter pi, pronounced "pie." We use π to denote the following constant:

$$\pi = \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$$



The value π is an **irrational number.** This means if we try to write it as a decimal, it neither ends nor repeats in a pattern.

Example 17

$\pi \approx 3.14159265$. Round π to the nearest hundredth.

Solution:

hundredths place 1 is less than 5.

$3.14\underbrace{159265}_{\text{Delete these digits.}}$

Thus, 3.14159265 rounded to the nearest hundredth is 3.14. In other words, $\pi \approx 3.14$.

Work Practice 17

Practice 17

$\pi \approx 3.14159265$. Round π to the nearest ten-thousandth.

Rounding often occurs with money amounts. Since there are 100 cents in a dollar, each cent is $\frac{1}{100}$ of a dollar. This means that if we want to round to the nearest cent, we round to the nearest hundredth of a dollar.

✓Concept Check 1756.0894 rounded to the nearest *ten* is

a. 1756.1 **b.** 1760.0894 **c.** 1760 **d.** 1750

Answers

16. −0.07 **17.** $\pi \approx 3.1416$

✓Concept Check Answer

c

Practice 18

Water bills in Mexia are always rounded to the nearest dollar. Round a water bill of $24.62 to the nearest dollar.

Answer

18. $25

Example 18 Determining State Taxable Income

A high school teacher's taxable income is $41,567.72. The tax tables in the teacher's state use amounts rounded to the nearest dollar. Round the teacher's income to the nearest whole dollar.

Solution: Rounding to the nearest whole dollar means rounding to the ones place.

ones place 7 is greater than 5.

$\$41{,}56\underset{\uparrow}{7}.\underbrace{72}$

Add 1. Delete these digits.

Thus, the teacher's income rounded to the nearest dollar is $41,568.

Work Practice 18

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

words
tens
decimals
circumference
tenths
and
after
standard form

1. The number "twenty and eight hundredths" is written in ____________ and "20.08" is written in ____________.
2. Another name for the distance around a circle is its ____________.
3. Like fractions, ____________ are used to denote part of a whole.
4. When writing a decimal number in words, the decimal point is written as "__________".
5. The place value ____________ is to the right of the decimal point while ____________ is to the left of the decimal point.
6. The decimal point in a whole number is ____________ the last digit.

Determine the place value for the digit 7 in each number.

7. 70
8. 700
9. 0.7
10. 0.07

Martin-Gay Interactive Videos

See Video 4.1

Watch the section lecture video and answer the following questions.

Objective A 11. In Example 1, how is the decimal point written?
Objective B 12. Why is 9.8 not the correct answer to Example 3? What is the correct answer?
Objective C 13. From Example 5, why does reading a decimal number correctly help you write it as an equivalent fraction?
Objective D 14. In Example 7, we compare place value by place value in which direction?
Objective E 15. Example 8 is being rounded to the nearest tenth, so why is the digit 7, which is not in the tenths place, looked at?

4.1 Exercise Set MyLab Math

Objective A *Write each decimal number in words. See Examples 1 through 3.*

1. 6.52

2. 7.59

3. 16.23

4. 47.65

5. −0.205

6. −0.495

7. 167.009

8. 233.056

9. 3000.04

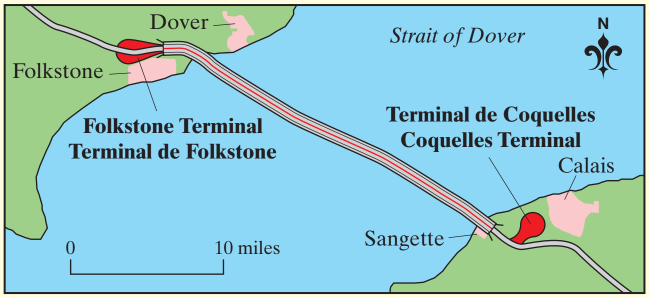
10. 5000.02

11. 105.6

12. 410.3

13. The Akashi Kaikyo Bridge, between Kobe and Awaji-Shima, Japan, is approximately 2.43 miles long.

14. The English Channel Tunnel is a 31.04-mile-long undersea rail tunnel connecting England and France. (*Source: Railway Directory & Year Book*)



Fill in each check for the described purchase. See Example 4.

15. Your monthly car loan of $321.42 to R.W. Financial.

Your Preprinted Name
Your Preprinted Address
60–8124/7233
1000613331
1407
DATE
PAY TO THE ORDER OF $
DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO
⑆621497260⑆ 1000613331⑈ 1407

16. Your part of the monthly apartment rent, which is $213.70. You pay this to Amanda Dupre.

Your Preprinted Name
Your Preprinted Address
60–8124/7233
1000613331
1408
DATE
PAY TO THE ORDER OF $
DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO
⑆621497260⑆ 1000613331⑈ 1408

17. Your bill of $91.68 to Verizon wireless.

Your Preprinted Name
Your Preprinted Address
60–8124/7233
1000613331
1409
DATE
PAY TO THE ORDER OF $
DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO
⑆621497260⑆ 1000613331⑈ 1409

18. Your grocery bill of $387.49 at Kroger.

Your Preprinted Name
Your Preprinted Address
60–8124/7233
1000613331
1410
DATE
PAY TO THE ORDER OF $
DOLLARS
FIRST STATE BANK
OF FARTHINGTON
FARTHINGTON, IL 64422
MEMO
⑆621497260⑆ 1000613331⑈ 1410

Objective B *Write each decimal number in standard form. See Examples 5 and 6.*

19. Six and five tenths

20. Three and nine tenths

21. Nine and eight hundredths

22. Twelve and six hundredths

23. Negative seven hundred five and six hundred twenty-five thousandths

24. Negative eight hundred four and three hundred ninety-nine thousandths

25. Forty-six ten-thousandths

26. Eighty-three ten-thousandths

Objective C *Write each decimal as a fraction or a mixed number. Write your answer in simplest form. See Examples 7 through 11.*

27. 0.3

28. 0.9

29. 0.27

30. 0.39

31. 0.2

32. 0.6

33. 5.4

34. 6.8

35. -0.058

36. -0.024

37. 7.008

38. 9.005

39. 15.802

40. 11.406

41. 0.3005

42. 0.2006

Objectives A B C Mixed Practice *Fill in the chart. The first row is completed for you. See Examples 1 through 11.*

	Decimal Number in Standard Form	In Words	Fraction
	0.37	thirty-seven hundredths	$\frac{37}{100}$
43.		eight tenths	
44.		five tenths	
45.	0.077		
46.	0.019		

Objective D *Insert <, >, or = between each pair of numbers to form a true statement. See Examples 12 through 14.*

47. 0.15 0.16

48. 0.12 0.15

49. -0.57 -0.54

50. -0.59 -0.52

51. 0.098 0.1

52. 0.0756 0.2

53. 0.54900 0.549

54. 0.98400 0.984

55. 167.908 167.980

56. 519.3405 519.3054

57. −1.062 −1.07

58. −18.1 −18.01

59. −7.052 7.0052

60. 0.01 −0.1

61. −0.023 −0.024

62. −0.562 −0.652

Objective E *Round each decimal to the given place value. See Examples 15 through 18.*

63. 0.57, nearest tenth

64. 0.68, nearest tenth

65. 98,207.23, nearest ten

66. 68,935.543, nearest ten

67. −0.234, nearest hundredth

68. −0.892, nearest hundredth

69. 0.5942, nearest thousandth

70. 63.4523, nearest thousandth

Recall that the number π, written as a decimal, neither ends nor repeats in a pattern. Given that $\pi \approx 3.14159265$, round π to the given place values below. (We study π further in Section 4.3.) See Example 17.

71. tenth

72. one

73. thousandth

74. hundred-thousandth

Round each monetary amount to the nearest cent or dollar as indicated. See Example 18.

75. $26.95, to the nearest dollar

76. $14,769.52, to the nearest dollar

77. $0.1992, to the nearest cent

78. $0.7633, to the nearest cent

Round each number to the given place value. See Example 18.

79. The latest generation Apple MacBook Air, at its thinnest point, measures 0.2794 cm. Round this number to the nearest tenth. (*Source:* Apple, Inc.)

80. A large tropical cockroach of the family Dictyoptera is the fastest-moving insect. This insect was clocked at a speed of 3.36 miles per hour. Round this number to the nearest tenth. (*Source:* University of California, Berkeley)

81. During the 2017 Boston Marathon, Manuela Schar of Switzerland was the first female wheelchair competitor to cross the finish line. Her time was 1.4714 hours. Round this time to the nearest hundredth of a minute. (*Source:* Boston Athletic Association)

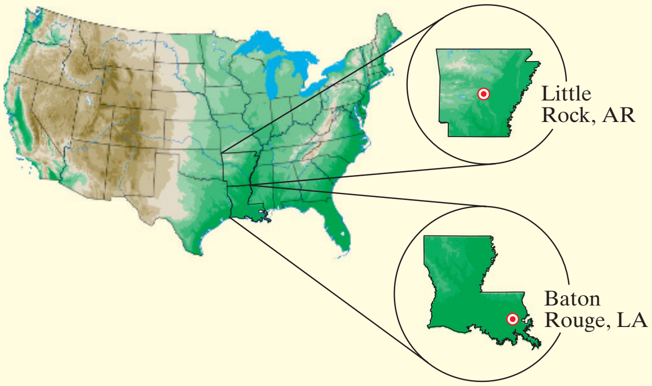
82. Mikaela Shiffrin of the U.S. ski team took first place in the women's giant slalom in the 2017 FIS World Cup in Squaw Valley, Idaho. Her winning time was 2.278 minutes. Round this time to the nearest tenth of a minute. (*Source:* International Ski Federation)

83. A used biology textbook is priced at $67.89. Round this price to the nearest dollar.

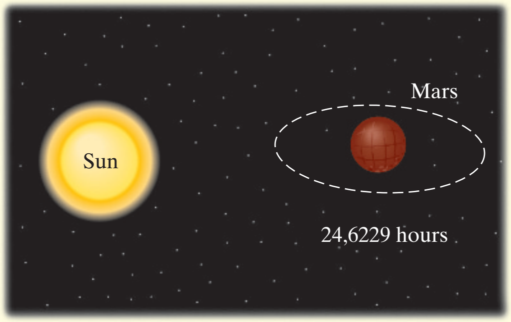
84. A used office desk is advertised at $19.95 by Drawley's Office Furniture. Round this price to the nearest dollar.

85. The 2016 estimated population density of the state of Louisiana is 89.3559 people per square mile. Round this population density to the nearest tenth. (*Source:* U.S. Census Bureau)

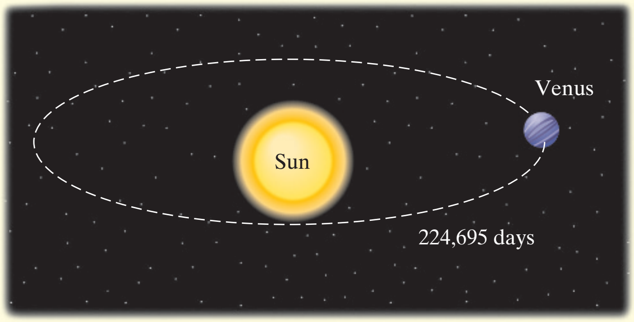
86. The 2016 estimated population density of the state of Arkansas is 56.0033 people per square mile. Round this population density to the nearest tenth. (*Source:* U.S. Census Bureau)



87. The length of a day on Mars is 24.6229 hours. Round this figure to the nearest thousandth. (*Source:* National Space Science Data Center)



88. Venus makes a complete orbit around the Sun every 224.695 days. Round this figure to the nearest whole day. (*Source:* National Space Science Data Center)



89. Kingda Ka is a hydraulic-launch roller coaster at Six Flags Great Adventure, an amusement park in Jackson, New Jersey. Currently it is the world's tallest roller coaster. A ride on the Kingda Ka lasts about 0.4667 minute. Round this figure to the nearest tenth. (*Source:* Roller Coaster DataBase)

90. During the 2016 NFL season, the average length of a Los Angeles Ram's punt was 47.8 yards. Round this figure to the nearest whole yard. (*Source:* National Football League)

Review

Perform each indicated operation. See Sections 1.3 and 1.4.

91. $3452 + 2314$

92. $8945 + 4536$

93. $82 - 47$

94. $4002 - 3897$

Concept Extensions

Solve. See the Concept Check in this section.

95. 2849.1738 rounded to the nearest hundred is
a. 2849.17 **b.** 2800 **c.** 2850 **d.** 2849.174

96. 146.059 rounded to the nearest ten is
a. 146.0 **b.** 146.1 **c.** 140 **d.** 150

97. 2849.1738 rounded to the nearest hundredth is
a. 2849.17 **b.** 2800 **c.** 2850 **d.** 2849.18

98. 146.059 rounded to the nearest tenth is
a. 146.0 **b.** 146.1 **c.** 140 **d.** 150

Solve.

99. In your own words, describe how to write a decimal as a fraction or a mixed number.

100. Explain how to identify the value of the 9 in the decimal 486.3297.

101. Write $7\frac{12}{100}$ as a decimal.

102. Write $17\frac{268}{1000}$ as a decimal.

103. Write 0.00026849577 as a fraction.

104. Write 0.00026849577 in words.

105. Write a 5-digit number that rounds to 1.7.

106. Write a 4-digit number that rounds to 26.3.

107. Write a decimal number that is greater than 8 but less than 9.

108. Write a decimal number that is greater than 48.1 but less than 48.2.

109. Which number(s) rounds to 0.26?
0.26559 0.26499 0.25786 0.25186

110. Which number(s) rounds to 0.06?
0.0612 0.066 0.0586 0.0506

For Exercises 111 and 112, write the numbers from smallest to largest.

111. 0.9
0.1038
0.10299
0.1037

112. 0.01
0.0839
0.09
0.1

Solve.

113. The all-time top six movies (those that have earned the most money in the United States) as of December 2017 along with the approximate amount of money they have earned are listed in the table. Estimate the total amount of money that these movies have earned by first rounding each earning to the nearest hundred million. (*Source:* The Nash Information Services, LLC: the-numbers.com) *Note:* Many of these movies are still earning a substantial amount of money.

All-Time Top American Box Office Movies as of December 2017

Movie	**Gross Domestic Earnings**
Star Wars Ep. VII: The Force Awakens (2015)	$936.7 million
Avatar (2009)	$760.5 million
Titanic (1997)	$659.4 million
Jurassic World (2015)	$652.2 million
The Avengers (2012)	$623.3 million
Star Wars Ep. VIII: The Last Jedi (2017)	$533.1 million

114. For the first half of 2017, there were 316.2 million singles downloaded at an average price of $1.20 each. Find an estimate of the total revenue from downloaded singles by answering parts **a** through **c**. (*Source:* Recording Industry Association of America)

a. Round 316.2 million to the nearest ten million.

b. Multiply the rounded value in part **a** by 12.

c. Move the decimal point in the product from part **b** one place to the left. This number is the total revenue in millions of dollars.

4.2 Adding and Subtracting Decimals

Objectives

- **A** Add Decimals.
- **B** Subtract Decimals.
- **C** Estimate When Adding or Subtracting Decimals.
- **D** Evaluate Expressions with Decimal Replacement Values.
- **E** Solve Problems That Involve Adding or Subtracting Decimals.

Objective A Adding Decimals

Adding decimals is similar to adding whole numbers. We add digits in corresponding place values from right to left, carrying if necessary. To make sure that digits in corresponding place values are added, we line up the decimal points vertically.

As we shall see later in this section, subtracting decimals is similar to subtracting whole numbers also.

Adding (or Subtracting) Decimals

Step 1: Write the decimals so that the decimal points line up vertically.

Step 2: Add or subtract as with whole numbers.

Step 3: Place the decimal point in the sum or difference so that it lines up vertically with the decimal points in the problem.

In this section, we will insert zeros in decimal numbers so that place value digits line up neatly. This is shown in Example 1.

Practice 1

Add.

a. 15.52 + 2.371

b. 20.06 + 17.612

c. 0.125 + 122.8

Example 1 Add: 23.85 + 1.604

Solution: First we line up the decimal points vertically.

$$\begin{array}{r} 23.850 \\ +\ \ 1.604 \\ \hline \end{array} \quad \text{Insert one 0 so that digits line up neatly.}$$

Line up decimal points.

Then we add the digits from right to left as for whole numbers.

$$\begin{array}{r} \overset{1}{2}3.850 \\ +\ \ 1.604 \\ \hline 25.454 \end{array}$$

Place the decimal point in the sum so that all decimal points line up.

Work Practice 1

Helpful Hint

Recall that 0s may be placed after the last digit to the right of the decimal point without changing the value of the decimal. This may be used to help line up place values when adding decimals.

$$\begin{array}{r} 3.2 \\ 15.567 \\ +\ \ 0.11 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 3.200 \\ 15.567 \\ +\ \ 0.110 \\ \hline 18.877 \end{array} \quad \begin{array}{l} \text{Insert two 0s.} \\ \\ \text{Insert one 0.} \\ \text{Add.} \end{array}$$

Answers

1. a. 17.891 **b.** 37.672 **c.** 122.925

Example 2 Add: 763.7651 + 22.001 + 43.89

Solution: First we line up the decimal points.

$$\begin{array}{rl} \overset{1}{7}6\overset{1}{3}.\overset{1}{7}651 & \\ 22.0010 & \text{Insert one 0.} \\ +\ \ 43.8900 & \text{Insert two 0s.} \\ \hline 829.6561 & \text{Add.} \end{array}$$

Work Practice 2

Practice 2

Add.

a. 34.567 + 129.43 + 2.8903

b. 11.21 + 46.013 + 362.526

Helpful Hint

Don't forget that the decimal point in a whole number is positioned after the last digit.

Example 3 Add: 45 + 2.06

Solution:

$$\begin{array}{rl} 45.00 & \text{Insert a decimal point and two 0s.} \\ +\ \ 2.06 & \text{Line up decimal points.} \\ \hline 47.06 & \text{Add.} \end{array}$$

Work Practice 3

Practice 3

Add: 119 + 26.072

✓Concept Check What is wrong with the following calculation of the sum of 7.03, 2.008, 19.16, and 3.1415?

$$\begin{array}{r} 7.03 \\ 2.008 \\ 19.16 \\ +\,3.1415 \\ \hline 3.6042 \end{array}$$

Example 4 Add: 3.62 + (−4.78)

Solution: Recall from Chapter 2 that to add two numbers with different signs, we find the difference of the larger absolute value and the smaller absolute value. The sign of the answer is the same as the sign of the number with the larger absolute value.

$$\begin{array}{rl} 4.78 & \\ -3.62 & \\ \hline 1.16 & \text{Subtract the absolute values.} \end{array}$$

Thus, 3.62 + (−4.78) = −1.16

The sign of the number with the larger absolute value; −4.78 has the larger absolute value.

Work Practice 4

Practice 4

Add: 8.1 + (−99.2)

Objective B Subtracting Decimals

Subtracting decimals is similar to subtracting whole numbers. We line up digits and subtract from right to left, borrowing when needed.

Answers

2. a. 166.8873 **b.** 419.749
3. 145.072 **4.** −91.1

✓Concept Check Answer

The decimal places are not lined up properly.

Practice 5

Subtract. Check your answers.

a. 5.8 − 3.92

b. 9.72 − 4.068

Example 5

Subtract: 3.5 − 0.068. Check your answer.

Solution:

$$\begin{array}{rl}
\scriptstyle 9 & \\
\scriptstyle 4\ \cancel{10}\ 10 & \\
3.\cancel{5}\cancel{0}\cancel{0} & \text{Insert two 0s.}\\
-0.068 & \text{Line up decimal points.}\\
\hline
3.432 & \text{Subtract.}
\end{array}$$

Check: Recall that we can check a subtraction problem by adding.

$$\begin{array}{rl}
3.432 & \text{Difference}\\
+0.068 & \text{Subtrahend}\\
\hline
3.500 & \text{Minuend}
\end{array}$$

Work Practice 5

Practice 6

Subtract. Check your answers.

a. 53 − 29.31

b. 120 − 68.22

Example 6

Subtract: 85 − 17.31. Check your answer.

Solution:

$$\begin{array}{r}
\scriptstyle 14\ \ 9\\
\scriptstyle 7\ \ \cancel{4}\ \ \cancel{10}\ 10\\
\cancel{8}\cancel{5}.\cancel{0}\cancel{0}\\
-17.31\\
\hline
67.69
\end{array}$$

Check:

$$\begin{array}{rl}
67.69 & \text{Difference}\\
+17.31 & \text{Subtrahend}\\
\hline
85.00 & \text{Minuend}
\end{array}$$

Work Practice 6

Practice 7

Subtract: 18 from 26.99

Example 7

Subtract 3 from 6.98. Check your answer.

Solution:

$$\begin{array}{rl}
6.98 & \\
-3.00 & \text{Insert two 0s.}\\
\hline
3.98 &
\end{array}$$

Check:

$$\begin{array}{rl}
3.98 & \text{Difference}\\
+3.00 & \text{Subtrahend}\\
\hline
6.98 & \text{Minuend}
\end{array}$$

Work Practice 7

Practice 8

Subtract: −3.4 − 9.6

Example 8

Subtract: −5.8 − 1.7

Solution: Recall from Chapter 2 that to subtract 1.7, we add the opposite of 1.7, or −1.7. Thus

$$-5.8 - 1.7 = -5.8 + (-1.7) \quad \text{To subtract, add the opposite of 1.7, which is −1.7.}$$

Add the absolute values.

$$= -7.5$$

Use the common negative sign.

Work Practice 8

Practice 9

Subtract: −1.05 − (−7.23)

Example 9

Subtract: −2.56 − (−4.01)

Solution: $-2.56 - (-4.01) = -2.56 + 4.01$ To subtract, add the opposite of −4.01, which is 4.01.

Subtract the absolute values.

$$= 1.45$$

The answer is positive since 4.01 has the larger absolute value.

Work Practice 9

Answers

5. a. 1.88 **b.** 5.652 **6. a.** 23.69 **b.** 51.78 **7.** 8.99 **8.** −13 **9.** 6.18

Objective C Estimating When Adding or Subtracting Decimals

To help avoid errors, we can also estimate to see if our answer is reasonable when adding or subtracting decimals. Although only one estimate is needed per operation, we show two for variety.

Example 10 Add or subtract as indicated. Then estimate to see if the answer is reasonable by rounding the given numbers and adding or subtracting the rounded numbers.

a. $27.6 + 519.25$

$$\begin{array}{rlrcr} \textbf{Exact} & & \textbf{Estimate 1} & & \textbf{Estimate 2}\\ \overset{1}{2}7.60 & \text{rounds to} & 30 & & 30 \\ \underline{+519.25} & \text{rounds to} & \underline{+500} & \text{or} & \underline{+520} \\ 546.85 & & 530 & & 550\end{array}$$

Since the exact answer is close to either estimate, it is reasonable. (In the first estimate, each number is rounded to the place value of the leftmost digit. In the second estimate, each number is rounded to the nearest ten.)

b. $11.01 - 0.862$

$$\begin{array}{rlrcr} \textbf{Exact} & & \textbf{Estimate 1} & & \textbf{Estimate 2}\\ 1\overset{0}{\cancel{1}}.\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{\overset{10}{\cancel{0}}}{\cancel{1}}\overset{10}{\cancel{0}} & \text{rounds to} & 10 & & 11 \\ \underline{-\ 0.862} & \text{rounds to} & \underline{-\ 1} & \text{or} & \underline{-\ 1} \\ 10.148 & & 9 & & 10\end{array}$$

In the first estimate, we rounded the first number to the nearest ten and the second number to the nearest one. In the second estimate, we rounded both numbers to the nearest one. Both estimates show us that our answer is reasonable.

Work Practice 10

Practice 10

Add or subtract as indicated. Then estimate to see if the answer is reasonable by rounding the given numbers and adding or subtracting the rounded numbers.

a. $48.1 + 326.97$

b. $18.09 - 0.746$

Helpful Hint Remember that estimates are used for our convenience to quickly check the reasonableness of an answer.

✓ **Concept Check** Why shouldn't the sum $21.98 + 42.36$ be estimated as $30 + 50 = 80$?

Objective D Using Decimals as Replacement Values

Let's review evaluating expressions with given replacement values. This time the replacement values are decimals.

Example 11 Evaluate $x - y$ for $x = 2.8$ and $y = 0.92$.

Solution: Replace x with 2.8 and y with 0.92 and simplify.

$$\begin{aligned} x - y &= 2.8 - 0.92 \\ &= 1.88 \end{aligned} \qquad \begin{array}{r} 2.80\\ \underline{-0.92}\\ 1.88\end{array}$$

Work Practice 11

Practice 11

Evaluate $y - z$ for $y = 11.6$ and $z = 10.8$.

Answers

10. a. Exact: 375.07 **b.** Exact: 17.344

11. 0.8

✓ **Concept Check Answer**

Each number is rounded incorrectly. The estimate is too high.

Objective E Solving Problems by Adding or Subtracting Decimals

Decimals are very common in real-life problems.

Practice 12

Find the total monthly cost of owning and operating a certain automobile given the expenses shown.

Monthly car payment:	$563.52
Monthly insurance cost:	$52.68
Average gasoline bill per month:	$127.50

Example 12 Calculating the Cost of Owning an Automobile

Find the total monthly cost of owning and operating a certain automobile given the expenses shown.

Monthly car payment:	$256.63
Monthly insurance cost:	$47.52
Average gasoline bill per month:	$195.33

Solution:

1. UNDERSTAND. Read and reread the problem. The phrase "total monthly cost" tells us to add.

2. TRANSLATE.

In words:	total monthly cost	is	car payment	plus	insurance cost	plus	gasoline bill
	↓	↓	↓	↓	↓	↓	↓
Translate:	total monthly cost	=	$256.63	+	$47.52	+	$195.33

3. SOLVE: Let's also estimate by rounding each number to the nearest ten.

$$\begin{array}{rlrl} \overset{1\,1\,1}{256.63} & \text{rounds to} & 260 & \\ 47.52 & \text{rounds to} & 50 & \\ \underline{+\ 195.33} & \text{rounds to} & \underline{200} & \\ 499.48 & \text{Exact} & 510 & \text{Estimate} \end{array}$$

4. INTERPRET. *Check* your work. Since our estimate is close to our exact answer, our answer is reasonable. *State* your conclusion: The total monthly cost is $499.48.

Work Practice 12

The bar graph in Example 13 has horizontal bars. To visualize the value represented by a bar, see how far it extends to the right. For this horizontal bar graph, the value of each bar is labeled. We will study bar graphs further in a later chapter.

Practice 13

Use the bar graph in Example 13. How much greater is the average male height in Australia than the average male height in China?

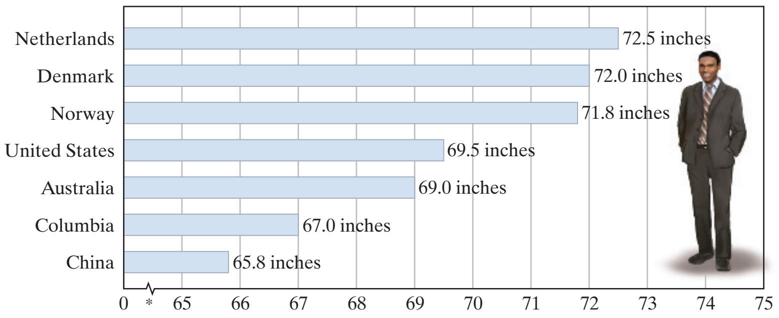
Example 13 Comparing Average Heights

The bar graph on the next page shows the current average heights for male adults in various countries. How much greater is the average male height in the Netherlands than the average male height in Norway?

Answers

12. $743.70 **13.** 3.2 in.

Average Male Adult Height in Various Countries



Source: averageheight.co

* The ⁓ means that some numbers are purposefully missing on the axis.

Solution:

1. UNDERSTAND. Read and reread the problem. Since we want to know "how much greater," we subtract.
2. TRANSLATE.

In words:	How much greater	is	Netherlands' average male height	minus	Norway's average male height
	↓	↓	↓	↓	↓
Translate:	How much greater	=	72.5	−	71.8

3. SOLVE: We also estimate by rounding each number to the nearest whole.

$$\begin{array}{rll} 7\overset{1}{\cancel{2}}.\overset{15}{\cancel{5}} & \text{rounds to} & 73 \\ \underline{-\,7\,1\,.\,8} & \text{rounds to} & \underline{-72} \\ 0\,.\,7 & \text{Exact} & 1 \text{ Estimate} \end{array}$$

4. INTERPRET. *Check* your work. Since our estimate is close to our exact answer, 0.7 inches is reasonable. *State* your conclusion: The average male height in the Netherlands is 0.7 inch greater than the average male height in Norway.

Work Practice 13

Calculator Explorations Decimals

Entering Decimal Numbers

To enter a decimal number, find the key marked $\boxed{\cdot}$. To enter the number 2.56, for example, press the keys $\boxed{2}\,\boxed{\cdot}\,\boxed{56}$.

The display will read $\boxed{\quad 2.56}$.

Operations on Decimal Numbers

Operations on decimal numbers are performed in the same way as operations on whole or signed numbers. For example, to find 8.625 − 4.29, press the keys $\boxed{8.625}$ $\boxed{-}$ $\boxed{4.29}$ and then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

The display will read $\boxed{\quad 4.335}$. (Although entering 8.625, for example, requires pressing more than one key, we group numbers together here for easier reading.)

Use a calculator to perform each indicated operation.

1. 315.782 + 12.96

2. 29.68 + 85.902

3. 6.249 − 1.0076

4. 5.238 − 0.682

5.
$$\begin{array}{r} 12.555 \\ 224.987 \\ 5.2 \\ \underline{+\,622.65} \end{array}$$

6.
$$\begin{array}{r} 47.006 \\ 0.17 \\ 313.259 \\ \underline{+\,139.088} \end{array}$$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

minuend	vertically	like	true	horizontally
difference	subtrahend	last	false	

1. The decimal point in a whole number is positioned after the ______________ digit.

2. In 89.2 − 14.9 = 74.3, the number 74.3 is called the ______________, 89.2 is the ______________, and 14.9 is the ______________.

3. To add or subtract decimals, we line up the decimal points ______________.

4. True or false: If we replace x with 11.2 and y with −8.6 in the expression $x - y$, we have 11.2 − 8.6. ______________

Mentally find the sum or difference.

5. $\begin{array}{r} 0.3 \\ +0.2 \\ \hline \end{array}$ **6.** $\begin{array}{r} 0.4 \\ +0.5 \\ \hline \end{array}$ **7.** $\begin{array}{r} 1.00 \\ +0.26 \\ \hline \end{array}$ **8.** $\begin{array}{r} 3.00 \\ +0.19 \\ \hline \end{array}$

9. $\begin{array}{r} 7.6 \\ +1.3 \\ \hline \end{array}$ **10.** $\begin{array}{r} 4.5 \\ +3.2 \\ \hline \end{array}$ **11.** $\begin{array}{r} 0.9 \\ -0.3 \\ \hline \end{array}$ **12.** $\begin{array}{r} 0.6 \\ -0.2 \\ \hline \end{array}$

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 4.2

Objectives A B **13.** From Examples 1–3, why do you think we line up decimal points?

Objective C **14.** In Example 4, estimating is used to check whether the answer to the subtraction problem is reasonable, but what is the best way to fully check?

Objective D **15.** In Example 5, why is the actual subtraction performed to the side?

Objective E **16.** In Example 6, to calculate the amount of border material needed, we are actually calculating the __________ of the triangle.

4.2 Exercise Set MyLab Math

Objectives A C Mixed Practice *Add. See Examples 1 through 4 and 10. For those exercises marked, also estimate to see if the answer is reasonable.*

1. 1.3 + 2.2 **2.** 2.5 + 4.1 **3.** 5.7 + 1.13

4. 2.31 + 6.4 **5.** 24.6 + 2.39 + 0.0678 **6.** 32.4 + 1.58 + 0.0934

7. $\begin{array}{r} 6.014 \\ 9.009 \\ +\ 76.181 \\ \hline \end{array}$ **8.** $\begin{array}{r} 3.003 \\ 8.098 \\ +\ 71.486 \\ \hline \end{array}$ **9.** −2.6 + (−5.97)

10. $-18.2 + (-10.8)$

11. $15.78 + (-4.62)$

12. $6.91 + (-7.03)$

13. $$\begin{array}{r} 234.89 \\ +\,230.67 \\ \hline \end{array}$$
Exact: Estimate:

14. $$\begin{array}{r} 734.89 \\ +\,640.56 \\ \hline \end{array}$$
Exact: Estimate:

15. $$\begin{array}{r} 100.009 \\ 6.08 \\ +\quad 9.034 \\ \hline \end{array}$$
Exact: Estimate:

16. $$\begin{array}{r} 200.89 \\ 7.49 \\ +\ \ 62.83 \\ \hline \end{array}$$
Exact: Estimate:

17. Find the sum of 45.023, 3.006, and 8.403.

18. Find the sum of 65.0028, 5.0903, and 6.9003.

Objectives **B** **C** Mixed Practice

Subtract. See Examples 5 through 10. For Exercises 19–30, check the subtraction. For those exercises marked, also estimate to see if the answer is reasonable.

19. $8.8 - 2.3$

20. $7.6 - 2.1$

21. $18 - 2.7$

22. $28 - 3.3$

▶ **23.** $$\begin{array}{r} 654.9 \\ -\ \ 56.67 \\ \hline \end{array}$$

24. $$\begin{array}{r} 863.23 \\ -\ \ 39.453 \\ \hline \end{array}$$

25. $5.9 - 4.07$
Exact:
Estimate:

26. $6.4 - 3.04$
Exact:
Estimate:

27. $923.5 - 61.9$

28. $845.93 - 45.8$

▶ **29.** $$\begin{array}{r} 1000 \\ -\ \ 123.4 \\ \hline \end{array}$$
Exact:

Estimate:

30. $$\begin{array}{r} 2000 \\ -\ \ 327.47 \\ \hline \end{array}$$
Exact:

Estimate:

31. $200 - 5.6$

32. $800 - 8.9$

▶ **33.** $-1.12 - 5.2$

34. $-8.63 - 5.6$

35. $7.7 - 14.1$

36. $10.25 - 21.76$

37. $-2.6 - (-5.7)$

38. $-9.4 - (-10.4)$

39. $3 - 0.0012$

40. $7 - 0.097$

41. Subtract 6.7 from 23.

42. Subtract 9.2 from 45.

Objectives **A** **B** Mixed Practice

Perform the indicated operation. See Examples 1 through 9.

43. $0.9 + 2.2$

44. $0.7 + 3.4$

45. $-6.06 + 0.44$

46. $-5.05 + 0.88$

47. $900.34 - 123.45$

48. $800.74 - 463.98$

49. $50.2 - 600$

50. $40.3 - 700$

51. Subtract 61.9 from 923.5.

52. Subtract 45.8 from 845.9.

53. Add 500.008 and 4.06 and 8.033.

54. Add 400.009 and 7.05 and 6.088.

55. $-0.003 + 0.091$

56. $-0.004 + 0.085$

57. $-102.4 - 78.04$ **58.** $-36.2 - 10.02$ **59.** $-2.9 - (-1.8)$ **60.** $-6.5 - (-3.3)$

Objective D *Evaluate each expression for* $x = 3.6$, $y = 5$, *and* $z = 0.21$. *See Example 11.*

61. $x + z$ **62.** $y + x$ **63.** $x - z$

64. $y - z$ **65.** $y - x + z$ **66.** $x + y + z$

Objective E *Solve. For Exercises 67 and 68, the solutions have been started for you. See Examples 12 and 13.*

67. Ann-Margaret Tober bought a book for $32.48. If she paid with two $20 bills, what was her change?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blank.)

change	is	two $20 bills	minus	cost of book
	↓	↓	↓	↓
change	=	40	−	____

Finish with

3. SOLVE and **4.** INTERPRET.

68. Phillip Guillot bought a car part for $18.26. If he paid with two $10 bills, what was his change?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blank.)

change	is	two $10 bills	minus	cost of car part
	↓	↓	↓	↓
change	=	20	−	____

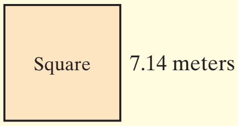
Finish with

3. SOLVE and **4.** INTERPRET.

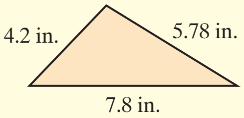
69. Microsoft stock opened the day at $85.95 per share, and the closing price the same day was $85.55. By how much did the price of each share change?

70. A pair of eyeglasses costs a total of $347.89. The frames of the glasses are $97.23. How much do the lenses of the eyeglasses cost?

△ **71.** Find the perimeter.



△ **72.** Find the perimeter.



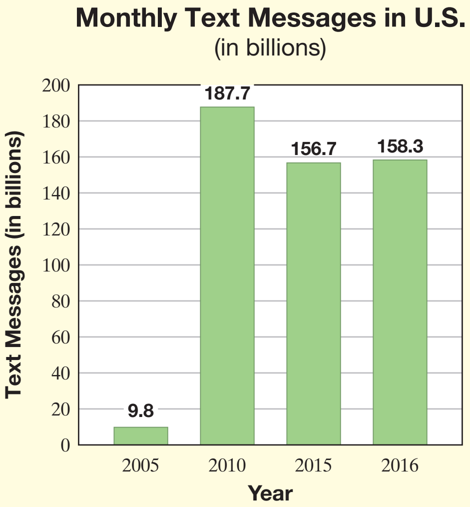
73. The Apple iPhone X was released in 2017. It measures 5.65 inches by 2.8 inches rounded. Find the perimeter of the rectangular face. (*Source:* Apple, Inc.)

74. The Google Pixel 2, released in 2017, is the newest Google phone (at this writing). It measures 5.74 inches by 2.7 inches rounded. Find the perimeter of the phone. (*Source:* Google.com)

75. The average wind speed at the weather station on Mt. Washington in New Hampshire is 35.2 miles per hour. The highest speed ever recorded at the station is 231.0 miles per hour. How much faster is the highest speed than the average wind speed? (*Source:* National Climatic Data Center)

76. The average annual rainfall in Omaha, Nebraska, is 30.08 inches. The average annual rainfall in New Orleans, Louisiana, is 64.16 inches. On average, how much more rain does New Orleans receive annually than Omaha? (*Source:* National Climatic Data Center)

The bar graph shows the average number of text messages sent each month in the United States for the years shown. Use this graph to answer Exercises 77 and 78. (Source: CTIA—The Wireless Association)



77. Find the decrease in monthly text messages from 2010 to 2016.

78. Find the increase in monthly text messages from 2005 to 2010.

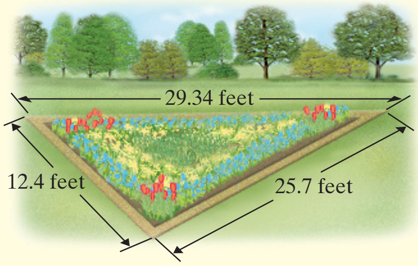
79. As of this writing, the three U.S. movies that have made the most money through movie ticket sales in the United States are *Star Wars Ep. VII: The Force Awakens* (2015), $936.7 million; *Avatar* (2009), $760.5 million; and *Titanic* (1997), $659.4 million. What is the total amount of ticket sales for these three movies? (*Source:* The Nash information Services, LLC: the-numbers.com)

80. In 2016, the average full-time American college student spent 3.65 hours on educational activities and 4.25 hours per day on leisure and sports. How much more time on average do American college students spend on leisure and sports activities than on educational activities? (*Source:* U.S. Bureau of Labor Statistics)

81. The snowiest city in the United States is Valdez, Alaska, which receives an average of 110.5 more inches of snow than the second snowiest city. The second snowiest city in the United States is Crested Butte, Colorado. Crested Butte receives an average of 215.8 inches annually. How much snow does Valdez receive on average each year? (*Source:* The Weather Channel)

82. The driest place in the world is the Atacama Desert in Chile, which receives an average of only 0.004 inch of rain per year. Yuma, Arizona, is the driest city in the United States. Yuma receives an average of 3.006 more inches of rain each year than the Atacama Desert. What is the average annual rainfall in Yuma? (*Source:* National Climatic Data Center)

▶ △ **83.** A landscape architect is planning a border for a flower garden shaped like a triangle. The sides of the garden measure 12.4 feet, 29.34 feet, and 25.7 feet. Find the amount of border material needed.



△ **84.** A contractor purchased enough railing to completely enclose the newly built deck shown below. Find the amount of railing purchased.

The table gives the average speed, in the kilometers per hour, for the winners of the 24 Hours of Le Mans for each of the years listed. Use the table to answer Exercises 85 and 86. (Source: lemans-history.com)

Year	Team	Average Speed (in kph)
2013	Audi Sport Team Joest	197.400
2014	Audi Sport Team Joest	214.927
2015	Porsche Team	224.200
2016	Porsche Team	217.503
2017	Porsche Team	244.795

85. How much slower was the average 24 Hours of Le Mans winner in 2013 than in 2017?

86. How much faster was the Porsche Team's average Le Mans winning speed in 2017 than its average winning speed in 2015?

The bar graph shows the top five chocolate-consuming nations in the world in 2016. Use this graph to answer Exercises 87 through 92.

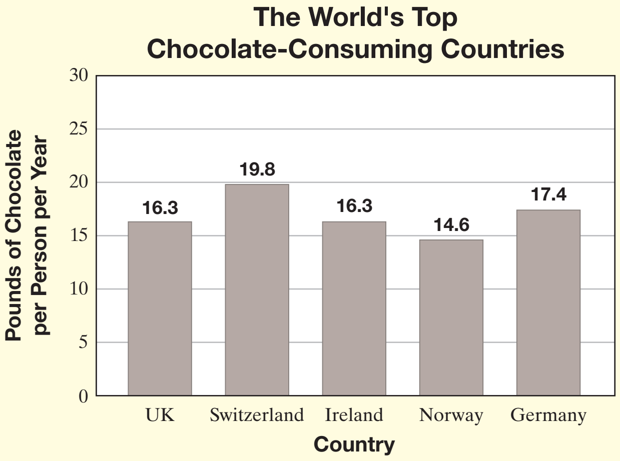
87. Which country in the bar graph has the greatest chocolate consumption per person?

88. Which country in the bar graph has the least chocolate consumption per person?

89. How much more is the greatest chocolate consumption than the least chocolate consumption shown in the bar graph?

90. How much more chocolate does the average German citizen consume per year than the average Irish?

91. Make a table listing the countries and their corresponding chocolate consumptions in order from greatest to least.



Source: Based on data from confectionarynews.com

92. Find the sum of the five bar heights shown in the graph. What type of company might be interested in this sum?

Review

Multiply. See Sections 1.6 and 3.3.

93. $23 \cdot 2$

94. $46 \cdot 3$

95. $\left(\frac{2}{3}\right)^2$

96. $\left(\frac{1}{5}\right)^3$

Concept Extensions

A friend asks you to check his calculations for Exercises 97 and 98. Are they correct? If not, explain your friend's errors and correct the calculations. See the first Concept Check in this section.

97.

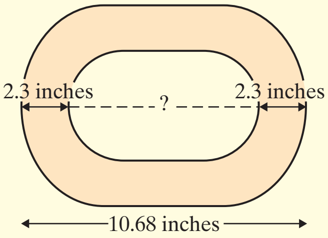
$$\begin{array}{r} \overset{1}{9}.2 \\ \overset{1}{8}.63 \\ +\ 4.005 \\ \hline 4.960 \end{array}$$

98.

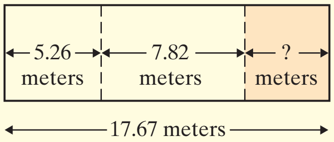
$$\begin{array}{r} \overset{8}{\cancel{9}}\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}.\overset{9}{\cancel{0}} \\ -\ \ 96.4 \\ \hline 803.5 \end{array}$$

Find the unknown length in each figure.

△ **99.**



△ **100.**



Let's review the values of these common U.S. coins in order to answer the following exercises.

For Exercises 101 and 102, write the value of each group of coins. To do so, it is usually easiest to start with the coin(s) of greatest value and end with the coin(s) of least value.

101.

102.

103. Name the different ways that coins can have a value of $0.17 given that you may use no more than 10 coins.

104. Name the different ways that coin(s) can have a value of $0.25 given that there are no pennies.

Solve.

105. Why shouldn't the sum

$$82.95 + 51.26$$

be estimated as $90 + 60 = 150$?
See the second Concept Check in this section.

106. Laser beams can be used to measure the distance to the moon. One measurement showed the distance to the moon to be 256,435.235 miles. A later measurement showed that the distance is 256,436.012 miles. Find how much farther away the moon is in the second measurement compared to the first.

107. Explain how adding or subtracting decimals is similar to adding or subtracting whole numbers.

108. Can the sum of two negative decimals ever be a positive decimal? Why or why not?

4.3 Multiplying Decimals and Circumference of a Circle

Objectives

- **A** Multiply Decimals.
- **B** Estimate When Multiplying Decimals.
- **C** Multiply Decimals by Powers of 10.
- **D** Evaluate Expressions with Decimal Replacement Values.
- **E** Find the Circumference of Circles.
- **F** Solve Problems by Multiplying Decimals.

Objective A Multiplying Decimals

Multiplying decimals is similar to multiplying whole numbers. The only difference is that we place a decimal point in the product. To discover where a decimal point is placed in the product, let's multiply 0.6 × 0.03. We first write each decimal as an equivalent fraction and then multiply.

$$\underset{\substack{\uparrow\\ \text{1 decimal}\\ \text{place}}}{0.6} \times \underset{\substack{\uparrow\\ \text{2 decimal}\\ \text{places}}}{0.03} = \frac{6}{10} \times \frac{3}{100} = \frac{18}{1000} = \underset{\substack{\uparrow\\ \text{3 decimal}\\ \text{places}}}{0.018}$$

Notice that $1 + 2 = 3$, the number of decimal places in the product. Now let's multiply 0.03 × 0.002.

$$\underset{\substack{\uparrow\\ \text{2 decimal}\\ \text{places}}}{0.03} \times \underset{\substack{\uparrow\\ \text{3 decimal}\\ \text{places}}}{0.002} = \frac{3}{100} \times \frac{2}{1000} = \frac{6}{100{,}000} = \underset{\substack{\uparrow\\ \text{5 decimal}\\ \text{places}}}{0.00006}$$

Again, we see that $2 + 3 = 5$, the number of decimal places in the product.

Instead of writing decimals as fractions each time we want to multiply, we notice a pattern from these examples and state a rule that we can use:

Multiplying Decimals

Step 1: Multiply the decimals as though they are whole numbers.

Step 2: The decimal point in the product is placed so that the number of decimal places in the product is equal to the *sum* of the number of decimal places in the factors.

Practice 1

Multiply: 45.9 × 0.42

Example 1 Multiply: 23.6 × 0.78

Solution:

$$\begin{array}{r l} 23.6 & \text{1 decimal place}\\ \times\ 0.78 & \text{2 decimal places}\\ \hline 1888 & \\ 16520 & \\ \hline 18.\underbrace{408} & \end{array}$$

Since 1 + 2 = 3, insert the decimal point in the product so that there are 3 decimal places.

Work Practice 1

Practice 2

Multiply: 0.0721 × 48

Example 2 Multiply: 0.0531 × 16

Solution:

$$\begin{array}{r l} 0.0531 & \text{4 decimal places}\\ \times\ \ \ 16 & \text{0 decimal places}\\ \hline 3186 & \\ 5310 & \\ \hline 0.\underbrace{8496} & \text{4 decimal places } (4 + 0 = 4) \end{array}$$

Work Practice 2

Answers

1. 19.278 **2.** 3.4608

✓Concept Check True or false? The number of decimal places in the product of 0.261 and 0.78 is 6. Explain.

Example 3 Multiply: $(-2.6)(0.8)$

Solution: Recall that the product of a negative number and a positive number is a negative number.

$$(-2.6)(0.8) = -2.08$$

■ **Work Practice 3**

Objective B Estimating When Multiplying Decimals

Just as for addition and subtraction, we can estimate when multiplying decimals to check the reasonableness of our answer.

Example 4 Multiply: 28.06×1.95. Then estimate to see whether the answer is reasonable by rounding each factor and then multiplying the rounded numbers.

Solution:

Exact	Estimate 1			Estimate 2	
28.06	28	Rounded to ones		30	Rounded to tens
× 1.95	× 2		or	× 2	Rounded to ones
14030	56			60	
252540					
280600					
54.7170					

The answer 54.7170 is reasonable.

■ **Work Practice 4**

As shown in Example 4, estimated results will vary depending on what estimates are used. Notice that estimating results is a good way to see whether the decimal point has been correctly placed.

Objective C Multiplying Decimals by Powers of 10

There are some patterns that occur when we multiply a number by a power of 10 such as 10, 100, 1000, 10,000, and so on.

$23.6951 \times \underset{\substack{\uparrow \\ \text{1 zero}}}{10} = 23\underset{\curvearrowright}{6.}951$ Move the decimal point *1 place* to the *right.*

$23.6951 \times \underset{\substack{\uparrow \\ \text{2 zeros}}}{100} = 2\underset{\curvearrowright}{369.}51$ Move the decimal point *2 places* to the *right.*

$23.6951 \times \underset{\substack{\uparrow \\ \text{5 zeros}}}{100{,}000} = 2{,}\underset{\curvearrowright}{369{,}510.}$ Move the decimal point *5 places* to the *right* (insert a 0).

Notice that we move the decimal point the same number of places as there are zeros in the power of 10.

Practice 3

Multiply: $(5.4)(-1.3)$

Practice 4

Multiply: 30.26×2.98. Then estimate to see whether the answer is reasonable.

Answers

3. -7.02
4. Exact: 90.1748

✓Concept Check Answer

false: 3 decimal places and 2 decimal places means 5 decimal places in the product

Multiplying Decimals by Powers of 10 Such as 10, 100, 1000, 10,000

Move the decimal point to the *right* the same number of places as there are *zeros* in the power of 10.

Practice 5–7

Multiply.

5. 46.8 × 10

6. 203.004 × 100

7. (−2.33)(1000)

Examples Multiply.

5. 7.68 × 10 = 76.8 7.68

6. 23.702 × 100 = 2370.2 23.702

7. (−76.3)(1000) = −76,300 76.300

Work Practice 5–7

There are also powers of 10 that are less than 1. The decimals 0.1, 0.01, 0.001, 0.0001, and so on, are examples of powers of 10 less than 1. Notice the pattern when we multiply by these powers of 10:

569.2 × 0.1 = 56.92 Move the decimal point *1 place* to the *left*.

↑
1 decimal place

569.2 × 0.01 = 5.692 Move the decimal point *2 places* to the *left*.

↑
2 decimal places

569.2 × 0.0001 = 0.05692 Move the decimal point *4 places* to the *left* (insert one 0).

↑
4 decimal places

Multiplying Decimals by Powers of 10 Such as 0.1, 0.01, 0.001, 0.0001

Move the decimal point to the *left* the same number of places as there are *decimal places* in the power of 10.

Practice 8–10

Multiply.

8. 6.94 × 0.1

9. 3.9 × 0.01

10. (−7682)(−0.001)

Examples Multiply.

8. 42.1 × 0.1 = 4.21 42.1

9. 76,805 × 0.01 = 768.05 76,805

10. (−9.2)(−0.001) = 0.0092 0009.2

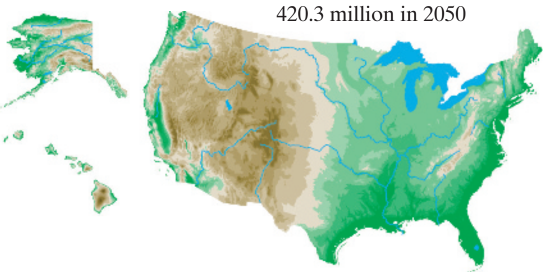
Work Practice 8–10

Many times we see large numbers written, for example, in the form 279.9 million rather than in the longer standard notation. The next example shows us how to interpret these numbers.

Answers

5. 468 **6.** 20,300.4 **7.** −2330
8. 0.694 **9.** 0.039 **10.** 7.682

Example 11 In 2050, the population of the United States is projected to be 420.3 million. Write this number in standard notation. (*Source:* U.S. Census Bureau)



Solution:

$$\begin{aligned} 420.3 \text{ million} &= 420.3 \times 1 \text{ million} \\ &= 420.3 \times 1{,}000{,}000 = 420{,}300{,}000 \end{aligned}$$

Work Practice 11

Objective D Using Decimals as Replacement Values

Now let's practice working with variables.

Example 12 Evaluate xy for $x = 2.3$ and $y = 0.44$.

Solution: Recall that xy means $x \cdot y$.

$$xy = (2.3)(0.44) \qquad \begin{array}{r} 2.3 \\ \times\ 0.44 \\ \hline 92 \\ 920 \\ \hline 1.012 \end{array}$$

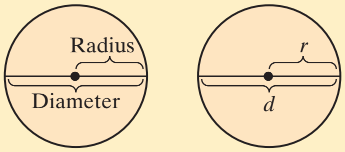
$$= 1.012 \longleftarrow$$

Work Practice 12

Objective E Finding the Circumference of a Circle

Recall from Section 1.3 that the distance around a polygon is called its perimeter. The distance around a circle is given the special name **circumference,** and this distance depends on the radius or the diameter of the circle.

Circumference of a Circle



Circumference $= 2 \cdot \pi \cdot$ **r**adius or **C**ircumference $= \pi \cdot$ **d**iameter

$$\boldsymbol{C} = 2\pi \boldsymbol{r} \qquad \text{or} \qquad \boldsymbol{C} = \pi \boldsymbol{d}$$

In Section 4.1, we learned about the symbol π as the Greek letter pi, pronounced "pie." It is a constant between 3 and 4.

Approximations for π

Two common approximations for π are:

$$\underbrace{\pi \approx 3.14}_{\text{a decimal approximation}} \qquad \text{or} \qquad \underbrace{\pi \approx \frac{22}{7}}_{\text{a fraction approximation}}$$

Practice 11

There were 60.25 million married couples in the United States in 2016. Write this number in standard notation. (*Source:* U.S. Census Bureau)

Practice 12

Evaluate $7y$ for $y = -0.028$.

Answers

11. 60,250,000 **12.** −0.196

Practice 13

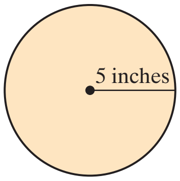
Find the circumference of a circle whose radius is 11 meters. Then use the approximation 3.14 for π to approximate this circumference.

Example 13 Circumference of a Circle

Find the circumference of a circle whose radius is 5 inches. Then use the approximation 3.14 for π to approximate the circumference.

Solution: Let $r = 5$ in the formula $C = 2\pi r$.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \cdot 5 \\ &= 10\pi \end{aligned}$$



Next, replace π with the approximation 3.14.

$$\begin{aligned} C &= 10\pi \\ (\text{is approximately}) \longrightarrow \quad &\approx 10(3.14) \\ &= 31.4 \end{aligned}$$

The **exact** circumference or distance around the circle is 10π inches, which is **approximately** 31.4 inches.

Work Practice 13

Objective F Solving Problems by Multiplying Decimals

The solutions to many real-life problems are found by multiplying decimals. We continue using our four problem-solving steps to solve such problems.

Practice 14

A biology major is fertilizing her personal garden. She uses 5.6 ounces of fertilizer per square yard. The garden measures 60.5 square yards. How much fertilizer does she need?

Example 14 Finding the Total Cost of Materials for a Job

A college student is hired to paint a billboard with paint costing \$2.49 per quart. If the job requires 3 quarts of paint, what is the total cost of the paint?

Solution:

1. UNDERSTAND. Read and reread the problem. The phrase "total cost" might make us think addition, but since this problem requires repeated addition, let's multiply.
2. TRANSLATE.

In words:	total cost	is	cost per quart of paint	times	number of quarts
	↓	↓	↓	↓	↓
Translate:	total cost	=	2.49	×	3

3. SOLVE. We can estimate to check our calculations. The number 2.49 rounds to 2 and $2 \times 3 = 6$.

$$\begin{array}{r} 2.49 \\ \times \quad 3 \\ \hline 7.47 \end{array}$$

4. INTERPRET. *Check* your work. Since 7.47 is close to our estimate of 6, our answer is reasonable. *State* your conclusion: The total cost of the paint is \$7.47.

Work Practice 14

Answers

13. 22π m $\approx$ 69.08 m
14. 338.8 oz

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

circumference
decimal places
left
right
sum
product
zeros
factor

1. When multiplying decimals, the number of decimal places in the product is equal to the ______________ of the number of decimal places in the factors.

2. In 8.6 × 5 = 43, the number 43 is called the ______________.

3. The distance around a circle is called its ______________.

4. In 8.6 × 5 = 43, the numbers 8.6 and 5 are each called a ______________.

5. When multiplying a decimal number by powers of 10 such as 10, 100, 1000, and so on, we move the decimal point in the number to the ______________ the same number of places as there are ______________ in the power of 10.

6. When multiplying a decimal number by powers of 10 such as 0.1, 0.01, and so on, we move the decimal point in the number to the ______________ the same number of places as there are ______________ in the power of 10.

Do not multiply. Just give the number of decimal places in the product. See the Concept Check in this section.

7. $\begin{array}{r} 0.46 \\ \times\ 0.81 \\ \hline \end{array}$ **8.** $\begin{array}{r} 57.9 \\ \times\ 0.36 \\ \hline \end{array}$ **9.** $\begin{array}{r} 0.0073 \\ \times\quad 21 \\ \hline \end{array}$ **10.** $\begin{array}{r} 0.428 \\ \times\quad 0.2 \\ \hline \end{array}$ **11.** $\begin{array}{r} 5.1296 \\ \times\ 7.3987 \\ \hline \end{array}$ **12.** $\begin{array}{r} 0.028 \\ \times\ 1.36 \\ \hline \end{array}$

Martin-Gay Interactive Videos

See Video 4.3

Watch the section lecture video and answer the following questions.

Objective A **13.** From the lecture before Example 1, what's the main difference between multiplying whole numbers and multiplying decimal numbers?

Objective B **14.** From Example 3, what does estimating especially help us with?

Objective C **15.** Why don't we do any actual multiplying in Example 5?

Objective D **16.** In Example 8, once all replacement values are inserted in the variable expression, what is the resulting expression to evaluate?

Objective E **17.** Why is 31.4 cm not the exact answer to Example 9?

Objective F **18.** In Example 10, why is 24.8 not the complete answer? What is the complete answer?

4.3 Exercise Set MyLab Math

Objectives A B Mixed Practice *Multiply. See Examples 1 through 4. For those exercises marked, also estimate to see if the answer is reasonable.*

1. 0.17 × 8 **2.** 0.19 × 6 **3.** $\begin{array}{r} 1.2 \\ \times 0.5 \\ \hline \end{array}$ **4.** $\begin{array}{r} 6.8 \\ \times 0.3 \\ \hline \end{array}$

5. $(-2.3)(7.65)$ **6.** $(4.7)(-9.02)$ **7.** $(-5.73)(-9.6)$ **8.** $(-7.84)(-3.5)$

9. 6.8×4.2
Exact:
Estimate:

10. 8.3×2.7
Exact:
Estimate:

11. $\begin{array}{r} 0.347 \\ \times\ \ 0.3 \\ \hline \end{array}$

12. $\begin{array}{r} 0.864 \\ \times\ \ 0.4 \\ \hline \end{array}$

13. $\begin{array}{r} 1.0047 \\ \times\ \ \ 8.2 \\ \hline \end{array}$
Exact: Estimate:

14. $\begin{array}{r} 2.0005 \\ \times\ \ \ 5.5 \\ \hline \end{array}$
Exact: Estimate:

15. $\begin{array}{r} 490.2 \\ \times 0.023 \\ \hline \end{array}$

16. $\begin{array}{r} 300.9 \\ \times 0.032 \\ \hline \end{array}$

Objective C *Multiply. See Examples 5 through 10.*

17. 6.5×10

18. 7.2×100

19. 8.3×0.1

20. 23.4×0.1

21. $(-7.093)(1000)$

22. $(-1.123)(1000)$

23. 0.7×100

24. 0.5×100

25. $(-9.83)(-0.01)$

26. $(-4.72)(-0.01)$

27. 25.23×0.001

28. 36.41×0.001

Objectives A B C Mixed Practice *Multiply. See Examples 1 through 10.*

29. 0.123×0.4

30. 0.216×0.3

31. $(147.9)(100)$

32. $(345.2)(100)$

33. 8.6×0.15

34. 0.42×5.7

35. $(937.62)(-0.01)$

36. $(-0.01)(562.01)$

37. 562.3×0.001

38. 993.5×0.001

39. $\begin{array}{r} 6.32 \\ \times\ 5.7 \\ \hline \end{array}$

40. $\begin{array}{r} 9.21 \\ \times\ 3.8 \\ \hline \end{array}$

Write each number in standard notation. See Example 11.

41. The total cost of the Cassini Solstice space mission was approximately \$3.26 billion. (*Source:* NASA)

42. About 60.2 million American households own at least one dog. (*Source:* American Pet Products Manufacturers Association)

43. The Blue Streak is the oldest roller coaster at Cedar Point, an amusement park in Sandusky, Ohio. Since 1964, it has given more than 49.8 million rides. (*Source:* Cedar Fair, L.P.)

44. In 2017, the restaurant industry had projected sales of \$798.7 billion. (*Source:* National Restaurant Association)

Objective D *Evaluate each expression for $x = 3$, $y = -0.2$, and $z = 5.7$. See Example 12.*

45. xy

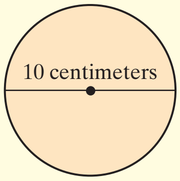
46. yz

47. $xz - y$

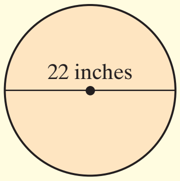
48. $-5y + z$

Objective E *Find the circumference of each circle. Then use the approximation 3.14 for π and approximate each circumference. See Example 13.*

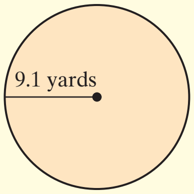
49.



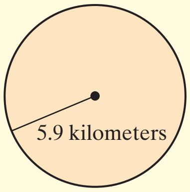
50.



51.



52.



Objectives E F Mixed Practice *Solve. For Exercises 53 and 54, the solutions have been started for you. See Examples 13 and 14. For circumference applications, find the exact circumference and then use 3.14 for π to approximate the circumference.*

53. An electrician for Central Power and Light worked 40 hours last week. Calculate his pay before taxes for last week if his hourly wage is $17.88.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

pay before taxes	is	hourly wage	times	hours worked
↓	↓	↓	↓	↓
pay before taxes	=	____	×	____

Finish with:

3. SOLVE and **4.** INTERPRET.

54. An assembly line worker worked 20 hours last week. Her hourly rate is $19.52 per hour. Calculate her pay before taxes.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

pay before taxes	is	hourly rate	times	hours worked
↓	↓	↓	↓	↓
pay before taxes	=	____	×	____

Finish with:

3. SOLVE and **4.** INTERPRET.

55. A 1-ounce serving of cream cheese contains 6.2 grams of saturated fat. How much saturated fat is in 4 ounces of cream cheese? (*Source: Home and Garden Bulletin No. 72;* U.S. Department of Agriculture)

56. A 3.5-ounce serving of lobster meat contains 0.1 gram of saturated fat. How much saturated fat do 3 servings of lobster meat contain? (*Source:* The National Institutes of Health)

57. Recall that the face of the Apple iPhone X (see Section 4.2) measures 5.65 inches by 2.8 inches rounded. Find the approximate area of the face of the Apple iPhone X.

58. Recall that the face of the Google Pixel 2 (see Section 4.2) measures 5.74 inches by 2.7 inches rounded. Find the approximate area of the face of the Google Pixel 2.

△ **59.** In 1893, the first ride called a Ferris wheel was constructed by Washington Gale Ferris. Its diameter was 250 feet. Find its circumference. Give the exact answer and an approximation using 3.14 for π. (*Source: The Handy Science Answer Book,* Visible Ink Press, 1994)

△ **60.** The radius of Earth is approximately 3950 miles. Find the distance around Earth at the equator. Give the exact answer and an approximation using 3.14 for π. (*Hint:* Find the circumference of a circle with radius 3950 miles.)

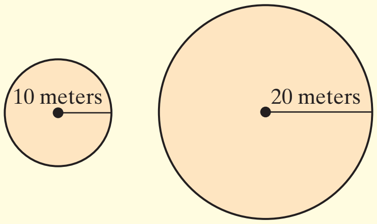
△ **61.** The London Eye, built for the Millennium celebration in London, resembles a gigantic Ferris wheel with a diameter of 135 meters. If Adam Hawn rides the Eye for one revolution, find how far he travels. Give the exact answer and an approximation using 3.14 for π. (*Source:* Londoneye.com)

△ **62.** The world's longest suspension bridge is the Akashi Kaikyo Bridge in Japan. This bridge has two circular caissons, which are underwater foundations. If the diameter of a caisson is 80 meters, find its circumference. Give the exact answer and an approximation using 3.14 for π. (*Source: Scientific American;* How Things Work Today)

63. A meter is a unit of length in the metric system that is approximately equal to 39.37 inches. Sophia Wagner is 1.65 meters tall. Find her approximate height in inches.

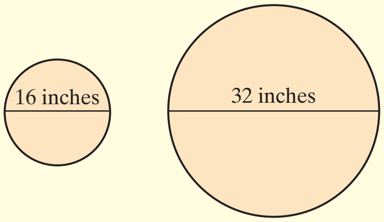
64. The doorway to a room is 2.15 meters tall. Approximate this height in inches. (*Hint:* See Exercise **63**.)

△ **65.** **a.** Approximate the circumference of each circle.



b. If the radius of a circle is doubled, is its corresponding circumference doubled?

△ **66.** **a.** Approximate the circumference of each circle.



b. If the diameter of a circle is doubled, is its corresponding circumference doubled?

67. On one day in 2017, the price of wheat was $4.27 per bushel. How much would 100 bushels of wheat cost at this price? (*Source:* National Agricultural Statistics Service)

68. On one day in 2017, the price of soybeans was $9.5625 per bushel. How much would a company pay for 10,000 bushels of soybeans? (*Source:* National Agricultural Statistics Service)

The table shows currency exchange rates for various countries on a particular day in 2018. To find the amount of foreign currency equivalent to an amount of U.S. dollars, multiply the U.S. dollar amount by the exchange rate listed in the table. Use this table to answer Exercises 69 through 72. Round answers for Exercises 71 and 72 to the nearest hundredth.

Foreign Currency Exchange Rates

Foreign Currency	Exchange Rate
Canadian dollar	1.25090
European Union euro	0.83019
New Zealand dollar	1.40822
Chinese yuan	6.49251
Japanese yen	112.30859
Swiss franc	0.97239

69. How many Canadian dollars were equivalent to $800 U.S.?

70. Suppose you exchanged 3000 American dollars for Chinese yuan. How much money, in Chinese yuan, did you receive?

71. The Scarpulla family traveled to New Zealand. How many New Zealand dollars did they “buy” with 300 U.S. dollars?

72. A French tourist to the United States spent $130 for souvenirs at the *Head of the Charles Regatta* in Boston. How much money did he spend in euros?

Review

Divide. See Sections 1.7 and 3.3.

73. $2916 \div 6$

74. $2920 \div 365$

75. $-\frac{24}{7} \div \frac{8}{21}$

76. $\frac{162}{25} \div -\frac{9}{75}$

Concept Extensions

Mixed Practice (*Sections 4.2 and 4.3*) *Perform the indicated operations.*

77. 3.6 + 0.04

78. 7.2 + 0.14 + 98.6

79. 3.6 − 0.04

80. 100 − 48.6

81. −0.221 × 0.5

82. −3.6 × 0.04

Solve.

83. Find how far radio waves travel in 20.6 seconds. (Radio waves travel at a speed of 1.86 × 100,000 miles per second.)

84. If it takes radio waves approximately 8.3 minutes to travel from the Sun to the Earth, find approximately how far it is from the Sun to the Earth. (*Hint:* See Exercise **83**.)

85. In your own words, explain how to find the number of decimal places in a product of decimal numbers.

86. In your own words, explain how to multiply by a power of 10.

87. Write down two decimal numbers whose product will contain 5 decimal places. Without multiplying, explain how you know your answer is correct.

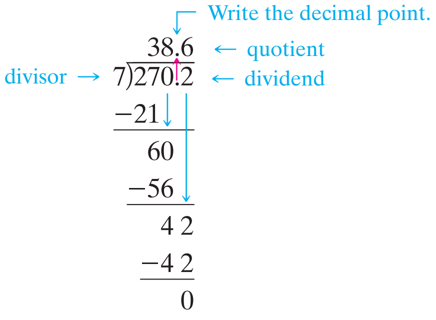
4.4 Dividing Decimals

Objective A Dividing Decimals

Dividing decimal numbers is similar to dividing whole numbers. The only difference is that we place a decimal point in the quotient. If the divisor is a whole number, we place the decimal point in the quotient directly above the decimal point in the dividend, and then we divide as with whole numbers. Recall that division can be checked by multiplication.

Example 1 Divide: 270.2 ÷ 7. Check your answer.

Solution: We divide as usual. The decimal point in the quotient is directly above the decimal point in the dividend.



Check:

$$\begin{array}{r} \scriptstyle 6\,4 \\ 38.6 \\ \times \quad 7 \\ \hline 270.2 \end{array}$$

The quotient is 38.6.

Work Practice 1

Objectives

A Divide Decimals.

B Estimate When Dividing Decimals.

C Divide Decimals by Powers of 10.

D Evaluate Expressions with Decimal Replacement Values.

E Solve Problems by Dividing Decimals.

Practice 1

Divide: 517.2 ÷ 6. Check your answer.

Answer

1. 86.2

Practice 2

Divide: $48\overline{)34.08}$. Check your answer.

Example 2

Divide: $32\overline{)8.32}$. Check your answer.

Solution: We divide as usual. The decimal point in the quotient is directly above the decimal point in the dividend.

$$\begin{array}{rrl}
 & 0.26 & \leftarrow \text{quotient} \\
\text{divisor} \rightarrow & 32\overline{)8.32} & \leftarrow \text{dividend} \\
 & \underline{-6\,4} & \\
 & 1\,92 & \\
 & \underline{-1\,92} & \\
 & 0 &
\end{array}
\qquad
\textbf{Check:}\quad
\begin{array}{rl}
0.26 & \text{quotient} \\
\underline{\times\ 32} & \text{divisor} \\
52 & \\
\underline{7\,80} & \\
8.32 & \text{dividend}
\end{array}$$

Work Practice 2

Sometimes to continue dividing we need to insert zeros after the last digit in the dividend.

Practice 3

Divide and check.

a. $-13.62 \div 12$

b. $-2.808 \div (-104)$

Example 3

Divide: $-5.98 \div 115$

Solution: Recall that a negative number divided by a positive number gives a negative quotient.

$$\begin{array}{rl}
0.052 & \\
115\overline{)5.980} & \leftarrow \text{Insert one 0.} \\
\underline{-5\,75} & \\
230 & \\
\underline{-230} & \\
0 &
\end{array}$$

Thus $-5.98 \div 115 = -0.052$.

Work Practice 3

If the divisor is not a whole number, before we divide we need to move the decimal point to the right until the divisor is a whole number.

$$\begin{array}{c}
1.5\overline{)64.85} \\
\text{divisor} \uparrow \quad \uparrow \text{dividend}
\end{array}$$

To understand how this works, let's rewrite

$$1.5\overline{)64.85} \quad \text{as} \quad \frac{64.85}{1.5}$$

and then multiply by 1 in the form of $\frac{10}{10}$. We use the form $\frac{10}{10}$ so that the denominator (divisor) becomes a whole number.

$$\frac{64.85}{1.5} = \frac{64.85}{1.5} \cdot 1 = \frac{64.85}{1.5} \cdot \frac{10}{10} = \frac{64.85 \cdot 10}{1.5 \cdot 10} = \frac{648.5}{15},$$

which can be written as $15.\overline{)648.5}$. Notice that

$1.\underset{\curvearrowright}{5}\overline{)64.\underset{\curvearrowright}{8}5}$ is equivalent to $15.\overline{)648.5}$.

Answers

2. 0.71 **3. a.** −1.135 **b.** 0.027

The decimal points in the dividend and the divisor were both moved one place to the right, and the divisor is now a whole number. This procedure is summarized next:

Dividing by a Decimal

Step 1: Move the decimal point in the divisor to the right until the divisor is a whole number.

Step 2: Move the decimal point in the dividend to the right the *same number of places* as the decimal point was moved in Step 1.

Step 3: Divide. Place the decimal point in the quotient directly over the moved decimal point in the dividend.

Example 4 Divide: $10.764 \div 2.3$

Solution: We move the decimal points in the divisor and the dividend one place to the right so that the divisor is a whole number.

$2.3\overline{)10.764}$ becomes

$$\begin{array}{r} 4.68 \\ 23.\overline{)107.64} \\ -92 \\ \hline 15\,6 \\ -13\,8 \\ \hline 1\,84 \\ -1\,84 \\ \hline 0 \end{array}$$

Work Practice 4

Practice 4

Divide: $166.88 \div 5.6$

Example 5 Divide: $5.264 \div 0.32$

Solution:

$0.32\overline{)5.264}$ becomes

$$\begin{array}{r} 16.45 \\ 32\overline{)526.40} \\ -32 \\ \hline 206 \\ -192 \\ \hline 14\,4 \\ -12\,8 \\ \hline 1\,60 \\ -1\,60 \\ \hline 0 \end{array}$$

Insert one 0.

Work Practice 5

Practice 5

Divide: $1.976 \div 0.16$

✓**Concept Check** Is it always true that the number of decimal places in a quotient equals the sum of the decimal places in the dividend and divisor?

Answers

4. 29.8 **5.** 12.35

✓**Concept Check Answer**

no

Practice 6

Divide: 23.4 ÷ 0.57. Round the quotient to the nearest hundredth.

Example 6

Divide: 17.5 ÷ 0.48. Round the quotient to the nearest hundredth.

Solution: First we move the decimal points in the divisor and the dividend two places. Then we divide and round the quotient to the nearest hundredth.

hundredths place

$$
\begin{array}{r}
36.458 \\
48.\overline{)1750.000} \\
\underline{-144} \\
310 \\
\underline{-288} \\
22\,0 \\
\underline{-19\,2} \\
2\,80 \\
\underline{-2\,40} \\
400 \\
\underline{-384} \\
16
\end{array}
\approx 36.46
$$

"is approximately"

When rounding to the nearest hundredth, carry the division process out to one more decimal place, the thousandths place.

Work Practice 6

✓**Concept Check** If a quotient is to be rounded to the nearest thousandth, to what place should the division be carried out? (Assume that the division carries out to your answer.)

Objective B Estimating When Dividing Decimals

Just as for addition, subtraction, and multiplication of decimals, we can estimate when dividing decimals to check the reasonableness of our answer.

Practice 7

Divide: 363.968 ÷ 37.6. Then estimate to see whether the proposed answer is reasonable.

Example 7

Divide: 272.356 ÷ 28.4. Then estimate to see whether the proposed result is reasonable.

Solution:

Exact:

$$
\begin{array}{r}
9.59 \\
284.\overline{)2723.56} \\
\underline{-2556} \\
167\,5 \\
\underline{-142\,0} \\
25\,56 \\
\underline{-25\,56} \\
0
\end{array}
$$

Estimate 1

$$
\begin{array}{r}
9 \\
30\overline{)270}
\end{array}
$$

or

Estimate 2

$$
\begin{array}{r}
10 \\
30\overline{)300}
\end{array}
$$

The estimate is 9 or 10, so 9.59 is reasonable.

Work Practice 7

Answers

6. 41.05 **7.** Exact: 9.68

✓**Concept Check Answer**

ten-thousandths place

Objective C Dividing Decimals by Powers of 10

As with multiplication, there are patterns that occur when we divide decimals by powers of 10 such as 10, 100, 1000, and so on.

$$\frac{569.2}{10} = 56.92$$ Move the decimal point *1 place* to the *left.*

1 zero

$$\frac{569.2}{10{,}000} = 0.05692$$ Move the decimal point *4 places* to the *left.*

4 zeros

This pattern suggests the following rule:

Dividing Decimals by Powers of 10 Such as 10, 100, or 1000

Move the decimal point of the dividend to the *left* the same number of places as there are *zeros* in the power of 10.

Examples Divide.

8. $$\frac{786.1}{1000} = 0.7861$$ Move the decimal point *3 places* to the *left.*

3 zeros

9. $$\frac{-0.12}{10} = -0.012$$ Move the decimal point *1 place* to the *left.*

1 zero

Work Practice 8–9

Objective D Using Decimals as Replacement Values

Example 10 Evaluate $x \div y$ for $x = 2.5$ and $y = 0.05$.

Solution: Replace x with 2.5 and y with 0.05.

$$x \div y = 2.5 \div 0.05 \qquad 0.05\overline{)2.5} \text{ becomes } \begin{array}{r} 50 \\ 5\overline{)250} \end{array}$$
$$= 50$$

Work Practice 10

Objective E Solving Problems by Dividing Decimals

Many real-life problems involve dividing decimals.

Example 11 Calculating Materials Needed for a Job

A gallon of paint covers a 250-square-foot area. If Betty Adkins wishes to paint a wall that measures 1450 square feet, how many gallons of paint does she need? If she can buy only gallon containers of paint, how many gallon containers does she need?

(*Continued on next page*)

Practice 8–9

Divide.

8. $\frac{128.3}{1000}$ **9.** $\frac{-0.56}{10}$

Practice 10

Evaluate $x \div y$ for $x = 0.035$ and $y = 0.02$.

(*Practice 11 is on the next page.*)

Answers

8. 0.1283 **9.** −0.056 **10.** 1.75

Practice 11

A bag of fertilizer covers 1250 square feet of lawn. Tim Parker's lawn measures 14,800 square feet. How many bags of fertilizer does he need? If he can buy only whole bags of fertilizer, how many whole bags does he need?

Solution:

1. UNDERSTAND. Read and reread the problem. We need to know how many 250s are in 1450, so we divide.
2. TRANSLATE.

In words:	number of gallons	is	square feet	divided by	square feet per gallon
	↓	↓	↓	↓	↓
Translate:	number of gallons	=	1450	÷	250

3. SOLVE. Let's see if our answer is reasonable by estimating. The dividend 1450 rounds to 1500 and the divisor 250 rounds to 300. Then 1500 ÷ 300 = 5.

$$\begin{array}{r} 5.8 \\ 250\overline{)1450.0} \\ -1250 \\ \hline 200\ 0 \\ -200\ 0 \\ \hline 0 \end{array}$$

4. INTERPRET. *Check* your work. Since our estimate is close to our answer of 5, our answer is reasonable. *State* your conclusion: Betty needs 5.8 gallons of paint. If she can buy only gallon containers of paint, she needs 6 gallon containers of paint to complete the job.

Work Practice 11

Answer
11. 11.84 bags; 12 bags

Calculator Explorations Estimation

Calculator errors can easily be made by pressing an incorrect key or by not pressing a correct key hard enough. Estimation is a valuable tool that can be used to check calculator results.

Example Use estimation to determine whether the calculator result is reasonable or not. (For example, a result that is not reasonable can occur if proper keys are not pressed.)

Simplify: 82.064 ÷ 23

Calculator display: $\boxed{35.68}$

Solution: Round each number to the nearest 10. Since 80 ÷ 20 = 4, the calculator display 35.68 is not reasonable.

Use estimation to determine whether each result is reasonable or not.

1. 102.62 × 41.8 Result: 428.9516
2. 174.835 ÷ 47.9 Result: 3.65
3. 1025.68 − 125.42 Result: 900.26
4. 562.781 + 2.96 Result: 858.781

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once and some not used at all.

dividend divisor quotient true

zeros left right false

1. In 6.5 ÷ 5 = 1.3, the number 1.3 is called the ___________, 5 is the ___________, and 6.5 is the ___________.

2. To check a division exercise, we can perform the following multiplication: quotient · ___________ = ___________.

3. To divide a decimal number by a power of 10 such as 10, 100, 1000, and so on, we move the decimal point in the number to the ___________ the same number of places as there are ___________ in the power of 10.

4. True or false: If we replace x with -12.6 and y with 0.3 in the expression $y \div x$, we have $0.3 \div (-12.6)$. ___________

Recall properties of division and simplify without the use of paper and pencil.

5. $\frac{5.9}{1}$

6. $\frac{0.7}{0.7}$

7. $\frac{0}{9.86}$

8. $\frac{2.36}{0}$

9. $\frac{7.261}{7.261}$

10. $\frac{8.25}{1}$

11. $\frac{11.1}{0}$

12. $\frac{0}{89.96}$

Martin-Gay Interactive Videos

See Video 4.4

Watch the section lecture video and answer the following questions.

Objective A **13.** From the lecture before Example 1, what must we make sure the divisor is before dividing decimals?

Objective B **14.** From Example 4, what does estimating especially help us with?

Objective C **15.** Why don't we do any actual dividing in Example 6?

Objective D **16.** In Example 7, 8 does not divide into 5. How does this affect the quotient?

Objective E **17.** In Example 8, why is the division carried to the hundredths place?

4.4 Exercise Set MyLab Math

Objectives A B Mixed Practice *Divide. See Examples 1 through 5 and 7. For those exercises marked, also estimate to see if the answer is reasonable.*

1. $3\overline{)13.8}$

2. $2\overline{)11.8}$

3. $5\overline{)0.47}$

4. $6\overline{)0.51}$

5. $0.06\overline{)18}$

6. $0.04\overline{)20}$

7. $0.54\overline{)1.404}$

8. $0.73\overline{)4.526}$

9. $5.5\overline{)36.3}$
Exact:
Estimate:

10. $2.2\overline{)21.78}$
Exact:
Estimate:

11. $7.434 \div 18$

12. $8.304 \div 16$

13. $36 \div (-0.06)$

14. $36 \div (-0.04)$

15. Divide -4.2 by -0.6.

16. Divide -3.6 by -0.9.

17. $0.27\overline{)1.296}$

18. $0.34\overline{)2.176}$

19. $0.02\overline{)42}$

20. $0.03\overline{)24}$

21. $4.756 \div 0.82$

22. $3.312 \div 0.92$

23. $-36.3 \div (-6.6)$

24. $-21.78 \div (-9.9)$

25. $7.2\overline{)70.56}$
Exact:
Estimate:

26. $6.3\overline{)54.18}$
Exact:
Estimate:

27. $5.4\overline{)51.84}$

28. $7.7\overline{)33.88}$

29. $\frac{1.215}{0.027}$

30. $\frac{3.213}{0.051}$

31. $0.25\overline{)13.648}$

32. $0.75\overline{)49.866}$

33. $3.78\overline{)0.02079}$

34. $2.96\overline{)0.01332}$

Divide. Round the quotients as indicated. See Example 6.

35. Divide: 429.34 by 2.4. Round the quotient to the nearest whole number.

36. Divide: 54.8 by 2.6. Round the quotient to the nearest whole number.

37. Divide: $0.549 \div 0.023$. Round the quotient to the nearest hundredth.

38. Divide: $0.0453 \div 0.98$. Round the quotient to the nearest thousandth.

39. Divide: $68.39 \div 0.6$. Round the quotient to the nearest tenth.

40. Divide: $98.83 \div 3.5$. Round the quotient to the nearest tenth.

Objective C *Divide. See Examples 8 and 9.*

41. $\frac{54.982}{100}$

42. $\frac{342.54}{100}$

43. $\frac{26.87}{10}$

44. $\frac{13.49}{10}$

45. $-12.9 \div 1000$

46. $-13.49 \div 10{,}000$

Objectives A C Mixed Practice *Divide. See Examples 1 through 5, 8, and 9.*

47. $7\overline{)88.2}$

48. $9\overline{)130.5}$

49. $\frac{13.1}{10}$

50. $\frac{17.7}{10}$

51. $\frac{456.25}{10{,}000}$

52. $\frac{986.11}{10{,}000}$

53. $1.239 \div 3$

54. $0.54 \div 12$

55. Divide 4.8 by -0.6.

56. Divide 4.9 by -0.7.

57. $-1.224 \div 0.17$

58. $-1.344 \div 0.42$

59. Divide 42 by 0.03.

60. Divide 27 by 0.03.

61. Divide −18 by −0.6.

62. Divide 20 by 0.4.

63. Divide 87 by −0.0015.

64. Divide 35 by −0.0007.

65. $-1.104 \div 1.6$

66. $-2.156 \div 0.98$

67. $-2.4 \div (-100)$

68. $-86.79 \div (-1000)$

69. $\dfrac{4.615}{0.071}$

70. $\dfrac{23.8}{0.035}$

Objective D

Evaluate each expression for $x = 5.65$, $y = -0.8$, *and* $z = 4.52$. *See Example 10.*

71. $z \div y$

72. $z \div x$

73. $x \div y$

74. $y \div 2$

Objective E

Solve. For Exercises 75 and 76, the solutions have been started for you. See Example 11.

75. A new homeowner is painting the walls of a room. The walls have a total area of 546 square feet. A quart of paint covers 52 square feet. If he must buy paint in whole quarts, how many quarts does he need?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

number of quarts	is	square feet	divided by	square feet per quart
↓	↓	↓	↓	↓
number of quarts	=	______	÷	______

3. SOLVE. Don’t forget to round up your quotient.
4. INTERPRET.

76. A shipping box can hold 36 books. If 486 books must be shipped, how many boxes are needed?

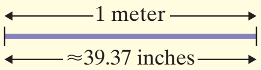
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into an equation. (Fill in the blanks.)

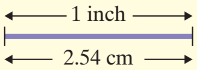
number of boxes	is	number of books	divided by	books per box
↓	↓	↓	↓	↓
number of boxes	=	______	÷	______

3. SOLVE. Don’t forget to round up your quotient.
4. INTERPRET.

77. There are approximately 39.37 inches in 1 meter. How many meters, to the nearest tenth of a meter, are there in 200 inches?



78. There are 2.54 centimeters in 1 inch. How many inches are there in 50 centimeters? Round to the nearest tenth.



79. In the United States, an average child will wear down 730 crayons by his or her tenth birthday. Find the number of boxes of 64 crayons this is equivalent to. Round to the nearest tenth. (*Source:* Binney & Smith Inc.)

80. In 2017, American farmers received an average of $98.80 per hundred pounds of turkey. What was the average price per pound for turkeys? Round to the nearest cent. (*Source:* National Agricultural Statistics Service)

A child is to receive a dose of 0.5 teaspoon of cough medicine every 4 hours. If the bottle contains 4 fluid ounces, answer Exercises 81 through 84.

81. A fluid ounce equals 6 teaspoons. How many teaspoons are in 4 fluid ounces?

82. The bottle of medicine contains how many doses for the child? (*Hint:* See Exercise **81**.)

83. If the child takes a dose every four hours, how many days will the medicine last?

84. If the child takes a dose every six hours, how many days will the medicine last?

Solve.

85. Americans ages 16–19 drive, on average, 7624 miles per year. About how many miles each week is that? Round to the nearest tenth. (*Note:* There are 52 weeks in a year.) (*Source:* Federal Highway Administration)

86. Drake Saucier was interested in the gas mileage on his "new" used car. He filled the tank, drove 423.8 miles, and filled the tank again. When he refilled the tank, it took 19.35 gallons of gas. Calculate the miles per gallon for Drake's car. Round to the nearest tenth.

87. During the 2016 Summer Olympics, Kenyan runner Vivian Cheruiyot took the gold medal in the women's 5000-meter event. Her time for the event was 866.28 seconds. What was her average speed in meters per second? Round to the nearest tenth. (*Source:* International Olympic Committee)

88. The book *Harry Potter and the Deathly Hallows* was released to the public on July 21, 2007. Booksellers in the United States sold approximately 8292 thousand copies in the first 24 hours after release. If the same number of books was sold each hour, calculate the number of books sold each hour in the United States for that first day.

Review

Write each decimal as a fraction. See Section 4.1.

89. 0.9

90. 0.7

91. 0.05

92. 0.08

Concept Extensions

Mixed Practice (*Sections 4.2, 4.3, and 4.4*) *Perform the indicated operation.*

93. $1.278 \div 0.3$

94. 1.278×0.3

95. $1.278 + 0.3$

96. $1.278 - 0.3$

97. $(-8.6)(3.1)$

98. $7.2 + 0.05 + 49.1$

99. $\begin{array}{r} 1000 \\ -\ \ 95.71 \\ \hline \end{array}$

100. $\dfrac{87.2}{-10{,}000}$

Choose the best estimate.

101. 8.62×41.7
- **a.** 36
- **b.** 32
- **c.** 360
- **d.** 3.6

102. $1.437 + 20.69$
- **a.** 34
- **b.** 22
- **c.** 3.4
- **d.** 2.2

103. $78.6 \div 97$
- **a.** 7.86
- **b.** 0.786
- **c.** 786
- **d.** 7860

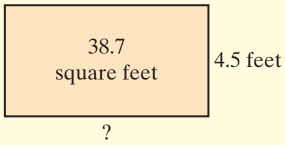
104. $302.729 - 28.697$
- **a.** 270
- **b.** 20
- **c.** 27
- **d.** 300

Recall from Section 1.7 that the average of a list of numbers is their total divided by how many numbers there are in the list. Use this procedure to find the average of the test scores listed in Exercises 105 and 106. If necessary, round to the nearest tenth.

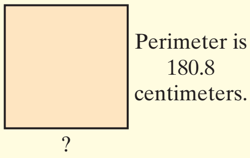
105. 86, 78, 91, 87

106. 56, 75, 80

△ **107.** The area of a rectangle is 38.7 square feet. If its width is 4.5 feet, find its length.



△ **108.** The perimeter of a square is 180.8 centimeters. Find the length of a side.



✎ **109.** When dividing decimals, describe the process you use to place the decimal point in the quotient.

✎ **110.** In your own words, describe how to quickly divide a number by a power of 10 such as 10, 100, 1000, etc.

To convert wind speeds in miles per hour to knots, divide by 1.15. Use this information and the Saffir-Simpson Hurricane Intensity chart below to answer Exercises 111 and 112. Round to the nearest tenth.

Saffir-Simpson Hurricane Intensity Scale

Category	Wind Speed	Barometric Pressure [inches of mercury (Hg)]	Storm Surge	Damage Potential
1 (Weak)	75–95 mph	≥28.94 in.	4–5 ft	Minimal damage to vegetation
2 (Moderate)	96–110 mph	28.50–28.93 in.	6–8 ft	Moderate damage to houses
3 (Strong)	111–130 mph	27.91–28.49 in.	9–12 ft	Extensive damage to small buildings
4 (Very Strong)	131–155 mph	27.17–27.90 in.	13–18 ft	Extreme structural damage
5 (Devastating)	>155 mph	<27.17 in.	>18 ft	Catastrophic building failures possible

111. The chart gives wind speeds in miles per hour. What is the range of wind speeds for a Category 1 hurricane in knots?

112. What is the range of wind speeds for a Category 4 hurricane in knots?

Solve.

△ **113.** A rancher is building a horse corral that's shaped like a rectangle with a width of 24.3 meters. He plans to make a four-wire fence; that is, he will string four wires around the corral. If he plans to use all of his 412.8 meters of wire, find the length of the corral he can construct.

114. A college student signed up for a new credit card that guarantees her no interest charges on transferred balances for a year. She transferred over a $2523.86 balance from her old credit card. Her minimum payment is $185.35 per month. If she pays only the minimum, will she pay off her balance before interest charges start again?

Integrated Review

Sections 4.1–4.4

Operations on Decimals

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______

Perform the indicated operations.

1. $1.6 + 0.97$

2. $3.2 + 0.85$

3. $9.8 - 0.9$

4. $10.2 - 6.7$

5. $\begin{array}{r} 0.8 \\ \times 0.2 \\ \hline \end{array}$

6. $\begin{array}{r} 0.6 \\ \times 0.4 \\ \hline \end{array}$

7. $8\overline{)2.16}$

8. $6\overline{)3.12}$

9. $(9.6)(-0.5)$

10. $(-8.7)(-0.7)$

11. $\begin{array}{r} 123.6 \\ -\ 48.04 \\ \hline \end{array}$

12. $\begin{array}{r} 325.2 \\ -\ 36.08 \\ \hline \end{array}$

13. $-25 + 0.026$

14. $0.125 + (-44)$

15. $29.24 \div (-3.4)$

16. $-10.26 \div (-1.9)$

17. -2.8×100

18. 1.6×1000

19. $\begin{array}{r} 96.21 \\ 7.028 \\ +121.7 \\ \hline \end{array}$

20. $\begin{array}{r} 0.268 \\ 1.93 \\ +142.881 \\ \hline \end{array}$

21. $-25.76 \div (-46)$

22. $-27.09 \div 43$

23. $\begin{array}{r} 12.004 \\ \times\ \ 2.3 \\ \hline \end{array}$

24. $\begin{array}{r} 28.006 \\ \times\ \ 5.2 \\ \hline \end{array}$

25. Subtract 4.6 from 10.

26. Subtract 18 from 0.26.

27. $-268.19 - 146.25$

28. $-860.18 - 434.85$

29. $\frac{2.958}{-0.087}$

30. $\frac{-1.708}{0.061}$

31. $160 - 43.19$

32. $120 - 101.21$

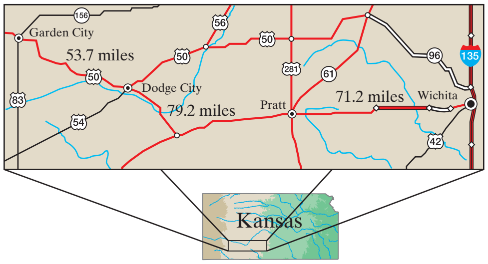
33. 15.62×10

34. $15.62 \div 10$

35. $15.62 + 10$

36. $15.62 - 10$

37. Estimate the distance in miles between Garden City, Kansas, and Wichita, Kansas, by rounding each given distance to the nearest ten.



38. It costs $7.15 to send a 5-lb package locally via a small flat-rate box at a U.S. Post Office. To send the same package Priority Mail costs $9.85. How much more does it cost to send the package Priority Mail? (*Source:* United States Postal Service)

39. In 2016, sales of physical movies (either DVD or Blu-ray) were $7.96 billion, while digital sales of movies were $10.32 billion. Write the total in billions of dollars and also in standard notation. (*Source: Los Angeles Times*)

25. __________

26. __________

27. __________

28. __________

29. __________

30. __________

31. __________

32. __________

33. __________

34. __________

35. __________

36. __________

37. __________

38. __________

39. __________

4.5 Fractions, Decimals, and Order of Operations

Objectives

A Write Fractions as Decimals.

B Compare Decimals and Fractions.

C Simplify Expressions Containing Decimals and Fractions Using Order of Operations.

D Solve Area Problems Containing Fractions and Decimals.

E Evaluate Expressions Given Decimal Replacement Values.

Practice 1

a. Write $\frac{2}{5}$ as a decimal.

b. Write $\frac{9}{40}$ as a decimal.

Practice 2

Write $-\frac{3}{8}$ as a decimal.

Practice 3

a. Write $\frac{5}{6}$ as a decimal.

b. Write $\frac{2}{9}$ as a decimal.

Answers

1. a. 0.4 **b.** 0.225 **2.** −0.375

3. a. $0.8\overline{3}$ **b.** $0.\overline{2}$

Objective A Writing Fractions as Decimals

To write a fraction as a decimal, we interpret the fraction bar to mean division and find the quotient.

Writing Fractions as Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

Example 1 Write $\frac{1}{4}$ as a decimal.

Solution: $\frac{1}{4} = 1 \div 4$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Thus, $\frac{1}{4}$ written as a decimal is 0.25.

Work Practice 1

Example 2 Write $-\frac{5}{8}$ as a decimal.

Solution: $-\frac{5}{8} = -(5 \div 8) = -0.625$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{-4\,8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Work Practice 2

Example 3 Write $\frac{2}{3}$ as a decimal.

Solution:

$$\begin{array}{rl} 0.666\ldots & \\ 3\overline{)2.000} & \text{This pattern will continue because } \frac{2}{3} = 0.6666\ldots \\ \underline{-1\,8} & \\ 20 & \text{Remainder is 2, and then 0 is brought down.} \\ \underline{-18} & \\ 20 & \text{Remainder is 2, and then 0 is brought down.} \\ \underline{-18} & \\ 2 & \text{Remainder is 2.} \end{array}$$

Notice that the digit 2 keeps occurring as the remainder. This will continue, and the digit 6 will keep repeating in the quotient. We place a bar over the digit 6 to indicate that it repeats.

$$\frac{2}{3} = 0.666\ldots \quad = 0.\overline{6}$$

We can also write a decimal approximation for $\frac{2}{3}$. For example, $\frac{2}{3}$ rounded to the nearest hundredth is 0.67. This can be written as $\frac{2}{3} \approx 0.67$.

Work Practice 3

Example 4 Write $\frac{22}{7}$ as a decimal. (Recall that the fraction $\frac{22}{7}$ is an approximation for π.) Round to the nearest hundredth.

Solution:

$$\begin{array}{r} 3.142 \\ 7\overline{)22.000} \\ -21 \\ \hline 1\;0 \\ -7 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 6 \end{array} \approx 3.14 \quad \text{Carry the division out to the thousandths place.}$$

The fraction $\frac{22}{7}$ in decimal form is approximately 3.14.

Work Practice 4

Practice 4

Write $\frac{28}{13}$ as a decimal. Round to the nearest thousandth.

Example 5 Write $2\frac{3}{16}$ as a decimal.

Solution:

Option 1. Write the fractional part only as a decimal.

$$\frac{3}{16} \longrightarrow \begin{array}{r} 0.1875 \\ 16\overline{)3.0000} \\ -1\;6\downarrow \\ \hline 1\;40 \\ -1\;28\downarrow \\ \hline 120 \\ -112\downarrow \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

Thus $2\frac{3}{16} = 2.1875$

Option 2. Write $2\frac{3}{16}$ as an improper fraction, and divide.

$$2\frac{3}{16} = \frac{35}{16} \longrightarrow \begin{array}{r} 2.1875 \\ 16\overline{)35.0000} \\ -32\downarrow \\ \hline 3\;0 \\ -1\;6\downarrow \\ \hline 1\;40 \\ -1\;28\downarrow \\ \hline 120 \\ -112\downarrow \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

Work Practice 5

Practice 5

Write $3\frac{5}{16}$ as a decimal.

Answers

4. 2.154 **5.** 3.3125

Practice 6

Write $\frac{3}{5}$ as a decimal.

Practice 7

Write $\frac{3}{50}$ as a decimal.

Practice 8

Insert $<$, $>$, or $=$ to form a true statement.

$\frac{1}{5}$ 0.25

Answers

6. 0.6 **7.** 0.06 **8.** $<$

✓Concept Check Answer

$\frac{9}{16}$ is less than 1 while 1.735 is greater than 1.

Some fractions may be written as decimals using our knowledge of decimals. From Section 4.1, we know that if the denominator of a fraction is 10, 100, 1000, or so on, we can immediately write the fraction as a decimal. For example,

$$\frac{4}{10} = 0.4, \qquad \frac{12}{100} = 0.12, \text{ and so on}$$

Example 6 Write $\frac{4}{5}$ as a decimal.

Solution: Let's write $\frac{4}{5}$ as an equivalent fraction with a denominator of 10.

$$\frac{4}{5} = \frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10} = 0.8$$

Work Practice 6

Example 7 Write $\frac{1}{25}$ as a decimal.

Solution: $\frac{1}{25} = \frac{1}{25} \cdot \frac{4}{4} = \frac{4}{100} = 0.04$

Work Practice 7

✓Concept Check Suppose you are writing the fraction $\frac{9}{16}$ as a decimal. How do you know you have made a mistake if your answer is 1.735?

Objective B Comparing Decimals and Fractions

Now we can compare decimals and fractions by writing fractions as equivalent decimals.

Example 8 Insert $<$, $>$, or $=$ to form a true statement.

$\frac{1}{8}$ 0.12

Solution: First we write $\frac{1}{8}$ as an equivalent decimal. Then we compare decimal places.

$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Original numbers	$\frac{1}{8}$	0.12
Decimals	0.125	0.120
Compare	0.125 > 0.12	

Thus, $\frac{1}{8} > 0.12$

Work Practice 8

Example 9 Insert <, >, or = to form a true statement.

$$0.\overline{7} \qquad \frac{7}{9}$$

Solution: We write $\frac{7}{9}$ as a decimal and then compare.

$$\begin{array}{r} 0.77\ldots \\ 9\overline{)7.00} \\ -6\;3 \\ \hline 70 \\ -63 \\ \hline 7 \end{array} \quad = 0.\overline{7}$$

Original numbers	$0.\overline{7}$	$\frac{7}{9}$
Decimals	$0.\overline{7}$	$0.\overline{7}$
Compare	$0.\overline{7} = 0.\overline{7}$	

Thus, $0.\overline{7} = \frac{7}{9}$

Work Practice 9

Practice 9

Insert <, >, or = to form a true statement.

a. $\frac{1}{2}$ 0.54 **b.** $0.\overline{4}$ $\frac{4}{9}$

c. $\frac{5}{7}$ 0.72

Example 10 Write the numbers in order from smallest to largest.

$$\frac{9}{20}, \frac{4}{9}, 0.456$$

Solution:

Original numbers	$\frac{9}{20}$	$\frac{4}{9}$	0.456
Decimals	0.450	0.444 . . .	0.456
Compare in order	2nd	1st	3rd

Written in order, we have

$$\begin{array}{ccc} \text{1st} & \text{2nd} & \text{3rd} \\ \downarrow & \downarrow & \downarrow \\ \frac{4}{9}, & \frac{9}{20}, & 0.456 \end{array}$$

Work Practice 10

Practice 10

Write the numbers in order from smallest to largest.

a. $\frac{1}{3}, 0.302, \frac{3}{8}$

b. $1.26, 1\frac{1}{4}, 1\frac{2}{5}$

c. $0.4, 0.41, \frac{3}{7}$

Objective C Simplifying Expressions with Decimals and Fractions

In the remaining examples, we will review the order of operations by simplifying expressions that contain decimals and fractions.

Order of Operations

1. Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars or square roots, starting with the inner most set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Answers

9. a. < **b.** = **c.** <

10. a. $0.302, \frac{1}{3}, \frac{3}{8}$ **b.** $1\frac{1}{4}, 1.26, 1\frac{2}{5}$

c. $0.4, 0.41, \frac{3}{7}$

Practice 11

Simplify: $897.8 \div 100 \times 10$

Example 11

Simplify: $723.6 \div 1000 \times 10$

Solution: Multiply or divide in order from left to right.

$$\begin{aligned} 723.6 \div 1000 \times 10 &= 0.7236 \times 10 && \text{Divide.} \\ &= 7.236 && \text{Multiply.} \end{aligned}$$

Work Practice 11

Practice 12

Simplify: $-8.69(3.2 - 1.8)$

Example 12

Simplify: $-0.5(8.6 - 1.2)$

Solution: According to the order of operations, we simplify inside the parentheses first.

$$\begin{aligned} -0.5(8.6 - 1.2) &= -0.5(7.4) && \text{Subtract.} \\ &= -3.7 && \text{Multiply.} \end{aligned}$$

Work Practice 12

Practice 13

Simplify: $\left(-\frac{7}{10}\right)^2 + 2.1$

Example 13

Simplify: $\left(-\frac{13}{10}\right)^2 + 2.4$

Solution: Recall the meaning of an exponent.

$$\begin{aligned} \left(-\frac{13}{10}\right)^2 + 2.4 &= \left(-\frac{13}{10}\right)\left(-\frac{13}{10}\right) + 2.4 && \text{Use the definition of an exponent.} \\ &= \frac{169}{100} + 2.4 && \text{Multiply. The product of two negative numbers is a positive number.} \\ &= 1.69 + 2.4 && \text{Write } \tfrac{169}{100} \text{ as a decimal.} \\ &= 4.09 && \text{Add.} \end{aligned}$$

Work Practice 13

Practice 14

Simplify: $\dfrac{20.06 - (1.2)^2 \div 10}{0.02}$

Example 14

Simplify: $\dfrac{5.68 + (0.9)^2 \div 100}{0.2}$

Solution: First we simplify the numerator of the fraction. Then we divide.

$$\begin{aligned} \frac{5.68 + (0.9)^2 \div 100}{0.2} &= \frac{5.68 + 0.81 \div 100}{0.2} && \text{Simplify } (0.9)^2. \\ &= \frac{5.68 + 0.0081}{0.2} && \text{Divide.} \\ &= \frac{5.6881}{0.2} && \text{Add.} \\ &= 28.4405 && \text{Divide.} \end{aligned}$$

Work Practice 14

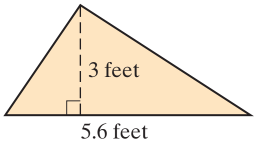
Answers

11. 89.78 **12.** −12.166 **13.** 2.59
14. 995.8

Objective D Solving Area Problems Containing Fractions and Decimals

Sometimes real-life problems contain both fractions and decimals. In the next example, we review the area of a triangle, and when values are substituted, the result may be an expression containing both fractions and decimals.

Example 15 The area of a triangle is Area $= \frac{1}{2} \cdot \text{base} \cdot \text{height}$. Find the area of the triangle shown.



Solution:

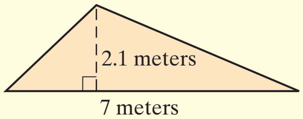
$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot \text{base} \cdot \text{height} \\ &= \frac{1}{2} \cdot 5.6 \cdot 3 \\ &= 0.5 \cdot 5.6 \cdot 3 && \text{Write } \frac{1}{2} \text{ as the decimal 0.5.} \\ &= 8.4 \end{aligned}$$

The area of the triangle is 8.4 square feet.

Work Practice 15

Practice 15

Find the area of the triangle.



Objective E Using Decimals as Replacement Values

Example 16 Evaluate $-2x + 5$ for $x = 3.8$.

Solution: Replace x with 3.8 in the expression $-2x + 5$ and simplify.

$$\begin{aligned} -2x + 5 &= -2(3.8) + 5 && \text{Replace } x \text{ with 3.8.} \\ &= -7.6 + 5 && \text{Multiply.} \\ &= -2.6 && \text{Add.} \end{aligned}$$

Work Practice 16

Practice 16

Evaluate $1.7y - 2$ for $y = 2.3$.

Answers

15. 7.35 sq m **16.** 1.91

Vocabulary, Readiness & Video Check

Answer each exercise "true" or "false."

1. The number $0.\overline{5}$ means 0.555.

2. To write $\frac{9}{19}$ as a decimal, perform the division $19\overline{)9}$.

3. $(-1.2)^2$ means $(-1.2)(-1.2)$ or -1.44.

4. To simplify $8.6(4.8 - 9.6)$, we first subtract.

Martin-Gay Interactive Videos

See Video 4.5

Watch the section lecture video and answer the following questions.

Objective A **5.** In Example 2, why is the bar placed over just the 6?

Objective B **6.** In Example 3, why do we write the fraction as a decimal rather than the decimal as a fraction?

Objective C **7.** In Example 4, besides meaning division, what other purpose does the fraction bar serve?

Objective D **8.** What formula is used to solve Example 5? What is the final answer?

Objective E **9.** In Example 6, once all replacement values are put into the variable expression, what is the resulting expression to evaluate?

4.5 Exercise Set MyLab Math

Objective A *Write each number as a decimal. See Examples 1 through 7.*

1. $\frac{1}{5}$ **2.** $\frac{1}{20}$ **3.** $\frac{17}{25}$ **4.** $\frac{13}{25}$ **5.** $\frac{3}{4}$

6. $\frac{1}{8}$ **7.** $-\frac{2}{25}$ **8.** $-\frac{3}{25}$ **9.** $\frac{9}{4}$ **10.** $\frac{8}{5}$

11. $\frac{11}{12}$ **12.** $\frac{5}{12}$ **13.** $\frac{17}{40}$ **14.** $\frac{19}{25}$ **15.** $\frac{9}{20}$

16. $\frac{31}{40}$ **17.** $-\frac{1}{3}$ **18.** $-\frac{7}{9}$ **19.** $\frac{7}{16}$ **20.** $\frac{9}{16}$

21. $\frac{7}{11}$ **22.** $\frac{9}{11}$ **23.** $5\frac{17}{20}$ **24.** $4\frac{7}{8}$ **25.** $\frac{78}{125}$ **26.** $\frac{159}{375}$

Round each number as indicated. See Example 4.

27. Round your answer to Exercise **17** to the nearest hundredth.

28. Round your answer to Exercise **18** to the nearest hundredth.

29. Round your answer to Exercise **19** to the nearest hundredth.

30. Round your answer to Exercise **20** to the nearest hundredth.

31. Round your answer to Exercise **21** to the nearest tenth.

32. Round your answer to Exercise **22** to the nearest tenth.

Write each fraction as a decimal. If necessary, round to the nearest hundredth. See Examples 1 through 7.

33. Of the U.S. mountains that are over 14,000 feet in elevation, $\frac{56}{91}$ are located in Colorado. (*Source:* U.S. Geological Survey)

34. About $\frac{21}{50}$ of all blood donors have type A blood. (*Source:* American Red Cross Biomedical Services)

35. In 2017, about $\frac{23}{25}$ of Americans were Internet users. (*Source:* Digitalcenter.org)

36. In 2017, about $\frac{41}{50}$ of Americans used the Internet through a mobile phone. (*Source:* Digitalcenter.org)

37. When first launched, the Hubble Space Telescope's primary mirror was out of shape on the edges by $\frac{1}{50}$ of a human hair. This very small defect made it difficult to focus faint objects being viewed. Because the HST was in low Earth orbit, it was serviced by a shuttle and the defect was corrected.

38. The two mirrors currently in use in the Hubble Space Telescope were ground so that they do not deviate from a perfect curve by more than $\frac{1}{800{,}000}$ of an inch. Do not round this number.

Objective B

Insert $<$, $>$, or $=$ to form a true statement. See Examples 8 and 9.

39. $0.562 \qquad 0.569$

40. $0.983 \qquad 0.988$

41. $0.215 \qquad \frac{43}{200}$

42. $\frac{29}{40} \qquad 0.725$

43. $-0.0932 \qquad -0.0923$

44. $-0.00563 \qquad -0.00536$

45. $0.\overline{6} \qquad \frac{5}{6}$

46. $0.\overline{1} \qquad \frac{2}{17}$

47. $\frac{51}{91} \qquad 0.56\overline{4}$

48. $0.58\overline{3} \qquad \frac{6}{11}$

49. $\frac{4}{7} \qquad 0.14$

50. $\frac{5}{9} \qquad 0.557$

51. $1.38 \qquad \frac{18}{13}$

52. $0.372 \qquad \frac{22}{59}$

53. $7.123 \qquad \frac{456}{64}$

54. $12.713 \qquad \frac{89}{7}$

Write the numbers in order from smallest to largest. See Example 10.

55. 0.34, 0.35, 0.32

56. 0.47, 0.42, 0.40

57. 0.49, 0.491, 0.498

58. 0.72, 0.727, 0.728

59. $5.23, \frac{42}{8}, 5.34$

60. $7.56, \frac{67}{9}, 7.562$

61. $\frac{5}{8}, 0.612, 0.649$

62. $\frac{5}{6}, 0.821, 0.849$

Objective C

Simplify each expression. See Examples 11 through 14.

63. $(0.3)^2 + 0.5$

64. $(-2.5)(3) - 4.7$

65. $\frac{1 + 0.8}{-0.6}$

66. $(-0.05)^2 + 3.13$

67. $(-2.3)^2(0.3 + 0.7)$

68. $(8.2)(100) - (8.2)(10)$

69. $(5.6 - 2.3)(2.4 + 0.4)$

70. $\frac{0.222 - 2.13}{12}$

71. $\frac{(4.5)^2}{100}$

72. $0.9(5.6 - 6.5)$

73. $\frac{7 + 0.74}{-6}$

74. $(1.5)^2 + 0.5$

Find the value of each expression. Give the result as a decimal. See Examples 11 through 14.

75. $\frac{1}{5} - 2(7.8)$

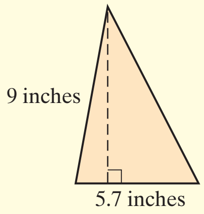
76. $\frac{3}{4} - (9.6)(5)$

77. $\frac{1}{4}(-9.6 - 5.2)$

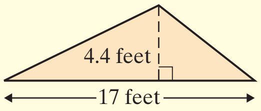
78. $\frac{3}{8}(4.7 - 5.9)$

Objective D *Find the area of each triangle or rectangle. See Example 15.*

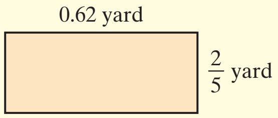
△ **79.**



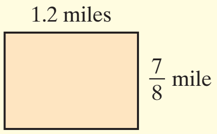
△ **80.**



▶ △ **81.**



△ **82.**



Objective E *Evaluate each expression for $x = 6$, $y = 0.3$, and $z = -2.4$. See Example 16.*

83. z^2

84. y^2

85. $x - y$

86. $x - z$

▶ **87.** $4y - z$

88. $\frac{x}{y} + 2z$

Review

Simplify. See Sections 1.9 and 3.3.

89. $6^2 \cdot 2$

90. $4 \cdot 3^4$

91. $\left(\frac{2}{5}\right)\left(\frac{5}{2}\right)^2$

92. $\left(\frac{2}{3}\right)^2\left(\frac{3}{2}\right)^3$

Concept Extensions

Without calculating, describe each number as < 1, $= 1$, or > 1. See the Concept Check in this section.

93. 1.0

94. 1.0000

95. 1.00001

96. $\frac{101}{99}$

97. $\frac{99}{100}$

98. $\frac{99}{99}$

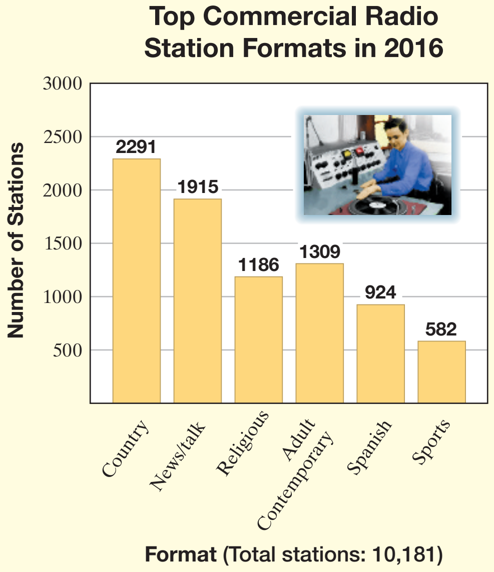
In 2016, there were 10,181 commercial radio stations in the United States. The most popular formats are shown in the graph along with their counts. Use this graph to answer Exercises 99 through 102.

99. Write the fraction of radio stations that are all sports as a decimal. Round to the nearest thousandth.

100. Write the number of radio stations with a Spanish format as a decimal. Round to the nearest hundredth.

101. Estimate, by rounding each number in the table to the nearest hundred, the total number of stations with the top six formats in 2016.

102. Use your estimate from Exercise **101** to write the fraction of radio stations accounted for by the top six formats as a decimal. Round to the nearest hundredth.



✎ **103.** Describe two ways to write fractions as decimals.

✎ **104.** Describe two ways to write mixed numbers as decimals.

4.6 Square Roots and the Pythagorean Theorem

Objective A Finding Square Roots

The square of a number is the number times itself. For example:

The square of 5 is 25 because 5^2 or $5 \cdot 5 = 25$.

The square of 4 is 16 because 4^2 or $4 \cdot 4 = 16$.

The square of 10 is 100 because 10^2 or $10 \cdot 10 = 100$.

Recall from Chapter 1 that the reverse process of squaring is finding a **square root**. For example:

A square root of 16 is 4 because $4^2 = 16$.

A square root of 25 is 5 because $5^2 = 25$.

A square root of 100 is 10 because $10^2 = 100$.

We use the symbol $\sqrt{}$, called a **radical sign,** to denote square roots. For example:

$$\sqrt{16} = 4 \quad \text{because } 4^2 = 16$$
$$\sqrt{25} = 5 \quad \text{because } 5^2 = 25$$

Square Root of a Number

A square root of a number a is a number b whose square is a. We use the radical sign $\sqrt{}$ to name square roots. In symbols,

$$\sqrt{a} = b \quad \text{if} \quad b^2 = a$$

Also,

$$\sqrt{0} = 0$$

Example 1 Find each square root.

a. $\sqrt{49}$ **b.** $\sqrt{1}$ **c.** $\sqrt{81}$

Solution:

a. $\sqrt{49} = 7$ because $7^2 = 49$

b. $\sqrt{1} = 1$ because $1^2 = 1$

c. $\sqrt{81} = 9$ because $9^2 = 81$

Work Practice 1

Example 2 Find: $\sqrt{\frac{1}{36}}$

Solution: $\sqrt{\frac{1}{36}} = \frac{1}{6}$ because $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Work Practice 2

Example 3 Find: $\sqrt{\frac{4}{25}}$

Solution: $\sqrt{\frac{4}{25}} = \frac{2}{5}$ because $\frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

Work Practice 3

Objectives

A Find the Square Root of a Number.

B Approximate Square Roots.

C Use the Pythagorean Theorem.

Practice 1

Find each square root.

a. $\sqrt{100}$ **b.** $\sqrt{64}$

c. $\sqrt{121}$ **d.** $\sqrt{0}$

Practice 2

Find: $\sqrt{\frac{1}{4}}$

Practice 3

Find: $\sqrt{\frac{9}{16}}$

Answers

1. a. 10 **b.** 8 **c.** 11 **d.** 0

2. $\frac{1}{2}$ **3.** $\frac{3}{4}$

Objective B Approximating Square Roots

Thus far, we have found square roots of perfect squares. Numbers like $\frac{1}{4}$, 36, $\frac{4}{25}$, and 1 are called **perfect squares** because their square root is a whole number or a fraction. A square root such as $\sqrt{5}$ cannot be written as a whole number or a fraction since 5 is not a perfect square.

Although $\sqrt{5}$ cannot be written as a whole number or a fraction, it can be approximated by estimating, by using a table (as in the appendix), or by using a calculator.

Practice 4

Use Appendix A.6 or a calculator to approximate each square root to the nearest thousandth.

a. $\sqrt{21}$ **b.** $\sqrt{52}$

Example 4

Use Appendix A.6 or a calculator to approximate each square root to the nearest thousandth.

a. $\sqrt{43} \approx 6.557$ is approximately

b. $\sqrt{80} \approx 8.944$

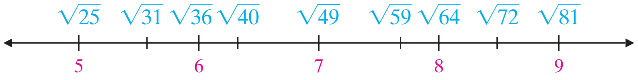
Work Practice 4

Helpful Hint

$\sqrt{80}$, above, is *approximately* 8.944. This means that if we multiply 8.944 by 8.944, the product is *close* to 80.

$$8.944 \times 8.944 \approx 79.995$$

It is possible to approximate a square root to the nearest whole number without the use of a calculator or table. To do so, study the number line below and look for patterns.



Above the number line, notice that as the numbers under the radical signs increase, their values, and thus their placement on the number line, increase also.

Practice 5

Without a calculator or table, approximate $\sqrt{62}$ to the nearest whole.

Example 5

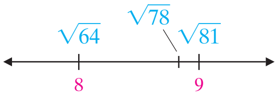
Without a calculator or table:

a. Determine which two whole numbers $\sqrt{78}$ is between.

b. Use part **a** to approximate $\sqrt{78}$ to the nearest whole.

Solution:

a. Review perfect squares and recall that $\sqrt{64} = 8$ and $\sqrt{81} = 9$. Since 78 is between 64 and 81, $\sqrt{78}$ is between $\sqrt{64}$ (or 8) and $\sqrt{81}$ (or 9).



Thus, $\sqrt{78}$ is between 8 and 9.

b. Since 78 is closer to 81, then (as our number line shows) $\sqrt{78}$ is closer to $\sqrt{81}$, or 9.

Work Practice 5

Objective C Using the Pythagorean Theorem

One important application of square roots has to do with right triangles. Recall that a **right triangle** is a triangle in which one of the angles is a right angle, or measures 90° (degrees). The **hypotenuse** of a right triangle is the side opposite the right angle.

Answers

4. a. 4.583 **b.** 7.211
5. 8

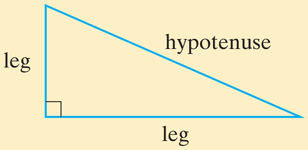
The **legs** of a right triangle are the other two sides. These are shown in the following figure. The right angle in the triangle is indicated by the small square drawn in that angle.

The following theorem is true for all right triangles:

Pythagorean Theorem

In any **right triangle,**

$$(\text{leg})^2 + (\text{other leg})^2 = (\text{hypotenuse})^2$$



Using the Pythagorean theorem, we can use one of the following formulas to find an unknown length of a right triangle:

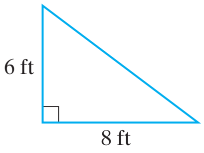
Finding an Unknown Length of a Right Triangle

$$\text{hypotenuse} = \sqrt{(\text{leg})^2 + (\text{other leg})^2}$$

or

$$\text{leg} = \sqrt{(\text{hypotenuse})^2 - (\text{other leg})^2}$$

Example 6 Find the length of the hypotenuse of the given right triangle.



Solution: Since we are finding the hypotenuse, we use the formula

$$\text{hypotenuse} = \sqrt{(\text{leg})^2 + (\text{other leg})^2}$$

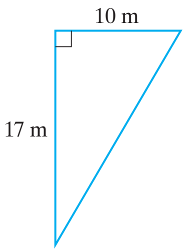
Putting the known values into the formula, we have

$$\begin{aligned} \text{hypotenuse} &= \sqrt{(6)^2 + (8)^2} && \text{The legs are 6 feet and 8 feet.} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

The hypotenuse is 10 feet long.

Work Practice 6

Example 7 Approximate the length of the hypotenuse of the given right triangle. Round the length to the nearest whole unit.



Solution:

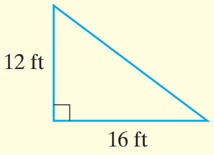
$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(17)^2 + (10)^2} && \text{The legs are 10 meters and 17 meters.} \\ &= \sqrt{289 + 100} \\ &= \sqrt{389} \\ &\approx 20 && \text{From Appendix A.6 or a calculator} \end{aligned}$$

The hypotenuse is exactly $\sqrt{389}$ meters, which is approximately 20 meters.

Work Practice 7

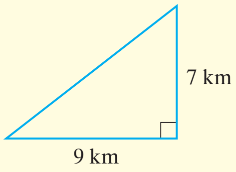
Practice 6

Find the length of the hypotenuse of the given right triangle.



Practice 7

Approximate the length of the hypotenuse of the given right triangle. Round to the nearest whole unit.

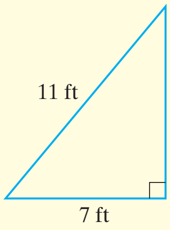


Answers

6. 20 ft **7.** 11 km

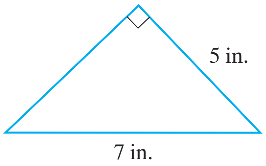
Practice 8

Find the length of the leg in the given right triangle. Give the exact length and a two-decimal-place approximation.



Example 8

Find the length of the leg in the given right triangle. Give the exact length and a two-decimal-place approximation.



Solution: Notice that the hypotenuse measures 7 inches and the length of one leg measures 5 inches. Since we are looking for the length of the other leg, we use the formula

$$\text{leg} = \sqrt{(\text{hypotenuse})^2 - (\text{other leg})^2}$$

Putting the known values into the formula, we have

$$\begin{aligned} \text{leg} &= \sqrt{(7)^2 - (5)^2} && \text{The hypotenuse is 7 inches, and the other leg is 5 inches.} \\ &= \sqrt{49 - 25} \\ &= \sqrt{24} && \text{Exact answer} \\ &\approx 4.90 && \text{From Appendix A.6 or a calculator} \end{aligned}$$

The length of the leg is exactly $\sqrt{24}$ inches, which is approximately 4.90 inches.

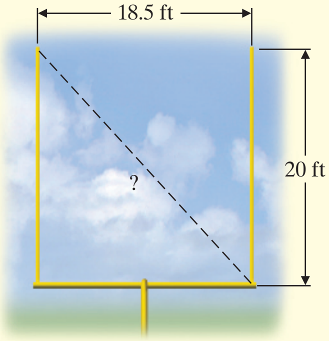
■ **Work Practice 8**

✓ **Concept Check** The following lists are the lengths of the sides of two triangles. Which set forms a right triangle? Explain.

a. 8, 15, 17 **b.** 24, 30, 40

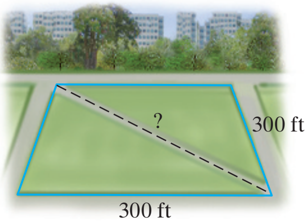
Practice 9

A football crossbar has posts that are 20 feet high. The posts for the National Football League are 18.5 feet wide. Find the length of the diagonal of the posts to the nearest foot.



Example 9 Finding the Dimensions of a Park

An inner-city park is in the shape of a square that measures 300 feet on a side. A sidewalk is to be constructed along the diagonal of the park. Find the length of the sidewalk rounded to the nearest foot.



Solution: The diagonal is the hypotenuse of a right triangle, so we use the formula

$$\text{hypotenuse} = \sqrt{(\text{leg})^2 + (\text{other leg})^2}$$

Putting the known values into the formula, we have

$$\begin{aligned} \text{hypotenuse} &= \sqrt{(300)^2 + (300)^2} && \text{The legs are both 300 feet.} \\ &= \sqrt{90{,}000 + 90{,}000} \\ &= \sqrt{180{,}000} \\ &\approx 424 && \text{From Appendix A.6 or a calculator} \end{aligned}$$

The length of the sidewalk is approximately 424 feet.

■ **Work Practice 9**

Answers

8. $\sqrt{72}$ ft $\approx$ 8.49 ft
9. 27 ft

✓ **Concept Check Answer**

Set **a** forms a right triangle.

Calculator Explorations Finding Square Roots

To simplify or approximate square roots using a calculator, locate the key marked $\boxed{\sqrt{}}$.

To simplify $\sqrt{64}$, for example, press the keys

$\boxed{64}\ \boxed{\sqrt{}}$ or $\boxed{\sqrt{}}\ \boxed{64}\ \boxed{\text{ENTER}}$

The display should read $\boxed{8}$. Then

$\sqrt{64} = 8$

To *approximate* $\sqrt{10}$, press the keys

$\boxed{10}\ \boxed{\sqrt{}}$ or $\boxed{\sqrt{}}\ \boxed{10}\ \boxed{\text{ENTER}}$

The display should read $\boxed{3.16227766}$. This is an *approximation* for $\sqrt{10}$. A three-decimal-place approximation is

$\sqrt{10} \approx 3.162$

Is this answer reasonable? Since 10 is between perfect squares 9 and 16, $\sqrt{10}$ is between $\sqrt{9} = 3$ and $\sqrt{16} = 4$. Our answer is reasonable since 3.162 is between 3 and 4.

Simplify.

1. $\sqrt{1024}$

2. $\sqrt{676}$

Approximate each square root. Round each answer to the nearest thousandth.

3. $\sqrt{31}$

4. $\sqrt{19}$

5. $\sqrt{97}$

6. $\sqrt{56}$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

squaring Pythagorean theorem radical leg
hypotenuse perfect squares 10

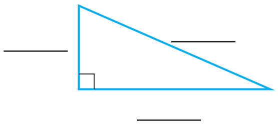
1. $\sqrt{100} =$ ________________ because $10 \cdot 10 = 100$.

2. The ________________ sign is used to denote the square root of a number.

3. The reverse process of ________________ a number is finding a square root of a number.

4. The numbers 9, 1, and $\frac{1}{25}$ are called ________________.

5. Label the parts of the right triangle.



6. The ________________ can be used for right triangles.

Martin-Gay Interactive Videos

See Video 4.6

Watch the section lecture video and answer the following questions.

Objective A **7.** From the lecture before Example 1, explain why $\sqrt{49} = 7$.

Objective B **8.** In Example 5, how do we know $\sqrt{15}$ is closer to 4 than to 3?

Objective C **9.** At the beginning of Example 6, what are we reminded about regarding the Pythagorean theorem?

4.6 Exercise Set MyLab Math

Objective A

Find each square root. See Examples 1 through 3.

1. $\sqrt{4}$

2. $\sqrt{9}$

3. $\sqrt{121}$

4. $\sqrt{144}$

5. $\sqrt{\frac{1}{81}}$

6. $\sqrt{\frac{1}{64}}$

7. $\sqrt{\frac{16}{64}}$

8. $\sqrt{\frac{36}{81}}$

Objective B

Use Appendix A.6 or a calculator to approximate each square root. Round the square root to the nearest thousandth. See Examples 4 and 5.

9. $\sqrt{3}$

10. $\sqrt{5}$

11. $\sqrt{15}$

12. $\sqrt{17}$

13. $\sqrt{47}$

14. $\sqrt{85}$

15. $\sqrt{26}$

16. $\sqrt{35}$

Determine what two whole numbers each square root is between without using a calculator or table. Then use a calculator or Appendix A.6 to check. See Example 5.

17. $\sqrt{38}$

18. $\sqrt{27}$

19. $\sqrt{101}$

20. $\sqrt{85}$

Objectives A B Mixed Practice

Find each square root. If necessary, round the square root to the nearest thousandth. See Examples 1 through 5.

21. $\sqrt{256}$

22. $\sqrt{625}$

23. $\sqrt{92}$

24. $\sqrt{18}$

25. $\sqrt{\frac{49}{144}}$

26. $\sqrt{\frac{121}{169}}$

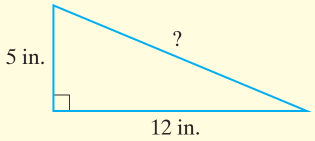
27. $\sqrt{71}$

28. $\sqrt{62}$

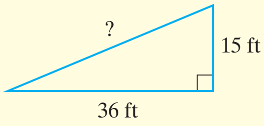
Objective C

Find the unknown length in each right triangle. If necessary, approximate the length to the nearest thousandth. See Examples 6 through 8.

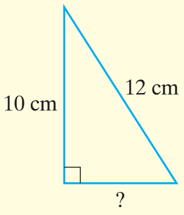
29.



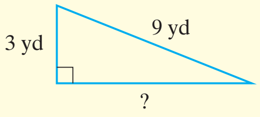
30.



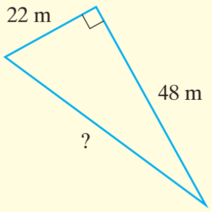
31.



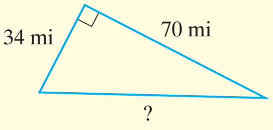
32.



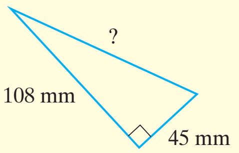
33.



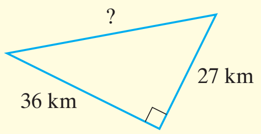
34.



35.



36.



Sketch each right triangle and find the length of the side not given. If necessary, approximate the length to the nearest thousandth. (Each length is in units.) See Examples 6 through 8.

37. leg = 3, leg = 4

38. leg = 9, leg = 12

39. leg = 5, hypotenuse = 13

40. leg = 6, hypotenuse = 10

41. leg = 10, leg = 14

42. leg = 2, leg = 16

43. leg = 35, leg = 28

44. leg = 30, leg = 15

45. leg = 30, leg = 30

46. leg = 21, leg = 21

47. hypotenuse = 2, leg = 1

48. hypotenuse = 9, leg = 8

49. leg = 7.5, leg = 4

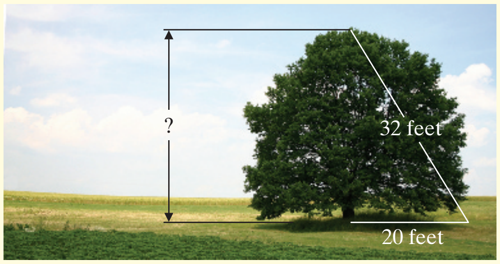
50. leg = 12, leg = 22.5

Solve. See Example 9.

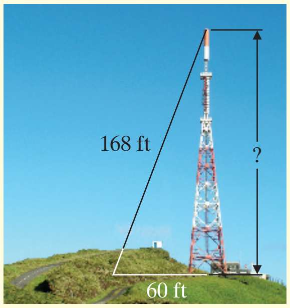
51. A standard city block is a square with each side measuring 100 yards. Find the length of the diagonal of a city block to the nearest hundredth yard.

52. A section of land is a square with each side measuring 1 mile. Find the length of the diagonal of the section of land to the nearest thousandth mile.

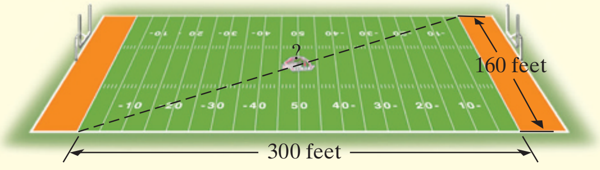
53. Find the height of the tree. Round the height to one decimal place.



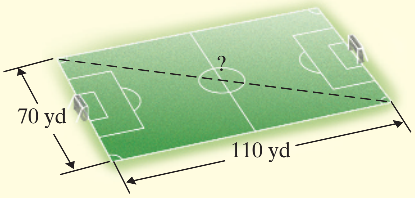
54. Find the height of the antenna. Round the height to one decimal place.



55. The playing field for football is a rectangle that is 300 feet long by 160 feet wide. Find, to the nearest foot, the length of a straight-line run that started at one corner and went diagonally to end at the opposite corner.



56. A soccer field is in the shape of a rectangle and its dimensions depend on the age of the players. The dimensions of the soccer field below are the minimum dimensions for international play. Find the length of the diagonal of this rectangle. Round the answer to the nearest tenth of a yard.



Review

Write each fraction in simplest form. See Section 3.2.

57. $\frac{10}{12}$

58. $\frac{10}{15}$

59. $\frac{24}{60}$

60. $\frac{35}{75}$

61. $\frac{30}{72}$

62. $\frac{18}{30}$

Concept Extensions

Use the results of Exercises 17 through 20 and approximate each square root to the nearest whole number without using a calculator or table. Then use a calculator or Appendix A.6 to check. See Example 5.

63. $\sqrt{38}$

64. $\sqrt{27}$

65. $\sqrt{101}$

66. $\sqrt{85}$

Solve.

67. Without using a calculator, explain how you know that $\sqrt{105}$ is *not* approximately 9.875.

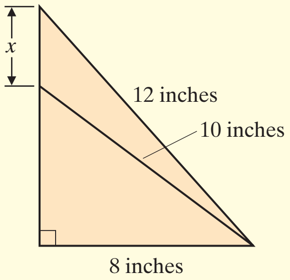
68. Without using a calculator, explain how you know that $\sqrt{27}$ is *not* approximately 3.296.

Does the set form the lengths of the sides of a right triangle? See the Concept Check in this section.

69. 25, 60, 65

70. 20, 45, 50

△ **71.** Find the exact length of x. Then give a two-decimal-place approximation.



Chapter 4 Group Activity

Maintaining a Checking Account

(Sections 4.1, 4.2)

This activity may be completed by working in groups or individually.

A checking account is a convenient way of handling money and paying bills. To open a checking account, the bank or savings and loan association requires a customer to make a deposit. Then the customer receives a checkbook that contains checks, deposit slips, and a register for recording checks written and deposits made. It is important to record all payments and deposits that affect the account. It is also important to keep the checkbook balance current by subtracting checks written and adding deposits made.

About once a month, checking customers receive a statement from the bank listing all activity that the account has had in the last month. The statement lists a beginning balance, all checks and deposits, any service charges made against the account, and an ending balance. Because it may take several days for checks that a customer has written to clear the banking system, the check register may list checks that do not appear on the monthly bank statement. These checks are called **outstanding checks**. Deposits that are recorded in the check register but do not appear on the statement are called **deposits in transit**. Because of these differences, it is important to balance, or reconcile, the checkbook against the monthly statement. The steps for doing so are listed below.

Balancing or Reconciling a Checkbook

Step 1: Place a check mark in the checkbook register next to each check and deposit listed on the monthly bank statement. Any entries in the register without a check mark are outstanding checks or deposits in transit.

Step 2: Find the ending checkbook register balance and add to it any outstanding checks and any interest paid on the account.

Step 3: From the total in Step 2, subtract any deposits in transit and any service charges.

Step 4: Compare the amount found in Step 3 with the ending balance listed on the bank statement. If they are the same, the checkbook balances with the bank statement. Be sure to update the check register with service charges and interest.

Step 5: If the checkbook does not balance, recheck the balancing process. Next, make sure that the running checkbook register balance was calculated correctly. Finally, compare the checkbook register with the statement to make sure that each check was recorded for the correct amount.

For the checkbook register and monthly bank statement given:

a. *update the checkbook register* **b.** *list the outstanding checks and deposits in transit and the totals of these*

c. *balance the checkbook*

Checkbook Register

#	Date	Description	Payment	✓	Deposit	Balance
						425.86
114	4/1	Market Basket	30.27			
115	4/3	May's Texaco	8.50			
	4/4	Cash at ATM	50.00			
116	4/6	UNO Bookstore	121.38			
	4/7	Deposit			100.00	
117	4/9	MasterCard	84.16			
118	4/10	Salle's Watch Repair	6.12			
119	4/12	Kroger	18.72			
120	4/14	Parking sticker	18.50			
	4/15	Direct deposit			294.36	
121	4/20	Rent	395.00			
122	4/25	Student fees	20.00			
	4/28	Deposit			75.00	

First National Bank Monthly Statement 4/30

BEGINNING BALANCE:		425.86
Date	Number	Amount
CHECKS AND ATM WITHDRAWALS		
4/3	114	30.27
4/4	ATM	50.00
4/11	117	84.16
4/13	115	8.50
4/15	119	18.72
4/22	121	395.00
DEPOSITS		
4/7		100.00
4/15	Direct deposit	294.36
SERVICE CHARGES		
Low balance fee		7.50
INTEREST		
Credited 4/30		1.15
ENDING BALANCE:		227.22

Chapter 4 Vocabulary Check

Fill in each blank with one of the words listed below.

vertically
sum
decimal
denominator
and
numerator
right triangle
square root
hypotenuse
standard form
legs

1. Like fractional notation, ________ notation is used to denote a part of a whole.

2. To write fractions as decimals, divide the ________ by the ________.

3. To add or subtract decimals, write the decimals so that the decimal points line up ________.

4. When writing decimals in words, write "________" for the decimal point.

5. When multiplying decimals, the decimal point in the product is placed so that the number of decimal places in the product is equal to the ________ of the number of decimal places in the factors.

6. The ________, $\sqrt{\ }$, of a positive number a is the positive number b whose square is a.

7. A ________ is a triangle with a right angle. The side opposite the right angle is called the ________, and the other two sides are called ________.

8. When 2 million is written as 2,000,000, we say it is written in ________.

Helpful Hint Are you preparing for your test? To help, don't forget to take these:

- Chapter 4 Getting Ready for the Test on page 358
- Chapter 4 Test on page 359

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

4 Chapter Highlights

Definitions and Concepts	Examples

Section 4.1 Introduction to Decimals

Place-Value Chart

		4	.	2	6	5		
hundreds	tens	ones	↑	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths
100	10	1	decimal point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$

4.265 means

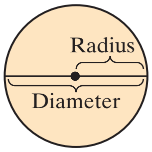
$$4 \cdot 1 + 2 \cdot \frac{1}{10} + 6 \cdot \frac{1}{100} + 5 \cdot \frac{1}{1000}$$

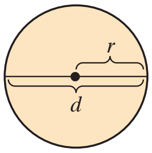
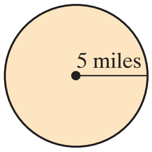
or

$$4 + \frac{2}{10} + \frac{6}{100} + \frac{5}{1000}$$

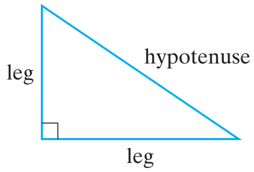
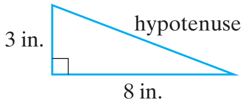
Definitions and Concepts	Examples
Section 4.1 Introduction to Decimals (*continued*)	
Writing (or Reading) a Decimal in Words **Step 1:** Write the whole number part in words. **Step 2:** Write “and” for the decimal point. **Step 3:** Write the decimal part in words as though it were a whole number, followed by the place value of the last digit.	Write 3.08 in words. Three and eight hundredths
A decimal written in words can be written in standard form by reversing the above procedure.	Write “negative four and twenty-one thousandths” in standard form. -4.021
To Round a Decimal to a Place Value to the Right of the Decimal Point **Step 1:** Locate the digit to the right of the given place value. **Step 2:** If this digit is 5 or greater, add 1 to the digit in the given place value and delete all digits to its right. If this digit is less than 5, delete all digits to the right of the given place value.	Round 86.1256 to the nearest hundredth. **Step 1:** $86.1\overset{\downarrow\text{ hundredths place}}{2}\,\underset{\uparrow\text{ digit to the right}}{\boxed{5}}\,6$ **Step 2:** Since the digit to the right is 5 or greater, we add 1 to the digit in the hundredths place and delete all digits to its right. 86.1256 rounded to the nearest hundredth is 86.13.
Section 4.2 Adding and Subtracting Decimals	
To Add or Subtract Decimals **Step 1:** Write the decimals so that the decimal points line up vertically. **Step 2:** Add or subtract as with whole numbers. **Step 3:** Place the decimal point in the sum or difference so that it lines up vertically with the decimal points in the problem.	Add: $4.6 + 0.28$ $\begin{array}{r} 4.60 \\ +\ 0.28 \\ \hline 4.88 \end{array}$ Subtract: $2.8 - 1.04$ $\begin{array}{r} 2.\overset{7}{\cancel{8}}\,\overset{10}{\cancel{0}} \\ -\ 1.0\,4 \\ \hline 1.7\,6 \end{array}$
Section 4.3 Multiplying Decimals and Circumference of a Circle	
To Multiply Decimals **Step 1:** Multiply the decimals as though they are whole numbers. **Step 2:** The decimal point in the product is placed so that the number of decimal places in the product is equal to the *sum* of the number of decimal places in the factors.	Multiply: 1.48×5.9 $\begin{array}{rl} 1.48 & \leftarrow \text{2 decimal places} \\ \times\ 5.9 & \leftarrow \text{1 decimal place} \\ \hline 1332 & \\ 7400 & \\ \hline 8.732 & \leftarrow \text{3 decimal places} \end{array}$

(continued)

<table>
<tr><th>Definitions and Concepts</th><th>Examples</th></tr>
<tr><td colspan="2">Section 4.3 Multiplying Decimals and Circumference of a Circle (<i>continued</i>)</td></tr>
<tr><td>The circumference of a circle is the distance around the circle.


$C = \pi d$ or $C = 2\pi r$
where $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$.
or

 </td><td>Find the exact circumference of a circle with radius 5 miles and an approximation by using 3.14 for π.


$\begin{aligned} C &= 2\pi r \\ &= 2\pi(5) \\ &= 10\pi \\ &\approx 10(3.14) \\ &= 31.4 \end{aligned}$
The circumference is exactly 10π miles and approximately 31.4 miles.</td></tr>
<tr><td colspan="2">Section 4.4 Dividing Decimals</td></tr>
<tr><td>To Divide Decimals
Step 1: If the divisor is not a whole number, move the decimal point in the divisor to the right until the divisor is a whole number.
Step 2: Move the decimal point in the dividend to the right the <i>same number of places</i> as the decimal point was moved in Step 1.
Step 3: Divide. The decimal point in the quotient is directly over the moved decimal point in the dividend.</td><td>Divide: $1.118 \div 2.6$
$\begin{array}{r} 0.43 \\ 2.6\overline{)1.118} \\ -104 \\ \hline 78 \\ -78 \\ \hline 0 \end{array}$</td></tr>
<tr><td colspan="2">Section 4.5 Fractions, Decimals, and Order of Operations</td></tr>
<tr><td>To write fractions as decimals, divide the numerator by the denominator.</td><td>Write $\frac{3}{8}$ as a decimal.
$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\,4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$</td></tr>
<tr><td>Order of Operations
1. Perform all operations within parentheses ( ), brackets [ ], or grouping symbols such as fraction bars or square roots, starting with the inner most set.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.</td><td>Simplify.
$-1.9(12.8 - 4.1) = -1.9(8.7)$ Subtract.
$= -16.53$ Multiply.</td></tr>
</table>

Definitions and Concepts	Examples
Section 4.6 Square Roots and the Pythagorean Theorem	
Square Root of a Number A **square root** of a number a is a number b whose square is a. We use the radical sign $\sqrt{}$ to indicate square roots.	$\sqrt{9} = 3,\ \sqrt{100} = 10,\ \sqrt{1} = 1$
Pythagorean Theorem $(\text{leg})^2 + (\text{other leg})^2 = (\text{hypotenuse})^2$  	Find the hypotenuse of the given triangle. 
To Find an Unknown Length of a Right Triangle $\text{hypotenuse} = \sqrt{(\text{leg})^2 + (\text{other leg})^2}$ $\text{leg} = \sqrt{(\text{hypotenuse})^2 - (\text{other leg})^2}$	$\text{hypotenuse} = \sqrt{(\text{leg})^2 + (\text{other leg})^2}$ $= \sqrt{(3)^2 + (8)^2}$ The legs are 3 and 8 inches. $= \sqrt{9 + 64}$ $= \sqrt{73}$ inches ≈ 8.5 inches

Chapter 4 Review

(4.1) *Determine the place value of the digit 4 in each decimal.*

1. 23.45

2. 0.000345

Write each decimal in words.

3. −23.45

4. 0.00345

5. 109.23

6. 200.000032

Write each decimal in standard form.

7. Two and seven hundredths

8. Negative five hundred three and one hundred two thousandths

9. Sixteen thousand twenty-five and fourteen ten-thousandths

10. Fourteen and eleven thousandths

Write each decimal as a fraction or a mixed number.

11. 0.16

12. 0.55

13. −12.023

14. 25.25

Insert <, >, or = between each pair of numbers to make a true statement.

15. 0.49 0.43

16. 0.973 0.9730

17. −402.00032 −402.000032

18. −0.230505 −0.23505

Round each decimal to the given place value.

19. 0.623, nearest tenth

20. 0.9384, nearest hundredth

21. −42.895, nearest hundredth

22. −16.34925, nearest thousandth

(4.2) *Add.*

23. $2.4 + 7.1$

24. $3.9 + 1.2$

25. $-6.4 + (-0.88)$

26. $-19.02 + 6.98$

27. $200.49 + 16.82 + 103.002$

28. $0.00236 + 100.45 + 48.29$

Subtract.

29. $4.9 - 3.2$

30. $5.23 - 2.74$

31. $-892.1 - 432.4$

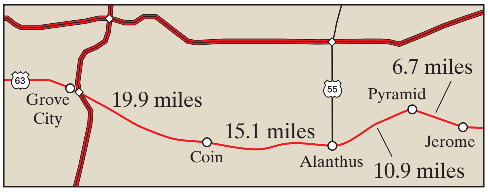
32. $0.064 - 10.2$

33. $100 - 34.98$

34. $200 - 0.00198$

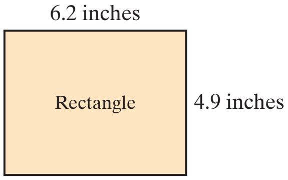
Solve.

35. Find the total distance between Grove City and Jerome.

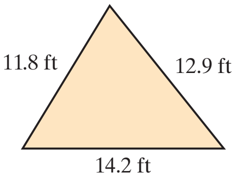


36. Evaluate $x - y$ for $x = 1.2$ and $y = 6.9$.

△ **37.** Find the perimeter.



△ **38.** Find the perimeter.



(4.3) *Multiply.*

39. 7.2×10

40. 9.345×1000

41. -34.02×2.3

42. $-839.02 \times (-87.3)$

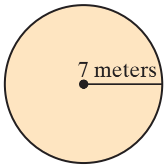
Write each number in standard notation.

43. Saturn is a distance of about 887 million miles from the Sun.

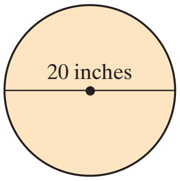
44. The tail of a comet can be over 600 thousand miles long.

Find the exact circumference of each circle. Then use the approximation 3.14 for π and approximate the circumference.

△ **45.**



△ **46.**



(4.4) *Divide. Round the quotient to the nearest thousandth if necessary.*

47. $3\overline{)0.2631}$

48. $20\overline{)316.5}$

49. $-21 \div (-0.3)$

50. $-0.0063 \div 0.03$

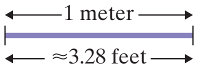
51. $0.34\overline{)2.74}$

52. $19.8\overline{)601.92}$

53. $\frac{23.65}{100}$

54. $\frac{-93}{10}$

55. There are approximately 3.28 feet in 1 meter. Find how many meters are in 24 feet to the nearest tenth of a meter.



56. George Strait pays $69.71 per month to pay back a loan of $3136.95. In how many months will the loan be paid off?

(4.5) *Write each fraction or mixed number as a decimal. Round to the nearest thousandth if necessary.*

57. $\frac{4}{5}$

58. $-\frac{12}{13}$

59. $2\frac{1}{3}$

60. $\frac{13}{60}$

Insert <, >, or = to make a true statement.

61. 0.392 0.39200

62. −0.0231 −0.0221

63. $\frac{4}{7}$ 0.625

64. 0.293 $\frac{5}{17}$

Write the numbers in order from smallest to largest.

65. 0.837, 0.839, 0.832

66. 0.685, 0.626, $\frac{5}{8}$

67. $\frac{3}{7}$, 0.42, 0.43

68. $\frac{18}{11}$, 1.63, $\frac{19}{12}$

Simplify each expression.

69. $-7.6 \times 1.9 + 2.5$

70. $(-2.3)^2 - 1.4$

71. $0.0726 \div 10 \times 1000$

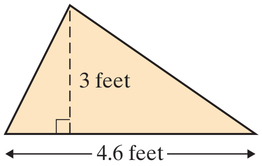
72. $0.6(2 - 0.65)$

73. $\frac{(1.5)^2 + 0.5}{0.05}$

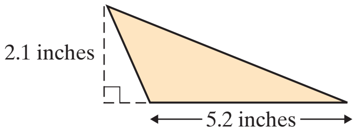
74. $\frac{5 + 2.74}{-0.06}$

Find each area.

△ **75.**



△ **76.**



(4.6) *Simplify.*

77. $\sqrt{64}$

78. $\sqrt{144}$

79. $\sqrt{\frac{4}{25}}$

80. $\sqrt{\frac{1}{100}}$

Find the unknown length of each given right triangle. If necessary, round to the nearest tenth.

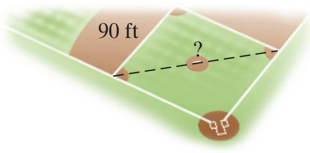
81. leg = 12, leg = 5

82. leg = 20, leg = 21

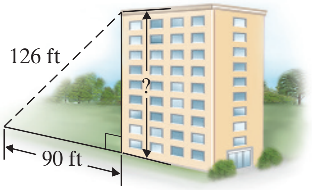
83. leg = 9, hypotenuse = 14

84. leg = 124, hypotenuse = 155

85. A baseball diamond is in the shape of a square and has sides of length 90 feet. Find the distance across the diamond from third base to first base, to the nearest tenth of a foot.



86. Find the height of the building rounded to the nearest tenth.



Mixed Review

87. Write 2.0032 in words.

88. Write negative sixteen and fourteen thousandths in standard form.

89. Write 0.00231 as a fraction or a mixed number.

90. Write the numbers $\frac{6}{7}, \frac{8}{9}$, 0.75 in order from smallest to largest.

Write each fraction as a decimal.

91. $-\frac{7}{100}$

92. $\frac{9}{80}$

Insert <, >, or = to make a true statement.

93. −402.000032 −402.00032

94. $\frac{6}{11}$ 0.55

Round each decimal to the given place value.

95. 42.895, nearest tenth

96. 16.34925, nearest hundredth

Round each money amount to the nearest dollar.

97. \$123.46

98. \$3645.52

Add or subtract as indicated.

99. $3.2 - 4.9$

100. $5.23 - 2.74$

101. $200.49 - 16.82 - 103.002$

102. $-0.00236 + (-100.45) + (-48.29)$

Multiply or divide as indicated. Round to the nearest thousandth, if necessary.

103. $\begin{array}{r} 2.54 \\ \times\ 3.2 \\ \hline \end{array}$

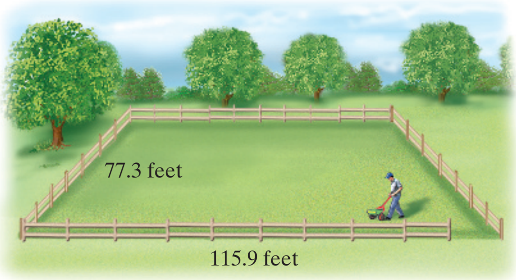
104. $(-3.45)(2.1)$

105. $0.005\overline{)24.5}$

106. $2.3\overline{)54.98}$

Solve.

107. Tomaso is going to fertilize his lawn, a rectangle that measures 77.3 feet by 115.9 feet. Approximate the area of the lawn by rounding each measurement to the nearest ten feet.



108. Estimate the cost of the items to see whether the groceries can be purchased with a \$10 bill.

Simplify each expression.

109. $\dfrac{(3.2)^2}{100}$

110. $(2.6 + 1.4)(4.5 - 3.6)$

Simplify.

111. $\sqrt{1}$

112. $\sqrt{36}$

113. $\sqrt{\dfrac{16}{81}}$

114. $\sqrt{\dfrac{1}{121}}$

Find the unknown length of each given right triangle. If necessary, round to the nearest tenth.

115. leg = 66, leg = 56

116. leg = 12, hypotenuse = 24

117. leg = 17, hypotenuse = 51

118. leg = 10, leg = 17

Chapter 4 Getting Ready for the Test LC

MATCHING Exercises 1 through 12 are **Matching** exercises.

For Exercises 1 through 4, the number 8603.2855 is rounded to different place values. ***Match*** *the rounded number in the left column to the correct place it is rounded to in the columns to the right.*

1. 8603.3

2. 8600

3. 8603.286

4. 8603.29

A. 8603.2855 rounded to ones

B. 8603.2855 rounded to tens

C. 8603.2855 rounded to tenths

D. 8603.2855 rounded to hundredths

E. 8603.2855 rounded to thousandths

For Exercises 5 through 8, ***Match*** *each fraction or mixed number with its equivalent decimal representation in the right column.*

5. $\frac{23}{1000}$

6. $2\frac{3}{10}$

7. $\frac{23}{100}$

8. $2\frac{3}{100}$

A. 2.03

B. 0.023

C. 0.23

D. 2.3

For Exercises 9 through 12, ***Match*** *the multiplication or division with the correct product or quotient in the right column.*

9. 23.6051×100

10. 23.6051×10

11. $\frac{23.6051}{10}$

12. $\frac{23.6051}{1000}$

A. 0.0236051

B. 236.051

C. 2360.51

D. 2.36051

MULTIPLE CHOICE Exercises 13 through 17 are **Multiple Choice**. Choose the correct answer.

13. Find $10 - 0.08$.

A. 2 **B.** 9.2 **C.** 9.02 **D.** 9.92

14. Find $10 + 0.08$.

A. 10.08 **B.** 10.8 **C.** 18 **D.** 10.008

15. Find $37 + 2.1$

A. 58 **B.** 39.1 **C.** 37.21 **D.** 3.91

16. A product of decimal numbers below is completed except for placement of the decimal point in the product. Choose the correct product.

$$\begin{array}{r} 2.326 \\ \times \quad 1.5 \\ \hline 11630 \\ \underline{23260} \\ 34890 \end{array}$$

A. 348.90 **B.** 34.890 **C.** 3.4890 **D.** 3489.0

17. A quotient of decimal numbers below is completed except for placement of the decimal point in the quotient. Choose the correct quotient.

$$\begin{array}{r} 186 \\ 0.38\overline{)7.068} \end{array}$$

A. 0.186 **B.** 1.86 **C.** 18.6 **D.** 186

For additional practice go to your study plan in MyLab Math.

MyLab Math

Test

Chapter 4

Write each decimal as indicated.

1. 45.092, in words

2. Three thousand and fifty-nine thousandths, in standard form

Perform each indicated operation. Round the result to the nearest thousandth if necessary.

3. $2.893 + 4.21 + 10.492$

4. $-47.92 - 3.28$

5. $9.83 - 30.25$

6. 10.2×4.01

7. $(-0.00843) \div (-0.23)$

Round each decimal to the indicated place value.

8. 34.8923, nearest tenth

9. 0.8623, nearest thousandth

Insert $<$, $>$, or $=$ between each pair of numbers to form a true statement.

10. 25.0909 25.9090

11. $\frac{4}{9}$ 0.445

Write each decimal as a fraction or a mixed number.

12. 0.345

13. -24.73

Write each fraction as a decimal. If necessary, round to the nearest thousandth.

14. $-\frac{13}{26}$

15. $\frac{16}{17}$

Simplify.

16. $(-0.6)^2 + 1.57$

17. $\frac{0.23 + 1.63}{-0.3}$

18. Subtract 8.6 from 20.

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. ______

18. ______

19. ________

20. ________

21. ________

22. ________

23. ________

24. ________

25. a. ________

b. ________

26. ________

Find each square root and simplify. Round to the nearest thousandth if necessary.

19. $\sqrt{49}$

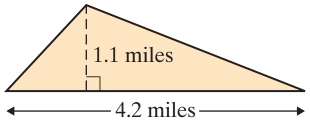
20. $\sqrt{157}$

21. $\sqrt{\frac{64}{100}}$

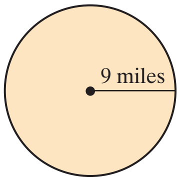
Solve.

22. At its farthest, Pluto is 4583 million miles from the Sun. Write this number using standard notation.

23. Find the area.



24. Find the exact circumference of the circle. Then use the approximation 3.14 for π and approximate the circumference.

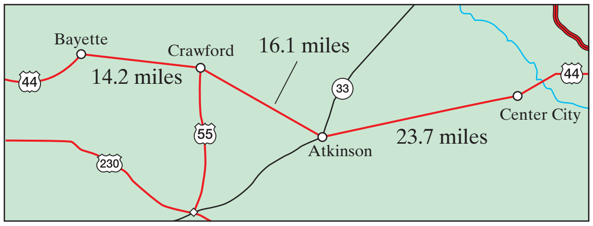


25. A botanist is going to put insecticide on her lawn to control grubworms. The lawn is a rectangle that measures 123.8 feet by 80 feet. The amount of insecticide required is 0.02 ounce per square foot.

a. Find the area of her lawn.

b. Find how much insecticide she needs to purchase.

26. Find the total distance from Bayette to Center City.



Cumulative Review Chapters 1–4

Write each number in words.

1. 85

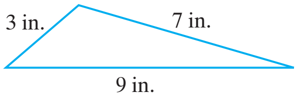
2. 107

3. 126

4. 5026

5. Add: 23 + 136

6. Find the perimeter.



7. Subtract: 543 − 29. Check by adding.

8. Divide: 3268 ÷ 27

9. Round 278,362 to the nearest thousand.

10. Write the prime factorization of 30.

11. Multiply: 236 × 86

12. Multiply: 236 × 86 × 0

13. Find each quotient. Check by multiplying.

a. $1\overline{)7}$

b. 12 ÷ 1

c. $\frac{6}{6}$

d. 9 ÷ 9

e. $\frac{20}{1}$

f. $19\overline{)19}$

14. Find the average of 25, 17, 19, and 39.

15. The Hudson River in New York State is 306 miles long. The Snake River in the northwestern United States is 732 miles longer than the Hudson River. How long is the Snake River? (*Source:* U.S. Department of the Interior)

16. Evaluate: $\sqrt{121}$

Answers

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

11. ____________

12. ____________

13. a. ____________

b. ____________

c. ____________

d. ____________

e. ____________

f. ____________

14. ____________

15. ____________

16. ____________

Evaluate.

17. 9^2 **18.** 5^3 **19.** 3^4 **20.** 10^3

21. Evaluate $\dfrac{x - 5y}{y}$ for $x = 21$ and $y = 3$.

22. Evaluate $\dfrac{2a + 4}{c}$ for $a = 7$ and $c = 3$.

23. Find the opposite of each number.
a. 11 **b.** −2 **c.** 0

24. Find the opposite of each number.
a. −7 **b.** 4 **c.** −1

25. Add: $-2 + (-21)$

26. Add: $-7 + (-15)$

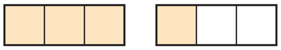
Find the value of each expression.

27. $5 \cdot 6^2$ **28.** $4 \cdot 2^3$ **29.** -7^2

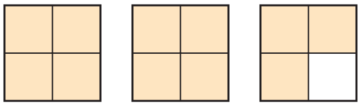
30. $(-2)^5$ **31.** $(-5)^2$ **32.** -3^2

Represent the shaded part as an improper fraction and a mixed number.

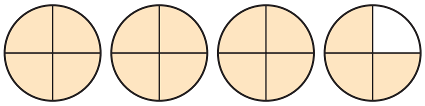
33.



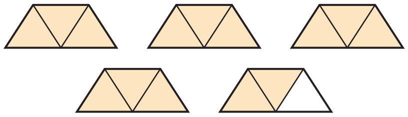
34.



35.



36.



17.
18.
19.
20.
21.
22.
23. a.
b.
c.
24. a.
b.
c.
25.
26.
27.
28.
29.
30.
31.
32.
33.
34.
35.
36.

37. Write the prime factorization of 252.

38. Find the difference of 87 and 25.

39. Write $-\frac{72}{26}$ in simplest form.

40. Write $9\frac{7}{8}$ as an improper fraction.

41. Determine whether $\frac{16}{40}$ and $\frac{10}{25}$ are equivalent.

42. Insert $<$ or $>$ to form a true statement. $\frac{4}{7} \quad \frac{5}{9}$

Multiply.

43. $\frac{2}{3} \cdot \frac{5}{11}$

44. $2\frac{5}{8} \cdot \frac{4}{7}$

45. $\frac{1}{4} \cdot \frac{1}{2}$

46. $7 \cdot 5\frac{2}{7}$

47. Add: $763.7651 + 22.001 + 43.89$

48. Add: $89.27 + 14.361 + 127.2318$

49. Multiply: 23.6×0.78

50. Multiply: 43.8×0.645

37. ____________

38. ____________

39. ____________

40. ____________

41. ____________

42. ____________

43. ____________

44. ____________

45. ____________

46. ____________

47. ____________

48. ____________

49. ____________

50. ____________

5

Ratio, Proportion, and Measurement

Having studied fractions in Chapter 3, we are ready to explore the useful notions of ratio and proportion. Ratio is another name for quotient and can be written in fraction form. A proportion is an equation with two equal ratios. In the second half of this chapter, we study the important U.S. and metric systems of measurement.

Sections

Check Your Progress

One of many 3-D cameras

Note: To date, there continue to be much research and controversy over 3-D films. The main controversy is focused on inferior conversions of 2-D filmed movies to 3-D. The main research is focused on watching 3-D movies with adequate light and glasses or without glasses at all.

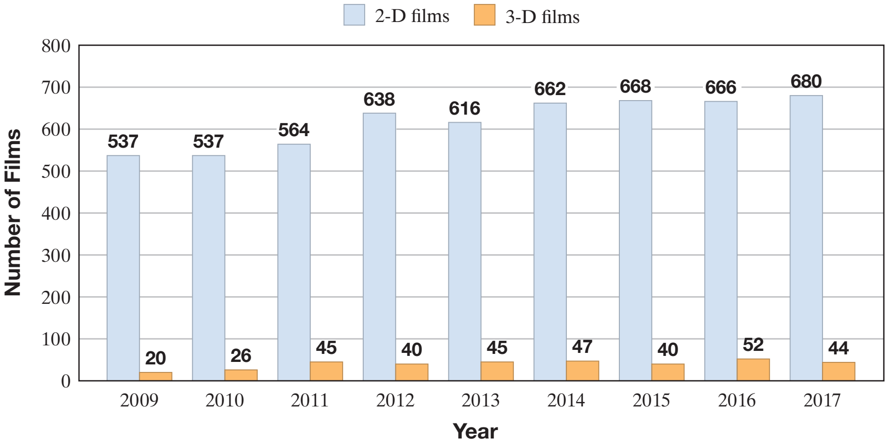
The 2-D Versus 3-D Film Controversy Continues . . .

A 3-D (three-dimensional) film is a film that enhances the illusion of depth perception. Believe it or not, 3-D films have existed in some form since 1890, but because of high cost and lack of a standardized format, these films are only now starting to be widely shown and produced.

By looking at the graph below, we see a decrease in the number of 3-D films released in 2017. Does this mean that more people are viewing them? Recently, opening weekend gross revenue for films in 2-D and 3-D shows that only 30% to 40% of the revenue comes from the 3-D version. The graph shows the trends in the releases of both 2-D and 3-D films in recent years.

In Section 5.1, Exercise 25, we calculate the ratio of 3-D to total films.

Films Released by U.S. Production Companies



Source: Motion Picture Association of America

5.1 Ratios

Objectives

- **A** Write Ratios as Fractions.
- **B** Write Rates as Fractions.
- **C** Find Unit Rates.
- **D** Find Unit Prices.

Objective A Writing Ratios as Fractions

A **ratio** is the quotient of two quantities. A ratio, in fact, is no different from a fraction, except that a ratio is sometimes written using notation other than fractional notation. For example, the ratio of 1 to 2 can be written as

$$1 \text{ to } 2 \quad \text{or} \quad \underset{\substack{\uparrow \\ \text{fractional notation}}}{\frac{1}{2}} \quad \text{or} \quad \underset{\substack{\uparrow \\ \text{colon notation}}}{1:2}$$

These ratios are all read as, "the ratio of 1 to 2."

✓Concept Check How should each ratio be read aloud?

a. $\frac{8}{5}$ **b.** $\frac{5}{8}$

In this section, we write ratios using fractional notation. If the fraction happens to be an improper fraction, do not write the fraction as a mixed number. Why? The mixed number form is not a ratio or quotient of two quantities.

Writing a Ratio as a Fraction

The order of the quantities is important when writing ratios. To write a ratio as a fraction, write the *first number* of the ratio as the *numerator* of the fraction and the *second number* as the *denominator.*

Helpful Hint

The ratio of 6 to 11 is $\frac{6}{11}$, *not* $\frac{11}{6}$.

Example 1 Write the ratio of 12 to 17 using fractional notation.

Solution: The ratio is $\frac{12}{17}$.

Helpful Hint Don't forget that order is important when writing ratios. The ratio $\frac{17}{12}$ is *not* the same as the ratio $\frac{12}{17}$.

Work Practice 1

Practice 1

Write the ratio of 20 to 23 using fractional notation.

To simplify a ratio, we just write the fraction in simplest form. Common factors as well as common units can be divided out.

Example 2 Write the ratio of \$15 to \$10 as a fraction in simplest form.

Solution:

$$\frac{\$15}{\$10} = \frac{15}{10} = \frac{3 \cdot \overset{1}{\cancel{5}}}{2 \cdot \underset{1}{\cancel{5}}} = \frac{3}{2}$$

Work Practice 2

Practice 2

Write the ratio of \$8 to \$6 as a fraction in simplest form.

Answers

1. $\frac{20}{23}$ **2.** $\frac{4}{3}$

✓Concept Check Answers

a. "eight to five" **b.** "five to eight"

Helpful Hint

The ratio answer to Example 2 is $\frac{3}{2}$. Although $\frac{3}{2} = 1\frac{1}{2}$, a ratio is a quotient of *two* quantities. For that reason, ratios are not written as mixed numbers.

If a ratio contains decimal numbers or mixed numbers, we simplify by writing the ratio as a ratio of whole numbers.

Practice 3

Write the ratio of 3.9 to 8.8 as a fraction in simplest form.

Example 3

Write the ratio of 2.6 to 3.1 as a fraction in simplest form.

Solution: The ratio in fraction form is

$$\frac{2.6}{3.1}$$

Now let's clear the ratio of decimals.

$$\frac{2.6}{3.1} = \frac{2.6}{3.1}\cdot 1 = \frac{2.6}{3.1}\cdot\frac{10}{10} = \frac{2.6\cdot 10}{3.1\cdot 10} = \frac{26}{31} \quad \text{Simplest form}$$

Work Practice 3

Practice 4

Write the ratio of $2\frac{2}{3}$ to $1\frac{13}{15}$ as a fraction in simplest form.

Example 4

Write the ratio of $1\frac{1}{5}$ to $2\frac{7}{10}$ as a fraction in simplest form.

Solution: The ratio in fraction form is $\dfrac{1\frac{1}{5}}{2\frac{7}{10}}$.

To simplify, remember that the fraction bar means division.

$$\frac{1\frac{1}{5}}{2\frac{7}{10}} = 1\frac{1}{5} \div 2\frac{7}{10} = \frac{6}{5} \div \frac{27}{10} = \frac{6}{5}\cdot\frac{10}{27} = \frac{6\cdot 10}{5\cdot 27} = \frac{2\cdot\overset{1}{\cancel{3}}\cdot 2\cdot\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\cdot\underset{1}{\cancel{3}}\cdot 3\cdot 3} = \frac{4}{9} \quad \text{Simplest form}$$

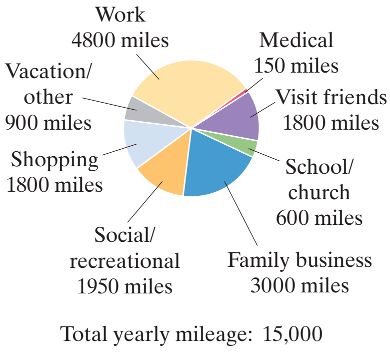
Work Practice 4

Practice 5

Use the circle graph for Example 5 to write the ratio of work miles to total miles as a fraction in simplest form.

Example 5 Writing a Ratio from a Circle Graph

The circle graph at the right shows the part of a car's total mileage that falls into a particular category. Write the ratio of family business miles to total miles as a fraction in simplest form.



Sources: The American Automobile Manufacturers Association and the National Automobile Dealers Association.

Solution:

$$\frac{\text{family business miles}}{\text{total miles}} = \frac{3000 \text{ miles}}{15{,}000 \text{ miles}}$$

$$= \frac{3000}{15{,}000}$$

$$= \frac{\overset{1}{\cancel{3000}}}{5\cdot\underset{1}{\cancel{3000}}}$$

$$= \frac{1}{5}$$

Work Practice 5

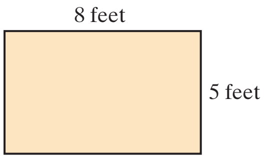
Answers

3. $\frac{39}{88}$ **4.** $\frac{10}{7}$ **5.** $\frac{8}{25}$

Example 6 Given the rectangle shown:

a. Find the ratio of its width to its length.

b. Find the ratio of its length to its perimeter.



Solution:

a. The ratio of its width to its length is

$$\frac{\text{width}}{\text{length}} = \frac{5\ \cancel{\text{feet}}}{8\ \cancel{\text{feet}}} = \frac{5}{8}$$

b. Recall that the perimeter of the rectangle is the distance around the rectangle: $8 + 5 + 8 + 5 = 26$ feet. The ratio of its length to its perimeter is

$$\frac{\text{length}}{\text{perimeter}} = \frac{8\ \cancel{\text{feet}}}{26\ \cancel{\text{feet}}} = \frac{8}{26} = \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\underset{1}{\cancel{2}} \cdot 13} = \frac{4}{13}$$

Work Practice 6

✓**Concept Check** Explain why the answer $\frac{8}{5}$ would be incorrect for part **a** of Example 6.

Objective B Writing Rates as Fractions

A special type of ratio is a rate. **Rates** are used to compare *different* kinds of quantities. For example, suppose that a recreational runner can run 3 miles in 33 minutes. If we write this rate as a fraction, we have

$$\frac{3 \text{ miles}}{33 \text{ minutes}} = \frac{1 \text{ mile}}{11 \text{ minutes}} \quad \text{In simplest form}$$

Helpful Hint

When comparing quantities with different units, write the units as part of the comparison. They do not divide out.

Same Units: $\frac{3\ \cancel{\text{inches}}}{12\ \cancel{\text{inches}}} = \frac{1}{4}$

Different Units: $\frac{2 \text{ miles}}{20 \text{ minutes}} = \frac{1 \text{ mile}}{10 \text{ minutes}}$ Units are still written.

Examples Write each rate as a fraction in simplest form.

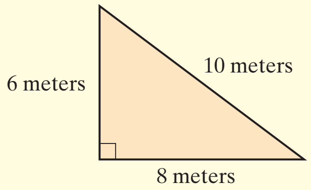
7. \$2160 for 12 weeks is $\frac{2160 \text{ dollars}}{12 \text{ weeks}} = \frac{180 \text{ dollars}}{1 \text{ week}}$

8. 360 miles on 16 gallons of gasoline is $\frac{360 \text{ miles}}{16 \text{ gallons}} = \frac{45 \text{ miles}}{2 \text{ gallons}}$

Work Practice 7–8

Practice 6

Given the triangle shown:



a. Find the ratio of the length of the shortest side to the length of the longest side.

b. Find the ratio of the length of the longest side to the perimeter of the triangle.

Practice 7–8

Write each rate as a fraction in simplest form.

7. \$1680 for 8 weeks

8. 236 miles on 12 gallons of gasoline

Answers

6. a. $\frac{3}{5}$ **b.** $\frac{5}{12}$ **7.** $\frac{\$210}{1 \text{ wk}}$ **8.** $\frac{59 \text{ mi}}{3 \text{ gal}}$

✓**Concept Check Answer**

$\frac{8}{5}$ would be the ratio of the rectangle's length to its width.

✓**Concept Check** True or false? $\frac{16 \text{ gallons}}{4 \text{ gallons}}$ is a rate. Explain.

Objective C Finding Unit Rates

A **unit rate** is a rate with a denominator of 1. A familiar example of a unit rate is 55 mph, read as "55 **miles per hour**." This means 55 miles per 1 hour or

$$\frac{55 \text{ miles}}{1 \text{ hour}}$$ Denominator of 1

Helpful Hint In this context, the word "per" translates to division.

Writing a Rate as a Unit Rate

To write a rate as a unit rate, divide the numerator of the rate by the denominator.

Practice 9

Write as a unit rate: 3200 feet every 8 seconds

Example 9

Write as a unit rate: $31,500 every 7 months

Solution:

$$\frac{31{,}500 \text{ dollars}}{7 \text{ months}} \qquad \begin{array}{r} 4\,500 \\ 7\overline{)31{,}500} \end{array}$$

The unit rate is

$$\frac{4500 \text{ dollars}}{1 \text{ month}} \text{ or } 4500 \text{ dollars/month}$$ Read as, "4500 dollars per month."

Work Practice 9

Practice 10

Write as a unit rate: 78 bushels of fruit from 12 trees

Example 10

Write as a unit rate: 337.5 miles every 15 gallons of gas

Solution:

$$\frac{337.5 \text{ miles}}{15 \text{ gallons}} \qquad \begin{array}{r} 22.5 \\ 15\overline{)337.5} \end{array}$$

The unit rate is

$$\frac{22.5 \text{ miles}}{1 \text{ gallon}} \text{ or } 22.5 \text{ miles/gallon}$$ Read as, "22.5 miles per gallon."

Work Practice 10

Objective D Finding Unit Prices

Rates are used extensively in sports, business, medicine, and science. One of the most common uses of rates is in consumer economics. When a unit rate is "money per item," it is also called a **unit price**.

$$\text{unit price} = \frac{\text{price}}{\text{number of units}}$$

Answers

9. $\frac{400 \text{ ft}}{1 \text{ sec}}$ or 400 ft/sec

10. $\frac{6.5 \text{ bushels}}{1 \text{ tree}}$ or 6.5 bushels/tree

✓**Concept Check Answer**

false; a rate compares different kinds of quantities

Example 11 Finding Unit Price

A store charges \$3.36 for a 16-ounce jar of picante sauce. What is the unit price in dollars per ounce?

Solution:

$$\text{unit price} = \frac{\text{price}}{\text{number of units}} = \frac{\$3.36}{16\text{ ounces}} = \frac{\$0.21}{1\text{ ounce}} \text{ or } \$0.21 \text{ per ounce}$$

Work Practice 11

Example 12 Finding the Best Buy

Approximate each unit price to decide which is the better buy: 4 bars of soap for \$3.99 or 5 bars of soap for \$4.59.

Solution:

$$\text{unit price} = \frac{\text{price}}{\text{no. of units}} = \frac{\$3.99}{4\text{ bars}} \approx \$1.00 \text{ per bar of soap} \qquad \begin{array}{r}0.997\\4\overline{)3.990}\end{array} \approx 1.00 \quad \text{(“is approximately”)}$$

$$\text{unit price} = \frac{\text{price}}{\text{no. of units}} = \frac{\$4.59}{5\text{ bars}} \approx \$0.92 \text{ per bar of soap} \qquad \begin{array}{r}0.918\\5\overline{)4.590}\end{array} \approx 0.92$$

Since the 5-bar package has a cheaper price per bar, it is the better buy.

Work Practice 12

Practice 11

An automobile rental agency charges \$170 for 5 days for a certain model car. What is the unit price in dollars per day?

Practice 12

Approximate each unit price to decide which is the better buy for a bag of nacho chips: 11 ounces for \$3.99 or 16 ounces for \$5.99.

Answers

11. \$34 per day **12.** 11-oz bag

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

rate
numerator
division
different
unit price
denominator
unit
ratio

1. A rate with a denominator of 1 is called a ______________ rate.

2. When a rate is written as money per item, a unit rate is called a ______________.

3. The word *per* translates to ______________.

4. Rates are used to compare ______________ types of quantities.

5. To write a rate as a unit rate, divide the ______________ of the rate by the ______________.

6. The quotient of two quantities is called a ______________.

Answer each statement true or false.

7. The ratio $\frac{7}{5}$ means the same as the ratio $\frac{5}{7}$. ________

8. The ratio $\frac{9}{10}$ is in simplest form. ________

9. The ratio $\frac{7.2}{8.1}$ is in simplest form. ________

10. The ratio $\frac{10 \text{ feet}}{30 \text{ feet}}$ is in simplest form. ________

11. The ratio 30 : 41 equals $\frac{30}{41}$ in fractional notation. ________

12. The ratio 2 to 5 equals $\frac{5}{2}$ in fractional notation. ________

Martin-Gay Interactive Videos

See Video 5.1

Watch the section lecture video and answer the following questions.

Objective A **13.** Based on the lecture before Example 1, what three notations can we use for a ratio? For your answer, use the ratio example given in the lecture.

Objective B **14.** Why can't we divide out the units in Example 6?

Objective C **15.** Why did we divide the first quantity of the rate in Example 8 by the second quantity?

Objective D **16.** From Example 9, unit prices can be especially helpful when?

5.1 Exercise Set MyLab Math

Objective A *Write each ratio as a ratio of whole numbers using fractional notation. Write the fraction in simplest form. See Examples 1 through 4.*

1. 16 to 24

2. 25 to 150

3. 7.7 to 10

4. 8.1 to 10

5. 4.63 to 8.21

6. 9.61 to 7.62

7. 9 inches to 12 inches

8. 14 centimeters to 20 centimeters

9. \$32 to \$100

10. \$46 to \$102

11. 24 days to 14 days

12. 80 miles to 120 miles

13. $3\frac{1}{2}$ to $12\frac{1}{4}$

14. $3\frac{1}{3}$ to $4\frac{1}{6}$

15. $7\frac{3}{5}$ hours to $1\frac{9}{10}$ hours

16. $25\frac{1}{2}$ days to $2\frac{5}{6}$ days

Write the ratio described in each exercise as a fraction in simplest form. See Examples 5 and 6.

17. **Average Weight of Mature Whales**

Blue Whale	Fin Whale
145 tons	50 tons

Use the table to find the ratio of the weight of an average mature fin whale to the weight of an average mature blue whale.

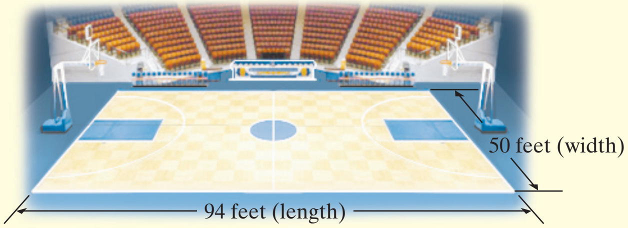
18. **Countries with Small Land Areas**

Tuvalu	San Marino
10 sq mi	24 sq mi

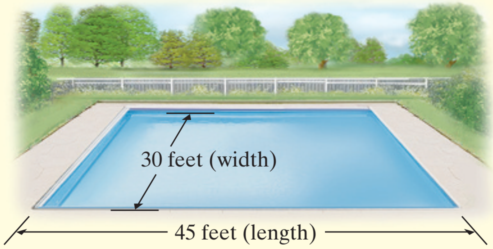
(*Source: World Almanac*)

Use the table to find the ratio of the land area of Tuvalu to the land area of San Marino.

△ **19.** Find the ratio of the width of a regulation size basketball court to its perimeter.



△ **20.** Find the ratio of the width to the perimeter shown of the swimming pool.



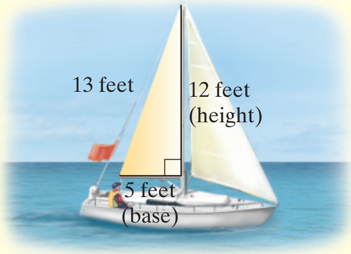
At the Hidalgo County School Board meeting one night, there were 125 women and 100 men present.

21. Find the ratio of women to men.

22. Find the ratio of men to the total number of people present.

△ **23.** Find the ratio of the longest side to the perimeter of the right-triangular-shaped billboard.

△ **24.** Find the ratio of the base to the perimeter of the triangular mainsail.



In 2017, 724 films by U.S. production companies were released. Use this information for Exercise 25.

25. In 2017, 724 total films were released. Of these, 44 were released in 3-D form. Find the ratio of 3-D films to total films. (*Source:* Motion Picture Association of America)

26. In 2016, 718 total films were released. Of these, 139 were released by production companies who were members of the Motion Picture Association of America (MPAA). Find the ratio of MPAA films to total films. (*Source:* Motion Picture Association of America)

27. Of the U.S. mountains that are over 14,000 feet in elevation, 57 are located in Colorado and 19 are located in Alaska. Find the ratio of the number of mountains over 14,000 feet found in Alaska to the number of mountains over 14,000 feet found in Colorado. (*Source:* U.S. Geological Survey)

28. Citizens of the United States eat an average of 25 pints of ice cream per year. Residents of the New England states eat an average of 39 pints of ice cream per year. Find the ratio of the amount of ice cream eaten by New Englanders to the amount eaten by the average U.S. citizen. (*Source:* International Dairy Foods Association)

Blood contains three types of cells: red blood cells, white blood cells, and platelets. For approximately every 600 red blood cells in healthy humans, there are 40 platelets and 1 white blood cell. Use this information for Exercises 29 and 30. (Source: American Red Cross Biomedical Services)

29. Write the ratio of red blood cells to platelet cells.

30. Write the ratio of white blood cells to red blood cells.

Objective B *Write each rate as a fraction in simplest form. See Examples 7 and 8.*

31. 5 shrubs every 15 feet

32. 14 lab tables for 28 students

33. 15 returns for 100 sales

34. 150 graduate students for 8 advisors

35. 8 phone lines for 36 employees

36. 6 laser printers for 28 computers

37. 18 gallons of pesticide for 4 acres of crops

38. 4 inches of rain in 18 hours

Objective C *Write each rate as a unit rate. See Examples 9 and 10.*

39. 375 riders in 5 subway cars

40. 275 miles in 11 hours

41. A hummingbird moves its wings at a rate of 5400 wingbeats a minute. Write this rate in wingbeats per second.

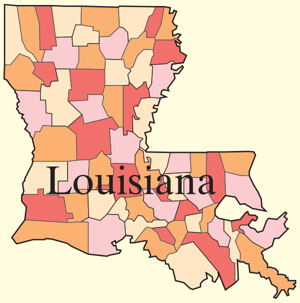
42. A bat moves its wings at a rate of 1200 wingbeats a minute. Write this rate in wingbeats per second.

43. $1,000,000 lottery winnings paid over 20 years

44. 400,000 library books for 8000 students

45. In 2018, the state of Delaware had 687,532 registered voters for two senators. (*Source:* Delaware.gov)

46. The 2020 projected population of Louisiana is approximately 4,758,720 residents for 64 parishes. (*Note:* Louisiana is the only U.S. state with parishes instead of counties.) (*Source:* Louisiana.gov)



47. 12,000 good assembly-line products to 40 defective products

48. 5,000,000 lottery tickets for 4 lottery winners

49. For fiscal year 2017, the Smithsonian National Zoological Park in Washington, D.C., requested an annual budget of roughly $26,000,000 for its 400 different species. (*Source:* Smithsonian Institution)

50. On average in 2016, business class passengers spent $2500 to travel internationally 5000 miles by plane. (*Source:* Airlines for America)

51. During the 2017 Boston Maranthon, the 32 charities certified to raise money by running the Boston Marathon brought in $264 million. (*Source:* WalletHub.com)

52. The top-grossing concert tour of all time was the 2009–2011 U2 360° Tour, which grossed approximately $737 million for 110 shows worldwide. (*Source:* Billboard)

53. Charlie Catlett can assemble 250 computer boards in an 8-hour shift while Suellen Catlett can assemble 402 computer boards in a 12-hour shift.

a. Find the unit rate of Charlie.

b. Find the unit rate of Suellen.

c. Who can assemble computer boards faster, Charlie or Suellen?

54. Jerry Stein laid 713 bricks in 46 minutes while his associate, Bobby Burns, laid 396 bricks in 30 minutes.

a. Find the unit rate of Jerry.

b. Find the unit rate of Bobby.

c. Who is the faster bricklayer?

For Exercises 55 and 56, round the rates to the nearest tenth.

55. One student drove 400 miles in his car on 14.5 gallons of gasoline. His sister drove 270 miles in her truck on 9.25 gallons of gasoline.

a. Find the unit rate of the car.

b. Find the unit rate of the truck.

c. Which vehicle gets better gas mileage?

56. Charlotte Leal is a grocery scanner who can scan an average of 100 items in 3.5 minutes while her cousin Leo can scan 148 items in 5.5 minutes.

a. Find the unit rate of Charlotte.

b. Find the unit rate of Leo.

c. Who is the faster scanner?

Objective D *Find each unit price. See Example 11.*

57. $57.50 for 5 DVDs

58. $0.87 for 3 apples

59. $1.19 for 7 bananas

60. $73.50 for 6 lawn chairs

Find each unit price and decide which is the better buy. Round to three decimal places. Assume that we are comparing different sizes of the same brand. See Examples 11 and 12.

61. Crackers:
$1.19 for 8 ounces
$1.59 for 12 ounces

62. Pickles:
$1.89 for 32 ounces
$0.89 for 18 ounces

63. Frozen orange juice:
$1.69 for 16 ounces
$0.69 for 6 ounces

64. Eggs:
$0.69 for a dozen
$2.10 for a flat $\left(2\frac{1}{2}\text{dozen}\right)$

65. Soy sauce:
12 ounces for $2.29
8 ounces for $1.49

66. Shampoo:
20 ounces for $1.89
32 ounces for $3.19

67. Napkins:
100 for $0.59
180 for $0.93

68. Crackers:
20 ounces for $2.39
8 ounces for $0.99

Review

Divide. See Section 4.4.

69. $9\overline{)20.7}$

70. $7\overline{)60.2}$

71. $3.7\overline{)0.555}$

72. $4.6\overline{)1.15}$

Concept Extensions

Solve.

73. Is the ratio $\frac{11}{15}$ the same as the ratio $\frac{15}{11}$? Explain your answer.

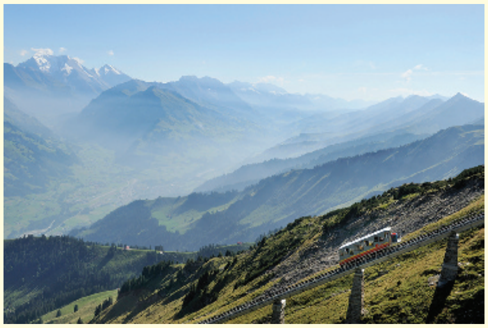
74. Explain why the ratio $\frac{40}{17}$ is incorrect for Exercise **23**.

Fill in the table to calculate miles per gallon.

	Beginning Odometer Reading	Ending Odometer Reading	Miles Driven	Gallons of Gas Used	Miles per Gallon (round to the nearest tenth)
75.	29,286	29,543		13.4	
76.	16,543	16,895		15.8	
77.	79,895	80,242		16.1	
78.	31,623	32,056		11.9	

For Exercises 79 and 80, find each unit rate.

79. The longest stairway is the service stairway for the Niesenbahn Cable railway near Spiez, Switzerland. It has 11,674 steps and rises to a height of 7759 feet. Find the unit rate of steps per foot rounded to the nearest tenth of a step. (*Source: Guinness World Records*)



80. In the United States in 2016, the total number of students enrolled in public schools was 50,400,000. There were 98,300 public schools. Write a unit rate in students per school. Round to the nearest whole. (*Source:* National Center for Education Statistics)

81. In your own words, define the phrase “unit rate.”

82. In your own words, define the phrase “unit price.”

83. Should the rate $\frac{3 \text{ lights}}{2 \text{ feet}}$ be written as $\frac{3}{2}$? Explain why or why not.

84. Find an item in the grocery store and calculate its unit price.

Decide whether each value is a ratio written as a fraction in simplest form. If not, write it as a fraction in simplest form.

85. $\frac{7.1}{4.3}$

86. $\frac{1 \text{ foot}}{30 \text{ inches}}$

87. $4\frac{1}{2}$

88. $\frac{12 \text{ inches}}{2 \text{ feet}}$

Solve.

89. A grocer will refuse a shipment of tomatoes if the ratio of bruised tomatoes to the total batch is at least 1 to 10. A sample is found to contain 3 bruised tomatoes and 33 good tomatoes. Determine whether the shipment should be refused.

90. A panty hose manufacturing machine will be repaired if the ratio of defective panty hose to good panty hose is at least 1 to 20. A quality control engineer found 10 defective panty hose in a batch of 200. Determine whether the machine should be repaired.

91. In 2017, 14 states had primary laws prohibiting all drivers from using a handheld cell phone while driving. These laws allow law enforcement officers to ticket a driver for using a handheld cell phone, even if no other traffic offense has occurred. (*Source:* Governors Highway Safety Association)

a. Find the ratio of states with primary handheld cell phone laws to total U.S. states.

b. Find the number of states with no primary law prohibiting handheld cell phone use while driving.

c. Find the ratio of states with primary handheld cell phone laws to states without such laws.

5.2 Proportions

Objectives

A Write Sentences as Proportions.

B Determine Whether Proportions Are True.

C Find an Unknown Number in a Proportion.

Objective A Writing Proportions

A **proportion** is a statement that two ratios or rates are equal. For example,

$$\frac{5}{6} = \frac{10}{12}$$

is a proportion. We can read this as, "5 is to 6 as 10 is to 12."

Practice 1

Write each sentence as a proportion.

a. 24 right is to 6 wrong as 4 right is to 1 wrong.

b. 32 Cubs fans is to 18 Mets fans as 16 Cubs fans is to 9 Mets fans.

Example 1 Write each sentence as a proportion.

a. 12 diamonds is to 15 rubies as 4 diamonds is to 5 rubies.

b. 5 hits is to 9 at bats as 20 hits is to 36 at bats.

Solution:

a. $\begin{matrix}\text{diamonds} \rightarrow \\ \text{rubies} \rightarrow\end{matrix} \quad \frac{12}{15} = \frac{4}{5} \quad \begin{matrix}\leftarrow \text{diamonds} \\ \leftarrow \text{rubies}\end{matrix}$

b. $\begin{matrix}\text{hits} \rightarrow \\ \text{at bats} \rightarrow\end{matrix} \quad \frac{5}{9} = \frac{20}{36} \quad \begin{matrix}\leftarrow \text{hits} \\ \leftarrow \text{at bats}\end{matrix}$

Work Practice 1

Helpful Hint

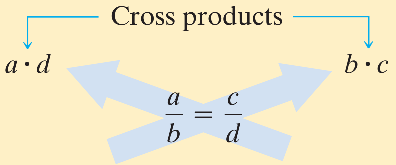
Notice in the above examples of proportions that the numerators contain the same units and the denominators contain the same units. In this text, proportions will be written so that this is the case.

Objective B Determining Whether Proportions Are True

Like other mathematical statements, a proportion may be either true or false. A proportion is true if its ratios are equal. Since ratios are fractions, one way to determine whether a proportion is true is to write both fractions in simplest form and compare them.

Another way is to compare cross products as we did in Section 3.2.

Using Cross Products to Determine Whether Proportions Are True or False



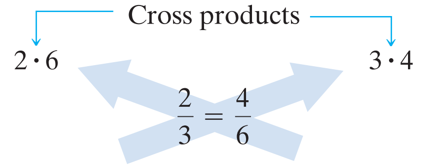
If cross products are *equal,* the proportion is *true*.
If cross products are *not equal,* the proportion is *false*.

Answers

1. a. $\frac{24}{6} = \frac{4}{1}$ **b.** $\frac{32}{18} = \frac{16}{9}$

Example 2 Is $\frac{2}{3} = \frac{4}{6}$ a true proportion?

Solution:



$2 \cdot 6 \stackrel{?}{=} 3 \cdot 4$ Are cross products equal?

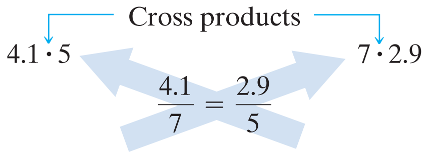
$12 = 12$ Equal, so proportion is true.

Since the cross products are equal, the proportion is true.

Work Practice 2

Example 3 Is $\frac{4.1}{7} = \frac{2.9}{5}$ a true proportion?

Solution:



$4.1 \cdot 5 \stackrel{?}{=} 7 \cdot 2.9$ Are cross products equal?

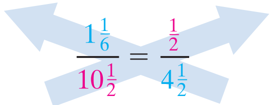
$20.5 \neq 20.3$ Not equal, so proportion is false.

Since the cross products are not equal, $\frac{4.1}{7} \neq \frac{2.9}{5}$. The proportion is false.

Work Practice 3

Example 4 Is $\dfrac{1\frac{1}{6}}{10\frac{1}{2}} = \dfrac{\frac{1}{2}}{4\frac{1}{2}}$ a true proportion?

Solution:



$1\frac{1}{6} \cdot 4\frac{1}{2} \stackrel{?}{=} 10\frac{1}{2} \cdot \frac{1}{2}$ Are cross products equal?

$\frac{7}{6} \cdot \frac{9}{2} \stackrel{?}{=} \frac{21}{2} \cdot \frac{1}{2}$ Write mixed numbers as improper fractions.

$\frac{21}{4} = \frac{21}{4}$ Equal, so proportion is true.

Since the cross products are equal, the proportion is true.

Work Practice 4

Practice 2

Is $\frac{3}{6} = \frac{4}{8}$ a true proportion?

Practice 3

Is $\frac{3.6}{6} = \frac{5.4}{8}$ a true proportion?

Practice 4

Is $\dfrac{4\frac{1}{5}}{2\frac{1}{3}} = \dfrac{3\frac{3}{10}}{1\frac{5}{6}}$ a true proportion?

Answers

2. yes **3.** no **4.** yes

✓Concept Check Think about cross products and write the true proportion $\frac{5}{8} = \frac{10}{16}$ in two other ways so that each result is also a true proportion. (*Note:* There are no units attached in this proportion.)

Objective C Finding Unknown Numbers in Proportions

When one number of a proportion is unknown, we can use cross products to find the unknown number. For example, to find the unknown number n in the proportion $\frac{n}{30} = \frac{2}{3}$, we first find the cross products.

$$n \cdot 3 \qquad \frac{n}{30} = \frac{2}{3} \qquad 30 \cdot 2$$ Find the cross products.

If the proportion is true, then the cross products are equal.

$n \cdot 3 = 30 \cdot 2$ Set the cross products equal to each other.

$n \cdot 3 = 60$ Write $2 \cdot 30$ as 60.

To find the unknown number n, we ask ourselves, "What number times 3 is 60?" The number is 20 and can be found by dividing 60 by 3.

$n = \frac{60}{3}$ Divide 60 by the number multiplied by n.

$n = 20$ Simplify.

Thus, the unknown number is 20.

Check: To *check*, let's replace n with this value, 20, and verify that a true proportion results.

$\frac{20}{30} \stackrel{?}{=} \frac{2}{3}$ ← Replace n with 20.

$$\frac{20}{30} \stackrel{?}{=} \frac{2}{3}$$

$$3 \cdot 20 \stackrel{?}{=} 2 \cdot 30$$

$60 = 60$ Cross products are equal.

Finding an Unknown Value *n* in a Proportion

Step 1: Set the cross products equal to each other.

Step 2: Divide the number not multiplied by n by the number multiplied by n.

✓Concept Check Answer

possible answers: $\frac{8}{5} = \frac{16}{10}$; $\frac{5}{10} = \frac{8}{16}$; $\frac{10}{5} = \frac{16}{8}$

Example 5 Find the unknown number n.

$$\frac{7}{n} = \frac{6}{5}$$

Solution:

Step 1:

$$\frac{7}{n} = \frac{6}{5}$$

$7 \cdot 5 = n \cdot 6$ Set the cross products equal to each other.

$35 = n \cdot 6$ Multiply.

Step 2:

$\frac{35}{6} = n$ Divide 35 by 6, the number multiplied by n.

$5\frac{5}{6} = n$

Check: Check to see that $5\frac{5}{6}$ is the unknown number.

Work Practice 5

Practice 5

Find the unknown number n.

$$\frac{8}{n} = \frac{5}{9}$$

Example 6 Find the value of the unknown number n.

$$\frac{51}{-34} = \frac{3}{n}$$

Solution:

Step 1:

$$\frac{51}{-34} = \frac{3}{n}$$

$51 \cdot n = -34 \cdot 3$ Set cross products equal.

$51 \cdot n = -102$ Multiply.

Step 2:

$n = \frac{-102}{51}$ Divide -102 by 51, the number multiplied by n.

$n = -2$ Simplify.

Check: $\frac{51}{-34} \stackrel{?}{=} \frac{3}{-2}$ Replace n with its value, -2.

$$\frac{51}{-34} \stackrel{?}{=} \frac{3}{-2}$$

$51 \cdot -2 \stackrel{?}{=} -34 \cdot 3$

$-102 = -102$ Cross products are equal, so the proportion is true.

Since the proportion is true, the unknown number, n, is -2.

Work Practice 6

Practice 6

Find the value of the unknown number n.

$$\frac{15}{-2} = \frac{60}{n}$$

Answers

5. $n = 14\frac{2}{5}$ **6.** $n = -8$

Practice 7

Find the unknown number n.

$$\frac{n}{6} = \frac{0.7}{1.2}$$

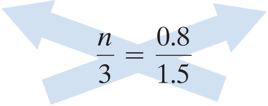
Example 7

Find the unknown number n.

$$\frac{n}{3} = \frac{0.8}{1.5}$$

Solution:

Step 1:



$$\frac{n}{3} = \frac{0.8}{1.5}$$

$n \cdot 1.5 = 3 \cdot 0.8$ Set the cross products equal to each other.

$n \cdot 1.5 = 2.4$ Multiply.

Step 2:

$n = \frac{2.4}{1.5}$ Divide 2.4 by 1.5, the number multiplied by n.

$n = 1.6$ Simplify.

Check: Check to see that 1.6 is the unknown number.

Work Practice 7

Practice 8

Find the unknown number n.

$$\frac{n}{4\frac{1}{3}} = \frac{4\frac{1}{2}}{1\frac{3}{4}}$$

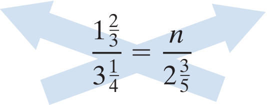
Example 8

Find the unknown number n.

$$\frac{1\frac{2}{3}}{3\frac{1}{4}} = \frac{n}{2\frac{3}{5}}$$

Solution:

Step 1:



$$\frac{1\frac{2}{3}}{3\frac{1}{4}} = \frac{n}{2\frac{3}{5}}$$

$1\frac{2}{3} \cdot 2\frac{3}{5} = 3\frac{1}{4} \cdot n$ Set the cross products equal to each other.

$\frac{13}{3} = 3\frac{1}{4} \cdot n$ Multiply. $1\frac{2}{3} \cdot 2\frac{3}{5} = \frac{5}{3} \cdot \frac{13}{5} = \frac{\overset{1}{\cancel{5}} \cdot 13}{3 \cdot \underset{1}{\cancel{5}}} = \frac{13}{3}$

$\frac{13}{3} = \frac{13}{4} \cdot n$ Write $3\frac{1}{4}$ as $\frac{13}{4}$.

Step 2:

$\frac{13}{3} \div \frac{13}{4} = n$ Divide $\frac{13}{3}$ by $\frac{13}{4}$, the number multiplied by n.

$n = \frac{13}{3} \cdot \frac{4}{13} = \frac{4}{3}$ or $1\frac{1}{3}$ Divide by multiplying by the reciprocal.

Check: Check to see that $1\frac{1}{3}$ is the unknown number.

Work Practice 8

Answers

7. $n = 3.5$ **8.** $n = 11\frac{1}{7}$

Vocabulary, Readiness & Video Check

Use the words and phrases below to fill in each blank.

ratio cross products true
false proportion

1. $\frac{4.2}{8.4} = \frac{1}{2}$ is called a ____________ while $\frac{7}{8}$ is called a(n) ____________.

2. In $\frac{a}{b} = \frac{c}{d}$, $a \cdot d$ and $b \cdot c$ are called ____________.

3. In a proportion, if cross products are equal, the proportion is ____________.

4. In a proportion, if cross products are not equal, the proportion is ____________.

Use cross products and mentally determine whether each proportion is true or false.

5. $\frac{2}{1} = \frac{6}{3}$ **6.** $\frac{3}{1} = \frac{15}{5}$ **7.** $\frac{1}{2} = \frac{3}{5}$ **8.** $\frac{2}{11} = \frac{1}{5}$ **9.** $\frac{2}{3} = \frac{40}{60}$ **10.** $\frac{3}{4} = \frac{6}{8}$

Martin-Gay Interactive Videos

See Video 5.2

Watch the section lecture video and answer the following questions.

Objective A **11.** From Example 1, what does "as" translate to in a proportion statement?

Objective B **12.** In Example 2, what are the cross products of the proportion? Is the proportion true or false?

Objective C **13.** As briefly mentioned in Example 4, what's another word for the unknown value n?

5.2 Exercise Set MyLab Math

Objective A *Write each sentence as a proportion. See Example 1.*

1. 10 diamonds is to 6 opals as 5 diamonds is to 3 opals.

2. 8 books is to 6 courses as 4 books is to 3 courses.

3. 3 printers is to 12 computers as 1 printer is to 4 computers.

4. 4 hit songs is to 16 releases as 1 hit song is to 4 releases.

5. 6 eagles is to 58 sparrows as 3 eagles is to 29 sparrows.

6. 12 errors is to 8 pages as 1.5 errors is to 1 page.

7. $2\frac{1}{4}$ cups of flour is to 24 cookies as $6\frac{3}{4}$ cups of flour is to 72 cookies.

8. $1\frac{1}{2}$ cups milk is to 10 bagels as $\frac{3}{4}$ cup milk is to 5 bagels.

9. 22 vanilla wafers is to 1 cup of cookie crumbs as 55 vanilla wafers is to 2.5 cups of cookie crumbs. (*Source:* Based on data from *Family Circle* magazine)

10. 1 cup of instant rice is to 1.5 cups cooked rice as 1.5 cups of instant rice is to 2.25 cups of cooked rice. (*Source:* Based on data from *Family Circle* magazine)

Objective B *Determine whether each proportion is a true proportion. See Examples 2 through 4.*

11. $\frac{15}{9} = \frac{5}{3}$

12. $\frac{8}{6} = \frac{20}{15}$

13. $\frac{8}{6} = \frac{9}{7}$

14. $\frac{7}{12} = \frac{4}{7}$

15. $\frac{9}{36} = \frac{2}{8}$

16. $\frac{8}{24} = \frac{3}{9}$

17. $\frac{5}{8} = \frac{625}{1000}$

18. $\frac{30}{50} = \frac{600}{1000}$

19. $\frac{0.8}{0.3} = \frac{0.2}{0.6}$

20. $\frac{0.7}{0.4} = \frac{0.3}{0.1}$

21. $\frac{8}{10} = \frac{5.6}{0.7}$

22. $\frac{4.2}{8.4} = \frac{5}{10}$

23. $\dfrac{\frac{3}{4}}{\frac{4}{3}} = \dfrac{\frac{1}{2}}{\frac{8}{9}}$

24. $\dfrac{\frac{2}{5}}{\frac{2}{7}} = \dfrac{\frac{1}{10}}{\frac{1}{3}}$

25. $\dfrac{2\frac{2}{5}}{\frac{2}{3}} = \dfrac{1\frac{1}{9}}{\frac{1}{4}}$

26. $\dfrac{5\frac{5}{8}}{\frac{5}{3}} = \dfrac{4\frac{1}{2}}{1\frac{1}{5}}$

27. $\dfrac{\frac{4}{5}}{6} = \dfrac{\frac{6}{5}}{9}$

28. $\dfrac{\frac{6}{7}}{3} = \dfrac{\frac{10}{7}}{5}$

Objectives A B Mixed Practice—Translating *Write each sentence as a proportion. Then determine whether the proportion is a true proportion. See Examples 1 through 4.*

29. Eight is to twelve as four is to six.

30. Six is to eight as nine is to twelve.

31. Five is to two as thirteen is to five.

32. Four is to three as seven is to five.

33. One and eight tenths is to two as four and five tenths is to five.

34. Fifteen hundredths is to three as thirty-five hundredths is to seven.

35. Two thirds is to one fifth as two fifths is to one ninth.

36. Ten elevenths is to three fourths as one fourth is to one half.

Objective C *For each proportion, find the unknown number n. See Examples 5 through 8.*

37. $\frac{n}{5} = \frac{6}{10}$

38. $\frac{n}{3} = \frac{12}{9}$

39. $\frac{-18}{54} = \frac{3}{n}$

40. $\frac{-25}{100} = \frac{7}{n}$

41. $\frac{n}{8} = \frac{50}{100}$

42. $\frac{n}{21} = \frac{12}{18}$

43. $\frac{8}{15} = \frac{n}{6}$

44. $\frac{12}{10} = \frac{n}{16}$

45. $\frac{24}{n} = \frac{60}{96}$

46. $\frac{26}{n} = \frac{28}{49}$

47. $\frac{3.5}{12.5} = \frac{7}{n}$

48. $\frac{0.2}{0.7} = \frac{8}{n}$

49. $\frac{0.05}{12} = \frac{n}{0.6}$

50. $\frac{7.8}{13} = \frac{n}{2.6}$

51. $\frac{8}{\frac{1}{3}} = \frac{24}{n}$

52. $\frac{12}{\frac{3}{4}} = \frac{48}{n}$

53. $\frac{\frac{1}{3}}{\frac{3}{8}} = \frac{\frac{2}{5}}{n}$

54. $\frac{\frac{7}{9}}{\frac{8}{27}} = \frac{\frac{1}{4}}{n}$

55. $\frac{12}{n} = \frac{\frac{2}{3}}{\frac{13}{18}}$

56. $\frac{24}{n} = \frac{\frac{8}{15}}{\frac{5}{9}}$

57. $\frac{n}{1\frac{1}{5}} = \frac{4\frac{1}{6}}{6\frac{2}{3}}$

58. $\frac{n}{3\frac{1}{8}} = \frac{7\frac{3}{5}}{2\frac{3}{8}}$

59. $\frac{25}{n} = \frac{3}{\frac{7}{30}}$

60. $\frac{9}{n} = \frac{5}{\frac{11}{15}}$

Review

Insert $<$ or $>$ to form a true statement. See Sections 3.7 and 4.1.

61. 8.01 8.1

62. 7.26 7.026

63. $2\frac{1}{2}$ $2\frac{1}{3}$

64. $9\frac{1}{5}$ $9\frac{1}{4}$

65. $5\frac{1}{3}$ $6\frac{2}{3}$

66. $1\frac{1}{2}$ $2\frac{1}{2}$

Concept Extensions

Think about cross products and write each proportion in two other ways so that each result is also a true proportion. See the Concept Check in this section.

67. $\frac{9}{15} = \frac{3}{5}$

68. $\frac{1}{4} = \frac{5}{20}$

69. $\frac{6}{18} = \frac{1}{3}$

70. $\frac{2}{7} = \frac{4}{14}$

Solve.

71. If the proportion $\frac{a}{b} = \frac{c}{d}$ is a true proportion, write two other true proportions using the same letters.

72. Write a true proportion.

73. Explain the difference between a ratio and a proportion.

74. Explain how to find the unknown number in a proportion such as $\frac{n}{18} = \frac{12}{8}$.

For each proportion, find the unknown number n. For Exercises 75 through 80, round your answer to the given place value.

75. $\frac{3.2}{0.3} = \frac{n}{1.4}$
Round to the nearest tenth.

76. $\frac{1.8}{n} = \frac{2.5}{8.4}$
Round to the nearest tenth.

77. $\frac{n}{5.2} = \frac{0.08}{6}$
Round to the nearest hundredth.

78. $\frac{4.25}{6.03} = \frac{5}{n}$
Round to the nearest hundredth.

79. $\frac{43}{17} = \frac{8}{n}$
Round to the nearest thousandth.

80. $\frac{n}{12} = \frac{18}{7}$
Round to the nearest thousandth.

81. $\frac{n}{7} = \frac{0}{8}$

82. $\frac{0}{2} = \frac{n}{3.5}$

83. $\frac{n}{1150} = \frac{588}{483}$

84. $\frac{585}{n} = \frac{117}{474}$

85. $\frac{222}{1515} = \frac{37}{n}$

86. $\frac{1425}{1062} = \frac{n}{177}$

5.3 Proportions and Problem Solving

Objective

A Solve Problems by Writing Proportions.

Objective A Solving Problems by Writing Proportions

Writing proportions is a powerful tool for solving problems in almost every field, including business, chemistry, biology, health sciences, and engineering, as well as in daily life. Given a specified ratio (or rate) of two quantities, a proportion can be used to determine an unknown quantity.

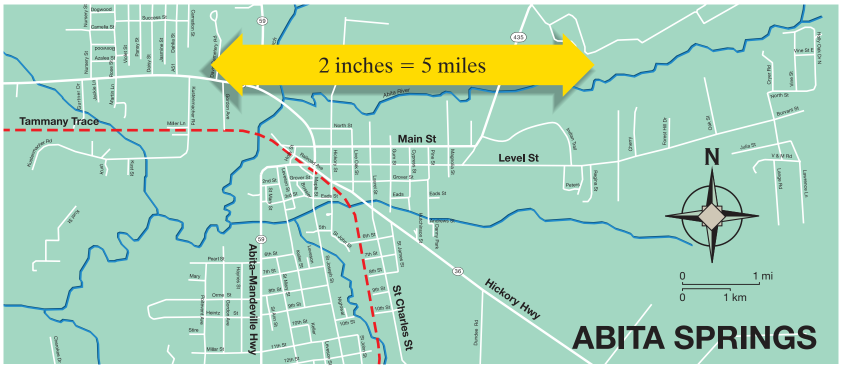
In this section, we use the same problem-solving steps that we have used earlier in this text.

Practice 1

On an architect's blueprint, 1 inch corresponds to 4 feet. How long is a wall represented by a $4\frac{1}{4}$-inch line on the blueprint?

Example 1 Determining Distances from a Map

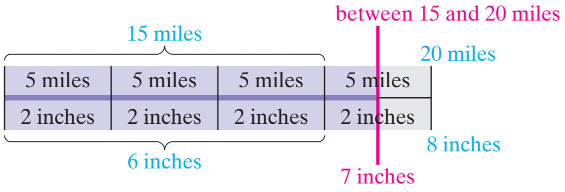
On a chamber of commerce map of Abita Springs, 5 miles corresponds to 2 inches. How many miles correspond to 7 inches?



Answer
1. 17 ft

Solution:

1. UNDERSTAND. Read and reread the problem. You may want to draw a diagram.



From the diagram we can see that a reasonable solution should be between 15 and 20 miles.

2. TRANSLATE. We will let n represent our unknown number. Since 5 miles corresponds to 2 inches as n miles corresponds to 7 inches, we have the proportion

$$\begin{array}{lcc}\text{miles} & \rightarrow & \dfrac{5}{2} = \dfrac{n}{7} & \leftarrow & \text{miles}\\ \text{inches} & \rightarrow & & \leftarrow & \text{inches}\end{array}$$

3. SOLVE: In earlier sections, we estimated to obtain a reasonable answer. Notice we did this in Step 1 above.

$$\frac{5}{2} = \frac{n}{7}$$

$$\begin{aligned} 5 \cdot 7 &= 2 \cdot n && \text{Set the cross products equal to each other.}\\ 35 &= 2 \cdot n && \text{Multiply.}\\ \frac{35}{2} &= n && \text{Divide 35 by 2, the number multiplied by } n.\\ n &= 17\frac{1}{2} \text{ or } 17.5 && \text{Simplify.} \end{aligned}$$

4. INTERPRET. *Check* your work. This result is reasonable since it is between 15 and 20 miles. *State* your conclusion: 7 inches corresponds to 17.5 miles.

Work Practice 1

Helpful Hint

We can also solve Example 1 by writing the proportion

$$\frac{2 \text{ inches}}{5 \text{ miles}} = \frac{7 \text{ inches}}{n \text{ miles}}$$

Although other proportions may be used to solve Example 1, we will solve by writing proportions so that the numerators have the same unit measures and the denominators have the same unit measures.

Example 2 Finding Medicine Dosage

The standard dose of an antibiotic is 4 cc (cubic centimeters) for every 25 pounds (lb) of body weight. At this rate, find the standard dose for a 140-lb woman.

Solution:

1. UNDERSTAND. Read and reread the problem. You may want to draw a diagram to estimate a reasonable solution.

(Continued on next page)

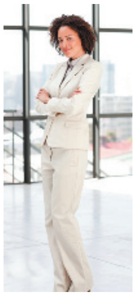
Practice 2

An auto mechanic recommends that 3 ounces of isopropyl alcohol be mixed with a tankful of gas (14 gallons) to increase the octane of the gasoline for better engine performance. At this rate, how many gallons of gas can be treated with a 16-ounce bottle of alcohol?

Answer

2. $74\frac{2}{3}$ or $74.\overline{6}$ gal

140-pound woman



25 pounds → 4 cc
25 pounds → 4 cc
25 pounds → 4 cc
25 pounds → 4 cc
25 pounds → 4 cc
15 pounds → ?
140 pounds over 20 cc

From the diagram, we can see that a reasonable solution is a little over 20 cc.

2. TRANSLATE. We will let n represent the unknown number. From the problem, we know that 4 cc is to 25 pounds as n cc is to 140 pounds, or

$$\begin{array}{rcccl} \text{cubic centimeters} & \rightarrow & \frac{4}{25} = \frac{n}{140} & \leftarrow & \text{cubic centimeters} \\ \text{pounds} & \rightarrow & & \leftarrow & \text{pounds} \end{array}$$

3. SOLVE:

$$\frac{4}{25} = \frac{n}{140}$$

$$4 \cdot 140 = 25 \cdot n$$ Set the cross products equal to each other.

$$560 = 25 \cdot n$$ Multiply.

$$\frac{560}{25} = n$$ Divide 560 by 25, the number multiplied by n.

$$n = 22\frac{2}{5} \text{ or } 22.4$$ Simplify.

4. INTERPRET. *Check* your work. This result is reasonable since it is a little over 20 cc. *State* your conclusion: The standard dose for a 140-lb woman is 22.4 cc.

Work Practice 2

Practice 3

If a gallon of paint covers 400 square feet, how many gallons are needed to paint a retaining wall that is 260 feet long and 4 feet high? Round the answer up to the nearest whole gallon.

Example 3 Calculating Supplies Needed to Fertilize a Lawn

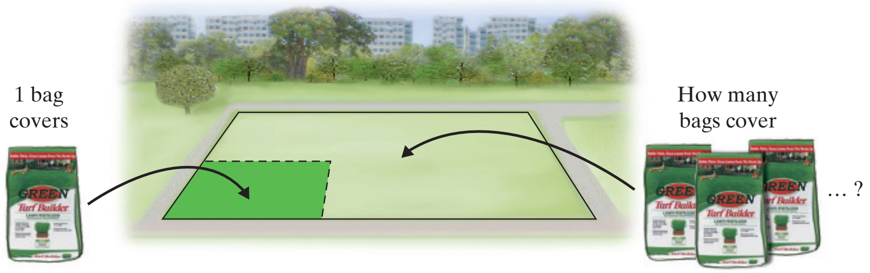
A 50-pound bag of fertilizer covers 2400 square feet of lawn. How many bags of fertilizer are needed to cover a town square containing 15,360 square feet of lawn? Round up to the next whole bag as only whole bags may be purchased.

Answer

3. 3 gal

Solution:

1. UNDERSTAND. Read and reread the problem. Draw a picture.



Since one bag covers 2400 square feet, let's see how many 2400s there are in 15,360. We will estimate. The number 15,360 rounded to the nearest thousand is 15,000 and 2400 rounded to the nearest thousand is 2000. Then

$$15{,}000 \div 2000 = 7\frac{1}{2} \text{ or } 7.5.$$

2. TRANSLATE. We'll let n represent the unknown number. From the problem, we know that 1 bag is to 2400 square feet as n bags is to 15,360 square feet.

$$\begin{array}{rcl} \text{bags} \rightarrow & \dfrac{1}{2400} = \dfrac{n}{15{,}360} & \leftarrow \text{bags} \\ \text{square feet} \rightarrow & & \leftarrow \text{square feet} \end{array}$$

3. SOLVE:

$$\frac{1}{2400} = \frac{n}{15{,}360}$$

$$1 \cdot 15{,}360 = 2400 \cdot n \quad \text{Set the cross products equal to each other.}$$

$$15{,}360 = 2400 \cdot n \quad \text{Multiply.}$$

$$\frac{15{,}360}{2400} = n \quad \text{Divide 15,360 by 2400, the number multiplied by } n.$$

$$n = 6.4 \quad \text{Simplify.}$$

4. INTERPRET. *Check* that replacing n with 6.4 makes the proportion true. Is the answer reasonable? Yes, because it's close to $7\frac{1}{2}$ or 7.5. Because we must buy whole bags of fertilizer, 7 bags are needed. *State* your conclusion: To cover 15,300 square feet of lawn, 7 bags are needed.

Work Practice 3

✓**Concept Check** You are told that 12 ounces of ground coffee will brew enough coffee to serve 20 people. How could you estimate how much ground coffee will be needed to serve 95 people?

✓**Concept Check Answer**

Find how much will be needed for 100 people (20×5) by multiplying 12 ounces by 5, which is 60 ounces.

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 5.3

Watch the section lecture video and answer the following question.

Objective A **1.** In Example 2, interpret the meaning of the answer 102.9.

5.3 Exercise Set MyLab Math

Objective A *Solve. For Exercises 1 and 2, the solutions have been started for you. See Examples 1 through 3.*

An NBA basketball player averages 45 baskets for every 100 attempts.

1. If he attempted 800 field goals, how many field goals did he make?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let

n = how many field goals he made

2. TRANSLATE into an equation.

$$\begin{array}{r} \text{baskets (field goals)} \rightarrow \\ \text{attempts} \rightarrow \end{array} \frac{45}{100} = \frac{n}{800} \begin{array}{l} \leftarrow \text{baskets (field goals)} \\ \leftarrow \text{attempts} \end{array}$$

3. SOLVE the equation. Set cross products equal to each other and solve.

$$\frac{45}{100} \times \frac{n}{800}$$

After SOLVING, then

4. INTERPRET.

2. If he made 225 baskets, how many did he attempt?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let

n = how many baskets attempted

2. TRANSLATE into an equation.

$$\begin{array}{r} \text{baskets} \rightarrow \\ \text{attempts} \rightarrow \end{array} \frac{45}{100} = \frac{225}{n} \begin{array}{l} \leftarrow \text{baskets} \\ \leftarrow \text{attempts} \end{array}$$

3. SOLVE the equation. Set cross products equal to each other and solve.

$$\frac{45}{100} \times \frac{225}{n}$$

After SOLVING, then

4. INTERPRET.

It takes a word processor 30 minutes to word process and spell check 4 pages.

3. Find how long it takes her to word process and spell check 22 pages.

4. Find how many pages she can word process and spell check in 4.5 hours.

University Law School accepts 2 out of every 7 applicants.

5. If the school accepted 180 students, find how many applications they received.

6. If the school accepted 150 students, find how many applications they received.

On an architect's blueprint, 1 inch corresponds to 8 feet.

7. Find the length of a wall represented by a line $2\frac{7}{8}$ inches long on the blueprint.

8. Find the length of a wall represented by a line $5\frac{1}{4}$ inches long on the blueprint.

A human-factors expert recommends that there be at least 9 square feet of floor space in a college classroom for every student in the class.

△ **9.** Find the minimum floor space that 30 students require.

△ **10.** Due to a lack of space, a university converts a 21-by-15-foot conference room into a classroom. Find the maximum number of students the room can accommodate.

A Honda Civic Hybrid car averages 627 miles on a 12.3-gallon tank of gas.

11. Manuel Lopez is planning a 1250-mile vacation trip in his Honda Civic Hybrid. Find how many gallons of gas he can expect to burn. Round to the nearest gallon.

12. Ramona Hatch has enough money to put 6.9 gallons of gas in her Honda Civic Hybrid. She is planning on driving home from college for the weekend. If her home is 290 miles away, should she make it home before she runs out of gas?

The scale on an Italian map states that 1 centimeter corresponds to 30 kilometers.

13. Find how far apart Milan and Rome are if their corresponding points on the map are 15 centimeters apart.

14. On the map, a small Italian village is located 0.4 centimeter from the Mediterranean Sea. Find the actual distance.

A bag of Scotts fertilizer covers 3000 square feet of lawn.

△ **15.** Find how many bags of fertilizer should be purchased to cover a rectangular lawn 260 feet by 180 feet.

△ **16.** Find how many bags of fertilizer should be purchased to cover a square lawn measuring 160 feet on each side.

A Cubs baseball player gets 3 hits every 8 times at bat.

17. If this Cubs player comes up to bat 40 times in the World Series, find how many hits he would be expected to get.

18. At this rate, if he got 12 hits, find how many times he batted.

A survey reveals that 2 out of 3 people prefer Coke to Pepsi.

19. In a room of 40 people, how many people are likely to prefer Coke? Round the answer to the nearest person.

20. In a college class of 36 students, find how many students are likely to prefer Pepsi.

A self-tanning lotion advertises that a 3-oz bottle will provide four applications.

21. Jen Haddad found a great deal on a 14-oz bottle of the self-tanning lotion she had been using. Based on the advertising claims, how many applications of the self-tanner should Jen expect? Round down to the smaller whole number.

22. The Community College thespians need fake tans for a play they are doing. If the play has a cast of 35, how many ounces of self-tanning lotion should the cast purchase? Round up to the next whole number of ounces.

The school's computer lab goes through 5 reams of printer paper every 3 weeks.

▶ **23.** Find out how long a case of printer paper is likely to last (a case of paper holds 8 reams of paper). Round to the nearest week.

24. How many cases of printer paper should be purchased to last the entire semester of 15 weeks? Round up to the next case.

A recipe for pancakes calls for 2 cups flour and $1\frac{1}{2}$ *cups milk to make a serving for four people.*

25. Ming has plenty of flour, but only 4 cups milk. How many servings can he make?

26. The swim team has a weekly breakfast after early practice. How much flour will it take to make pancakes for 18 swimmers?

Solve.

27. In the Seattle Space Needle, the elevators whisk you to the revolving restaurant at a speed of 800 feet in 60 seconds. If the revolving restaurant is 500 feet up, how long will it take you to reach the restaurant by elevator? (*Source:* Seattle Space Needle)

28. A 16-oz grande Tazo Black Iced Tea at Starbucks has 80 calories. How many calories are there in a 24-oz venti Tazo Black Iced Tea? (*Source:* Starbucks Coffee Company)

29. Mosquitos are annoying insects. To eliminate mosquito larvae, a certain granular substance can be applied to standing water in a ratio of 1 tsp per 25 sq ft of standing water.

- **a.** At this rate, find how many teaspoons of granules must be used for 450 square feet.
- **b.** If 3 tsp = 1 tbsp, how many tablespoons of granules must be used?

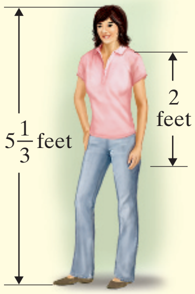
30. Another type of mosquito control is liquid, where 3 oz of pesticide is mixed with 100 oz of water. This mixture is sprayed on roadsides to control mosquito breeding grounds hidden by tall grass.

- **a.** If one mixture of water with this pesticide can treat 150 feet of roadway, how many ounces of pesticide are needed to treat one mile? (*Hint:* 1 mile = 5280 feet)
- **b.** If 8 liquid ounces equals one cup, write your answer to part a in cups. Round to the nearest cup.

31. The daily supply of oxygen for one person is provided by 625 square feet of lawn. A total of 3750 square feet of lawn would provide the daily supply of oxygen for how many people? (*Source:* Professional Lawn Care Association of America)

32. In 2017, approximately \$26 billion of the \$70 billion Americans spent on their pets was spent on pet food. Petco Animal Supplies had \$4,100,000,000 in net sales that year. How much of Petsmart's net sales would you expect to have been spent on pet food? Round to the nearest thousand. (*Source:* American Pet Products Manufacturers Association and Petsmart)

33. A student would like to estimate the height of the Statue of Liberty in New York City's harbor. The length of the Statue of Liberty's right arm is 42 feet. The student's right arm is 2 feet long and her height is $5\frac{1}{3}$ feet. Use this information to estimate the height of the Statue of Liberty. How close is your estimate to the statue's actual height of 111 feet, 1 inch from heel to top of head? (*Source:* National Park Service)



34. The length of the Statue of Liberty's index finger is 8 feet while the height to the top of the head is about 111 feet. Suppose your measurements are proportionally the same as this statue and your height is 5 feet.

- **a.** Use this information to find the proposed length of your index finger. Give an exact measurement and then a decimal rounded to the nearest hundredth.
- **b.** Measure your index finger and write it as a decimal in feet rounded to the nearest hundredth. How close is the length of your index finger to the answer to part **a**? Explain why.

35. There are 72 milligrams of cholesterol in a 3.5-ounce serving of lobster. How much cholesterol is in 5 ounces of lobster? Round to the nearest tenth of a milligram. (*Source:* The National Institutes of Health)

36. There are 76 milligrams of cholesterol in a 3-ounce serving of skinless chicken. How much cholesterol is in 8 ounces of chicken? (*Source:* USDA)

37. The Comcast Building in New York City is 850 feet tall and contains 69 stories. The Empire State Building contains 102 stories. If the Empire State Building has the same number of feet per floor as the Comcast Building, approximate its height rounded to the nearest foot. (*Source:* skyscrapers.com)

38. In 2017, approximately 125 million of the 152 million U.S. workers worked in service industries. In a town of 19,000 workers, how many would be expected to work in service-industry jobs? (*Source:* U.S. Bureau of Labor Statistics)

39. Medication is prescribed in 7 out of every 10 hospital emergency room visits that involve an injury. If a large urban hospital had 620 emergency room visits involving an injury in the past month, how many of these visits would you expect included a prescription for medication? (*Source:* National Center for Health Statistics)

40. One pound of firmly packed brown sugar yields $2\frac{1}{4}$ cups. How many pounds of brown sugar will be required in a recipe that calls for 6 cups of firmly packed brown sugar? (*Source:* Based on data from *Family Circle* magazine)

41. In 2016, six out every 20 vehicles sold in the United States were SUV crossovers. Approximately 7 million vehicles were sold in the United States in 2016. How many of them were SUV crossovers? Round to the nearest million. (*Source: Wall Street Journal*)

42. In the first few months of 2017, three out of every 40 computers sold were Apple products. Approximately 65,000 computers were sold in the United States in February 2017. How many of these were Apple products? (*Source:* Gartner.com)

When making homemade ice cream in a hand-cranked freezer, the tub containing the ice cream mix is surrounded by a brine (water/salt) solution. To freeze the ice cream mix rapidly so that smooth and creamy ice cream results, the brine solution should combine crushed ice and rock salt in a ratio of 5 to 1. Use this information for Exercises 43 and 44. (Source: White Mountain Freezers, The Rival Company)

43. A small ice cream freezer requires 12 cups of crushed ice. How much rock salt should be mixed with the ice to create the necessary brine solution?

44. A large ice cream freezer requires $18\frac{3}{4}$ cups of crushed ice. How much rock salt will be needed to create the necessary brine solution?

45. The gas/oil ratio for a certain chainsaw is 50 to 1.

a. How much oil (in gallons) should be mixed with 5 gallons of gasoline?

b. If 1 gallon equals 128 fluid ounces, write the answer to part **a** in fluid ounces. Round to the nearest whole ounce.

46. The gas/oil ratio for a certain tractor mower is 20 to 1.

a. How much oil (in gallons) should be mixed with 10 gallons of gas?

b. If 1 gallon equals 4 quarts, write the answer to part **a** in quarts.

47. The adult daily dosage for a certain medicine is 150 mg (milligrams) of medicine for every 20 pounds of body weight.

a. At this rate, find the daily dose for a man who weighs 275 pounds.

b. If the man is to receive 500 mg of this medicine every 8 hours, is he receiving the proper dosage?

48. The adult daily dosage for a certain medicine is 80 mg (milligrams) for every 25 pounds of body weight.

a. At this rate, find the daily dose for a woman who weighs 190 pounds.

b. If she is to receive this medicine every 6 hours, find the amount to be given every 6 hours.

Review

Find the prime factorization of each number. See Section 3.2.

49. 15 **50.** 21 **51.** 20 **52.** 24 **53.** 200 **54.** 300 **55.** 32 **56.** 81

Concept Extensions

As we have seen, proportions are often used in medicine dosage calculations. The exercises below have to do with liquid drug preparations, where the weight of the drug is contained in a volume of solution. The descriptions of mg and ml below will help. We will study metric units further in Sections 5.4 through 5.7.

mg means milligram (A paper clip weighs about a gram. A milligram is about the weight of $\frac{1}{1000}$ *of a paper clip.)*

ml means milliliter (A liter is about a quart. A milliliter is about the amount of liquid in $\frac{1}{1000}$ *of a quart.)*

One way to solve the applications below is to set up the proportion $\frac{\text{mg}}{\text{ml}} = \frac{\text{mg}}{\text{ml}}$.

A solution strength of 15 mg of medicine in 1 ml of solution is available.

57. If a patient needs 12 mg of medicine, how many ml do you administer?

58. If a patient needs 33 mg of medicine, how many ml do you administer?

A solution strength of 8 mg of medicine in 1 ml of solution is available.

59. If a patient needs 10 mg of medicine, how many ml do you administer?

60. If a patient needs 6 mg of medicine, how many ml do you administer?

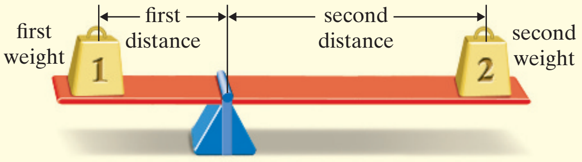
Estimate the following. See the Concept Check in this section.

61. It takes 1.5 cups of milk to make 11 muffins. Estimate the amount of milk needed to make 8 dozen muffins. Explain your calculation.

62. A favorite chocolate chip recipe calls for $2\frac{1}{2}$ cups of flour to make 2 dozen cookies. Estimate the amount of flour needed to make 50 cookies. Explain your calculation.

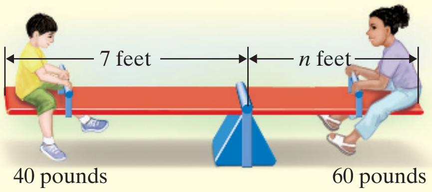
A board such as the one pictured below will balance if the following proportion is true:

$$\frac{\text{first weight}}{\text{second distance}} = \frac{\text{second weight}}{\text{first distance}}$$

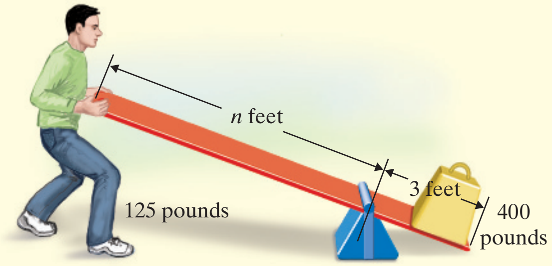


Use this proportion to solve Exercises 63 and 64.

63. Find the distance n that will allow the board to balance.



64. Find the length n needed to lift the weight below.



65. Describe a situation in which writing a proportion might solve a problem related to driving a car.

Integrated Review

Sections 5.1–5.3

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. a. ____________
b. ____________
14. ____________

Ratio and Proportion

Write each ratio as a ratio of whole numbers using fractional notation. Write the fraction in simplest form.

1. 18 to 20

2. 36 to 100

3. 8.6 to 10

4. 1.6 to 4.6

5. \$8.65 to \$6.95

6. 7.2 ounces to 8.4 ounces

7. $3\frac{1}{2}$ to 13

8. $1\frac{2}{3}$ to $2\frac{3}{4}$

9. 8 inches to 12 inches

10. 3 hours to 24 hours

Find the ratio described in each problem.

11. During the 2016–2017 academic year, a full college professor in Massachusetts Institute of Technology earned \$185.9 thousand on average. By contrast, an assistant professor averaged \$123.7 thousand. Find the ratio of full professor salary to assistant professor salary at MIT. (*Source:* American Association of University Professors)

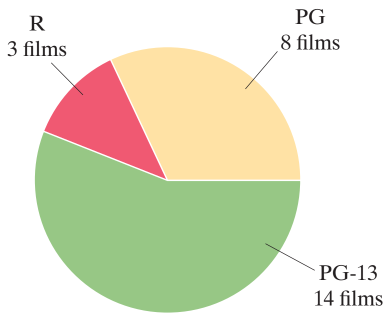
12. The Montreal Canadiens are a dynastic hockey powerhouse. They won 24 out of the 100 Stanley Cup finals from 1915 through 2016. (*Source:* National Hockey League)

13. The circle graph below shows how the top 25 movies of 2016 were rated. Use this graph to answer the questions.

a. How many top 25 movies were rated R?

b. Find the ratio of top 25 PG-rated movies to PG-13-rated movies for 2016.

Top 25 Movies of 2016



Source: MPAA

14. Find the ratio of the width to the length of the sign below.

Write each rate as a fraction in simplest form.

15. 5 offices for every 20 graduate assistants

16. 6 lights every 15 feet

17. 64 computers for every 100 households

18. 45 students for every 10 computers

Write each rate as a unit rate.

19. 165 miles in 3 hours

20. 560 feet in 4 seconds

21. 115 miles every 5 gallons

22. 112 teachers for 7 computers

Write each unit price, rounded to the nearest cent, and decide which is the better buy.

23. Microwave popcorn:
3 packs for \$2.39
8 packs for \$5.99

24. AA batteries:
4 for \$3.69
10 for \$9.89

Determine whether each proportion is true.

25. $\frac{7}{4} = \frac{5}{3}$

26. $\frac{8.2}{2} = \frac{16.4}{4}$

Find the unknown number n in each proportion.

27. $\frac{5}{3} = \frac{40}{n}$

28. $\frac{n}{10} = \frac{13}{4}$

29. $\frac{6}{11} = \frac{n}{5}$

30. $\frac{21}{n} = \frac{\frac{7}{2}}{3}$

15. ____________

16. ____________

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

22. ____________

23. ____________

24. ____________

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

5.4 Length: U.S. and Metric Systems of Measurement

Objectives

A Define U.S. Units of Length and Convert from One Unit to Another.

B Use Mixed U.S. Units of Length.

C Perform Arithmetic Operations on U.S. Units of Length.

D Define Metric Units of Length and Convert from One Unit to Another.

E Perform Arithmetic Operations on Metric Units of Length.

Objective A Defining and Converting U.S. System Units of Length

In the United States, two systems of measurement are commonly used. They are the **United States (U.S.), or English, measurement system** and the **metric system.** The U.S. measurement system is familiar to most Americans. Units such as feet, miles, ounces, and gallons are used. However, the metric system is also commonly used in fields such as medicine, sports, international marketing, and certain physical sciences. We are accustomed to buying 2-liter bottles of soft drinks, watching televised coverage of the 100-meter dash at the Olympic Games, or taking a 200-milligram dose of pain reliever.

The U.S. system of measurement uses the **inch, foot, yard,** and **mile** to measure **length.** The following is a summary of equivalencies between units of length:

U.S. Units of Length

$$\begin{aligned} 12 \text{ inches (in.)} &= 1 \text{ foot (ft)} \\ 3 \text{ feet} &= 1 \text{ yard (yd)} \\ 36 \text{ inches} &= 1 \text{ yard} \\ 5280 \text{ feet} &= 1 \text{ mile (mi)} \end{aligned}$$

To convert from one unit of length to another, we will use **unit fractions.** We define a unit fraction to be a fraction that is equivalent to 1. Examples of unit fractions are as follows:

Unit Fractions

$$\frac{12 \text{ in.}}{1 \text{ ft}} = 1 \text{ or } \frac{1 \text{ ft}}{12 \text{ in.}} = 1 \text{ (since } 12 \text{ in.} = 1 \text{ ft)}$$

$$\frac{3 \text{ ft}}{1 \text{ yd}} = 1 \text{ or } \frac{1 \text{ yd}}{3 \text{ ft}} = 1 \text{ (since } 3 \text{ ft} = 1 \text{ yd)}$$

$$\frac{5280 \text{ ft}}{1 \text{ mi}} = 1 \text{ or } \frac{1 \text{ mi}}{5280 \text{ ft}} = 1 \text{ (since } 5280 \text{ ft} = 1 \text{ mi)}$$

Remember that multiplying a number by 1 does not change the value of the number.

Practice 1

Convert 6 feet to inches.

Example 1 Convert 8 feet to inches.

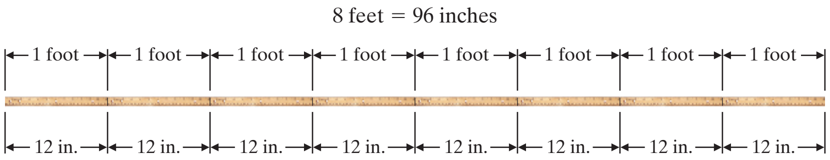
Solution: We multiply 8 feet by a unit fraction that uses the equality 12 inches = 1 foot. The unit fraction should be in the form $\frac{\text{units to convert to}}{\text{original units}}$ or, in this case, $\frac{12 \text{ inches}}{1 \text{ foot}}$. We do this so that like units will divide out to 1, as shown.

$$\begin{aligned} 8 \text{ ft} &= \frac{8 \text{ ft}}{1} \cdot 1 \\ &= \frac{8 \cancel{\text{ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} && \text{Multiply by 1 in the form of } \frac{12 \text{ in.}}{1 \text{ ft}}. \\ &= 8 \cdot 12 \text{ in.} \\ &= 96 \text{ in.} && \text{Multiply.} \end{aligned}$$

Answer

1. 72 in.

Thus, 8 ft = 96 in., as shown in the diagram:



Work Practice 1

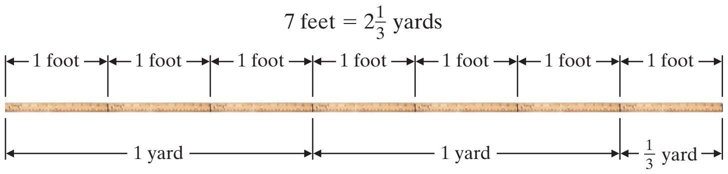
Example 2

Convert 7 feet to yards.

Solution: We multiply by a unit fraction that compares 1 yard to 3 feet.

$$\begin{aligned} 7\text{ ft} &= \frac{7\text{ ft}}{1}\cdot 1 \\ &= \frac{7\,\cancel{\text{ft}}}{1}\cdot\frac{1\text{ yd}}{3\,\cancel{\text{ft}}} && \begin{aligned}&\leftarrow \text{Units to convert to}\\ &\leftarrow \text{Original units}\end{aligned} \\ &= \frac{7}{3}\text{ yd} \\ &= 2\frac{1}{3}\text{ yd} && \text{Divide.} \end{aligned}$$

Thus, 7 ft $= 2\frac{1}{3}$ yd, as shown in the diagram.



Work Practice 2

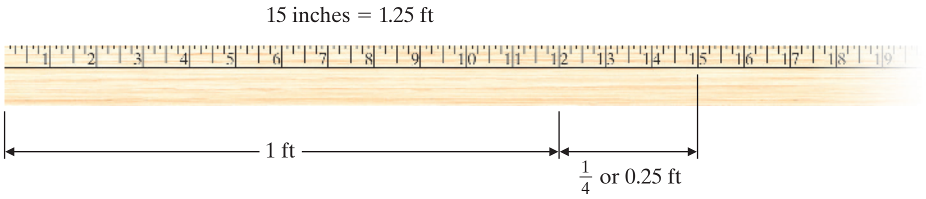
Example 3 Finding the Length of a Pelican's Bill

The Australian pelican has the longest bill, measuring from 13 to 18.5 inches long. The pelican in the photo has a 15-inch bill. Convert 15 inches to feet, using decimals in your final answer.

Solution:

$$\begin{aligned} 15\text{ in.} &= \frac{15\,\cancel{\text{in.}}}{1}\cdot\frac{1\text{ ft}}{12\,\cancel{\text{in.}}} && \begin{aligned}&\leftarrow \text{Units to convert to}\\ &\leftarrow \text{Original units}\end{aligned} \\ &= \frac{15}{12}\text{ ft} \\ &= \frac{5}{4}\text{ ft} && \text{Simplify } \frac{15}{12}. \\ &= 1.25\text{ ft} && \text{Divide.} \end{aligned}$$

Thus, 15 in. = 1.25 ft, as shown in the diagram.



Work Practice 3

Practice 2

Convert 8 yards to feet.

Helpful Hint When converting from one unit to another, select a unit fraction with the properties below:

$$\frac{\text{units you are converting to}}{\text{original units}}$$

By using this unit fraction, the original units will divide out, as wanted.

Practice 3

Suppose the pelican's bill (in the photo) measures 18 inches. Convert 18 inches to feet, using decimals.

Answers

2. 24 ft **3.** 1.5 ft

Objective B Using Mixed U.S. System Units of Length

Sometimes it is more meaningful to express a measurement of length with mixed units such as 1 ft and 5 in. We usually condense this and write 1 ft 5 in.

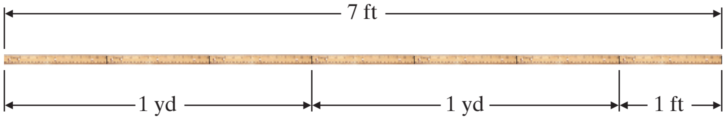
In Example 2, we found that 7 feet is the same as $2\frac{1}{3}$ yards. The measurement can also be written as a mixture of yards and feet. That is,

7 ft = ____ yd ____ ft

Because 3 ft = 1 yd, we divide 3 into 7 to see how many whole yards are in 7 feet. The quotient is the number of yards, and the remainder is the number of feet.

$$\begin{array}{r} 2\text{ yd } 1\text{ ft} \\ 3\overline{)7}\phantom{\text{ yd } 1\text{ ft}} \\ -6\phantom{\text{ yd } 1\text{ ft}} \\ \hline 1\phantom{\text{ yd } 1\text{ ft}} \end{array}$$

Thus, 7 ft = 2 yd 1 ft, as seen in the diagram:



Practice 4

Convert:
68 in. = ______ ft ______ in

Example 4

Convert: 134 in. = _____________ ft _____________ in.

Solution: Because 12 in. = 1 ft, we divide 12 into 134. The quotient is the number of feet. The remainder is the number of inches. To see why we divide 12 into 134, notice that

$$134\text{ in.} = \frac{134\ \cancel{\text{in.}}}{1}\cdot\frac{1\text{ ft}}{12\ \cancel{\text{in.}}} = \frac{134}{12}\text{ ft}$$

$$\begin{array}{r} 11\text{ ft } 2\text{ in.} \\ 12\overline{)134}\phantom{\text{ ft } 2\text{ in.}} \\ -12\phantom{\text{ ft } 2\text{ in.}} \\ \hline 14\phantom{\text{ ft } 2\text{ in.}} \\ -12\phantom{\text{ ft } 2\text{ in.}} \\ \hline 2\phantom{\text{ ft } 2\text{ in.}} \end{array}$$

Thus, 134 in. = 11 ft 2 in.

Work Practice 4

Practice 5

Convert 5 yards 2 feet to feet.

Example 5

Convert 3 feet 7 inches to inches.

Solution: First, we convert 3 feet to inches. Then we add 7 inches.

$$3\text{ ft} = \frac{3\ \cancel{\text{ft}}}{1}\cdot\frac{12\text{ in.}}{1\ \cancel{\text{ft}}} = 36\text{ in.}$$

Then

$$3\text{ ft } 7\text{ in.} = 36\text{ in.} + 7\text{ in.} = 43\text{ in.}$$

Work Practice 5

Answers
4. 5 ft 8 in. 5. 17 ft

Objective C Performing Operations on U.S. System Units of Length

Finding sums or differences of measurements often involves converting units, as shown in the next example. Just remember that, as usual, only like units can be added or subtracted.

Example 6 Add 3 ft 2 in. and 5 ft 11 in.

Solution: To add, we line up the similar units.

$$\begin{array}{r} 3\text{ ft }\ \ 2\text{ in.} \\ +\ 5\text{ ft }11\text{ in.} \\ \hline 8\text{ ft }13\text{ in.} \end{array}$$

Since 13 inches is the same as 1 ft 1 in., we have

$$\begin{aligned} 8\text{ ft }13\text{ in.} &= 8\text{ ft} + \overbrace{1\text{ ft }1\text{ in.}} \\ &= 9\text{ ft }1\text{ in.} \end{aligned}$$

Work Practice 6

✓Concept Check How could you estimate the following sum?

$$\begin{array}{r} 7\text{ yd }\ \ 4\text{ in.} \\ +\ 3\text{ yd }27\text{ in.} \\ \hline \end{array}$$

Example 7 Multiply 8 ft 9 in. by 3.

Solution: By the distributive property, we multiply 8 ft by 3 and 9 in. by 3.

$$\begin{array}{r} 8\text{ ft }\ \ 9\text{ in.} \\ \times\qquad\quad 3 \\ \hline 24\text{ ft }27\text{ in.} \end{array}$$

Since 27 in. is the same as 2 ft 3 in., we simplify the product as

$$\begin{aligned} 24\text{ ft }27\text{ in.} &= 24\text{ ft} + \overbrace{2\text{ ft }3\text{ in.}} \\ &= 26\text{ ft }3\text{ in.} \end{aligned}$$

Work Practice 7

We divide in a similar manner as above.

Example 8 Finding the Length of a Piece of Rope

A rope of length 6 yd 1 ft has 2 yd 2 ft cut from one end. Find the length of the remaining rope.

Solution: Subtract 2 yd 2 ft from 6 yd 1 ft.

$$\begin{array}{rcr} \text{beginning length} & \rightarrow & 6\text{ yd }1\text{ ft} \\ -\qquad \text{amount cut} & \rightarrow & -\,2\text{ yd }2\text{ ft} \\ \hline \text{remaining length} & & \end{array}$$

We cannot subtract 2 ft from 1 ft, so we borrow 1 yd from the 6 yd. One yard is converted to 3 ft and combined with the 1 ft already there.

(Continued on next page)

Practice 6

Add 4 ft 8 in. to 8 ft 11 in.

Practice 7

Multiply 4 ft 7 in. by 4.

Practice 8

A carpenter cuts 1 ft 9 in. from a board of length 5 ft 8 in. Find the length of the remaining board.

Answers

6. 13 ft 7 in. **7.** 18 ft 4 in. **8.** 3 ft 11 in.

✓Concept Check Answer

round each to the nearest yard:
7 yd + 4 yd = 11 yd

Borrow 1 yd $=$ 3 ft

$5\text{ yd} + \boxed{1\text{ yd}}\ \boxed{3\text{ ft}}$

$$\begin{array}{rcr} \cancel{6\text{ yd}}\ 1\text{ ft} & = & 5\text{ yd } 4\text{ ft} \\ -2\text{ yd } 2\text{ ft} & = & -2\text{ yd } 2\text{ ft} \\ \hline & & 3\text{ yd } 2\text{ ft} \end{array}$$

The remaining rope is 3 yd 2 ft long.

Work Practice 8

Objective D Defining and Converting Metric System Units of Length

The basic unit of length in the metric system is the **meter**. A meter is slightly longer than a yard. It is approximately 39.37 inches long. Recall that a yard is 36 inches long.

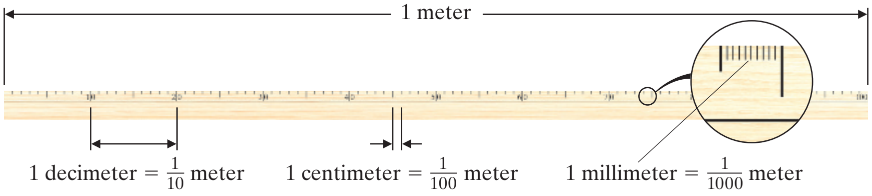
1 yard = 36 inches

1 meter ≈ 39.37 inches

All units of length in the metric system are based on the meter. The following is a summary of the prefixes used in the metric system. Also shown are equivalencies between units of length. Like the decimal system, the metric system uses powers of 10 to define units.

Metric Units of Length
1 **kilo**meter (km) = 1000 meters (m)
1 **hecto**meter (hm) = 100 m
1 **deka**meter (dam) = 10 m
1 meter (m) = 1 m
1 **deci**meter (dm) = 1/10 m or 0.1 m
1 **centi**meter (cm) = 1/100 m or 0.01 m
1 **milli**meter (mm) = 1/1000 m or 0.001 m

The figure below will help you with decimeters, centimeters, and millimeters.



Helpful Hint

Study the figure above for other equivalencies between metric units of length.

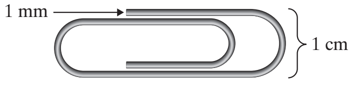
10 decimeters = 1 meter
100 centimeters = 1 meter
1000 millimeters = 1 meter
10 millimeters = 1 centimeter
10 centimeters = 1 decimeter

These same prefixes are used in the metric system for mass and capacity. The most commonly used measurements of length in the metric system are the **meter, millimeter, centimeter,** and **kilometer.**

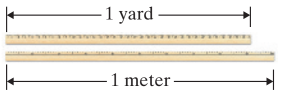
✓Concept Check Is this statement reasonable? "The screen of a home television set has a 30-meter diagonal." Why or why not?

Being comfortable with the metric units of length means gaining a "feeling" for metric lengths, just as you have a "feeling" for the lengths of an inch, a foot, and a mile. To help you accomplish this, study the following examples:

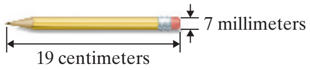
Examples of Metric System—Length



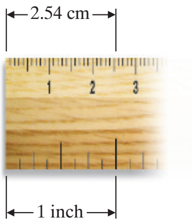
- A millimeter ia about the thickness of a large paper clip.
- A centimeter is about the width of a large paper clip.



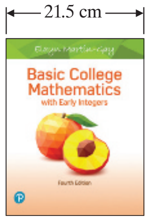
- A meter is slightly longer than a yard.



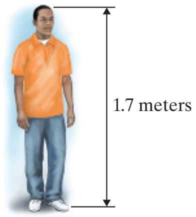
- Pencil dimensions



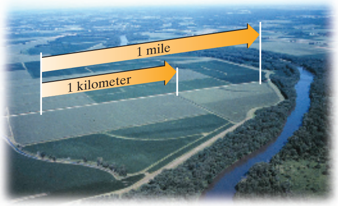
- 1 in. = 2.54 cm



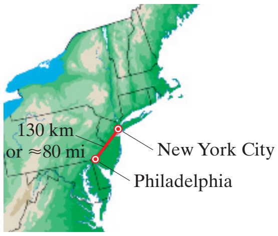
- The width of this book is approximately 21.5 centimeters.



- Man's height



- A kilometer is about two-thirds of a mile.



- The distance between New York City and Philiadelphia is about 130 kilometers by air.

As with the U.S. system of measurement, unit fractions may be used to convert from one unit of length to another. For example, let's convert 1200 meters to kilometers. To do so, we will multiply by 1 in the form of the unit fraction

$$\frac{1\text{ km}}{1000\text{ m}} \begin{array}{l} \leftarrow \text{Units to convert to} \\ \leftarrow \text{Original units} \end{array}$$

$$1200\text{ m} = \frac{1200\text{ m}}{1} \cdot 1 = \frac{1200\ \cancel{\text{m}}}{1} \cdot \overbrace{\frac{1\text{ km}}{1000\ \cancel{\text{m}}}}^{\text{Unit fraction}} = \frac{1200\text{ km}}{1000} = 1.2\text{ km}$$

The metric system does, however, have a distinct advantage over the U.S. system of measurement: the ease of converting from one unit of length to another. Since all units of length are powers of 10 of the meter, converting from one unit of length to another is as simple as moving the decimal point. Listing units of length in order from largest to smallest helps to keep track of how many places to move the decimal point when converting.

✓Concept Check Answer
no; answers may vary

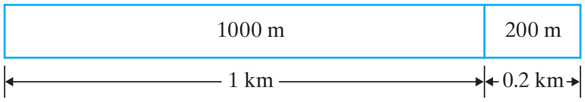
Let's again convert 1200 meters to kilometers. This time, to convert from meters to kilometers, we move along the chart shown, 3 units to the left, from meters to kilometers. This means that we move the decimal point 3 places to the left.

km hm dam **m** dm cm mm

3 units to the left

$1200\text{ m} = 1.200\text{ km}$

3 places
to the left



Thus, $1200\text{ m} = 1.2\text{ km}$, as shown in the diagram.

Practice 9

Convert 2.5 m to millimeters.

Example 9 Convert 2.3 m to centimeters.

Solution: First we will convert by using a unit fraction.

$$2.3\text{ m} = \frac{2.3\cancel{\text{ m}}}{1}\cdot\overbrace{\frac{100\text{ cm}}{1\cancel{\text{ m}}}}^{\text{Unit fraction}} = 230\text{ cm}$$

Now we will convert by listing the units of length in order from left to right and moving from meters to centimeters.

km hm dam m dm cm mm

2 units to the right

$2.30\text{ m} = 230.\text{ cm}$

2 places to the right

With either method, we get 230 cm.

Work Practice 9

Practice 10

Convert 3500 m to kilometers.

Example 10 Convert 450,000 mm to meters.

Solution: We list the units of length in order from left to right and move from millimeters to meters.

km hm dam m dm cm mm

3 units to the left

$450{,}000\text{ mm} = 450.000\text{ m}$ or 450 m

Work Practice 10

✓**Concept Check** What is wrong with the following conversion of 150 cm to meters?

~~$150.00\text{ cm} = 15{,}000\text{ m}$~~

Objective E Performing Operations on Metric System Units of Length

To add, subtract, multiply, or divide with metric measurements of length, we write all numbers using the same unit of length and then add, subtract, multiply, or divide as with decimals.

Answers

9. 2500 mm **10.** 3.5 km

✓**Concept Check Answer**

decimal point should be moved two places to the left: 1.5 m

Example 11

Subtract 430 m from 1.3 km.

Solution: First we convert both measurements to kilometers or both to meters.

$$430 \text{ m} = 0.43 \text{ km} \qquad \text{or} \qquad 1.3 \text{ km} = 1300 \text{ m}$$

$$\begin{array}{r} 1.30 \text{ km} \\ -0.43 \text{ km} \\ \hline 0.87 \text{ km} \end{array} \qquad\qquad \begin{array}{r} 1300 \text{ m} \\ -430 \text{ m} \\ \hline 870 \text{ m} \end{array}$$

The difference is 0.87 km or 870 m.

Work Practice 11

Example 12

Multiply 5.7 mm by 4.

Solution: Here we simply multiply the two numbers. Note that the unit of measurement remains the same.

$$\begin{array}{r} 5.7 \text{ mm} \\ \times \quad 4 \\ \hline 22.8 \text{ mm} \end{array}$$

Work Practice 12

Example 13 Finding a Person's Height

Fritz Martinson was 1.2 meters tall on his last birthday. Since then, he has grown 14 centimeters. Find his current height in meters.

Solution:

$$\begin{array}{rcrl} \text{original height} & \rightarrow & 1.20 \text{ m} & \\ + \text{ height grown} & \rightarrow & + \ 0.14 \text{ m} & (\text{Since } 14 \text{ cm} = 0.14 \text{ m}) \\ \hline \text{current height} & & 1.34 \text{ m} & \end{array}$$

Fritz is now 1.34 meters tall.

Work Practice 13

Example 14 Finding a Crocodile's Length

A newly hatched Nile crocodile averages 26 centimeters in length. This type of crocodile normally grows 4.74 meters to reach its adult length. What is the adult length of this type of crocodile?

Solution:

$$\begin{array}{rcrl} \text{original length} & \rightarrow & 0.26 \text{ m} & (\text{Since } 26 \text{ cm} = 0.26 \text{ m}) \\ + \text{ length grown} & \rightarrow & + \ 4.74 \text{ m} & \\ \hline \text{adult length} & & 5.00 \text{ m} & \end{array}$$

The adult length is 5 meters.

Work Practice 14

Practice 11

Subtract 640 m from 2.1 km.

Practice 12

Multiply 18.3 hm by 5.

Practice 13

A child was 55 centimeters at birth. Her adult height was 1.72 meters. Find how much she grew from birth to adult height.

Practice 14

A newly born male giraffe is 205 centimeters tall. If this male grows another 3.5 meters to adulthood, find the adult height of this giraffe.

Answers

11. 1.46 km or 1460 m **12.** 91.5 hm
13. 117 cm or 1.17 m
14. 555 cm or 5.55 m

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

inches yard unit fraction
feet meter

1. The basic unit of length in the metric system is the __________.

2. The expression $\frac{1 \text{ foot}}{12 \text{ inches}}$ is an example of a(n) __________.

3. A meter is slightly longer than a(n) __________.

4. One foot equals 12 __________.

5. One yard equals 3 __________.

6. One yard equals 36 __________.

7. One mile equals 5280 __________.

8. One foot equals $\frac{1}{3}$ __________.

Martin-Gay Interactive Videos

See Video 5.4

Watch the section lecture video and answer the following questions.

Objective A **9.** In Example 3, what units are used in the denominator of the unit fraction and why was this decided?

Objective B **10.** In Example 4, how is a mixed unit similar to a mixed number? Use examples in your answer.

Objective C **11.** In Example 5, why is the original sum of the addition problem not the final answer? Reference the sum in your answer.

Objective D **12.** Based on the lecture before Example 6, explain how to convert metric units to other metric units.

Objective E **13.** What two answers did we get for Example 8? Explain why both answers are correct.

5.4 Exercise Set MyLab Math

Objective A *Convert each measurement as indicated. See Examples 1 through 3.*

1. 60 in. to feet

2. 84 in. to feet

3. 12 yd to feet

4. 18 yd to feet

5. 42,240 ft to miles

6. 36,960 ft to miles

7. $8\frac{1}{2}$ ft to inches

8. $12\frac{1}{2}$ ft to inches

9. 10 ft to yards

10. 25 ft to yards

11. 6.4 mi to feet

12. 3.8 mi to feet

13. 162 in. to yd (Write answer as a decimal.)

14. 7216 yd to mi (Write answer as a decimal.)

15. 3 in. to ft (Write answer as a decimal.)

16. 129 in. to ft (Write answer as a decimal.)

Objective B *Convert each measurement as indicated. See Examples 4 and 5.*

17. 40 ft = ______ yd ______ ft

18. 100 ft = ______ yd ______ ft

19. 85 in. = ______ ft ______ in.

20. 47 in. = ______ ft ______ in.

21. 10,000 ft = ______ mi ______ ft

22. 25,000 ft = ______ mi ______ ft

23. 5 ft 2 in. = ______ in.

24. 4 ft 11 in. = ______ in.

25. 8 yd 2 ft = ______ ft

26. 4 yd 1 ft = ______ ft

27. 2 yd 1 ft = ______ in.

28. 1 yd 2 ft = ______ in.

Objective C *Perform each indicated operation. Simplify the result if possible. See Examples 6 through 8.*

29. 3 ft 10 in. + 7 ft 4 in.

30. 12 ft 7 in. + 9 ft 11 in.

31. 12 yd 2 ft + 9 yd 2 ft

32. 16 yd 2 ft + 8 yd 2 ft

33. 22 ft 8 in. − 16 ft 3 in.

34. 15 ft 5 in. − 8 ft 2 in.

35. 18 ft 3 in. − 10 ft 9 in.

36. 14 ft 8 in. − 3 ft 11 in.

37. 28 ft 8 in. ÷ 2

38. 34 ft 6 in. ÷ 2

39. 16 yd 2 ft × 5

40. 15 yd 1 ft × 8

Objective D *Convert as indicated. See Examples 9 and 10.*

41. 60 m to centimeters

42. 46 m to centimeters

43. 40 mm to centimeters

44. 14 mm to centimeters

45. 500 m to kilometers

46. 400 m to kilometers

47. 1700 mm to meters

48. 6400 mm to meters

49. 1500 cm to meters

50. 6400 cm to meters

51. 0.42 km to centimeters

52. 0.95 km to centimeters

53. 7 km to meters

54. 5 km to meters

55. 8.3 cm to millimeters

56. 4.6 cm to millimeters

57. 20.1 mm to decimeters

58. 140.2 mm to decimeters

59. 0.04 m to millimeters

60. 0.2 m to millimeters

Objective E *Perform each indicated operation. Remember to insert units when writing your answers. See Examples 11 through 14.*

61. 8.6 m + 0.34 m

62. 14.1 cm + 3.96 cm

63. 2.9 m + 40 mm

64. 30 cm + 8.9 m

65. 24.8 mm − 1.19 cm

66. 45.3 m − 2.16 dam

67. 15 km − 2360 m

68. 14 cm − 15 mm

69. 18.3 m × 3

70. 14.1 m × 4

71. 6.2 km ÷ 4

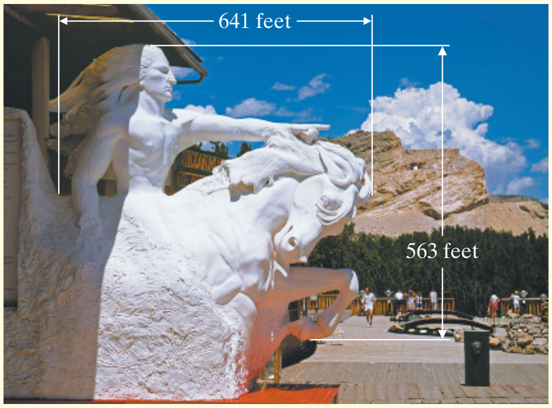
72. 9.6 m ÷ 5

Objectives A C D E Mixed Practice *Solve. Remember to insert units when writing your answers. For Exercises 73 through 82, complete the charts. See Examples 1 through 14.*

		Yards	Feet	Inches
73.	Chrysler Building in New York City		1046	
74.	4-story building			792
75.	Python length		35	
76.	Ostrich height			108

		Meters	Millimeters	Kilometers	Centimeters
77.	Length of elephant	5			
78.	Height of grizzly bear	3			
79.	Tennis ball diameter				6.5
80.	Golf ball diameter				4.6
81.	Distance from London to Paris			342	
82.	Distance from Houston to Dallas			396	

A massive structure of Crazy Horse is currently being carved into the Black Hills about 8 miles from Mt. Rushmore. Use the dimensions in the photo for Exercises 83 and 84.



83. The total width of the Crazy Horse carving is 641 feet. Convert this width to

a. yards

b. inches

84. The total height of the Crazy Horse carving is 563 feet. Convert this height to

a. yards

b. inches

85. The National Zoo maintains a small patch of bamboo, which it grows as a food supply for its pandas. Two weeks ago, the bamboo was 6 ft 10 in. tall. Since then, the bamboo has grown 3 ft 8 in. How tall is the bamboo now?

86. While exploring in the Mariana Trench, a submarine probe was lowered to a point 1 mile 1400 feet below the ocean's surface. Later it was lowered an additional 1 mile 4000 feet below this point. How far was the probe below the surface of the Pacific?

87. At its deepest point, the Grand Canyon of the Colorado River in Arizona is about 6000 ft. The Grand Canyon of the Yellowstone River, which is in Yellowstone National Park in Wyoming, is at most 900 feet deep. How much deeper is the Grand Canyon of the Colorado River than the Grand Canyon of the Yellowstone River? (*Source:* National Park Service)

88. The Black Canyon of the Gunnison River is only 1150 ft wide at its narrowest point. At its narrowest, the Grand Canyon of the Yellowstone is $\frac{1}{2}$ mile wide. Find the difference in width between the Grand Canyon of the Yellowstone and the Black Canyon of the Gunnison. (*Note:* Notice that the dimensions are different.) (*Source:* National Park Service)

89. The tallest man in the world is recorded as Robert Pershing Wadlow of Alton, Illinois. Born in 1918, he measured 8 ft 11 in. at his tallest. The shortest man in the world is Chandra Bahadur Dangi of Nepal, who measures 21.5 in. How many times taller than Chandra was Robert? Round to one decimal place. (*Source: Guinness World Records*)

90. A 3.4-m rope is attached to a 5.8-m rope. However, when the ropes are tied, 8 cm of length is lost to form the knot. What is the length of the tied ropes?

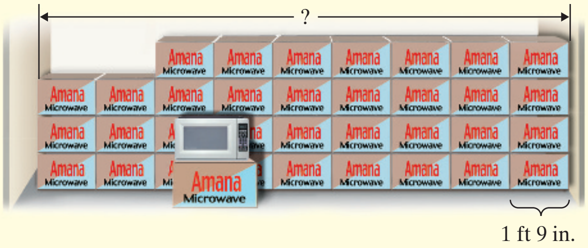
91. The length of one of the Statue of Liberty's hands is 16 ft 5 in. One of the statue's eyes is 2 ft 6 in. across. How much longer is a hand than the width of an eye? (*Source:* National Park Service)

92. The width of the Statue of Liberty's tablet is 13 ft 7 in. The height of the statue's head from chin to cranium is 17 ft 3 in. How much longer is the statue's head than the width of its tablet? (*Source:* National Park Service)

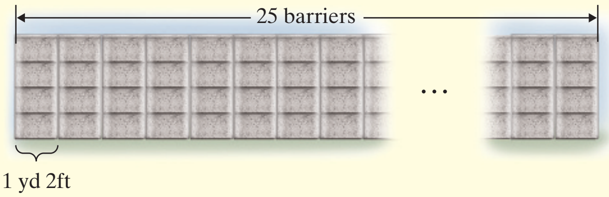
93. The ice on a pond is 5.33 cm thick. For safe skating, the owner of the pond insists that it be 80 mm thick. How much thicker must the ice be before skating is allowed?

94. The sediment on the bottom of the Towamencin Creek is normally 14 cm thick, but the recent flood washed away 22 mm of sediment. How thick is it now?

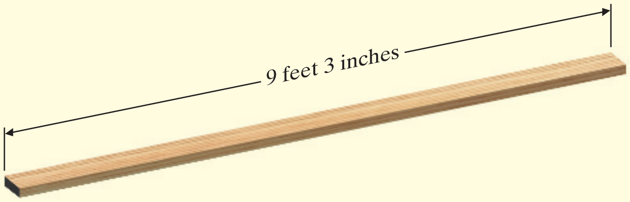
95. The Amana Corporation stacks up its microwave ovens in a distribution warehouse. Each stack is 1 ft 9 in. wide. How far from the wall would 9 of these stacks extend?



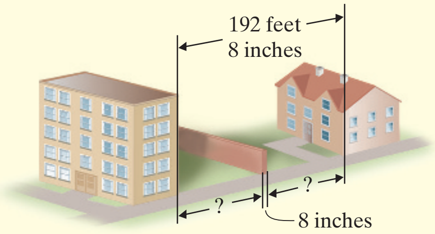
96. The highway commission is installing concrete sound barriers along a highway. Each barrier is 1 yd 2 ft long. Find the total length of 25 barriers placed end to end.



97. A carpenter needs to cut a board into thirds. If the board is 9 ft 3 in. long originally, how long will each cut piece be?



98. A wall is erected exactly halfway between two buildings that are 192 ft 8 in. apart. If the wall is 8 in. wide, how far is it from the wall to either of the buildings?



99. An art class is learning how to make kites. The two sticks used for each kite have lengths of 1 m and 65 cm. What total length of wood must be ordered for the sticks if 25 kites are to be built?

100. The total pages of a hardbound economics text are 3.1 cm thick. The front and back covers are each 2 mm thick. How high would a stack of 10 of these texts be?

101. A logging firm needs to cut a 67-m-long redwood log into 20 equal pieces before loading it onto a truck for shipment. How long will each piece be?

102. An 18.3-m-tall flagpole is mounted on a 65-cm-high pedestal. How far is the top of the flagpole from the ground?

103. A 2.15-m-long sash cord has become frayed at both ends. To correct this, 1 cm is trimmed from each end. How long is the remaining cord?

104. A 112.5-foot-tall dead pine tree is removed by starting at the top and cutting off 9-foot-long sections. How many whole sections are removed?

105. The giant Coca-Cola billboard in Times Square is lit by more than 2.6 million LEDs, which are completely powered by wind energy. This huge advertisement measures approximately 44 feet by 65 feet. Find the perimeter of the sign in yards. (*Source:* Coca-Cola Company)

106. The longest truck in the world is operated by Gould Transport in Australia, and is the 182-ft Road Train. How many *yards* long are 2 of these trucks? (*Source:* Guinness World Records)

107. A floor tile is 22.86 cm wide. How many tiles in a row are needed to cross a room 3.429 m wide?

△ **108.** A standard postcard is 1.6 times longer than it is wide. If it is 9.9 cm wide, what is its length?

Review

Write each decimal as a fraction and each fraction as a decimal. See Sections 4.1 and 4.5.

109. 0.21 **110.** 0.86 **111.** $\frac{13}{100}$ **112.** $\frac{47}{100}$ **113.** $\frac{1}{4}$ **114.** $\frac{3}{20}$

Concept Extensions

Determine whether the measurement in each statement is reasonable. See the second Concept Check in this section.

115. The width of a twin-size bed is 20 meters.

116. A window measures 1 meter by 0.5 meter.

117. A drinking glass is made of glass 2 millimeters thick.

118. A paper clip is 4 kilometers long.

119. The distance across the Colorado River is 50 kilometers.

120. A model's hair is 30 centimeters long.

Estimate each sum or difference. See the first Concept Check in this section.

121.
$$\begin{array}{r} 5\text{ yd}\ \ \ 2\text{ in.} \\ \underline{+\ 7\text{ yd}\ 30\text{ in.}} \end{array}$$

122.
$$\begin{array}{r} 45\text{ ft}\ \ \ 1\text{ in.} \\ \underline{-\ 10\text{ ft}\ 11\text{ in.}} \end{array}$$

Solve.

123. Using a unit other than the foot, write a length that is equivalent to 4 feet. (*Hint:* There are many possibilities.)

124. Using a unit other than the meter, write a length that is equivalent to 7 meters. (*Hint:* There are many possibilities.)

125. To convert from meters to centimeters, the decimal point is moved two places to the right. Explain how this relates to the fact that the prefix *centi* means $\frac{1}{100}$.

126. Explain why conversions in the metric system are easier to make than conversions in the U.S. system of measurement.

127. An advertisement sign outside Fenway Park in Boston measures 18.3 m by 18.3 m. What is the area of this sign?

5.5 Weight and Mass: U.S. and Metric Systems of Measurement

Objective A Defining and Converting U.S. System Units of Weight

Whenever we talk about how heavy an object is, we are concerned with the object's **weight.** We discuss weight when we refer to a 12-ounce box of Rice Krispies, a 15-pound tabby cat, and a barge hauling 24 tons of garbage.

12 ounces

15 pounds

24 tons of garbage

Objectives

- **A** Define U.S. Units of Weight and Convert from One Unit to Another.
- **B** Perform Arithmetic Operations on U.S. Units of Weight.
- **C** Define Metric Units of Mass and Convert from One Unit to Another.
- **D** Perform Arithmetic Operations on Metric Units of Mass.

The most common units of weight in the U.S. measurement system are the **ounce,** the **pound,** and the **ton.** The following is a summary of equivalencies between units of weight:

U.S. Units of Weight	**Unit Fractions**
16 ounces (oz) = 1 pound (lb)	$\frac{16\text{ oz}}{1\text{ lb}} = \frac{1\text{ lb}}{16\text{ oz}} = 1$
2000 pounds = 1 ton	$\frac{2000\text{ lb}}{1\text{ ton}} = \frac{1\text{ ton}}{2000\text{ lb}} = 1$

✓Concept Check If you were describing the weight of a fully loaded semi-trailer, which type of unit would you use: ounce, pound, or ton? Why?

Unit fractions, which equal 1, are used to convert between units of weight in the U.S. system. When converting using unit fractions, recall that the numerator of a unit fraction should contain the units we are converting to and the denominator should contain the original units.

Practice 1

Convert 6500 pounds to tons.

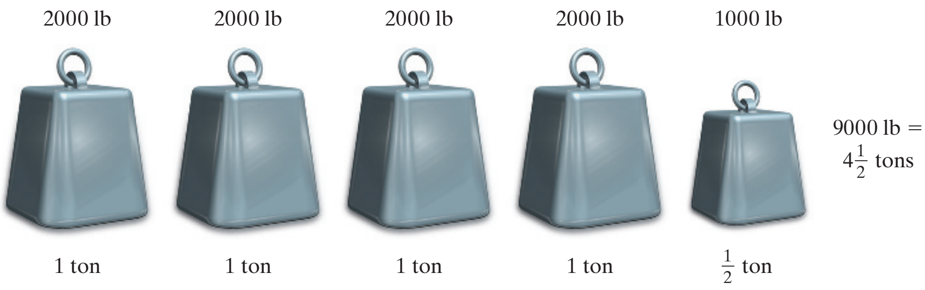
Example 1 Convert 9000 pounds to tons.

Solution: We multiply 9000 lb by a unit fraction that uses the equality

2000 pounds = 1 ton

Remember, the unit fraction should be $\frac{\text{units to convert to}}{\text{original units}}$ or $\frac{1\text{ ton}}{2000\text{ lb}}$.

$$9000\text{ lb} = \frac{9000\text{ lb}}{1}\cdot 1 = \frac{9000\ \cancel{\text{lb}}}{1}\cdot\frac{1\text{ ton}}{2000\ \cancel{\text{lb}}} = \frac{9000\text{ tons}}{2000} = \frac{9}{2}\text{ tons or }4\frac{1}{2}\text{ tons}$$



Work Practice 1

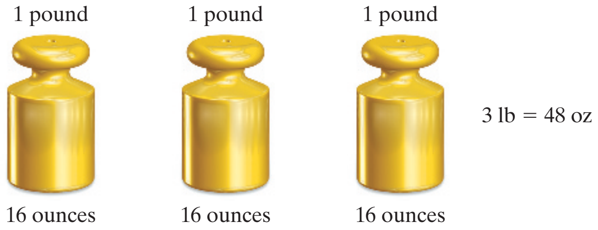
Practice 2

Convert 72 ounces to pounds.

Example 2 Convert 3 pounds to ounces.

Solution: We multiply by the unit fraction $\frac{16\text{ oz}}{1\text{ lb}}$ to convert from pounds to ounces.

$$3\text{ lb} = \frac{3\text{ lb}}{1}\cdot 1 = \frac{3\ \cancel{\text{lb}}}{1}\cdot\frac{16\text{ oz}}{1\ \cancel{\text{lb}}} = 3\cdot 16\text{ oz} = 48\text{ oz}$$



Work Practice 2

Answers

1. $3\frac{1}{4}$ tons **2.** $4\frac{1}{2}$ lb

✓Concept Check Answer

ton

As with length, it is sometimes useful to simplify a measurement of weight by writing it in terms of mixed units.

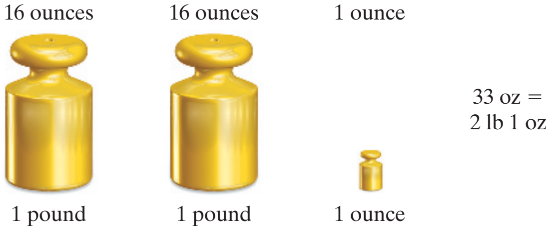
Example 3 Convert: 33 ounces = _____ lb _____ oz

Solution: Because 16 oz = 1 lb, divide 16 into 33 to see how many pounds are in 33 ounces. The quotient is the number of pounds, and the remainder is the number of ounces. To see why we divide 16 into 33, notice that

$$33 \text{ oz} = 33 \cancel{\text{oz}} \cdot \frac{1 \text{ lb}}{16 \cancel{\text{oz}}} = \frac{33}{16} \text{ lb}$$

$$\begin{array}{r} 2 \text{ lb } 1 \text{ oz} \\ 16\overline{)33} \\ \underline{-32} \\ 1 \end{array}$$

Thus, 33 ounces is the same as 2 lb 1 oz.



Work Practice 3

Practice 3

Convert:

47 ounces = _____ lb _____ oz

Objective B Performing Operations on U.S. System Units of Weight

Performing arithmetic operations on units of weight works the same way as performing arithmetic operations on units of length.

Example 4 Subtract 3 tons 1350 lb from 8 tons 1000 lb.

Solution: To subtract, we line up similar units.

$$\begin{array}{r} 8 \text{ tons } 1000 \text{ lb} \\ \underline{-\,3 \text{ tons } 1350 \text{ lb}} \end{array}$$

Since we cannot subtract 1350 lb from 1000 lb, we borrow 1 ton from the 8 tons. To do so, we write 1 ton as 2000 lb and combine it with the 1000 lb.

$$\underbrace{7 \text{ tons} + \boxed{1 \text{ ton}}}\ 2000 \text{ lb}$$

$$\begin{array}{rcr} \cancel{8} \text{ tons } 1000 \text{ lb} & = & 7 \text{ tons } 3000 \text{ lb} \\ \underline{-\,3 \text{ tons } 1350 \text{ lb}} & = & \underline{-\,3 \text{ tons } 1350 \text{ lb}} \\ & & 4 \text{ tons } 1650 \text{ lb} \end{array}$$

To check, see that the sum of 4 tons 1650 lb and 3 tons 1350 lb is 8 tons 1000 lb.

Work Practice 4

Practice 4

Subtract 5 tons 1200 lb from 8 tons 100 lb.

Answers

3. 2 lb 15 oz **4.** 2 tons 900 lb

Practice 5

Divide 5 lb 8 oz by 4.

Example 5

Divide 9 lb 6 oz by 2.

Solution: We divide each of the units by 2.

$$\begin{array}{rrr}
 & 4\text{ lb} & 11\text{ oz}\\
2\big) & 9\text{ lb} & 6\text{ oz}\\
 & -8 & \\
\hline
 & 1\text{ lb} = & 16\text{ oz}\\
\hline
 & & 22\text{ oz}
\end{array}$$

Divide 2 into 22 oz to get 11 oz.

To check, multiply 4 pounds 11 ounces by 2. The result is 9 pounds 6 ounces.

Work Practice 5

Practice 6

A 5-lb 14-oz batch of cookies is packed into a 6-oz container before it is mailed. Find the total weight.

Example 6 Finding the Weight of a Child

Bryan weighed 8 lb 8 oz at birth. By the time he was 1 year old, he had gained 11 lb 14 oz. Find his weight at age 1 year.

Solution:

$$\begin{array}{rcr}
\text{birth weight} & \rightarrow & 8\text{ lb }\ 8\text{ oz}\\
+\ \text{weight gained} & \rightarrow & +\ 11\text{ lb }14\text{ oz}\\
\hline
\text{total weight} & \rightarrow & 19\text{ lb }22\text{ oz}
\end{array}$$

Since 22 oz equals 1 lb 6 oz,

$$\begin{aligned}
19\text{ lb }22\text{ oz} &= 19\text{ lb} + \overbrace{1\text{ lb }6\text{ oz}}\\
&= 20\text{ lb }6\text{ oz}
\end{aligned}$$

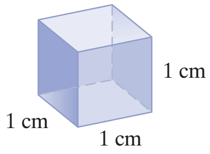
Bryan weighed 20 lb 6 oz on his first birthday.

Work Practice 6

Objective C Defining and Converting Metric System Units of Mass

In scientific and technical areas, a careful distinction is made between **weight** and **mass. Weight** is really a measure of the pull of gravity. The farther from Earth an object gets, the less it weighs. However, **mass** is a measure of the amount of substance in the object and does not change. Astronauts orbiting Earth weigh much less than they weigh on Earth, but they have the same mass in orbit as they do on Earth. Here on Earth, weight and mass are the same, so either term may be used.

The basic unit of mass in the metric system is the **gram.** It is defined as the mass of water contained in a cube 1 centimeter (cm) on each side.



The following examples may help you get a feeling for metric masses:

Answers

5. 1 lb 6 oz **6.** 6 lb 4 oz

Examples of Metric System—Weight and Mass

 • A tablet contains 200 milligrams of ibuprofen.	 • A large paper clip weighs approximately 1 gram.
 • NET WEIGHT 16.5 OZ (468 g)	 • A kilogram is slightly over 2 pounds. • An adult woman may weigh 60 kilograms.

The prefixes for units of mass in the metric system are the same as for units of length, as shown in the following table:

Metric Units of Mass
1 **kilo**gram (kg) = 1000 grams (g)
1 **hecto**gram (hg) = 100 g
1 **deka**gram (dag) = 10 g
1 gram (g) = 1 g
1 **deci**gram (dg) = 1/10 g or 0.1 g
1 **centi**gram (cg) = 1/100 g or 0.01 g
1 **milli**gram (mg) = 1/1000 g or 0.001 g

✓Concept Check True or false? A decigram is larger than a dekagram. Explain.

The **milligram,** the **gram,** and the **kilogram** are the three most commonly used units of mass in the metric system.

As with lengths, all units of mass are powers of 10 of the gram, so converting from one unit of mass to another only involves moving the decimal point. To convert from one unit of mass to another in the metric system, list the units of mass in order from largest to smallest.

Let's convert 4300 milligrams to grams. To convert from milligrams to grams, we move along the list 3 units to the left.

kg   hg   dag   **g**   dg   cg   **mg**

3 units to the left

This means that we move the decimal point 3 places to the left to convert from milligrams to grams.

$4300\text{ mg} = 4.3\text{ g}$

Don't forget that the same conversion can be done with unit fractions.

✓Concept Check Answer

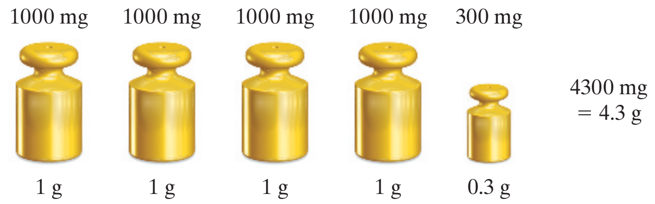
false

$$4300\text{ mg} = \frac{4300\text{ mg}}{1}\cdot 1 = \frac{4300\ \cancel{\text{mg}}}{1}\cdot\frac{0.001\text{ g}}{1\ \cancel{\text{mg}}}$$

$$= 4300\cdot 0.001\text{ g}$$

$$= 4.3\text{ g}$$ To multiply by 0.001, move the decimal point 3 places to the left.

To see that this is reasonable, study the diagram:



Thus, 4300 mg = 4.3 g.

Practice 7

Convert 3.41 g to milligrams.

Example 7

Convert 3.2 kg to grams.

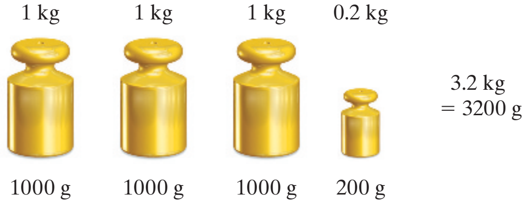
Solution: First we convert by using a unit fraction.

$$3.2\text{ kg} = 3.2\text{ kg}\cdot 1 = 3.2\ \cancel{\text{kg}}\cdot\overbrace{\frac{1000\text{ g}}{1\ \cancel{\text{kg}}}}^{\text{Unit fraction}} = 3200\text{ g}$$

Now let's list the units of mass in order from left to right and move from kilograms to grams.

$$\underbrace{\text{kg}\quad\text{hg}\quad\text{dag}\quad\text{g}}_{\text{3 units to the right}}\quad\text{dg}\quad\text{cg}\quad\text{mg}$$

$$3.\underbrace{200}_{\text{3 places to the right}}\text{ kg} = 3200.\text{ g}$$



Work Practice 7

Practice 8

Convert 56.2 cg to grams.

Example 8

Convert 2.35 cg to grams.

Solution: We list the units of mass in a chart and move from centigrams to grams.

$$\text{kg}\quad\text{hg}\quad\text{dag}\quad\underbrace{\text{g}\quad\text{dg}\quad\text{cg}}_{\text{2 units to the left}}\quad\text{mg}$$

$$\underbrace{02.}_{\text{2 places to the left}}35\text{ cg} = 0.0235\text{ g}$$

Work Practice 8

Objective D Performing Operations on Metric System Units of Mass

Arithmetic operations can be performed with metric units of mass just as we performed operations with metric units of length. We convert each number to the same unit of mass and add, subtract, multiply, or divide as with decimals.

Answers

7. 3410 mg **8.** 0.562 g

Example 9 Subtract 5.4 dg from 1.6 g.

Solution: We convert both numbers to decigrams or to grams before subtracting.

$$\begin{array}{rcl} 5.4\text{ dg} = 0.54\text{ g} & \quad\text{or}\quad & 1.6\text{ g} = 16\text{ dg} \\ 1.60\text{ g} & & 16.0\text{ dg} \\ \underline{-0.54\text{ g}} & & \underline{-5.4\text{ dg}} \\ 1.06\text{ g} & & 10.6\text{ dg} \end{array}$$

The difference is 1.06 g or 10.6 dg.

Work Practice 9

Example 10 Calculating Allowable Weight in an Elevator

An elevator has a weight limit of 1400 kg. A sign posted in the elevator indicates that the maximum capacity of the elevator is 17 persons. What is the average allowable weight for each passenger, rounded to the nearest kilogram?

Solution: To solve, notice that the total weight of 1400 kilograms $\div$ 17 $=$ average weight.

$$\begin{array}{r} 82.3\text{ kg} \approx 82\text{ kg} \\ 17\overline{)\ 1400.0\text{ kg}} \\ \underline{-136}\phantom{.0\text{ kg}} \\ 40\phantom{.0\text{ kg}} \\ \underline{-34}\phantom{.0\text{ kg}} \\ 6\,0\phantom{\text{ kg}} \\ \underline{-5\,1}\phantom{\text{ kg}} \\ 9\phantom{\text{ kg}} \end{array}$$

Each passenger can weigh an average of 82 kg. (Recall that a kilogram is slightly over 2 pounds, so 82 kilograms is over 164 pounds.)

Work Practice 10

Practice 9

Subtract 3.1 dg from 2.5 g.

Practice 10

Twenty-four bags of cement weigh a total of 550 kg. Find the average weight of 1 bag, rounded to the nearest kilogram.

Answers

9. 2.19 g or 21.9 dg **10.** 23 kg

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

mass weight gram

1. ____________ is a measure of the amount of substance in an object. This measure does not change.

2. ____________ is the measure of the pull of gravity.

3. The basic unit of mass in the metric system is the ____________.

Fill in these blanks with the correct number. Choices for these blanks are not shown in the list of terms above.

4. One pound equals ____________ ounces.

5. One ton equals ____________ pounds.

Convert without pencil or paper.

6. 3 tons to pounds **7.** 32 ounces to pounds **8.** 2 pounds to ounces **9.** 4000 pounds to tons **10.** 1 ton to pounds

Martin-Gay Interactive Videos

See Video 5.5

Watch the section lecture video and answer the following questions.

Objective A **11.** In Example 2, what units are used in the numerator of the unit fraction and why was this decided?

Objective B **12.** In Example 4, explain the first step taken to solve the problem.

Objective C **13.** In Example 5, how many places is the decimal point moved and in what direction? What is the final conversion?

Objective D **14.** What is the answer to Example 7 in decigrams?

5.5 Exercise Set MyLab Math

Objective A *Convert as indicated. See Examples 1 through 3.*

1. 2 pounds to ounces

2. 5 pounds to ounces

3. 5 tons to pounds

4. 7 tons to pounds

5. 18,000 pounds to tons

6. 28,000 pounds to tons

7. 60 ounces to pounds

8. 90 ounces to pounds

9. 3500 pounds to tons

10. 11,000 pounds to tons

11. 12.75 pounds to ounces

12. 9.5 pounds to ounces

13. 4.9 tons to pounds

14. 8.3 tons to pounds

15. $4\frac{3}{4}$ pounds to ounces

16. $9\frac{1}{8}$ pounds to ounces

17. 2950 pounds to the nearest tenth of a ton

18. 51 ounces to the nearest tenth of a pound

19. $\frac{4}{5}$ oz to pounds

20. $\frac{1}{4}$ oz to pounds

21. $5\frac{3}{4}$ lb to ounces

22. $2\frac{1}{4}$ lb to ounces

23. 10 lb 1 oz to ounces

24. 7 lb 6 oz to ounces

25. 89 oz = _____ lb _____ oz

26. 100 oz = _____ lb _____ oz

Objective B *Perform each indicated operation. See Examples 4 through 6.*

27. 34 lb 12 oz + 18 lb 14 oz

28. 6 lb 10 oz + 10 lb 8 oz

29. 3 tons 1820 lb + 4 tons 930 lb

30. 1 ton 1140 lb + 5 tons 1200 lb

31. 5 tons 1050 lb − 2 tons 875 lb

32. 4 tons 850 lb − 1 ton 260 lb

33. 12 lb 4 oz − 3 lb 9 oz

34. 45 lb 6 oz − 26 lb 10 oz

35. 5 lb 3 oz × 6

36. 2 lb 5 oz × 5

37. 6 tons 1500 lb ÷ 5

38. 5 tons 400 lb ÷ 4

Objective C *Convert as indicated. See Examples 7 and 8.*

39. 500 g to kilograms

40. 820 g to kilograms

41. 4 g to milligrams

42. 9 g to milligrams

43. 25 kg to grams

44. 18 kg to grams

45. 48 mg to grams

46. 112 mg to grams

47. 6.3 g to kilograms

48. 4.9 g to kilograms

49. 15.14 g to milligrams

50. 16.23 g to milligrams

51. 6.25 kg to grams

52. 3.16 kg to grams

53. 35 hg to centigrams

54. 4.26 cg to dekagrams

Objective D *Perform each indicated operation. Remember to insert units when writing your answers. See Examples 9 and 10.*

55. 3.8 mg + 9.7 mg

56. 41.6 g + 9.8 g

57. 205 mg + 5.61 g

58. 2.1 g + 153 mg

59. 9 g − 7150 mg

60. 6.13 g − 418 mg

61. 1.61 kg − 250 g

62. 4 kg − 2410 g

63. 5.2 kg × 2.6

64. 4.8 kg × 9.3

65. 17 kg ÷ 8

66. 8.25 g ÷ 6

Objectives A B C D Mixed Practice *Solve. Remember to insert units when writing your answers. For Exercises 67 through 74, complete the charts. See Examples 1 through 10.*

	Object	Tons	Pounds	Ounces
67.	Statue of Liberty–weight of copper sheeting	100		
68.	Statue of Liberty–weight of steel	125		
69.	 A 12-inch cube of osmium (heaviest metal)		1345	
70.	 A 12-inch cube of lithium (lightest metal)		32	

	Object	Grams	Kilograms	Milligrams	Centigrams
71.	Capsule of amoxicillin (antibiotic)			500	
72.	Tablet of Topamax (epilepsy and migraine uses)			25	
73.	A six-year-old boy		21		
74.	A golf ball	45			

75. A can of 7-Up weighs 336 grams. Find the weight in kilograms of 24 cans.

77. Elizabeth Hamtini was amazed when she grew a 28-lb 10-oz zucchini in her garden, but later she learned that the heaviest zucchini ever grown weighed 64 lb 8 oz in Llanharry, Wales, by B. Lavery in 1990. How far below the record weight was Elizabeth's zucchini? (*Source:* Guinness World Records)

79. The supermarket prepares hamburger in 450-gram market packages. When Leo Gonzalas gets home, he divides the package in fourths before refrigerating the meat. How much will each divided package weigh?

81. A chef has two open containers of whole grain brown rice. If she combines 1 lb 10 oz from one container with 3 lb 14 oz from the other container, how much total rice does she have?

83. The smallest surviving baby weighed only 8 ounces, less than an average grapefruit. She was born in Witten, Germany, in December 2015. How much lighter was she than an average European baby, who weighs about 7 lb 7.5 oz?

85. Three milligrams of preservatives are added to a 0.5-kg box of dried fruit. How many milligrams of preservatives are in 3 cartons of dried fruit if each carton contains 16 boxes?

87. A carton of 12 boxes of Quaker Oats Oatmeal weighs 6.432 kg. Each box includes 26 grams of packaging material. What is the actual weight of the oatmeal in the carton?

89. The Shop 'n Bag supermarket chain ships hamburger meat by placing 10 packages of hamburger in a box, with each package weighing 3 lb 4 oz. How much will 4 boxes of hamburger weigh?

91. A carton of Del Monte Pineapple weighs 55 lb 4 oz, but 2 lb 8 oz of this weight is due to packaging. Find the actual weight of the pineapple in 4 cartons.

76. Guy Green normally weighs 73 kg, but he lost 2800 grams after being sick with the flu. Find Guy's new weight.

78. A small can of Planters sunflower seeds weighs 177 g. If each can contains 6 servings, find the weight of one serving.

80. Sudafed is a decongestant that comes in two strengths. Regular strength contains 60 mg of medication. Extra strength contains 0.09 g of medication. How much extra medication is in the extra-strength tablet?

82. Dru Mizel maintains the records of the amount of coal delivered to his department in the steel mill. In January, 3 tons 1500 lb were delivered. In February, 2 tons 1200 lb were delivered. Find the total amount delivered in these two months.

84. The heaviest baby born in good health weighed an incredible 22 lb 8 oz. He was born in Italy in September 1955. How much heavier is this than a 7-lb 12-oz baby? (*Source:* Guinness World Records)

86. A large bottle of Hire's Root Beer weighs 1900 grams. If a carton contains 6 large bottles of root beer, find the weight in kilograms of 5 cartons.

88. One box of Swiss Miss Cocoa Mix weighs 0.385 kg, but 39 grams of this weight is the packaging. Find the actual weight of the cocoa in 8 boxes.

90. The Quaker Oats Company ships its 1-lb 2-oz boxes of oatmeal in cartons containing 12 boxes of oatmeal. How much will 3 such cartons weigh?

92. The Hormel Corporation ships cartons of canned ham weighing 43 lb 2 oz each. Of this weight, 3 lb 4 oz is due to packaging. Find the actual weight of the ham found in 3 cartons.

Review

Write each fraction as a decimal. See Section 4.5.

93. $\frac{4}{25}$

94. $\frac{3}{5}$

95. $\frac{7}{8}$

96. $\frac{3}{16}$

Concept Extensions

Determine whether the measurement in each statement is reasonable.

97. The doctor prescribed a pill containing 2 kg of medication.

98. A full-grown cat weighs approximately 15 g.

99. A bag of flour weighs 4.5 kg.

100. A staple weighs 15 mg.

101. A professor weighs less than 150 g.

102. A car weighs 2000 mg.

Solve.

103. Use a unit other than centigram and write a mass that is equivalent to 25 centigrams. (*Hint:* There are many possibilities.)

104. Use a unit other than pound and write a weight that is equivalent to 4000 pounds. (*Hint:* There are many possibilities.)

True or false? See the second Concept Check in this section.

105. A kilogram is larger than a gram.

106. A decigram is larger than a milligram.

107. Why is the decimal point moved to the right when grams are converted to milligrams?

108. To change 8 pounds to ounces, multiply by 16. Why is this the correct procedure?

5.6 Capacity: U.S. and Metric Systems of Measurement

Objectives

A Define U.S. Units of Capacity and Convert from One Unit to Another.

B Perform Arithmetic Operations on U.S. Units of Capacity.

C Define Metric Units of Capacity and Convert from One Unit to Another.

D Perform Arithmetic Operations on Metric Units of Capacity.

Objective A Defining and Converting U.S. System Units of Capacity

Units of **capacity** are generally used to measure liquids. The number of gallons of gasoline needed to fill a gas tank in a car, the number of cups of water needed in a bread recipe, and the number of quarts of milk sold each day at a supermarket are all examples of using units of capacity. The following summary shows equivalencies between units of capacity:

U.S. Units of Capacity

$$8 \text{ fluid ounces (fl oz)} = 1 \text{ cup (c)}$$
$$2 \text{ cups} = 1 \text{ pint (pt)}$$
$$2 \text{ pints} = 1 \text{ quart (qt)}$$
$$4 \text{ quarts} = 1 \text{ gallon (gal)}$$

Just as with units of length and weight, we can form unit fractions to convert between different units of capacity. For instance,

$$\frac{2\text{ c}}{1\text{ pt}} = \frac{1\text{ pt}}{2\text{ c}} = 1 \quad \text{and} \quad \frac{2\text{ pt}}{1\text{ qt}} = \frac{1\text{ qt}}{2\text{ pt}} = 1$$

Practice 1

Convert 43 pints to quarts.

Example 1

Convert 9 quarts to gallons.

Solution: We multiply by the unit fraction $\frac{1\text{ gal}}{4\text{ qt}}$.

$$\begin{aligned} 9\text{ qt} &= \frac{9\text{ qt}}{1} \cdot 1 \\ &= \frac{9\text{ }\cancel{\text{qt}}}{1} \cdot \frac{1\text{ gal}}{4\text{ }\cancel{\text{qt}}} \\ &= \frac{9\text{ gal}}{4} \\ &= 2\frac{1}{4}\text{ gal} \end{aligned}$$

Thus, 9 quarts is the same as $2\frac{1}{4}$ gallons, as shown in the diagram:

Work Practice 1

Practice 2

Convert 26 quarts to cups.

Example 2

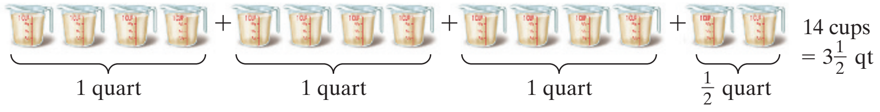
Convert 14 cups to quarts.

Solution: Our equivalency table contains no direct conversion from cups to quarts. However, from this table we know that

$$1\text{ qt} = 2\text{ pt} = \frac{2\text{ pt}}{1} \cdot 1 = \frac{2\text{ pt}}{1} \cdot \frac{2\text{ c}}{1\text{ pt}} = 4\text{ c}$$

so 1 qt = 4 c. Now we have the unit fraction $\frac{1\text{ qt}}{4\text{ c}}$. Thus,

$$14\text{ c} = \frac{14\text{ c}}{1} \cdot 1 = \frac{14\text{ }\cancel{\text{c}}}{1} \cdot \frac{1\text{ qt}}{4\text{ }\cancel{\text{c}}} = \frac{14\text{ qt}}{4} = \frac{7}{2}\text{ qt} \quad \text{or} \quad 3\frac{1}{2}\text{ qt}$$



Work Practice 2

Answers

1. $21\frac{1}{2}$ qt **2.** 104 c

✓Concept Check Answer

less than 50; answers may vary

✓Concept Check If 50 cups is converted to quarts, will the equivalent number of quarts be less than or greater than 50? Explain.

Objective B Performing Operations on U.S. System Units of Capacity

As is true of units of length and weight, units of capacity can be added, subtracted, multiplied, and divided.

Example 3 Subtract 3 qt from 4 gal 2 qt.

Solution: To subtract, we line up similar units.

$$\begin{array}{r} 4\text{ gal } 2\text{ qt} \\ -\qquad 3\text{ qt} \\ \hline \end{array}$$

We cannot subtract 3 qt from 2 qt. We need to borrow 1 gallon from the 4 gallons, convert it to 4 quarts, and then combine it with the 2 quarts.

$$\begin{array}{rcr} \underbrace{3\text{ gal} + \boxed{1\text{ gal}}}\ \curvearrowright\ 4\text{ qt} & & \\ 4\text{ gal } 2\text{ qt} & = & 3\text{ gal } 6\text{ qt} \\ -\qquad 3\text{ qt} & = & -\qquad 3\text{ qt} \\ \hline & & 3\text{ gal } 3\text{ qt} \end{array}$$

To check, see that the sum of 3 gal 3 qt and 3 qt is 4 gal 2 qt.

Work Practice 3

Example 4 Divide 3 gal 2 qt by 2.

Solution: We divide each unit of capacity by 2.

$$\begin{array}{rll} 1\text{ gal} & 3\text{ qt} & \\ 2\overline{)3\text{ gal}} & \overline{2\text{ qt}} & \\ \underline{-2} & & \\ 1\text{ gal} = & \underline{4\text{ qt}} & \text{Convert 1 gallon to 4 qt and add to 2 qt before continuing.} \\ & 6\text{ qt} & 6\text{ qt} \div 2 = 3\text{ qt} \end{array}$$

Work Practice 4

Example 5 Finding the Amount of Water in an Aquarium

An aquarium contains 6 gal 3 qt of water. If 2 gal 2 qt of water is added, what is the total amount of water in the aquarium?

Solution:

$$\begin{array}{rcr} \text{beginning water} & \rightarrow & 6\text{ gal } 3\text{ qt} \\ +\quad \underline{\text{water added}} & \rightarrow & \underline{+\ 2\text{ gal } 2\text{ qt}} \\ \text{total water} & \rightarrow & 8\text{ gal } 5\text{ qt} \end{array}$$

Since 5 qt = 1 gal 1 qt, we have

$$\begin{aligned} &= \overbrace{8\text{ gal}}^{8\text{ gal}} + \overbrace{1\text{ gal } 1\text{ qt}}^{5\text{ qt}} \\ &= 9\text{ gal } 1\text{ qt} \end{aligned}$$

The total amount of water is 9 gal 1 qt.

Work Practice 5

Practice 3

Subtract 2 qt from 1 gal 1 qt.

Practice 4

Divide 7 gal 2 qt by 3.

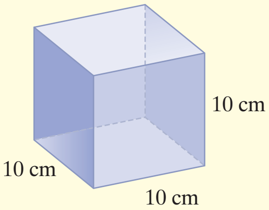
Practice 5

A large oil drum contains 15 gal 3 qt of oil. How much will be in the drum if an additional 4 gal 3 qt of oil is poured into it?

Answers

3. 3 qt **4.** 2 gal 2 qt **5.** 20 gal 2 qt

Objective C Defining and Converting Metric System Units of Capacity



Thus far, we know that the basic unit of length in the metric system is the meter and that the basic unit of mass in the metric system is the gram. What is the basic unit of capacity? The **liter.** By definition, a **liter** is the capacity or volume of a cube measuring 10 centimeters on each side.

The following examples may help you get a feeling for metric capacities:

Examples of Metric System—Capacity

- One liter of liquid is slightly more than one quart.

2 liters

- Many soft drinks are packaged in 2-liter bottles.

- Liquid infant pain reliever doses in milliliters (ml or mL)

The metric system was designed to be a consistent system. Once again, the prefixes for metric units of capacity are the same as for metric units of length and mass, as summarized in the table to the right:

Metric Units of Capacity		
1 **kilo**liter (kl)	=	1000 liters (L)
1 **hecto**liter (hl)	=	100 L
1 **deka**liter (dal)	=	10 L
1 liter (L)	**=**	**1 L**
1 **deci**liter (dl)	=	1/10 L or 0.1 L
1 **centi**liter (cl)	=	1/100 L or 0.01 L
1 **milli**liter (ml)	=	1/1000 L or 0.001 L

The **milliliter** and the **liter** are the two most commonly used metric units of capacity.

Converting from one unit of capacity to another involves multiplying by powers of 10 or moving the decimal point to the left or to the right. Listing units of capacity in order from largest to smallest helps to keep track of how many places to move the decimal point when converting.

Let's convert 2.6 liters to milliliters. To convert from liters to milliliters, we move along the chart 3 units to the right.

$$\text{kl} \quad \text{hl} \quad \text{dal} \quad \underbrace{\textbf{L} \quad \text{dl} \quad \text{cl} \quad \textbf{ml}}_{\text{3 units to the right}}$$

This means that we move the decimal point 3 places to the right to convert from liters to milliliters.

$$2.\underbrace{600}\ \text{L} = 2600.\ \text{ml}$$

This same conversion can be done with unit fractions.

$$\begin{aligned} 2.6\ \text{L} &= \frac{2.6\ \text{L}}{1} \cdot 1 \\ &= \frac{2.6\ \cancel{\text{L}}}{1} \cdot \frac{1000\ \text{ml}}{1\ \cancel{\text{L}}} \\ &= 2.6 \cdot 1000\ \text{ml} \\ &= 2600\ \text{ml} \quad \text{To multiply by 1000, move the decimal point 3 places to the right.} \end{aligned}$$

To visualize the result, study the diagram below:

Thus, 2.6 L = 2600 ml.

Example 6 Convert 3210 ml to liters.

Solution: Let's use the unit fraction method first.

$$3210\text{ ml} = \frac{3210\text{ ml}}{1} \cdot 1 = 3210\,\cancel{\text{ml}} \cdot \overbrace{\frac{1\text{ L}}{1000\,\cancel{\text{ml}}}}^{\text{Unit fraction}} = 3.21\text{ L}$$

Now let's list the unit measures in order from left to right and move from milliliters to liters.

kl hl dal L dl cl ml

3 units to the left

3210 ml = 3.210 L, the same results as before and shown below in the diagram.

3 places to the left

Work Practice 6

Practice 6

Convert 2100 ml to liters.

Example 7 Convert 0.185 dl to milliliters.

Solution: We list the unit measures in order from left to right and move from deciliters to milliliters.

kl hl dal L dl cl ml

2 units to the right

0.185 dl = 18.5 ml

2 places to the right

Work Practice 7

Practice 7

Convert 2.13 dal to liters.

Objective D Performing Operations on Metric System Units of Capacity

As was true for length and weight, arithmetic operations involving metric units of capacity can also be performed. Make sure that the metric units of capacity are the same before adding or subtracting.

Answers

6. 2.1 L **7.** 21.3 L

Practice 8

Add 1250 ml to 2.9 L.

Example 8 Add 2400 ml to 8.9 L.

Solution: We must convert both to liters or both to milliliters before adding the capacities together.

$$2400\text{ ml} = 2.4\text{ L} \qquad \text{or} \qquad 8.9\text{ L} = 8900\text{ ml}$$

$$\begin{array}{r} 2.4\text{ L} \\ +\ 8.9\text{ L} \\ \hline 11.3\text{ L} \end{array} \qquad\qquad \begin{array}{r} 2400\text{ ml} \\ +\ 8900\text{ ml} \\ \hline 11{,}300\text{ ml} \end{array}$$

The total is 11.3 L or 11,300 ml. They both represent the same capacity.

Work Practice 8

✓Concept Check How could you estimate the following operation? Subtract 950 ml from 7.5 L.

Practice 9

If 28.6 L of water can be pumped every minute, how much water can be pumped in 85 minutes?

Example 9 Finding the Amount of Medication a Person Receives

A patient hooked up to an IV unit in the hospital is to receive 12.5 ml of medication every hour. How much medication does the patient receive in 3.5 hours?

Solution: We multiply 12.5 ml by 3.5.

$$\begin{array}{rcr} \text{medication per hour} & \rightarrow & 12.5\text{ ml} \\ \times \qquad\qquad \text{hours} & \rightarrow & \times\ 3.5 \\ \hline \text{total medication} & & 625 \\ & & 3750 \\ \hline & & 43.75\text{ ml} \end{array}$$

The patient receives 43.75 ml of medication.

Work Practice 9

Answers

8. 4150 ml or 4.15 L **9.** 2431 L

✓Concept Check Answer

950 ml = 0.95 L; round 0.95 to 1; 7.5 − 1 = 6.5 L

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

cups pints liter
quarts fluid ounces capacity

1. Units of ______________ are generally used to measure liquids.
2. The basic unit of capacity in the metric system is the ______________.
3. One cup equals 8 ______________.
4. One quart equals 2 ______________.
5. One pint equals 2 ______________.
6. One quart equals 4 ______________.
7. One gallon equals 4 ______________.

Convert as indicated without pencil or paper or calculator.

8. 2 c to pints

9. 4 c to pints

10. 4 qt to gallons

11. 8 qt to gallons

12. 2 pt to quarts

13. 6 pt to quarts

14. 8 fl oz to cups

15. 24 fl oz to cups

16. 3 pt to cups

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 5.6

Objective A **17.** Complete this statement based on Example 1: When using a unit fraction, we are not changing the ______________; we are changing the ______________.

Objective B **18.** In Example 4, explain the first step taken to solve the problem.

Objective C **19.** In Example 5, how many places is the decimal point moved and in what direction? What is the final conversion?

Objective D **20.** What is the answer to Example 7 in dekaliters?

5.6 Exercise Set MyLab Math

Objective A *Convert each measurement as indicated. See Examples 1 and 2.*

1. 32 fluid ounces to cups

2. 16 quarts to gallons

3. 8 quarts to pints

4. 9 pints to quarts

5. 14 quarts to gallons

6. 11 cups to pints

7. 80 fluid ounces to pints

8. 18 pints to gallons

9. 2 quarts to cups

10. 3 pints to fluid ounces

11. 120 fluid ounces to quarts

12. 20 cups to gallons

13. 42 cups to quarts

14. 7 quarts to cups

15. $4\frac{1}{2}$ pints to cups

16. $6\frac{1}{2}$ gallons to quarts

17. 5 gal 3 qt to quarts

18. 4 gal 1 qt to quarts

19. $\frac{1}{2}$ cup to pints

20. $\frac{1}{2}$ pint to quarts

21. 58 qt = _____ gal _____ qt

22. 70 qt = _____ gal _____ qt

23. 39 pt = _____ gal _____ qt _____ pt

24. 29 pt = _____ gal _____ qt _____ pt

25. $2\frac{3}{4}$ gallons to pints

26. $3\frac{1}{4}$ quarts to cups

Objective B *Perform each indicated operation. See Examples 3 through 5.*

27. 5 gal 3 qt + 7 gal 3 qt

28. 2 gal 2 qt + 9 gal 3 qt

29. 1 c 5 fl oz + 2 c 7 fl oz

30. 2 c 3 fl oz + 2 c 6 fl oz

▶ **31.** 3 gal − 1 gal 3 qt

32. 2 pt − 1 pt 1 c

33. 3 gal 1 qt − 1 qt 1 pt

34. 3 qt 1 c − 1 c 4 fl oz

35. 8 gal 2 qt × 2

36. 6 gal 1 pt × 2

37. 9 gal 2 qt ÷ 2

38. 5 gal 2 qt ÷ 2

Objective C *Convert as indicated. See Examples 6 and 7.*

39. 5 L to milliliters

40. 8 L to milliliters

41. 0.16 L to kiloliters

42. 0.127 L to kiloliters

▶ **43.** 5600 ml to liters

44. 1500 ml to liters

45. 3.2 L to centiliters

46. 1.7 L to centiliters

47. 410 L to kiloliters

48. 250 L to kiloliters

49. 64 ml to liters

50. 39 ml to liters

▶ **51.** 0.16 kl to liters

52. 0.48 kl to liters

53. 3.6 L to milliliters

54. 1.9 L to milliliters

Objective D *Perform each indicated operation. Remember to insert units when writing your answers. See Examples 8 and 9.*

55. 3.4 L + 15.9 L

56. 18.5 L + 4.6 L

▶ **57.** 2700 ml + 1.8 L

58. 4.6 L + 1600 ml

59. 8.6 L − 190 ml

60. 4.8 L − 283 ml

61. 17,500 ml − 0.9 L

62. 6850 ml − 0.3 L

63. 480 ml × 8

64. 290 ml × 6

65. 81.2 L ÷ 0.5

66. 5.4 L ÷ 3.6

Objectives A B C D Mixed Practice *Solve. Remember to insert units when writing your answers. For Exercises 67 through 70, complete the chart. See Examples 1 through 9.*

	Capacity	Cups	Gallons	Quarts	Pints
67.	An average-size bath of water		21		
68.	A dairy cow's daily milk yield				38
69.	Your kidneys filter about this amount of blood every minute	4			
70.	The amount of water needed in a punch recipe	2			

71. A student poured 410 ml of Mountain Dew from a 2-liter bottle. How much Mountain Dew remains in the bottle?

72. The Werners' Volvo has a 54.5-L gas tank. Only 38 deciliters of gasoline still remain in the tank. How many liters will be needed to fill it?

73. A mechanic added 354 ml of Prestone gas treatment to the 18.6 L of gasoline in his client's car's tank. Find the total amount of gasoline in the tank.

74. Eight friends are equally sharing a 2-L bottle of Coca-Cola. How much will each person get?

75. A garden tool engine requires a 30-to-1 gas-to-oil mixture. This means that $\frac{1}{30}$ of a gallon of oil should be mixed with 1 gallon of gas. Convert $\frac{1}{30}$ gallon to fluid ounces. Round to the nearest tenth.

76. Henning's Supermarket sells homemade soup in 1 qt 1 pt containers. How much soup is contained in three such containers?

77. Can 5 pt 1 c of fruit punch and 2 pt 1 c of ginger ale be poured into a 1-gal container without it overflowing?

78. Three cups of prepared Jell-O are poured into 6 dessert dishes. How many fluid ounces of Jell-O are in each dish?

79. Stanley Fisher paid $30 to fill his car with 44.3 liters of gasoline. Find the price per liter of gasoline to the nearest thousandth of a dollar.

80. A student carelessly misread the scale on a cylinder in the chemistry lab and added 40 cl of water to a mixture instead of 40 ml. Find the excess amount of water.

Review

Write each fraction in simplest form. See Section 3.2.

81. $\frac{20}{25}$

82. $\frac{75}{100}$

83. $\frac{27}{45}$

84. $\frac{56}{60}$

85. $\frac{72}{80}$

86. $\frac{18}{20}$

Concept Extensions

Determine whether the measurement in each statement is reasonable.

87. Clair took a dose of 2 L of cough medicine to cure her cough.

88. John drank 250 ml of milk for lunch.

89. Jeannie likes to relax in a tub filled with 3000 ml of hot water.

90. Sarah pumped 20 L of gasoline into her car yesterday.

Solve. See the Concept Checks in this section.

91. If 70 pints are converted to gallons, will the equivalent number of gallons be less than or greater than 70? Explain why.

92. If 30 gallons are converted to quarts, will the equivalent number of quarts be less than or greater than 30? Explain why.

93. Explain how to estimate the following operation: Add 986 ml to 6.9 L.

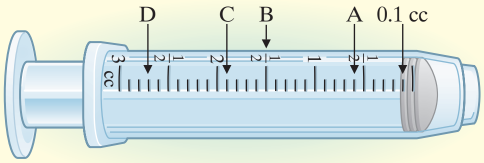
94. Explain how to borrow in order to subtract 1 gal 2 qt from 3 gal 1 qt.

95. Find the number of fluid ounces in 1 gallon.

96. Find the number of fluid ounces in 1.5 gallons.

A cubic centimeter (cc) is the amount of space that a volume of 1 ml occupies. Because of this, we will say that 1 cc = 1 ml.

A common syringe is one with a capacity of 3 cc. Use the diagram and give the measurement indicated by each arrow.



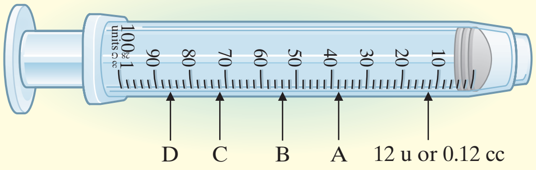
97. B

98. A

99. D

100. C

In order to measure small dosages, such as for insulin, u-100 syringes are used. For these syringes, 1 cc has been divided into 100 equal units (u). Use the diagram and give the measurement indicated by each arrow in units (u) and then in cubic centimeters. Use 100 u = 1 cc.



101. B

102. A

103. D

104. C

5.7 Conversions Between the U.S. and Metric Systems

Objective

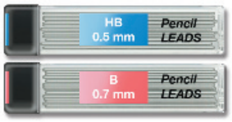
A Convert Between the U.S. and Metric Systems.

Objective A Converting Between the U.S. and Metric Systems

The metric system probably had its beginnings in France in the 1600s, but it was the Metric Act of 1866 that made the use of this system legal (but not mandatory) in the United States. Other laws have followed that allow for a slow, but deliberate, transfer to the modernized metric system. In April 2001, for example, the U.S. Stock Exchanges completed their change to decimal trading instead of fractions. By the end of 2009, all products sold in Europe (with some exceptions) were required to have only metric units on their labels. (*Source:* U.S. Metric Association and National Institute of Standards and Technology)

You may be surprised at the number of everyday items we use that are already manufactured in metric units. We easily recognize 1-L and 2-L soda bottles, but what about the following?

- Pencil leads (0.5 mm or 0.7 mm)
- Sporting events (5-km or 10-km races)
- Medicines (500-mg capsules)
- Labels on retail goods (dual-labeled since 1994)



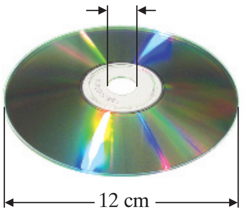
Since the United States has not completely converted to the metric system, we need to practice converting from one system to the other. Below is a table of mostly approximate conversions.

Length:			**Capacity:**			**Weight (mass):**		
Metric		U.S. System	Metric		U.S. System	Metric		U.S. System
1 m	≈	1.09 yd	1 L	≈	1.06 qt	1 kg	≈	2.20 lb
1 m	≈	3.28 ft	1 L	≈	0.26 gal	1 g	≈	0.04 oz
1 km	≈	0.62 mi	3.79 L	≈	1 gal	0.45 kg	≈	1 lb
2.54 cm	=	1 in.	0.95 L	≈	1 qt	28.35 g	≈	1 oz
0.30 m	≈	1 ft	29.57 ml	≈	1 fl oz			
1.61 km	≈	1 mi						

There are many ways to perform these metric-to-U.S. conversions. We will do so by using unit fractions.

Example 1 Compact Discs

Standard-sized compact discs are 12 centimeters in diameter. Convert this length to inches. Round the result to two decimal places. (*Source:* usByte.com)



Solution: From our length conversion table, we know that 2.54 cm = 1 in. This fact gives us two unit fractions: $\frac{2.54\text{ cm}}{1\text{ in.}}$ and $\frac{1\text{ in.}}{2.54\text{ cm}}$. We use the unit fraction with cm in the denominator so that these units divide out.

$$12\text{ cm} = \frac{12\text{ cm}}{1}\cdot 1 = \frac{12\ \cancel{\text{cm}}}{1}\cdot \overbrace{\frac{1\text{ in.}}{2.54\ \cancel{\text{cm}}}}^{\text{Unit fraction}} \begin{array}{l}\leftarrow \text{Units to convert to}\\ \leftarrow \text{Original units}\end{array}$$

$$= \frac{12\text{ in.}}{2.54}$$

$$\approx 4.72\text{ in.} \quad \text{Divide.}$$

Practice 1

The center hole of a standard-sized compact disc is 1.5 centimeters in diameter. Convert this length to inches. Round the result to 2 decimal places.

Answer

1. 0.59 in.

Thus, the diameter of a standard compact disc is exactly 12 cm or approximately 4.72 inches. For a dimension this size, you can use a ruler to check. Another method is to approximate. Our result, 4.72 in., is close to 5 inches. Since 1 in. is about 2.5 cm, then 5 in. is about 5(2.5 cm) = 12.5 cm, which is close to 12 cm.

Work Practice 1

Example 2 Liver

The liver is your largest internal organ. It weighs about 3.5 pounds in a grown man. Convert this weight to kilograms. Round to the nearest tenth. (*Source: Some Body!* by Dr. Pete Rowan)

Solution:

$$3.5\text{ lb} \approx \frac{3.5\,\cancel{\text{lb}}}{1}\cdot\overbrace{\frac{0.45\text{ kg}}{1\,\cancel{\text{lb}}}}^{\text{Unit fraction}} = 3.5(0.45\text{ kg}) \approx 1.6\text{ kg}$$

Thus 3.5 pounds are approximately 1.6 kilograms. From the table of conversions, we know that 1 kg ≈ 2.2 lb. So that means 0.5 kg ≈ 1.1 lb and after adding, we have 1.5 kg ≈ 3.3 lb. Our result is reasonable.

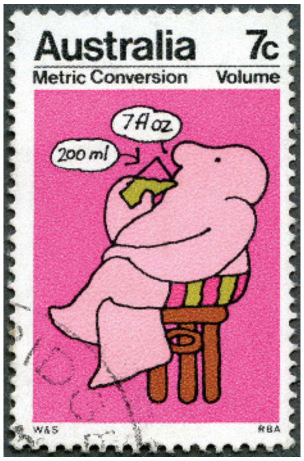
Work Practice 2

Practice 2

A full-grown human heart weighs about 8 ounces. Convert this weight to grams. If necessary, round your result to the nearest tenth of a gram.

Example 3 Postage Stamp

Australia converted to the metric system in 1973. In that year, four postage stamps were issued to publicize this conversion. One such stamp is shown. Let's check the mathematics on the stamp by converting 7 fluid ounces to milliliters. Round to the nearest hundred.



Solution:

$$\begin{aligned} 7\text{ fl oz} &\approx \frac{7\,\cancel{\text{fl oz}}}{1}\cdot\overbrace{\frac{29.57\text{ ml}}{1\,\cancel{\text{fl oz}}}}^{\text{Unit fraction}} \\ &= 7(29.57\text{ ml}) = 206.99\text{ ml} \end{aligned}$$

Rounded to the nearest hundred, 7 fl oz ≈ 200 ml.

Work Practice 3

Practice 3

Convert 237 ml to fluid ounces. Round to the nearest whole fluid ounce.

Answers

2. 226.8 g **3.** 8 fl oz

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 5.7

Watch the section lecture video and answer the following questions.

Objective A

1. Write two conversions that may be used to solve Example 2.

2. Why isn't 0.1125 kg the final answer to Example 3?

5.7 Exercise Set MyLab Math

Note: Because approximations are used, your answers may vary slightly from the answers given in the back of the book.

Objective A *Convert as indicated. If necessary, round answers to two decimal places. See Examples 1 through 3.*

1. 756 milliliters to fluid ounces

2. 18 liters to quarts

▶ **3.** 86 inches to centimeters

4. 86 miles to kilometers

5. 1000 grams to ounces

6. 100 kilograms to pounds

7. 93 kilometers to miles

8. 9.8 meters to feet

▶ **9.** 14.5 liters to gallons

10. 150 milliliters to fluid ounces

11. 30 pounds to kilograms

12. 15 ounces to grams

Fill in the chart. Give exact answers or round to one decimal place. See Examples 1 through 3.

		Meters	Yards	Centimeters	Feet	Inches
13.	The height of a woman				5	
14.	Statue of Liberty length of nose	1.37				
15.	Leaning Tower of Pisa		60			
16.	Blue whale		36			

Solve. If necessary, round answers to two decimal places. See Examples 1 through 3.

17. The balance beam for female gymnasts is 10 centimeters wide. Convert this width to inches.

18. In men's gymnastics, the rings are 250 centimeters from the floor. Convert this height to inches and then to feet.

19. In many states, the maximum speed limit for recreational vehicles is 50 miles per hour. Convert this to kilometers per hour.

20. In some states, the speed limit is 70 miles per hour. Convert this to kilometers per hour.

21. Ibuprofen comes in 200-milligram tablets. Convert this to ounces. (Round your answer to this exercise to 3 decimal places.)

22. Vitamin C tablets come in 500-milligram caplets. Convert this to ounces.

The 70-meter-diameter antenna is the largest and most sensitive Deep Space Network antenna. Use this for Exercises 23 through 26. (For more information about Deep Space Network, see the Chapter 9 Opener.)

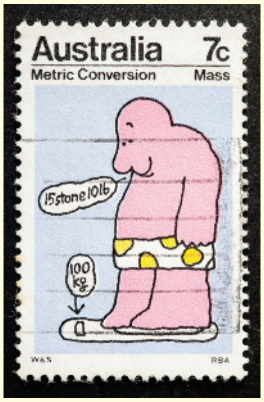
23. Convert 70 meters to feet.

24. The Deep Space Network sites also have a 26-meter antenna. Convert 26 meters to feet.

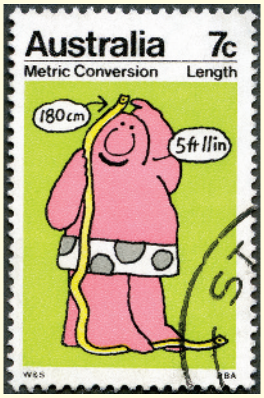
25. The 70-meter-diameter antenna can track a spacecraft traveling more than 16 billion kilometers from Earth. Convert this distance to miles.

26. The dish reflector and the mount atop the concrete pedestal of the 70-meter antenna weigh nearly 2.7 million kilograms. Convert this number to tons.

27. A stone is a unit in the British customary system. Use the conversion 14 pounds = 1 stone to check the equivalencies in this 1973 Australian stamp. Is 100 kilograms approximately 15 stone 10 pounds?



28. Convert 5 feet 11 inches to centimeters and check the conversion on this 1973 Australian stamp. Is it correct?



29. The Monarch butterfly migrates annually between the northern United States and central Mexico. The trip is about 4500 km long. Convert this to miles.

30. There is a species of African termite that builds nests up to 18 ft high. Convert this to meters.

31. A $3\frac{1}{2}$-inch diskette is not really $3\frac{1}{2}$ inches. To find its actual width, convert this measurement to centimeters and then to millimeters. Round the result to the nearest ten.

32. The average two-year-old is 84 centimeters tall. Convert this to feet and inches.

33. For an average adult, the weight of the right lung is greater than the weight of the left lung. If the right lung weighs 1.5 pounds and the left lung weighs 1.25 pounds, find the difference in grams. (*Source: Some Body!*)

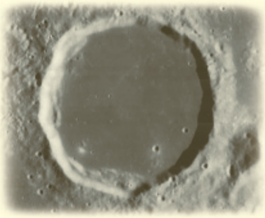
34. The skin of an average adult weighs 9 pounds and is the heaviest organ. Find the weight in grams. (*Source: Some Body!*)

35. A fast sneeze has been clocked at about 167 kilometers per hour. Convert this to miles per hour. Round to the nearest whole.

36. A Boeing 747 has a cruising speed of about 980 kilometers per hour. Convert this to miles per hour. Round to the nearest whole.

37. The General Sherman giant sequoia tree has a diameter of about 8 meters at its base. Convert this to feet. (*Source: Fantastic Book of Comparisons*)

38. The largest crater on the near side of the moon is Billy Crater. It has a diameter of 303 kilometers. Convert this to miles. (*Source: Fantastic Book of Comparisons*)



39. The total length of the track on a CD is about 4.5 kilometers. Convert this to miles. Round to the nearest whole mile.

40. The distance between Mackinaw City, Michigan, and Cheyenne, Wyoming, is 2079 kilometers. Convert this to miles. Round to the nearest whole mile.

41. A doctor orders a dosage of 5 ml of medicine every 4 hours for 1 week. How many fluid ounces of medicine should be purchased? Round up to the next whole fluid ounce.

42. A doctor orders a dosage of 12 ml of medicine every 6 hours for 10 days. How many fluid ounces of medicine should be purchased? Round up to the next whole fluid ounce.

Without actually converting, choose the most reasonable answer.

43. This math book has a height of about ______.

a. 28 mm **b.** 28 cm
c. 28 m **d.** 28 km

44. A mile is ______ a kilometer.

a. shorter than **b.** longer than
c. the same length as

45. A liter has ______ capacity than a quart.

a. less **b.** greater
c. the same

46. A foot is ______ a meter.

a. shorter than **b.** longer than
c. the same length as

47. A kilogram weighs ______ a pound.

a. the same as **b.** less than
c. more than

48. A football field is 100 yards, which is about ______.

a. 9 m **b.** 90 m
c. 900 m **d.** 9000 m

49. An $8\frac{1}{2}$-ounce glass of water has a capacity of about ______.

a. 250 L **b.** 25 L
c. 2.5 L **d.** 250 ml

50. A 5-gallon gasoline can has a capacity of about ______.

a. 19 L **b.** 1.9 L
c. 19 ml **d.** 1.9 ml

51. The weight of an average man is about ______.

a. 700 kg **b.** 7 kg
c. 0.7 kg **d.** 70 kg

52. The weight of a pill is about ______.

a. 200 kg **b.** 20 kg
c. 2 kg **d.** 200 mg

Review

Perform the indicated operations. See Section 1.9.

53. $6 \cdot 4 + 5 \div 1$

54. $10 \div 2 + 9(8)$

55. $\dfrac{10 + 8}{10 - 8}$

56. $\dfrac{14 + 1}{5(3)}$

57. $3 + 5(19 - 17) - 8$

58. $1 + 4(19 - 9) + 5$

59. $3[(1 + 5) \cdot (8 - 6)]$

60. $5[(18 - 8) - 9]$

Concept Extensions

Body surface area (BSA) is often used to calculate dosages for some drugs. BSA is calculated in square meters using a person's weight and height.

$$\text{BSA} = \sqrt{\frac{(\text{weight in kg}) \times (\text{height in cm})}{3600}}$$

For Exercises 61 through 66, calculate the BSA for each person. Round to the nearest hundredth. You will need to use the square root key on your calculator.

61. An adult whose height is 182 cm and weight is 90 kg.

62. An adult whose height is 157 cm and weight is 63 kg.

63. A child whose height is 50 in. and weight is 40 kg. (*Hint:* Don't forget to first convert inches to centimeters)

64. A child whose height is 26 in. and weight is 13 kg.

65. An adult whose height is 60 in. and weight is 150 lb.

66. An adult whose height is 69 in. and weight is 172 lb.

Solve.

67. Suppose the adult from Exercise **61** is to receive a drug that has a recommended dosage range of 10–12 mg per sq meter. Find the dosage range for the adult.

68. Suppose the child from Exercise **64** is to receive a drug that has a recommended dosage of 30 mg per sq meter. Find the dosage for the child.

69. A handball court is a rectangle that measures 20 meters by 40 meters. Find its area in square meters and square feet.

70. A backpack measures 16 inches by 13 inches by 5 inches. Find the volume of a box with these dimensions. Find the volume in cubic inches and cubic centimeters. Round the cubic centimeters to the nearest whole cubic centimeter.

Chapter 5 Group Activity

Consumer Price Index

Sections 5.1–5.3

Do you remember when the regular price of a candy bar was 5¢, 10¢, or 25¢? It is certainly difficult to find a candy bar for that price these days. The reason is inflation: the tendency for the price of a given product to increase over time. Businesses and government agencies use the Consumer Price Index (CPI) to track inflation. The CPI measures the change in prices of basic consumer goods and services over time.

The CPI is very useful for comparing the prices of fixed items in various years. For instance, suppose an insurance company customer submits a claim for the theft of a fishing boat purchased in 1990. Because the customer's policy includes replacement cost coverage, the insurance company must calculate how much it would cost to replace the boat at the time of the theft. (Let's assume the theft took place in August 2017.) The customer has a receipt for the boat showing that it cost \$2058 in 1990. The insurance company can use the following proportion to calculate the replacement cost:

$$\frac{\text{price in earlier year}}{\text{price in later year}} = \frac{\text{CPI value in earlier year}}{\text{CPI value in later year}}$$

The CPI value is 130.7 for 1990. In August 2017, the CPI value was 246.524. The insurance company would then use the following proportion for this situation. (We will let n represent the unknown price in August 2017.)

$$\begin{aligned}\frac{\text{price in 1990}}{\text{price in 2017}} &= \frac{\text{CPI value in 1990}}{\text{CPI value in 2017}}\\ \frac{2058}{n} &= \frac{130.7}{246.524}\\ 130.7 \cdot n &= 2058(246.524)\\ \frac{130.7 \cdot n}{130.7} &= \frac{2058(246.524)}{130.7}\\ n &\approx 3882\end{aligned}$$

The replacement cost of the fishing boat in August 2017 prices is approximately \$3882.

Critical Thinking

1. What trends do you see in the CPI values in the table? Do you think these trends make sense? Explain.
2. A piece of jewelry cost \$800 in 1985. What is its 2017 replacement value? Round to the nearest dollar.
3. In 2000, the cost of a loaf of bread was about \$1.89. What would an equivalent loaf of bread have cost in 1950? Round to the nearest dollar.
4. Suppose a couple purchased a house for \$45,000 in 1970. At what price could they have been expected to sell the house in 2000? Round to the nearest dollar.
5. An original Ford Model T cost about \$850 in 1915. What is the equivalent cost of a Model T in 2010 dollars? Round to the nearest dollar.

Consumer Price Index

Year	CPI
1915	10.1
1920	20.0
1925	17.5
1930	16.7
1935	13.7
1940	14.0
1945	18.0
1950	24.1
1955	26.8
1960	29.6
1965	31.5
1970	38.8
1975	53.8
1980	82.4
1985	107.6
1990	130.7
1995	152.4
2000	172.2
2005	195.3
2010	218.1
2011	224.9
2012	229.6
2013	233.0
2014	236.7
2015	237.2
2016	240.0
2017	245.1

(*Source:* Bureau of Labor Statistics, U.S. Department of Labor)

Chapter 5 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

not equal
ratio
equal
unit price
cross products
proportion
rate
meter
mass
liter
unit fractions
weight
unit rate
gram

1. A ______________ is the quotient of two numbers. It can be written as a fraction, using a colon, or using the word *to*.

2. $\frac{x}{2} = \frac{7}{16}$ is an example of a ______________.

3. A ______________ is a rate with a denominator of 1.

4. A ______________ is a "money per item" unit rate.

5. A ______________ is used to compare different kinds of quantities.

6. In the proportion $\frac{x}{2} = \frac{7}{16}$, $x \cdot 16$ and $2 \cdot 7$ are called ______________.

7. If cross products are ______________, the proportion is true.

8. If cross products are ______________, the proportion is false.

9. ______________ is a measure of the pull of gravity.

10. ______________ is a measure of the amount of substance in an object. This measure does not change.

11. The basic unit of length in the metric system is the ______________.

12. To convert from one unit of length to another, ______________ may be used.

13. The ______________ is the basic unit of capacity in the metric system.

14. The basic unit of mass in the metric system is the ______________.

Helpful Hint

Are you preparing for your test? To help, don't forget to take these:

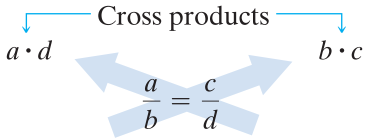
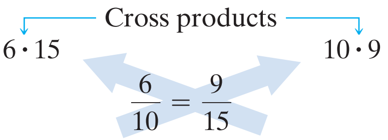
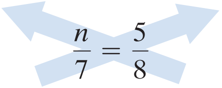
- Chapter 5 Getting Ready for the Test on page 444
- Chapter 5 Test on page 446

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

5 Chapter Highlights

Definitions and Concepts	Examples
Section 5.1 Ratios	
A **ratio** is the quotient of two quantities.	The ratio of 3 to 4 can be written as $\underset{\uparrow \atop \text{fraction notation}}{\frac{3}{4}}$ or $\underset{\uparrow \atop \text{colon notation}}{3:4}$
Rates are used to compare different kinds of quantities.	Write the rate 12 spikes every 8 inches as a fraction in simplest form. $\frac{12 \text{ spikes}}{8 \text{ inches}} = \frac{3 \text{ spikes}}{2 \text{ inches}}$
A **unit rate** is a rate with a denominator of 1.	Write as a unit rate: 117 miles on 5 gallons of gas $\frac{117 \text{ miles}}{5 \text{ gallons}} = \frac{23.4 \text{ miles}}{1 \text{ gallon}}$ or 23.4 miles per gallon or 23.4 miles/gallon
A **unit price** is a "money per item" unit rate.	Write as a unit price: \$5.88 for 42 ounces of detergent $\frac{\$5.88}{42 \text{ ounces}} = \frac{\$0.14}{1 \text{ ounce}} = \$0.14$ per ounce

Section 5.2 Proportions

Definitions and Concepts	Examples
A **proportion** is a statement that two ratios or rates are equal.	$\frac{1}{2} = \frac{4}{8}$ is a proportion.
Using Cross Products to Determine Whether Proportions Are True or False  If cross products are equal, the proportion is true. If $ad = bc$, then the proportion is true. If cross products are not equal, the proportion is false. If $ad \neq bc$, then the proportion is false.	Is $\frac{6}{10} = \frac{9}{15}$ a true proportion?  $6 \cdot 15 \stackrel{?}{=} 10 \cdot 9$ Are cross products equal? $90 = 90$ Since cross products are equal, the proportion is a true proportion.
Finding an Unknown Value *n* in a Proportion **Step 1.** Set the cross products equal to each other.	Find n: $\frac{n}{7} = \frac{5}{8}$ **Step 1.**  $n \cdot 8 = 7 \cdot 5$ Set the cross products equal to each other. $n \cdot 8 = 35$ Multiply.
Step 2. Divide the number not multiplied by n by the number multiplied by n.	**Step 2.** $n = \frac{35}{8}$ Divide 35 by 8, the number multiplied by n. $n = 4\frac{3}{8}$

Section 5.3 Proportions and Problem Solving

Given a specified ratio (or rate) of two quantities, a proportion can be used to determine an unknown quantity.	On a map, 50 miles corresponds to 3 inches. How many miles correspond to 10 inches? **1.** UNDERSTAND. Read and reread the problem. **2.** TRANSLATE. We let n represent the unknown number. We are given that 50 miles is to 3 inches as n miles is to 10 inches. $\begin{matrix}\text{miles} \rightarrow \\ \text{inches} \rightarrow\end{matrix} \frac{50}{3} = \frac{n}{10} \begin{matrix}\leftarrow \text{miles} \\ \leftarrow \text{inches}\end{matrix}$

Definitions and Concepts | **Examples**

Section 5.3 Proportions and Problem Solving (*Continued*)

3. SOLVE:

$$\frac{50}{3} = \frac{n}{10}$$

$$\begin{aligned} 50 \cdot 10 &= 3 \cdot n && \text{Set the cross products equal to each other.} \\ 500 &= 3 \cdot n && \text{Multiply.} \\ \frac{500}{3} &= \frac{3n}{3} && \text{Divide 500 by 3, the number multiplied by } n. \\ n &= 166\frac{2}{3} \end{aligned}$$

4. INTERPRET. *Check* your work. *State* your conclusion: On the map, $166\frac{2}{3}$ miles corresponds to 10 inches.

Section 5.4 Length: U.S. and Metric Systems of Measurement

To convert from one unit of length to another, multiply by a **unit fraction** in the form

$$\frac{\text{units to convert to}}{\text{original units}}.$$

Length: U.S. System of Measurement

12 inches (in.) = 1 foot (ft)
3 feet = 1 yard (yd)
5280 feet = 1 mile (mi)

The basic unit of length in the metric system is the **meter.** A meter is slightly longer than a yard.

Length: Metric System of Measurement

Metric Units of Length

1 **kilo**meter (km) = 1000 meters (m)
1 **hecto**meter (hm) = 100 m
1 **deka**meter (dam) = 10 m
1 meter (m) = 1 m
1 **deci**meter (dm) = 1/10 m or 0.1 m
1 **centi**meter (cm) = 1/100 m or 0.01 m
1 **milli**meter (mm) = 1/1000 m or 0.001 m

$$\frac{12 \text{ inches}}{1 \text{ foot}}, \frac{1 \text{ foot}}{12 \text{ inches}}, \frac{3 \text{ feet}}{1 \text{ yard}}$$

Convert 6 feet to inches.

$$\begin{aligned} 6 \text{ ft} &= \frac{6 \text{ ft}}{1} \cdot 1 \\ &= \frac{6 \cancel{\text{ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \begin{array}{l} \leftarrow \text{units to convert to} \\ \leftarrow \text{original units} \end{array} \\ &= 6 \cdot 12 \text{ in.} \\ &= 72 \text{ in.} \end{aligned}$$

Convert 3650 centimeters to meters.

$$\begin{aligned} 3650 \text{ cm} &= 3650 \text{ cm} \cdot 1 \\ &= \frac{3650 \cancel{\text{cm}}}{1} \cdot \frac{0.01 \text{ m}}{1 \cancel{\text{cm}}} = 36.5 \text{ m} \end{aligned}$$

or

$$\text{km} \quad \text{hm} \quad \text{dam} \quad \underbrace{\text{m} \quad \text{dm} \quad \text{cm}}_{\text{2 units to the left}} \quad \text{mm}$$

$$\underset{\text{2 places to the left}}{3650} \text{ cm} = 36.5 \text{ m}$$

Definitions and Concepts | **Examples**

Section 5.5 Weight and Mass: U.S. and Metric Systems of Measurement

Weight is really a measure of the pull of gravity. **Mass** is a measure of the amount of substance in an object and does not change.

Weight: U.S. System of Measurement

16 ounces (oz) = 1 pound (lb)

2000 pounds = 1 ton

A **gram** is the basic unit of mass in the metric system. It is the mass of water contained in a cube 1 centimeter on each side. A paper clip weighs about 1 gram.

Mass: Metric System of Measurement

Metric Units of Mass

1 kilogram (kg) = 1000 grams (g)
1 hectogram (hg) = 100 g
1 dekagram (dag) = 10 g
1 gram (g) = 1 g
1 decigram (dg) = 1/10 g or 0.1 g
1 centigram (cg) = 1/100 g or 0.01 g
1 milligram (mg) = 1/1000 g or 0.001 g

Convert 5 pounds to ounces.

$$5\text{ lb} = 5\text{ lb}\cdot 1 = \frac{5\ \cancel{\text{lb}}}{1}\cdot\frac{16\text{ oz}}{1\ \cancel{\text{lb}}} = 80\text{ oz}$$

Convert 260 grams to kilograms.

$$260\text{ g} = \frac{260\text{ g}}{1}\cdot 1 = \frac{260\ \cancel{\text{g}}}{1}\cdot\frac{1\text{ kg}}{1000\ \cancel{\text{g}}} = 0.26\text{ kg}$$

or

kg hg dag g dg cg mg

3 units to the left

260 g = 0.260 kg

3 places to the left

Section 5.6 Capacity: U.S. and Metric Systems of Measurement

Capacity: U.S. System of Measurement

8 fluid ounces (fl oz) = 1 cup (c)
2 cups = 1 pint (pt)
2 pints = 1 quart (qt)
4 quarts = 1 gallon (gal)

The **liter** is the basic unit of capacity in the metric system. It is the capacity or volume of a cube measuring 10 centimeters on each side. A liter of liquid is slightly more than 1 quart.

Capacity: Metric System of Measurement

Metric Units of Capacity

1 kiloliter (kl) = 1000 liters (L)
1 hectoliter (hl) = 100 L
1 dekaliter (dal) = 10 L
1 liter (L) = 1 L
1 deciliter (dl) = 1/10 L or 0.1 L
1 centiliter (cl) = 1/100 L or 0.01 L
1 milliliter (ml) = 1/1000 L or 0.001 L

Convert 5 pints to gallons.

1 gal = 4 qt = 8 pt

$$5\text{ pt} = 5\text{ pt}\cdot 1 = \frac{5\ \cancel{\text{pt}}}{1}\cdot\frac{1\text{ gal}}{8\ \cancel{\text{pt}}} = \frac{5}{8}\text{ gal}$$

Convert 1.5 liters to milliliters.

$$1.5\text{ L} = \frac{1.5\text{ L}}{1}\cdot 1 = \frac{1.5\ \cancel{\text{L}}}{1}\cdot\frac{1000\text{ ml}}{1\ \cancel{\text{L}}} = 1500\text{ ml}$$

or

kl hl dal L dl cl ml

3 units to the right

1.500 L = 1500 ml

3 places to the right

Definitions and Concepts	Examples
Section 5.7 Conversions Between the U.S. and Metric Systems	
To convert between systems, use approximate unit fractions from Section 5.7.	Convert 7 feet to meters. $7 \text{ ft} \approx \frac{7 \cancel{\text{ft}}}{1} \cdot \frac{0.30 \text{ m}}{1 \cancel{\text{ft}}} = 2.1 \text{ m}$ Convert 8 liters to quarts. $8 \text{ L} \approx \frac{8 \cancel{\text{L}}}{1} \cdot \frac{1.06 \text{ qt}}{1 \cancel{\text{L}}} = 8.48 \text{ qt}$ Convert 363 grams to ounces. $363 \text{ g} \approx \frac{363 \cancel{\text{g}}}{1} \cdot \frac{0.04 \text{ oz}}{1 \cancel{\text{g}}} = 14.52 \text{ oz}$

Chapter 5 Review

(5.1) *Write each ratio as a fraction in simplest form.*

1. 23 to 37

2. 14 to 51

3. 6000 people to 4800 people

4. \$121 to \$143

5. 3.5 centimeters to 7.5 centimeters

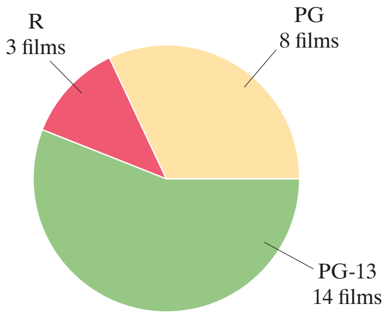
6. 4.25 yards to 8.75 yards

7. $2\frac{1}{4}$ to $4\frac{3}{8}$

8. $3\frac{1}{2}$ to $2\frac{7}{10}$

The circle graph below shows how the top 25 movies (or films) of 2016 were rated. Use this graph to answer the questions.

Top 25 Movies of 2016



Source: MPAA

9. a. How many top 25 movies were rated R?
b. Find the ratio of top 25 R-rated movies to total movies for that year.

10. a. How many top 25 movies were rated PG-13?
b. Find the ratio of top 25 PG-13-rated movies to total movies for that year.

Write each rate as a fraction in simplest form.

11. 15 word processing pages printed in 6 minutes

12. 8 computers assembled in 6 hours

Write each rate as a unit rate.

13. 468 miles in 9 hours

14. 180 feet in 12 seconds

15. \$27.84 for 4 CDs

16. 8 gallons of pesticide for 6 acres of crops

Find each unit price and decide which is the better buy. Round to three decimal places. Assume that we are comparing different sizes of the same brand.

17. Taco sauce: 8 ounces for \$0.99 or 12 ounces for \$1.69

18. Peanut butter: 18 ounces for \$1.49 or 28 ounces for \$2.39

(5.2) *Write each sentence as a proportion.*

19. 16 sandwiches is to 8 players as 2 sandwiches is to 1 player.

20. 12 tires is to 3 cars as 4 tires is to 1 car.

Determine whether each proportion is true.

21. $\frac{21}{8} = \frac{14}{6}$

22. $\frac{3}{5} = \frac{60}{100}$

23. $\frac{3.75}{3} = \frac{7.5}{6}$

24. $\frac{3.1}{6.2} = \frac{0.8}{0.16}$

Find the unknown number n in each proportion.

25. $\frac{n}{6} = \frac{-15}{18}$

26. $\frac{n}{-9} = \frac{5}{3}$

27. $\frac{9}{2} = \frac{n}{\frac{3}{2}}$

28. $\frac{6}{\frac{5}{2}} = \frac{n}{3}$

29. $\frac{0.4}{n} = \frac{2}{4.7}$

30. $\frac{7.2}{n} = \frac{6}{0.3}$

31. $\frac{n}{4\frac{1}{2}} = \frac{2\frac{1}{10}}{8\frac{2}{5}}$

32. $\frac{n}{4\frac{2}{7}} = \frac{3\frac{1}{9}}{9\frac{1}{3}}$

(5.3) *Solve.*

The ratio of a quarterback's completed passes to attempted passes is 3 to 7.

33. If he attempted 32 passes, find how many passes he completed. Round to the nearest whole pass.

34. If he completed 15 passes, find how many passes he attempted.

One bag of pesticide covers 4000 square feet of garden.

△ **35.** Find how many bags of pesticide should be purchased to cover a rectangular garden that is 180 feet by 175 feet.

△ **36.** Find how many bags of pesticide should be purchased to cover a square garden that is 250 feet on each side.

On an architect's blueprint, 1 *inch* = 12 *feet.*

37. Find the length of a wall represented by a $3\frac{3}{8}$-inch line on the blueprint.

38. If an exterior wall is 99 feet long, find how long the blueprint measurement should be.

(5.4) *Convert.*

39. 108 in. to feet

40. $\frac{1}{2}$ yd to inches

41. 52 ft = ______ yd ______ ft

42. 46 in. = ______ ft ______ in.

43. 42 m to centimeters

44. 2.31 m to kilometers

Perform each indicated operation.

45. 4 yd 2 ft + 16 yd 2 ft

46. 12 ft 1 in. − 4 ft 8 in.

47. 8 cm + 15 mm

48. 19.6 km ÷ 8

Solve.

49. The trip from Philadelphia to Washington, D.C., is 217 km each way. Four friends agree to share the driving equally. How far must each drive on this round-trip vacation?

△ **50.** The college has ordered that NO SMOKING signs be placed above the doorway of each classroom. Each sign is 0.8 m long and 30 cm wide. Find the area of each sign. (*Hint:* Recall that the area of a rectangle = width · length.)

(5.5) *Convert.*

51. 66 oz to pounds

52. 2.3 tons to pounds

53. 52 oz = ______ lb ______ oz

54. 10,300 lb = ______ tons ______ lb

55. 27 mg to grams

56. 40 kg to grams

Perform each indicated operation.

57. 6 lb 5 oz − 2 lb 12 oz

58. 8 lb 6 oz × 4

Solve.

59. Donshay Berry ordered 1 lb 12 oz of soft-center candies and 2 lb 8 oz of chewy-center candies for his party. Find the total weight of the candy ordered.

60. Four local townships jointly purchase 38 tons 300 lb of cinders to spread on their roads during an ice storm. Determine the weight of the cinders each township receives if they share the purchase equally.

(5.6) *Convert.*

61. 16 pints to quarts

62. 40 fluid ounces to cups

63. 3 qt 1 pt to pints

64. 18 quarts to cups

65. 9 pt = ______ qt ______ pt

66. 15 qt = ______ gal ______ qt

67. 3.8 L to milliliters

68. 4.2 ml to deciliters

Perform each indicated operation.

69. 1 qt 1 pt + 3 qt 1 pt

70. 0.946 L − 210 ml

Solve.

71. Each bottle of Kiwi liquid shoe polish holds 85 ml of the polish. Find the number of liters of shoe polish contained in 8 boxes if each box contains 16 bottles.

72. Ivan Miller wants to pour three separate containers of saline solution into a single vat with a capacity of 10 liters. Will 6 liters of solution in the first container combined with 1300 milliliters in the second container and 2.6 liters in the third container fit into the vat?

(5.7) *Note: Because approximations are used in this section, your answers may vary slightly from the answers given in the back of the book.*

Convert as indicated. If necessary, round to two decimal places.

73. 7 meters to feet

74. 11.5 yards to meters

75. 17.5 liters to gallons

76. 7.8 liters to quarts

77. 15 ounces to grams

78. 23 pounds to kilograms

79. A 100-meter dash is being held today. How many yards is this?

80. If a person weighs 82 kilograms, how many pounds is this?

81. How many quarts are contained in a 3-liter bottle of cola?

82. A compact disc is 1.2 mm thick. Find the height (in inches) of 50 discs.

Mixed Review

Write each ratio as a fraction in simplest form.

83. 15 to 25

84. 6 pints to 48 pints

Write each rate as a fraction in simplest form.

85. 2 teachers for 18 students

86. 6 nurses for 24 patients

Write each rate as a unit rate.

87. 136 miles in 4 hours

88. 12 gallons of milk from 6 cows

Find each unit price and decide which is the better buy. Round to three decimal places. Assume that we are comparing different sizes of the same brand.

89. cold medicine:
\$4.94 for 4 oz,
\$9.98 for 8 oz

90. juice:
12 oz for \$0.65,
64 oz for \$2.98

Write each sentence as a proportion.

91. 2 cups of cookie dough is to 30 cookies as 4 cups of cookie dough is to 60 cookies.

92. 5 nickels is to 3 dollars as 20 nickels is to 12 dollars.

Find the unknown number n in each proportion.

93. $\frac{3}{n} = \frac{15}{8}$

94. $\frac{5}{4} = \frac{n}{20}$

Convert the following.

95. 2.5 mi to feet

96. 129 in. to feet

97. 8200 lb = ______ tons ______ lb

98. 5 m to centimeters

99. 1400 mg to grams

100. 286 mm to kilometers

Perform the indicated operations and simplify.

101. 9.3 km − 183 m

102. 6 gal 1 qt + 2 gal 1 qt

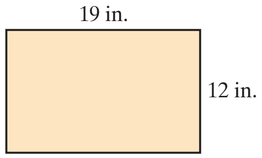
Chapter 5 Getting Ready for the Test

MULTIPLE CHOICE. Exercises 1 through 18 are **Multiple Choice**. Choose the correct letter for each exercise.

1. The ratio of $\frac{9}{15}$ is the same as what ratio?

A. $\frac{15}{9}$ **B.** $\frac{3}{5}$ **C.** $\frac{5}{3}$ **D.** 15 : 9

2. Given the rectangle, find the ratio of its width to its *perimeter*.



A. $\frac{12}{19}$ **B.** $\frac{19}{12}$ **C.** $\frac{12}{62} = \frac{6}{31}$ **D.** $\frac{19}{31}$

3. The rate 3 trees every 45 feet in simplest form is:

A. $\frac{1\text{ tree}}{15\text{ ft}}$ **B.** $\frac{45\text{ ft}}{3\text{ trees}}$ **C.** $\frac{15\text{ ft}}{1\text{ tree}}$

For Exercises 4 through 6, determine whether the proportions are true or false. Answer choices are

A. True **B.** False

4. $\frac{5}{6} = \frac{4}{5}$

5. $\frac{15}{6} = \frac{5}{2}$

6. $\frac{\frac{8}{11}}{4} = \frac{\frac{10}{11}}{5}$

For Exercises 7 and 8, choose the correct letter.

7. The phrase "6 is to 7 as 18 is to 21" translates to

A. $\frac{6}{7} = \frac{18}{21}$ **B.** $\frac{6}{18} = \frac{21}{7}$ **C.** $\frac{6}{21} = \frac{7}{18}$

8. Use cross products to determine which proportion is *not* equivalent to the proportion $\frac{3}{n} = \frac{5}{11}$

A. $\frac{3}{11} = \frac{n}{5}$ **B.** $\frac{n}{3} = \frac{11}{5}$ **C.** $\frac{3}{5} = \frac{n}{11}$ **D.** $\frac{11}{n} = \frac{5}{3}$

For Exercises 9 through 14, choose tbe best response.

9. An inch is about how many centimeters?

A. 10 cm **B.** 100 cm **C.** 2.5 cm **D.** 1 cm

10. A meter is about how many yards?

A. 10 yd **B.** 100 yd **C.** 2.5 yd **D.** 1 yd

11. A ton is how many pounds?
A. 10 lb **B.** 100 lb **C.** 1000 lb **D.** 2000 lb

12. Which is longer, a mile or a kilometer?
A. a mile **B.** a kilometer

13. A kilogram is about how many pounds?
A. 1 lb **B.** 2 lb **C.** 10 lb **D.** 100 lb

14. A liter is about how many quarts?
A. 1 qt **B.** 2 qt **C.** 10 qt **D.** 100 qt

For Exercises 15 through 18, choose the most reasonable answer.

15. A man is how tall?
A. 16 centimeters **B.** 1.6 feet **C.** 16 inches **D.** 1.6 meters

16. Choose a reasonable pencil lead widths for mechanical pencils.
A. 7 millimeters **B.** 7 centimeters **C.** 7 decimeters **D.** 7 meters

17. Choose a reasonable mass for an antibiotic tablet.
A. 500 grams **B.** 500 kilograms **C.** 500 milligrams

18. A man weighs how much?
A. 75 kilograms **B.** 75 grams **C.** 75 milligrams

Chapter 5 Test

MyLab Math For additional practice go to your study plan in MyLab Math.

Answers

1. ______________

2. ______________

3. ______________

4. ______________

5. ______________

6. ______________

7. ______________

8. ______________

9. ______________

10. ______________

11. ______________

12. ______________

13. ______________

14. ______________

15. ______________

16. ______________

Write each ratio or rate as a fraction in simplest form.

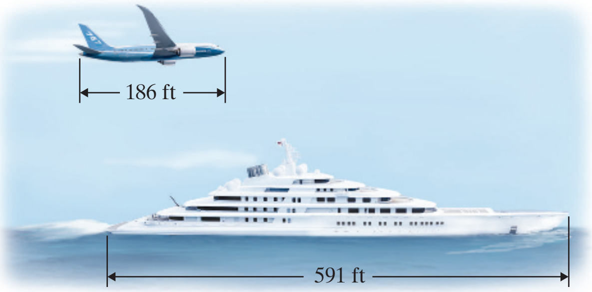
1. \$75 to \$10

2. 9 inches of rain in 30 days

3. 8.6 to 10

4. $5\frac{7}{8}$ to $9\frac{3}{4}$

5. The world's largest yacht, the *Azzam*, measures in at 591 feet long. A Boeing 787-8 Dreamliner measures 186 feet long. Find the ratio of the length of the *Azzam* to the length of a 787-8. (*Source:* Superyachts.com, Boeing)



Find each unit rate.

6. 650 kilometers in 8 hours

7. 140 students for 5 teachers

8. QRIO (Quest for Curiosity) is the world's first bipedal robot capable of running (moving with both legs off the ground at the same time) at a rate of 108 inches each 12 seconds. (*Source: Guinness World Records*)

Find each unit price and compare them to decide which is the better buy.

9. Steak sauce:
8 ounces for \$1.19
12 ounces for \$1.89

Determine whether the proportion is true.

10. $\frac{28}{16} = \frac{14}{8}$

Find the unknown number n in each proportion.

11. $\frac{n}{3} = \frac{15}{9}$

12. $\frac{8}{n} = \frac{11}{6}$

13. $\frac{-1.5}{5} = \frac{2.4}{n}$

14. $\frac{n}{2\frac{5}{8}} = \frac{1\frac{1}{6}}{3\frac{1}{2}}$

Solve.

15. On an architect's drawing, 2 inches corresponds to 9 feet. Find the length of a home represented by a line that is 11 inches long.

16. If a car can be driven 80 miles in 3 hours, how long will it take to travel 100 miles?

17. The standard dose of medicine for a dog is 10 grams for every 15 pounds of body weight. What is the standard dose for a dog that weighs 80 pounds?

Convert.

18. 280 in. = ________ ft ________ in.

19. $2\frac{1}{2}$ gal to quarts

20. 30 oz to pounds

21. 40 mg to grams

22. 3.6 cm to millimeters

23. 0.83 L to milliliters

Perform each indicated operation.

24. 8 lb 6 oz − 4 lb 9 oz

25. 5 gal 2 qt ÷ 2

26. 1.8 km + 456 m

Solve.

27. The sugar maples in front of Bette MacMillan's house are 8.4 meters tall. Because they interfere with the phone lines, the telephone company plans to remove the top third of the trees. How tall will the maples be after they are shortened?

28. A total of 15 gal 1 qt of oil has been removed from a 20-gallon drum. How much oil still remains in the container?

29. The engineer in charge of bridge construction said that the span of a certain bridge would be 88 m. But the actual construction required it to be 340 cm longer. Find the span of the bridge, in meters.

30. If 2 ft 9 in. of material is used to manufacture one scarf, how much material is needed for 6 scarves?

31. The Vietnam Veterans Memorial, inscribed with the names of 58,226 deceased and missing U.S. soldiers from the Vietnam War, is located on the National Mall in Washington, D.C. This memorial is formed from two straight sections of wall that meet at an angle at the center of the monument. Each wall is 246 ft 9 in. long. What is the total length of the Vietnam Veterans Memorial's wall? (*Source:* National Park Service)

32. Each panel making up the wall of the Vietnam Veterans Memorial is 101.6 cm wide. There are a total of 148 panels making up the wall. What is the total length of the wall in meters? (*Source:* National Park Service)

33. A 5-kilometer race is being held today. How many miles is this?

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

22. ____________

23. ____________

24. ____________

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

Chapters 1–5 Cumulative Review

Answers

1. a. ____

b. ____

c. ____

d. ____

2. a. **b.** ____

c. **d.** ____

3. ____

4. ____

5. a. ____

b. ____

6. ____

7. ____

8. ____

9. ____

10. ____

11. ____

12. ____

13. ____

14. ____

15. ____

16. ____

17. ____

18. ____

19. ____

20. ____

21. ____

22. ____

1. Subtract. Check each answer by adding.

a. $12 - 9$
b. $22 - 7$
c. $35 - 35$
d. $70 - 0$

2. Multiply:

a. $20 \cdot 0$
b. $20 \cdot 1$
c. $0 \cdot 20$
d. $1 \cdot 20$

3. Round 248,982 to the nearest hundred.

4. Round 248,982 to the nearest thousand.

5. Multiply:

a. $\begin{array}{r} 25 \\ \times\ 8 \\ \hline \end{array}$ **b.** $\begin{array}{r} 246 \\ \times\ 5 \\ \hline \end{array}$

6. Divide: $10{,}468 \div 28$

7. The director of a computer lab at a local state college is working on next year's budget. Thirty-three new desktop computers are needed at a cost of $487 each. What is the total cost of these desktops?

8. A study is being conducted for erecting soundproof walls along the interstate of a metropolitan area. The following feet of walls are part of the proposal. Find their total: 4800 feet, 3270 feet, 2761 feet, 5760 feet.

9. Write the prime factorization of 80.

10. Find $\sqrt{64}$.

11. Write $\frac{12}{20}$ in simplest form.

12. Find $9^2 \cdot \sqrt{9}$.

Multiply.

13. $-\frac{1}{4} \cdot \frac{1}{2}$

14. $3\frac{3}{8} \cdot 4\frac{5}{9}$

15. $\left(-\frac{6}{13}\right)\left(-\frac{26}{30}\right)$

16. $\frac{2}{11} \cdot \frac{5}{8} \cdot \frac{22}{27}$

Perform the indicated operation and simplify.

17. $\frac{2}{7} + \frac{3}{7}$

18. $\frac{26}{30} - \frac{7}{30}$

19. $\frac{7}{8} + \frac{6}{8} + \frac{3}{8}$

20. $\frac{7}{10} - \frac{3}{10} + \frac{4}{10}$

21. Find the LCD of $\frac{3}{7}$ and $\frac{5}{14}$.

22. Add: $\frac{17}{25} + \frac{3}{10}$

23. Write $\frac{3}{4}$ as an equivalent fraction with a denominator of 20.

24. Determine whether these fractions are equivalent.

$\frac{10}{55}$ $\frac{6}{33}$

25. Subtract: $\frac{2}{3} - \frac{10}{11}$

26. Subtract: $17\frac{5}{24} - 9\frac{5}{9}$

27. A flight from Tucson to Phoenix, Arizona, requires $\frac{5}{12}$ of an hour. If the plane has been flying $\frac{1}{4}$ of an hour, find how much time remains before landing.

28. Simplify: $80 \div 8 \cdot 2 + 7$

29. Add: $2\frac{1}{3} + 5\frac{3}{8}$

30. Find the average of $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{11}{15}$.

31. Insert $<$ or $>$ to form a true statement.

$\frac{3}{4}$ $\frac{9}{11}$

32. Multiply: $28{,}000 \times 500$

33. Write the decimal -50.82 in words.

34. Write "seventy-five thousandths" in standard form.

35. Round 736.2359 to the nearest tenth.

36. Round 736.2359 to the nearest thousandth.

37. Add: $23.85 + 1.604$

38. Subtract: $700 - 18.76$

39. Multiply: 0.0531×16

40. Write $\frac{3}{8}$ as a decimal.

41. Divide: $-5.98 \div 115$

42. Write 7.9 as an improper fraction.

43. Simplify: $-0.5(8.6 - 1.2)$

44. Find the unknown number n.

$$\frac{n}{4} = \frac{12}{16}$$

45. Write the numbers in order from smallest to largest.

$\frac{9}{20}$, $\frac{4}{9}$, 0.456

46. Write the rate as a unit rate. 700 meters in 5 seconds

Write each ratio as a fraction in simplest form.

47. The ratio of \$15 to \$10

48. The ratio of 7 to 21

49. The ratio of 2.6 to 3.1

50. The ratio of 900 to 9000

23. ______
24. ______
25. ______
26. ______
27. ______
28. ______
29. ______
30. ______
31. ______
32. ______
33. ______
34. ______
35. ______
36. ______
37. ______
38. ______
39. ______
40. ______
41. ______
42. ______
43. ______
44. ______
45. ______
46. ______
47. ______
48. ______
49. ______
50. ______

6

Percent

This chapter is devoted to percent, a concept used virtually every day in ordinary and business life. Understanding percent and using it efficiently depend on understanding ratios because a percent is a ratio whose denominator is 100. We present techniques to write percents as fractions and as decimals and then solve problems related to sales tax, commission, discounts, interest, and other real-life situations that use percents.

Sections

Check Your Progress

What Does Iceland Have to Do with Hydroelectricity and Geothermal Heat?

Renewable energy is energy that is collected from renewable resources, which are naturally replenished, such as sunlight, wind, rain, tides, waves, waterfalls, and geothermal heat. Renewable energy often provides energy in electricity generation, air and water heating/cooling, and transportation.

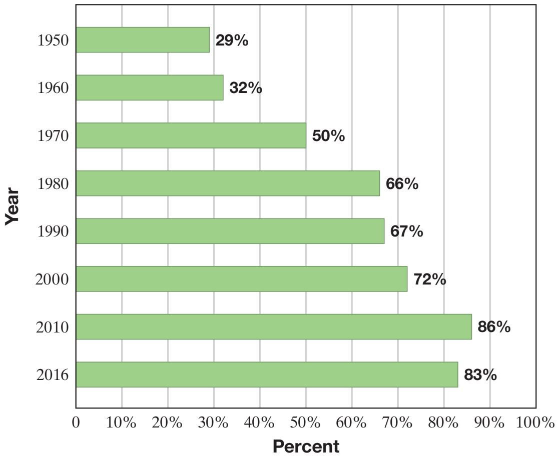
Hydroelectricity is electricity produced from hydropower, or "water power." **Geothermal energy** or geothermal heat is heat energy generated and stored in the Earth.

Iceland, known as the land of fire and ice, is a small island between North America and Europe. It has massive glaciers, bubbling hot springs, green valleys, and spectacular waterfalls, among many other natural beauties.

Iceland is also unique in that a major effort has been made to use renewable energy when possible. Nearly 90% of Icelandic homes enjoy heating by geothermal energy, and even the streets of downtown Reykjavik are kept snow free by heated pipes running under the pavement.

Below, the graph shows the increase in percent of Iceland's energy needs met by renewable energy. In Section 6.1, Exercises 77 and 78, we will explore the percentage of energy Iceland obtains from geothermal and hydro forces.

Iceland's Percent of Energy Needs by Renewable Energy

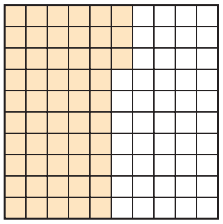


Source: Orkustofnun National Energy Authority

6.1 Percents, Decimals, and Fractions

Objective A Understanding Percent

The word **percent** comes from the Latin phrase *per centum*, which means **"per 100."** For example, 53% (percent) means 53 per 100. In the square below, 53 of the 100 squares are shaded. Thus, 53% of the figure is shaded.



53 of 100 squares
are shaded
or
53% is shaded.

Since 53% means 53 per 100, 53% is the ratio of 53 to 100, or $\frac{53}{100}$.

$$53\% = \frac{53}{100}$$

Also,

$$7\% = \frac{7}{100} \quad \text{7 parts per 100 parts}$$

$$73\% = \frac{73}{100} \quad \text{73 parts per 100 parts}$$

$$109\% = \frac{109}{100} \quad \text{109 parts per 100 parts}$$

Percent

Percent means **per one hundred.** The "%" symbol is used to denote percent.

Percent is used in a variety of everyday situations. For example,

- 88.5% of the U.S. population uses the Internet.
- The store is having a 25%-off sale.
- The enrollment in community colleges is predicted to increase 1.8% each year.
- The South is the home of 49% of all frequent paintball participants.
- 66% of chocolate consumed is milk chocolate.

Example 1 Since 2011, white has been the world's most popular color for cars. For 2017 model cars, 25 out of every 100 were painted white. What percent of model-year 2017 cars were white? (*Source:* PPG Industries)

Solution: Since 25 out of 100 cars were painted white, the fraction is $\frac{25}{100}$. Then

$$\frac{25}{100} = 25\%$$

Work Practice 1

Objectives

A Understand Percent.

B Write Percents as Decimals or Fractions.

C Write Decimals or Fractions as Percents.

D Convert Percents, Decimals, and Fractions.

Practice 1

Of 100 students in a club, 23 are freshmen. What percent of the students are freshmen?

Answer

1. 23%

Practice 2

29 out of 100 executives are in their forties. What percent of executives are in their forties?

Example 2 46 out of every 100 college students live at home. What percent of students live at home? (*Source:* Independent Insurance Agents of America)

Solution:

$$\frac{46}{100} = 46\%$$

■ Work Practice 2

Objective B Writing Percents as Decimals or Fractions

Since percent means "per hundred," we have that

$$1\% = \frac{1}{100} = 0.01$$

In other words, the percent symbol means "per hundred" or, equivalently, "$\frac{1}{100}$" or "0.01." Thus

Write 87% as a fraction: $87\% = 87 \times \frac{1}{100} = \frac{87}{100}$

or

Write 87% as a decimal: $87\% = 87 \times (0.01) = 0.87$

Results are the same.

Of course, we know that the end results are the same, that is,

$$\frac{87}{100} = 0.87$$

The above gives us two options for converting percents. We can replace the percent symbol, %, by $\frac{1}{100}$ or 0.01 and then multiply.

For consistency, when we

- convert from a percent to a *decimal*, we will drop the % symbol and multiply by 0.01
- convert from a percent to a *fraction*, we will drop the % symbol and multiply by $\frac{1}{100}$

Let's practice writing percents as decimals and then writing percents as fractions.

Writing a Percent as a Decimal

Replace the percent symbol with its decimal equivalent, 0.01; then multiply.

$$43\% = 43(0.01) = 0.\underbrace{43}$$

Helpful Hint

If it helps, think of writing a percent as a decimal by

Percent → | Remove the % symbol and move the decimal point 2 places to the left. | → Decimal

Answer

2. 29%

Examples Write each percent as a decimal.

3. $23\% = 23(0.01) = 0.23$ Replace the percent symbol with 0.01. Then multiply.

4. $4.6\% = 4.6(0.01) = 0.046$ Replace the percent symbol with 0.01. Then multiply.

5. $190\% = 190(0.01) = 1.90$ or 1.9

6. $0.74\% = 0.74(0.01) = 0.0074$

7. $100\% = 100(0.01) = 1.00$ or 1

Helpful Hint We just learned that $100\% = 1$.

Work Practice 3–7

Practice 3–7

Write each percent as a decimal.

3. 89%

4. 2.7%

5. 150%

6. 0.69%

7. 800%

✓Concept Check Why is it incorrect to write the percent 0.033% as 3.3 in decimal form?

Now let's write percents as fractions.

Writing a Percent as a Fraction

Replace the percent symbol with its fraction equivalent, $\frac{1}{100}$; then multiply. Don't forget to simplify the fraction if possible.

$$43\% = 43 \cdot \frac{1}{100} = \frac{43}{100}$$

Examples Write each percent as a fraction or mixed number in simplest form.

8. $40\% = 40 \cdot \frac{1}{100} = \frac{40}{100} = \frac{2 \cdot \overset{1}{\cancel{20}}}{5 \cdot \underset{1}{\cancel{20}}} = \frac{2}{5}$

9. $1.9\% = 1.9 \cdot \frac{1}{100} = \frac{1.9}{100}$. We don't want the numerator of the fraction to contain a decimal, so we multiply by 1 in the form of $\frac{10}{10}$.

$$= \frac{1.9}{100} \cdot \frac{10}{10} = \frac{1.9 \cdot 10}{100 \cdot 10} = \frac{19}{1000}$$

10. $125\% = 125 \cdot \frac{1}{100} = \frac{125}{100} = \frac{5 \cdot \overset{1}{\cancel{25}}}{4 \cdot \underset{1}{\cancel{25}}} = \frac{5}{4}$ or $1\frac{1}{4}$

11. $33\frac{1}{3}\% = \underbrace{33\frac{1}{3}}_{\text{Write as an improper fraction.}} \cdot \frac{1}{100} = \frac{100}{3} \cdot \frac{1}{100} = \frac{\overset{1}{\cancel{100}} \cdot 1}{3 \cdot \underset{1}{\cancel{100}}} = \frac{1}{3}$

12. $100\% = 100 \cdot \frac{1}{100} = \frac{100}{100} = 1$

Helpful Hint Just as we did in Example 7, we confirm that $100\% = 1$.

Work Practice 8–12

Practice 8–12

Write each percent as a fraction or mixed number in simplest form.

8. 25%

9. 2.3%

10. 225%

11. $66\frac{2}{3}\%$

12. 8%

Answers

3. 0.89 **4.** 0.027 **5.** 1.5 **6.** 0.0069

7. 8.00 or 8 **8.** $\frac{1}{4}$ **9.** $\frac{23}{1000}$

10. $\frac{9}{4}$ or $2\frac{1}{4}$ **11.** $\frac{2}{3}$ **12.** $\frac{2}{25}$

✓Concept Check Answer

To write a percent as a decimal, the decimal point should be moved two places to the left, not to the right. So the correct answer is 0.00033.

Objective C Writing Decimals or Fractions as Percents

To write a decimal as a percent, we use the result of Example 7 or 12 on the previous page. In these examples, we found that $1 = 100\%$.

Write 0.38 as a percent: $0.38 = 0.38(1) = 0.38(100\%) = \underset{\smile\smile}{38}.\%$

Write $\frac{1}{4}$ as a percent: $\frac{1}{4} = \frac{1}{4}(1) = \frac{1}{4} \cdot 100\% = \frac{100}{4}\% = 25\%$

First, let's practice writing decimals as percents.

Writing a Decimal as a Percent

Multiply by 1 in the form of 100%.

$0.27 = 0.27(100\%) = \underset{\smile\smile}{27}.\%$

Helpful Hint

If it helps, think of writing a decimal as a percent by reversing the steps in the Helpful Hint on page 452.

Percent ← Move the decimal point 2 places to the right and attach a % symbol. ← Decimal

Practice 13–16

Write each decimal as a percent.

13. 0.19
14. 1.75
15. 0.044
16. 0.7

Examples Write each decimal as a percent.

13. $0.65 = 0.65(100\%) = \underset{\smile\smile}{65}.\%$ or 65% Multiply by 100%.

14. $1.25 = 1.25(100\%) = 1\underset{\smile\smile}{25}.\%$ or 125%

15. $0.012 = 0.012(100\%) = 00\underset{\smile\smile}{1.}2\%$ or 1.2%

16. $0.6 = 0.6(100\%) = 0\underset{\smile\smile}{60}.\%$ or 60%

Helpful Hint A zero was inserted as a placeholder.

Work Practice 13–16

✓ **Concept Check** Why is it incorrect to write the decimal 0.0345 as 34.5% in percent form?

Now let's write fractions as percents.

Writing a Fraction as a Percent

Multiply by 1 in the form of 100%.

$\frac{1}{8} = \frac{1}{8} \cdot 100\% = \frac{1}{8} \cdot \frac{100}{1}\% = \frac{100}{8}\% = 12\frac{1}{2}\%$ or 12.5%

Answers

13. 19% **14.** 175% **15.** 4.4%
16. 70%

✓ **Concept Check Answer**

To change a decimal to a percent, multiply by 100%, or move the decimal point *only* two places to the right. So the correct answer is 3.45%.

Helpful Hint

From Examples 7 and 12, we know that

$$100\% = 1$$

Recall that when we multiply a number by 1, we are not changing the value of that number. This means that when we multiply a number by 100%, we are not changing its value but rather writing the number as an equivalent percent.

Examples

Write each fraction or mixed number as a percent.

17. $\frac{9}{20} = \frac{9}{20} \cdot 100\% = \frac{9}{20} \cdot \frac{100}{1}\% = \frac{900}{20}\% = 45\%$

18. $\frac{2}{3} = \frac{2}{3} \cdot 100\% = \frac{2}{3} \cdot \frac{100}{1}\% = \frac{200}{3}\% = 66\frac{2}{3}\%$

19. $1\frac{1}{2} = \frac{3}{2} \cdot 100\% = \frac{3}{2} \cdot \frac{100}{1}\% = \frac{300}{2}\% = 150\%$

Helpful Hint

$\frac{200}{3} = 66.\overline{6}.$ Thus, another way to write $\frac{200}{3}\%$ is $66.\overline{6}\%$.

Work Practice 17–19

Practice 17–19

Write each fraction or mixed number as a percent.

17. $\frac{1}{2}$ **18.** $\frac{7}{40}$ **19.** $2\frac{1}{4}$

✓Concept Check Which digit in the percent 76.4582% represents

a. A tenth percent?

b. A thousandth percent?

c. A hundredth percent?

d. A whole percent?

Example 20

Write $\frac{1}{12}$ as a percent. Round to the nearest hundredth percent.

Solution:

$$\frac{1}{12} = \frac{1}{12} \cdot 100\% = \frac{1}{12} \cdot \frac{100}{1}\% = \frac{100}{12}\% \overset{\text{“approximately”}}{\approx} 8.33\%$$

```
      8.333 ≈ 8.33
12)100.000
   - 96
      4 0
    - 3 6
        40
      - 36
          40
        - 36
           4
```

Thus, $\frac{1}{12}$ is approximately 8.33%.

Work Practice 20

Practice 20

Write $\frac{3}{17}$ as a percent. Round to the nearest hundredth percent.

Answers

17. 50% **18.** $17\frac{1}{2}\%$ **19.** 225%

20. 17.65%

✓Concept Check Answers

a. 4 **b.** 8 **c.** 5 **d.** 6

Objective D Converting Percents, Decimals, and Fractions

Let's summarize what we have learned so far about percents, decimals, and fractions:

Summary of Converting Percents, Decimals, and Fractions

- *To write a percent as a decimal,* replace the % symbol with its decimal equivalent, 0.01; then multiply.
- *To write a percent as a fraction,* replace the % symbol with its fraction equivalent, $\frac{1}{100}$; then multiply.
- *To write a decimal or fraction as a percent,* multiply by 100%.

If we let p represent a number, below we summarize using symbols.

Write a percent as a decimal:	Write a percent as a fraction:	Write a number as a percent:
$p\% = p(0.01)$	$p\% = p \cdot \frac{1}{100}$	$p = p \cdot 100\%$

Practice 21

A family decides to spend no more than 22.5% of its monthly income on rent. Write 22.5% as a decimal and as a fraction.

Example 21 40.4% of automobile thefts in 2016 in the continental United States occur in the West, the greatest percent. Write this percent as a decimal and as a fraction. (*Source:* Federal Bureau of Investigation's [FBI] Uniform Crime Report)

Solution:

As a decimal: $40.4\% = 40.4(0.01) = 0.\underbrace{40}4$

As a fraction: $40.4\% = 40.4 \cdot \frac{1}{100} = \frac{40.4}{100} = \frac{40.4}{100} \cdot \frac{10}{10} = \frac{404}{1000} = \frac{\overset{1}{\cancel{4}} \cdot 101}{\underset{1}{\cancel{4}} \cdot 250} = \frac{101}{250}$

Thus, 40.4% written as a decimal is 0.404 and written as a fraction is $\frac{101}{250}$.

Work Practice 21

Practice 22

Provincetown's budget for waste disposal increased by $1\frac{1}{4}$ times over the budget from last year. What percent increase is this?

Example 22 An advertisement for a stereo system reads "$\frac{1}{4}$ off." What percent off is this?

Solution: Write $\frac{1}{4}$ as a percent.

$$\frac{1}{4} = \frac{1}{4} \cdot 100\% = \frac{1}{4} \cdot \frac{100}{1}\% = \frac{100}{4}\% = 25\%$$

Thus, "$\frac{1}{4}$ off" is the same as "25% off."

Work Practice 22

Note: It is helpful to know a few basic percent conversions. Appendix A.4 contains a handy reference of percent, decimal, and fraction equivalencies.

Also, Appendix A.5 shows how to find common percents of a number.

Answers

21. 0.225, $\frac{9}{40}$ **22.** 125%

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

$\frac{1}{100}$ 0.01 100% percent

1. __________ means "per hundred."

2. __________ = 1.

3. The % symbol is read as __________.

4. To write a decimal or a fraction as a *percent*, multiply by 1 in the form of __________.

5. To write a percent as a *decimal*, drop the % symbol and multiply by __________.

6. To write a percent as a *fraction*, drop the % symbol and multiply by __________.

Write each fraction as a percent.

7. $\frac{13}{100}$ **8.** $\frac{92}{100}$ **9.** $\frac{87}{100}$ **10.** $\frac{71}{100}$ **11.** $\frac{1}{100}$ **12.** $\frac{2}{100}$

Martin-Gay Interactive Videos

See Video 6.1

Watch the section lecture video and answer the following questions.

Objective A **13.** From the lecture before Example 1, what is the most important thing to remember about percent? ▶

Objective B **14.** In Example 7, since the % symbol is replaced with $\frac{1}{100}$, why doesn't the final answer have a denominator of 100? ▶

Objective C **15.** From the lecture before Example 14, how is writing a fraction as a percent similar to writing a decimal as a percent? ▶

Objective D **16.** From Example 17, what is the main difference between writing a percent as an equivalent decimal and writing a percent as an equivalent fraction? ▶

6.1 Exercise Set MyLab Math

Objective A *Solve. See Examples 1 and 2.*

▶ **1.** In a survey of 100 college students, 96 use the Internet. What percent use the Internet?

2. A basketball player makes 81 out of 100 attempted free throws. What percent of free throws are made?

3. Michigan leads the United States in tart cherry production, producing 75 out of every 100 tart cherries each year. (*Source:* Cherry Marketing Institute)

a. What percent of tart cherries are produced in Michigan?

b. What percent of tart cherries are *not* produced in Michigan?

4. The United States is the world's second-largest producer of apples. Twenty-five out of every 100 apples harvested in the United States are exported (shipped to other countries). (*Source:* U.S. Apple Association)

a. What percent of U.S.-grown apples are exported?

b. What percent of U.S.-grown apples are *not* exported?

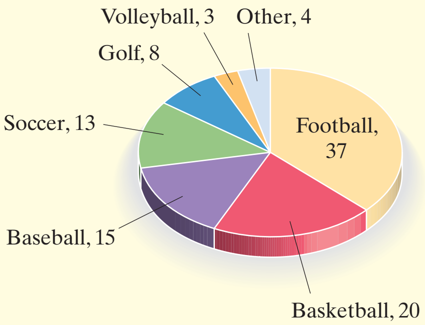
One hundred adults were asked to name their favorite sport, and the results are shown in the circle graph.

5. What sport was preferred by most adults? What percent preferred this sport?

6. What sport was preferred by the least number of adults? What percent preferred this sport?

7. What percent of adults preferred football or soccer?

8. What percent of adults preferred basketball or baseball?



Objective B *Write each percent as a decimal. See Examples 3 through 7.*

9. 41%
10. 64%
11. 6%
12. 9%
13. 100%
14. 136%
15. 61.3%
16. 52.7%
17. 2.8%
18. 1.7%
19. 0.6%
20. 0.9%
21. 300%
22. 700%
23. 32.58%
24. 72.18%

Write each percent as a fraction or mixed number in simplest form. See Examples 8 through 12.

25. 12%
26. 24%
27. 4%
28. 2%
29. 4.5%
30. 7.5%
31. 175%
32. 250%
33. 6.25%
34. 3.75%
35. $10\frac{1}{3}\%$
36. $7\frac{3}{4}\%$
37. $22\frac{3}{8}\%$
38. $15\frac{5}{8}\%$

Objective C *Write each decimal as a percent. See Examples 13 through 16.*

39. 0.003
40. 0.006
41. 0.22
42. 0.45
43. 5.3
44. 1.6
45. 0.056
46. 0.027
47. 0.3328
48. 0.1115
49. 3
50. 5
51. 0.7
52. 0.8

Write each fraction or mixed number as a percent. See Examples 17 through 19.

53. $\frac{7}{10}$
54. $\frac{3}{10}$
55. $\frac{2}{5}$
56. $\frac{4}{5}$
57. $\frac{17}{50}$
58. $\frac{47}{50}$
59. $\frac{3}{8}$
60. $\frac{5}{16}$
61. $\frac{7}{9}$
62. $\frac{1}{3}$
63. $2\frac{1}{2}$
64. $2\frac{1}{5}$
65. $1\frac{9}{10}$
66. $2\frac{7}{10}$

Write each fraction as a percent. Round to the nearest hundredth percent. See Example 20.

67. $\frac{7}{11}$
68. $\frac{5}{12}$
69. $\frac{4}{15}$
70. $\frac{10}{11}$

Objective D *Complete each table. See Examples 21 and 22.*

71.

Percent	Decimal	Fraction
35%		
		$\frac{1}{5}$
	0.5	
70%		
		$\frac{3}{8}$

72.

Percent	Decimal	Fraction
50%		
		$\frac{2}{5}$
	0.25	
12.5%		
		$\frac{5}{8}$
		$\frac{7}{50}$

73.

Percent	Decimal	Fraction
40%		
	0.235	
		$\frac{4}{5}$
$33\frac{1}{3}\%$		
		$\frac{7}{8}$
7.5%		

74.

Percent	Decimal	Fraction
	0.525	
		$\frac{3}{4}$
$66\frac{2}{3}\%$		
		$\frac{5}{6}$
100%		

75.

Percent	Decimal	Fraction
200%		
	2.8	
705%		
		$4\frac{27}{50}$

76.

Percent	Decimal	Fraction
800%		
	3.2	
608%		
		$9\frac{13}{50}$

Solve. See Examples 21 and 22.

77. In 2016, 66% of Iceland's primary energy was geothermal. Write this percent as a decimal and a fraction. (*Source:* National Energy Authority of Iceland)

78. In 2016, 20% of Iceland's primary energy was hydroelectric. Write this percent as a decimal and a fraction. (*Source:* National Energy Authority of Iceland)

79. In 2016, 59.2% of all veterinarians in the United States were female. Write this percent as a decimal and a fraction. (*Source:* American Veterinary Medical Association)

80. The U.S. penny is 97.5% zinc. Write this percent as a decimal and a fraction. (*Source:* Americans for Common Cents)

81. In 2016, the restaurant industry accounted for a $\frac{12}{25}$ share of U.S. dollars spent on food. Write this fraction as a percent. (*Source:* National Restaurant Association)

82. Of all U.S. veterinarians in private practice in 2016, $\frac{3}{50}$ focused exclusively on horses. Write this fraction as a percent. (*Source:* American Veterinary Medical Association)

83. People take aspirin for a variety of reasons. The most common use of aspirin is to prevent heart disease, accounting for 38% of all aspirin use. Write this percent as a decimal. (*Source:* Bayer Market Research)

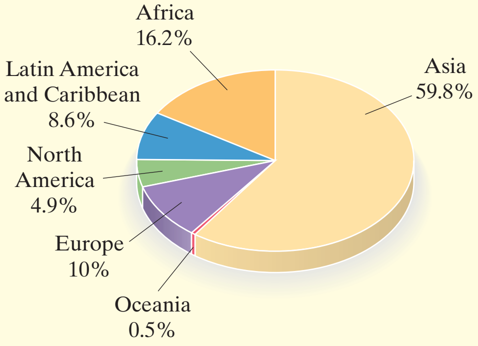
84. In 2017, China accounted for 28% of all motorcycle exports in the world. Write this percent as a decimal. (*Source:* China International Trade center)

85. The average American wastes $\frac{9}{50}$ of all grain products brought into the home. Write this fraction as a percent. (*Source:* Natural Resources Defense Council)

86. Canada produces $\frac{1}{4}$ of the uranium produced in the world. Write this fraction as a percent. (*Source:* World Nuclear Association)

In Exercises 87 through 92, write the percent from the circle graph as a decimal and a fraction.

World Population by Continent



87. Oceania: 0.5%

88. Europe: 10%

89. Africa: 16.2%

90. Asia: 59.8%

91. North America: 4.9%

92. Latin America/Caribbean: 8.6%

Review

Find the value of n. See Section 5.2.

93. $3 \cdot n = 45$

94. $2 \cdot n = 16$

95. $6 \cdot n = 72$

96. $5 \cdot n = 35$

Concept Extensions

97. Write 0.7682 as a percent rounded to the nearest percent.

98. Write 0.2371 as a percent rounded to the nearest percent.

99. Write 1.07835 as a percent rounded to the nearest tenth of a percent.

100. Write 1.25348 as a percent rounded to the nearest tenth of a percent.

Solve. See the Concept Checks in this section.

101. Given the percent 52.8647%, round as indicated.

a. Round to a tenth of a percent.

b. Round to a hundredth of a percent.

102. Given the percent 0.5269%, round as indicated.

a. Round to a tenth of a percent.

b. Round to a hundredth of a percent.

103. Which of the following are correct?

a. 6.5% = 0.65 **b.** 7.8% = 0.078

c. 120% = 0.12 **d.** 0.35% = 0.0035

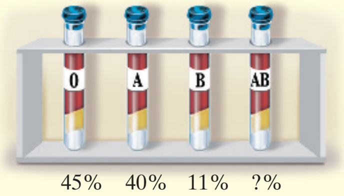
104. Which of the following are correct?

a. 0.231 = 23.1% **b.** 5.12 = 0.0512%

c. 3.2 = 320% **d.** 0.0175 = 0.175%

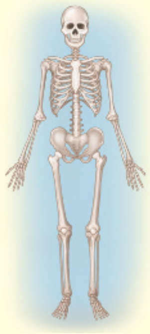
Recall that 1 = 100%. This means that 1 whole is 100%. Use this for Exercises 105 and 106. (Source: Some Body! *by Dr. Pete Rowan)*

105. The four blood types are A, B, O, and AB. (Each blood type can also be further classified as Rh-positive or Rh-negative depending upon whether your blood contains protein or not.) Given the percent blood types for the United States below, calculate the percent of the U.S. population with AB blood type.

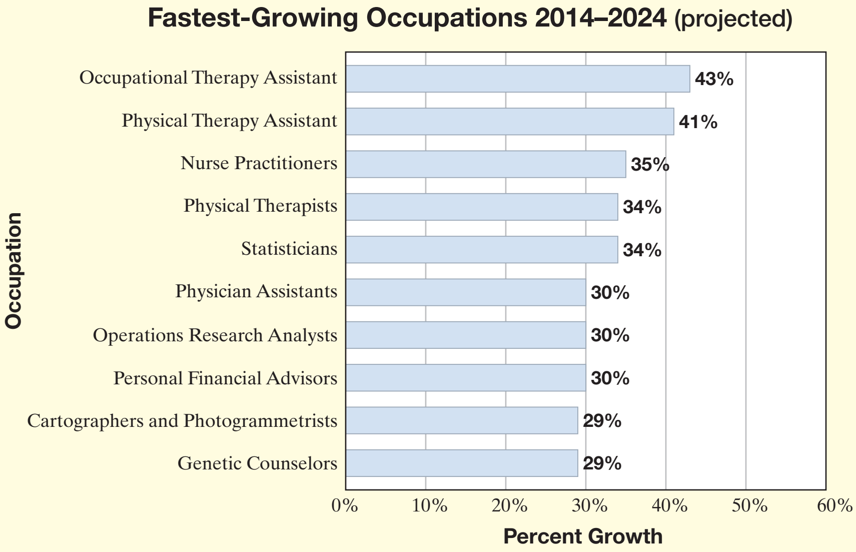


106. The components of bone are all listed in the categories below. Find the missing percent.

1. Minerals–45%
2. Living tissue–30%
3. Water–20%
4. Other–?



The bar graph shows the predicted fastest-growing occupations by percent that require an associate degree or more education. Use this graph for Exercises 107 through 110. (Source: Bureau of Labor Statistics)



Source: Bureau of Labor Statistics

107. What occupation is predicted to be the fastest growing?

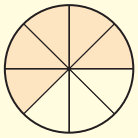
108. What occupation is predicted to be the second fastest growing?

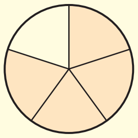
109. Write the percent change for physician assistants as a decimal.

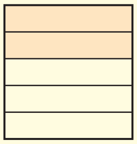
110. Write the percent change for statisticians as a decimal.

What percent of the figure is shaded?

111.


112.


113.


114.


Fill in the blanks.

115. A fraction written as a percent is greater than 100% when the numerator is ________ than the denominator. (greater/less)

116. A decimal written as a percent is less than 100% when the decimal is ________ than 1. (greater/less)

117. In your own words, explain how to write a percent as a fraction.

118. In your own words, explain how to write a fraction as a decimal.

Write each fraction as a decimal and then write each decimal as a percent. Round the decimal to three decimal places (nearest thousandth) and the percent to the nearest tenth of a percent.

119. $\frac{21}{79}$

120. $\frac{56}{102}$

121. $\frac{850}{736}$

122. $\frac{506}{248}$

6.2 Solving Percent Problems Using Equations

Note: Sections **6.2** and **6.3** introduce two methods for solving percent problems. It is not necessary that you study both sections. You may want to check with your instructor for further advice.

Throughout this text, we have written mathematical statements such as $3 + 10 = 13$, or area $=$ length $\cdot$ width. These statements are called "equations." An **equation** is a mathematical statement that contains an equal sign. To solve percent problems in this section, we translate the problems into such mathematical statements, or equations.

Objective A Writing Percent Problems as Equations

Recognizing key words in a percent problem is helpful in writing the problem as an equation. Three key words in the statement of a percent problem and their meanings are as follows:

of means **multiplication** $(\cdot)$
is means **equals** $(=)$
what (or some equivalent) means **the unknown number**

In our examples, we let the letter n stand for the unknown number.

Helpful Hint

Any letter of the alphabet can be used to represent the unknown number. In this section, we mostly use the letter n.

Example 1 Translate to an equation.

5 is what percent of 20?

Solution:

$$\begin{array}{ccccc} 5 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 20? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 5 & = & n & \cdot & 20 \end{array}$$

Work Practice 1

Helpful Hint

Remember that an equation is simply a mathematical statement that contains an equal sign $(=)$.

$$5 \underset{\substack{\uparrow \\ \text{equal sign}}}{=} n \cdot 20$$

Example 2 Translate to an equation.

1.2 is 30% of what number?

Solution:

$$\begin{array}{ccccc} 1.2 & \text{is} & 30\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1.2 & = & 30\% & \cdot & n \end{array}$$

Work Practice 2

Objectives

A Write Percent Problems as Equations.

B Solve Percent Problems.

Practice 1

Translate: 6 is what percent of 24?

Practice 2

Translate: 1.8 is 20% of what number?

Answers

1. $6 = n \cdot 24$ **2.** $1.8 = 20\% \cdot n$

Practice 3

Translate: What number is 40% of 3.6?

Example 3 Translate to an equation.

What number is 25% of 0.008?

Solution:

$$\begin{array}{ccccc} \underbrace{\text{What number}} & \text{is} & 25\% & \text{of} & 0.008? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & = & 25\% & \cdot & 0.008 \end{array}$$

Work Practice 3

Practice 4–6

Translate each to an equation.

4. 42% of 50 is what number?
5. 15% of what number is 9?
6. What percent of 150 is 90?

Examples Translate each of the following to an equation:

4.

$$\begin{array}{ccccc} 38\% & \text{of} & 200 & \text{is} & \underbrace{\text{what number?}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 38\% & \cdot & 200 & = & n \end{array}$$

5.

$$\begin{array}{ccccc} 40\% & \text{of} & \underbrace{\text{what number}} & \text{is} & 80? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 40\% & \cdot & n & = & 80 \end{array}$$

6.

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & 85 & \text{is} & 34? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & \cdot & 85 & = & 34 \end{array}$$

Work Practice 4–6

✓Concept Check In the equation $2 \cdot n = 10$, what step should be taken to solve the equation for n?

Objective B Solving Percent Problems

You may have noticed by now that each percent problem has contained three numbers—in our examples, two are known and one is unknown. Each of these numbers is given a special name.

$$\begin{array}{ccccc} 15\% & \text{of} & 60 & \text{is} & \underbrace{9} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \boxed{\begin{array}{c} 15\% \\ \text{percent} \end{array}} & \cdot & \boxed{\begin{array}{c} 60 \\ \text{base} \end{array}} & = & \boxed{\begin{array}{c} 9 \\ \text{amount} \end{array}} \end{array}$$

We call this equation the **percent equation.**

Percent Equation

percent · base = amount

Helpful Hint

Notice that the percent equation given above is a true statement. To see this, simplify the left side as shown:

$$\begin{aligned} 15\% \cdot 60 &= 9 \\ 0.15 \cdot 60 &= 9 \quad \text{Write 15\% as 0.15.} \\ 9 &= 9 \quad \text{Multiply.} \end{aligned}$$

The statement 9 = 9 is true.

Answers

3. $n = 40\% \cdot 3.6$ **4.** $42\% \cdot 50 = n$
5. $15\% \cdot n = 9$ **6.** $n \cdot 150 = 90$

✓Concept Check Answer

If $2 \cdot n = 10$, then $n = \frac{10}{2}$, or $n = 5$.

After a percent problem has been written as a percent equation, we can use the equation to find the unknown number. This is called **solving** the equation.

Example 7 Solving a Percent Equation for the Amount

$$\begin{array}{ccccc} \underbrace{\text{What number}} & \text{is} & 35\% & \text{of} & 40? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

Solution:

$$\begin{array}{lll} n & = 35\% \cdot 40 & \text{Translate to an equation.} \\ n & = 0.35 \cdot 40 & \text{Write 35\% as 0.35.} \\ n & = 14 & \text{Multiply } 0.35 \cdot 40 = 14. \end{array}$$

Thus, 14 is 35% of 40.

Is this reasonable? To see, round 35% to 40%. Then 40% of 40 or 0.40(40) is 16. Our result is reasonable since 16 is close to 14.

Work Practice 7

Practice 7

What number is 20% of 85?

Helpful Hint

When solving a percent equation, write the percent as a decimal (or fraction).

Example 8 Solving a Percent Equation for the Amount

$$\begin{array}{cccccc} 85\% & \text{of} & 300 & \text{is} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

Solution:

$$\begin{array}{lll} 85\% \cdot 300 = & n & \text{Translate to an equation.} \\ 0.85 \cdot 300 = & n & \text{Write 85\% as 0.85.} \\ 255 = & n & \text{Multiply } 0.85 \cdot 300 = 255. \end{array}$$

Thus, 85% of 300 is 255.

Is this result reasonable? To see, round 85% to 90%. Then 90% of 300 or $0.90(300) = 270$, which is close to 255.

Work Practice 8

Practice 8

90% of 150 is what number?

Example 9 Solving a Percent Equation for the Base

$$\begin{array}{ccccc} 12\% & \text{of} & \underbrace{\text{what number}} & \text{is} & 0.6? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

Solution:

$$\begin{array}{lll} 12\% \cdot n & = 0.6 & \text{Translate to an equation.} \\ 0.12 \cdot n & = 0.6 & \text{Write 12\% as 0.12.} \end{array}$$

Recall from Section 5.2 that if "0.12 times some number is 0.6," then that number is 0.6 divided by 0.12.

$$n = \frac{0.6}{0.12} \quad \text{Divide 0.6 by 0.12, the number multiplied by } n.$$

$$n = 5$$

Thus, 12% of 5 is 0.6.

Is this reasonable? To see, round 12% to 10%. Then 10% of 5 or $0.10(5) = 0.5$, which is close to 0.6.

Work Practice 9

Practice 9

15% of what number is 1.2?

Answers

7. 17 **8.** 135 **9.** 8

Practice 10

27 is $4\frac{1}{2}\%$ of what number?

Example 10 Solving a Percent Equation for the Base

$$\begin{array}{lccccl}
 & 13 & \text{is} & 6\frac{1}{2}\% & \text{of} & \underbrace{\text{what number}}? \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\textbf{Solution:} & 13 & & 6\frac{1}{2}\% & \cdot & n \quad \text{Translate to an equation.} \\
 & 13 & = & 0.065 & \cdot & n \quad 6\frac{1}{2}\% = 6.5\% = 0.065. \\
 & \frac{13}{0.065} & = & n & & \quad \text{Divide 13 by 0.065, the number multiplied by } n. \\
 & 200 & = & n & &
\end{array}$$

Thus, 13 is $6\frac{1}{2}\%$ of 200.

Check to see if this result is reasonable.

Work Practice 10

Practice 11

What percent of 80 is 8?

Example 11 Solving a Percent Equation for the Percent

$$\begin{array}{lccccc}
 & \underbrace{\text{What percent}} & \text{of} & 12 & \text{is} & 9? \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\textbf{Solution:} & n & \cdot & 12 & = & 9 \quad \text{Translate to an equation.} \\
 & & & n & = & \frac{9}{12} \quad \text{Divide 9 by 12, the number multiplied by } n. \\
 & & & n & = & 0.75
\end{array}$$

Next, since we are looking for percent, we write 0.75 as a percent.

$n = 75\%$

So, 75% of 12 is 9. To check, see that $75\% \cdot 12 = 9$.

Work Practice 11

Helpful Hint

If your unknown in the percent equation is the percent, don't forget to convert your answer to a percent.

Practice 12

35 is what percent of 25?

Example 12 Solving a Percent Equation for the Percent

$$\begin{array}{lccccc}
 & 78 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 65? \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\textbf{Solution:} & 78 & = & n & \cdot & 65 \quad \text{Translate to an equation.} \\
 & \frac{78}{65} & = & n & & \quad \text{Divide 78 by 65, the number multiplied by } n. \\
 & 1.2 & = & n & & \\
 & 120\% & = & n & & \quad \text{Write 1.2 as a percent.}
\end{array}$$

So, 78 is 120% of 65. Check this result.

Work Practice 12

Answers

10. 600 **11.** 10% **12.** 140%

✓**Concept Check** Consider these problems.

1. 75% of 50 =

a. 50 **b.** a number greater than 50 **c.** a number less than 50

2. 40% of a number is 10. Is the number

a. 10? **b.** less than 10? **c.** greater than 10?

3. 800 is 120% of what number? Is the number

a. 800? **b.** less than 800? **c.** greater than 800?

Helpful Hint

Use the following to see if your answers are reasonable.

$$(100\%) \text{ of a number} = \text{the number}$$

$$\begin{pmatrix}\text{a percent}\\ \text{greater than}\\ 100\%\end{pmatrix} \text{ of a number} = \begin{array}{l}\text{a number greater}\\ \text{than the original number}\end{array}$$

$$\begin{pmatrix}\text{a percent}\\ \text{less than } 100\%\end{pmatrix} \text{ of a number} = \begin{array}{l}\text{a number less}\\ \text{than the original number}\end{array}$$

✓**Concept Check Answers**

1. c **2.** c **3.** b

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

percent
base
amount
the number
of
is
less
greater

1. The word __________ translates to "=".

2. The word __________ usually translates to "multiplication."

3. In the statement "10% of 90 is 9," the number 9 is called the __________, 90 is called the __________, and 10 is called the __________.

4. 100% of a number = __________.

5. Any "percent greater than 100%" of "a number" = "a number __________ than the original number."

6. Any "percent less than 100%" of "a number" = "a number __________ than the original number."

Identify the percent, the base, and the amount in each equation. Recall that percent · base = amount.

7. $42\% \cdot 50 = 21$

8. $30\% \cdot 65 = 19.5$

9. $107.5 = 125\% \cdot 86$

10. $99 = 110\% \cdot 90$

Martin-Gay Interactive Videos

See Video 6.2

Watch the section lecture video and answer the following questions.

Objective A **11.** From the lecture before Example 1, what are the key words and their translations that we need to remember?

Objective B **12.** What is the difference between the translated equation in Example 5 and those in Examples 4 and 6?

6.2 Exercise Set MyLab Math

Objective A **Translating** *Translate each to an equation. Do not solve. See Examples 1 through 6.*

1. 18% of 81 is what number?

2. 36% of 72 is what number?

3. 20% of what number is 105?

4. 40% of what number is 6?

5. 0.6 is 40% of what number?

6. 0.7 is 20% of what number?

7. What percent of 80 is 3.8?

8. 9.2 is what percent of 92?

9. What number is 9% of 43?

10. What number is 25% of 55?

11. What percent of 250 is 150?

12. What percent of 375 is 300?

Objective B *Solve. See Examples 7 and 8.*

13. 10% of 35 is what number?

14. 25% of 68 is what number?

15. What number is 14% of 205?

16. What number is 18% of 425?

Solve. See Examples 9 and 10.

17. 1.2 is 12% of what number?

18. 0.22 is 44% of what number?

19. $8\frac{1}{2}\%$ of what number is 51?

20. $4\frac{1}{2}\%$ of what number is 45?

Solve. See Examples 11 and 12.

21. What percent of 80 is 88?

22. What percent of 40 is 60?

23. 17 is what percent of 50?

24. 48 is what percent of 50?

Objectives A B Mixed Practice *Solve. See Examples 1 through 12.*

25. 0.1 is 10% of what number?

26. 0.5 is 5% of what number?

27. 150% of 430 is what number?

28. 300% of 56 is what number?

29. 82.5 is $16\frac{1}{2}$% of what number?

30. 7.2 is $6\frac{1}{4}$% of what number?

31. 2.58 is what percent of 50?

32. 2.64 is what percent of 25?

33. What number is 42% of 60?

34. What number is 36% of 80?

35. What percent of 184 is 64.4?

36. What percent of 120 is 76.8?

37. 120% of what number is 42?

38. 160% of what number is 40?

39. 2.4% of 26 is what number?

40. 4.8% of 32 is what number?

41. What percent of 600 is 3?

42. What percent of 500 is 2?

43. 6.67 is 4.6% of what number?

44. 9.75 is 7.5% of what number?

45. 1575 is what percent of 2500?

46. 2520 is what percent of 3500?

47. 2 is what percent of 50?

48. 2 is what percent of 40?

Review

Find the value of n in each proportion. See Section 5.2.

49. $\frac{27}{n} = \frac{9}{10}$

50. $\frac{35}{n} = \frac{7}{5}$

51. $\frac{n}{5} = \frac{8}{11}$

52. $\frac{n}{3} = \frac{6}{13}$

Write each phrase as a proportion. See Section 5.2.

53. 17 is to 12 as n is to 20

54. 20 is to 25 as n is to 10

55. 8 is to 9 as 14 is to n

56. 5 is to 6 as 15 is to n

Concept Extensions

For each equation in Exercises 57 through 60, determine the next step taken to find the value of n. See the first Concept Check in this section.

57. $5 \cdot n = 32$

a. $n = 5 \cdot 32$ **b.** $n = \frac{5}{32}$ **c.** $n = \frac{32}{5}$ **d.** none of these

58. $n = 0.7 \cdot 12$

a. $n = 8.4$ **b.** $n = \frac{12}{0.7}$ **c.** $n = \frac{0.7}{12}$ **d.** none of these

59. $0.06 = n \cdot 7$

a. $n = 0.06 \cdot 7$ **b.** $n = \frac{0.06}{7}$ **c.** $n = \frac{7}{0.06}$ **d.** none of these

60. $0.01 = n \cdot 8$

a. $n = 0.01 \cdot 8$ **b.** $n = \frac{8}{0.01}$ **c.** $n = \frac{0.01}{8}$ **d.** none of these

61. Write a word statement for the equation $20\% \cdot n = 18.6$. Use the phrase "some number" for "n."

62. Write a word statement for the equation $n = 33\frac{1}{3}\% \cdot 24$. Use the phrase "some number" for "n."

For each exercise, determine whether the percent, n, is **a.** *100%,* **b.** *greater than 100%, or* **c.** *less than 100%. See the second Concept Check in this section.*

63. n% of 20 is 30

64. n% of 98 is 98

65. n% of 120 is 85

66. n% of 35 is 50

For each exercise, determine whether the number, n, is **a.** *equal to 45,* **b.** *greater than 45, or* **c.** *less than 45.*

67. 55% of 45 is n

68. 230% of 45 is n

69. 100% of 45 is n

70. 30% of n is 45

71. 100% of n is 45

72. 180% of n is 45

Solve.

73. In your own words, explain how to solve a percent equation.

74. Write a percent problem that uses the percent 50%.

75. 1.5% of 45,775 is what number?

76. What percent of 75,528 is 27,945.36?

77. 22,113 is 180% of what number?

6.3 Solving Percent Problems Using Proportions

Objectives

A Write Percent Problems as Proportions.

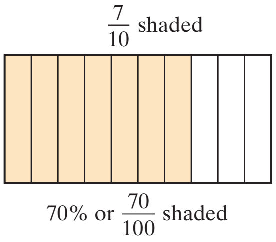
B Solve Percent Problems.

There is more than one method that can be used to solve percent problems. (See the note at the beginning of Section **6.2**.) In the last section, we used the percent equation. In this section, we will use proportions.

Objective A Writing Percent Problems as Proportions

To understand the proportion method, recall that 70% means the ratio of 70 to 100, or $\frac{70}{100}$.

$$70\% = \frac{70}{100} = \frac{7}{10}$$



Since the ratio $\frac{70}{100}$ is equal to the ratio $\frac{7}{10}$, we have the proportion

$$\frac{7}{10} = \frac{70}{100}$$

We call this proportion the "percent proportion." In general, we can name the parts of this proportion as follows:

Percent Proportion

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100} \quad \leftarrow \text{always } 100$$

or

$$\begin{matrix}\text{amount} \rightarrow \\ \text{base} \rightarrow\end{matrix} \frac{a}{b} = \frac{p}{100} \leftarrow \text{percent}$$

When we translate percent problems to proportions, the **percent,** p, can be identified by looking for the symbol % or the word *percent.* The **base,** b, usually follows the word *of.* The **amount,** a, is the part compared to the whole.

Helpful Hint

Part of Proportion	How It's Identified
Percent	% or percent
Base	Appears after *of*
Amount	Part compared to whole

Example 1 Translate to a proportion.

$$12\% \text{ of } \underbrace{\text{what number}} \text{ is } 47?$$

Solution:

percent	base (It appears after the word *of*.)	amount (It is the part compared to the whole.)

$$\begin{matrix}\text{amount} \rightarrow \\ \text{base} \rightarrow\end{matrix} \frac{47}{b} = \frac{12}{100} \leftarrow \text{percent}$$

Work Practice 1

Practice 1

Translate to a proportion.
15% of what number is 55?

Example 2 Translate to a proportion.

$$101 \text{ is } \underbrace{\text{what percent}} \text{ of } 200?$$

Solution:

amount (It is the part compared to the whole.)	percent	base (It appears after the word *of*.)

$$\begin{matrix}\text{amount} \rightarrow \\ \text{base} \rightarrow\end{matrix} \frac{101}{200} = \frac{p}{100} \leftarrow \text{percent}$$

Work Practice 2

Practice 2

Translate to a proportion.
35 is what percent of 70?

Answers

1. $\frac{55}{b} = \frac{15}{100}$ **2.** $\frac{35}{70} = \frac{p}{100}$

Practice 3

Translate to a proportion.
What number is 25% of 68?

Example 3

Translate to a proportion.

$$\begin{array}{ccccc} \underbrace{\text{What number}} & \text{is} & 90\% & \text{of} & 45? \\ \downarrow & & \downarrow & & \downarrow \\ \begin{array}{c}\text{amount}\\ \text{(It is the part}\\ \text{compared to}\\ \text{the whole.)}\end{array} & & \text{percent} & & \begin{array}{c}\text{base}\\ \text{(It appears}\\ \text{after the}\\ \text{word } of\text{.)}\end{array} \end{array}$$

Solution:

$$\begin{array}{r}\text{amount}\rightarrow\\ \text{base}\rightarrow\end{array} \frac{a}{45}=\frac{90}{100} \leftarrow \text{percent}$$

Work Practice 3

Practice 4

Translate to a proportion.
520 is 65% of what number?

Example 4

Translate to a proportion.

$$\begin{array}{ccccc} 238 & \text{is} & 40\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount} & & \text{percent} & & \text{base} \end{array}$$

Solution:

$$\frac{238}{b}=\frac{40}{100}$$

Work Practice 4

Practice 5

Translate to a proportion.
What percent of 50 is 65?

Example 5

Translate to a proportion.

$$\begin{array}{cccc} \underbrace{\text{What percent}} & \text{of} \quad 30 & \text{is} & 75? \\ \downarrow & \downarrow & & \downarrow \\ \text{percent} & \text{base} & & \text{amount} \end{array}$$

Solution:

$$\frac{75}{30}=\frac{p}{100}$$

Work Practice 5

Practice 6

Translate to a proportion.
36% of 80 is what number?

Example 6

Translate to a proportion.

$$\begin{array}{ccccc} 45\% & \text{of} & 105 & \text{is} & \underbrace{\text{what number}}? \\ \downarrow & & \downarrow & & \downarrow \\ \text{percent} & & \text{base} & & \text{amount} \end{array}$$

Solution:

$$\frac{a}{105}=\frac{45}{100}$$

Work Practice 6

Objective B Solving Percent Problems

The proportions that we have written in this section contain three values that can change: the percent, the base, and the amount. If any two of these values are known, we can find the third (the unknown value). To do this, we write a percent proportion and find the unknown value as we did in Section 5.2.

Answers

3. $\frac{a}{68}=\frac{25}{100}$ **4.** $\frac{520}{b}=\frac{65}{100}$
5. $\frac{65}{50}=\frac{p}{100}$ **6.** $\frac{a}{80}=\frac{36}{100}$

Example 7 Solving a Percent Proportion for the Amount

Solution:

$$\begin{array}{ccccc} \underbrace{\text{What number}} & \text{is} & 30\% & \text{of} & 9? \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount} & & \text{percent} & & \text{base} \end{array}$$

$$\frac{a}{9} = \frac{30}{100}$$

To solve, we set cross products equal to each other.

$$\frac{a}{9} = \frac{30}{100}$$

$$a \cdot 100 = 9 \cdot 30 \quad \text{Set cross products equal.}$$

$$a \cdot 100 = 270 \quad \text{Multiply.}$$

Recall from Section 5.2 that if "some number times 100 is 270," then the number is 270 divided by 100.

$$a = \frac{270}{100} \quad \text{Divide 270 by 100, the number multiplied by } a.$$

$$a = 2.7 \quad \text{Simplify.}$$

Thus, 2.7 is 30% of 9.

Work Practice 7

Practice 7

What number is 8% of 120?

Helpful Hint

The proportion in Example 7 contains the ratio $\frac{30}{100}$. A ratio in a proportion may be simplified before solving the proportion. The unknown number in both $\frac{a}{9} = \frac{30}{100}$ and $\frac{a}{9} = \frac{3}{10}$ is 2.7.

✓Concept Check Consider the statement: "78 is what percent of 350?" Which part of the percent proportion is unknown?

a. the amount **b.** the base **c.** the percent

Consider another statement: "14 is 10% of some number." Which part of the percent proportion is unknown?

a. the amount **b.** the base **c.** the percent

Example 8 Solving a Percent Proportion for the Base

Solution:

$$\begin{array}{ccccc} 150\% & \text{of} & \underbrace{\text{what number}} & \text{is} & 30? \\ \downarrow & & \downarrow & & \downarrow \\ \text{percent} & & \text{base} & & \text{amount} \end{array}$$

$$\frac{30}{b} = \frac{150}{100} \quad \text{Write the proportion.}$$

$$\frac{30}{b} = \frac{3}{2} \quad \text{Write } \frac{150}{100} \text{ as } \frac{3}{2}.$$

$$30 \cdot 2 = b \cdot 3 \quad \text{Set cross products equal.}$$

$$60 = b \cdot 3 \quad \text{Multiply.}$$

$$\frac{60}{3} = b \quad \text{Divide 60 by 3, the number multiplied by } b.$$

$$20 = b \quad \text{Simplify.}$$

Thus, 150% of 20 is 30.

Work Practice 8

Practice 8

75% of what number is 60?

Answers

7. 9.6 **8.** 80

✓Concept Check Answers

c, b

✓**Concept Check** When solving a percent problem by using a proportion, describe how you can check the result.

Practice 9

15.2 is 5% of what number?

Example 9 Solving a Percent Proportion for the Base

$$\begin{array}{ccccc} 20.8 & \text{is} & 40\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount} & & \text{percent} & & \text{base} \end{array}$$

Solution:

$$\begin{aligned} \frac{20.8}{b} &= \frac{40}{100} \text{ or } \frac{20.8}{b} = \frac{2}{5} && \text{Write the proportion and simplify } \tfrac{40}{100}. \\ 20.8 \cdot 5 &= b \cdot 2 && \text{Set cross products equal.} \\ 104 &= b \cdot 2 && \text{Multiply.} \\ \frac{104}{2} &= b && \text{Divide 104 by 2, the number multiplied by } b. \\ 52 &= b && \text{Simplify.} \end{aligned}$$

So, 20.8 is 40% of 52.

Work Practice 9

Practice 10

What percent of 40 is 6?

Example 10 Solving a Percent Proportion for the Percent

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & 50 & \text{is} & 8? \\ \downarrow & & \downarrow & & \downarrow \\ \text{percent} & & \text{base} & & \text{amount} \end{array}$$

Solution:

$$\begin{aligned} \frac{8}{50} &= \frac{p}{100} \text{ or } \frac{4}{25} = \frac{p}{100} && \text{Write the proportion and simplify } \tfrac{8}{50}. \\ 4 \cdot 100 &= 25 \cdot p && \text{Set cross products equal.} \\ 400 &= 25 \cdot p && \text{Multiply.} \\ \frac{400}{25} &= p && \text{Divide 400 by 25, the number multiplied by } p. \\ 16 &= p && \text{Simplify.} \end{aligned}$$

So, 16% of 50 is 8.

Work Practice 10

Helpful Hint

Recall from our percent proportion that this number already is a percent. Just keep the number as is and attach a % symbol.

Practice 11

336 is what percent of 160?

Example 11 Solving a Percent Proportion for the Percent

$$\begin{array}{ccccc} 504 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 360? \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount} & & \text{percent} & & \text{base} \end{array}$$

Solution:

$$\frac{504}{360} = \frac{p}{100}$$

Answers

9. 304 **10.** 15% **11.** 210%

✓**Concept Check Answer**

by putting the result into the proportion and checking that the proportion is true

Let's choose not to simplify the ratio $\frac{504}{360}$.

$$\begin{aligned} 504 \cdot 100 &= 360 \cdot p && \text{Set cross products equal.} \\ 50{,}400 &= 360 \cdot p && \text{Multiply.} \\ \frac{50{,}400}{360} &= p && \text{Divide 50,400 by 360, the number multiplied by } p. \\ 140 &= p && \text{Simplify.} \end{aligned}$$

Notice that by choosing not to simplify $\frac{504}{360}$, we had larger numbers in our equation. Either way, we find that 504 is 140% of 360.

Work Practice 11

You may have noticed the following while working the examples.

Helpful Hint

Use the following to see whether your answers are reasonable.

$$\begin{aligned} 100\% \text{ of a number} &= \text{the number} \\ \begin{pmatrix}\text{a percent} \\ \text{greater than} \\ 100\%\end{pmatrix} \text{ of a number} &= \begin{matrix}\text{a number larger} \\ \text{than the original number}\end{matrix} \\ \begin{pmatrix}\text{a percent} \\ \text{less than } 100\%\end{pmatrix} \text{ of a number} &= \begin{matrix}\text{a number less} \\ \text{than the original number}\end{matrix} \end{aligned}$$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. These choices will be used more than once.

amount base percent

1. When translating the statement "20% of 15 is 3" to a proportion, the number 3 is called the ______________, 15 is the ______________, and 20 is the ______________.

2. In the question "50% of what number is 28?," which part of the percent proportion is unknown? ______________

3. In the question "What number is 25% of 200?," which part of the percent proportion is unknown? ______________

4. In the question "38 is what percent of 380?," which part of the percent proportion is unknown? ______________

Identify the amount, the base, and the percent in each equation. Recall that $\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$.

5. $\frac{12.6}{42} = \frac{30}{100}$

6. $\frac{201}{300} = \frac{67}{100}$

7. $\frac{20}{100} = \frac{102}{510}$

8. $\frac{40}{100} = \frac{248}{620}$

Martin-Gay Interactive Videos

See Video 6.3

Watch the section lecture video and answer the following questions.

Objective A **9.** In Example 1, how did we identify what part of the percent proportion 45 is?

Objective B **10.** From Examples 4–6, what number is *always* part of the cross product equation of a percent proportion?

6.3 Exercise Set MyLab Math

Objective A Translating *Translate each to a proportion. Do not solve. See Examples 1 through 6.*

1. 98% of 45 is what number?

2. 92% of 30 is what number?

3. What number is 4% of 150?

4. What number is 7% of 175?

5. 14.3 is 26% of what number?

6. 1.2 is 47% of what number?

7. 35% of what number is 84?

8. 85% of what number is 520?

9. What percent of 400 is 70?

10. What percent of 900 is 216?

11. 8.2 is what percent of 82?

12. 9.6 is what percent of 96?

Objective B *Solve. See Example 7.*

13. 40% of 65 is what number?

14. 25% of 84 is what number?

15. What number is 18% of 105?

16. What number is 60% of 29?

Solve. See Examples 8 and 9.

17. 15% of what number is 90?

18. 55% of what number is 55?

19. 7.8 is 78% of what number?

20. 1.1 is 44% of what number?

Solve. See Examples 10 and 11.

21. What percent of 35 is 42?

22. What percent of 98 is 147?

23. 14 is what percent of 50?

24. 24 is what percent of 50?

Objectives A B Mixed Practice *Solve. See Examples 1 through 11.*

25. 3.7 is 10% of what number?

26. 7.4 is 5% of what number?

27. 2.4% of 70 is what number?

28. 2.5% of 90 is what number?

29. 160 is 16% of what number?

30. 30 is 6% of what number?

31. 394.8 is what percent of 188?

32. 550.4 is what percent of 172?

33. What number is 89% of 62?

34. What number is 53% of 130?

▶ **35.** What percent of 6 is 2.7?

36. What percent of 5 is 1.6?

37. 140% of what number is 105?

38. 170% of what number is 221?

39. 1.8% of 48 is what number?

40. 7.8% of 24 is what number?

41. What percent of 800 is 4?

42. What percent of 500 is 3?

43. 3.5 is 2.5% of what number?

44. 9.18 is 6.8% of what number?

▶ **45.** 20% of 48 is what number?

46. 75% of 14 is what number?

47. 2486 is what percent of 2200?

48. 9310 is what percent of 3800?

Review

Add or subtract the fractions. See Sections 3.4, 3.5, and 3.7.

49. $-\frac{11}{16}+\left(-\frac{3}{16}\right)$

50. $\frac{7}{12}-\frac{5}{8}$

51. $3\frac{1}{2}-\frac{11}{30}$

52. $2\frac{2}{3}+4\frac{1}{2}$

Add or subtract the decimals. See Section 4.2.

53.
$$\begin{array}{r} 0.41 \\ +0.29 \\ \hline \end{array}$$

54.
$$\begin{array}{r} 10.78 \\ 4.3 \\ +\ 0.21 \\ \hline \end{array}$$

55.
$$\begin{array}{r} 2.38 \\ -0.19 \\ \hline \end{array}$$

56.
$$\begin{array}{r} 16.37 \\ -\ 2.61 \\ \hline \end{array}$$

Concept Extensions

57. Write a word statement for the proportion $\frac{x}{28}=\frac{25}{100}$. Use the phrase "what number" for "x."

58. Write a percent statement that translates to $\frac{16}{80}=\frac{20}{100}$.

Suppose you have finished solving four percent problems using proportions that you set up correctly. Check each answer to see if each makes the proportion a true proportion. If any proportion is not true, solve it to find the correct solution. See the Concept Checks in this section.

59. $\frac{a}{64}=\frac{25}{100}$
Is the amount equal to 17?

60. $\frac{520}{b}=\frac{65}{100}$
Is the base equal to 800?

61. $\frac{p}{100}=\frac{13}{52}$
Is the percent equal to 25 (25%)?

62. $\frac{36}{12}=\frac{p}{100}$
Is the percent equal to 50 (50%)?

Solve.

63. In your own words, describe how to identify the percent, the base, and the amount in a percent problem.

64. In your own words, explain how to use a proportion to solve a percent problem.

Solve. Round to the nearest tenth.

65. What number is 22.3% of 53,862?

66. What percent of 110,736 is 88,542?

67. 8652 is 119% of what number?

Percent and Percent Problems

Write each number as a percent.

1. 0.12

2. 0.68

3. $\frac{1}{8}$

4. $\frac{5}{2}$

5. 5.2

6. 8

7. $\frac{3}{50}$

8. $\frac{11}{25}$

9. $7\frac{1}{2}$

10. $3\frac{1}{4}$

11. 0.03

12. 0.05

Write each percent as a decimal.

13. 65%

14. 31%

15. 8%

16. 7%

17. 142%

18. 400%

19. 2.9%

20. 6.6%

Write each percent as a decimal and as a fraction or mixed number in simplest form. (If necessary when writing as a decimal, round to the nearest thousandth.)

21. 3%

22. 5%

23. 5.25%

24. 12.75%

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. ______

18. ______

19. ______

20. ______

21. ______

22. ______

23. ______

24. ______

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

25. 38% **26.** 45% **27.** $12\frac{1}{3}\%$ **28.** $16\frac{2}{3}\%$

Solve each percent problem.

29. 12% of 70 is what number?

30. 36 is 36% of what number?

31. 212.5 is 85% of what number?

32. 66 is what percent of 55?

33. 23.8 is what percent of 85?

34. 38% of 200 is what number?

35. What number is 25% of 44?

36. What percent of 99 is 128.7?

37. What percent of 250 is 215?

38. What number is 45% of 84?

39. 42% of what number is 63?

40. 95% of what number is 58.9?

6.4 Applications of Percent

Objective A Solving Applications Involving Percent

Percent is used in a variety of everyday situations. The next four examples show just a few ways that percent occurs in real-life settings. (Each of these examples shows two ways of solving these problems. If you studied Section **6.2** only, see *Method 1.* If you studied Section **6.3** only, see *Method 2.*)

The first example has to do with the Appalachian Trail, a hiking trail conceived by a forester in 1921 and diagrammed to the right.

Example 1 The circle graph in the margin shows the Appalachian Trail mileage by state. If the total mileage of the trail is 2174, use the circle graph to determine the number of miles in the state of New York. Round to the nearest whole mile.

Solution: *Method 1.* First, we state the problem in words.

$$\begin{array}{llccccc} \text{In words:} & \underbrace{\text{What number}} & \text{is} & 4\% & \text{of} & 2174? \\ & \quad\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translate:} & \quad n & = & 4\% & \cdot & 2174 \end{array}$$

To solve for n, we find $4\% \cdot 2174$.

$$\begin{aligned} n &= 0.04 \cdot 2174 && \text{Write 4\% as a decimal.} \\ n &= 86.96 && \text{Multiply.} \\ n &\approx 87 && \text{Round to the nearest whole.} \end{aligned}$$

Rounded to the nearest whole mile, we have that approximately 87 miles of the Appalachian Trail are in New York state.

Method 2. State the problem in words; then translate.

$$\begin{array}{llccccc} \text{In words:} & \underbrace{\text{What number}} & \text{is} & 4\% & \text{of} & 2174? \\ & \quad\downarrow & & \downarrow & & \downarrow \\ & \boxed{\text{amount}} & & \boxed{\text{percent}} & & \boxed{\text{base}} \end{array}$$

$$\text{Translate:} \quad \begin{array}{r} \text{amount} \rightarrow \\ \text{base} \rightarrow \end{array} \frac{a}{2174} = \frac{4}{100} \leftarrow \text{percent}$$

Next, we solve for a.

$$\begin{aligned} a \cdot 100 &= 2174 \cdot 4 && \text{Set cross products equal.} \\ a \cdot 100 &= 8696 && \text{Multiply.} \\ \frac{a \cdot 100}{100} &= \frac{8696}{100} && \text{Divide both sides by 100.} \\ a &= 86.96 && \text{Simplify.} \\ a &\approx 87 && \text{Round to the nearest whole.} \end{aligned}$$

Rounded to the nearest whole mile, we have that approximately 87 miles of the Appalachian Trail are in New York state.

Work Practice 1

Objectives

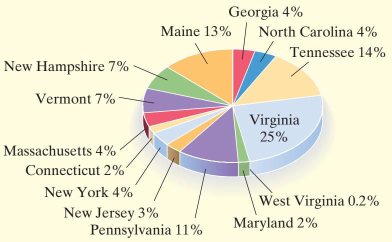
A Solve Applications Involving Percent.

B Find Percent of Increase and Percent of Decrease.

Practice 1

If the total mileage of the Appalachian Trail is 2174, use the circle graph to determine the number of miles in the state of Virginia.

Appalachian Trail Mileage by State Percent



Total miles: 2174
(*Due to rounding, these percents have a sum greater than 100%.)
Source: purebound.com

Answer

1. 543.5 mi

Practice 2

From 2014 to 2024, it is projected that the number of employed nurses will grow by 439,300. If the number of nurses employed in 2014 was 2,751,000, find the percent increase in nurses employed from 2014 to 2024. Round to nearest whole percent. (*Source:* Bureau of Labor Statistics)

Example 2 Finding Percent of Nursing School Applications Accepted

There continues to be a shortage of nursing school facilities. In 2015, of the 266,000 applications to bachelor degree nursing schools, 120,000 of these were accepted. What percent of these applications were accepted? Round to the nearest percent. (*Source:* American Association of Colleges of Nursing)

Solution: *Method 1.* First, we state the problem in words.

$$\begin{array}{lccccc} \text{In words:} & 120{,}000 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 266{,}000? \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translate:} & 120{,}000 & = & n & \cdot & 266{,}000 \end{array}$$

Next, solve for *n*.

$$\frac{120{,}000}{266{,}000} = n \qquad \text{Divide 120,000 by 266,000, the number multiplied by } n.$$

$$0.45 \approx n \qquad \text{Divide and round to the nearest hundredth.}$$

$$45\% \approx n \qquad \text{Write as a percent.}$$

About 45% of nursing school applications were accepted.

Method 2.

$$\begin{array}{lccccc} \text{In words:} & 120{,}000 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 266{,}000? \\ & \downarrow & & \downarrow & & \downarrow \\ & \text{amount} & & \text{percent} & & \text{base} \end{array}$$

$$\text{Translate:} \quad \begin{array}{r} \text{amount} \rightarrow \\ \text{base} \rightarrow \end{array} \frac{120{,}000}{266{,}000} = \frac{p}{100} \leftarrow \text{percent}$$

Next, solve for *p*.

$$120{,}000 \cdot 100 = 266{,}000 \cdot p \qquad \text{Set cross products equal.}$$

$$12{,}000{,}000 = 266{,}000 \cdot p \qquad \text{Multiply.}$$

$$\frac{12{,}000{,}000}{266{,}000} = p \qquad \text{Divide 12,000,000 by 266,000, the number multiplied by } p.$$

$$45 \approx p$$

About 45% of nursing school applications were accepted.

Work Practice 2

Practice 3

The freshmen class of 775 students is 31% of all students at Euclid University. How many students go to Euclid University?

Example 3 Finding the Base Number of Absences

Mr. Buccaran, the principal at Slidell High School, counted 31 freshmen absent during a particular day. If this is 4% of the total number of freshmen, how many freshmen are there at Slidell High School?

Solution: *Method 1.* First we state the problem in words; then we translate.

$$\begin{array}{lccccc} \text{In words:} & 31 & \text{is} & 4\% & \text{of} & \underbrace{\text{what number}}? \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translate:} & 31 & = & 4\% & \cdot & n \end{array}$$

Answers
2. 16% **3.** 2500

Next, we solve for n.

$$\begin{aligned} 31 &= 0.04 \cdot n && \text{Write 4\% as a decimal.} \\ \frac{31}{0.04} &= n && \text{Divide 31 by 0.04, the number multiplied by } n. \\ 775 &= n && \text{Simplify.} \end{aligned}$$

There are 775 freshmen at Slidell High School.

Method 2. First we state the problem in words; then we translate.

In words: $$\underset{\text{amount}}{\underset{\downarrow}{31}} \quad \text{is} \quad \underset{\text{percent}}{\underset{\searrow}{4\%}} \quad \text{of} \quad \underset{\text{base}}{\underset{\downarrow}{\underbrace{\text{what number}}}}?$$

Translate: $$\begin{matrix}\text{amount}\rightarrow \\ \text{base}\rightarrow\end{matrix} \frac{31}{b} = \frac{4}{100} \leftarrow \text{percent}$$

Next, we solve for b.

$$\begin{aligned} 31 \cdot 100 &= b \cdot 4 && \text{Set cross products equal.} \\ 3100 &= b \cdot 4 && \text{Multiply.} \\ \frac{3100}{4} &= b && \text{Divide 3100 by 4, the number multiplied by } b. \\ 775 &= b && \text{Simplify.} \end{aligned}$$

There are 775 freshmen at Slidell High School.

Work Practice 3

Example 4 Finding the Base Increase in Licensed Drivers

From 2015 to 2016, the number of licensed drivers on the road in the United States increased by 5.8%. In 2015, there were about 210 million licensed drivers on the road. (*Source:* Federal Highway Administration)

a. Find the increase in licensed drivers from 2015 to 2016.

b. Find the number of licensed drivers on the road in 2016.

Solution: *Method 1.* First we find the increase in the number of licensed drivers.

In words: $$\underset{n}{\underset{\downarrow}{\underbrace{\text{What number}}}} \quad \underset{=}{\underset{\downarrow}{\text{is}}} \quad \underset{5.8\%}{\underset{\downarrow}{5.8\%}} \quad \underset{\cdot}{\underset{\downarrow}{\text{of}}} \quad \underset{210}{\underset{\downarrow}{210?}}$$

Translate: $n = 5.8\% \cdot 210$

(*Continued on next page*)

Practice 4

From 2015 to 2016, the number of registered cars and light trucks on the road in the United States increased by 1.5%. In 2015, the number of registered cars and light trucks on the road was 260 million. (*Source:* Hedges Company and Federal Highway Administration)

a. Find the increase in the number of registered cars and light trucks on the road in 2016.

b. Find the total number of registered cars and light trucks on the road in 2016.

Answers

4. a. 3.9 million **b.** 263.9 million

Next, we solve for *n*.

$$\begin{aligned} n &= 0.058 \cdot 210 && \text{Write 5.8\% as a decimal.} \\ n &= 12.18 && \text{Multiply.} \end{aligned}$$

a. The increase in the number of licensed drivers was 12.18 million.

b. This means that the number of licensed drivers in 2016 was

$$\begin{aligned} \begin{array}{c}\text{Number of}\\ \text{licensed drivers}\\ \text{in 2016}\end{array} &= \begin{array}{c}\text{Number of}\\ \text{licensed drivers}\\ \text{in 2015}\end{array} + \begin{array}{c}\text{Increase}\\ \text{in number of}\\ \text{licensed drivers}\end{array} \\ &= 210 \text{ million} + 12.18 \text{ million} \\ &= 222.18 \text{ million} \end{aligned}$$

Method 2. First we find the increase in the number of licensed drivers.

$$\text{In words: } \underbrace{\text{What number}}_{\downarrow\atop\boxed{\text{amount}}} \quad \text{is} \quad \underset{\downarrow\atop\boxed{\text{percent}}}{5.8\%} \quad \text{of} \quad \underset{\downarrow\atop\boxed{\text{base}}}{210?}$$

$$\text{Translate: } \begin{array}{r}\text{amount} \rightarrow\\ \text{base} \rightarrow\end{array} \frac{a}{210} = \frac{5.8}{100} \leftarrow \text{percent}$$

Next, we solve for *a*.

$$\begin{aligned} a \cdot 100 &= 210 \cdot 5.8 && \text{Set cross products equal.} \\ a \cdot 100 &= 1218 && \text{Multiply.} \\ a &= \frac{1218}{100} && \text{Divide 1218 by 100, the number multiplied by } a. \\ a &= 12.18 && \text{Simplify.} \end{aligned}$$

a. The increase in the number of licensed drivers was 12.18 million.

b. This means that the number of licensed drivers in 2016 was

$$\begin{aligned} \begin{array}{c}\text{Number of}\\ \text{licensed drivers}\\ \text{in 2016}\end{array} &= \begin{array}{c}\text{Number of}\\ \text{licensed drivers}\\ \text{in 2015}\end{array} + \begin{array}{c}\text{Increase}\\ \text{in number of}\\ \text{licensed drivers}\end{array} \\ &= 210 \text{ million} + 12.18 \text{ million} \\ &= 222.18 \text{ million} \end{aligned}$$

Work Practice 4

Objective B Finding Percent of Increase and Percent of Decrease

We often use percents to show how much an amount has increased or decreased.

Suppose that the population of a town is 10,000 people and then it increases by 2000 people. The **percent of increase** is

$$\begin{array}{r}\text{amount of increase} \rightarrow\\ \text{original amount} \rightarrow\end{array} \frac{2000}{10{,}000} = 0.2 = 20\%$$

In general, we have the following.

Percent of Increase

$$\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

Then write the quotient as a percent.

Example 5 Finding Percent of Increase

The number of applications for a mathematics scholarship at Yale increased from 34 to 45 in one year. What is the percent of increase? Round to the nearest whole percent.

Solution: First we find the amount of increase by subtracting the original number of applicants from the new number of applicants.

$$\text{amount of increase} = 45 - 34 = 11$$

The amount of increase is 11 applicants. To find the percent of increase,

$$\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}} = \frac{11}{34} \approx 0.32 = 32\%$$

The number of applications increased by about 32%.

Work Practice 5

Practice 5

The number of people attending the local play, *Peter Pan,* increased from 285 on Friday to 333 on Saturday. Find the percent of increase in attendance. Round to the nearest tenth percent.

Helpful Hint Make sure that this number is the original number and not the new number.

✓Concept Check A student is calculating the percent of increase in enrollment from 180 students one year to 200 students the next year. Explain what is wrong with the following calculations:

$$\xcancel{\begin{array}{l} \text{Amount of increase} = 200 - 180 = 20 \\ \text{Percent of increase} = \dfrac{20}{200} = 0.1 = 10\% \end{array}}$$

Suppose that your income was \$300 a week and then it decreased by \$30. The **percent of decrease** is

$$\begin{array}{r} \text{amount of decrease} \rightarrow \\ \text{original amount} \rightarrow \end{array} \frac{\$30}{\$300} = 0.1 = 10\%$$

Percent of Decrease

$$\text{percent of decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

Then write the quotient as a percent.

Example 6 Finding Percent of Decrease

In response to a decrease in sales, a company with 1500 employees reduces the number of employees to 1230. What is the percent of decrease?

Solution: First we find the amount of decrease by subtracting 1230 from 1500.

$$\text{amount of decrease} = 1500 - 1230 = 270$$

The amount of decrease is 270. To find the percent of decrease,

$$\text{percent of decrease} = \frac{\text{amount of decrease}}{\text{original amount}} = \frac{270}{1500} = 0.18 = 18\%$$

The number of employees decreased by 18%.

Work Practice 6

Practice 6

A town's population of 20,200 in 1995 decreased to 18,483 in 2005. What was the percent of decrease?

✓Concept Check An ice cream stand sold 6000 ice cream cones last summer. This year the same stand sold 5400 cones. Was there a 10% increase, a 10% decrease, or neither? Explain.

Answers

5. 16.8% **6.** 8.5%

✓Concept Check Answers

To find the percent of increase, you have to divide the amount of increase (20) by the original amount (180); 10% decrease

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 6.4

Watch the section lecture video and answer the following questions.

Objective A **1.** How do we interpret the answer 175,000 in Example 1?

Objective B **2.** In Example 3, what does the improper fraction tell us?

6.4 Exercise Set MyLab Math

Objective A *Solve. For Exercises 1 and 2, the solutions have been started for you. See Examples 1 through 4. If necessary, round percents to the nearest tenth and all other answers to the nearest whole.*

1. An inspector found 24 defective bolts during an inspection. If this is 1.5% of the total number of bolts inspected, how many bolts were inspected?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

Go to *Method 1* or *Method 2*.

Method 1.

2. TRANSLATE into an equation. (Fill in the boxes.)

$$\begin{array}{ccccc} 24 & \text{is} & 1.5\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 24 & \square & 1.5\% & \square & n \end{array}$$

3. SOLVE for n. (See Example 3, Method 1, for help.)

4. INTERPRET. The total number of bolts inspected was ____________.

Method 2.

2. TRANSLATE into a proportion. (Fill in the blanks with "amount" or "base.")

$$\begin{array}{ccccc} 24 & \text{is} & 1.5\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & & \searrow & & \downarrow \\ \underline{\qquad} & & \text{percent} & & \underline{\qquad} \end{array}$$

$$\begin{array}{r} \text{amount} \rightarrow \\ \text{base} \rightarrow \end{array} \frac{\underline{\qquad}}{\underline{\qquad}} = \frac{1.5}{100} \leftarrow \text{percent}$$

3. SOLVE the proportion. (See Example 3, Method 2, for help.)

4. INTERPRET. The total number of bolts inspected was ____________.

2. A day care worker found 28 children absent one day during an epidemic of chicken pox. If this was 35% of the total number of children attending the day care center, how many children attend this day care center?

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.

Go to *Method 1* or *Method 2*.

Method 1.

2. TRANSLATE into an equation. (Fill in the boxes.)

$$\begin{array}{ccccc} 28 & \text{is} & 35\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 28 & \square & 35\% & \square & n \end{array}$$

3. SOLVE for n. (See Example 3, Method 1, for help.)

4. INTERPRET. The total number of children attending the day care center is ____________.

Method 2.

2. TRANSLATE into a proportion. (Fill in the blanks with "amount" or "base.")

$$\begin{array}{ccccc} 28 & \text{is} & 35\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & & \searrow & & \downarrow \\ \underline{\qquad} & & \text{percent} & & \underline{\qquad} \end{array}$$

$$\begin{array}{r} \text{amount} \rightarrow \\ \text{base} \rightarrow \end{array} \frac{\underline{\qquad}}{\underline{\qquad}} = \frac{35}{100} \leftarrow \text{percent}$$

3. SOLVE the proportion. (See Example 3, Method 2, for help.)

4. INTERPRET. The total number of children attending the day care center is ____________.

3. The Total Gym provides weight resistance through adjustments of incline. The minimum weight resistance is 4% of the weight of the person using the Total Gym. Find the minimum weight resistance possible for a 220-pound man. (*Source:* Total Gym)

5. A student's cost for last semester at her community college was $2700. She spent $378 of that on books. What percent of last semester's college costs was spent on books?

7. The United States' motion picture and television industry is made up of over 108,000 businesses. About 85% of these are small businesses with fewer than 10 employees. How many motion picture and television industry businesses have fewer than 10 employees? (*Source:* Motion Picture Association of America)

9. In 2016, there were approximately 36,900 McDonald's restaurants worldwide, with about 14,250 of them located in the United States. Determine the percent of McDonald's restaurants that are in the United States. (*Source:* McDonald's Corporation)

11. A furniture company currently produces 6200 chairs per month. If production decreases by 8%, find the decrease and the new number of chairs produced each month.

13. From 2014 to 2024, the number of people employed as occupational therapy assistants in the United States is expected to increase by 43%. The number of people employed as occupational therapy assistants in 2014 was 41,900. Find the predicted number of occupational therapy assistants in 2024. (*Source:* Bureau of Labor Statistics)

4. The maximum weight resistance for the Total Gym is 60% of the weight of the person using it. Find the maximum weight resistance possible for a 220-pound man. (See Exercise **3** if needed.)

6. Pierre Sampeau belongs to his local food cooperative, where he receives a percentage of what he spends each year as a dividend. He spent $3850 last year at the food cooperative store and received a dividend of $154. What percent of his total spending at the food cooperative did he receive as a dividend?

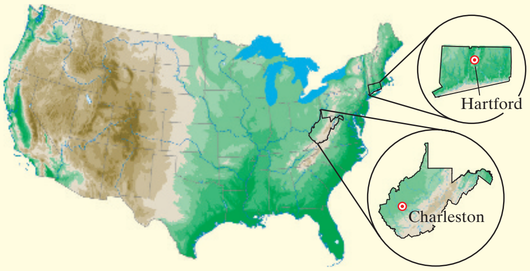
8. In 2016, there were approximately 43,500 cinema screens in the United States and Canada. If about 38.5% of the total screens in the United States and Canada were digital 3-D screens, find the approximate number of digital 3-D screens.

10. Of the 66,800 veterinarians in private practice in the United States in 2015, approximately 30,100 are men. Determine the percent of male veterinarians in private practice in the United States in 2015. (*Source:* American Veterinary Medical Association)

12. The enrollment at a local college decreased by 5% over last year's enrollment of 7640. Find the decrease in enrollment and the current enrollment.

14. From the London Summer Olympics 2012 to the Rio Summer Olympics 2016, the number of medals awarded increased by 118.5%. If 962 medals were awarded in London, how many were awarded four years later in Rio? (*Source:* BBC Sports)

Two States, West Virginia and Connecticut, decreased in population from 2014 to 2017. Their locations are shown on the partial U.S. map below. Round each answer to the nearest thousand. (Source: United States Census Bureau)



15. In 2014, the population of West Virginia was approximately 1853 thousand. If the population decrease was about 0.97%, find the population of West Virginia in 2017.

16. In 2014, the population of Connecticut was approximately 3595 thousand. If the population decrease was about 0.22%, find the population of Connecticut in 2017.

A popular extreme sport is snowboarding. Ski trails are marked with difficulty levels of easy ●, intermediate ■, difficult ◆, expert ◆◆, and other variations. Use this information for Exercises 17 and 18. Round each percent to the nearest whole. See Example 2.

17. At Keystone ski area in Colorado, 38 of the 131 total ski runs are rated intermediate. What percent of the runs are intermediate? (*Source:* Vail Resorts Management Company)

18. At Telluride ski area in Colorado, 29 of the 127 total ski trails are rated easy. What percent of the trails are easy? (*Source:* Telluride Ski & Golf Resort)

For each food described, find the percent of total calories from fat. If necessary, round to the nearest tenth percent. See Example 2.

19. Ranch dressing serving size of 2 tablespoons

	Calories
Total	40
From fat	20

20. Unsweetened cocoa powder serving size of 1 tablespoon

	Calories
Total	20
From fat	5

21.

Nutrition Facts
Serving Size 1 pouch (20g)
Servings Per Container 6

Amount Per Serving	
Calories	80
Calories from fat	10
	% Daily Value*
Total Fat 1g	**2%**
Sodium 45mg	**2%**
Total Carbohydrate 17g	**6%**
Sugars 9g	
Protein 0g	
Vitamin C	25%

Not a significant source of saturated fat, cholesterol, dietary fiber, vitamin A, calcium and iron.

*Percent Daily Values are based on a 2,000 calorie diet.

Artificial Fruit Snacks

22.

Nutrition Facts
Serving Size $\frac{1}{4}$ cup (33g)
Servings Per Container About 9

Amount Per Serving
Calories 190 **Calories from Fat** 130

	% Daily Value
Total Fat 16g	**24%**
Saturated Fat 3g	**16%**
Cholesterol 0mg	**0%**
Sodium 135mg	**6%**
Total Carbohydrate 9g	**3%**
Dietary Fiber 1g	**5%**
Sugars 2g	
Protein 5g	

Vitamin A 0% • Vitamin C 0%
Calcium 0% • Iron 8%

Peanut Mixture

23.

Nutrition Facts
Serving Size 18 crackers (29g)
Servings Per Container About 9

Amount Per Serving

Calories 120 **Calories from Fat** 35

	% Daily Value*
Total Fat 4g	**6%**
Saturated Fat 0.5g	**3%**
Polyunsaturated Fat 0g	
Monounsaturated Fat 1.5g	
Cholesterol 0mg	**0%**
Sodium 220mg	**9%**
Total Carbohydrate 21g	**7%**
Dietary Fiber 2g	**7%**
Sugars 3g	
Protein 2g	

Vitamin A 0% • Vitamin C 0%
Calcium 2% • Iron 4%
Phosphorus 10%

Snack Crackers

24.

Nutrition Facts
Serving Size 28 crackers (31g)
Servings Per Container About 6

Amount Per Serving

Calories 130 **Calories from Fat** 35

	% Daily Value*
Total Fat 4g	**6%**
Saturated Fat 2g	**10%**
Polyunsaturated Fat 1g	
Monounsaturated Fat 1g	
Cholesterol 0mg	**0%**
Sodium 470mg	**20%**
Total Carbohydrate 23g	**8%**
Dietary Fiber 1g	**4%**
Sugars 4g	
Protein 2g	

Vitamin A 0% • Vitamin C 0%
Calcium 0% • Iron 2%

Snack Crackers

Solve. If necessary, round money amounts to the nearest cent and all other amounts to the nearest tenth. See Examples 1 through 4.

25. A family paid $26,250 as a down payment for a home. If this represents 15% of the price of the home, find the price of the home.

26. A banker learned that $842.40 is withheld from his monthly check for taxes and insurance. If this represents 18% of his total pay, find the total pay.

27. An owner of a repair service company estimates that for every 40 hours a repairperson is on the job, he can bill for only 78% of the hours. The remaining hours, the repairperson is idle or driving to or from a job. Determine the number of hours per 40-hour week the owner can bill for a repairperson.

28. A manufacturer of electronic components expects 1.04% of its products to be defective. Determine the number of defective components expected in a batch of 28,350 components. Round to the nearest whole component.

29. A car manufacturer announced that next year the price of a certain model of car will increase by 4.5%. This year the price is $19,286. Find the increase in price and the new price.

30. A union contract calls for a 6.5% salary increase for all employees. Determine the increase and the new salary that a worker currently making $58,500 under this contract can expect.

A popular extreme sport is artificial wall climbing. The photo shown is an artificial climbing wall. Exercises 31 and 32 are about the Footsloggers Climbing Tower in Boone, North Carolina.

31. A climber is resting at a height of 24 feet while on the Footsloggers Climbing Tower. If this is 60% of the tower's total height, find the height of the tower.

32. A group plans to climb the Footsloggers Climbing Tower at the group rate, once they save enough money. Thus far, $175 has been saved. If this is 70% of the total amount needed for the group, find the total price.

Solve.

33. Tuition for an Ohio resident at the Columbus campus of Ohio State University was $8994 in 2010. The tuition increased by 11.6% during the period from 2010 to 2016. Find the increase and the tuition for the 2016–2017 school year. Round the increase to the nearest whole dollar. (*Source:* Ohio State University)

34. The population of Americans aged 65 and older was 46 million in 2014. That population is projected to increase by 57% by 2030. Find the increase and the projected 2030 population. (*Source:* Administration for Community Living, U.S. Department of Health and Human Services)

35. From 2014–2015 to 2024–2025, the number of students enrolled in an associate degree program is projected to increase by 22.3%. If the enrollment in associate degree programs in 2014–2015 was 6,700,000, find the increase and the projected number of students enrolled in an associate degree program in 2024–2025. (*Source:* National Center for Educational Statistics)

36. From 2010–2011 to 2021–2022, the number of bachelor degrees awarded is projected to increase by 17.4%. If the number of bachelor degrees awarded in 2010–2011 was 1,703,000, find the increase and the projected number of bachelor degrees awarded in the 2021–2022 school year. (*Source:* National Center for Educational Statistics)

Objective B *Find the amount of increase and the percent of increase. See Example 5.*

	Original Amount	New Amount	Amount of Increase	Percent of Increase
37.	50	80		
38.	8	12		
39.	65	117		
40.	68	170		

Find the amount of decrease and the percent of decrease. See Example 6.

	Original Amount	New Amount	Amount of Decrease	Percent of Decrease
41.	8	6		
42.	25	20		
43.	160	40		
44.	200	162		

Solve. Round percents to the nearest tenth, if necessary. See Examples 5 and 6.

45. There are 150 calories in a cup of whole milk and only 84 in a cup of skim milk. In switching to skim milk, find the percent of decrease in the number of calories per cup.

46. In reaction to a slow economy, the number of employees at a soup company decreased from 530 to 477. What was the percent of decrease in the number of employees?

47. The number of cable TV systems recently decreased from 10,845 to 10,700. Find the percent of decrease.

48. Before taking a typing course, Geoffry Landers could type 32 words per minute. By the end of the course, he was able to type 76 words per minute. Find the percent of increase.

49. In 1940, the average size of a privately owned farm in the United States was 174 acres. In a recent year, the average size of a privately owned farm in the United States had increased to 421 acres. Find the percent of increase. (*Source:* National Agricultural Statistics Service)

50. In 2012, there were 2109 thousand farms in the United States. In 2016, the number of farms in the United States had decreased to 2060 thousand farms. Find the percent of decrease. (*Source:* U.S. Dept. of Agriculture)

51. In 2016, there were approximately 71 million virtual reality devices in use worldwide. This is expected to grow to 337 million in 2020. Find the projected percent of increase. (*Source:* CTIA—The Wireless Association)

52. Between 2014 and 2015, permanent digital downloads of singles decreased from approximately 1199 million to approximately 1021 million. Find the percent of decrease. (*Source:* Recording Industry Association of America)

53. In 2014, there were 3782 thousand elementary and secondary teachers employed in the United States. This number is expected to increase to 4151 thousand teachers in 2021. Find the percent of increase. (*Source:* National Center for Educational Statistics)

54. In 2014, approximately 475 thousand correctional officers were employed in the United States. By 2024, this number is expected to increase to 493 thousand correctional officers. Find the percent of increase. (*Source:* Bureau of Labor Statistics)

55. In 2015, total revenue from U.S. music industry was \$6.7 billion. By 2017, this revenue had increased to \$8.7 billion. Find this percent of increase in music revenue. (*Source:* Recording Industry Association of America)

56. As the largest health care occupation, registered nurses held about 3.1 million jobs in 2015. The number of registered nurses is expected to be 3.9 million by 2025. Find the percent of increase. (*Source:* American Association of Colleges of Nursing)

57. The average U.S. movie theater ticket price in 2007 was \$6.88. For 2017, it was estimated to be \$8.87. Find the percent of increase in average move theater ticket price for this 10–year period. (*Source:* Motion Picture Association of America)

58. The average temperature for the contiguous United States during February 2017 was 41.2° Fahrenheit. The 20th-century average temperature for February is 33.9° Fahrenheit. What is the percent of increase in average temperature for February? (*Source:* National Centers for Environmental Information)

59. The number of cell phone tower sites in the United States was 253,086 in 2010. By 2016, the number of cell sites had increased to 307,626. Find the percent of increase. (*Source:* CTIA—The Wireless Association)

60. In 2014, Ford Motor Company sold 2386 thousand automobiles and trucks. In 2016, its sales were 2464 thousand vehicles. Find the percent of increase from 2014 to 2016. (*Source:* Ford Motor Company)

Review

Perform each indicated operation. See Sections 4.2 and 4.3.

61. $\begin{array}{r} 0.12 \\ \times\ 38 \\ \hline \end{array}$

62. $\begin{array}{r} 42 \\ \times\ 0.7 \\ \hline \end{array}$

63. $9.20 + 1.98$

64. $46 + 7.89$

65. $78 - 19.46$

66. $64.80 - 10.72$

Concept Extensions

67. If a number is increased by 100%, how does the increased number compare with the original number? Explain your answer.

68. In your own words, explain what is wrong with the following statement: "Last year we had 80 students attend. This year we have a 50% increase or a total of 160 students attending."

For Exercises 69 through 71, explain what errors were made by each student when solving percent of increase or decrease problems. Then correct the errors. See the Concept Checks in this section.

The population of a certain rural town was 150 in 1980, 180 in 1990, and 150 in 2000.

69. Find the percent of increase in population from 1980 to 1990.

Miranda's solution: Percent of increase $= \frac{30}{180} = 0.1\overline{6} \approx 16.7\%$

70. Find the percent of decrease in population from 1990 to 2000.

Jeremy's solution: Percent of decrease $= \frac{30}{150} = 0.20 = 20\%$

71. The percent of increase from 1980 to 1990 is the same as the percent of decrease from 1990 to 2000. True or false?

Chris's answer: True because they had the same amount of increase as the amount of decrease.

72. Refer to Exercises **49** and **50**. They are both about U.S. farms except that one asks us to find a percent of increase and one asks us to find a percent of decrease. In your own words, explain how these can both be correct.

6.5 Percent and Problem Solving: Sales Tax, Commission, and Discount

Objectives

A Calculate Sales Tax and Total Price.

B Calculate Commissions.

C Calculate Discount and Sale Price.

Objective A Calculating Sales Tax and Total Price

Percents are frequently used in the retail trade. For example, most states charge a tax on certain items when purchased. This tax is called a **sales tax,** and retail stores collect it for the state. Sales tax is almost always stated as a percent of the purchase price.

A 9% sales tax rate on a purchase of a $10 calculator gives a sales tax of

$$\text{sales tax} = 9\% \text{ of } \$10 = 0.09 \cdot \$10.00 = \$0.90$$

The total price to the customer would be

$$\underbrace{\text{purchase price}}_{\$10.00} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{sales tax}}_{\$0.90} = \$10.90$$

This example suggests the following equations:

Sales Tax and Total Price

$$\begin{aligned} \text{sales tax} &= \text{tax rate} \cdot \text{purchase price} \\ \text{total price} &= \text{purchase price} + \text{sales tax} \end{aligned}$$

In this section we round dollar amounts to the nearest cent.

Example 1 Finding Sales Tax and Purchase Price

Find the sales tax and the total price on the purchase of an $85.50 atlas in a city where the sales tax rate is 7.5%.

Solution: The purchase price is $85.50 and the tax rate is 7.5%.

$$\begin{aligned} \text{sales tax} &= \text{tax rate} \cdot \text{purchase price} \\ \downarrow \quad & \quad \downarrow \qquad \swarrow \\ \text{sales tax} &= 7.5\% \cdot \$85.50 \\ &= 0.075 \cdot \$85.50 && \text{Write 7.5\% as a decimal.} \\ &\approx \$6.41 && \text{Round to the nearest cent.} \end{aligned}$$

Thus, the sales tax is $6.41. Next, find the total price.

$$\begin{aligned} \text{total price} &= \text{purchase price} + \text{sales tax} \\ \downarrow \quad & \quad \downarrow \qquad \swarrow \\ \text{total price} &= \$85.50 + \$6.41 \\ &= \$91.91 \end{aligned}$$

The sales tax on $85.50 is $6.41, and the total price is $91.91.

Work Practice 1

✓Concept Check The purchase price of a textbook is $50 and sales tax is 10%. If you are told by the cashier that the total price is $75, how can you tell that a mistake has been made?

Practice 1

If the sales tax rate is 8.5%, what is the sales tax and the total amount due on a $59.90 Goodgrip tire?

Answer

1. tax: $5.09; total: $64.99

✓Concept Check Answer

Since $10\% = \frac{1}{10}$, the sales tax is $\frac{\$50}{10} = \$5$. The total price should have been $55.

Practice 2

The sales tax on an $18,500 automobile is $1665. Find the sales tax rate.

Example 2 Finding a Sales Tax Rate

The sales tax on a $406 Sony flat-screen, digital, 27-inch television is $34.51. Find the sales tax rate.

Solution: Let r represent the unknown sales tax rate. Then

$$\begin{array}{ccccc} \text{sales tax} & = & \text{tax rate} & \cdot & \text{purchase price} \\ \downarrow & & \swarrow & & \swarrow \\ \$34.51 & = & r \cdot \$406 & & \end{array}$$

$$\frac{34.51}{406} = \frac{r \cdot 406}{406}$$ Divide both sides by 406.

$$0.085 = r$$ Simplify.

$$8.5\% = r$$ Write 0.085 as a percent.

The sales tax rate is 8.5%.

Work Practice 2

Objective B Calculating Commissions

A **wage** is payment for performing work. Hourly wage, commissions, and salary are some of the ways wages can be paid. Many people who work in sales are paid a commission. An employee who is paid a **commission** is paid a percent of his or her total sales.

Commission

commission = commission rate · sales

Practice 3

A sales representative for Office Product Copiers sold $47,632 worth of copier equipment and supplies last month. What is his commission for the month if he is paid a commission rate of 6.6% of his total sales for the month?

Example 3 Finding the Amount of Commission

Sherry Souter, a real estate broker for Wealth Investments, sold a house for $214,000 last week. If her commission rate is 1.5% of the selling price of the home, find the amount of her commission.

Solution:

$$\begin{array}{ccccc} \text{commission} & = & \text{commission rate} & \cdot & \text{sales} \\ \downarrow & & \downarrow & & \downarrow \\ \text{commission} & = & 1.5\% & \cdot & \$214{,}000 \\ & = & 0.015 & \cdot & \$214{,}000 \\ & = & \$3210 & & \end{array}$$

Write 1.5% as 0.015.

Multiply.

Her commission on the house is $3210.

Work Practice 3

Answers
2. 9% **3.** $3143.71

Example 4 Finding a Commission Rate

A salesperson earned \$1560 for selling \$13,000 worth of electronics equipment. Find the commission rate.

Solution: Let r stand for the unknown commission rate. Then

$$\begin{array}{ccccl} \text{commission} & = & \text{commission rate} & \cdot & \text{sales} \\ \downarrow & & \downarrow & & \downarrow \\ \$1560 & = & r & \cdot & \$13{,}000 \\ \dfrac{1560}{13{,}000} & = & r & & \text{Divide 1560 by 13,000, the number multiplied by } r. \\ 0.12 & = & r & & \text{Simplify.} \\ 12\% & = & r & & \text{Write 0.12 as a percent.} \end{array}$$

The commission rate is 12%.

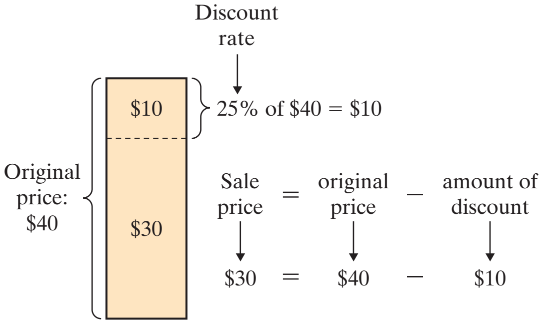
Work Practice 4

Practice 4

A salesperson earns \$645 for selling \$4300 worth of appliances. Find the commission rate.

Objective C Calculating Discount and Sale Price

Suppose that an item that normally sells for \$40 is on sale for 25% off. This means that the **original price** of \$40 is reduced, or **discounted,** by 25% of \$40, or \$10. The **discount rate** is 25%, the **amount of discount** is \$10, and the **sale price** is \$40 − \$10, or \$30. Study the diagram below to visualize these terms.



To calculate discounts and sale prices, we can use the following equations:

Discount and Sale Price

$$\begin{aligned} \text{amount of discount} &= \text{discount rate} \cdot \text{original price} \\ \text{sale price} &= \text{original price} - \text{amount of discount} \end{aligned}$$

Example 5 Finding a Discount and a Sale Price

An electric rice cooker that normally sells for \$65 is on sale for 25% off. What is the amount of discount and what is the sale price?

Solution: First we find the amount of discount, or simply the discount.

$$\begin{array}{ccccl} \text{amount of discount} & = & \text{discount rate} & \cdot & \text{original price} \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount of discount} & = & 25\% & \cdot & \$65 \\ & = & 0.25 \cdot \$65 & & \text{Write 25\% as 0.25.} \\ & = & \$16.25 & & \text{Multiply.} \end{array}$$

(*Continued on next page*)

Practice 5

A discontinued washer and dryer combo is advertised on sale for 35% off the regular price of \$700. Find the amount of discount and the sale price.

Answers

4. 15% **5.** \$245; \$455

The discount is $16.25. Next, find the sale price.

$$\begin{array}{ccccc} \text{sale price} & = & \text{original price} & - & \text{discount} \\ \downarrow & & \downarrow & & \downarrow \\ \text{sale price} & = & \$65 & - & \$16.25 \\ & = & \$48.75 & \text{Subtract.} & \end{array}$$

The sale price is $48.75.

■ Work Practice 5

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

amount of discount   sale price   sales tax
commission   total price

1. ________________ = tax rate · purchase price
2. ________________ = purchase price + sales tax
3. ________________ = commission rate · sales
4. ________________ = discount rate · original price
5. ________________ = original price − amount of discount
6. sale price = original price − ________________

Martin-Gay Interactive Videos

See Video 6.5

Watch the section lecture video and answer the following questions.

Objective A **7.** In Example 1, what is our first step after translating the problem into an equation?

Objective B **8.** What is our final step in solving Example 2?

Objective C **9.** In the lecture before Example 3, since both equations shown involve the "amount of discount," how can the two equations be combined into one equation?

6.5 Exercise Set MyLab Math

Objective A *Solve. See Examples 1 and 2.*

1. What is the sales tax on a jacket priced at $150 if the sales tax rate is 5%?

2. If the sales tax rate is 6%, find the sales tax on a microwave oven priced at $188.

3. The purchase price of a camcorder is $799. What is the total price if the sales tax rate is 7.5%?

4. A stereo system has a purchase price of $426. What is the total price if the sales tax rate is 8%?

5. A new large-screen television has a purchase price of $4790. If the sales tax on this purchase is $335.30, find the sales tax rate.

6. The sales tax on the purchase of a $6800 used car is $374. Find the sales tax rate.

7. The sales tax on a table saw is $22.95.

a. What is the purchase price of the table saw (before tax) if the sales tax rate is 8.5%? (*Hint:* Use the sales tax equation and insert the replacement values.)

b. Find the total price of the table saw.

8. The sales tax on a one-half-carat diamond ring is $76.

a. Find the purchase price of the ring (before tax) if the sales tax rate is 9.5%. (See the hint for Exercise **7a**.)

b. Find the total price of the ring.

9. A gold-plated and diamond bracelet sells for $1800. Find the sales tax and the total price if the sales tax rate is 6.5%.

10. The purchase price of a laptop computer is $1890. If the sales tax rate is 8%, what is the sales tax and the total price?

11. The sales tax on the purchase of a futon is $24.25. If the tax rate is 5%, find the purchase price of the futon (before tax).

12. The sales tax on the purchase of an LED television combination is $59.85. If the tax rate is 9%, find the purchase price of the LED TV (before tax).

13. The sales tax is $98.70 on a stereo sound system purchase of $1645. Find the sales tax rate.

14. The sales tax is $103.50 on a necklace purchase of $1150. Find the sales tax rate.

15. A cell phone costs $210, a battery charger costs $15, and batteries cost $5. What is the sales tax and total price for purchasing these items if the sales tax rate is 7%?

16. Ms. Warner bought a blouse for $35, a skirt for $55, and a blazer for $95. Find the sales tax and the total price she paid, given a sales tax rate of 6.5%.

Objective B *Solve. See Examples 3 and 4.*

17. A sales representative for a large furniture warehouse is paid a commission rate of 4%. Find her commission if she sold $1,329,401 worth of furniture last year.

18. Rosie Davis-Smith is a beauty consultant for a home cosmetic business. She is paid a commission rate of 12.8%. Find her commission if she sold $1638 in cosmetics last month.

19. A salesperson earned a commission of $1380.40 for selling $9860 worth of paper products. Find the commission rate.

20. A salesperson earned a commission of $3575 for selling $32,500 worth of books to various bookstores. Find the commission rate.

21. How much commission will Jack Pruet make on the sale of a $325,900 house if he receives 1.5% of the selling price?

22. Frankie Lopez sold $9638 of jewelry this week. Find her commission for the week if she receives a commission rate of 5.6%.

23. A real estate agent earned a commission of $5565 for selling a house. If his rate is 3%, find the selling price of the house. (*Hint:* Use the commission equation and insert the replacement values.)

24. A salesperson earned $1750 for selling fertilizer. If her commission rate is 7%, find the selling price of the fertilizer. (See the hint for Exercise **23**.)

Objective C *Find the amount of discount and the sale price. See Example 5.*

	Original Price	Discount Rate	Amount of Discount	Sale Price
25.	$89	10%		
26.	$74	20%		
27.	$196.50	50%		
28.	$110.60	40%		
29.	$410	35%		
30.	$370	25%		
31.	$21,700	15%		
32.	$17,800	12%		

▶ **33.** A $300 fax machine is on sale for 15% off. Find the amount of discount and the sale price.

34. A $4295 designer dress is on sale for 30% off. Find the amount of discount and the sale price.

Objectives A B Mixed Practice *Complete each table.*

	Purchase Price	Tax Rate	Sales Tax	Total Price
35.	$305	9%		
36.	$243	8%		
37.	$56	5.5%		
38.	$65	8.4%		

	Sale	Commission Rate	Commission
39.	$235,800	3%	
40.	$195,450	5%	
41.	$17,900		$1432
42.	$25,600		$2304

Review

Multiply. See Sections 4.3 and 4.5.

43. $2000 \cdot \frac{3}{10} \cdot 2$

44. $500 \cdot \frac{2}{25} \cdot 3$

45. $400 \cdot \frac{3}{100} \cdot 11$

46. $1000 \cdot \frac{1}{20} \cdot 5$

47. $600 \cdot 0.04 \cdot \frac{2}{3}$

48. $6000 \cdot 0.06 \cdot \frac{3}{4}$

Concept Extensions

Solve. See the Concept Check in this section.

49. Your purchase price is \$68 and the sales tax rate is 9.5%. Round each amount and use the rounded amounts to estimate the total price. Choose the best estimate.

a. \$105 **b.** \$58 **c.** \$93 **d.** \$77

50. Your purchase price is \$200 and the tax rate is 10%. Choose the best estimate of the total price.

a. \$190 **b.** \$210 **c.** \$220 **d.** \$300

Tipping

One very useful application of percent is mentally calculating a tip. Recall that to find 10% of a number, simply move the decimal point one place to the left. To find 20% of a number, just double 10% of the number. To find 15% of a number, find 10% and then add to that number half of the 10% amount. Mentally fill in the chart below. To do so, start by rounding the bill amount to the nearest dollar.

Tipping Chart

	Bill Amount	10%	15%	20%
51.	\$40.21			
52.	\$15.89			
53.	\$72.17			
54.	\$9.33			

55. Suppose that the original price of a shirt is \$50. Which is better, a 60% discount or a discount of 30% followed by a discount of 35% of the reduced price? Explain your answer.

56. Which is better, a 30% discount followed by an additional 25% off or a 20% discount followed by an additional 40% off? To see, suppose an item costs \$100 and calculate each discounted price. Explain your answer.

57. A diamond necklace sells for \$24,966. If the tax rate is 7.5%, find the total price.

58. A house recently sold for \$562,560. The commission rate on the sale is 5.5%. If the real estate agent is to receive 60% of the commission, find the amount received by the agent.

6.6 Percent and Problem Solving: Interest

Objectives

A Calculate Simple Interest.

B Calculate Compound Interest.

C Calculate Monthly Payments.

Objective A Calculating Simple Interest

Interest is money charged for using other people's money. When you borrow money, you pay interest. When you loan or invest money, you earn interest. The money borrowed, loaned, or invested is called the **principal amount,** or simply **principal.** Interest is normally stated in terms of a percent of the principal for a given period of time. The **interest rate** is the percent used in computing the interest. Unless stated otherwise, *the rate is understood to be per year*. When the interest is computed on the original principal, it is called **simple interest.** Simple interest is calculated using the following equation:

Simple Interest

$$\text{Simple Interest} = \text{Principal} \cdot \text{Rate} \cdot \text{Time}$$

$$I = P \cdot R \cdot T$$

where the rate is understood to be per year and time is in years.

Practice 1

Find the simple interest after 5 years on \$875 at an interest rate of 7%.

Example 1 Finding Simple Interest

Find the simple interest after 2 years on \$500 at an interest rate of 12%.

Solution: In this example, $P = \$500$, $R = 12\%$, and $T = 2$ years. Replace the variables in the formula $I = PRT$ with values.

$$\begin{aligned} I &= P \cdot R \cdot T \\ I &= \$500 \cdot 12\% \cdot 2 && \text{Let } P = \$500,\ R = 12\%, \text{ and } T = 2. \\ &= \$500 \cdot (0.12) \cdot 2 && \text{Write 12\% as a decimal.} \\ &= \$120 && \text{Multiply.} \end{aligned}$$

The simple interest is \$120.

Work Practice 1

If time is not given in years, we need to convert the given time to years.

Practice 2

A student borrowed \$1500 for 9 months on her credit card at a simple interest rate of 20%. How much interest did she pay?

Example 2 Finding Simple Interest

Ivan Borski borrowed \$2400 at 10% simple interest for 8 months to buy a used Toyota Corolla. Find the simple interest he paid.

Solution: Since there are 12 months in a year, we first find what part of a year 8 months is.

$$8 \text{ months} = \frac{8}{12} \text{ year} = \frac{2}{3} \text{ year}$$

Now we find the simple interest.

$$\begin{array}{ccccccc} \text{simple interest} & = & \text{principal} & \cdot & \text{rate} & \cdot & \text{time} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{simple interest} & = & \$2400 & \cdot & 10\% & \cdot & \frac{2}{3} \\ & = & \$2400 & \cdot & 0.10 & \cdot & \frac{2}{3} \\ & = & \$160 & & & & \end{array}$$

The interest on Ivan's loan is \$160.

Work Practice 2

✓**Concept Check** Suppose in Example 2 you had obtained an answer of \$16,000. How would you know that you had made a mistake in this problem?

When money is borrowed, the borrower pays the original amount borrowed, or the principal, as well as the interest. When money is invested, the investor receives the original amount invested, or the principal, as well as the interest. In either case, the **total amount** is the sum of the principal and the interest.

Finding the Total Amount of a Loan or Investment

total amount (paid or received) = principal + interest

Answers

1. \$306.25 **2.** \$225

✓**Concept Check Answer**

\$16,000 is too much interest.

Example 3 Finding the Total Amount of an Investment

An accountant invested $2000 at a simple interest rate of 10% for 2 years. What total amount of money will she have from her investment in 2 years?

Solution: First we find her interest.

$$\begin{aligned} I &= P \cdot R \cdot T \\ &= \$2000 \cdot (0.10) \cdot 2 \qquad \text{Let } P = \$2000,\ R = 10\% \text{ or } 0.10, \text{ and } T = 2. \\ &= \$400 \end{aligned}$$

The interest is $400.

Next, we add the interest to the principal.

$$\begin{array}{ccccc} \text{total amount} & = & \text{principal} & + & \text{interest} \\ \downarrow & & \downarrow & & \downarrow \\ \text{total amount} & = & \$2000 & + & \$400 \\ & = & \$2400 & & \end{array}$$

After 2 years, she will have a total amount of $2400.

Work Practice 3

Concept Check Which investment would earn more interest: an amount of money invested at 8% interest for 2 years, or the same amount of money invested at 8% for 3 years? Explain.

Objective B Calculating Compound Interest

Recall that simple interest depends on the original principal only. Another type of interest is compound interest. **Compound interest** is computed not only on the principal, but also on the interest already earned in previous compounding periods. Compound interest is used more often than simple interest.

Let's see how compound interest differs from simple interest. Suppose that $2000 is invested at 7% interest **compounded annually** for 3 years. This means that interest is added to the principal at the end of each year and that next year's interest is computed on this new amount. In this section, we round dollar amounts to the nearest cent.

	Amount at Beginning of Year	Principal	·	Rate	·	Time	= Interest	Amount at End of Year
1st year	$2000	$2000	·	0.07	·	1	= $140	$2000 + 140 = $2140
2nd year	$2140	$2140	·	0.07	·	1	= $149.80	$2140 + 149.80 = $2289.80
3rd year	$2289.80	$2289.80	·	0.07	·	1	= $160.29	$2289.80 + 160.29 = $2450.09

The compound interest earned can be found by

$$\begin{array}{ccccc} \text{total amount} & - & \text{original principal} & = & \text{compound interest} \\ \downarrow & & \downarrow & & \downarrow \\ \$2450.09 & - & \$2000 & = & \$450.09 \end{array}$$

The simple interest earned would have been

$$\begin{array}{ccccccc} \text{principal} & \cdot & \text{rate} & \cdot & \text{time} & = & \text{interest} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \$2000 & \cdot & 0.07 & \cdot & 3 & = & \$420 \end{array}$$

Practice 3

If $2100 is borrowed at a simple interest rate of 13% for 6 months, find the total amount paid.

Answer

3. $2236.50

Concept Check Answer

8% for 3 years. Since the interest rate is the same, the longer you keep the money invested, the more interest you earn.

Since compound interest earns "interest on interest," compound interest earns more than simple interest.

Computing compound interest using the method just shown can be tedious. We can use a calculator and the compound interest formula below to compute compound interest more quickly.

Compound Interest Formula

The total amount A in an account is given by

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

where P is the principal, r is the interest rate written as a decimal, t is the length of time in years, and n is the number of times compounded per year.

Practice 4

\$3000 is invested at 4% interest compounded annually. Find the total amount after 6 years.

Example 4 \$1800 is invested at 2% interest compounded annually. Find the total amount after 3 years.

Solution: "Compounded annually" means 1 time a year, so $n = 1$. Also, $P = \$1800$, $r = 2\% = 0.02$, and $t = 3$ years.

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t} \\ &= 1800\left(1 + \frac{0.02}{1}\right)^{1 \cdot 3} \\ &= 1800(1.02)^3 \\ &\approx 1910.17 \quad \text{Round to 2 decimal places.} \end{aligned}$$

Helpful Hint Remember order of operations. **First** evaluate $(1.02)^3$; then multiply by 1800.

The total amount at the end of 3 years is about \$1910.17.

Work Practice 4

Practice 5

\$5500 is invested at $6\frac{1}{4}\%$ compounded *daily* for 5 years. Find the total amount at the end of 5 years. (Use 1 year = 365 days.)

Example 5 Finding Total Amount Received from an Investment

\$4000 is invested at 5.3% compounded quarterly for 10 years. Find the total amount at the end of 10 years.

Solution: "Compounded quarterly" means 4 times a year, so $n = 4$. Also, $P = \$4000$, $r = 5.3\% = 0.053$, and $t = 10$ years.

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t} \\ &= 4000\left(1 + \frac{0.053}{4}\right)^{4 \cdot 10} \\ &= 4000(1.01325)^{40} \\ &\approx 6772.12 \end{aligned}$$

The total amount after 10 years is about \$6772.12.

Work Practice 5

Note: Part of the compound interest formula, $\left(1 + \frac{r}{n}\right)^{n \cdot t}$, is called the **compound interest factor.** Appendix A.7 contains a table of various calculated compound interest factors. Another way to calculate the total amount, A, in the compound interest formula is to multiply the principal, P, by the appropriate compound interest factor found in Appendix A.7.

Answers

4. \$3795.96 **5.** \$7517.41

The Calculator Explorations box below shows how compound interest factors are calculated.

Objective C Calculating a Monthly Payment

We conclude this section with a method to find the monthly payment on a loan.

Finding the Monthly Payment on a Loan

$$\text{monthly payment} = \frac{\text{principal} + \text{interest}}{\text{total number of payments}}$$

Example 6 Finding a Monthly Payment

Find the monthly payment on a \$2000 loan for 2 years. The interest on the 2-year loan is \$435.88.

Solution: First we determine the total number of monthly payments. The loan is for 2 years. Since there are 12 months per year, the number of payments is $2 \cdot 12$, or 24. Now we calculate the monthly payment.

$$\text{monthly payment} = \frac{\text{principal} + \text{interest}}{\text{total number of payments}}$$

$$\text{monthly payment} = \frac{\$2000 + \$435.88}{24}$$

$$\approx \$101.50.$$

The monthly payment is about \$101.50.

Work Practice 6

Practice 6

Find the monthly payment on a \$3000 3-year loan if the interest on the loan is \$1123.58.

Answer

6. \$114.54

Calculator Explorations Compound Interest Factor

A compound interest factor may be found by using your calculator and evaluating the formula

$$\textbf{compound interest factor} = \left(1 + \frac{r}{n}\right)^{n \cdot t}$$

where r is the interest rate, t is the time in years, and n is the number of times compounded per year. For example, the compound interest factor for 10 years at 8% compounded semiannually is about 2.19112. Let's find this factor by evaluating the compound interest factor formula when $r = 8\%$ or 0.08, $t = 10$, and $n = 2$ (compounded semiannually means 2 times per year). Thus,

$$\text{compound interest factor} = \left(1 + \frac{0.08}{2}\right)^{2 \cdot 10}$$

$$\text{or} \quad \left(1 + \frac{0.08}{2}\right)^{20}$$

To evaluate, press the keys $\boxed{(}$ $\boxed{1}$ $\boxed{+}$ $\boxed{0.08}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{)}$ $\boxed{y^x}$ or $\boxed{\wedge}$ then $\boxed{20}$ and then $\boxed{=}$ or $\boxed{\text{ENTER}}$. The display will read $\boxed{2.1911231}$. Rounded to 5 decimal places, this is 2.19112.

Find the compound interest factors. Use the table in Appendix A.7 to check your answers for Exercises 1–4. For Exercises 1–4, round to 5 decimal places. For Exercises 5 and 6, round to 2 decimal places.

1. 5 years, 9%, compounded quarterly
2. 15 years, 14%, compounded daily
3. 20 years, 11%, compounded annually
4. 1 year, 7%, compounded semiannually
5. Find the total amount after 4 years when \$500 is invested at 6% compounded quarterly. (Multiply the appropriate compound interest factor by \$500.)
6. Find the total amount for 19 years when \$2500 is invested at 5% compounded daily.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Choices may be used more than once.

total amount simple principal amount compound

1. To calculate _______________ interest, use $I = P \cdot R \cdot T$.

2. To calculate _______________ interest, use $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$.

3. _______________ interest is computed not only on the original principal, but also on interest already earned in previous compounding periods.

4. When interest is computed on the original principal only, it is called _______________ interest.

5. _______________ (paid or received) = principal + interest.

6. The _______________ is the money borrowed, loaned, or invested.

Martin-Gay Interactive Videos

See Video 6.6

Watch the section lecture video and answer the following questions.

Objective A **7.** Complete this statement based on the lecture before Example 1: Simple interest is charged on the ____________ only.

Objective B **8.** In Example 2, how often is the interest compounded and what number does this translate to in the formula?

Objective C **9.** In Example 3, how was the denominator of 48 determined?

6.6 Exercise Set MyLab Math

Objective A *Find the simple interest. See Examples 1 and 2.*

	Principal	Rate	Time
1.	\$200	8%	2 years
3.	\$160	11.5%	4 years
5.	\$5000	10%	$1\frac{1}{2}$ years
7.	\$375	18%	6 months
9.	\$2500	16%	21 months

	Principal	Rate	Time
2.	\$800	9%	3 years
4.	\$950	12.5%	5 years
6.	\$1500	14%	$2\frac{1}{4}$ years
8.	\$775	15%	8 months
10.	\$1000	10%	18 months

Solve. See Examples 1 through 3.

11. A company borrows \$162,500 for 5 years at a simple interest rate of 12.5%. Find the interest paid on the loan and the total amount paid back.

12. \$265,000 is borrowed to buy a house. If the simple interest rate on the 30-year loan is 8.25%, find the interest paid on the loan and the total amount paid back.

13. A money market fund advertises a simple interest rate of 9%. Find the total amount received on an investment of \$5000 for 15 months.

14. The Real Service Company takes out a 270-day (9-month) short-term, simple interest loan of \$4500 to finance the purchase of some new equipment. If the interest rate is 14%, find the total amount that the company pays back.

15. Marsha borrows $8500 and agrees to pay it back in 4 years. If the simple interest rate is 17%, find the total amount she pays back.

16. An 18-year-old is given a high school graduation gift of $2000. If this money is invested at 8% simple interest for 5 years, find the total amount.

Objective B *Find the total amount in each compound interest account. See Examples 4 and 5.*

17. $6150 is compounded semiannually at a rate of 14% for 15 years.

18. $2060 is compounded annually at a rate of 15% for 10 years.

19. $1560 is compounded daily at a rate of 8% for 5 years.

20. $1450 is compounded quarterly at a rate of 10% for 15 years.

21. $10,000 is compounded semiannually at a rate of 9% for 20 years.

22. $3500 is compounded daily at a rate of 8% for 10 years.

23. $2675 is compounded annually at a rate of 9% for 1 year.

24. $6375 is compounded semiannually at a rate of 10% for 1 year.

25. $2000 is compounded annually at a rate of 8% for 5 years.

26. $2000 is compounded semiannually at a rate of 8% for 5 years.

27. $2000 is compounded quarterly at a rate of 8% for 5 years.

28. $2000 is compounded daily at a rate of 8% for 5 years.

Objective C *Solve. See Example 6.*

29. A college student borrows $1500 for 6 months to pay for a semester of school. If the interest is $61.88, find the monthly payment.

30. Jim Tillman borrows $1800 for 9 months. If the interest is $148.90, find his monthly payment.

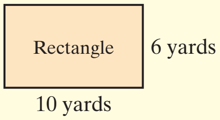
31. $20,000 is borrowed for 4 years. If the interest on the loan is $10,588.70, find the monthly payment.

32. $105,000 is borrowed for 15 years. If the interest on the loan is $181,125, find the monthly payment.

Review

Find the perimeter of each figure. See Section 1.3.

33.



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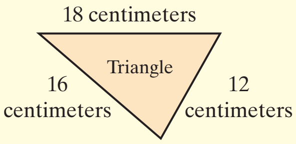

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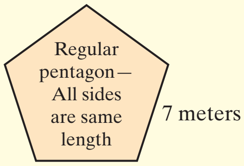

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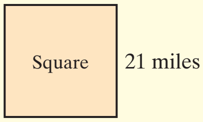


Concept Extensions

37. Explain how to look up a compound interest factor in the compound interest table.

38. Explain how to find the amount of interest in a compounded account.

39. Compare the following accounts: Account 1: $1000 is invested for 10 years at a simple interest rate of 6%. Account 2: $1000 is compounded semiannually at a rate of 6% for 10 years. Discuss how the interest is computed for each account. Determine which account earns more interest. Why?

Chapter 6 Group Activity

Fastest-Growing Occupations

According to U.S. Bureau of Labor Statistics projections, the careers listed below are the top ten fastest-growing jobs ranked by expected percent of increase through the year 2024. (*Source:* Bureau of Labor Statistics)

	Occupation	**Employment in 2014**	**Percent of Increase**	**Expected Employment in 2024**
1	Wind turbine technician	4400	108%	
2	Occupational therapy assistants and aides	41,900	40%	
3	Physical therapist assistants and aides	128,700	40%	
4	Home health aides	913,500	38%	
5	Commercial divers	3450	37%	
6	Nurse practitioners	170,400	35%	
7	Physical therapists	210,900	34%	
8	Statisticians	30,000	34%	
9	Ambulance drivers and attendants but not EMTs	19,950	33%	
10	Physician assistants	94,400	30%	

What do most of these fast-growing occupations have in common? They require knowledge of math! For some careers, such as nurse practitioners, and statisticians, the ways math is used on the job may be obvious. For other occupations, the use of math may not be quite as apparent. However, tasks common to many jobs—filling in a time sheet, writing up an expense or mileage report, planning a budget, figuring a bill, ordering supplies, and even making a work schedule—all require math.

This activity may be completed by working in groups or individually.

1. List the top five occupations by order of employment figures for 2014.
2. Using the 2014 employment figures and the percent of increase from 2014 to 2024, find the expected 2024 employment figure for each occupation listed in the table. Round to the nearest hundred.
3. List the top five occupations by order of employment figures for 2024. Did the order change at all from 2014? Explain.

Chapter 6 Vocabulary Check

Fill in each blank with one of the words or phrases listed below. Some choices may be used more than once.

percent	sales tax	is	0.01	$\frac{1}{100}$	amount of discount	percent of decrease	total price
base	of	amount	100%	compound interest	percent of increase	sale price	commission

1. In a mathematical statement, ________________ usually means "multiplication."

2. In a mathematical statement, ________________ means "equals."

3. ________________ means "per hundred."

4. ________________ is computed not only on the principal, but also on interest already earned in previous compounding periods.

5. In the percent proportion, $\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \frac{\text{percent}}{100}$.

6. To write a decimal or fraction as a percent, multiply by ______________.

7. The decimal equivalent of the % symbol is ______________.

8. The fraction equivalent of the % symbol is ______________.

9. The percent equation is ______________ · percent = ______________.

10. ______________ $= \frac{\text{amount of decrease}}{\text{original amount}}$.

11. ______________ $= \frac{\text{amount of increase}}{\text{original amount}}$.

12. ______________ = tax rate · purchase price.

13. ______________ = purchase price + sales tax.

14. ______________ = commission rate · sales.

15. ______________ = discount rate · original price.

16. ______________ = original price − amount of discount.

Helpful Hint

Are you preparing for your test? To help, don't forget to take these:

- Chapter 6 Getting Ready for the Test on page 514.
- Chapter 6 Test on page 515

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of the exercises are located in MyLab Math.

6 Chapter Highlights

Definitions and Concepts	Examples
Section 6.1 Percents, Decimals, and Fractions	
Percent means "per hundred." The % symbol denotes percent.	$51\% = \frac{51}{100}$ 51 per 100 $7\% = \frac{7}{100}$ 7 per 100
To write a percent as a decimal, replace the % symbol with its decimal equivalent, 0.01, and multiply.	$32\% = 32(0.01) = 0.\underset{\smile\smile}{32}$
To write a decimal as a percent, multiply by 100%.	$0.08 = 0.08(100\%) = \underset{\smile\smile}{08}.\% = 8\%$
To write a percent as a fraction, replace the % symbol with its fraction equivalent, $\frac{1}{100}$, and multiply.	$25\% = \frac{25}{100} = \frac{\overset{1}{\cancel{25}}}{4 \cdot \underset{1}{\cancel{25}}} = \frac{1}{4}$
To write a fraction as a percent, multiply by 100%.	$\frac{1}{6} = \frac{1}{6} \cdot 100\% = \frac{1}{6} \cdot \frac{100}{1}\% = \frac{100}{6}\% = 16\frac{2}{3}\%$

Definitions and Concepts

Examples

Section 6.2 Solving Percent Problems Using Equations

Three key words in the statement of a percent problem are

of, which means **multiplication** (·)

is, which means **equals** (=)

what (or some equivalent word or phrase), which stands for **the unknown number**

Solve:

$$\begin{array}{ccccc} 6 & \text{is} & 12\% & \text{of} & \text{what number?} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 6 & = & 12\% & \cdot & n \end{array}$$

$$6 = 0.12 \cdot n \quad \text{Write 12\% as a decimal.}$$

$$\frac{6}{0.12} = n \quad \text{Divide 6 by 0.12, the number multiplied by } n.$$

$$50 = n$$

Thus, 6 is 12% of 50.

Section 6.3 Solving Percent Problems Using Proportions

Percent Proportion

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100} \leftarrow \text{always 100}$$

or

$$\frac{\text{amount} \rightarrow a}{\text{base} \rightarrow b} = \frac{p \leftarrow \text{percent}}{100}$$

Solve:

$$\begin{array}{ccccc} 20.4 & \text{is what} & \text{percent} & \text{of} & 85? \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount} & & \text{percent} & & \text{base} \end{array}$$

$$\frac{\text{amount} \rightarrow 20.4}{\text{base} \rightarrow 85} = \frac{p \leftarrow \text{percent}}{100}$$

$$20.4 \cdot 100 = 85 \cdot p \quad \text{Set cross products equal.}$$

$$2040 = 85 \cdot p \quad \text{Multiply.}$$

$$\frac{2040}{85} = p \quad \text{Divide 2040 by 85, the number multiplied by } p.$$

$$24 = p \quad \text{Simplify.}$$

Thus, 20.4 is 24% of 85.

Section 6.4 Applications of Percent

Percent of Increase

$$\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

Percent of Decrease

$$\text{percent of decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

A town's population of 16,480 decreased to 13,870 over a 12-year period. Find the percent of decrease. Round to the nearest whole percent.

$$\begin{aligned} \text{amount of decrease} &= 16{,}480 - 13{,}870 \\ &= 2610 \\ \text{percent of decrease} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{2610}{16{,}480} \approx 0.16 \\ &= 16\% \end{aligned}$$

The town's population decreased by 16%.

Section 6.5 Percent and Problem Solving: Sales Tax, Commission, and Discount

Sales Tax and Total Price

sales tax = sales tax rate · purchase price

total price = purchase price + sales tax

Find the sales tax and the total price on a purchase of $42 if the sales tax rate is 9%.

$$\begin{array}{ccccc} \text{sales tax} & = & \text{sales tax rate} & \cdot & \text{purchase price} \\ \downarrow & & \downarrow & & \downarrow \\ \text{sales tax} & = & 9\% & \cdot & \$42 \\ & = & 0.09 \cdot \$42 & & \\ & = & \$3.78 & & \end{array}$$

Definitions and Concepts	Examples

Section 6.5 Percent and Problem Solving: Sales Tax, Commission, and Discount (*continued*)

The total price is

$$\begin{array}{ccccc} \text{total price} & = & \text{purchase price} & + & \text{sales tax} \\ \downarrow & & \downarrow & & \downarrow \\ \text{total price} & = & \$42 & + & \$3.78 \\ & = & \$45.78 & & \end{array}$$

Commission

$$\text{commission} = \text{commission rate} \cdot \text{total sales}$$

A salesperson earns a commission of 3%. Find the commission from sales of $12,500 worth of appliances.

$$\begin{array}{ccccc} \text{commission} & = & \text{commission rate} & \cdot & \text{sales} \\ \downarrow & & \downarrow & & \downarrow \\ \text{commission} & = & 3\% & \cdot & \$12{,}500 \\ & = & 0.03 \cdot \$12{,}500 & & \\ & = & \$375 & & \end{array}$$

Discount and Sale Price

$$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$

$$\text{sale price} = \text{original price} - \text{amount of discount}$$

A suit is priced at $320 and is on sale today for 25% off. What is the sale price?

$$\begin{array}{ccccc} \text{amount of discount} & = & \text{discount rate} & \cdot & \text{original price} \\ \downarrow & & \downarrow & & \downarrow \\ \text{amount of discount} & = & 25\% & \cdot & \$320 \\ & = & 0.25 \cdot \$320 & & \\ & = & \$80 & & \end{array}$$

$$\begin{array}{ccccc} \text{sale price} & = & \text{original price} & - & \text{amount of discount} \\ \downarrow & & \downarrow & & \downarrow \\ \text{sale price} & = & \$320 & - & \$80 \\ & = & \$240 & & \end{array}$$

The sale price is $240.

Section 6.6 Percent and Problem Solving: Interest

Simple Interest

$$\text{interest} = \text{principal} \cdot \text{rate} \cdot \text{time}$$

where the rate is understood to be per year.

Find the simple interest after 3 years on $800 at an interest rate of 5%.

$$\begin{array}{ccccccc} \text{interest} & = & \text{principal} & \cdot & \text{rate} & \cdot & \text{time} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{interest} & = & \$800 & \cdot & 5\% & \cdot & 3 \\ & = & \$800 \cdot 0.05 \cdot 3 & & \text{Write 5\% as 0.05.} & & \\ & = & \$120 & & \text{Multiply.} & & \end{array}$$

The interest is $120.

Compound interest is computed not only on the principal, but also on interest already earned in previous compounding periods. (See Appendix A.7 for various compound interest factors.)

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

where n is the number of times compounded per year.

$800 is invested at 5% compounded quarterly for 10 years. Find the total amount at the end of 10 years.

$$\begin{aligned} A &= \$800\left(1 + \frac{0.05}{4}\right)^{4 \cdot 10} \\ &= \$800(1.0125)^{40} \\ &\approx \$1314.90 \end{aligned}$$

Chapter 6 Review

(6.1) *Solve.*

1. In a survey of 100 adults, 37 preferred pepperoni on their pizzas. What percent preferred pepperoni?

2. A basketball player made 77 out of 100 attempted free throws. What percent of free throws were made?

Write each percent as a decimal.

3. 83%

4. 75%

5. 73.5%

6. 1.5%

7. 125%

8. 145%

9. 0.5%

10. 0.7%

11. 200%

12. 400%

13. 26.25%

14. 85.34%

Write each decimal as a percent.

15. 2.6

16. 1.02

17. 0.35

18. 0.55

19. 0.725

20. 0.252

21. 0.076

22. 0.085

23. 0.71

24. 0.65

25. 4

26. 9

Write each percent as a fraction or mixed number in simplest form.

27. 1%

28. 10%

29. 25%

30. 8.5%

31. 10.2%

32. $16\frac{2}{3}\%$

33. $33\frac{1}{3}\%$

34. 110%

Write each fraction or mixed number as a percent.

35. $\frac{1}{5}$

36. $\frac{7}{10}$

37. $\frac{5}{6}$

38. $\frac{3}{5}$

39. $1\frac{1}{4}$

40. $1\frac{2}{3}$

41. $\frac{1}{16}$

42. $\frac{5}{8}$

(6.2) *Translate each to an equation and solve.*

43. 1250 is 1.25% of what number?

44. 22.9 is 20% of what number?

45. 124.2 is what percent of 540?

46. 693 is what percent of 462?

47. What number is 40% of 7500?

48. What number is $33\frac{1}{3}\%$ of 24,000?

(6.3) *Translate each to a proportion and solve.*

49. 104.5 is 25% of what number?

50. 16.5 is 5.5% of what number?

51. What number is 36% of 180?

52. What number is 33% of 500?

53. 93.5 is what percent of 85?

54. 63 is what percent of 35?

(6.4) *Solve.*

55. In a survey of 2000 people, it was found that 1320 have a microwave oven. Find the percent of people who own microwaves.

56. Of the 12,360 freshmen entering County College, 2000 are enrolled in basic college mathematics. Find the percent of entering freshmen who are enrolled in basic college mathematics. Round to the nearest whole percent.

57. The number of violent crimes in a city decreased from 675 to 534. Find the percent of decrease. Round to the nearest tenth of a percent.

58. The current charge for dumping waste in a local landfill is $16 per cubic foot. To cover new environmental costs, the charge will increase to $33 per cubic foot. Find the percent of increase.

59. This year the fund drive for a charity collected $215,000. Next year, a 4% decrease is expected. Find how much is expected to be collected in next year's drive.

60. A local union negotiated a new contract that increases the hourly pay 15% over last year's pay. The old hourly rate was $11.50. Find the new hourly rate rounded to the nearest cent.

(6.5) *Solve.*

61. If the sales tax rate is 9.5%, what is the total amount charged for a $250 coat?

62. Find the sales tax paid on a $32 purchase if the sales tax rate is 10.5%.

63. Russ James is a sales representative for a chemical company and is paid a commission rate of 5% on all sales. Find his commission if he sold $100,000 worth of chemicals last month.

64. Carol Sell is a sales clerk in a clothing store. She receives a commission of 7.5% on all sales. Find her commission for the week if her sales for the week were $4005. Round to the nearest cent.

65. A $3000 mink coat is on sale for 30% off. Find the discount and the sale price.

66. A $90 calculator is on sale for 10% off. Find the discount and the sale price.

(6.6) *Solve.*

67. Find the simple interest due on $4000 loaned for 4 months at 12% interest.

68. Find the simple interest due on $6500 loaned for 3 months at 20%.

69. Find the total amount in an account if $5500 is compounded annually at 12% for 15 years.

70. Find the total amount in an account if $6000 is compounded semiannually at 11% for 10 years.

71. Find the compound interest earned if $100 is compounded quarterly at 12% for 5 years.

72. Find the compound interest earned if $1000 is compounded quarterly at 18% for 20 years.

Mixed Review

Write each percent as a decimal.

73. 3.8%

74. 24.5%

75. 0.9%

Write each decimal as a percent.

76. 0.54

77. 95.2

78. 0.3

Write each percent as a fraction or mixed number in simplest form.

79. 47%

80. $6\frac{2}{5}\%$

81. 5.6%

Write each fraction or mixed number as a percent.

82. $\frac{3}{8}$

83. $\frac{2}{13}$

84. $\frac{6}{5}$

Translate each into an equation and solve.

85. 43 is 16% of what number?

86. 27.5 is what percent of 25?

87. What number is 36% of 1968?

88. 67 is what percent of 50?

Translate each into a proportion and solve.

89. 75 is what percent of 25?

90. What number is 16% of 240?

91. 28 is 5% of what number?

92. 52 is what percent of 16?

Solve.

93. The total number of cans in a soft drink machine is 300. If 78 soft drinks have been sold, find the percent of soft drink cans that have been sold.

94. A home valued at $96,950 last year has lost 7% of its value this year because of flooding. Find the loss in value.

95. A used printer sells for $568.00. If the sales tax rate is 8.75%, find the purchase price of the printer.

96. The original price to buy and download a video game is $63.00. It is on sale for 15% off. What is the amount of the discount?

97. A regional candy salesman makes a commission of $1.60 from each case of candy he sells. If a case of candy costs $12.80, what is his rate of commission?

98. Find the total amount due on a 6-month loan of $1400 at a simple interest rate of 13%.

99. $8800 is invested at 8% interest compounded quarterly. Find the total amount after 9 years.

100. Find the total amount due on a loan of $5500 for 9 years at 12.5% simple interest.

Chapter 6 Getting Ready for the Test LC

MULTIPLE CHOICE All of the exercises below are **Multiple Choice**. Choose the correct letter.

1. Since "percent" means "per hundred," choose the number that does *not* equal 12%.

A. $\frac{12}{100}$ **B.** 0.12 **C.** $\frac{3}{25}$ **D.** 1.2

2. Choose the number that does *not* equal 100%.

A. 10 **B.** 1 **C.** 1.00 **D.** $\frac{100}{100}$

3. Choose the number that does *not* equal 50%.

A. 0.5 **B.** 5 **C.** $\frac{1}{2}$ **D.** $\frac{50}{100}$

4. Choose the number that does *not* equal 80%.

A. 0.80 **B.** 0.8 **C.** 8 **D.** $\frac{4}{5}$

Use the information below for Exercises 5 through 8.

- 100% of a number is that original number.
- 50% of a number is half that number.
- 25% of a number is $\frac{1}{4}$ of that number.
- 10% of a number is $\frac{1}{10}$ of that number.

For Exercises 5 through 8, choose the letter that correctly fills in each blank.

A. 100% **B.** 50% **C.** 25% **D.** 10%

5. ______________ of 70 is 35.

6. ______________ of 88 is 8.8.

7. ______________ of 47 is 47.

8. ______________ of 28 is 7.

For Exercise 9 , choose the correct letter.

9. Your bill at a restaurant is $24.86. If you want to leave a 20% tip, which tip amount is closest to 20%?

A. $2.48 **B.** $20 **C.** $5 **D.** $10

For Exercises 10 and 11, an amount of $150 is to be discounted by 10%.

10. Find the amount of discount.

A. $15 **B.** $45 **C.** $30 **D.** $10

11. Find the new discounted price (original price − discount).

A. $120 **B.** $10 **C.** $140 **D.** $135

For Exercise 12 , choose the correct letter.

12. If the original price of a pair of shoes is $40 and the shoe price is to be discounted by 25% at the register, choose the closest amount for your shoes before tax.

A. $10 **B.** $20 **C.** $30 **D.** $40

Write each percent as a decimal.

1. 85% **2.** 500% **3.** 0.8%

Write each decimal as a percent.

4. 0.056 **5.** 6.1 **6.** 0.39

Write each percent as a fraction or mixed number in simplest form.

7. 120% **8.** 38.5% **9.** 0.2%

Write each fraction or mixed number as a percent.

10. $\frac{11}{20}$ **11.** $\frac{3}{8}$ **12.** $1\frac{5}{9}$

Solve.

13. What number is 42% of 80? **14.** 0.6% of what number is 7.5?

15. 567 is what percent of 756?

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______

16. ________

17. ________

18. ________

19. ________

20. ________

21. ________

22. ________

23. ________

24. ________

25. ________

Solve. Round all dollar amounts to the nearest cent.

16. An alloy is 12% copper. How much copper is contained in 320 pounds of this alloy?

17. A farmer in Nebraska estimates that 20% of his potential crop, or $11,350, has been lost to a hard freeze. Find the total value of his potential crop.

18. If the local sales tax rate is 1.25%, find the total amount charged for a stereo system priced at $354.

19. A town's population increased from 25,200 to 26,460. Find the percent of increase.

20. A $120 framed picture is on sale for 15% off. Find the discount and the sale price.

21. Randy Nguyen is paid a commission rate of 4% on all sales. Find Randy's commission if his sales were $9875.

22. A sales tax of $1.53 is added to an item's price of $152.99. Find the sales tax rate. Round to the nearest whole percent.

23. Find the simple interest earned on $2000 saved for $3\frac{1}{2}$ years at an interest rate of 9.25%.

24. $1365 is compounded annually at 8%. Find the total amount in the account after 5 years.

25. A couple borrowed $400 from a bank at 13.5% simple interest for 6 months for car repairs. Find the total amount due the bank at the end of the 6-month period.

Cumulative Review Chapters 1–6

1. How many cases can be filled with 9900 cans of jalapeños if each case holds 48 cans? How many cans will be left over? Will there be enough cases to fill an order for 200 cases?

2. Multiply: 409×76

3. Write each fraction as a mixed number or a whole number.

a. $\frac{30}{7}$ **b.** $\frac{16}{15}$ **c.** $\frac{84}{6}$

4. Write each mixed number as an improper fraction.

a. $2\frac{5}{7}$ **b.** $10\frac{1}{10}$ **c.** $5\frac{3}{8}$

5. Write $-\frac{10}{27}$ in simplest form.

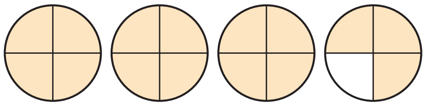
6. Find the average of 28, 34, and 70.

7. Multiply and simplify: $\frac{23}{32} \cdot \frac{4}{7}$

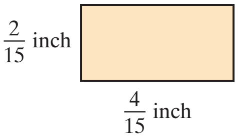
8. Round 76,498 to the nearest ten.

9. Divide and simplify: $\frac{2}{5} \div \frac{1}{2}$

10. Write the shaded part of the figure as an improper fraction and as a mixed number.



11. Find the perimeter of the rectangle.



12. Find $2 \cdot 5^2$.

13. Find the LCD of $\frac{11}{12}$ and $\frac{7}{20}$.

14. Subtract $\frac{7}{9}$ from $\frac{10}{9}$.

15. Add: $\frac{2}{5} + \frac{4}{15}$

16. Find $\frac{2}{3}$ of 510.

17. Subtract: $7\frac{3}{14} - 3\frac{6}{7}$

18. Simplify: $9 \cdot \sqrt{25} - 6 \cdot \sqrt{4}$

Perform each indicated operation.

19. $\frac{3}{4} \div 5$

20. $20\frac{4}{5} + 12\frac{7}{8}$

21. $\left(-\frac{1}{4}\right)^2$

22. $1\frac{7}{8} \cdot 3\frac{2}{5}$

Answers

1. ______
2. ______
3. a. ______
b. ______
c. ______
4. a. ______
b. ______
c. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
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40.
41.
42.
43.
44. a.
b.
c.
45.
46.

Write each fraction as a decimal.

23. $-\frac{5}{8}$

24. $\frac{9}{100}$

25. $\frac{22}{7}$ (Round to the nearest hundredth.)

26. $\frac{48}{10{,}000}$

27. A high school teacher's taxable income is $41,567.72. The tax tables in the teacher's state use amounts rounded to the nearest dollar. Round the teacher's income to the nearest whole dollar.

28. Subtract: $38 - 10.06$

29. Add: $763.7651 + 22.001 + 43.89$

30. 12.483×100

31. Multiply: 23.6×0.78

32. 76.3×1000

Divide.

33. $\frac{786.1}{1000}$

34. $0.5\overline{)0.638}$

35. $\frac{-0.12}{10}$

36. $0.23\overline{)11.6495}$

37. Simplify: $723.6 \div 1000 \times 10$

38. Simplify: $\frac{3.19 - 0.707}{13}$

39. Write $\frac{1}{4}$ as a decimal.

40. Write $\frac{5}{9}$ as a decimal. Give the exact answer and a three-decimal-place approximation.

41. Is $\frac{4.1}{7} = \frac{2.9}{5}$ a true proportion?

42. Find each unit rate and decide on the better buy.
$0.93 for 18 flour tortillas
$1.40 for 24 flour tortillas

43. On a chamber of commerce map of Abita Springs, 5 miles corresponds to 2 inches. How many miles correspond to 7 inches?

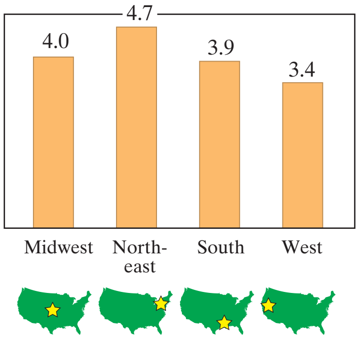
44. Write each percent as a decimal.
a. 7% **b.** 200% **c.** 0.5%

45. Translate to an equation: What number is 25% of 0.008?

46. Write $\frac{3}{8}$ as a percent.

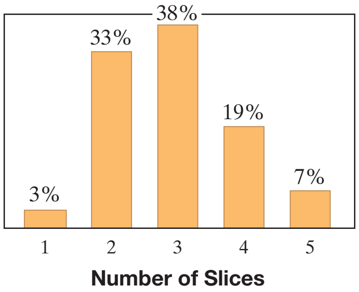
Reading Graphs and Introduction to Statistics and Probability

How Many Times per Month Do You Usually Eat Pizza?



The **average** is 4 times a month.

How Many Slices Do You Usually Eat When Eating Pizza?



The **average** is about 3 slices.

All About Pizza!

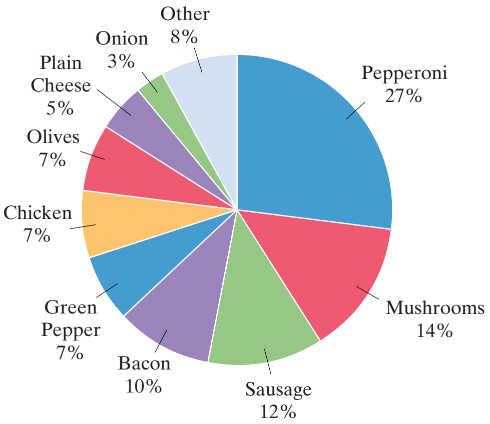
The word *pizza* was first documented in AD 997 in Italy, and it literally means "pie." Foods similar to pizza can be traced back to ancient Greece, Persia, and other countries, although modern-day pizza is believed to have been invented in Naples, Italy. Pizza was mainly eaten in Italy until immigrants brought the idea of pizza to the United States.

Surveys show that pizza is ranked number 1 among comfort foods. In this chapter, we study all types of graphs as well as measures of central tendency, such as average. For example,

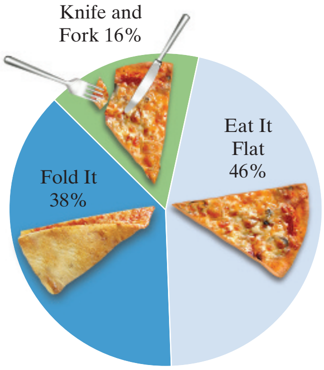
- the average price of a slice of pizza is $3.26, and
- the average cost of a pie (or whole pizza) is $16.73.
- Depending on the survey, New York City is usually ranked first as the city with the best pizza, followed by Chicago.

Throughout this chapter, we study all types of graphs as well as average, shown on the graphs above and below.

Favorite Pizza Topping



How Do You Eat Your Pizza?



Sources: Zagat, Statista, Harris Poll

7

We often need to make decisions based on known statistics or the probability of an event occurring. For example, we decide whether to bring an umbrella to work based on the probability of rain. We choose an investment based on its mean, or average, return. We can predict which football team will win based on the trend in its previous wins and losses. This chapter reviews presenting data in a usable form on a graph and the basic ideas of statistics and probability.

Sections

Check Your Progress

7.1 Pictographs, Bar Graphs, Histograms, and Line Graphs

Objectives

A Read Pictographs.

B Read and Construct Bar Graphs.

C Read and Construct Histograms (or Frequency Distribution Graphs).

D Read Line Graphs.

Often data are presented visually in a graph. In this section, we practice reading several kinds of graphs including pictographs, bar graphs, and line graphs.

Objective A Reading Pictographs

A **pictograph** such as the one below is a graph in which pictures or symbols are used. This type of graph contains a key that explains the meaning of the symbol used. An advantage of using a pictograph to display information is that comparisons can easily be made. A disadvantage of using a pictograph is that it is often hard to tell what fractional part of a symbol is shown. For example, in the pictograph below, Portuguese shows a part of a symbol, but it's hard to read with any accuracy what fractional part of a symbol is shown.

Practice 1

Use the pictograph shown in Example 1 to answer the following questions:

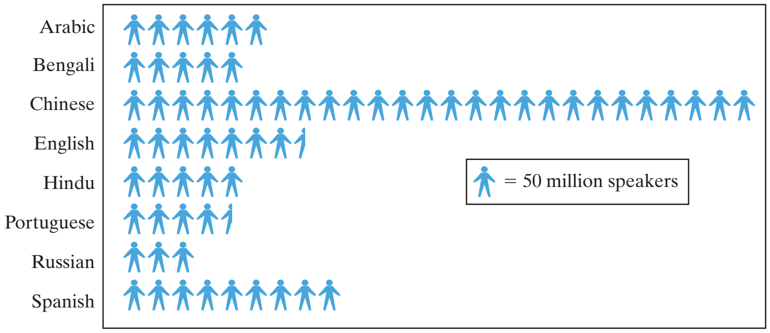
a. Approximate the number of people who primarily speak Spanish.

b. Approximate how many more people primarily speak Spanish than Arabic.

Example 1 Calculating Languages Spoken

The following pictograph shows the top eight most-spoken (primary) languages. Use this pictograph to answer the questions.

Top 8 Most-Spoken (Primary) Languages



Source: www.ethnologue.com

a. Approximate the number of people who primarily speak Russian.

b. Approximate how many more people primarily speak English than Russian.

Solution:

a. Russian corresponds to 3 symbols, and each symbol represents 50 million speakers. This means that the number of people who primarily speak Russian is approximately $3 \cdot (50 \text{ million})$ or 150 million people.

b. English shows $4\frac{1}{2}$ more symbols than Russian. This means that $4\frac{1}{2} \cdot (50 \text{ million})$ or 225 million more people primarily speak English than Russian.

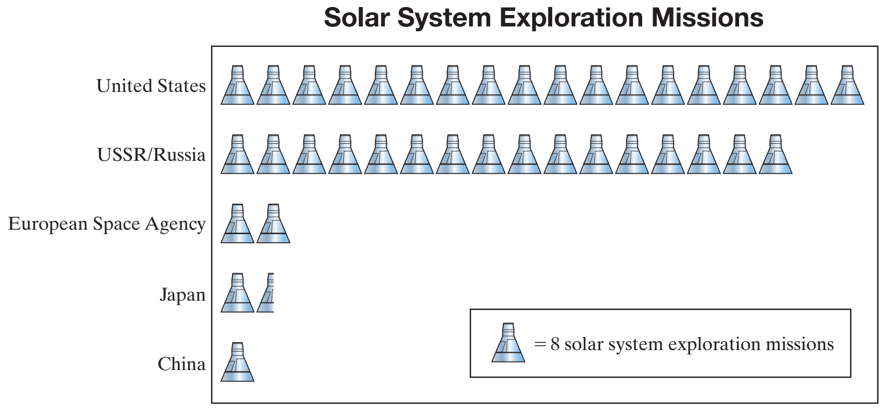
Work Practice 1

Answers

1. a. 450 million people
b. 150 million people

Example 2 Calculating Solar System Exploration

The following pictograph shows the approximate number of solar system exploration missions by various countries or space consortia from 1957 through 2017. Use this pictograph to answer the questions.



a. Approximate the number of solar system exploration missions undertaken by the United States.

b. Approximate how many more solar system exploration missions have been undertaken by the United States than by the USSR/Russia.

Solution:

a. The United States corresponds to 18 symbols, and each symbol represents 8 solar system exploration missions. This means that the United States has undertaken approximately $18 \cdot 8 = 144$ missions for solar system exploration.

b. The USSR/Russia shows 16 symbols, or 2 fewer than the United States. This means that the United States has undertaken $2 \cdot 8 = 16$ more solar system exploration missions than the USSR/Russia.

Work Practice 2

Objective B Reading and Constructing Bar Graphs

Another way to visually present data is with a **bar graph.** Bar graphs can appear with vertical bars or horizontal bars. Although we have studied bar graphs in previous sections, we now practice reading the height or length of the bars contained in a bar graph. An advantage to using bar graphs is that a scale is usually included for greater accuracy. Care must be taken when reading bar graphs, as well as other types of graphs—they may be misleading, as shown later in this section.

Practice 2

Use the pictograph shown in Example 2 to answer the following questions:

a. Approximate the number of solar system exploration missions undertaken by the European Space Agency.

b. Approximate how many more solar system exploration missions have been undertaken by the European Space Agency than by the Chinese.

Answers

2. a. 16 **b.** 8

Practice 3

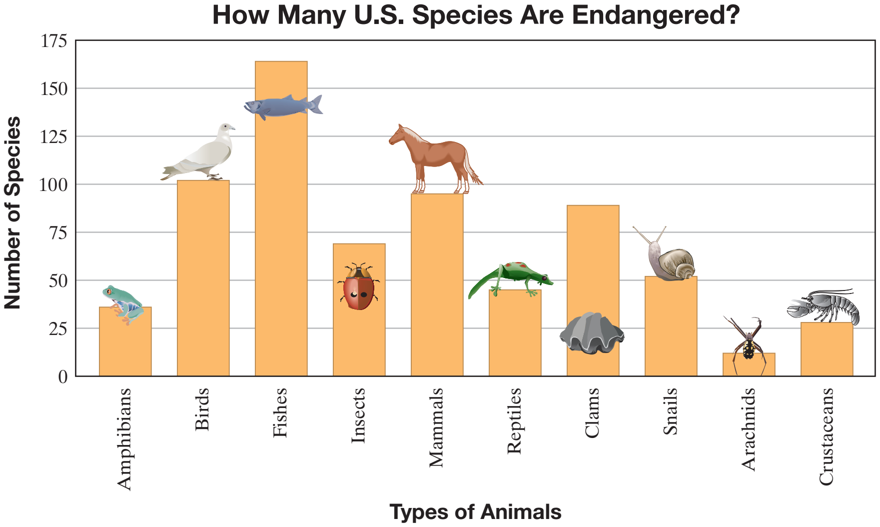
Use the bar graph in Example 3 to answer the following questions:

a. Approximate the number of endangered species that are fishes.

b. Which category shows the fewest endangered species?

Example 3 Finding the Number of Endangered Species

The following bar graph shows the number of endangered species in the United States in 2017. Use this graph to answer the questions.



Source: U.S. Fish and Wildlife Service

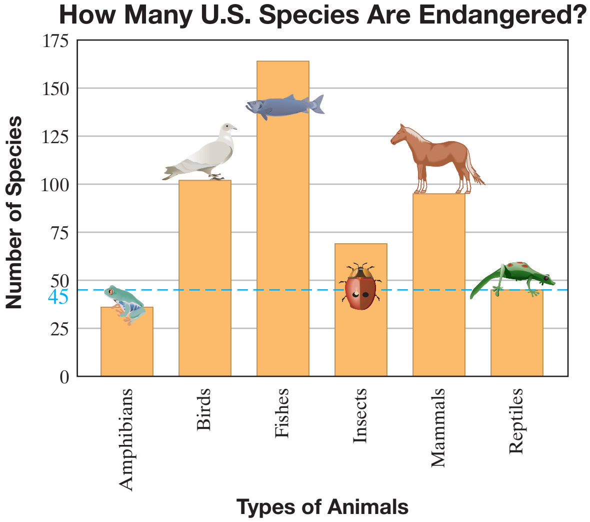
a. Approximate the number of endangered species that are reptiles.

b. Which category has the most endangered species?

Solution:

a. To approximate the number of endangered species that are reptiles, we go to the top of the bar that represents reptiles. From the top of this bar, we move horizontally to the left until the scale is reached. We read the height of the bar on the scale as approximately 45. There are approximately 45 reptile species that are endangered, as shown. (See the graph below.)

b. The most endangered species is represented by the tallest (longest) bar. The tallest bar corresponds to fishes.



Source: U.S. Fish and Wildlife Service

Work Practice 3

Answers

3. a. 164 (exact number) **b.** arachnids

Next, we practice constructing a bar graph.

Example 4 Draw a vertical bar graph using the information in the table below, which gives the caffeine content of selected foods.

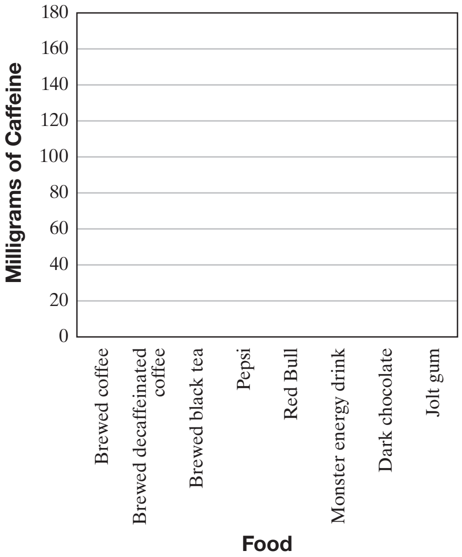
Average Caffeine Content of Selected Foods

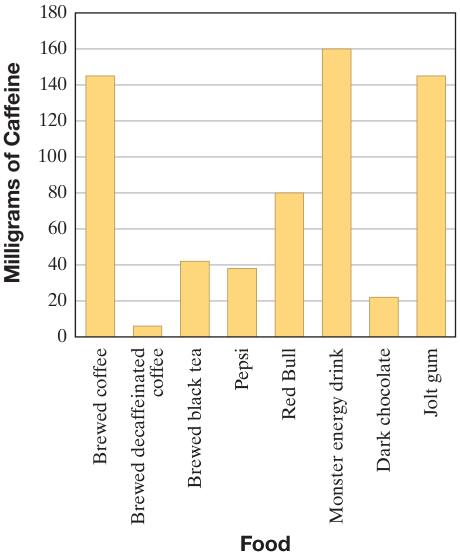
Food	Milligrams	Food	Milligrams
Brewed Coffee (8 ounces)	145	Brewed decaffeinated coffee (8 ounces)	6
Brewed black tea (8 ounces)	42	Pepsi Cola (12 ounces)	38
Red Bull (8.46 ounces)	80	Monster energy drink (16 ounces)	160
Dark Chocolate (semisweet, 1 ounce)	22	Jolt gum (2 pieces)	145

(*Source:* Caffeineinformer.com)

Solution: We draw and label a vertical line and a horizontal line as shown below on the left. These lines are also called axes. We place the different food categories along the horizontal axis. Along the vertical axis, we place a scale.

There are many choices of scales that would be appropriate. Notice that the milligrams range from a low of 6 to a high of 160. From this information, we use a scale that starts at 0 and then shows multiples of 20 so that the scale is not too cluttered. The scale stops at 180, the smallest multiple of 20 that is greater that the high of 160 milligrams. It may also be helpful to draw horizontal lines along the scale markings to help draw the vertical bars at the correct height. The finished bar graph is shown below on the right.





Work Practice 4

As mentioned previously, graphs can be misleading. Both graphs on the next page show the same information but with different scales. Special care should be taken when forming conclusions from the appearance of a graph.

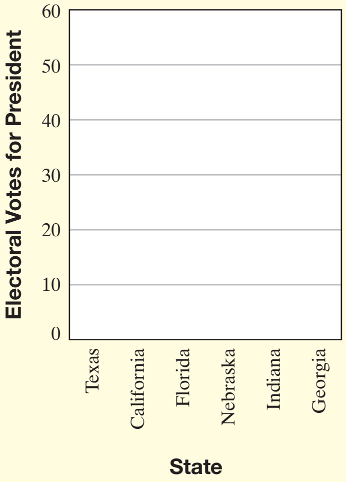
Practice 4

Draw a vertical bar graph using the information in the table about electoral votes for selected states.

Total Electoral Votes for Selected States

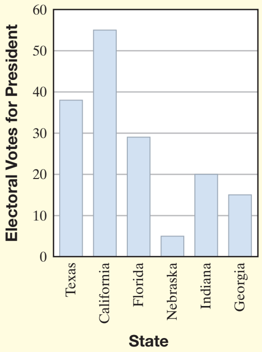
State	Electoral Votes
Texas	38
California	55
Florida	29
Nebraska	5
Indiana	20
Georgia	15

(*Source:* World Almanac)

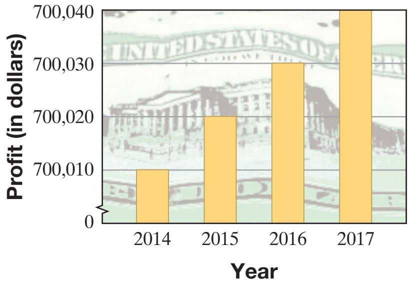


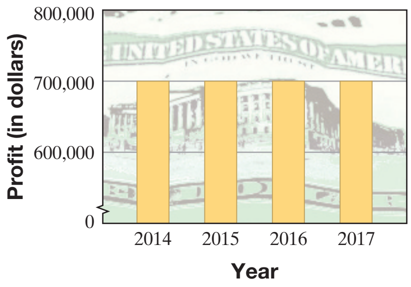
Answer

4.



Notice the ⌇ symbol on each vertical scale on the graphs below. This symbol alerts us that numbers are missing from that scale





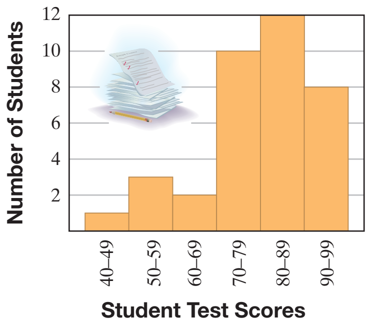
Are profits shown in the graphs above greatly increasing, or are they remaining about the same?

Objective C Reading and Constructing Histograms

Suppose that the test scores of 36 students are summarized in the table below. We call this table a **frequency distribution table** since one column gives the frequency or number of times the event in the other column occurred.

Student Scores	Frequency (Number of Students)
40–49	1
50–59	3
60–69	2
70–79	10
80–89	12
90–99	8

The results in this frequency distribution table can be displayed in a histogram. A **histogram** is a special bar graph. The width of each bar represents a range of numbers called a **class interval.** The height of each bar corresponds to how many times a number in the class interval occurs and is called the **class frequency.** The bars in a histogram lie side by side with no space between them. Note: Another name for this histogram is a **frequency distribution graph**.



Practice 5

Use the histogram above Example 5 to determine how many students scored 80–89 on the test.

Answer

5. 12

Example 5 Reading a Histogram on Student Test Scores

Use the preceding histogram to determine how many students scored 50–59 on the test.

Solution: We find the bar representing 50–59. The height of this bar is 3, which means 3 students scored 50–59 on the test.

Work Practice 5

Example 6 Reading a Histogram on Student Test Scores

Use the histogram above Example 5 to determine how many students scored 80 or above on the test.

Solution: We see that two different bars fit this description. There are 12 students who scored 80–89 and 8 students who scored 90–99. The sum of these two categories is 12 + 8 or 20 students. Thus, 20 students scored 80 or above on the test.

Work Practice 6

Now we will look at a way to construct histograms.

The daily high temperatures for 1 month in New Orleans, Louisiana, are recorded in the following list:

85°	90°	95°	89°	88°	94°
87°	90°	95°	92°	95°	94°
82°	92°	96°	91°	94°	92°
89°	89°	90°	93°	95°	91°
88°	90°	88°	86°	93°	89°

The data in this list have not been organized and can be hard to interpret. One way to organize the data is to place them in a **frequency distribution table.** We will do this in Example 7.

Example 7 Completing a Frequency Distribution on Temperature

Complete the frequency distribution table for the preceding temperature data.

Solution: Go through the data and place a tally mark in the second column of the table next to the class interval. Then count the tally marks and write each total in the third column of the table.

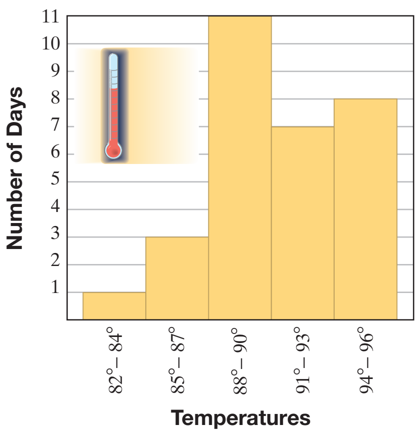
Class Intervals (Temperatures)	Tally	Class Frequency (Number of Days)
82°–84°	|	1
85°–87°	|||	3
88°–90°	卌 卌 |	11
91°–93°	卌 ||	7
94°–96°	卌 |||	8

Work Practice 7

Example 8 Constructing a Histogram

Construct a histogram from the frequency distribution table in Example 7.

Solution:



Work Practice 8

Practice 6

Use the histogram above Example 5 to determine how many students scored less than 80 on the test.

Practice 7

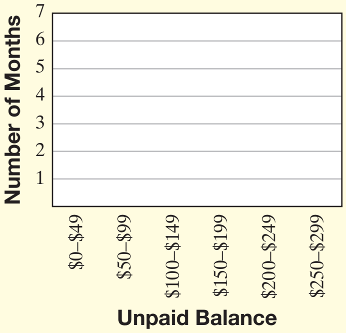
Complete the frequency distribution table for the data below. Each number represents a credit card owner's unpaid balance for one month.

0	53	89	125
265	161	37	76
62	201	136	42

Class Intervals (Credit Card Balances)	Tally	Class Frequency (Number of Months)
$0–$49	______	______
$50–$99	______	______
$100–$149	______	______
$150–$199	______	______
$200–$249	______	______
$250–$299	______	______

Practice 8

Construct a histogram from the frequency distribution table for Practice 7.



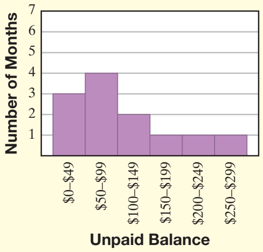
Answers

6. 16

7.

Tally	Class Frequency (Number Months)	Tally	Class Frequency (Number Months)
|||	3	|	1
||||	4	|	1
||	2	|	1

8.



✓Concept Check Which of the following sets of data is better suited to representation by a histogram? Explain.

Set 1		Set 2	
Grade on Final	# of Students	Section Number	Avg. Grade on Final
51–60	12	150	78
61–70	18	151	83
71–80	29	152	87
81–90	23	153	73
91–100	25		

Objective D Reading Line Graphs

Another common way to display information with a graph is by using a **line graph.** An advantage of a line graph is that it can be used to visualize relationships between two quantities. A line graph can also be very useful in showing changes over time.

Practice 9

Use the temperature graph in Example 9 to answer the following questions:

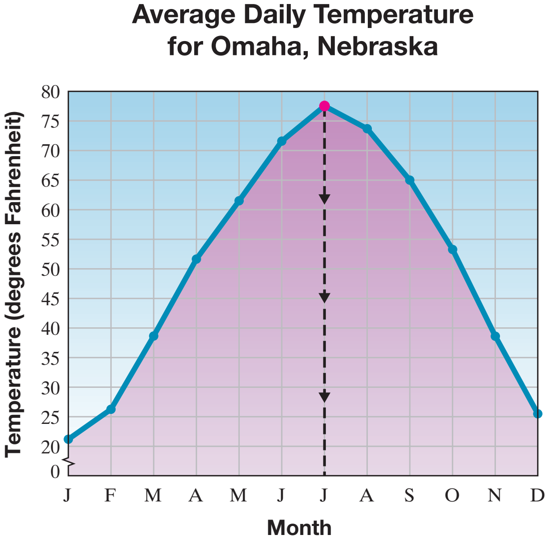
a. During what month is the average daily temperature the lowest?

b. During what month is the average daily temperature 25°F?

c. During what months is the average daily temperature greater than 70°F?

Example 9 Reading Temperatures from a Line Graph

The following line graph shows the average daily temperature for each month in Omaha, Nebraska. Use this graph to answer the questions below.



Source: National Climatic Data Center

a. During what month is the average daily temperature the highest?

b. During what month, from July through December, is the average daily temperature 65°F?

c. During what months is the average daily temperature less than 30°F?

Solution:

a. The month with the highest temperature corresponds to the highest point. This is the red point shown on the graph above. We follow this highest point downward to the horizontal month scale and see that this point corresponds to July.

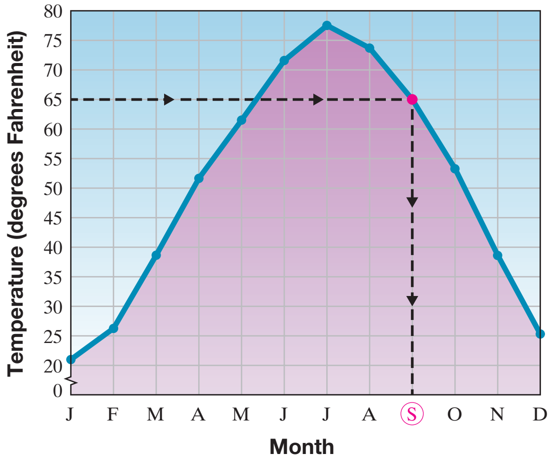
Answers

9. a. January **b.** December
c. June, July, and August

✓Concept Check Answer

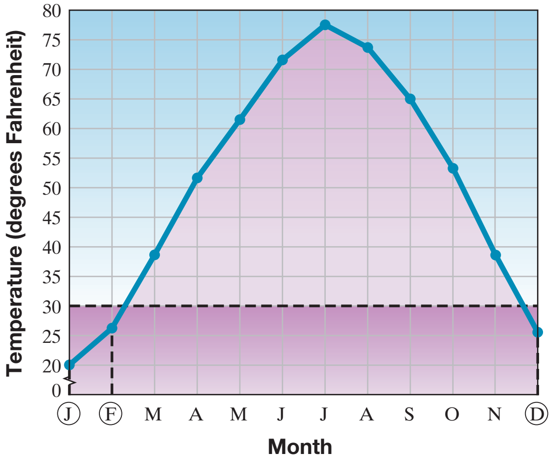
Set 1; the grades are arranged in ranges of scores.

b. The months July through December correspond to the right side of the graph. We find the 65°F mark on the vertical temperature scale and move to the right until a point on the right side of the graph is reached. From that point, we move downward to the horizontal month scale and read the corresponding month. During the month of September, the average daily temperature is 65°F.



Source: National Climatic Data Center

c. To see what months the temperature is less than 30°F, we find what months correspond to points that fall below the 30°F mark on the vertical scale. These months are January, February, and December.



Source: National Climatic Data Center

Work Practice 9

Vocabulary, Readiness & Video Check

Fill in each blank with one of the choices below.

pictograph	bar	class frequency
histogram	line	class interval

1. A(n) ______________ graph presents data using vertical or horizontal bars.

2. A(n) ______________ is a graph in which pictures or symbols are used to visually present data.

3. A(n) ______________ graph displays information with a line that connects data points.

4. A(n) ______________ is a special bar graph in which the width of each bar represents a(n) ______________ and the height of each bar represents the ______________.

Martin-Gay Interactive Videos

See Video 7.1

Watch the section lecture video and answer the following questions.

Objective A 5. From the pictograph in Example 1, how would you approximate the number of wildfires for any given year?

Objective B 6. From Example 5, what is one advantage of displaying data in a bar graph?

Objective C 7. Complete this statement based on the lecture before Example 6: A histogram is a special kind of __________.

Objective D 8. From the line graph in Examples 10–13, what year averaged the greatest number of goals per game average and what was this average?

7.1 Exercise Set MyLab Math

Objective A *The following pictograph shows the number of acres devoted to wheat production in selected states in 2017. Use this graph to answer Exercises 1 through 8. See Examples 1 and 2. (Source: U.S. Department of Agriculture)*

1. Which state plants the greatest acreage in wheat?

2. Which of the states shown plant the least amount of wheat acreage?

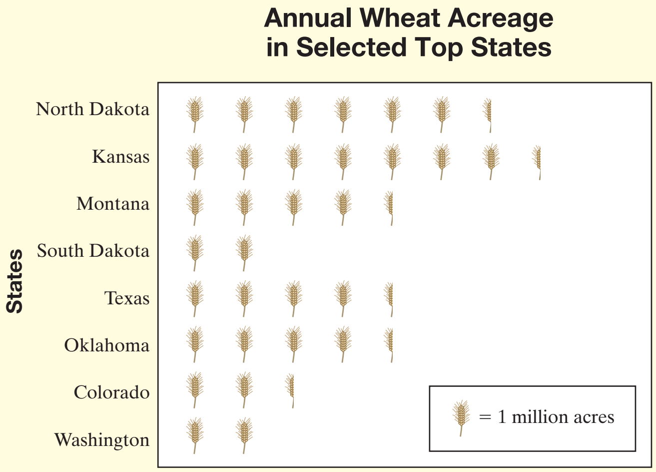
3. Approximate the number of acres of wheat planted in Oklahoma.

4. Approximate the number of acres of wheat planted in Kansas.

5. Which state(s) plant less than 3,000,000 acres of wheat?

6. Which state(s) plant more than 7,000,000 acres of wheat?

7. Which state plants more wheat: North Dakota or Oklahoma?



Annual Wheat Acreage in Selected Top States

8. Which state plants more wheat: Colorado or Washington?

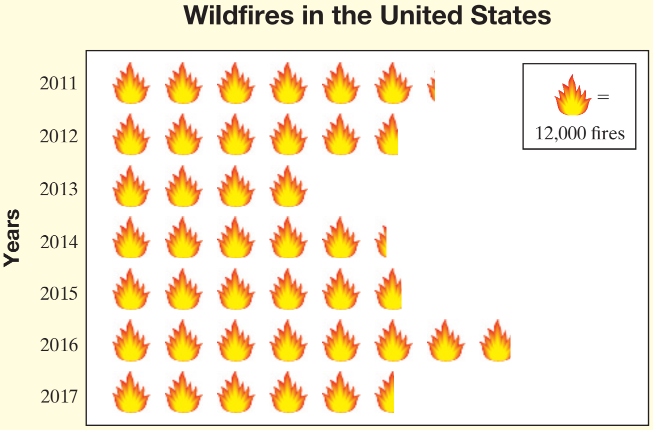
The following pictograph shows the average number of wildfires in the United States between 2011 and 2017. Use this graph to answer Exercises 9 through 16. See Examples 1 and 2. (Source: National Interagency Fire Center)

9. Approximate the number of wildfires in 2013.

10. Approximately how many wildfires were there in 2012?

11. Which year, of the years shown, had the most wildfires?

12. In what years were the number of wildfires greater than 72,000?



Wildfires in the United States

13. What was the amount of decrease in wildfires from 2012 to 2013?

14. What was the amount of increase in wildfires from 2015 to 2016?

15. What was the average annual number of wildfires from 2013 to 2015? (*Hint:* How do you calculate the average?)

16. Give a possible explanation for the sharp increase in the number of wildfires in 2016.

Objective B *The National Weather Service has exacting definitions for hurricanes; they are tropical storms with winds in excess of 74 mph. The following bar graph shows the number of hurricanes, by month, that have made landfall on the mainland United States between 1851 and 2017. Use this graph to answer Exercises 17 through 22. See Example 3. (Source: National Weather Service: National Hurricane Center)*

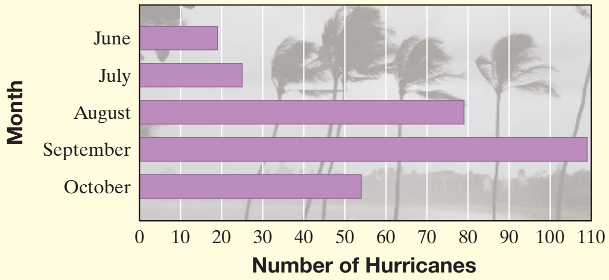
17. In which month did the most hurricanes make landfall in the United States?

18. In which month did the fewest hurricanes make landfall in the United States?

19. Approximate the number of hurricanes that made landfall in the United States during the month of August.

20. Approximate the number of hurricanes that made landfall in the United States in September.

Hurricanes Making Landfall in the United States, by Month, 1851–2017



21. In 2005 alone, three hurricanes made landfall during the month of October. What fraction of all the 54 hurricanes that made landfall during October is this?

22. In 2007, only one hurricane made landfall in the United States during the entire season, in the month of September. If there have been 109 hurricanes to make landfall in the month of September since 1851, approximately what percent of these arrived in 2007?

The following horizontal bar graph shows the approximate 2017 population of the world's largest cities (including their suburbs). Use this graph to answer Exercises 23 through 28. See Example 3. (Source: World Atlas)

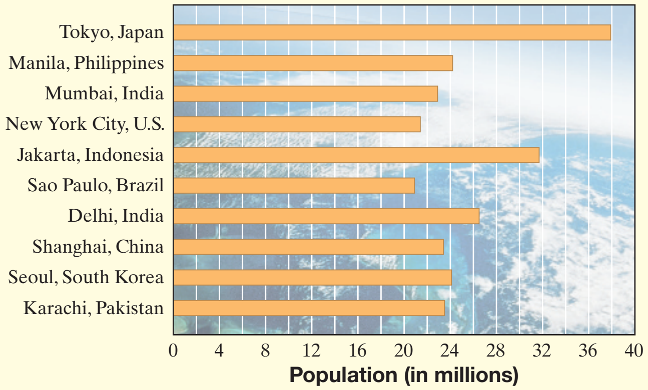
23. Name the city with the largest population, and estimate its population.

24. Name the city whose population is between 26 million and 27 million.

25. Name the city in the United States with the largest population, and estimate its population.

26. Name the two cities that have approximately a population of 24 million.

World's Largest Cities
(including Suburbs)



27. How much larger (in terms of population) is Manila, Philippines, than Sao Paulo, Brazil?

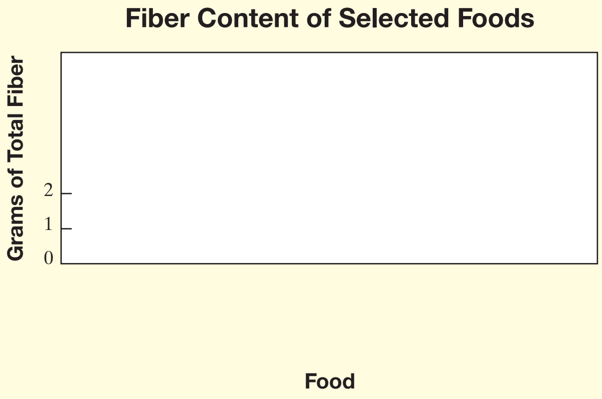
28. How much larger (in terms of population) is Jakarta, Indonesia, than Delhi, India?

Use the information given to draw a vertical bar graph. Clearly label the bars. See Example 4.

29.

Fiber Content of Selected Foods	
Food	**Grams of Total Fiber**
Kidney beans $\left(\frac{1}{2}\text{ c}\right)$	4.5
Oatmeal, cooked $\left(\frac{3}{4}\text{ c}\right)$	3.0
Peanut butter, chunky (2 tbsp)	1.5
Popcorn (1 c)	1.0
Potato, baked with skin (1 med)	4.0
Whole wheat bread (1 slice)	2.5

(*Sources:* American Dietetic Association and National Center for Nutrition and Dietetics)



30.

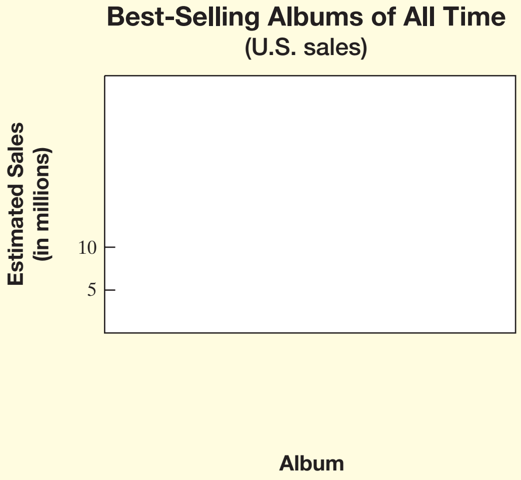
U.S. Annual Food Sales	
Year	**Sales in Billions of Dollars**
2012	1345
2013	1410
2014	1462
2015	1512
2016	1584
2017	2123

(*Source:* U.S. Department of Agriculture)

31.

Best-Selling Albums of All Time (U.S. Sales)	
Album	**Estimated Sales (in millions)**
Pink Floyd: *The Wall* (1979)	23
Michael Jackson: *Thriller* (1982)	29
Billy Joel: *Greatest Hits Volumes I&II* (1985)	23
Eagles: *Their Greatest Hits* (1976)	29
Led Zeppelin: *Led Zeppelin IV* (1971)	23

(*Source:* Recording Industry Association of America)

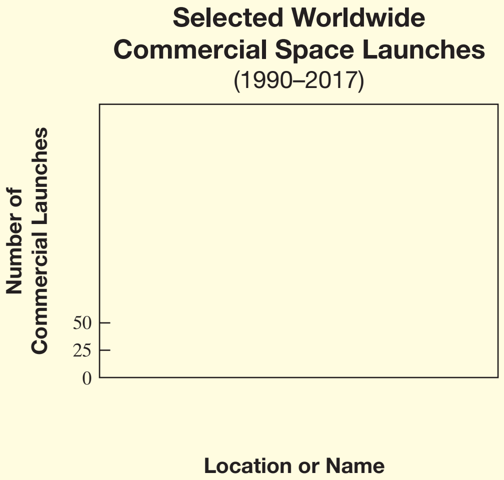


32.

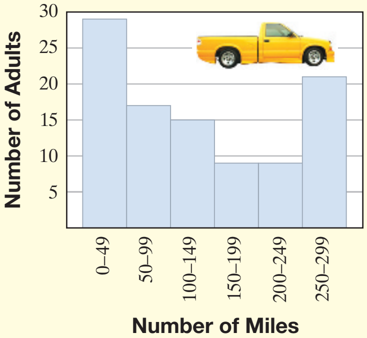
Selected Worldwide Commercial Space Launches	
Location or Name	**Total Commercial Space Launches 1990–2017**
United States	231
Russia	200
Europe	187
China	60
Sea Launch*	48

*Sea Launch is an international venture involving four countries that uses its own launch facility outside national borders.

(*Source:* Space Launch Report and NASA)



Objective C *The following histogram shows the number of miles that each adult, from a survey of 100 adults, drives per week. Use this histogram to answer Exercises 33 through 42. See Examples 5 and 6.*



33. How many adults drive 100–149 miles per week?

34. How many adults drive 200–249 miles per week?

35. How many adults drive fewer than 150 miles per week?

36. How many adults drive 200 miles or more per week?

37. How many adults drive 100–199 miles per week?

38. How many adults drive 150–249 miles per week?

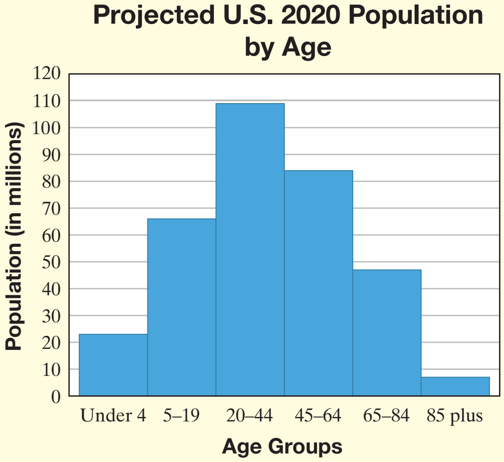
39. How many more adults drive 250–299 miles per week than 200–249 miles per week?

40. How many more adults drive 0–49 miles per week than 50–99 miles per week?

41. What is the ratio of adults who drive 150–199 miles per week to the total number of adults surveyed?

42. What is the ratio of adults who drive 50–99 miles per week to the total number of adults surveyed?

The following histogram shows the projected population (in millions), by age groups, for the United States for the year 2020. Use this histogram to answer Exercises 43 through 50. For Exercises 45 through 48, estimate to the nearest whole million. See Examples 5 and 6.



43. What age range will be the largest population group in 2020?

44. What age range will be the smallest population group in 2020?

45. How large is the population of 20- to 44-year-olds expected to be in 2020?

46. How large is the population of 45- to 64-year-olds expected to be in 2020?

47. How large is the population of those less than 4 years old expected to be in 2020?

48. How large is the population of 5- to 19-year-olds expected to be in 2020?

49. Which bar represents the age range you expect to be in during 2020?

50. How many more 20- to 44-year-olds are there expected to be than 45- to 64-year-olds in 2020?

The following list shows the golf scores for an amateur golfer. Use this list to complete the frequency distribution table to the right. See Example 7.

78	84	91	93	97
97	95	85	95	96
101	89	92	89	100

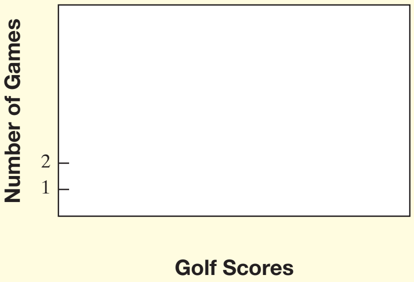
	Class Intervals (Scores)	Tally	Class Frequency (Number of Games)
51.	70–79		
52.	80–89		
53.	90–99		
54.	100–109		

Twenty-five people in a survey were asked to give their current checking account balances. Use the balances shown in the following list to complete the frequency distribution table to the right. See Example 7.

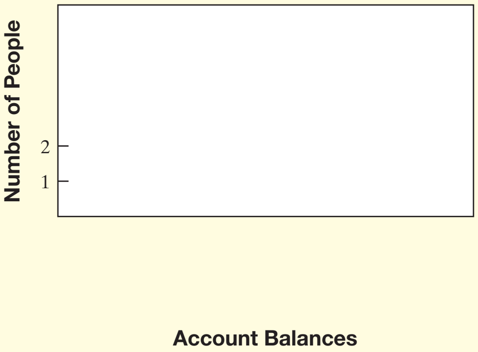
$53	$105	$162	$443	$109
$468	$47	$259	$316	$228
$207	$357	$15	$301	$75
$86	$77	$512	$219	$100
$192	$288	$352	$166	$292

	Class Intervals (Account Balances)	Tally	Class Frequency (Number of People)
55.	$0–$99		
56.	$100–$199		
57.	$200–$299		
58.	$300–$399		
59.	$400–$499		
60.	$500–$599		

61. Use the frequency distribution table from Exercises **51** through **54** to construct a histogram. See Example 8.



62. Use the frequency distribution table from Exercises **55** through **60** to construct a histogram. See Example 8.



Objective D *Beach Soccer World Cup is now held every two years. The following line graph shows the World Cup goals per game average for beach soccer during the years shown. Use this graph to answer Exercises 63 through 70. See Example 9.*

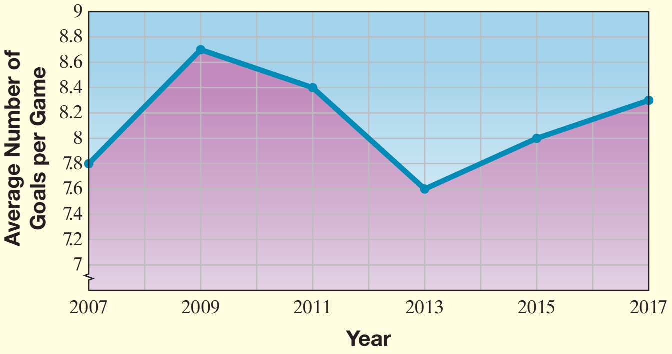
63. Find the average number of goals per game in 2017.

64. Find the average number of goals per game in 2013.

65. During what year shown was the average number of goals per game the highest?

66. During what year shown was the average number of goals per game the lowest?

Beach Soccer World Cup Goals per Game Average



Source: FIVB.org

67. From 2013 to 2015, did the average number of goals per game increase or decrease?

68. From 2011 to 2013, did the average number of goals per game increase or decrease?

69. During what year(s) shown were the average goals per game less than 8?

70. During what year(s) shown were the average goals per game greater than 8?

Review

Find each percent. See Section 6.2 or 6.3.

71. 30% of 12

72. 45% of 120

73. 10% of 62

74. 95% of 50

Write each fraction as a percent. See Section 6.1.

75. $\frac{1}{4}$

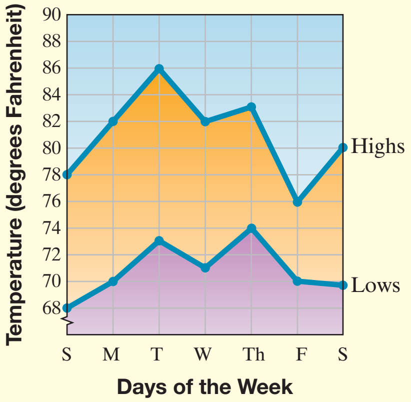
76. $\frac{2}{5}$

77. $\frac{17}{50}$

78. $\frac{9}{10}$

Concept Extensions

The following double line graph shows temperature highs and lows for a week. Use this graph to answer Exercises 79 through 84.



79. What was the high temperature reading on Thursday?

80. What was the low temperature reading on Thursday?

81. What day was the temperature the lowest? What was this low temperature?

82. What day of the week was the temperature the highest? What was this high temperature?

83. On what day of the week was the difference between the high temperature and the low temperature the greatest? What was this difference in temperature?

84. On what day of the week was the difference between the high temperature and the low temperature the least? What was this difference in temperature?

85. True or false? With a bar graph, the width of the bar is just as important as the height of the bar. Explain your answer.

86. Kansas plants about 24% of the wheat acreage in the United States. About how many acres of wheat are planted in the United States in 2017, according to the pictograph for Exercises **1** through **8**? Round to the nearest million acres.

7.2 Circle Graphs

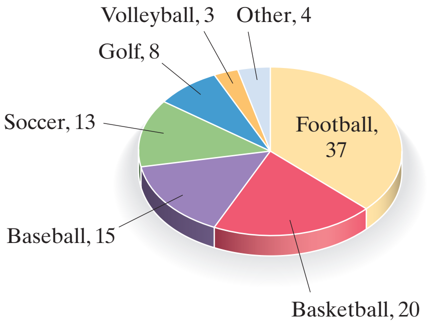
Objectives

A Read Circle Graphs.

B Draw Circle Graphs.

Objective A Reading Circle Graphs

In Exercise Set 6.1, the following **circle graph** was shown. This particular graph shows the favorite sport for 100 adults.



Each sector of the graph (shaped like a piece of pie) shows a category and the relative size of the category. In other words, the most popular sport is football, and it is represented by the largest sector.

Practice 1

Find the ratio of adults preferring golf to total adults. Write the ratio as a fraction in simplest form.

Example 1 Find the ratio of adults preferring basketball to total adults. Write the ratio as a fraction in simplest form.

Solution: The ratio is

$$\frac{\text{people preferring basketball}}{\text{total adults}} = \frac{20}{100} = \frac{1}{5}$$

Work Practice 1

A circle graph is often used to show percents in different categories, with the whole circle representing 100%.

Practice 2

Using the circle graph shown in Example 2, determine the percent of visitors to the United States that came from Europe, Asia, and South America.

Example 2 Using a Circle Graph

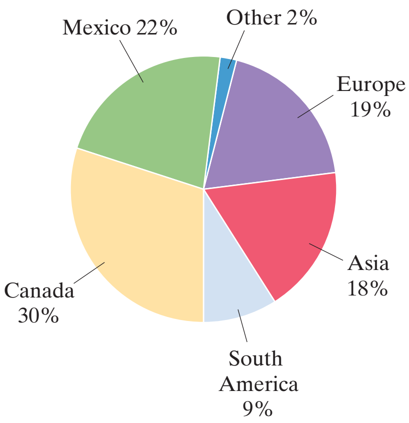
The following graph shows the percent of visitors to the United States in a recent year by various regions. Using the circle graph shown, determine the percent of visitors who came to the United States from Mexico or Canada.

Solution: To find this percent, we add the percents corresponding to Mexico and Canada. The percent of visitors to the United States that came from Mexico or Canada is

$$22\% + 30\% = 52\%$$

Work Practice 2

Visitors to U.S. by Region



Source: ITA National Travel and Tourism Office

Answers

1. $\frac{2}{25}$ **2.** 46%

Helpful Hint

Since a circle graph represents a whole, the percents should add to 100% or 1. Notice this is true for Example 2.

Example 3 Finding Percent of Population

The U.S. Department of Commerce forecasts 89 million international visitors to the United States in 2022. Use the circle graph from Example 2 and predict the number of tourists that might be from Europe.

Solution: We use the percent equation.

$$\begin{aligned} \text{amount} &= \text{percent} \cdot \text{base} \\ \text{amount} &= 0.19 \cdot 89{,}000{,}000 \\ &= 0.19(89{,}000{,}000) \\ &= 16{,}910{,}000 \end{aligned}$$

Thus, 16,910,000 tourists might come from Europe in 2022.

Work Practice 3

Practice 3

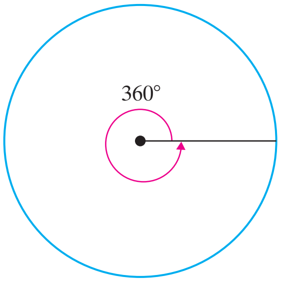
Use the information in Example 3 and the circle graph from Example 2 to predict the number of tourists from Mexico in 2022.

✓Concept Check Can the following data be represented by a circle graph? Why or why not?

Responses to the Question "In Which Activities Are You Involved?"	
Intramural sports	60%
On-campus job	42%
Fraternity/sorority	27%
Academic clubs	21%
Music programs	14%

Objective B Drawing Circle Graphs

To draw a circle graph, we use the fact that a whole circle contains 360° (degrees).



Answer

3. 19,580,000 tourists from Mexico

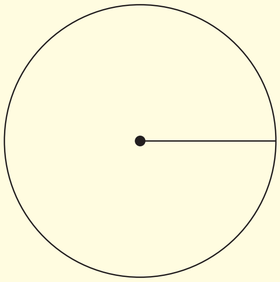
✓Concept Check Answer

no; the percents add up to more than 100%

Practice 4

Use the data shown to draw a circle graph.

Freshmen	30%
Sophomores	27%
Juniors	25%
Seniors	18%



Example 4 Drawing a Circle Graph for U.S. Armed Forces Personnel

The following table shows the percent of U.S. armed forces personnel that were in each branch of service in 2017. (*Source:* U.S. Department of Defense)

Branch of Service	Percent
Army	37
Navy	25
Marine Corps	14
Air Force	24
(Note: The Coast Guard is now under the Department of Homeland Security.)	

Draw a circle graph showing this data.

Solution: First we find the number of degrees in each sector representing each branch of service. Remember that the whole circle contains 360°. (We will round degrees to the nearest whole degree.)

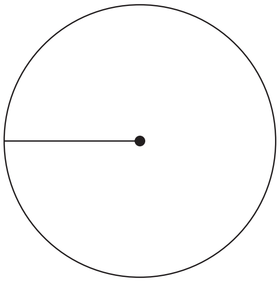
Sector	Degrees in Each Sector
Army	$37\% \times 360° = 0.37 \times 360° = 133.2° \approx 133°$
Navy	$25\% \times 360° = 0.25 \times 360° = 90°$
Marine Corps	$14\% \times 360° = 0.14 \times 360° = 50.4° \approx 50°$
Air Force	$24\% \times 360° = 0.24 \times 360° = 86.4° \approx 86°$

Helpful Hint

Check your calculations by finding the sum of the degrees.

$$133° + 90° + 50° + 86° = 359°$$

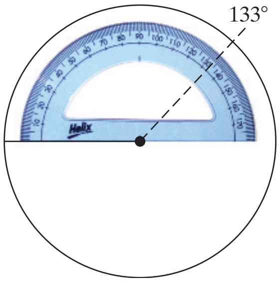
The sum should be 360°. (Our sum varies slightly because of rounding.)

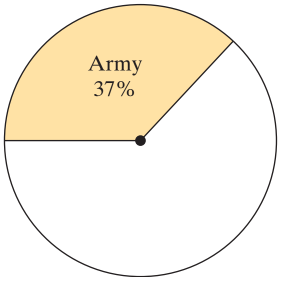


Next we draw a circle and mark its center. Then we draw a line from the center of the circle to the circle itself.

To construct the sectors, we will use a **protractor.** A protractor measures the number of degrees in an angle. We place the hole in the protractor over the center of the circle. Then we adjust the protractor so that 0° on the protractor is aligned with the line that we drew.

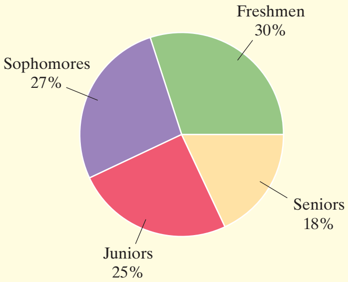
It makes no difference which sector we draw first. To construct the "Army" sector, we find 133° on the protractor and mark our circle. Then we remove the protractor and use this mark to draw a second line from the center to the circle itself.



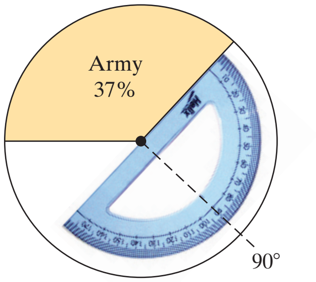


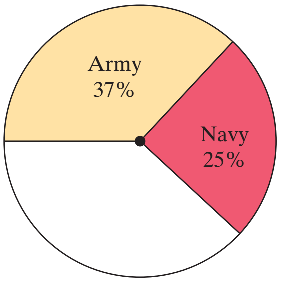
Answer

4.

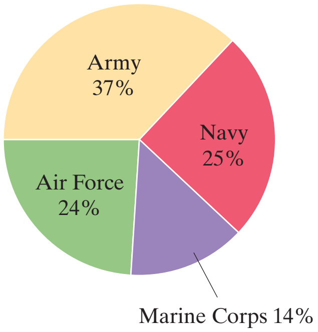


To construct the "Navy" sector, we follow the same procedure as above, except that we line up 0° with the second line we drew and mark the protractor at 90°.





We continue in this manner until the circle graph is complete.



Work Practice 4

✓**Concept Check** True or false? The larger a sector in a circle graph, the larger the percent of the total it represents. Explain your answer.

✓**Concept Check Answer**
true

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

sector circle 100 360

1. In a ________ graph, each section (shaped like a piece of pie) shows a category and the relative size of the category.

2. A circle graph contains pie-shaped sections, each called a ________.

3. The number of degrees in a whole circle is ________.

4. If a circle graph has percent labels, the percents should add up to ________.

Martin-Gay Interactive Videos

See Video 7.2

Watch the section lecture video and answer the following questions.

Objective A **5.** From Example 3, when a circle graph shows different parts or percents of some whole category, what is the sum of the percents in the whole circle graph?

Objective B **6.** From Example 6, when looking at the sector degree measures of a circle graph, the whole circle graph corresponds to what degree measure?

7.2 Exercise Set MyLab Math

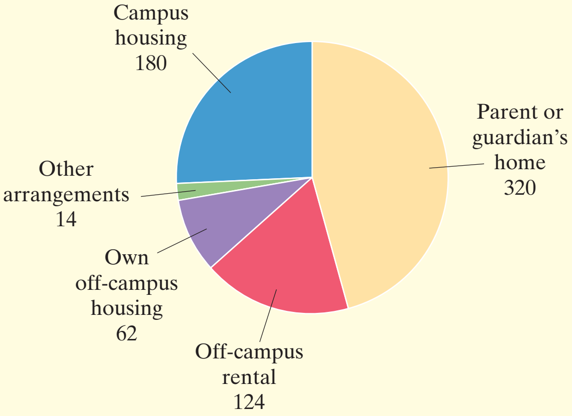
Objective A *The following circle graph is a result of surveying 700 college students. They were asked where they live while attending college. Use this graph to answer Exercises 1 through 6. Write all ratios as fractions in simplest form. See Example 1.*

1. Where do most of these college students live?

2. Besides the category "Other arrangements," where do the fewest of these college students live?

3. Find the ratio of students living in campus housing to total students.

4. Find the ratio of students living in off-campus rentals to total students.



5. Find the ratio of students living in campus housing to students living in a parent or guardian's home.

6. Find the ratio of students living in off-campus rentals to students living in a parent or guardian's home.

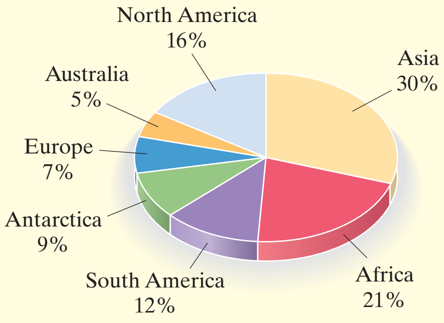
The following circle graph shows the percent of the land area of the continents of Earth. Use this graph for Exercises 7 through 14. See Example 2.

7. Which continent is the largest?

8. Which continent is the smallest?

9. What percent of the land on Earth is accounted for by Asia and Europe together?

10. What percent of the land on Earth is accounted for by North and South America?



Source: National Geographic Society

The total amount of land from the continents is approximately 57,000,000 square miles. Use the graph to find the area of the continents given in Exercises 11 through 14. See Example 3.

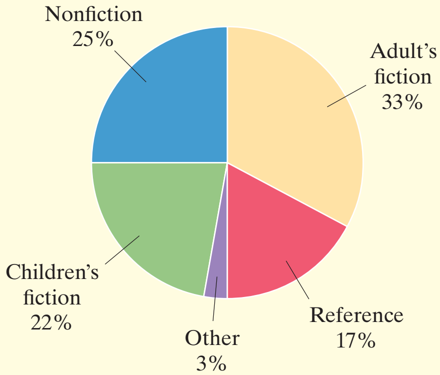
11. Asia

12. South America

13. Australia

14. Europe

The following circle graph shows the percent of the types of books available at Midway Memorial Library. Use this graph for Exercises 15 through 24. See Example 2.



15. What percent of books are classified as some type of fiction?

16. What percent of books are nonfiction or reference?

17. What is the second-largest category of books?

18. What is the third-largest category of books?

If this library has 125,600 books, find how many books are in each category given in Exercises 19 through 24. See Example 3.

19. Nonfiction

20. Reference

21. Children's fiction

22. Adult's fiction

23. Reference or other

24. Nonfiction or other

Objective B *Fill in each table. Round to the nearest degree. Then draw a circle graph to represent the information given in each table. (Remember: The total of "Degrees in Sector" column should equal 360° or very close to 360° because of rounding.) See Example 4.*

25. **Types of Apples Grown in Washington State**

Type of Apple	Percent	Degrees in Sector
Red Delicious	37%	
Golden Delicious	13%	
Fuji	14%	
Gala	15%	
Granny Smith	12%	
Other varieties	6%	
Braeburn	3%	

(*Source:* U.S. Apple Association)

26. **Color Distribution of M&M's Milk Chocolate**

Color	Percent	Degrees in Sector
Blue	24%	
Orange	20%	
Green	16%	
Yellow	14%	
Red	13%	
Brown	13%	

(*Source:* M&M Mars)

27.

Distribution of Large Dams by Continent

Continent	Percent	Degrees in Sector
Europe	19%	
North America	32%	
South America	3%	
Asia	39%	
Africa	5%	
Australia	2%	

(*Source:* International Commission on Large Dams)

28.

Distribution of Department of the Interior Public Lands by Management

Bureau of Management	Percent	Degrees in Sector
Bureau of Indian Affairs	9%	
National Park Service	11%	
Fish and Wildlife	20%	
U.S. Forest Service	31%	
Bureau of Land Management	29%	

(*Source:* Department of the Interior: National Park Service)

Review

Write the prime factorization of each number. See Section 3.2.

29. 20

30. 25

31. 40

32. 16

33. 85

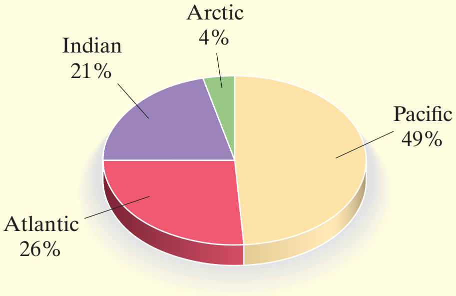
34. 105

Concept Extensions

The following circle graph shows the relative sizes of the great oceans. Use this graph for Exercises 35 through 40.

35. Without calculating, determine which ocean is the largest. How can you answer this question by looking at the circle graph?

36. Without calculating, determine which ocean is the smallest. How can you answer this question by looking at the circle graph?



Source: Philip's World Atlas

These oceans together make up 264,489,800 square kilometers of Earth's surface. Find the square kilometers for each ocean.

37. Pacific Ocean

38. Atlantic Ocean

39. Indian Ocean

40. Arctic Ocean

The following circle graph summarizes the results of online spending in America. Let's use these results to make predictions about the online spending behavior of a community of 2800 Internet users age 18 and over. Use this graph for Exercises 41 through 46. Round to the nearest whole. (Note: Because of rounding, these percents do not have a sum of 100%.)

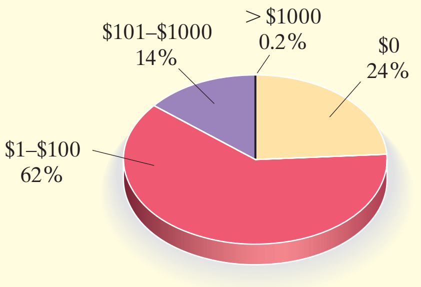
41. How many of the survey respondents said that they spend $0 online each month?

42. How many of the survey repondents said that they spend $1–$100 online each month?

43. How many of the survey respondents said that they spend $0 to $100 online each month?

44. How many of the survey respondents said that they spend $1 to $1000 online each month?

Online Spending per Month



Source: The Digital Future Report, 2013

45. Find the ratio of *number* of respondents who spend $0 online to *number* of respondents who spend $1–$100 online. Write the ratio as a fraction. Simplify the fraction if possible.

46. Find the ratio of *percent* of respondents who spend $101–$1000 online to *percent* of those who spend $1–$100. Write the ratio as a fraction with integers in the numerator and denominator. Simplify the fraction if possible.

See the Concept Checks in this section.

47. Can the data below be represented by a circle graph? Why or why not?

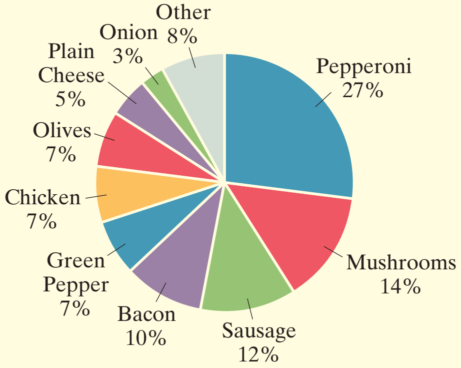
Responses to the Question "What Classes Are You Taking?"	
Math	80%
English	72%
History	37%
Biology	21%
Chemistry	14%

48. True or false? The smaller a sector in a circle graph, the smaller the percent of the total it represents. Explain why.

Study the Chapter Opener circle graphs below and conduct surveys with at least 30 people.

49. Using the "Favorite Pizza Topping" circle graph as a guide, ask each person in your survey to choose his or her favorite pizza topping. Tally the results, draw a circle graph and compare your circle graph to the one shown.

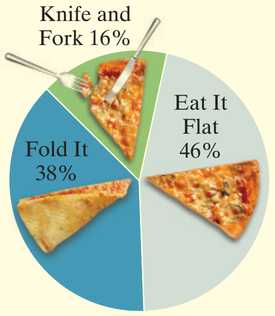
Favorite Pizza Topping



Sources: Zagot, Statista, Harris Poll

50. Using the "How Do You Eat Your Pizza?" circle graph as a guide, ask each person in your survey to choose how he or she eats pizza. Then follow the directions in Exercise **49**.

How Do You Eat Your Pizza?



Sources: Zagot, Statista, Harris Poll

Integrated Review

Sections 7.1–7.2

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

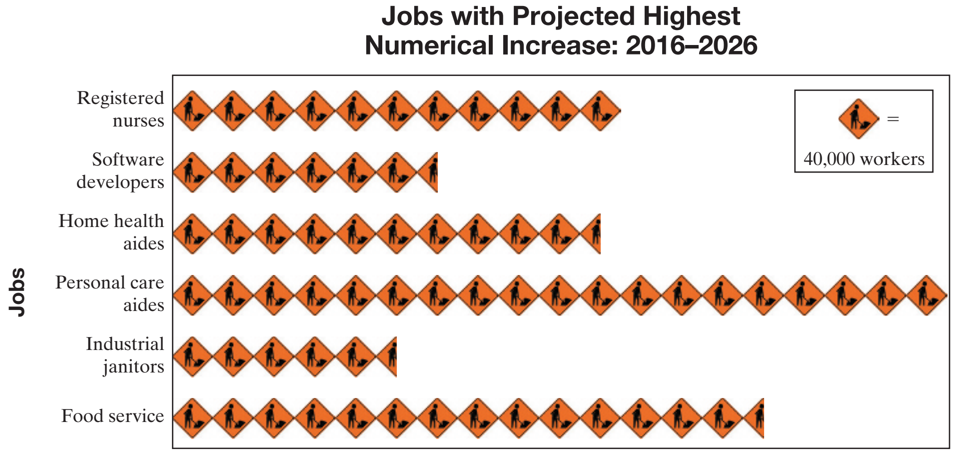
6. ______

7. ______

8. ______

Reading Graphs

The following pictograph shows the six occupations with the largest estimated numerical increase in employment in the United States between 2016 and 2026. Use this graph to answer Exercises 1 through 4.



Jobs with Projected Highest Numerical Increase: 2016–2026

Source: Bureau of Labor Statistics

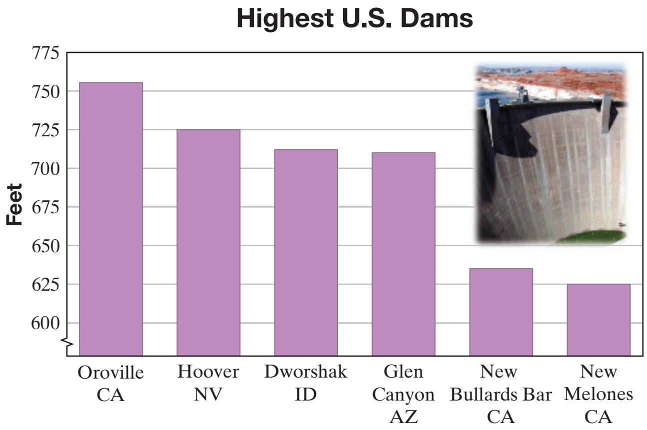
1. Approximate the increase in the number of software developers from 2016 to 2026.

2. Approximate the increase in the number of registered nurses from 2016 to 2026.

3. Which occupation is expected to show the greatest increase in number of employees between the years shown?

4. Which of the listed occupations is expected to show the smallest increase in number of employees between the years shown?

The following bar graph shows the highest U.S. dams. Use this graph to answer Exercises 5 through 8.



Highest U.S. Dams

Source: Committee on Register of Dams

5. Name the U.S. dam with the greatest height and estimate its height.

6. Name the U.S. dam whose height is between 625 and 650 feet and estimate its height.

7. Estimate how much higher the Hoover Dam is than the Glen Canyon Dam.

8. How many U.S. dams have heights over 700 feet?

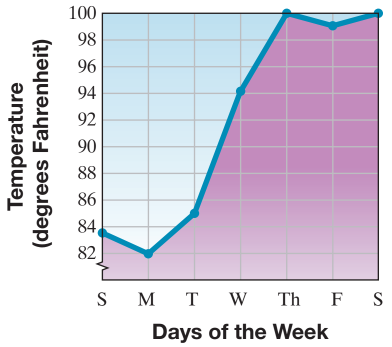
The following line graph shows the daily high temperatures for 1 week in Annapolis, Maryland. Use this graph to answer Exercises 9 through 12.

9. Name the day(s) of the week with the highest temperature and give that high temperature.

10. Name the day(s) of the week with the lowest temperature and give that low temperature.

11. On what days of the week was the temperature less than 90° Fahrenheit?

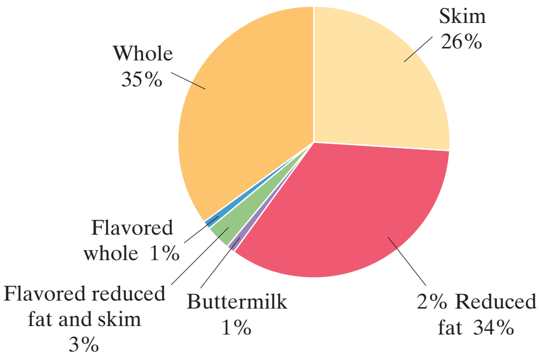
12. On what days of the week was the temperature greater than 90° Fahrenheit?



The following circle graph shows the types of milk beverages consumed in the United States. Use this graph for Exercises 13 through 16.

If a store in Kerrville, Texas, sells 200 quart containers of milk per week, estimate how many quart containers are sold in each category below.

Types of Beverage Milk Consumed



Source: U.S. Department of Agriculture

13. Whole milk

14. Skim milk

15. Buttermilk

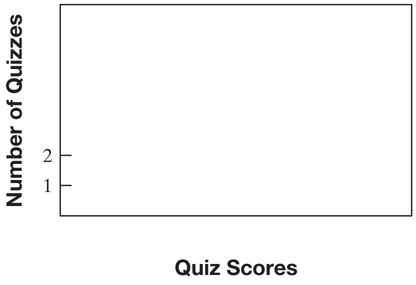
16. Flavored reduced fat and skim milk

The following list shows weekly quiz scores for a student in basic college mathematics. Use this list to complete the frequency distribution table.

50	80	71	83	86
67	89	93	88	97
75	80	78	93	99
	53	90		

	Class Intervals (Scores)	Tally	Class Frequency (Number of Quizzes)
17.	50–59		
18.	60–69		
19.	70–79		
20.	80–89		
21.	90–99		

22. Use the table from Exercises **17** through **21** to construct a histogram.



9. ____________

10. ____________

11. ____________

12. ____________

13. ____________

14. ____________

15. ____________

16. ____________

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

22. ____________

Objectives

A Find the Mean (or Average) of a List of Numbers.

B Find the Median of a List of Numbers.

C Find the Mode of a List of Numbers.

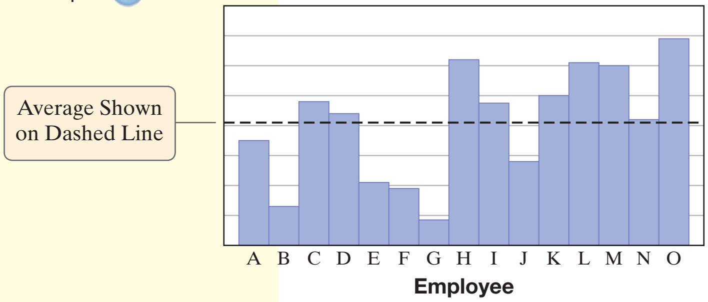
D Calculate Range, Mean, Median, and Mode from a Frequency Distribution Table or Graph.

7.3 Mean, Median, Mode, and Range

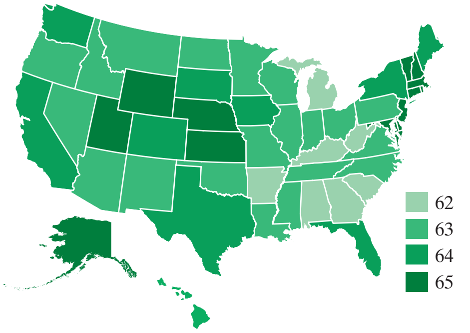
We are certainly familiar with the word "average." The next examples show real-life averages. For example,

- The average cost of a pie (or whole pizza) is $16.73.
- Adults employed in the United States report working an **average** of 47 hours per week, according to a Gallup poll.
- The U.S. miles per gallon **average** for light vehicles is 25.5, according to autonews.com (*Automotive News*).

Average Annual Sales for Employees A Through O



The Average Retirement Age in the U.S.



Based on U.S. Census Bureau labor force participation data.

As our accumulation of data increases, our ability to gather, store, and present these tremendous amounts of data increases. Sometimes it is desirable to be able to describe a set of data by a single "middle" number or a measure of central tendency. Three of the most common **measures of central tendency** are the **mean** (or average), the **median**, and the **mode**.

Objective A Finding the Mean

The most common measure of central tendency is the mean (sometimes called the "arithmetic mean" or the "average"). Recall that we first introduced finding the average of a list of numbers in Section 1.7.

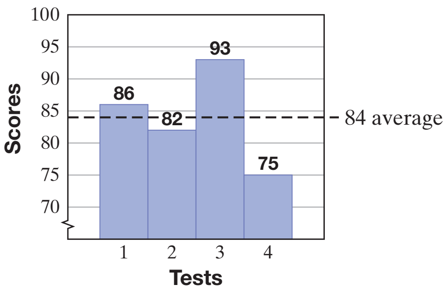
The **mean (average)** of a set of number items is the sum of the items divided by the number of items.

$$\text{mean} = \frac{\text{sum of items}}{\text{number of items}}$$

For example: To find the mean of four test scores—86, 82, 93, and 75—we find the sum of the scores and then divide by the number of scores, 4.

$$\text{mean} = \frac{86 + 82 + 93 + 75}{4} = \frac{336}{4} = 84$$

Notice that by looking at a bar graph of the scores with a dashed line at the mean, it is reasonable that 84 is one *measure of central tendency*.



Example 1 Finding the Mean Time in an Experiment

Seven students in a psychology class conducted an experiment on mazes. Each student was given a pencil and asked to successfully complete the same maze. The timed results are below:

Student	Ann	Thanh	Carlos	Jesse	Melinda	Ramzi	Dayni
Time (Seconds)	13.2	11.8	10.7	16.2	15.9	13.8	18.5

a. Who completed the maze in the shortest time? Who completed the maze in the longest time?

b. Find the mean time.

c. How many students took longer than the mean time? How many students took shorter than the mean time?

Solution:

a. Carlos completed the maze in 10.7 seconds, the shortest time. Dayni completed the maze in 18.5 seconds, the longest time.

b. To find the mean (or average), we find the sum of the items and divide by 7, the number of items.

$$\text{mean} = \frac{\text{sum of items}}{\text{number of items}} = \frac{13.2 + 11.8 + 10.7 + 16.2 + 15.9 + 13.8 + 18.5}{7}$$

$$= \frac{100.1}{7} = 14.3$$

c. Three students, Jesse, Melinda, and Dayni, had times longer than the mean time. Four students, Ann, Thanh, Carlos, and Ramzi, had times shorter than the mean time.

Work Practice 1

Practice 1

Find the mean of the following test scores: 87, 75, 96, 91, and 78.

✓Concept Check Estimate the mean of the following set of data:

5, 10, 10, 10, 10, 15

The mean has one main disadvantage. This measure of central tendency can be greatly affected by *outliers*. (Outliers are values that are especially large or small when compared with the rest of the data set.) Let's see an example of this next.

Example 2

The table lists the rounded salary of 10 staff numbers.

Staff	A	B	C	D	E	F	G	H	I	J
Salary (in thousands)	\$32	\$34	\$46	\$38	\$42	\$95	\$102	\$50	\$42	\$41

a. Find the mean of all 10 staff members.

b. Find the mean of all staff members except F and G.

Solution

a. $\text{mean} = \frac{32 + 34 + 46 + 38 + 42 + 95 + 102 + 50 + 42 + 41}{10} = \frac{522}{10}$

$= 52.2$

The mean salary is \$52.2 thousand or \$52,200.

(Continued on next page)

Practice 2

Use the table in Example 2 and find the mean salary of all staff members except G. Round thousands of dollars to 2 decimal places.

Answers

1. 85.4 **2.** \$46.67 thousand or \$46,670

✓Concept Check Answer

10

b. mean $= \frac{32 + 34 + 46 + 38 + 42 + 50 + 42 + 41}{8} = \frac{325}{8}$

$= 40.625$

Now, the mean salary is \$40.625 thousand or \$40,625.

Work Practice 2

The mean in part **a.** does not appear to be a measure of central tendency because this mean, \$52.2 thousand, is greater than all salaries except 2 of the 10. Also, notice the difference in the means for parts **a.** and **b.** By removing the 2 outliers, the mean was greatly reduced.

Although the mean was calculated correctly each time, parts **a.** and **b.** of Example 2 show one disadvantage of the mean. That is, a few numerical outliers can greatly affect the mean.

Later in this section, we will discuss the range of a data set as well as calculate measures of central tendency from frequency distribution tables and graphs.

Helpful Hint

Remember an important disadvantage of the mean:
If our data set has a few outliers, the mean may not be the best measure of central tendency.

Often in college, the calculation of a **grade point average** (GPA) is a **weighted mean** and is calculated as shown in Example 3.

Practice 3

Find the grade point average if the following grades were earned in one semester. Round to 2 decimal places.

Grade	Credit Hours
A	2
B	4
C	5
D	2
A	2

Example 3 Calculating Grade Point Average (GPA)

The following grades were earned by a student during one semester. Find the student's grade point average.

Course	Grade	Credit Hours
College mathematics	A	3
Biology	B	3
English	A	3
PE	C	1
Social studies	D	2

Solution: To calculate the grade point average, we need to know the point values for the different possible grades. The point values of grades commonly used in colleges and universities are given below:

A: 4, B: 3, C: 2, D: 1, F: 0

Now, to find the grade point average, we multiply the number of credit hours for each course by the point value of each grade. The grade point average is the sum of these products divided by the sum of the credit hours.

Course	Grade	Point Value of Grade	Credit Hours	Point Value of Credit Hours
College mathematics	A	4	3	12
Biology	B	3	3	9
English	A	4	3	12
PE	C	2	1	2
Social studies	D	1	2	2
		Totals:	12	37

Answer

3. 2.67

$$\text{grade point average} = \frac{37}{12} \approx 3.08 \text{ rounded to two decimal places}$$

The student earned a grade point average of 3.08.

Work Practice 3

Objective B Finding the Median

You may have noticed that a very low number or a very high number can affect the mean of a list of numbers. Because of this, you may sometimes want to use another measure of central tendency. A second measure of central tendency is called the **median.** The median of a list of numbers is not affected by a low or high number in the list.

> The **median** of a set of numbers in numerical order is the middle number. If the number of items is odd, the median is the middle number. If the number of items is even, the median is the mean of the two middle numbers.

Example 4 Find the median of the following list of numbers:

25, 54, 56, 57, 60, 71, 98

Solution: Because this list is in numerical order, the median is the middle number, 57.

Work Practice 4

Practice 4

Find the median of the list of numbers: 5, 11, 14, 23, 24, 35, 38, 41, 43

Example 5 Find the median of the following list of scores: 67, 91, 75, 86, 55, 91

Solution: First we list the scores in numerical order and then we find the middle number.

55, 67, 75, 86, 91, 91

Since there is an even number of scores, there are two middle numbers, 75 and 86. The median is the mean of the two middle numbers.

$$\text{median} = \frac{75 + 86}{2} = 80.5$$

The median is 80.5.

Work Practice 5

Helpful Hint Don't forget to write the numbers in order from smallest to largest before finding the median.

Practice 5

Find the median of the list of scores:
36, 91, 78, 65, 95, 95, 88, 71

Objective C Finding the Mode

The last common measure of central tendency is called the **mode.**

> The **mode** of a set of numbers is the number that occurs most often. (It is possible for a set of numbers to have more than one mode or to have no mode.)

Example 6 Find the mode of the list of numbers:

11, 14, 14, 16, 31, 56, 65, 77, 77, 78, 79

Solution: There are two numbers that occur the most often. They are 14 and 77. This list of numbers has two modes, 14 and 77.

Work Practice 6

Practice 6

Find the mode of the list of numbers:
14, 10, 10, 13, 15, 15, 15, 17, 18, 18, 20

Answers

4. 24 **5.** 83 **6.** 15

Practice 7

Find the median and the mode of the list of numbers:
26, 31, 15, 15, 26, 30, 16, 18, 15, 35

Example 7

Find the median and the mode of the following set of numbers. These numbers were high temperatures for 14 consecutive days in a city in Montana.

76, 80, 85, 86, 89, 87, 82, 77, 76, 79, 82, 89, 89, 92

Solution: First we write the numbers in numerical order.

76, 76, 77, 79, 80, 82, 82, 85, 86, 87, 89, 89, 89, 92

Since there is an even number of items, the median is the mean of the two middle numbers, 82 and 85.

$$\text{median} = \frac{82 + 85}{2} = 83.5$$

The mode is 89, since 89 occurs most often.

Work Practice 7

✓Concept Check True or false? Every set of numbers *must* have a mean, median, and mode. Explain your answer.

Helpful Hint

Don't forget that it is possible for a list of numbers to have no mode. For example, the list

2, 4, 5, 6, 8, 9

has no mode. There is no number or numbers that occur more often than the others.

Objective D Finding the Range of a Data Set and Reviewing Mean, Median, and Mode

In this objective, we study one way to describe the dispersion of a data set, and we review mean, median, and mode. What is dispersion? In statistics, **dispersion** is a way to describe the degree to which the data values are scattered.

Range

The range of a data set is the difference between the largest data value and the smallest data value.

$$\text{range} = \text{largest data value} - \text{smallest data value}$$

Practice 8

The table lists the rounded salary of 10 staff members. Find the range.

Staff	*Salary (in thousands)*
A	\$32
B	\$34
C	\$46
D	\$38
E	\$42
F	\$95
G	\$102
H	\$50
J	\$42

Example 8

The following pulse rates (for 1 minute) were recorded for a group of 15 students. Find the range.

78, 80, 66, 68, 71, 64, 82, 71, 70, 65, 70, 75, 77, 86, 72.

Solution:
$$\begin{aligned}\text{range} &= \text{largest data value} - \text{smallest data value}\\ &= 86 - 64\\ &= 22\end{aligned}$$

The range of this data set is 22.

Work Practice 8

Answers

7. median: 22; mode: 15 **8.** \$70 thousand

✓Concept Check Answer

false; a set of numbers may have no mode

Let's recall a few facts about the median, and then we will introduce a formula for finding the *position* of the median.

- The **median** of a set of numbers in numerical order is the middle number.
- If the number of items is odd, the median is the middle number.
- If the number of items is even, the median is the *mean* (average) of the two middle numbers.

For a long list of data items, this formula gives us the **position** of the median.

Position of the Median

For n data items in order from smallest to largest, the median is the item in the

$$\frac{n+1}{2} \text{ position.}$$

Note:

If n is an even number, then the position formula, $\frac{n+1}{2}$, will not be a whole number.

In this case, simply find the average of the two data items whose positions are closest to, but before and after $\frac{n+1}{2}$.

Helpful Hint

The formula above, $\frac{n+1}{2}$ does not give the *value* of the median, just the **position of the median**.

Example 9

Find the (a) range, (b) mean, (c) median, and (d) mode from this frequency distribution table of retirement ages. If needed, round answers to 1 decimal place.

Age	*Frequency*
60	3
61	1
62	1
63	2
64	2
65	2

Solution: Study the table for a moment. From the frequency column, we see that there are 11 data items $(3 + 1 + 1 + 2 + 2 + 2)$.

a. range = largest data value − smallest data value

$$= 65 - 60$$
$$= 5$$

The range of this data set is 5.

(Continued on next page)

Practice 9

One state with a young retirement age is Michigan. The table below is from a poll of retirement ages from that state.

Age	*Frequency*
50	1
59	3
60	3
62	5
63	2
67	1

Find the (a) range, (b) mean, (c) median, and (d) mode of these data. If needed, round answers to 1 decimal place.

Answers

9. a. range: 17 **b.** mean: 60.7, **c.** median: 62, **d.** mode: 62

Helpful Hint Since there are three 60’s, for example, we can either use:

60 + 60 + 60 or 3 · 60.

b. To find the mean, we use our mean formula:

$$\text{mean} = \frac{\text{sum of items}}{\text{number of items}} = \frac{\overbrace{3 \cdot 60} + 61 + 62 + 2 \cdot 63 + 2 \cdot 64 + 2 \cdot 65}{11}$$

$$= \frac{687}{11} \approx 62.5$$

The mean of the data set is approximately 62.5.

c. Since there are 11 data items and the items are arranged in numerical order in the table, we find $\frac{n+1}{2}$ to locate the middle item. This is $\frac{11+1}{2} = \frac{12}{2} = 6$, or the sixth item.

The median is the sixth number, or 63.

d. The mode has the greatest frequency, so the mode is 60.

Work Practice 9

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

mean	mode	grade point average
median	range	average

1. Another word for “mean” is ________________.

2. The number that occurs most often in a set of numbers is called the ________________.

3. The ________________ of a set of number items is $\frac{\text{sum of items}}{\text{number of items}}$.

4. The ________________ of a set of numbers is the middle number. If the number of numbers is even, it is the ________________ of the two middle numbers.

5. An example of weighted mean is a calculation of ________________.

6. ________________ = greatest data value − smallest data value

7. The formula $\frac{n+1}{2}$ can be used to find the position of the ________________.

Martin-Gay Interactive Videos

See Video 7.3

Watch the section lecture video and answer the following questions.

Objective A **8.** Why is the ≈ symbol used in 🎞 Example 1? ▶

Objective B **9.** From 🎞 Example 3, what is always the first step when finding the median of a set of data numbers? ▶

Objective C **10.** From 🎞 Example 4, why do you think it is helpful to have data numbers in numerical order when finding the mode? ▶

Objective D **11.** Based on the results of 🎞 Example 5, give an example of a data set of four data items whose range is 0. ▶

7.3 Exercise Set MyLab Math

Objectives A B C Mixed Practice *For each set of numbers, find the mean, median, and mode. If necessary, round the mean to one decimal place. See Examples 1, 2, and 4 through 6.*

1. 15, 23, 24, 18, 25

2. 45, 36, 28, 46, 52

3. 7.6, 8.2, 8.2, 9.6, 5.7, 9.1

4. 4.9, 7.1, 6.8, 6.8, 5.3, 4.9

5. 0.5, 0.2, 0.2, 0.6, 0.3, 1.3, 0.8, 0.1, 0.5

6. 0.6, 0.6, 0.8, 0.4, 0.5, 0.3, 0.7, 0.8, 0.1

7. 231, 543, 601, 293, 588, 109, 334, 268

8. 451, 356, 478, 776, 892, 500, 467, 780

The 10 tallest buildings in the world, completed as of 2017, are listed in the following table. Use this table to answer Exercises 9 through 14. If necessary, round results to one decimal place. See Examples 1, 2, and 4 through 6.

Building	Height in Feet
Burj Khalifa, Dubai	2717
Shanghai Tower, Shanghai	2073
Makkah Royal Clock Tower, Mecca	1972
Ping An Finance Center	1965
Lotte World Tower	1819
One World Trade Center, New York City	1776
Guangzhou CTF Finance Center, Guangzhou	1739
Taipei 101, Taipei	1667
Shanghai World Financial Center, Shanghai	1614
International Commerce Center, Hong Kong	1588

(*Source:* Council on Tall Buildings and Urban Habitat)

9. Find the mean height of the five tallest buildings.

10. Find the median height of the five tallest buildings.

11. Find the median height of the six tallest buildings.

12. Find the mean height of the six tallest buildings.

13. Given the building heights, explain how you know, without calculating, that the answer to Exercise **10** is greater than the answer to Exercise **11.**

14. Given the building heights, explain how you know, without calculating, that the answer to Exercise **12** is less than the answer to Exercise **9.**

For Exercises 15 through 18, the grades are given for a student for a particular semester. Find the grade point average. If necessary, round the grade point average to the nearest hundredth. See Example 3.

15.

Grade	Credit Hours
B	3
C	3
A	4
C	4

16.

Grade	Credit Hours
D	1
F	1
C	4
B	5

17.

Grade	Credit Hours
A	3
A	3
A	4
B	3
C	1

18.

Grade	Credit Hours
B	2
B	2
C	3
A	3
B	3

During an experiment, the following times (in seconds) were recorded:

7.8, 6.9, 7.5, 4.7, 6.9, 7.0.

19. Find the mean.

20. Find the median.

21. Find the mode.

In a mathematics class, the following test scores were recorded for a student:

93, 85, 89, 79, 88, 91.

22. Find the mean.

23. Find the median.

24. Find the mode.

The following pulse rates were recorded for a group of 15 students:

78, 80, 66, 68, 71, 64, 82, 71, 70, 65, 70, 75, 77, 86, 72.

25. Find the mean.

26. Find the median.

27. Find the mode.

28. How many pulse rates were higher than the mean?

29. How many pulse rates were lower than the mean?

30. Explain how to find the position of the median.

Below are lengths for the six longest rivers in the world.

Name	*Length (miles)*
Nile	4160
Amazon	4000
Yangtze	3915
Mississippi-Missouri	3709
Ob-Irtysh	3459
Huang Ho	3395

Find the mean and the median for each of the following.

31. the six longest rivers

32. the three longest rivers

Objective D *Find the range for each data set. See Example 8.*

33. 14, 16, 8, 10, 20

34. 25, 15, 11, 40, 37

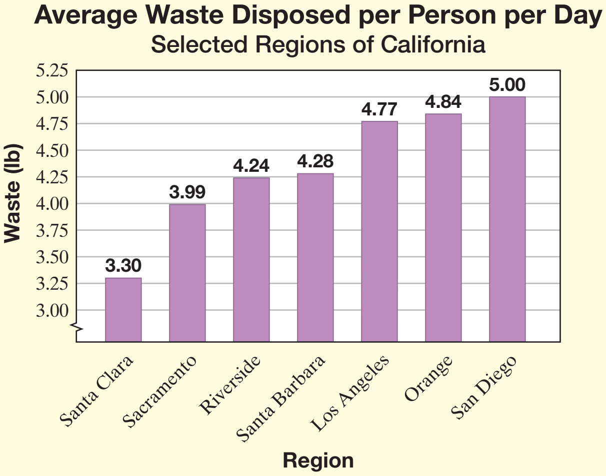
35. 206, 206, 555, 556

36. 129, 188, 188, 276

37. 9, 9, 9, 9, 11

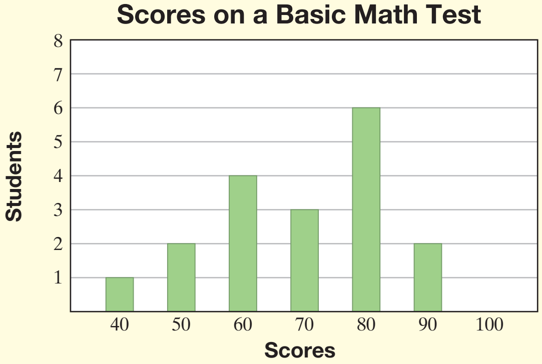
38. 7, 7, 7, 7, 10

39.



Sources: Equinox Center, 2013; Calrecycle, 2013

40.



Use each frequency distribution table to find the **a.** *mean,* **b.** *median, and* **c.** *mode. If needed, round the mean to 1 decimal place. See Example 9.*

41.

Data Item	*Frequency*
5	1
6	1
7	2
8	5
9	6
10	2

42.

Data Item	*Frequency*
3	2
4	1
5	4
6	7
7	2
8	1

43.

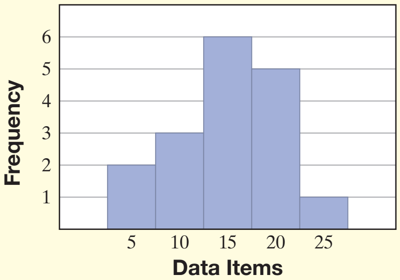
Data Item	*Frequency*
2	5
3	7
4	4
5	7
6	8
7	8
8	8
9	6
10	5

44.

Data Item	*Frequency*
4	3
5	8
6	5
7	8
8	2

Use each graph of data items to find the **a.** *mean,* **b.** *median, and* **c.** *mode. If needed round the mean to one decimal place.*

45.



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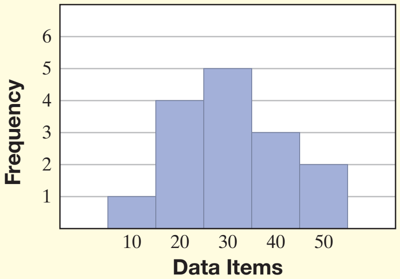

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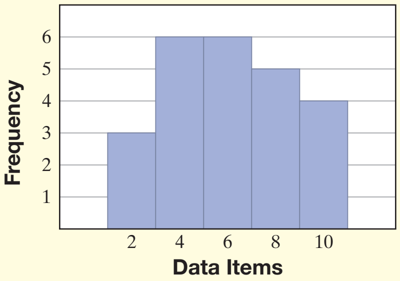
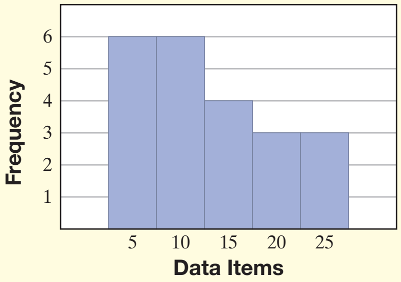


48.



Review

Write each fraction in simplest form. See Section 3.2.

49. $\frac{12}{20}$

50. $\frac{6}{18}$

51. $\frac{4}{36}$

52. $\frac{18}{30}$

53. $\frac{35}{100}$

54. $\frac{55}{75}$

Concept Extensions

Find the missing numbers in each set of numbers.

55. _____, _____, _____, 40, _____. The mode is 35. The median is 37. The mean is 38.

56. 16, 18, _____, _____, _____. The mode is 21. The median is 20.

57. Without making any computations, decide whether the median of the following list of numbers will be a whole number. Explain your reasoning.

36, 77, 29, 58, 43

58. Write a list of numbers for which you feel the median would be a better measure of central tendency than the mean.

7.4 Counting and Introduction to Probability

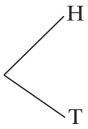
Objective A Using a Tree Diagram

In our daily conversations, we often talk about the likelihood or **probability** of a given result occurring. For example:

> The *chance* of thundershowers is 70 percent.
> What are the *odds* that the New Orleans Saints will go to the Super Bowl?
> What is the *probability* that you will finish cleaning your room today?

Each of these chance happenings—thundershowers, the New Orleans Saints playing in the Super Bowl, and finishing cleaning your room today—is called an **experiment.** The possible results of an experiment are called **outcomes.** For example, flipping a coin is an experiment, and the possible outcomes are heads (H) or tails (T).

One way to picture the outcomes of an experiment is to draw a **tree diagram.** Each outcome is shown on a separate branch. For example, the outcomes of flipping a coin are

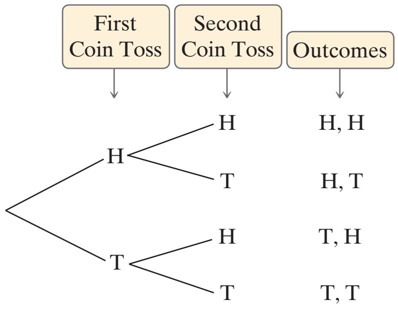



Heads


Tails

Example 1 Draw a tree diagram for tossing a coin twice. Then use the diagram to find the number of possible outcomes.

Solution:



There are 4 possible outcomes when tossing a coin twice.

Work Practice 1

Example 2 Draw a tree diagram for an experiment consisting of rolling a die and then tossing a coin. Then use the diagram to find the number of possible outcomes.


Die

(Continued on next page)

Objectives

A Use a Tree Diagram to Count Outcomes.

B Find the Probability of an Event.

Practice 1

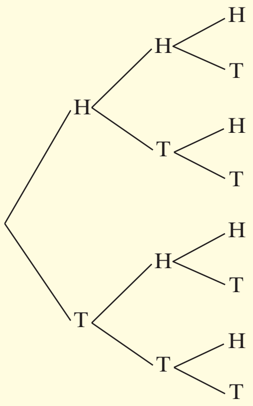
Draw a tree diagram for tossing a coin three times. Then use the diagram to find the number of possible outcomes.

Practice 2

Draw a tree diagram for an experiment consisting of tossing a coin and then rolling a die. Then use the diagram to find the number of possible outcomes. (Answer appears on the next page.)

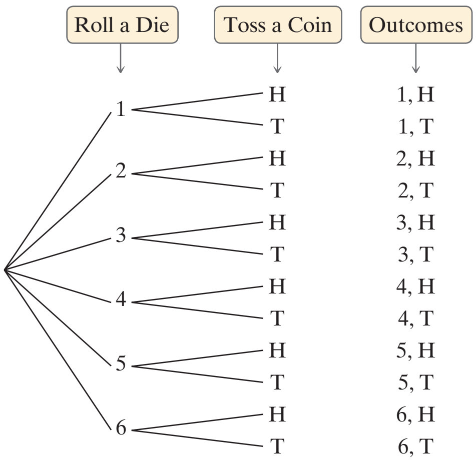
Answer

1.



8 outcomes

Solution: Recall that a die has six sides and that each side represents a number, 1 through 6.



There are 12 possible outcomes for rolling a die and then tossing a coin.

Work Practice 2

Any number of outcomes considered together is called an **event.** For example, when tossing a coin twice, H, H is an event. The event is tossing heads first and tossing heads second. Another event would be tossing tails first and then heads (T, H), and so on.

Objective B Finding the Probability of an Event

As we mentioned earlier, the **probability of an event is a measure of the chance or likelihood of it occurring.** For example, if a coin is tossed, what is the probability that heads occurs? Since one of two equally likely possible outcomes is heads, the probability is $\frac{1}{2}$.

The Probability of an Event

$$\text{probability of an event} = \frac{\text{number of ways that the event can occur}}{\text{number of possible outcomes}}$$

Note from the definition of probability that the probability of an event is always between 0 and 1, inclusive (i.e., including 0 and 1). A probability of 0 means that an event won't occur, and a probability of 1 means that an event is certain to occur.

Example 3 If a coin is tossed twice, find the probability of tossing heads on the first toss and then heads again on the second toss (H, H).

Solution:

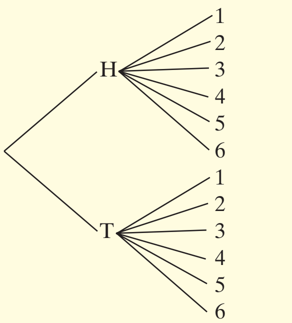
$$\underbrace{\text{H,T,}\quad \overset{\text{1 way the event can occur}}{\overset{\downarrow}{\overbrace{\text{H,H,}}}}\quad \text{T,H,}\quad \text{T,T}}_{\text{4 possible outcomes}}$$

Practice 3

If a coin is tossed three times, find the probability of tossing tails, then heads, then tails (T, H, T).

Answers

2.



12 outcomes

3. $\frac{1}{8}$

$$\text{probability} = \frac{1}{4} \quad \begin{matrix}\text{Number of ways the event can occur}\\ \text{Number of possible outcomes}\end{matrix}$$

The probability of tossing heads and then heads is $\frac{1}{4}$.

Work Practice 3

Example 4 If a die is rolled one time, find the probability of rolling a 3 or a 4.

Solution: Recall that there are 6 possible outcomes when rolling a die.

$$\text{possible outcomes: } \underbrace{1,\ 2,\ \overset{\swarrow}{3},\ \overset{\searrow}{4},\ 5,\ 6}_{6\text{ possible outcomes}} \quad \text{2 ways that the event can occur}$$

$$\text{probability of a 3 or a 4} = \frac{2}{6} \quad \begin{matrix}\text{Number of ways the event can occur}\\ \text{Number of possible outcomes}\end{matrix}$$

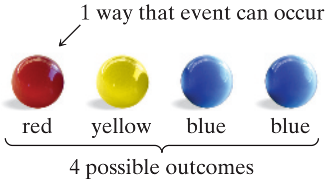
$$= \frac{1}{3} \quad \text{Simplest form}$$

Work Practice 4

✓**Concept Check** Suppose you have calculated a probability of $\frac{11}{9}$. How do you know that you have made an error in your calculation?

Example 5 Find the probability of choosing a red marble from a box containing 1 red, 1 yellow, and 2 blue marbles.

Solution:



$$\text{probability} = \frac{1}{4}$$

Work Practice 5

Practice 4

If a die is rolled one time, find the probability of rolling a 2 or a 5.

Practice 5

Use the diagram and information in Example 5 and find the probability of choosing a blue marble from the box.

Answers

4. $\frac{1}{3}$ **5.** $\frac{1}{2}$

✓**Concept Check Answer**

The number of ways an event can occur can't be larger than the number of possible outcomes.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Choices may be used more than once.

0	probability	tree diagram
1	outcome	

1. A possible result of an experiment is called a(n) ____________.

2. A(n) ____________ shows each outcome of an experiment as a separate branch.

3. The ____________ of an event is a measure of the likelihood of it occurring.

4. __________ is calculated by the number of ways that an event can occur divided by the number of possible outcomes.

5. A probability of __________ means that an event won't occur.

6. A probability of __________ means that an event is certain to occur.

Martin-Gay Interactive Videos

See Video 7.4

Watch the section lecture video and answer the following questions.

Objective A **7.** In Example 1, how was the possible number of outcomes for the experiment determined from the tree diagram?

Objective B **8.** In Example 2, what is the probability of getting a 7? Explain this result.

7.4 Exercise Set MyLab Math

Objective A *Draw a tree diagram for each experiment. Then use the diagram to find the number of possible outcomes. See Examples 1 and 2.*

1. Choosing a letter in the word MATH and then a number (1, 2, or 3)

2. Choosing a number (1 or 2) and then a vowel (a, e, i, o, or u)

Spinner A

Spinner B

3. Spinning Spinner A once

4. Spinning Spinner B once

5. Spinning Spinner B twice

6. Spinning Spinner A twice

7. Spinning Spinner A and then Spinner B

8. Spinning Spinner B and then Spinner A

9. Tossing a coin and then spinning Spinner B

10. Spinning Spinner A and then tossing a coin

Objective B *If a single die is tossed once, find the probability of each event. See Examples 3 through 5.*

11. A 5

12. A 9

13. A 1 or a 6

14. A 2 or a 3

15. An even number

16. An odd number

17. A number greater than 2

18. A number less than 6

Suppose the spinner shown is spun once. Find the probability of each event. See Examples 3 through 5.

19. The result of the spin is 2.

20. The result of the spin is 3.

21. The result of the spin is 1, 2, or 3.

22. The result of the spin is not 3.

23. The result of the spin is an odd number.

24. The result of the spin is an even number.

If a single choice is made from the bag of marbles shown, find the probability of each event. See Examples 3 through 5.

25. A red marble is chosen.

26. A blue marble is chosen.

27. A yellow marble is chosen.

28. A green marble is chosen.

29. A green or red marble is chosen.

30. A blue or yellow marble is chosen.

A new drug is being tested that is supposed to lower blood pressure. This drug was given to 200 people, and the results are shown below.

Lower Blood Pressure	Higher Blood Pressure	Blood Pressure Not Changed
152	38	10

31. If a person is testing this drug, what is the probability that his or her blood pressure will be higher?

32. If a person is testing this drug, what is the probability that his or her blood pressure will be lower?

33. If a person is testing this drug, what is the probability that his or her blood pressure will not change?

34. What is the sum of the answers to Exercises **31**, **32**, and **33**? In your own words, explain why.

Review

Perform each indicated operation. See Sections 3.3 and 3.5.

35. $\frac{1}{2}+\frac{1}{3}$

36. $\frac{7}{10}-\frac{2}{5}$

37. $\frac{1}{2}\cdot\frac{1}{3}$

38. $\frac{7}{10}\div\frac{2}{5}$

39. $5\div\frac{3}{4}$

40. $\frac{3}{5}\cdot 10$

Concept Extensions

Recall that a deck of cards contains 52 cards. These cards consist of four suits (hearts, spades, clubs, and diamonds) of each of the following: 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, and ace. If a card is chosen from a deck of cards, find the probability of each event.

41. The king of hearts

42. The 10 of spades

43. A king

44. A 10

45. A heart

46. A club

47. A card in black ink

48. A queen or ace

Two dice are tossed. Find the probability of each sum of the dice. (Hint: Draw a tree diagram of the possibilities of two tosses of a die, and then find the sum of the numbers on each branch.)

49. A sum of 6

50. A sum of 10

51. A sum of 13

52. A sum of 2

Solve. See the Concept Check in this section.

53. In your own words, explain why the probability of an event cannot be greater than 1.

54. In your own words, explain when the probability of an event is 0.

Chapter 7 Group Activity

Sections 7.1, 7.2, 7.3

This activity may be completed by working in groups or individually.

How often have you read an article in a newspaper or in a magazine that included results from a survey or poll? Surveys seem to have become very popular ways of getting feedback on anything from a political candidate, to a new product, to services offered by a health club. In this activity, you will conduct a survey and analyze the results.

1. Conduct a survey of 30 students in one of your classes. Ask each student to report his or her age.
2. Classify each age according to the following categories: under 20, 20 to 24, 25 to 29, 30 to 39, 40 to 49, and 50 or over. Tally the number of your survey respondents that fall into each category. Make a histogram of your results. What does this graph tell you about the ages of your survey respondents?
3. Find the average age of your survey respondents.
4. Find the median age of your survey respondents.
5. Find the mode of the ages of your survey respondents.
6. Compare the mean, median, and mode of your age data. Are these measures similar? Which is largest? Which is smallest? If there is a noticeable difference between any of these measures, can you explain why?
7. Conduct another survey with at least 30 people. Follow the directions of Exercises **49** and **50** of Section 7.2.

Chapter 7 Vocabulary Check

Fill in each blank with one of the words or phrases listed below. Some choices may be used more than once.

outcomes	bar	experiment	mean	tree diagram
pictograph	line	class interval	median	probability
histogram	circle	class frequency	range	mode

1. A(n) ______________ graph presents data using vertical or horizontal bars.

2. The ______________ of a set of number items is $\frac{\text{sum of items}}{\text{number of items}}$.

3. The possible results of an experiment are the ______________.

4. A(n) ______________ is a graph in which pictures or symbols are used to visually present data.

5. The ______________ of a set of numbers is the number that occurs most often.

6. A(n) ______________ graph displays information with a line that connects data points.

7. The ______________ of an ordered set of numbers is the middle number.

8. A(n) ______________ is one way to picture and count outcomes.

9. A(n) ______________ is an activity being considered, such as tossing a coin or rolling a die.

10. In a(n) ______________ graph, each section (shaped like a piece of pie) shows a category and the relative size of the category.

11. The ______________ of an event is $\frac{\text{number of ways that the event can occur}}{\text{number of possible outcomes}}$.

12. A(n) ______________ is a special bar graph in which the width of each bar represents a(n) ______________ and the height of each bar represents the ______________.

13. ______________ = greatest data value − smallest data value

14. The formula $\frac{n+1}{2}$ can be used to find the position of the ______________.

Helpful Hint

Are you preparing for your test? To help, don't forget to take these:

- Chapter 7 Getting Ready for the Test on page 571
- Chapter 7 Test on page 573

Then check all of your answers at the back of the text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

7 Chapter Highlights

Definitions and Concepts | **Examples**

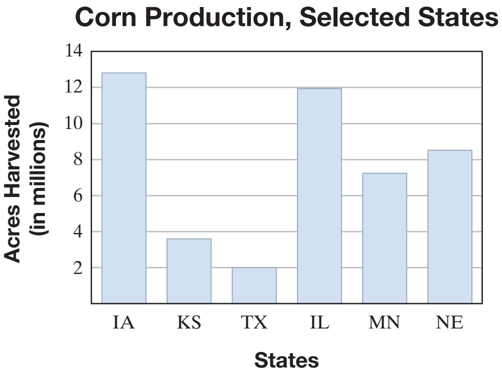
Section 7.1 Pictographs, Bar Graphs, Histograms, and Line Graphs

A **pictograph** is a graph in which pictures or symbols are used to visually present data.

A **line graph** displays information with a line that connects data points.

A **bar graph** presents data using vertical or horizontal bars.

The bar graph on the right shows the number of acres of corn harvested in a recent year for selected states.



Source: U.S. Department of Agriculture

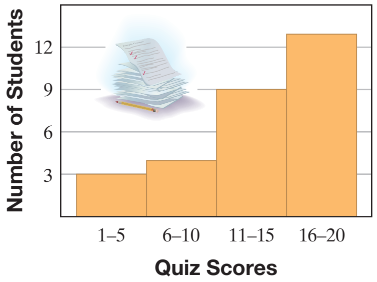
1. Approximately how many acres of corn were harvested in Iowa?

   12,800,000 acres

2. About how many more acres of corn were harvested in Illinois than Nebraska?

$$\begin{array}{r} 12\ \ \ \text{million} \\ -\ 8.5\ \text{million} \\ \hline 3.5\ \text{million} \end{array} \quad \text{or } 3{,}500{,}000 \text{ acres}$$

A **histogram** is a special bar graph in which the width of each bar represents a **class interval** and the height of each bar represents the **class frequency.** The histogram on the right shows student quiz scores.

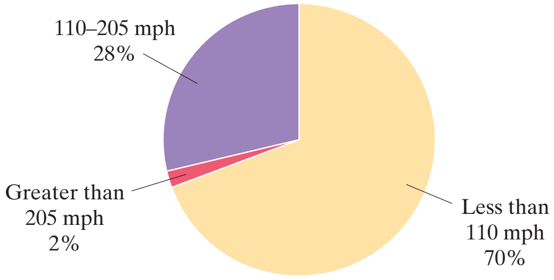


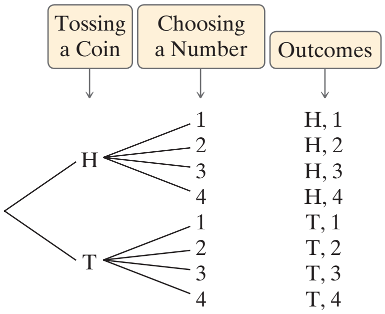
1. How many students received a score of 6–10?

   4 students

2. How many students received a score of 11–20?

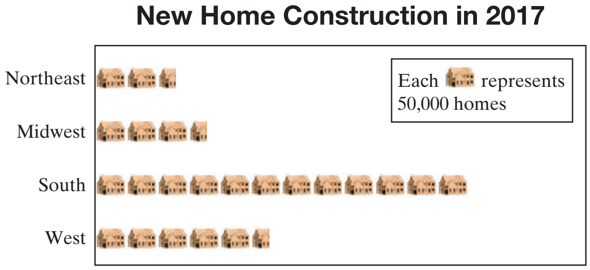
   9 + 13 = 22 students

Definitions and Concepts	Examples
Section 7.2 Circle Graphs	
In a **circle graph,** each section (shaped like a piece of pie) shows a category and the relative size of the category. The circle graph on the right classifies tornadoes by wind speed.	**Tornado Wind Speeds**  *Source:* National Oceanic and Atmospheric Administration **1.** What percent of tornadoes have wind speeds of 110 mph or greater? $28\% + 2\% = 30\%$ **2.** If there were 1059 tornadoes in the United States in 2016, how many of these might we expect to have had wind speeds less than 110 mph? Find 70% of 1059. $70\%(1059) = 0.70(1059) = 741.3 \approx 741$ Around 741 tornadoes would be expected to have had wind speeds of less than 110 mph.
Section 7.3 Mean, Median, Mode, and Range	
The **mean** (or **average**) of a set of number items is $\text{mean} = \dfrac{\text{sum of items}}{\text{number of items}}$	Find the mean, median, and mode of the following set of numbers: 33, 35, 35, 43, 68, 68 $\text{mean} = \dfrac{33 + 35 + 35 + 43 + 68 + 68}{6} = 47$
The **median** of a set of numbers in numerical order is the middle number. If the number of items is even, the median is the mean of the two middle numbers.	The median is the mean of the two middle numbers, 35 and 43 $\text{median} = \dfrac{35 + 43}{2} = 39$
The **mode** of a set of numbers is the number that occurs most often. (A set of numbers may have no mode or more than one mode.)	There are two modes because there are two numbers that occur twice: 35 and 68
Range The range of a data set is the difference between the largest data value and the smallest data value. $\text{range} = \text{largest data value} - \text{smallest data value}$	The range of data set 5, 7, 9, 11, 21, 21 is: $\text{range} = 21 - 5 = 16$
Position of the Median For n data items in order from smallest to largest, the median is the item in the $\dfrac{n + 1}{2}$ position.	If n is odd: For $n = 71$ data items (in order from smallest to largest), the median is the item in the $\dfrac{71 + 1}{2} = 36^{\text{th}}$ position.
Note: If n is an even number, then the position formula, $\dfrac{n + 1}{2}$, will not be a whole number. In this case, simply find the average of the two data items whose positions are closest to, but before and after $\dfrac{n + 1}{2}$.	If n is even: For $n = 56$ data items (in order from smallest to largest), the position formula gives us $\dfrac{56 + 1}{2} = 28.5$. Thus, the median is the average of the 28^{th} and 29^{th} data items.

Definitions and Concepts	Examples
Section 7.4 Counting and Introduction to Probability	
An **experiment** is an activity being considered, such as tossing a coin or rolling a die. The possible results of an experiment are the **outcomes.** A **tree diagram** is one way to picture and count outcomes.	Draw a tree diagram for tossing a coin and then choosing a number from 1 to 4. 
Any number of outcomes considered together is called an **event.** The **probability** of an event is a measure of the chance or likelihood of it occurring. $\text{probability of an event} = \dfrac{\text{number of ways that the event can occur}}{\text{number of possible outcomes}}$	Find the probability of tossing a coin twice and tails occurring each time. $\underbrace{(\text{H}, \text{H}), (\text{H}, \text{T}), (\text{T}, \text{H}), \overset{\text{1 way the event can occur}}{(\text{T}, \text{T})}}_{\text{4 possible outcomes}}$ $\text{probability} = \dfrac{1}{4}$

Chapter 7 Review

(7.1) *The following pictograph shows the number of new homes whose construction was completed in 2017 by region. Use this graph to answer Exercises 1 through 6.*



Source: U.S. Census Bureau

1. How many new homes were constructed in the Midwest during 2017?

2. How many new homes were constructed in the South during 2017?

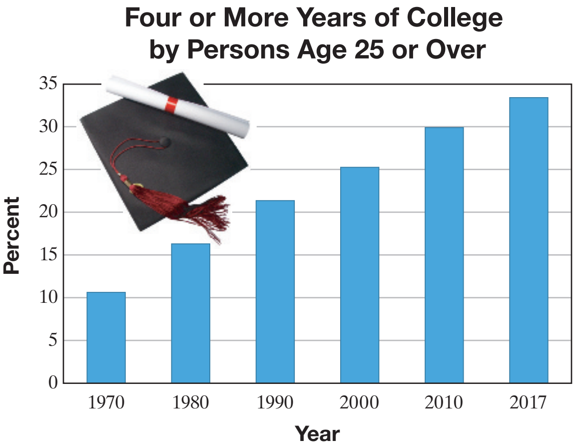
3. Which region had the most new homes constructed?

4. Which region had the fewest new homes constructed?

5. Which region(s) had 250,000 or more new homes constructed?

6. Which region(s) had fewer than 200,000 new homes constructed?

The following bar graph shows the percent of persons age 25 or over who completed four or more years of college. Use this graph to answer Exercises 7 through 10.



Source: U.S. Census Bureau

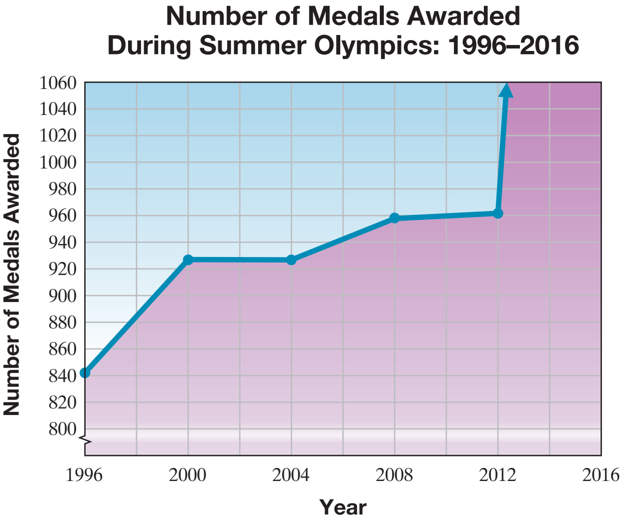
7. Approximate the percent of persons who had completed four or more years of college by 2010.

8. What year shown had the greatest percent of persons completing four or more years of college?

9. What years shown had 20% or more of persons completing four or more years of college?

10. Describe any patterns you notice in this graph.

The following line graph shows the total number of Olympic medals awarded during the Summer Olympics since 1996. Use this graph to answer Exercises 11 through 16.



Source: International Olympic Committee

11. Approximate the number of medals awarded during the Summer Olympics of 2012.

12. Approximate the number of medals awarded during the Summer Olympics of 2000.

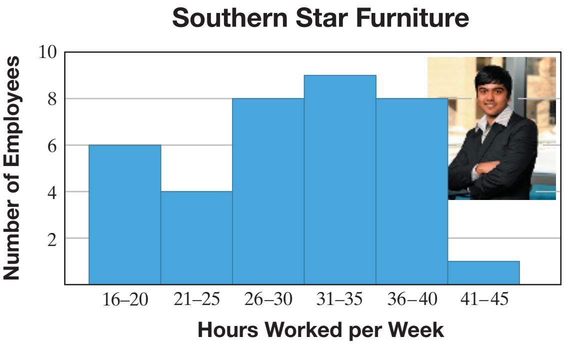
13. Approximate the number of medals awarded during the Summer Olympics of 2008.

14. Approximate the number of medals awarded during the Summer Olympics of 1996.

15. How many more medals were awarded at the Summer Olympics of 2008 than at the Summer Olympics of 2004?

16. The number of medals awarded at the Summer Olympics of 2016 was 2102. This was more than twice the number of medals awarded at previous Summer Olympics. Why do you think this is so? Give your explanation in complete sentences.

The following histogram shows the hours worked per week by the employees of Southern Star Furniture. Use this histogram to answer Exercises 17 through 20.



17. How many employees work 41–45 hours per week?

18. How many employees work 21–25 hours per week?

19. How many employees work 30 hours or less per week?

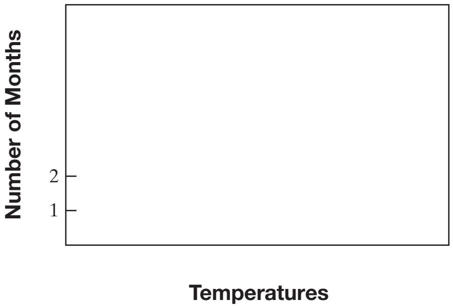
20. How many employees work 36 hours or more per week?

Following is a list of monthly record high temperatures for New Orleans, Louisiana. Use this list to complete the frequency distribution table below.

83	96	101	92
85	100	92	102
89	101	87	84

	Class Intervals (Temperatures)	Tally	Class Frequency (Number of Months)
21.	80°–89°		
22.	90°–99°		
23.	100°–109°		

24. Use the table from Exercises **21** through **23** to draw a histogram.



(7.2) *The following circle graph shows a family's $4000 monthly budget. Use this graph to answer Exercises 25 through 30. Write all ratios as fractions in simplest form.*

25. What is the largest budget item?

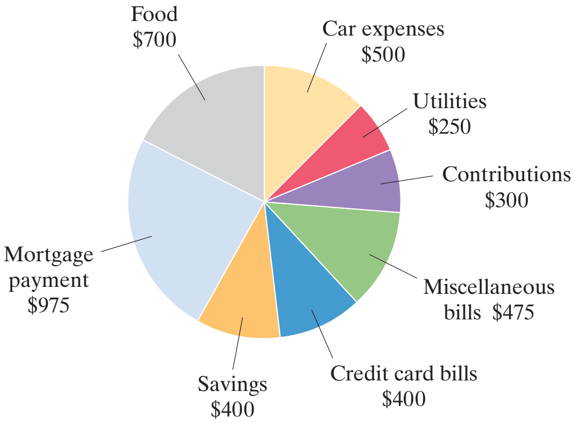
26. What is the smallest budget item?

27. How much money is budgeted for the mortgage payment and utilities?

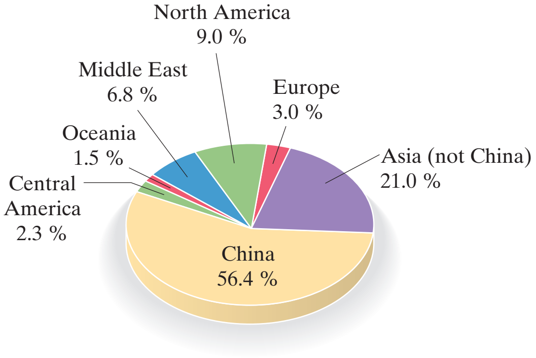
28. How much money is budgeted for savings and contributions?

29. Find the ratio of the mortage payment to the total monthly budget.

30. Find the ratio of food to the total monthly budget.



In 2017, there were 133 buildings 200 meters or taller completed in the world. The following circle graph shows the percent of these buildings by region. Use this graph to answer Exercises 31 through 34. Round each answer to the nearest whole.


Percent of Tall Buildings Completed in 2017 200 Meters or Tall by Region

(*Source:* Council on Tall Buildings and Urban Habitat)

31. How many completed tall buildings were located in China?

32. How many completed tall buildings were located in the rest of Asia?

33. How many completed tall buildings were located in Oceania?

34. How many completed tall buildings were located in the Middle East?

(7.3) *Find the mean, median, and any mode(s) for each list of numbers. If necessary, round to the nearest tenth.*

35. 13, 23, 33, 14, 6

36. 45, 86, 21, 60, 86, 64, 45

37. 14,000, 20,000, 12,000, 20,000, 36,000, 45,000

38. 560, 620, 123, 400, 410, 300, 400, 780, 430, 450

For Exercises 39 and 40, the grades are given for a student for a particular semester. Find each grade point average. If necessary, round the grade point average to the nearest hundredth.

39.

Grade	Credit Hours
A	3
A	3
C	2
B	3
C	1

40.

Grade	Credit Hours
B	3
B	4
C	2
D	2
B	3

Find the range for each data set.

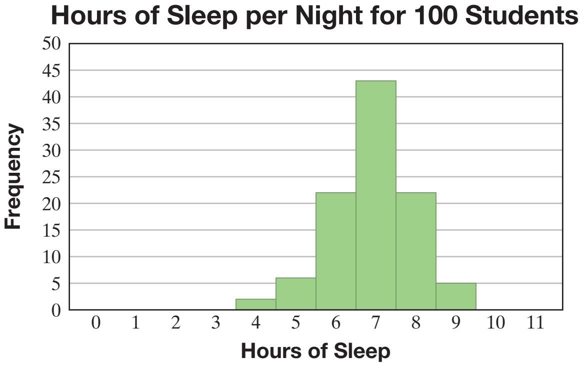
41. 4, 4, 4, 3, 3, 5, 6

42. 1, 1, 1, 8, 8

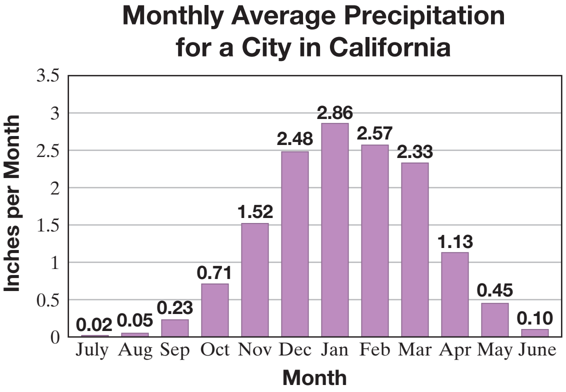
43. 70, 75, 95, 60, 88

44. 80, 87, 97, 99, 85

45.



46.



Use each frequency distribution table to find the ***a.*** *mean,* ***b.*** *median, and* ***c.*** *mode. If needed, round the mean to 1 decimal place.*

47.

Data Item	*Frequency*
60	2
61	10
62	5
63	11
64	3

48.

Data Item	*Frequency*
60	5
61	7
62	8
63	10
64	15

49.

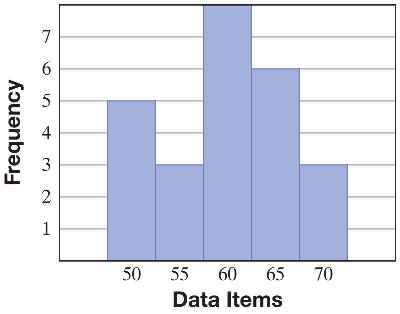
Data Item	*Frequency*
11	5
12	5
13	3
14	1
16	3
17	1

50.

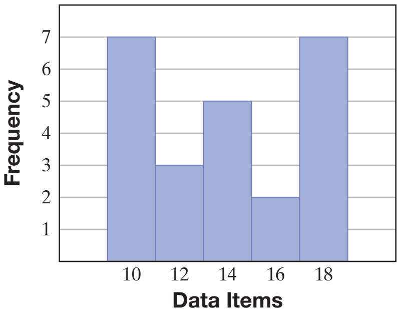
Data Item	*Frequency*
15	3
16	2
17	1
18	6
19	4
20	6

Use each graph of data items to find the **a.** *mean,* **b.** *median, and* **c.** *mode. If needed, round the mean to 1 decimal place.*

51.



52.



(7.4) *Draw a tree diagram for each experiment. Then use the diagram to determine the number of outcomes.*

Spinner 1 Spinner 2

53. Tossing a coin and then spinning Spinner 1

54. Spinning Spinner 2 and then tossing a coin

55. Spinning Spinner 1 twice

56. Spinning Spinner 2 twice

57. Spinning Spinner 1 and then Spinner 2

Find the probability of each event.


Die

58. Rolling a 4 on a die

59. Rolling a 3 on a die

60. Spinning a 4 on Spinner 1

61. Spinning a 3 on Spinner 1

62. Spinning either a 1, 3, or 5 on Spinner 1

63. Spinning either a 2 or a 4 on Spinner 1

64. Rolling an even number on a die

65. Rolling a number greater than 3 on a die

Mixed Review

Find the mean, median, and any mode(s) for each list of numbers. If needed, round answers to two decimal places.

66. 73, 82, 95, 68, 54

67. 25, 27, 32, 98, 62

68. 750, 500, 427, 322, 500, 225

69. 952, 327, 566, 814, 327, 729

Given a bag containing 2 red marbles, 2 blue marbles, 3 yellow marbles, and 1 green marble, find the following:

70. The probability of choosing a blue marble from the bag

71. The probability of choosing a yellow marble from the bag

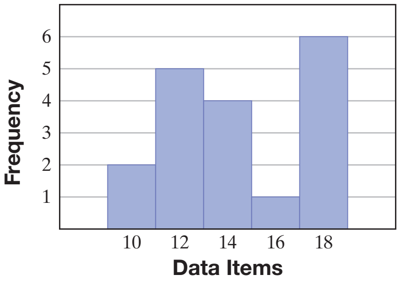
72. The probability of choosing a red marble from the bag

73. The probability of choosing a green marble from the bag

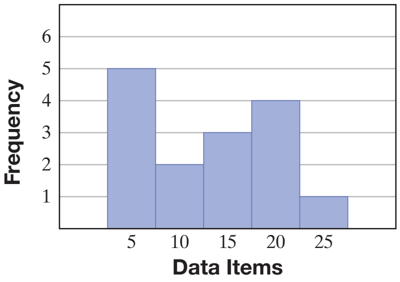
For each graph, calculate parts ***a.–d.*** *Round the mean to two decimal places, if necessary.*

a. mean **b.** median **c.** mode **d.** range

74.



75.

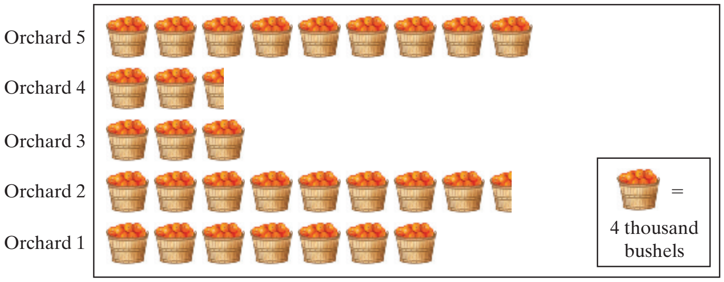


Chapter 7 Getting Ready for the Test

MULTIPLE CHOICE Exercises 1 through 13 are **Multiple Choice.** Choose the correct letter.

For Exercises 1 through 4, use the graph below.

Bushels of Oranges Picked



1. How many bushels of oranges did Orchard 1 produce?

A. 7 bushels **B.** 28 bushels **C.** 28,000 bushels

2. Which orchard above produced the most bushels?

A. Orchard 1 **B.** Orchard 2 **C.** Orchard 5 **D.** Orchards 2 and 5 produced the same.

3. How many bushels of oranges did Orchard 4 produce?

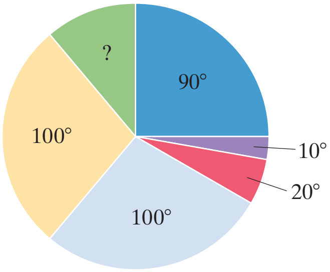
A. 3 bushels **B.** $2\frac{1}{2}$ bushels **C.** 10 bushels **D.** 10 thousand bushels

4. How many more bushels did Orchard 2 produce than Orchard 4?

A. 24 bushels **B.** $6\frac{1}{2}$ bushels **C.** 24,000 bushels **D.** 6000 bushels

For Exercises 5 through 7, choose the correct letter.

5. Choose the degrees in the unknown sector of this circle graph.



A. 60° **B.** 50° **C.** 40° **D.** 80°

6. The five colored marbles are placed in a bag. What is the probability of choosing a red marble?

A. $\frac{1}{5}$ **B.** $\frac{2}{5}$ **C.** $\frac{3}{5}$ **D.** $\frac{4}{5}$

7. For the marbles in Exercise **6**, What is the probability of choosing a green marble?

A. 0 **B.** 1 **C.** $\frac{2}{5}$ **D.** $\frac{3}{5}$

For Exercises 8 through 10, choose the correct directions that lead to the given correct answer for the data set: 7, 9, 10, 13, 13

A. Find the mean.

B. Find the median.

C. Find the mode.

8. answer: 10

9. answer: 13

10. answer: 10.4

For Exercises 11 through 13, use the data sets and choices below to answer.

A. data set: 10, 10, 10, 10, 10

B. data set: 6, 8, 10, 12, 14

C. data set: 8, 9, 10, 11, 12

D. they are the same

11. Which data set has the greatest range?

12. Which data set has the greatest median?

13. Which data set has the greatest mean?

MATCHING Exercises 14 through 17 are **Matching** exercises.

*Choose **two** data sets (two letters) from the right column that make each statement in the left column true. Data sets may be used more than once or not at all.*

14. equal means

15. equal number of modes

16. equal ranges

17. equal medians

A. 10, 20, 30, 30, 40, 50

B. 9, 11, 11, 30, 48

C. 20, 25, 30, 35, 40

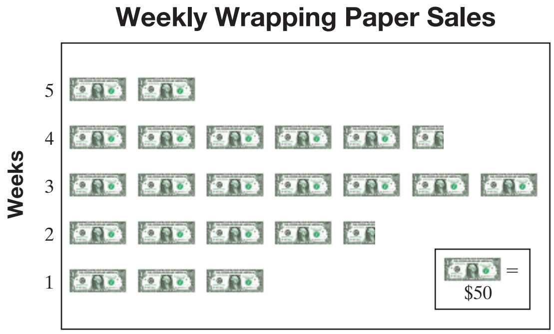
D. 50, 55, 60, 65, 70

For additional practice go to your study plan in MyLab Math.

MyLab Math

Test Chapter 7

The following pictograph shows the money collected each week from a wrapping paper fundraiser. Use this graph to answer Exercises 1 through 3.

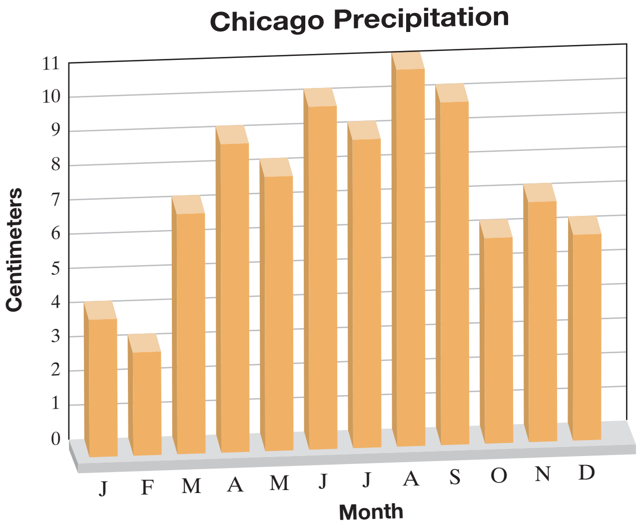


1. How much money was collected during the second week?

2. During which week was the most money collected? How much money was collected during that week?

3. What was the total money collected for the fundraiser?

The bar graph shows the normal monthly precipitation in centimeters for Chicago, Illinois. Use this graph to answer Exercises 4 through 6.



Source: U.S. National Oceanic and Atmospheric Administration, *Climatography of the United States*, No. 81

4. During which month(s) does Chicago normally have more than 9 centimeters of precipitation?

5. During which month does Chicago normally have the least amount of precipitation? How much precipitation occurs during that month?

Answers

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

11. ____________

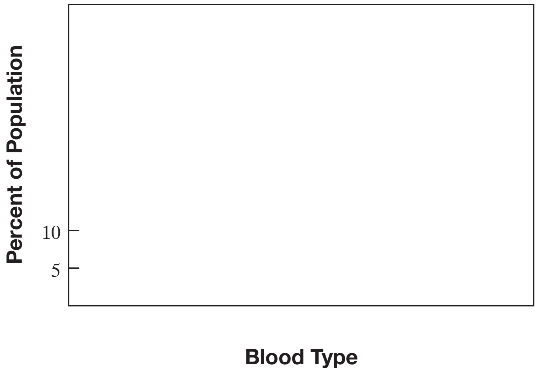
12. ____________

6. During which month(s) does 7 centimeters of precipitation normally occur?

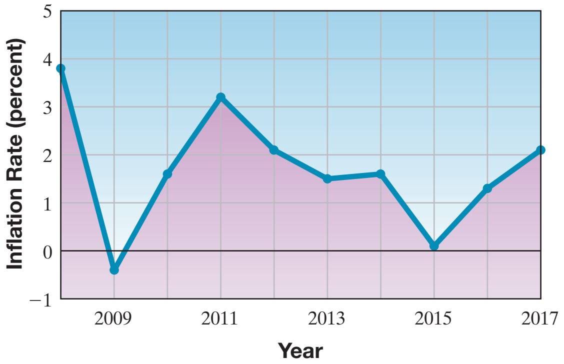
7. Use the information in the table to draw a bar graph. Clearly label each bar.

Most Common Blood Types

Blood Type	% of Population with This Blood Type
O+	38%
A+	34%
B+	9%
O−	7%
A−	6%
AB+	3%
B−	2%
AB−	1%


Most Common Blood Types by Percent in the Population

The following line graph shows the annual inflation rate in the United States for the years 2008–2017. Use this graph to answer Exercises 8 through 10.


U.S. Annual Inflation Rate

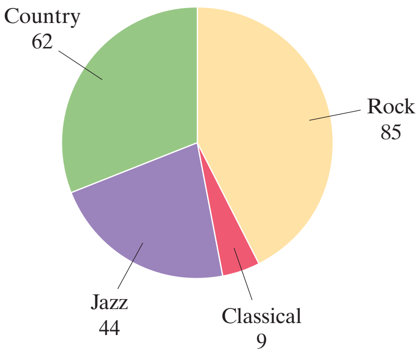
Source: Bureau of Labor Statistics

8. Approximate the annual inflation rate in 2014.

9. During which of the years shown was the inflation rate greater than 3%?

10. During which sets of years was the inflation rate decreasing?

The result of a survey of 200 people is shown in the following circle graph. Each person was asked to tell his or her favorite type of music. Use this graph to answer Exercises 11 and 12.

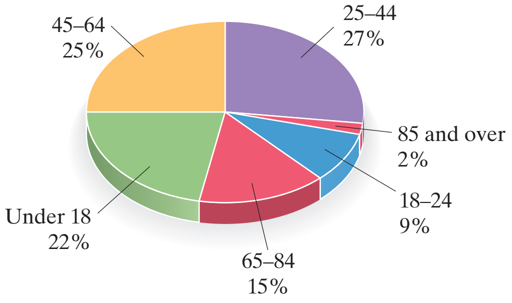


11. Find the ratio of those who prefer rock music to the total number surveyed.

12. Find the ratio of those who prefer country music to those who prefer jazz.

The following circle graph shows the projected age distribution of the population of the United States in 2020. There are projected to be 335 million people in the United States in 2020. Use the graph to find how many people are expected to be in the age groups given.

U.S. Population in 2020 by Age Groups



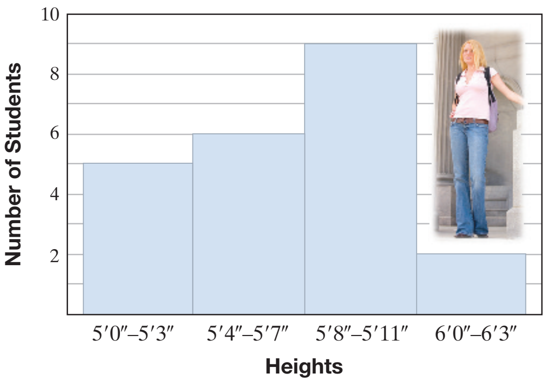
Source: U.S. Census Bureau

13. Under 18 (Round to nearest whole million.)

14. 25–44 (Round to nearest whole million.)

A professor measures the heights of the students in her class. The results are shown in the following histogram. Use this histogram to answer Exercises 15 and 16.

Student Heights



15. How many students are 5′8″–5′11″ tall?

16. How many students are 5′7″ or shorter?

17. The history test scores of 25 students are shown below. Use these scores to complete the frequency distribution table.

70	86	81	65	92
43	72	85	69	97
82	51	75	50	68
88	83	85	77	99
77	63	59	84	90

Class Intervals (Scores)	Tally	Class Frequency (Number of Students)
40–49		
50–59		
60–69		
70–79		
80–89		
90–99		

13. ____

14. ____

15. ____

16. ____

17. ____

18. ______

19. ______

20. ______

21. ______

22. ______

23. ______

24. a. ______

b. ______

c. ______

d. ______

18. Use the results of Exercise **17** to draw a histogram.

Find the mean, median, and mode of each list of numbers.

19. 26, 32, 42, 43, 49

20. 8, 10, 16, 16, 14, 12, 12, 13

Find the grade point average. If necessary, round to the nearest hundredth.

21.

Grade	Credit Hours
A	3
B	3
C	3
B	4
A	1

22. *Given the data items:* 10, 18, 13, 16, 13. Find the range.

23. Use the data for Exercise **17** and find the range.

24. Use the frequency distribution table to find the following. Round the mean to 1 decimal place.

a. mean

b. median

c. mode

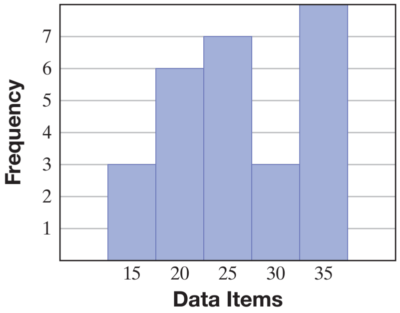
d. range

Data Item	*Frequency*
90	3
91	1
92	2
93	8
94	8

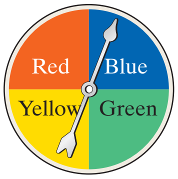
25. Use the graph of data items to find the following. Round the mean to 1 decimal place.

a. mean **b.** median

c. mode **d.** range



26. Draw a tree diagram for the experiment of spinning the spinner twice. State the number of outcomes.



27. Draw a tree diagram for the experiment of tossing a coin twice. State the number of outcomes.

Suppose that the numbers 1 to 10 are each written on same-size pieces of paper and placed in a bag. You then select one piece of paper from the bag.

28. What is the probability of choosing a 6 from the bag?

29. What is the probability of choosing a 3 or a 4 from the bag?

25. a. ______

b. ______

c. ______

d. ______

26. ______

27. ______

28. ______

29. ______

Chapters 1–7

Cumulative Review

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. a. ______

b. ______

c. ______

18. a. ______

b. ______

19. ______

20. ______

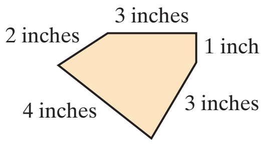
21. ______

22. ______

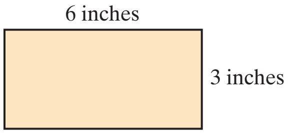
1. Write 106,052,447 in words.

2. Write 276,004 in words.

3. Find the perimeter of the polygon shown.



4. Find the perimeter of the rectangle shown.



5. Subtract: $900 - 174$. Check by adding.

6. Subtract: $17{,}801 - 8216$. Check by adding.

7. Round 248,982 to the nearest hundred.

8. Round 844,497 to the nearest thousand.

9. Multiply: 25×8

10. Multiply: 395×74

11. Divide and check: $1872 \div 9$

12. Divide and check: $3956 \div 46$

13. Simplify: $2 \cdot 4 - 3 \div 3$

14. Simplify: $8 \cdot 4 + 9 \div 3$

15. Evaluate $x^2 + z - 3$ for $x = 5$ and $z = 4$.

16. Evaluate $2a^2 + 5 - c$ for $a = 2$ and $c = 3$.

17. Insert $<$ or $>$ between each pair of numbers to make a true statement.

a. $-7 \qquad 7$

b. $0 \qquad -4$

c. $-9 \qquad -11$

18. Insert $<$ or $>$ between each pair of numbers to make a true statement.

a. $-14 \qquad 0$

b. $-(-7) \qquad -8$

19. Add using a number line: $5 + (-2)$

20. Add using a number line: $-3 + (-4)$

Add.

21. $-5 + (-10)$

22. $3 + (-7)$

23. $2 + 6$

24. $21 + 15 + (-19)$

Subtract.

25. $-4 - 10$

26. $-2 - 3$

27. $6 - (-5)$

28. $19 - (-10)$

29. $-11 - (-7)$

30. $-16 - (-13)$

Divide.

31. $\frac{-12}{6}$

32. $\frac{-30}{-5}$

33. $-20 \div (-4)$

34. $26 \div (-2)$

35. $\frac{48}{-3}$

36. $\frac{-120}{12}$

37. Add: $2\frac{4}{5} + 5 + 1\frac{1}{2}$

38. Multiply: $5\frac{1}{3} \cdot 2\frac{1}{8}$

Write each rate as a fraction in simplest form.

39. $2160 for 12 weeks

40. 340 miles every 5 hours

41. Convert 7 feet to yards.

42. Convert 2.5 tons to pounds.

43. Convert 2.35 cg to grams.

44. Convert 106 cm to millimeters.

45. Find: $\sqrt{\frac{1}{36}}$

46. Find: $\sqrt{\frac{1}{25}}$

47. Find the mode of the list of numbers: 11, 14, 14, 16, 31, 56, 65, 77, 77, 78, 79

48. Find the median of the numbers in Exercise **47**.

49. If a coin is tossed twice, find the probability of tossing heads on the first toss and then heads again on the second toss (H, H).

50. A bag contains 3 red marbles and 2 blue marbles. Find the probability of choosing a red marble.

23. ______
24. ______
25. ______
26. ______
27. ______
28. ______
29. ______
30. ______
31. ______
32. ______
33. ______
34. ______
35. ______
36. ______
37. ______
38. ______
39. ______
40. ______
41. ______
42. ______
43. ______
44. ______
45. ______
46. ______
47. ______
48. ______
49. ______
50. ______

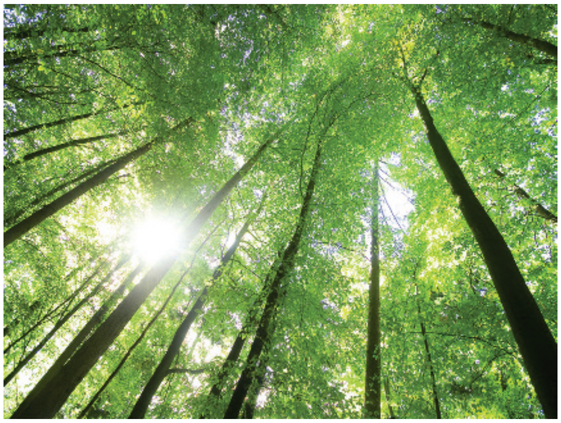
8

Introduction to Algebra

In this chapter we make the transition from arithmetic to algebra. In algebra, letters are used to stand for unknown quantities. Using variables is a very powerful tool for solving problems that cannot be solved with arithmetic alone. This chapter introduces variables, algebraic expressions, and solving variable equations.

Sections

Check Your Progress



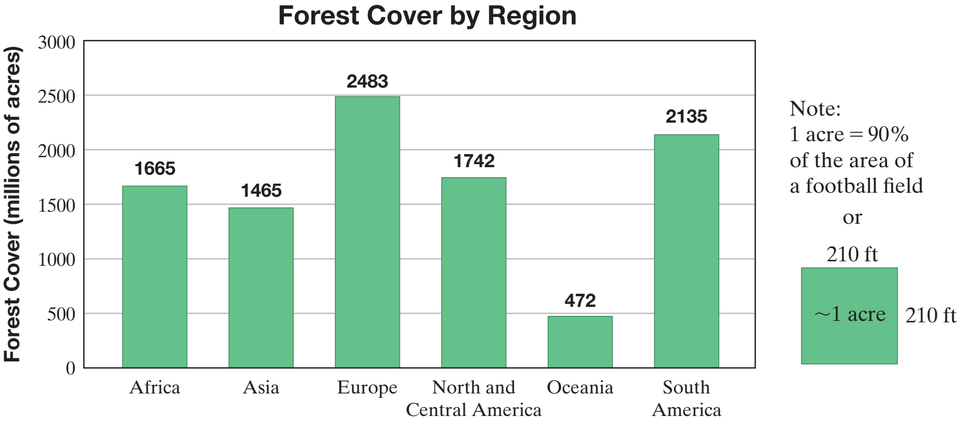
A Canopy of Trees

The Rings of a Tree

What Are the Largest and Longest-Living Plants on Our Planet?

Trees are not only the largest plants on our planet, but they are the longest-living species. They absorb carbon dioxide, store the carbon, and release oxygen back into the air. Trees also prevent soil erosion and flooding as well as offer food and shelter to birds, insects, fungi, and other living plants and animals. Thousands of things are made from trees—from medicines to toothpaste to furniture. Since some trees live thousands of years, they also give us a glimpse into the past through the study of their rings. (See the photo above to the right.)

The photo above to the left shows a canopy of trees, which act as a filter. This filter provides shade, reduces noise, traps dust, and absorbs pollutants that are present in the air. The graph below shows the forest cover in regions of the world. In Section 8.1, Exercises 117 and 118, we examine the size of some giant trees.



Source: Earth Policy Institute

8.1 Introduction to Variables

Objective A Evaluating Algebraic Expressions

Perhaps the most important quality of mathematics is that it is a science of pattern. Communicating about patterns is made possible by using a letter to represent all the numbers fitting a pattern. We call such a letter a **variable.** For example, in Section 1.3 we presented the addition property of 0, which states that the sum of 0 and any whole number is that number. We might write

$$\begin{aligned} 0 + 1 &= 1 \\ 0 + 2 &= 2 \\ 0 + 3 &= 3 \\ 0 + 4 &= 4 \\ 0 + 5 &= 5 \\ 0 + 6 &= 6 \\ &\vdots \end{aligned}$$

continuing indefinitely. This is a pattern, and all whole numbers fit the pattern. We can communicate this pattern for all whole numbers by letting a letter, such as a, represent all whole numbers. We can then write

$$0 + a = a$$

Helpful Hint

Variables have been used in previous chapters, although we have not called them that. For example, in the ratio and proportion chapter, we wrote equations such as

$$\frac{4}{n} = \frac{6}{12}$$

Here, the letter n is a variable.

Learning to use variable notation is a primary goal of learning **algebra.** We now take some important beginning steps in learning to use variable notation.

A combination of numbers, letters (variables), and operation symbols is called an **algebraic expression** or simply an **expression.** For example,

$$3 + x, \qquad 5 \cdot y, \qquad \text{and} \qquad 2 \cdot z - 1 + x$$

are expressions.

If two variables or a number and a variable are next to each other, with no operation sign between them, the indicated operation is multiplication. For example,

$$2x \qquad \text{means} \qquad 2 \cdot x$$

and

$$xy \text{ or } x(y) \qquad \text{means} \qquad x \cdot y$$

Also, the meaning of an exponent remains the same when the base is a variable. For example,

$$x^2 = \underbrace{x \cdot x}_{\text{2 factors of } x} \quad \text{and} \quad y^5 = \underbrace{y \cdot y \cdot y \cdot y \cdot y}_{\text{5 factors of } y}$$

Algebraic expressions have different values depending on the replacement values for the variable(s). Replacing a variable in an expression by a number and then finding the value of the expression is called **evaluating the expression.** When finding the value of an expression, remember to follow the order of operations.

Objectives

A Evaluate Algebraic Expressions for Given Replacement Values for the Variables.

B Use Properties of Numbers to Combine Like Terms.

C Use Properties of Numbers to Multiply Expressions.

D Simplify Expressions by Multiplying and Then Combining Like Terms.

E Find the Perimeter and Area of Figures.

Practice 1

Evaluate: $5x - 12$ when $x = 2$

Example 1 Evaluate: $3x - 7$ when $x = 2$

Solution:

$$\begin{aligned} 3x - 7 &= 3 \cdot 2 - 7 && \text{Replace } x \text{ with 2.} \\ &= 6 - 7 && \text{Multiply.} \\ &= -1 && \text{Subtract.} \end{aligned}$$

Work Practice 1

Practice 2

Evaluate: $5x - y$ when $x = 2$ and $y = -3$

Example 2 Evaluate: $2x + y$ when $x = 8$ and $y = -7$

Solution: Replace x with 8 and y with -7 in $2x + y$.

$$\begin{aligned} 2x + y &= 2 \cdot 8 + (-7) && \text{Replace } x \text{ with 8 and } y \text{ with } -7. \\ &= 16 + (-7) && \text{Multiply first because of the order of operations.} \\ &= 9 && \text{Add.} \end{aligned}$$

Work Practice 2

Practice 3

Evaluate: $\dfrac{5r - 2s}{-3q}$ when $r = 3, s = 3$, and $q = 1$

Example 3 Evaluate: $\dfrac{3m - 2n}{-2q}$ when $m = 8, n = 4$, and $q = 1$

Solution:

$$\begin{aligned} \frac{3m - 2n}{-2q} &= \frac{3 \cdot 8 - 2 \cdot 4}{-2 \cdot 1} && \text{Replace } m \text{ with 8, } n \text{ with 4, and } q \text{ with 1.} \\ &= \frac{24 - 8}{-2} && \text{Multiply.} \\ &= \frac{16}{-2} && \text{Subtract in the numerator.} \\ &= -8 && \text{Divide.} \end{aligned}$$

Work Practice 3

Practice 4

Evaluate: $13 - (3a + 8)$ when $a = -2$

Example 4 Evaluate: $8 - (6a - 5)$ when $a = -3$

Solution:

$$\begin{aligned} 8 - (6a - 5) &= 8 - (6 \cdot (-3) - 5) && \text{Replace } a \text{ with } -3. \\ &= 8 - (-18 - 5) && \text{Multiply.} \\ &= 8 - (-23) && \text{Simplify inside the parentheses.} \\ &= 8 + 23 \\ &= 31 && \text{Add.} \end{aligned}$$

Work Practice 4

Practice 5

Evaluate: $a^2 - 0.7b$ when $a = 6$ and $b = -2$

Example 5 Evaluate: $x^3 - 1.1y$ when $x = 4$ and $y = -1$

Solution:

$$\begin{aligned} x^3 - 1.1y &= 4^3 - 1.1(-1) && \text{Replace } x \text{ with 4 and } y \text{ with } -1. \\ &= 64 - 1.1(-1) && \text{Evaluate } 4^3. \\ &= 64 - (-1.1) && \text{Multiply.} \\ &= 64 + 1.1 \\ &= 65.1 && \text{Add.} \end{aligned}$$

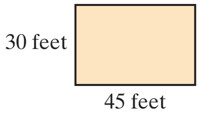
Work Practice 5

Answers

1. −2 **2.** 13 **3.** −3 **4.** 11 **5.** 37.4

Example 6 Finding the Area of a Rectangle

The formula for finding the area of a rectangle is $A = l \cdot w$ (which we now know can be written as $A = lw$), where l is the length of the rectangle and w is the width. Find the area of a rectangular floor that is 45 feet long and 30 feet wide.



Solution:

$$A = \underset{\downarrow}{l} \cdot \underset{\downarrow}{w}$$

$$= (45\text{ feet}) \cdot (30\text{ feet}) \quad \text{Replace } l \text{ with 45 feet and } w \text{ with 30 feet.}$$

$$= 1350\text{ square feet}$$

The area of the floor is 1350 square feet.

Work Practice 6

Objective B Combining Like Terms

The addends of an algebraic expression are called the **terms** of the expression.

$$\underbrace{x + 3}_{\text{2 terms}}$$

$$\underbrace{3y^2 + (-6y) + 4}_{\text{3 terms}}$$

A term that is only a number has a special name. It is called a **constant term,** or simply a **constant.** A term that contains a variable is called a **variable term.**

x +	3	$3y^2 + (-6y) +$	4
variable term	constant term	variable terms	constant term

The number factor of a variable term is called the **numerical coefficient.** A numerical coefficient of 1 is usually not written.

$5x$	x or $1x$	$3y^2$	$-6y$
Numerical coefficient is 5.	Understood numerical coefficient is 1.	Numerical coefficient is 3.	Numerical coefficient is −6.

Terms with the same variable factors, except that they may have different numerical coefficients, are called **like terms**.

Like Terms	Unlike Terms
$3x, \frac{1}{2}x$	$5x, x^2$
$-6y, 2y, y$	$7x, 7y$

✓**Concept Check** True or false? The terms $-7xy$ and $-7yx$ are like terms. Explain.

A sum or difference of like terms can be simplified using the **distributive property.** Recall from Section 1.6 that the distributive property says that multiplication distributes over addition (and subtraction). Using variables, we can write the distributive property as follows:

$$(a + b)c = ac + bc.$$

Practice 6

The formula for finding the perimeter of a rectangle is $P = 2 \cdot l + 2 \cdot w$ (which we now know can be written as $P = 2l + 2w$). Find the perimeter of a rectangular garden that is 25 meters wide and 40 meters long.

Helpful Hint

Recall that $1 \cdot$ any number = that number. This means that

$1 \cdot x = x$ or that $1x = x$.

Thus x can always be replaced by $1x$ or $1 \cdot x$.

Answer

6. 130 m

✓**Concept Check Answer**

true; the order of the variable factors does not make a difference

If we write the right side of the equation first, then the left side, we have the following:

Distributive Property

If a, b, and c are numbers, then

$$ac + bc = (a + b)c$$

Also,

$$ac - bc = (a - b)c$$

The distributive property guarantees that, no matter what number x is, $7x + 5x$ (for example) has the same value as $(7 + 5)x$, or $12x$. We then have that

$$7x + 5x = (7 + 5)x = 12x$$

This is an example of **combining like terms.** An algebraic expression is **simplified** when all like terms have been combined.

Practice 7

Simplify each expression by combining like terms.

a. $8m - 11m$

b. $5a + a$

Example 7

Simplify each expression by combining like terms.

a. $3x + 2x$ **b.** $y - 7y$

Solution: We add or subtract like terms.

a. $3x + 2x = (3 + 2)x$

$= 5x$

Understood 1

b. $y - 7y = 1y - 7y$

$= (1 - 7)y$

$= -6y$

Work Practice 7

The commutative and associative properties of addition and multiplication can also help us simplify expressions. We presented these properties in Sections 1.3 and 1.6 and state them again using variables.

Properties of Addition and Multiplication

If a, b, and c are numbers, then

$a + b = b + a$ Commutative property of addition

$a \cdot b = b \cdot a$ Commutative property of multiplication

That is, the **order** of adding or multiplying two numbers can be changed without changing their sum or product.

$(a + b) + c = a + (b + c)$ Associative property of addition

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ Associative property of multiplication

That is, the **grouping** of numbers in addition or multiplication can be changed without changing their sum or product.

Answers

7. a. $-3m$ **b.** $6a$

Helpful Hint

- Examples of the comutative and associative properties are

$2 + 3 = 3 + 2$	Commutative property of addition
$7 \cdot 9 = 9 \cdot 7$	Commutative property of multiplication
$(1 + 8) + 10 = 1 + (8 + 10)$	Associative property of addition
$(4 \cdot 2) \cdot 3 = 4 \cdot (2 \cdot 3)$	Associative property of multiplication

- These properties are not true for subtraction or division.

Example 8 Simplify: $2y - 6 + 4y + 8$

Solution: We begin by writing subtraction as the addition of opposites.

$2y - 6 + 4y + 8 = 2y + (-6) + 4y + 8$

$= 2y + 4y + (-6) + 8$ Apply the commutative property of addition.

$= (2 + 4)y + (-6) + 8$ Apply the distributive property.

$= 6y + 2$ Simplify.

Work Practice 8

Examples Simplify each expression by combining like terms.

9. $6x + 2x - 5 = 8x - 5$

10. $4x + 3 - 5x + 2x = 4x - 5x + 2x + 3$

$= 1x + 3$ or $x + 3$

11. $1.2y + 10 - 5.7y - 9 = 1.2y - 5.7y + 10 - 9$

$= -4.5y + 1$

12. $2x - 5 + 3y + 4x - 10y + 11 = 6x - 7y + 6$

Work Practice 9–12

Objective C Multiplying Expressions

We can also use properties of numbers to multiply expressions such as $3(2x)$. By the associative property of multiplication, we can write the product $3(2x)$ as $(3 \cdot 2)x$, which simplifies to $6x$.

Examples Multiply.

13. $5(3y) = (5 \cdot 3)y$ Apply the associative property of multiplication.

$= 15y$ Multiply.

14. $-2(4x) = (-2 \cdot 4)x$ Apply the associative property of multiplication.

$= -8x$ Multiply.

Work Practice 13–14

We can use the distributive property to combine like terms, which we have done, and also to multiply expressions such as $2(3 + x)$. By the distributive property, we have that

$2(3 + x) = 2 \cdot 3 + 2 \cdot x$ Apply the distributive property.

$= 6 + 2x$ Multiply.

Practice 8

Simplify: $8m + 5 + m - 4$

Practice 9–12

Simplify each expression by combining like terms.

9. $7y + 11y - 8$

10. $2y - 6 + y + 7y$

11. $3.7x + 5 - 4.2x + 15$

12. $-9y + 2 - 4y - 8x + 12 - x$

Practice 13–14

Multiply.

13. $7(8a)$

14. $-5(9x)$

Answers

8. $9m + 1$ **9.** $18y - 8$ **10.** $10y - 6$
11. $-0.5x + 20$ **12.** $-13y - 9x + 14$
13. $56a$ **14.** $-45x$

Practice 15

Use the distributive property to multiply: $7(y + 2)$

Example 15 Use the distributive property to multiply: $6(x + 4)$

Solution: By the distributive property,

$$\begin{aligned} 6(x + 4) &= 6 \cdot x + 6 \cdot 4 && \text{Apply the distributive property.} \\ &= 6x + 24 && \text{Multiply.} \end{aligned}$$

Work Practice 15

✓Concept Check What's wrong with the following?

$\xcancel{8(a - b) = 8a - b}$

Practice 16

Multiply: $4(7a - 5)$

Example 16 Multiply: $-3(5a + 2)$

Solution: By the distributive property,

$$\begin{aligned} -3(5a + 2) &= -3(5a) + (-3)(2) && \text{Apply the distributive property.} \\ &= (-3 \cdot 5)a + (-6) && \text{Apply the associative property. Also, write } (-3)(2) \text{ as } -6. \\ &= -15a - 6 && \text{Multiply.} \end{aligned}$$

Work Practice 16

Objective D Simplifying Expressions

Next, we will **simplify** expressions containing parentheses by first using the distributive property to multiply and then **combining** any like terms.

Practice 17

Simplify: $5(2y - 3) - 8$

Example 17 Simplify: $2(3 + 7x) - 15$

Solution: First we use the distributive property to remove parentheses.

$$\begin{aligned} \underbrace{2(3 + 7x)} - 15 &= 2(3) + 2(7x) - 15 && \text{Apply the distributive property.} \\ &= 6 + 14x - 15 && \text{Multiply.} \\ &= 14x + (-9) \quad \text{or} \quad 14x - 9 && \text{Combine like terms.} \end{aligned}$$

Helpful Hint

2 is *not* distributed to the −15 since it is not within the parentheses.

Work Practice 17

Practice 18

Simplify:
$-7(x - 1) + 5(2x + 3)$

Example 18 Simplify: $-2(x - 5) + 4(2x + 2)$

Solution: First we use the distributive property to remove parentheses.

$$\begin{aligned} -2(x - 5) + 4(2x + 2) &= -2(x) - (-2)(5) + 4(2x) + 4(2) && \text{Apply the distributive property.} \\ &= -2x + 10 + 8x + 8 && \text{Multiply.} \\ &= 6x + 18 && \text{Combine like terms.} \end{aligned}$$

Work Practice 18

Answers

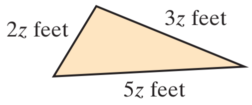
15. $7y + 14$ **16.** $28a - 20$
17. $10y - 23$ **18.** $3x + 22$

✓Concept Check Answer

did not distribute the 8

Objective E Finding Perimeter and Area

Example 19 Find the perimeter of the triangle.



Solution: Recall that the perimeter of a figure is the distance around the figure. To find the perimeter, then, we find the sum of the lengths of the sides. We use the letter P to represent perimeter.

$$
\begin{aligned}
P &= 2z + 3z + 5z \\
&= 10z
\end{aligned}
$$

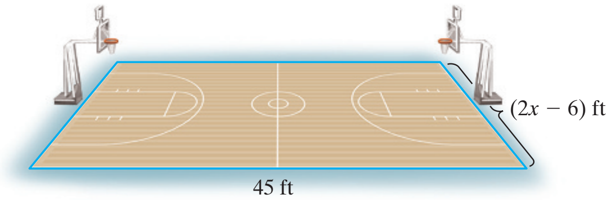
The perimeter is $10z$ feet.

Helpful Hint Don't forget to insert proper units.

Work Practice 19

Example 20 Finding the Area of a Basketball Court

Find the area of this YMCA basketball court.



Solution: Recall how to find the area of a rectangle. **A**rea = **L**ength · **W**idth, or if A represents area, l represents length, and w represents width, we have $A = l \cdot w$.

$$
\begin{aligned}
A &= l \cdot w \\
&= 45(2x - 6) && \text{Let } l = 45 \text{ and } w = (2x - 6). \\
&= 90x - 270 && \text{Multiply.}
\end{aligned}
$$

The area is $(90x - 270)$ *square* feet.

Work Practice 20

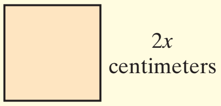
Helpful Hint

Don't forget…

Area:	Perimeter:
• surface enclosed • measured in square units	• distance around • measured in units

Practice 19

Find the perimeter of the square.



Practice 20

Find the area of the rectangular garden.

Answers

19. $8x$ cm **20.** $(36y + 27)$ sq yd

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

numerical coefficient
constant
combine like terms
expression
like
unlike
term
distributive
variable
commutative
associative

1. $14y^2 + 2x - 23$ is called a(n) ________ while $14y^2$, $2x$, and -23 are each called a(n) ________.
2. To multiply $3(-7x + 1)$, we use the ________ property.
3. To simplify an expression like $y + 7y$, we ________.
4. By the ________ properties, the *order* of adding or multiplying two numbers can be changed without changing their sum or product.
5. The term $5x$ is called a(n) ________ term while the term 7 is called a(n) ________ term.
6. The term z has an understood ________ of 1.
7. By the ________ properties, the grouping of numbers in addition or multiplication can be changed without changing their sum or product.
8. The terms $-x$ and $5x$ are ________ terms.
9. For the term $-3x^2y$, -3 is called the ________.
10. The terms $5x$ and $5y$ are ________ terms.

Identify each pair of terms as like terms or unlike terms.

11. $5x$ and $5y$
12. $-3a$ and $-3b$
13. x and $-2x$
14. $7y$ and y
15. $-5n$ and $6n^2$
16. $4m^2$ and $2m$
17. $8b$ and $-6b$
18. $12a$ and $-11a$

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 8.1

Objective A **19.** Complete this statement based on the lecture before Example 1: When a letter and a variable are next to each other, the operation is an understood ________.

Objective B **20.** From Example 3, what is the numerical coefficient of the term $-x$?

Objective C **21.** In Example 6, what two properties are used to multiply the expressions and in what order?

Objective D **22.** In Example 7, why can't we add 3 to 6?

Objective E **23.** In Example 8, what operation is used to find P? What operation is used to find A? What do P and A stand for?

8.1 Exercise Set MyLab Math

Objective A *Evaluate each expression when $x = -2$, $y = 5$, and $z = -3$. See Examples 1 through 6.*

1. $3 + 2z$

2. $7 + 3z$

3. $-y - z$

4. $-y - x$

5. $z - x + y$

6. $x + y - z$

7. $3x - z$

8. $y + 5z$

9. $8 - (5y - 7)$

10. $5 + (2x - 1)$

11. $y^3 - 4x$

12. $y^2 - 2z$

13. $\frac{6xy}{4}$

14. $\frac{8yz}{15}$

15. $\frac{2y - 2}{x}$

16. $\frac{6 + 3x}{z}$

17. $\frac{x + 2y}{2z}$

18. $\frac{2z - y}{3x}$

19. $\frac{5x}{y} - 10$

20. $7 - \frac{3y}{z}$

21. $\frac{xz}{y} + \frac{3}{10}$

22. $\frac{x}{yz} + \frac{31}{30}$

23. $|x| - |y| - 7.6$

24. $|z| - |y| - 12.7$

Objective B *Simplify each expression by combining like terms. See Examples 7 through 12.*

25. $3x + 5x$

26. $8y + 3y$

27. $5n - 9n$

28. $7z - 10z$

29. $4c + c - 7c$

30. $5b - 8b - b$

31. $5x - 7x + x - 3x$

32. $8y + y - 2y - y$

33. $4a + 3a + 6a - 8$

34. $5b - 4b + b - 15$

35. $1.7x + 3.4 - 2.6x + 7.8$

36. $-8.6y + 1.3 - 2.9y - 14.7$

37. $3x + 7 - x - 14$

38. $9x - 6 + x - 10$

39. $4x + 5y + 2 - y - 9x - 7$

40. $a + 4b + 3 - 7a - 5b - 10$

41. $\frac{5}{6} - \frac{7}{12}x - \frac{1}{3} - \frac{3}{10}x$

42. $-\frac{2}{5} + \frac{4}{9}y - \frac{4}{15} + \frac{1}{6}y$

43. $-5m - 2.3m + 11 + 2.5m - 15.1$

44. $-13n - 4.8n + 13 + 6.9n - 13.6$

Objective C *Multiply. See Examples 13 through 16.*

45. $6(5x)$

46. $4(4x)$

47. $-2(11y)$

48. $-3(21z)$

49. $-0.6(7a)$

50. $-0.4(9a)$

51. $\frac{2}{3}(-6a)$

52. $\frac{3}{4}(-8a)$

53. $2(y + 2)$

54. $3(x + 1)$

55. $5(3a - 8)$

56. $4(5y - 6)$

▶ **57.** $-4(3x + 7)$

58. $-8(8y + 10)$

59. $1.2(5x - 0.1)$

60. $3.1(7x - 0.3)$

61. $\frac{1}{2}(-8x - 3)$

62. $\frac{1}{5}(-20x - 7)$

Objective D *Simplify each expression. Use the distributive property to remove parentheses first. See Examples 17 and 18.*

63. $2(x + 4) - 17$

64. $5(6 + y) - 2$

65. $4(6n - 5) + 3n$

66. $3(5 - 2b) - 4b$

▶ **67.** $3 + 6(w + 2) + w$

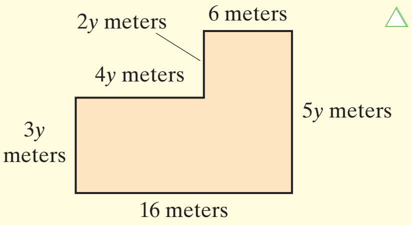
68. $8z + 5(6 + z) + 20$

69. $-2(3x + 1) - 5(x - 2)$

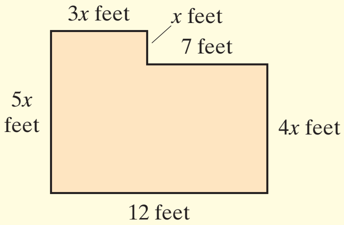
70. $-3(5x - 2) - 2(3x + 1)$

Objective E *Find the perimeter of each figure. See Example 19.*

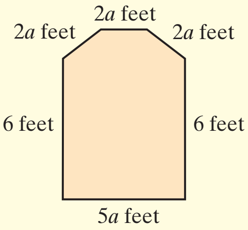
△ **71.**



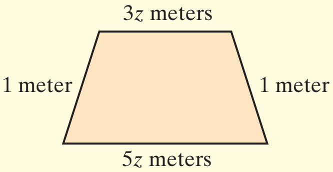
△ **72.**



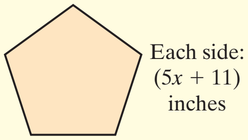
△ **73.**



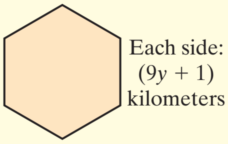
△ **74.**



△ **75.**

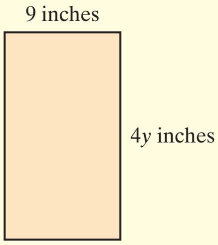


△ **76.**

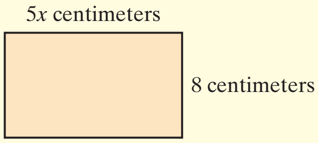


Find the area of each rectangle. For Exercises 81 and 82, find the perimeter and the area. See Examples 19 and 20.

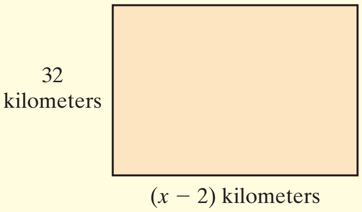
△ **77.**



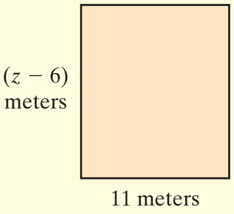
△ **78.**



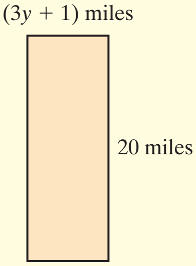
△ **79.**



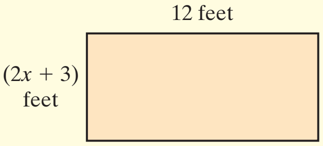
△ **80.**



▶ △ **81.**



△ **82.**



Objectives A B C D E Mixed Practice *Solve. See Examples 1 through 20.*

83. Find the area of a regulation NCAA basketball court that is 94 feet long and 50 feet wide.

84. Find the area of a rectangular movie screen that is 50 feet long and 40 feet high.

85. A decorator wishes to put a wallpaper border around a rectangular room that measures 14 feet by 18 feet. Find the room's perimeter.

86. How much fencing will a rancher need for a rectangular cattle lot that measures 80 feet by 120 feet?

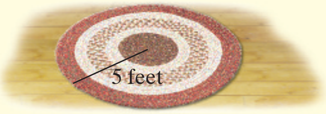
87. Find the perimeter of a triangular garden that measures 5 feet by x feet by $(2x + 1)$ feet.

88. Find the perimeter of a triangular picture frame that measures x inches by x inches by $(x - 14)$ inches.

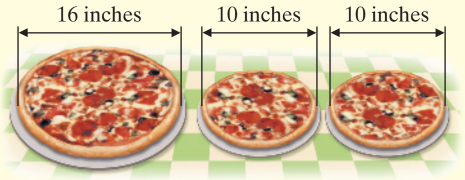
89. How much interest will $3000 in a passbook savings account earn in 2 years at Money Bank, which pays 6% simple interest? Use $I = PRT$.

90. How much interest will a $12,000 certificate of deposit earn in 1 year at a rate of 8% simple interest? Use $I = PRT$.

91. A store sells one circular braided rug with a radius of 5 feet for the same price as two circular rugs, each with a radius of 3 feet. Which deal gives you a greater area of rug? Use $A = \pi r^2$ and $\pi \approx 3.14$. [*Hint:* Compare the area of one 5-foot-radius circle with the area of two 3-foot-radius circles.]



92. Mario's Pizza sells one 16″ cheese pizza or two 10″ cheese pizzas for $9.99. Which deal gives you more pizza? Use $A = \pi r^2$ with $\pi \approx 3.14$.



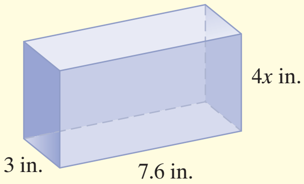
93. Convert Paris, France's, low temperature of -5°C to Fahrenheit. Use $F = \frac{9}{5}C + 32$.

94. Convert Nome, Alaska's, 18°F high temperature to Celsius. Use $C = \frac{5}{9}(F - 32)$.

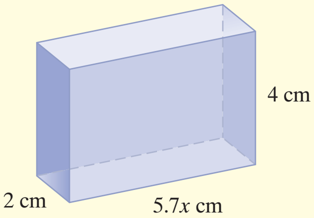
95. Find the volume of a box that measures 12 inches by 6 inches by 4 inches. Use $V = lwh$.

96. How many cubic meters does a space shuttle cargo compartment have if its dimensions are 8 meters long by 4 meters wide by 3 meters high? Use $V = lwh$.

97. Find the volume. Use $V = lwh$.



98. Find the volume. Use $V = lwh$.



Review

Perform each indicated operation. See Sections 2.3 and 2.4.

99. $-13 + 10$

100. $-15 + 23$

101. $-4 - (-12)$

102. $-7 - (-4)$

103. $-4 + 4$

104. $8 + (-8)$

Concept Extensions

If the expression on the left side of the equal sign is equivalent to the right, write "correct." If not, write "incorrect" and then write an expression that is equivalent to the left side. See the second Concept Check in this section.

105. $5(3x - 2) \stackrel{?}{=} 15x - 2$

106. $2(xy) \stackrel{?}{=} 2x \cdot 2y$

107. $7x - (x + 2) \stackrel{?}{=} 7x - x + 2$

108. $4(y - 3) + 11 \stackrel{?}{=} 4y - 3 + 11$

Review the commutative, associative, and distributive properties. Then identify which property allows us to write the equivalent expression on the right side of the equal sign.

109. $6(2x - 3) + 5 = 12x - 18 + 5$

110. $9 + 7x + (-2) = 7x + 9 + (-2)$

111. $-7 + (4 + y) = (-7 + 4) + y$

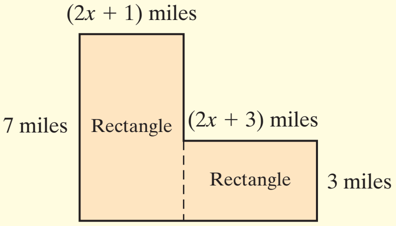
112. $(x + y) + 11 = 11 + (x + y)$

113. If x is a whole number, which expression is the largest: $2x$, $5x$, or $\frac{1}{3}x$? Explain your answer.

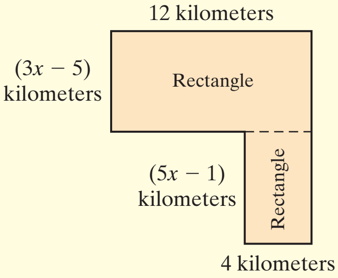
114. If x is a whole number, which expression is the smallest: $2x$, $5x$, or $\frac{1}{3}x$? Explain your answer.

Find the area of each figure.

115.



116.



To appraise the value of a large tree in landscaping, the trunk area is used. The trunk area A is calculated with the formula $A = 0.7854d^2$, *where d is the diameter of the tree.*

117. *Tane Mahuta* is the name of New Zealand's largest known living Kauri tree. Its trunk has a diameter of 173 inches. Use the formula to find the trunk area of this tree. Round your result to the nearest tenth. (*Source*: New Zealand Parks)

118. *Lost Monarch,* which was only discovered in 1998, is a coast redwood tree in northern California. It is located among other giant redwoods in "The Grove of Titans," in Jedediah Smith Redwoods State Park. This mammoth tree has confirmed diameter of 312 inches. Use the formula to find the trunk area of this tree. Round your result to the nearest tenth. (*Source:* Sierra Club)

Simplify.

119. $9684q - 686 - 4860q + 12{,}960$

120. $76(268x + 592) - 2960$

8.2 Solving Equations: The Addition Property

Objectives

A Determine Whether a Given Number Is a Solution of an Equation.

B Use the Addition Property of Equality to Solve Equations.

Frequently in this text we have written statements like
$7 + 4 = 11$ or Area $=$ length $\cdot$ width.
Each of these statements is called an **equation.** An equation is of the form

expression $=$ expression

An equation can be labeled as

$$\underbrace{x + 7}_{\text{left side}} \overset{\text{equal sign}}{=} \underset{\text{right side}}{10}$$

It is very important to know the difference between an **expression** and an **equation.** An equation contains an equal sign and an expression does not.

Equations	Expressions
$7x = 6x + 4$	$7x - 6x + 4$
$3(3y - 5) = 10y$	$y - 1 + 11y - 21$

equal signs

no equal signs

Objective A Determining Whether a Number Is a Solution

When an equation contains a variable, finding which values of the variable make an equation a true statement is called **solving** the equation for the variable. A **solution** of an equation is a value for the variable that makes the equation a true statement. For example, 2 is a solution of the equation $x + 5 = 7$ since replacing x with 2 results in the *true* statement $2 + 5 = 7$. Similarly, 3 is not a solution of $x + 5 = 7$ since replacing x with 3 results in the *false* statement $3 + 5 = 7$.

Example 1 Determine whether 6 is a solution of the equation $4(x - 3) = 12$.

Solution: We replace x with 6 in the equation.

$$4(x - 3) = 12$$
$$4(6 - 3) \stackrel{?}{=} 12 \quad \text{Replace } x \text{ with 6.}$$
$$4(3) \stackrel{?}{=} 12$$
$$12 \stackrel{?}{=} 12 \quad \text{True}$$

Since $12 = 12$ is a true statement, 6 *is* a solution of the equation.

Work Practice 1

Practice 1

Determine whether 4 is a solution of the equation $3(y - 6) = 6$.

Example 2 Determine whether -1 is a solution of the equation $3y + 1 = 3$.

Solution:

$$3y + 1 = 3$$
$$3(-1) + 1 \stackrel{?}{=} 3$$
$$-3 + 1 \stackrel{?}{=} 3$$
$$-2 \stackrel{?}{=} 3 \quad \text{False}$$

Since $-2 = 3$ is false, -1 is *not* a solution of the equation.

Work Practice 2

Practice 2

Determine whether -2 is a solution of the equation $-4x - 3 = 5$.

Answers
1. no **2.** yes

Objective B Using the Addition Property to Solve Equations

To solve an equation, we use properties of equality to write simpler equations, all equivalent to the original equation, until the final equation has the form

$x =$ **number** or **number** $= x$

Equivalent equations have the same solution, so the word "number" above represents the solution of the original equation. The first property of equality to help us write simpler, equivalent, equations is the **addition property of equality**.

Addition Property of Equality

Let a, b, and c represent numbers. Then

$a = b$
and $a + c = b + c$
are equivalent equations.

Also, $a = b$
and $a - c = b - c$
are equivalent equations.

In other words, the **same number** may be **added to or subtracted from both sides** of an equation without changing the solution of the equation.

A good way to visualize a true equation is to picture a balanced scale. Since it is balanced, each side of the scale weighs the same amount. Similarly, in a true equation the expressions on each side have the same value. Picturing our balanced scale, if we add the same weight to each side, the scale remains balanced.

Practice 3

Solve the equation for y:
$y - 5 = -3$

Example 3 Solve: $x - 2 = 1$ for x.

Solution: To solve the equation for x, we need to rewrite the equation in the form $x =$ number. In other words, our goal is to get x alone on one side of the equation. To do so, we add 2 to both sides of the equation.

$x - 2 = 1$
$x - 2 + 2 = 1 + 2$ Add 2 to both sides of the equation.
$x + 0 = 3$ Replace $-2 + 2$ with 0.
$x = 3$ Simplify by replacing $x + 0$ with x.

Check: To check, we replace x with 3 in the *original* equation.

$x - 2 = 1$ Original equation
$3 - 2 \stackrel{?}{=} 1$ Replace x with 3.
$1 \stackrel{?}{=} 1$ True

Since $1 = 1$ is a true statement, 3 is the solution of the equation.

Work Practice 3

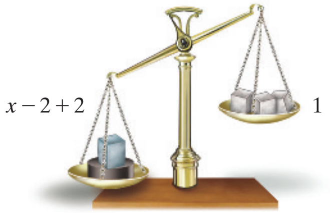
Helpful Hint

Note that it is always a good idea to check the solution in the *original* equation to see that it makes the equation a true statement.

Answer
3. 2

Let's visualize how we used the addition property of equality to solve the equation in Example 3. Picture the original equation, $x - 2 = 1$, as a balanced scale. The left side of the equation has the same value as the right side.

If the same weight is added to each side of a scale, the scale remains balanced. Likewise, if the same number is added to each side of an equation, the left side continues to have the same value as the right side.



Example 4 Solve: $-8 = x + 1$

Solution: To get x alone on one side of the equation, we subtract 1 from both sides of the equation.

$$-8 = x + 1$$

$$-8 - 1 = x + 1 - 1 \quad \text{Subtract 1 from both sides.}$$

$$-9 = x + 0 \quad \text{Replace } 1 - 1 \text{ with 0.}$$

$$-9 = x \quad \text{Simplify.}$$

Check:

$$-8 = x + 1$$

$$-8 \stackrel{?}{=} -9 + 1 \quad \text{Replace } x \text{ with } -9.$$

$$-8 \stackrel{?}{=} -8 \quad \text{True}$$

The solution is −9.

Work Practice 4

Helpful Hint Remember that we can get the variable alone on either side of the equation. For example, the equations $x = 2$ and $2 = x$ both have the solution of 2.

Practice 4

Solve: $-1 = z + 9$

Example 5 Solve: $y - 1.2 = -3.2 - 6.6$

Solution: First we simplify the right side of the equation.

$$y - 1.2 = -3.2 - 6.6$$

$$y - 1.2 = -9.8$$

Next, we get y alone on the left side by adding 1.2 to both sides of the equation.

$$y - 1.2 + 1.2 = -9.8 + 1.2 \quad \text{Add 1.2 to both sides.}$$

$$y = -8.6 \quad \text{Simplify.}$$

Check to see that −8.6 is the solution.

Work Practice 5

Practice 5

Solve: $x - 2.6 = -1.8 - 5.9$

✓ **Concept Check** What number should be added to or subtracted from both sides of the equation in order to solve the equation $-3.75 = y + 2.1$?

Answers

4. −10 **5.** −5.1

✓ **Concept Check Answer**

subtract 2.1 from both sides

Practice 6

Solve:
$-6y + 1 + 7y = 6 - 11$

Example 6 Solve: $5x + 2 - 4x = 7 - 9$

Solution: First we simplify each side of the equation separately.

$$5x + 2 - 4x = 7 - 9$$
$$\underbrace{5x - 4x} + 2 = \underbrace{7 - 9}$$
$$1x + 2 = -2$$

To get x alone on the left side, we subtract 2 from both sides.

$$1x + 2 - 2 = -2 - 2$$
$$1x = -4 \text{ or } x = -4$$

Check to verify that -4 is the solution.

Work Practice 6

Practice 7

Solve: $\frac{2}{3} = x - \frac{4}{9}$

Example 7 Solve: $\frac{7}{8} = y - \frac{1}{2}$

Solution: We use the addition property of equality to add $\frac{1}{2}$ to both sides.

$$\frac{7}{8} = y - \frac{1}{2}$$
$$\frac{7}{8} + \frac{1}{2} = y - \frac{1}{2} + \frac{1}{2} \quad \text{Add } \frac{1}{2} \text{ to both sides.}$$
$$\frac{7}{8} + \frac{4}{8} = y \quad \text{Simplify.}$$
$$\frac{11}{8} = y \quad \text{Simplify.}$$

Check to see that $\frac{11}{8}$ is the solution. (Although $\frac{11}{8} = 1\frac{3}{8}$, we will leave solutions as improper fractions.)

Work Practice 7

Practice 8

Solve: $13x = 4(3x - 1)$

Example 8 Solve: $3(3x - 5) = 10x$

Solution: First we multiply on the left side to remove the parentheses.

$$3(3x - 5) = 10x$$
$$3 \cdot 3x - 3 \cdot 5 = 10x \quad \text{Use the distributive property.}$$
$$9x - 15 = 10x$$

Now we subtract $9x$ from both sides.

$$9x - 15 - 9x = 10x - 9x \quad \text{Subtract } 9x \text{ from both sides.}$$
$$-15 = 1x \quad \text{or} \quad x = -15 \quad \text{Simplify.}$$

Check to verify that -15 is the soluation.

Work Practice 8

Recall that the addition property of equality allows us to add or subtract the same number to or from both sides of an equation. Let's see how adding the same number to both sides of an equation also allows us to subtract the same number from both sides. To do so, let's add $(-c)$ to both sides of $a = b$. Then we have

$$a + (-c) = b + (-c)$$

which is the same as $a - c = b - c$, and there we have it.

Answers

6. -6 **7.** $\frac{10}{9}$ **8.** -4

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

equation	addition	simplifying
solving	equivalent	expression

1. The equations $x + 6 = 10$ and $x + 6 - 6 = 10 - 6$ are called ____________ equations.

2. The difference between an equation and an expression is that a(n) ____________ contains an equal sign, while a(n) ____________ does not.

3. The process of writing $-3x + 10x$ as $7x$ is called ____________ the expression.

4. For the equation $x - 1 = -21$, the process of finding that -20 is the solution is called ____________ the equation.

5. By the ____________ property of equality, $x = -2$ and $x + 7 = -2 + 7$ are equivalent equations.

Martin-Gay Interactive Videos *Watch the section lecture video and answer the following questions.*

See Video 8.2

Objective A **6.** From the lecture before Example 1, what does an equation have that an expression does not?

Objective B **7.** In the lecture before Example 2, what does the addition property of equality mean in words?

8. When solving Example 7, what must be done before applying the addition property of equality?

8.2 Exercise Set MyLab Math

Objective A *Decide whether the given number is a solution of the given equation. See Examples 1 and 2.*

1. Is 10 a solution of $x - 8 = 2$?

2. Is 9 a solution of $y - 2 = 7$?

3. Is -5 a solution of $x + 12 = 17$?

4. Is -7 a solution of $a + 23 = -16$?

5. Is -8 a solution of $-9f = 64 - f$?

6. Is -6 a solution of $-3k = 12 - k$?

7. Is 7 a solution of $5(c - 5) = -10$?

8. Is 1 a solution of $2(b - 3) = 10$?

Objective B *Solve. Check each solution. See Examples 3 through 7.*

9. $a + 5 = 23$

10. $f + 4 = -6$

11. $d - 9 = -17$

12. $s - 7 = 15$

13. $7 = y - 2$

14. $1 = y + 7$

15. $-12 = x + 4$

16. $-10 = z - 15$

17. $x + \frac{1}{2} = \frac{7}{2}$

18. $x + \frac{1}{3} = \frac{4}{3}$

19. $y - \frac{3}{4} = -\frac{5}{8}$

20. $y - \frac{5}{6} = -\frac{11}{12}$

21. $x - 3 = -1 + 4$

22. $y - 8 = -5 - 1$

23. $-7 + 10 = m - 5$

24. $1 - 8 = n + 2$

25. $x - 0.6 = 4.7$

26. $y - 1.2 = 7.5$

27. $-2 - 3 = -4 + x$

28. $7 - (-10) = x - 5$

29. $y + 2.3 = -9.2 - 8.6$

30. $x + 4.7 = -7.5 - 3.4$

31. $-8x + 4 + 9x = -1 + 7$

32. $3x - 2x + 5 = 5 - 2$

33. $5 + (-12) = 5x - 7 - 4x$

34. $11 + (-15) = 6x - 4 - 5x$

35. $7x + 14 - 6x = -4 + (-10)$

36. $-10x + 11x + 5 = -9 + (-5)$

Solve. First multiply to remove parentheses. See Example 8.

37. $2(5x - 3) = 11x$

38. $6(3x + 1) = 19x$

39. $3y = 2(y + 12)$

40. $17x = 4(4x - 6)$

41. $21y = 5(4y - 6)$

42. $28z = 9(3z - 2)$

43. $-3(-4 - 2z) = 7z$

44. $-2(-1 - 3y) = 7y$

Review

Perform each indicated operation. See Section 3.3.

45. $\frac{-7}{-7}$

46. $\frac{4.2}{4.2}$

47. $\frac{1}{3} \cdot 3$

48. $\frac{1}{5} \cdot 5$

49. $-\frac{2}{3} \cdot -\frac{3}{2}$

50. $-\frac{7}{2} \cdot -\frac{2}{7}$

Concept Extensions

What number should be added to or subtracted from both sides of each equation in order to solve the equation? See the Concept Check in this section.

51. $\frac{2}{3} + x = \frac{1}{12}$

52. $12.5 = -3.75 + x$

53. $-\frac{1}{7} = -\frac{4}{5} + x$

54. $9.1 = 5.9 + x$

55. In your own words, explain what is meant by the phrase “a number is a solution of an equation.”

56. In your own words, explain how to check a possible solution of an equation.

Solve.

57. $x - 76{,}862 = 86{,}102$

58. $-968 + 432 = 86y - 508 - 85y$

A football team’s total offense T is found by adding the total passing yardage P to the total rushing yardage R: $T = P + R$.

59. During the 2017 regular football season, the New England Patriots’ total offense was 6307 yards. The Patriots’ passing yardage for the season was 4418 yards. How many yards did the Patriots gain by rushing during the regular season? (*Source:* National Football League)

60. During the 2017 regular football season, the Philadelphia Eagles’ total offense was 5852 yards. The Eagles’ rushing yardage for the season was 2115 yards. How many yards did the Eagles gain by passing during the regular season? (*Source:* National Football League)

In accounting, a company’s annual net income I can be computed using the relation $I = R - E$, *where R is the company’s total revenues for the year and E is the company’s total expenses for the year.*

61. At the end of fiscal year 2017, Kohl’s has a net income of \$6,742,000,000. During the year, Kohl’s has total expenses of \$11,944,000,000. What were Kohl’s total revenues for the year? (*Source:* Kohl’s Corporation)

62. At the end of fiscal year 2017, Target had a net income of \$20,623,000,000. During the year, Target had total expenses of \$48,872,000,000. What were Target’s total revenues for the year? (*Source:* Target Corporation)

8.3 Solving Equations: The Multiplication Property

Objective

A Use the Multiplication Property to Solve Equations.

Objective A Using the Multiplication Property to Solve Equations

Although the addition property of equality is a powerful tool for helping us solve equations, it cannot help us solve all types of equations. For example, it cannot help us solve an equation such as $2x = 6$. To solve this equation, we use a second property of equality called the **multiplication property of equality**.

Multiplication Property of Equality

Let a, b, and c represent numbers and let $c \neq 0$. Then

$$a = b$$

$$\text{and } a \cdot c = b \cdot c$$

are equivalent equations.

$$\text{Also, } a = b$$

$$\text{and } \frac{a}{c} = \frac{b}{c}$$

are equivalent equations.

In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.

Picturing again our balanced scale, if we multiply or divide the weight on each side by the same nonzero number, the scale (or equation) remains balanced.

To solve $2x = 6$ for x, we use the multiplication property of equality to divide both sides of the equation by 2, and simplify as follows:

$$\begin{aligned} 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} && \text{Divide both sides by 2.} \\ \frac{2}{2}\cdot x &= 3 \\ 1\cdot x &= 3 \\ x &= 3 \end{aligned}$$

Practice 1

Solve: $-3y = 18$

Example 1

Solve: $-5x = 15$

Solution: To get x by itself, we divide both sides by -5.

$$\begin{aligned} -5x &= 15 && \text{Original equation} \\ \frac{-5x}{-5} &= \frac{15}{-5} && \text{Divide both sides by } -5. \\ \frac{-5}{-5}\cdot x &= \frac{15}{-5} \\ 1\cdot x &= -3 && \text{Simplify.} \\ x &= -3 \end{aligned}$$

Check: To check, we replace x with -3 in the original equation.

$$\begin{aligned} -5x &= 15 && \text{Original equation} \\ -5(-3) &\stackrel{?}{=} 15 && \text{Let } x = -3. \\ 15 &\stackrel{?}{=} 15 && \text{True} \end{aligned}$$

The solution is -3.

■ Work Practice 1

Practice 2

Solve: $-16 = 8x$

Example 2

Solve: $-8 = 2y$

Solution: To get y alone, we divide both sides of the equation by 2.

$$\begin{aligned} -8 &= 2y \\ \frac{-8}{2} &= \frac{2y}{2} && \text{Divide both sides by 2.} \\ -4 &= 1\cdot y \quad \text{or} \quad y = -4 \end{aligned}$$

Check to see that -4 is the solution.

■ Work Practice 2

Answers

1. -6 **2.** -2

Example 3

Solve: $-1.2x = -36$

Solution: We divide both sides of the equation by the numerical coefficient of x, which is -1.2.

$$\begin{aligned} -1.2x &= -36 \\ \frac{-1.2x}{-1.2} &= \frac{-36}{-1.2} \\ 1 \cdot x &= 30 \\ x &= 30 \end{aligned}$$

Check to see that 30 is the solution.

Work Practice 3

Example 4

Solve: $\frac{3}{5}a = 9$

Solution: Recall that the product of a number and its reciprocal is 1. To get a alone then, we multiply both sides by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$.

$$\frac{3}{5}a = 9$$

$$\frac{5}{3} \cdot \frac{3}{5}a = \frac{5}{3} \cdot 9 \qquad \text{Multiply both sides by } \frac{5}{3}.$$

$$1 \cdot a = \frac{5 \cdot \overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}} \cdot 1} \qquad \text{Multiply.}$$

$$a = 15 \qquad \text{Simplify.}$$

Check: To check, we replace a with 15 in the original equation.

$$\frac{3}{5}a = 9 \qquad \text{Original equation}$$

$$\frac{3}{5} \cdot 15 \stackrel{?}{=} 9 \qquad \text{Replace } a \text{ with 15.}$$

$$\frac{3}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{3}{\cancel{15}}}{1} \stackrel{?}{=} 9 \qquad \text{Multiply.}$$

$$9 \stackrel{?}{=} 9 \qquad \text{True}$$

Since $9 = 9$ is true, 15 is the solution of $\frac{3}{5}a = 9$.

Work Practice 4

Practice 3

Solve: $-0.3y = -27$

Practice 4

Solve: $\frac{5}{7}b = 25$

Answers

3. 90 **4.** 35

Practice 5

Solve: $-\frac{7}{10}x = \frac{2}{5}$

Example 5

Solve: $-\frac{1}{4}x = \frac{1}{8}$

Solution: We multiply both sides of the equation by $-\frac{4}{1}$, the reciprocal of $-\frac{1}{4}$.

$$-\frac{1}{4}x = \frac{1}{8}$$

$$-\frac{4}{1}\cdot -\frac{1}{4}x = -\frac{4}{1}\cdot\frac{1}{8} \qquad \text{Multiply both sides by } -\frac{4}{1}.$$

$$1\cdot x = -\frac{\overset{1}{\cancel{4}}\cdot 1}{1\cdot \underset{2}{\cancel{8}}} \qquad \text{Multiply.}$$

$$x = -\frac{1}{2} \qquad \text{Simplify.}$$

Check to see that $-\frac{1}{2}$ is the solution.

Work Practice 5

✓Concept Check Which operation is appropriate for solving each of the following equations, addition or division?

a. $6 = -4x$ **b.** $6 = x - 4$

We often need to simplify one or both sides of an equation before applying the properties of equality to get the variable alone.

Practice 6

Solve: $2m - 4m = 10$

Example 6

Solve: $3y - 7y = 12$

Solution: First we combine like terms.

$$3y - 7y = 12$$

$$-4y = 12 \qquad \text{Combine like terms.}$$

$$\frac{-4y}{-4} = \frac{12}{-4} \qquad \text{Divide both sides by } -4.$$

$$y = -3 \qquad \text{Simplify.}$$

Check: We replace y with -3.

$$3y - 7y = 12$$

$$3(-3) - 7(-3) \stackrel{?}{=} 12$$

$$-9 + 21 \stackrel{?}{=} 12$$

$$12 \stackrel{?}{=} 12 \qquad \text{True}$$

The solution is -3.

Work Practice 6

Answers

5. $-\frac{4}{7}$ **6.** -5

✓Concept Check Answers

a. division **b.** addition

Example 7 Solve: $-2z + z = 11 - 5$

Solution: We simplify both sides of the equation first.

$$-2z + z = 11 - 5$$

$$-1z = 6 \quad \text{Combine like terms.}$$

$$\frac{-1z}{-1} = \frac{6}{-1} \quad \text{Divide both sides by } -1.$$

$$z = -6 \quad \text{Simplify.}$$

Check to see that −6 is the solution.

Work Practice 7

Practice 7

Solve: $-3a + 2a = -8 + 6$

Example 8 Solve: $\frac{z}{-4} = 11 - 5$

Solution: Simplify the right side of the equation first.

$$\frac{z}{-4} = 11 - 5$$

$$\frac{z}{-4} = 6$$

Next, to get z alone, multiply both sides by −4.

$$-4 \cdot \frac{z}{-4} = -4 \cdot 6 \quad \text{Multiply both sides by } -4.$$

$$\frac{-4}{-4} \cdot z = -4 \cdot 6$$

$$1z = -24 \quad \text{or} \quad z = -24$$

Check to see that −24 is the solution.

Work Practice 8

Practice 8

Solve: $-8 + 6 = \frac{a}{3}$

Answers

7. 2 **8.** −6

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

equation multiplication simplifying
solving equivalent expression

1. The equations $-3x = 51$ and $\frac{-3x}{-3} = \frac{51}{-3}$ are called ______________ equations.

2. The difference between an equation and an expression is that a(n) ______________ contains an equal sign, while a(n) ______________ does not.

3. The process of writing $-3x + x$ as $2x$ is called ______________ the expression.

4. For the equation $-5x = -20$, the process of finding that 4 is the solution is called ______________ the equation.

5. By the ______________ property of equality, $y = 8$ and $3 \cdot y = 3 \cdot 8$ are equivalent equations.

Martin-Gay Interactive Videos

See Video 8.3

Watch the section lecture video and answer the following questions.

Objective A **6.** In solving Example 1, how is the multiplication property of equality used?

7. When solving Example 5, what was done before applying the multiplication property of equality?

8.3 Exercise Set MyLab Math

Objective A *Solve. See Examples 1 through 5.*

1. $5x = 20$

2. $6y = 48$

3. $-3z = 12$

4. $-2x = 26$

5. $0.4y = -12$

6. $0.8x = -8$

7. $2z = -34$

8. $7y = -21$

9. $-0.3x = -15$

10. $-0.4z = -16$

11. $10 = \frac{2}{5}x$

12. $27 = \frac{3}{7}x$

13. $\frac{1}{6}y = -5$

14. $\frac{1}{8}y = -3$

15. $\frac{5}{6}x = \frac{5}{18}$

16. $\frac{4}{7}y = \frac{8}{21}$

17. $-\frac{2}{9}z = \frac{4}{27}$

18. $-\frac{3}{4}v = \frac{9}{14}$

Solve. First combine any like terms on each side of the equation. See Examples 6 and 7.

19. $2w - 12w = 40$

20. $-8y + y = 35$

21. $16 = 10t - 8t$

22. $100 = 15y - 5y$

23. $2z = 1.2 + 1.4$

24. $3x = 1.1 + 0.7$

25. $4 - 10 = -3z$

26. $12 - 20 = -4x$

Mixed Practice *Solve. See Examples 1 through 8.*

27. $-7x = 0$

28. $-20y = 0$

29. $0.4 = -8z$

30. $0.5 = -20x$

31. $\frac{8}{5}t = -\frac{3}{8}$

32. $\frac{7}{4}r = -\frac{2}{7}$

33. $-\frac{3}{5}x = -\frac{6}{15}$

34. $-\frac{6}{7}y = -\frac{1}{14}$

35. $-3.6 = -0.9u + 0.3u$

36. $-5.4 = -1.4y + 1.3y$

37. $5 - 5 = 2x + 7x$

38. $12 + (-12) = 7x + 8x$

39. $-42 + 20 = -2x + 13x$

40. $-4y + 9y = -20 + 15$

41. $-3x - 3x = 50 - 2$

42. $5y - 9y = -14 + (-14)$

43. $23x - 25x = 7 - 9$

44. $6x - 8x = 12 - 22$

45. $\frac{1}{4}x - \frac{5}{8}x = 20 - 47$

46. $\frac{1}{2}x - \frac{4}{5}x = 10 - 19$

47. $18 - 11 = \frac{x}{5}$

48. $14 - 9 = \frac{x}{12}$

49. $\frac{x}{-4} = 1 - (-6)$

50. $\frac{y}{-6} = 6 - (-1)$

Review

Evaluate each expression when $x = 5$. See Section 8.1.

51. $3x + 10$

52. $40x$

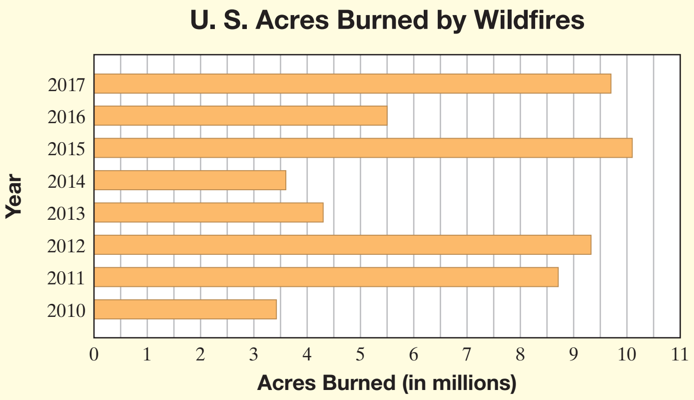
53. $\frac{x - 3}{2}$

54. $7x - 20$

55. $\frac{3x + 5}{x - 7}$

56. $\frac{2x - 1}{x - 8}$

The bar graph shows the number of acres burned by wildfires in the United States in recent years. Use the bar graph to answer Exercises 57 through 60. See Section 7.1.



Source: National Interagency Fire Center

57. During which year shown is the number of acres burned by wildfires the greatest?

58. During which year shown is the number of acres burned by wildfires the least?

59. Use the length of the bar to estimate the number of acres burned by wildfires in 2017.

60. Describe any trends shown in this graph.

Concept Extensions

What operation is appropriate for solving each equation: addition or division? See the Concept Check in this section.

61. $12 = x - 5$

62. $12 = -5x$

63. $-7x = 21$

64. $-7 + x = 21$

65. Why does the multiplication property of equality not allow us to divide both sides of an equation by zero?

66. Is the equation $-x = 6$ solved for the variable? Explain why or why not.

The equation $d = r \cdot t$ describes the relationship between distance d in miles, rate r in miles per hour, and time t in hours. If necessary, round answers to the nearest tenth.

67. The distance between New Orleans, Louisiana, and Memphis, Tennessee, by road is 390 miles. How long will it take to drive from New Orleans to Memphis if the driver maintains a speed of 60 miles per hour? (*Source: World Almanac*)

68. The distance between Boston, Massachusetts, and Milwaukee, Wisconsin, by road is 1050 miles. How long will it take to drive from Boston to Milwaukee if the driver maintains a speed of 55 miles per hour? (*Source: World Almanac*)

69. The distance between Cleveland, Ohio, and Indianapolis, Indiana, by road is 294 miles. At what speed should a driver drive if he or she would like to make the trip in 5 hours? (*Source: World Almanac*)

70. The distance between St. Louis, Missouri, and Minneapolis, Minnesota, by road is 552 miles. If it took 9 hours to drive from St. Louis to Minneapolis, what was the driver's average speed? (*Source: World Almanac*)

Solve.

71. $-0.025x = 91.2$

72. $3.6y = -1.259 - 3.277$

73. $\frac{y}{72} = -86 - (-1029)$

74. $\frac{x}{-13} = 4^6 - 5^7$

75. $\frac{x}{-2} = 5^2 - |-10| - (-9)$

76. $\frac{y}{10} = (-8)^2 - |20| + (-2)^2$

Expressions and Equations

For the table below, identify each as an expression or an equation.

	Expression or Equation	
1.	$7x - 5y + 14$	
2.	$7x = 35 + 14$	
3.	$3(x - 2) = 5(x + 1) - 17$	
4.	$-9(2x + 1) - 4(x - 2) + 14$	

Fill in each blank with "simplify" or "solve."

5. To ________ an expression, we combine any like terms.

6. To ________ an equation, we use the properties of equality to find any value of the variable that makes the equation a true statement.

Evaluate each expression when $x = -1$ and $y = 3$.

7. $y - x$

8. $\frac{8y}{4x}$

9. $5x + 2y$

10. $\frac{y^2 + x}{2x}$

Simplify each expression by combining like terms.

11. $7x + x$

12. $6y - 10y$

13. $2a + 5a - 9a - 2$

14. $3x - y + 4 - 5x + 4y - 11$

Multiply and simplify if possible.

15. $-2(4x + 7)$

16. $-3(2x - 10)$

Answers

1. ________

2. ________

3. ________

4. ________

5. ________

6. ________

7. ________

8. ________

9. ________

10. ________

11. ________

12. ________

13. ________

14. ________

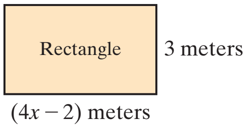
15. ________

16. ________

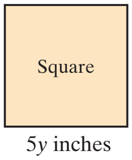
17. $5(y+2)-20$

18. $12x+3(x-6)-13$

△ **19.** Find the area.



△ **20.** Find the perimeter.



Solve and check.

21. $x+7=20$

22. $-11=x-2$

23. $n-\frac{2}{5}=\frac{3}{10}$

24. $-7y=0$

25. $12=11x-14x$

26. $\frac{3}{5}x=15$

27. $x-1.2=-4.5+2.3$

28. $8y+7y=-45$

29. $6-(-5)=x+5$

30. $-0.2m=-1.6$

31. $-\frac{2}{3}n=\frac{6}{11}$

32. $11x=55$

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

22. ____________

23. ____________

24. ____________

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

8.4 Solving Equations Using Addition and Multiplication Properties

Objective A Solving Equations Using Addition and Multiplication Properties

We will now solve equations using more than one property of equality. To solve an equation such as $2x - 6 = 18$, we will first get the variable term $2x$ alone on one side of the equation.

Example 1 Solve: $2x - 6 = 18$

Solution: We start by adding 6 to both sides to get the variable term $2x$ alone.

$$2x - 6 = 18$$

$$2x - 6 + 6 = 18 + 6 \quad \text{Add 6 to both sides.}$$

$$2x = 24 \quad \text{Simplify.}$$

To finish solving, we divide both sides by 2.

$$\frac{2x}{2} = \frac{24}{2} \quad \text{Divide both sides by 2.}$$

$$1 \cdot x = 12 \quad \text{Simplify.}$$

$$x = 12$$

Check:

$$2x - 6 = 18$$

$$2(12) - 6 \stackrel{?}{=} 18 \quad \text{Replace } x \text{ with 12 and simplify.}$$

$$24 - 6 \stackrel{?}{=} 18$$

$$18 \stackrel{?}{=} 18 \quad \text{True}$$

The solution is 12.

Work Practice 1

Helpful Hint

Make sure you understand which property to use to solve an equation.

Addition

$$x + 2 = 10$$

To undo addition of 2, we subtract 2 from both sides.

$$x + 2 - 2 = 10 - 2 \quad \text{Use addition property of equality.}$$

$$x = 8$$

Check:

$$x + 2 = 10$$

$$8 + 2 \stackrel{?}{=} 10$$

$$10 \stackrel{?}{=} 10 \quad \text{True}$$

Understood multiplication

$$2x = 10$$

To undo multiplication of 2, we divide both sides by 2.

$$\frac{2x}{2} = \frac{10}{2} \quad \text{Use multiplication property of equality.}$$

$$x = 5$$

Check:

$$2x = 10$$

$$2 \cdot 5 \stackrel{?}{=} 10$$

$$10 \stackrel{?}{=} 10 \quad \text{True}$$

Objectives

A Solve Equations Using Addition and Multiplication Properties.

B Solve Equations Containing Parentheses.

C Write Numerical Sentences as Equations.

Practice 1

Solve: $5y - 8 = 17$

Answer

1. 5

Practice 2

Solve: $7 - y + 3 = 20 - (-25)$

Example 2 Solve: $17 - x + 3 = 15 - (-6)$

Solution: First we simplify each side of the equation.

$$\begin{aligned} 17 - x + 3 &= 15 - (-6) \\ 20 - x &= 21 && \text{Combine like terms on each side of the equation.} \end{aligned}$$

Next, we get the variable term alone on one side of the equation.

$$\begin{aligned} 20 - x - 20 &= 21 - 20 && \text{Subtract 20 from both sides.} \\ -1x &= 1 && \text{Simplify. Recall that } -x \text{ means } -1x. \\ \frac{-1x}{-1} &= \frac{1}{-1} && \text{Divide both sides by } -1. \\ 1 \cdot x &= -1 && \text{Simplify.} \\ x &= -1 \end{aligned}$$

Check:

$$\begin{aligned} 17 - x + 3 &= 15 - (-6) \\ 17 - (-1) + 3 &\stackrel{?}{=} 15 - (-6) \\ 21 &\stackrel{?}{=} 21 && \text{True} \end{aligned}$$

The solution is -1.

Work Practice 2

Practice 3

Solve: $11 = \frac{3}{4}y + 20$

Example 3 Solve: $1 = \frac{2}{3}x + 7$

Solution: Subtract 7 from both sides to get the variable term alone.

$$\begin{aligned} 1 - 7 &= \frac{2}{3}x + 7 - 7 && \text{Subtract 7 from both sides.} \\ -6 &= \frac{2}{3}x && \text{Simplify.} \\ \frac{3}{2} \cdot -6 &= \frac{3}{2} \cdot \frac{2}{3}x && \text{Multiply both sides by } \frac{3}{2}. \\ \frac{3}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{-3}{\cancel{-6}}}{1} &= 1 \cdot x && \text{Simplify.} \\ -9 &= x && \text{Simplify.} \end{aligned}$$

Helpful Hint Don't forget that we can get the variable alone on either side of the equation.

Check to see that the solution is -9.

Work Practice 3

If an equation contains variable terms on both sides, we use the addition property of equality to get all the variable terms on one side and all the constants, or numbers, on the other side.

Answers

2. -35 **3.** -12

Example 4 Solve: $3a - 6 = a + 4$

Solution:

$$\begin{aligned} 3a - 6 &= a + 4 \\ 3a - 6 + 6 &= a + 4 + 6 && \text{Add 6 to both sides.} \\ 3a &= a + 10 && \text{Simplify.} \\ 3a - a &= a + 10 - a && \text{Subtract } a \text{ from both sides.} \\ 2a &= 10 && \text{Simplify.} \\ \frac{2a}{2} &= \frac{10}{2} && \text{Divide both sides by 2.} \\ a &= 5 && \text{Simplify.} \end{aligned}$$

Check to see that the solution is 5.

Work Practice 4

Example 5 Solve: $7x + 3.2 = 4x - 1.6$

Solution:

$$\begin{aligned} 7x + 3.2 &= 4x - 1.6 \\ 7x + 3.2 - 3.2 &= 4x - 1.6 - 3.2 && \text{Subtract 3.2 from both sides.} \\ 7x &= 4x - 4.8 && \text{Simplify.} \\ 7x - 4x &= 4x - 4.8 - 4x && \text{Subtract } 4x \text{ from both sides.} \\ 3x &= -4.8 && \text{Simplify.} \\ \frac{3x}{3} &= \frac{-4.8}{3} && \text{Divide both sides by 3.} \\ x &= -1.6 && \text{Simplify.} \end{aligned}$$

Check to see that -1.6 is the solution.

Work Practice 5

Objective B Solving Equations Containing Parentheses

If an equation contains parentheses, we must first use the distributive property to remove them.

Example 6 Solve: $7(x - 2) = 9x - 6$

Solution: First we apply the distributive property.

$$\begin{aligned} 7(x - 2) &= 9x - 6 \\ 7x - 14 &= 9x - 6 && \text{Apply the distributive property.} \end{aligned}$$

Next, we move variable terms to one side of the equation and constants to the other side.

(*Continued on next page*)

Practice 4

Solve: $9x - 12 = x + 4$

Practice 5

Solve: $8x + 4.2 = 10x - 11.6$

Practice 6

Solve: $6(a - 5) = 8a - 12$

Answers

4. 2 **5.** 7.9 **6.** -9

$$\begin{aligned}
7x - 14 - 9x &= 9x - 6 - 9x && \text{Subtract } 9x \text{ from both sides.}\\
-2x - 14 &= -6 && \text{Simplify.}\\
-2x - 14 + 14 &= -6 + 14 && \text{Add 14 to both sides.}\\
-2x &= 8 && \text{Simplify.}\\
\frac{-2x}{-2} &= \frac{8}{-2} && \text{Divide both sides by } -2.\\
x &= -4 && \text{Simplify.}
\end{aligned}$$

Check to see that −4 is the solution.

Work Practice 6

You may want to use the steps shown below to solve equations.

Steps for Solving an Equation

Step 1: If parentheses are present, use the distributive property.

Step 2: Combine any like terms on each side of the equation.

Step 3: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.

Step 4: Use the multiplication property of equality to divide both sides by the numerical coefficient of the variable to solve.

Step 5: Check the solution in the *original equation*.

Practice 7

Solve: $4(2x - 3) + 4 = 0$

Example 7 Solve: $3(2x - 6) + 6 = 0$

Solution:

$$\begin{aligned}
&& 3(2x - 6) + 6 &= 0 &&\\
&\textbf{Step 1:} & 6x - 18 + 6 &= 0 && \text{Apply the distributive property.}\\
&\textbf{Step 2:} & 6x - 12 &= 0 && \text{Combine like terms on the left side of the equation.}\\
&\textbf{Step 3:} & 6x - 12 + 12 &= 0 + 12 && \text{Add 12 to both sides.}\\
&& 6x &= 12 && \text{Simplify.}\\
&\textbf{Step 4:} & \frac{6x}{6} &= \frac{12}{6} && \text{Divide both sides by 6.}\\
&& x &= 2 && \text{Simplify.}
\end{aligned}$$

Step 5:

$$\begin{aligned}
&\textbf{Check:} & 3(2x - 6) + 6 &= 0 &&\\
&& 3(2 \cdot 2 - 6) + 6 &\stackrel{?}{=} 0 &&\\
&& 3(4 - 6) + 6 &\stackrel{?}{=} 0 &&\\
&& 3(-2) + 6 &\stackrel{?}{=} 0 &&\\
&& -6 + 6 &\stackrel{?}{=} 0 &&\\
&& 0 &\stackrel{?}{=} 0 && \text{True}
\end{aligned}$$

The solution is 2.

Work Practice 7

Answer
7. 1

Objective C Writing Sentences as Equations

Next, we practice translating sentences into equations. Below are key words and phrases that translate to an equal sign:

Key Words and Phrases	Examples	Symbols
equals	3 equals 2 plus 1	$3 = 2 + 1$
gives	the quotient of 10 and −5 gives −2	$\frac{10}{-5} = -2$
is/was	17 minus 12 is 5	$17 - 12 = 5$
yields	11 plus 2 yields 13	$11 + 2 = 13$
amounts to	twice −15 amounts to −30	$2(-15) = -30$
is equal to	−24 is equal to 2 times −12	$-24 = 2(-12)$

Example 8 Translate each sentence into an equation.

a. The product of 7 and 6 is 42.

b. Twice the sum of 3 and 5 is equal to 16.

c. The quotient of −45 and 5 yields −9.

Solution:

$$\begin{array}{lccc} \textbf{a.}\ \text{In words:} & \text{the product of 7 and 6} & \text{is} & 42 \\ & \downarrow & \downarrow & \downarrow \\ \text{Translate:} & 7\cdot 6 & = & 42 \end{array}$$

$$\begin{array}{lcccc} \textbf{b.}\ \text{In words:} & \text{twice} & \text{the sum of 3 and 5} & \text{is equal to} & 16 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translate:} & 2 & (3 + 5) & = & 16 \end{array}$$

$$\begin{array}{lccc} \textbf{c.}\ \text{In words:} & \text{the quotient of } -45 \text{ and } 5 & \text{yields} & -9 \\ & \downarrow & \downarrow & \downarrow \\ \text{Translate:} & \frac{-45}{5} & = & -9 \end{array}$$

Work Practice 8

Practice 8

Translate each sentence into an equation.

a. The difference of 110 and 80 is 30.

b. The product of 3 and the sum of −9 and 11 amounts to 6.

c. The quotient of 24 and −6 yields −4.

Answers

8. a. $110 - 80 = 30$ **b.** $3(-9 + 11) = 6$ **c.** $\frac{24}{-6} = -4$

Calculator Explorations Checking Possible Solutions

A calculator can be used to check possible solutions of equations. To do this, replace the variable by the possible solution and evaluate each side of the equation separately. For example, to see whether 7 is a solution of the equation $52x = 15x + 259$, replace x with 7 and use your calculator to evaluate each side separately.

Equation: $52x = 15x + 259$

$52 \cdot 7 \overset{?}{=} 15 \cdot 7 + 259$

Evaluate left side: $\boxed{52}$ $\boxed{\times}$ $\boxed{7}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$.

Display: $\boxed{364}$.

Evaluate right side: $\boxed{15}$ $\boxed{\times}$ $\boxed{7}$ $\boxed{+}$ $\boxed{259}$ then $\boxed{=}$ or $\boxed{\text{ENTER}}$. Display: $\boxed{364}$.

Since the left side equals the right side, 7 is a solution of the equation $52x = 15x + 259$.

Use a calculator to determine whether the numbers given are solutions of each equation.

1. $76(x - 25) = -988;\quad 12$
2. $-47x + 862 = -783;\quad 35$
3. $x + 562 = 3x + 900;\quad -170$
4. $55(x + 10) = 75x + 910;\quad -18$
5. $29x - 1034 = 61x - 362;\quad -21$
6. $-38x + 205 = 25x + 120;\quad 25$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once.

addition	multiplication	combine like terms
$5(2x + 6) - 1 = 39$	$3x - 9 + x - 16$	distributive

1. An example of an expression is ________________ while an example of an equation is ________________.

2. To solve $\frac{x}{-7} = -10$, we use the ________________ property of equality.

3. To solve $x - 7 = -10$, we use the ________________ property of equality.

Use the order of the Steps for Solving an Equation in this section to answer Exercises 4 through 6.

4. To solve $9x - 6x = 10 + 6$, first ________________.

5. To solve $5(x - 1) = 25$, first use the ________________ property.

6. To solve $4x + 3 = 19$, first use the ________________ property of equality.

Martin-Gay Interactive Videos

See Video 8.4

Watch the section lecture video and answer the following questions.

Objective A **7.** In Example 1, the number 1 is subtracted from the left side of the equation. What property tells us we must also subtract 1 from the right side? Why is it important to do the same thing to both sides?

Objective B **8.** From Example 3, what is the first step when solving an equation that contains parentheses? What property do we use to perform this step?

Objective C **9.** What word or phrase translates to "equals" in Example 5? In Example 6?

8.4 Exercise Set MyLab Math

Objective A *Solve each equation. See Examples 1 through 5.*

1. $2x - 6 = 0$

2. $3y - 12 = 0$

3. $3n + 3.6 = 9.3$

4. $4z + 0.8 = 5.2$

5. $6 - n = 10$

6. $7 - y = 9$

7. $-\frac{2}{5}x + 19 = -21$

8. $-\frac{3}{7}y - 14 = 7$

9. $1.7 = 2y + 9.5$

10. $-5.1 = 3x + 2.4$

11. $2n + 8 = 0$

12. $8w + 40 = 0$

13. $3x - 7 = 4x + 5$

14. $7x - 1 = 8x + 4$

15. $10x + 15 = 6x + 3$

16. $5x - 3 = 2x - 18$

17. $9 - 3x = 14 + 2x$

18. $4 - 7m = -3m + 4$

19. $-1.4x - 2 = -1.2x + 7$

20. $5.7y + 14 = 5.4y - 10$

21. $x + 20 + 2x = -10 - 2x - 15$

22. $2x + 10 + 3x = -12 - x - 20$

23. $40 + 4y - 16 = 13y - 12 - 3y$

24. $19x - 2 - 7x = 31 + 6x - 15$

Objective B *Solve each equation. See Examples 6 and 7.*

25. $-2(y + 4) = 2$

26. $-1(y + 3) = 10$

27. $3(x - 1) - 12 = 0$

28. $2(x + 5) + 8 = 0$

29. $35 - 17 = 3(x - 2)$

30. $22 - 42 = 4(x - 1)$

31. $2(y - 3) = y - 6$

32. $3(z + 2) = 5z + 6$

33. $2t - 1 = 3(t + 7)$

34. $-4 + 3c = 4(c + 2)$

35. $3(5c + 1) - 12 = 13c + 3$

36. $4(3t + 4) - 20 = 3 + 5t$

Mixed Practice (*Sections 8.1, 8.2, 8.3, and 8.4*) *Solve each equation. See Examples 1 through 7.*

37. $-4x = 44$

38. $-3x = 51$

39. $x + 9 = 2$

40. $y - 6 = -11$

41. $8 - b = 13$

42. $7 - z = 15$

43. $3r + 4 = 19$

44. $5m + 1 = 46$

45. $2x - 1 = -7$

46. $3t - 2 = -11$

47. $7 = 4c - 1$

48. $9 = 2b - 5$

49. $9a + 29 = -7$

50. $10 + 4v = -6$

51. $0 = 4x + 4$

52. $0 = 5y + 5$

53. $11(x - 2) = 22$

54. $5(a - 4) = 20$

55. $-7c + 1 = -20$

56. $-2b + 5 = -7$

57. $3(x - 5) = -7 - 11$

58. $4(x - 2) = -20 - 4$

59. $-5 + 7k = -13 + 8k$

60. $-7 + 9d = -17 + 10d$

61. $4x + 3 = 2x + 11$

62. $6y - 8 = 3y + 7$

63. $-8(n + 2) + 17 = -6n - 5$

64. $-10(x + 1) + 2 = -x + 10$

65. $\frac{3}{8}x + 14 = \frac{5}{8}x - 2$

66. $\frac{2}{7}x - 9 = \frac{5}{7}x - 15$

67. $10 + 5(z - 2) = -4z + 1$

68. $20 + 4(w - 5) = 5 - 2w$

69. $\frac{5}{8}a = \frac{1}{8}a + \frac{3}{4}$

70. $\frac{4}{9}a = \frac{1}{9}a + \frac{5}{6}$

71. $7(6 + w) = 6(w - 2)$

72. $6(5 + c) = 5(c - 4)$

73. $3 + 2(2n - 5) = 1$

74. $5 + 4(3x - 2) = 21$

75. $2(3z - 2) - 2(5 - 2z) = 4$

76. $2(3w + 7) - 4(5 - 2w) = 6$

77. $-20 - (-50) = \frac{x}{9}$

78. $-2 - 10 = \frac{z}{10}$

79. $12 + 5t = 6(t + 2)$

80. $4 + 3c = 2(c + 2)$

81. $3(5c - 1) - 2 = 13c + 3$

82. $4(2t + 5) - 21 = 7t - 6$

83. $10 + 5(z - 2) = 4z + 1$

84. $14 + 4(w - 5) = 6 - 2w$

Objective C *Write each sentence as an equation. See Example 8.*

85. The sum of −42 and 16 is −26.

86. The difference of −30 and 10 equals −40.

87. The product of −5 and −29 gives 145.

88. The quotient of −16 and 2 yields −8.

89. Three times the difference of −14 and 2 amounts to −48.

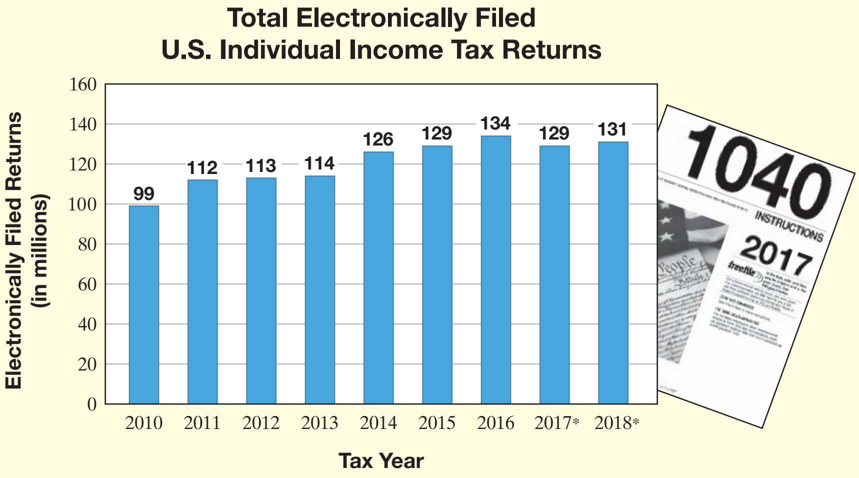
90. The product of −2 and the sum of 3 and 12 is −30.

91. The quotient of 100 and twice 50 is equal to 1.

92. Seventeen subtracted from −12 equals −29.

Review

The following bar graph shows the number of U.S. federal individual income tax returns that are filed electronically during the years shown using efile. Use this graph to answer Exercises 93 through 96. Write number answers in millions and in standard form. See Section 7.1.



Source: efile; *projected

93. Determine the number of electronically filed returns for 2014.

94. Determine the number of electronically filed returns for 2012.

95. By how much did the number of electronically filed returns increase from 2013 to 2016?

96. Describe any trends shown in this graph.

Concept Extensions

Using the steps for solving an equation, choose the next operation for solving the given equation.

97. $2x - 5 = -7$

- **a.** Add 7 to both sides.
- **b.** Add 5 to both sides.
- **c.** Divide both sides by 2.

98. $3x + 2x = -x - 4$

- **a.** Add 4 to both sides.
- **b.** Subtract $2x$ from both sides.
- **c.** Add $3x$ and $2x$.

99. $-3x = -12$

- **a.** Divide both sides by -3.
- **b.** Add 12 to both sides.
- **c.** Add $3x$ to both sides.

100. $9 - 5x = 15$

- **a.** Divide both sides by -5.
- **b.** Subtract 15 from both sides.
- **c.** Subtract 9 from both sides.

A classmate shows you steps for solving an equation. The solution does not check, but the classmate is unable to find the error. For each set of steps, check the solution, find the error, and correct it.

101.

$$\begin{aligned} 2(3x - 5) &= 5x - 7 \\ 6x - 5 &= 5x - 7 \\ 6x - 5 + 5 &= 5x - 7 + 5 \\ 6x &= 5x - 2 \\ 6x - 5x &= 5x - 2 - 5x \\ x &= -2 \end{aligned}$$

102.

$$\begin{aligned} 37x + 1 &= 9(4x - 7) \\ 37x + 1 &= 36x - 7 \\ 37x + 1 - 1 &= 36x - 7 - 1 \\ 37x &= 36x - 8 \\ 37x - 36x &= 36x - 8 - 36x \\ x &= -8 \end{aligned}$$

Solve.

103. $(-8)^2 + 3x = 5x + 4^3$

104. $3^2 \cdot x = (-9)^3$

105. $2^3(x + 4) = 3^2(x + 4)$

106. $x + 45^2 = 54^2$

107. A classmate tries to solve $3x = 39$ by subtracting 3 from both sides of the equation. Will this step solve the equation for x? Why or why not?

108. A classmate tries to solve $2 + x = 20$ by dividing both sides by 2. Will this step solve the equation for x? Why or why not?

The equation $C = \frac{5}{9}(F - 32)$ *gives the relationship between Celsius temperatures C and Fahrenheit temperatures F.*

109. The highest recorded temperature in Australia occurred in January 1960 at Oodnadatta, South Australia. The temperature reached 50.7°C. Use the given equation to convert this temperature to degrees Fahrenheit. (*Source:* World Weather Centre at Perth)

110. The highest recorded temperature in Africa occurred in July 1931 at Kebili, Tunisia. The temperature reached 55.0°C. Use the given equation to convert this temperature to degrees Fahrenheit. (*Source:* World Meteorological Organization)

111. The lowest recorded temperature in Australia occurred in June 1994 at Charlotte Pass, New South Wales. The temperature plummeted to −23.0°C. Use the given equation to convert this temperature to degrees Fahrenheit. (*Source:* World Weather Centre at Perth)

112. The lowest recorded temperature in North America occurred in February 1947 at Snag, Canada. The temperature plummeted to −63.0°C. Use the given equation to convert this temperature to degrees Fahrenheit. (*Source:* World Weather Centre at Perth)

8.5 Equations and Problem Solving

Objectives

A Write Phrases as Algebraic Expressions.

B Write Sentences as Equations.

C Use Problem-Solving Steps to Solve Problems.

Objective A Writing Phrases as Algebraic Expressions

Now that we have practiced solving equations for a variable, we can extend considerably our problem-solving skills. We begin by writing phrases as algebraic expressions using the following key words and phrases as a guide:

Addition	Subtraction	Multiplication	Division	Equal Sign
sum	difference	product	quotient	equals
plus	minus	times	divided by	gives
added to	subtracted from	multiply	into	is/was
more than	less than	twice	per	yields
increased by	decreased by	of		amounts to
total	less	double		is equal to

Practice 1

Write each phrase as an algebraic expression. Use x to represent "a number."

a. twice a number

b. 8 increased by a number

c. 10 minus a number

d. 10 subtracted from a number

e. the quotient of 3 and a number

f. the sum of 14 and triple a number

Example 1

Write each phrase as an algebraic expression. Use x to represent "a number."

a. 7 increased by a number

b. 15 decreased by a number

c. the product of 2 and a number

d. the quotient of a number and 5

e. 2 subtracted from a number

f. the sum of 9 and twice a number

Solution:

a.

In words:	7	increased by	a number
	↓	↓	↓
Translate:	7	+	x

b.

In words:	15	decreased by	a number
	↓	↓	↓
Translate:	15	−	x

c.

In words:		the product of		
	2	and	a number	
	↓	↓	↓	
Translate:	2	·	x	or $2x$

d.

In words:		the quotient of		
	a number	and	5	
	↓	↓	↓	
Translate:	x	÷	5	or $\frac{x}{5}$

e.

In words:	2	subtracted from	a number
Translate:	x	−	2

Answers

1. a. $2x$ **b.** $8 + x$ **c.** $10 - x$ **d.** $x - 10$ **e.** $3 \div x$ or $\frac{3}{x}$ **f.** $14 + 3x$

f. In words: the sum of

In words:	9	and	twice a number
Translate:	9	+	$2x$

Work Practice 1

Objective B Writing Sentences as Equations

Now that we have practiced writing phrases as algebraic expressions, let's write sentences as equations. You may want to first study the key words and phrases chart to review some key words and phrases that translate to an equal sign.

Example 2 Write each sentence as an equation. Use x to represent "a number."

a. Nine increased by a number is 5.

b. Twice a number equals −10.

c. A number minus 6 amounts to 168.

d. Three times the sum of a number and 5 is −30.

e. The quotient of twice a number and 8 is equal to 2.

Solution:

a.

In words:	nine	increased by	a number	is	5
Translate:	9	+	x	=	5

b.

In words:	twice a number	equals	−10
Translate:	$2x$	=	−10

c.

In words:	a number	minus	6	amounts to	168
Translate:	x	−	6	=	168

d.

In words:	three times	the sum of a number and 5	is	−30
Translate:	3	$(x + 5)$	=	−30

e. In words: the quotient of

In words:	twice a number	and	8	is equal to	2
Translate:	$2x$	÷	8	=	2

or $\frac{2x}{8} = 2$

Work Practice 2

Practice 2

Write each sentence as an equation. Use x to represent "a number."

a. Five times a number is 20.

b. The sum of a number and −5 yields 14.

c. Ten subtracted from a number amounts to −23.

d. Five times the difference of a number and 7 is equal to −8.

e. The quotient of triple a number and 5 gives 1.

Answers

2. a. $5x = 20$ **b.** $x + (-5) = 14$
c. $x - 10 = -23$ **d.** $5(x - 7) = -8$
e. $\frac{3x}{5} = 1$ or $3x \div 5 = 1$

Objective C Using Problem-Solving Steps to Solve Problems

Our main purpose for studying arithmetic and algebra is to solve problems. The same problem-solving steps that have been used throughout this text are used in this section also. Those steps are next.

Problem-Solving Steps

1. UNDERSTAND the problem. During this step, become comfortable with the problem. Some ways of doing this are as follows:
    - Read and reread the problem.
    - Construct a drawing.
    - Propose a solution and check. Pay careful attention to how you check your proposed solution. This will help when writing an equation to model the problem.
    - Choose a variable to represent an unknown. Use this variable to represent any other unknowns.
2. TRANSLATE the problem into an equation.
3. SOLVE the equation.
4. INTERPRET the results: *Check* the proposed solution in the stated problem and *state* your conclusion.

The first problem that we solve consists of finding an unknown number.

Practice 3

Translate "The difference of a number and 2 equals 6 added to three times the number" into an equation and solve.

Example 3 Finding an Unknown Number

Twice a number plus 3 is the same as the number minus 6. Find the unknown number.

Solution:

1. UNDERSTAND the problem. To do so, we read and reread the problem.

Let's propose a solution to help us understand. Suppose the unknown number is 5. Twice this number plus 3 is $2 \cdot 5 + 3$ or 13. Is this the same as the number minus 6, or $5 - 6$, or -1? Since 13 is not the same as -1, we know that 5 is not the solution. However, remember that the purpose of proposing a solution is not to guess correctly, but to better understand the problem.

Now let's choose a variable to represent the unknown. Let's let

x = unknown number

2. TRANSLATE the problem into an equation.

In words:	twice a number	plus 3	is the same as	the number minus 6
	↓	↓	↓	↓
Translate:	$2x$	$+ 3$	$=$	$x - 6$

3. SOLVE the equation. To solve the equation, we first subtract x from both sides.

$$\begin{aligned} 2x + 3 &= x - 6 \\ 2x + 3 - x &= x - 6 - x \\ x + 3 &= -6 && \text{Simplify.} \\ x + 3 - 3 &= -6 - 3 && \text{Subtract 3 from both sides.} \\ x &= -9 && \text{Simplify.} \end{aligned}$$

Answer

3. $x - 2 = 6 + 3x; -4$

4. INTERPRET. **Check:** To Check, use the proposed solution in the stated problem. Twice "-9" is -18 and $-18 + 3$ is -15. This is equal to the number minus 6, or "-9" $- 6$, or -15. **State:** The unknown number is -9.

Work Practice 3

✓ **Concept Check** Suppose you have solved an equation involving perimeter to find the length of a rectangular table. Explain why you would want to recheck your math if you obtain the result of -5.

Example 4 Determining Distances

The distance by road from Chicago, Illinois, to Los Angeles, California, is 1091 miles *more* than the distance from Chicago to Boston, Massachusetts. If the total of these two distances is 3017 miles, find the distance from Chicago to Boston. (*Source: World Almanac*)

Solution:

1. UNDERSTAND the problem. We read and reread the problem.

Let's propose and check a solution to help us better understand the problem. Suppose the distance from Chicago to Boston is 600 miles. Since the distance from Chicago to Los Angeles is 1091 *more* miles, then this distance is $600 + 1091 = 1691$ miles. With these numbers, the total of the distances is $600 + 1691 = 2291$ miles. This is less than the given total of 3017 miles, so we are incorrect. But not only do we have a better understanding of this exercise, we also know that the distance from Boston to Chicago is greater than 600 miles since this proposed solution led to a total too small. Now let's choose a variable to represent an unknown. Then we'll use this variable to represent any other unknown quantities. Let

x = distance from Chicago to Boston

Then

$x + 1091$ = distance from Chicago to Los Angeles

since that distance is 1091 more miles.

2. TRANSLATE the problem into an equation.

In words:	Chicago to Boston distance	+	Chicago to Los Angeles distance	=	total miles
	↓		↓		↓
Translate:	x	+	$\overbrace{x + 1091}$	=	3017

3. SOLVE the equation:

$$\begin{aligned} x + x + 1091 &= 3017 \\ 2x + 1091 &= 3017 && \text{Combine like terms.} \\ 2x + 1091 - 1091 &= 3017 - 1091 && \text{Subtract 1091 from both sides.} \\ 2x &= 1926 && \text{Simplify.} \\ \frac{2x}{2} &= \frac{1926}{2} && \text{Divide both sides by 2.} \\ x &= 963 && \text{Simplify.} \end{aligned}$$

4. INTERPRET.

Check: Since x represents the distance from Chicago to Boston, this is 963 miles. The distance from Chicago to Los Angeles is $x + 1091 = 963 + 1091 = 2054$ miles. To check, notice that the total number of

(*Continued on next page*)

Practice 4

The distance by road from Cincinnati, Ohio, to Denver, Colorado, is 71 miles *less* than the distance from Denver to San Francisco, California. If the total of these two distances is 2399 miles, find the distance from Denver to San Francisco.

Answer

4. 1235 miles

✓ **Concept Check Answer**

Length cannot be negative.

miles is 963 + 2054 = 3017 miles, the given total of miles. Also, 2054 is 1091 more miles than 963, so the solution checks.

State: The distance from Chicago to Boston is 963 miles.

Work Practice 4

Practice 5

A woman's $57,000 estate is to be divided so that her husband receives twice as much as her son. How much will each receive?

Example 5 Calculating Separate Costs

A salesperson at an electronics store sold a computer system and software for $2100, receiving four times as much money for the computer system as for the software. Find the price of each.

Solution:

1. UNDERSTAND the problem. We read and reread the problem. Then we choose a variable to represent an unknown. We use this variable to represent any other unknown quantities. We let

   x = the software price
   $4x$ = the computer system price

2. TRANSLATE the problem into an equation.

In words:	software price	and	computer price	is	2100
	↓	↓	↓	↓	↓
Translate:	x	$+$	$4x$	$=$	2100

3. SOLVE the equation:

$$
\begin{aligned}
x + 4x &= 2100 \\
5x &= 2100 && \text{Combine like terms.} \\
\frac{5x}{5} &= \frac{2100}{5} && \text{Divide both sides by 5.} \\
x &= 420 && \text{Simplify.}
\end{aligned}
$$

4. INTERPRET.

   Check: The software sold for $420. The computer system sold for $4x = 4(\$420) = \$1680$. Since $420 + $1680 = $2100, the total price, and $1680 is four times $420, the solution checks.

   State: The software sold for $420, and the computer system sold for $1680.

Work Practice 5

Answer

5. husband: $38,000; son: $19,000

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 8.5

Watch the section lecture video and answer the following questions.

Objective A 1. In Example 2, what phrase translates to subtraction?

Objective B 2. In Example 4, why does the left side of the equation translate to $-20 - x$ and not $x - (-20)$?

Objective C 3. Why are parentheses used in the translation of the left side of the equation in Example 6?

4. In Example 7, the solution to the equation is $x = 37$. Why is this not the solution to the application?

8.5 Exercise Set MyLab Math

Objective A Translating *Write each phrase as a variable expression. Use x to represent "a number." See Example 1.*

1. The sum of a number and five

2. Ten plus a number

3. The total of a number and eight

4. The difference of a number and five hundred

5. Twenty decreased by a number

6. A number less thirty

7. The product of 512 and a number

8. A number times twenty

9. A number divided by 2

10. The quotient of six and a number

11. The sum of seventeen, a number, and the product of five and the number

12. The difference of twice a number, and four

Objective B Translating *Write each sentence as an equation. Use x to represent "a number." See Example 2.*

13. A number added to -5 is -7.

14. Five subtracted from a number equals 10.

15. Three times a number yields 27.

16. The quotient of 8 and a number is -2.

17. A number subtracted from -20 amounts to 104.

18. Two added to twice a number gives -14.

Objectives A B Mixed Practice Translating *Write each phrase as a variable expression or each sentence as an equation. Use x to represent "a number." See Examples 1 and 2.*

19. The product of five and a number

20. The quotient of twenty and a number, decreased by three

21. A number subtracted from 11

22. Twelve subtracted from a number

23. Twice a number gives 108.

24. Five times a number is equal to -75.

25. Fifty decreased by eight times a number

26. Twenty decreased by twice a number

27. The product of 5 and the sum of -3 and a number is -20.

28. Twice the sum of -17 and a number is -14.

Objective C *Translate each to an equation. Then solve the equation. See Example 3.*

29. Three times a number, added to 9, is 33. Find the number.

30. Twice a number, subtracted from 60, is 20. Find the number.

31. The sum of 3, 4, and a number amounts to 16. Find the number.

32. The sum of 7, 9, and a number is 40. Find the number.

33. The difference of a number and 3 is equal to the quotient of 10 and 5. Find the number.

34. Eight decreased by a number equals the quotient of 15 and 5. Find the number.

35. Thirty less a number is equal to the product of 3 and the sum of the number and 6. Find the number.

36. The product of a number and 3 is twice the sum of that number and 5. Find the number.

37. 40 subtracted from five times a number is 8 more than the number. Find the number.

38. Five times the sum of a number and 2 is 11 less than the number times 8. Find the number.

39. Three times the difference of a number and 5 amounts to the quotient of 108 and 12. Find the number.

40. Seven times the difference of a number and 1 gives the quotient of 70 and 10. Find the number.

41. The product of 4 and a number is the same as 30 less twice that same number. Find the number.

42. Twice a number equals 25 less triple that same number. Find the number.

Solve. For Exercises 43 and 44, the solutions have been started for you. See Examples 4 and 5.

43. The Stanley Cup is the trophy given to the Championship National Hockey League Team each year. The Montreal Canadiens hockey team holds the record for the most Stanley Cup wins. The Toronto Maple Leafs have 11 fewer Stanley Cup wins than the Canadiens. If the total number of Stanley Cup wins for these two teams is 37, find the number of wins for each team. (*Source:* National Hockey League)

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let

x = number of Stanley Cup wins for Canadiens

then

$x - 11$ = number of Stanley Cup wins for Maple Leafs

2. TRANSLATE into an equation. (Fill in the blanks below.)

$$\begin{array}{ccccc}\text{wins for Canadiens} & + & \text{wins for Maple Leafs} & = 37\\ \downarrow & & \downarrow & \\ \underline{\qquad} & + & \underline{\qquad} & = 37\end{array}$$

Now, you finish with

3. SOLVE the equation.

4. INTERPRET the results.

44. The New York Yankees hold the record for the most World Series titles. The St. Louis Cardinals have won 16 fewer World Series titles than the Yankees. If the total number of World Series titles for these two teams is 38, find the number of World Series titles for each team.

Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed. Let's let

x = number of World Series titles for Yankees

then

$x - 16$ = number of World Series titles for Cardinals

2. TRANSLATE into an equation. (Fill in the blanks below.)

$$\begin{array}{ccccc}\text{titles for Yankees} & + & \text{titles for Cardinals} & = 38\\ \downarrow & & \downarrow & \\ \underline{\qquad} & + & \underline{\qquad} & = 38\end{array}$$

Now, you finish with

3. SOLVE the equation.

4. INTERPRET the results.

45. A falcon, when diving, can travel five times as fast as a pheasant's top speed. If the total speed for these two birds is 222 miles per hour, find the fastest speed of the falcon and the fastest speed of the pheasant. (*Source: Fantastic Book of Comparisons*)

46. Norway has had three times as many rulers as Liechtenstein. If the total number of rulers for both countries is 56, find the number of rulers for Norway and the number for Liechtenstein.

47. The largest university (by enrollment) is Indira Gandhi National Open University in India, followed by Anadolu University in Turkey. If the enrollment in the Indian university is 1.5 million more students than the Turkish university and their combined enrollment is 5.5 million students, find the enrollment for each university. (*Source:* India Today Magazine)

48. In 2017, McDonald's spent the most money in music-based radio station advertising. This company spent \$12 million more than Home Depot. If the total amount of money spent for these two companies was \$80 million, find the amount of money spent on radio ads by McDonald's and the amount of money spent by Home Depot. (*Source*: Neilsen)

49. The two top-selling Xbox 360 games are *Kinect Adventures* and *Grand Theft Auto V.* A price for *Grand Theft Auto V* is \$18 more than a price for *Kinect Adventures.* If the total of these two prices is \$42, find the price of each game. (*Source:* Gamestop.com and Amazon.com)

50. The U.S. newspaper with the greatest circulation is *USA Today,* followed by *The Wall Street Journal.* If the average daily circulation for *USA Today* is 226 thousand more than the average daily circulation of *The Wall Street Journal* and their combined circulation is 4340 thousand, find the circulation for each paper. (*Source:* USA Today)

51. By air, the distance from New York City to London is 2001 miles *less* than the distance from Los Angeles to Tokyo. If the total of these two distances is 8939 miles, find the distance from Los Angeles to Tokyo.

52. By air, the distance from Melbourne, Australia, to Cairo, Egypt, is 2338 miles *more* than the distance from Madrid, Spain, to Bangkok, Thailand. If the total of these distances is 15,012 miles, find the distance from Madrid to Bangkok.

53. The two NCAA stadiums with the largest capacities are Michigan Stadium (Univ. of Michigan) and Beaver Stadium (Penn State). Michigan Stadium has a capacity of 3329 more than Beaver Stadium. If the combined capacity for the two stadiums is 216,473, find the capacity for each stadium. (*Source:* collegexpress.com)

54. In 2020, China is projected to be the country with the greatest number of visiting tourists. This number is twice the number of tourists projected for Spain. If the total number of tourists for these two countries is projected to be 210 million, find the number projected for each. (*Source: The State of the World Atlas* by Dan Smith)

55. California contains the largest state population of native Americans. This population is five times the native American population of Washington state. If the total of these two populations is 792 thousand, find the native American population in each of these two states. (*Source:* U.S. Census Bureau)

56. During 2017, twice as many motor vehicles were manufactured in the United States than in Germany. If the total number of vehicles manufactured in these two countries was approximately 16,800,000, find the number manufactured in the United States and the number manufactured in Germany. (*Source:* Organization of Motor Vehicle Manufacturers)

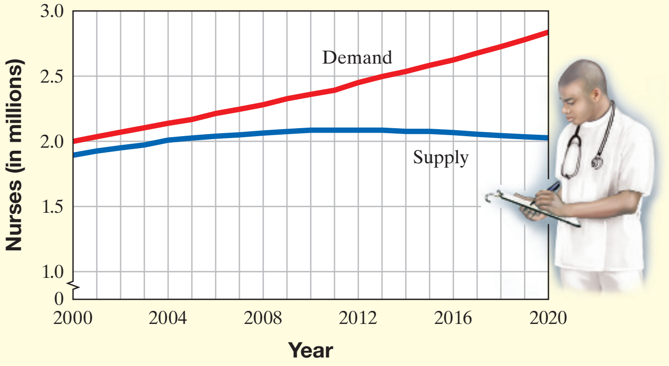
57. During the 2017 Men's NCAA Division 1 basketball championship game, the North Carolina Tar Heels scored 6 points more than the Gonzaga Bulldogs. Together, both teams scored 136 points. How many points did the 2017 Champion North Carolina Tar Heels score during the game? (*Source:* National Collegiate Athletic Association)

58. During the 2017 Women's NCAA Division 1 basketball championship game, the Mississippi State Bulldogs scored 12 points fewer than the South Carolina Gamecocks. Together, both teams scored a total of 122 points. How many points did the 2017 Champion South Carolina Gamecocks score during this game?

59. In 2020, the shortage of nurses is projected to be 533,201 more nurses than the shortage in 2010. If the total number of nurse shortages for these two years is 1,083,631, find the nurse shortage for each year.

60. The percent shortage of nurses in 2019 is predicted to be three times the percent shortage in 2007. If the total percent shortages for these two years is 36%, find the percent shortage in 2007 and the percent shortage in 2019.

National Supply and Demand Projections for Full-Time Equivalent Registered Nurses: 2000 to 2020



Source: Bureau of Health Professions, RN Supply and Demand Projections

61. The cheetah is known to be the fastest animal for short distances. Its speed is 25 miles per hour faster than a lion's speed. If the total of their speeds is 123 miles per hour, find the fastest speed of a cheetah and the fastest speed of a lion.

62. The average life expectancy for an elephant is 24 years longer than the life expectancy for a chimpanzee. If the total of these life expectancies is 130 years, find the life expectancy of each.

63. A biker sold his used mountain bike and accessories for $270. If he received five times as much money for the bike as he did for the accessories, find how much money he received for the bike.

64. A tractor and a plow attachment are worth $1200. The tractor is worth seven times as much money as the plow. Find the value of the tractor and the value of the plow.

65. In the beginning of 2018, Asia had the highest number of Internet users, followed by Europe. If Asia had 1319 million more Internet users than Europe and the total number of Internet users for both continents was 2727 million, find the number of Internet users for each continent. (*Source:* internetworldstats.com)

66. In the beginning of 2018, North America had 181 million more Internet users than the Middle East. If these two regions together had 509 million Internet users, how many Internet users did each region have? (*Source:* internetworldstats.com)

Review

Round each number to the given place value. See Section 1.5.

67. 586 to the nearest ten

68. 82 to the nearest ten

69. 1026 to the nearest hundred

70. 52,333 to the nearest thousand

71. 2986 to the nearest thousand

72. 101,552 to the nearest hundred

Concept Extensions

73. Solve Exercise **43** again, but this time let x be the number of Stanley Cup wins for Maple Leafs. Did you get the same results? Explain why or why not.

74. Solve Exercise **44** again, but this time let x be the number of World Series wins for the Cardinals. Did you get the same results? Explain why or why not.

In real estate, a house's selling price P is found by adding the real estate agent's commission C to the amount A that the seller of the house receives: $P = A + C$.

75. A house sold for $230,000. The owner's real estate agent received a commission of $13,800. How much did the seller receive? (*Hint:* Substitute the known values into the equation, then solve the equation for the remaining unknown.)

76. A homeowner plans to use a real estate agent to sell a plot of land. He hopes to sell the land for $165,000 and keep $156,750 of that. If everything goes as he has planned, how much will his real estate agent receive as a commission?

In retailing, the retail price P of an item can be computed using the equation $P = C + M$, where C is the wholesale cost of the item and M is the amount of markup.

77. The retail price of a computer system is $999 after a markup of $450. What is the wholesale cost of the computer system? (*Hint:* Substitute the known values into the equation, then solve the equation for the remaining unknown.)

78. Slidell Feed and Seed sells a bag of cat food for $12. If the store paid $7 for the cat food, what is the markup on the cat food?

Chapter 8 Group Activity

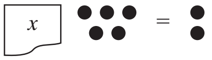
Modeling Equation Solving with Addition and Subtraction

Sections 8.1–8.4

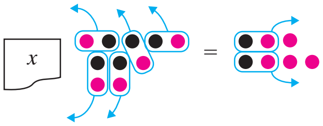
We can use positive counters • and negative counters • to help us model the equation-solving process. We also need to use an object that represents a variable. We use small slips of paper with the variable name written on them.

Recall that taking a • and • together creates a neutral or zero pair. After a neutral pair has been formed, it can be removed from or added to an equation model without changing the overall value. We also need to remember that we can add or remove the same number of positive or negative counters from both sides of an equation without changing the overall value.

We can represent the equation $x + 5 = 2$ as follows:

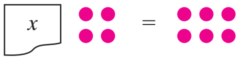


To get the variable by itself, we must remove 5 black counters from the left side of the model. To do so, we must add 5 negative counters to both sides of the model. Then we can remove neutral pairs: 5 from the left side and 2 from the right side (since there are only 2 black counters on the right side).

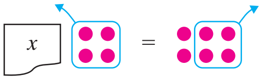


We are left with the following model, which represents the solution, $x = -3$.

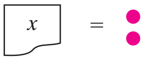
Similarly, we can represent the equation $x - 4 = -6$ as follows:



To get the variable by itself, we must remove 4 red counters from both sides of the model



We are left with the following model, which represents the solution, $x = -2$.



Use the counter model to solve each equation.

1. $x - 3 = -7$
2. $x - 1 = -9$
3. $x + 2 = 8$
4. $x + 4 = 5$
5. $x + 8 = 3$
6. $x - 5 = -1$
7. $x - 2 = 1$
8. $x - 5 = 10$
9. $x + 3 = -7$
10. $x + 8 = -2$

Chapter 8 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

variable
terms
like
addition
simplified
combined
constant
multiplication
numerical coefficient
algebraic expression
evaluating the expression
distributive
equation
solution

1. An algebraic expression is ______________ when all like terms have been ______________.
2. Terms that are exactly the same, except that they may have different numerical coefficients, are called ______________ terms.
3. A letter used to represent a number is called a(n) ______________.
4. A combination of operations on variables and numbers is called a(n) ______________.
5. The addends of an algebraic expression are called the ______________ of the expression.
6. The number factor of a variable term is called the ______________.
7. Replacing a variable in an expression by a number and then finding the value of the expression is called ______________ for the variable.
8. A term that is a number only is called a(n) ______________.
9. A(n) ______________ is of the form expression = expression.
10. A(n) ______________ of an equation is a value for the variable that makes the equation a true statement.
11. To multiply $-3(2x + 1)$, we use the ______________ property.
12. By the ______________ property of equality, we may multiply or divide both sides of an equation by any nonzero number without changing the solution of the equation.
13. By the ______________ property of equality, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.

Helpful Hint

Are you preparing for your test?
To help, don't forget to take these:

- Chapter 8 Getting Ready for the Test on page 637
- Chapter 8 Test on page 638

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

8 Chapter Highlights

Definitions and Concepts	Examples
Section 8.1 Introduction to Variables	
A letter used to represent a number is called a **variable.** A combination of numbers, letters (variables), and operation symbols is called an **algebraic expression,** or **expression.**	x, y, z, a, b $3 + x, 7y, x^3 + y - 10$
Replacing a variable in an expression by a number and then finding the value of the expression is called **evaluating the expression.**	Evaluate: $2x + y$ when $x = 22$ and $y = 4$ $2x + y = 2 \cdot 22 + 4$ Replace x with 22 and y with 4. $= 44 + 4$ Multiply. $= 48$ Add.
The addends of an algebraic expression are called the **terms** of the expression.	$\underbrace{5x^2 + (-4x) + (-2)}_{\text{3 terms}}$

(Continued)

Definitions and Concepts	Examples
Section 8.1 Introduction to Variables (*continued*)	
The number factor of a variable term is called the **numerical coefficient.**	**Term** **Numerical Coefficient** $7x$ 7 $-6y$ -6 x or $1x$ 1
Terms that are exactly the same, except that they may have different numerical coefficients, are called **like terms.**	$5x + 11x = (5 + 11)x = 16x$ like terms $y - 6y = (1 - 6)y = -5y$
An algebraic expression is **simplified** when all like terms have been **combined.** Use the distributive property to multiply an algebraic expression within parentheses by a term. Once any like terms are then combined, the algebraic expression is simplified.	Simplify: $-4(x + 2) + 3(5x - 7)$ $= -4(x) + (-4)(2) + 3(5x) - 3(7)$ $= -4x + (-8) + 15x - (21)$ $= -4x + 15x + (-8) + (-21)$ $= 11x + (-29)$ or $11x - 29$
Section 8.2 Solving Equations: The Addition Property	
Addition Property of Equality Let *a*, *b*, and *c* represent numbers. Then $a = b$ and $a + c = b + c$ are equivalent equations. Also, $a = b$ and $a - c = b - c$ are equivalent equations. In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.	Solve for x: $x + 8 = 2 + (-1)$ $x + 8 = 1$ $x + 8 - 8 = 1 - 8$ Subtract 8 from both sides. $x = -7$ Simplify. The solution is -7.
Section 8.3 Solving Equations: The Multiplication Property	
Multiplication Property of Equality Let *a*, *b*, and *c* represent numbers and let $c \neq 0$. then $a = b$ and $a \cdot c = b \cdot c$ are equivalent equations. Also, $a = b$ and $\frac{a}{c} = \frac{b}{c}$ are equivalent equations. In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.	Solve: $-7x = 42$ $\frac{-7x}{-7} = \frac{42}{-7}$ Divide both sides by -7. $x = -6$ Simplify. Solve: $\frac{2}{3}x = -10$ $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} x = \frac{3}{2} \cdot -10$ Multiply both sides by $\frac{3}{2}$. $x = -15$ Simplify.

Definitions and Concepts	Examples
Section 8.4 Solving Equations Using Addition and Multiplication Properties	
Steps for Solving an Equation	Solve for x: $5(3x-1)+15=-5$
Step 1: If parentheses are present, use the distributive property.	**Step 1:** $15x-5+15=-5$ Apply the distributive property.
Step 2: Combine any like terms on each side of the equation.	**Step 2:** $15x+10=-5$ Combine like terms.
Step 3: Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.	**Step 3:** $15x+10-10=-5-10$ Subtract 10 from both sides. $15x=-15$
Step 4: Use the multiplication property of equality to divide both sides by the numerical coefficient of the variable to solve.	**Step 4:** $\frac{15x}{15}=\frac{-15}{15}$ Divide both sides by 15. $x=-1$
Step 5: Check the solution in the *original equation*.	**Step 5:** Check to see that -1 is the solution.
Section 8.5 Equations and Problem Solving	
Problem-Solving Steps	The incubation period for a golden eagle is three times the incubation period for a hummingbird. If the total of their incubation periods is 60 days, find the incubation period for each bird. (*Source: Wildlife Fact File,* International Masters Publishers)
1. UNDERSTAND the problem. Some ways of doing this are Read and reread the problem. Construct a drawing. Choose a variable to represent an unknown in the problem.	**1.** UNDERSTAND the problem. Then choose a variable to represent an unknown. Let x = incubation period of a hummingbird $3x$ = incubation period of a golden eagle
2. TRANSLATE the problem into an equation.	**2.** TRANSLATE. $\begin{array}{ccccc} \text{incubation of hummingbird} & + & \text{incubation of golden eagle} & \text{is} & 60 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & + & 3x & = & 60 \end{array}$
3. SOLVE the equation.	**3.** SOLVE: $x+3x=60$ $4x=60$ $\frac{\overset{1}{\cancel{4}}x}{\underset{1}{\cancel{4}}}=\frac{60}{4}$ $x=15$
4. INTERPRET the results. *Check* the proposed solution in the stated problem and *state* your conclusion.	**4.** INTERPRET the solution in the stated problem. The incubation period for a hummingbird is 15 days. The incubation period for a golden eagle is $3x=3\cdot 15=45$ days. Since 15 days + 45 days = 60 days and 45 is 3(15), the solution checks. *State* your conclusion: The incubation period for a hummingbird is 15 days. The incubation period for a golden eagle is 45 days.

Chapter 8 Review

(8.1) *Evaluate each expression when* $x = 5, y = 0,$ *and* $z = -2$.

1. $\frac{2x}{z}$

2. $4x - 3$

3. $\frac{x + 7}{y}$

4. $\frac{y}{5x}$

5. $x^3 - 2z$

6. $\frac{7 + x}{3z}$

△ **7.** Find the volume of a storage cube whose sides measure 2 feet. Use $V = s^3$.

△ **8.** Find the volume of a wooden crate in the shape of a cube 4 feet on each side. Use $V = s^3$.

9. Lamar deposited his \$5000 bonus into an account paying 6% annual interest. How much interest will he earn in 6 years? Use $I = PRT$.

10. Jennifer Lewis borrowed \$2000 from her grandmother and agreed to pay her 5% simple interest. How much interest will she owe after 3 years? Use $I = PRT$.

Simplify each expression by combining like terms.

11. $-6x - 9x$

12. $\frac{2}{3}x - \frac{9}{10}x$

13. $2y - 10 - 8y$

14. $8a + a - 7 - 15a$

15. $y + 3 - 9y - 1$

16. $1.7x - 3.2 + 2.9x - 8.7$

Multiply.

17. $-2(4y)$

18. $3(5y - 8)$

Simplify.

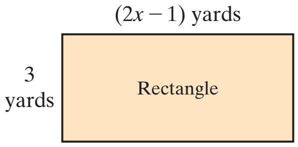
19. $7x + 3(x - 4) + x$

20. $4(x - 7) + 21$

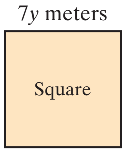
21. $3(5a - 2) + 10(-2a + 1)$

22. $6y + 3 + 2(3y - 6)$

△ **23.** Find the area.



△ **24.** Find the perimeter.



(8.2)

25. Is 4 a solution of $5(2 - x) = -10$?

26. Is 0 a solution of $6y + 2 = 23 + 4y$?

Solve.

27. $z - 5 = -7$

28. $x + 1 = 8$

29. $x + \frac{7}{8} = \frac{3}{8}$

30. $y + \frac{4}{11} = -\frac{2}{11}$

31. $n + 18 = 10 - (-2)$

32. $15 = 8x + 35 - 7x$

33. $m - 3.9 = -2.6$

34. $z - 4.6 = -2.2$

(8.3) *Solve.*

35. $-3y = -21$

36. $-8x = 72$

37. $-5n = -5$

38. $-3a = 15$

39. $\frac{2}{3}x = -\frac{8}{15}$

40. $-\frac{7}{8}y = 21$

41. $-1.2x = 144$

42. $-0.8y = -10.4$

43. $-5x = 100 - 120$

44. $18 - 30 = -4x$

(8.4) *Solve.*

45. $3x - 4 = 11$

46. $6y + 1 = 73$

47. $-\frac{5}{9}x + 23 = -12$

48. $-\frac{2}{3}x - 11 = \frac{2}{3}x - 55$

49. $6.8 + 4y = -2.2$

50. $-9.6 + 5y = -3.1$

51. $2x + 7 = 6x - 1$

52. $5x - 18 = -4x + 36$

53. $5(n - 3) = 7 + 3n$

54. $7(2 + x) = 4x - 1$

55. $2(4n - 11) + 8 = 5n + 4$

56. $3(5x - 6) + 9 = 13x + 7$

Write each sentence as an equation.

57. The difference of 20 and −8 is 28.

58. Nineteen subtracted from −2 amounts to −21.

59. The quotient of −75 and the sum of 5 and 20 is equal to −3.

60. The product of −5 and the sum of −2 and 6 yields −20.

(8.5) *Write each phrase as an algebraic expression. Use x to represent "a number."*

61. The quotient of 70 and a number

62. The difference of a number and 13

63. A number subtracted from 85.

64. Eleven added to twice a number

Write each sentence as an equation. Use x to represent "a number."

65. A number increased by 8 is 40.

66. Twelve subtracted from twice a number is 10.

Solve.

67. Five times a number subtracted from 40 is the same as three times the number. Find the number.

68. The product of a number and 3 is twice the difference of that number and 8. Find the number.

69. Bamboo and Pacific kelp, a kind of seaweed, are two fast-growing plants. Bamboo grows twice as fast as kelp. If in one day both can grow a total of 54 inches, find how many inches each plant can grow in one day.

Bamboo

Kelp

70. In an election between the incumbent and a challenger, the incumbent received 11,206 more votes than the challenger. If a total of 18,298 votes were cast, find the number of votes for each candidate.

Mixed Review

Evaluate each expression when $x = 4$, $y = -3$, *and* $z = 5$.

71. $18 - (9 - 5x)$

72. $\frac{z}{100} + \frac{y}{10}$

Simplify.

73. $9x - 20x$

74. $-5(7x)$

75. $12x + 5(2x - 3) - 4$

76. $-7(x + 6) - 2(x - 5)$

Write each phrase as an algebraic expression. Use x to represent "a number."

77. Seventeen less than a number

78. Three times the sum of a number and five

Write each sentence as an equation using x to represent "a number."

79. The difference of a number and 3 is the quotient of the number and 4.

80. The product of a number and 6 is equal to the sum of the number and 2.

81. Is 3 a solution of $4y + 2 - 6y = 5 + 7$?

82. Is 7 a solution of $4(z - 8) + 12 = 8$?

Solve.

83. $c - 5 = -13 + 7$

84. $7x + 5 - 6x = -20$

85. $-7x + 3x = -50 - 2$

86. $-x + 8x = -38 - 4$

87. $14 - y = -3$

88. $7 - z = 0$

89. $9x + 12 - 8x = -6 + (-4)$

90. $-17x + 14 + 20x - 2x = 5 - (-3)$

91. $\frac{4}{9}x = -\frac{1}{3}$

92. $-\frac{5}{24}x = \frac{5}{6}$

93. $2y + 6y = 24 - 8$

94. $13x - 7x = -4 - 12$

95. $\frac{2}{3}x - 12 = -4$

96. $\frac{7}{8}x + 5 = -2$

97. $-5z + 3z - 7 = 8z - 7$

98. $4x - 3 + 6x = 5x - 3$

99. Three times a number added to twelve is 27. Find the number.

100. Twice the sum of a number and four is ten. Find the number.

Advertisers in the United States use many media: newspapers, magazines, mailings, Internet, and TV. See Exercises 101 and 102.

101. In 2017, the money spent on Internet advertising in the United States first exceeded the amount spent on TV advertising. If an estimated 0.6 billion dollars more was spent on Internet advertising in 2017 and the total amount spent on Internet and TV advertising was $150 billion, find the amount of money spent on each medium. *(Source: USA Today)*

102. By 2020, the money spent on Internet advertising in the United States is projected to continue to exceed the amount spent on TV advertising. If it is estimated that 11.8 billion dollars more will be spent on Internet advertising in 2020 and the total amount spent on Internet and TV advertising is expected to be about $175 billion, find the amount of money projected to be spent on each medium. *(Source: USA Today)*

Chapter 8 Getting Ready for the Test LC

MULTIPLE CHOICE Exercises 1 through 16 are **Multiple Choice**. Choose the correct letter.

For Exercises 1 through 4, let $a = 35$ *and* $b = 5$*. Choose the expression that gives each answer.*

A. $a - b$ **B.** $a \div b$ **C.** $a + b$ **D.** ab

1. answer: 7

2. answer: 175

3. answer: 30

4. answer: 40

Simplify each expression in Exercises 5 through 10. Then choose the simplified form: A, B, C, D, or E. Choices may be used more than once or not at all.

A. $-6x$ **B.** $6x$ **C.** $4x + 2$ **D.** $4x + 1$ **E.** $4x + 6$

5. $x - 7x$

6. $-2x + 3 + 8x - 3$

7. $-3(2x)$

8. $2(2x + 1)$

9. $-(x + 2) + 5x + 4$

10. $3(1 + 2x) - 3$

For Exercises 11 through 14, choose the solution of each equation as

A. 2 **B.** -2 **C.** 18 **D.** -18

11. $-3x = 6$

12. $x - 2 = -4$

13. $\frac{x}{-3} = 6$

14. $x + 2 = 4$

For Exercises 15 and 16, choose the correct letter.

15. To solve $5x - 10 = 0$, we will first add 10 to each side of the equation. Once this is done, the equivalent equation is

A. $5x = 0$ **B.** $5x = 10$ **C.** $5x = -10$

16. To solve $4(2x + 1) - 8 = 14x - 10$, we will first simplify the left side of the equation. Once this is done, the equivalent equation is

A. $8x - 4 = 14x - 10$ **B.** $8x - 7 = 14x - 10$ **C.** $2x - 7 = 14x - 10$

MATCHING *Let x represent "a number." Match each phrase in the first column with its translated variable expression in the second column.*

17. the sum of -3 and a number

18. the product of -3 and a number

19. 5 subtracted from a number

20. 5 minus a number

A. $-3x$

B. $3 + x$

C. $-3 + x$

D. $x - 5$

E. $5 - x$

F. $5x$

MATCHING *Let x represent "a number." Match each sentence in the first column with its translated equation in the second or third column.*

21. Twice a number gives 14.

22. A number divided by 6 yields 14.

23. Two subtracted from a number equals 14.

24. Six times a number, added to 2 is 14.

A. $6x = 14$

B. $2x = 14$

C. $\frac{x}{6} = 14$

D. $2 - x = 14$

E. $x - 2 = 14$

F. $2 + 6x = 14$

G. $6(2 + x) = 14$

Chapter 8 Test MyLab Math

For additional practice go to your study plan in MyLab Math.

Answers

1. ________________

2. ________________

3. ________________

4. ________________

5. ________________

6. ________________

7. ________________

8. ________________

9. ________________

10. ________________

11. ________________

12. ________________

13. ________________

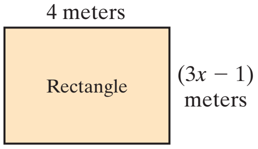
1. Evaluate $\frac{3x - 5}{2y}$ when $x = 7$ and $y = -8$.

2. Simplify $7x - 5 - 12x + 10$ by combining like terms.

3. Multiply: $-2(3y + 7)$

4. Simplify: $5(3z + 2) - z - 18$

5. Write a product that represents the area of the rectangle. Then multiply.



Solve.

6. $x - 17 = -10$

7. $y + \frac{3}{4} = \frac{1}{4}$

8. $-4x = 48$

9. $-\frac{5}{8}x = -25$

10. $5x + 12 - 4x - 14 = 22$

11. $2 - c + 2c = 5$

12. $3x - 5 = -11$

13. $-4x + 7 = 15$

14. $3.6 - 2x = -5.4$

15. $12 = 3(4 + 2y)$

16. $5x - 2 = x - 10$

17. $10y - 1 = 7y + 21$

18. $6 + 2(3n - 1) = 28$

19. $4(5x + 3) = 2(7x + 6)$

Solve.

△ **20.** A lawn is in the shape of a trapezoid with a height of 60 feet and bases of 70 feet and 130 feet. Find the area of the lawn. Use $A = \frac{1}{2}h(B + b)$.

△ **21.** If the height of a triangular-shaped jib sail is 12 feet and its base is 5 feet, find the area of the sail. Use $A = \frac{1}{2}bh$.

22. Translate the following phrases into mathematical expressions. Use x to represent "a number."

a. The product of a number and 17

b. Twice a number subtracted from 20

23. The difference of three times a number and five times the same number is 4. Find the number.

24. In a championship basketball game, Paula Zimmerman scored twice as many points as Maria Kaminsky. If the total number of points made by both women was 51, find how many points Paula scored.

25. In a 10-kilometer race, there are 112 more men entered than women. Find the number of women runners if the total number of runners in the race is 600.

14. ____

15. ____

16. ____

17. ____

18. ____

19. ____

20. ____

21. ____

22. a. ____

b. ____

23. ____

24. ____

25. ____

Chapters 1–8 Cumulative Review

Answers

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11. a.

b.

12.

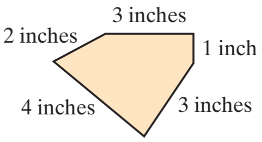
1. Find the place value of the digit 3 in the whole number 396,418.

2. Write 2036 in words.

3. Add: 34,285 + 149,761

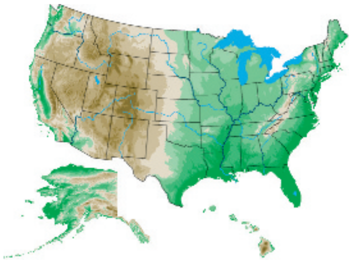
4. Find the average of 56, 18, and 43.

5. Find the perimeter of the polygon shown.



6. Subtract 8 from 25.

7. In 2015, a total of 9,879,465 trucks were sold in the United States. In 2016, total truck sales in the United States had increased by 712,397 vehicles. Find the total number of trucks sold in the United States in 2016. (*Source:* Alliance of Automobile Manufacturers)

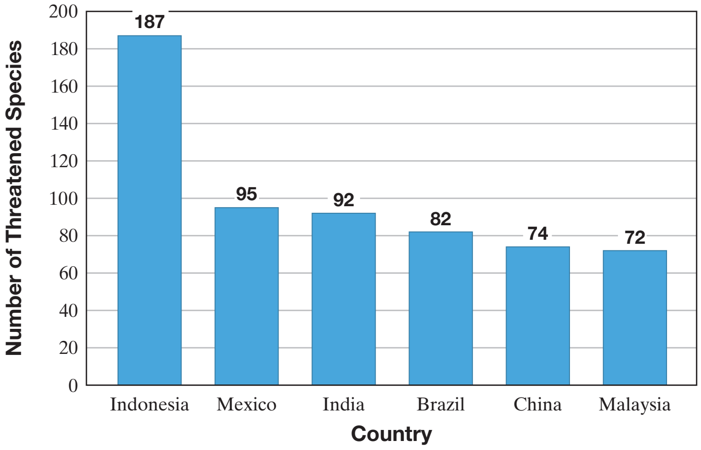


8. Find $\sqrt{25}$.

9. Subtract 7826 − 505. Check by adding.

10. Find 8^2.

11. In the following graph, each bar represents a country and the height of each bar represents the number of threatened mammal species identified in that country.


Number of Threatened Mammal Species, 2016

Source: International Union for Conservation of Nature

a. Which country shown has the greatest number of threatened mammal species?

b. Find the total number of threatened mammal species for Malaysia, China, and Indonesia.

12. Evaluate: $\left(-\frac{1}{2}\right)^3$

Simplify.

13. $-(-4)$

14. $-|20|$

15. $-|-5|$

16. $|0|$

Add.

17. $-2 + (-21)$

18. $-8.2 + 4.6$

19. $(-3) + 4 + (-11)$

20. $\frac{2}{5} + \left(-\frac{3}{10}\right)$

Subtract.

21. $8 - 15$

22. $4.6 - (-1.2)$

23. $-4 - (-5)$

24. $\frac{7}{10} - \frac{23}{24}$

Multiply.

25. $-3(-5)$

26. $-8(1.2)$

27. $(-1)(-2)(-3)(-4)$

28. $-2\frac{2}{9}\left(1\frac{4}{5}\right)$

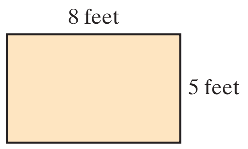
29. Simplify: $(-3) \cdot |-5| - (-2) + 4^2$

30. Solve: $4x - 7.1 = 3x + 2.6$

31. Multiply: 0.0531×16

32. Multiply: 0.0531×1000

33. Given the rectangle shown:



a. Find the ratio of its width to its length.

b. Find the ratio of its length to its perimeter.

34. Add: $\frac{5}{12} + \frac{2}{9}$

13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______
25. ______
26. ______
27. ______
28. ______
29. ______
30. ______
31. ______
32. ______
33. a. ______
b. ______
34. ______

35.
36.
37.
38.
39.
40.
41.
42.
43.
44.
45.
46.
47.
48.
49.
50.
51.
52.

35. 12% of what number is 0.6?

36. Multiply: $\frac{7}{8} \cdot \frac{2}{3}$

37. What percent of 12 is 9?

38. Divide: $1\frac{4}{5} \div 2\frac{3}{10}$

39. Convert 3 pounds to ounces.

40. Round 23,781 to the nearest thousand.

41. Add 2400 ml to 8.9 L.

42. Round 0.02351 to the nearest thousandth.

43. Is $\frac{1\frac{1}{6}}{10\frac{1}{2}} = \frac{\frac{1}{2}}{4\frac{1}{2}}$ a true proportion?

44. Is $\frac{7.8}{3} = \frac{5.2}{2}$ a true proportion?

45. The standard dose of an antibiotic is 4 cc (cubic centimeters) for every 25 pounds (lb) of body weight. At this rate, find the standard dose for a 140-lb woman.

46. On a certain map, 2 inches represents 75 miles. How many miles are represented by 7 inches?

Write each percent as a decimal.

47. 4.6%

48. 452%

Write each percent as a fraction in simplest form.

49. $33\frac{1}{3}\%$

50. 27%

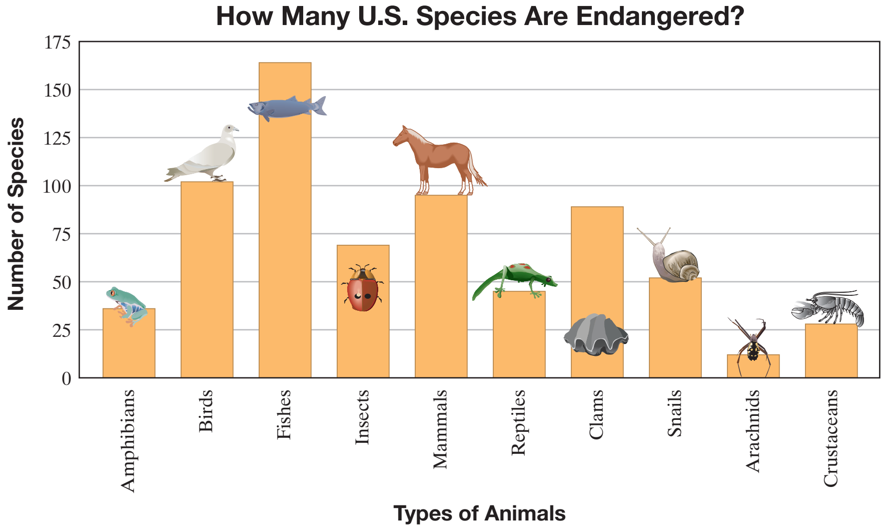
51. Translate to an equation: Five is what percent of 20?

52. Translate to a proportion: Five is what percent of 20?

53. Find the sales tax and the total price on the purchase of an $85.50 atlas in a city where the sales tax rate is 7.5%.

54. A salesperson makes a 7% commission rate on her total sales. If her total sales are $23,000, what is her commission?

55. The following bar graph shows the number of endangered species in 2017. Use this graph to answer the questions.



Source: U.S. Fish and Wildlife Service

a. Approximate the number of endangered species that are reptiles.

b. Which category has the most endangered species?

56. Find the mean, median, and mode of 1, 7, 8, 10, 11, 11.

53. ____________

54. ____________

55. a. ____________

b. ____________

56. ____________

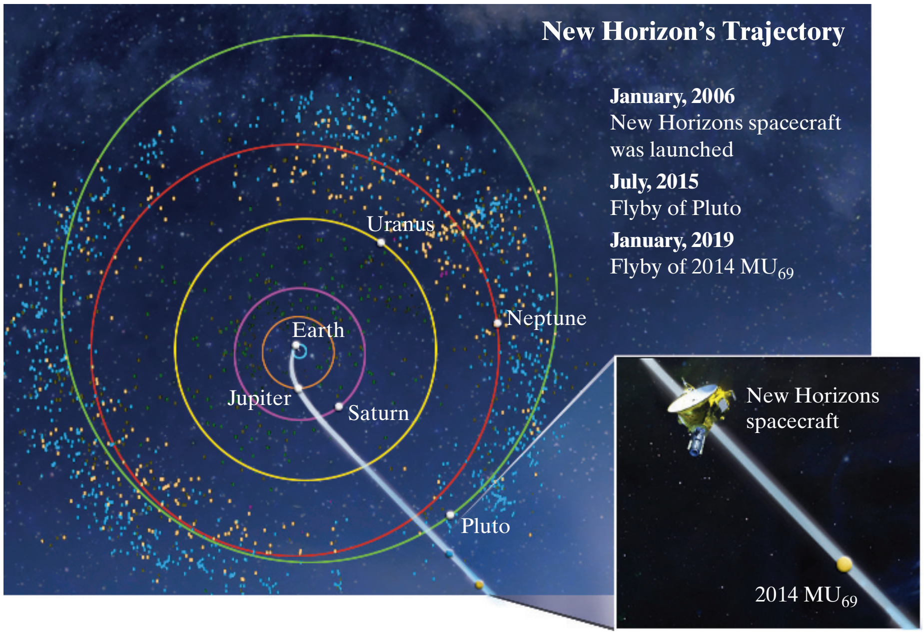
9

Geometry

The word *geometry* is formed from the Greek words *geo*, meaning Earth, and *metron*, meaning measure. Geometry literally means to measure the Earth. In this chapter we learn about various geometric figures and their properties such as perimeter, area, and volume. Knowledge of geometry can help us solve practical problems in real-life situations. For instance, knowing certain measures of a circular swimming pool allows us to calculate how much water it can hold.

Sections

Check Your Progress



New Horizon's Trajectory

January, 2006
New Horizons spacecraft was launched

July, 2015
Flyby of Pluto

January, 2019
Flyby of 2014 MU_{69}

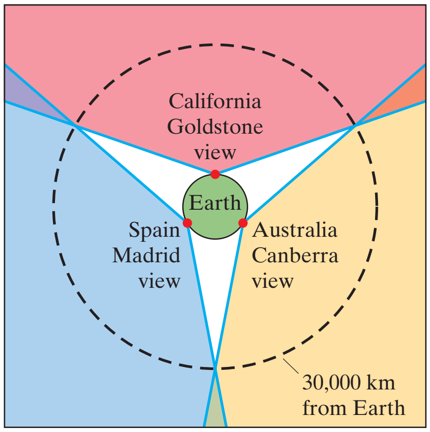
2014 MU_{69}
- Diameter: 10–30 miles
- 4 billion miles from earth
- An object in the Kuiper belt

Where Is New Horizons Spacecraft Now, and How Do We Receive Data Collected?

New Horizons is NASA's robotic spacecraft mission. This spacecraft is about the size and shape of a grand piano with a satellite dish attached. It was launched in January 2006 and is now headed for a January 1, 2019, flyby past 2014 MU_{69}, a small object in the Kuiper asteroid belt. How do we receive the images of this spacecraft when it is so far into deep space?

The Deep Space Network (DSN) is a worldwide network of large antennas and communication facilities. When a mission is in deep space, fewer sites are needed for sending and receiving transmissions; thus, the DSN uses only three sites, shown below. The diagram below shows an overview of Earth from the vantage point of the North Pole and the location of these three sites.

We study some geometry of the DSN in Section 9.1, Exercises 65 and 66.



9.1 Lines and Angles

Objectives

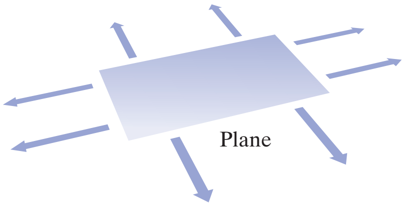
- **A** Identify Lines, Line Segments, Rays, and Angles.
- **B** Classify Angles as Acute, Right, Obtuse, or Straight.
- **C** Identify Complementary and Supplementary Angles.
- **D** Find Measures of Angles.

Objective A Identifying Lines, Line Segments, Rays, and Angles

Let's begin with a review of two important concepts—space and plane.

Space extends in all directions indefinitely. Examples of objects in space are houses, grains of salt, bushes, your *Basic College Mathematics with Early Integers* textbook, and you.

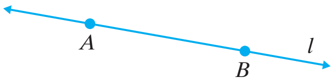
A **plane** is a flat surface that extends indefinitely. Surfaces like a plane are a classroom floor or a blackboard or whiteboard.



The most basic concept of geometry is the idea of a point in space. A **point** has no length, no width, and no height, but it does have location. We represent a point by a dot, and we usually label points with capital letters.

Point P

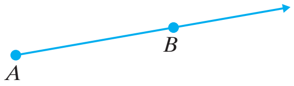
A **line** is a set of points extending indefinitely in two directions. A line has no width or height, but it does have length. We can name a line by any two of its points or by a single lowercase letter. A **line segment** is a piece of a line with two endpoints.



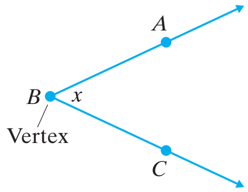
Line AB, $\overleftrightarrow{AB}$, or line l*

Line segment AB or $\overline{AB}$*

A **ray** is a part of a line with one endpoint. A ray extends indefinitely in one direction. An **angle** is made up of two rays that share the same endpoint. The common endpoint is called the **vertex.**



Ray AB or $\overrightarrow{AB}$



The angle in the figure above can be named

$\angle ABC$ $\angle CBA$ $\angle B$ or $\angle x$

↑ ↑

The vertex is the middle point.

Rays BA and BC are **sides** of the angle.

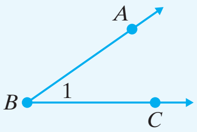
---

*Although line l is also line BA or $\overleftrightarrow{BA}$, we will use only one order of points to name a line or line segment.

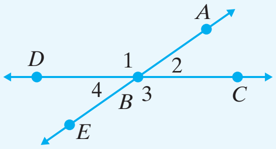
Helpful Hint

Naming an Angle

When there is no confusion as to what angle is being named, you may use the vertex alone.



Name of $\angle B$ is all right.
There is no confusion. $\angle B$ means $\angle 1$.

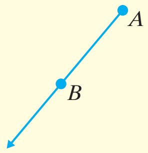


Name of $\angle B$ is *not* all right.
There is confusion. Does $\angle B$ mean $\angle 1$, $\angle 2$, $\angle 3$, or $\angle 4$?

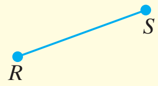
Practice 1

Identify each figure as a line, a ray, a line segment, or an angle. Then name the figure using the given points.

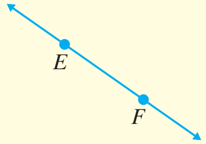
a.



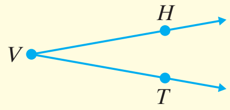
b.



c.



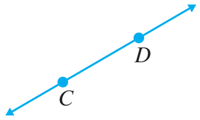
d.



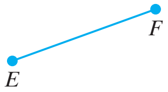
Example 1

Identify each figure as a line, a ray, a line segment, or an angle. Then name the figure using the given points.

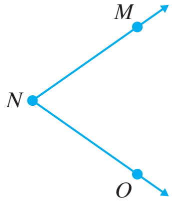
a.



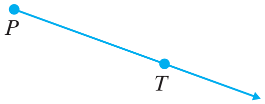
b.



c.



d.



Solution:

Figure **(a)** extends indefinitely in two directions. It is line *CD* or $\overleftrightarrow{CD}$.
Figure **(b)** has two endpoints. It is line segment *EF* or $\overline{EF}$.
Figure **(c)** has two rays with a common endpoint. It is $\angle MNO$, $\angle ONM$, or $\angle N$.
Figure **(d)** is part of a line with one endpoint. It is ray *PT* or $\overrightarrow{PT}$.

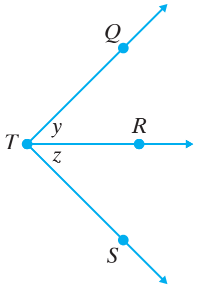
Work Practice 1

Practice 2

Use the figure in Example 2 to list other ways to name $\angle z$.

Example 2

List other ways to name $\angle y$.



Solution: Two other ways to name $\angle y$ are $\angle QTR$ and $\angle RTQ$. We may *not* use the vertex alone to name this angle because three different angles have *T* as their vertex.

Work Practice 2

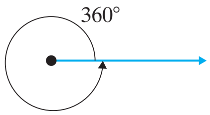
Answers

1. a. ray; ray *AB* or $\overrightarrow{AB}$ **b.** line segment; line segment *RS* or $\overline{RS}$ **c.** line; line *EF* or $\overleftrightarrow{EF}$ **d.** angle; $\angle TVH$ or $\angle HVT$ or $\angle V$

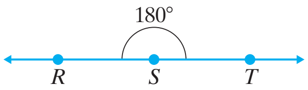
2. $\angle RTS$, $\angle STR$

Objective B Classifying Angles as Acute, Right, Obtuse, or Straight

An angle can be measured in **degrees.** The symbol for degrees is a small, raised circle, °. There are 360° in a full revolution, or a full circle.

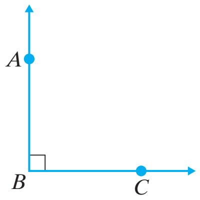


$\frac{1}{2}$ of a revolution measures $\frac{1}{2}(360°) = 180°$. An angle that measures 180° is called a **straight angle.**



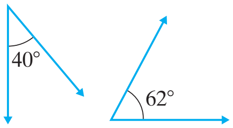
∠*RST* is a straight angle.

$\frac{1}{4}$ of a revolution measures $\frac{1}{4}(360°) = 90°$. An angle that measures 90° is called a **right angle.** The symbol ∟ is used to denote a right angle.



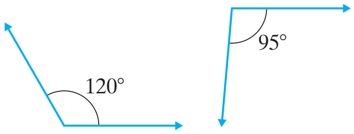
∠*ABC* is a right angle.

An angle whose measure is between 0° and 90° is called an **acute angle.**



Acute angles

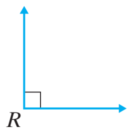
An angle whose measure is between 90° and 180° is called an **obtuse angle.**



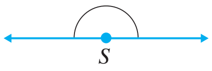
Obtuse angles

Example 3 Classify each angle as acute, right, obtuse, or straight.

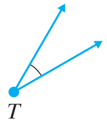
a.



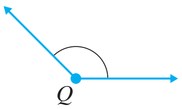
b.



c.



d.



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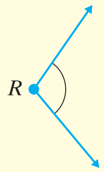
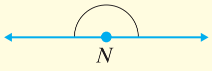
Practice 3

Classify each angle as acute, right, obtuse, or straight.

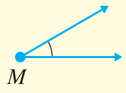
a.



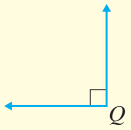
b.



c.



d.



Answers

3. a. obtuse **b.** straight **c.** acute **d.** right

Solution:

a. ∠R is a right angle, denoted by ∟. It measures 90°.

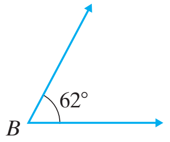
b. ∠S is a straight angle. It measures 180°.

c. ∠T is an acute angle. It measures between 0° and 90°.

d. ∠Q is an obtuse angle. It measures between 90° and 180°.

Work Practice 3

Let's look at ∠B below, whose measure is 62°.



There is a shorthand notation for writing the measure of this angle. To write "The measure of ∠B is 62°," we can write

$$m\angle B = 62°.$$

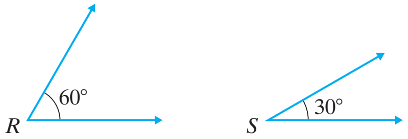
By the way, note that ∠B is an acute angle because $m\angle B$ is between 0° and 90°.

Objective C Identifying Complementary and Supplementary Angles

Two angles that have a sum of 90° are called **complementary angles.** We say that each angle is the **complement** of the other.

∠R and ∠S are complementary angles because

$$m\angle R + m\angle S = 60° + 30° = 90°$$

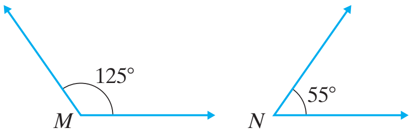


Complementary angles
60° + 30° = 90°

Two angles that have a sum of 180° are called **supplementary angles.** We say that each angle is the **supplement** of the other.

∠M and ∠N are supplementary angles because

$$m\angle M + m\angle N = 125° + 55° = 180°$$



Supplementary angles
125° + 55° = 180°

Practice 4

Find the complement of a 29° angle.

Example 4 Find the complement of a 48° angle.

Solution: Two angles that have a sum of 90° are complementary. This means that the complement of an angle that measures 48° is an angle that measures 90° − 48° = 42°.

Work Practice 4

Answer

4. 61°

Example 5 Find the supplement of a 107° angle.

Solution: Two angles that have a sum of 180° are supplementary. This means that the supplement of an angle that measures 107° is an angle that measures $180° - 107° = 73°$.

Work Practice 5

✓Concept Check True or false? The supplement of a 48° angle is 42°. Explain.

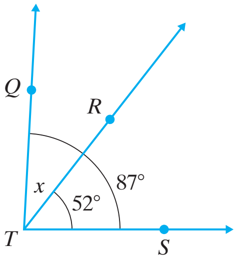
Objective D Finding Measures of Angles

Measures of angles can be added or subtracted to find measures of related angles.

Example 6 Find the measure of $\angle x$. Then classify $\angle x$ as an acute, obtuse, or right angle.

Solution:

$$\begin{aligned} m\angle x &= m\angle QTS - m\angle RTS \\ &= 87° - 52° \\ &= 35° \end{aligned}$$

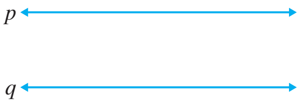


Thus, the measure of $\angle x$ $(m\angle x)$ is 35°.

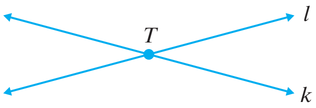
Since $\angle x$ measures between 0° and 90°, it is an acute angle.

Work Practice 6

Two lines in a plane can be either parallel or intersecting. **Parallel lines** never meet. **Intersecting lines** meet at a point. The symbol $\|$ is used to indicate "is parallel to." For example, in the figure, $p \| q$.

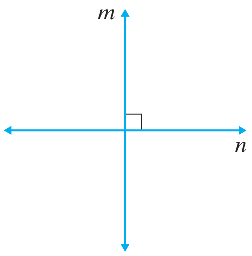


Parallel lines



Intersecting lines

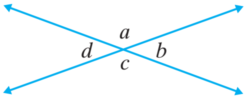
Some intersecting lines are perpendicular. Two lines are **perpendicular** if they form right angles when they intersect. The symbol $\perp$ is used to denote "is perpendicular to." For example, in the figure below, $m \perp n$.



Perpendicular lines

When two lines intersect, four angles are formed. Two angles that are opposite each other are called **vertical angles.** Vertical angles have the same measure.

Two angles that share a common side are called **adjacent angles.** Adjacent angles formed by intersecting lines are supplementary. That is, the sum of their measures is 180°.



Vertical angles:
$\angle a$ and $\angle c$
$\angle d$ and $\angle b$

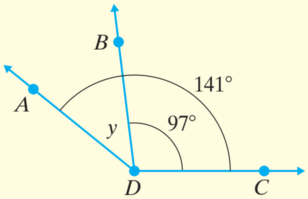
Adjacent angles:
$\angle a$ and $\angle b$
$\angle b$ and $\angle c$
$\angle c$ and $\angle d$
$\angle d$ and $\angle a$

Practice 5

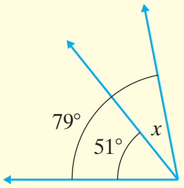
Find the supplement of a 67° angle.

Practice 6

a. Find the measure of $\angle y$.



b. Find the measure of $\angle x$.



c. Classify $\angle x$ and $\angle y$ as acute, obtuse, or right angles.

Answers

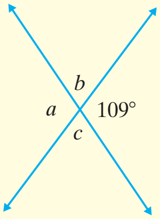
5. 113° **6. a.** 44° **b.** 28°
c. both acute

✓Concept Check Answer

false; the *complement* of a 48° angle is 42°; the *supplement* of a 48° angle is 132°

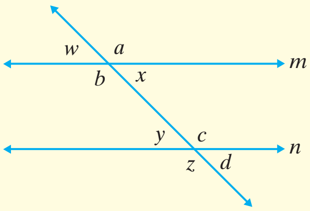
Practice 7

Find the measures of ∠a, ∠b, and ∠c.



Practice 8

Given that $m \parallel n$ and that the measure of $\angle w = 45°$, find the measures of all the angles shown.



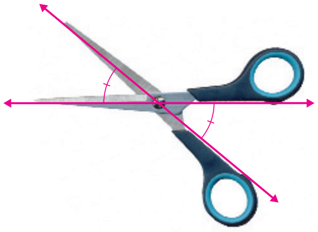
Answers

7. $m\angle a = 109°$; $m\angle b = 71°$; $m\angle c = 71°$

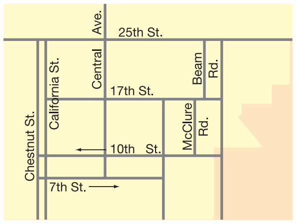
8. $m\angle x = 45°$; $m\angle y = 45°$; $m\angle z = 135°$; $m\angle a = 135°$; $m\angle b = 135°$; $m\angle c = 135°$; $m\angle d = 45°$

Here are a few real-life examples of the lines we just discussed.

Parallel lines



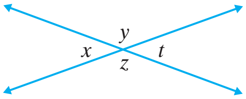
Vertical angles



Perpendicular lines

Example 7

Find the measures of ∠x, ∠y, and ∠z if the measure of ∠t is 42°.



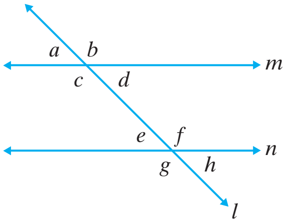
Solution: Since ∠t and ∠x are vertical angles, they have the same measure, so ∠x measures 42°.

Since ∠t and ∠y are adjacent angles, their measures have a sum of 180°. So ∠y measures $180° - 42° = 138°$.

Since ∠y and ∠z are vertical angles, they have the same measure. So ∠z measures 138°.

Work Practice 7

A line that intersects two or more lines at different points is called a **transversal.** Line l is a transversal that intersects lines m and n. The eight angles formed have special names. Some of these names are:



Corresponding angles: ∠a and ∠e, ∠c and ∠g, ∠b and ∠f, ∠d and ∠h

Alternate interior angles: ∠c and ∠f, ∠d and ∠e

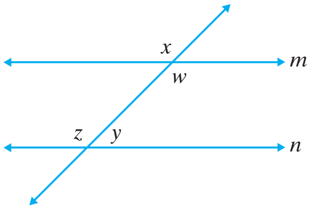
When two lines cut by a transversal are *parallel,* the following statement is true:

Parallel Lines Cut by a Transversal

If two parallel lines are cut by a transversal, then the measures of **corresponding angles are equal** and the measures of the **alternate interior angles are equal.**

Example 8

Given that $m \parallel n$ and that the measure of ∠w is 100°, find the measures of ∠x, ∠y, and ∠z.



Solution:

$m\angle x = 100°$ ∠x and ∠w are vertical angles.

$m\angle z = 100°$ ∠x and ∠z are corresponding angles.

$m\angle y = 180° - 100° = 80°$ ∠z and ∠y are supplementary angles.

Work Practice 8

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

acute
obtuse
right
straight
space
angle
degrees
plane
ray
adjacent
point
line
parallel
vertical
perpendicular
intersecting
vertex
transversal

1. A(n) ______________ is a flat surface that extends indefinitely.
2. A(n) ______________ has no length, no width, and no height.
3. ______________ extends in all directions indefinitely.
4. A(n) ______________ is a set of points extending indefinitely in two directions.
5. A(n) ______________ is part of a line with one endpoint.
6. A(n) ______________ is made up of two rays that share a common endpoint. The common endpoint is called the ______________.
7. A(n) ______________ angle measures 180°.
8. A(n) ______________ angle measures 90°.
9. A(n) ______________ angle measures between 0° and 90°.
10. A(n) ______________ angle measures between 90° and 180°.
11. ______________ lines never meet and ______________ lines meet at a point.
12. Two intersecting lines are ______________ if they form right angles when they intersect.
13. An angle can be measured in ______________.
14. A line that intersects two or more lines at different points is called a(n) ______________.
15. When two lines intersect, four angles are formed. The angles that are opposite each other are called ______________ angles.
16. Two angles that share a common side are called ______________ angles.

Martin-Gay Interactive Videos

See Video 9.1

Watch the section lecture video and answer the following questions.

Objective A **17.** In the lecture after Example 2, what are the four ways we can name the angle shown?

Objective B **18.** In the lecture before Example 3, what type of angle forms a line? What is its measure?

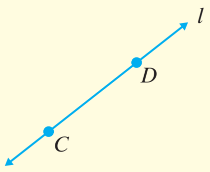
Objective C **19.** What calculation is used to find the answer to Example 6?

Objective D **20.** In the lecture before Example 7, two lines in a plane that aren't parallel must what?

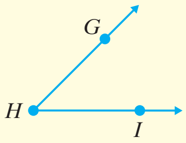
9.1 Exercise Set MyLab Math

Objective A *Identify each figure as a line, a ray, a line segment, or an angle. Then name the figure using the given points. See Examples 1 and 2.*

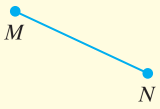
1.



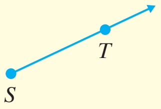
2.



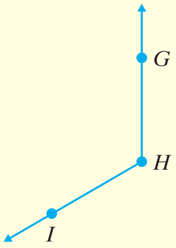
3.



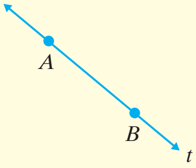
4.



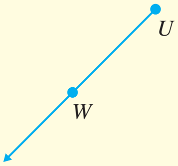
5.



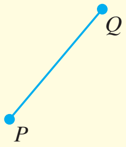
6.



▶ **7.**



8.



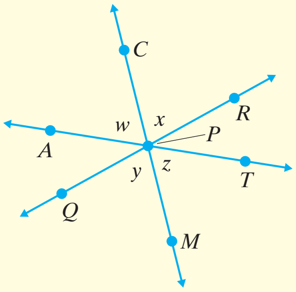
List two other ways to name each angle. See Example 2.

9. $\angle x$

10. $\angle w$

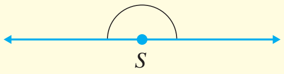
11. $\angle z$

12. $\angle y$



Objective B *Classify each angle as acute, right, obtuse, or straight. See Example 3.*

13.



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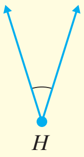
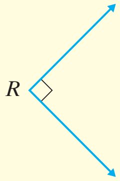


▶ **15.**



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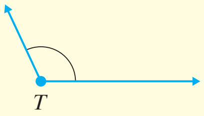
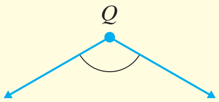


▶ **17.**



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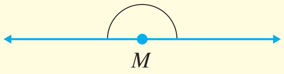

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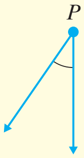

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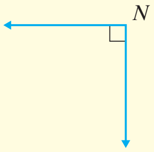


Objective C *Find each complementary or supplementary angle as indicated. See Examples 4 and 5.*

▶ **21.** Find the complement of a 23° angle.

22. Find the complement of a 77° angle.

▶ **23.** Find the supplement of a 17° angle.

24. Find the supplement of a 77° angle.

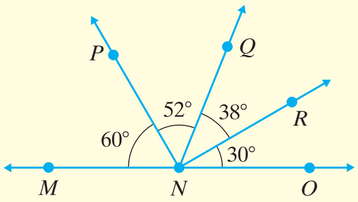
25. Find the complement of a 58° angle.

26. Find the complement of a 22° angle.

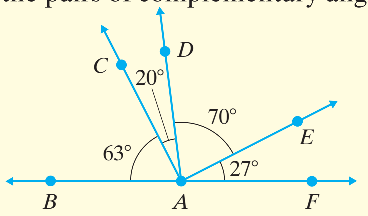
27. Find the supplement of a 150° angle.

28. Find the supplement of a 130° angle.

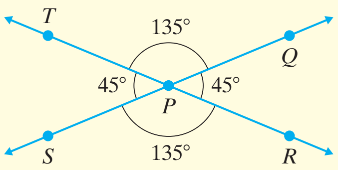
29. Identify the pairs of complementary angles.



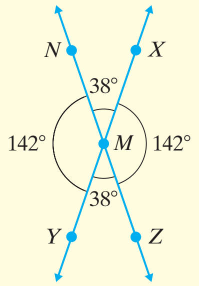
30. Identify the pairs of complementary angles.



31. Identify the pairs of supplementary angles.

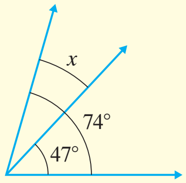


32. Identify the pairs of supplementary angles.



Objective **D** *Find the measure of ∠x in each figure. See Example 6.*

33.



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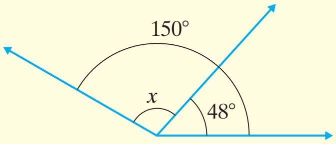

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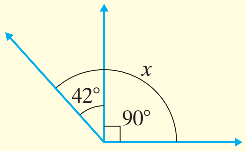
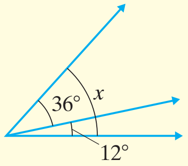
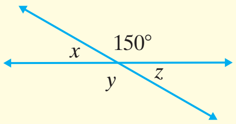


36.

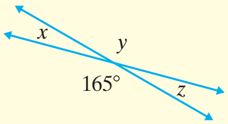


Find the measures of angles x, y, and z in each figure. See Examples 7 and 8.

37.



38.



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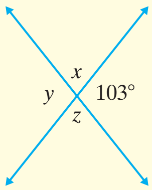
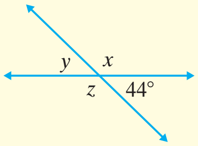
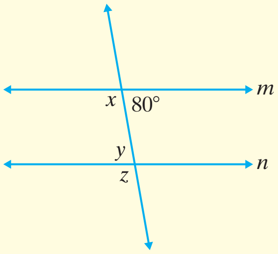


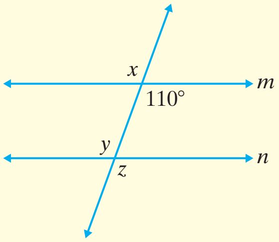
40.



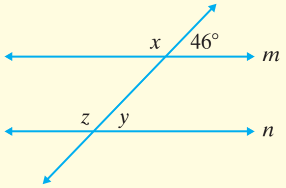
41. $m \parallel n$



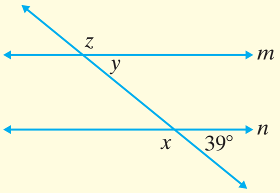
42. $m \parallel n$



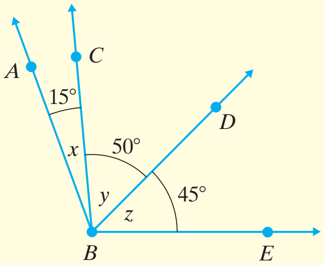
43. $m \parallel n$



44. $m \parallel n$



Objectives A D Mixed Practice *Find two other ways of naming each angle. See Example 2.*



45. $\angle x$

46. $\angle y$

47. $\angle z$

48. $\angle ABE$ (just name one other way)

Find the measure of each angle in the figure above. See Example 6.

49. $\angle ABC$

50. $\angle EBD$

51. $\angle CBD$

52. $\angle CBA$

53. $\angle DBA$

54. $\angle EBC$

55. $\angle CBE$

56. $\angle ABE$

Review

Perform each indicated operation. See Sections 3.3, 3.5, and 3.7.

57. $\frac{7}{8} + \frac{1}{4}$

58. $\frac{7}{8} - \frac{1}{4}$

59. $\frac{7}{8} \cdot \frac{1}{4}$

60. $\frac{7}{8} \div \frac{1}{4}$

61. $3\frac{1}{3} - 2\frac{1}{2}$

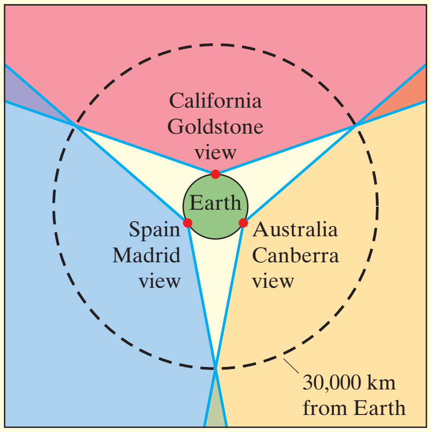
62. $3\frac{1}{3} + 2\frac{1}{2}$

63. $3\frac{1}{3} \div 2\frac{1}{2}$

64. $3\frac{1}{3} \cdot 2\frac{1}{2}$

Concept Extensions

Use this North Pole overhead view of the three sites of the Deep Space Network to answer Exercises 65 and 66. (See the Chapter Opener.)



65. How many degrees are there around the Earth at the equator?

66. If the three sites of the Deep Space Network (red dots shown) are about the same number of degrees apart, how many degrees apart are they?

67. The angle between the two walls of the Vietnam Veterans Memorial in Washington, D.C., is 125.2°. Find the supplement of this angle. (*Source:* National Park Service)

68. The faces of Khafre's Pyramid at Giza, Egypt, are inclined at an angle of 53.13°. Find the complement of this angle. (*Source:* PBS *NOVA* Online)

69. One great pyramid at Chichen Itza, Mexico, was the Temple of Kukulkan. The four faces of this pyramid have protruding stairways that rise at a 45° angle. Find the complement of this angle.

70. The UNESCO World Heritage Site of the Cahokia Mounds is located in Cahokia, Illinois. This site was once populated by the Mississippian People of North America and was built up on a series of mounds. These mounds were completely constructed of silt and dirt. The largest and best known of these is the Monk's Mound, which is considered a truncated pyramid, with the top being a flat base rather than extending to a point. On one side of this mound, modern archeologists have measured a 35° angle. Find the complement of this angle.

Answer true or false for Exercises 71 through 74. See the Concept Check in this section. If false, explain why.

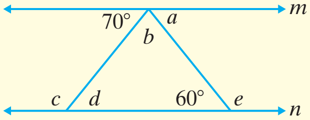
71. The complement of a 100° angle is an 80° angle.

72. It is possible to find the complement of a 120° angle.

73. It is possible to find the supplement of a 120° angle.

74. The supplement of a 5° angle is a 175° angle.

75. If lines m and n are parallel, find the measures of angles a through e.



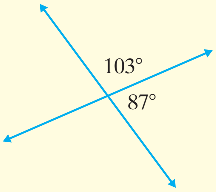
76. Below is a rectangle. List which segments, if extended, would be parallel lines.

77. Can two supplementary angles both be acute? Explain why or why not.

78. In your own words, describe how to find the complement and the supplement of a given angle.

79. Find two complementary angles with the same measure.

80. Is the figure below possible? Why or why not?



9.2 Plane Figures and Solids

Objectives

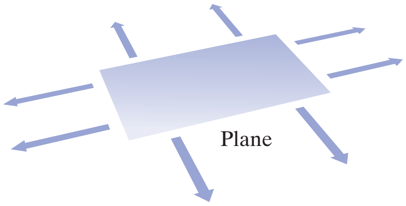
A Identify Plane Figures.

B Identify Solids.

In order to prepare for the sections ahead in this chapter, we first review plane figures and solids.

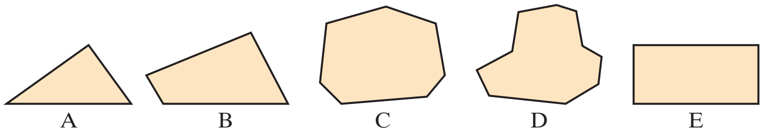
Objective A Identifying Plane Figures

Recall from Section 9.1 that a **plane** is a flat surface that extends indefinitely.

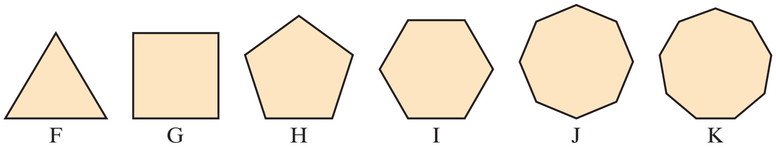


A **plane figure** is a figure that lies on a plane. Plane figures, like planes, have length and width but no thickness or depth.

A **polygon** is a closed plane figure that basically consists of three or more line segments that meet at their endpoints.



A **regular polygon** is one whose sides are all the same length and whose angles are the same measure.



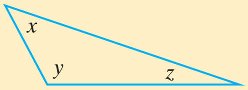
A polygon is named according to the number of its sides.

Polygons

Number of Sides	Name	Figure Examples
3	Triangle	A, F
4	Quadrilateral	B, E, G
5	Pentagon	H
6	Hexagon	I
7	Heptagon	C
8	Octagon	J
9	Nonagon	K
10	Decagon	D

Some triangles and quadrilaterals are given special names, so let's study these polygons further. We begin with triangles.

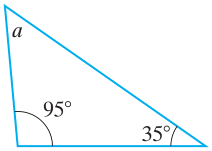
The sum of the measures of the angles of a triangle is 180°.



$$m\angle x + m\angle y + m\angle z = 180^\circ$$

Example 1

Find the measure of $\angle a$.



Solution: Since the sum of the measures of the three angles is 180°, we have

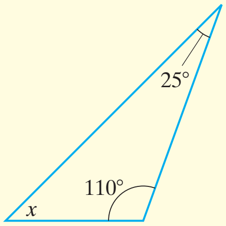
$$\text{measure of } \angle a, \text{ or } m\angle a = 180^\circ - 95^\circ - 35^\circ = 50^\circ$$

To check, see that $95^\circ + 35^\circ + 50^\circ = 180^\circ$.

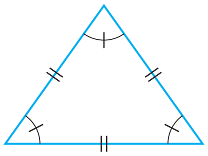
Work Practice 1

Practice 1

Find the measure of $\angle x$.

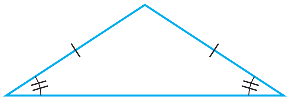


We can classify triangles according to the lengths of their sides. (We will use tick marks to denote the sides and angles of a figure that are equal.)



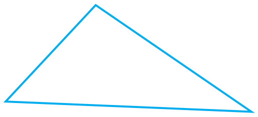
Equilateral triangle

All three sides are the same length. Also, all three angles have the same measure.



Isosceles triangle

Two sides are the same length. Also, the angles opposite the equal sides have equal measure.



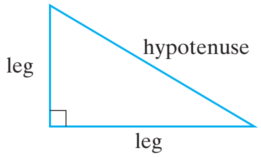
Scalene triangle

No sides are the same length. No angles have the same measure.

Answer

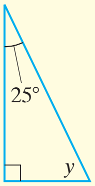
1. 45°

One other important type of triangle is a right triangle. A **right triangle** is a triangle with a right angle. The side opposite the right angle is called the **hypotenuse**, and the other two sides are called **legs**.

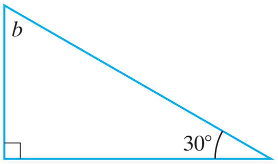


Practice 2

Find the measure of $\angle y$.



Example 2 Find the measure of $\angle b$.



Solution: We know that the measure of the right angle, ∟, is 90°. Since the sum of the measures of the angles is 180°, we have

$$\text{measure of } \angle b\text{, or } m\angle b = 180° - 90° - 30° = 60°$$

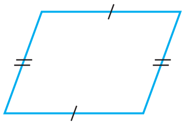
Work Practice 2

Helpful Hint

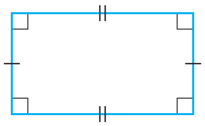
From the previous example, can you see that in a right triangle, the sum of the other two acute angles is 90°? This is because

$$\underset{\substack{\uparrow\\ \text{right}\\ \text{angle's}\\ \text{measure}}}{90°} + \underset{\substack{\uparrow\\ \text{sum of}\\ \text{other two}\\ \text{angles'}\\ \text{measures}}}{90°} = \underset{\substack{\uparrow\\ \text{sum of}\\ \text{angles'}\\ \text{measures}}}{180°}$$

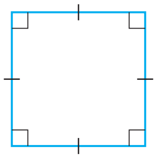
Now we review some special quadrilaterals. A **parallelogram** is a special quadrilateral with opposite sides parallel and equal in length.



A **rectangle** is a special **parallelogram** that has four right angles.



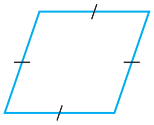
A **square** is a special **rectangle** that has all four sides equal in length.



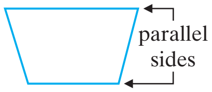
Answer

2. 65°

A **rhombus** is a special **parallelogram** that has all four sides equal in length.

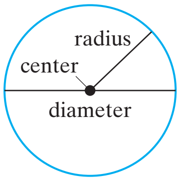


A **trapezoid** is a quadrilateral with exactly one pair of opposite sides parallel.



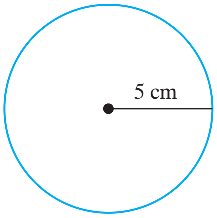
✓Concept Check True or false? All quadrilaterals are parallelograms. Explain.

In addition to triangles, quadrilaterals, and other polygons, circles are also plane figures. A **circle** is a plane figure that consists of all points that are the same fixed distance from a point *c*. The point *c* is called the **center** of the circle. The **radius** of a circle is the distance from the center of the circle to any point on the circle. The **diameter** of a circle is the distance across the circle passing through the center. Notice that the diameter is twice the radius, and the radius is half the diameter.



$$\begin{array}{ccccc} \text{diameter} & = & 2 & \cdot & \text{radius} \\ \downarrow & & \downarrow & & \downarrow \\ d & = & 2 & \cdot & r \end{array} \qquad \begin{array}{ccc} \text{radius} & = & \dfrac{\text{diameter}}{2} \\ \downarrow & & \downarrow \\ r & = & \dfrac{d}{2} \end{array}$$

Example 3 Find the diameter of the circle.



Solution: The diameter is twice the radius.

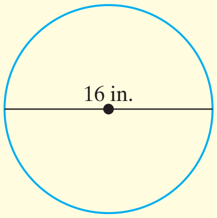
$$d = 2 \cdot r$$
$$d = 2 \cdot 5 \text{ cm} = 10 \text{ cm}$$

The diameter is 10 centimeters.

Work Practice 3

Practice 3

Find the radius of the circle.



Objective B Identifying Solids

Recall from Section 9.1 that space extends in all directions indefinitely.

A **solid** is a figure that lies in space. Solids have length, width, and height or depth.

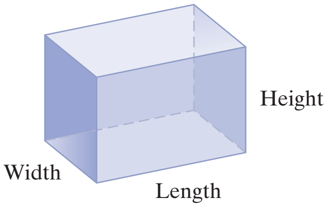
Answer

3. 8 in.

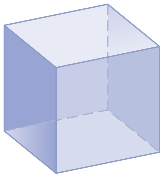
✓Concept Check Answer

false

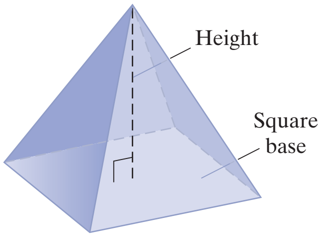
A **rectangular solid** is a solid that consists of six sides, or faces, all of which are rectangles.



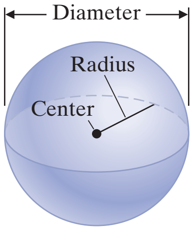
A **cube** is a rectangular solid whose six sides are squares.



A **pyramid** is shown below. The pyramids we will study have square bases and heights that are perpendicular to their base.



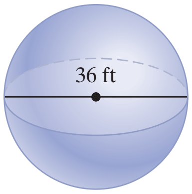
A **sphere** consists of all points in space that are the same distance from a point c. The point c is called the **center** of the sphere. The **radius** of a sphere is the distance from the center to any point on the sphere. The **diameter** of a sphere is the distance across the sphere passing through the center.



The radius and diameter of a sphere are related in the same way as the radius and diameter of a circle.

$$d = 2 \cdot r \quad \text{or} \quad r = \frac{d}{2}$$

Example 4 Find the radius of the sphere.



Solution: The radius is half the diameter.

$$r = \frac{d}{2}$$

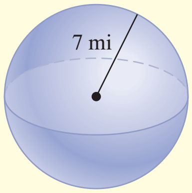
$$r = \frac{36 \text{ feet}}{2} = 18 \text{ feet}$$

The radius is 18 feet.

Work Practice 4

Practice 4

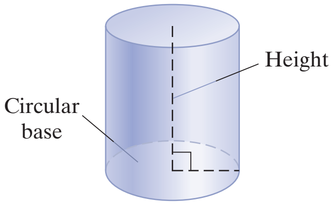
Find the diameter of the sphere.



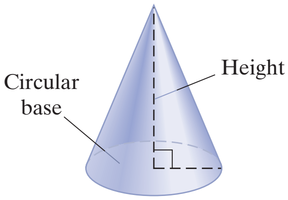
Answer

4. 14 mi

The **cylinders** we will study have bases that are in the shape of circles and heights that are perpendicular to their base.



The **cones** we will study have bases that are circles and heights that are perpendicular to their base.



Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 9.2

Watch the section lecture video and answer the following questions.

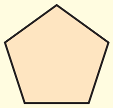
Objective A **1.** From the lecture after Example 2, since all angles of an equilateral triangle have the same measure, what is the measure of each angle?

Objective B **2.** What solid is identified in Example 6? What two real-life examples of the solid are given?

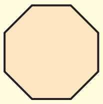
9.2 Exercise Set MyLab Math

Objective A *Identify each polygon. See the table at the beginning of this section.*

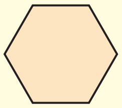
1.



2.



3.



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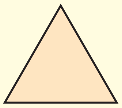

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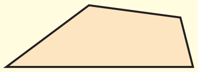
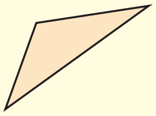


6.



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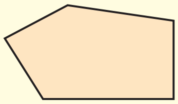

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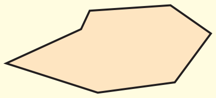
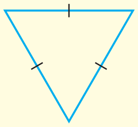
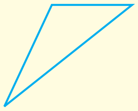


Classify each triangle as equilateral, isosceles, or scalene. Also identify any triangles that are also right triangles. See the triangle classification after Example 1.

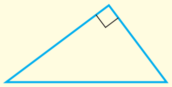
9.



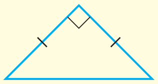
10.



11.



12.



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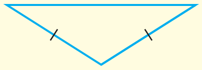

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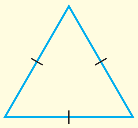
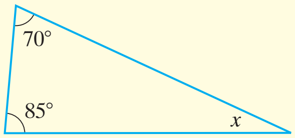


Find the measure of ∠x in each figure. See Examples 1 and 2.

15.



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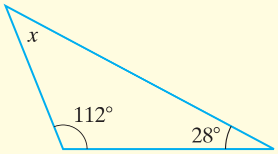

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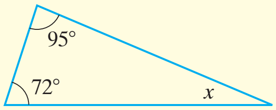

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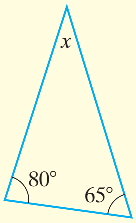
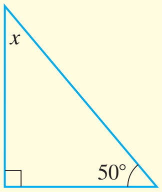


19.



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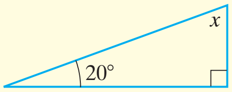


Fill in each blank.

21. Twice the radius of a circle is its _____________.

22. A rectangle with all four sides equal is a(n) _____________.

23. A parallelogram with four right angles is a(n) _____________.

24. Half the diameter of a circle is its _____________.

25. A quadrilateral with opposite sides parallel is a(n) _____________.

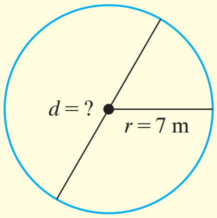
26. A quadrilateral with exactly one pair of opposite sides parallel is a(n) _____________.

27. The side opposite the right angle of a right triangle is called the _____________.

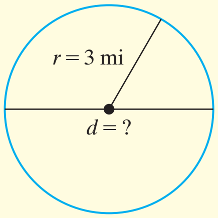
28. A triangle with no equal sides is a(n) _____________.

Find the unknown diameter or radius in each figure. See Example 3.

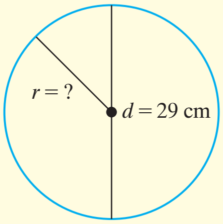
29.



30.



31.



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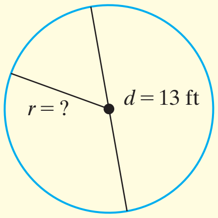
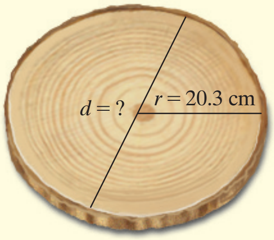
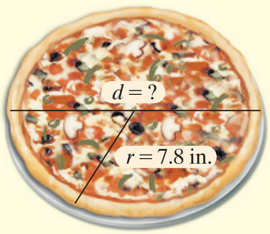


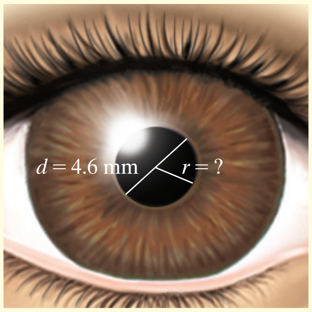
33.



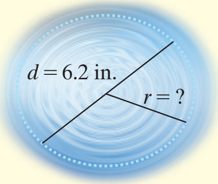
34.



35. The normal pupil size in adults varies and even changes as one ages. The average diameter of a pupil in a 20-year-old is 4.6 mm. (*Source: National Geographic*)

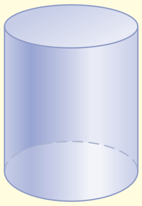


36. A ripple in the water has a diameter of 6.2 inches.

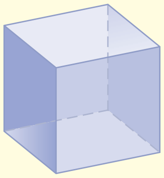


Objective B *Identify each solid.*

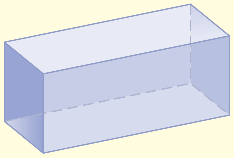
37.



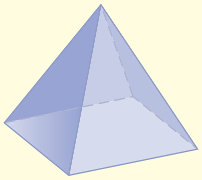
38.



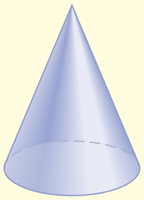
39.



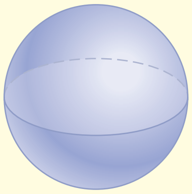
40.



41.



42.



Identify the basic shape of each item.

43.

44.

45.

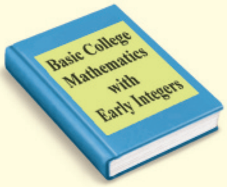
46.

47.

48.

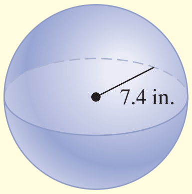
49.

50.

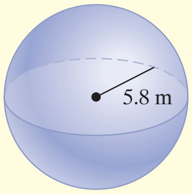


Find each unknown radius or diameter. See Example 4.

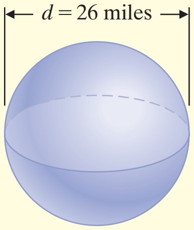
51. The radius of a sphere is 7.4 inches. Find its diameter.



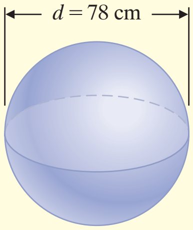
52. The radius of a sphere is 5.8 meters. Find its diameter.



53. Find the radius of the sphere.



54. Find the radius of the sphere.



55. Saturn has a radius of approximately 36,184 miles. What is its diameter?

56. A sphere-shaped wasp nest found in Japan had a radius of approximately 15 inches. What was its diameter? (*Source: Guinness World Records*)

Review

Perform each indicated operation. See Sections 1.3, 1.6, 4.2, and 4.3.

57. $2(18) + 2(36)$

58. $4(87)$

59. $4(3.14)$

60. $2(7.8) + 2(9.6)$

Concept Extensions

Determine whether each statement is true or false. See the Concept Check in this section.

61. A square is also a rhombus.

62. A square is also a regular polygon.

63. A rectangle is also a parallelogram.

64. A trapezoid is also a parallelogram.

65. A pentagon is also a quadrilateral.

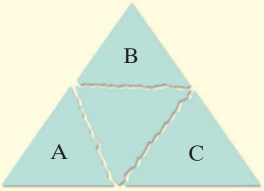
66. A rhombus is also a parallelogram.

67. Is an isosceles right triangle possible? If so, draw one.

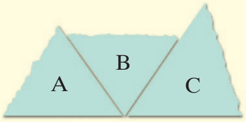
68. In your own words, explain whether a rhombus is always a square.

69. The following demonstration is credited to the mathematician Pascal, who is said to have developed it as a young boy.

Cut a triangle from a piece of paper. The length of the sides and the size of the angles are unimportant. Tear the points off the triangle as shown in the top right figure.



Place the points of the triangle together, as shown in the bottom right figure. Notice that a straight line is formed. What was Pascal trying to show?

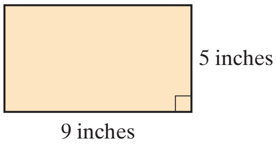


9.3 Perimeter

Objective A Using Formulas to Find Perimeters

Recall from Section 1.3 that the perimeter of a polygon is the distance around the polygon. This means that the perimeter of a polygon is the sum of the lengths of its sides.

Example 1 Find the perimeter of the rectangle below.



Solution:

$$\begin{aligned}\text{perimeter} &= 9\text{ inches} + 9\text{ inches} + 5\text{ inches} + 5\text{ inches}\\ &= 28\text{ inches}\end{aligned}$$

Work Practice 1

Notice that the perimeter of the rectangle in Example 1 can be written as $2\cdot(\underset{\substack{\uparrow\\ \text{length}}}{9\text{ inches}}) + 2\cdot(\underset{\substack{\uparrow\\ \text{width}}}{5\text{ inches}})$.

In general, we can say that the perimeter of a rectangle is always

$2\cdot\text{length} + 2\cdot\text{width}$

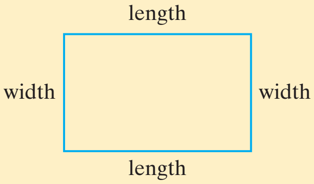
As we have just seen, the perimeters of some special figures such as rectangles form patterns. These patterns are given as **formulas.** The formula for the perimeter of a rectangle is shown next:

Perimeter of a Rectangle

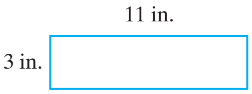
$$\text{perimeter} = 2\cdot\text{length} + 2\cdot\text{width}$$

In symbols, this can be written as

$$P = 2\cdot l + 2\cdot w$$



Example 2 Find the perimeter of a rectangle with a length of 11 inches and a width of 3 inches.



Solution: We use the formula for perimeter and replace the letters by their known lengths.

$$\begin{aligned}P &= 2\cdot l + 2\cdot w\\ &= 2\cdot 11\text{ in.} + 2\cdot 3\text{ in.} \quad \text{Replace } l \text{ with 11 in. and } w \text{ with 3 in.}\\ &= 22\text{ in.} + 6\text{ in.}\\ &= 28\text{ in.}\end{aligned}$$

The perimeter is 28 inches.

Work Practice 2

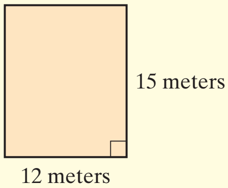
Objectives

A Use Formulas to Find Perimeters.

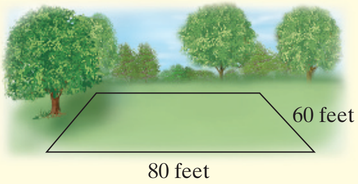
B Use Formulas to Find Circumferences.

Practice 1

a. Find the perimeter of the rectangle.



b. Find the perimeter of the rectangular lot shown below:



Practice 2

Find the perimeter of a rectangle with a length of 22 centimeters and a width of 10 centimeters.

Answers

1. a. 54 m **b.** 280 ft

2. 64 cm

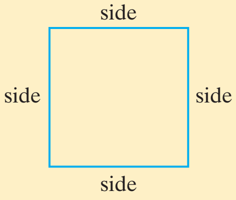
Recall that a square is a special rectangle with all four sides the same length. The formula for the perimeter of a square is shown next:

Perimeter of a Square

Perimeter = side + side + side + side
= 4 · side

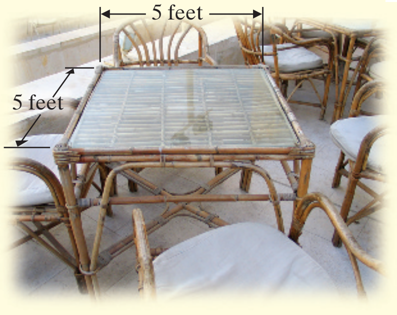
In symbols,

$P = 4 \cdot s$



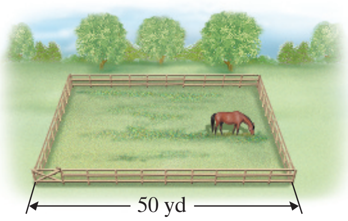
Practice 3

Find the perimeter of a square tabletop if each side is 5 feet long.



Example 3 Finding the Perimeter of a Field

How much fencing is needed to enclose a square field 50 yards on a side?



Solution: To find the amount of fencing needed, we find the distance around, or perimeter. The formula for the perimeter of a square is $P = 4 \cdot s$. We use this formula and replace s by 50 yards.

$$\begin{aligned} P &= 4 \cdot s \\ &= 4 \cdot 50 \text{ yd} \\ &= 200 \text{ yd} \end{aligned}$$

The amount of fencing needed is 200 yards.

Work Practice 3

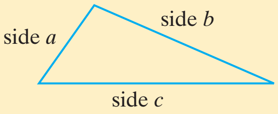
The formula for the perimeter of a triangle with sides of lengths a, b, and c is given next:

Perimeter of a Triangle

Perimeter = side a + side b + side c

In symbols,

$P = a + b + c$

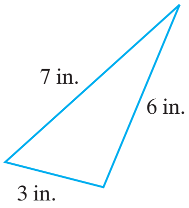


Practice 4

Find the perimeter of a triangle if the sides are 5 centimeters, 10 centimeters, and 6 centimeters in length.

Example 4

Find the perimeter of a triangle if the sides are 3 inches, 7 inches, and 6 inches.



Answers

3. 20 ft **4.** 21 cm

Solution: The formula for the perimeter is $P = a + b + c$, where a, b, and c are the lengths of the sides. Thus,

$$
\begin{aligned}
P &= a + b + c \\
&= 3 \text{ in.} + 7 \text{ in.} + 6 \text{ in.} \\
&= 16 \text{ in.}
\end{aligned}
$$

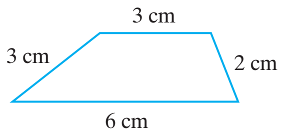
The perimeter of the triangle is 16 inches.

Work Practice 4

Recall that to find the perimeter of other polygons, we find the sum of the lengths of their sides.

Example 5

Find the perimeter of the trapezoid shown below:



Solution: To find the perimeter, we find the sum of the lengths of its sides.

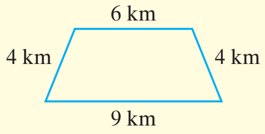
$$\text{perimeter} = 3 \text{ cm} + 2 \text{ cm} + 6 \text{ cm} + 3 \text{ cm} = 14 \text{ cm}$$

The perimeter is 14 centimeters.

Work Practice 5

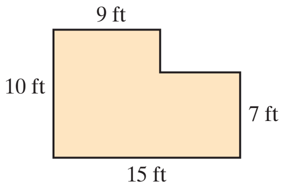
Practice 5

Find the perimeter of the trapezoid shown.

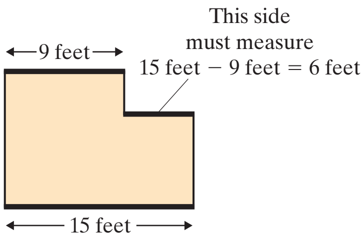


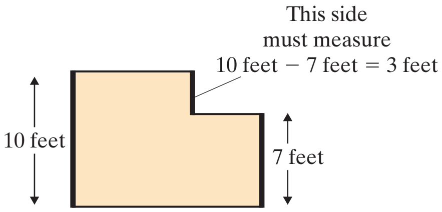
Example 6 Finding the Perimeter of a Room

Find the perimeter of the room shown below:



Solution: To find the perimeter of the room, we first need to find the lengths of all sides of the room.

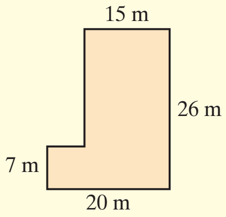




(*Continued on next page*)

Practice 6

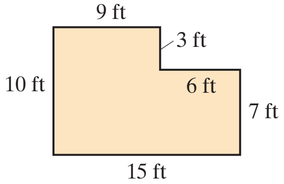
Find the perimeter of the room shown.



Answers

5. 23 km **6.** 92 m

Now that we know the measures of all sides of the room, we can add the measures to find the perimeter.



$$\begin{aligned} \text{perimeter} &= 10 \text{ ft} + 9 \text{ ft} + 3 \text{ ft} + 6 \text{ ft} + 7 \text{ ft} + 15 \text{ ft} \\ &= 50 \text{ ft} \end{aligned}$$

The perimeter of the room is 50 feet.

Work Practice 6

Practice 7

A rectangular lot measures 60 feet by 120 feet. Find the cost to install fencing around the lot if the cost of fencing is \$1.90 per foot.

Example 7 Calculating the Cost of Wallpaper Border

A rectangular room measures 10 feet by 12 feet. Find the cost to hang a wallpaper border on the walls close to the ceiling if the cost of the wallpaper border is \$1.09 per foot.

Solution: First we find the perimeter of the room.

$$\begin{aligned} P &= 2 \cdot l + 2 \cdot w \\ &= 2 \cdot 12 \text{ ft} + 2 \cdot 10 \text{ ft} && \text{Replace } l \text{ with 12 feet and } w \text{ with 10 feet.} \\ &= 24 \text{ ft} + 20 \text{ ft} \\ &= 44 \text{ ft} \end{aligned}$$

The cost of the wallpaper is

$$\text{cost} = \$1.09 \cdot 44 \text{ ft} = 47.96$$

The cost of the wallpaper is \$47.96.

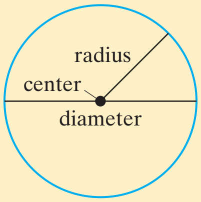
Work Practice 7

Objective B Using Formulas to Find Circumferences

Recall from Section 4.3 that the distance around a circle is called the **circumference.** This distance depends on the radius or the diameter of the circle.

The formulas for circumference are shown next:

Circumference of a Circle



$$\text{Circumference} = 2 \cdot \pi \cdot \text{radius} \quad \text{or} \quad \text{Circumference} = \pi \cdot \text{diameter}$$

In symbols,

$$C = 2 \cdot \pi \cdot r \quad \text{or} \quad C = \pi \cdot d,$$

where $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$.

Answer

7. \$684

To better understand circumference and π (pi), try the following experiment. Take any can and measure its circumference and its diameter.

The can in the figure above has a circumference of 23.5 centimeters and a diameter of 7.5 centimeters. Now divide the circumference by the diameter.

$$\frac{\text{circumference}}{\text{diameter}} = \frac{23.5\text{ cm}}{7.5\text{ cm}} \approx 3.13$$

Try this with other sizes of cylinders and circles–you should always get a number close to 3.1. The exact ratio of circumference to diameter is π. (Recall that $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$.)

Example 8 Finding Circumference of Spa

A homeowner plans to install a border of new tiling around the circumference of her circular spa. If her spa has a diameter of 14 feet, find its exact circumference. Then use the approximation 3.14 for π to approximate the circumference.

Solution: Because we are given the diameter, we use the formula $C = \pi \cdot d$.

$$\begin{aligned} C &= \pi \cdot d \\ &= \pi \cdot 14\text{ ft} \quad \text{Replace } d \text{ with 14 feet.} \\ &= 14\pi\text{ ft} \end{aligned}$$

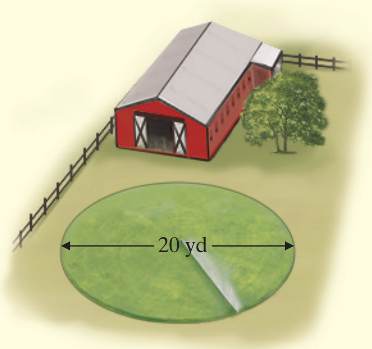
The circumference of the spa is *exactly* 14π feet. By replacing π with the *approximation* 3.14, we find that the circumference is *approximately* 14 feet · 3.14 = 43.96 feet.

Work Practice 8

✓Concept Check The distance around which figure is greater: a square with side length 5 inches or a circle with radius 3 inches?

Practice 8

a. An irrigation device waters a circular region with a diameter of 20 yards. Find the exact circumference of the watered region, then use $\pi \approx 3.14$ to give an approximation.



b. A manufacturer of clocks is designing a new model. To help the designer calculate the cost of materials to make the new clock, calculate the circumference of a clock with a face diameter of 12 inches. Give the exact circumference; then use $\pi \approx 3.14$ to approximate.

Answers

8. a. exactly 20π yd $\approx$ 62.8 yd
b. exactly 12π in. $\approx$ 37.68 in.

✓Concept Check Answer

a square with side length 5 in.

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

circumference radius π $\frac{22}{7}$

diameter perimeter 3.14

1. The ______________ of a polygon is the sum of the lengths of its sides.

2. The distance around a circle is called the ______________.

3. The exact ratio of circumference to diameter is ______________.

4. The diameter of a circle is double its ______________.

5. Both ______________ and ______________ are approximations for π.

6. The radius of a circle is half its ______________.

Martin-Gay Interactive Videos

See Video 9.3

Watch the section lecture video and answer the following questions.

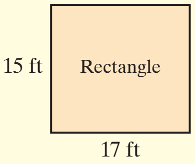
Objective A 7. In Example 1, how can the perimeter be found if we forget the formula?

Objective B 8. From the lecture before Example 6, circumference is a special name for what?

9.3 Exercise Set MyLab Math

Objective A *Find the perimeter of each figure. See Examples 1 through 6.*

1.



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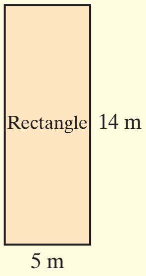

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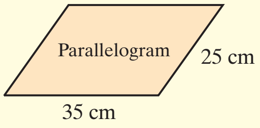

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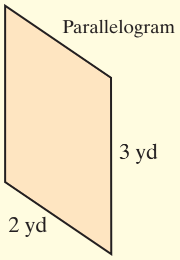

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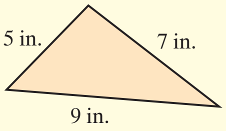

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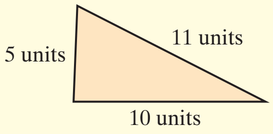

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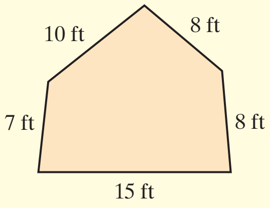

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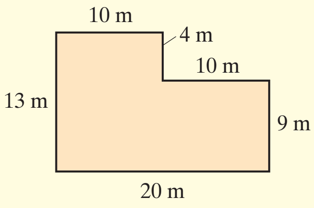
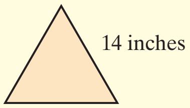


Find the perimeter of each regular polygon. (The sides of a regular polygon have the same length.)

9.



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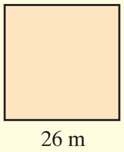

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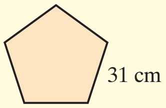

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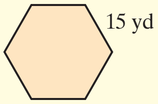


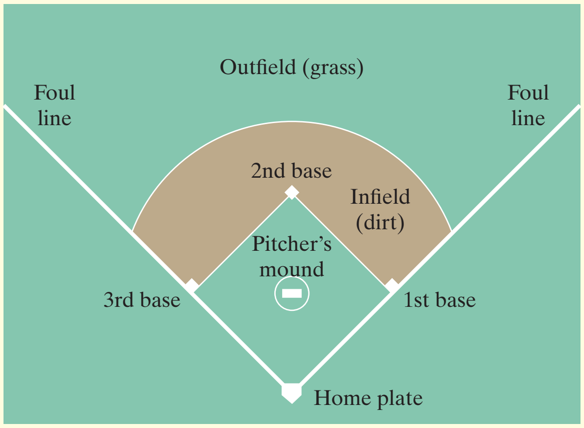
Solve. See Examples 1 through 7.

13. A polygon has sides of length 5 feet, 3 feet, 2 feet, 7 feet, and 4 feet. Find its perimeter.

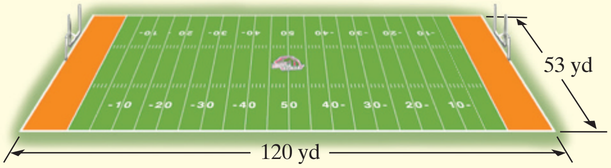
15. A line-marking machine lays down lime powder to mark both foul lines on a baseball field. If each foul line for this field measures 312 feet, how many feet of lime powder will be deposited?

16. A baseball diamond has 4 sides, with each side length 90 feet. If a baseball player hits a home run, how far does the player run (home plate, around the bases, then back to home plate)?

14. A triangle has sides of length 8 inches, 12 inches, and 10 inches. Find its perimeter.



17. If a football field is 53 yards wide and 120 yards long, what is the perimeter?



18. A stop sign has eight equal sides of length 12 inches. Find its perimeter.

19. A metal strip is being installed around a workbench that is 8 feet long and 3 feet wide. Find how much stripping is needed for this project.

20. Find how much fencing is needed to enclose a rectangular garden 70 feet by 21 feet.

21. If the stripping in Exercise **19** costs $2.50 per foot, find the total cost of the stripping.

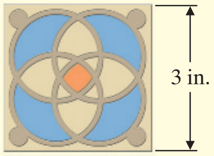
22. If the fencing in Exercise **20** costs $2 per foot, find the total cost of the fencing.

23. A regular octagon has a side length of 9 inches. Find its perimeter.

24. A regular pentagon has a side length of 14 meters. Find its perimeter.

25. Find the perimeter of the top of a square compact disc case if the length of one side is 7 inches.

26. Find the perimeter of a square ceramic tile with a side of length 3 inches.

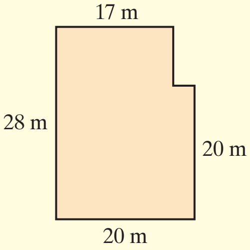


27. A rectangular room measures 10 feet by 11 feet. Find the cost of installing a strip of wallpaper around the room if the wallpaper costs \$0.86 per foot.

28. A rectangular house measures 85 feet by 70 feet. Find the cost of installing gutters around the house if the cost is \$2.36 per foot.

Find the perimeter of each figure. See Example 6.

29.



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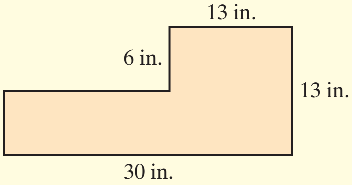

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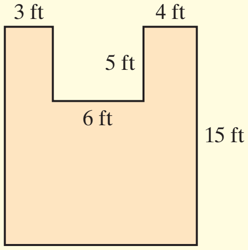

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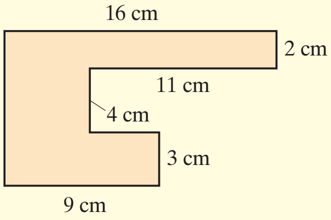

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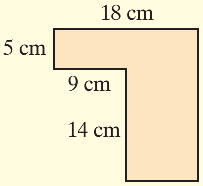

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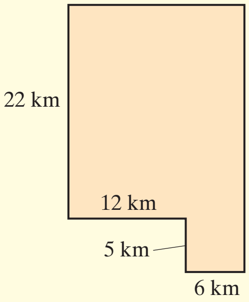
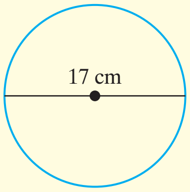


Objective B

Find the circumference of each circle. Give the exact circumference and then an approximation. Use $\pi \approx 3.14$. *See Example 8.*

35.



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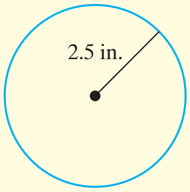

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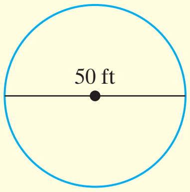

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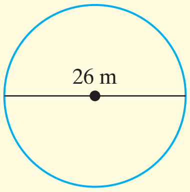

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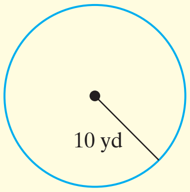


Solve.

41. The largest round barn in the world is located at the Marshfield Fairgrounds in Wisconsin. The barn has a diameter of 150 ft. What is the circumference of the barn? Give the exact circumference and then an approximation using $\pi \approx 3.14$. (*Source: The Milwaukee Journal Sentinel*)

42. A father just bought a trampoline for his children to use. The trampoline has a diameter of 15 feet. If he wishes to buy netting to go around the outside of the trampoline, how many feet of netting does he need? Give the exact circumference and then an approximation using $\pi \approx 3.14$.

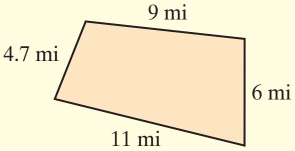
43. Meteor Crater, near Winslow, Arizona, is 4000 feet in diameter. Approximate the distance around the crater. Use 3.14 for π. (*Source: The Handy Science Answer Book*)

44. The largest pearl, the *Pearl of Lao-tze,* has a diameter of $5\frac{1}{2}$ inches. Approximate the distance around the pearl. Use $\frac{22}{7}$ for π. (*Source: The Guinness Book of World Records*)

Objectives A B Mixed Practice

Find the distance around each figure. For circles, give the exact circumference and then an approximation. Use $\pi \approx 3.14$. See Examples 1 through 8.

45.



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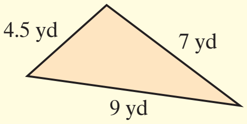

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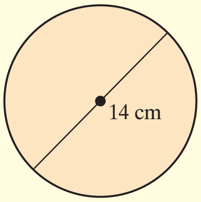

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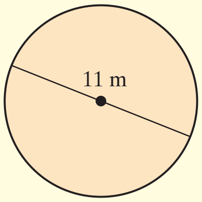

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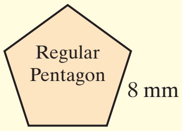

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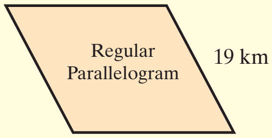

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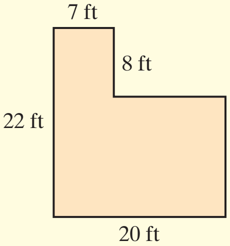

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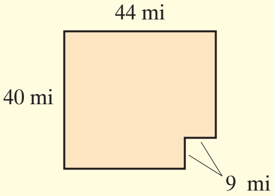


Review

Simplify. See Section 1.9.

53. $5 + 6 \cdot 3$

54. $25 - 3 \cdot 7$

55. $(20 - 16) \div 4$

56. $6 \cdot (8 + 2)$

57. $72 \div (2 \cdot 6)$

58. $(72 \div 2) \cdot 6$

59. $(18 + 8) - (12 + 4)$

60. $4^1 \cdot (2^3 - 8)$

Concept Extensions

There are a number of factors that determine the dimensions of a rectangular soccer field. Use the table below to answer Exercises 61 and 62.

Soccer Field Width and Length

Age	Width Min–Max	Length Min–Max
Under 6/7:	15–20 yards	25–30 yards
Under 8:	20–25 yards	30–40 yards
Under 9:	30–35 yards	40–50 yards
Under 10:	40–50 yards	60–70 yards
Under 11:	40–50 yards	70–80 yards
Under 12:	40–55 yards	100–105 yards
Under 13:	50–60 yards	100–110 yards
International:	70–80 yards	110–120 yards

61. **a.** Find the minimum length and width of a soccer field for 8-year-old children. (Carefully consider the age.)
b. Find the perimeter of this field.

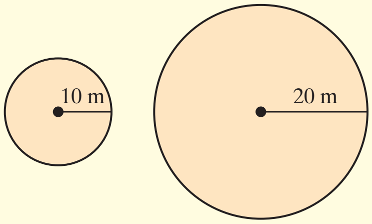
62. **a.** Find the maximum length and width of a soccer field for 12-year-old children.
b. Find the perimeter of this field.

Solve. See the Concept Check in this section. Choose the figure that has the greater distance around.

63. **a.** A square with side length 3 inches
b. A circle with diameter 4 inches

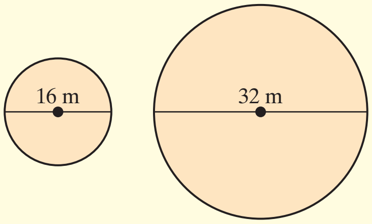
64. **a.** A circle with diameter 7 inches
b. A square with side length 7 inches

65. **a.** Find the circumference of each circle. Approximate the circumference by using 3.14 for π.



b. If the radius of a circle is doubled, is its corresponding circumference doubled?

66. **a.** Find the circumference of each circle. Approximate the circumference by using 3.14 for π.



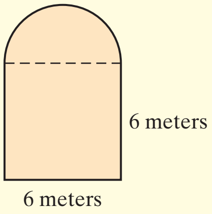
b. If the diameter of a circle is doubled, is its corresponding circumference doubled?

67. In your own words, explain how to find the perimeter of any polygon.

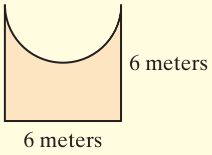
68. In your own words, explain how perimeter and circumference are the same and how they are different.

Find the perimeter. Round your results to the nearest tenth.

69.

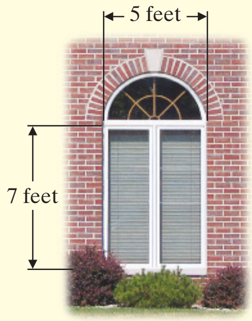


70.

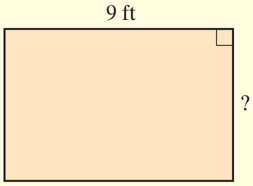


71.

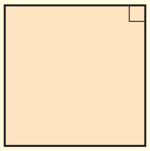
72.



73. The perimeter of this rectangle is 31 feet. Find its width.



74. The perimeter of this square is 18 inches. Find the length of a side.



9.4 Area

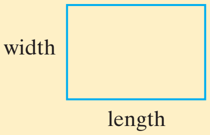
Objective A Finding Areas of Geometric Figures

Recall that area measures the amount of surface of a region. Thus far, we know how to find the area of a rectangle and a square. These formulas, as well as formulas for finding the areas of other common geometric figures, are given next:

Area Formulas of Common Geometric Figures

Geometric Figure | **Area Formula**

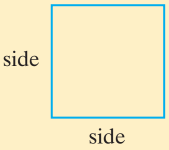
RECTANGLE



Area of a rectangle:

$$\text{Area} = \text{length} \cdot \text{width}$$
$$A = l \cdot w$$

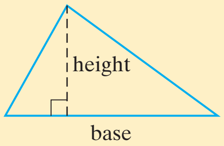
SQUARE



Area of a square:

$$\text{Area} = \text{side} \cdot \text{side}$$
$$A = s \cdot s = s^2$$

TRIANGLE



Area of a triangle:

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$
$$A = \frac{1}{2} \cdot b \cdot h$$

(*Continued on next page*)

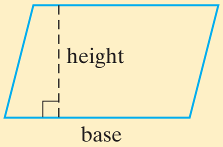
Objective

A Find the Areas of Geometric Figures.

Area Formulas of Common Geometric Figures (*continued*)

Geometric Figure | **Area Formula**

PARALLELOGRAM

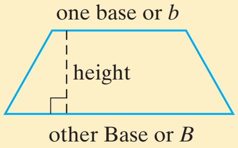


Area of a parallelogram:

Area = **b**ase · **h**eight

$A = b \cdot h$

TRAPEZOID



Area of a trapezoid:

Area $= \frac{1}{2} \cdot$ (one **b**ase + other **B**ase) · **h**eight

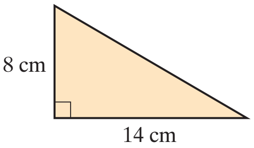
$$A = \frac{1}{2} \cdot (b + B) \cdot h$$

Use these formulas for the following examples.

Helpful Hint

Area is always measured in square units.

Example 1 Find the area of the triangle.



Solution:

$$\begin{aligned} A &= \frac{1}{2} \cdot b \cdot h \\ &= \frac{1}{2} \cdot 14 \text{ cm} \cdot 8 \text{ cm} \\ &= \frac{\overset{1}{\cancel{2}} \cdot 7 \cdot 8}{\underset{1}{\cancel{2}}} \text{ sq cm} \\ &= 56 \text{ square cm} \end{aligned}$$

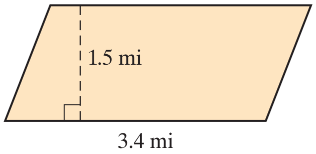
The area is 56 square centimeters.

Helpful Hint

You may see 56 sq cm, for example, written with the notation 56 cm^2. Both of these notations mean the same quantity.

Work Practice 1

Example 2 Find the area of the parallelogram.



Solution:

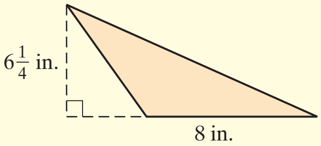
$$\begin{aligned} A &= b \cdot h \\ &= 3.4 \text{ miles} \cdot 1.5 \text{ miles} \\ &= 5.1 \text{ square miles} \end{aligned}$$

The area is 5.1 square miles.

Work Practice 2

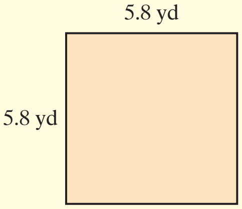
Practice 1

Find the area of the triangle.



Practice 2

Find the area of the square.



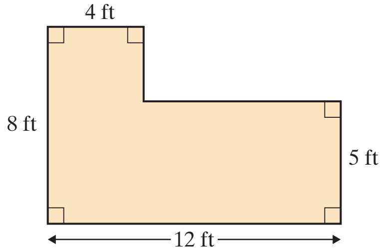
Answers

1. 25 sq in. **2.** 33.64 sq yd

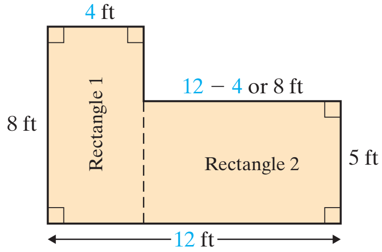
Helpful Hint

When finding the area of figures, be sure all measurements are changed to the same unit before calculations are made.

Example 3 Find the area of the figure.



Solution: Split the figure into two rectangles. To find the area of the figure, we find the sum of the areas of the two rectangles.



$$\text{Area of Rectangle 1} = l \cdot w$$
$$= 8 \text{ feet} \cdot 4 \text{ feet}$$
$$= 32 \text{ square feet}$$

Notice that the length of Rectangle 2 is 12 feet − 4 feet, or 8 feet.

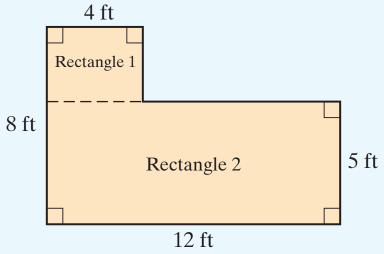
$$\text{Area of Rectangle 2} = l \cdot w$$
$$= 8 \text{ feet} \cdot 5 \text{ feet}$$
$$= 40 \text{ square feet}$$

$$\text{Area of the Figure} = \text{Area of Rectangle 1} + \text{Area of Rectangle 2}$$
$$= 32 \text{ square feet} + 40 \text{ square feet}$$
$$= 72 \text{ square feet}$$

Work Practice 3

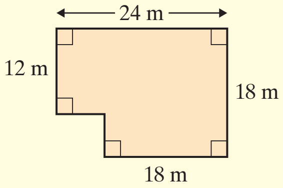
Helpful Hint

The figure in Example 3 can also be split into two rectangles as shown:



Practice 3

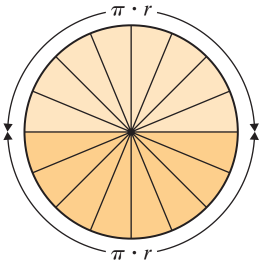
Find the area of the figure.



Answer

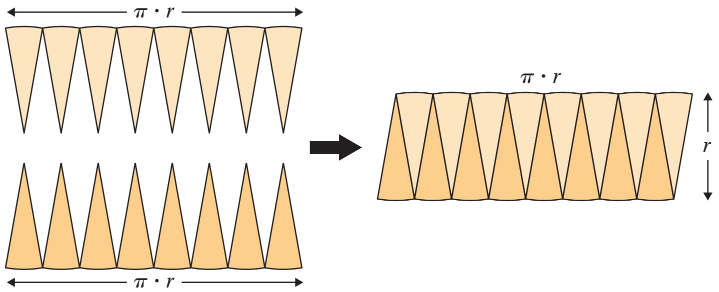
3. 396 sq m

To better understand the formula for area of a circle, try the following. Cut a circle into many pieces as shown:



The circumference of a circle is $2 \cdot \pi \cdot r$. This means that the circumference of half a circle is half of $2 \cdot \pi \cdot r$, or $\pi \cdot r$.

Then unfold the two halves of the circle and place them together as shown:



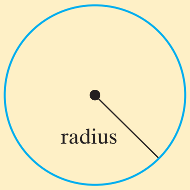
The figure on the right is almost a parallelogram with a base of $\pi \cdot r$ and a height of r. The area is

$$\begin{aligned} A &= \text{base} \cdot \text{height} \\ &\qquad\downarrow \qquad\quad \downarrow \\ &= (\pi \cdot r) \cdot \quad r \\ &= \pi \cdot r^2 \end{aligned}$$

This is the formula for area of a circle.

Area Formula of a Circle

CIRCLE



Area of a circle:

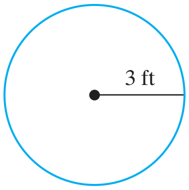
$$\text{Area} = \pi \cdot (\text{radius})^2$$

$$A = \pi \cdot r^2$$

(A fraction approximation for π is $\frac{22}{7}$.)

(A decimal approximation for π is 3.14.)

Example 4 Find the area of a circle with a radius of 3 feet. Find the exact area and an approximation. Use 3.14 as an approximation for π.

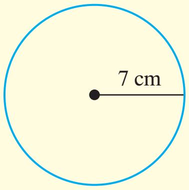


Solution: We let $r = 3$ ft and use the formula.

$$\begin{aligned} A &= \pi \cdot r^2 \\ &= \pi \cdot (3\text{ ft})^2 \\ &= \pi \cdot 9 \text{ square ft, or } 9 \cdot \pi \text{ square ft} \end{aligned}$$

Practice 4

Find the area of the given circle. Find the exact area and an approximation. Use 3.14 as an approximation for π.



Answer

4. 49π sq cm $\approx$ 153.86 sq cm

To approximate this area, we substitute 3.14 for π.

$$9 \cdot \pi \text{ square feet} \approx 9 \cdot 3.14 \text{ square feet}$$
$$= 28.26 \text{ square feet}$$

The *exact* area of the circle is 9π square feet, which is *approximately* 28.26 square feet.

Work Practice 4

✓**Concept Check** Use diagrams to decide which figure would have a larger area: a circle of diameter 10 inches or a square 10 inches long on each side.

✓**Concept Check Answer**

a square 10 in. long on each side

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 9.4

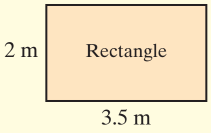
Watch the section lecture video and answer the following question.

Objective A **1.** What formula was used twice and why did we use it twice to solve Example 3?

9.4 Exercise Set MyLab Math

Objective A *Find the area of the geometric figure. If the figure is a circle, give the exact area and then use the given approximation for π to approximate the area. See Examples 1 through 4.*

1.



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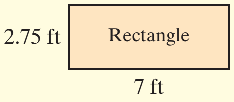

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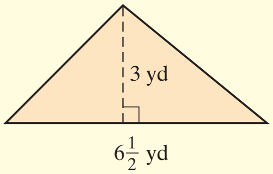

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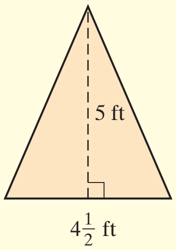

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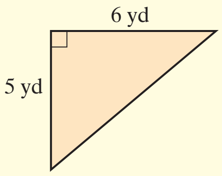

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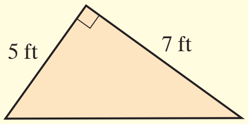
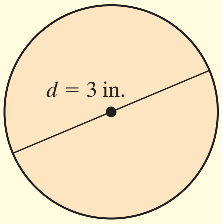
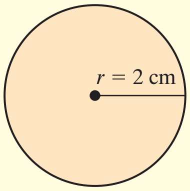


7. Use 3.14 for π.



8. Use $\frac{22}{7}$ for π.



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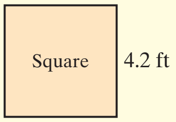

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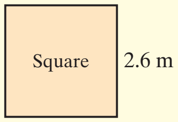

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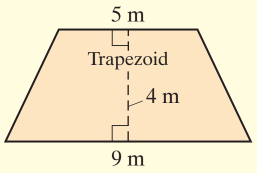

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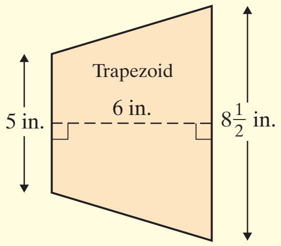

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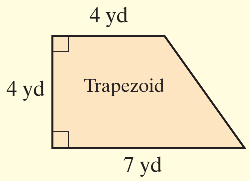

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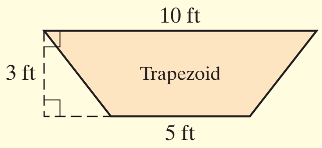

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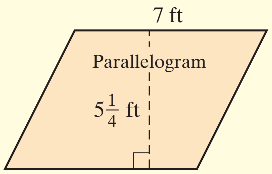

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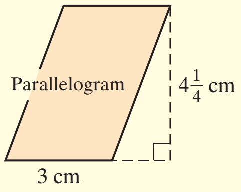

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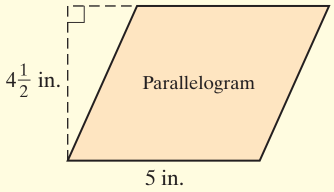

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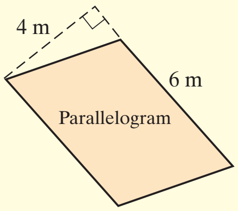

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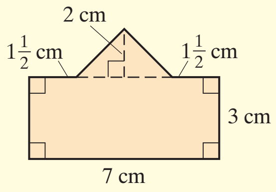

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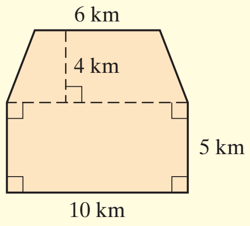

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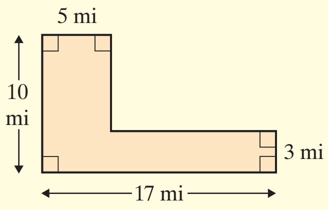

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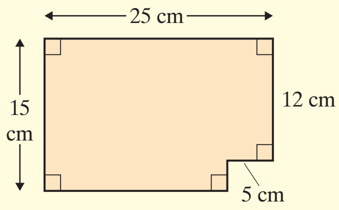

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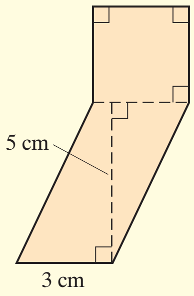

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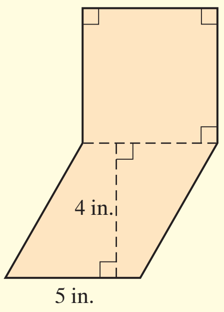
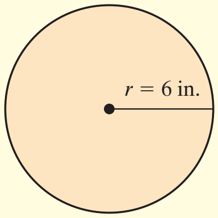
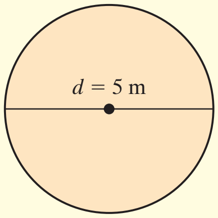


25. Use $\frac{22}{7}$ for π.



26. Use 3.14 for π.

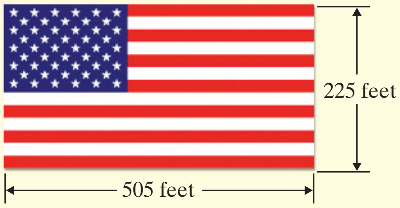


Solve. See Examples 1 through 4.

27. A $10\frac{1}{2}$-foot by 16-foot concrete wall is to be built using concrete blocks. Find the area of the wall.

28. The floor of an attic is 24 feet by 35 feet. Find how many square feet of insulation are needed to cover the attic floor.

29. The world's largest U.S. flag is the "Superflag," which measures 505 feet by 255 feet. Find its area. (*Source:* Superflag.com)



30. The world's largest illuminated indoor advertising sign is located in the Dubai International Airport in Dubai, UAE. It measures 28.0 meters in length by 6.2 meters in height. Find its area. (*Source:* World Record Academy)

31. The face of a watch has a diameter of 2 centimeters. What is its area? Give the exact answer then an approximation using 3.14 for π.

32. The world's largest commercially available pizza is sold by Big Mama's & Papa's Pizzeria in Los Angeles, CA. This huge square pizza, called "The Giant Sicilian," measures 54 inches on each side and sells for $199.99 plus tax. Find the area of the top of the pizza. (*Source: Guinness World Records*, Big Mama's & Big Papa's Pizzeria Inc.)

33. One side of a concrete block measures 8 inches by 16 inches. Find the area of the side in square inches. Find the area in square feet (144 sq in. = 1 sq ft).

34. A standard *double* roll of wallpaper is $6\frac{5}{6}$ feet wide and 33 feet long. Find the area of the *double* roll.

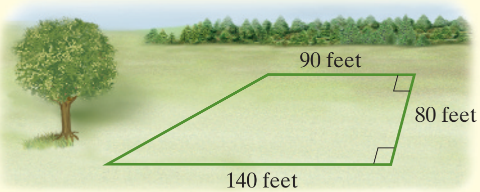
35. A picture frame measures 20 inches by $25\frac{1}{2}$ inches. Find how many square inches of glass the frame requires.

36. A mat to go under a tablecloth is made to fit a round dining table with a 4-foot diameter. Approximate how many square feet of mat there are. Use 3.14 as an approximation for π.

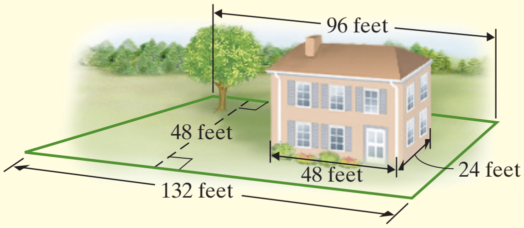
37. A drapery panel measures 6 feet by 7 feet. Find how many square feet of material are needed for *four* panels.

38. A page in a book measures 27.5 centimeters by 20.5 centimeters. Find its area.

39. Find how many square feet of land are in the plot shown:

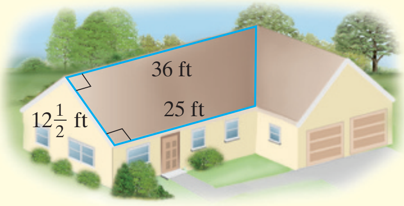


40. For a homeowner to determine how much grass seed he needs to buy, he must know the size of his yard. Use the drawing to determine how many square feet are in his yard.



41. The outlined part of the roof shown is in the shape of a trapezoid and needs to be shingled. The number of shingles to buy depends on the area.

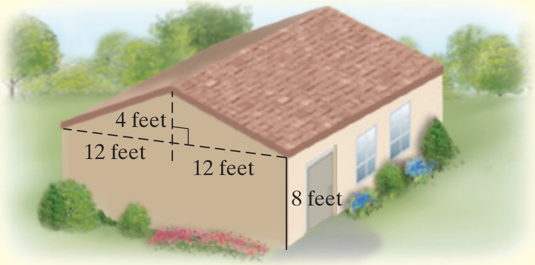
a. Use the dimensions given to find the area of the outlined part of the roof to the nearest whole square foot.



b. Shingles are packaged in a unit called a "square." If a "square" covers 100 square feet, how many whole squares need to be purchased to shingle this part of the roof?

42. The entire side of the building shaded in the drawing is to be bricked. The number of bricks to buy depends on the area.

a. Find the area.

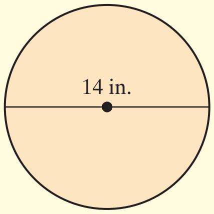


b. If the side area of each brick (including mortar room) is $\frac{1}{6}$ square foot, find the number of bricks needed to brick the end of the building.

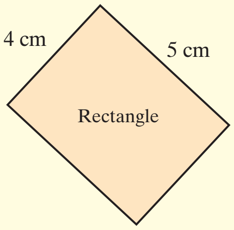
Review

Find the perimeter or circumference of each geometric figure. See Section 9.3.

43. Give the exact circumference and an approximation. Use 3.14 for π.



44.



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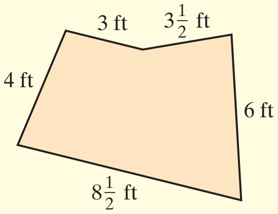

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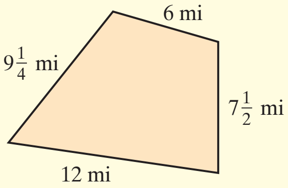

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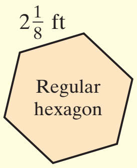

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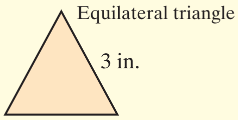


Concept Extensions

Given the following situations, tell whether you are more likely to be concerned with area or perimeter.

49. ordering fencing to fence a yard

50. ordering grass seed to plant in a yard

51. buying carpet to install in a room

52. buying gutters to install on a house

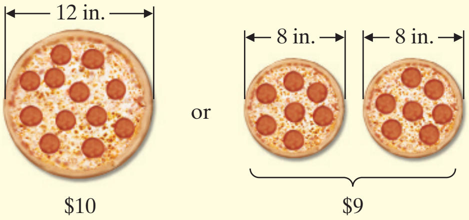
53. ordering paint to paint a wall

54. ordering baseboards to install in a room

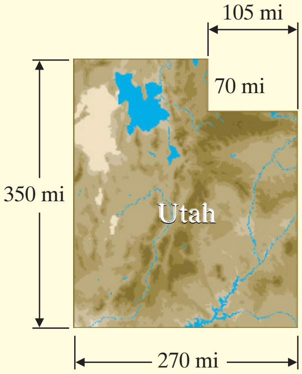
55. buying a wallpaper border to go on the walls around a room

56. buying fertilizer for your yard

57. A pizza restaurant recently advertised two specials. The first special was a 12-inch diameter pizza for $10. The second special was two 8-inch diameter pizzas for $9. Determine the better buy. (*Hint:* First find and compare the areas of the pizzas in the two specials. Then find a price per square inch for the pizzas in both specials.)



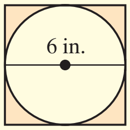
58. Find the approximate area of the state of Utah.



59. Find the area of a rectangle that measures 2 *feet* by 8 *inches*. Give the area in square feet and in square inches.

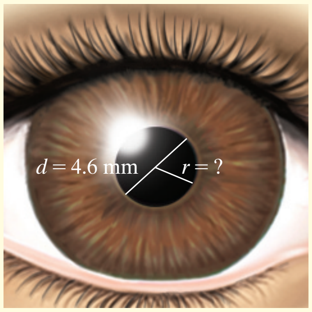
60. In your own words, explain why perimeter is measured in units and area is measured in square units. (*Hint:* See Section 1.6 for an introduction to the meaning of area.)

61. Find the area of the shaded region. Use the approximation 3.14 for π.

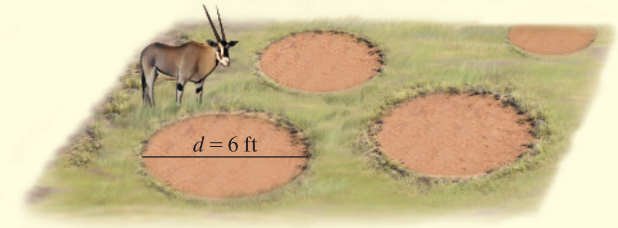


62. Estimate the cost of a piece of carpet for a rectangular room 10 feet by 15 feet. The cost of the carpet is $6.50 per square foot.

63. The average pupil of a 20-year-old is 4.6 mm in diameter. Find the exact area of the average pupil and an approximation. Use $\pi \approx 3.14$. (*Source: National Geographic*)



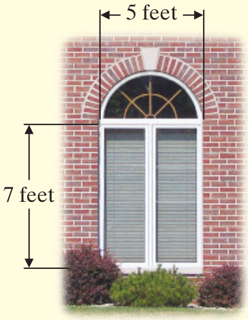
64. The desert grasslands of Namibia are dotted with tens of thousands of grassless spots, roughly circular in shape. These are naturally occurring patches, and the smallest of them measures about 6 feet in diameter. Calculate the exact area of this circle and an approximation. Use $\pi \approx 3.14$. (*Source: National Geographic*)



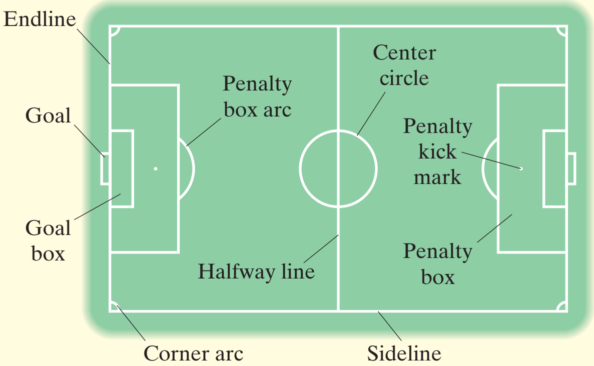
Find the area of each figure. Use $\pi \approx 3.14$ and round results to the nearest tenth.

65. Find the skating area.

66.



There are a number of factors that determine the dimensions of a rectangular soccer field. Use the table below to answer Exercises 67 and 68.



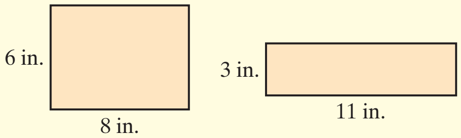
Soccer Field Width and Length

Age	Width Min–Max	Length Min–Max
Under 6/7:	15–20 yards	25–30 yards
Under 8:	20–25 yards	30–40 yards
Under 9:	30–35 yards	40–50 yards
Under 10:	40–50 yards	60–70 yards
Under 11:	40–50 yards	70–80 yards
Under 12:	40–55 yards	100–105 yards
Under 13:	50–60 yards	100–110 yards
International:	70–80 yards	110–120 yards

67. **a.** Find the minimum length and width of a soccer field for 9-year-old children. (Carefully consider the age.)
b. Find the area of this field.

68. **a.** Find the maximum length and width of a soccer field for 11-year-old children.
b. Find the area of this field.

69. Do two rectangles with the same perimeter have the same area? To see, find the perimeter and the area of each rectangle.



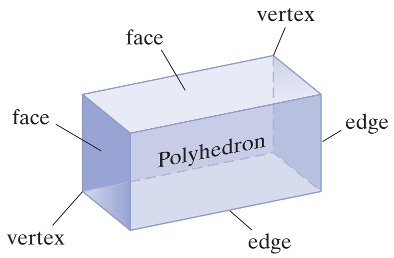
9.5 Volume and Surface Area

Objective A Finding Volume and Surface Area of Solids

Objective

A Find the Volume and Surface Area of Solids.

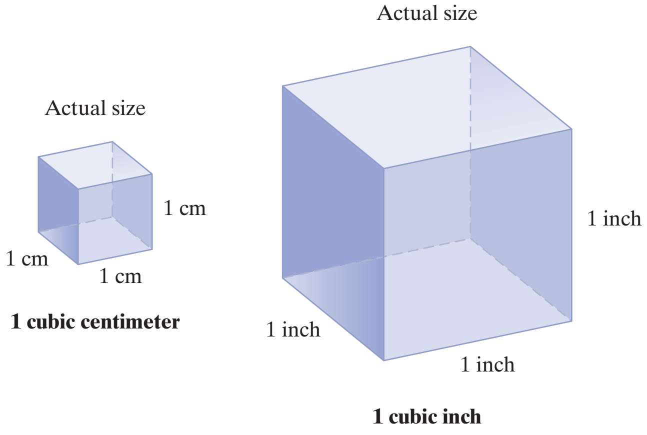
A **convex solid** is a set of points, *S*, not all in one plane, such that for any two points *A* and *B* in *S*, all points between *A* and *B* are also in *S*. In this section, we will find the volume and surface area of special types of solids called polyhedrons. A solid formed by the intersection of a finite number of planes is called a **polyhedron.** The box below is an example of a polyhedron.



Each of the plane regions of a polyhedron is called a **face** of the polyhedron. If the intersection of two faces is a line segment, this line segment is an **edge** of the polyhedron. The intersections of the edges are the **vertices** of the polyhedron.

Volume is a measure of the space of a region. The volume of a box or can, for example, is the amount of space inside. Volume can be used to describe the amount of juice in a pitcher or the amount of concrete needed to pour a foundation for a house.

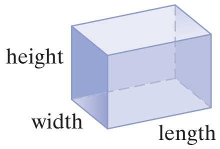
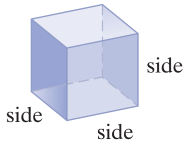
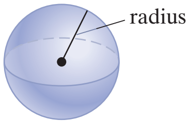
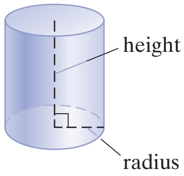
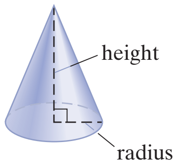
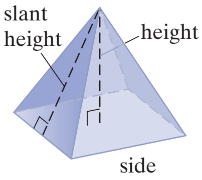
The volume of a solid is the number of **cubic units** in the solid. A cubic centimeter and a cubic inch are illustrated.



The **surface area** of a polyhedron is the sum of the areas of the faces of the polyhedron. For example, each face of the cube on the left above has an area of 1 square centimeter. Since there are 6 faces of the cube, the sum of the areas of the faces is 6 square centimeters. Surface area can be used to describe the amount of material needed to cover a solid. Surface area is measured in square units.

Formulas for finding the volumes, V, and surface areas, SA, of some common solids are given next.

Volume and Surface Area Formulas of Common Solids

Solid	Formulas
Rectangular Solid  	$V = lwh$ $SA = 2lh + 2wh + 2lw$ where h = height, w = width, l = length
Cube  	$V = s^3$ $SA = 6s^2$ where s = side
Sphere  	$V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$ where r = radius
Circular Cylinder  	$V = \pi r^2 h$ $SA = 2\pi rh + 2\pi r^2$ where h = height, r = radius
Cone  	$V = \frac{1}{3}\pi r^2 h$ $SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$ where h = height, r = radius
Square-Based Pyramid  	$V = \frac{1}{3}s^2 h$ $SA = B + \frac{1}{2}pl$ where B = area of base, p = perimeter of base, h = height, s = side, l = slant height

Example 1 Find the volume and surface area of a rectangular box that is 12 inches long, 6 inches wide, and 3 inches high.

Practice 1

Find the volume and surface area of a rectangular box that is 7 feet long, 3 feet wide, and 4 feet high.

Answer

1. $V = 84$ cu ft; $SA = 122$ sq ft

Solution: Use the volume and surface area formulas for a rectangular solid.

Let $h = 3$ in., $l = 12$ in., and $w = 6$ in.

$$V = lwh$$

$$V = 12 \text{ inches} \cdot 6 \text{ inches} \cdot 3 \text{ inches} = 216 \text{ cubic inches}$$

The volume of the rectangular box is 216 cubic inches.

$$\begin{aligned} SA &= 2lh + 2wh + 2lw \\ &= 2(12 \text{ in.})(3 \text{ in.}) + 2(6 \text{ in.})(3 \text{ in.}) + 2(12 \text{ in.})(6 \text{ in.}) \\ &= 72 \text{ sq in.} + 36 \text{ sq in.} + 144 \text{ sq in.} \\ &= 252 \text{ sq in.} \end{aligned}$$

The surface area of the rectangular box is 252 square inches.

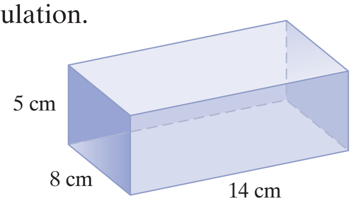
Work Practice 1

✓Concept Check A student is calculating the volume of the following rectangular solid. Find the student's error in his calculation.

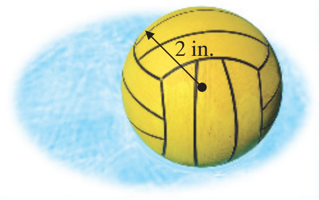
~~Volume $= l + w + h$~~

~~$= 14 \text{ cm} + 8 \text{ cm} + 5 \text{ cm}$~~

~~$= 27 \text{ cu cm}$~~



Example 2 Find the volume and surface area of a ball of radius 2 inches. Give the exact volume and surface area. Then use the approximation $\frac{22}{7}$ for π.



Solution: Use the volume and surface area formulas for a sphere.

$V = \frac{4}{3}\pi r^3$ Formula for volume of a sphere

$V = \frac{4}{3} \cdot \pi (2 \text{ in.})^3$ Let $r = 2$ inches.

$= \frac{32}{3}\pi$ cu in. Exact volume

$\approx \frac{32}{3} \cdot \frac{22}{7}$ cu in. Approximate π with $\frac{22}{7}$.

$= \frac{704}{21}$ or $33\frac{11}{21}$ cu in. Approximate volume

(Continued on next page)

Practice 2

Find the volume and surface area of a ball of radius $\frac{1}{2}$ centimeter. Give the exact volume and surface area. Then use $\frac{22}{7}$ for π and approximate the values.

Answer

2. $V = \frac{1}{6}\pi$ cu cm $\approx \frac{11}{21}$ cu cm; $SA = \pi$ sq cm $\approx 3\frac{1}{7}$ sq cm

✓Concept Check Answer

Volume $= l \cdot w \cdot h$

$= 14 \text{ cm} \cdot 8 \text{ cm} \cdot 5 \text{ cm}$

$= 560 \text{ cu cm}$

The volume of the sphere is exactly $\frac{32}{3}\pi$ cubic inches or approximately $33\frac{11}{21}$ cubic inches.

$$\begin{aligned} SA &= 4\pi r^2 && \text{Formula for surface area} \\ SA &= 4 \cdot \pi(2 \text{ in.})^2 && \text{Let } r = 2 \text{ inches.} \\ &= 16\pi \text{ sq in.} && \text{Exact surface area} \\ &\approx 16 \cdot \frac{22}{7} \text{ sq in.} && \text{Approximate } \pi \text{ with } \frac{22}{7}. \\ &= \frac{352}{7} \text{ or } 50\frac{2}{7} \text{ sq in.} && \text{Approximate surface area} \end{aligned}$$

The surface area of the sphere is exactly 16π square inches or approximately $50\frac{2}{7}$ square inches.

Work Practice 2

Practice 3

Find the volume of a cylinder of radius 5 inches and height 9 inches. Use 3.14 for π. Give the exact answer and an approximate answer.

Example 3

Find the volume of a can that has a $3\frac{1}{2}$-inch radius and a height of 6 inches. Use $\frac{22}{7}$ for π. Give the exact volume and an approximate volume.

Solution: Using the formula for a circular cylinder, we have

$$\begin{aligned} V &= \pi \cdot r^2 \cdot h \qquad 3\frac{1}{2} = \frac{7}{2} \\ &= \pi \cdot \left(\frac{7}{2} \text{ in.}\right)^2 \cdot 6 \text{ in.} \\ &= \pi \cdot \frac{49}{4} \text{ sq in.} \cdot 6 \text{ in.} \\ &= \frac{\pi \cdot 49 \cdot \overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 2} \text{ cu in.} \\ &= 73\frac{1}{2}\pi \text{ cu in. or } 73.5\pi \text{ cu in.} \end{aligned}$$

This is the exact volume. To approximate the volume, use the approximation $\frac{22}{7}$ for π.

$$\begin{aligned} V &= 73\frac{1}{2}\pi \text{ or } \approx \frac{147}{2} \cdot \frac{22}{7} \text{ cu in.} && \text{Replace } \pi \text{ with } \frac{22}{7}. \\ &= \frac{21 \cdot \overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{2}} \cdot 11}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{7}}} \text{ cu in.} \\ &= 231 \text{ cubic in.} \end{aligned}$$

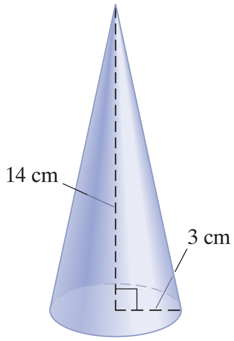
The volume is exactly $73\frac{1}{2}\pi$ cu in. or approximately 231 cubic inches.

Work Practice 3

Answer

3. 225π cu in. ≈ 706.5 cu in.

Example 4 Find the volume of a cone that has a height of 14 centimeters and a radius of 3 centimeters. Use 3.14 for π. Give the exact answer and an approximate answer.



Solution: Using the formula for volume of a cone, we have

$$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$$

$$= \frac{1}{3} \cdot \pi \cdot (3 \text{ cm})^2 \cdot 14 \text{ cm} \quad \text{Replace } r \text{ with 3 cm and } h \text{ with 14 cm.}$$

$$= 42\pi \text{ cu cm}$$

Thus, 42π cubic centimeters is the exact volume. To approximate the volume, use the approximation 3.14 for π.

$$V \approx 42 \cdot 3.14 \text{ cu cm} \quad \text{Replace } \pi \text{ with 3.14.}$$

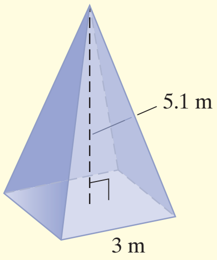
$$= 131.88 \text{ cu cm}$$

The volume is exactly 42π cu cm or approximately 131.88 cubic centimeters.

Work Practice 4

Practice 4

Find the volume of a square-based pyramid that has a 3-meter side and a height of 5.1 meters.



Answer

4. 15.3 cu m

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some exercises are based on Section 9.3.

units	volume	square
perimeter	surface area	cubic

1. The __________ of a polyhedron is the sum of the areas of its faces.

2. The measure of the amount of space inside a solid is its __________.

3. Volume is measured in __________units.

4. Surface area is measured in __________units.

5. The __________ of a polygon is the sum of the lengths of its sides.

6. Perimeter is measured in __________.

Martin-Gay Interactive Videos

See Video 9.5

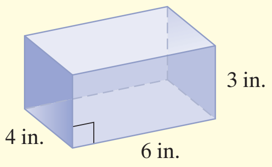
Watch the section lecture video and answer the following questions.

Objective A **7.** In Examples 2 and 3, explain the difference in the two answers found for each.

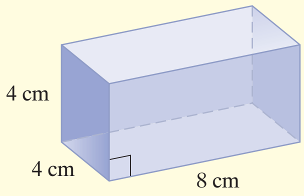
9.5 Exercise Set MyLab Math

Objective A *Find the volume and surface area of each solid. See Examples 1 through 4. For formulas containing π, give the exact answer and then approximate using $\frac{22}{7}$ for π.*

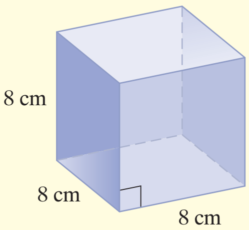
1.



2.



3.



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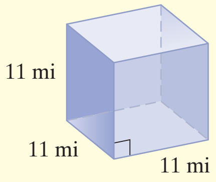
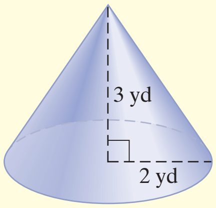
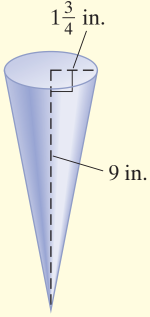


5. For surface area, round the approximation to 2 decimal places.



6. For surface area, round the approximation to 2 decimal places.



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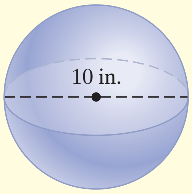

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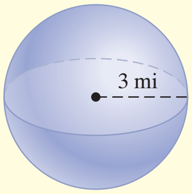
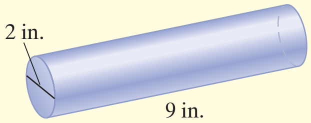
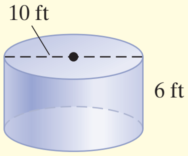


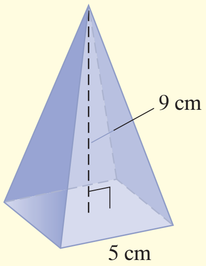
9. Find the volume only.



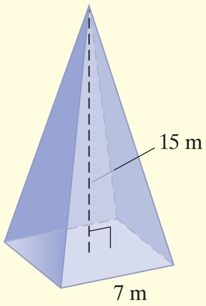
10. Find the volume only.



11. Find the volume only.

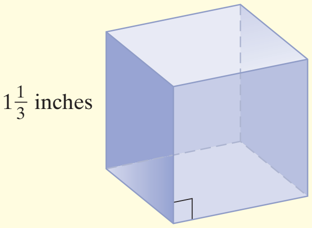


12. Find the volume only.

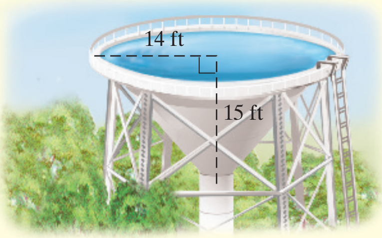


Solve. See Examples 1 through 4.

13. Find the volume of a cube with edges of $1\frac{1}{3}$ inches.



14. A water storage tank is in the shape of a cone with the pointed end down. If the radius is 14 ft and the depth of the tank is 15 ft, approximate the volume of the tank in cubic feet. Use $\frac{22}{7}$ for π.



15. Find the volume and surface area of a rectangular box 2 ft by 1.4 ft by 3 ft.

16. Find the volume and surface area of a box in the shape of a cube that is 5 ft on each side.

17. A paperweight is in the shape of a square-based pyramid 20 centimeters tall. If an edge of the base is 12 centimeters, find the volume of the paperweight.

18. A birdbath is made in the shape of a hemisphere (half-sphere). If its radius is 10 inches, approximate its volume. Use $\frac{22}{7}$ for π.

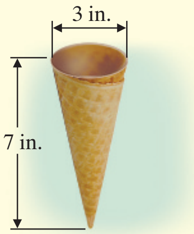
19. Find the exact volume and surface area of a sphere with a radius of 7 inches.

20. A tank is in the shape of a cylinder 8 feet tall and 3 feet in radius. Find the exact volume and surface area of the tank.

21. Find the volume of a rectangular block of ice 2 feet by $2\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

22. Find the capacity (volume in cubic feet) of a rectangular ice chest with inside measurements of 3 feet by $1\frac{1}{2}$ feet by $1\frac{3}{4}$ feet.

23. Find the exact volume of a waffle ice cream cone with a 3-in. diameter and a height of 7 inches.



24. A snow globe has a diameter of 6 inches. Find its exact volume. Then approximate its volume using 3.14 for π.

25. Zorbing is an extreme sport invented by two New Zealanders who joke that they were looking for a way to walk on water. A Zorb is a large sphere inside a second sphere with the space between the spheres pumped full of air. There is a tunnel-like opening so a person can crawl into the inner sphere. You are strapped in and sent down a Zorbing hill. A standard Zorb is approximately 3 m in diameter. Find the exact volume of a Zorb, and approximate the volume using 3.14 for π.

26. Mount Fuji, in Japan, is considered the most beautiful composite volcano in the world. The mountain is in the shape of a cone whose height is about 3.5 kilometers and whose base radius is about 3 kilometers. Approximate the volume of Mt. Fuji in cubic kilometers. Use $\frac{22}{7}$ for π.

27. Find the volume of a pyramid with a square base 5 inches on a side and a height of $1\frac{3}{10}$ inches.

28. Approximate to the nearest hundredth the volume of a sphere with a radius of 2 centimeters. Use 3.14 for π.

29. In 2013, the largest free-floating soap bubble made with a wand had a diameter between 11 and 12 feet. Calculate the exact volume of a sphere with a diameter of 12 feet. (*Source: Guinness World Records*)

30. The largest inflatable beach ball was created in Poland in 2012. It has a diameter of just under 54 feet. Calculate the exact volume of a sphere with a diameter of 54 feet. (*Source: Guinness World Records*)

31. An ice cream cone with a 4-centimeter diameter and 3-centimeter depth is filled exactly level with the top of the cone. Approximate how much ice cream (in cubic centimeters) is in the cone. Use $\frac{22}{7}$ for π.

32. A child's toy is in the shape of a square-based pyramid 10 inches tall. If an edge of the base is 7 inches, find the volume of the toy.

The Space Cube is supposed to be the world's smallest computer, with dimensions of 2 inches by 2 inches by 2.2 inches.

33. Find the volume of the Space Cube.

34. Find the volume of an actual cube that measures 2 inches by 2 inches by 2 inches.

35. Find the volume of an actual cube that measures 2.2 inches by 2.2 inches by 2.2 inches.

36. Comment on the results of Exercises **33–35**. Were you surprised when you compared volumes? Why or why not?

Review

Evaluate. See Section 1.9.

37. 5^2

38. 7^2

39. 3^2

40. 20^2

41. $1^2 + 2^2$

42. $5^2 + 3^2$

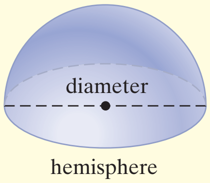
43. $4^2 + 2^2$

44. $1^2 + 6^2$

Concept Extensions

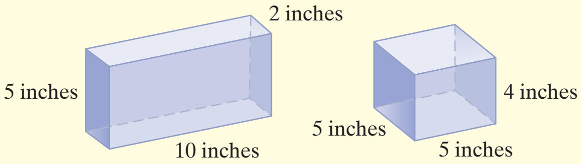
Solve.

45. The Hayden Planetarium, at the Museum of Natural History in New York City, boasts a dome that has a diameter of 20 m. The dome is a hemisphere, or half a sphere. What is the volume enclosed by the dome at the Hayden Planetarium? Use 3.14 for π and round to the nearest hundredth. (*Source:* Hayden Planetarium)

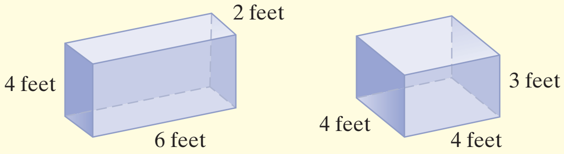


46. The Adler Museum in Chicago recently added a new planetarium, its StarRider Theater, which has a diameter of 55 feet. Find the volume of its hemispheric (half a sphere) dome. Use 3.14 for π and round to the nearest hundredth. (*Source:* The Adler Museum)

47. Do two rectangular solids with the same volume have the same shape? To see, find the volume of each rectangular solid.



48. Do two rectangular solids with the same volume have the same surface area? To see, find the volume and surface area of each rectangular solid.



49. Two kennels are offered at a hotel. The kennels measure

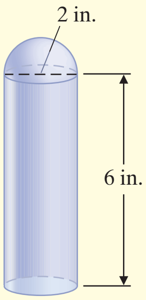
a. 2′1″ by 1′8″ by 1′7″ and

b. 1′1″ by 2′ by 2′8″

What is the volume of each kennel rounded to the nearest tenth of a cubic foot? Which is larger?

50. The centerpiece of the New England Aquarium in Boston is its Giant Ocean Tank. This exhibit is a multi-story cylindrical saltwater tank containing a coral reef and hundreds of Caribbean reef animals. The radius of the tank is 20 feet, and its height is 30 feet. What is the volume of the Giant Ocean Tank? Use $\pi \approx 3.14$. (*Source:* New England Aquarium)

51. Find the volume of the figure shown. Give the exact measure and then a whole number approximation.



52. Can you compute the volume of a rectangle? Why or why not

Integrated Review

Sections 9.1–9.5

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

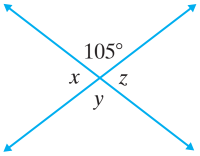
16. ______

Geometry Concepts

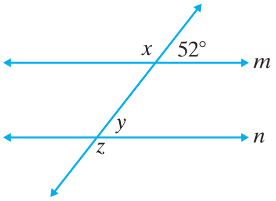
△ **1.** Find the supplement and the complement of a 27° angle.

Find the measures of angles x, y, and z in each figure.

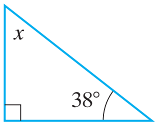
2.



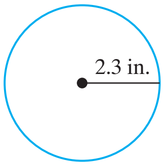
3. $m \parallel n$



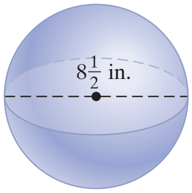
4. Find the measure of $\angle x$.



5. Find the diameter.

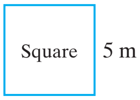


6. Find the radius.

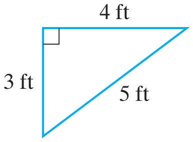


For Exercises 7 through 11, find the perimeter (or circumference) and area of each figure. For the circle give the exact circumference and area. Then use $\pi \approx 3.14$ *to approximate each. Don't forget to attach correct units.*

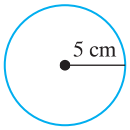
7.



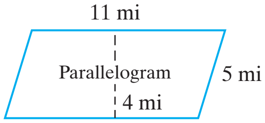
8.



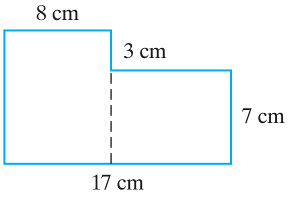
9.



10.



11.



12. The smallest cathedral is in Highlandville, Missouri. The rectangular floor of the cathedral measures 14 feet by 17 feet. Find its perimeter and its area. (*Source: The Guinness Book of Records*)

Find the volume of each solid. Don't forget to attach correct units.

13. A cube with edges of 4 inches each.

14. A rectangular box 2 feet by 3 feet by 5.1 feet.

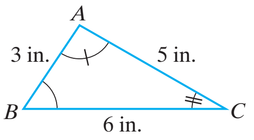
15. A pyramid with a square base 10 centimeters on a side and a height of 12 centimeters.

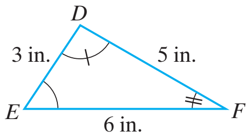
16. A sphere with a diameter of 3 miles. Give the exact volume and then use $\pi \approx \frac{22}{7}$ to approximate.

9.6 Congruent and Similar Triangles

Objective A Deciding Whether Two Triangles Are Congruent

Congruent angles are angles that have the same measure. Two triangles are **congruent** when they have the same shape and the same size. In congruent triangles, the measures of corresponding angles are equal and the lengths of corresponding sides are equal. The following triangles are congruent:





Since these triangles are congruent, the measures of corresponding angles are equal.

Angles with equal measure: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$. Also, the lengths of corresponding sides are equal.

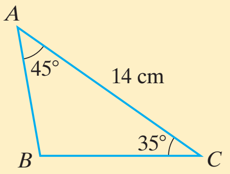
Equal corresponding sides: $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, $\overline{CA}$ and $\overline{FD}$

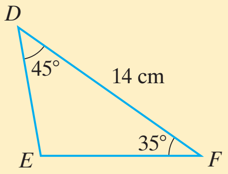
Any one of the following may be used to determine whether two triangles are congruent:

Congruent Triangles

Angle-Side-Angle (ASA)

If the measures of two angles of a triangle equal the measures of two angles of another triangle, and the lengths of the sides between each pair of angles are equal, the triangles are congruent.

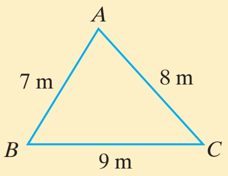


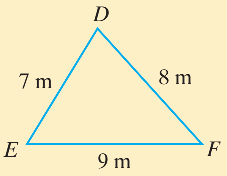


For example, these two triangles are congruent by Angle-Side-Angle.

Side-Side-Side (SSS)

If the lengths of the three sides of a triangle equal the lengths of the corresponding sides of another triangle, the triangles are congruent.





For example, these two triangles are congruent by Side-Side-Side.

Side-Angle-Side (SAS)

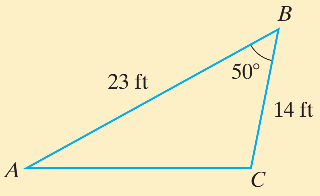
If the lengths of two sides of a triangle equal the lengths of corresponding sides of another triangle, and the measures of the angles between each pair of sides are equal, the triangles are congruent. (See illustration on next page.)

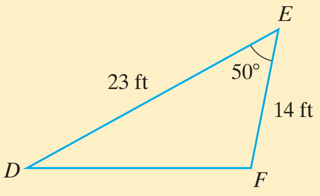
Objectives

A Decide Whether Two Triangles Are Congruent.

B Find the Ratio of Corresponding Sides in Similar Triangles.

C Find Unknown Lengths of Sides in Similar Triangles.

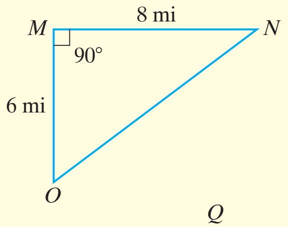


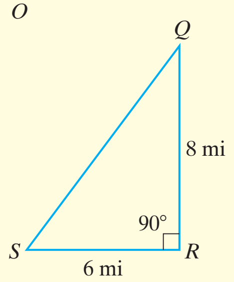


For example, these two triangles are congruent by Side-Angle-Side.

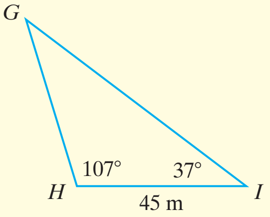
Practice 1

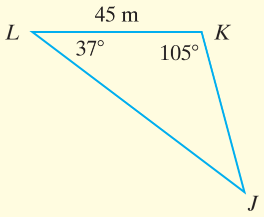
a. Determine whether triangle *MNO* is congruent to triangle *RQS*.





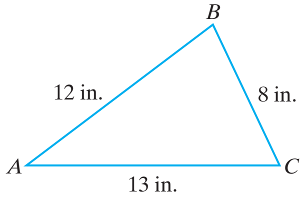
b. Determine whether triangle *GHI* is congruent to triangle *JKL*.

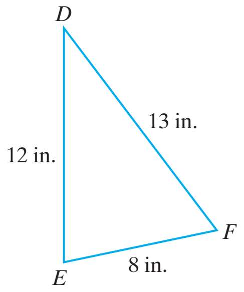




Example 1

Determine whether triangle *ABC* is congruent to triangle *DEF*.





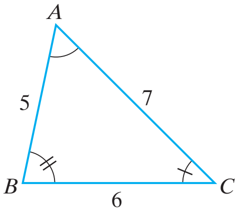
Solution: Since the lengths of all three sides of triangle *ABC* equal the lengths of all three sides of triangle *DEF*, the triangles are congruent.

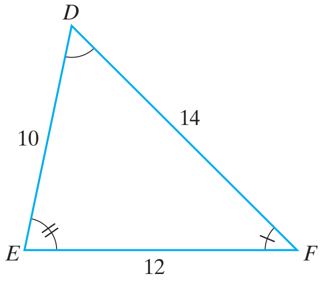
Work Practice 1

In Example 1, notice that as soon as we know that the two triangles are congruent, we know that all three corresponding angles are congruent.

Objective B Finding the Ratio of Corresponding Sides in Similar Triangles

Two triangles are **similar** when they have the same shape but not necessarily the same size. In similar triangles, the measures of corresponding angles are equal and corresponding sides are in proportion. The following triangles are similar:





Since these triangles are similar, the measures of corresponding angles are equal.

Angles with equal measure: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$. Also, the lengths of corresponding sides are in proportion.

Answers

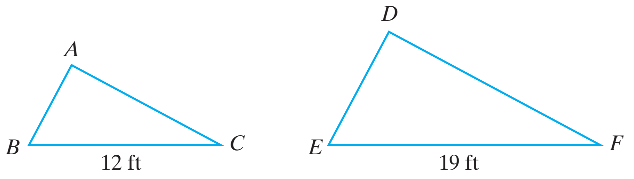
1. a. congruent **b.** not congruent

Sides in proportion: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ or, in this particular case,

$$\frac{AB}{DE} = \frac{5}{10} = \frac{1}{2}, \frac{BC}{EF} = \frac{6}{12} = \frac{1}{2}, \frac{CA}{FD} = \frac{7}{14} = \frac{1}{2}$$

The ratio of corresponding sides is $\frac{1}{2}$.

Example 2 Find the ratio of corresponding sides for the similar triangles *ABC* and *DEF*.



Solution: We are given the lengths of two corresponding sides. Their ratio is

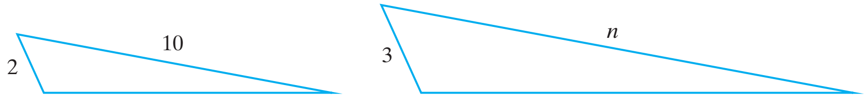
$$\frac{12\ \cancel{\text{feet}}}{19\ \cancel{\text{feet}}} = \frac{12}{19}$$

Work Practice 2

Objective C Finding Unknown Lengths of Sides in Similar Triangles

Because the ratios of lengths of corresponding sides are equal, we can use proportions to find unknown lengths in similar triangles.

Example 3 Given that the triangles are similar, find the missing length *n*.



Solution: Since the triangles are similar, corresponding sides are in proportion. Thus, the ratio of 2 to 3 is the same as the ratio of 10 to *n*, or

$$\frac{2}{3} = \frac{10}{n}$$

To find the unknown length *n*, we set cross products equal.

$$\frac{2}{3} = \frac{10}{n}$$

$2 \cdot n = 3 \cdot 10$ Set cross products equal.

$2 \cdot n = 30$ Multiply.

$n = \frac{30}{2}$ Divide 30 by 2, the number multiplied by *n*.

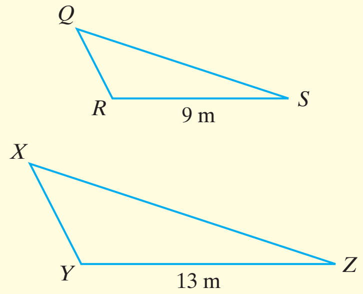
$n = 15$

The missing length is 15 units.

Work Practice 3

Practice 2

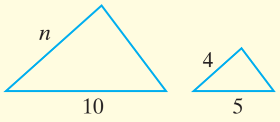
Find the ratio of corresponding sides for the similar triangles *QRS* and *XYZ*.



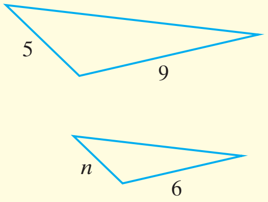
Practice 3

Given that the triangles are similar, find the missing length *n*.

a.



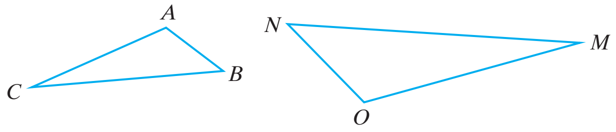
b.



Answers

2. $\frac{9}{13}$ **3. a.** $n = 8$ **b.** $n = \frac{10}{3}$ or $3\frac{1}{3}$

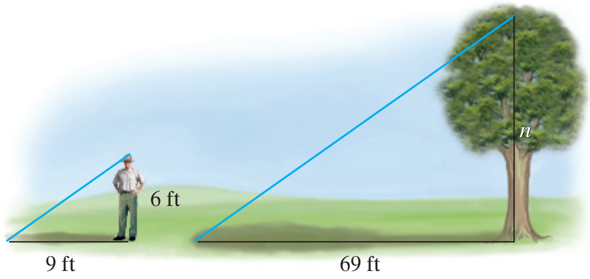
✓**Concept Check** The following two triangles are similar. Which vertices of the first triangle appear to correspond to which vertices of the second triangle?



Many applications involve diagrams containing similar triangles. Surveyors, astronomers, and many other professionals continually use similar triangles in their work.

Example 4 Finding the Height of a Tree

A 6-foot-tall park ranger needs to know the height of a particular tree. He measures the shadow of the tree to be 69 feet long when his own shadow is 9 feet long. Find the height of the tree.



Solution:

1. UNDERSTAND. Read and reread the problem. Notice that the triangle formed by the Sun's rays, the ranger, and his shadow is similar to the triangle formed by the Sun's rays, the tree, and its shadow.

2. TRANSLATE. Write a proportion from the similar triangles formed.

$$\frac{\text{ranger's height}}{\text{height of tree}} \begin{matrix}\rightarrow\\ \rightarrow\end{matrix} \frac{6}{n} = \frac{9}{69} \begin{matrix}\leftarrow\\ \leftarrow\end{matrix} \frac{\text{length of ranger's shadow}}{\text{length of tree's shadow}}$$

$$\text{or } \frac{6}{n} = \frac{3}{23} \quad \text{Simplify } \frac{9}{69}. \text{ (ratio in lowest terms)}$$

3. SOLVE for n:

$$\frac{6}{n} = \frac{3}{23}$$

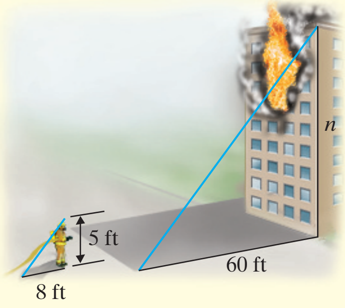
$$\begin{aligned} 6 \cdot 23 &= n \cdot 3 && \text{Set cross products equal.} \\ 138 &= n \cdot 3 && \text{Multiply.} \\ \frac{138}{3} &= n && \text{Divide 138 by 3, the number multiplied by } n. \\ 46 &= n \end{aligned}$$

4. INTERPRET. *Check* to see that replacing n with 46 in the proportion makes the proportion true. *State* your conclusion: The height of the tree is 46 feet.

Work Practice 4

Practice 4

A firefighter, needs to estimate the height of a burning building. She estimates the length of her shadow to be 8 feet long and the length of the building's shadow to be 60 feet long. Find the approximate height of the building if she is 5 feet tall.



Answer

4. approximately 37.5 ft

✓**Concept Check Answer**

A corresponds to *O*; *B* corresponds to *N*; *C* corresponds to *M*

Vocabulary, Readiness & Video Check

Answer each question true or false.

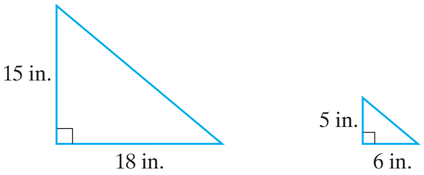
1. Two triangles that have the same shape but not necessarily the same size are congruent.

2. Two triangles are congruent if they have the same shape and size.

3. Congruent triangles are also similar.

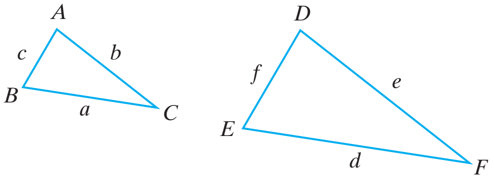
4. Similar triangles are also congruent.

5. For the two similar triangles, the ratio of corresponding sides is $\frac{5}{6}$.

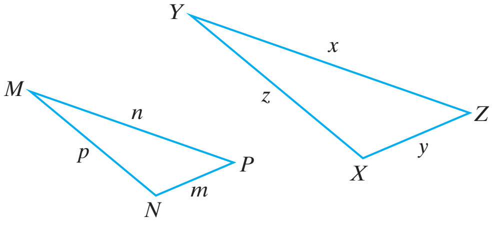


Each pair of triangles is similar. Name the congruent angles and the corresponding sides that are proportional.

6.



7.



Martin-Gay Interactive Videos

See Video 9.6

Watch the section lecture video and answer the following questions.

Objective A **8.** How did we decide which congruency rule to use to determine if the two triangles in Example 1 are congruent?

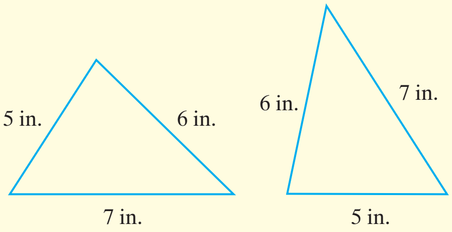
Objective B **9.** From Example 2, what does "corresponding sides are in proportion" mean?

Objective C **10.** In Example 3, what is another proportion discussed that we could have used to solve the application?

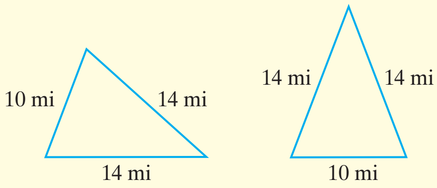
9.6 Exercise Set MyLab Math

Objective A *Determine whether each pair of triangles is congruent. If congruent, state the reason why, such as SSS, SAS, or ASA. See Example 1.*

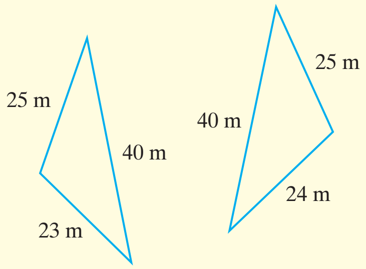
1.



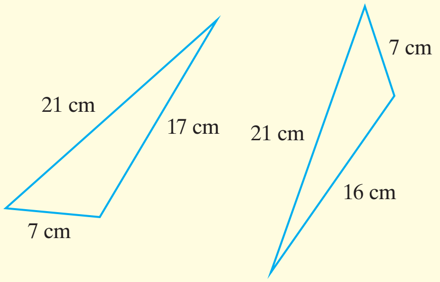
2.



3.



4.



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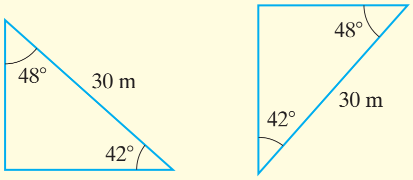
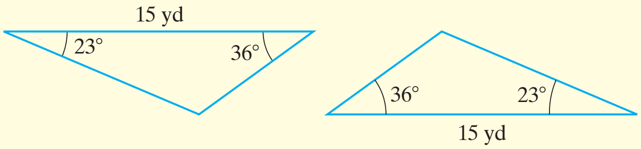


6.



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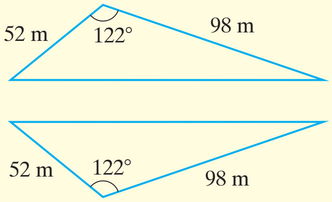

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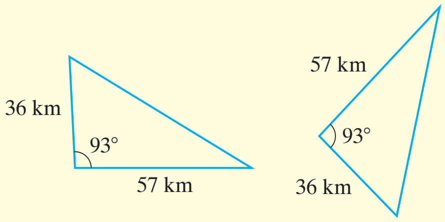
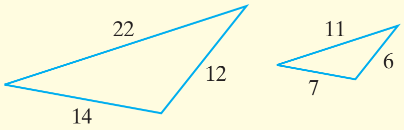
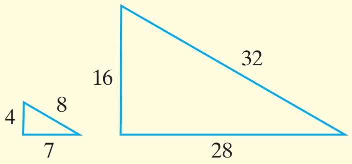


Objective B *Find each ratio of the corresponding sides of the given similar triangles. See Example 2.*

9.



10.



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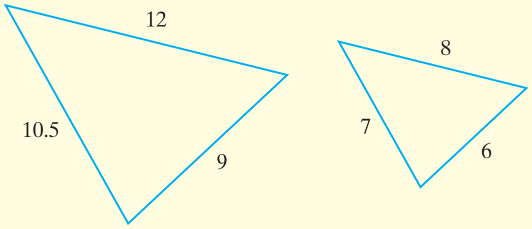

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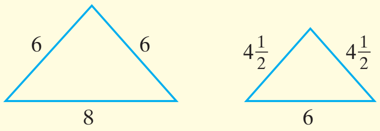
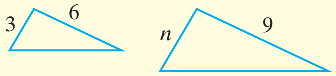
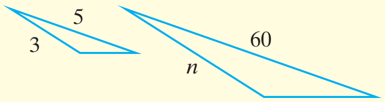


Objective C *Given that the pairs of triangles are similar, find the unknown length of the side labeled n. See Example 3.*

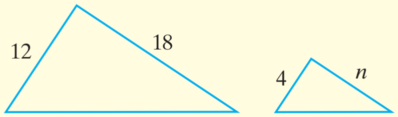
13.



14.



15.



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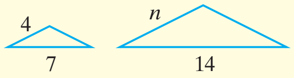

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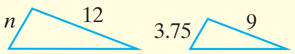

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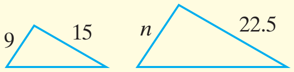

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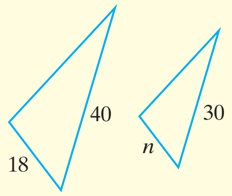

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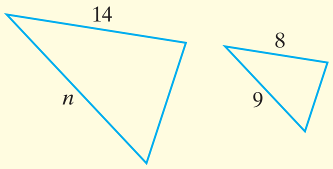

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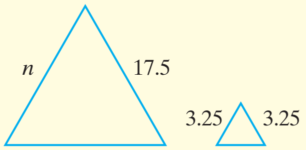

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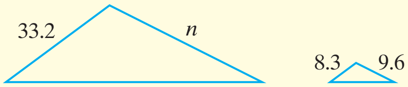

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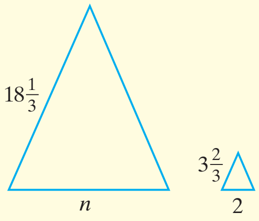

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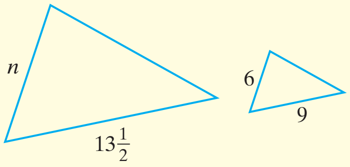

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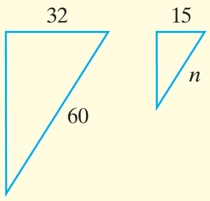

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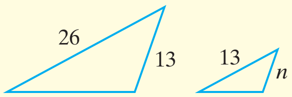
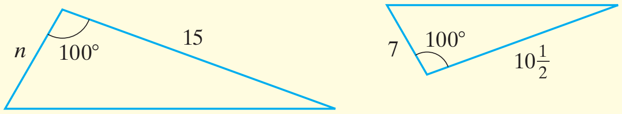


27.



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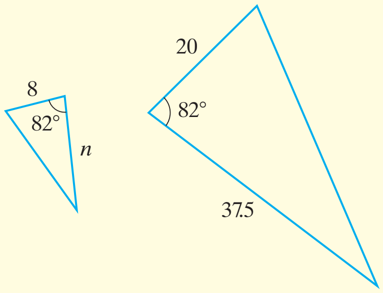
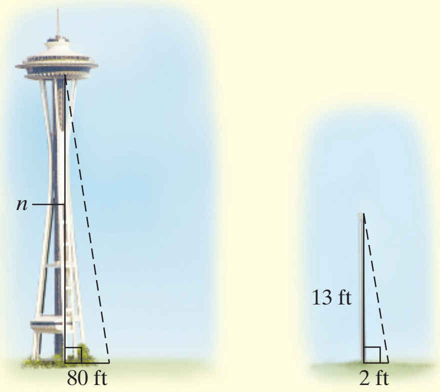
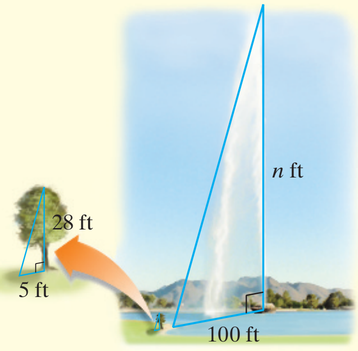


Solve. For Exercises 29 and 30, the solutions have been started for you on the next page. See Example 4.

29. Given the following diagram, approximate the height of the observation deck in the Seattle Space Needle in Seattle, Washington. (*Source:* Seattle Space Needle)



30. A fountain in Fountain Hills, Arizona, sits in a 28-acre lake and shoots up a column of water every hour. Based on the diagram below, what is the height of the fountain?



Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into a proportion using the similar triangles formed. (Fill in the blanks.)

$$\begin{array}{rcl} \text{height of observation deck} \rightarrow & \dfrac{n}{13} = \dfrac{\_\_}{\_\_} & \leftarrow \text{length of Space Needle shadow} \\ \text{height of pole} \rightarrow & & \leftarrow \text{length of pole shadow} \end{array}$$

3. SOLVE by setting cross products equal.
4. INTERPRET.

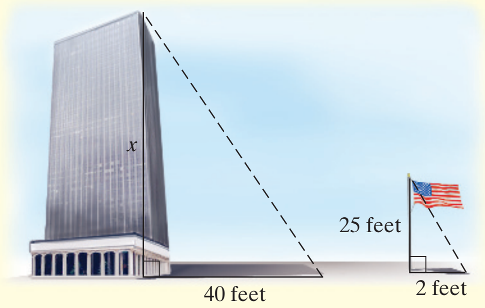
Start the solution:

1. UNDERSTAND the problem. Reread it as many times as needed.
2. TRANSLATE into a proportion using the similar triangles formed. (Fill in the blanks.)

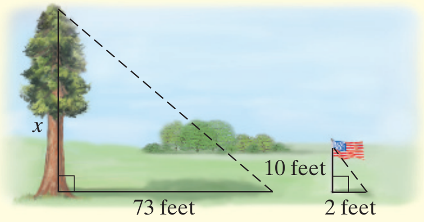
$$\begin{array}{rcl} \text{height of tree} \rightarrow & \dfrac{28}{n} = \dfrac{\_\_}{\_\_} & \leftarrow \text{length of tree shadow} \\ \text{height of fountain} \rightarrow & & \leftarrow \text{length of fountain shadow} \end{array}$$

3. SOLVE by setting cross products equal.
4. INTERPRET.

31. Given the following diagram, approximate the height of the Chase Tower in Oklahoma City, Oklahoma. Here, we use x to represent the unknown number. (*Source:* Council on Tall Buildings and Urban Habitat)



32. The tallest tree standing today is a redwood located in the Humboldt Redwoods State Park near Ukiah, California. Given the following diagram, approximate its height. Here, we use x to represent the unknown number. (*Source: Guinness World Records*)



33. A 5-foot-tall park ranger, needs to know the height of a tree. She notices that when the shadow of the tree is 48 feet long, her shadow is 4 feet long. Find the height of the tree.

34. Lance Cusson, a firefighter, needs to estimate the height of a burning building. He estimates the length of his shadow to be 9 feet long and the length of the building's shadow to be 75 feet long. Find the approximate height of the building if he is 6 feet tall.

35. If a 30-foot tree casts an 18-foot shadow, find the length of the shadow cast by a 24-foot tree.

36. If a 24-foot flagpole casts a 32-foot shadow, find the length of the shadow cast by a 44-foot antenna. Round to the nearest tenth.

Review

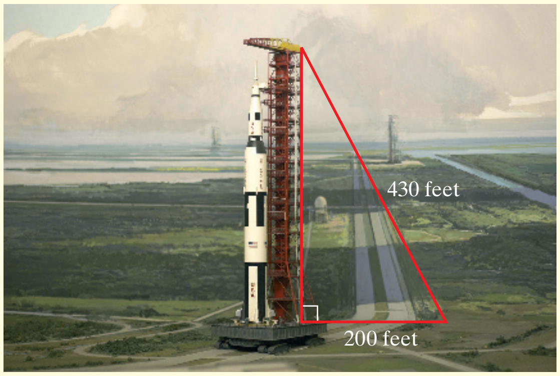
Solve. See Section 5.3.

37. For the health of his fish, the owner of Pete's Sea World uses the standard that a 20-gallon tank should house only 19 neon tetras. Find the number of neon tetras that Pete would place into a 55-gallon tank.

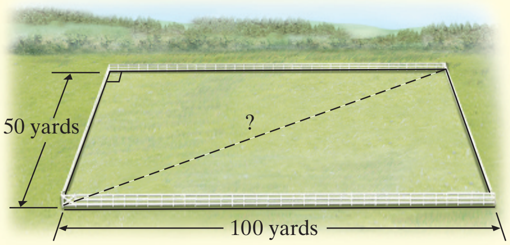
38. A local package express deliveryman is traveling the city expressway at 45 mph when he is forced to slow down due to traffic ahead. His truck slows at the rate of 3 mph every 5 seconds. Find his speed 8 seconds after braking.

Solve. See Section 4.6.

39. Launch Umbilical Tower 1 is the name of the gantry used for the *Apollo* launch that took Neil Armstrong and Buzz Aldrin to the moon. Find the height of the gantry to the nearest whole foot.



40. Arena polo, popular in the United States and England, is played on a field that is 100 yards long and usually 50 yards wide. Find the length, to the nearest yard, of the diagonal of this field.



Perform the indicated operation. See Sections 4.2 through 4.4.

41. $3.6 + 0.41$

42. $3.6 - 0.41$

43. $(0.41)(3)$

44. $0.48 \div 3$

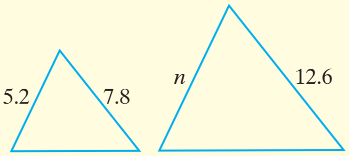
Concept Extensions

45. The print area on a particular page measures 7 inches by 9 inches. A printing shop is to copy the page and reduce the print area so that its length is 5 inches. What will its width be? Will the print now fit on a 3-by-5-inch index card?

46. The art sample for a banner measures $\frac{1}{3}$ foot in width by $1\frac{1}{2}$ feet in length. If the completed banner is to have a length of 9 feet, find its width.

Given that the pairs of triangles are similar, find the length of the side labeled n. Round your results to 1 decimal place.

47.



48.

49. In your own words, describe any differences in similar triangles and congruent triangles.

50. Describe a situation where similar triangles would be useful for a contractor building a house.

51. A triangular park is planned and waiting to be approved by the city zoning commission. A drawing of the park shows sides of length 5 inches, $7\frac{1}{2}$ inches, and $10\frac{5}{8}$ inches. If the scale on the drawing is $\frac{1}{4}$ in. $=$ 10 ft, find the actual proposed dimensions of the park.

52. John and Robyn Costello draw a triangular deck on their house plans. Robyn measures sides of the deck drawing on the plans to be 3 inches, $4\frac{1}{2}$ inches, and 6 inches. If the scale on the drawing is $\frac{1}{4}$ in. $=$ 1 foot, find the lengths of the sides of the deck they want built.

Chapter 9 Group Activity

The Cost of Road Signs

Sections 9.1, 9.2, 9.4

There are nearly 4 million miles of streets and roads in the United States. With streets, roads, and highways comes the need for traffic control, guidance, warning, and regulation. Road signs perform many of these tasks. Just in our routine travels, we see a wide variety of road signs every day. Think how many road signs must exist on the 4 million miles of roads in the United States. Have you ever wondered how much signs like these cost?

The cost of a road sign generally depends on the type of sign. Costs for several types of signs and signposts are listed in the table. Examples of various types of signs are shown below.

Road Sign Costs

Type of Sign	Cost
Regulatory, warning, marker	\$15–\$18 per square foot
Large guide	\$20–\$25 per square foot
Type of Post	**Cost**
U-channel	\$125–\$200 each
Square tube	\$10–\$15 per foot
Steel breakaway posts	\$15–\$25 per foot

The cost of a sign is based on its area. For diamond, square, or rectangular signs, the area is found by multiplying the length (in feet) times the width (in feet). Then the area is multiplied by the cost per square foot. For signs with irregular shapes, costs are generally figured *as if* the sign were a rectangle, multiplying the height and width at the tallest and widest parts of the sign.

Group Activity

Locate four different kinds of road signs on or near your campus. Measure the dimensions of each sign, including the height of the post on which it is mounted. Using the cost data given in the table, find the minimum and maximum costs of each sign, including its post. Summarize your results in a table, and include a sketch of each sign.

Chapter 9 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

transversal
right
right triangle
similar
line
line segment
volume
perimeter
congruent
complementary
obtuse
area
hypotenuse
plane
vertex
straight
legs
vertical
ray
adjacent
acute
supplementary
angle

1. A(n) ______________ is a triangle with a right angle. The side opposite the right angle is called the ______________, and the other two sides are called ______________.

2. A(n) ______________ is a piece of a line with two endpoints.

3. Two angles that have a sum of 90° are called ______________ angles.

4. A(n) ______________ is a set of points extending indefinitely in two directions.

5. The ______________ of a polygon is the distance around the polygon.

6. A(n) ______________ is made up of two rays that share the same endpoint. The common endpoint is called the ______________.

7. ______________ triangles have the same shape and the same size.

8. ______________ measures the amount of surface of a region.

9. A(n) ______________ is a part of a line with one endpoint. A ray extends indefinitely in one direction.

10. A(n) ______________ is a flat surface that extends indefinitely in all directions.

11. A line that intersects two or more lines at different points is called a(n) ______________.

12. A angle that measures 180° is called a(n) ______________ angle.

13. The measure of the space of a solid is called its ______________.

14. When two lines intersect, four angles are formed. The angles that are opposite each other are called ______________ angles.

15. When two of the four angles from Exercise **14** share a common side, they are called ______________ angles.

16. An angle whose measure is between 90° and 180° is called a(n) ______________ angle.

17. An angle that measures 90° is called a(n) ______________ angle.

18. An angle whose measure is between 0° and 90° is called a(n) ______________ angle.

19. Two angles that have a sum of 180° are called ______________ angles.

20. ______________ triangles have exactly the same shape but not necessarily the same size.

Helpful Hint Are you preparing for your test? To help, don't forget to take these:

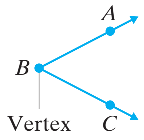
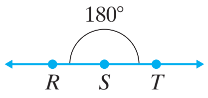
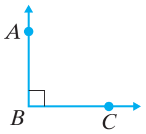
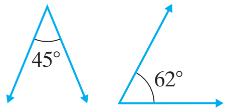
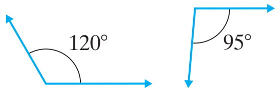
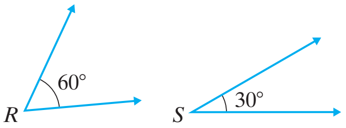
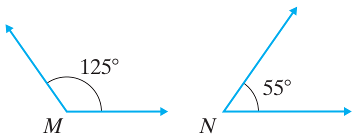
- Chapter 9 Getting Ready for the Test on page 715
- Chapter 9 Test on page 716

Then check all of your answers at the back of this text. For further review, the step-by-step video solutions to any of these exercises are located in MyLab Math.

9 Chapter Highlights

Definitions and Concepts	Examples
Section 9.1 Lines and Angles	
A **line** is a set of points extending indefinitely in two directions. A line has no width or height, but it does have length. We name a line by any two of its points.	 Line AB or $\overleftrightarrow{AB}$

(continued)

Definitions and Concepts	Examples
Section 9.1 Lines and Angles (*continued*)	
A **line segment** is a piece of a line with two endpoints.	Line segment AB or $\overline{AB}$ 
A **ray** is a part of a line with one endpoint. A ray extends indefinitely in one direction.	Ray AB or $\overrightarrow{AB}$ 
An **angle** is made up of two rays that share the same endpoint. The common endpoint is called the **vertex.**	Angle ABC, $\angle ABC$, $\angle CBA$, or $\angle B$ 
An angle that measures 180° is called a **straight angle.**	$\angle RST$ is a straight angle. 
An angle that measures 90° is called a **right angle.** The symbol ∟ is used to denote a right angle.	$\angle ABC$ is a right angle. 
An angle whose measure is between 0° and 90° is called an **acute angle.**	 Acute angles
An angle whose measure is between 90° and 180° is called an **obtuse angle.**	 Obtuse angles
Two angles that have a sum of 90° are called **complementary angles.** We say that each angle is the **complement** of the other.	 Complementary angles $60° + 30° = 90°$
Two angles that have a sum of 180° are called **supplementary angles.** We say that each angle is the **supplement** of the other.	 Supplementary angles $125° + 55° = 180°$

Definitions and Concepts | **Examples**

Section 9.1 Lines and Angles (*continued*)

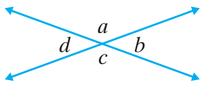
When two lines intersect, four angles are formed. Two of these angles that are opposite each other are called **vertical angles.** Vertical angles have the same measure.

Two of these angles that share a common side are called **adjacent angles.** Adjacent angles formed by intersecting lines are supplementary.

Vertical angles:

$\angle a$ and $\angle c$

$\angle d$ and $\angle b$



Adjacent angles:

$\angle a$ and $\angle b$

$\angle b$ and $\angle c$

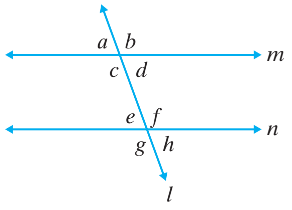
$\angle c$ and $\angle d$

$\angle d$ and $\angle a$

A line that intersects two or more lines at different points is called a **transversal.** Line l is a transversal that intersects lines m and n. The eight angles formed have special names. Some of these names are:

Corresponding angles: $\angle a$ and $\angle e$, $\angle c$ and $\angle g$, $\angle b$ and $\angle f$, $\angle d$ and $\angle h$

Alternate interior angles: $\angle c$ and $\angle f$, $\angle d$ and $\angle e$



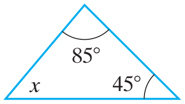
Parallel Lines Cut by a Transversal

If two parallel lines are cut by a transversal, then the measures of **corresponding angles are equal** and the measures of **alternate interior angles are equal.**

Section 9.2 Plane Figures and Solids

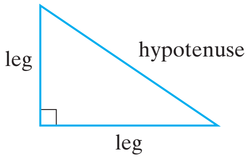
The **sum of the measures** of the angles of a triangle is 180°.

Find the measure of $\angle x$.



The measure of $\angle x = 180° - 85° - 45° = 50°$

A **right triangle** is a triangle with a right angle. The side opposite the right angle is called the **hypotenuse,** and the other two sides are called **legs.**

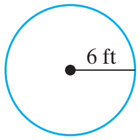


For a circle or a sphere:

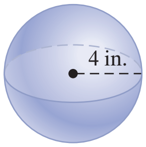
$$\boxed{\text{diameter}} = \boxed{2} \cdot \boxed{\text{radius}}$$
$$\searrow \quad \downarrow \quad \swarrow$$
$$d = 2 \cdot r$$

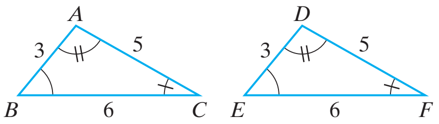
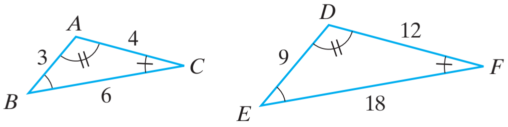
$$\boxed{\text{radius}} = \boxed{\frac{\text{diameter}}{2}}$$
$$\downarrow \qquad \swarrow$$
$$r = \frac{d}{2}$$

Find the diameter of the circle.



$$\begin{aligned} d &= 2 \cdot r \\ &= 2 \cdot 6 \text{ feet} = 12 \text{ feet} \end{aligned}$$

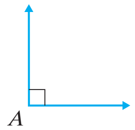
Definitions and Concepts	Examples
Section 9.3 Perimeter	
Perimeter Formulas **Rectangle:** $P = 2\cdot l + 2\cdot w$ **Square:** $P = 4\cdot s$ **Triangle:** $P = a + b + c$ **Circumference of a Circle:** $C = 2\cdot\pi\cdot r$ or $C = \pi\cdot d$, where $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$	Find the perimeter of a rectangle with length 28 meters and width 15 meters. $P = 2\cdot l + 2\cdot w$ $= 2\cdot 28\text{ m} + 2\cdot 15\text{ m}$ $= 56\text{ m} + 30\text{ m}$ $= 86\text{ m}$ The perimeter is 86 meters.
Section 9.4 Area	
Area Formulas **Rectangle:** $A = l\cdot w$ **Square:** $A = s^2$ **Triangle:** $A = \frac{1}{2}\cdot b\cdot h$ **Parallelogram:** $A = b\cdot h$ **Trapezoid:** $A = \frac{1}{2}\cdot(b + B)\cdot h$ **Circle:** $A = \pi\cdot r^2$	Find the area of a square with side length 8 centimeters. $A = s^2$ $= (8\text{ cm})^2$ $= 64$ square centimeters The area of the square is 64 square centimeters.
Section 9.5 Volume and Surface Area	
Volume Formulas **Rectangular Solid:** $V = l\cdot w\cdot h$ **Cube:** $V = s^3$ **Sphere:** $V = \frac{4}{3}\cdot\pi\cdot r^3$ **Right Circular Cylinder:** $V = \pi\cdot r^2\cdot h$ **Cone:** $V = \frac{1}{3}\cdot\pi\cdot r^2\cdot h$ **Square-Based Pyramid:** $V = \frac{1}{3}\cdot s^2\cdot h$ **Surface Area Formulas** See page 686.	Find the volume of the sphere. Use $\frac{22}{7}$ for π.  $V = \frac{4}{3}\cdot\pi\cdot r^3$ $\approx \frac{4}{3}\cdot\frac{22}{7}\cdot(4\text{ inches})^3$ $= \frac{4\cdot 22\cdot 64}{3\cdot 7}$ cubic inches $= \frac{5632}{21}$ or $268\frac{4}{21}$ cubic inches

Definitions and Concepts	Examples
Section 9.6 Congruent and Similar Triangles	
Congruent triangles have the same shape and the same size. Corresponding angles are equal, and corresponding sides are equal.	 Congruent triangles
Similar triangles have exactly the same shape but not necessarily the same size. Corresponding angles are equal, and the ratios of the lengths of corresponding sides are equal.	 Similar triangles $\frac{AB}{DE} = \frac{3}{9} = \frac{1}{3}, \frac{BC}{EF} = \frac{6}{18} = \frac{1}{3},$ $\frac{CA}{FD} = \frac{4}{12} = \frac{1}{3}$

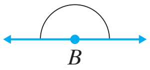
Chapter 9 Review

(9.1) *Classify each angle as acute, right, obtuse, or straight.*

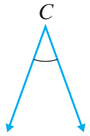
1.



2.



3.



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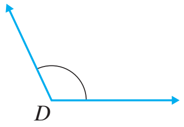
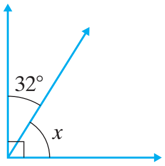


5. Find the complement of a 25° angle.

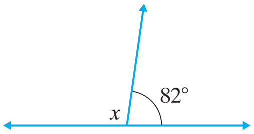
6. Find the supplement of a 105° angle.

Find the measure of angle x in each figure.

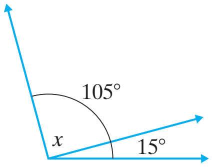
7.



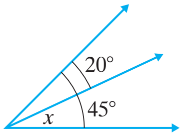
8.



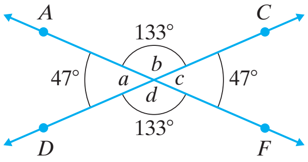
9.



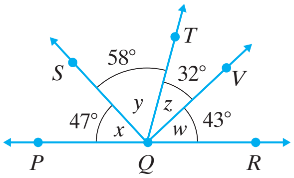
10.



11. Identify the pairs of supplementary angles.

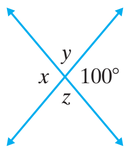


12. Identify the pairs of complementary angles.

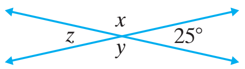


Find the measures of angles x, y, and z in each figure.

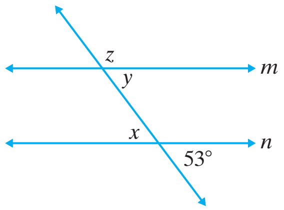
13.



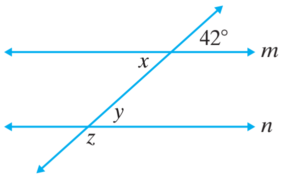
14.



15. Given that $m \parallel n$.

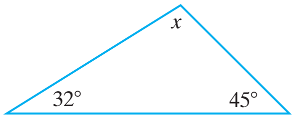


16. Given that $m \parallel n$.

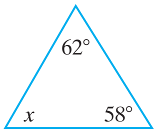


(9.2) *Find the measure of ∠x in each figure.*

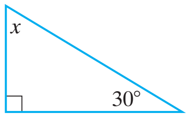
17.



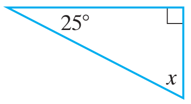
18.



19.

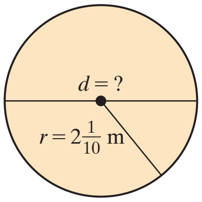


20.

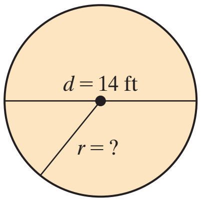


Find the unknown diameter or radius as indicated.

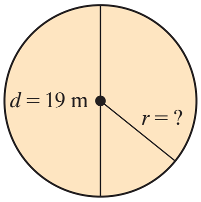
21.



22.



23.



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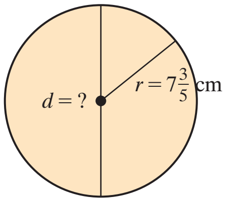
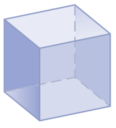
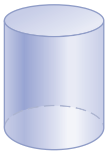


Identify each solid.

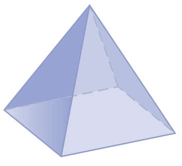
25.



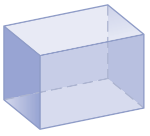
26.



27.



28.



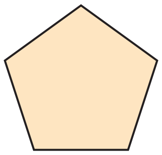
Find the unknown radius or diameter as indicated.

29. The radius of a sphere is 9 inches. Find its diameter.

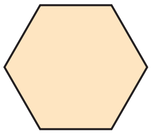
30. The diameter of a sphere is 4.7 meters. Find its radius.

Identify each regular polygon.

31.

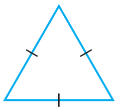


32.



Identify each triangle as equilateral, isosceles, or scalene. Also identify any triangle that is a right triangle.

33.



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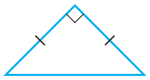
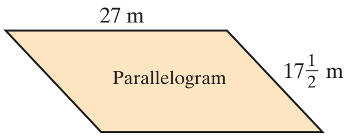
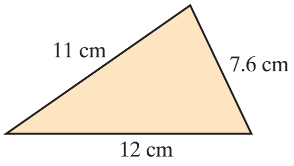


(9.3) *Find the perimeter of each figure.*

35.



36.



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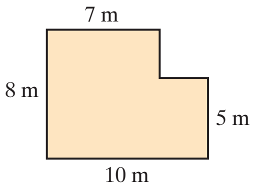
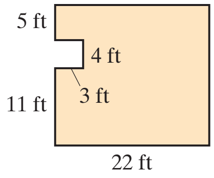


38.



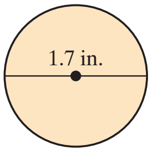
Solve.

39. Find the perimeter of a rectangular sign that measures 6 feet by 10 feet.

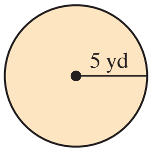
40. Find the perimeter of a town square that measures 110 feet on a side.

Find the circumference of each circle. Use $\pi \approx 3.14$.

41.

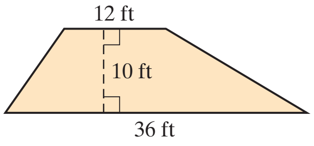


42.

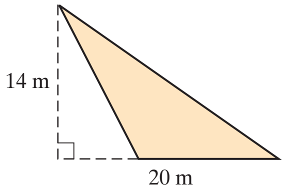


(9.4) *Find the area of each figure. For the circles, find the exact area and then use* $\pi \approx 3.14$ *to approximate the area.*

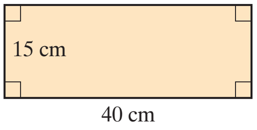
43.



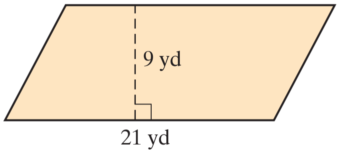
44.



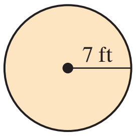
45.



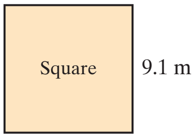
46.



47.



48.



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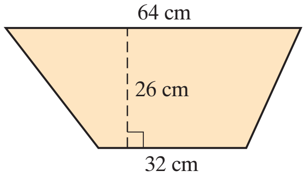
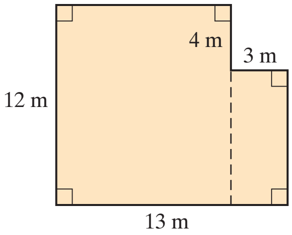
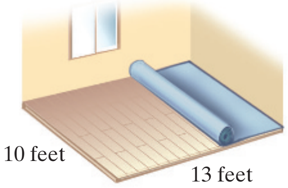


50.



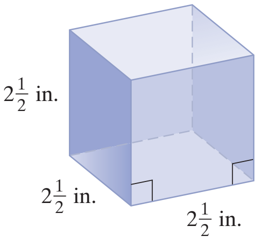
51. The amount of sealant necessary to seal a driveway depends on the area. Find the area of a rectangular driveway 36 feet by 12 feet.

52. Find how much carpet is necessary to cover the floor of the room shown.

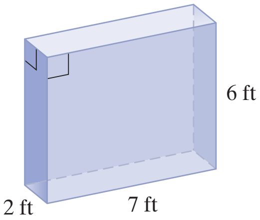


(9.5) *Find the volume and surface area of the solids in Exercises 53 and 54. For Exercises 55 and 56, give the exact volume and an approximation.*

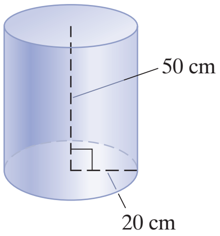
53.



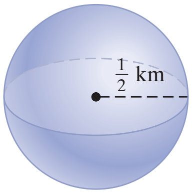
54.



55. Use $\pi \approx 3.14$.



56. Use $\pi \approx \frac{22}{7}$.



57. Find the volume of a pyramid with a square base 2 feet on a side and a height of 2 feet.

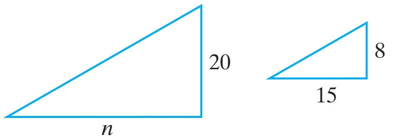
58. Approximate the volume of a tin can 8 inches high and 3.5 inches in radius. Use 3.14 for π.

59. A chest has 3 drawers. If each drawer has inside measurements of $2\frac{1}{2}$ feet by $1\frac{1}{2}$ feet by $\frac{2}{3}$ foot, find the total volume of the 3 drawers.

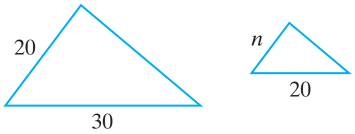
60. A cylindrical canister for a shop vacuum is 2 feet tall and 1 foot in *diameter*. Find its exact volume.

(9.6) *Given that the pairs of triangles are similar, find the unknown length n.*

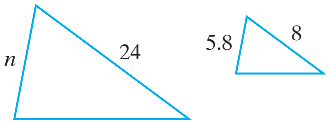
61.



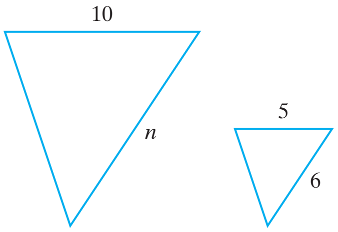
62.



63.



64.



Solve.

65. A housepainter needs to estimate the height of a condominium. He estimates the length of his shadow to be 7 feet long and the length of the building's shadow to be 42 feet long. Find the approximate height of the building if the housepainter is $5\frac{1}{2}$ feet tall.

66. A toy company is making a triangular sail for a toy sailboat. The toy sail is to be the same shape as a real sailboat's sail. Use the following diagram to find the unknown lengths x and y.

Mixed Review

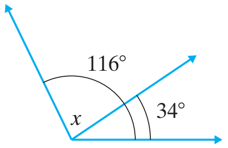
Find the following.

67. The supplement of a 72° angle

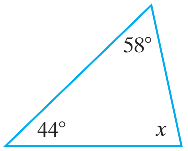
68. The complement of a 1° angle

Find the measure of angle x in each figure.

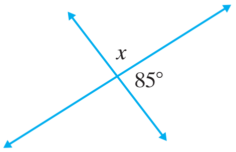
69.



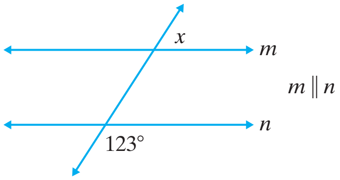
70.



71.

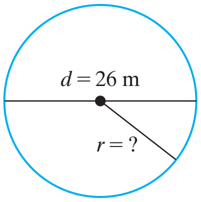


72.

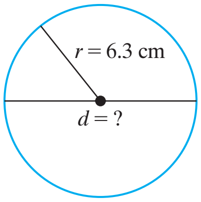


Find the unknown diameter or radius as indicated.

73.

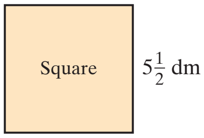


74.

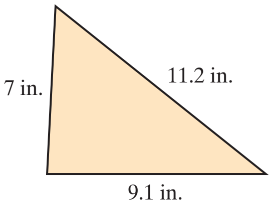


Find the perimeter of each figure.

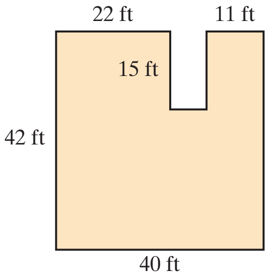
75.



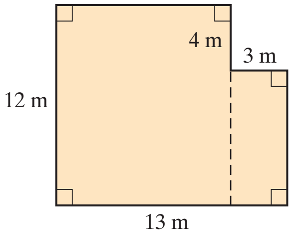
76.



77.

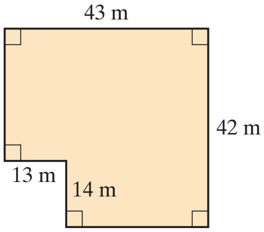


78.

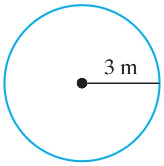


Find the area of each figure. For the circle, find the exact area and then use $\pi \approx 3.14$ to approximate the area.

79.

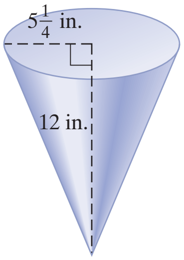


80.

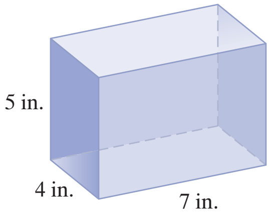


Find the volume of each solid.

81. Give an approximation using $\frac{22}{7}$ for π.



82.



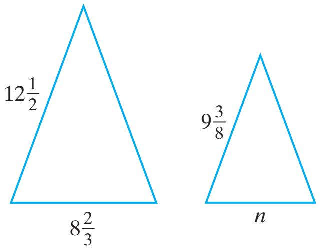
Solve.

83. Find the volume of air in a rectangular room 15 feet by 12 feet with a 7-foot ceiling.

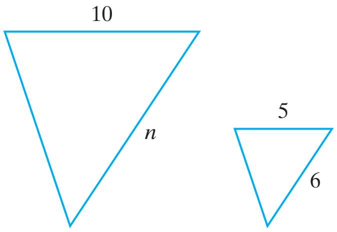
84. A mover has two boxes left for packing. Both are cubical, one 3 feet on a side and the other 1.2 feet on a side. Find their combined volume.

Given that the pairs of triangles are similar, find the unknown length n.

85.



86.



Chapter 9 Getting Ready for the Test LC

MATCHING **Match** each word in the first column with its illustration in the columns to the right.

1. line

2. line segment

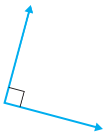
3. ray

4. right angle

5. acute angle

6. obtuse angle

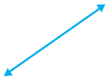
A.



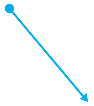
B.

C.

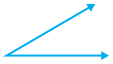
D.



E.

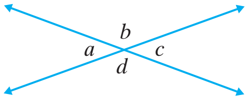


F.



MULTIPLE CHOICE Exercises 7 through 16 are **Multiple Choice**. Choose the correct letter for each exercise.

Use the given figure for Exercises 7 and 8.



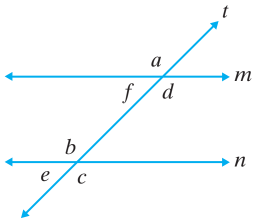
7. Choose two angles that have a sum of 180°.

A. $\angle a$ and $\angle c$ **B.** $\angle a$ and $\angle d$ **C.** $\angle b$ and $\angle d$

8. Choose two angles that have the same measure.

A. $\angle a$ and $\angle b$ **B.** $\angle c$ and $\angle d$ **C.** $\angle b$ and $\angle d$

Use the given figure for Exercises 9 and 10. For this figure, $m \parallel n$.



9. Choose two angles that have a sum of 180°.

A. $\angle a$ and $\angle e$ **B.** $\angle a$ and $\angle d$ **C.** $\angle a$ and $\angle b$

10. Choose two angles that have the same measure.

A. $\angle a$ and $\angle e$ **B.** $\angle a$ and $\angle c$ **C.** $\angle a$ and $\angle f$

For Exercises 11 through 16, the choices are below. Exercises 11, 13, and 15 have two correct choices.

A. perimeter **B.** area **C.** volume **D.** circumference **E.** surface area

11. Which calculation is measured in square units?

12. Which calculation is measured in cubic units?

13. Which calculation is measured in units?

For Exercises 14 through 16 name the calculation (choices **A.**, **B.**, **C.**, *or* **D.** *above) to be used to solve each exercise.*

14. The amount of material needed for a rectangular tablecloth.

15. The amount of trim needed to go around the edge of a tablecloth.

16. The amount of soil needed to fill in a hole in the ground.

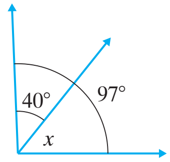
Chapter 9 Test MyLab Math

For additional practice go to your study plan in MyLab Math.

1. Find the complement of a 78° angle.

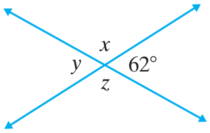
2. Find the supplement of a 124° angle.

3. Find the measure of $\angle x$.

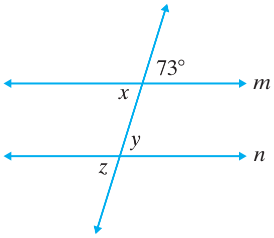


Find the measures of x, y, and z in each figure.

4.

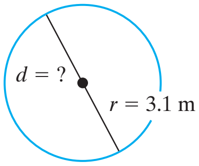


5. Given: $m \parallel n$.

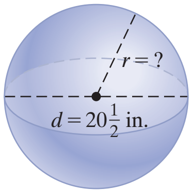


Find the unknown diameter or radius as indicated.

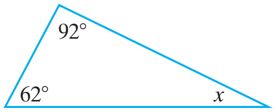
6.



7.

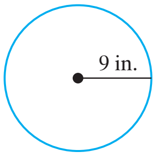


8. Find the measure of $\angle x$.

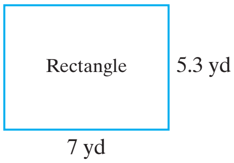


Find the perimeter (or circumference) and area of each figure. For the circle, give the exact value and then use $\pi \approx 3.14$ for an approximation.

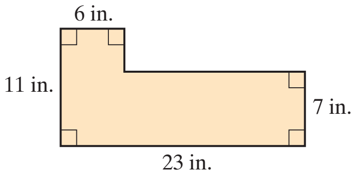
9.



10.



11.

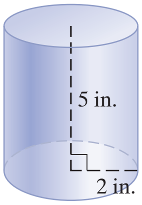


Answers

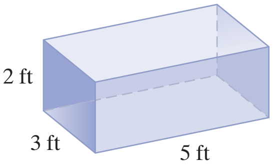
1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____
11. ____

Find the volume of each solid. For the cylinder, use $\pi \approx \frac{22}{7}$.

12.



13. Find the surface area also.



Solve.

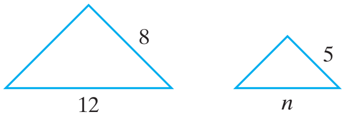
14. Find the perimeter of a square photo with a side length of 4 inches.

15. How much soil is needed to fill a rectangular hole 3 feet by 3 feet by 2 feet?

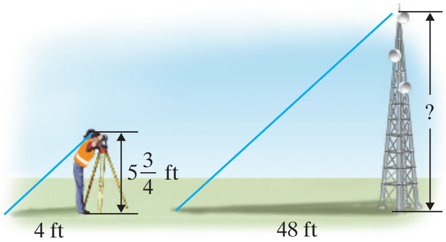
16. Find how much baseboard is needed to go around a rectangular room that measures 18 feet by 13 feet. If baseboard costs $1.87 per foot, also calculate the total cost needed for materials.

17. Vivian Thomas is going to put insecticide on her lawn. The lawn is a rectangle measuring 123.8 feet by 80 feet. The amount of insecticide required is 0.02 ounces per square foot. Find the area of Vivian's lawn and then find how much insecticide she needs to purchase.

18. Given that the triangles are similar, find the missing length n.



19. Tamara Watford, a surveyor, needs to estimate the height of a tower. She estimates the length of her shadow to be 4 feet long and the length of the tower's shadow to be 48 feet long. Find the approximate height of the tower if she is $5\frac{3}{4}$ feet tall.



12. ______________

13. ______________

14. ______________

15. ______________

16. ______________

17. ______________

18. ______________

19. ______________

Chapters 1–9 Cumulative Review

Answers

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.
16.
17.
18.
19.
20.
21.
22.

1. Write the decimal -50.82 in words.

2. Add: $\frac{7}{11} + \frac{1}{6}$

3. Round 736.2359 to the nearest tenth.

4. Round 736.2359 to the nearest hundred.

5. Add: $45 + 2.06$

6. Divide: $-3\frac{1}{3} \div 1\frac{5}{6}$

Multiply.

7. 7.68×10

8. $\frac{7}{11} \cdot \frac{1}{6}$

9. $(-76.3)(1000)$

10. $5\frac{1}{2} \cdot 2\frac{1}{11}$

11. Divide: $270.2 \div 7$. Check your answer.

12. Divide: $\frac{56.7}{100}$

13. Simplify: $-0.5(8.6 - 1.2)$

14. Simplify: $\frac{5 + 2(8 - 3)}{30 \div 6 \cdot 5}$

15. Insert $<$, $>$, or $=$ to form a true statement. $\frac{1}{8}$ 0.12

16. Insert $<$, $>$, or $=$ to form a true statement. 0.75 $\frac{13}{16}$

17. Write the ratio of 2.6 to 3.1 as a fraction in simplest form.

18. Find: $\frac{2}{9} + \frac{7}{15} - \frac{1}{3}$

19. Is $\frac{2}{3} = \frac{4}{6}$ a true proportion?

20. Solve for n: $\frac{7}{8} = \frac{n}{20}$

21. For 2017 model cars, 25 out of every 100 were painted white. What percent of model-year 2017 cars were white?

22. Solve for x: $4x - 7x = -30$

Write each percent as a fraction or mixed number in simplest form.

23. 1.9%

24. 26%

25. 125%

26. 560%

27. 85% of 300 is what number?

28. What percent of 16 is 2.4?

29. 20.8 is 40% of what number?

30. Find: $(7 - \sqrt{16})^2$

31. Mr. Buccaran, the principal at Slidell High School, counted 31 freshmen absent during a particular day. If this is 4% of the total number of freshmen, how many freshmen are there at Slidell High School?

32. Flooring tiles cost $90 for a box with 40 tiles. Each tile is 1 square foot. Find the unit price in dollars per square foot.

33. Sherry Souter, a real estate broker for Wealth Investments, sold a house for $214,000 last week. If her commission rate is 1.5% of the selling price of the home, find the amount of her commission.

34. A student can complete 7 exercises in 6 minutes. At this rate, how many exercises can be completed in 30 minutes?

35. Convert 8 feet to inches.

36. 100 inches = _____ yd _____ ft _____ in.

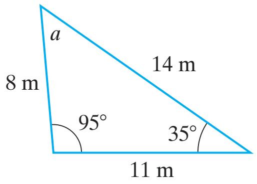
37. Convert 3.2 kilograms to grams.

38. Convert 70 mm to meters.

39. Subtract 3 quarts from 4 gallons 2 quarts.

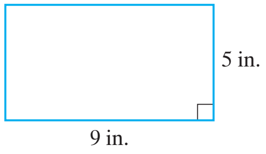
40. Write seventy thousand, fifty-two in standard form.

41. Find the measure of $\angle a$.



42. Find the perimeter of the triangle in Exercise **41**.

43. Find the perimeter of the rectangle below:



44. Solve for x: $7(x - 2) = 9x - 6$

45. Find $\sqrt{\frac{4}{25}}$.

46. Find $\sqrt{\frac{9}{16}}$.

23. ______________

24. ______________

25. ______________

26. ______________

27. ______________

28. ______________

29. ______________

30. ______________

31. ______________

32. ______________

33. ______________

34. ______________

35. ______________

36. ______________

37. ______________

38. ______________

39. ______________

40. ______________

41. ______________

42. ______________

43. ______________

44. ______________

45. ______________

46. ______________

Appendix

A

Tables

A.1 Addition Table and One Hundred Addition Facts

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

One Hundred Addition Facts

Knowledge of the basic addition facts found above is an important prerequisite for a course in basic college mathematics with early integers. Study the table above and then perform the additions. Check your answers either by comparing them with those found in the back-of-the-book answer section or by using the table. Review any facts that you missed.

1. $\begin{array}{r} 1 \\ +4 \\ \hline \end{array}$ **2.** $\begin{array}{r} 5 \\ +6 \\ \hline \end{array}$ **3.** $\begin{array}{r} 2 \\ +3 \\ \hline \end{array}$ **4.** $\begin{array}{r} 7 \\ +8 \\ \hline \end{array}$ **5.** $\begin{array}{r} 3 \\ +9 \\ \hline \end{array}$ **6.** $\begin{array}{r} 6 \\ +1 \\ \hline \end{array}$

7. $\begin{array}{r} 4 \\ +4 \\ \hline \end{array}$ **8.** $\begin{array}{r} 0 \\ +6 \\ \hline \end{array}$ **9.** $\begin{array}{r} 9 \\ +5 \\ \hline \end{array}$ **10.** $\begin{array}{r} 8 \\ +2 \\ \hline \end{array}$ **11.** $\begin{array}{r} 5 \\ +7 \\ \hline \end{array}$ **12.** $\begin{array}{r} 3 \\ +2 \\ \hline \end{array}$

13. $\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$ **14.** $\begin{array}{r} 1 \\ +1 \\ \hline \end{array}$ **15.** $\begin{array}{r} 8 \\ +1 \\ \hline \end{array}$ **16.** $\begin{array}{r} 6 \\ +6 \\ \hline \end{array}$ **17.** $\begin{array}{r} 2 \\ +9 \\ \hline \end{array}$ **18.** $\begin{array}{r} 3 \\ +5 \\ \hline \end{array}$

19. $\begin{array}{r} 9 \\ +9 \\ \hline \end{array}$ **20.** $\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$ **21.** $\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$ **22.** $\begin{array}{r} 0 \\ +0 \\ \hline \end{array}$ **23.** $\begin{array}{r} 1 \\ +9 \\ \hline \end{array}$ **24.** $\begin{array}{r} 3 \\ +7 \\ \hline \end{array}$

25. $\begin{array}{r} 9 \\ +8 \\ \hline \end{array}$ **26.** $\begin{array}{r} 0 \\ +8 \\ \hline \end{array}$ **27.** $\begin{array}{r} 4 \\ +9 \\ \hline \end{array}$ **28.** $\begin{array}{r} 3 \\ +0 \\ \hline \end{array}$ **29.** $\begin{array}{r} 7 \\ +5 \\ \hline \end{array}$ **30.** $\begin{array}{r} 8 \\ +9 \\ \hline \end{array}$

31. $\begin{array}{r}9\\+7\\\hline\end{array}$ **32.** $\begin{array}{r}2\\+6\\\hline\end{array}$ **33.** $\begin{array}{r}4\\+3\\\hline\end{array}$ **34.** $\begin{array}{r}8\\+5\\\hline\end{array}$ **35.** $\begin{array}{r}3\\+1\\\hline\end{array}$ **36.** $\begin{array}{r}0\\+3\\\hline\end{array}$

37. $\begin{array}{r}7\\+1\\\hline\end{array}$ **38.** $\begin{array}{r}3\\+4\\\hline\end{array}$ **39.** $\begin{array}{r}8\\+0\\\hline\end{array}$ **40.** $\begin{array}{r}6\\+3\\\hline\end{array}$ **41.** $\begin{array}{r}2\\+4\\\hline\end{array}$ **42.** $\begin{array}{r}0\\+9\\\hline\end{array}$

43. $\begin{array}{r}8\\+8\\\hline\end{array}$ **44.** $\begin{array}{r}5\\+3\\\hline\end{array}$ **45.** $\begin{array}{r}3\\+6\\\hline\end{array}$ **46.** $\begin{array}{r}6\\+9\\\hline\end{array}$ **47.** $\begin{array}{r}4\\+8\\\hline\end{array}$ **48.** $\begin{array}{r}0\\+1\\\hline\end{array}$

49. $\begin{array}{r}2\\+5\\\hline\end{array}$ **50.** $\begin{array}{r}6\\+0\\\hline\end{array}$ **51.** $\begin{array}{r}2\\+0\\\hline\end{array}$ **52.** $\begin{array}{r}4\\+2\\\hline\end{array}$ **53.** $\begin{array}{r}8\\+3\\\hline\end{array}$ **54.** $\begin{array}{r}7\\+4\\\hline\end{array}$

55. $\begin{array}{r}1\\+7\\\hline\end{array}$ **56.** $\begin{array}{r}4\\+6\\\hline\end{array}$ **57.** $\begin{array}{r}0\\+5\\\hline\end{array}$ **58.** $\begin{array}{r}9\\+1\\\hline\end{array}$ **59.** $\begin{array}{r}8\\+6\\\hline\end{array}$ **60.** $\begin{array}{r}5\\+1\\\hline\end{array}$

61. $\begin{array}{r}6\\+7\\\hline\end{array}$ **62.** $\begin{array}{r}4\\+0\\\hline\end{array}$ **63.** $\begin{array}{r}1\\+6\\\hline\end{array}$ **64.** $\begin{array}{r}4\\+5\\\hline\end{array}$ **65.** $\begin{array}{r}0\\+7\\\hline\end{array}$ **66.** $\begin{array}{r}5\\+8\\\hline\end{array}$

67. $\begin{array}{r}7\\+6\\\hline\end{array}$ **68.** $\begin{array}{r}7\\+0\\\hline\end{array}$ **69.** $\begin{array}{r}4\\+1\\\hline\end{array}$ **70.** $\begin{array}{r}5\\+4\\\hline\end{array}$ **71.** $\begin{array}{r}0\\+4\\\hline\end{array}$ **72.** $\begin{array}{r}1\\+2\\\hline\end{array}$

73. $\begin{array}{r}7\\+9\\\hline\end{array}$ **74.** $\begin{array}{r}3\\+8\\\hline\end{array}$ **75.** $\begin{array}{r}7\\+7\\\hline\end{array}$ **76.** $\begin{array}{r}9\\+4\\\hline\end{array}$ **77.** $\begin{array}{r}1\\+0\\\hline\end{array}$ **78.** $\begin{array}{r}4\\+7\\\hline\end{array}$

79. $\begin{array}{r}2\\+2\\\hline\end{array}$ **80.** $\begin{array}{r}1\\+3\\\hline\end{array}$ **81.** $\begin{array}{r}2\\+8\\\hline\end{array}$ **82.** $\begin{array}{r}5\\+9\\\hline\end{array}$ **83.** $\begin{array}{r}6\\+2\\\hline\end{array}$ **84.** $\begin{array}{r}9\\+6\\\hline\end{array}$

85. $\begin{array}{r}5\\+0\\\hline\end{array}$ **86.** $\begin{array}{r}8\\+7\\\hline\end{array}$ **87.** $\begin{array}{r}7\\+3\\\hline\end{array}$ **88.** $\begin{array}{r}0\\+2\\\hline\end{array}$ **89.** $\begin{array}{r}9\\+2\\\hline\end{array}$ **90.** $\begin{array}{r}3\\+3\\\hline\end{array}$

91. $\begin{array}{r}9\\+3\\\hline\end{array}$ **92.** $\begin{array}{r}1\\+5\\\hline\end{array}$ **93.** $\begin{array}{r}2\\+7\\\hline\end{array}$ **94.** $\begin{array}{r}6\\+5\\\hline\end{array}$ **95.** $\begin{array}{r}7\\+2\\\hline\end{array}$ **96.** $\begin{array}{r}1\\+8\\\hline\end{array}$

97. $\begin{array}{r}6\\+8\\\hline\end{array}$ **98.** $\begin{array}{r}8\\+4\\\hline\end{array}$ **99.** $\begin{array}{r}9\\+0\\\hline\end{array}$ **100.** $\begin{array}{r}2\\+1\\\hline\end{array}$

A.2 Multiplication Table and One Hundred Multiplication Facts

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

One Hundred Multiplication Facts

Knowledge of the basic multiplication facts found above is an important prerequisite for a course in basic college mathematics with early integers. Study the table above and then perform the multiplications. Check your answers either by comparing them with those found in the back-of-the-book answer section or by using the table. Review any facts that you missed.

1. $\begin{array}{r} 1 \\ \times 1 \\ \hline \end{array}$
2. $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$
3. $\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$
4. $\begin{array}{r} 3 \\ \times 3 \\ \hline \end{array}$
5. $\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$
6. $\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$

7. $\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$
8. $\begin{array}{r} 7 \\ \times 1 \\ \hline \end{array}$
9. $\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array}$
10. $\begin{array}{r} 0 \\ \times 5 \\ \hline \end{array}$
11. $\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$
12. $\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$

13. $\begin{array}{r} 3 \\ \times 2 \\ \hline \end{array}$
14. $\begin{array}{r} 6 \\ \times 0 \\ \hline \end{array}$
15. $\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}$
16. $\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$
17. $\begin{array}{r} 4 \\ \times 6 \\ \hline \end{array}$
18. $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$

19. $\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$
20. $\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$
21. $\begin{array}{r} 5 \\ \times 8 \\ \hline \end{array}$
22. $\begin{array}{r} 7 \\ \times 2 \\ \hline \end{array}$
23. $\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$
24. $\begin{array}{r} 1 \\ \times 2 \\ \hline \end{array}$

25. $\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$
26. $\begin{array}{r} 3 \\ \times 1 \\ \hline \end{array}$
27. $\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$
28. $\begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$
29. $\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$
30. $\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$

31. $\begin{array}{r} 2 \\ \times 1 \\ \hline \end{array}$
32. $\begin{array}{r} 8 \\ \times 0 \\ \hline \end{array}$
33. $\begin{array}{r} 4 \\ \times 9 \\ \hline \end{array}$
34. $\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$
35. $\begin{array}{r} 6 \\ \times 2 \\ \hline \end{array}$
36. $\begin{array}{r} 4 \\ \times 5 \\ \hline \end{array}$

37. $\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$ **38.** $\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$ **39.** $\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$ **40.** $\begin{array}{r} 1 \\ \times 6 \\ \hline \end{array}$ **41.** $\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$ **42.** $\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$

43. $\begin{array}{r} 1 \\ \times 8 \\ \hline \end{array}$ **44.** $\begin{array}{r} 5 \\ \times 1 \\ \hline \end{array}$ **45.** $\begin{array}{r} 9 \\ \times 0 \\ \hline \end{array}$ **46.** $\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$ **47.** $\begin{array}{r} 9 \\ \times 3 \\ \hline \end{array}$ **48.** $\begin{array}{r} 0 \\ \times 3 \\ \hline \end{array}$

49. $\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$ **50.** $\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$ **51.** $\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$ **52.** $\begin{array}{r} 2 \\ \times 6 \\ \hline \end{array}$ **53.** $\begin{array}{r} 1 \\ \times 0 \\ \hline \end{array}$ **54.** $\begin{array}{r} 3 \\ \times 9 \\ \hline \end{array}$

55. $\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$ **56.** $\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$ **57.** $\begin{array}{r} 0 \\ \times 6 \\ \hline \end{array}$ **58.** $\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array}$ **59.** $\begin{array}{r} 5 \\ \times 0 \\ \hline \end{array}$ **60.** $\begin{array}{r} 6 \\ \times 1 \\ \hline \end{array}$

61. $\begin{array}{r} 9 \\ \times 2 \\ \hline \end{array}$ **62.** $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$ **63.** $\begin{array}{r} 1 \\ \times 3 \\ \hline \end{array}$ **64.** $\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$ **65.** $\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$ **66.** $\begin{array}{r} 4 \\ \times 0 \\ \hline \end{array}$

67. $\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$ **68.** $\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$ **69.** $\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$ **70.** $\begin{array}{r} 2 \\ \times 0 \\ \hline \end{array}$ **71.** $\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$ **72.** $\begin{array}{r} 0 \\ \times 8 \\ \hline \end{array}$

73. $\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$ **74.** $\begin{array}{r} 2 \\ \times 4 \\ \hline \end{array}$ **75.** $\begin{array}{r} 0 \\ \times 1 \\ \hline \end{array}$ **76.** $\begin{array}{r} 3 \\ \times 8 \\ \hline \end{array}$ **77.** $\begin{array}{r} 9 \\ \times 1 \\ \hline \end{array}$ **78.** $\begin{array}{r} 7 \\ \times 0 \\ \hline \end{array}$

79. $\begin{array}{r} 5 \\ \times 3 \\ \hline \end{array}$ **80.** $\begin{array}{r} 4 \\ \times 4 \\ \hline \end{array}$ **81.** $\begin{array}{r} 1 \\ \times 5 \\ \hline \end{array}$ **82.** $\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$ **83.** $\begin{array}{r} 3 \\ \times 0 \\ \hline \end{array}$ **84.** $\begin{array}{r} 1 \\ \times 4 \\ \hline \end{array}$

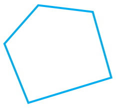
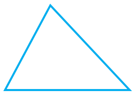
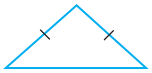
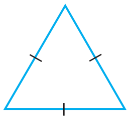
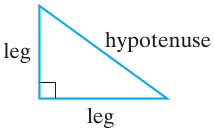
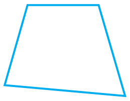
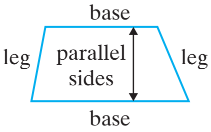
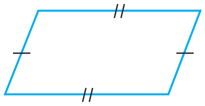
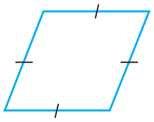
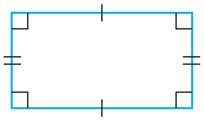
85. $\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$ **86.** $\begin{array}{r} 4 \\ \times 2 \\ \hline \end{array}$ **87.** $\begin{array}{r} 0 \\ \times 2 \\ \hline \end{array}$ **88.** $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$ **89.** $\begin{array}{r} 8 \\ \times 2 \\ \hline \end{array}$ **90.** $\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$

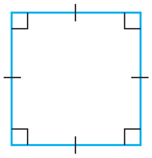
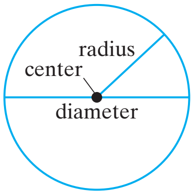
91. $\begin{array}{r} 0 \\ \times 0 \\ \hline \end{array}$ **92.** $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$ **93.** $\begin{array}{r} 4 \\ \times 1 \\ \hline \end{array}$ **94.** $\begin{array}{r} 0 \\ \times 4 \\ \hline \end{array}$ **95.** $\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$ **96.** $\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$

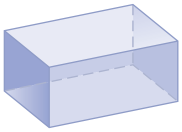
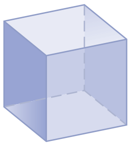
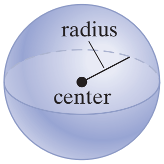
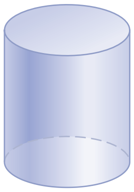
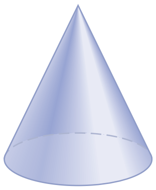
97. $\begin{array}{r} 3 \\ \times 6 \\ \hline \end{array}$ **98.** $\begin{array}{r} 5 \\ \times 2 \\ \hline \end{array}$ **99.** $\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$ **100.** $\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$

A.3 Tables of Geometric Figures

Plane Figures Have Length and Width but No Thickness or Depth

Name	Description	Figure
Polygon	Union of three or more coplanar line segments that intersect with each other only at each endpoint, with each endpoint shared by two segments.	
Triangle	Polygon with three sides (sum of measures of three angles is 180°).	
Scalene Triangle	Triangle with no sides of equal length.	
Isosceles Triangle	Triangle with two sides of equal length.	
Equilateral Triangle	Triangle with all sides of equal length.	
Right Triangle	Triangle that contains a right angle.	
Quadrilateral	Polygon with four sides (sum of measures of four angles is 360°).	
Trapezoid	Quadrilateral with exactly one pair of opposite sides parallel.	
Isosceles Trapezoid	Trapezoid with legs of equal length.	
Parallelogram	Quadrilateral with both pairs of opposite sides parallel.	
Rhombus	Parallelogram with all sides of equal length.	
Rectangle	Parallelogram with four right angles.	

Plane Figures Have Length and Width but No Thickness or Depth (*continued*)		
Name	Description	Figure
Square	Rectangle with all sides of equal length.	
Circle	All points in a plane the same distance from a fixed point called the center.	

Solid Figures Have Length, Width, and Height or Depth		
Name	Description	Figure
Rectangular Solid	A solid with six sides, all of which are rectangles.	
Cube	A rectangular solid whose six sides are squares.	
Sphere	All points the same distance from a fixed point called the center.	
Right Circular Cylinder	A cylinder having two circular bases that are perpendicular to its altitude.	
Right Circular Cone	A cone with a circular base that is perpendicular to its altitude.	

A.4 Table of Percents, Decimals, and Fraction Equivalents

Percent	Decimal	Fraction
1%	0.01	$\frac{1}{100}$
5%	0.05	$\frac{1}{20}$
10%	0.1	$\frac{1}{10}$
12.5% or $12\frac{1}{2}\%$	0.125	$\frac{1}{8}$
$16.\overline{6}\%$ or $16\frac{2}{3}\%$	$0.1\overline{6}$	$\frac{1}{6}$
20%	0.2	$\frac{1}{5}$
25%	0.25	$\frac{1}{4}$
30%	0.3	$\frac{3}{10}$
$33.\overline{3}\%$ or $33\frac{1}{3}\%$	$0.\overline{3}$	$\frac{1}{3}$
37.5% or $37\frac{1}{2}\%$	0.375	$\frac{3}{8}$
40%	0.4	$\frac{2}{5}$
50%	0.5	$\frac{1}{2}$
60%	0.6	$\frac{3}{5}$
62.5% or $62\frac{1}{2}\%$	0.625	$\frac{5}{8}$
$66.\overline{6}\%$ or $66\frac{2}{3}\%$	$0.\overline{6}$	$\frac{2}{3}$
70%	0.7	$\frac{7}{10}$
75%	0.75	$\frac{3}{4}$
80%	0.8	$\frac{4}{5}$
$83.\overline{3}\%$ or $83\frac{1}{3}\%$	$0.8\overline{3}$	$\frac{5}{6}$
87.5% or $87\frac{1}{2}\%$	0.875	$\frac{7}{8}$
90%	0.9	$\frac{9}{10}$
100%	1.0	1
110%	1.1	$1\frac{1}{10}$
125%	1.25	$1\frac{1}{4}$
$133.\overline{3}\%$ or $133\frac{1}{3}\%$	$1.\overline{3}$	$1\frac{1}{3}$
150%	1.5	$1\frac{1}{2}$
$166.\overline{6}\%$ or $166\frac{2}{3}\%$	$1.\overline{6}$	$1\frac{2}{3}$
175%	1.75	$1\frac{3}{4}$
200%	2.0	2

A.5 Table on Finding Common Percents of a Number

Common Percent Equivalences*	Shortcut Method for Finding Percent	Example
$1\% = 0.01 \left(\text{or } \frac{1}{100}\right)$	To find 1% of a number, multiply by 0.01. To do so, move the decimal point two places to the left.	1% of 210 is 2.10 or 2.1. 1% of 1500 is 15. 1% of 8.6 is 0.086.
$10\% = 0.1 \left(\text{or } \frac{1}{10}\right)$	To find 10% of a number, multiply by 0.1, or move the decimal point of the number one place to the left.	10% of 140 is 14. 10% of 30 is 3. 10% of 17.6 is 1.76.
$25\% = \frac{1}{4}$	To find 25% of a number, find $\frac{1}{4}$ of the number, or divide the number by 4.	25% of 20 is $\frac{20}{4}$ or 5. 25% of 8 is 2. 25% of 10 is $\frac{10}{4}$ or $2\frac{1}{2}$.
$50\% = \frac{1}{2}$	To find 50% of a number, find $\frac{1}{2}$ of the number, or divide the number by 2.	50% of 64 is $\frac{64}{2}$ or 32. 50% of 1000 is 500. 50% of 9 is $\frac{9}{2}$ or $4\frac{1}{2}$.
$100\% = 1$	To find 100% of a number, multiply the number by 1. In other words, 100% of a number is the number.	100% of 98 is 98. 100% of 1407 is 1407. 100% of 18.4 is 18.4.
$200\% = 2$	To find 200% of a number, multiply the number by 2.	200% of 31 is $31 \cdot 2$ or 62. 200% of 750 is 1500. 200% of 6.5 is 13.

*See Appendix A.4.

A.6 Table of Squares and Square Roots

n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$
1	1	1.000	51	2601	7.141
2	4	1.414	52	2704	7.211
3	9	1.732	53	2809	7.280
4	16	2.000	54	2916	7.348
5	25	2.236	55	3025	7.416
6	36	2.449	56	3136	7.483
7	49	2.646	57	3249	7.550
8	64	2.828	58	3364	7.616
9	81	3.000	59	3481	7.681
10	100	3.162	60	3600	7.746
11	121	3.317	61	3721	7.810
12	144	3.464	62	3844	7.874
13	169	3.606	63	3969	7.937
14	196	3.742	64	4096	8.000
15	225	3.873	65	4225	8.062
16	256	4.000	66	4356	8.124
17	289	4.123	67	4489	8.185
18	324	4.243	68	4624	8.246
19	361	4.359	69	4761	8.307
20	400	4.472	70	4900	8.367
21	441	4.583	71	5041	8.426
22	484	4.690	72	5184	8.485
23	529	4.796	73	5329	8.544
24	576	4.899	74	5476	8.602
25	625	5.000	75	5625	8.660
26	676	5.099	76	5776	8.718
27	729	5.196	77	5929	8.775
28	784	5.292	78	6084	8.832
29	841	5.385	79	6241	8.888
30	900	5.477	80	6400	8.944
31	961	5.568	81	6561	9.000
32	1024	5.657	82	6724	9.055
33	1089	5.745	83	6889	9.110
34	1156	5.831	84	7056	9.165
35	1225	5.916	85	7225	9.220
36	1296	6.000	86	7396	9.274
37	1369	6.083	87	7569	9.327
38	1444	6.164	88	7744	9.381
39	1521	6.245	89	7921	9.434
40	1600	6.325	90	8100	9.487
41	1681	6.403	91	8281	9.539
42	1764	6.481	92	8464	9.592
43	1849	6.557	93	8649	9.644
44	1936	6.633	94	8836	9.695
45	2025	6.708	95	9025	9.747
46	2116	6.782	96	9216	9.798
47	2209	6.856	97	9409	9.849
48	2304	6.928	98	9604	9.899
49	2401	7.000	99	9801	9.950
50	2500	7.071	100	10,000	10.000

A.7 Compound Interest Table

Compounded Annually

	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
1 year	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000	1.13000	1.14000	1.15000	1.16000	1.17000	1.18000
5 years	1.27628	1.33823	1.40255	1.46933	1.53862	1.61051	1.68506	1.76234	1.84244	1.92541	2.01136	2.10034	2.19245	2.28776
10 years	1.62889	1.79085	1.96715	2.15892	2.36736	2.59374	2.83942	3.10585	3.39457	3.70722	4.04556	4.41144	4.80683	5.23384
15 years	2.07893	2.39656	2.75903	3.17217	3.64248	4.17725	4.78459	5.47357	6.25427	7.13794	8.13706	9.26552	10.53872	11.97375
20 years	2.65330	3.20714	3.86968	4.66096	5.60441	6.72750	8.06231	9.64629	11.52309	13.74349	16.36654	19.46076	23.10560	27.39303

Compounded Semiannually

	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
1 year	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250	1.11303	1.12360	1.13423	1.14490	1.15563	1.16640	1.17723	1.18810
5 years	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889	1.70814	1.79085	1.87714	1.96715	2.06103	2.15892	2.26098	2.36736
10 years	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	2.91776	3.20714	3.52365	3.86968	4.24785	4.66096	5.11205	5.60441
15 years	2.09757	2.42726	2.80679	3.24340	3.74532	4.32194	4.98395	5.74349	6.61437	7.61226	8.75496	10.06266	11.55825	13.26768
20 years	2.68506	3.26204	3.95926	4.80102	5.81636	7.03999	8.51331	10.28572	12.41607	14.97446	18.04424	21.72452	26.13302	31.40942

Compounded Quarterly

	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
1 year	1.05095	1.06136	1.07186	1.08243	1.09308	1.10381	1.11462	1.12551	1.13648	1.14752	1.15865	1.16986	1.18115	1.19252
5 years	1.28204	1.34686	1.41478	1.48595	1.56051	1.63862	1.72043	1.80611	1.89584	1.98979	2.08815	2.19112	2.29891	2.41171
10 years	1.64362	1.81402	2.00160	2.20804	2.43519	2.68506	2.95987	3.26204	3.59420	3.95926	4.36038	4.80102	5.28497	5.81636
15 years	2.10718	2.44322	2.83182	3.28103	3.80013	4.39979	5.09225	5.89160	6.81402	7.87809	9.10513	10.51963	12.14965	14.02741
20 years	2.70148	3.29066	4.00639	4.87544	5.93015	7.20957	8.76085	10.64089	12.91828	15.67574	19.01290	23.04980	27.93091	33.83010

Compounded Daily

	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
1 year	1.05127	1.06183	1.07250	1.08328	1.09416	1.10516	1.11626	1.12747	1.13880	1.15024	1.16180	1.17347	1.18526	1.19716
5 years	1.28400	1.34983	1.41902	1.49176	1.56823	1.64861	1.73311	1.82194	1.91532	2.01348	2.11667	2.22515	2.33918	2.45906
10 years	1.64866	1.82203	2.01362	2.22535	2.45933	2.71791	3.00367	3.31946	3.66845	4.05411	4.48031	4.95130	5.47178	6.04696
15 years	2.11689	2.45942	2.85736	3.31968	3.85678	4.48077	5.20569	6.04786	7.02625	8.16288	9.48335	11.01738	12.79950	14.86983
20 years	2.71810	3.31979	4.05466	4.95216	6.04831	7.38703	9.02202	11.01883	13.45751	16.43582	20.07316	24.51533	29.94039	36.56577

Appendix

B

Exponents and Polynomials

B.1 Adding and Subtracting Polynomials

Objectives

- **A** Add Polynomials.
- **B** Subtract Polynomials.
- **C** Evaluate Polynomials at Given Replacement Values.

Before we add and subtract polynomials, let's first review some definitions presented in Section 8.1 . Recall that the *addends* of an algebraic expression are the *terms* of the expression.

Expression

$$\underbrace{3x + 5}_{\text{2 terms}} \qquad \underbrace{7y^2 + (-6y) + 4}_{\text{3 terms}}$$

Also, recall that *like terms* can be added or subtracted by using the distributive property. For example,

$$7x + 3x = (7 + 3)x = 10x$$

Objective A Adding Polynomials

Some terms are also **monomials**. A term is a monomial if the term contains only whole number exponents and no variable in the denominator.

Monomials	**Not Monomials**	
$3x^2$	$\frac{2}{y}$	Variable in denominator
$-\frac{1}{2}a^2bc^3$	$-2x^{-5}$	Not a whole number exponent
7		

A monomial or a sum and/or difference of monomials is called a **polynomial**.

Polynomial

A **polynomial** is a monomial or a sum and/or difference of monomials.

Examples of Polynomials

$$5x^3 - 6x^2 + 2x + 10, \quad -1.2y^3 + 0.7y, \quad z, \quad \frac{1}{3}r - \frac{1}{2}, \quad 0$$

Some polynomials are given special names depending on their number of terms.

Types of Polynomials

A **monomial** is a polynomial with exactly one term.
A **binomial** is a polynomial with exactly two terms.
A **trinomial** is a polynomial with exactly three terms.

Below are examples of monomials, binomials, and trinomials. Each of these examples is also a polynomial.

Polynomials

Monomials	Binomials	Trinomials	More than Three Terms
z	$x + 2$	$x^2 - 2x + 1$	$5x^3 - 6x^2 + 2x - 10$
4	$\frac{1}{3}r - \frac{1}{2}$	$y^5 + 3y^2 - 1.7$	$t^7 - t^5 + t^3 - t + 1$
$0.2x^2$	$-1.2y^3 + 0.7y$	$-a^3 + 2a^2 - 5a$	$z^8 - z^4 + 3z^2 - 2z$

↑ 1 term

↑ 2 terms

↑ 3 terms

To add polynomials, we use the commutative and associative properties to rearrange and group like terms. Then, we combine like terms.

Adding Polynomials

To add polynomials, combine like terms.

Example 1 Add: $(3x - 1) + (-6x + 2)$

Solution:

$$\begin{aligned}(3x - 1) + (-6x + 2) &= (3x - 6x) + (-1 + 2) && \text{Group like terms.}\\ &= (-3x) + (1) && \text{Combine like terms.}\\ &= -3x + 1\end{aligned}$$

Work Practice 1

Example 2 Add: $(9y^2 - 6y) + (7y^2 + 10y + 2)$

Solution:

$$\begin{aligned}(9y^2 - 6y) + (7y^2 + 10y + 2) &= 9y^2 + 7y^2 - 6y + 10y + 2 && \text{Group like terms.}\\ &= 16y^2 + 4y + 2\end{aligned}$$

Work Practice 2

Example 3 Find the sum of $(-y^2 + 2y + 1.7)$ and $(12y^2 - 6y - 3.6)$.

Solution: Recall that "sum" means addition.

$$\begin{aligned}&(-y^2 + 2y + 1.7) + (12y^2 - 6y - 3.6)\\ &= \underbrace{-y^2 + 12y^2} + \underbrace{2y - 6y} + \underbrace{1.7 - 3.6} && \text{Group like terms.}\\ &= 11y^2 - 4y - 1.9 && \text{Combine like terms.}\end{aligned}$$

Work Practice 3

Polynomials can also be added vertically. To do this, line up like terms underneath one another. Let's vertically add the polynomials in Example 3.

Practice 1

Add: $(2y + 7) + (9y - 14)$

Practice 2

Add:
$(5x^2 + 4x - 3) + (x^2 - 6x)$

Practice 3

Find the sum of $(7z^2 - 4.2z + 11)$ and $(-9z^2 - 1.9z + 4)$.

Answers

1. $11y - 7$
2. $6x^2 - 2x - 3$
3. $-2z^2 - 6.1z + 15$

Practice 4

Add the polynomials in Practice 3 vertically.

Example 4 Find the sum of $(-y^2 + 2y + 1.7)$ and $(12y^2 - 6y - 3.6)$. Use a vertical format.

Solution: Line up like terms underneath one another.

$$\begin{array}{r} -y^2 + 2y + 1.7 \\ +12y^2 - 6y - 3.6 \\ \hline 11y^2 - 4y - 1.9 \end{array}$$

Work Practice 4

Notice that we are finding the same sum in Example 4 as we found in Example 3. Of course, the results are the same.

Objective B Subtracting Polynomials

To subtract one polynomial from another, recall how we subtract numbers. To subtract a number, we add its opposite: $a - b = a + (-b)$.

For example,

$$\begin{aligned} 7 - 10 &= 7 + (-10) \\ &= -3 \end{aligned}$$

To subtract a polynomial, we also add its opposite. Just as the opposite of 3 is -3, the opposite of $(2x^2 - 5x + 1)$ is $-(2x^2 - 5x + 1)$. Let's practice simplifying the opposite of a polynomial.

Practice 5

Simplify: $-(7y^2 + 4y - 6)$

Example 5 Simplify: $-(2x^2 - 5x + 1)$

Solution: Rewrite $-(2x^2 - 5x + 1)$ as $-1(2x^2 - 5x + 1)$ and use the distributive property.

$$\begin{aligned} -(2x^2 - 5x + 1) &= -1(2x^2 - 5x + 1) \\ &= -1(2x^2) + (-1)(-5x) + (-1)(1) \\ &= -2x^2 + 5x - 1 \end{aligned}$$

Work Practice 5

Notice the result of Example 5.

$$-(2x^2 - 5x + 1) = -2x^2 + 5x - 1$$

This means that **the opposite of a polynomial can be found by changing the signs of the terms of the polynomial.** This leads to the following.

Subtracting Polynomials

To subtract polynomials, change the signs of the terms of the polynomial being subtracted, and then add.

Practice 6

Subtract:
$(3b - 2) - (7b + 23)$

Example 6 Subtract: $(5a + 7) - (2a - 10)$

Solution:

$$\begin{aligned} (5a + 7) - (2a - 10) &= (5a + 7) + (-2a + 10) && \text{Add the opposite of } 2a - 10. \\ &= 5a - 2a + 7 + 10 && \text{Group like terms.} \\ &= 3a + 17 \end{aligned}$$

Work Practice 6

Answers
4. $-2z^2 - 6.1z + 15$
5. $-7y^2 - 4y + 6$
6. $-4b - 25$

Example 7 Subtract: $(8x^2 - 4x + 1) - (10x^2 + 4)$

Solution:

$$(8x^2 - 4x + 1) - (10x^2 + 4) = (8x^2 - 4x + 1) + (-10x^2 - 4)$$ Add the opposite of $10x^2 + 4$.

$$= 8x^2 - 10x^2 - 4x + 1 - 4$$ Group like terms.

$$= -2x^2 - 4x - 3$$

Work Practice 7

Example 8 Subtract $(-6z^2 - 2z + 13)$ from $(4z^2 - 20z)$.

Solution: Be careful when arranging the polynomials in this example.

$$(4z^2 - 20z) - (-6z^2 - 2z + 13) = (4z^2 - 20z) + (6z^2 + 2z - 13)$$

$$= 4z^2 + 6z^2 - 20z + 2z - 13$$ Group like terms.

$$= 10z^2 - 18z - 13$$

Work Practice 8

✓Concept Check Find the error in the following subtraction.

$(3x^2 + 4) - (x^2 - 3x)$

~~$= (3x^2 + 4) + (-x^2 - 3x)$~~

~~$= 3x^2 - x^2 - 3x + 4$~~

~~$= 2x^2 - 3x + 4$~~

Just as with adding polynomials, we can subtract polynomials using a vertical format. Let's subtract the polynomials in Example 8 using a vertical format.

Example 9 Subtract $(-6z^2 - 2z + 13)$ from $(4z^2 - 20z)$. Use a vertical format.

Solution: Line up like terms underneath one another.

$$\begin{array}{r} 4z^2 - 20z \\ \underline{-(-6z^2 - 2z + 13)} \end{array} \qquad \begin{array}{r} 4z^2 - 20z \phantom{{}-13} \\ \underline{+6z^2 + 2z - 13} \\ 10z^2 - 18z - 13 \end{array}$$

Work Practice 9

Objective C Evaluating Polynomials

Polynomials have different values depending on the replacement values for the variables.

Example 10 Find the value of the polynomial $3t^3 - 2t + 5$ when $t = 1$.

Solution: Replace t with 1 and simplify.

$$3t^3 - 2t + 5 = 3(1)^3 - 2(1) + 5$$ Let $t = 1$.

$$= 3(1) - 2 + 5$$ $(1)^3 = 1$.

$$= 3 - 2 + 5$$

$$= 6$$

The value of $3t^3 - 2t + 5$ when $t = 1$ is 6.

Work Practice 10

Practice 7

Subtract:
$(11x^2 + 7x + 2) - (15x^2 + 4x)$

Practice 8

Subtract $(3x^2 - 12x)$ from $(-4x^2 + 20x + 17)$.

Practice 9

Subtract $(3x^2 - 12x)$ from $(-4x^2 + 20x + 17)$. Use a vertical format.

Practice 10

Find the value of the polynomial $2y^3 + y^2 - 6$ when $y = 3$.

Answers

7. $-4x^2 + 3x + 2$ **8.** $-7x^2 + 32x + 17$

9. $-7x^2 + 32x + 17$ **10.** 57

✓Concept Check Answer

$(3x^2 + 4) - (x^2 - 3x)$

$= (3x^2 + 4) + (-x^2 + 3x)$

$= 3x^2 - x^2 + 3x + 4$

$= 2x^2 + 3x + 4$

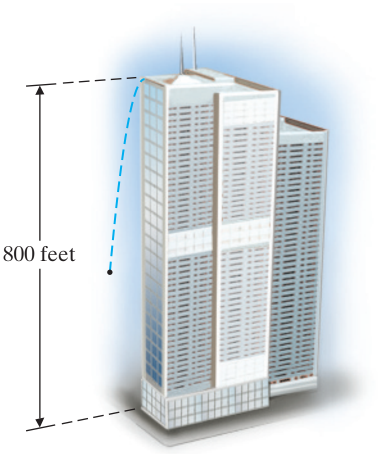
Many real-world applications are modeled by polynomials.

Practice 11

An object is dropped from the top of a 530-foot cliff. Its height in feet at time t seconds is given by the polynomial $-16t^2 + 530$. Find the height of the object when $t = 1$ second and when $t = 4$ seconds.

Example 11 Finding the Height of an Object

An object is dropped from the top of an 800-foot-tall building. Its height at time t seconds is given by the polynomial $-16t^2 + 800$. Find the height of the object when $t = 1$ second and when $t = 3$ seconds.



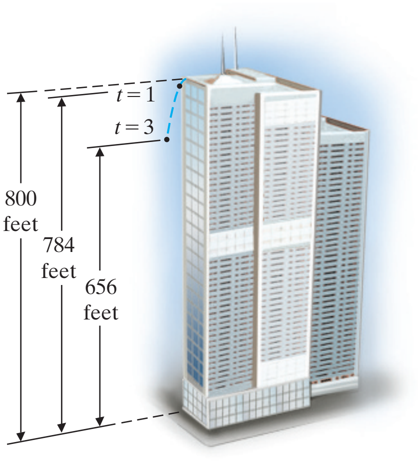
Solution: To find each height, we evaluate the polynomial when $t = 1$ and when $t = 3$.

$$\begin{aligned} -16t^2 + 800 &= -16(1)^2 + 800 \\ &= -16 + 800 \\ &= 784 \end{aligned}$$

Helpful Hint Don't forget to insert units, if appropriate.

The height of the object at 1 second is 784 feet.

$$\begin{aligned} -16t^2 + 800 &= -16(3)^2 + 800 \\ &= -16(9) + 800 \\ &= -144 + 800 \\ &= 656 \end{aligned}$$



The height of the object at 3 seconds is 656 feet.

Work Practice 11

Answer

11. 514 feet; 274 feet

B.1 Exercise Set MyLab Math

Objective A *Add the polynomials. See Examples 1 through 4.*

1. $(2x + 3) + (-7x - 27)$

2. $(9y - 16) + (-43y + 16)$

3. $(-4z^2 - 6z + 1) + (-5z^2 + 4z + 5)$

4. $(17a^2 - 6a + 3) + (16a^2 - 6a - 10)$

5. $(12y - 20) + (9y^2 + 13y - 20)$

6. $(5x^2 - 6) + (-3x^2 + 17x - 2)$

7. $(4.3a^4 + 5) + (-8.6a^4 - 2a^2 + 4)$

8. $(-12.7z^3 - 14z) + (-8.9z^3 + 12z + 2)$

Objective B *Subtract the polynomials. See Examples 5 through 9.*

9. $(5a - 6) - (a + 2)$

10. $(12b + 7) - (-b - 5)$

11. $(3x^2 - 2x + 1) - (5x^2 - 6x)$

12. $(-9z^2 + 6z + 2) - (3z^2 + 1)$

13. $(10y^2 - 7) - (20y^3 - 2y^2 - 3)$

14. $(11x^3 + 15x - 9) - (-x^3 + 10x^2 - 9)$

15. Subtract $(3x - 4)$ from $(2x + 12)$.

16. Subtract $(6a + 1)$ from $(-7a + 7)$.

17. Subtract $(5y^2 + 4y - 6)$ from $(13y^2 - 6y - 14)$.

18. Subtract $(16x^2 - x + 1)$ from $(12x^2 - 3x - 12)$.

Objectives A B Mixed Practice *Perform each indicated operation. See Examples 1 through 9.*

19. $(25x - 5) + (-20x - 7)$

20. $(14x + 2) + (-7x - 1)$

21. $(4y + 4) - (3y + 8)$

22. $(6z - 3) - (8z + 5)$

23. $(9x^2 - 6) + (-5x^2 + x - 10)$

24. $(12a^2 - 4a - 4) + (-5a - 5)$

25. $(10x + 4.5) + (-x - 8.6)$

26. $(20x - 0.8) + (x + 1.2)$

27. $(12a - 5) - (-3a + 2)$

28. $(8t + 9) - (-2t + 6)$

29. $(21y - 4.6) - (36y - 8.2)$

30. $(8.6x + 4) - (9.7x - 93)$

31. $(18t^2 - 4t + 2) - (-t^2 + 7t - 1)$

32. $(35x^2 + x - 5) - (17x^2 - x + 5)$

33. $(b^3 - 2b^2 + 10b + 11) + (b^2 - 3b - 12)$

34. $(-2z^3 + 5z^2 - 13z + 6) + (3z^2 - 7z - 7)$

35. Add $(6x^2 - 7)$ and $(-11x^2 - 11x + 20)$.

36. Add $(-2x^2 + 3x)$ and $(9x^2 - x + 14)$.

37. Subtract $\left(3z - \frac{3}{7}\right)$ from $\left(-3z + \frac{6}{7}\right)$.

38. Subtract $\left(8y^2 - \frac{7}{10}y\right)$ from $\left(-5y^2 + \frac{3}{10}y\right)$.

Objective C *Find the value of each polynomial when $x = 2$. See Examples 10 and 11.*

39. $-3x + 7$

40. $-5x - 7$

41. $x^2 - 6x + 3$

42. $5x^2 + 4x - 100$

43. $\frac{3x^2}{2} - 14$

44. $\frac{7x^3}{14} - x + 5$

Find the value of each polynomial when $x = 5$. See Examples 10 and 11.

45. $2x + 10$

46. $-5x - 6$

47. x^2

48. x^3

49. $2x^2 + 4x - 20$

50. $4x^2 - 5x + 10$

Solve. See Example 11.

The distance in feet traveled by a free-falling object in t seconds is given by the polynomial

$16t^2$

Use this polynomial for Exercises 51 and 52.

51. Find the distance traveled by an object that falls for 6 seconds.

52. It takes 8 seconds for a hard hat to fall from the top of a building. How tall is the building?

Office Supplies, Inc. manufactures office products. They determine that the total cost for manufacturing x file cabinets is given by the polynomial

$3000 + 20x$

Use this polynomial for Exercises 53 and 54.

53. Find the total cost to manufacture 10 file cabinets.

54. Find the total cost to manufacture 100 file cabinets.

An object is dropped from the deck of the Royal Gorge Bridge, which stretches across Royal Gorge at a height of 1053 feet above the Arkansas River. The height of the object above the river after t seconds is given by the polynomial

$$1053 - 16t^2$$

Use this polynomial for Exercises 55 and 56. (Source: Royal Gorge Bridge Co.)

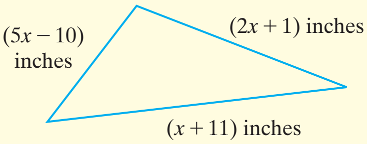
55. How far above the river is an object that has been falling for 3 seconds?

56. How far above the river is an object that has been falling for 7 seconds?

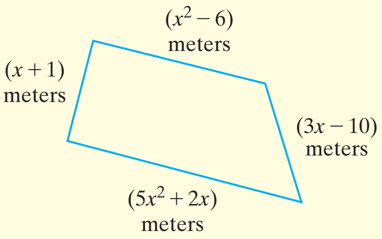
Concept Extensions

Find the perimeter of each figure.

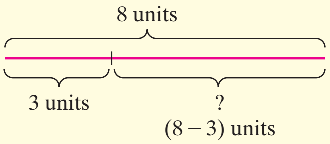
△ **57.**



△ **58.**



Given the lengths in the figure below, we find the unknown length by subtracting. Use the information to find the unknown lengths in Exercises 59 and 60.



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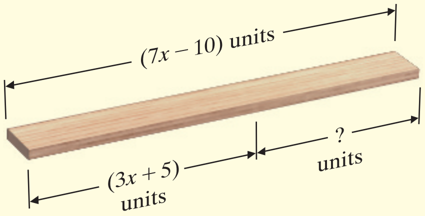
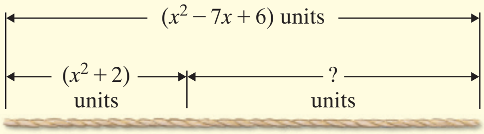


60.



Fill in the blanks.

61. $(3x^2 + \_\_\_\_ x - \_\_\_\_) + (\_\_\_\_ x^2 - 6x + 2) = 5x^2 + 14x - 4$

62. $(\_\_\_\_ y^2 + 4y - 3) + (8y^2 - \_\_\_\_ y + \_\_\_\_) = 9y^2 + 2y + 7$

63. Find the value of $7a^4 - 6a^2 + 2a - 1$ when $a = 1.2$.

64. Find the value of $3b^3 + 4b^2 - 100$ when $b = -2.5$.

65. For Exercises 55 and 56, the polynomial $1053 - 16t^2$ was used to give the height of an object above the river after t seconds. Find the height when $t = 8$ seconds and $t = 9$ seconds. Explain what happened and why.

B.2 Multiplication Properties of Exponents

Objectives

A Use the Product Rule for Exponents.

B Use the Power Rule for Exponents.

C Use the Power of a Product Rule for Exponents.

Objective A Using the Product Rule

Recall from Section 8.1 that an exponent has the same meaning whether the base is a number or a variable. For example,

$$5^3 = \underbrace{5 \cdot 5 \cdot 5}_{\text{3 factors of 5}} \quad \text{and} \quad x^3 = \underbrace{x \cdot x \cdot x}_{\text{3 factors of } x}$$

We can use this definition of an exponent to discover properties that will help us to simplify products and powers of exponential expressions.

For example, let's use the definition of an exponent to find the product of x^3 and x^4.

$$\begin{aligned} x^3 \cdot x^4 &= (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) \\ &= \underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}_{\text{7 factors of } x} \\ &= x^7 \end{aligned}$$

Notice that the result is the same if we add the exponents.

$$x^3 \cdot x^4 = x^{3+4} = x^7$$

This suggests the following product rule or property for exponents.

Product Property for Exponents

If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}$$

In other words, to multiply two exponential expressions with the same base, keep the base and add the exponents.

Practice 1

Multiply: $z^4 \cdot z^8$

Example 1 Multiply: $y^7 \cdot y^2$

Solution:

$$\begin{aligned} y^7 \cdot y^2 &= y^{7+2} && \text{Use the product property for exponents.} \\ &= y^9 && \text{Simplify.} \end{aligned}$$

Work Practice 1

Practice 2

Multiply: $7y^5 \cdot 4y^9$

Example 2 Multiply: $3x^5 \cdot 6x^3$

Solution:

$$\begin{aligned} 3x^5 \cdot 6x^3 &= (3 \cdot 6)(x^5 \cdot x^3) && \text{Apply the commutative and associative properties.} \\ &= 18x^{5+3} && \text{Use the product property for exponents.} \\ &= 18x^8 && \text{Simplify.} \end{aligned}$$

Work Practice 2

Answers

1. z^{12} **2.** $28y^{14}$

Example 3 Multiply: $(-2a^4b^{10})(9a^5b^3)$

Solution: Use properties of multiplication to group numbers and like variables together.

$$\begin{aligned}(-2a^4b^{10})(9a^5b^3) &= (-2\cdot 9)(a^4\cdot a^5)(b^{10}\cdot b^3)\\ &= -18a^{4+5}b^{10+3}\\ &= -18a^9b^{13}\end{aligned}$$

Work Practice 3

Example 4 Multiply: $2x^3\cdot 3x\cdot 5x^6$

Solution: First notice the factor $3x$. Since there is one factor of x in $3x$, it can also be written as $3x^1$.

$$\begin{aligned}2x^3\cdot 3x^1\cdot 5x^6 &= (2\cdot 3\cdot 5)(x^3\cdot x^1\cdot x^6)\\ &= 30x^{10}\end{aligned}$$

Work Practice 4

Helpful Hint

These examples will remind you of the difference between adding and multiplying terms.

Addition

$5x^3 + 3x^3 = (5 + 3)x^3 = 8x^3$

$7x + 4x^2 = 7x + 4x^2$

Multiplication

$(5x^3)(3x^3) = 5\cdot 3\cdot x^3\cdot x^3 = 15x^{3+3} = 15x^6$

$(7x)(4x^2) = 7\cdot 4\cdot x\cdot x^2 = 28x^{1+2} = 28x^3$

Objective B Using the Power Rule

Next suppose that we want to simplify an exponential expression raised to a power. To see how we simplify $(x^2)^3$, we again use the definition of an exponent.

$$(x^2)^3 = \underbrace{(x^2)\cdot(x^2)\cdot(x^2)}_{\text{3 factors of } x^2} \qquad \text{Apply the definition of an exponent.}$$

$$= x^{2+2+2} \qquad \text{Use the product property for exponents.}$$

$$= x^6 \qquad \text{Simplify.}$$

Notice the result is exactly the same if we multiply the exponents.

$$(x^2)^3 = x^{2\cdot 3} = x^6$$

This suggests the following power rule or property for exponents.

Power Property for Exponents

If m and n are positive integers and a is a real number, then

$$(a^m)^n = a^{m\cdot n}$$

Practice 3

Multiply: $(-7r^6s^2)(-3r^2s^5)$

Practice 4

Multiply: $9y^4\cdot 3y^2\cdot y$. (Recall that $y = y^1$.)

Helpful Hint Don't forget that if an exponent is not written, it is assumed to be 1.

Answers

3. $21r^8s^7$ **4.** $27y^7$

In other words, to raise a power to a power, keep the base and multiply the exponents.

Helpful Hint

Take a moment to make sure that you understand when to apply the product rule and when to apply the power rule.

Product Property → Add Exponents	**Power Property → Multiply Exponents**
$x^5 \cdot x^7 = x^{5+7} = x^{12}$	$(x^5)^7 = x^{5 \cdot 7} = x^{35}$
$y^6 \cdot y^2 = y^{6+2} = y^8$	$(y^6)^2 = y^{6 \cdot 2} = y^{12}$

Practice 5

Simplify: $(z^3)^{10}$

Example 5 Simplify: $(y^8)^2$

Solution:

$$(y^8)^2 = y^{8 \cdot 2} \quad \text{Use the power property.}$$
$$= y^{16}$$

Work Practice 5

Practice 6

Simplify: $(z^4)^5 \cdot (z^3)^7$

Example 6 Simplify: $(a^3)^4 \cdot (a^2)^9$

Solution:

$$(a^3)^4 \cdot (a^2)^9 = a^{12} \cdot a^{18} \quad \text{Use the power property.}$$
$$= a^{12+18} \quad \text{Use the product property.}$$
$$= a^{30} \quad \text{Simplify.}$$

Work Practice 6

Objective C Using the Power of a Product Rule

Next, let's simplify the power of a product.

$$(xy)^3 = xy \cdot xy \cdot xy \quad \text{Apply the definition of an exponent.}$$
$$= (x \cdot x \cdot x)(y \cdot y \cdot y) \quad \text{Group like bases.}$$
$$= x^3y^3 \quad \text{Simplify.}$$

Notice that the power of a product can be written as the product of powers. This leads to the following power of a product rule or property.

Power of a Product Property for Exponents

If n is a positive integer and a and b are real numbers, then

$$(ab)^n = a^nb^n$$

In other words, to raise a product to a power, raise each factor to the power.

Answers

5. z^{30} **6.** z^{41}

✓Concept Check Which property is needed to simplify $(x^6)^3$? Explain.

a. Product property for exponents

b. Power property for exponents

c. Power of a product property for exponents

Example 7 Simplify: $(5t)^3$

Solution:

$$(5t)^3 = 5^3 t^3 \quad \text{Apply the power of a product property.}$$
$$= 125t^3 \quad \text{Write } 5^3 \text{ as } 125.$$

Work Practice 7

Practice 7

Simplify: $(3b)^4$

Example 8 Simplify: $(2a^5b^3)^3$

Solution:

$$(2a^5b^3)^3 = 2^3(a^5)^3(b^3)^3 \quad \text{Apply the power of a product property.}$$
$$= 8a^{15}b^9 \quad \text{Apply the power property.}$$

Work Practice 8

Practice 8

Simplify: $(4x^2y^6)^3$

Example 9 Simplify: $(3y^4z^2)^4(2y^3z^5)^5$

Solution:

$$(3y^4z^2)^4(2y^3z^5)^5 = 3^4(y^4)^4(z^2)^4 \cdot 2^5(y^3)^5(z^5)^5 \quad \text{Apply the power of a product property.}$$
$$= 81y^{16}z^8 \cdot 32y^{15}z^{25} \quad \text{Apply the power property.}$$
$$= (81 \cdot 32)(y^{16} \cdot y^{15})(z^8 \cdot z^{25}) \quad \text{Group like bases.}$$
$$= 2592y^{31}z^{33} \quad \text{Apply the product property.}$$

Work Practice 9

Practice 9

Simplify: $(2x^2y^4)^4(3x^6y^9)^2$

Answers

7. $81b^4$ **8.** $64x^6y^{18}$ **9.** $144x^{20}y^{34}$

✓Concept Check Answer

b

B.2 Exercise Set MyLab Math

Objective A *Multiply. See Examples 1 through 4.*

1. $x^5 \cdot x^9$

2. $y^4 \cdot y^7$

3. $a^6 \cdot a$

4. $b \cdot b^8$

5. $3z^3 \cdot 5z^2$

6. $8r^2 \cdot 2r^{15}$

7. $-4x \cdot 10x$

8. $-9y \cdot 3y$

9. $(-5x^2y^3)(-5x^4y)$

10. $(-2xy^4)(-6x^3y^7)$

11. $(7ab)(4a^4b^5)$

12. $(3a^3b^6)(12a^2b^9)$

13. $2x \cdot 3x \cdot 7x$

14. $4y \cdot 3y \cdot 5y$

15. $a \cdot 4a^{11} \cdot 3a^5$

16. $b \cdot 7b^{10} \cdot 5b^8$

Objectives A B C Mixed Practice *Simplify. See Examples 1 through 9.*

17. $(x^5)^3$

18. $(y^4)^7$

19. $(z^2)^{10}$

20. $(a^6)^9$

21. $(b^7)^6(b^2)^{10}$

22. $(x^2)^9 \cdot (x^5)^3$

23. $(3a)^4$

24. $(2y)^5$

25. $(a^{11}b^8)^3$

26. $(x^7y^4)^8$

27. $(11x^3y^6)^2$

28. $(9a^4b^3)^2$

29. $(-3y)(2y^7)^3$

30. $(-2x)(5x^2)^4$

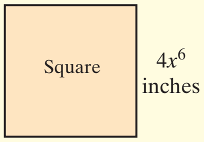
31. $(4xy)^3(2x^3y^5)^2$

32. $(2xy)^4(3x^4y^3)^3$

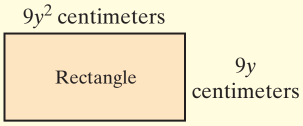
Concept Extensions

Find the area of each figure.

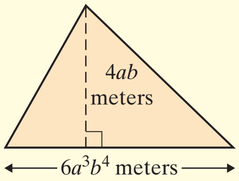
△ **33.**



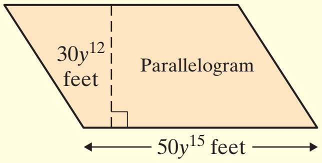
△ **34.**



△ **35.**



△ **36.**



(*Hint:* Area = base · height)

Multiply and simplify.

37. $(14a^7b^6)^3(9a^6b^3)^4$

38. $(5x^{14}y^6)^7(3x^{20}y^{19})^5$

39. $(8.1x^{10})^5$

40. $(4.6a^{14})^4$

41. $(a^{20}b^{10}c^5)^5(a^9b^{12})^3$

42. $(x^{90}y^{72})^3$

43. In your own words, explain why $x^2 \cdot x^3 = x^5$ and $(x^2)^3 = x^6$.

B.3 Multiplying Polynomials

Objective A Multiplying a Monomial and a Polynomial

Recall that a polynomial that consists of one term is called a **monomial**. For example, $5x$ is a monomial. To multiply a monomial and any polynomial, we use the distributive property

$$a(b + c) = a \cdot b + a \cdot c$$

and apply properties of exponents.

Example 1 Multiply: $5x(3x^2 + 2)$

Solution:

$$5x(3x^2 + 2) = 5x \cdot 3x^2 + 5x \cdot 2 \quad \text{Apply the distributive property.}$$
$$= 15x^3 + 10x$$

Work Practice 1

Example 2 Multiply: $2z(4z^2 + 6z - 9)$

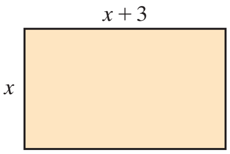
Solution:

$$2z(4z^2 + 6z - 9) = 2z \cdot 4z^2 + 2z \cdot 6z + 2z(-9)$$
$$= 8z^3 + 12z^2 - 18z$$

Work Practice 2

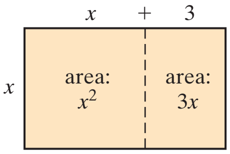
To visualize multiplication by a monomial, let's look at two ways we can represent the area of the same rectangle.

The width of the rectangle is x and its length is $x + 3$. One way to calculate the area of the rectangle is



$$\text{area} = \text{width} \cdot \text{length}$$
$$= x(x + 3)$$

Another way to calculate the area of the rectangle is to find the sum of the areas of the smaller figures.



$$\text{area} = x^2 + 3x$$

Since the areas must be equal, we have that

$$x(x + 3) = x^2 + 3x \quad \text{As expected by the distributive property}$$

Objectives

A Multiply a Monomial and Any Polynomial.

B Multiply Two Binomials.

C Square a Binomial.

D Use the FOIL Order to Multiply Binomials.

E Multiply Any Two Polynomials.

Practice 1

Multiply: $3y(7y^2 + 5)$

Practice 2

Multiply: $5r(8r^2 - r + 11)$

Answers

1. $21y^3 + 15y$ **2.** $40r^3 - 5r^2 + 55r$

Objective B Multiplying Binomials

Recall from Appendix B.1 that a polynomial that consists of exactly two terms is called a **binomial.** To multiply two binomials, we use a version of the distributive property:

$$(b + c)a = b \cdot a + c \cdot a$$

Practice 3

Multiply: $(b + 7)(b + 5)$

Example 3 Multiply: $(x + 2)(x + 3)$

Solution:

$$\begin{aligned} (x + 2)(x + 3) &= x(x + 3) + 2(x + 3) && \text{Apply the distributive property.} \\ &= x \cdot x + x \cdot 3 + 2 \cdot x + 2 \cdot 3 && \text{Apply the distributive property.} \\ &= x^2 + 3x + 2x + 6 && \text{Multiply.} \\ &= x^2 + 5x + 6 && \text{Combine like terms.} \end{aligned}$$

Work Practice 3

Practice 4

Multiply: $(5x - 1)(5x + 4)$

Example 4 Multiply: $(4y + 9)(3y - 2)$

Solution:

$$\begin{aligned} (4y + 9)(3y - 2) &= 4y(3y - 2) + 9(3y - 2) && \text{Apply the distributive property.} \\ &= 4y \cdot 3y + 4y(-2) + 9 \cdot 3y + 9(-2) && \text{Apply the distributive property.} \\ &= 12y^2 - 8y + 27y - 18 && \text{Multiply.} \\ &= 12y^2 + 19y - 18 && \text{Combine like terms} \end{aligned}$$

Work Practice 4

Objective C Squaring a Binomial

Raising a binomial to the power of 2 is also called squaring a binomial. To square a binomial, we use the definition of an exponent, and then multiply.

Practice 5

Multiply: $(6y - 1)^2$

Example 5 Multiply: $(2x + 1)^2$

Solution:

$$\begin{aligned} (2x + 1)^2 &= (2x + 1)(2x + 1) && \text{Apply the definition of an exponent.} \\ &= 2x(2x + 1) + 1(2x + 1) && \text{Apply the distributive property.} \\ &= 2x \cdot 2x + 2x \cdot 1 + 1 \cdot 2x + 1 \cdot 1 && \text{Apply the distributive property.} \\ &= 4x^2 + 2x + 2x + 1 && \text{Multiply.} \\ &= 4x^2 + 4x + 1 && \text{Combine like terms.} \end{aligned}$$

Work Practice 5

Objective D Using the FOIL Order to Multiply Binomials

Recall from Example 3 that

$$\begin{aligned} (x + 2)(x + 3) &= x \cdot x + x \cdot 3 + 2 \cdot x + 2 \cdot 3 \\ &= x^2 + 5x + 6 \end{aligned}$$

Answers

3. $b^2 + 12b + 35$ **4.** $25x^2 + 15x - 4$

5. $36y^2 - 12y + 1$

One way to remember these products—$x\cdot x$, $x\cdot 3$, $2\cdot x$, and $2\cdot 3$—is to use a special order for multiplying binomials called the FOIL order. Of course, the product is the same no matter what order or method you choose to use.

FOIL stands for the products of the First terms, Outer terms, Inner terms, and then Last terms. For example,

$$(x+2)(x+3) = \overset{F}{x\cdot x} + \overset{O}{x\cdot 3} + \overset{I}{2\cdot x} + \overset{L}{2\cdot 3} = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Helpful Hint The product is the same no matter what order or method you choose to use.

Examples Use the FOIL order to multiply.

6. $(3x-6)(2x+1) = \overset{F}{3x\cdot 2x} + \overset{O}{3x\cdot 1} + \overset{I}{(-6)(2x)} + \overset{L}{(-6)(1)}$

$= 6x^2 + 3x - 12x - 6$ Multiply.

$= 6x^2 - 9x - 6$ Combine like terms.

7. $(3x-5)^2 = (3x-5)(3x-5)$

$= \overset{F}{3x\cdot 3x} + \overset{O}{3x(-5)} + \overset{I}{(-5)(3x)} + \overset{L}{(-5)(-5)}$

$= 9x^2 - 15x - 15x + 25$ Multiply.

$= 9x^2 - 30x + 25$ Combine like terms.

Work Practice 6–7

Practice 6–7

Use the FOIL order to multiply.

6. $(10x-7)(x+3)$

7. $(3x+2)^2$

Helpful Hint

Remember that the FOIL order can only be used to multiply **two binomials.**

Objective E Multiplying Polynomials

Recall from Appendix B.1 that a polynomial that consists of exactly three terms is called a **trinomial.** Next, we multiply a binomial by a trinomial.

Example 8 Multiply: $(3a+2)(a^2-6a+3)$

Solution: Use the distributive property to multiply $3a$ by the trinomial (a^2-6a+3) and then 2 by the trinomial.

$(3a+2)(a^2-6a+3) = 3a(a^2-6a+3) + 2(a^2-6a+3)$ Apply the distributive property.

$= 3a\cdot a^2 + 3a(-6a) + 3a\cdot 3 +$
$2\cdot a^2 + 2(-6a) + 2\cdot 3$ Apply the distributive property.

$= 3a^3 - 18a^2 + 9a + 2a^2 - 12a + 6$ Multiply.

$= 3a^3 - 16a^2 - 3a + 6$ Combine like terms.

Work Practice 8

Practice 8

Multiply:
$(2x+5)(x^2+4x-1)$

Answers

6. $10x^2 + 23x - 21$ **7.** $9x^2 + 12x + 4$
8. $2x^3 + 13x^2 + 18x - 5$

In general, we have the following.

To Multiply Two Polynomials

Multiply each term of the first polynomial by each term of the second polynomial, and then combine like terms.

✓Concept Check True or false? When a trinomial is multiplied by a trinomial, the result will have at most nine terms. Explain.

A convenient method of multiplying polynomials is to use a vertical format similar to multiplying real numbers.

Practice 9

Multiply $(x^2 + 4x - 1)$ and $(2x + 5)$ vertically.

Example 9 Find the product of $(a^2 - 6a + 3)$ and $(3a + 2)$ vertically.

Solution:

$$\begin{array}{rrrrl}
 & a^2 & -\ 6a & +\ 3 & \\
\times & & 3a & +\ 2 & \\
\hline
 & 2a^2 & -\ 12a & +\ 6 & \text{Multiply } a^2 - 6a + 3 \text{ by } 2. \\
3a^3 & -\ 18a^2 & +\ \ 9a & & \text{Multiply } a^2 - 6a + 3 \text{ by } 3a. \text{ Line up like terms.} \\
\hline
3a^3 & -\ 16a^2 & -\ \ 3a & +\ 6 & \\
\uparrow & \uparrow & \uparrow & \uparrow & \text{Combine like terms.}
\end{array}$$

Notice that this example is the same as Example 8 and of course the products are the same.

Work Practice 9

Answer

9. $2x^3 + 13x^2 + 18x - 5$

✓Concept Check Answer

True

B.3 Exercise Set MyLab Math

Objective A *Multiply. See Examples 1 and 2.*

1. $3x(9x^2 - 3)$

2. $4y(10y^3 + 2y)$

3. $-5a(4a^2 - 6a + 1)$

4. $-2b(3b^2 - 2b + 5)$

5. $7x^2(6x^2 - 5x + 7)$

6. $6z^2(-3z^2 - z + 4)$

Objectives B C D Mixed Practice *Multiply. See Examples 3 through 7.*

7. $(x + 3)(x + 10)$

8. $(y + 5)(y + 9)$

9. $(2x - 6)(x + 4)$

10. $(7z + 1)(z - 6)$

11. $(6a + 4)^2$

12. $(8b - 3)^2$

Objective E *Multiply. See Examples 8 and 9.*

13. $(a + 6)(a^2 - 6a + 3)$

14. $(y + 4)(y^2 + 8y - 2)$

15. $(4x - 5)(2x^2 + 3x - 10)$

16. $(9z - 2)(2z^2 + z + 1)$

17. $(x^3 + 2x + x^2)(3x + 1 + x^2)$

18. $(y^2 - 2y + 5)(y^3 + 2 + y)$

Objectives A B C D E Mixed Practice *Multiply. See Examples 1 through 9.*

19. $10r(-3r + 2)$

20. $5x(4x^2 + 5)$

21. $-2y^2(3y + y^2 - 6)$

22. $3z^3(4z^4 - 2z + z^3)$

23. $(x + 2)(x + 12)$

24. $(6s + 1)(3s - 1)$

25. $(2a + 3)(2a - 3)$

26. $(y + 7)(y - 7)$

27. $(x + 5)^2$

28. $(x + 3)^2$

29. $\left(b + \frac{3}{5}\right)\left(b + \frac{4}{5}\right)$

30. $\left(a - \frac{7}{10}\right)\left(a + \frac{3}{10}\right)$

31. $(6x + 1)(x^2 + 4x + 1)$

32. $(9y - 1)(y^2 + 3y - 5)$

33. $(7x + 5)^2$

34. $(5x + 9)^2$

35. $(2x - 1)^2$

36. $(4a - 3)^2$

37. $(2x^2 - 3)(4x^3 + 2x - 3)$

38. $(3y^2 + 2)(5y^2 - y + 2)$

39. $(x^3 + x^2 + x)(x^2 + x + 1)$

40. $(a^4 + a^2 + 1)(a^4 + a^2 - 1)$

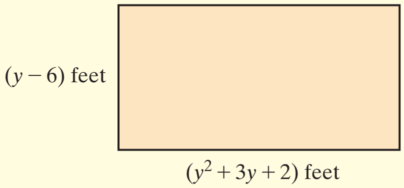
41. $(2z^2 - z + 1)(5z^2 + z - 2)$

42. $(2b^2 - 4b + 3)(b^2 - b + 2)$

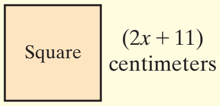
Concept Extensions

Find the area of each figure.

△ **43.**

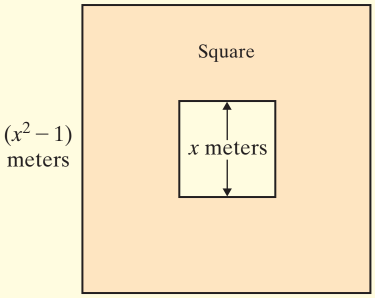


△ **44.**

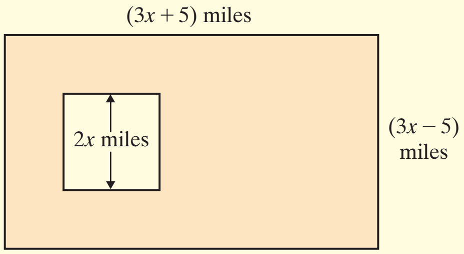


Find the area of the shaded figure. To do so, subtract the area of the smaller square from the area of the larger geometric figure.

△ **45.**



△ **46.**



47. Suppose that a classmate asked you why $(2x + 1)^2$ is not $4x^2 + 1$. Write down your response to this classmate.

Appendix C

Inductive and Deductive Reasoning

Logic and logical reasoning have applications in many fields, including science, law, psychology, and mathematics. For example, computers must have logic built into their circuits in order to process information correctly. We begin our study of logic by examining inductive reasoning.

Objectives

A Understand and Use Inductive Reasoning.

B Understand and Use Deductive Reasoning.

Objective A Using Inductive Reasoning

Inductive Reasoning

This is the process of forming a general conclusion based on observing a number of specific examples or outcomes.

Specific Observations $\xrightarrow{\text{lead to a}}$ general conclusion

Example 1 Find the next number in the sequence, or listing, of numbers.

1, 5, 25, 125

Solution: Each number after the first is obtained by multiplying the previous number by 5. If we assume that this pattern continues, the next number is $125 \times 5 = 625$.

$$\begin{array}{ccccccccc}
1, & & 5, & & 25, & & 125, & & 625 \\
\searrow & & \nearrow\ \searrow & & \nearrow\ \searrow & & \nearrow\ \searrow & & \nearrow \\
 & 1\cdot 5 & & 5\cdot 5 & & 25\cdot 5 & & 125\cdot 5 &
\end{array}$$

Work Practice 1

Practice 1

Find the next number in the sequence
2, 6, 18, 54

Example 2 Find the next two numbers in the given sequence.

1, 1, 2, 3, 5, 8

Solution: Each number after the first two numbers is obtained by adding the two previous numbers in the list. Notice that $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$, and so on. If this pattern is to continue, the next number in the sequence is $5 + 8 = 13$, and the next number is $8 + 13 = 21$.

Work Practice 2

Practice 2

Find the next two numbers in the sequence
2, 4, 6, 10, 16

The sequence described in Example 2 is called the *Fibonacci sequence*. There are many examples of this sequence found in nature. This sequence also has many applications in science, business, economics, operations research, archeology, fine arts, architecture, and poetry.

Answers

1. 162 **2.** 26, 42

Practice 3

Find the next letter in each sequence.

a. S, M, T, W, T

b. A, F, D, I, G

Example 3

Find the next letter in each sequence.

a. J, F, M, A, M
b. A, E, D, H, G

Solution:

a. Each letter is the first letter of some of the months of the year, in order: January, February, March, April, May. The next month is June, so the letter is J.

b. Let's look for a pattern by corresponding each letter of the alphabet to a number, according to order.

$$\underbrace{\overset{A}{1} + 4 = \overset{E}{5}}_{\text{Add 4.}} \quad \underbrace{5 - 1 = \overset{D}{4}}_{\text{Subtract 1.}} \quad \underbrace{4 + 4 = \overset{H}{8}}_{\text{Add 4.}} \quad \underbrace{8 - 1 = \overset{G}{7}}_{\text{Subtract 1.}}$$

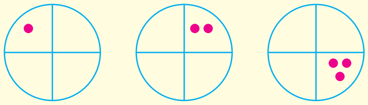
From this pattern, the next letter is

$$\overset{G}{7} + 4 = \overset{K}{11}$$

Work Practice 3

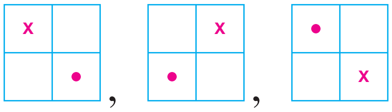
Practice 4

Use inductive reasoning to find the next shape in the sequence.

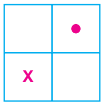


Example 4

Use inductive reasoning to find the next shape in the sequence.



Solution: Notice that the "x" rotates clockwise in each square. Also, there is a dot always diagonally across from each x. Thus, we might reason that the next shape is the following.



Work Practice 4

Next, let's study deductive reasoning. We begin with a definition.

Objective B Using Deductive Reasoning

Deductive Reasoning

This is the process of forming a specific conclusion based on accepted assumptions.

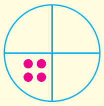
$$\text{General Assumptions} \xrightarrow{\text{lead to a}} \text{specific conclusion}$$

In short, with inductive reasoning, we reason from observed specific examples to a general conclusion; with deductive reasoning, we reason logically from general statements or assumptions to a specific conclusion.

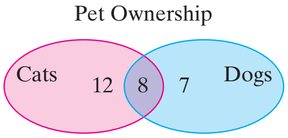
Answers

3. a. F (each letter is the first letter of the days of the week) **b.** L

4.



Diagrams called *Venn diagrams* can help us reason deductively. Let's discuss the diagram below, which has to do with pet ownership.



Since 8 people are in both regions, these 8 people have both cats and dogs as pets. See if you understand each answer below.

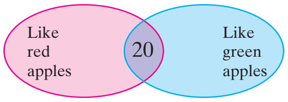
How many people have cats? 12 + 8 = 20
How many people have cats and no dogs? 12
How many people have dogs? 8 + 7 = 15
How many people have dogs and no cats? 7

Example 5 The results of a survey of 50 people are as follows.

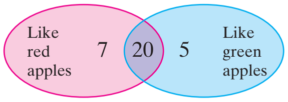
27 people like red apples.
25 people like green apples.
20 people like both red and green apples.

How many people like neither red nor green apples?

Solution: We draw a Venn diagram to organize the information in the survey. This survey concerns red apples and green apples, which indicates that our diagram will consist of two circles, one representing people who like red apples and one representing people who like green apples. Since there are 20 people who like both red and green apples, we draw two overlapping circles and place 20 in the overlapping section.



A total of 27 people like red apples. This means that $27 - 20 = 7$ is the number of people who like red apples only. Since a total of 25 people like green apples, $25 - 20 = 5$ people like green apples only.



This means that $7 + 20 + 5 = 32$ people like red or green apples. Since 50 people were polled, $50 - 32 = 18$ people like neither red nor green apples.

Work Practice 5

Practice 5

The results of a survey of 30 people are as follows.

20 people like potato chips.
17 people like tortilla chips.
13 people like both potato and tortilla chips.

How many people like neither potato nor tortilla chips?

Answer
5. 6 people

Sometimes a grid may be useful in organizing information.

Practice 6

Persons A, B, C, and D sit in the front row of their mathematics class. Use the statements below to determine their order.

1. Person D is sitting between persons A and B.
2. Person C is sitting next to person B only.
3. Person C is not on the far left.

Example 6 Jim, Max, Michael, and Dong Ming have careers as an accountant, a mathematician, a computer programmer, and a manager, not necessarily in the given order. Use the statements below to determine which person is the mathematician.

1. Jim and Max went to lunch with the mathematician.
2. The accountant and the computer programmer taught Jim in college.
3. Max and Dong Ming went to Florida with the computer programmer.

Solution: Draw a grid to organize the information. Then record the information given in each statement. From statement 1, we know that both Jim and Max are not the mathematician. To indicate this on the grid, place X's in the appropriate places. (See below on the left.)

	math.	comp. prog.	acct.	manager
Jim	X			
Max	X			
Michael				
Dong Ming				

	math.	comp. prog.	acct.	manager
Jim	X	X	X	
Max	X			
Michael				
Dong Ming				

From statement 2, we know Jim is not the accountant and he is not the computer programmer, so place an X in the grid corresponding to Jim/accountant and an X in the grid corresponding to Jim/computer programmer. (See above on the right.)

At this point, notice that Jim must be the manager. Place a check mark in the grid corresponding to Jim/manager. Since Jim is the manager, no one else is and we can place X's in the rest of the manager column to indicate that no one else is the manager. (See below on the left.)

	math.	comp. prog.	acct.	manager
Jim	X	X	X	✓
Max	X			X
Michael				X
Dong Ming				X

	math.	comp. prog.	acct.	manager
Jim	X	X	X	✓
Max	X	X		X
Michael				X
Dong Ming		X		X

From statement 3, we know Max and Dong Ming are not the computer programmer. Record this information in the grid. (See above on the right.) Notice that Max must be the accountant. Place a check mark in the grid under Max/accountant and mark the rest of that column with X's. (See below on the left.) **Now Dong Ming must be the mathematician** and Michael must be the computer programmer. (See below on the right.)

	math.	comp. prog.	acct.	manager
Jim	X	X	X	✓
Max	X	X	✓	X
Michael			X	X
Dong Ming		X	X	X

	math.	comp. prog.	acct.	manager
Jim	X	X	X	✓
Max	X	X	✓	X
Michael	X	✓	X	X
Dong Ming	✓	X	X	X

Work Practice 6

Answer

6. A, D, B, C

C Exercise Set MyLab Math

Objectives A B *Determine whether each is an example of inductive or deductive reasoning.*

1. My coat is red. My neighbor's coat is red. Therefore, all coats are red.

2. All typewriters type the letter *b*. I have a typewriter. Therefore, my typewriter will type the letter *b*.

3. Rabbits do not lay eggs. Therefore, my pet rabbit will not lay an egg.

4. The last two times Ken flew on a commercial jet, his luggage was lost. Ken reasons that the next time he flies on a commercial jet, his luggage will again be lost.

5. A scientist holds a piece of salt over a burning candle and notices that it burns with a yellow flame. She does this again with another piece of salt and notices that it also burns with a yellow flame. She therefore reasons that all salt burns with a yellow flame.

6. Sherlock Holmes knows that the murderer was the butler, the maid, or the cook. The night of the murder, the cook catered a party in a nearby town and has plenty of witnesses who saw him. The butler was in the hospital with pneumonia. Therefore, Detective Holmes reasons that the maid did it.

Use inductive reasoning to determine the next number, letter, or figure in each sequence. See Examples 1 through 4 .

7. 1, 3, 9, 27

8. 2, 4, 6, 8

9. A, Z, B, Y

10. A, C, E, G

11. 1, 4, 9, 16, 25

12. 2, 4, 8, 16

13. O, T, T, F, F, S, S

14. S, S, M, T, W, T

15. 9, 99, 999, 9999

16. 1, 10, 100, 1000

17. 2, 7, 4, 8, 6

18. 3, 1, 4, 2, 5

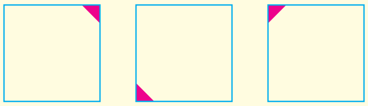
19. 9, 12, 20, 33, 51

20. 4, 13, 29, 52, 82

21. 1, 11, 38, 84, 151

22. 2, 12, 24, 41, 66

23.



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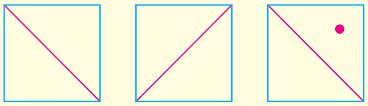

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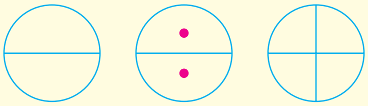

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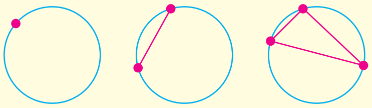

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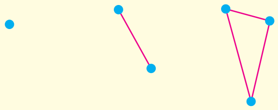

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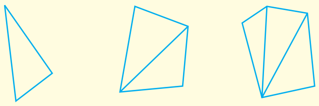

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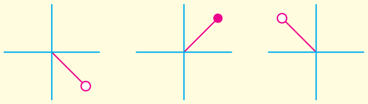

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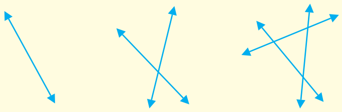

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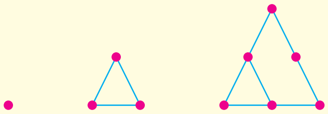

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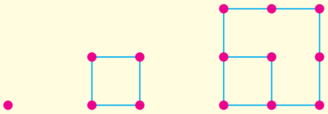

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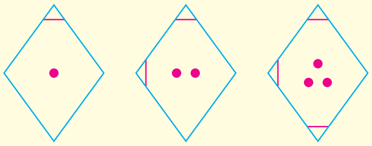

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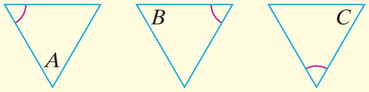


35. Give an example occurring outside the classroom where you used inductive reasoning.

36. Give an example occurring outside the classroom where you used deductive reasoning.

Draw a Venn diagram illustrating each of the following statements. Draw a region representing birds and let a point represent Wendy. See Example 5.

37. Wendy is a bird.

38. Wendy is not a bird.

Draw a Venn diagram illustrating each of the following statements. See Example 5.

39. No bicycles are tricycles.

40. All whales are mammals.

41. Some roosters crow.

42. No trees have wheels.

43. All politicians make promises.

44. Some dryers are heated by electricity.

Use diagrams and deductive reasoning to solve each problem. See Example 6.

45. Four men in a wheelchair race finished 1st, 2nd, 3rd, and 4th.

Mark beat Bob.
Joey finished between Mark and Bob.
Sal beat Mark.

Who finished 3rd in the race?

46. Four women in a marathon finished 1st, 2nd, 3rd, and 4th.

Wynonna beat Mary.
Sally beat Wynonna.
June did not beat Mary.

Who finished 1st in the race?

47. 75 people in a health club were surveyed and the following information was gathered.

55 people drink water after exercising.
40 people drink Gatorade after exercising.
35 people drink water and Gatorade after exercising.

How many people drink neither water nor Gatorade after exercising?

48. In a poll of 200 people, the following information was gathered.

80 people listen to country music.
100 people listen to rock ‘n’ roll music.
50 people listen to both country and rock ‘n’ roll music.

How many people listen to country music only?

49. Aaron, Marty, Donna, and Maria have careers as a civil engineer, an electrical engineer, a mechanical engineer, and a nautical engineer, not necessarily in the given order. Given the information below, determine who is the civil engineer.

- **a.** Aaron and Marty wrote a research paper with the electrical engineer.
- **b.** Donna carpools with the electrical engineer.
- **c.** Marty works on the same floor as the civil engineer and the nautical engineer.
- **d.** Aaron and the civil engineer are second cousins.

50. Celeste, Bryan, Clay, and Eric are majoring in business, elementary education, art, and computer science but not necessarily in that order. Given the information below, determine who is majoring in art.

- **a.** Celeste, Bryan, and the elementary education major are in a class together.
- **b.** Bryan and the art major had lunch with the business major.
- **c.** Clay and the elementary education major are cousins.
- **d.** Celeste and the art major carpool together.

51. John, Leon, Alberto, and Julio sit in a theater according to the following.

Alberto is on the far left.
If Leon is on the far right, then John is not sitting next to him.

If Leon is on the far right, who is sitting next to him on his left?

52. Ralph, Anoa, Tumulish, and Pier sit on a bench during graduation ceremonies according to the following.

Tumulish is sitting on the far left.
If Anoa is sitting on the far right, then Ralph is not sitting next to her.

If Anoa is indeed sitting on the far right, list their seating order starting with the person sitting on the far left.

53. 100 people were surveyed and the results are as follows.

65 people said they drink Coke.
40 people said they drink Pepsi.
10 people said they drink both Coke and Pepsi.

How many people drink neither Coke nor Pepsi?

54. A poll was taken in Gotham City one day about the type of transportation used to commute to work. One hundred fifty-five people were polled.

70 people ride the bus.
95 people ride the subway.
30 people ride both the bus and the subway.

How many people ride neither the bus nor the subway?

Contents of Student Resources

Student Resources

Study Skills Builders

Attitude and Study Tips

Study Skills Builder 1

Have You Decided to Complete This Course Successfully?

Ask yourself if one of your current goals is to complete this course successfully.

If it is not a goal of yours, ask yourself why. One common reason is fear of failure. Amazingly enough, fear of failure alone can be strong enough to keep many of us from doing our best in any endeavor.

Another common reason is that you simply haven't taken the time to think about or write down your goals for this course. To help accomplish this, answer the questions below.

Exercises

1. Write down your goal(s) for this course.
2. Now list steps you will take to make sure your goal(s) in Exercise **1** are accomplished.
3. Rate your commitment to this course with a number between 1 and 5. Use the diagram below to help.

High Commitment		Average Commitment	Not Committed at All	
5	4	3	2	1

4. If you have rated your personal commitment level (from the exercise above) as a 1, 2, or 3, list the reasons why this is so. Then determine whether it is possible to increase your commitment level to a 4 or 5.

Good luck, and don't forget that a positive attitude will make a big difference.

Study Skills Builder 2

Tips for Studying for an Exam

To prepare for an exam, try the following study techniques:

- Start the study process days before your exam.
- Make sure that you are up to date on your assignments.
- If there is a topic that you are unsure of, use one of the many resources that are available to you. For example,

  See your instructor.
  View a lecture video on the topic.
  Visit a learning resource center on campus.
  Read the textbook material and examples on the topic.

- Reread your notes and carefully review the Chapter Highlights at the end of any chapter.
- Work the review exercises at the end of the chapter.
- Find a quiet place to take the Chapter Test found at the end of the chapter. Do not use any resources when taking this sample test. This way, you will have a clear indication of how prepared you are for your exam. Check your answers and use the Chapter Test Prep Videos to make sure that you correct any missed exercises.

Good luck, and keep a positive attitude.

Exercises

Let's see how you did on your last exam.

1. How many days before your last exam did you start studying for that exam?

2. Were you up to date on your assignments at that time or did you need to catch up on assignments?

3. List the most helpful text supplement (if you used one).

4. List the most helpful campus resource (if you used one).

5. List your process for preparing for a mathematics test.

6. Was this process helpful? In other words, were you satisfied with your performance on your exam?

7. If not, what changes can you make in your process that will make it more helpful to you?

Study Skills Builder 3

What to Do the Day of an Exam

Your first exam may be soon. On the day of an exam, don't forget to try the following:

- Allow yourself plenty of time to arrive.
- Read the directions on the test carefully.
- Read each problem carefully as you take your test. Make sure that you answer the question asked.
- Watch your time and pace yourself so that you may attempt each problem on your test.
- Check your work and answers.
- ***Do not turn your test in early.*** If you have extra time, spend it double-checking your work.

Good luck!

Exercises

Answer the following questions based on your most recent mathematics exam, whenever that was.

1. How soon before class did you arrive?

2. Did you read the directions on the test carefully?

3. Did you make sure you answered the question asked for each problem on the exam?

4. Were you able to attempt each problem on your exam?

5. If your answer to Exercise **4** is no, list reasons why.

6. Did you have extra time on your exam?

7. If your answer to Exercise **6** is yes, describe how you spent that extra time.

Study Skills Builder 4

Are You Satisfied with Your Performance on a Particular Quiz or Exam?

If not, don't forget to analyze your quiz or exam and look for common errors. Were most of your errors a result of:

- *Carelessness?* Did you turn in your quiz or exam before the allotted time expired? If so, resolve to use any extra time to check your work.
- *Running out of time?* Answer the questions you are sure of first. Then attempt the questions you are unsure of, and delay checking your work until all questions have been answered.
- *Not understanding a concept?* If so, review that concept and correct your work so that you make sure you understand it before the next quiz or the final exam.
- *Test conditions?* When studying for a quiz or exam, make sure you place yourself in conditions similar to test conditions. For example, before your next quiz or exam, take a sample test without the aid of your notes or text.

(For a sample test, see your instructor or use the Chapter Test at the end of each chapter.)

Exercises

1. Have you corrected all your previous quizzes and exams?
2. List any errors you have found common to two or more of your graded papers.
3. Is one of your common errors not understanding a concept? If so, are you making sure you understand all the concepts for the next quiz or exam?
4. Is one of your common errors making careless mistakes? If so, are you now taking all the time allotted to check over your work so that you can minimize the number of careless mistakes?
5. Are you satisfied with your grades thus far on quizzes and tests?
6. If your answer to Exercise **5** is no, are there any more suggestions you can make to your instructor or yourself to help? If so, list them here and share these with your instructor.

Study Skills Builder 5

How Are You Doing?

If you haven't done so yet, take a few moments and think about how you are doing in this course. Are you working toward your goal of successfully completing this course? Is your performance on homework, quizzes, and tests satisfactory? If not, you might want to see your instructor to see if he/she has any suggestions on how you can improve your performance. Reread Section 1.1 for ideas on places to get help with your mathematics course.

Exercises

Answer the following.

1. List any textbook supplements you are using to help you through this course.
2. List any campus resources you are using to help you through this course.
3. Write a short paragraph describing how you are doing in your mathematics course.
4. If improvement is needed, list ways that you can work toward improving your situation as described in Exercise **3**.

Study Skills Builder 6

Are You Preparing for Your Final Exam?

To prepare for your final exam, try the following study techniques:

- Review the material that you will be responsible for on your exam. This includes material from your textbook, your notebook, and any handouts from your instructor.
- Review any formulas that you may need to memorize.
- Check to see if your instructor or mathematics department will be conducting a final exam review.
- Check with your instructor to see whether previous final exams are available to students for review.
- Use your previously taken exams as a practice final exam. To do so, rewrite the test questions in mixed order on blank sheets of paper. This will help you prepare for exam conditions.
- If you are unsure of a few concepts, see your instructor or visit a learning lab for assistance. Also, view the video segments of any troublesome sections.
- If you need further exercises to work, try the Cumulative Reviews at the end of the chapters.
- When you feel you are prepared for your final exam, take the Practice Final Exam on page 770. Make sure you check your answers in the answer section of the text. Also, video solutions are available.

Organizing Your Work

Study Skills Builder 7

Learning New Terms

Many of the terms used in this text may be new to you. It will be helpful to make a list of new mathematical terms and symbols as you encounter them and to review them frequently. Placing these new terms (including page references) on 3×5 index cards might help you later when you're preparing for a quiz.

Exercises

1. Name one way you might place a word and its definition on a 3×5 card.
2. How do new terms stand out in this text so that they can be found?

Study Skills Builder 8

Are You Organized?

Have you ever had trouble finding a completed assignment? When it's time to study for a test, are your notes neat and organized? Have you ever had trouble reading your own mathematics handwriting? (Be honest—I have.)

When any of these things happens, it's time to get organized. Here are a few suggestions:

- Write your notes and complete your homework assignments in a notebook with pockets (spiral or ring binder).
- Take class notes in this notebook, and then follow the notes with your completed homework assignment.
- When you receive graded papers or handouts, place them in the notebook pocket so that you will not lose them.
- Mark (possibly with an exclamation point) any note(s) that seem extra important to you.
- Mark (possibly with a question mark) any notes or homework that you are having trouble with.
- See your instructor or a math tutor to help you with the concepts or exercises that you are having trouble understanding.
- If you are having trouble reading your own handwriting, *slow down* and write your mathematics work clearly!

Exercises

1. Have you been completing your assignments on time?
2. Have you been correcting any exercises you may be having difficulty with?
3. If you are having trouble with a mathematical concept or correcting any homework exercises, have you visited your instructor, a tutor, or your campus math lab?
4. Are you taking lecture notes in your mathematics course? (By the way, these notes should include worked-out examples solved by your instructor.)
5. Is your mathematics course material (handouts, graded papers, lecture notes) organized?
6. If your answer to Exercise **5** is no, take a moment and review your course material. List at least two ways that you might better organize it.

Study Skills Builder 9

Organizing a Notebook

It's never too late to get organized. If you need ideas about organizing a notebook for your mathematics course, try some of these:

- Use a spiral or ring binder notebook with pockets and use it for mathematics only.
- Start each page by writing the book's section number you are working on at the top.
- When your instructor is lecturing, take notes. *Always* include any examples your instructor works for you.
- Place your worked-out homework exercises in your notebook immediately after the lecture notes from that section. This way, a section's worth of material is together.
- Homework exercises: Attempt and check all assigned homework.
- Place graded quizzes in the pockets of your notebook or a special section of your binder.

Exercises

Check your notebook organization by answering the following questions.

1. Do you have a spiral or ring binder notebook for your mathematics course only?
2. Have you ever had to flip through several sheets of notes and work in your mathematics notebook to determine what section's work you are in?
3. Are you now writing the textbook's section number at the top of each notebook page?
4. Have you ever lost or had trouble finding a graded quiz or test?
5. Are you now placing all your graded work in a dedicated place in your notebook?
6. Are you attempting all of your homework and placing all of your work in your notebook?
7. Are you checking and correcting your homework in your notebook? If not, why not?
8. Are you writing in your notebook the examples your instructor works for you in class?

Study Skills Builder 10

How Are Your Homework Assignments Going?

It is very important in mathematics to keep up with homework. Why? Many concepts build on each other. Often your understanding of a day's concepts depends on an understanding of the previous day's material.

Remember that completing your homework assignment involves a lot more than attempting a few of the problems assigned.

To complete a homework assignment, remember these four things:

- Attempt all of it.
- Check it.
- Correct it.
- If needed, ask questions about it.

Exercises

Take a moment and review your completed homework assignments. Answer the questions below based on this review.

1. Approximate the fraction of your homework you have attempted.
2. Approximate the fraction of your homework you have checked (if possible).
3. If you are able to check your homework, have you corrected it when errors have been found?
4. When working homework, if you do not understand a concept, what do you do?

MyLab Math and MathXL

Study Skills Builder 11

Tips for Turning In Your Homework on Time

It is very important to keep up with your mathematics homework assignments. Why? Many concepts in mathematics build upon each other.

Remember these 4 tips to help ensure your work is completed on time:

- Know the assignments and due dates set by your instructor.
- Do not wait until the last minute to submit your homework.
- Set a goal to submit your homework 6–8 hours before the scheduled due date in case you have unexpected technology trouble.
- Schedule enough time to complete each assignment.

Following the tips above will also help you avoid potentially losing points for late or missed assignments.

Exercises

Take a moment to consider your work on your homework assignments to date and answer the following questions:

1. What percentage of your assignments have you turned in on time?
2. Why might it be a good idea to submit your homework 6–8 hours before the scheduled deadline?
3. If you have missed submitting any homework by the due date, list some of the reasons why this occurred.
4. What steps do you plan to take in the future to ensure your homework is submitted on time?

Study Skills Builder 12

Tips for Doing Your Homework Online

Practice is one of the main keys to success in any mathematics course. Did you know that MyLab Math/MathXL provides you with **immediate feedback** for each exercise? If you are incorrect, you are given hints to work the exercise correctly. You have **unlimited practice opportunities** and can rework any exercises you have trouble with until you master them, and can submit homework assignments unlimited times before the deadline.

Remember these success tips when doing your homework online:

- Attempt all assigned exercises.
- Write down (neatly) your step-by-step work for each exercise before entering your answer.
- Use the immediate feedback provided by the program to help you check and correct your work for each exercise.
- Rework any exercises you have trouble with until you master them.
- Work through your homework assignment as many times as necessary until you are satisfied.

Exercises

Take a moment to think about your homework assignments to date and answer the following:

1. Have you attempted all assigned exercises?
2. Of the exercises attempted, have you also written out your work before entering your answer—so that you can check it?
3. Are you familiar with how to enter answers using the MathXL player so that you avoid answer entry–type errors?
4. List some ways the immediate feedback and practice supports have helped you with your homework. If you have not used these supports, how do you plan to use them with the success tips above on your next assignment?

Study Skills Builder 13

Organizing Your Work

Have you ever used any readily available paper (such as the back of a flyer, another course assignment, Post-its, etc.) to work out homework exercises before entering the answer in MathXL? To save time, have you ever entered answers directly into MathXL without working the exercises on paper? When it's time to study, have you ever been unable to find your completed work or read and follow your own mathematics handwriting?

When any of these things happen, it's time to get organized. Here are some suggestions:

- Write your step-by-step work for each homework exercise (neatly) on lined, loose-leaf paper and keep this in a three-ring binder.
- Refer to your step-by-step work when you receive feedback that your answer is incorrect in MathXL. Double-check against the steps and hints provided by the program and correct your work accordingly.
- Keep your written homework with your class notes for that section.
- Identify any exercises you are having trouble with and ask questions about them.
- Keep all graded quizzes and tests in this binder as well to study later.

If you follow the suggestions above, you and your instructor or tutor will be able to follow your steps and correct any mistakes. You will have a written copy of your work to refer to later to ask questions and study for tests.

Exercises

1. Why is it important that you write out your step-by-step work on homework exercises and keep a hard copy of all work submitted online?
2. If you have gotten an incorrect answer, are you able to follow your steps and find your error?
3. If you were asked today to review your previous homework assignments and first test, could you find them? If not, list some ways you might better organize your work.

Study Skills Builder 14

Getting Help with Your Homework Assignments

There are many helpful resources available to you through MathXL to help you work through any homework exercises you may have trouble with. It is important that you know what these resources are and know when and how to use them.

Let's review these features found in the homework exercises:

- **Help Me Solve This**—provides step-by-step help for the exercise you are working. You must work an additional exercise of the same type (without this help) before you can get credit for having worked it correctly.
- **View an Example**—allows you to view a correctly worked exercise similar to the one you are having trouble with. You can go back to your original exercise and work it on your own.
- **E-Book**—allows you to read examples from your text and find similar exercises.
- **Video**—your text author, Elayn Martin-Gay, works an exercise similar to the one you need help with.

  **Not all exercises have an accompanying video clip.
- **Ask My Instructor**—allows you to e-mail your instructor for help with an exercise.

Exercises

1. How does the "Help Me Solve This" feature work?
2. If the "View an Example" feature is used, is it necessary to work an additional problem before continuing the assignment?
3. When might be a good time to use the "Video" feature? Do all exercises have an accompanying video clip?
4. Which of the features above have you used? List those you found the most helpful to you.
5. If you haven't used the features discussed, list those you plan to try on your next homework assignment.

Study Skills Builder 15

Tips for Preparing for an Exam

Did you know that you can rework your previous homework assignments in MyLab Math and MathXL? This is a great way to prepare for tests. To do this, open a previous homework assignment and click "similar exercise." This will generate new exercises similar to the homework you have submitted. You can then rework the exercises and assignments until you feel confident that you understand them.

To prepare for an exam, follow these tips:

- Review your written work for your previous homework assignments along with your class notes.
- Identify any exercises or topics that you have questions on or have difficulty understanding.
- Rework your previous assignments in MyLab Math and MathXL until you fully understand them and can do them without help.
- Get help for any topics you feel unsure of or for which you have questions.

Exercises

1. Are your current homework assignments up to date and is your written work for them organized in a binder or notebook? If the answer is no, it's time to get organized. For tips on this, see Study Skills Builder 13—Organizing Your Work.
2. How many days in advance of an exam do you usually start studying?
3. List some ways you think that practicing previous homework assignments can help you prepare for your test.
4. List two or three resources you can use to get help for any topics you are unsure of or have questions on.

Good luck!

Study Skills Builder 16

How Well Do You Know the Resources Available to You in MyLab Math?

There are many helpful resources available to you in MyLab Math. Let's take a moment to locate and explore a few of them now. Go into your MyLab Math course, and visit the Multimedia Library, Tools for Success, and E-Book.

Let's see what you found.

Exercises

1. List the resources available to you in the Multimedia Library.
2. List the resources available to you in the Tools for Success folder.
3. Where did you find the English/Spanish Audio Glossary?
4. Can you view videos from the E-Book?
5. Did you find any resources you did not know about? If so, which ones?
6. Which resources have you used most often or found most helpful?

Additional Help Inside and Outside Your Textbook

Study Skills Builder 17

How Well Do You Know Your Textbook?

The questions below will help determine whether you are familiar with your textbook. For additional information, see Section 1.1 in this text.

Exercises

1. What does the ▶ icon mean?
2. What does the ✏ icon mean?
3. What does the △ icon mean?
4. Where can you find a review for each chapter? What answers to this review can be found in the back of your text?
5. Each chapter contains an overview of the chapter along with examples. What is this feature called?
6. Each chapter contains a review of vocabulary. What is this feature called?
7. There are practice exercises that are contained in this text. Where are they and how can they be used?
8. This text contains a student section in the back entitled Student Resources. List the contents of this section and how they might be helpful.
9. What exercise answers are available in this text? Where are they located?

Study Skills Builder 18

Are You Familiar with Your Textbook Supplements?

Below is a review of some of the student supplements available for additional study. Check to see if you are using the ones most helpful to you.

- **Getting Ready for the Test Videos.** These videos provide video clip solutions to the Getting Ready for the Test exercises in this text. These exercises and video solutions are extremely useful when studying for tests or exams.
- **Chapter Test Prep Videos.** These videos provide video clip solutions to the Chapter Test exercises in this text. You will find this extremely useful when studying for tests or exams.
- **Interactive Lecture Series in MyLab Math.** These are keyed to each section of the text. The material is presented by me, Elayn Martin-Gay, and I have placed a ▶ by the exercises in the text that I have worked on the video.
- ***The Student Solutions Manual.*** This contains worked-out solutions to odd-numbered exercises as well as every exercise in the Integrated Reviews, Chapter Reviews, Getting Ready for the Tests, Chapter Tests, and Cumulative Reviews.
- **MyLab Math and MathXL.** MyLab Math is a text-specific online course. MathXL is an online homework, tutorial, and assessment system. Take a moment and determine whether these are available to you.

As usual, your instructor is your best source of information.

Exercises

Let's see how you are doing with textbook supplements.

1. Name one way the Lecture Videos can be helpful to you.
2. Name one way the Chapter Test Prep Videos can help you prepare for a chapter test.
3. List any textbook supplements that you have found useful.
4. Have you located and visited a learning resource lab on your campus?
5. List the textbook supplements that are currently housed in your campus's learning resource lab.

Study Skills Builder 19

Are You Getting All the Mathematics Help That You Need?

Remember that, in addition to your instructor, there are many places to get help with your mathematics course. For example:

- This text has an accompanying video lesson for every section. There are also worked-out solutions to every Getting Ready for the Test exercise, Chapter Test exercise, and Practice Final Exam exercise.
- The back of the book contains answers to odd-numbered section exercises as well as answers to all exercises in the Chapter Reviews, Getting Ready for the Tests, Chapter Tests, Cumulative Reviews, and the Practice Final Exam.
- A *Student Solutions Manual* is available that contains worked-out solutions to odd-numbered exercises as well as solutions to every exercise in the Integrated Reviews, Chapter Reviews, Getting Ready for the Tests, Chapter Tests, and Cumulative Reviews.
- Don't forget to check with your instructor for other local resources available to you, such as a tutoring center.

Exercises

1. List items you find helpful in the text and all student supplements to this text.
2. List all the campus help that is available to you for this course.
3. List any help (besides the textbook) from Exercises **1** and **2** above that you are using.
4. List any help (besides the textbook) that you feel you should try.
5. Write a goal for yourself that includes trying everything you listed in Exercise **4** during the next week.

Bigger Picture— Study Guide Outline

I. Operations on Sets of Numbers

A. Whole Numbers

1. **Add or Subtract:**

$$\begin{array}{r} 14 \\ +\ 39 \\ \hline 53 \end{array} \qquad \begin{array}{r} 300 \\ -\ 27 \\ \hline 273 \end{array}$$

2. **Multiply or Divide:**

$$\begin{array}{r} 238 \\ \times\ 47 \\ \hline 1666 \\ 9520 \\ \hline 11{,}186 \end{array} \qquad \begin{array}{r} 127\text{ R2} \\ 7\overline{)891} \\ -7 \\ \hline 19 \\ -14 \\ \hline 51 \\ -49 \\ \hline 2 \end{array}$$

3. **Exponent:**

$$3^4 = \overbrace{3\cdot3\cdot3\cdot3}^{\text{4 factors of 3}} = 81$$

4. **Square Root:**

$\sqrt{25} = 5$ because $5\cdot5 = 25$ and 5 is a positive number.

5. **Order of Operations:**

$$\begin{aligned} 24 \div 3\cdot2 - (2+8) &= 24 \div 3\cdot2 - (10) && \text{Simplify within parentheses.} \\ &= 8\cdot2 - 10 && \text{Multiply or divide from left to right.} \\ &= 16 - 10 && \text{Multiply or divide from left to right.} \\ &= 6 && \text{Add or subtract from left to right.} \end{aligned}$$

B. Integers

1. **Add:** $-5 + (-2) = -7$ Adding like signs
   Add absolute values. Attach the common sign.

   $-5 + 2 = -3$ Adding unlike signs
   Subtract absolute values. Attach the sign of the number with the larger absolute value.

2. **Subtract:** Add the first number to the opposite of the second number.

   $7 - 10 = 7 + (-10) = -3$

3. **Multiply or Divide:** Multiply or divide as usual. If the signs of the two numbers are the same, the answer is positive. If the signs of the two numbers are different, the answer is negative.

   $-5\cdot5 = -25, \dfrac{-32}{-8} = 4$

C. Fractions

1. **Simplify:** Factor the numerator and denominator. Then divide out factors of 1 by dividing out common factors in the numerator and denominator.

   Simplify: $\dfrac{20}{28} = \dfrac{4\cdot5}{4\cdot7} = \dfrac{5}{7}$

2. **Multiply:** Numerator times numerator over denominator times denominator.

   $\dfrac{5}{9}\cdot\dfrac{2}{7} = \dfrac{10}{63}$

3. **Divide:** First fraction times the reciprocal of the second fraction.

$$\frac{2}{11} \div \frac{3}{4} = \frac{2}{11} \cdot \frac{4}{3} = \frac{8}{33}$$

4. **Add or Subtract:** Must have same denominators. If not, find the LCD, and write each fraction as an equivalent fraction with the LCD as denominator.

$$\frac{2}{5} + \frac{1}{15} = \frac{2}{5} \cdot \frac{3}{3} + \frac{1}{15} = \frac{6}{15} + \frac{1}{15} = \frac{7}{15}$$

D. Decimals

1. **Add or Subtract:** Line up decimal points.

$$\begin{array}{r} 1.27 \\ +\,0.6 \\ \hline 1.87 \end{array}$$

2. **Multiply:**

$$\begin{array}{r} 2.56 \\ \times 3.2 \\ \hline 512 \\ 768 \\ \hline 8.192 \end{array}$$

2 decimal places

1 decimal place

2 + 1 = 3

3 decimal places

3. **Divide:**

$$\begin{array}{r} 0.7 \\ 8\overline{)5.6} \end{array} \qquad \begin{array}{r} 1.31 \\ 0.6\overline{)0.786} \end{array}$$

II. Solving Equations

A. Proportions: Set cross products equal to each other. Then solve.

$$\frac{14}{3} = \frac{2}{n} \text{ or } 14 \cdot n = 3 \cdot 2 \text{ or } 14 \cdot n = 6 \text{ or } n = \frac{6}{14} = \frac{3}{7}$$

B. Percent Problems

1. **Solved by Equations:** Remember that "of" means multiplication and "is" means equals.

"12% of some number is 6" translates to

$$12\% \cdot n = 6 \text{ or } 0.12 \cdot n = 6 \text{ or } n = \frac{6}{0.12} \text{ or } n = 50$$

2. **Solved by Proportions:** Remember that percent, p, is identified by % or "percent"; base, b, usually appears after "of"; and amount, a, is the part compared to the whole.

"12% of some number is 6" translates to

$$\frac{6}{b} = \frac{12}{100} \text{ or } 6 \cdot 100 = b \cdot 12 \text{ or } \frac{600}{12} = b \text{ or } 50 = b$$

C. Equations in General: Simplify both sides of the equation by removing parentheses and adding any like terms. Then use the addition property to write variable terms on one side and constants (or numbers) on the other side. Then use the multiplication property to solve for the variable by dividing both sides of the equation by the coefficient of the variable.

Solve: $2(x - 5) = 80$

$2x - 10 = 80$ Use the distributive property.

$2x - 10 + 10 = 80 + 10$ Add 10 to both sides.

$2x = 90$ Simplify.

$\frac{2x}{2} = \frac{90}{2}$ Divide both sides by 2.

$x = 45$ Simplify.

Practice Final Exam

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____
11. ____
12. ____
13. ____
14. ____
15. ____
16. ____
17. ____
18. ____
19. ____
20. ____
21. ____
22. ____
23. ____

Simplify by performing the indicated operations.

1. $600 - 487$

2. $(2^4 - 5) \cdot 3$

3. $-\frac{16}{3} \div -\frac{3}{12}$

4. $\frac{11}{12} - \frac{3}{8} + \frac{5}{24}$

5. $\frac{64 \div 8 \cdot 2}{(\sqrt{9} - \sqrt{4})^2 + 1}$

6. 10.2×4.01

7. $\frac{0.23 + 1.63}{-0.3}$

8. $\begin{array}{r} 19 \\ -\ 2\frac{3}{11} \\ \hline \end{array}$

9. $3\frac{1}{3} \cdot 6\frac{3}{4}$

10. $9.83 - 30.25$

11. $\left(-\frac{3}{4}\right)^2 \div \left(\frac{2}{3} + \frac{5}{6}\right)$

12. $(-5)^3 - 24 \div (-3)$

13. $-7 - (-19)$

14. $\frac{-3(-2) + 12}{-1(-4 - 5)}$

15. $-\frac{2}{7} \cdot \left(6 - \frac{1}{6}\right)$

16. Round 52,369 to the nearest thousand.

17. Round 34.8923 to the nearest tenth.

18. Write $\frac{16}{17}$ as a decimal. Round to the nearest thousandth.

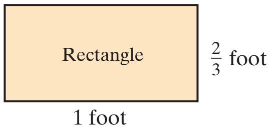
19. Write 85% as a decimal.

20. Write 6.1 as a percent.

21. Write $\frac{3}{8}$ as a percent.

22. Write 0.2% as a fraction in simplest form.

23. Find the perimeter and the area of the rectangle below.



Write each ratio or rate as a fraction in simplest form.

24. $75 to $10

25. 9 inches of rain in 30 days

26. Find the unit rate:
650 kilometers in 8 hours

27. Find each unit price and decide which is the better buy.

Steak sauce:
8 ounces for $1.19
12 ounces for $1.89

28. Find the unknown number, n, in the proportion:

$$\frac{8}{n} = \frac{11}{6}$$

Solve.

29. Subtract 8.6 from 20.

30. During a 258-mile trip, a car used $10\frac{3}{4}$ gallons of gas. How many miles would we expect the car to travel on 1 gallon of gas?

31. The standard dose of medicine for a dog is 10 grams for every 15 pounds of body weight. What is the standard dose for a dog that weighs 80 pounds?

32. Twenty-nine cans of Sherwin-Williams paint cost $493. How much was each can?

33. 0.6% of what number is 7.5?

34. 567 is what percent of 756?

35. An alloy is 12% copper. How much copper is contained in 320 pounds of this alloy?

36. A $120 framed picture is on sale for 15% off. Find the discount and the sale price.

Convert.

37. 40 mg to grams

38. $2\frac{1}{2}$ gal to quarts

Solve.

39. If 2 ft 9 in. of material is used to manufacture one scarf, how much material is needed for 6 scarves?

Find the mean, median, and mode of the list of numbers.

40. 26, 32, 42, 43, 49

24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________
38. ____________
39. ____________
40. ____________

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

47. ____________________

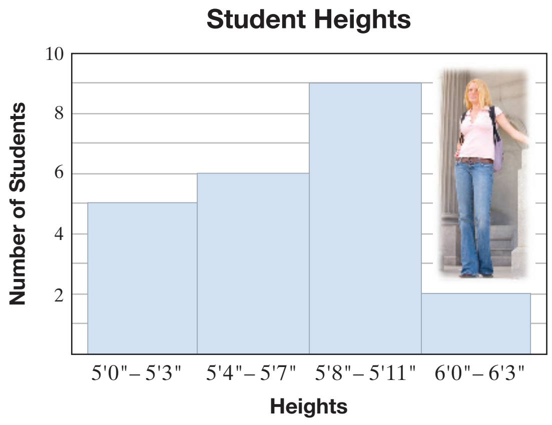
48. ____________________

49. ____________________

50. ____________________

51. ____________________

A professor measures the heights of the students in her class. The results are shown in the following histogram. Use this histogram to answer Exercise 41.



41. How many students are 5′7″ or shorter?

42. Evaluate: $\frac{3x - 5}{2y}$ when $x = 7$ and $y = -8$

43. Simplify: $5(3z + 2) - z - 18$

Solve.

44. $-\frac{5}{8}x = -25$

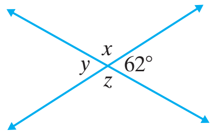
45. $5x + 12 - 4x - 14 = 22$

46. $3x - 5 = -11$

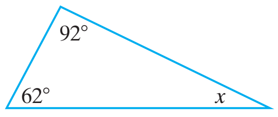
47. In a 10-kilometer race, there are 112 more men entered than women. Find the number of women runners if the total number of runners in the race is 600.

48. Find the complement of a 78° angle.

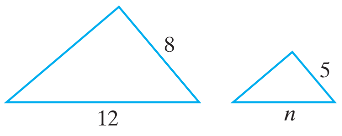
49. Find the measures of angles x, y, and z.



50. Find the measure of $\angle x$.



51. Given that the following triangles are similar, find the missing length n.



Answers to Selected Exercises

Chapter 1 The Whole Numbers

Section 1.2

Vocabulary, Readiness & Video Check **1.** whole **3.** words **5.** period **7.** hundreds **9.** 80,000

Exercise Set 1.2 **1.** tens **3.** thousands **5.** hundred-thousands **7.** millions **9.** three hundred fifty-four **11.** eight thousand, two hundred seventy-nine **13.** twenty-six thousand, nine hundred ninety **15.** two million, three hundred eighty-eight thousand **17.** twenty-four million, three hundred fifty thousand, one hundred eighty-five **19.** three hundred twenty-two thousand, six hundred fifty-three **21.** two thousand, seven hundred seventeen **23.** one hundred one million, five hundred thousand **25.** fourteen thousand, four hundred thirty-three **27.** twenty-two million, three hundred thirty-eight thousand, six hundred eighteen **29.** 6587 **31.** 59,800 **33.** 13,601,011 **35.** 7,000,017 **37.** 260,997 **39.** 418 **41.** 16,732 **43.** $119,119,000 **45.** 108,000 **47.** 400 + 6 **49.** 3000 + 400 + 70 **51.** 80,000 + 700 + 70 + 4 **53.** 60,000 + 6000 + 40 + 9 **55.** 30,000,000 + 9,000,000 + 600,000 + 80,000 **57.** 5532; five thousand, five hundred thirty-two **59.** 5000 + 400 + 90 + 2 **61.** Mt. Washington **63.** National Gallery **65.** six million, two thousand **67.** 3 **69.** 9861 **71.** no; one hundred five **73.** answers may vary **75.** 93,000,000,000,000,000 **77.** Canton

Section 1.3

Calculator Explorations **1.** 134 **3.** 340 **5.** 2834

Vocabulary, Readiness & Video Check **1.** number **3.** sum; addend **5.** grouping; associative **7.** place; right; left **9.** increased by

Exercise Set 1.3 **1.** 36 **3.** 292 **5.** 49 **7.** 5399 **9.** 117 **11.** 512 **13.** 209,078 **15.** 25 **17.** 62 **19.** 212 **21.** 94 **23.** 910 **25.** 8273 **27.** 11,926 **29.** 1884 **31.** 16,717 **33.** 1110 **35.** 8999 **37.** 35,901 **39.** 632,389 **41.** 42 in. **43.** 25 ft **45.** 24 in. **47.** 8 yd **49.** 29 in. **51.** 44 m **53.** 2093 **55.** 266 **57.** 544 **59.** 3452 **61.** 22,434 thousand **63.** 6684 ft **65.** 340 ft **67.** 2425 ft **69.** 262,191 **71.** 115,310 F-Series trucks and Silverados **73.** 124 ft **75.** 3275 **77.** California **79.** 2282 **81.** Florida and Georgia **83.** 6358 mi **85.** answers may vary **87.** answers may vary **89.** 1,044,473,765 **91.** correct **93.** incorrect: 530

Section 1.4

Calculator Explorations **1.** 770 **3.** 109 **5.** 8978

Vocabulary, Readiness & Video Check **1.** 0 **3.** minuend; subtrahend **5.** 0 **7.** 600 **9.** We cannot take 7 from 2 in the ones place, so we borrow one ten from the tens place and move it over to the ones place to give us 10 + 2 or 12.

Exercise Set 1.4 **1.** 44 **3.** 265 **5.** 135 **7.** 2254 **9.** 5545 **11.** 600 **13.** 25 **15.** 45 **17.** 146 **19.** 288 **21.** 168 **23.** 106 **25.** 447 **27.** 5723 **29.** 504 **31.** 89 **33.** 79 **35.** 39,914 **37.** 32,711 **39.** 5041 **41.** 31,213 **43.** 4 **45.** 20 **47.** 7 **49.** 72 **51.** 88 **53.** 264 pages **55.** 5 million sq km **57.** 264,000 sq mi **59.** 283,000 sq mi **61.** 6065 ft **63.** 28 ft **65.** 358 mi **67.** $619 **69.** 1609 thousand **71.** 100 dB **73.** 58 dB **75.** 30 **77.** 5920 sq ft **79.** Hartsfield-Jackson Atlanta International **81.** 12 million **83.** Jo; by 271 votes **85.** 1034 **87.** 9 **89.** 8518 **91.** 22,876 **93.** minuend: 48; subtrahend: 1 **95.** minuend: 70; subtrahend: 7 **97.** incorrect: 685 **99.** correct **101.** 5269 − 2385 = 2884 **103.** no; answers may vary **105.** no: 1089 more pages

Section 1.5

Vocabulary, Readiness & Video Check **1.** graph **3.** 70; 60 **5.** 3 is in the place value we're rounding to (tens), and the digit to the right of this place value is 5 or greater, so we need to add 1 to the 3. **7.** Each circled digit is to the right of the place value being rounded to and is used to determine whether or not we add 1 to the digit in the place value being rounded to.

Exercise Set 1.5 **1.** 420 **3.** 640 **5.** 2800 **7.** 500 **9.** 21,000 **11.** 34,000 **13.** 328,500 **15.** 36,000 **17.** 39,990 **19.** 30,000,000 **21.** 5280; 5300; 5000 **23.** 9440; 9400; 9000 **25.** 14,880; 14,900; 15,000 **27.** 27,000 students **29.** 38,000 points **31.** $150,000,000,000 **33.** $4,200,000 **35.** 220,000,000 smart phone users **37.** 130 **39.** 80 **41.** 5700 **43.** 300 **45.** 11,400 **47.** incorrect **49.** correct **51.** correct **53.** $3400 **55.** 900 mi **57.** 6000 ft **59.** 1,000,000,000 pieces **61.** 182,000 children **63.** $12,140,000,000; $12,100,000,000; $12,000,000,000 **65.** $2,110,000,000; $2,100,000,000; $2,000,000,000 **67.** 5723, for example **69.** 1000 **71.** 0 **73.** 8550 **75.** answers may vary **77.** 140 m

Section 1.6

Calculator Explorations **1.** 3456 **3.** 15,322 **5.** 272,291

Vocabulary, Readiness & Video Check **1.** 0 **3.** product; factor **5.** grouping; associative **7.** length **9.** distributive property **11.** Think of the problem as 50 times 9 and then attach the two zeros from 900, or think of the problem as 5 times 9 and then attach the three zeros at the end of 50 and 900. Both approaches give us 45,000. **13.** Multiplication is also an application of addition since it is addition of the same addend.

Exercise Set 1.6 **1.** 24 **3.** 0 **5.** 0 **7.** 87 **9.** $6 \cdot 3 + 6 \cdot 8$ **11.** $4 \cdot 3 + 4 \cdot 9$ **13.** $20 \cdot 14 + 20 \cdot 6$ **15.** 512 **17.** 3678 **19.** 1662 **21.** 6444 **23.** 1157 **25.** 24,418 **27.** 24,786 **29.** 15,600 **31.** 0 **33.** 6400 **35.** 48,126 **37.** 142,506 **39.** 2,369,826 **41.** 64,790 **43.** 3,949,935 **45.** 800 **47.** 11,000 **49.** 74,060 **51.** 24,000 **53.** 45,000 **55.** 3,280,000 **57.** area: 63 sq m; perimeter: 32 m **59.** area: 680 sq ft; perimeter: 114 ft **61.** 240,000 **63.** 300,000 **65.** **c** **67.** **c** **69.** 880 **71.** 4200 **73.** 4480 **75.** 375 cal **77.** \$3290 **79.** **a.** 20 boxes **b.** 100 boxes **c.** 2000 lb **81.** 8800 sq ft **83.** 56,000 sq ft **85.** 5828 pixels **87.** 2100 characters **89.** 1360 cal **91.** \$10, \$60; \$10, \$200; \$12, \$36, \$12, \$36; total cost: \$372 **93.** 1,440,000 tea bags **95.** 135 **97.** 2144 **99.** 23 **101.** 15 **103.** $4 \cdot 7$ or $7 \cdot 4$ **105.** **a.** $5 + 5 + 5$ or $3 + 3 + 3 + 3 + 3$ **b.** answers may vary **107.** $\begin{array}{r} 203 \\ \times\ 14 \\ \hline 812 \\ 2030 \\ \hline 2842 \end{array}$ **109.** $\begin{array}{r} \underline{4}2 \\ \times\ \underline{9}3 \\ \hline \end{array}$ **111.** answers may vary **113.** 506 windows

Section 1.7

Calculator Explorations **1.** 53 **3.** 62 **5.** 261 **7.** 0

Vocabulary, Readiness & Video Check **1.** quotient; dividend; divisor **3.** 1 **5.** undefined **7.** 0 **9.** $202 \cdot 102 + 15 = 20{,}619$ **11.** addition and division

Exercise Set 1.7 **1.** 6 **3.** 12 **5.** 0 **7.** 31 **9.** 1 **11.** 8 **13.** undefined **15.** 1 **17.** 0 **19.** 9 **21.** 29 **23.** 74 **25.** 338 **27.** undefined **29.** 9 **31.** 25 **33.** 68 R 3 **35.** 236 R 5 **37.** 38 R 1 **39.** 326 R 4 **41.** 13 **43.** 49 **45.** 97 R 8 **47.** 209 R 11 **49.** 506 **51.** 202 R 7 **53.** 54 **55.** 99 R 100 **57.** 202 R 15 **59.** 579 R 72 **61.** 17 **63.** 511 R 3 **65.** 2132 R 32 **67.** 6080 **69.** 23 R 2 **71.** 5 R 25 **73.** 20 R 2 **75.** 33 students **77.** 165 lb **79.** 310 yd **81.** 89 bridges **83.** 11 light poles **85.** 5 mi **87.** 1760 yd **89.** 20 **91.** 387 **93.** 79 **95.** 74° **97.** 9278 **99.** 15,288 **101.** 679 **103.** undefined **105.** 9 R 12 **107.** **c** **109.** **b** **111.** 77 **113.** increase; answers may vary **115.** no; answers may vary **117.** 12 ft **119.** answers may vary **121.** 5 R 1

Integrated Review **1.** 148 **2.** 6555 **3.** 1620 **4.** 562 **5.** 79 **6.** undefined **7.** 9 **8.** 1 **9.** 0 **10.** 0 **11.** 0 **12.** 3 **13.** 2433 **14.** 9826 **15.** 213 R 3 **16.** 79,317 **17.** 27 **18.** 9 **19.** 138 **20.** 276 **21.** 1099 R 2 **22.** 111 R 1 **23.** 663 R 6 **24.** 1076 R 60 **25.** 1024 **26.** 9899 **27.** 30,603 **28.** 47,500 **29.** 65 **30.** 456 **31.** 6 R 8 **32.** 53 **33.** 183 **34.** 231 **35.** 9740; 9700; 10,000 **36.** 1430; 1400; 1000 **37.** 20,800; 20,800; 21,000 **38.** 432,200; 432,200; 432,000 **39.** perimeter: 24 ft; area: 36 sq ft **40.** perimeter: 42 in.; area: 98 sq in. **41.** 28 mi **42.** 26 m **43.** 24 **44.** 124 **45.** Lake Pontchartrain Bridge; 2175 ft **46.** 730 qt

Section 1.8

Vocabulary, Readiness & Video Check **1.** The George Washington Bridge has a length of 3500 feet.

Exercise Set 1.8 **1.** 49 **3.** 237 **5.** 42 **7.** 600 **9.** **a.** 400 ft **b.** 9600 sq ft **11.** \$15,500 **13.** 168 hr **15.** 3500 ft **17.** 141 yr **19.** 372 billion bricks **21.** 719 towns **23.** \$26 **25.** 55 cal **27.** 21 hot dogs **29.** 3,219,600 visitors **31.** 741,000 people **33.** 3987 mi **35.** 13 paychecks **37.** \$239 **39.** \$1045 **41.** **b** will be cheaper by \$3 **43.** Asia **45.** 1846 million **47.** 66 million **49.** 951 million **51.** \$14,754 **53.** 16,800 mg **55.** **a.** 3750 sq ft **b.** 375 sq ft **c.** 3375 sq ft **57.** \$10 **59.** answers may vary

Section 1.9

Calculator Explorations **1.** 4096 **3.** 3125 **5.** 2048 **7.** 2526 **9.** 4295 **11.** 8

Vocabulary, Readiness & Video Check **1.** base; exponent **3.** addition **5.** division **7.** exponent; base **9.** Because $8 \cdot 8 = 64$. **11.** The area of a rectangle is length · width. A square is a special rectangle where length = width. Thus, the area of a square is side · side or $(\text{side})^2$.

Exercise Set 1.9 **1.** 4^3 **3.** 7^6 **5.** 12^3 **7.** $6^2 \cdot 5^3$ **9.** $9 \cdot 8^2$ **11.** $3 \cdot 2^4$ **13.** $3 \cdot 2^4 \cdot 5^5$ **15.** 64 **17.** 125 **19.** 32 **21.** 1 **23.** 7 **25.** 128 **27.** 256 **29.** 256 **31.** 729 **33.** 144 **35.** 100 **37.** 20 **39.** 729 **41.** 192 **43.** 162 **45.** 3 **47.** 8 **49.** 12 **51.** 4 **53.** 21 **55.** 7 **57.** 5 **59.** 16 **61.** 46 **63.** 8 **65.** 64 **67.** 83 **69.** 2 **71.** 48 **73.** 4 **75.** undefined **77.** 59 **79.** 52 **81.** 44 **83.** 12 **85.** 21 **87.** 24 **89.** 28 **91.** 3 **93.** 25 **95.** 23 **97.** 13 **99.** area: 49 sq m; perimeter: 28 m **101.** area: 529 sq mi; perimeter: 92 mi **103.** true **105.** false **107.** $(2 + 3) \cdot 6 - 2$ **109.** $24 \div (3 \cdot 2) + 2 \cdot 5$ **111.** 1260 ft **113.** 6,384,814 **115.** answers may vary; sample answer: $(20 - 10) \cdot 5 \div 25 + 3$

Chapter 1 Vocabulary Check **1.** whole numbers **2.** perimeter **3.** place value **4.** exponent **5.** area **6.** square root **7.** digits **8.** average **9.** divisor **10.** dividend **11.** quotient **12.** factor **13.** product **14.** minuend **15.** subtrahend **16.** difference **17.** addend **18.** sum

Chapter 1 Review **1.** tens **2.** ten-millions **3.** seven thousand, six hundred forty **4.** forty-six million, two hundred thousand, one hundred twenty **5.** $3000 + 100 + 50 + 8$ **6.** $400{,}000{,}000 + 3{,}000{,}000 + 200{,}000 + 20{,}000 + 5000$ **7.** 81,900 **8.** 6,304,000,000 **9.** 636,831,820 **10.** 326,975,340 **11.** Asia **12.** Oceania/Australia **13.** 63 **14.** 67 **15.** 48 **16.** 77 **17.** 956 **18.** 840 **19.** 7950 **20.** 7250 **21.** 4211 **22.** 1967 **23.** 1326 **24.** 886 **25.** 27,346 **26.** 39,300 **27.** 8032 mi **28.** \$197,699 **29.** 276 ft **30.** 66 km

31. 14 **32.** 34 **33.** 65 **34.** 304 **35.** 3914 **36.** 7908 **37.** 17,897 **38.** 34,658 **39.** 141,934 **40.** 36,746 **41.** 397 pages **42.** \$25,626 **43.** May **44.** August **45.** \$110 **46.** \$240 **47.** 90 **48.** 50 **49.** 470 **50.** 500 **51.** 4800 **52.** 58,000 **53.** 50,000,000 **54.** 800,000 **55.** 18,000,000 **56.** 98,000 **57.** 7400 **58.** 4100 **59.** 2500 mi **60.** 800,000 **61.** 1911 **62.** 1396 **63.** 1410 **64.** 2898 **65.** 800 **66.** 900 **67.** 3696 **68.** 1694 **69.** 0 **70.** 0 **71.** 16,994 **72.** 8954 **73.** 113,634 **74.** 44,763 **75.** 411,426 **76.** 636,314 **77.** 375,000 **78.** 108,000 **79.** 12,000 **80.** 35,000 **81.** 5,100,000 **82.** 7,600,000 **83.** 1150 **84.** 4920 **85.** 108 **86.** 112 **87.** 24 g **88.** \$158,980 **89.** 60 sq mi **90.** 500 sq cm **91.** 3 **92.** 4 **93.** 6 **94.** 7 **95.** 5 R 2 **96.** 4 R 2 **97.** undefined **98.** 0 **99.** 1 **100.** 10 **101.** 0 **102.** undefined **103.** 33 R 2 **104.** 19 R 7 **105.** 24 R 2 **106.** 35 R 15 **107.** 506 R 10 **108.** 907 R 40 **109.** 2793 R 140 **110.** 2012 R 60 **111.** 18 R 2 **112.** 21 R 2 **113.** 458 ft **114.** 13 mi **115.** 51 **116.** 59 **117.** 27 boxes **118.** \$192 **119.** \$1,700,000,000 **120.** 75¢ **121.** \$898 **122.** 23,150 sq ft **123.** 49 **124.** 125 **125.** 45 **126.** 400 **127.** 13 **128.** 10 **129.** 15 **130.** 7 **131.** 12 **132.** 9 **133.** 42 **134.** 33 **135.** 9 **136.** 2 **137.** 1 **138.** 0 **139.** 6 **140.** 29 **141.** 40 **142.** 72 **143.** 5 **144.** 7 **145.** 49 sq m **146.** 9 sq in. **147.** 307 **148.** 682 **149.** 2169 **150.** 2516 **151.** 901 **152.** 1411 **153.** 458 R 8 **154.** 237 R 1 **155.** 70,848 **156.** 95,832 **157.** 1644 **158.** 8481 **159.** 740 **160.** 258,000 **161.** 2000 **162.** 40,000 **163.** thirty-six thousand, nine hundred eleven **164.** one hundred fifty-four thousand, eight hundred sixty-three **165.** 70,943 **166.** 43,401 **167.** 64 **168.** 125 **169.** 12 **170.** 10 **171.** 12 **172.** 1 **173.** 2 **174.** 6 **175.** 4 **176.** 24 **177.** 24 **178.** 14 **179.** \$190,000 **180.** \$1,289,000 **181.** 53 full boxes with 18 left over **182.** \$86

Chapter 1 Getting Ready for the Test **1.** D **2.** C **3.** B **4.** E **5.** C **6.** A **7.** B, E **8.** D **9.** B **10.** C **11.** A **12.** D **13.** B **14.** A **15.** E **16.** C **17.** C **18.** A **19.** B **20.** A

Chapter 1 Test **1.** eighty-two thousand, four hundred twenty-six **2.** 402,550 **3.** 141 **4.** 113 **5.** 14,880 **6.** 766 R 42 **7.** 200 **8.** 10 **9.** 0 **10.** undefined **11.** 33 **12.** 21 **13.** 8 **14.** 36 **15.** 5,698,000 **16.** 11,200,000 **17.** 52,000 **18.** 13,700 **19.** 1600 **20.** 92 **21.** 122 **22.** 1605 **23.** 7 R 2 **24.** \$17 **25.** \$126 **26.** 360 cal **27.** \$7905 **28.** 20 cm; 25 sq cm **29.** 60 yd; 200 sq yd

Chapter 2 Integers and Introduction to Variables

Section 2.1

Vocabulary, Readiness & Video Check **1.** expression **3.** expression; variables **5.** multiplication

Exercise Set 2.1 **1.** 28; 14; 147; 3 **3.** 152; 152; 0; undefined **5.** 57; 55; 56; 56 **7.** 9 **9.** 26 **11.** 6 **13.** 3 **15.** 117 **17.** 94 **19.** 5 **21.** 626 **23.** 20 **25.** 4 **27.** 4 **29.** 0 **31.** 33 **33.** 121 **35.** 121 **37.** 100 **39.** 60 **41.** 4 **43.** 16, 64, 144, 256 **45.** $x + 5$ **47.** $x + 8$ **49.** $20 - x$ **51.** $5x$ **53.** $x \div 2$ or $\frac{x}{2}$ **55.** $5x + 17$ **57.** $\frac{x}{5} - 12$ **59.** $11 - x$ **61.** $x - 5$ **63.** $6 \div x$ or $\frac{6}{x}$ **65.** $50 - 8x$ **67.** hundreds **69.** thousands **71.** incorrect; $2(0) + 3(7) = 0 + 21 = 21$ **73.** correct **75.** 274,657 **77.** 777 **79.** $5x$ **81.** As t gets larger, $16t^2$ gets larger.

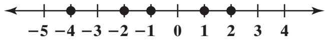
Section 2.2

Vocabulary, Readiness & Video Check **1.** integers **3.** inequality symbols **5.** is less than; is greater than **7.** absolute value **9.** number of feet a miner works underground **11.** negative **13.** opposite of

Exercise Set 2.2 **1.** −1445 **3.** +14,433 **5.** +120 **7.** −11,810 **9.** −9800 million **11.** −160, −147; Guillermo **13.** −10

15.

17.



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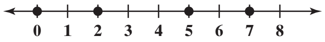

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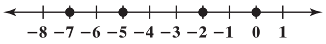


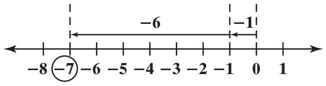
23. > **25.** < **27.** > **29.** < **31.** 5 **33.** 8 **35.** 0 **37.** 5 **39.** −5 **41.** 4 **43.** −23 **45.** 85 **47.** 7 **49.** −20 **51.** −3 **53.** 43 **55.** −15 **57.** 33 **59.** 6 **61.** −2 **63.** 32 **65.** −7 **67.** < **69.** < **71.** = **73.** < **75.** > **77.** < **79.** > **81.** < **83.** 31; −31 **85.** 28; 28 **87.** Caspian Sea **89.** Lake Superior **91.** iodine **93.** oxygen **95.** 13 **97.** 35 **99.** 360 **101.** $-|-8|, -|3|, 2^2, -(-5)$ **103.** $-|-6|, -|1|, |-1|, -(-6)$ **105.** $-10, -|-9|, -(-2), |-12|, 5^2$ **107.** **d** **109.** 5 **111.** false **113.** true **115.** false **117.** answers may vary **119.** no; answers may vary

Section 2.3

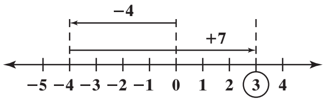
Calculator Explorations **1.** −159 **3.** 44 **5.** −894,855

Vocabulary, Readiness & Video Check **1.** 0 **3.** a **5.** 5 **7.** −35 **9.** 0 **11.** 0 **13.** Negative; the numbers have different signs and the sign of the sum is the same as the sign of the number with the larger absolute value, −6. **15.** The diver's current depth is 231 feet below the surface.

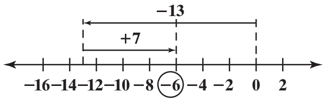
Exercise Set 2.3 **1.**



3.



5.



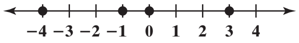
7. 35 **9.** −8 **11.** 0 **13.** 4 **15.** 2 **17.** −2 **19.** −9 **21.** −24 **23.** −57 **25.** −223 **27.** 0 **29.** 7 **31.** −3 **33.** −9 **35.** 30 **37.** 20 **39.** 51 **41.** −33 **43.** −20 **45.** −125 **47.** −7 **49.** −246 **51.** 16 **53.** 13 **55.** −33 **57.** −21 **59.** 21 **61.** −45 **63.** 9 **65.** 0 **67.** 0 **69.** −103 **71.** −70 **73.** 3 **75.** −21 **77.** $-8 + 25; 17$ **79.** $-31 + (-9) + 30; -10$ **81.** $0 + (-215) + (-16) = -231$; 231 ft below the surface **83.** Fowler: −11; Woods: −4 **85.** \$48,351,000,000 **87.** \$11,314,000,000 **89.** 2°C **91.** −\$12,198 **93.** −2°F **95.** −7679 m **97.** 44 **99.** 141 **101.** answers may vary **103.** −3 **105.** −22 **107.** true **109.** false **111.** answers may vary

Section 2.4

Vocabulary, Readiness & Video Check **1.** $a + (-b)$; **b** **3.** $-10 - (-14)$; **d** **5.** 0 **7.** 0 **9.** additive inverse **11.** to follow the order of operations

Exercise Set 2.4 **1.** 0 **3.** 5 **5.** −5 **7.** 14 **9.** 3 **11.** −18 **13.** −14 **15.** −33 **17.** 402 **19.** −14 **21.** −4 **23.** −7 **25.** −42 **27.** −17 **29.** 13 **31.** −38 **33.** −11 **35.** −127 **37.** −11 **39.** 2 **41.** 0 **43.** −1 **45.** −27 **47.** 40 **49.** −22 **51.** −8 **53.** 14 **55.** −11 **57.** 31 **59.** 12 **61.** 20 **63.** 389°F **65.** 7°F **67.** 263°F **69.** −\$16 **71.** −12°C **73.** 154 ft **75.** 69 ft **77.** 652 ft **79.** 144 ft **81.** −\$49 billion **83.** $-5 + x$ **85.** $-20 - x$ **87.** 5 **89.** 1058 **91.** answers may vary **93.** 16 **95.** −20 **97.** −4 **99.** 0 **101.** −12 **103.** false **105.** answers may vary

Integrated Review **1.** +29,028 **2.** −35,840 **3.** −7 **4.**



5. > **6.** < **7.** < **8.** > **9.** 1 **10.** −4 **11.** 8 **12.** 5 **13.** −6 **14.** 3 **15.** −89 **16.** 0 **17.** 5 **18.** −20 **19.** −10 **20.** −2 **21.** 52 **22.** −3 **23.** 84 **24.** 6 **25.** 1 **26.** −19 **27.** 12 **28.** −4 **29.** −44 **30.** **b, c, d** **31.** **a, b, c, d** **32.** 10 **33.** −12 **34.** 12 **35.** 10 **36.** 56 **37.** −34

Section 2.5

Vocabulary, Readiness & Video Check **1.** negative **3.** positive **5.** 0 **7.** undefined **9.** multiplication **11.** The phrase "lost four yards" in the example translates to the negative number −4.

Exercise Set 2.5 **1.** 12 **3.** −36 **5.** −81 **7.** 0 **9.** 48 **11.** −12 **13.** 80 **15.** 0 **17.** −15 **19.** −9 **21.** 9 **23.** −36 **25.** −64 **27.** −8 **29.** −5 **31.** 7 **33.** 0 **35.** undefined **37.** −14 **39.** 0 **41.** −15 **43.** −63 **45.** 42 **47.** −24 **49.** 49 **51.** −5 **53.** −9 **55.** −6 **57.** 120 **59.** −1080 **61.** undefined **63.** −6 **65.** −7 **67.** 3 **69.** −1 **71.** −32 **73.** 180 **75.** 1 **77.** −30 **79.** −1104 **81.** −2870 **83.** −56 **85.** −18 **87.** 35 **89.** −1 **91.** undefined **93.** 6 **95.** 16; 4 **97.** 0; 0 **99.** $-54 \div 9; -6$ **101.** $-42(-6); 252$ **103.** $-71 \cdot x$ or $-71x$ **105.** $-16 - x$ **107.** $-29 + x$ **109.** $\frac{x}{-33}$ or $x \div (-33)$ **111.** $3 \cdot (-4) = -12$; a loss of 12 yd **113.** $5 \cdot (-20) = -100$; a depth of 100 ft **115.** −210°C **117.** −189°C **119.** −45 thousand bee colonies per year **121 a.** −23,940 movie screens **b.** −3990 movie screens per year **123.** 109 **125.** 8 **127.** −19 **129.** −28 **131.** −8 **133.** negative **135.** $(-5)^{17}, (-2)^{17}, (-2)^{12}, (-5)^{12}$ **137.** answers may vary

Section 2.6

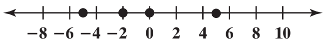
Calculator Explorations **1.** 48 **3.** −258

Vocabulary, Readiness & Video Check **1.** division **3.** average **5.** subtraction **7.** base: 3, exponent: 2 **9.** base: 4, exponent: 1; base: 2, exponent: 3 **11.** base: −7, exponent: 5 **13.** base: 5, exponent: 7; base: 10, exponent: 1 **15.** A fraction bar means "divided by" and it is a grouping symbol. **17.** Finding the average is a good application of both order of operations and adding and dividing integers.

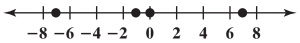
Exercise Set 2.6 **1.** −64 **3.** −64 **5.** 24 **7.** −1 **9.** −7 **11.** −14 **13.** −43 **15.** −8 **17.** −13 **19.** −1 **21.** 4 **23.** −3 **25.** −55 **27.** 8 **29.** 16 **31.** 13 **33.** −77 **35.** 64 **37.** 452 **39.** 117 **41.** 3 **43.** −4 **45.** 4 **47.** 16 **49.** −27 **51.** 34 **53.** 65 **55.** −59 **57.** −7 **59.** −61 **61.** −11 **63.** 36 **65.** −117 **67.** 30 **69.** −3 **71.** −59 **73.** −30 **75.** 1 **77.** −12 **79.** 0 **81.** −20 **83.** 9 **85.** −16 **87.** 1 **89.** −50 **91.** −2 **93.** −19 **95.** 21 **97.** −3 **99.** no; answers may vary **101.** 4050 **103.** 45 **105.** 32 in. **107.** 30 ft **109.** $2 \cdot (7 - 5) \cdot 3$ **111.** $-6 \cdot (10 - 4)$ **113.** no; answers may vary **115.** answers may vary **117.** 20,736 **119.** 8900 **121.** 9

Chapter 2 Vocabulary Check **1.** opposites **2.** signed **3.** absolute value **4.** integers **5.** variable **6.** negative **7.** positive

Chapter 2 Review **1.** 5 **2.** 17 **3.** undefined **4.** 0 **5.** 121 **6.** 2 **7.** 4 **8.** 20 **9.** $x - 5$ **10.** $x + 7$ **11.** $10 \div x$ or $\frac{10}{x}$ **12.** $5x$ **13.** −1435 **14.** +11,239 **15.**



16.



17. 12 **18.** 0 **19.** −6 **20.** 9 **21.** −9 **22.** 2 **23.** > **24.** < **25.** > **26.** > **27.** 12 **28.** −3 **29.** false **30.** true **31.** true **32.** true **33.** 2 **34.** 14 **35.** 4 **36.** 17 **37.** −23 **38.** −22 **39.** −21 **40.** −70 **41.** 0 **42.** 0 **43.** −151 **44.** −606 **45.** −20° C **46.** −150 ft **47.** −9 **48.** 11 **49.** 99°F **50.** 110°F **51.** 8 **52.** −16 **53.** −11 **54.** −27 **55.** 20 **56.** 8 **57.** 0 **58.** −32 **59.** 0 **60.** −7 **61.** −10 **62.** −9 **63.** −\$25 **64.** 692 ft **65.** −3°F **66.** −4°F **67.** true **68.** false **69.** true **70.** true **71.** 21 **72.** −18 **73.** −64 **74.** 60 **75.** 25 **76.** −1 **77.** 0 **78.** 24 **79.** −5 **80.** 3 **81.** 0 **82.** undefined **83.** −20 **84.** −9 **85.** 38 **86.** −5 **87.** $(-5)(2) = -10$ **88.** $(-50)(4) = -200$ **89.** −18°F **90.** −3°F **91.** 28°F **92.** −26°F **93.** 49 **94.** −49 **95.** −32 **96.** −32 **97.** −21 **98.** −8 **99.** −16 **100.** 35 **101.** −28 **102.** −44 **103.** 3 **104.** −1 **105.** 7 **106.** −17 **107.** 39 **108.** −26 **109.** 7 **110.** 46 **111.** −2 **112.** −12 **113.** −3 **114.** −35 **115.** −5 **116.** 5 **117.** −1 **118.** −7 **119.** 4 **120.** −4 **121.** 3 **122.** 108 **123.** 16 **124.** −16

125. −15 **126.** −19 **127.** 48 **128.** −21 **129.** 21 **130.** −5 **131.** Elevator D **132.** Elevator B **133.** 13,118 ft **134.** −27°C **135.** 2 **136.** 4 **137.** 3 **138.** 37 **139.** −5 **140.** −25 **141.** −20 **142.** 17

Chapter 2 Getting Ready for the Test **1.** A **2.** A **3.** A **4.** B **5.** C **6.** C **7.** A **8.** D **9.** C **10.** D **11.** B **12.** C **13.** B **14.** D **15.** A **16.** C **17.** B **18.** A **19.** C **20.** C **21.** D **22.** B **23.** C **24.** D

Chapter 2 Test **1.** 3 **2.** −6 **3.** −100 **4.** 4 **5.** −30 **6.** 12 **7.** 65 **8.** 5 **9.** 12 **10.** −6 **11.** 50 **12.** −2 **13.** −11 **14.** −46 **15.** −117 **16.** 3456 **17.** 28 **18.** −213 **19.** −1 **20.** −2 **21.** 2 **22.** −5 **23.** −32 **24.** −12 **25.** −3 **26.** 5 **27.** −1 **28.** −54 **29.** 1 **30.** −17 **31.** 88 ft below sea level **32.** 45 or $45 **33.** 31,642 or 31,642 ft **34.** 3820 ft below sea level **35.** −4 **36. a.** $17x$ **b.** $20 - 2x$

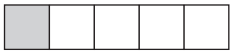
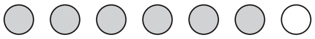
Cumulative Review **1.** hundred-thousands; Sec. 1.2, Ex. 1 **2.** hundreds; Sec. 1.2 **3.** thousands; Sec. 1.2, Ex. 2 **4.** thousands; Sec. 1.2 **5.** ten-millions; Sec. 1.2, Ex. 3 **6.** hundred-thousands; Sec. 1.2 **7. a.** $<$ **b.** $>$ **c.** $>$; Sec. 2.2, Ex. 3 **8. a.** $>$ **b.** $<$ **c.** $>$; Sec. 2.2 **9.** 39; Sec. 1.3, Ex. 3 **10.** 39; Sec. 1.3 **11.** 7321; Sec. 1.4, Ex. 2 **12.** 3013; Sec. 1.4 **13.** 36,184 mi; Sec. 1.4, Ex. 5 **14.** $525; Sec. 1.4 **15.** 570; Sec. 1.5, Ex. 1 **16.** 600; Sec. 1.5 **17.** 1800; Sec. 1.5, Ex. 5 **18.** 5000; Sec. 1.5 **19. a.** $3 \cdot 4 + 3 \cdot 5$ **b.** $10 \cdot 6 + 10 \cdot 8$ **c.** $2 \cdot 7 + 2 \cdot 3$; Sec. 1.6, Ex. 2 **20. a.** $5 \cdot 2 + 5 \cdot 12$ **b.** $9 \cdot 3 + 9 \cdot 6$ **c.** $4 \cdot 8 + 4 \cdot 1$; Sec. 1.6 **21.** 78,875; Sec. 1.6, Ex. 5 **22.** 31,096; Sec. 1.6 **23. a.** 6 **b.** 8 **c.** 7; Sec. 1.7, Ex. 1 **24. a.** 7 **b.** 8 **c.** 12; Sec. 1.7 **25.** 741; Sec. 1.7, Ex. 4 **26.** 456; Sec. 1.7 **27.** 12 cards; 10 cards left over; Sec. 1.7, Ex. 11 **28.** $9; Sec. 1.7 **29.** 81; Sec. 1.9, Ex. 5 **30.** 125; Sec. 1.9 **31.** 6; Sec. 1.9, Ex. 6 **32.** 4; Sec. 1.9 **33.** 180; Sec. 1.9, Ex. 8 **34.** 56; Sec. 1.9 **35.** 2; Sec. 1.9, Ex. 16 **36.** 5; Sec. 1.9 **37.** 15; Sec. 2.1, Ex. 1 **38.** 14; Sec. 2.1 **39. a.** 2 **b.** 8 **c.** 0; Sec. 2.2, Ex. 4 **40. a.** 4 **b.** 7; Sec. 2.2 **41.** 21; Sec. 2.3, Ex. 7 **42.** 5; Sec. 2.3 **43.** 22; Sec. 2.4, Ex. 12 **44.** 5; Sec. 2.4 **45.** −21; Sec. 2.5, Ex. 1 **46.** −10; Sec. 2.5 **47.** 0; Sec. 2.5, Ex. 3 **48.** −54; Sec. 2.5 **49.** −16; Sec. 2.6, Ex. 8 **50.** −27; Sec. 2.6

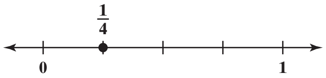
Chapter 3 Fractions and Mixed Numbers

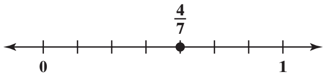
Section 3.1

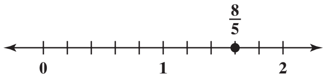
Vocabulary, Readiness & Video Check **1.** fraction; denominator; numerator **3.** improper; proper; mixed number **5.** equal; improper **7.** whole number; fraction **9.** The fraction is equal to 1.

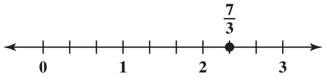
Exercise Set 3.1 **1.** numerator: 1; denominator: 2; proper **3.** numerator: 10; denominator: 3; improper **5.** numerator: 15; denominator: 15; improper **7.** $\frac{1}{3}$ **9. a.** $\frac{11}{4}$ **b.** $2\frac{3}{4}$ **11. a.** $\frac{23}{6}$ **b.** $3\frac{5}{6}$ **13.** $\frac{7}{12}$ **15.** $\frac{3}{7}$ **17.** $\frac{4}{9}$ **19. a.** $\frac{4}{3}$ **b.** $1\frac{1}{3}$ **21. a.** $\frac{11}{2}$ **b.** $5\frac{1}{2}$ **23.** $\frac{1}{6}$ **25.** $\frac{5}{8}$

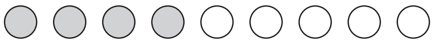
27.  **29.**  **31.**  **33.** $\frac{42}{131}$ of the students **35. a.** 89 students **b.** $\frac{89}{131}$ of the students **37.** $\frac{7}{44}$ of the U.S. presidents **39.** $\frac{3}{10}$ of the hurricanes **41.** $\frac{11}{31}$ of the month **43.** $\frac{10}{31}$ of the class **45. a.** $\frac{33}{50}$ of the states **b.** 17 states **c.** $\frac{17}{50}$ of the states **47. a.** $\frac{21}{50}$ of the marbles **b.** 29 marbles **c.** $\frac{29}{50}$ of the marbles

49. 

51. 

53. 

55. 

57. 1 **59.** −5 **61.** 0 **63.** 1 **65.** undefined **67.** 3 **69.** 9 **71.** 125 **73.** 7^5 **75.** $2^3 \cdot 3$ **77.** $\frac{-11}{2}$; $\frac{11}{-2}$ **79.** $\frac{13}{-15}$; $-\frac{13}{15}$ **81.** answers may vary **83.**  **85.** 7 **87.** $\frac{56}{355}$ of the stores **89.** $\frac{1500}{1580}$ of the affiliates

Section 3.2

Calculator Explorations **1.** $\frac{4}{7}$ **3.** $\frac{20}{27}$ **5.** $\frac{8}{15}$ **7.** $\frac{2}{9}$

Vocabulary, Readiness & Video Check **1.** prime factorization **3.** prime **5.** equivalent **7.** yes, yes, yes **9.** $3 \cdot 5$ **11.** $2 \cdot 3$ **13.** 2^2 **15.** Check that every factor is a prime number and check that the product of the factors is the original number. **17.** You can simplify the two fractions and then compare them. $\frac{3}{9}$ and $\frac{6}{18}$ both simplify to $\frac{1}{3}$, so the original fractions are equivalent.

Exercise Set 3.2 **1.** $2^2 \cdot 5$ **3.** $2^4 \cdot 3$ **5.** $2 \cdot 5^2$ **7.** $2 \cdot 3^4$ **9.** $2 \cdot 5 \cdot 11$ **11.** $5 \cdot 17$ **13.** $2^4 \cdot 3 \cdot 5$ **15.** $2^2 \cdot 3^2 \cdot 23$ **17.** $\frac{1}{4}$ **19.** $\frac{2}{21}$ **21.** $\frac{7}{8}$ **23.** $\frac{2}{3}$ **25.** $\frac{7}{10}$ **27.** $-\frac{7}{9}$ **29.** $\frac{3}{5}$ **31.** $\frac{27}{64}$ **33.** $\frac{5}{8}$ **35.** $-\frac{5}{8}$ **37.** $\frac{3}{2}$ **39.** $\frac{3}{4}$ **41.** $\frac{5}{14}$ **43.** $\frac{3}{14}$ **45.** $-\frac{11}{17}$ **47.** $\frac{7}{8}$ **49.** 14 **51.** equivalent **53.** not equivalent **55.** equivalent **57.** equivalent **59.** not equivalent **61.** not equivalent **63.** $\frac{1}{4}$ of a shift **65.** $\frac{1}{2}$ mi

67. a. $\frac{4}{9}$ **b.** 15 **c.** $\frac{5}{9}$ **69.** $\frac{5}{12}$ of the wall **71. a.** 13 states **b.** $\frac{13}{50}$ **73.** $\frac{1}{4}$ of the movies **75.** 364 **77.** 2322 **79.** 2520 **81.** answers may vary **83.** $\frac{3}{5}$ **85.** $\frac{9}{25}$ **87.** $\frac{1}{25}$ **89.** $2^2 \cdot 3^5 \cdot 5 \cdot 7$ **91.** answers may vary **93.** no; answers may vary **95.** $\frac{3}{50}$ **97.** answers may vary **99.** $\frac{1}{25}$ **101.** answers may vary **103.** 786, 222, 900, 1470 **105.** 6; answers may vary

Section 3.3

Vocabulary, Readiness & Video Check **1.** $\frac{a \cdot c}{b \cdot d}$ **3.** $\frac{2 \cdot 2 \cdot 2}{7}; \frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7}$ **5.** $\frac{a \cdot d}{b \cdot c}$ **7.** $\frac{2}{15}$ **9.** $\frac{6}{35}$ **11.** $\frac{9}{8}$ **13.** We have a negative fraction times a negative fraction, and a negative number times a negative number is a positive number. **15.** numerator; denominator **17.** radius $= \frac{1}{2} \cdot$ diameter

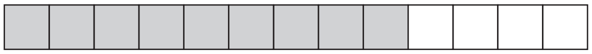
Exercise Set 3.3 **1.** $\frac{12}{77}$ **3.** $-\frac{9}{50}$ **5.** $\frac{1}{15}$ **7.** $\frac{6}{35}$ **9.** $\frac{5}{28}$ **11.** $\frac{1}{70}$ **13.** 0 **15.** $\frac{18}{55}$ **17.** $\frac{1}{56}$ **19.** $\frac{1}{125}$ **21.** $\frac{4}{9}$ **23.** $-\frac{4}{27}$ **25.** $\frac{4}{5}$ **27.** $-\frac{1}{6}$ **29.** $-\frac{16}{9}$ **31.** $-\frac{1}{6}$ **33.** $\frac{1}{100}$ **35.** $\frac{35}{36}$ **37.** $\frac{8}{45}$ **39.** undefined **41.** 0 **43.** $\frac{10}{27}$ **45.** $-\frac{1}{4}$ **47.** $\frac{3}{4}$ **49.** $\frac{9}{16}$ **51.** $\frac{77}{2}$ **53.** $\frac{64}{75}$ **55.** $-\frac{1}{36}$ **57. a.** $\frac{1}{3}$ **b.** $\frac{12}{25}$ **59. a.** $-\frac{36}{55}$ **b.** $-\frac{44}{45}$ **61.** 50 **63.** 20 **65.** 112 **67.** 120 million **69.** 868 mi **71.** $\frac{3}{16}$ in. **73.** \$1838 **75.** 50 libraries **77.** $\frac{1}{14}$ sq ft **79.** 3840 mi **81.** 2400 mi **83.** 201 **85.** 196 **87.** answers may vary **89.** $\frac{2}{5}$ **91.** 5 **93.** 40,718,500 people **95.** 247,568,480 Americans

Section 3.4

Vocabulary, Readiness & Video Check **1.** like; unlike **3.** $-\frac{a}{b}$ **5.** least common denominator (LCD) **7.** unlike **9.** like **11.** like **13.** unlike **15.** numerators; denominator **17.** $P = \frac{5}{12} + \frac{7}{12} + \frac{5}{12} + \frac{7}{12}$; 2 meters **19.** Multiplying by 1 does not change the value of the fraction.

Exercise Set 3.4 **1.** $\frac{3}{7}$ **3.** $\frac{1}{5}$ **5.** $\frac{2}{3}$ **7.** $-\frac{1}{4}$ **9.** $-\frac{1}{2}$ **11.** $\frac{13}{11}$ **13.** $\frac{7}{13}$ **15.** $-\frac{1}{9}$ **17.** $\frac{6}{11}$ **19.** $\frac{3}{5}$ **21.** 1 **23.** $\frac{3}{4}$ **25.** $-\frac{10}{3}$ **27.** $\frac{4}{5}$ **29.** $-\frac{19}{33}$ **31.** $\frac{13}{21}$ **33.** $\frac{9}{10}$ **35.** $-\frac{13}{14}$ **37.** $-\frac{3}{4}$ **39.** $\frac{5}{4}$ **41.** $\frac{2}{5}$ **43.** 1 in. **45.** 2 m **47.** $\frac{7}{10}$ mi **49.** $\frac{13}{50}$ **51.** $\frac{7}{25}$ **53.** $\frac{1}{50}$ **55.** 45 **57.** 72 **59.** 150 **61.** 168 **63.** 126 **65.** 168 **67.** 14 **69.** 20 **71.** 25 **73.** 56 **75.** 8 **77.** 20 **79.** $\frac{86}{100}; \frac{80}{100}; \frac{58}{100}; \frac{68}{100}; \frac{12}{100}; \frac{84}{100}; \frac{68}{100}; \frac{81}{100}; \frac{90}{100}; \frac{79}{100}; \frac{74}{100}; \frac{78}{100}$ **81.** drugs, health and beauty aids **83.** 9 **85.** 81 **87.** 24 **89.** $\frac{2}{7} + \frac{9}{7} = \frac{11}{7}$ **91.** answers may vary **93.** 1; answers may vary **95.** 814 **97.** answers may vary **99. a**, **b**, and **d**

Integrated Review **1.** $\frac{3}{7}$ **2.** $\frac{5}{4}$ or $1\frac{1}{4}$ **3.** $\frac{73}{85}$ **4.**



5. -1 **6.** 17 **7.** 0 **8.** undefined **9.** $5 \cdot 13$ **10.** $2 \cdot 5 \cdot 7$ **11.** $3^2 \cdot 5 \cdot 7$ **12.** $3^2 \cdot 7^2$ **13.** $\frac{1}{7}$ **14.** $\frac{6}{5}$ **15.** $-\frac{14}{15}$ **16.** $-\frac{9}{10}$ **17.** $\frac{2}{5}$ **18.** $\frac{3}{8}$ **19.** $\frac{11}{14}$ **20.** $\frac{7}{11}$ **21.** not equivalent **22.** equivalent **23. a.** $\frac{1}{25}$ **b.** 48 **c.** $\frac{24}{25}$ **24. a.** $\frac{43}{121}$ **b.** 234 **c.** $\frac{78}{121}$ **25.** 30 **26.** 14 **27.** 90 **28.** $\frac{28}{36}$ **29.** $\frac{55}{75}$ **30.** $\frac{40}{48}$ **31.** $\frac{6}{5}$ **32.** $\frac{3}{5}$ **33.** $\frac{3}{5}$ **34.** $\frac{27}{20}$ **35.** $\frac{9}{35}$ **36.** $\frac{12}{35}$ **37.** $\frac{98}{5}$ **38.** $\frac{9}{250}$ **39.** $-\frac{28}{45}$ **40.** $\frac{10}{11}$ **41.** $-\frac{2}{3}$ **42.** $-\frac{2}{45}$ **43.** 24 lots **44.** $\frac{3}{4}$ ft

Section 3.5

Calculator Explorations **1.** $\frac{37}{80}$ **3.** $\frac{95}{72}$ **5.** $\frac{394}{323}$

Vocabulary, Readiness & Video Check **1.** equivalent; least common denominator **3.** $\frac{4}{24}; \frac{15}{24}; \frac{19}{24}$ **5.** expression **7.** 6 **9.** 12 **11.** 56 **13.** 12 **15.** The fractions are unlike, so the numerators cannot be combined. **17.** $\frac{3}{2} + \frac{1}{3}$; 6

Exercise Set 3.5 **1.** $\frac{5}{6}$ **3.** $\frac{1}{6}$ **5.** $-\frac{4}{33}$ **7.** $-\frac{3}{14}$ **9.** $\frac{3}{5}$ **11.** $\frac{19}{12}$ **13.** $\frac{11}{36}$ **15.** $\frac{12}{7}$ **17.** $\frac{89}{99}$ **19.** $\frac{1}{2}$ **21.** $\frac{13}{27}$ **23.** $-\frac{8}{33}$ **25.** $\frac{3}{14}$ **27.** $\frac{1}{35}$ **29.** $-\frac{11}{36}$ **31.** $\frac{1}{20}$ **33.** $\frac{33}{56}$ **35.** $\frac{17}{16}$ **37.** $\frac{8}{9}$ **39.** $-\frac{11}{30}$ **41.** $-\frac{53}{42}$ **43.** $\frac{11}{18}$ **45.** $\frac{98}{143}$ **47.** $-\frac{11}{60}$ **49.** $\frac{19}{20}$ **51.** $\frac{56}{45}$ **53.** $-\frac{5}{24}$ **55.** $-\frac{9}{1000}$ **57.** $<$ **59.** $>$ **61.** $>$ **63.** $\frac{13}{12}$ **65.** $\frac{1}{4}$ **67.** $\frac{11}{6}$ **69.** $\frac{34}{15}$ cm **71.** $\frac{17}{10}$ m **73.** $x + \frac{1}{2}$ **75.** $-\frac{3}{8} - x$ **77.** $\frac{7}{100}$ mph **79.** $\frac{5}{8}$ in. **81.** $\frac{49}{100}$ of students **83.** $\frac{47}{32}$ in. **85.** $\frac{19}{25}$ **87.** $\frac{3}{40}$ **89.** $\frac{63}{100}$ **91.** 20 **93.** -6 **95.** $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$ **97.** $\frac{223}{540}$ **99.** $\frac{49}{44}$
101. answers may vary **103.** marketing mail

Section 3.6

Vocabulary, Readiness & Video Check **1.** complex **3.** division **5.** addition **7.** distributive property **9.** Since x is squared and the replacement value is negative, we use parentheses to make sure the whole value of x is squared. Without parentheses, the exponent would not apply to the negative sign. **11.** division

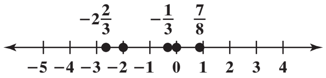
Exercise Set 3.6 **1.** $\frac{1}{6}$ **3.** $\frac{7}{3}$ **5.** $\frac{1}{6}$ **7.** $\frac{23}{22}$ **9.** $\frac{2}{13}$ **11.** $\frac{17}{60}$ **13.** $\frac{5}{8}$ **15.** $\frac{35}{9}$ **17.** $-\frac{17}{45}$ **19.** $\frac{11}{8}$ **21.** $\frac{29}{10}$ **23.** $\frac{27}{32}$ **25.** $\frac{1}{100}$ **27.** $\frac{9}{64}$ **29.** $\frac{7}{6}$ **31.** $-\frac{2}{5}$ **33.** $-\frac{2}{9}$ **35.** $\frac{11}{9}$ **37.** $\frac{5}{2}$ **39.** $\frac{10}{27}$ **41.** $\frac{9}{20}$ **43.** $-\frac{13}{2}$ **45.** $\frac{9}{25}$ **47.** $-\frac{5}{32}$ **49.** 1 **51.** $-\frac{2}{5}$ **53.** $-\frac{11}{40}$ **55.** $\frac{7}{3}$ **57.** $\frac{18}{5}$ **59.** $\frac{53}{8}$ **61.** $\frac{83}{7}$ **63.** $\frac{187}{20}$ **65.** $\frac{500}{3}$ **67.** $3\frac{2}{5}$ **69.** $4\frac{5}{8}$ **71.** $3\frac{2}{15}$ **73.** 15 **75.** $1\frac{7}{175}$ **77.** $6\frac{65}{112}$ **79.** $\frac{7}{2}$ or $3\frac{1}{2}$ **81.** $\frac{49}{6}$ or $8\frac{1}{6}$ **83.** answers may vary **85.** no; answers may vary **87.** $\frac{5}{8}$ **89.** $\frac{11}{56}$ **91.** halfway between a and b **93.** false **95.** true **97.** true **99.** addition; answers may vary **101.** subtraction, multiplication, addition, division **103.** division, multiplication, subtraction, addition **105.** $-\frac{77}{16}$ **107.** $-\frac{55}{16}$

Section 3.7

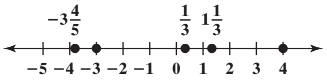
Calculator Explorations **1.** $\frac{280}{11}$ **3.** $\frac{3776}{35}$ **5.** $26\frac{1}{14}$ **7.** $92\frac{3}{10}$

Vocabulary, Readiness & Video Check **1.** mixed number **3.** round **5.** The denominator of the mixed number we're graphing, $-3\frac{4}{5}$, is 5. **7.** The fractional part of a mixed number should always be a proper fraction. **9.** We're adding two mixed numbers with unlike signs, so the answer has the sign of the mixed number with the larger absolute value, which in this case is negative.

Exercise Set 3.7 **1.**



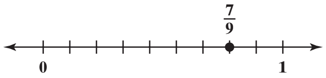
3.



5. **b** **7.** **a** **9.** $\frac{8}{21}$ **11.** $4\frac{3}{8}$ **13.** $7\frac{7}{10}$; 8 **15.** $23\frac{31}{35}$; 24 **17.** $12\frac{1}{2}$ **19.** $5\frac{1}{2}$ **21.** $18\frac{2}{3}$ **23.** **a** **25.** **b** **27.** $6\frac{2}{3}$; 7 **29.** $13\frac{11}{14}$; 14 **31.** $17\frac{7}{25}$ **33.** $47\frac{53}{84}$ **35.** $25\frac{5}{14}$ **37.** $13\frac{13}{24}$ **39.** $2\frac{3}{5}$; 3 **41.** $7\frac{5}{14}$; 7 **43.** $\frac{24}{25}$ **45.** $3\frac{5}{9}$ **47.** $15\frac{3}{4}$ **49.** 4 **51.** $5\frac{11}{14}$ **53.** $6\frac{2}{9}$ **55.** $\frac{25}{33}$ **57.** $35\frac{13}{18}$ **59.** $2\frac{1}{2}$ **61.** $73\frac{7}{30}$ **63.** $\frac{11}{14}$ **65.** $5\frac{4}{7}$ **67.** $13\frac{16}{33}$ **69.** $-5\frac{2}{7} - x$ **71.** $1\frac{9}{10} \cdot x$ **73.** $3\frac{3}{16}$ mi **75.** $9\frac{2}{5}$ in. **77.** $7\frac{13}{20}$ in. **79.** $3\frac{1}{2}$ sq yd **81.** $\frac{15}{16}$ sq in. **83.** $21\frac{5}{24}$ m **85.** no; it will be $\frac{1}{12}$ ft short **87.** $4\frac{2}{3}$ m **89.** $9\frac{23}{60}$ min **91.** $2\frac{1}{6}$ min **93.** $-10\frac{3}{25}$ **95.** $-24\frac{7}{8}$ **97.** $-13\frac{59}{60}$ **99.** $4\frac{2}{7}$ **101.** $-1\frac{23}{24}$ **103.** $\frac{73}{1000}$ **105.** 4 **107.** 167 **109.** **a**, **b**, **c** **111.** Incorrect; to divide mixed numbers, first write each mixed number as an improper fraction. **113.** answers may vary **115.** answers may vary **117.** answers may vary

Chapter 3 Vocabulary Check **1.** reciprocals **2.** composite number **3.** equivalent **4.** improper fraction **5.** prime number **6.** simplest form **7.** proper fraction **8.** mixed number **9.** numerator; denominator **10.** prime factorization **11.** undefined **12.** 0 **13.** like **14.** least common denominator **15.** complex fraction **16.** cross products

Chapter 3 Review **1.** proper fraction **2.** improper fraction **3.** proper fraction **4.** mixed number **5.** $\frac{2}{6}$ **6.** $\frac{4}{7}$ **7.** $\frac{7}{3}$ or $2\frac{1}{3}$ **8.** $\frac{13}{4}$ or $3\frac{1}{4}$ **9.** $\frac{11}{12}$ **10.** **a.** 108 **b.** $\frac{108}{131}$ **11.** -1 **12.** 1 **13.** 0 **14.** undefined **15.**



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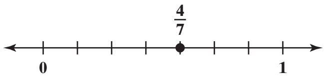

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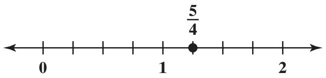

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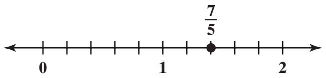


19. $2^2 \cdot 17$ **20.** $2 \cdot 3^2 \cdot 5$ **21.** $3 \cdot 5^2 \cdot 11$ **22.** $3 \cdot 5 \cdot 17$ **23.** $\frac{3}{7}$ **24.** $\frac{5}{9}$ **25.** $-\frac{1}{3}$ **26.** $-\frac{1}{2}$ **27.** $\frac{29}{32}$ **28.** $\frac{18}{23}$ **29.** 8 **30.** 6 **31.** $\frac{2}{3}$ of a foot **32.** $\frac{3}{5}$ of the cars **33.** no **34.** yes **35.** $-\frac{3}{10}$ **36.** $\frac{5}{14}$ **37.** 9 **38.** $\frac{5}{3}$ **39.** $-\frac{1}{27}$ **40.** $\frac{25}{144}$ **41.** $\frac{2}{15}$ **42.** $-\frac{63}{10}$ **43.** $\frac{1}{7}$ **44.** $\frac{23}{14}$ **45.** -2 **46.** $\frac{15}{4}$ **47.** $-\frac{5}{6}$ **48.** $\frac{27}{2}$ **49.** $\frac{12}{7}$ **50.** $-\frac{15}{2}$ **51.** $\frac{77}{48}$ sq ft **52.** $\frac{4}{9}$ sq m **53.** $\frac{10}{11}$ **54.** $\frac{2}{3}$ **55.** $-\frac{1}{3}$ **56.** $\frac{4}{5}$ **57.** $\frac{1}{21}$ **58.** $-\frac{1}{15}$ **59.** 21 **60.** 24 **61.** 20 **62.** 35 **63.** 49 **64.** 45 **65.** 40 **66.** 10 **67.** $\frac{3}{4}$ of his homework **68.** $\frac{3}{2}$ mi **69.** $\frac{11}{18}$ **70.** $\frac{7}{26}$ **71.** $-\frac{1}{12}$ **72.** $-\frac{5}{12}$ **73.** $-\frac{15}{14}$ **74.** $\frac{7}{36}$ **75.** $\frac{91}{150}$ **76.** $\frac{5}{18}$ **77.** $\frac{19}{9}$ m **78.** $\frac{3}{2}$ ft **79.** $\frac{21}{50}$ of the donors **80.** $\frac{1}{4}$ yd **81.** $<$ **82.** $>$ **83.** $>$ **84.** $<$ **85.** $\frac{4}{7}$ **86.** $\frac{3}{11}$ **87.** -2 **88.** -7 **89.** $\frac{4}{9}$ **90.** $-\frac{3}{10}$ **91.** $3\frac{3}{4}$ **92.** 3 **93.** 1 **94.** $31\frac{1}{4}$ **95.** $\frac{11}{5}$ **96.** $\frac{35}{9}$ **97.** $\frac{8}{13}$ **98.** $-\frac{1}{27}$ **99.** $\frac{29}{110}$ **100.** $-\frac{1}{7}$ **101.** $45\frac{16}{21}$ **102.** $20\frac{7}{24}$ **103.** $4\frac{19}{35}$ **104.** $2\frac{51}{55}$ **105.** Exact: $5\frac{1}{5}$ Estimate: 6 **106.** Exact: $5\frac{5}{11}$ Estimate: 8 **107.** $5\frac{1}{4}$ **108.** $2\frac{29}{46}$ **109.** 22 mi **110.** $36\frac{2}{3}$ g **111.** Each measurement is $4\frac{1}{4}$ in. **112.** $\frac{7}{10}$ yd **113.** $-27\frac{5}{14}$ **114.** $-\frac{33}{40}$ **115.** $1\frac{5}{27}$ **116.** $-3\frac{15}{16}$ **117.** $\frac{7}{12}$ **118.** $\frac{1}{4}$ **119.** 9 **120.** $\frac{27}{2}$ or $13\frac{1}{2}$ **121.** $\frac{1}{6}$ **122.** $\frac{1}{5}$ **123.** $\frac{11}{12}$ **124.** $\frac{27}{55}$ **125.** Exact: 10 Estimate: 8 **126.** Exact: $12\frac{3}{4}$ Estimate: 12 **127.** $2\frac{1}{3}$ **128.** $6\frac{2}{5}$ **129.** $13\frac{5}{12}$ **130.** $12\frac{3}{8}$ **131.** $3\frac{16}{35}$ **132.** $8\frac{1}{21}$ **133.** $\frac{11}{25}$ **134.** $\frac{1}{144}$ **135.** $-\frac{1}{12}$ **136.** $6\frac{7}{20}$ lb **137.** $\frac{47}{61}$ in. **138.** $44\frac{1}{2}$ yd **139.** $40\frac{1}{2}$ sq ft

Chapter 3 Getting Ready for the Test **1.** A **2.** B **3.** D **4.** C **5.** C **6.** B **7.** C **8.** D **9.** B **10.** A **11.** F **12.** H **13.** B **14.** D **15.** C **16.** A **17.** D **18.** C **19.** C **20.** B

Chapter 3 Test **1.** $\frac{7}{16}$ **2.** $\frac{23}{3}$ **3.** $18\frac{3}{4}$ **4.** $\frac{4}{35}$ **5.** $-\frac{3}{5}$ **6.** not equivalent **7.** equivalent **8.** $2^2 \cdot 3 \cdot 7$ **9.** $3^2 \cdot 5 \cdot 11$ **10.** 72 **11.** $\frac{8}{9}$ **12.** $-\frac{2}{3}$ **13.** $\frac{4}{3}$ or $1\frac{1}{3}$ **14.** $-\frac{4}{3}$ or $-1\frac{1}{3}$ **15.** $\frac{8}{21}$ **16.** $\frac{13}{24}$ **17.** $\frac{16}{45}$ **18.** 16 **19.** $\frac{1}{7}$ **20.** $\frac{7}{50}$ **21.** $\frac{4}{11}$ **22.** 9 **23.** $\frac{3}{4}$ **24.** $14\frac{1}{40}$ **25.** $16\frac{8}{11}$ **26.** $\frac{64}{3}$ or $21\frac{1}{3}$ **27.** $22\frac{1}{2}$ **28.** $-\frac{5}{3}$ or $-1\frac{2}{3}$ **29.** $\frac{9}{16}$ **30.** $\frac{3}{8}$ **31.** $\frac{11}{12}$ **32.** $\frac{76}{21}$ or $3\frac{13}{21}$ **33.** $\frac{5}{2}$ or $2\frac{1}{2}$ **34.** $\frac{4}{31}$ **35.** $3\frac{3}{4}$ ft **36.** $\frac{23}{50}$ **37.** $\frac{13}{50}$ **38.** \$2820 **39.** perimeter: $3\frac{1}{3}$ ft; area: $\frac{2}{3}$ sq ft **40.** 24 mi

Cumulative Review **1.** one hundred twenty-six; Sec. 1.2, Ex. 5 **2.** one hundred fifteen; Sec. 1.2 **3.** twenty-seven thousand thirty-our; Sec. 1.2, Ex. 6 **4.** six thousand five hundred seventy-three; Sec. 1.2 **5.** 159; Sec. 1.3, Ex. 1 **6.** 631; Sec. 1.3 **7.** 514; Sec. 1.4, Ex. 3 **8.** 933; Sec. 1.4 **9.** 278,000; Sec. 1.5, Ex. 2 **10.** 1440; Sec. 1.5 **11.** 57,600 megabytes; Sec. 1.6, Ex. 11 **12.** 1305 mi; Sec. 1.6 **13.** 7089 R 5; Sec. 1.7, Ex. 7 **14.** 379 R 10; Sec. 1.7 **15.** 7^3; Sec. 1.9, Ex. 1 **16.** 7^2; Sec. 1.9 **17.** $3^4 \cdot 17^3$; Sec. 1.9, Ex. 4 **18.** $9^4 \cdot 5^2$; Sec. 1.9 **19.** 8; Sec. 2.1, Ex. 2 **20.** 52; Sec. 2.1 **21.** -7188; Sec. 2.2, Ex. 1 **22.** -21; Sec. 2.2 **23.** -4; Sec. 2.3, Ex. 3 **24.** 5; Sec. 2.3 **25.** 3; Sec. 2.4, Ex. 9 **26.** 10; Sec. 2.4 **27.** 25; Sec. 2.5, Ex. 8 **28.** -16; Sec. 2.5 **29.** -2; Sec. 2.6, Ex. 5 **30.** 25; Sec. 2.6 **31.** $3 \cdot 3 \cdot 5$ or $3^2 \cdot 5$; Sec. 3.2, Ex. 1 **32.** $2 \cdot 2 \cdot 23$ or $2^2 \cdot 23$; Sec. 3.2 **33.** $\frac{10}{33}$; Sec. 3.3, Ex. 1 **34.** $\frac{2}{35}$; Sec. 3.3 **35.** $\frac{1}{8}$; Sec. 3.3, Ex. 2 **36.** $\frac{3}{25}$; Sec. 3.3 **37.** $\frac{2}{5}$; Sec. 3.1, Ex. 3 **38.** $2^2 \cdot 3 \cdot 13$; Sec. 3.2 **39. a.** $\frac{38}{9}$ **b.** $\frac{19}{11}$; Sec. 3.6, Ex. 8 **40.** $\frac{39}{5}$; Sec. 3.6 **41.** $\frac{7}{11}$; Sec. 3.2, Ex. 5 **42.** $\frac{2}{3}$; Sec. 3.2 **43.** $2\frac{11}{12}$; Sec. 3.7, Ex. 2 **44.** $\frac{8}{3}$ or $2\frac{2}{3}$; Sec. 3.7 **45.** $\frac{5}{12}$; Sec. 3.3, Ex. 10 **46.** $\frac{11}{56}$; Sec. 3.7

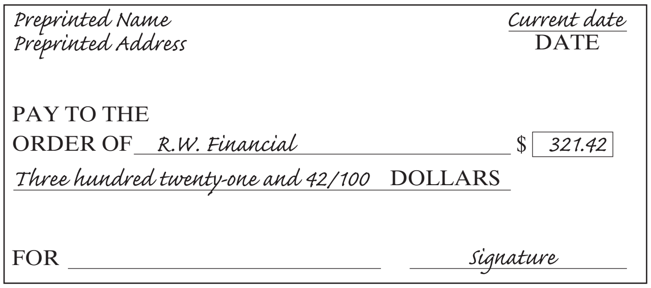
Chapter 4 Decimals

Section 4.1

Vocabulary, Readiness & Video Check **1.** words; standard form **3.** decimals **5.** tenths; tens **7.** tens **9.** tenths **11.** as "and" **13.** Reading a decimal correctly gives you the correct place value, which tells you the denominator of your equivalent fraction. **15.** When rounding, we look at the digit to the right of the place value we're rounding to. In this case, we look at the hundredths-place digit, which is 7.

Exercise Set 4.1 **1.** six and fifty-two hundredths **3.** sixteen and twenty-three hundredths **5.** negative two hundred five thousandths **7.** one hundred sixty-seven and nine thousandths **9.** three thousand and four hundredths **11.** one hundred five and six tenths **13.** two and forty-three hundredths

15.

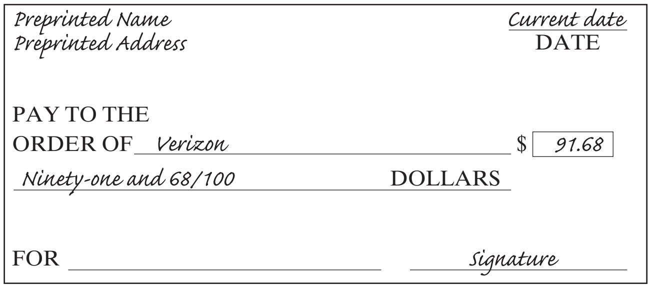


Preprinted Name
Preprinted Address
Current date
DATE

PAY TO THE
ORDER OF R.W. Financial $ 321.42
Three hundred twenty-one and 42/100 DOLLARS

FOR
Signature

17.



Preprinted Name
Preprinted Address
Current date
DATE

PAY TO THE
ORDER OF Verizon $ 91.68
Ninety-one and 68/100 DOLLARS

FOR
Signature

19. 6.5 **21.** 9.08 **23.** -705.625 **25.** 0.0046 **27.** $\frac{3}{10}$ **29.** $\frac{27}{100}$ **31.** $\frac{1}{5}$ **33.** $5\frac{2}{5}$ **35.** $-\frac{29}{500}$ **37.** $7\frac{1}{125}$ **39.** $15\frac{401}{500}$ **41.** $\frac{601}{2000}$ **43.** 0.8; $\frac{8}{10}$ or $\frac{4}{5}$ **45.** seventy-seven thousandths; $\frac{77}{1000}$ **47.** $<$ **49.** $<$ **51.** $<$ **53.** $=$ **55.** $<$ **57.** $>$ **59.** $<$ **61.** $>$ **63.** 0.6 **65.** 98,210 **67.** -0.23 **69.** 0.594 **71.** 3.1 **73.** 3.142 **75.** \$27 **77.** \$0.20 **79.** 0.3 cm **81.** 1.47 hr **83.** \$68 **85.** 89.4 people per sq mi **87.** 24.623 hr **89.** 0.5 min **91.** 5766 **93.** 35 **95.** b **97.** a **99.** answers may vary **101.** 7.12 **103.** $\frac{26{,}849{,}577}{100{,}000{,}000{,}000}$ **105.** answers may vary **107.** answers may vary **109.** 0.26499, 0.25786 **111.** 0.10299, 0.1037, 0.1038, 0.9 **113.** \$4200 million

Section 4.2

Calculator Explorations **1.** 328.742 **3.** 5.2414 **5.** 865.392

Vocabulary, Readiness & Video Check **1.** last **3.** vertically **5.** 0.5 **7.** 1.26 **9.** 8.9 **11.** 0.6 **13.** Lining up the decimal points also lines up place values, so we only add or subtract digits in the same place values. **15.** So that the subtraction can be written vertically with decimal points lined up

Exercise Set 4.2 **1.** 3.5 **3.** 6.83 **5.** 27.0578 **7.** 91.204 **9.** -8.57 **11.** 11.16 **13.** Exact: 465.56; Estimate: $\begin{array}{r} 230 \\ +\ 230 \\ \hline 460 \end{array}$ **15.** Exact: 115.123; Estimate: $\begin{array}{r} 100 \\ 6 \\ +\ \ 9 \\ \hline 115 \end{array}$ **17.** 56.432 **19.** 6.5 **21.** 15.3 **23.** 598.23 **25.** Exact: 1.83; Estimate: $6 - 4 = 2$ **27.** 861.6 **29.** Exact: 876.6; Estimate: $\begin{array}{r} 1000 \\ -\ 100 \\ \hline 900 \end{array}$ **31.** 194.4 **33.** -6.32 **35.** -6.4 **37.** 3.1 **39.** 2.9988 **41.** 16.3 **43.** 3.1 **45.** -5.62 **47.** 776.89 **49.** -549.8 **51.** 861.6 **53.** 512.101 **55.** 0.088 **57.** -180.44 **59.** -1.1 **61.** 3.81 **63.** 3.39 **65.** 1.61 **67.** \$7.52 **69.** $-$\$0.40 **71.** 28.56 m **73.** 16.9 in. **75.** 195.8 mph **77.** 29.4 billion or 29,400,000,000 **79.** \$2356.6 million **81.** 326.3 in. **83.** 67.44 ft **85.** 47.395 kph **87.** Switzerland **89.** 5.2 lb **91.**

Country	Pounds of Chocolate per Person
Switzerland	19.8
Germany	17.4
Ireland	16.3
UK	16.3
Norway	14.6

93. 46 **95.** $\frac{4}{9}$ **97.** incorrect; $\begin{array}{r} 9.200 \\ 8.630 \\ +\ 4.005 \\ \hline 21.835 \end{array}$

99. 6.08 in. **101.** \$1.20 **103.** 1 nickel, 1 dime, and 2 pennies; 3 nickels and 2 pennies; 1 dime and 7 pennies; 2 nickels and 7 pennies **105.** answers may vary **107.** answers may vary

Section 4.3

Vocabulary, Readiness & Video Check **1.** sum **3.** circumference **5.** right; zeros **7.** 4 **9.** 4 **11.** 8 **13.** We need to learn where to place the decimal point in the product. **15.** We just need to know how to move the decimal point. 100 has two zeros, so we move the decimal point two places to the right. **17.** We used an approximation for π. The exact answer is 10π cm.

Exercise Set 4.3 **1.** 1.36 **3.** 0.6 **5.** -17.595 **7.** 55.008 **9.** Exact: 28.56; Estimate: $7 \times 4 = 28$ **11.** 0.1041 **13.** Exact: 8.23854; Estimate: $\begin{array}{r}1\\ \times\ 8\\ \hline 8\end{array}$ **15.** 11.2746 **17.** 65 **19.** 0.83 **21.** -7093 **23.** 70 **25.** 0.0983 **27.** 0.02523 **29.** 0.0492 **31.** 14,790 **33.** 1.29 **35.** -9.3762 **37.** 0.5623 **39.** 36.024 **41.** 3,260,000,000 **43.** 49,800,000 **45.** -0.6 **47.** 17.3 **49.** 10π cm $\approx$ 31.4 cm **51.** 18.2π yd $\approx$ 57.148 yd **53.** \$715.20 **55.** 24.8 g **57.** 15.82 sq in. **59.** 250π ft $\approx$ 785 ft **61.** 135π m $\approx$ 423.9 m **63.** 64.9605 in. **65. a.** 62.8 m and 125.6 m **b.** yes **67.** \$427 **69.** 1000.72 Canadian dollars **71.** 422.47 New Zealand dollars **73.** 486 **75.** -9 **77.** 3.64 **79.** 3.56 **81.** -0.1105 **83.** 3,831,600 mi **85.** answers may vary **87.** answers may vary

Section 4.4

Calculator Explorations **1.** not reasonable **3.** reasonable

Vocabulary, Readiness & Video Check **1.** quotient; divisor; dividend **3.** left; zeros **5.** 5.9 **7.** 0 **9.** 1 **11.** undefined **13.** a whole number **15.** We just need to know how to move the decimal point. 1000 has three zeros, so we move the decimal point in the decimal number three places to the left. **17.** We want the answer rounded to the nearest tenth, so we go to one extra place value, to the hundredths place, in order to round.

Exercise Set 4.4 **1.** 4.6 **3.** 0.094 **5.** 300 **7.** 2.6 **9.** Exact: 6.6; Estimate: $\begin{array}{r}6\\ 6\overline{)36}\end{array}$ **11.** 0.413 **13.** -600 **15.** 7 **17.** 4.8 **19.** 2100 **21.** 5.8 **23.** 5.5 **25.** Exact: 9.8; Estimate: $\begin{array}{r}10\\ 7\overline{)70}\end{array}$ **27.** 9.6 **29.** 45 **31.** 54.592 **33.** 0.0055 **35.** 179 **37.** 23.87 **39.** 114.0 **41.** 0.54982 **43.** 2.687 **45.** -0.0129 **47.** 12.6 **49.** 1.31 **51.** 0.045625 **53.** 0.413 **55.** -8 **57.** -7.2 **59.** 1400 **61.** 30 **63.** $-58{,}000$ **65.** -0.69 **67.** 0.024 **69.** 65 **71.** -5.65 **73.** -7.0625 **75.** 11 qt **77.** 5.1 m **79.** 11.4 boxes **81.** 24 tsp **83.** 8 days **85.** 146.6 mi per week **87.** 5.8 mps **89.** $\frac{9}{10}$ **91.** $\frac{1}{20}$ **93.** 4.26 **95.** 1.578 **97.** -26.66 **99.** 904.29 **101.** **c** **103.** **b** **105.** 85.5 **107.** 8.6 ft **109.** answers may vary **111.** 65.2–82.6 knots **113.** 27.3 m

Integrated Review **1.** 2.57 **2.** 4.05 **3.** 8.9 **4.** 3.5 **5.** 0.16 **6.** 0.24 **7.** 0.27 **8.** 0.52 **9.** -4.8 **10.** 6.09 **11.** 75.56 **12.** 289.12 **13.** -24.974 **14.** -43.875 **15.** -8.6 **16.** 5.4 **17.** -280 **18.** 1600 **19.** 224.938 **20.** 145.079 **21.** 0.56 **22.** -0.63 **23.** 27.6092 **24.** 145.6312 **25.** 5.4 **26.** -17.74 **27.** -414.44 **28.** -1295.03 **29.** -34 **30.** -28 **31.** 116.81 **32.** 18.79 **33.** 156.2 **34.** 1.562 **35.** 25.62 **36.** 5.62 **37.** 200 mi **38.** \$2.70 **39.** \$18.28 billion, or \$18,280,000,000

Section 4.5

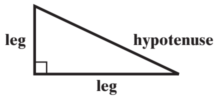
Vocabulary, Readiness & Video Check **1.** false **3.** false **5.** We place a bar over just the repeating digits, and only 6 repeats in our decimal answer. **7.** The fraction bar serves as a grouping symbol. **9.** $4(0.3) - (-2.4)$

Exercise Set 4.5 **1.** 0.2 **3.** 0.68 **5.** 0.75 **7.** -0.08 **9.** 2.25 **11.** $0.91\overline{6}$ **13.** 0.425 **15.** 0.45 **17.** $-0.\overline{3}$ **19.** 0.4375 **21.** $0.\overline{63}$ **23.** 5.85 **25.** 0.624 **27.** -0.33 **29.** 0.44 **31.** 0.6 **33.** 0.62 **35.** 0.92 **37.** 0.02 **39.** $<$ **41.** $=$ **43.** $<$ **45.** $<$ **47.** $<$ **49.** $>$ **51.** $<$ **53.** $<$ **55.** 0.32, 0.34, 0.35 **57.** 0.49, 0.491, 0.498 **59.** $5.23, \frac{42}{8}, 5.34$ **61.** $0.612, \frac{5}{8}, 0.649$ **63.** 0.59 **65.** -3 **67.** 5.29 **69.** 9.24 **71.** 0.2025 **73.** -1.29 **75.** -15.4 **77.** -3.7 **79.** 25.65 sq in. **81.** 0.248 sq yd **83.** 5.76 **85.** 5.7 **87.** 3.6 **89.** 72 **91.** $\frac{5}{2}$ **93.** $= 1$ **95.** > 1 **97.** < 1 **99.** 0.057 **101.** 8200 stations **103.** answers may vary

Section 4.6

Calculator Explorations **1.** 32 **3.** 5.568 **5.** 9.849

Vocabulary, Readiness & Video Check **1.** 10 **3.** squaring **5.**



7. $\sqrt{49} = 7$ because 7^2 or $7 \cdot 7 = 49$. **9.** The Pythagorean theorem works only for right triangles.

Exercise Set 4.6 **1.** 2 **3.** 11 **5.** $\frac{1}{9}$ **7.** $\frac{4}{8} = \frac{1}{2}$ **9.** 1.732 **11.** 3.873 **13.** 6.856 **15.** 5.099 **17.** 6, 7 **19.** 10, 11 **21.** 16 **23.** 9.592 **25.** $\frac{7}{12}$ **27.** 8.426 **29.** 13 in. **31.** 6.633 cm **33.** 52.802 m **35.** 117 mm **37.** 5 **39.** 12 **41.** 17.205 **43.** 44.822 **45.** 42.426 **47.** 1.732 **49.** 8.5 **51.** 141.42 yd **53.** 25.0 ft **55.** 340 ft **57.** $\frac{5}{6}$ **59.** $\frac{2}{5}$ **61.** $\frac{5}{12}$ **63.** 6 **65.** 10 **67.** answers may vary **69.** yes **71.** $(\sqrt{80} - 6)$ in. $\approx$ 2.94 in.

Chapter 4 Vocabulary Check **1.** decimal **2.** numerator; denominator **3.** vertically **4.** and **5.** sum **6.** square root **7.** right triangle; hypotenuse; legs **8.** standard form

Chapter 4 Review **1.** tenths **2.** hundred-thousandths **3.** negative twenty-three and forty-five hundredths **4.** three hundred forty-five hundred-thousandths **5.** one hundred nine and twenty-three hundredths **6.** two hundred and thirty-two millionths **7.** 2.07 **8.** -503.102 **9.** 16,025.0014 **10.** 14.011 **11.** $\frac{4}{25}$ **12.** $\frac{11}{20}$ **13.** $-12\frac{23}{1000}$ **14.** $25\frac{1}{4}$ **15.** $>$ **16.** $=$ **17.** $<$ **18.** $>$ **19.** 0.6 **20.** 0.94 **21.** -42.90 **22.** -16.349 **23.** 9.5 **24.** 5.1 **25.** -7.28 **26.** -12.04 **27.** 320.312 **28.** 148.74236 **29.** 1.7 **30.** 2.49 **31.** -1324.5 **32.** -10.136 **33.** 65.02 **34.** 199.99802 **35.** 52.6 mi **36.** -5.7 **37.** 22.2 in. **38.** 38.9 ft **39.** 72 **40.** 9345 **41.** -78.246 **42.** 73,246.446 **43.** 887,000,000 **44.** 600,000 **45.** 14π m $\approx$ 43.96 m **46.** 20π in. $\approx$ 62.8 in. **47.** 0.0877 **48.** 15.825 **49.** 70 **50.** -0.21 **51.** 8.059 **52.** 30.4 **53.** 0.2365 **54.** -9.3 **55.** 7.3 m **56.** 45 months **57.** 0.8 **58.** -0.923 **59.** $2.\overline{3}$ or 2.333 **60.** $0.21\overline{6}$ or 0.217 **61.** $=$ **62.** $<$ **63.** $<$ **64.** $<$ **65.** 0.832, 0.837, 0.839 **66.** $\frac{5}{8}$, 0.626, 0.685 **67.** 0.42, $\frac{3}{7}$, 0.43 **68.** $\frac{19}{12}$, 1.63, $\frac{18}{11}$ **69.** -11.94 **70.** 3.89 **71.** 7.26 **72.** 0.81 **73.** 55 **74.** -129 **75.** 6.9 sq ft **76.** 5.46 sq in. **77.** 8 **78.** 12 **79.** $\frac{2}{5}$ **80.** $\frac{1}{10}$ **81.** 13 **82.** 29 **83.** 10.7 **84.** 93 **85.** 127.3 ft **86.** 88.2 ft **87.** two and thirty-two ten-thousandths **88.** -16.014 **89.** $\frac{231}{100{,}000}$ **90.** 0.75, $\frac{6}{7}$, $\frac{8}{9}$ **91.** -0.07 **92.** 0.1125 **93.** $>$ **94.** $<$ **95.** 42.9 **96.** 16.35 **97.** \$123.00 **98.** \$3646.00 **99.** -1.7 **100.** 2.49 **101.** 80.668 **102.** -148.74236 **103.** 8.128 **104.** -7.245 **105.** 4900 **106.** 23.904 **107.** 9600 sq ft **108.** yes **109.** 0.1024 **110.** 3.6 **111.** 1 **112.** 6 **113.** $\frac{4}{9}$ **114.** $\frac{1}{11}$ **115.** 86.6 **116.** 20.8 **117.** 48.1 **118.** 19.7

Chapter 4 Getting Ready for the Test **1.** C **2.** B **3.** E **4.** D **5.** B **6.** D **7.** C **8.** A **9.** C **10.** B **11.** D **12.** A **13.** D **14.** A **15.** B **16.** C **17.** C

Chapter 4 Test **1.** forty-five and ninety-two thousandths **2.** 3000.059 **3.** 17.595 **4.** -51.2 **5.** -20.42 **6.** 40.902 **7.** 0.037 **8.** 34.9 **9.** 0.862 **10.** $<$ **11.** $<$ **12.** $\frac{69}{200}$ **13.** $-24\frac{73}{100}$ **14.** -0.5 **15.** 0.941 **16.** 1.93 **17.** -6.2 **18.** 11.4 **19.** 7 **20.** 12.530 **21.** $\frac{8}{10} = \frac{4}{5}$ **22.** 4,583,000,000 **23.** 2.31 sq mi **24.** 18π mi $\approx$ 56.52 mi **25.** **a.** 9904 sq ft **b.** 198.08 oz **26.** 54 mi

Cumulative Review **1.** eighty-five; Sec. 1.2, Ex. 4 **2.** one hundred seven; Sec. 1.2 **3.** one hundred twenty-six; Sec. 1.2, Ex. 5 **4.** five thousand, twenty-six; Sec. 1.2 **5.** 159; Sec. 1.3, Ex. 1 **6.** 19 in.; Sec. 1.3 **7.** 514; Sec. 1.4, Ex. 3 **8.** 121 R 1; Sec. 1.7 **9.** 278,000; Sec. 1.5, Ex. 2 **10.** $2 \cdot 3 \cdot 5$; Sec. 3.2 **11.** 20,296; Sec. 1.6, Ex. 4 **12.** 0; Sec. 1.6 **13.** **a.** 7 **b.** 12 **c.** 1 **d.** 1 **e.** 20 **f.** 1; Sec. 1.7, Ex. 2 **14.** 25; Sec. 1.7 **15.** 1038 mi; Sec. 1.8, Ex. 1 **16.** 11; Sec. 1.9 **17.** 81; Sec. 1.9, Ex. 5 **18.** 125; Sec. 1.9 **19.** 81; Sec. 1.9, Ex. 7 **20.** 1000; Sec. 1.9 **21.** 2; Sec. 2.1, Ex. 3 **22.** 6; Sec. 2.1 **23.** **a.** -11 **b.** 2 **c.** 0; Sec. 2.2, Ex. 5 **24.** **a.** 7 **b.** -4 **c.** 1; Sec. 2.2 **25.** -23; Sec. 2.3, Ex. 4 **26.** -22; Sec. 2.3 **27.** 180; Sec. 1.9, Ex. 8 **28.** 32; Sec. 1.9 **29.** -49; Sec. 2.5, Ex. 9 **30.** -32; Sec. 2.5 **31.** 25; Sec. 2.5, Ex. 8 **32.** -9; Sec. 2.5 **33.** $\frac{4}{3}$; $1\frac{1}{3}$; Sec. 3.1, Ex. 11 **34.** $\frac{11}{4}$; $2\frac{3}{4}$; Sec. 3.1 **35.** $\frac{15}{4}$; $3\frac{3}{4}$; Sec. 3.1, Ex. 12 **36.** $\frac{14}{3}$; $4\frac{2}{3}$; Sec. 3.1 **37.** $2^2 \cdot 3^2 \cdot 7$; Sec. 3.2, Ex. 3 **38.** 62; Sec. 1.4 **39.** $-\frac{36}{13}$; Sec. 3.2, Ex. 8 **40.** $\frac{79}{8}$; Sec. 3.6 **41.** equivalent; Sec. 3.2, Ex. 10 **42.** $>$; Sec. 3.5 **43.** $\frac{10}{33}$; Sec. 3.3, Ex. 1 **44.** $1\frac{1}{2}$; Sec. 3.7 **45.** $\frac{1}{8}$; Sec. 3.3, Ex. 2 **46.** 37; Sec. 3.7 **47.** 829.6561; Sec. 4.2, Ex. 2 **48.** 230.8628; Sec. 4.2 **49.** 18.408; Sec. 4.3, Ex. 1 **50.** 28.251; Sec. 4.3

Chapter 5 Ratio, Proportion, and Measurement

Section 5.1

Vocabulary, Readiness & Video Check **1.** unit **3.** division **5.** numerator; denominator **7.** false **9.** false **11.** true **13.** We can use "to" as in 1 to 2, a colon as in 1 : 2, or a fraction as in $\frac{1}{2}$. **15.** We want a unit rate, which is a rate with a denominator of 1. A unit rate tells us how much of the first quantity (\$) will occur in 1 of the second quantity (years).

Exercise Set 5.1 **1.** $\frac{2}{3}$ **3.** $\frac{77}{100}$ **5.** $\frac{463}{821}$ **7.** $\frac{3}{4}$ **9.** $\frac{8}{25}$ **11.** $\frac{12}{7}$ **13.** $\frac{2}{7}$ **15.** $\frac{4}{1}$ **17.** $\frac{10}{29}$ **19.** $\frac{25}{144}$ **21.** $\frac{5}{4}$ **23.** $\frac{17}{40}$ **25.** $\frac{11}{181}$ **27.** $\frac{1}{3}$ **29.** $\frac{15}{1}$ **31.** $\frac{1 \text{ shrub}}{3 \text{ ft}}$ **33.** $\frac{3 \text{ returns}}{20 \text{ sales}}$ **35.** $\frac{2 \text{ phone lines}}{9 \text{ employees}}$ **37.** $\frac{9 \text{ gal}}{2 \text{ acres}}$ **39.** 75 riders/car **41.** 90 wingbeats/sec **43.** \$50,000/yr **45.** 343,766 voters/senator **47.** 300 good/defective **49.** \$65,000/species **51.** \$8.25 million/charity **53.** **a.** 31.25 computer boards/hr **b.** 33.5 computer boards/hr **c.** Suellen **55.** **a.** $\approx$27.6 miles/gal **b.** $\approx$29.2 miles/gal **c.** the truck **57.** \$11.50 per DVD **59.** \$0.17 per banana **61.** 8 oz: \$0.149 per oz; 12 oz: \$0.133 per oz; 12 oz **63.** 16 oz: \$0.106 per oz; 6 oz: \$0.115 per oz; 16 oz **65.** 12 oz: \$0.191 per oz; 8 oz: \$0.186 per oz; 8 oz **67.** 100: \$0.006 per napkin; 180: \$0.005 per napkin; 180 napkins **69.** 2.3 **71.** 0.15

73. no; answers may vary **75.** 257; 19.2 **77.** 347; 21.6 **79.** 1.5 steps/ft **81.** answers may vary **83.** no; answers may vary **85.** no; $\frac{71}{43}$ **87.** no; $\frac{9}{2}$ **89.** No, the shipment should not be refused. **91.** **a.** $\frac{7}{25}$ **b.** 36 states **c.** $\frac{7}{18}$

Section 5.2

Vocabulary, Readiness & Video Check **1.** proportion; ratio **3.** true **5.** true **7.** false **9.** true **11.** equals or = **13.** a variable

Exercise Set 5.2 **1.** $\frac{10\text{ diamonds}}{6\text{ opals}} = \frac{5\text{ diamonds}}{3\text{ opals}}$ **3.** $\frac{3\text{ printers}}{12\text{ computers}} = \frac{1\text{ printer}}{4\text{ computers}}$ **5.** $\frac{6\text{ eagles}}{58\text{ sparrows}} = \frac{3\text{ eagles}}{29\text{ sparrows}}$ **7.** $\frac{2\frac{1}{4}\text{ cups flour}}{24\text{ cookies}} = \frac{6\frac{3}{4}\text{ cups flour}}{72\text{ cookies}}$ **9.** $\frac{22\text{ vanilla wafers}}{1\text{ cup cookie crumbs}} = \frac{55\text{ vanilla wafers}}{2.5\text{ cups cookie crumbs}}$ **11.** true **13.** false **15.** true **17.** true **19.** false **21.** false **23.** true **25.** false **27.** true **29.** $\frac{8}{12} = \frac{4}{6}$; true **31.** $\frac{5}{2} = \frac{13}{5}$; false **33.** $\frac{1.8}{2} = \frac{4.5}{5}$; true **35.** $\frac{\frac{2}{3}}{\frac{1}{5}} = \frac{\frac{2}{5}}{\frac{1}{9}}$; false **37.** 3 **39.** −9 **41.** 4 **43.** 3.2 **45.** 38.4 **47.** 25 **49.** 0.0025 **51.** 1 **53.** $\frac{9}{20}$ **55.** 13 **57.** $\frac{3}{4}$ **59.** $\frac{35}{18}$ **61.** $<$ **63.** $>$ **65.** $<$ **67.** possible answers: $\frac{9}{3} = \frac{15}{5}$; $\frac{5}{15} = \frac{3}{9}$; $\frac{15}{9} = \frac{5}{3}$ **69.** possible answers: $\frac{6}{1} = \frac{18}{3}$; $\frac{3}{18} = \frac{1}{6}$; $\frac{18}{6} = \frac{3}{1}$ **71.** possible answers: $\frac{d}{b} = \frac{c}{a}$; $\frac{a}{c} = \frac{b}{d}$; $\frac{b}{a} = \frac{d}{c}$ **73.** answers may vary **75.** 14.9 **77.** 0.07 **79.** 3.163 **81.** 0 **83.** 1400 **85.** 252.5

Section 5.3

Vocabulary, Readiness & Video Check **1.** There are approximately 102.9 mg of cholesterol in a 5-ounce serving of lobster.

Exercise Set 5.3 **1.** 360 baskets **3.** 165 min **5.** 630 applications **7.** 23 ft **9.** 270 sq ft **11.** 25 gal **13.** 450 km **15.** 16 bags **17.** 15 hits **19.** 27 people **21.** 18 applications **23.** 5 weeks **25.** $10\frac{2}{3}$ servings **27.** 37.5 sec **29.** **a.** 18 tsp **b.** 6 tbsp **31.** 6 people **33.** 112 ft; 11-in. difference **35.** 102.9 mg **37.** 1257 ft; The actual height of the Empire State Building is 1248 ft. **39.** 434 emergency room visits **41.** 2 million SUV crossovers **43.** 2.4 c **45.** **a.** 0.1 gal **b.** 13 fl oz **47.** **a.** 2062.5 mg **b.** no **49.** $3 \cdot 5$ **51.** $2^2 \cdot 5$ **53.** $2^3 \cdot 5^2$ **55.** 2^5 **57.** 0.8 ml **59.** 1.25 ml **61.** $11 \approx 12$ or 1 dozen; $1.5 \times 8 = 12$; 12 cups of milk **63.** $4\frac{2}{3}$ ft **65.** answers may vary

Integrated Review **1.** $\frac{9}{10}$ **2.** $\frac{9}{25}$ **3.** $\frac{43}{50}$ **4.** $\frac{8}{23}$ **5.** $\frac{173}{139}$ **6.** $\frac{6}{7}$ **7.** $\frac{7}{26}$ **8.** $\frac{20}{33}$ **9.** $\frac{2}{3}$ **10.** $\frac{1}{8}$ **11.** $\frac{1859}{1237}$ **12.** $\frac{6}{25}$ **13.** **a.** 3 **b.** $\frac{4}{7}$ **14.** $\frac{2}{3}$ **15.** $\frac{1\text{ office}}{4\text{ graduate assistants}}$ **16.** $\frac{2\text{ lights}}{5\text{ ft}}$ **17.** $\frac{16\text{ computers}}{25\text{ households}}$ **18.** $\frac{9\text{ students}}{2\text{ computers}}$ **19.** 55 mi/hr **20.** 140 ft/sec **21.** 23 mi/gal **22.** 16 teachers/computer **23.** 3 packs: $0.80 per pack; 8 packs: $0.75 per pack; 8 packs **24.** 4: $0.92 per battery; 10: $0.99 per battery; 4 batteries **25.** no **26.** yes **27.** 24 **28.** 32.5 **29.** $2.\overline{72}$ or $2\frac{8}{11}$ **30.** 18

Section 5.4

Vocabulary, Readiness & Video Check **1.** meter **3.** yard **5.** feet **7.** feet **9.** feet; Feet are the original units and we want them to divide out. **11.** The sum of 21 yd 4 ft is correct but is not in a good format since there is a yard in 4 feet. Convert 4 feet = 1 yd 1 ft and add again: 21 yd + 1 yd + 1 ft = 22 yd 1 ft. **13.** 1.29 cm and 12.9 mm; These two different-unit lengths are equal.

Exercise Set 5.4 **1.** 5 ft **3.** 36 ft **5.** 8 mi **7.** 102 in. **9.** $3\frac{1}{3}$ yd **11.** 33,792 ft **13.** 4.5 yd **15.** 0.25 ft **17.** 13 yd 1 ft **19.** 7 ft 1 in. **21.** 1 mi 4720 ft **23.** 62 in. **25.** 26 ft **27.** 84 in. **29.** 11 ft 2 in. **31.** 22 yd 1 ft **33.** 6 ft 5 in. **35.** 7 ft 6 in. **37.** 14 ft 4 in. **39.** 83 yd 1 ft **41.** 6000 cm **43.** 4 cm **45.** 0.5 km **47.** 1.7 m **49.** 15 m **51.** 42,000 cm **53.** 7000 m **55.** 83 mm **57.** 0.201 dm **59.** 40 mm **61.** 8.94 m **63.** 2.94 m or 2940 mm **65.** 1.29 cm or 12.9 mm **67.** 12.64 km or 12,640 m **69.** 54.9 m **71.** 1.55 km **73.** $348\frac{2}{3}$; 12,552 **75.** $11\frac{2}{3}$; 420 **77.** 5000; 0.005; 500 **79.** 0.065; 65; 0.000065 **81.** 342,000; 342,000,000; 34,200,000 **83.** **a.** $213\frac{2}{3}$ yd **b.** 7692 in. **85.** 10 ft 6 in. **87.** 5100 ft **89.** 5.0 times **91.** 13 ft 11 in. **93.** 26.7 mm **95.** 15 ft 9 in. **97.** 3 ft 1 in. **99.** 41.25 m or 4125 cm **101.** 3.35 m **103.** 2.13 m **105.** $72\frac{2}{3}$ yd **107.** 15 tiles **109.** $\frac{21}{100}$ **111.** 0.13 **113.** 0.25 **115.** no **117.** yes **119.** no **121.** Estimate: 13 yd **123.** answers may vary; for example, $1\frac{1}{3}$ yd or 48 in. **125.** answers may vary **127.** 334.89 sq m

Section 5.5

Vocabulary, Readiness & Video Check **1.** Mass **3.** gram **5.** 2000 **7.** 2 lb **9.** 2 tons **11.** pounds; Pounds are the units we're converting to. **13.** 3 places to the right; 4 g = 4000 mg

Exercise Set 5.5 **1.** 32 oz **3.** 10,000 lb **5.** 9 tons **7.** $3\frac{3}{4}$ lb **9.** $1\frac{3}{4}$ tons **11.** 204 oz **13.** 9800 lb **15.** 76 oz **17.** 1.5 tons **19.** $\frac{1}{20}$ lb **21.** 92 oz **23.** 161 oz **25.** 5 lb 9 oz **27.** 53 lb 10 oz **29.** 8 tons 750 lb **31.** 3 tons 175 lb **33.** 8 lb 11 oz **35.** 31 lb 2 oz **37.** 1 ton 700 lb **39.** 0.5 kg **41.** 4000 mg **43.** 25,000 g **45.** 0.048 g **47.** 0.0063 kg **49.** 15,140 mg **51.** 6250 g **53.** 350,000 cg **55.** 13.5 mg **57.** 5.815 g or 5815 mg **59.** 1850 mg or 1.85 g **61.** 1360 g or 1.36 kg **63.** 13.52 kg **65.** 2.125 kg **67.** 200,000; 3,200,000 **69.** $\frac{269}{400}$ or 0.6725; 21,520 **71.** 0.5; 0.0005; 50 **73.** 21,000; 21,000,000; 2,100,000 **75.** 8.064 kg **77.** 35 lb 14 oz **79.** 112.5 g **81.** 5 lb 8 oz **83.** 6 lb 15.4 oz **85.** 144 mg **87.** 6.12 kg **89.** 130 lb **91.** 211 lb **93.** 0.16 **95.** 0.875 **97.** no **99.** yes **101.** no **103.** answers may vary; for example, 250 mg or 0.25 g **105.** true **107.** answers may vary

Section 5.6

Vocabulary, Readiness & Video Check **1.** capacity **3.** fluid ounces **5.** cups **7.** quarts **9.** 2 pt **11.** 2 gal **13.** 3 qt **15.** 3 c **17.** amount; units **19.** 3 places to the left; 5600 ml = 5.6 L

Exercise Set 5.6 **1.** 4 c **3.** 16 pt **5.** $3\frac{1}{2}$ gal **7.** 5 pt **9.** 8 c **11.** $3\frac{3}{4}$ qt **13.** $10\frac{1}{2}$ qt **15.** 9 c **17.** 23 qt **19.** $\frac{1}{4}$ pt **21.** 14 gal 2 qt **23.** 4 gal 3 qt 1 pt **25.** 22 pt **27.** 13 gal 2 qt **29.** 4 c 4 fl oz **31.** 1 gal 1 qt **33.** 2 gal 3 qt 1 pt **35.** 17 gal **37.** 4 gal 3 qt **39.** 5000 ml **41.** 0.00016 kl **43.** 5.6 L **45.** 320 cl **47.** 0.41 kl **49.** 0.064 L **51.** 160 L **53.** 3600 ml **55.** 19.3 L **57.** 4.5 L or 4500 ml **59.** 8410 ml or 8.41 L **61.** 16,600 ml or 16.6 L **63.** 3840 ml **65.** 162.4 L **67.** 336; 84; 168 **69.** $\frac{1}{4}$; 1; 2 **71.** 1.59 L **73.** 18.954 L **75.** 4.3 fl oz **77.** yes **79.** \$0.677 **81.** $\frac{4}{5}$ **83.** $\frac{3}{5}$ **85.** $\frac{9}{10}$ **87.** no **89.** no **91.** less than; answers may vary **93.** answers may vary **95.** 128 fl oz **97.** 1.5 cc **99.** 2.7 cc **101.** 54 u or 0.54 cc **103.** 86 u or 0.86 cc

Section 5.7

Vocabulary, Readiness & Video Check **1.** 1 L ≈ 0.26 gal or 3.79 L ≈ 1 gal

Exercise Set 5.7 **1.** 25.57 fl oz **3.** 218.44 cm **5.** 40 oz **7.** 57.66 mi **9.** 3.77 gal **11.** 13.5 kg **13.** 1.5; $1\frac{2}{3}$; 150; 60 **15.** 55; 5500; 180; 2160 **17.** 3.94 in. **19.** 80.5 kph **21.** 0.008 oz **23.** 229.6 ft **25.** 9.92 billion mi **27.** yes **29.** 2790 mi **31.** 90 mm **33.** 112.5 g **35.** 104 mph **37.** 26.24 ft **39.** 3 mi **41.** 8 fl oz **43. b** **45. b** **47. c** **49. d** **51. d** **53.** 29 **55.** 9 **57.** 5 **59.** 36 **61.** 2.13 sq m **63.** 1.19 sq m **65.** 1.69 sq m **67.** 21.3 mg–25.56 mg **69.** 800 sq m or 8606.72 sq ft

Chapter 5 Vocabulary Check **1.** ratio **2.** proportion **3.** unit rate **4.** unit price **5.** rate **6.** cross products **7.** equal **8.** not equal **9.** Weight **10.** Mass **11.** meter **12.** unit fractions **13.** liter **14.** gram

Chapter 5 Review **1.** $\frac{23}{37}$ **2.** $\frac{14}{51}$ **3.** $\frac{5}{4}$ **4.** $\frac{11}{13}$ **5.** $\frac{7}{15}$ **6.** $\frac{17}{35}$ **7.** $\frac{18}{35}$ **8.** $\frac{35}{27}$ **9. a.** 3 **b.** $\frac{3}{25}$ **10. a.** 14 **b.** $\frac{14}{25}$ **11.** $\frac{5\text{ pages}}{2\text{ min}}$ **12.** $\frac{4\text{ computers}}{3\text{ hr}}$ **13.** 52 mi/hr **14.** 15 ft/sec **15.** \$6.96/CD **16.** $1\frac{1}{3}$ gal/acre **17.** 8 oz: \$0.124 per oz; 12 oz: \$0.141 per oz; 8-oz size **18.** 18 oz: \$0.083; 28 oz: \$0.085; 18-oz size **19.** $\frac{16\text{ sandwiches}}{8\text{ players}} = \frac{2\text{ sandwiches}}{1\text{ player}}$ **20.** $\frac{12\text{ tires}}{3\text{ cars}} = \frac{4\text{ tires}}{1\text{ car}}$ **21.** no **22.** yes **23.** yes **24.** no **25.** −5 **26.** −15 **27.** $6\frac{3}{4}$ **28.** $7\frac{1}{5}$ **29.** 0.94 **30.** 0.36 **31.** $1\frac{1}{8}$ **32.** $1\frac{3}{7}$ **33.** 14 passes **34.** 35 attempts **35.** 8 bags **36.** 16 bags **37.** $40\frac{1}{2}$ ft **38.** $8\frac{1}{4}$ in. **39.** 9 ft **40.** 18 in. **41.** 17 yd 1 ft **42.** 3 ft 10 in. **43.** 4200 cm **44.** 0.00231 km **45.** 21 yd 1 ft **46.** 7 ft 5 in. **47.** 9.5 cm or 95 mm **48.** 2.45 km **49.** 108.5 km **50.** 0.24 sq m **51.** 4.125 lb **52.** 4600 lb **53.** 3 lb 4 oz **54.** 5 tons 300 lb **55.** 0.027 g **56.** 40,000 g **57.** 3 lb 9 oz **58.** 33 lb 8 oz **59.** 4 lb 4 oz **60.** 9 tons 1075 lb **61.** 8 qt **62.** 5 c **63.** 7 pt **64.** 72 c **65.** 4 qt 1 pt **66.** 3 gal 3 qt **67.** 3800 ml **68.** 0.042 dl **69.** 1 gal 1 qt **70.** 736 ml or 0.736 L **71.** 10.88 L **72.** yes **73.** 22.96 ft **74.** 10.55 m **75.** 4.55 gal **76.** 8.27 qt **77.** 425.25 g **78.** 10.35 kg **79.** 109 yd **80.** 180.4 lb **81.** 3.18 qt **82.** 2.36 in. **83.** $\frac{3}{5}$ **84.** $\frac{1}{8}$ **85.** $\frac{1\text{ teacher}}{9\text{ students}}$ **86.** $\frac{1\text{ nurse}}{4\text{ patients}}$ **87.** 34 miles/hour **88.** 2 gallons/cow **89.** 4 oz: \$1.235; 8 oz: \$1.248; 4-oz size **90.** 12 oz: \$0.054; 64 oz: \$0.047; 64-oz size **91.** $\frac{2\text{ cups cookie dough}}{30\text{ cookies}} = \frac{4\text{ cups cookie dough}}{60\text{ cookies}}$ **92.** $\frac{5\text{ nickels}}{3\text{ dollars}} = \frac{20\text{ nickels}}{12\text{ dollars}}$ **93.** 1.6 **94.** 25 **95.** 13,200 ft **96.** 10.75 ft **97.** 4 tons 200 lb **98.** 500 cm **99.** 1.4 g **100.** 0.000286 km **101.** 9117 m or 9.117 km **102.** 8 gal 2 qt

Chapter 5 Getting Ready for the Test **1.** B **2.** C **3.** A **4.** B **5.** A **6.** A **7.** A **8.** A **9.** C **10.** D **11.** D **12.** A **13.** B **14.** A **15.** D **16.** A **17.** C **18.** A

Chapter 5 Test **1.** $\frac{15}{2}$ **2.** $\frac{3 \text{ in.}}{10 \text{ days}}$ **3.** $\frac{43}{50}$ **4.** $\frac{47}{78}$ **5.** $\frac{197}{62}$ **6.** 81.25 km/hr **7.** 28 students/teacher **8.** 9 in./sec **9.** 8-oz size **10.** true **11.** 5 **12.** $4\frac{4}{11}$ **13.** −8 **14.** $\frac{7}{8}$ **15.** $49\frac{1}{2}$ ft **16.** $3\frac{3}{4}$ hr **17.** $53\frac{1}{3}$ g **18.** 23 ft 4 in. **19.** 10 qt **20.** 1.875 lb **21.** 0.04 g **22.** 36 mm **23.** 830 ml **24.** 3 lb 13 oz **25.** 2 gal 3 qt **26.** 2.256 km or 2256 m **27.** 5.6 m **28.** 4 gal 3 qt **29.** 91.4 m **30.** 16 ft 6 in. **31.** 493 ft 6 in. **32.** 150.368 m **33.** 3.1 mi

Cumulative Review **1.** **a.** 3 **b.** 15 **c.** 0 **d.** 70; Sec. 1.4, Ex. 1 **2.** **a.** 0 **b.** 20 **c.** 0 **d.** 20; Sec. 1.6 **3.** 249,000; Sec. 1.5, Ex. 3 **4.** 249,000; Sec. 1.5 **5** **a.** 200 **b.** 1230; Sec. 1.6, Ex. 3 **6.** 373 R 24; Sec. 1.7 **7.** $16,071; Sec. 1.8, Ex. 3 **8.** 16,591 feet; Sec. 1.8 **9.** $2^4 \cdot 5$; Sec. 3.2, Ex. 2 **10.** 8; Sec. 1.9 **11.** $\frac{3}{5}$; Sec. 3.2, Ex. 4 **12.** 243; Sec. 1.9 **13.** $-\frac{1}{8}$; Sec. 3.3, Ex. 5 **14.** $15\frac{3}{8}$; Sec. 3.7 **15.** $\frac{2}{5}$; Sec. 3.3, Ex. 6 **16.** $\frac{5}{54}$; Sec. 3.3 **17.** $\frac{5}{7}$; Sec. 3.4, Ex. 1 **18.** $\frac{19}{30}$; Sec. 3.4 **19.** 2; Sec. 3.4, Ex. 3 **20.** $\frac{4}{5}$; Sec. 3.4 **21.** 14; Sec. 3.4, Ex. 12 **22.** $\frac{49}{50}$; Sec. 3.5 **23.** $\frac{15}{20}$; Sec. 3.4, Ex. 16 **24.** yes; Sec. 3.2 **25.** $-\frac{8}{33}$; Sec. 3.5, Ex. 5 **26.** $7\frac{47}{72}$; Sec. 3.7 **27.** $\frac{1}{6}$ hr; Sec. 3.5, Ex. 11 **28.** 27; Sec. 1.9 **29.** $7\frac{17}{24}$; Sec. 3.7, Ex. 9 **30.** $\frac{16}{27}$; Sec. 3.6 **31.** <; Sec. 3.5, Ex. 7 **32.** 14,000,000; Sec. 1.6 **33.** negative fifty and eighty-two hundredths; Sec. 4.1, Ex. 1b **34.** 0.075; Sec. 4.1 **35.** 736.2; Sec. 4.1, Ex. 15 **36.** 736.236; Sec. 4.1 **37.** 25.454; Sec. 4.2, Ex. 1 **38.** 681.24; Sec. 4.2 **39.** 0.8496; Sec. 4.3, Ex. 2 **40.** 0.375; Sec. 4.5 **41.** −0.052; Sec. 4.4, Ex. 3 **42.** $\frac{79}{10}$; Sec. 4.1 **43.** −3.7; Sec. 4.5, Ex. 12 **44.** 3; Sec. 5.2 **45.** $\frac{4}{9}, \frac{9}{20}$, 0.456; Sec. 4.5, Ex. 10 **46.** 140 m/sec; Sec. 5.1 **47.** $\frac{3}{2}$; Sec. 5.1, Ex. 2 **48.** $\frac{1}{3}$; Sec. 5.1 **49.** $\frac{26}{31}$; Sec. 5.1, Ex. 3 **50.** $\frac{1}{10}$; Sec. 5.1

Chapter 6 Percent

Section 6.1

Vocabulary, Readiness & Video Check **1.** Percent **3.** percent **5.** 0.01 **7.** 13% **9.** 87% **11.** 1% **13.** Percent means "per 100." **15.** In both cases, we multiply the number by 1 in the form of 100%.

Exercise Set 6.1 **1.** 96% **3.** **a.** 75% **b.** 25% **5.** football; 37% **7.** 50% **9.** 0.41 **11.** 0.06 **13.** 1.00 or 1 **15.** 0.613 **17.** 0.028 **19.** 0.006 **21.** 3.00 or 3 **23.** 0.3258 **25.** $\frac{3}{25}$ **27.** $\frac{1}{25}$ **29.** $\frac{9}{200}$ **31.** $\frac{7}{4}$ or $1\frac{3}{4}$ **33.** $\frac{1}{16}$ **35.** $\frac{31}{300}$ **37.** $\frac{179}{800}$ **39.** 0.3% **41.** 22% **43.** 530% **45.** 5.6% **47.** 33.28% **49.** 300% **51.** 70% **53.** 70% **55.** 40% **57.** 34% **59.** $37\frac{1}{2}\%$ **61.** $77\frac{7}{9}\%$ **63.** 250% **65.** 190% **67.** 63.64% **69.** 26.67% **71.** 0.35, $\frac{7}{20}$; 20%, 0.2; 50%, $\frac{1}{2}$; 0.7, $\frac{7}{10}$; 37.5%, 0.375 **73.** 0.4, $\frac{2}{5}$; 23.5%, $\frac{47}{200}$; 80%, 0.8; $0.33\overline{3}$, $\frac{1}{3}$; 87.5%, 0.875; 0.075, $\frac{3}{40}$ **75.** 2, 2; 280%, $2\frac{4}{5}$; 7.05, $7\frac{1}{20}$; 454%, 4.54 **77.** 0.66; $\frac{33}{50}$ **79.** 0.592; $\frac{74}{125}$ **81.** 48% **83.** 0.38 **85.** 18% **87.** 0.005; $\frac{1}{200}$ **89.** 0.162; $\frac{81}{500}$ **91.** 0.049; $\frac{49}{1000}$ **93.** $n = 15$ **95.** $n = 12$ **97.** 77% **99.** 107.8% **101.** **a.** 52.9% **b.** 52.86% **103.** **b, d** **105.** 4% **107.** occupational therapy assistant **109.** 0.30 **111.** 75% **113.** 80% **115.** greater **117.** answers may vary **119.** 0.266; 26.6% **121.** 1.155; 115.5%

Section 6.2

Vocabulary, Readiness & Video Check **1.** is **3.** amount; base; percent **5.** greater **7.** percent: 42%; base: 50; amount: 21 **9.** percent: 125%; base: 86; amount: 107.5 **11.** "of" means multiplication; "is" means equals; "what" (or some equivalent) means the unknown number

Exercise Set 6.2 **1.** $18\% \cdot 81 = n$ **3.** $20\% \cdot n = 105$ **5.** $0.6 = 40\% \cdot n$ **7.** $n \cdot 80 = 3.8$ **9.** $n = 9\% \cdot 43$ **11.** $n \cdot 250 = 150$ **13.** 3.5 **15.** 28.7 **17.** 10 **19.** 600 **21.** 110% **23.** 34% **25.** 1 **27.** 645 **29.** 500 **31.** 5.16% **33.** 25.2 **35.** 35% **37.** 35 **39.** 0.624 **41.** 0.5% **43.** 145 **45.** 63% **47.** 4% **49.** $n = 30$ **51.** $n = 3\frac{7}{11}$ **53.** $\frac{17}{12} = \frac{n}{20}$ **55.** $\frac{8}{9} = \frac{14}{n}$ **57.** **c** **59.** **b** **61.** Twenty percent of some number is eighteen and six-tenths. **63.** **b** **65.** **c** **67.** **c** **69.** **a** **71.** **a** **73.** answers may vary **75.** 686.625 **77.** 12,285

Section 6.3

Vocabulary, Readiness & Video Check **1.** amount; base; percent **3.** amount **5.** amount: 12.6; base: 42; percent: 30 **7.** amount: 102; base: 510; percent: 20 **9.** 45 follows the word "of," so it is the base.

Exercise Set 6.3 **1.** $\frac{a}{45} = \frac{98}{100}$ **3.** $\frac{a}{150} = \frac{4}{100}$ **5.** $\frac{14.3}{b} = \frac{26}{100}$ **7.** $\frac{84}{b} = \frac{35}{100}$ **9.** $\frac{70}{400} = \frac{p}{100}$ **11.** $\frac{8.2}{82} = \frac{p}{100}$ **13.** 26 **15.** 18.9 **17.** 600 **19.** 10 **21.** 120% **23.** 28% **25.** 37 **27.** 1.68 **29.** 1000 **31.** 210% **33.** 55.18 **35.** 45% **37.** 75 **39.** 0.864 **41.** 0.5% **43.** 140 **45.** 9.6 **47.** 113% **49.** $-\frac{7}{8}$ **51.** $3\frac{2}{15}$ **53.** 0.7 **55.** 2.19 **57.** answers may vary **59.** no; $a = 16$ **61.** yes **63.** answers may vary **65.** 12,011.2 **67.** 7270.6

Integrated Review **1.** 12% **2.** 68% **3.** 12.5% **4.** 250% **5.** 520% **6.** 800% **7.** 6% **8.** 44% **9.** 750% **10.** 325% **11.** 3% **12.** 5% **13.** 0.65 **14.** 0.31 **15.** 0.08 **16.** 0.07 **17.** 1.42 **18.** 4 **19.** 0.029 **20.** 0.066 **21.** 0.03; $\frac{3}{100}$ **22.** 0.05; $\frac{1}{20}$ **23.** 0.0525; $\frac{21}{400}$ **24.** 0.1275; $\frac{51}{400}$ **25.** 0.38; $\frac{19}{50}$ **26.** 0.45; $\frac{9}{20}$ **27.** 0.123; $\frac{37}{300}$ **28.** 0.167; $\frac{1}{6}$ **29.** 8.4 **30.** 100 **31.** 250 **32.** 120% **33.** 28% **34.** 76 **35.** 11 **36.** 130% **37.** 86% **38.** 37.8 **39.** 150 **40.** 62

Section 6.4

Vocabulary, Readiness & Video Check **1.** The price of the home was $175,000.

Exercise Set 6.4 **1.** 1600 bolts **3.** 8.8 lb **5.** 14% **7.** 91,800 businesses **9.** 38.6% **11.** 496 chairs; 5704 chairs **13.** 59,917 occupational therapy assistants **15.** 1835 thousand **17.** 29% **19.** 50% **21.** 12.5% **23.** 29.2% **25.** $175,000 **27.** 31.2 hr **29.** increase: $867.87; new price: $20,153.87 **31.** 40 ft **33.** increase: $1043; tuition in 2016–2017: $10,037 **35.** increase: 1,494,100 enrolled in associate degree programs; projected enrollment in 2024–2025: 8,194,100 **37.** 30; 60% **39.** 52; 80% **41.** 2; 25% **43.** 120; 75% **45.** 44% **47.** 1.3% **49.** 142.0% **51.** 374.6% **53.** 9.8% **55.** 29.9% **57.** 28.9% **59.** 21.5% **61.** 4.56 **63.** 11.18 **65.** 58.54 **67.** The increased number is double the original number. **69.** percent of increase $= \frac{30}{150} = 20\%$ **71.** False; the percents are different since the original amounts are different.

Section 6.5

Vocabulary, Readiness & Video Check **1.** sales tax **3.** commission **5.** sale price **7.** We rewrite the percent as an equivalent decimal. **9.** Replace "amount of discount" in the second equation with "discount rate · original price": sale price = original price − (discount rate · orginal price).

Exercise Set 6.5 **1.** $7.50 **3.** $858.93 **5.** 7% **7. a.** $270 **b.** $292.95 **9.** $117; $1917 **11.** $485 **13.** 6% **15.** $16.10; $246.10 **17.** $53,176.04 **19.** 14% **21.** $4888.50 **23.** $185,500 **25.** $8.90; $80.10 **27.** $98.25; $98.25 **29.** $143.50; $266.50 **31.** $3255; $18,445 **33.** $45; $255 **35.** $27.45; $332.45 **37.** $3.08; $59.08 **39.** $7074 **41.** 8% **43.** 1200 **45.** 132 **47.** 16 **49. d** **51.** $4.00; $6.00; $8.00 **53.** $7.20; $10.80; $14.40 **55.** a discount of 60% is better; answers may vary **57.** $26,838.45

Section 6.6

Calculator Explorations **1.** 1.56051 **3.** 8.06231 **5.** $634.49

Vocabulary, Readiness & Video Check **1.** simple **3.** Compound **5.** Total amount **7.** principal **9.** The denominator is the total number of payments. We are asked to find the monthly payment for a 4-year loan, and since there are 48 months in 4 years, there are 48 total payments.

Exercise Set 6.6 **1.** $32 **3.** $73.60 **5.** $750 **7.** $33.75 **9.** $700 **11.** $101,562.50; $264,062.50 **13.** $5562.50 **15.** $14,280 **17.** $46,815.37 **19.** $2327.14 **21.** $58,163.65 **23.** 2915.75 **25.** $2938.66 **27.** $2971.89 **29.** $260.31 **31.** $637.26 **33.** 32 yd **35.** 35 m **37.** answers may vary **39.** answers may vary

Chapter 6 Vocabulary Check **1.** of **2.** is **3.** Percent **4.** Compound interest **5.** $\frac{\text{amount}}{\text{base}}$ **6.** 100% **7.** 0.01 **8.** $\frac{1}{100}$ **9.** base; amount **10.** Percent of decrease **11.** Percent of increase **12.** Sales tax **13.** Total price **14.** Commission **15.** Amount of discount **16.** Sale price

Chapter 6 Review **1.** 37% **2.** 77% **3.** 0.83 **4.** 0.75 **5.** 0.735 **6.** 0.015 **7.** 1.25 **8.** 1.45 **9.** 0.005 **10.** 0.007 **11.** 2.00 or 2 **12.** 4.00 or 4 **13.** 0.2625 **14.** 0.8534 **15.** 260% **16.** 102% **17.** 35% **18.** 55% **19.** 72.5% **20.** 25.2% **21.** 7.6% **22.** 8.5% **23.** 71% **24.** 65% **25.** 400% **26.** 900% **27.** $\frac{1}{100}$ **28.** $\frac{1}{10}$ **29.** $\frac{1}{4}$ **30.** $\frac{17}{200}$ **31.** $\frac{51}{500}$ **32.** $\frac{1}{6}$ **33.** $\frac{1}{3}$ **34.** $1\frac{1}{10}$ **35.** 20% **36.** 70% **37.** $83\frac{1}{3}\%$ **38.** 60% **39.** 125% **40.** $166\frac{2}{3}\%$ **41.** 6.25% **42.** 62.5% **43.** 100,000 **44.** 114.5 **45.** 23% **46.** 150%

47. 3000 **48.** 8000 **49.** 418 **50.** 300 **51.** 64.8 **52.** 165 **53.** 110% **54.** 180% **55.** 66% **56.** 16% **57.** 20.9% **58.** 106.25% **59.** $206,400 **60.** $13.23 **61.** $273.75 **62.** $3.36 **63.** $5000 **64.** $300.38 **65.** discount: $900; sale price: $2100 **66.** discount: $9; sale price: $81 **67.** $160 **68.** $325 **69.** $30,104.61 **70.** $17,506.54 **71.** $80.61 **72.** $32,830.10 **73.** 0.038 **74.** 0.245 **75.** 0.009 **76.** 54% **77.** 9520% **78.** 30% **79.** $\frac{47}{100}$ **80.** $\frac{8}{125}$ **81.** $\frac{7}{125}$ **82.** $37\frac{1}{2}\%$ **83.** $15\frac{5}{13}\%$ **84.** 120% **85.** 268.75 **86.** 110% **87.** 708.48 **88.** 134% **89.** 300% **90.** 38.4 **91.** 560 **92.** 325% **93.** 26% **94.** $6786.50 **95.** $617.70 **96.** $9.45 **97.** 12.5% **98.** $1491 **99.** $17,951.01 **100.** $11,687.50

Chapter 6 Getting Ready for the Test **1.** D **2.** A **3.** B **4.** C **5.** B **6.** D **7.** A **8.** C **9.** C **10.** A **11.** D **12.** C

Chapter 6 Test **1.** 0.85 **2.** 5 **3.** 0.008 **4.** 5.6% **5.** 610% **6.** 39% **7.** $\frac{6}{5}$ or $1\frac{1}{5}$ **8.** $\frac{77}{200}$ **9.** $\frac{1}{500}$ **10.** 55% **11.** 37.5% **12.** $155\frac{5}{9}\%$ **13.** 33.6 **14.** 1250 **15.** 75% **16.** 38.4 lb **17.** $56,750 **18.** $358.43 **19.** 5% **20.** discount: $18; sale price: $102 **21.** $395 **22.** 1% **23.** $647.50 **24.** $2005.63 **25.** $427

Cumulative Review **1.** 206 cases; 12 cans; yes; Sec. 1.8, Ex. 2 **2.** 31,084; Sec. 1.6 **3. a.** $4\frac{2}{7}$ **b.** $1\frac{1}{15}$ **c.** 14; Sec. 3.6, Ex. 9 **4. a.** $\frac{19}{7}$ **b.** $\frac{101}{10}$ **c.** $\frac{43}{8}$; Sec. 3.6 **5.** $-\frac{10}{27}$; Sec. 3.2, Ex. 6 **6.** 44; Sec. 1.7 **7.** $\frac{23}{56}$; Sec. 3.3, Ex. 4 **8.** 76,500; Sec. 1.5 **9.** $\frac{4}{5}$; Sec. 3.3, Ex. 11 **10.** $\frac{15}{4}$; $3\frac{3}{4}$; Sec. 3.1 **11.** $\frac{4}{5}$ in.; Sec. 3.4, Ex. 10 **12.** 50; Sec. 1.9 **13.** 60; Sec. 3.4, Ex. 13 **14.** $\frac{1}{3}$; Sec. 3.4 **15.** $\frac{2}{3}$; Sec. 3.5, Ex. 1 **16.** 340; Sec. 3.3 **17.** $3\frac{5}{14}$; Sec. 3.7, Ex. 13 **18.** 33; Sec. 1.9 **19.** $\frac{3}{20}$; Sec. 3.7, Ex. 6 **20.** $33\frac{27}{40}$; Sec. 3.7 **21.** $\frac{1}{16}$; Sec. 3.3, Ex. 8b **22.** $6\frac{3}{8}$; Sec. 3.7 **23.** −0.625; Sec. 4.5, Ex. 2 **24.** 0.09; Sec. 4.5 **25.** 3.14; Sec. 4.5, Ex. 4 **26.** 0.0048; Sec. 4.5 **27.** $41,568; Sec. 4.1, Ex. 18 **28.** 27.94; Sec. 4.2 **29.** 829.6561; Sec. 4.2, Ex. 2 **30.** 1248.3; Sec. 4.3 **31.** 18.408; Sec. 4.3, Ex. 1 **32.** 76,300; Sec. 4.3 **33.** 0.7861; Sec. 4.4, Ex. 8 **34.** 1.276; Sec. 4.4 **35.** −0.012; Sec. 4.4, Ex. 9 **36.** 50.65; Sec. 4.4 **37.** 7.236; Sec. 4.5, Ex. 11 **38.** 0.191; Sec. 4.5 **39.** 0.25; Sec. 4.5, Ex. 1 **40.** $0.\overline{5} \approx 0.556$; Sec. 4.5 **41.** no; Sec. 5.2, Ex. 3 **42.** 0.052 per tortilla; 0.058 per tortilla; 18-tortilla pkg is better buy; Sec. 5.1 **43.** $17\frac{1}{2}$ mi; Sec. 5.3, Ex. 1 **44. a.** 0.07 **b.** 2 **c.** 0.005; Sec. 6.1 **45.** $n = 25\% \cdot 0.008$; Sec. 6.2, Ex. 3 **46.** 37.5% or $37\frac{1}{2}\%$; Sec. 6.1

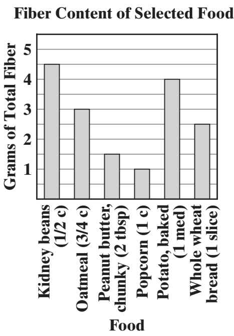
Chapter 7 Reading Graphs and Introduction to Statistics and Probability

Section 7.1

Vocabulary, Readiness & Video Check **1.** bar **3.** line **5.** Count the number of symbols and multiply this number by how much each symbol stands for (from the key). **7.** bar graph

Exercise Set 7.1 **1.** Kansas **3.** 4.5 million or 4,500,000 acres **5.** South Dakota, Colorado, and Washington **7.** North Dakota **9.** 48,000 **11.** 2016 **13.** 18,000 **15.** 60,000 wildfires/year **17.** September **19.** 79 (exact); or approximately 80 **21.** $\frac{1}{18}$ **23.** Tokyo, Japan; about 38 million or 38,000,000 **25.** New York; 21.4 million or 21,400,000 **27.** approximately 3 million

29.



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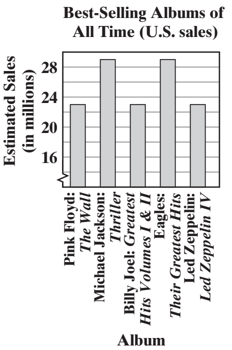
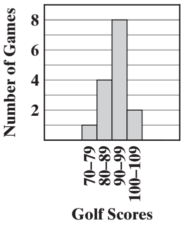


33. 15 adults **35.** 61 adults **37.** 24 adults **39.** 12 adults **41.** $\frac{9}{100}$ **43.** 20 to 44 **45.** 109 million **47.** 23 million **49.** answers may vary **51.** |; 1 **53.** 𝍸 |||; 8 **55.** 𝍸 |; 6 **57.** 𝍸 |; 6 **59.** ||; 2 **61.**

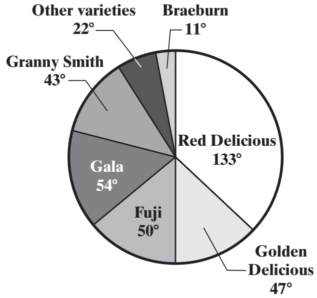


63. 8.3 goals/game **65.** 2009 **67.** increase **69.** 2007, 2013 **71.** 3.6 **73.** 6.2 **75.** 25% **77.** 34% **79.** 83°F **81.** Sunday; 68°F **83.** Tuesday; 13°F **85.** answers may vary

Section 7.2

Vocabulary, Readiness & Video Check **1.** circle **3.** 360 **5.** 100%

Exercise Set 7.2 **1.** parent or guardian's home **3.** $\frac{9}{35}$ **5.** $\frac{9}{16}$ **7.** Asia **9.** 37% **11.** 17,100,000 sq mi **13.** 2,850,000 sq mi **15.** 55% **17.** nonfiction **19.** 31,400 books **21.** 27,632 books **23.** 25,120 books **25.**



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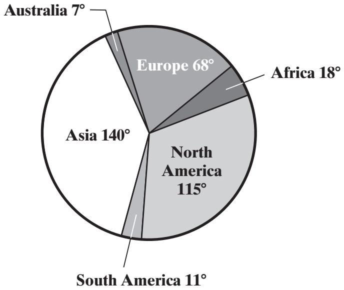
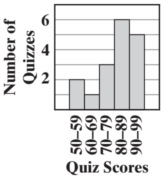


29. $2^2 \times 5$ **31.** $2^3 \times 5$ **33.** 5×17 **35.** Pacific; answers may vary **37.** 129,600,002 sq km **39.** 55,542,858 sq km **41.** 672 respondents **43.** 2408 respondents **45.** $\frac{12}{31}$ **47.** no; answers may vary **49.** answers may vary

Integrated Review **1.** 260,000 **2.** 440,000 **3.** personal care aides **4.** industrial janitors **5.** Oroville Dam; 755 ft **6.** New Bullards Bar Dam; 635 ft **7.** 15 ft **8.** 4 dams **9.** Thursday and Saturday; 100°F **10.** Monday; 82°F **11.** Sunday, Monday, and Tuesday **12.** Wednesday, Thursday, Friday, and Saturday **13.** 70 qt containers **14.** 52 qt containers **15.** 2 qt containers **16.** 6 qt containers **17.** | |; 2 **18.** |; 1 **19.** | | |; 3 **20.** ~~||||~~ |; 6 **21.** ~~||||~~; 5 **22.**



Section 7.3

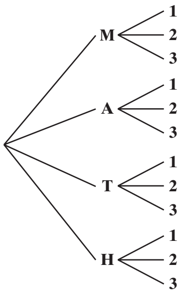
Vocabulary, Readiness & Video Check **1.** average **3.** mean (or average) **5.** grade point average **7.** median **9.** Place the data numbers in numerical order (or verify that they already are). **11.** answers may vary; For example: 6, 6, 6, 6

Exercise Set 7.3 **1.** mean: 21; median: 23; no mode **3.** mean: 8.1; median: 8.2; mode: 8.2 **5.** mean: 0.5; median: 0.5; mode: 0.2 and 0.5 **7.** mean: 370.9; median: 313.5: no mode **9.** 2109.2 ft **11.** 1968.5 ft **13.** answers may vary **15.** 2.79 **17.** 3.64 **19.** 6.8 **21.** 6.9 **23.** 88.5 **25.** 73 **27.** 70 and 71 **29.** 9 rates **31.** mean: 3773 mi; median: 3812 mi **33.** 12 **35.** 350 **37.** 2 **39.** 1.7 **41.** **a.** 8.2 **b.** 8 **c.** 9 **43.** **a.** 6.1 **b.** 6 **c.** 6, 7, 8 **45.** **a.** 15 **b.** 15 **c.** 15 **47.** **a.** 6.1 **b.** 6 **c.** 4, 6 **49.** $\frac{3}{5}$ **51.** $\frac{1}{9}$ **53.** $\frac{7}{20}$ **55.** 35, 35, 37, 43 **57.** yes; answers may vary

Section 7.4

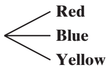
Vocabulary, Readiness & Video Check **1.** outcome **3.** probability **5.** 0 **7.** The number of outcomes equals the ending number of branches drawn.

Exercise Set 7.4 **1.**



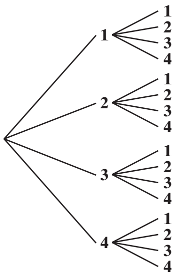
12 outcomes

3.



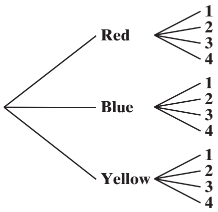
3 outcomes

5.



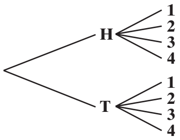
16 outcomes

7.



12 outcomes

9.



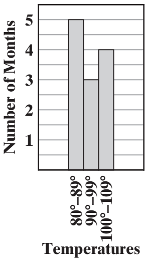
8 outcomes

11. $\frac{1}{6}$ **13.** $\frac{1}{3}$ **15.** $\frac{1}{2}$ **17.** $\frac{2}{3}$ **19.** $\frac{1}{3}$ **21.** 1 **23.** $\frac{2}{3}$ **25.** $\frac{1}{7}$ **27.** $\frac{2}{7}$ **29.** $\frac{4}{7}$ **31.** $\frac{19}{100}$ **33.** $\frac{1}{20}$ **35.** $\frac{5}{6}$ **37.** $\frac{1}{6}$ **39.** $\frac{20}{3}$ or $6\frac{2}{3}$ **41.** $\frac{1}{52}$ **43.** $\frac{1}{13}$ **45.** $\frac{1}{4}$ **47.** $\frac{1}{2}$ **49.** $\frac{5}{36}$ **51.** 0 **53.** answers may vary

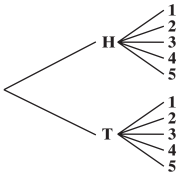
Chapter 7 Vocabulary Check **1.** bar **2.** mean **3.** outcomes **4.** pictograph **5.** mode **6.** line **7.** median **8.** tree diagram **9.** experiment **10.** circle **11.** probability **12.** histogram; class interval; class frequency **13.** range **14.** median

Chapter 7 Review **1.** 175,000 **2.** 600,000 **3.** South **4.** Northeast **5.** South, West **6.** Northeast, Midwest **7.** 30% **8.** 2017 **9.** 1990, 2000, 2010, 2017 **10.** answers may vary **11.** 962 (exact number) **12.** 927 (exact number) **13.** 958 (exact number) **14.** 842 (exact number) **15.** 31 **16.** answers may vary **17.** 1 employee **18.** 4 employees

19. 18 employees **20.** 9 employees **21.** 𝍸; 5 **22.** |||; 3 **23.** ||||; 4 **24.**

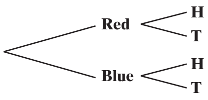


25. mortgage payment **26.** utilities **27.** $1225 **28.** $700 **29.** $\frac{39}{160}$ **30.** $\frac{7}{40}$ **31.** 75 **32.** 28 **33.** 2 **34.** 9 **35.** mean: 17.8; median: 14; no mode **36.** mean: 58.1; median: 60; mode: 45 and 86 **37.** mean: 24,500; median: 20,000; mode: 20,000 **38.** mean: 447.3; median: 420; mode: 400 **39.** 3.25 **40.** 2.57 **41.** 3 **42.** 7 **43.** 35 **44.** 19 **45.** 5 **46.** 2.84 **47.** **a.** 62.1 **b.** 62 **c.** 63 **48.** **a.** 62.5 **b.** 63 **c.** 64 **49.** **a.** 12.9 **b.** 12 **c.** 11 and 12 **50.** **a.** 18.1 **b.** 18 **c.** 18 and 20 **51.** **a.** 59.8 **b.** 60 **c.** 60 **52.** **a.** 13.9 **b.** 14 **c.** 10 and 18 **53.**



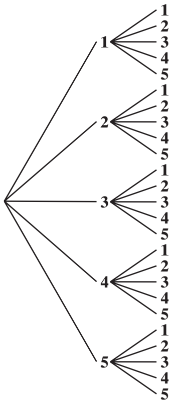
10 outcomes

54.



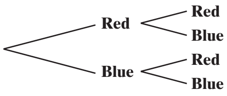
4 outcomes

55.



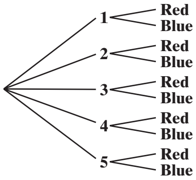
25 outcomes

56.



4 outcomes

57.



10 outcomes

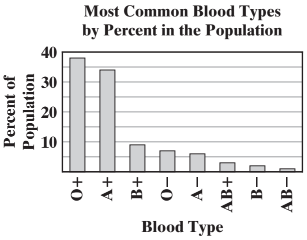
58. $\frac{1}{6}$ **59.** $\frac{1}{6}$ **60.** $\frac{1}{5}$ **61.** $\frac{1}{5}$ **62.** $\frac{3}{5}$ **63.** $\frac{2}{5}$ **64.** $\frac{1}{2}$ **65.** $\frac{1}{2}$ **66.** mean: 74.4; median: 73; mode: none **67.** mean: 48.8; median: 32; mode: none **68.** mean: 454; median: 463.5; mode: 500 **69.** mean: 619.17; median: 647.5; mode: 327 **70.** $\frac{1}{4}$ **71.** $\frac{3}{8}$ **72.** $\frac{1}{4}$ **73.** $\frac{1}{8}$ **74.** **a.** 14.44 **b.** 14 **c.** 18 **d.** 8 **75.** **a.** 13 **b.** 15 **c.** 5 **d.** 20

Chapter 7 Getting Ready for the Test

1. C **2.** C **3.** D **4.** C **5.** C **6.** C **7.** A **8.** B **9.** C **10.** A **11.** B **12.** D **13.** D **14.** A and C **15.** A and B **16.** C and D **17.** A and C

Chapter 7 Test

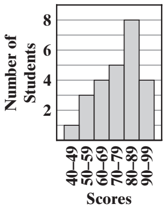
1. $225 **2.** 3rd week; $350 **3.** $1100 **4.** June, August, September **5.** February; 3 cm **6.** March and November **7.**



8. 1.6% **9.** 2008, 2011 **10.** 2008–2009, 2011–2012, 2012–2013, 2014–2015 **11.** $\frac{17}{40}$ **12.** $\frac{31}{22}$ **13.** 74 million **14.** 90 million **15.** 9 students **16.** 11 students **17.**

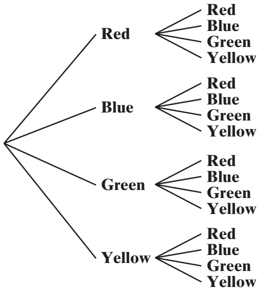
Class Intervals (Scores)	Tally	Class Frequency (Number of Students)
40–49	|	1
50–59	|||	3
60–69	||||	4
70–79	𝍸	5
80–89	𝍸|||	8
90–99	||||	4

18.



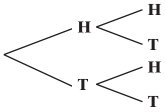
19. mean: 38.4; median: 42; no mode **20.** mean: 12.625; median: 12.5; mode: 12 and 16 **21.** 3.07 **22.** 8 **23.** 56 **24.** **a.** 92.8 **b.** 93 **c.** 93 and 94 **d.** 4 **25.** **a.** 26.3 **b.** 25 **c.** 35 **d.** 20

26.



16 outcomes

27.



4 outcomes

28. $\frac{1}{10}$ **29.** $\frac{1}{5}$

Cumulative Review **1.** one hundred six million, fifty-two thousand, four hundred forty-seven; Sec. 1.2, Ex. 7 **2.** two hundred seventy-six thousand, four; Sec. 1.2 **3.** 13 in.; Sec. 1.3, Ex. 5 **4.** 18 in.; Sec. 1.3 **5.** 726; Sec. 1.4, Ex. 4 **6.** 9585; Sec. 1.4 **7.** 249,000; Sec. 1.5, Ex. 3 **8.** 844,000; Sec. 1.5 **9.** 200; Sec. 1.6, Ex. 3a **10.** 29,230; Sec. 1.6 **11.** 208; Sec. 1.7, Ex. 5 **12.** 86; Sec. 1.7 **13.** 7; Sec. 1.9, Ex. 12 **14.** 35; Sec. 1.9 **15.** 26; Sec. 2.1, Ex. 4 **16.** 10; Sec. 2.1 **17. a.** $<$ **b.** $>$ **c.** $>$ Sec. 2.2, Ex. 3 **18. a.** $<$ **b.** $>$; Sec. 2.2 **19.** 3; Sec. 2.3, Ex. 1 **20.** -7; Sec. 2.3 **21.** -15; Sec. 2.3, Ex. 5 **22.** -4; Sec. 2.3 **23.** 8; Sec. 2.3, Ex. 6 **24.** 17; Sec. 2.3 **25.** -14; Sec. 2.4, Ex. 2 **26.** -5; Sec. 2.4 **27.** 11; Sec. 2.4, Ex. 3 **28.** 29; Sec. 2.4 **29.** -4; Sec. 2.4, Ex. 4 **30.** -3; Sec. 2.4 **31.** -2; Sec. 2.5, Ex. 10 **32.** 6; Sec. 2.5 **33.** 5; Sec. 2.5, Ex. 11 **34.** -13; Sec. 2.5 **35.** -16; Sec. 2.5, Ex. 12 **36.** -10; Sec. 2.5 **37.** $9\frac{3}{10}$; Sec. 3.7, Ex. 11 **38.** $11\frac{1}{3}$; Sec. 3.7 **39.** $\frac{\$180}{1\text{ week}}$; Sec. 5.1, Ex. 7 **40.** $\frac{68\text{ mi}}{1\text{ hr}}$ or 68 mi/hr; Sec. 5.1 **41.** $2\frac{1}{3}$ yd; Sec. 5.4, Ex. 2 **42.** 5000 pounds; Sec. 5.5 **43.** 0.0235 g; Sec. 5.5, Ex. 8 **44.** 1060 mm; Sec. 5.4 **45.** $\frac{1}{6}$; Sec. 4.6, Ex. 2 **46.** $\frac{1}{5}$; Sec. 4.6 **47.** 14 and 77; Sec. 7.3, Ex. 6 **48.** 56; Sec. 7.3 **49.** $\frac{1}{4}$; Sec. 7.4, Ex. 3 **50.** $\frac{3}{5}$; Sec. 7.4

Chapter 8 Introduction to Algebra

Section 8.1

Vocabulary, Readiness & Video Check **1.** expression; term **3.** combine like terms **5.** variable; constant **7.** associative **9.** numerical coefficient **11.** unlike **13.** like **15.** unlike **17.** like **19.** multiplication **21.** the distributive property first, then the associative property of multiplication **23.** addition; multiplication; P = perimeter, A = area

Exercise Set 8.1 **1.** -3 **3.** -2 **5.** 4 **7.** -3 **9.** -10 **11.** 133 **13.** -15 **15.** -4 **17.** $-\frac{4}{3}$ or $-1\frac{1}{3}$ **19.** -12 **21.** $\frac{3}{2}$ or $1\frac{1}{2}$ **23.** -10.6 **25.** $8x$ **27.** $-4n$ **29.** $-2c$ **31.** $-4x$ **33.** $13a - 8$ **35.** $-0.9x + 11.2$ **37.** $2x - 7$ **39.** $-5x + 4y - 5$ **41.** $\frac{1}{2} - \frac{53}{60}x$ **43.** $-4.8m - 4.1$ **45.** $30x$ **47.** $-22y$ **49.** $-4.2a$ **51.** $-4a$ **53.** $2y + 4$ **55.** $15a - 40$ **57.** $-12x - 28$ **59.** $6x - 0.12$ **61.** $-4x - \frac{3}{2}$ **63.** $2x - 9$ **65.** $27n - 20$ **67.** $15 + 7w$ **69.** $-11x + 8$ **71.** $(14y + 22)$ m **73.** $(11a + 12)$ ft **75.** $(25x + 55)$ in. **77.** $36y$ sq in. **79.** $(32x - 64)$ sq km **81.** P = $(6y + 42)$ mi; A = $(60y + 20)$ sq mi **83.** 4700 sq ft **85.** 64 ft **87.** $(3x + 6)$ ft **89.** \$360 **91.** one 5-ft rug **93.** 23°F **95.** 288 cu in. **97.** $91.2x$ cu in. **99.** -3 **101.** 8 **103.** 0 **105.** incorrect; $15x - 10$ **107.** incorrect; $7x - x - 2$ or $6x - 2$ **109.** distributive **111.** associative **113.** answers may vary **115.** $(20x + 16)$ sq mi **117.** 23,506.2 sq in. **119.** $4824q + 12{,}274$

Section 8.2

Vocabulary, Readiness & Video Check **1.** equivalent **3.** simplifying **5.** addition **7.** We can add the same number to both sides of an equation or subtract the same number from both sides of an equation and we'll have an equivalent equation.

Exercise Set 8.2 **1.** yes **3.** no **5.** yes **7.** no **9.** 18 **11.** -8 **13.** 9 **15.** -16 **17.** 3 **19.** $\frac{1}{8}$ **21.** 6 **23.** 8 **25.** 5.3 **27.** -1 **29.** -20.1 **31.** 2 **33.** 0 **35.** -28 **37.** -6 **39.** 24 **41.** -30 **43.** 12 **45.** 1 **47.** 1 **49.** 1 **51.** subtract $\frac{2}{3}$ from both sides **53.** add $\frac{4}{5}$ to both sides **55.** answers may vary **57.** 162,964 **59.** 1889 yd **61.** \$18,686,000,000

Section 8.3

Vocabulary, Readiness & Video Check **1.** equivalent **3.** simplifying **5.** multiplication **7.** Simplified each side of the equation by combining like terms.

Exercise Set 8.3 **1.** 4 **3.** -4 **5.** -30 **7.** -17 **9.** 50 **11.** 25 **13.** -30 **15.** $\frac{1}{3}$ **17.** $-\frac{2}{3}$ **19.** -4 **21.** 8 **23.** 1.3 **25.** 2 **27.** 0 **29.** -0.05 **31.** $-\frac{15}{64}$ **33.** $\frac{2}{3}$ **35.** 6 **37.** 0 **39.** -2 **41.** -8 **43.** 1 **45.** 72 **47.** 35 **49.** -28 **51.** 25 **53.** 1 **55.** -10 **57.** 2015 **59.** 9.7 million acres or 9,700,000 acres **61.** addition **63.** division **65.** answers may vary **67.** 6.5 hr **69.** 58.8 mph **71.** -3648 **73.** 67,896 **75.** -48

Integrated Review **1.** expression **2.** equation **3.** equation **4.** expression **5.** simplify **6.** solve **7.** 4 **8.** -6 **9.** 1 **10.** -4 **11.** $8x$ **12.** $-4y$ **13.** $-2a - 2$ **14.** $-2x + 3y - 7$ **15.** $-8x - 14$ **16.** $-6x + 30$ **17.** $5y - 10$ **18.** $15x - 31$ **19.** $(12x - 6)$ sq m **20.** $20y$ in. **21.** 13 **22.** -9 **23.** $\frac{7}{10}$ **24.** 0 **25.** -4 **26.** 25 **27.** -1 **28.** -3 **29.** 6 **30.** 8 **31.** $-\frac{9}{11}$ **32.** 5

Section 8.4

Calculator Explorations **1.** yes **3.** no **5.** yes

Vocabulary, Readiness & Video Check **1.** $3x - 9 + x - 16$; $5(2x + 6) - 1 = 39$ **3.** addition **5.** distributive **7.** the addition property of equality; to make sure we get an equivalent equation **9.** gives; amounts to

Exercise Set 8.4 **1.** 3 **3.** 1.9 **5.** −4 **7.** 100 **9.** −3.9 **11.** −4 **13.** −12 **15.** −3 **17.** −1 **19.** −45 **21.** −9 **23.** 6 **25.** −5 **27.** 5 **29.** 8 **31.** 0 **33.** −22 **35.** 6 **37.** −11 **39.** −7 **41.** −5 **43.** 5 **45.** −3 **47.** 2 **49.** −4 **51.** −1 **53.** 4 **55.** 3 **57.** −1 **59.** 8 **61.** 4 **63.** 3 **65.** 64 **67.** $\frac{1}{9}$ **69.** $\frac{3}{2}$ **71.** −54 **73.** 2 **75.** $\frac{9}{5}$ **77.** 270 **79.** 0 **81.** 4 **83.** 1 **85.** $-42 + 16 = -26$ **87.** $-5(-29) = 145$ **89.** $3(-14 - 2) = -48$ **91.** $\frac{100}{2(50)} = 1$ **93.** 126 million or 126,000,000 **95.** 20 million or 20,000,000 **97.** b **99.** a **101.** $\begin{aligned} 6x - 10 &= 5x - 7 \\ 6x &= 5x + 3 \\ x &= 3 \end{aligned}$ **103.** 0 **105.** −4 **107.** no; answers may vary **109.** 123.26°F **111.** −9.4°F

Section 8.5

Vocabulary, Readiness & Video Check **1.** decreased by **3.** The phrase is "three times the difference of a number and 5." The "difference of a number and 5" translates to the expression $x - 5$, and in order to multiply 3 times this expression, we insert parentheses around the expression.

Exercise Set 8.5 **1.** $x + 5$ **3.** $x + 8$ **5.** $20 - x$ **7.** $512 \cdot x$ or $512x$ **9.** $x \div 2$ or $\frac{x}{2}$ **11.** $17 + x + 5x$ **13.** $-5 + x = -7$ **15.** $3x = 27$ **17.** $-20 - x = 104$ **19.** $5x$ **21.** $11 - x$ **23.** $2x = 108$ **25.** $50 - 8x$ **27.** $5(-3 + x) = -20$ **29.** $9 + 3x = 33$; 8 **31.** $3 + 4 + x = 16$; 9 **33.** $x - 3 = \frac{10}{5}$; 5 **35.** $30 - x = 3(x + 6)$; 3 **37.** $5x - 40 = x + 8$; 12 **39.** $3(x - 5) = \frac{108}{12}$; 8 **41.** $4x = 30 - 2x$; 5 **43.** Canadiens: 24 wins; Maple Leafs: 13 wins **45.** falcon: 185 mph; pheasant: 37 mph **47.** India: 3.5 million students; Turkey: 2.0 million students **49.** *Grand Theft Auto V*: \$30; *Kinect Adventures:* \$12 **51.** 5470 miles **53.** Michigan Stadium: 109,901; Beaver Stadium: 106,572 **55.** California: 660 thousand; Washington: 132 thousand **57.** 71 points **59.** 2010: 275,215; 2020: 808,416 **61.** cheetah: 74 mph; lion: 49 mph **63.** \$225 **65.** Asia: 2023 million; Europe: 704 million **67.** 590 **69.** 1000 **71.** 3000 **73.** yes; answers may vary **75.** \$216,200 **77.** \$549

Chapter 8 Vocabulary Check **1.** simplified; combined **2.** like **3.** variable **4.** algebraic expression **5.** terms **6.** numerical coefficient **7.** evaluating the expression **8.** constant **9.** equation **10.** solution **11.** distributive **12.** multiplication **13.** addition

Chapter 8 Review **1.** −5 **2.** 17 **3.** undefined **4.** 0 **5.** 129 **6.** −2 **7.** 8 cu ft **8.** 64 cu ft **9.** \$1800 **10.** \$300 **11.** $-15x$ **12.** $-\frac{7}{30}x$ **13.** $-6y - 10$ **14.** $-6a - 7$ **15.** $-8y + 2$ **16.** $4.6x - 11.9$ **17.** $-8y$ **18.** $15y - 24$ **19.** $11x - 12$ **20.** $4x - 7$ **21.** $-5a + 4$ **22.** $12y - 9$ **23.** $(6x - 3)$ sq yd **24.** $28y$ m **25.** yes **26.** no **27.** −2 **28.** 7 **29.** $-\frac{1}{2}$ **30.** $-\frac{6}{11}$ **31.** −6 **32.** −20 **33.** 1.3 **34.** 2.4 **35.** 7 **36.** −9 **37.** 1 **38.** −5 **39.** $-\frac{4}{5}$ **40.** −24 **41.** −120 **42.** 13 **43.** 4 **44.** 3 **45.** 5 **46.** 12 **47.** 63 **48.** 33 **49.** −2.25 **50.** 1.3 **51.** 2 **52.** 6 **53.** 11 **54.** −5 **55.** 6 **56.** 8 **57.** $20 - (-8) = 28$ **58.** $-2 - 19 = -21$ **59.** $\frac{-75}{5 + 20} = -3$ **60.** $-5(-2 + 6) = -20$ **61.** $\frac{70}{x}$ **62.** $x - 13$ **63.** $85 - x$ **64.** $2x + 11$ **65.** $x + 8 = 40$ **66.** $2x - 12 = 10$ **67.** 5 **68.** −16 **69.** bamboo: 36 inches; kelp: 18 inches **70.** incumbent: 14,752 votes; challenger: 3546 votes **71.** 29 **72.** $-\frac{1}{4}$ **73.** $-11x$ **74.** $-35x$ **75.** $22x - 19$ **76.** $-9x - 32$ **77.** $x - 17$ **78.** $3(x + 5)$ **79.** $x - 3 = \frac{x}{4}$ **80.** $6x = x + 2$ **81.** no **82.** yes **83.** −1 **84.** −25 **85.** 13 **86.** −6 **87.** 17 **88.** 7 **89.** −22 **90.** −6 **91.** $-\frac{3}{4}$ **92.** −4 **93.** 2 **94.** $-\frac{8}{3}$ **95.** 12 **96.** −8 **97.** 0 **98.** 0 **99.** 5 **100.** 1 **101.** TV: \$74.7 billion; Internet: \$75.3 billion **102.** TV: \$81.6 billion; Internet: \$93.4 billion

Chapter 8 Getting Ready for the Test **1.** B **2.** D **3.** A **4.** C **5.** A **6.** B **7.** A **8.** C **9.** C **10.** B **11.** B **12.** B **13.** D **14.** A **15.** B **16.** A **17.** C **18.** A **19.** D **20.** E **21.** B **22.** C **23.** E **24.** F

Chapter 8 Test **1.** −1 **2.** $-5x + 5$ **3.** $-6y - 14$ **4.** $14z - 8$ **5.** $4(3x - 1)$ sq m $= (12x - 4)$ sq m **6.** 7 **7.** $-\frac{1}{2}$ **8.** −12 **9.** 40 **10.** 24 **11.** 3 **12.** −2 **13.** −2 **14.** 4.5 **15.** 0 **16.** −2 **17.** $\frac{22}{3}$ **18.** 4 **19.** 0 **20.** 6000 sq ft **21.** 30 sq ft **22.** **a.** $17x$ **b.** $20 - 2x$ **23.** −2 **24.** 34 points **25.** 244 women

Cumulative Review **1.** hundred-thousands; Sec. 1.2, Ex. 1 **2.** two thousand thirty-six; Sec. 1.2 **3.** 184,046; Sec. 1.3, Ex. 2 **4.** 39; Sec. 1.7 **5.** 13 in.; Sec. 1.3, Ex. 5 **6.** 17; Sec. 1.4 **7.** 10,591,862; Sec. 1.3, Ex. 7 **8.** 5; Sec. 1.9 **9.** 7321; Sec. 1.4, Ex. 2 **10.** 64; Sec. 1.9 **11.** **a.** Indonesia **b.** 333; Sec. 1.3, Ex. 8 **12.** $-\frac{1}{8}$; Sec. 3.3 **13.** 4; Sec. 2.2, Ex. 6a **14.** −20; Sec. 2.2 **15.** −5; Sec. 2.2, Ex. 6b **16.** 0; Sec. 2.2 **17.** −23; Sec. 2.3, Ex. 4 **18.** −3.6; Sec. 4.2 **19.** −10; Sec. 2.3, Ex. 14

20. $\frac{1}{10}$; Sec. 3.5 **21.** −7; Sec. 2.4, Ex. 6 **22.** 5.8; Sec. 4.2 **23.** 1; Sec. 2.4, Ex. 7 **24.** $-\frac{31}{120}$; Sec. 3.5 **25.** 15; Sec. 2.5, Ex. 2 **26.** −9.6; Sec. 4.3 **27.** 24; Sec. 2.5, Ex. 7 **28.** −4; Sec. 3.7 **29.** 3; Sec. 2.6, Ex. 9 **30.** 9.7; Sec. 8.2 **31.** 0.8496; Sec. 4.3, Ex. 2 **32.** 53.1; Sec. 4.3 **33.** **a.** $\frac{5}{8}$ **b.** $\frac{4}{13}$; Sec. 5.1, Ex. 6 **34.** $\frac{23}{36}$; Sec. 3.5 **35.** 5; Sec. 6.2, Ex. 9 **36.** $\frac{7}{12}$; Sec. 3.3 **37.** 75%; Sec. 6.2, Ex. 11 **38.** $\frac{18}{23}$; Sec. 3.7 **39.** 48 oz; Sec. 5.5, Ex. 2 **40.** 24,000; Sec. 1.5 **41.** 11.3 L or 11,300 ml; Sec. 5.6, Ex. 8 **42.** 0.024; Sec. 4.1 **43.** yes; Sec. 5.2, Ex. 4 **44.** yes; Sec. 5.2 **45.** 22.4 cc; Sec. 5.3, Ex. 2 **46.** 262.5 mi; Sec. 5.3 **47.** 0.046; Sec. 6.1, Ex. 4 **48.** 4.52; Sec. 6.1 **49.** $\frac{1}{3}$; Sec. 6.1, Ex. 11 **50.** $\frac{27}{100}$; Sec. 6.1 **51.** $5 = n \cdot 20$; Sec. 6.2, Ex. 1 **52.** $\frac{5}{20} = \frac{p}{100}$; Sec. 6.3 **53.** sales tax: \$6.41; total price: \$91.91; Sec. 6.5, Ex. 1 **54.** \$1610; Sec. 6.5 **55.** **a.** 45 reptile species **b.** fishes; Sec. 7.1, Ex. 3 **56.** mean: 8; median: 9; mode: 11; Sec. 7.3

Chapter 9 Geometry

Section 9.1

Vocabulary, Readiness & Video Check **1.** plane **3.** Space **5.** ray **7.** straight **9.** acute **11.** Parallel; intersecting **13.** degrees **15.** vertical **17.** $\angle WUV$, $\angle VUW$, $\angle U$, $\angle x$ **19.** $180° - 17° = 163°$

Exercise Set 9.1 **1.** line; line CD or line l or $\overleftrightarrow{CD}$ **3.** line segment; line segment MN or $\overline{MN}$ **5.** angle; $\angle GHI$ or $\angle IHG$ or $\angle H$ **7.** ray; ray UW or $\overrightarrow{UW}$ **9.** $\angle CPR$, $\angle RPC$ **11.** $\angle TPM$, $\angle MPT$ **13.** straight **15.** right **17.** obtuse **19.** acute **21.** 67° **23.** 163° **25.** 32° **27.** 30° **29.** $\angle MNP$ and $\angle RNO$; $\angle PNQ$ and $\angle QNR$ **31.** $\angle SPT$ and $\angle TPQ$; $\angle SPR$ and $\angle RPQ$; $\angle SPT$ and $\angle SPR$; $\angle TPQ$ and $\angle QPR$ **33.** 27° **35.** 132° **37.** $m\angle x = 30°$; $m\angle y = 150°$; $m\angle z = 30°$ **39.** $m\angle x = 77°$; $m\angle y = 103°$; $m\angle z = 77°$ **41.** $m\angle x = 100°$; $m\angle y = 80°$; $m\angle z = 100°$ **43.** $m\angle x = 134°$; $m\angle y = 46°$; $m\angle z = 134°$ **45.** $\angle ABC$ or $\angle CBA$ **47.** $\angle DBE$ or $\angle EBD$ **49.** 15° **51.** 50° **53.** 65° **55.** 95° **57.** $\frac{9}{8}$ or $1\frac{1}{8}$ **59.** $\frac{7}{32}$ **61.** $\frac{5}{6}$ **63.** $\frac{4}{3}$ or $1\frac{1}{3}$ **65.** 360° **67.** 54.8° **69.** 45° **71.** false; answers may vary **73.** true **75.** $m\angle a = 60°$; $m\angle b = 50°$; $m\angle c = 110°$; $m\angle d = 70°$; $m\angle e = 120°$ **77.** no; answers may vary **79.** 45°, 45°

Section 9.2

Vocabulary, Readiness & Video Check **1.** Because the sum of the measures of the angles of a triangle equals 180°, each angle in an equilateral triangle must measure 60°.

Exercise Set 9.2 **1.** pentagon **3.** hexagon **5.** quadrilateral **7.** pentagon **9.** equilateral **11.** scalene; right **13.** isosceles **15.** 25° **17.** 13° **19.** 40° **21.** diameter **23.** rectangle **25.** parallelogram **27.** hypotenuse **29.** 14 m **31.** 14.5 cm **33.** 40.6 cm **35.** 2.3 mm **37.** cylinder **39.** rectangular solid **41.** cone **43.** cube **45.** rectangular solid **47.** sphere **49.** pyramid **51.** 14.8 in. **53.** 13 mi **55.** 72,368 mi **57.** 108 **59.** 12.56 **61.** true **63.** true **65.** false **67.** yes; answers may vary **69.** answers may vary

Section 9.3

Vocabulary, Readiness & Video Check **1.** perimeter **3.** π **5.** $\frac{22}{7}$ (or 3.14); 3.14 $\left(\text{or } \frac{22}{7}\right)$ **7.** Opposite sides of a rectangle have the same length, so we can just find the sum of the measures of all four sides.

Exercise Set 9.3 **1.** 64 ft **3.** 120 cm **5.** 21 in. **7.** 48 ft **9.** 42 in. **11.** 155 cm **13.** 21 ft **15.** 624 ft **17.** 346 yd **19.** 22 ft **21.** \$55 **23.** 72 in. **25.** 28 in. **27.** \$36.12 **29.** 96 m **31.** 66 ft **33.** 74 cm **35.** 17π cm; 53.38 cm **37.** 16π mi; 50.24 mi **39.** 26π m; 81.64 m **41.** 150π ft; 471 ft **43.** 12,560 ft **45.** 30.7 mi **47.** 14π cm $\approx$ 43.96 cm **49.** 40 mm **51.** 84 ft **53.** 23 **55.** 1 **57.** 6 **59.** 10 **61.** **a.** width: 30 yd; length: 40 yd **b.** 140 yd **63.** b **65.** **a.** 62.8 m; 125.6 m **b.** yes **67.** answers may vary **69.** 27.4 m **71.** 75.4 m **73.** 6.5 ft

Section 9.4

Vocabulary, Readiness & Video Check **1.** The formula for the area of a rectangle; we split the L-shaped figure into two rectangles, used the area formula twice to find the area of each, and then added these two areas.

Exercise Set 9.4 **1.** 7 sq m **3.** $9\frac{3}{4}$ sq yd **5.** 15 sq yd **7.** 2.25π sq in. $\approx$ 7.065 sq in. **9.** 17.64 sq ft **11.** 28 sq m **13.** 22 sq yd **15.** $36\frac{3}{4}$ sq ft **17.** $22\frac{1}{2}$ sq in. **19.** 25 sq cm **21.** 86 sq mi **23.** 24 sq cm **25.** 36π sq in. $\approx 113\frac{1}{7}$ sq in. **27.** 168 sq ft **29.** 128,775 sq ft **31.** 1π sq cm $\approx$ 3.14 sq cm **33.** 128 sq in.; $\frac{8}{9}$ sq ft **35.** 510 sq in. **37.** 168 sq ft **39.** 9200 sq ft **41.** **a.** 381 sq ft **b.** 4 squares **43.** 14π in. $\approx$ 43.96 in. **45.** 25 ft **47.** $12\frac{3}{4}$ ft **49.** perimeter **51.** area **53.** area **55.** perimeter **57.** 12-in. pizza

59. $1\frac{1}{3}$ sq ft; 192 sq in. **61.** 7.74 sq in. **63.** 5.29π sq mm; 16.6106 sq mm **65.** 298.5 sq m **67.** **a.** width: 40 yd; length: 60 yd **b.** 2400 sq yd **69.** no; answers may vary

Section 9.5

Vocabulary, Readiness & Video Check **1.** surface area **3.** cubic **5.** perimeter **7.** Exact answers are in terms of π, and approximate answers use an approximation for π.

Exercise Set 9.5 **1.** $V = 72$ cu in.; $SA = 108$ sq in. **3.** $V = 512$ cu cm; $SA = 384$ sq cm **5.** $V = 4\pi$ cu yd $\approx 12\frac{4}{7}$ cu yd; $SA = (2\sqrt{13}\pi + 4\pi)$ sq yd ≈ 35.23 sq yd **7.** $V = \frac{500}{3}\pi$ cu in. $\approx 523\frac{17}{21}$ cu in.; $SA = 100\pi$ sq in. $\approx 314\frac{2}{7}$ sq in. **9.** $V = 9\pi$ cu in. $\approx 28\frac{2}{7}$ cu in. **11.** V = 75 cu cm **13.** $2\frac{10}{27}$ cu in. **15.** $V = 8.4$ cu ft; $SA = 26$ sq ft **17.** 960 cu cm **19.** $V = \frac{1372}{3}\pi$ cu in. or $457\frac{1}{3}\pi$ cu in.; $SA = 196\pi$ sq in. **21.** $7\frac{1}{2}$ cu ft **23.** 5.25π cu in. **25.** 4.5π cu m; 14.13 cu m **27.** $10\frac{5}{6}$ cu in. **29.** 288π cu ft **31.** $12\frac{4}{7}$ cu cm **33.** 8.8 cu in. **35.** 10.648 cu in. **37.** 25 **39.** 9 **41.** 5 **43.** 20 **45.** 2093.33 cu m **47.** no; answers may vary **49.** 5.5 cu ft; 5.8 cu ft; **(b)** is larger **51.** $6\frac{2}{3}\pi$ cu in. $\approx$ 21 cu in.

Integrated Review **1.** 153°; 63° **2.** $m\angle x = 75°$; $m\angle y = 105°$; $m\angle z = 75°$ **3.** $m\angle x = 128°$; $m\angle y = 52°$; $m\angle z = 128°$ **4.** $m\angle x = 52°$ **5.** 4.6 in. **6.** $4\frac{1}{4}$ in. **7.** 20 m; 25 sq m **8.** 12 ft; 6 sq ft **9.** 10π cm $\approx$ 31.4 cm; 25π sq cm $\approx$ 78.5 sq cm **10.** 32 mi; 44 sq mi **11.** 54 cm; 143 sq cm **12.** 62 ft; 238 sq ft **13.** 64 cu in. **14.** 30.6 cu ft **15.** 400 cu cm **16.** $4\frac{1}{2}\pi$ cu mi $\approx 14\frac{1}{7}$ cu mi

Section 9.6

Vocabulary, Readiness & Video Check **1.** false **3.** true **5.** false **7.** $\angle M$ and $\angle Y$, $\angle N$ and $\angle X$, $\angle P$ and $\angle Z$; $\frac{p}{z} = \frac{m}{y} = \frac{n}{x}$ **9.** The ratios of corresponding sides are the same.

Exercise Set 9.6 **1.** congruent; SSS **3.** not congruent **5.** congruent; ASA **7.** congruent; SAS **9.** $\frac{2}{1}$ **11.** $\frac{3}{2}$ **13.** 4.5 **15.** 6 **17.** 5 **19.** 13.5 **21.** 17.5 **23.** 10 **25.** 28.125 **27.** 10 **29.** 520 ft **31.** 500 ft **33.** 60 ft **35.** 14.4 ft **37.** 52 neon tetras **39.** 381 ft **41.** 4.01 **43.** 1.23 **45.** $3\frac{8}{9}$ in.; no **47.** 8.4 **49.** answers may vary **51.** 200 ft, 300 ft, 425 ft

Chapter 9 Vocabulary Check **1.** right triangle; hypotenuse; legs **2.** line segment **3.** complementary **4.** line **5.** perimeter **6.** angle; vertex **7.** Congruent **8.** Area **9.** ray **10.** plane **11.** transversal **12.** straight **13.** volume **14.** vertical **15.** adjacent **16.** obtuse **17.** right **18.** acute **19.** supplementary **20.** Similar

Chapter 9 Review **1.** right **2.** straight **3.** acute **4.** obtuse **5.** 65° **6.** 75° **7.** 58° **8.** 98° **9.** 90° **10.** 25° **11.** $\angle a$ and $\angle b$; $\angle b$ and $\angle c$; $\angle c$ and $\angle d$; $\angle d$ and $\angle a$ **12.** $\angle x$ and $\angle w$; $\angle y$ and $\angle z$ **13.** $m\angle x = 100°$; $m\angle y = 80°$; $m\angle z = 80°$ **14.** $m\angle x = 155°$; $m\angle y = 155°$; $m\angle z = 25°$ **15.** $m\angle x = 53°$; $m\angle y = 53°$; $m\angle z = 127°$ **16.** $m\angle x = 42°$; $m\angle y = 42°$; $m\angle z = 138°$ **17.** 103° **18.** 60° **19.** 60° **20.** 65° **21.** $4\frac{1}{5}$ m **22.** 7 ft **23.** 9.5 m **24.** $15\frac{1}{5}$ cm **25.** cube **26.** cylinder **27.** pyramid **28.** rectangular solid **29.** 18 in. **30.** 2.35 m **31.** pentagon **32.** hexagon **33.** equilateral **34.** isosceles, right **35.** 89 m **36.** 30.6 cm **37.** 36 m **38.** 90 ft **39.** 32 ft **40.** 440 ft **41.** 5.338 in. **42.** 31.4 yd **43.** 240 sq ft **44.** 140 sq m **45.** 600 sq cm **46.** 189 sq yd **47.** 49π sq ft $\approx$ 153.86 sq ft **48.** 82.81 sq m **49.** 1248 sq cm **50.** 144 sq m **51.** 432 sq ft **52.** 130 sq ft **53.** $V = 15\frac{5}{8}$ cu in.; $SA = 37\frac{1}{2}$ sq in. **54.** $V = 84$ cu ft; $SA = 136$ sq ft **55.** $V = 20{,}000\pi$ cu cm $\approx$ 62,800 cu cm **56.** $V = \frac{1}{6}\pi$ cu km $\approx \frac{11}{21}$ cu km **57.** $2\frac{2}{3}$ cu ft **58.** 307.72 cu in. **59.** $7\frac{1}{2}$ cu ft **60.** 0.5π cu ft or $\frac{1}{2}\pi$ cu ft **61.** $37\frac{1}{2}$ **62.** $13\frac{1}{3}$ **63.** 17.4 **64.** 12 **65.** 33 ft **66.** $x = \frac{5}{6}$ in.; $y = 2\frac{1}{6}$ in. **67.** 108° **68.** 89° **69.** 82° **70.** 78° **71.** 95° **72.** 57° **73.** 13 m **74.** 12.6 cm **75.** 22 dm **76.** 27.3 in. **77.** 194 ft **78.** 50 m **79.** 1624 sq m **80.** 9π sq m $\approx$ 28.26 sq m **81.** $346\frac{1}{2}$ cu in. **82.** 140 cu in. **83.** 1260 cu ft **84.** 28.728 cu ft **85.** $6\frac{1}{2}$ **86.** 12

Chapter 9 Getting Ready for the Test **1.** D **2.** B **3.** E **4.** A **5.** F **6.** C **7.** B **8.** C **9.** A **10.** B **11.** B and E **12.** C **13.** A and D **14.** B **15.** A or D **16.** C

Chapter 9 Test **1.** 12° **2.** 56° **3.** 57° **4.** $m\angle x = 118°$; $m\angle y = 62°$; $m\angle z = 118°$ **5.** $m\angle x = 73°$; $m\angle y = 73°$; $m\angle z = 73°$

6. 6.2m **7.** $10\frac{1}{4}$ in. **8.** 26° **9.** circumference $= 18\pi$ in. $\approx$ 56.52 in.; area $= 81\pi$ sq in. $\approx$ 254.34 sq in. **10.** perimeter = 24.6 yd; area = 37.1 sq yd **11.** perimeter = 68 in.; area = 185 sq in. **12.** $62\frac{6}{7}$ cu in. **13.** V = 30 cu ft; SA = 62 sq ft **14.** 16 in. **15.** 18 cu ft **16.** 62 ft; $115.94 **17.** 9904 sq ft; 198.08 oz **18.** 7.5 **19.** 69 ft

Cumulative Review **1.** negative fifty and eighty-two hundredths; Sec. 4.1, Ex. 1b **2.** $\frac{53}{66}$; Sec. 3.5 **3.** 736.2; Sec. 4.1, Ex. 15 **4.** 700; Sec. 4.1 **5.** 47.06; Sec. 4.2, Ex. 3 **6.** $-1\frac{9}{11}$; Sec. 3.7 **7.** 76.8; Sec. 4.3, Ex. 5 **8.** $\frac{7}{66}$; Sec. 3.3 **9.** −76,300; Sec. 4.3, Ex. 7 **10.** $11\frac{1}{2}$; Sec. 3.7 **11.** 38.6; Sec. 4.4, Ex. 1 **12.** 0.567; Sec. 4.4 **13.** −3.7; Sec. 4.5, Ex. 12 **14.** $\frac{3}{5}$ or 0.6; Sec. 4.5 **15.** >; Sec. 4.5, Ex. 8 **16.** <; Sec. 4.5 **17.** $\frac{26}{31}$; Sec. 5.1, Ex. 3 **18.** $\frac{16}{45}$; Sec. 3.5 **19.** yes; Sec. 5.2, Ex. 2 **20.** $\frac{35}{2}$ or $17\frac{1}{2}$; Sec. 5.2 **21.** 25%; Sec. 6.1, Ex. 1 **22.** 10; Sec. 8.3 **23.** $\frac{19}{1000}$; Sec. 6.1, Ex. 9 **24.** $\frac{13}{50}$; Sec. 6.1 **25.** $\frac{5}{4}$ or $1\frac{1}{4}$; Sec. 6.1, Ex. 10 **26.** $\frac{28}{5}$ or $5\frac{3}{5}$; Sec. 6.1 **27.** 255; Sec. 6.2, Ex. 8 **28.** 15%; Sec. 6.2 or 6.3 **29.** 52; Sec. 6.3, Ex. 9 **30.** 9; Sec. 1.9 **31.** 775 freshmen; Sec. 6.4, Ex. 3 **32.** $2.25/sq ft; Sec. 5.1 **33.** $3210; Sec. 6.5, Ex. 3 **34.** 35 exercises; Sec. 5.3 **35.** 96 in.; Sec. 5.4, Ex. 1 **36.** 2 yd 2 ft 4 in.; Sec. 5.4 **37.** 3200 g; Sec. 5.5, Ex. 7 **38.** 0.07 m; Sec. 5.4 **39.** 3 gal 3 qt; Sec. 5.6, Ex. 3 **40.** 70,052; Sec. 1.2 **41.** 50°; Sec. 9.2, Ex. 1 **42.** 33 m; Sec. 9.3 **43.** 28 in.; Sec. 9.3, Ex. 1 **44.** −4; Sec. 8.4 **45.** $\frac{2}{5}$; Sec. 4.6, Ex. 3 **46.** $\frac{3}{4}$; Sec. 4.6

Appendix A Tables

Exercise Set A.1 **1.** 5 **3.** 5 **5.** 12 **7.** 8 **9.** 14 **11.** 12 **13.** 10 **15.** 9 **17.** 11 **19.** 18 **21.** 10 **23.** 10 **25.** 17 **27.** 13 **29.** 12 **31.** 16 **33.** 7 **35.** 4 **37.** 8 **39.** 8 **41.** 6 **43.** 16 **45.** 9 **47.** 12 **49.** 7 **51.** 2 **53.** 11 **55.** 8 **57.** 5 **59.** 14 **61.** 13 **63.** 7 **65.** 7 **67.** 13 **69.** 5 **71.** 4 **73.** 16 **75.** 14 **77.** 1 **79.** 4 **81.** 10 **83.** 8 **85.** 5 **87.** 10 **89.** 11 **91.** 12 **93.** 9 **95.** 9 **97.** 1
4 **99.** 9

Exercise Set A.2 **1.** 1 **3.** 56 **5.** 32 **7.** 28 **9.** 4 **11.** 63 **13.** 6 **15.** 30 **17.** 24 **19.** 18 **21.** 40 **23.** 32 **25.** 54 **27.** 56 **29.** 54 **31.** 2 **33.** 36 **35.** 12 **37.** 36 **39.** 12 **41.** 48 **43.** 8 **45.** 0 **47.** 27 **49.** 15 **51.** 45 **53.** 0 **55.** 81 **57.** 0 **59.** 0 **61.** 18 **63.** 3 **65.** 36 **67.** 63 **69.** 35 **71.** 42 **73.** 40 **75.** 0 **77.** 9 **79.** 15 **81.** 5 **83.** 0 **85.** 21 **87.** 0 **89.** 16 **91.** 0 **93.** 4 **95.** 6 **97.** 18 **99.** 0

Appendix B Exponents and Polynomials

Exercise Set B.1 **1.** $-5x - 24$ **3.** $-9z^2 - 2z + 6$ **5.** $9y^2 + 25y - 40$ **7.** $-4.3a^4 - 2a^2 + 9$ **9.** $4a - 8$ **11.** $-2x^2 + 4x + 1$ **13.** $-20y^3 + 12y^2 - 4$ **15.** $-x + 16$ **17.** $8y^2 - 10y - 8$ **19.** $5x - 12$ **21.** $y - 4$ **23.** $4x^2 + x - 16$ **25.** $9x - 4.1$ **27.** $15a - 7$ **29.** $-15y + 3.6$ **31.** $19t^2 - 11t + 3$ **33.** $b^3 - b^2 + 7b - 1$ **35.** $-5x^2 - 11x + 13$ **37.** $-6z + \frac{9}{7}$ **39.** 1 **41.** −5 **43.** −8 **45.** 20 **47.** 25 **49.** 50 **51.** 576 ft **53.** $3200 **55.** 909 ft **57.** $(8x + 2)$ in. **59.** $(4x - 15)$ units **61.** 20; 6; 2 **63.** 7.2752 **65.** 29 ft; −243 ft; answers may vary

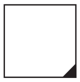
Exercise Set B.2 **1.** x^{14} **3.** a^7 **5.** $15z^5$ **7.** $-40x^2$ **9.** $25x^6y^4$ **11.** $28a^5b^6$ **13.** $42x^3$ **15.** $12a^{17}$ **17.** x^{15} **19.** z^{20} **21.** b^{62} **23.** $81a^4$ **25.** $a^{33}b^{24}$ **27.** $121x^6y^{12}$ **29.** $-24y^{22}$ **31.** $256x^9y^{13}$ **33.** $16x^{12}$ sq in. **35.** $12a^4b^5$ sq m **37.** $18{,}003{,}384a^{45}b^{30}$ **39.** $34{,}867.84401x^{50}$ **41.** $a^{127}b^{86}c^{25}$ **43.** answers may vary

Exercise Set B.3 **1.** $27x^3 - 9x$ **3.** $-20a^3 + 30a^2 - 5a$ **5.** $42x^4 - 35x^3 + 49x^2$ **7.** $x^2 + 13x + 30$ **9.** $2x^2 + 2x - 24$ **11.** $36a^2 + 48a + 16$ **13.** $a^3 - 33a + 18$ **15.** $8x^3 + 2x^2 - 55x + 50$ **17.** $x^5 + 4x^4 + 6x^3 + 7x^2 + 2x$ **19.** $-30r^2 + 20r$ **21.** $-6y^3 - 2y^4 + 12y^2$ **23.** $x^2 + 14x + 24$ **25.** $4a^2 - 9$ **27.** $x^2 + 10x + 25$ **29.** $b^2 + \frac{7}{5}b + \frac{12}{25}$ **31.** $6x^3 + 25x^2 + 10x + 1$ **33.** $49x^2 + 70x + 25$ **35.** $4x^2 - 4x + 1$ **37.** $8x^5 - 8x^3 - 6x^2 - 6x + 9$ **39.** $x^5 + 2x^4 + 3x^3 + 2x^2 + x$ **41.** $10z^4 - 3z^3 + 3z - 2$ **43.** $(y^3 - 3y^2 - 16y - 12)$ sq ft **45.** $(x^4 - 3x^2 + 1)$ sq m **47.** answers may vary

Appendix C Inductive and Deductive Reasoning

Exercise Set C **1.** inductive **3.** deductive **5.** inductive **7.** 81 **9.** C **11.** 36 **13.** E **15.** 99,999 **17.** 9 **19.** 74 **21.** 241

23.



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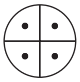

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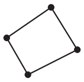

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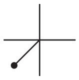

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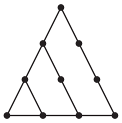

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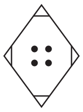
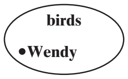


35. answers may vary **37.**



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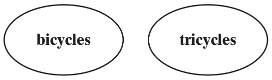

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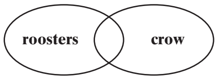

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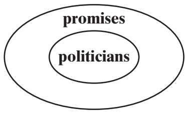


45. Joey **47.** 15 people **49.** Donna **51.** Julio **53.** 5 people

Practice Final Exam **1.** 113 **2.** 33 **3.** $\frac{64}{3}$ or $21\frac{1}{3}$ **4.** $\frac{3}{4}$ **5.** 8 **6.** 40.902 **7.** -6.2 **8.** $16\frac{8}{11}$ **9.** $\frac{45}{2}$ or $22\frac{1}{2}$ **10.** -20.42 **11.** $\frac{3}{8}$ **12.** -117 **13.** 12 **14.** 2 **15.** $-\frac{5}{3}$ or $-1\frac{2}{3}$ **16.** 52,000 **17.** 34.9 **18.** 0.941 **19.** 0.85 **20.** 610% **21.** 37.5% **22.** $\frac{1}{500}$ **23.** perimeter: $3\frac{1}{3}$ ft; area: $\frac{2}{3}$ sq ft **24.** $\frac{15}{2}$ **25.** $\frac{3\text{ in.}}{10\text{ days}}$ **26.** 81.25 km/hr **27.** 8-oz size **28.** $\frac{48}{11}$ or $4\frac{4}{11}$ **29.** 11.4 **30.** 24 mi **31.** $53\frac{1}{3}$ g **32.** \$17 **33.** 1250 **34.** 75% **35.** 38.4 lb **36.** discount: \$18; sale price: \$102 **37.** 0.04 g **38.** 10 qt **39.** 16 ft 6 in. **40.** mean: 38.4; median: 42; no mode **41.** 11 students **42.** -1 **43.** $14z - 8$ **44.** 40 **45.** 24 **46.** -2 **47.** 244 women **48.** 12° **49.** $m\angle x = 118°; m\angle y = 62°; m\angle z = 118°$ **50.** 26° **51.** 7.5 or $7\frac{1}{2}$

Solutions to Selected Exercises

Chapter 1

Exercise Set 1.2

1. The place value of the 5 in 657 is tens.

5. The place value of the 5 in 43,526,000 is hundred-thousands.

9. 354 is written as three hundred fifty-four.

13. 26,990 is written as twenty-six thousand, nine hundred ninety.

17. 24,350,185 is written as twenty-four million, three hundred fifty thousand, one hundred eighty-five.

21. 2717 is written as two thousand, seven hundred seventeen.

25. 14,433 is written as fourteen thousand, four hundred thirty-three.

29. Six thousand, five hundred eighty-seven in standard form is 6587.

33. Thirteen million, six hundred one thousand, eleven in standard form is 13,601,011.

37. Two hundred sixty thousand, nine hundred ninety-seven in standard form is 260,997.

41. Sixteen thousand, seven hundred thirty-two in standard form is 16,732.

45. One hundred eight thousand in standard form is 108,000.

49. $3470 = 3000 + 400 + 70$

53. $66{,}049 = 60{,}000 + 6000 + 40 + 9$

57. 5532 is written as five thousand, five hundred thirty-two.

61. The tallest mountain in New England is Mt. Washington.

65. The number of visitors to the Vatican Museums was 6,002,000, which is written as six million, two thousand.

69. The largest number is 9861.

73. answers may vary

77. The Pro Football Hall of Fame is located in the town of Canton.

Exercise Set 1.3

1.
$$\begin{array}{r} 14 \\ +\ 22 \\ \hline 36 \end{array}$$

5.
$$\begin{array}{r} 12 \\ 13 \\ +\ 24 \\ \hline 49 \end{array}$$

9.
$$\begin{array}{r} 53 \\ +\ 64 \\ \hline 117 \end{array}$$

13.
$$\begin{array}{r} \overset{1}{}2\overset{1}{2},\overset{1}{7}81 \\ +186{,}297 \\ \hline 209{,}078 \end{array}$$

17.
$$\begin{array}{r} \overset{2}{}6 \\ 21 \\ 14 \\ 9 \\ +\ 12 \\ \hline 62 \end{array}$$

21.
$$\begin{array}{r} \overset{1}{6}2 \\ 18 \\ +\ 14 \\ \hline 94 \end{array}$$

25.
$$\begin{array}{r} \overset{1}{7}\overset{1}{5}\overset{1}{4}2 \\ 49 \\ +\ 682 \\ \hline 8273 \end{array}$$

29.
$$\begin{array}{r} 6\overset{2}{2}7 \\ 628 \\ +\ 629 \\ \hline 1884 \end{array}$$

33.
$$\begin{array}{r} \overset{1}{5}\overset{1}{0}7 \\ 593 \\ +\ \ 10 \\ \hline 1110 \end{array}$$

37.
$$\begin{array}{r} \overset{1}{}\overset{1}{}\overset{2}{}\overset{2}{4}9 \\ 628 \\ 5\ 762 \\ +29{,}462 \\ \hline 35{,}901 \end{array}$$

41.
$$\begin{array}{r} \overset{2}{}9 \\ 12 \\ 9 \\ +\ 12 \\ \hline 42 \end{array}$$

The perimeter is 42 inches.

45. Opposite sides of a rectangle have the same length.
$$\begin{array}{r} \overset{2}{}4 \\ 4 \\ 8 \\ +\ 8 \\ \hline 24 \end{array}$$

The perimeter is 24 inches.

49.
$$\begin{aligned} 8 + 3 + 5 + 7 + 5 + 1 &= 8 + 1 + 3 + 7 + 5 + 5 \\ &= 9 + 10 + 10 \\ &= 29 \end{aligned}$$

The perimeter is 29 inches.

53.
$$\begin{array}{r} \overset{1}{}\overset{1}{2}\overset{1}{9}7 \\ +\ 1796 \\ \hline 2093 \end{array}$$

57. "Increased by" indicates addition.
$$\begin{array}{r} \overset{1}{4}52 \\ +\ 92 \\ \hline 544 \end{array}$$

452 increased by 92 is 544.

61.
$$\begin{array}{r} \scriptstyle 1\,1 \\ 20{,}148 \\ +\ 2286 \\ \hline 22{,}434 \end{array}$$
The population of Florida is projected to be 22,434 thousand in 2025.

65.
$$\begin{array}{r} \scriptstyle 21 \\ 78 \\ 90 \\ 102 \\ +\ 70 \\ \hline 340 \end{array}$$
He needs 340 feet of wiring.

69.
$$\begin{array}{r} \scriptstyle 1\ \ 1 \\ 161{,}839 \\ +100{,}352 \\ \hline 262{,}191 \end{array}$$
262,191 Harley-Davidson motorcycles were sold in 2016.

73. The sides of a square are all the same length.
$$\begin{array}{r} \scriptstyle 1 \\ 31 \\ 31 \\ 31 \\ +\ 31 \\ \hline 124 \end{array}$$
The perimeter of the board is 124 feet.

77. California had the most CVS pharmacies.

81. Pennsylvania and New York:
$$\begin{array}{r} \scriptstyle 1 \\ 408 \\ +486 \\ \hline 894 \end{array}$$
Florida and Georgia:
$$\begin{array}{r} \scriptstyle 1 \\ 756 \\ +\ 313 \\ \hline 1069 \end{array}$$
Florida and Georgia had more CVS pharmacies.

85. answers may vary

89.
$$\begin{array}{r} \scriptstyle 1111\ 121\ 1 \\ 56{,}468{,}980 \\ 1{,}236{,}785 \\ +\ 986{,}768{,}000 \\ \hline 1{,}044{,}473{,}765 \end{array}$$

93.
$$\begin{array}{r} \scriptstyle 22 \\ 14 \\ 173 \\ 86 \\ +\ 257 \\ \hline 530 \end{array}$$
The given sum is incorrect; the correct sum is 530.

Exercise Set 1.4

1.
$$\begin{array}{r} 67 \\ -\ 23 \\ \hline 44 \end{array} \qquad \textit{Check:}\ \begin{array}{r} 44 \\ +\ 23 \\ \hline 67 \end{array}$$

5.
$$\begin{array}{r} 167 \\ -\ 32 \\ \hline 135 \end{array} \qquad \textit{Check:}\ \begin{array}{r} 135 \\ +\ 32 \\ \hline 167 \end{array}$$

9.
$$\begin{array}{r} 6998 \\ -\ 1453 \\ \hline 5545 \end{array} \qquad \textit{Check:}\ \begin{array}{r} 5545 \\ +\ 1453 \\ \hline 6998 \end{array}$$

13.
$$\begin{array}{r} 62 \\ -\ 37 \\ \hline 25 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1 \\ 25 \\ +\ 37 \\ \hline 62 \end{array}$$

17.
$$\begin{array}{r} 938 \\ -\ 792 \\ \hline 146 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1 \\ 146 \\ +\ 792 \\ \hline 938 \end{array}$$

21.
$$\begin{array}{r} 600 \\ -\ 432 \\ \hline 168 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1\,1 \\ 168 \\ +\ 432 \\ \hline 600 \end{array}$$

25.
$$\begin{array}{r} 923 \\ -\ 476 \\ \hline 447 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1\,1 \\ 447 \\ +\ 476 \\ \hline 923 \end{array}$$

29.
$$\begin{array}{r} 533 \\ -\ 29 \\ \hline 504 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1 \\ 504 \\ +\ 29 \\ \hline 533 \end{array}$$

33.
$$\begin{array}{r} 1983 \\ -\ 1904 \\ \hline 79 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1 \\ 79 \\ +\ 1904 \\ \hline 1983 \end{array}$$

37.
$$\begin{array}{r} 50{,}000 \\ -\ 17{,}289 \\ \hline 32{,}711 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1\,1\ 1\,1 \\ 32{,}711 \\ +\ 17{,}289 \\ \hline 50{,}000 \end{array}$$

41.
$$\begin{array}{r} 51{,}111 \\ -\ 19{,}898 \\ \hline 31{,}213 \end{array} \qquad \textit{Check:}\ \begin{array}{r} \scriptstyle 1\,1\ 1\,1 \\ 31{,}213 \\ +\ 19{,}898 \\ \hline 51{,}111 \end{array}$$

45.
$$\begin{array}{r} 41 \\ -\ 21 \\ \hline 20 \end{array}$$
The difference of 41 and 21 is 20.

49.
$$\begin{array}{r} 108 \\ -\ 36 \\ \hline 72 \end{array}$$
108 less 36 is 72.

53.
$$\begin{array}{r} 503 \\ -\ 239 \\ \hline 264 \end{array}$$
She has 264 more pages to read.

57.
$$\begin{array}{r} 189{,}000 \\ +\ 75{,}000 \\ \hline 264{,}000 \end{array}$$
The total land area drained by the Upper Mississippi and Lower Mississippi sub-basins is 264,000 square miles.

61.
$$\begin{array}{r} 20{,}320 \\ -\ 14{,}255 \\ \hline 6065 \end{array}$$
The peak of Denali is 6065 feet higher than the peak of Long’s Peak.

65.
$$\begin{array}{r} 645 \\ -\ 287 \\ \hline 358 \end{array}$$
The distance between Hays, Kansas, and Denver, Colorado, is 358 miles.

69.
$$\begin{array}{r} 8536 \\ -\ 6927 \\ \hline 1609 \end{array}$$
The projected increase in the population of Arizona is 1609 thousand.

73.
$$\begin{array}{r} 88 \\ -\ 30 \\ \hline 58 \end{array}$$
Snoring is 58 dB louder than normal conversation.

77.
$$\begin{array}{r} 100{,}000 \\ -\ 94{,}080 \\ \hline 5920 \end{array}$$
The Dole Plantation maze is 5920 square feet larger than the Ruurlo maze.

81.
$$\begin{array}{r} 78 \\ -\ 66 \\ \hline 12 \end{array}$$
Chicago O'Hare International Airport had 12 million more passengers than Dallas/Ft. Worth International Airport in 2016.

85.
$$\begin{array}{r} \overset{1}{9}\overset{1}{8}6 \\ +\ \ 48 \\ \hline 1034 \end{array}$$

89.
$$\begin{array}{r} 9000 \\ -\ \ 482 \\ \hline 8518 \end{array}$$

93. In $48 - 1$, 48 is the minuend and 1 is the subtrahend.

97.
$$\begin{array}{r} 741 \\ -\ \ 56 \\ \hline 685 \end{array}$$
The given answer is incorrect; the correct answer is 685.

101.
$$\begin{array}{r} 526\underline{9} \\ -2\underline{3}85 \\ \hline 28\underline{8}4 \end{array}$$

105.
$$\begin{array}{r} 289{,}462 \\ 369{,}477 \\ 218{,}287 \\ +121{,}685 \\ \hline 998{,}911 \end{array} \qquad \begin{array}{r} 1{,}000{,}000 \\ -\ \ 998{,}911 \\ \hline 1089 \end{array}$$
No, they have not reached their goal. They have 1089 more pages to read.

Exercise Set 1.5

1. To round 423 to the nearest ten, observe that the digit in the ones place is 3. Since this digit is less than 5, we do not add 1 to the digit in the tens place. The number 423 rounded to the nearest ten is 420.

5. To round 2791 to the nearest hundred, observe that the digit in the tens place is 9. Since this digit is 5 or greater, we add 1 to the digit in the hundreds place. The number 2791 rounded to the nearest hundred is 2800.

9. To round 21,094 to the nearest thousand, observe that the digit in the hundreds place is 0. Since this digit is less than 5, we do not add 1 to the digit in the thousands place. The number 21,094 rounded to the nearest thousand is 21,000.

13. To round 328,495 to the nearest hundred, observe that the digit in the tens place is 9. Since this digit is 5 or greater, we add 1 to the digit in the hundreds place. The number 328,495 rounded to the nearest hundred is 328,500.

17. To round 39,994 to the nearest ten, observe that the digit in the ones place is 4. Since this digit is less than 5, we do not add 1 to the digit in the tens place. The number 39,994 rounded to the nearest ten is 39,990.

21. Estimate 5281 to a given place value by rounding it to that place value. 5281 rounded to the tens place is 5280, to the hundreds place is 5300, and to the thousands place is 5000.

25. Estimate 14,876 to a given place value by rounding it to that place value. 14,876 rounded to the tens place is 14,880, to the hundreds place is 14,900, and to the thousands place is 15,000.

29. To round 38,387 to the nearest thousand, observe that the digit in the hundreds place is 3. Since this digit is less than 5, we do not add 1 to the digit in the thousands place. Therefore, 38,387 points rounded to the nearest thousand is 38,000 points.

33. To round 4,155,907 to the nearest hundred-thousand, observe that the digit in the ten-thousands place is 5. Since this digit 5 or greater, add 1 to the digit in the hundred-thousands place. Therefore, $4,155,907 rounded to the nearest hundred-thousand is $4,200,000.

37.
$$\begin{array}{rlr} 39 & \text{rounds to} & 40 \\ 45 & \text{rounds to} & 50 \\ 22 & \text{rounds to} & 20 \\ \underline{+\ 17} & \text{rounds to} & \underline{+\ 20} \\ & & 130 \end{array}$$

41.
$$\begin{array}{rlr} 1913 & \text{rounds to} & 1900 \\ 1886 & \text{rounds to} & 1900 \\ \underline{+\ 1925} & \text{rounds to} & \underline{+\ 1900} \\ & & 5700 \end{array}$$

45.
$$\begin{array}{rlr} 3995 & \text{rounds to} & 4000 \\ 2549 & \text{rounds to} & 2500 \\ \underline{+\ 4944} & \text{rounds to} & \underline{+\ 4900} \\ & & 11{,}400 \end{array}$$

49. $229 + 443 + 606$ is approximately $230 + 440 + 610 = 1280$.

The answer of 1278 is correct.

53.
$$\begin{array}{rlr} 899 & \text{rounds to} & 900 \\ 1499 & \text{rounds to} & 1500 \\ \underline{+\ 999} & \text{rounds to} & \underline{+\ 1000} \\ & & 3400 \end{array}$$
The total cost is approximately $3400.

57.
$$\begin{array}{rlr} 20{,}320 & \text{rounds to} & 20{,}000 \\ \underline{-\ 14{,}410} & \text{rounds to} & \underline{-\ 14{,}000} \\ & & 6000 \end{array}$$
The difference in elevation is approximately 6000 feet.

61.
$$\begin{array}{rlr} 1{,}128{,}030 & \text{rounds to} & 1{,}128{,}000 \\ \underline{-\ \ 946{,}357} & \text{rounds to} & \underline{-\ \ 946{,}000} \\ & & 182{,}000 \end{array}$$
The decrease in enrollment was approximately 182,000 children.

65. \$2110 million is \$2,110,000,000 in standard form.
\$2,110,000,000 rounded to the nearest hundred-million is \$2,100,000,000.
\$2,110,000,000 rounded to the nearest billion is \$2,000,000,000.

69. 999 rounded to the nearest hundred is 1000.

73. The smallest possible number that rounds to 8600 is 8550.

77. 54 rounds to 50
17 rounds to 20
$50 + 20 + 50 + 20 = 140$
The perimeter is approximately 140 meters.

Exercise Set 1.6

1. $1 \cdot 24 = 24$

5. $8 \cdot 0 \cdot 9 = 0$

9. $6(3 + 8) = 6 \cdot 3 + 6 \cdot 8$

13. $20(14 + 6) = 20 \cdot 14 + 20 \cdot 6$

17.
$$\begin{array}{r} 6\overset{1}{1}3 \\ \times \quad 6 \\ \hline 3678 \end{array}$$

21.
$$\begin{array}{r} 1\overset{4}{0}\overset{2}{7}4 \\ \times \quad 6 \\ \hline 6444 \end{array}$$

25.
$$\begin{array}{r} 421 \\ \times \ 58 \\ \hline 3\,368 \\ 21\,050 \\ \hline 24{,}418 \end{array}$$

29.
$$\begin{array}{r} 780 \\ \times \ 20 \\ \hline 15{,}600 \end{array}$$

33. $(640)(1)(10) = (640)(10) = 6400$

37.
$$\begin{array}{r} 609 \\ \times \ 234 \\ \hline 2\,436 \\ 18\,270 \\ 121\,800 \\ \hline 142{,}506 \end{array}$$

41.
$$\begin{array}{r} 589 \\ \times \ 110 \\ \hline 5\,890 \\ 58\,900 \\ \hline 64{,}790 \end{array}$$

45. $8 \times 100 = 800$

49. $7406 \cdot 10 = 74{,}060$

53. $50 \cdot 900 = 45{,}000$

57. Area = (length)(width)
= (9 meters)(7 meters)
= 63 square meters
Perimeter = (9 + 7 + 9 + 7) meters
= 32 meters

61.
$$\begin{array}{rcr} 576 & \text{rounds to} & 600 \\ \times\ 354 & \text{rounds to} & \times \quad 400 \\ \cline{3-3} & & 240{,}000 \end{array}$$

65. 38×42 is approximately 40×40, which is 1600.
The best estimate is **c**.

69.
$$\begin{aligned} 80 \times 11 &= (8 \times 10) \times 11 \\ &= 8 \times (10 \times 11) \\ &= 8 \times 110 \\ &= 880 \end{aligned}$$

73.
$$\begin{array}{r} 2240 \\ \times \quad 2 \\ \hline 4480 \end{array}$$

77.
$$\begin{array}{r} 94 \\ \times\ 35 \\ \hline 470 \\ 2820 \\ \hline 3290 \end{array}$$
The total cost is \$3290.

81. Area = (length)(width)
= (110 feet)(80 feet)
= 8800 square feet
The area is 8800 square feet.

85.
$$\begin{array}{r} 94 \\ \times\ 62 \\ \hline 188 \\ 5640 \\ \hline 5828 \end{array}$$
There are 5828 pixels on the screen.

89.
$$\begin{array}{r} 170 \\ \times \quad 8 \\ \hline 1360 \end{array}$$
There are 1360 calories in 8 ounces.

93. There are 60 minutes in one hour, so there are 24×60 minutes in one day.
$$\begin{aligned} 24 \times 60 \times 1000 &= 24 \times 6 \times 10 \times 1000 \\ &= 144 \times 10{,}000 \\ &= 1{,}440{,}000 \end{aligned}$$
They produce 1,440,000 tea bags in one day.

97.
$$\begin{array}{r} 134 \\ \times\ 16 \\ \hline 804 \\ 1340 \\ \hline 2144 \end{array}$$

101.
$$\begin{array}{r} 19 \\ -\ 4 \\ \hline 15 \end{array}$$
The difference of 19 and 4 is 15.

105. **a.** $3 \cdot 5 = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$
b. answers may vary

109. $42 \times 3 = 126$
$42 \times 90 = 3780$
The problem is 42×93.

113. On a side with 7 windows per row, there are $7 \times 23 = 161$ windows. On a side with 4 windows per row, there are $4 \times 23 = 92$ windows.
$161 + 161 + 92 + 92 = 506$
There are 506 windows on the building.

Exercise Set 1.7

1. $54 \div 9 = 6$ because $6 \cdot 9 = 54$

5. $0 \div 8 = 0$ because $0 \cdot 8 = 0$

9. $\frac{18}{18} = 1$ because $1 \cdot 18 = 18$

13. $26 \div 0$ is undefined.

17. $0 \div 14 = 0$ because $0 \cdot 14 = 0$

21.
$$\begin{array}{r} 29 \\ 3\overline{)87} \\ -6 \\ \hline 27 \\ -27 \\ \hline 0 \end{array}$$
Check: $3 \cdot 29 = 87$

25.
$$\begin{array}{r} 338 \\ 3\overline{)1014} \\ -9 \\ \hline 11 \\ -9 \\ \hline 24 \\ -24 \\ \hline 0 \end{array}$$
Check: $3 \cdot 338 = 1014$

29.
$$\begin{array}{r} 9 \\ 7\overline{)63} \\ -63 \\ \hline 0 \end{array}$$
Check: $7 \cdot 9 = 63$

33.
$$\begin{array}{r} 68 \text{ R } 3 \\ 7\overline{)479} \\ -42 \\ \hline 59 \\ -56 \\ \hline 3 \end{array}$$
Check: $7 \cdot 68 + 3 = 479$

37.
$$\begin{array}{r} 38 \text{ R } 1 \\ 8\overline{)305} \\ -24 \\ \hline 65 \\ -64 \\ \hline 1 \end{array}$$
Check: $8 \cdot 38 + 1 = 305$

41.
$$\begin{array}{r} 13 \\ 55\overline{)715} \\ -55 \\ \hline 165 \\ -165 \\ \hline 0 \end{array}$$
Check: $55 \cdot 13 = 715$

45.
$$\begin{array}{r} 97 \text{ R } 8 \\ 97\overline{)9417} \\ -873 \\ \hline 687 \\ -679 \\ \hline 8 \end{array}$$
Check: $97 \cdot 97 + 8 = 9417$

49.
$$\begin{array}{r} 506 \\ 13\overline{)6578} \\ -65 \\ \hline 07 \\ -0 \\ \hline 78 \\ -78 \\ \hline 0 \end{array}$$
Check: $13 \cdot 506 = 6578$

53.
$$\begin{array}{r} 54 \\ 236\overline{)12744} \\ -1180 \\ \hline 944 \\ -944 \\ \hline 0 \end{array}$$
Check: $236 \cdot 54 = 12{,}744$

57.
$$\begin{array}{r} 202 \text{ R } 15 \\ 102\overline{)20619} \\ -204 \\ \hline 21 \\ -0 \\ \hline 219 \\ -204 \\ \hline 15 \end{array}$$
Check: $102 \cdot 202 + 15 = 20{,}619$

61.
$$\begin{array}{r} 17 \\ 7\overline{)119} \\ -7 \\ \hline 49 \\ -49 \\ \hline 0 \end{array}$$

65.
$$\begin{array}{r} 2132 \text{ R } 32 \\ 40\overline{)85312} \\ -80 \\ \hline 53 \\ -40 \\ \hline 131 \\ -120 \\ \hline 112 \\ -80 \\ \hline 32 \end{array}$$

69.
$$\begin{array}{r} 23 \text{ R } 2 \\ 5\overline{)117} \\ -10 \\ \hline 17 \\ -15 \\ \hline 2 \end{array}$$
The quotient of 117 and 5 is 23 R 2.

73.
$$\begin{array}{r} 20 \text{ R } 2 \\ 3\overline{)62} \\ -6 \\ \hline 02 \\ -0 \\ \hline 2 \end{array}$$
The quotient of 62 and 3 is 20 R 2.

77.
$$\begin{array}{r} 165 \\ 318\overline{)52470} \\ \underline{-318} \\ 2067 \\ \underline{-1908} \\ 1590 \\ \underline{-1590} \\ 0 \end{array}$$
The person weighs 165 pounds on Earth.

81.
$$\begin{array}{r} 88 \text{ R } 1 \\ 3\overline{)265} \\ \underline{-24} \\ 25 \\ \underline{-24} \\ 1 \end{array}$$
There are 88 bridges every 3 miles over the 265 miles, plus the first bridge for a total of 89 bridges.

85.
$$\begin{array}{r} 5 \\ 5280\overline{)26400} \\ \underline{-26400} \\ 0 \end{array}$$
Broad Peak is 5 miles tall.

89.
$$\begin{array}{r} \overset{2}{1}0 \\ 24 \\ 35 \\ 22 \\ 17 \\ \underline{+\ 12} \\ 120 \end{array} \qquad \begin{array}{r} 20 \\ 6\overline{)120} \\ \underline{-12} \\ 0 \end{array}$$
$$\text{Average} = \frac{120}{6} = 20$$

93.
$$\begin{array}{r} \overset{2}{8}6 \\ 79 \\ 81 \\ 69 \\ \underline{+\ 80} \\ 395 \end{array} \qquad \begin{array}{r} 79 \\ 5\overline{)395} \\ \underline{-35} \\ 45 \\ \underline{-45} \\ 0 \end{array}$$
$$\text{Average} = \frac{395}{5} = 79$$

97.
$$\begin{array}{r} {}^{1\,1\,1} \\ 82 \\ 463 \\ 29 \\ \underline{+8704} \\ 9278 \end{array}$$

101.
$$\begin{array}{r} 722 \\ \underline{-\ 43} \\ 679 \end{array}$$

105.
$$\begin{array}{r} 9 \text{ R } 12 \\ 24\overline{)228} \\ \underline{-216} \\ 12 \end{array}$$

109. 200 divided by 20 is 200 ÷ 20, which is choice **b**.

113. The average will increase; answers may vary.

117. Since Area = length · width, length $= \dfrac{\text{Area}}{\text{width}}$.
$$\text{length} = \frac{60 \text{ square feet}}{5 \text{ feet}} = \frac{60}{5} \text{ feet} = 12 \text{ feet}$$
The length is 12 feet.

121.
$$\begin{array}{r} 26 \\ \underline{-\ 5} \\ 21 \\ \underline{-\ 5} \\ 16 \\ \underline{-\ 5} \\ 11 \\ \underline{-\ 5} \\ 6 \\ \underline{-\ 5} \\ 1 \end{array}$$
Thus 26 ÷ 5 = 5 R 1.

Exercise Set 1.8

1.
$$\begin{array}{ccccc} 41 & \text{increased by} & 8 & \text{is} & \text{what number} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 41 & + & 8 & = & \text{what number} \end{array}$$
$$\begin{array}{r} 41 \\ \underline{+\ 8} \\ 49 \end{array}$$
41 increased by 8 is 49.

5.
$$\begin{array}{cccccc} \text{The total of} & 35 & \text{and} & 7 & \text{is} & \text{what number} \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 35 & + & 7 & = & \text{what number} \end{array}$$
$$\begin{array}{r} 35 \\ \underline{+\ 7} \\ 42 \end{array}$$
The total of 35 and 7 is 42.

9. a.
$$\begin{array}{ccccccccc} \text{Perimeter} & \text{is} & \text{side} & + & \text{side} & + & \text{side} & + & \text{side} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Perimeter} & = & 120 & + & 80 & + & 120 & + & 80 \\ & = & 400 & & & & & & \end{array}$$
The perimeter is 400 feet.

b.
$$\begin{array}{ccccc} \text{Area} & \text{is} & \text{length} & \text{times} & \text{width} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Area} & = & 120 & \times & 80 \end{array}$$
$$\begin{array}{r} 120 \\ \underline{\times\ 80} \\ 9600 \end{array}$$
The area is 9600 square feet.

13.
$$\begin{array}{ccccc} \text{Hours per week} & \text{is} & \text{hours per day} & \text{times} & \text{days per week} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Hours per week} & = & 24 & \times & 7 \end{array}$$
$$\begin{array}{r} 24 \\ \underline{\times\ 7} \\ 168 \end{array}$$
There are 168 hours in a week.

17.

Difference in years	is	First State National Monument	minus	Yellowstone National Park
↓	↓	↓	↓	↓
Difference	=	2013	−	1872

$$\begin{array}{r}2013\\ -1872\\ \hline 141\end{array}$$

Yellowstone National Park is 141 years older than First State National Monument.

21.

Total	is	number of Fairview	plus	number of Midway	plus	number of Riverside
↓	↓	↓	↓	↓	↓	↓
Total	=	287	+	252	+	180

$= 287 + 252 + 180 = 719$

There are a total of 719 towns named Fairview, Midway, or Riverside.

25.

Calories in 1 ounce	is	calories	per	ounces
↓	↓	↓	↓	↓
Calories in 1 ounce	=	165	÷	3

$$\begin{array}{r}55\\ 3\overline{)\,165}\\ -15\\ \hline 15\\ -15\\ \hline 0\end{array}$$

There are 55 calories in 1 ounce of canned tuna.

29.

Total	is	visitors each month	multiplied	number of months
↓	↓	↓	↓	↓
Total	=	268,300	·	12

$$\begin{array}{r}268{,}300\\ \times\quad 12\\ \hline 536\,600\\ \underline{2683\,000}\\ 3{,}219{,}600\end{array}$$

The number of visitors in one year was 3,219,600.

33.

Northern length	is	Southern length	plus	2054
↓	↓	↓	↓	↓
Length	=	1933	+	2054

$= 1933 + 2054 = 3987$

The northern boundary of the conterminous United States is 3987 miles long.

37.

Total cost	is	number of sweaters	times	cost of sweater	plus	number of shirts	times	cost of shirt
↓	↓	↓	↓	↓	↓	↓	↓	↓
Total	=	3	·	38	+	5	·	25

$= 3 \cdot 38 + 5 \cdot 25 = 114 + 125 = 239$

The total cost is $239.

41. Option **a:** a hamburger, onion rings, a candy bar, and a soda cost \$4 + \$3 + \$2 + \$1 = \$10.

Option **b:** a hot dog, an apple, french fries, and a soda cost \$3 + \$1 + \$2 + \$1 = \$7.

Thus option b is cheaper by \$10 − \$7, or \$3.

45. The region with the most Internet users (Asia) had 1874 million users. The region with the fewest Internet users (Oceania/Australia) had 28 million users.

$$\begin{array}{r}1874\\ -\quad 28\\ \hline 1846\end{array}$$

Asia had 1846 million more users than Oceania/Australia.

49.

$$\begin{array}{r}1874\\ +\quad 28\\ \hline 1902\end{array} \qquad \begin{array}{r}951\\ 2\overline{)1902}\\ -18\\ \hline 10\\ -10\\ \hline 02\\ -\ 2\\ \hline 0\end{array}$$

The average number of Internet users per region in these two world regions was 951 million.

53. There are 7 days in a week.

$7 \cdot 2400 = 16{,}800$

No more than 16,800 milligrams of sodium should be consumed in one week.

57. 4,893,000,000 rounds to 5,000,000,000.

49,906,000,000 rounds to 50,000,000,000.

$$\begin{array}{r}10\\ 5000000000\overline{)\ 50000000000}\\ -\ 5000000000\\ \hline 00\end{array}$$

The average revenue generated by each package was $10.

Exercise Set 1.9

1. $4 \cdot 4 \cdot 4 = 4^3$

5. $12 \cdot 12 \cdot 12 = 12^3$

9. $9 \cdot 8 \cdot 8 = 9 \cdot 8^2$

13. $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3 \cdot 2^4 \cdot 5^5$

17. $5^3 = 5 \cdot 5 \cdot 5 = 125$

21. $1^{10} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$

25. $2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

29. $4^4 = 4 \cdot 4 \cdot 4 \cdot 4 = 256$

33. $12^2 = 12 \cdot 12 = 144$

37. $20^1 = 20$

41. $3 \cdot 2^6 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 192$

45. $\sqrt{9} = 3$ since $3 \cdot 3 = 9$

49. $\sqrt{144} = 12$ since $12 \cdot 12 = 144$

53. $15 + 3 \cdot 2 = 15 + 6 = 21$

57. $32 \div 4 - 3 = 8 - 3 = 5$

61. $6 \cdot 5 + 8 \cdot 2 = 30 + 16 = 46$

65.
$$\begin{aligned}(7 + 5^2) \div 4 \cdot 2^3 &= (7 + 25) \div 4 \cdot 2^3\\ &= 32 \div 4 \cdot 2^3\\ &= 32 \div 4 \cdot 8\\ &= 8 \cdot 8\\ &= 64\end{aligned}$$

69. $\dfrac{18 + 6}{2^4 - 2^2} = \dfrac{24}{16 - 4} = \dfrac{24}{12} = 2$

73. $\frac{7(9-6)+3}{3^2-3} = \frac{7(3)+3}{9-3} = \frac{21+3}{6} = \frac{24}{6} = 4$

77.
$$\begin{aligned} 2^4\cdot4-(25\div5) &= 2^4\cdot4-5 \\ &= 16\cdot4-5 \\ &= 64-5 \\ &= 59 \end{aligned}$$

81.
$$\begin{aligned} (7\cdot5)+[9\div(3\div3)] &= (7\cdot5)+[9\div(1)] \\ &= 35+9 \\ &= 44 \end{aligned}$$

85.
$$\begin{aligned} \frac{9^2+2^2-1^2}{8\div2\cdot3\cdot1\div3} &= \frac{81+4-1}{4\cdot3\cdot1\div3} \\ &= \frac{85-1}{12\cdot1\div3} \\ &= \frac{84}{12\div3} \\ &= \frac{84}{4} \\ &= 21 \end{aligned}$$

89.
$$\begin{aligned} 4\cdot\sqrt{49}-0\div\sqrt{100} &= 4\cdot7-0\div10 \\ &= 28-0\div10 \\ &= 28-0 \\ &= 28 \end{aligned}$$

93.
$$\begin{aligned} \sqrt{81}\div\sqrt{9}+4^2\cdot2-10 &= 9\div3+16\cdot2-10 \\ &= 3+16\cdot2-10 \\ &= 3+32-10 \\ &= 35-10 \\ &= 25 \end{aligned}$$

97. $7^2-\left\{18-\left[40\div(4\cdot2)+\sqrt{4}\right]+5^2\right\}$
$$\begin{aligned} &= 7^2-\left\{18-\left[40\div8+\sqrt{4}\right]+5^2\right\} \\ &= 7^2-\left\{18-\left[40\div8+2\right]+5^2\right\} \\ &= 7^2-\left\{18-\left[5+2\right]+5^2\right\} \\ &= 7^2-\left\{18-7+5^2\right\} \\ &= 7^2-\left\{18-7+25\right\} \\ &= 7^2-\{11+25\} \\ &= 7^2-36 \\ &= 49-36 \\ &= 13 \end{aligned}$$

101.
$$\begin{aligned} \text{Area of a square} &= (\text{side})^2 \\ &= (23\text{ miles})^2 \\ &= 529\text{ square miles} \end{aligned}$$
$$\begin{aligned} \text{Perimeter} &= (23+23+23+23)\text{ miles} \\ &= 92\text{ miles} \end{aligned}$$

105. $2^5 = 2\cdot2\cdot2\cdot2\cdot2$

The statement is false.

109. $24\div(3\cdot2)+2\cdot5 = 24\div6+2\cdot5 = 4+10 = 14$

113.
$$\begin{aligned} (7+2^4)^5-(3^5-2^4)^2 &= (7+16)^5-(243-16)^2 \\ &= 23^5-227^2 \\ &= 6{,}436{,}343-51{,}529 \\ &= 6{,}384{,}814 \end{aligned}$$

Chapter 1 Test

1. 82,426 is written as eighty-two thousand, four hundred twenty-six.

5.
$$\begin{array}{r} 496 \\ \times\quad 30 \\ \hline 14{,}880 \end{array}$$

9. $0\div49 = 0$

13.
$$\begin{aligned} \frac{64\div8\cdot2}{(\sqrt{9}-\sqrt{4})^2+1} &= \frac{8\cdot2}{(3-2)^2+1} \\ &= \frac{16}{1^2+1} \\ &= \frac{16}{1+1} \\ &= \frac{16}{2} \\ &= 8 \end{aligned}$$

17. To round 52,369 to the nearest thousand, observe that the digit in the hundreds place is 3. Since this digit is less than 5, we do not add 1 to the digit in the thousands place. The number 52,369 rounded to the nearest thousand is 52,000.

21.
$$\begin{array}{r} \overset{1}{1}5 \\ +\ 107 \\ \hline 122 \end{array}$$

The sum of 15 and 107 is 122.

25.
$$\begin{array}{r} 725 \\ -599 \\ \hline 126 \end{array}$$

The more expensive refrigerator costs $126 more than the less expensive one.

29.
$$\begin{aligned} \text{Perimeter} &= 20\text{ yards}+20\text{ yards}+10\text{ yards}+10\text{ yards} \\ &= 60\text{ yards} \end{aligned}$$

The perimeter is 60 yards.

$$\begin{aligned} \text{Area} &= (\text{length})\cdot(\text{width}) \\ &= (20\text{ yards})\cdot(10\text{ yards}) \\ &= 200\text{ square yards} \end{aligned}$$

The area is 200 square yards.

Chapter 2

Exercise Set 2.1

1. Replace a with 21 and b with 7 in each expression.

$a+b = 21+7 = 28$

$a-b = 21-7 = 14$

$a\cdot b = 21\cdot7 = 147$

$a\div b = 21\div7 = 3$

5. Replace a with 56 and b with 1 in each expression.

$a+b = 56+1 = 57$

$a-b = 56-1 = 55$

$a\cdot b = 56\cdot1 = 56$

$a\div b = 56\div1 = 56$

9. Replace x with 2 and z with 3.

$6xz-5x = 6(2)(3)-5(2) = 36-10 = 26$

13. Replace x with 2 and z with 3.

$3x-z = 3(2)-3 = 6-3 = 3$

17. Replace x with 2 and y with 5.

$2xy^2-6 = 2(2)(5)^2-6 = 2(2)(25)-6 = 100-6 = 94$

21. Replace x with 2, y with 5, and z with 3.
$$\begin{aligned} y^4+(z-x) &= 5^4+(3-2) \\ &= 625+(3-2) \\ &= 625+1 \\ &= 626 \end{aligned}$$

25. Replace x with 2 and y with 5.

$\frac{2y-2}{x} = \frac{2(5)-2}{2} = \frac{10-2}{2} = \frac{8}{2} = 4$

29. Replace x with 2 and y with 5.

$$\frac{5x}{y} - \frac{10}{y} = \frac{5(2)}{5} - \frac{10}{5} = \frac{10}{5} - \frac{10}{5} = 2 - 2 = 0$$

33. Replace x with 2 and y with 5.

$$(3y - 2x)^2 = [3(5) - 2(2)]^2 = [15 - 4]^2 = 11^2 = 121$$

37. Replace x with 2, y with 5, and z with 3.

$$2y(4z - x) = 2(5)[4(3) - 2] = 2(5)[12 - 2] = 2(5)(10) = 100$$

41. Replace x with 2 and y with 5.

$$\frac{7x + 2y}{3x} = \frac{7(2) + 2(5)}{3(2)} = \frac{14 + 10}{6} = \frac{24}{6} = 4$$

45. "The sum of a number and five" translates as $x + 5$.

49. "Twenty decreased by a number" translates as $20 - x$.

53. "A number divided by 2" translates as $x \div 2$ or $\frac{x}{2}$.

57. "The quotient of a number and five, decreased by 12" translates as $\frac{x}{5} - 12$.

61. "A number less 5" translates as $x - 5$.

65. "Fifty decreased by eight times a number" translates as $50 - 8 \cdot x$ or $50 - 8x$.

69. The place value of the 7 in 67,522 is thousands.

73. The expression is evaluated correctly.

77. Replace x with 23 and y with 72.

$$\begin{aligned} x^2 + 5y - 112 &= 23^2 + 5(72) - 112 \\ &= 529 + 5(72) - 112 \\ &= 529 + 360 - 112 \\ &= 777 \end{aligned}$$

81. As t gets larger, $16t^2$ gets larger.

Exercise Set 2.2

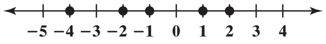
1. If 0 represents the surface of Earth, then 1445 feet underground can be represented by -1445.

5. If 0 represents zero degrees Fahrenheit, then 120 degrees above zero can be represented by $+120$.

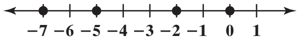
9. If 0 represents a debt of \$0, then a debt of \$9800 million can be represented by -9800 million.

13. If 0 represents a drop of 0¢, the a drop of 10¢ can be represented by -10.

17.



21.



25. -7 is to the left of -5 on a number line, so $-7 < -5$.

29. -26 is to the left of 26 on a number line, so $-26 < 26$.

33. $|-8| = 8$, because -8 is 8 units from 0.

37. $|-5| = 5$, because -5 is 5 units from 0.

41. The opposite of -4 is $-(-4) = 4$.

45. The opposite of -85 is $-(-85) = 85$.

49. $-|20| = -20$
The opposite of the absolute value of 20 is the opposite of 20.

53. $-(-43) = 43$
The opposite of negative 43 is 43.

57. $-(-33) = 33$
The opposite of negative 33 is 33.

61. Replace x with 2.
$-|-x| = -|-2| = -2$

65. Replace x with 7.
$-|x| = -|7| = -7$

69. $|-8| = 8$
$|-11| = 11$
Since 8 is to the left of 11 on a number line, $8 < 11$, so $|-8| < |-11|$.

73. $-|-12| = -12$
$-(-12) = 12$
Since -12 is to the left of 12 on a number line, $-12 < 12$, so $-|-12| < -(-12)$.

77. $|0| = 0$
$|-9| = 9$
Since 0 is to the left of 9 on a number line, $0 < 9$, so $|0| < |-9|$.

81. $-(-12) = 12$
$-(-18) = 18$
Since 12 is to the left of 18 on a number line, $12 < 18$, so $-(-12) < -(-18)$.

85. If -28 is the opposite of a number, then the original number is $-(-28) = 28$. The absolute value of 28 is $|28| = 28$.

89. The lake with the highest elevation is the lake that corresponds to the bar that extends the farthest in the positive direction. The lake with the highest elevation is Lake Superior.

93. The element with a boiling temperature closest to −200°C is the element with boiling temperature of −186°C, which is oxygen.

97. $15 + 20 = 35$

101. $2^2 = 4$, $-|3| = -3$, $-(-5)$, and $-|-8| = -8$, so the integers in order from least to greatest are $-|-8|, -|3|, 2^2, -(-5)$.

105. $-(-2) = 2$, $5^2 = 25$, $-10 = -10$, $-|-9| = -9$, $|-12| = 12$, so the integers in order from least to greatest are $-10, -|-9|, -(-2), |-12|, 5^2$.

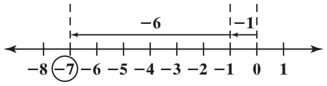
109. $-(-|-5|) = -(-5) = 5$

113. True; consider the values on a number line.

117. answers may vary

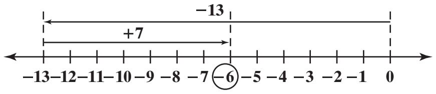
Exercise Set 2.3

1.



$-1 + (-6) = -7$

5.



$-13 + 7 = -6$

9. $|-6| + |-2| = 8$
Common sign is negative, so answer is -8.

13. $|6| - |-2| = 4$
$6 > 2$ so answer is $+4$.

17. $|-5| - |3| = 2$
$5 > 3$ so answer is -2.

21. $|-12| + |-12| = 24$
Common sign is negative, so answer is -24.

25. $|-123| + |-100| = 223$
Common sign is negative, so answer is -223.

29. $|12| - |-5| = 7$
$12 > 5$, so answer is $+7$.

33. $|-12| - |3| = 9$
$12 > 3$, so answer is -9.

37. $|57| - |-37| = 20$
$57 > 37$, so answer is $+20$.

41. $|-67| - |34| = 33$
$67 > 34$, so answer is -33.

45. $|-82| + |-43| = 125$
Common sign is negative, so answer is -125.

49. $$\begin{aligned} -52 + (-77) + (-117) &= -129 + (-117) \\ &= -246 \end{aligned}$$

53. $$\begin{aligned} -10 + 14 + 25 + (-16) &= 4 + 25 + (-16) \\ &= 29 + (-16) \\ &= 13 \end{aligned}$$

57. $-26 + 5 = -21$

61. $-14 + (-31) = -45$

65. $-87 + 87 = 0$

69. $0 + (-103) = -103$

73. Replace x with 2 and y with -3.
$3x + y = 3(2) + (-3) = 6 + (-3) = 3$

77. "The sum of -8 and 25" is $-8 + 25 = 17$.

81. $0 + (-215) + (-16) = -215 + (-16) = -231$
The diver is 231 feet below the surface.

85. The height of the bar for 2017 is marked as 48,351, so the net income for Apple, Inc. in 2017 was \$48,351 million or \$48,351,000,000.

89. $-10 + 12 = 2$
The temperature at 11 p.m. was 2°C.

93. $-23 + 21 = -2$
Florida's record low temperature is -2°F.

97. $44 - 0 = 44$

101. answers may vary

105. Since the numbers have the same sign, the absolute values should be added. The answer has the common sign.
$-10 + (-12) = -22$

109. False; since a positive number and a negative number have different signs, the sign of the sum will be the sign of the number with the larger absolute value. This could be the positive number or the negative number.

Exercise 2.4

1. $-5 - (-5) = -5 + 5 = 0$

5. $3 - 8 = 3 + (-8) = -5$

9. $-5 - (-8) = -5 + 8 = 3$

13. $2 - 16 = 2 + (-16) = -14$

17. $362 - (-40) = 362 + 40 = 402$

21. $-7 - (-3) = -7 + 3 = -4$

25. $-25 - 17 = -25 + (-17) = -42$

29. $2 - (-11) = 2 + 11 = 13$

33. $9 - 20 = 9 + (-20) = -11$

37. $48 - 59 = 48 + (-59) = -11$

41. $$\begin{aligned} 12 - 5 - 7 &= 12 + (-5) + (-7) \\ &= 7 + (-7) \\ &= 0 \end{aligned}$$

45. $$\begin{aligned} -10 + (-5) - 12 &= -10 + (-5) + (-12) \\ &= -15 + (-12) \\ &= -27 \end{aligned}$$

49. $$\begin{aligned} -(-6) - 12 + (-16) &= 6 + (-12) + (-16) \\ &= -6 + (-16) \\ &= -22 \end{aligned}$$

53. $$\begin{aligned} -3 + 4 - (-23) - 10 &= -3 + 4 + 23 + (-10) \\ &= 1 + 23 + (-10) \\ &= 24 + (-10) \\ &= 14 \end{aligned}$$

57. Replace x with 8 and y with -23.
$x - y = 8 - (-23) = 8 + 23 = 31$

61. Replace x with 1 and y with -18.
$2x - y = 2(1) - (-18) = 2 - (-18) = 2 + 18 = 20$

65. The two planets with the lowest temperatures correspond to the two bars that extend the farthest in the negative direction. Neptune's average daytime surface temperature is -330°F and Uranus' average daytime surface temperature is -323°F.
$-323 - (-330) = -323 + 330 = 7$
The difference in average daytime surface temperatures is 7°F.

69. Subtract check amounts from the checking account, and add the deposit.
$$\begin{aligned} &125 - 117 + 45 - 69 \\ &= 125 + (-117) + 45 + (-69) \\ &= 8 + 45 + (-69) \\ &= 53 + (-69) \\ &= -16 \end{aligned}$$
His balance is $-\$16$, so Aaron has overdrawn his checking account by \$16.

73. $$\begin{aligned} -282 - (-436) &= -282 + 436 \\ &= 154 \end{aligned}$$
The difference in elevation is 154 feet.

77. $600 - (-52) = 600 + 52 = 652$
The difference in elevation between Lake Superior and Lake Eyre is 652 feet.

81. $196 - 245 = 196 + (-245) = -49$
The U.S. trade balance in October 2017 was $-\$49$ billion.

85. "Subtract a number from -20" translates as $-20 - x$.

89. $$\begin{array}{r} 23 \\ \times\, 46 \\ \hline 138 \\ 920 \\ \hline 1058 \end{array}$$

93. $9 - (-7) = 9 + 7 = 16$

97. $|-3| - |-7| = 3 - 7 = 3 + (-7) = -4$

101. $|-17| - |-29| = 17 - 29 = 17 + (-29) = -12$

105. answers may vary

Exercise Set 2.5

1. $-6(-2) = 12$

5. $9(-9) = -81$

9. $6(-2)(-4) = -12(-4) = 48$

13. $-4(4)(-5) = -16(-5) = 80$

17. $$\begin{aligned} -5(3)(-1)(-1) &= -15(-1)(-1) \\ &= 15(-1) \\ &= -15 \end{aligned}$$

21. $(-3)^2 = (-3)(-3) = 9$

25. $(-4)^3 = (-4)(-4)(-4) = 16(-4) = -64$

29. $\frac{-30}{6} = -5$

33. $\frac{0}{-21} = 0$

37. $\frac{56}{-4} = -14$

41. $-5(3) = -15$

45. $-7(-6) = 42$

49. $(-7)^2 = (-7)(-7) = 49$

53. $-\frac{72}{8} = -9$

57. $4(-10)(-3) = -40(-3) = 120$

61. $\frac{-25}{0}$ is undefined.

65. $280 \div (-40) = -7$

69. $-1^4 = -(1 \cdot 1 \cdot 1 \cdot 1) = -1$

73.
$$\begin{aligned} -2(3)(5)(-6) &= (-6)(5)(-6) \\ &= (-30)(-6) \\ &= 180 \end{aligned}$$

77. $-2(-3)(-5) = 6(-5) = -30$

81.
$$\begin{array}{r} 35 \\ \times\ 82 \\ \hline 70 \\ 2800 \\ \hline 2870 \end{array}$$
$35 \cdot (-82) = -2870$

85. Replace a with 9 and b with -2.
$ab = 9(-2) = -18$

89. Replace x with 5 and y with -5.
$\frac{x}{y} = \frac{5}{-5} = -1$

93. Replace x with -36 and y with -6.
$\frac{x}{y} = \frac{-36}{-6} = 6$

97. Replace x with 0 and y with -8.
$xy = 0(-8) = 0$
$\frac{x}{y} = \frac{0}{-8} = 0$

101. "The product of -42 and -6" translates as $-42(-6)$.
$-42(-6) = 252$

105. "Subtract a number from -16" translates as $-16 - x$.

109. "Divide a number by -33" translates as $x \div (-33)$ or $\frac{x}{-33}$.

113. $5 \cdot (-20) = -100$
The diver is at a depth of 100 feet

117. $-3(63) = -189$
The melting point of argon is -189°C.

121. **a.** $870 - 24{,}810 = 870 + (-24{,}810) = -23{,}940$
The change was $-23{,}940$ screens (a decrease of 23,940 screens).

b. 2010 to 2016 is 6 years.
$\frac{-23{,}940}{6} = -3990$
The average change over this period was -3990 screens per year.

125. $12 \div 4 - 2 + 7 = 3 - 2 + 7 = 1 + 7 = 8$

129. $-8 - 20 = -8 + (-20) = -28$

133. The product of an odd number of negative numbers is a negative number. Thus, the product of seven negative numbers is a negative number.

137. answers may vary

Exercise Set 2.6

1.
$$\begin{aligned} (-4)^3 &= (-4)(-4)(-4) \\ &= 16(-4) \\ &= -64 \end{aligned}$$

5. $6 \cdot 2^2 = 6 \cdot 4 = 24$

9. $9 - 12 - 4 = 9 + (-12) + (-4) = -3 + (-4) = -7$

13. $5(-9) + 2 = -45 + 2 = -43$

17. $6 + 7 \cdot 3 - 40 = 6 + 21 - 40 = 27 - 40 = -13$

21. $\frac{24}{10 + (-4)} = \frac{24}{6} = 4$

25. $(-19) - 12(3) = (-19) - 36 = -19 + (-36) = -55$

29. $[8 + (-4)]^2 = [4]^2 = 16$

33. $4 - (-3)^4 = 4 - 81 = 4 + (-81) = -77$

37. $7 \cdot 8^2 + 4 = 7 \cdot 64 + 4 = 448 + 4 = 452$

41. $|3 - 12| \div 3 = |-9| \div 3 = 9 \div 3 = 3$

45. $(5 - 9)^2 \div (4 - 2)^2 = (-4)^2 \div (2)^2 = 16 \div 4 = 4$

49.
$$\begin{aligned} (-12 - 20) \div 16 - 25 &= (-32) \div 16 - 25 \\ &= -2 - 25 \\ &= -27 \end{aligned}$$

53. $(2 - 7) \cdot (6 - 19) = (-5) \cdot (-13) = 65$

57. $(-36 \div 6) - (4 \div 4) = (-6) - (1) = -7$

61. $(-5)^2 - 6^2 = 25 - 36 = -11$

65.
$$\begin{aligned} &2(8 - 10)^2 - 5(1 - 6)^2 \\ &= 2(-2)^2 - 5(-5)^2 \\ &= 2(4) - 5(25) \\ &= 8 - 125 \\ &= -117 \end{aligned}$$

69.
$$\begin{aligned} \frac{(-7)(-3) - (4)(3)}{3[7 \div (3 - 10)]} &= \frac{(-7)(-3) - (4)(3)}{3[7 \div (3 + (-10))]} \\ &= \frac{21 + (-12)}{3[7 \div (-7)]} \\ &= \frac{9}{3(-1)} \\ &= \frac{9}{-3} \\ &= -3 \end{aligned}$$

73.
$$\begin{aligned} -3[5 + 2(-4 + 9)] + 15 &= -3[5 + 2(5)] + 15 \\ &= -3[5 + 10] + 15 \\ &= -3[15] + 15 \\ &= -45 + 15 \\ &= -30 \end{aligned}$$

77. Replace x with -2, y with 4, and z with -1.
$$\begin{aligned} 2x - 3y - 4z &= 2(-2) - 3(4) - 4(-1) \\ &= -4 - 12 - (-4) \\ &= -16 + 4 \\ &= -12 \end{aligned}$$

81. Replace y with 4 and z with -1.
$\frac{5y}{z} = \frac{5(4)}{-1} = \frac{20}{-1} = -20$

85. Replace z with -4.
$-z^2 = -(-4)^2 = -16$

89. Replace x with -3 and z with -4.
$$\begin{aligned} 2x^3 - z = 2(-3)^3 - (-4) &= 2(-27) - (-4) \\ &= -54 + 4 \\ &= -50 \end{aligned}$$

93. $$\begin{aligned} \text{Average} &= \frac{-17 + (-26) + (-20) + (-13)}{4} \\ &= \frac{-76}{4} \\ &= -19 \end{aligned}$$

97. Average $= \dfrac{-11 + (-5) + 0 + 4}{4} = \dfrac{-12}{4} = -3$
The average score for Kaufman, Ryu, Shadoff, and Yan was -3.

101. $45 \cdot 90 = 4050$

105. Perimeter $= 8 + 8 + 8 + 8 = 32$
The perimeter of the square is 32 inches.

109. $2 \cdot (7 - 5) \cdot 3 = 2 \cdot 2 \cdot 3 = 12$

113. no; answers may vary

117. $(-12)^4$
$=(-12)(-12)(-12)(-12)$
$=(144)(144) = 20.736$

121. Replace x with 2, y with -5, and z with 7.
$(xy + z)^x = [2(-5) + 7]^2 = [-10 + 7]^2 = [-3]^2 = 9$

Chapter 2 Test

1. $-5 + 8 = 3$

5. $(-18) + (-12) = -30$

9. $|-25| + (-13) = 25 + (-13) = 12$

13. $(-8) + 9 \div (-3) = -8 + (-3) = -11$

17. $$\begin{aligned} -(-7)^2 \div 7 \cdot (-4) &= -49 \div 7 \cdot (-4) \\ &= -7 \cdot (-4) \\ &= 28 \end{aligned}$$

21. $\dfrac{(-3)(-2) + 12}{-1(-4 - 5)} = \dfrac{6 + 12}{-1(-9)} = \dfrac{18}{9} = 2$

25. Replace x with 0 and y with -3.
$3x + y = 3(0) + (-3) = 0 + (-3) = -3$

29. Replace y with -3.
$10 - y^2 = 10 - (-3)^2 = 10 - 9 = 1$

33. $6288 - (-25{,}354) = 6288 + 25{,}354 = 31{,}642$
The difference in elevation is 31,642 feet. As an integer, the answer is 31,642.

Chapter 3

Exercise Set 3.1

1. In $\frac{1}{2}$, the numerator is 1 and the denominator is 2. Since 1 is less than 2, the fraction is proper.

5. In $\frac{15}{15}$, the numerator is 15 and the denominator is 15. Since 15 is greater than or equal to 15, the fraction is improper.

9. Each part is $\frac{1}{4}$ of a whole and there are 11 parts shaded, or 2 whole circles and 3 more parts.

a. The improper fraction is $\frac{11}{4}$.

b. The mixed number is $2\frac{3}{4}$.

13. 7 of the 12 equal parts are shaded. The fraction is $\frac{7}{12}$.

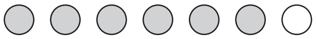
17. 4 of the 9 circles are shaded. The fraction is $\frac{4}{9}$.

21. Each part is $\frac{1}{2}$ of a whole and there are 11 parts shaded, or 5 whole ovals and 1 more part.

a. The improper fraction is $\frac{11}{2}$.

b. The mixed number is $5\frac{1}{2}$.

25. 5 of the 8 equal parts are shaded. The fraction is $\frac{5}{8}$.

29.


33. freshman $\rightarrow$ 42
students $\rightarrow$ 131
$\frac{42}{131}$ of the students are freshmen.

37. born in Ohio $\rightarrow$ 7
U.S. presidents $\rightarrow$ 44
$\frac{7}{44}$ of U.S. presidents were born in Ohio.

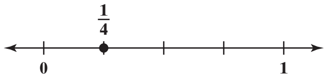
41. 11 of the 31 days of March is $\frac{11}{31}$ of the month.

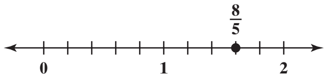
45. There are 50 states total. 33 states contain federal Indian reservations.

a. $\frac{33}{50}$ of the states contain federal Indian reservations.

b. $50 - 33 = 17$
17 states do not contain federal Indian reservations.

c. $\frac{17}{50}$ of the states do not contain federal Indian reservations.

49.


53.


57. $\frac{12}{12} = 12 \div 12 = 1$

61. $\frac{0}{-2} = 0 \div -2 = 0$

65. $\frac{-9}{0} = -9 \div 0$ is undefined

69. $3^2 = 3 \cdot 3 = 9$

73. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^5$

77. $-\frac{11}{2} = \frac{-11}{2} = \frac{11}{-2}$

81. answers may vary

85. 1 is not close to 8, so $\frac{1}{8}$ is closer to 0 than to 1. $7\frac{1}{8}$ rounded to the nearest whole number is 7.

89. There are $1500 + 80 = 1580$ total affiliates. 1500 of the 1580 affiliates are located in the United States: $\frac{1500}{1580}$

Exercise Set 3.2

1. $$\begin{array}{l} 20 = 2 \cdot 10 \\ \quad\;\; \downarrow \;\; \downarrow \searrow \\ \quad\;\; 2 \cdot 2 \cdot 5 = 2^2 \cdot 5 \end{array}$$

5. $$\begin{array}{l} 50 = 10 \cdot 5 \\ \quad\; \swarrow\searrow \;\; \searrow \\ \quad 2 \cdot 5 \cdot 5 = 2 \cdot 5^2 \end{array}$$

9. $$\begin{array}{l} 110 = 2 \cdot 55 \\ \quad\;\; \downarrow \;\; \downarrow \searrow \\ \quad\;\; 2 \cdot 5 \cdot 11 = 2 \cdot 5 \cdot 11 \end{array}$$

13. $$\begin{array}{l} 240 = 2 \cdot 120 \\ \quad\;\; \downarrow \quad \downarrow \searrow \\ \quad\;\; 2 \cdot 2 \cdot 60 \\ \quad\;\; \downarrow \;\; \downarrow \;\; \downarrow \searrow \\ \quad\;\; 2 \cdot 2 \cdot 2 \cdot 30 \\ \quad\;\; \downarrow \;\; \downarrow \;\; \downarrow \;\; \downarrow \searrow \\ \quad\;\; 2 \cdot 2 \cdot 2 \cdot 2 \cdot 15 \\ \quad\;\; \downarrow \;\; \downarrow \;\; \downarrow \;\; \downarrow \;\; \downarrow \searrow \\ \quad\;\; 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^4 \cdot 3 \cdot 5 \end{array}$$

17. $\frac{3}{12} = \frac{3 \cdot 1}{3 \cdot 4} = \frac{1}{4}$

21. $\frac{14}{16} = \frac{2 \cdot 7}{2 \cdot 8} = \frac{7}{8}$

25. $\frac{35}{50} = \frac{5 \cdot 7}{5 \cdot 10} = \frac{7}{10}$

29. $\frac{24}{40} = \frac{8 \cdot 3}{8 \cdot 5} = \frac{3}{5}$

33. $\frac{25}{40} = \frac{5 \cdot 5}{5 \cdot 8} = \frac{5}{8}$

37. $\frac{36}{24} = \frac{12 \cdot 3}{12 \cdot 2} = \frac{3}{2}$

41. $\frac{70}{196} = \frac{14 \cdot 5}{14 \cdot 14} = \frac{5}{14}$

45. $-\frac{55}{85} = -\frac{5 \cdot 11}{5 \cdot 17} = -\frac{11}{17}$

49. $\frac{224}{16} = \frac{16 \cdot 14}{16 \cdot 1} = \frac{14}{1} = 14$

53. Not equivalent, since the cross products are not equal: $7 \cdot 8 = 56$ and $5 \cdot 11 = 55$

57. Equivalent, since the cross products are equal: $3 \cdot 18 = 54$ and $9 \cdot 6 = 54$

61. Not equivalent, since the cross products are not equal: $8 \cdot 24 = 192$ and $12 \cdot 18 = 216$

65. $\frac{2640 \text{ feet}}{5280 \text{ feet}} = \frac{2640 \cdot 1}{2640 \cdot 2} = \frac{1}{2}$

2640 feet represents $\frac{1}{2}$ of a mile.

69. $\frac{10 \text{ inches}}{24 \text{ inches}} = \frac{2 \cdot 5}{2 \cdot 12} = \frac{5}{12}$

$\frac{5}{12}$ of the wall is concrete.

73. $\frac{5 \text{ movies}}{20 \text{ movies}} = \frac{5 \cdot 1}{5 \cdot 4} = \frac{1}{4}$

$\frac{1}{4}$ of the movies were rated PG.

77. $$\begin{array}{r} 387 \\ \times\ 6 \\ \hline 2322 \end{array}$$

81. answers may vary

85. $\frac{36 \text{ donors}}{100 \text{ donors}} = \frac{4 \cdot 9}{4 \cdot 25} = \frac{9}{25}$

$\frac{9}{25}$ of blood donors have blood type A Rh-positive.

89. $$\begin{array}{r} 7 \\ 5\overline{)35} \\ 3\overline{)105} \\ 3\overline{)315} \\ 3\overline{)945} \\ 3\overline{)2835} \\ 3\overline{)8505} \\ 2\overline{)17{,}010} \\ 2\overline{)34{,}020} \end{array}$$

$34{,}020 = 2^2 \cdot 3^5 \cdot 5 \cdot 7$

93. no; answers may vary

97. answers may vary

101. answers may vary

105. 6; answers may vary

Exercise Set 3.3

1. $\frac{2}{7} \cdot \frac{6}{11} = \frac{2 \cdot 6}{7 \cdot 11} = \frac{12}{77}$

5. $\left(-\frac{1}{2}\right)\left(-\frac{2}{15}\right) = \frac{1 \cdot 2}{2 \cdot 15} = \frac{1}{15}$

9. $\frac{2}{7} \cdot \frac{5}{8} = \frac{2 \cdot 5}{7 \cdot 2 \cdot 2 \cdot 2} = \frac{5}{7 \cdot 2 \cdot 2} = \frac{5}{28}$

13. $0 \cdot \frac{8}{9} = 0$

17. $\frac{11}{20} \cdot \frac{1}{7} \cdot \frac{5}{22} = \frac{11 \cdot 1 \cdot 5}{20 \cdot 7 \cdot 22} = \frac{11 \cdot 1 \cdot 5}{5 \cdot 2 \cdot 2 \cdot 7 \cdot 11 \cdot 2} = \frac{1}{2 \cdot 2 \cdot 7 \cdot 2} = \frac{1}{56}$

21. $\left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = \frac{2 \cdot 2}{3 \cdot 3} = \frac{4}{9}$

25. $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{2 \cdot 6}{3 \cdot 5} = \frac{2 \cdot 2 \cdot 3}{3 \cdot 5} = \frac{2 \cdot 2}{5} = \frac{4}{5}$

29. $\frac{8}{9} \div -\frac{1}{2} = \frac{8}{9} \cdot -\frac{2}{1} = -\frac{8 \cdot 2}{9 \cdot 1} = -\frac{16}{9}$

33. $\frac{1}{10} \div \frac{10}{1} = \frac{1}{10} \cdot \frac{1}{10} = \frac{1 \cdot 1}{10 \cdot 10} = \frac{1}{100}$

37. $\frac{3}{25} \div \frac{27}{40} = \frac{3}{25} \cdot \frac{40}{27} = \frac{3 \cdot 5 \cdot 2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 3 \cdot 3 \cdot 3} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 3 \cdot 3} = \frac{8}{45}$

41. $0 \div \frac{7}{8} = 0 \cdot \frac{8}{7} = 0$

45. $-\frac{5}{28} \cdot \frac{35}{25} = -\frac{5 \cdot 35}{28 \cdot 25} = -\frac{5 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 7 \cdot 5 \cdot 5} = -\frac{1}{2 \cdot 2} = -\frac{1}{4}$

49. $\left(-\frac{3}{4}\right)^2 = \left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$

53. $\frac{4}{25} \div \frac{3}{16} = \frac{4}{25} \cdot \frac{16}{3} = \frac{4 \cdot 16}{25 \cdot 3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 3} = \frac{64}{75}$

57. a. $xy = \frac{2}{5} \cdot \frac{5}{6} = \frac{2 \cdot 5}{5 \cdot 6} = \frac{2 \cdot 5}{5 \cdot 2 \cdot 3} = \frac{1}{3}$

b. $x \div y = \frac{2}{5} \div \frac{5}{6} = \frac{2}{5} \cdot \frac{6}{5} = \frac{2 \cdot 2 \cdot 3}{5 \cdot 5} = \frac{12}{25}$

61. $\frac{1}{4}$ of $200 = \frac{1}{4}\cdot 200 = \frac{1}{4}\cdot\frac{200}{1} = \frac{1\cdot 200}{4\cdot 1} = \frac{1\cdot 4\cdot 50}{4\cdot 1} = \frac{50}{1} = 50$

65. $\frac{7}{50}$ of $800 = \frac{7}{50}\cdot\frac{800}{1} = \frac{7\cdot 800}{50\cdot 1} = \frac{7\cdot 16\cdot 50}{50\cdot 1} = \frac{7\cdot 16}{1} = 112$

We project that 112 of the freshmen might be majoring in business.

69. $\frac{2}{5}$ of $2170 = \frac{2}{5}\cdot\frac{2170}{1} = \frac{2\cdot 2170}{5\cdot 1} = \frac{2\cdot 5\cdot 434}{5\cdot 1} = \frac{2\cdot 434}{1} = 868$

He has hiked 868 miles.

73. $\frac{2}{3}$ of $2757 = \frac{2}{3}\cdot 2757 = \frac{2}{3}\cdot\frac{2757}{1} = \frac{2\cdot 3\cdot 919}{3\cdot 1} = \frac{2\cdot 919}{1} = 1838$

The sale price of the cruise is \$1838.

77. Area = length · width $= \frac{5}{14}\cdot\frac{1}{5} = \frac{5\cdot 1}{14\cdot 5} = \frac{1}{14}$

The area is $\frac{1}{14}$ square foot.

81. $\frac{1}{5}$ of $12{,}000 = \frac{1}{5}\cdot\frac{12{,}000}{1} = \frac{1\cdot 12{,}000}{5\cdot 1} = \frac{1\cdot 5\cdot 2400}{5\cdot 1} = 2400$

The family drove 2400 miles for family business.

85.
$$\begin{array}{r} 968 \\ -772 \\ \hline 196 \end{array}$$

89. $\left(\frac{1}{2}\cdot\frac{2}{3}\right) \div \frac{5}{6} = \left(\frac{1\cdot 2}{2\cdot 3}\right) \div \frac{5}{6} = \frac{1}{3} \div \frac{5}{6} = \frac{1}{3}\cdot\frac{6}{5} = \frac{1\cdot 6}{3\cdot 5}$

$= \frac{1\cdot 2\cdot 3}{3\cdot 5} = \frac{1\cdot 2}{5} = \frac{2}{5}$

93. $\frac{1}{8}$ of $325{,}748{,}000 = \frac{1}{8}\cdot\frac{325{,}748{,}000}{1} = \frac{1\cdot 325{,}748{,}000}{8\cdot 1}$

$= \frac{1\cdot 8\cdot 40{,}718{,}500}{8\cdot 1} = 40{,}718{,}500$

The approximate population of California is 40,718,500.

Exercise Set 3.4

1. $\frac{1}{7} + \frac{2}{7} = \frac{1+2}{7} = \frac{3}{7}$

5. $\frac{2}{9} + \frac{4}{9} = \frac{2+4}{9} = \frac{6}{9} = \frac{3\cdot 2}{3\cdot 3} = \frac{2}{3}$

9. $-\frac{3}{14} + \left(-\frac{4}{14}\right) = \frac{-3+(-4)}{14} = \frac{-7}{14} = -\frac{7}{14} = -\frac{7\cdot 1}{7\cdot 2} = -\frac{1}{2}$

13. $\frac{4}{13} + \frac{2}{13} + \frac{1}{13} = \frac{4+2+1}{13} = \frac{7}{13}$

17. $\frac{10}{11} - \frac{4}{11} = \frac{10-4}{11} = \frac{6}{11}$

21. $\frac{7}{4} - \frac{3}{4} = \frac{7-3}{4} = \frac{4}{4} = 1$

25. $-\frac{25}{12} - \frac{15}{12} = \frac{-25-15}{12} = \frac{-40}{12} = -\frac{40}{12} = -\frac{4\cdot 10}{4\cdot 3} = -\frac{10}{3}$

29. $-\frac{27}{33} - \left(-\frac{8}{33}\right) = -\frac{27}{33} + \frac{8}{33} = \frac{-27+8}{33} = \frac{-19}{33} = -\frac{19}{33}$

33. $\frac{99}{100} - \frac{9}{100} = \frac{99-9}{100} = \frac{90}{100} = \frac{10\cdot 9}{10\cdot 10} = \frac{9}{10}$

37. $-\frac{3}{16} + \left(-\frac{7}{16}\right) + \left(-\frac{2}{16}\right) = \frac{-3+(-7)+(-2)}{16}$

$= \frac{-12}{16} = -\frac{12}{16} = -\frac{4\cdot 3}{4\cdot 4} = -\frac{3}{4}$

41. $x - y = -\frac{1}{5} - \left(-\frac{3}{5}\right) = -\frac{1}{5} + \frac{3}{5} = \frac{-1+3}{5} = \frac{2}{5}$

45. The perimeter is the distance around. Opposite sides of a rectangle have equal length.

Perimeter $= \frac{7}{12} + \frac{5}{12} + \frac{7}{12} + \frac{5}{12} = \frac{7+5+7+5}{12} = \frac{24}{12}$

$= \frac{12\cdot 2}{12\cdot 1} = \frac{2}{1} = 2$

The perimeter is 2 meters.

49. To find the number of states with speed limits less than 70 mph, subtract the number of states with 70 mph speed limits $\left(\frac{20}{50}\right)$ from the number of states with speed limits up to and including 70 mph $\left(\frac{33}{50}\right)$.

$\frac{33}{50} - \frac{20}{50} = \frac{33-20}{50} = \frac{13}{50}$

$\frac{13}{50}$ of states have maximum speed limits less than 70 mph.

53. Antarctica takes up $\frac{9}{100}$ of the world's surface area, while Europe takes up $\frac{7}{100}$ of the world's surface area.

$\frac{9}{100} - \frac{7}{100} = \frac{9-7}{100} = \frac{2}{100} = \frac{1\cdot 2}{2\cdot 50} = \frac{1}{50}$

Antarctica takes up $\frac{1}{50}$ more of the world's surface area than Europe.

57. Multiples of 36:

$36\cdot 1 = 36$ Not a multiple of 24

$36\cdot 2 = 72$ A multiple of 24

The LCD is 72.

61. $24 = \boxed{2}\cdot\boxed{2}\cdot\boxed{2}\cdot\boxed{3}$

$7 = \boxed{7}$

The LCD is $2\cdot 2\cdot 2\cdot 3\cdot 7 = 168$.

65. $3 = \boxed{3}$

$21 = 3\cdot\boxed{7}$

$56 = \boxed{2}\cdot\boxed{2}\cdot\boxed{2}\cdot 7$

The LCD is $2\cdot 2\cdot 2\cdot 3\cdot 7 = 168$.

69. $\frac{4}{7} = \frac{4\cdot 5}{7\cdot 5} = \frac{20}{35}$

73. $\frac{14}{17} = \frac{14\cdot 4}{17\cdot 4} = \frac{56}{68}$

77. $\frac{5}{9} = \frac{5\cdot 4}{9\cdot 4} = \frac{20}{36}$

81. $\frac{12}{100}$ is the smallest fraction, so drugs, health and beauty aids had the smallest fraction sold online.

85. $3^4 = 3\cdot 3\cdot 3\cdot 3 = 81$

89. When adding fractions with a common denominator, keep the common denominator and add the numerators.

$\frac{2}{7} + \frac{9}{7} = \frac{2+9}{7} = \frac{11}{7}$

93. 1; answers may vary

97. answers may vary

Exercise Set 3.5

1. The LCD of 3 and 6 is 6.

$$\frac{2}{3}+\frac{1}{6}=\frac{2\cdot 2}{3\cdot 2}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$$

5. The LCD of 11 and 33 is 33.

$$-\frac{2}{11}+\frac{2}{33}=-\frac{2\cdot 3}{11\cdot 3}+\frac{2}{33}=-\frac{6}{33}+\frac{2}{33}=\frac{-6+2}{33}=\frac{-4}{33}=-\frac{4}{33}$$

9. The LCD of 35 and 7 is 35.

$$\frac{11}{35}+\frac{2}{7}=\frac{11}{35}+\frac{2\cdot 5}{7\cdot 5}=\frac{11}{35}+\frac{10}{35}=\frac{11+10}{35}=\frac{21}{35}=\frac{7\cdot 3}{7\cdot 5}=\frac{3}{5}$$

13. The LCD of 12 and 9 is 36.

$$\frac{5}{12}-\frac{1}{9}=\frac{5\cdot 3}{12\cdot 3}-\frac{1\cdot 4}{9\cdot 4}=\frac{15}{36}-\frac{4}{36}=\frac{15-4}{36}=\frac{11}{36}$$

17. The LCD of 11 and 9 is 99.

$$\frac{5}{11}+\frac{4}{9}=\frac{5\cdot 9}{11\cdot 9}+\frac{4\cdot 11}{9\cdot 11}=\frac{45}{99}+\frac{44}{99}=\frac{45+44}{99}=\frac{89}{99}$$

21. The LCD of 3, 9, and 27 is 27.

$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}=\frac{1\cdot 9}{3\cdot 9}+\frac{1\cdot 3}{9\cdot 3}+\frac{1}{27}=\frac{9}{27}+\frac{3}{27}+\frac{1}{27}$$

$$=\frac{9+3+1}{27}=\frac{13}{27}$$

25. The LCD of 14 and 7 is 14.

$$\frac{9}{14}-\frac{3}{7}=\frac{9}{14}-\frac{3\cdot 2}{7\cdot 2}=\frac{9}{14}-\frac{6}{14}=\frac{9-6}{14}=\frac{3}{14}$$

29. The LCD of 9 and 12 is 36.

$$\frac{1}{9}-\frac{5}{12}=\frac{1\cdot 4}{9\cdot 4}-\frac{5\cdot 3}{12\cdot 3}=\frac{4}{36}-\frac{15}{36}=\frac{4-15}{36}=\frac{-11}{36}=-\frac{11}{36}$$

33. The LCD of 7 and 8 is 56.

$$\frac{5}{7}-\frac{1}{8}=\frac{5\cdot 8}{7\cdot 8}-\frac{1\cdot 7}{8\cdot 7}=\frac{40}{56}-\frac{7}{56}=\frac{40-7}{56}=\frac{33}{56}$$

37. $\frac{5}{9}+\frac{3}{9}=\frac{5+3}{9}=\frac{8}{9}$

41. The LCD of 6 and 7 is 42.

$$-\frac{5}{6}-\frac{3}{7}=-\frac{5\cdot 7}{6\cdot 7}-\frac{3\cdot 6}{7\cdot 6}=-\frac{35}{42}-\frac{18}{42}=\frac{-35-18}{42}=\frac{-53}{42}=-\frac{53}{42}$$

45. The LCD of 11 and 13 is 143.

$$\frac{5}{11}+\frac{3}{13}=\frac{5\cdot 13}{11\cdot 13}+\frac{3\cdot 11}{13\cdot 11}=\frac{65}{143}+\frac{33}{143}=\frac{65+33}{143}=\frac{98}{143}$$

49. The LCD of 5, 4, and 2 is 20.

$$\frac{6}{5}-\frac{3}{4}+\frac{1}{2}=\frac{6\cdot 4}{5\cdot 4}-\frac{3\cdot 5}{4\cdot 5}+\frac{1\cdot 10}{2\cdot 10}=\frac{24}{20}-\frac{15}{20}+\frac{10}{20}$$

$$=\frac{24-15+10}{20}=\frac{19}{20}$$

53. The LCD of 12, 24, and 6 is 24.

$$-\frac{9}{12}+\frac{17}{24}-\frac{1}{6}=-\frac{9\cdot 2}{12\cdot 2}+\frac{17}{24}-\frac{1\cdot 4}{6\cdot 4}=-\frac{18}{24}+\frac{17}{24}-\frac{4}{24}$$

$$=\frac{-18+17-4}{24}=\frac{-5}{24}=-\frac{5}{24}$$

57. The LCD of these fractions is 70. Let's write each fraction as an equivalent fraction with a denominator of 70.

$$\frac{2}{7}=\frac{2\cdot 10}{7\cdot 10}=\frac{20}{70}\quad\text{and}\quad\frac{3}{10}=\frac{3\cdot 7}{10\cdot 7}=\frac{21}{70}$$

Since $20 < 21$, then $\frac{20}{70} < \frac{21}{70}$ or $\frac{2}{7} < \frac{3}{10}$

61. The LCD of these fractions is 28. Let's write each fraction as an equivalent fraction with a denominator of 28.

$$-\frac{3}{4}=-\frac{3\cdot 7}{4\cdot 7}=-\frac{21}{28}\quad\text{and}\quad-\frac{11}{14}=-\frac{11\cdot 2}{14\cdot 2}=-\frac{22}{28}$$

Since $-21 > -22$, then $-\frac{21}{28} > -\frac{22}{28}$ or $-\frac{3}{4} > -\frac{11}{14}$

65. $xy=\frac{1}{3}\cdot\frac{3}{4}=\frac{1\cdot 3}{3\cdot 4}=\frac{1}{4}$

69.

$$\frac{1}{3}+\frac{4}{5}+\frac{1}{3}+\frac{4}{5}=\frac{2}{3}+\frac{8}{5}=\frac{2\cdot 5}{3\cdot 5}+\frac{8\cdot 3}{5\cdot 3}$$

$$=\frac{10}{15}+\frac{24}{15}=\frac{10+24}{15}=\frac{34}{15}$$

The perimeter is $\frac{34}{15}$ centimeters.

73. "The sum of a number and $\frac{1}{2}$" is represented by $x+\frac{1}{2}$.

77. Subtract the ground speed of the sloth $\left(\frac{1}{10}\right)$ from the speed of the sloth in trees $\left(\frac{17}{100}\right)$.

$$\frac{17}{100}-\frac{1}{10}=\frac{17}{100}-\frac{1}{10}\cdot\frac{10}{10}=\frac{17}{100}-\frac{10}{100}=\frac{17-10}{100}=\frac{7}{100}$$

The sloth can travel $\frac{7}{100}$ mile per hour faster in the trees.

81. Subtract the fraction of students who name art as their favorite subject $\left(\frac{4}{25}\right)$ from the fraction of students who name math, science, or art as their favorite subject $\left(\frac{13}{20}\right)$.

$$\frac{13}{20}-\frac{4}{25}=\frac{13}{20}\cdot\frac{5}{5}-\frac{4}{25}\cdot\frac{4}{4}=\frac{65}{100}-\frac{16}{100}=\frac{65-16}{100}=\frac{49}{100}$$

$\frac{49}{100}$ of students name math or science as their favorite subject.

85. The Pacific Ocean takes up $\frac{1}{2}$ of the world's water area, while the Atlantic Ocean takes up $\frac{13}{50}$ of the water area. The LCD of 2 and 50 is 50.

$$\frac{1}{2}+\frac{13}{50}=\frac{1}{2}\cdot\frac{25}{25}+\frac{13}{50}=\frac{25}{50}+\frac{13}{50}=\frac{25+13}{50}=\frac{38}{50}=\frac{2\cdot 19}{2\cdot 25}=\frac{19}{25}$$

$\frac{19}{25}$ of the world's water area is within the Pacific Ocean and the Atlantic Ocean.

89. $1-\frac{37}{100}=\frac{100}{100}-\frac{37}{100}=\frac{100-37}{100}=\frac{63}{100}$

$\frac{63}{100}$ of the areas maintained by the National Park Service are not national monuments.

93. $(8-6)\cdot(4-7)=(2)\cdot(-3)=-6$

97. The LCD of 3, 4, and 540 is 540.

$$\frac{2}{3}-\frac{1}{4}-\frac{2}{540}=\frac{2}{3}\cdot\frac{180}{180}-\frac{1}{4}\cdot\frac{135}{135}-\frac{2}{540}=\frac{360}{540}-\frac{135}{540}-\frac{2}{540}$$

$$=\frac{360-135-2}{540}=\frac{223}{540}$$

101. answers may vary

Exercise Set 3.6

1. $\dfrac{\frac{1}{8}}{\frac{3}{4}}=\frac{1}{8}\div\frac{3}{4}=\frac{1}{8}\cdot\frac{4}{3}=\frac{1\cdot 4}{8\cdot 3}=\frac{1\cdot 4}{2\cdot 4\cdot 3}=\frac{1}{6}$

5. $\dfrac{\frac{2}{27}}{\frac{4}{9}}=\frac{2}{27}\div\frac{4}{9}=\frac{2}{27}\cdot\frac{9}{4}=\frac{2\cdot 9}{27\cdot 4}=\frac{2\cdot 3\cdot 3}{3\cdot 3\cdot 3\cdot 2\cdot 2}=\frac{1}{3\cdot 2}=\frac{1}{6}$

9. $$\frac{\frac{3}{4}}{5-\frac{1}{8}} = \frac{8\cdot\left(\frac{3}{4}\right)}{8\cdot\left(\frac{5}{1}-\frac{1}{8}\right)} = \frac{6}{8\cdot\left(\frac{5}{1}\right)-8\cdot\left(\frac{1}{8}\right)}$$
$$= \frac{6}{40-1} = \frac{6}{39} = \frac{3\cdot 2}{3\cdot 13} = \frac{2}{13}$$

13. $$\frac{5}{6}\div\frac{1}{3}\cdot\frac{1}{4} = \frac{5}{6}\cdot\frac{3}{1}\cdot\frac{1}{4} = \frac{5\cdot 3\cdot 1}{2\cdot 3\cdot 1\cdot 2\cdot 2} = \frac{5}{2\cdot 2\cdot 2} = \frac{5}{8}$$

17. $$\left(\frac{2}{9}+\frac{4}{9}\right)\left(\frac{1}{3}-\frac{9}{10}\right)$$
$$= \left(\frac{2+4}{9}\right)\left(\frac{1}{3}\cdot\frac{10}{10}-\frac{9}{10}\cdot\frac{3}{3}\right) = \left(\frac{6}{9}\right)\left(\frac{10}{30}-\frac{27}{30}\right) = \left(\frac{6}{9}\right)\left(\frac{-17}{30}\right)$$
$$= -\frac{6\cdot 17}{9\cdot 30} = -\frac{3\cdot 2\cdot 17}{3\cdot 3\cdot 5\cdot 2\cdot 3} = -\frac{17}{3\cdot 3\cdot 5} = -\frac{17}{45}$$

21. $$2\cdot\left(\frac{1}{4}+\frac{1}{5}\right)+2 = 2\cdot\left(\frac{1}{4}\cdot\frac{5}{5}+\frac{1}{5}\cdot\frac{4}{4}\right)+2 = 2\cdot\left(\frac{5}{20}+\frac{4}{20}\right)+2$$
$$= 2\cdot\left(\frac{5+4}{20}\right)+2 = 2\cdot\frac{9}{20}+2 = \frac{2}{1}\cdot\frac{9}{20}+2 = \frac{2\cdot 9}{1\cdot 2\cdot 10}+2$$
$$= \frac{9}{10}+\frac{2}{1}\cdot\frac{10}{10} = \frac{9}{10}+\frac{20}{10} = \frac{9+20}{10} = \frac{29}{10}$$

25. $$\left(\frac{2}{5}-\frac{3}{10}\right)^2 = \left(\frac{2}{5}\cdot\frac{2}{2}-\frac{3}{10}\right)^2$$
$$= \left(\frac{4}{10}-\frac{3}{10}\right)^2$$
$$= \left(\frac{4-3}{10}\right)^2$$
$$= \left(\frac{1}{10}\right)^2$$
$$= \frac{1}{10}\cdot\frac{1}{10}$$
$$= \frac{1}{100}$$

29. $$5y-z = 5\left(\frac{2}{5}\right)-\frac{5}{6} = 2-\frac{5}{6} = \frac{2}{1}\cdot\frac{6}{6}-\frac{5}{6} = \frac{12}{6}-\frac{5}{6} = \frac{7}{6}$$

33. $$x^2-yz = \left(-\frac{1}{3}\right)^2-\left(\frac{2}{5}\right)\left(\frac{5}{6}\right) = \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)-\frac{2\cdot 5}{5\cdot 2\cdot 3} = \frac{1}{9}-\frac{1}{3}$$
$$= \frac{1}{9}-\frac{1}{3}\cdot\frac{3}{3} = \frac{1}{9}-\frac{3}{9} = \frac{1-3}{9} = \frac{-2}{9} = -\frac{2}{9}$$

37. $$\frac{\frac{5}{24}}{\frac{1}{12}} = \frac{5}{24}\div\frac{1}{12} = \frac{5}{24}\cdot\frac{12}{1} = \frac{5\cdot 12}{24\cdot 1} = \frac{5\cdot 12}{2\cdot 12} = \frac{5}{2}$$

41. $$\left(-\frac{1}{2}\right)^2+\frac{1}{5} = \frac{1}{4}+\frac{1}{5} = \frac{1\cdot 5}{4\cdot 5}+\frac{1\cdot 4}{5\cdot 4} = \frac{5}{20}+\frac{4}{20} = \frac{9}{20}$$

45. $$\left(1-\frac{2}{5}\right)^2 = \left(\frac{5}{5}-\frac{2}{5}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

49. $$\left(-\frac{2}{9}-\frac{7}{9}\right)^4 = \left(-\frac{9}{9}\right)^4 = (-1)^4 = 1$$

53. $$\left(\frac{3}{4}\div\frac{6}{5}\right)-\left(\frac{3}{4}\cdot\frac{6}{5}\right) = \left(\frac{3}{4}\cdot\frac{5}{6}\right)-\left(\frac{3\cdot 6}{4\cdot 5}\right)$$
$$= \left(\frac{3\cdot 5}{4\cdot 6}\right)-\left(\frac{3\cdot 2\cdot 3}{2\cdot 2\cdot 5}\right)$$
$$= \left(\frac{3\cdot 5}{4\cdot 2\cdot 3}\right)-\left(\frac{3\cdot 3}{2\cdot 5}\right)$$
$$= \left(\frac{5}{4\cdot 2}\right)-\frac{9}{10}$$
$$= \frac{5}{8}-\frac{9}{10}$$
$$= \frac{5\cdot 5}{8\cdot 5}-\frac{9\cdot 4}{10\cdot 4}$$
$$= \frac{25}{40}-\frac{36}{40}$$
$$= -\frac{11}{40}$$

57. $$3\frac{3}{5} = \frac{5\cdot 3+3}{5} = \frac{15+3}{5} = \frac{18}{5}$$

61. $$11\frac{6}{7} = \frac{7\cdot 11+6}{7} = \frac{77+6}{7} = \frac{83}{7}$$

65. $$166\frac{2}{3} = \frac{3\cdot 166+2}{3} = \frac{498+2}{3} = \frac{500}{3}$$

69. $$\begin{array}{r} 4\text{ R }5 \\ 8\overline{)37} \\ \underline{-32} \\ 5 \end{array}$$
$$\frac{37}{8} = 4\frac{5}{8}$$

73. $$\begin{array}{r} 15 \\ 15\overline{)225} \\ \underline{-15} \\ 75 \\ \underline{-75} \\ 0 \end{array}$$
$$\frac{225}{15} = 15$$

77. $$\begin{array}{r} 6\text{ R }65 \\ 112\overline{)737} \\ \underline{-672} \\ 65 \end{array}$$
$$\frac{737}{112} = 6\frac{65}{112}$$

81. $$9-\frac{5}{6} = \frac{9}{1}\cdot\frac{6}{6}-\frac{5}{6}$$
$$= \frac{54}{6}-\frac{5}{6}$$
$$= \frac{49}{6} \text{ or } 8\frac{1}{6}$$

85. no; answers may vary

89. $$\frac{\frac{1}{4}+\frac{2}{14}}{2} = \frac{28\left(\frac{1}{4}+\frac{2}{14}\right)}{28(2)} = \frac{28\cdot\frac{1}{4}+28\cdot\frac{2}{14}}{28\cdot 2} = \frac{7+4}{56} = \frac{11}{56}$$

93. False; the average of two numbers should be halfway between the two numbers.

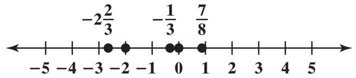
97. True; for instance $\frac{1}{2}+\frac{1}{2} = \frac{2}{2} = 1$.

101. The subtraction is within the innermost grouping symbols. The result of the subtraction then gets multiplied and the product is added within the outer grouping symbols. The result is then divided by the fraction outside the grouping symbols. The order is subtraction, multiplication, addition, division.

105. $$\frac{2+x}{y} = \frac{2+\frac{3}{4}}{-\frac{4}{7}} = \frac{28\left(2+\frac{3}{4}\right)}{28\left(-\frac{4}{7}\right)} = \frac{28\cdot 2 + 28\cdot\frac{3}{4}}{-28\cdot\frac{4}{7}}$$
$$= \frac{56+21}{-16} = \frac{77}{-16} = -\frac{77}{16}$$

Exercise Set 3.7

1.



5. $2\frac{11}{12}$ rounds to 3, $1\frac{1}{4}$ rounds to 1, and $3\cdot 1 = 3$, so the best estimate is 3, choice **b.**

9. $2\frac{2}{3}\cdot\frac{1}{7} = \frac{8}{3}\cdot\frac{1}{7} = \frac{8}{21}$

13. Exact: $2\frac{1}{5}\cdot 3\frac{1}{2} = \frac{11}{5}\cdot\frac{7}{2} = \frac{77}{10}$ or $7\frac{7}{10}$

Estimate: $2\frac{1}{5}$ rounds to 2, $3\frac{1}{2}$ rounds to 4.

$2\cdot 4 = 8$ so the answer is reasonable.

17. $5\cdot 2\frac{1}{2} = \frac{5}{1}\cdot\frac{5}{2} = \frac{25}{2}$ or $12\frac{1}{2}$

21. $2\frac{2}{3} \div \frac{1}{7} = \frac{8}{3}\cdot\frac{7}{1} = \frac{56}{3}$ or $18\frac{2}{3}$

25. $8\frac{1}{3}$ rounds to 8, $1\frac{1}{2}$ rounds to 2, and $8 + 2 = 10$, so the best estimate is 10, choice **b.**

29. Exact: $10\frac{3}{14} + 3\frac{4}{7} = 10\frac{3}{14} + 3\frac{8}{14} = 13\frac{11}{14}$

Estimate $10\frac{3}{14}$ rounds to 10, $3\frac{4}{7}$ rounds to 4.

$10 + 4 = 14$ so the answer is reasonable.

33. $$\begin{array}{r} 12\frac{3}{14} \\ 10 \\ +25\frac{5}{12} \\ \hline \end{array} \qquad \begin{array}{r} 12\frac{18}{84} \\ 10 \\ +\,25\frac{35}{84} \\ \hline 47\frac{53}{84} \end{array}$$

37. $$\begin{array}{r} 3\frac{5}{8} \\ 2\frac{1}{6} \\ +7\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} 3\frac{15}{24} \\ 2\frac{4}{24} \\ +7\frac{18}{24} \\ \hline \end{array}$$
$$12\frac{37}{24} = 12 + 1\frac{13}{24} = 13\frac{13}{24}$$

41. $$\begin{array}{r} 10\frac{13}{14} \\ -3\frac{4}{7} \\ \hline \end{array} \qquad \begin{array}{r} 10\frac{13}{14} \\ -3\frac{8}{14} \\ \hline 7\frac{5}{14} \end{array}$$

Exact: $7\frac{5}{14}$

Estimate: $10\frac{13}{14}$ rounds to 11, $3\frac{4}{7}$ rounds to 4.

$11 - 4 = 7$ so the answer is reasonable.

45. $$\begin{array}{r} 6 \\ -2\frac{4}{9} \\ \hline \end{array} \qquad \begin{array}{r} 5\frac{9}{9} \\ -2\frac{4}{9} \\ \hline 3\frac{5}{9} \end{array}$$

49. $$\begin{array}{r} 2\frac{3}{4} \\ +\,1\frac{1}{4} \\ \hline 3\frac{4}{4} \end{array} = 3 + 1 = 4$$

53. $3\frac{1}{9}\cdot 2 = \frac{28}{9}\cdot\frac{2}{1} = \frac{56}{9} = 6\frac{2}{9}$

57. $22\frac{4}{9} + 13\frac{5}{18} = 22\frac{8}{18} + 13\frac{5}{18} = 35\frac{13}{18}$

61. $$\begin{array}{r} 15\frac{4}{5} \\ 20\frac{3}{10} \\ +\,37\frac{2}{15} \\ \hline \end{array} \qquad \begin{array}{r} 15\frac{24}{30} \\ 20\frac{9}{30} \\ +\,37\frac{4}{30} \\ \hline \end{array}$$
$$72\frac{37}{30} = 72 + 1\frac{7}{30} = 73\frac{7}{30}$$

65. $4\frac{2}{7}\cdot 1\frac{3}{10} = \frac{30}{7}\cdot\frac{13}{10} = \frac{30\cdot 13}{7\cdot 10} = \frac{3\cdot 10\cdot 13}{7\cdot 10} = \frac{3\cdot 13}{7} = \frac{39}{7} = 5\frac{4}{7}$

69. "$-5\frac{2}{7}$ decreased by a number" translates as $-5\frac{2}{7} - x$.

73. $12\frac{3}{4} \div 4 = \frac{51}{4} \div \frac{4}{1} = \frac{51}{4}\cdot\frac{1}{4} = \frac{51\cdot 1}{4\cdot 4} = \frac{51}{16} = 3\frac{3}{16}$

The patient walked $3\frac{3}{16}$ miles each day.

77. Subtract Yuma's average rainfall from Tucson's average rainfall.

$$\begin{array}{r} 11\frac{1}{4} \\ -\,3\frac{3}{5} \\ \hline \end{array} \qquad \begin{array}{r} 11\frac{5}{20} \\ -\,3\frac{12}{20} \\ \hline \end{array} \qquad \begin{array}{r} 10\frac{25}{20} \\ -\,3\frac{12}{20} \\ \hline 7\frac{13}{20} \end{array}$$

Tucson gets an average of $7\frac{13}{20}$ inches more rainfall than Yuma.

81. $A = l\cdot w = 1\frac{1}{4}\cdot\frac{3}{4} = \frac{5}{4}\cdot\frac{3}{4} = \frac{3\cdot 5}{4\cdot 4} = \frac{15}{16}$

The area of the chip is $\frac{15}{16}$ square inch.

85. $$15\frac{2}{3} - 2\frac{1}{2} - 3\frac{1}{4} = 15\frac{8}{12} - 2\frac{6}{12} - 3\frac{3}{12}$$
$$= 14\frac{20}{12} - 2\frac{6}{12} - 3\frac{3}{12}$$
$$= 9\frac{11}{12}$$

The remaining piece is $9\frac{11}{12}$ feet, which is $\frac{1}{12}$ foot short of the 10-foot length she needs.

89.
$$\begin{array}{rr} 4\frac{7}{15} & 4\frac{28}{60} \\ 2\frac{3}{10} & 2\frac{18}{60} \\ +\,2\frac{37}{60} & +\,2\frac{37}{60} \\ \hline & 8\frac{83}{60} = 8 + 1\frac{23}{60} = 9\frac{23}{60} \end{array}$$

The total duration of the three eclipses is $9\frac{23}{60}$ minutes.

93. $-4\frac{2}{5}\cdot 2\frac{3}{10} = -\frac{22}{5}\cdot\frac{23}{10} = -\frac{2\cdot 11\cdot 23}{5\cdot 2\cdot 5} = -\frac{253}{25} = -10\frac{3}{25}$

97. $-31\frac{2}{15} + 17\frac{3}{20} = -31\frac{8}{60} + 17\frac{9}{60} = -30\frac{68}{60} + 17\frac{9}{60} = -13\frac{59}{60}$

101. $11\frac{7}{8} - 13\frac{5}{6} = 11\frac{21}{24} - 13\frac{20}{24} = -\left(13\frac{20}{24} - 11\frac{21}{24}\right)$

$= -\left(12\frac{44}{24} - 11\frac{21}{24}\right) = -1\frac{23}{24}$

105. $20 \div 10\cdot 2 = 2\cdot 2 = 4$

109. **a.** $9\frac{5}{5} = 9 + 1 = 10$

b. $9\frac{100}{100} = 9 + 1 = 10$

c. $6\frac{44}{11} = 6 + 4 = 10$

d. $8\frac{13}{13} = 8 + 1 = 9$

a, **b**, and **c** are equivalent to 10.

113. answers may vary

117. answers may vary

Chapter 3 Test

1. 7 out of 16 equal parts are shaded: $\frac{7}{16}$

5. $-\frac{42}{70} = -\frac{2\cdot 3\cdot 7}{2\cdot 5\cdot 7} = -\frac{3}{5}$

9.
$$\begin{array}{l} 495 = 5\cdot 99 \\ \quad\ \ \downarrow\ \ \downarrow\searrow \\ \quad\ \ 5\cdot 9\cdot 11 \\ \quad\ \ \downarrow\ \ \downarrow\searrow\ \searrow \\ \quad\ \ 5\cdot 3\cdot 3\cdot 11 = 3^2\cdot 5\cdot 11 \end{array}$$

13. $\frac{4}{4} \div \frac{3}{4} = \frac{4}{4}\cdot\frac{4}{3} = \frac{4\cdot 4}{4\cdot 3} = \frac{4}{3}$ or $1\frac{1}{3}$

17. $-\frac{2}{3}\cdot -\frac{8}{15} = \frac{2}{3}\cdot\frac{8}{15} = \frac{16}{45}$

21. $\frac{3}{8}\cdot\frac{16}{6}\cdot\frac{4}{11} = \frac{3\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 11}$

$= \frac{2\cdot 2}{11} = \frac{4}{11}$

25.
$$\begin{array}{rr} 19 & 18\frac{11}{11} \\ -\,2\frac{3}{11} & -\,2\frac{3}{11} \\ \hline & 16\frac{8}{11} \end{array}$$

29. $\frac{1}{2} \div \frac{2}{3}\cdot\frac{3}{4} = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{4} = \frac{1\cdot 3\cdot 3}{2\cdot 2\cdot 4} = \frac{9}{16}$

33. $-5x = -5\left(-\frac{1}{2}\right) = -\frac{5}{1}\cdot -\frac{1}{2} = \frac{5}{2}$ or $2\frac{1}{2}$

37. Add the fractions for education $\left(\frac{1}{50}\right)$, transportation $\left(\frac{1}{5}\right)$, and clothing $\left(\frac{1}{25}\right)$. The LCD is 50.

$\frac{1}{50} + \frac{1}{5} + \frac{1}{25} = \frac{1}{50} + \frac{10}{50} + \frac{2}{50} = \frac{13}{50}$

$\frac{13}{50}$ of consumer spending goes for education, transportation, and clothing.

Chapter 4

Exercise Set 4.1

1. 6.52 in words is six and fifty-two hundredths.

5. −0.205 in words is negative two hundred five thousandths.

9. 3000.04 in words is three thousand and four hundredths.

13. 2.43 in words is two and forty-three hundredths.

17. The check should be paid to “Verizon” for the amount of “91.68,” which is written in words as “Ninety-one and $\frac{68}{100}$.”

21. Nine and eight hundredths is 9.08.

25. Forty-six ten-thousandths is 0.0046.

29. $0.27 = \frac{27}{100}$

33. $5.4 = 5\frac{4}{10} = 5\frac{2}{5}$

37. $7.008 = 7\frac{8}{1000} = 7\frac{1}{125}$

41. $0.3005 = \frac{3005}{10{,}000} = \frac{601}{2000}$

45. In words, 0.077 is seventy-seven thousandths. As a fraction, $0.077 = \frac{77}{1000}$.

49.
$$\begin{array}{l} 0.57\quad 0.54 \\ \quad\uparrow\qquad\ \uparrow \\ \quad 7 > \ \ 4 \end{array}$$
so $0.57 > 0.54$

Thus $-0.57 < -0.54$

53.
$$\begin{array}{l} 0.54900\quad 0.549 \\ \quad\ \ \uparrow\qquad\quad\ \uparrow \\ \quad\ \ 9\quad = \quad 9 \end{array}$$
so $0.54900 = 0.549$

57.
$$\begin{array}{l} 1.062\quad 1.07 \\ \quad\uparrow\qquad\ \uparrow \\ \quad 6\ \ <\ \ 7 \end{array}$$
so $1.062 < 1.07$

Thus, $-1.062 > -1.07$.

61. $$\begin{array}{ccc} 0.023 & & 0.024 \\ \uparrow & & \uparrow \\ 3 & < & 4 \end{array}$$
so $0.023 < 0.024$
Thus, $-0.023 > -0.024$.

65. To round 98,207.23 to the nearest ten, observe that the digit in the ones place is 7. Since this digit is 5 or greater, add 1 to the digit in the tens place. The number 98,207.23 rounded to the nearest ten is 98,210.

69. To round 0.5942 to the nearest thousandth, observe that the digit in the ten-thousandths place is 2. Since this digit is less than 5, we do not add 1 to the digit in the thousandths place. The number 0.5942 rounded to the nearest thousandth is 0.594.

73. To round 3.14159265 to the nearest thousandth, observe that the digit in the ten-thousandths place is 5. Since this digit is 5 or greater, we add 1 to the digit in the thousandths place. The number 3.14159265 rounded to the nearest thousandth is 3.142.

77. To round 0.1992 to the nearest hundredth, observe that the digit in the thousandths place is 9. Since this digit is 5 or greater, we add 1 to the digit in the hundredths place. The number 0.1992 rounded to the nearest hundredth is 0.2. The amount is \$0.20.

81. To round 1.4714 to the nearest hundredth, observe that the digit in the thousandths place is 1. Since this digit is less than 5, we do not add 1 to the digit in the hundredths place. The number 1.4714 rounded to the nearest hundredth is 1.47. The time is 1.47 hours.

85. To round 89.3559 to the nearest tenth, observe that the digit in the hundredths place is 5. Since this digit is 5 or greater, we add 1 to the digit in the tenths place. The number 89.3559 rounded to the nearest tenth is 89.4. The population density is 89.4 people per square mile.

89. To round 0.4667 to the nearest tenth, observe that the digit in the hundredths place is 6. Since this digit is 5 or greater, we add 1 to the digit in the tenths place. The number 0.4667 rounded to the nearest tenth is 0.5. The ride lasts about 0.5 minute.

93. $$\begin{array}{r} 82 \\ -47 \\ \hline 35 \end{array}$$

97. To round 2849.1738 to the nearest hundredth, observe that the digit in the thousandths place is 3. Since this digit is less than 5, we do not add 1 to the digit in the hundredths place. 2849.1738 rounded to the nearest hundredth is 2849.17, which is choice **a**

101. $7\frac{12}{100} = 7.12$

105. answers may vary

109. 0.26559 rounded to the nearest hundredth is 0.27. 0.26499 rounds to 0.26. 0.25786 rounds to 0.26. 0.25186 rounds to 0.25. Thus, 0.26499 and 0.25786 round to 0.26.

113. $$\begin{array}{lcr} 936.7 & \text{rounds to} & 900 \\ 760.5 & \text{rounds to} & 800 \\ 659.4 & \text{rounds to} & 700 \\ 652.2 & \text{rounds to} & 700 \\ 623.3 & \text{rounds to} & 600 \\ 533.1 & \text{rounds to} & +\quad 500 \\ \cline{3-3} & & 4200 \end{array}$$

The total amount that these movies earned was about \$4200 million.

Exercise Set 4.2

1. $$\begin{array}{r} 1.3 \\ +\,2.2 \\ \hline 3.5 \end{array}$$

5. $$\begin{array}{r} 24.6000 \\ 2.3900 \\ +\,0.0678 \\ \hline 27.0578 \end{array}$$

9. $-2.6 + (-5.97)$
Add the absolute values.
$$\begin{array}{r} 2.60 \\ +\,5.97 \\ \hline 8.57 \end{array}$$
Attach the common sign.
$-2.6 + (-5.97) = -8.57$

13. Exact: $$\begin{array}{r} 23\overset{1}{4}.\overset{1}{8}9 \\ +230.67 \\ \hline 465.56 \end{array}$$ Estimate: $$\begin{array}{r} 230 \\ +230 \\ \hline 460 \end{array}$$

17. $$\begin{array}{r} \overset{1}{4}5.0\overset{1}{2}3 \\ 3.006 \\ +\,8.403 \\ \hline 56.432 \end{array}$$

21. $$\begin{array}{r} 18.0 \\ -2.7 \\ \hline 15.3 \end{array}$$ Check: $$\begin{array}{r} 1\overset{1}{5}.3 \\ +\;\,2.7 \\ \hline 18.0 \end{array}$$

25. Exact: $$\begin{array}{r} 5.90 \\ -4.07 \\ \hline 1.83 \end{array}$$ Estimate: $$\begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array}$$ Check: $$\begin{array}{r} 1.\overset{1}{8}3 \\ +\,4.07 \\ \hline 5.90 \end{array}$$

29. Exact: $$\begin{array}{r} 1000.0 \\ -123.4 \\ \hline 876.6 \end{array}$$ Estimate: $$\begin{array}{r} 1000 \\ -100 \\ \hline 900 \end{array}$$ Check: $$\begin{array}{r} \overset{1}{8}\overset{1}{7}\overset{1}{6}.6 \\ +123.4 \\ \hline 1000.0 \end{array}$$

33. $-1.12 - 5.2 = -1.12 + (-5.2)$
Add the absolute values.
$$\begin{array}{r} 1.12 \\ +\,5.20 \\ \hline 6.32 \end{array}$$
Attach the common sign.
$-1.12 - 5.2 = -6.32$

37. $-2.6 - (-5.7) = -2.6 + 5.7$
Subtract the absolute values.
$$\begin{array}{r} 5.7 \\ -2.6 \\ \hline 3.1 \end{array}$$
Attach the sign of the larger absolute value.
$-2.6 - (-5.7) = 3.1$

41. $$\begin{array}{r} 23.0 \\ -\;\,6.7 \\ \hline 16.3 \end{array}$$ Check: $$\begin{array}{r} \overset{1}{1}\overset{1}{6}.3 \\ +\;\,6.7 \\ \hline 23.0 \end{array}$$

45. $-6.06 + 0.44$

Subtract the absolute values.

$$\begin{array}{r} 6.06 \\ -0.44 \\ \hline 5.62 \end{array}$$

Attach the sign of the larger absolute value.

$-6.06 + 0.44 = -5.62$

49. $50.2 - 600 = 50.2 + (-600)$

Subtract the absolute values.

$$\begin{array}{r} 600.0 \\ -\ 50.2 \\ \hline 549.8 \end{array}$$

Attach the sign of the larger absolute value.

$50.2 - 600 = -549.8$

53.

$$\begin{array}{r} 500.008 \\ 4.060 \\ +\ \ 8.033 \\ \hline 512.101 \end{array}$$

57. $-102.4 - 78.04 = -102.4 + (-78.04)$

Add the absolute values.

$$\begin{array}{r} 102.40 \\ +\ 78.04 \\ \hline 180.44 \end{array}$$

Attach the common sign.

$-102.4 - 78.04 = -180.44$

61. $x + z = 3.6 + 0.21 = 3.81$

65. $$\begin{aligned} y - x + z &= 5 - 3.6 + 0.21 \\ &= 5.00 - 3.60 + 0.21 \\ &= 1.40 + 0.21 \\ &= 1.61 \end{aligned}$$

69. To see how much the price changed, we subtract.

$85.55 - 85.95 = 85.55 + (-85.95)$

$$\begin{array}{r} 85.95 \\ -85.55 \\ \hline 0.40 \end{array}$$

Since the larger absolute value is negative, $85.55 - 85.95 = -0.40$. The price of each share changed $-\$0.40$.

73. The perimeter is the sum of the lengths of the sides.

$$\begin{array}{r} 5.65 \\ 2.80 \\ 5.65 \\ +\ 2.80 \\ \hline 16.90 \end{array}$$

The perimeter of the Apple iPhone X is approximately 16.9 inches.

77. To find the decrease, we subtract.

$$\begin{array}{r} 187.7 \\ -158.3 \\ \hline 29.4 \end{array}$$

The decrease in monthly text message from 2010 to 2016 was 29.4 billion text messages, or 29,400,000,000 text messages.

81. To see how much snow Valdez receives, we add.

$$\begin{array}{r} 215.8 \\ +\ 110.5 \\ \hline 326.3 \end{array}$$

Valdez receives an average of 326.3 inches of snow annually.

85. The phrase "How much slower" indicates subtraction.

$$\begin{array}{r} 244.795 \\ -197.400 \\ \hline 47.395 \end{array}$$

The average 24 hours of Le Mans speed was 47.395 kph slower in 2013 than in 2017.

89. Subtract the least chocolate consumption shown in the bar graph (14.6 pounds per person per year) from the greatest chocolate consumption shown (19.8 pounds per person per year).

$$\begin{array}{r} 19.8 \\ -14.6 \\ \hline 5.2 \end{array}$$

The chocolate consumption in Switzerland is 5.2 pounds per person per year greater than the chocolate consumption in Norway.

93. $23 \cdot 2 = 46$

97. The calculations are incorrect – the decimal points should be lined up.

$$\begin{array}{r} 9.200 \\ 8.630 \\ +\ 4.005 \\ \hline 21.835 \end{array}$$

101. There are 3 quarters, 3 dimes, and 3 nickels.

$$\begin{array}{r} 0.25 \\ 0.25 \\ 0.25 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.05 \\ 0.05 \\ +\ 0.05 \\ \hline 1.20 \end{array}$$

The value of the coins is \$1.20.

105. answers may vary

Exercise Set 4.3

1.

$$\begin{array}{r} 0.17 \\ \times\quad 8 \\ \hline 1.36 \end{array}$$

5. The product of a negative number and a positive number is negative.

$$\begin{array}{r} 7.65 \\ \times\ \ 2.3 \\ \hline 2295 \\ \underline{15300} \\ 17.595 \end{array}$$

$(-2.3)(7.65) = -17.595$

9. Exact: $$\begin{array}{r} 6.8 \\ \times\ \ 4.2 \\ \hline 136 \\ \underline{2720} \\ 28.56 \end{array}$$ Estimate: $$\begin{array}{r} 7 \\ \times\quad 4 \\ \hline 28 \end{array}$$

13. Exact: $$\begin{array}{r} 1.0047 \\ \times\ \ 8.2 \\ \hline 20094 \\ \underline{803760} \\ 8.23854 \end{array}$$ Estimate: $$\begin{array}{r} 1 \\ \times\quad 8 \\ \hline 8 \end{array}$$

17. $6.5 \times 10 = 65$

21. $(-7.093)(1000) = -7093$

25. $(-9.83)(-0.01) = 0.0983$

29.
$$\begin{array}{r} 0.123 \\ \times\ 0.4 \\ \hline 0.0492 \end{array}$$

33.
$$\begin{array}{r} 8.6 \\ \times 0.15 \\ \hline 430 \\ 860 \\ \hline 1.290 \text{ or } 1.29 \end{array}$$

37. $562.3 \times 0.001 = 0.5623$

41.
$$\begin{aligned} 3.26\text{ billion} &= 3.26 \times 1\text{ billion} \\ &= 3.26 \times 1{,}000{,}000{,}000 \\ &= 3{,}260{,}000{,}000 \end{aligned}$$

45. Replace x with 3 and y with -0.2.
$xy = (3)(-0.2) = -0.6$

49. Circumference $= \pi \cdot$ diameter
$C = \pi \cdot 10 = 10\pi$
$C \approx 10(3.14) = 31.4$
The circumference is 10π centimeters, which is approximately 31.4 centimeters.

53. Multiply his hourly wage by the number of hours.
$$\begin{array}{r} 17.88 \\ \times\ 40 \\ \hline 715.20 \end{array}$$
His pay before taxes that week was \$715.20.

57. Area $=$ length $\cdot$ width
$$\begin{array}{r} 5.65 \\ \times\quad 2.8 \\ \hline 4520 \\ 11300 \\ \hline 15.820 \end{array}$$
The area of the Apple iPhone X is approximately 15.82 square inches.

61. Circumference $= \pi \cdot$ diameter
$C = \pi \cdot 135 = 135\pi$
$C \approx 135(3.14) = 423.9$
In one revolution, he travels exactly 135π meters, which is approximately 423.9 meters.

65. a. Circumference $= 2 \cdot \pi \cdot$ radius
Smaller circle:
$C = 2 \cdot \pi \cdot 10 = 20\pi$
$C \approx 20(3.14) = 62.8$
The circumference of the smaller circle is approximately 62.8 meters.
Larger circle:
$C = 2 \cdot \pi \cdot 20 = 40\pi$
$C \approx 40(3.14) = 125.6$
The circumference of the larger circle is approximately 125.6 meters.

b. Yes, the circumference gets doubled when the radius is doubled.

69.
$$\begin{array}{r} 1.25090 \\ \times\qquad 800 \\ \hline 1000.72000 \end{array}$$
On that day, \$800 U.S. was equivalent to 1000.72 Canadian dollars.

73.
$$\begin{array}{r} 486 \\ 6\overline{)2916} \\ -24 \\ \hline 51 \\ -48 \\ \hline 36 \\ -36 \\ \hline 0 \end{array}$$
$2916 \div 6 = 486$

77.
$$\begin{array}{r} 3.60 \\ +\ 0.04 \\ \hline 3.64 \end{array}$$

81. The product of a negative number and a positive number is a negative number.
$$\begin{array}{r} 0.221 \\ \times\quad 0.5 \\ \hline 0.1105 \end{array}$$
$-0.221 \times 0.5 = -0.1105$

85. answers may vary

Exercise Set 4.4

1.
$$\begin{array}{r} 4.6 \\ 3\overline{)\ 13.8} \\ -12 \\ \hline 18 \\ 18 \\ \hline 0 \end{array}$$

5. $0.06\overline{)18}$ becomes
$$\begin{array}{r} 300 \\ 6\overline{)\ 1800} \\ -18 \\ \hline 000 \end{array}$$

9. Exact: $5.5\overline{)36.3}$ becomes
$$\begin{array}{r} 6.6 \\ 55\overline{)\ 363.0} \\ -330 \\ \hline 33\ 0 \\ -33\ 0 \\ \hline 0 \end{array}$$
Estimate: $\begin{array}{r} 6 \\ 6\overline{)36} \end{array}$

13. A positive number divided by a negative number is a negative number.
$0.06\overline{)36}$ becomes
$$\begin{array}{r} 600 \\ 6\overline{)\ 3600} \\ -36 \\ \hline 000 \end{array}$$
$36 \div (-0.06) = -600$

17. $0.27\overline{)1.296}$ becomes
$$\begin{array}{r} 4.8 \\ 27\overline{)\ 129.6} \\ -108 \\ \hline 21\ 6 \\ -21\ 6 \\ \hline 0 \end{array}$$

21. $0.82\overline{)4.756}$ becomes
$$\begin{array}{r} 5.8 \\ 82\overline{)\ 475.6} \\ -410 \\ \hline 65\ 6 \\ -65\ 6 \\ \hline 0 \end{array}$$
$4.756 \div 0.82 = 5.8$

25. Exact: $7.2\overline{)70.56}$ becomes $\begin{array}{r} 9.8 \\ 72\overline{)\;705.6} \\ \underline{-648} \\ 57\,6 \\ \underline{-57\,6} \\ 0 \end{array}$

Estimate: $\begin{array}{r} 10 \\ 7\overline{)70} \end{array}$

29. $0.027\overline{)1.215}$ becomes $\begin{array}{r} 45 \\ 27\overline{)\;1215} \\ \underline{-108} \\ 135 \\ \underline{-135} \\ 0 \end{array}$

$$\frac{1.215}{0.027} = 45$$

33. $3.78\overline{)0.02079}$ becomes $\begin{array}{r} 0.0055 \\ 378\overline{)\;2.0790} \\ \underline{-1\,890} \\ 1\,890 \\ \underline{-1\,890} \\ 0 \end{array}$

37. $0.023\overline{)0.549}$ becomes $\begin{array}{r} 23.869 \approx 23.87 \\ 23\overline{)\;549.000} \\ \underline{-46} \\ 89 \\ \underline{-69} \\ 20\,0 \\ \underline{-18\,4} \\ 1\,60 \\ \underline{-1\,38} \\ 220 \\ \underline{-207} \\ 13 \end{array}$

41. $\dfrac{54.982}{100} = 0.54982$

45. $-12.9 \div 1000 = -0.0129$

49. $\dfrac{13.1}{10} = 1.31$

53. $\begin{array}{r} 0.413 \\ 3\overline{)\;1.239} \\ \underline{-1\,2} \\ 03 \\ \underline{-3} \\ 09 \\ \underline{-9} \\ 0 \end{array}$

$1.239 \div 3 = 0.413$

57. A negative number divided by a positive number is a negative number.

$0.17\overline{)1.224}$ becomes $\begin{array}{r} 7.2 \\ 17\overline{)\;122.4} \\ \underline{-119} \\ 3\,4 \\ \underline{-3\,4} \\ 0 \end{array}$

$-1.224 \div 0.17 = -7.2$

61. The quotient of two negative numbers is a positive number.

$0.6\overline{)18}$ becomes $\begin{array}{r} 30 \\ 6\overline{)\;180} \\ \underline{-18} \\ 00 \end{array}$

-18 divided by -0.6 is 30.

65. A negative number divided by a positive number is a negative number.

$1.6\overline{)1.104}$ becomes $\begin{array}{r} 0.69 \\ 16\overline{)\;11.04} \\ \underline{-9\,6} \\ 1\,44 \\ \underline{-1\,44} \\ 0 \end{array}$

$-1.104 \div 1.6 = -0.69$

69. $0.071\overline{)4.615}$ becomes $\begin{array}{r} 65 \\ 71\overline{)\;4615} \\ \underline{-426} \\ 355 \\ \underline{-355} \\ 0 \end{array}$

$$\frac{4.615}{0.071} = 65$$

73. $x \div y = 5.65 \div (-0.8)$

$0.8\overline{)5.65}$ becomes $\begin{array}{r} 7.0625 \\ 8\overline{)\;56.5000} \\ \underline{-56} \\ 0\,5 \\ \underline{-\;0} \\ 50 \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$

$x \div y = 5.65 \div (-0.8) = -7.0625$

77. Divide 200 by 39.37.

$39.37\overline{)200}$ becomes $\begin{array}{r} 5.08 \approx 5.1 \\ 3937\overline{)\;20000.00} \\ \underline{-19685} \\ 315\,0 \\ \underline{-\quad 0} \\ 315\,00 \\ \underline{-31496} \\ 4 \end{array}$

There are approximately 5.1 meters in 200 inches.

81. $4 \times 6 = 24$

There are 24 teaspoons in 4 fluid ounces.

85. Divide the number of miles by 52, the number of weeks in one year.

$$\begin{array}{r} 146.61 \approx 146.6 \\ 52\overline{)\,7624.00} \\ \underline{-52} \\ 242 \\ \underline{-208} \\ 344 \\ \underline{-312} \\ 32\,0 \\ \underline{-31\,2} \\ 80 \\ \underline{-52} \\ 28 \end{array}$$

Americans 16–19 drive approximately 146.6 miles each week.

89. $0.9 = \frac{9}{10}$

93. $0.3\overline{)1.278}$ becomes

$$\begin{array}{r} 4.26 \\ 3\overline{)\,12.78} \\ \underline{-12} \\ 0\,7 \\ \underline{-\;6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$$

$1.278 \div 0.3 = 4.26$

97. A negative number times a positive number is a negative number.

$$\begin{array}{r} 8.6 \\ \underline{\times\;3.1} \\ 86 \\ \underline{2580} \\ 26.66 \end{array}$$

$(-8.6)(3.1) = -26.66$

101. 8.62×41.7 is approximately $9 \times 40 = 360$, which is choice **c**.

105. Add the numbers, then divide by 4.

$$\begin{array}{r} \overset{2}{8}6 \\ 78 \\ 91 \\ \underline{+\;87} \\ 342 \end{array}$$

$$\begin{array}{r} 85.5 \\ 4\overline{)\,342.0} \\ \underline{-32} \\ 22 \\ \underline{-20} \\ 2\,0 \\ \underline{2\,0} \\ 0 \end{array}$$

The average is 85.5.

109. answers may vary

113. Divide the total length of wire by 4 to get the length of one wire around the corral, which is the perimeter.

$$\begin{array}{r} 103.2 \\ 4\overline{)\,412.8} \\ \underline{-4} \\ 01 \\ \underline{-0} \\ 12 \\ \underline{-12} \\ 0\,8 \\ \underline{-8} \\ 0 \end{array}$$

The perimeter of the corral is 103.2 meters.

Perimeter $= 2(\text{length}) + 2(\text{width})$ and the width of the corral is 24.3 meters.

$$\begin{aligned} 103.2 &= 2(\text{length}) + 2(24.3) \\ 103.2 &= 2(\text{length}) + 48.6 \\ 103.2 - 48.6 &= 2(\text{length}) + 48.6 - 48.6 \\ 54.6 &= 2(\text{length}) \\ \frac{54.6}{2} &= \frac{2(\text{length})}{2} \\ 27.3 &= \text{length} \end{aligned}$$

The length of the corral is 27.3 meters.

Exercise Set 4.5

1.
$$\begin{array}{r} 0.2 \\ 5\overline{)\,1.0} \\ \underline{-1\,0} \\ 0 \end{array} \quad \frac{1}{5} = 0.2$$

5.
$$\begin{array}{r} 0.75 \\ 4\overline{)\,3.00} \\ \underline{-2\,8} \\ 20 \\ \underline{-20} \\ 0 \end{array} \quad \frac{3}{4} = 0.75$$

9.
$$\begin{array}{r} 2.25 \\ 4\overline{)\,9.00} \\ \underline{-8} \\ 10 \\ \underline{-\;8} \\ 20 \\ \underline{-20} \\ 0 \end{array} \quad \frac{9}{4} = 2.25$$

13.
$$\begin{array}{r} 0.425 \\ 40\overline{)\,17.000} \\ \underline{-16\,0} \\ 1\,00 \\ \underline{-\,80} \\ 200 \\ \underline{200} \\ 0 \end{array} \quad \frac{17}{40} = 0.425$$

17. $$\begin{array}{r} 0.333\ldots \\ 3\overline{)1.000} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{9} \\ 1 \end{array} \qquad -\frac{1}{3} = -0.\overline{3}$$

21. $$\begin{array}{r} 0.636363\ldots \\ 11\overline{)\ 7.000000} \\ \underline{-6\ 6} \\ 40 \\ \underline{-33} \\ 70 \\ \underline{-66} \\ 40 \\ \underline{-33} \\ 70 \\ \underline{-66} \\ 40 \\ \underline{-33} \\ 7 \end{array} \qquad \frac{7}{11} = 0.\overline{63}$$

25. $$\begin{array}{r} 0.624 \\ 125\overline{)\ 78.000} \\ \underline{-75\ 0} \\ 3\ 00 \\ \underline{-2\ 50} \\ 500 \\ \underline{-500} \\ 0 \end{array} \qquad \frac{78}{125} = 0.624$$

29. $\frac{7}{16} = 0.4375 \approx 0.44$

33. $$\begin{array}{r} 0.615 \\ 91\overline{)\ 56.000} \\ \underline{-54\ 6} \\ 1\ 40 \\ \underline{-91} \\ 490 \\ \underline{-455} \\ 35 \end{array} \qquad \frac{56}{91} \approx 0.62$$

37. $$\begin{array}{r} 0.02 \\ 50\overline{)1.00} \\ \underline{-100} \\ 0 \end{array} \qquad \frac{1}{50} = 0.02$$

41. $$\begin{array}{r} 0.215 \\ 200\overline{)\ 43.000} \\ \underline{-40\ 0} \\ 3\ 00 \\ \underline{-2\ 00} \\ 1\ 000 \\ \underline{-1\ 000} \\ 0 \end{array}$$

$0.215 = \frac{43}{200}$

45. $$\begin{array}{r} 0.833\ldots \\ 6\overline{)\ 5.000} \\ \underline{-4\ 8} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

$\frac{5}{6} = 0.8\overline{3}$ and $0.\overline{6} < 0.8\overline{3}$, or $0.\overline{6} < \frac{5}{6}$

49. $$\begin{array}{r} 0.571 \\ 7\overline{)\ 4.000} \\ \underline{-3\ 5} \\ 50 \\ \underline{-49} \\ 10 \\ \underline{-7} \\ 3 \end{array}$$

$\frac{4}{7} \approx 0.57$ and $0.57 > 0.14$, so $\frac{4}{7} > 0.14$.

53. $$\begin{array}{r} 7.125 \\ 64\overline{)\ 456.000} \\ \underline{-448} \\ 8\ 0 \\ \underline{-6\ 4} \\ 1\ 60 \\ \underline{-1\ 28} \\ 320 \\ \underline{-320} \\ 0 \end{array}$$

$\frac{456}{64} = 7.125$ and $7.123 < 7.125$, so

$7.123 < \frac{456}{64}$

57. $0.49 = 0.490$

$0.49, 0.491, 0.498$

61. $\frac{5}{8} = 0.625$

$0.612, \frac{5}{8}, 0.649$

65. $\frac{1 + 0.8}{-0.6} = \frac{1.8}{-0.6} = \frac{18}{-6} = -3$

69. $(5.6 - 2.3)(2.4 + 0.4) = (3.3)(2.8) = 9.24$

73. $\frac{7 + 0.74}{-6} = \frac{7.74}{-6} = -1.29$

77. $\frac{1}{4}(-9.6 - 5.2) = \frac{1}{4}(-14.8) = -3.7$

81. $$\begin{aligned} \text{Area} &= l \cdot w \\ &= (0.62)\left(\frac{2}{5}\right) \\ &= (0.62)(0.4) \\ &= 0.248 \end{aligned}$$

The area is 0.248 square yard.

85. $x - y = 6 - 0.3 = 5.7$

89. $6^2 \cdot 2 = 36 \cdot 2 = 72$

93. $1.0 = 1$

97. $\frac{99}{100} = 0.99$

$0.99 < 1$

101.

$$\begin{array}{rcr} 2291 & \text{rounds to} & 2300 \\ 1915 & \text{rounds to} & 1900 \\ 1186 & \text{rounds to} & 1200 \\ 1309 & \text{rounds to} & 1300 \\ 924 & \text{rounds to} & 900 \\ \underline{+\ 582} & \text{rounds to} & \underline{+\ 600} \\ & & 8200 \end{array}$$

The total number of stations with the top six formats was approximately 8200.

Exercise Set 4.6

1. $\sqrt{4} = 2$ because $2^2 = 4$.

5. $\sqrt{\frac{1}{81}} = \frac{1}{9}$ because $\frac{1}{9} \cdot \frac{1}{9} = \frac{1}{81}$.

9. $\sqrt{3} \approx 1.732$

13. $\sqrt{47} \approx 6.856$

17. Since $\sqrt{36} = 6$, $\sqrt{49} = 7$, and 38 is between 36 and 49, $\sqrt{38}$ is between 6 and 7.

21. $\sqrt{256} = 16$ because $16^2 = 256$.

25. $\sqrt{\frac{49}{144}} = \frac{7}{12}$ because $\frac{7}{12} \cdot \frac{7}{12} = \frac{49}{144}$.

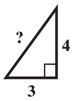
29.
$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(5)^2 + (12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

The hypotenuse is 13 inches.

33.
$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(22)^2 + (48)^2} \\ &= \sqrt{484 + 2304} \\ &= \sqrt{2788} \\ &\approx 52.802 \end{aligned}$$

The hypotenuse is about 52.802 meters.

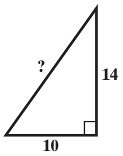
37.



$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(3)^2 + (4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

The hypotenuse has length 5 units.

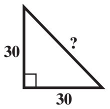
41.



$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(10)^2 + (14)^2} \\ &= \sqrt{100 + 196} \\ &= \sqrt{296} \\ &\approx 17.205 \end{aligned}$$

The hypotenuse is about 17.205 units.

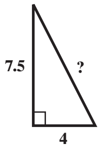
45.



$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(30)^2 + (30)^2} \\ &= \sqrt{900 + 900} \\ &= \sqrt{1800} \\ &\approx 42.426 \end{aligned}$$

The hypotenuse is about 42.426 units.

49.



$$\begin{aligned} \text{hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{other leg})^2} \\ &= \sqrt{(7.5)^2 + (4)^2} \\ &= \sqrt{56.25 + 16} \\ &= \sqrt{72.25} \\ &= 8.5 \end{aligned}$$

The hypotenuse has length 8.5 units.

53.
$$\begin{aligned} \text{leg} &= \sqrt{(\text{hypotenuse})^2 - (\text{other leg})^2} \\ &= \sqrt{(32)^2 - (20)^2} \\ &= \sqrt{1024 - 400} \\ &= \sqrt{624} \\ &\approx 25.0 \end{aligned}$$

The tree is about 25 feet tall.

57. $\frac{10}{12} = \frac{5}{6}$

61. $\frac{30}{72} = \frac{5}{12}$

65. Since $\sqrt{100} = 10$, $\sqrt{121} = 11$, and 101 is between 100 and 121, $\sqrt{101}$ is between 10 and 11.
Since 101 is closer to 100 than to 121, $\sqrt{101}$ is close to 10.

69.
$$\begin{aligned} \sqrt{(25)^2 + (60)^2} &= \sqrt{625 + 3600} \\ &= \sqrt{4225} \\ &= 65 \end{aligned}$$

Yes, the set forms the lengths of the sides of a right triangle.

Chapter 4 Test

1. 45.092 is forty-five and ninety-two thousandths.

5. $9.83 - 30.25 = 9.83 + (-30.25)$
Subtract the absolute values.

$$\begin{array}{r} 30.25 \\ -\ \ 9.83 \\ \hline 20.42 \end{array}$$

$9.83 - 30.25 = -20.42$

9. To round 0.8623 to the nearest thousandth, observe that the digit in the ten-thousandths place is 3. Since this digit is less than 5, we do not add 1 to the digit in the thousandths place. The number 0.8623 rounded to the nearest thousandth is 0.862.

13. $-24.73 = -24\frac{73}{100}$

17. $\frac{0.23 + 1.63}{-0.3} = \frac{1.86}{-0.3} = -6.2$

21. $\sqrt{\frac{64}{100}} = \frac{8}{10} = \frac{4}{5}$ since $\left(\frac{8}{10}\right)^2 = \frac{64}{100}$.

25. **a.**

$$\begin{array}{r} 123.8 \\ \times \quad 80 \\ \hline 9904.0 \end{array}$$

The area of the lawn is 9904 square feet.

b. Multiply the number of square feet by the number of ounces per square foot.

$$\begin{array}{r} 9904 \\ \times \quad 0.02 \\ \hline 198.08 \end{array}$$

198.08 ounces of insecticide are needed.

Chapter 5

Exercise Set 5.1

1. The ratio of 16 to 24 is $\frac{16}{24} = \frac{8 \cdot 2}{8 \cdot 3} = \frac{2}{3}$.

5. The ratio of 4.63 to 8.21 is $\frac{4.63}{8.21} = \frac{4.63 \cdot 100}{8.21 \cdot 100} = \frac{463}{821}$.

9. The ratio of \$32 to \$100 is $\frac{\$32}{\$100} = \frac{32}{100} = \frac{4 \cdot 8}{4 \cdot 25} = \frac{8}{25}$.

13. The ratio of $3\frac{1}{2}$ to $12\frac{1}{4}$ is

$$\begin{aligned} \frac{3\frac{1}{2}}{12\frac{1}{4}} &= 3\frac{1}{2} \div 12\frac{1}{4} \\ &= \frac{7}{2} \div \frac{49}{4} \\ &= \frac{7}{2} \cdot \frac{4}{49} \\ &= \frac{7 \cdot 2 \cdot 2}{2 \cdot 7 \cdot 7} \\ &= \frac{2}{7} \end{aligned}$$

17. The ratio of the average weight of a mature fin whale to the average weight of a mature blue whale is $\frac{50 \text{ tons}}{145 \text{ tons}} = \frac{50}{145} = \frac{5 \cdot 10}{5 \cdot 29} = \frac{10}{29}$.

21. The ratio of women to men is $\frac{125 \text{ women}}{100 \text{ men}} = \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{5}{2 \cdot 2} = \frac{5}{4}$

25. The ratio of 3-D films to total films is $\frac{44 \text{ films}}{724 \text{ films}} = \frac{44}{724} = \frac{4 \cdot 11}{4 \cdot 181} = \frac{11}{181}$.

29. The ratio of red blood cells to platelet cells is $\frac{600 \text{ cells}}{40 \text{ cells}} = \frac{600}{40} = \frac{15 \cdot 40}{1 \cdot 40} = \frac{15}{1}$.

33. The rate of 15 returns for 100 sales is $\frac{15 \text{ returns}}{100 \text{ sales}} = \frac{15}{100} = \frac{5 \cdot 3}{5 \cdot 20} = \frac{3}{20} = \frac{3 \text{ returns}}{20 \text{ sales}}$

37. The rate of 18 gallons for 4 acres is $\frac{18}{4} = \frac{2 \cdot 9}{2 \cdot 2} = \frac{9 \text{ gal}}{2 \text{ acres}}$

41. 1 minute = 60 seconds

$$\begin{array}{r} 90 \\ 60\overline{)\;5400} \\ \underline{-540} \\ 00 \end{array}$$

$$\begin{aligned} \frac{5400 \text{ wingbeats}}{1 \text{ minute}} &= \frac{5400 \text{ wingbeats}}{60 \text{ seconds}} \\ &= \frac{90 \text{ wingbeats}}{1 \text{ second}} \\ &= 90 \text{ wingbeats/second} \end{aligned}$$

45.

$$\begin{array}{r} 343{,}766 \\ 2\overline{)\;687{,}532} \\ \underline{-6}\phantom{00{,}000} \\ 08\phantom{0{,}000} \\ \underline{-\;8}\phantom{0{,}000} \\ 07\phantom{{,}000} \\ \underline{-\;6}\phantom{{,}000} \\ 15 \\ \underline{-14} \\ 13 \\ \underline{-12} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

$\frac{687{,}532 \text{ voters}}{2 \text{ senators}} = \frac{343{,}766 \text{ voters}}{1 \text{ senator}} = 343{,}766 \text{ voters/senator}$

49.

$$\begin{array}{r} 65{,}000 \\ 400\overline{)\;26{,}000{,}000} \\ \underline{-24\;00}\phantom{0{,}000} \\ 2\;000\phantom{{,}000} \\ \underline{-2\;000}\phantom{{,}000} \\ 0000 \end{array}$$

$\frac{\$26{,}000{,}000}{400 \text{ species}} = \frac{\$65{,}000}{1 \text{ species}} = \$65{,}000/\text{species}$

53. **a.**

$$\begin{array}{r} 31.25 \\ 8\overline{)\;250.00} \\ \underline{-24} \\ 10 \\ \underline{-\;8} \\ 2\,0 \\ \underline{-1\,6} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Charlie can assemble 31.25 computer boards/hour.

b.

$$\begin{array}{r} 33.5 \\ 12\overline{)\;402.0} \\ \underline{-36} \\ 42 \\ \underline{-36} \\ 6\,0 \\ \underline{-6\,0} \\ 0 \end{array}$$

Suellen can assemble 33.5 computer boards/hour.

c. Suellen can assemble computer boards faster.

57.
$$\begin{array}{r}
11.50\\
5\overline{)\,57.50}\\
\underline{-5}\\
07\\
\underline{-\ 5}\\
2\,5\\
\underline{-2\,5}\\
00
\end{array}$$

The unit price is $11.50 per DVD.

61.
$$\begin{array}{r}
0.1487 \approx 0.149\\
8\overline{)\,1.1900}\\
\underline{-\ 8}\\
39\\
\underline{-32}\\
70\\
\underline{-64}\\
60\\
\underline{-56}\\
4
\end{array}$$

The 8-ounce size costs approximately $0.149 per ounce.

$$\begin{array}{r}
0.1325 \approx 0.133\\
12\overline{)\,1.5900}\\
\underline{-1\,2}\\
39\\
\underline{-36}\\
30\\
\underline{-24}\\
60\\
\underline{-60}\\
0
\end{array}$$

The 12-ounce size costs approximately $0.133 per ounce. The 12-ounce size is the better buy.

65.
$$\begin{array}{r}
0.1908 \approx 0.191\\
12\overline{)\,2.2900}\\
\underline{-1\,2}\\
1\,09\\
\underline{-1\,08}\\
10\\
\underline{-\ 0}\\
100\\
\underline{-\ 96}\\
4
\end{array}$$

The 12-ounce size costs approximately $0.191 per ounce.

$$\begin{array}{r}
0.1862 \approx 0.186\\
8\overline{)\,1.4900}\\
\underline{-8}\\
69\\
\underline{-64}\\
50\\
\underline{-48}\\
20\\
\underline{-16}\\
4
\end{array}$$

The 8-ounce size costs approximately $0.186 per ounce. The 8-ounce size is the better buy.

69.
$$\begin{array}{r}
2.3\\
9\overline{)\,20.7}\\
\underline{-18}\\
2\,7\\
\underline{-2\,7}\\
0
\end{array}$$

73. no; answers may vary

77. miles driven = ending odometer reading − beginning odometer reading
$$\begin{aligned}
&= 80{,}242 - 79{,}895\\
&= 347
\end{aligned}$$

$$16.1\overline{)347} \text{ becomes } \begin{array}{r}
21.55 \approx 21.6\\
161\overline{)\,3470.00}\\
\underline{-322}\\
250\\
\underline{-161}\\
89\,0\\
\underline{-80\,5}\\
8\,50\\
\underline{-8\,05}\\
45
\end{array}$$

There were 347 miles driven, and the miles per gallon was approximately 21.6 miles per gallon.

81. answers may vary

85. $\frac{7.1}{4.3}$ is not in simplest form.

$\frac{7.1}{4.3} = \frac{7.1 \cdot 10}{4.3 \cdot 10} = \frac{71}{43}$

89. The ratio of bruised tomatoes to the total batch is $\frac{3}{3+33} = \frac{3}{36} = \frac{3 \cdot 1}{3 \cdot 12} = \frac{1}{12}$. This is less than the ratio 1 to 10, since $\frac{1}{12} = \frac{5 \cdot 1}{5 \cdot 12} = \frac{5}{60} < \frac{6}{60} = \frac{6 \cdot 1}{6 \cdot 10} = \frac{1}{10}$. The shipment should not be refused.

Exercise Set 5.2

1. $\frac{\text{10 diamonds}}{\text{6 opals}} = \frac{\text{5 diamonds}}{\text{3 opals}}$

5. $\frac{\text{6 eagles}}{\text{58 sparrows}} = \frac{\text{3 eagles}}{\text{29 sparrows}}$

9. $\frac{\text{22 vanilla wafers}}{\text{1 cup cookie crumbs}} = \frac{\text{55 vanilla wafers}}{\text{2.5 cup cookie crumbs}}$

13. $\frac{8}{6} \stackrel{?}{=} \frac{9}{7}$

$8 \cdot 7 \stackrel{?}{=} 9 \cdot 6$

$56 \neq 54$

The proportion is false.

17. $\frac{5}{8} \stackrel{?}{=} \frac{625}{1000}$

$5 \cdot 1000 \stackrel{?}{=} 8 \cdot 625$

$5000 = 5000$

The proportion is true.

21. $\frac{8}{10} \stackrel{?}{=} \frac{5.6}{0.7}$

$8(0.7) \stackrel{?}{=} 10(5.6)$

$5.6 \neq 56$

The proportion is false.

25.
$$\frac{2\frac{2}{5}}{\frac{2}{3}} \stackrel{?}{=} \frac{1\frac{1}{9}}{\frac{1}{4}}$$
$$2\frac{2}{5}\cdot\frac{1}{4} \stackrel{?}{=} 1\frac{1}{9}\cdot\frac{2}{3}$$
$$\frac{12}{5}\cdot\frac{1}{4} \stackrel{?}{=} \frac{10}{9}\cdot\frac{2}{3}$$
$$\frac{3}{5} \neq \frac{20}{27}$$
The proportion is false.

29.
$$\frac{8}{12} \stackrel{?}{=} \frac{4}{6}$$
$$8\cdot 6 \stackrel{?}{=} 12\cdot 4$$
$$48 = 48$$
The proportion is true.

33.
$$\frac{1.8}{2} \stackrel{?}{=} \frac{4.5}{5}$$
$$1.8(5) \stackrel{?}{=} 4.5(2)$$
$$9 = 9$$
The proportion is true.

37.
$$\begin{aligned} \frac{n}{5} &= \frac{6}{10}\\ n\cdot 10 &= 5\cdot 6\\ n\cdot 10 &= 30\\ n &= \frac{30}{10}\\ n &= 3 \end{aligned}$$

41.
$$\begin{aligned} \frac{n}{8} &= \frac{50}{100}\\ n\cdot 100 &= 8\cdot 50\\ n\cdot 100 &= 400\\ n &= \frac{400}{100}\\ n &= 4 \end{aligned}$$

45.
$$\begin{aligned} \frac{24}{n} &= \frac{60}{96}\\ 24\cdot 96 &= n\cdot 60\\ 2304 &= n\cdot 60\\ \frac{2304}{60} &= n\\ 38.4 &= n \end{aligned}$$

49.
$$\begin{aligned} \frac{0.05}{12} &= \frac{n}{0.6}\\ 0.05\cdot 0.6 &= 12\cdot n\\ 0.03 &= 12\cdot n\\ \frac{0.03}{12} &= n\\ 0.0025 &= n \end{aligned}$$

53.
$$\begin{aligned} \frac{\frac{1}{3}}{\frac{3}{8}} &= \frac{\frac{2}{5}}{n}\\ \frac{1}{3}\cdot n &= \frac{3}{8}\cdot\frac{2}{5}\\ \frac{1}{3}\cdot n &= \frac{3}{20}\\ n &= \frac{3}{20}\div\frac{1}{3}\\ n &= \frac{3}{20}\cdot\frac{3}{1}\\ n &= \frac{9}{20} \end{aligned}$$

57.
$$\begin{aligned} \frac{n}{1\frac{1}{5}} &= \frac{4\frac{1}{6}}{6\frac{2}{3}}\\ n\cdot 6\frac{2}{3} &= \left(1\frac{1}{5}\right)\left(4\frac{1}{6}\right)\\ n\cdot\frac{20}{3} &= \frac{6}{5}\cdot\frac{25}{6}\\ n\cdot\frac{20}{3} &= \frac{5}{1}\\ n &= \frac{5}{1}\div\frac{20}{3}\\ n &= \frac{5}{1}\cdot\frac{3}{20}\\ n &= \frac{3}{4} \end{aligned}$$

61.
$$\begin{array}{ccc} 8.01 & & 8.1\\ \uparrow & & \uparrow\\ 0 & < & 1 \text{ so} \end{array}$$
$$8.01 < 8.1$$

65.
$$5\frac{1}{3} = \frac{16}{3}$$
$$6\frac{2}{3} = \frac{20}{3}$$
$16 < 20$, so $5\frac{1}{3} < 6\frac{2}{3}$

69.
$$\frac{6}{18} = \frac{1}{3}$$
$$\frac{6}{1} = \frac{18}{3}$$
$$\frac{3}{18} = \frac{1}{6}$$
$$\frac{18}{6} = \frac{3}{1}$$

73. answers may vary

77.
$$\begin{aligned} \frac{n}{5.2} &= \frac{0.08}{6}\\ n\cdot 6 &= 5.2\cdot 0.08\\ n\cdot 6 &= 0.416\\ n &= \frac{0.416}{6}\\ n &\approx 0.07 \end{aligned}$$

81.
$$\begin{aligned} \frac{n}{7} &= \frac{0}{8}\\ n\cdot 8 &= 7\cdot 0\\ n\cdot 8 &= 0\\ n &= \frac{0}{8}\\ n &= 0 \end{aligned}$$

85.
$$\begin{aligned} \frac{222}{1515} &= \frac{37}{n}\\ 222\cdot n &= 1515\cdot 37\\ 222\cdot n &= 56{,}055\\ n &= \frac{56{,}055}{222}\\ n &= 252.5 \end{aligned}$$

Exercise Set 5.3

1. Let n represent the number of field goals made.

$$\begin{array}{r}\text{field goals made}\rightarrow\\ \text{attempts}\rightarrow\end{array}\frac{45}{100}=\frac{n}{800}\begin{array}{l}\leftarrow\text{field goals made}\\ \leftarrow\text{attempts}\end{array}$$

$$\begin{aligned}45\cdot 800&=100\cdot n\\ 36{,}000&=100\cdot n\\ \frac{36{,}000}{100}&=n\\ 360&=n\end{aligned}$$

The player made 360 field goals.

5. Let n represent the number of applications received.

$$\begin{array}{r}\text{applications}\rightarrow\\ \text{accepted}\rightarrow\end{array}\frac{7}{2}=\frac{n}{180}\begin{array}{l}\leftarrow\text{applications}\\ \leftarrow\text{accepted}\end{array}$$

$$\begin{aligned}7\cdot 180&=2\cdot n\\ 1260&=2\cdot n\\ \frac{1260}{2}&=n\\ 630&=n\end{aligned}$$

The school received 630 applications.

9. Let n represent the square feet of floor space needed.

$$\begin{array}{r}\text{floor space}\rightarrow\\ \text{students}\rightarrow\end{array}\frac{n}{30}=\frac{9}{1}\begin{array}{l}\leftarrow\text{floor space}\\ \leftarrow\text{students}\end{array}$$

$$\begin{aligned}1\cdot n&=9\cdot 30\\ n&=270\end{aligned}$$

270 square feet of floor space are needed.

13. Let n represent the number of kilometers between Milan and Rome.

$$\begin{array}{r}\text{kilometers}\rightarrow\\ \text{cm on map}\rightarrow\end{array}\frac{n}{15}=\frac{30}{1}\begin{array}{l}\leftarrow\text{kilometers}\\ \leftarrow\text{cm on map}\end{array}$$

$$\begin{aligned}1\cdot n&=15\cdot 30\\ n&=450\end{aligned}$$

Milan and Rome are 450 kilometers apart.

17. Let n represent the number of hits expected.

$$\begin{array}{r}\text{hits}\rightarrow\\ \text{at bats}\rightarrow\end{array}\frac{3}{8}=\frac{n}{40}\begin{array}{l}\leftarrow\text{hits}\\ \leftarrow\text{at bats}\end{array}$$

$$\begin{aligned}3\cdot 40&=8\cdot n\\ 120&=8\cdot n\\ \frac{120}{8}&=n\\ 15&=n\end{aligned}$$

The player would be expected to get 15 hits.

21. Let n represent the number of applications expected.

$$\begin{array}{r}\text{ounces}\rightarrow\\ \text{applications}\rightarrow\end{array}\frac{3}{4}=\frac{14}{n}\begin{array}{l}\leftarrow\text{ounces}\\ \leftarrow\text{applications}\end{array}$$

$$\begin{aligned}3\cdot n&=4\cdot 14\\ 3\cdot n&=56\\ n&=\frac{56}{3}\\ n&=18\frac{2}{3}\end{aligned}$$

Jen should expect 18 applications of the self-tanner.

25. Let n represent the number of servings he can make.

$$\begin{array}{r}\text{milk}\rightarrow\\ \text{servings}\rightarrow\end{array}\frac{1\frac{1}{2}}{4}=\frac{4}{n}\begin{array}{l}\leftarrow\text{milk}\\ \leftarrow\text{servings}\end{array}$$

$$\begin{aligned}\left(1\frac{1}{2}\right)\cdot n&=4\cdot 4\\ \frac{3}{2}\cdot n&=16\\ n&=16\div\frac{3}{2}\\ n&=16\cdot\frac{2}{3}\\ n&=\frac{32}{3}\text{ or }10\frac{2}{3}\end{aligned}$$

He can make $10\frac{2}{3}$ servings of pancakes.

29. **a.** Let n represent the teaspoons of granules needed.

$$\begin{array}{r}\text{teaspoons}\rightarrow\\ \text{square feet}\rightarrow\end{array}\frac{1}{25}=\frac{n}{450}\begin{array}{l}\leftarrow\text{teaspoons}\\ \leftarrow\text{square feet}\end{array}$$

$$\begin{aligned}1\cdot 450&=25\cdot n\\ \frac{450}{25}&=n\\ 18&=n\end{aligned}$$

18 teaspoons of granules are needed to treat 450 square feet of standing water.

b. Let n represent the number of tablespoons.

$$\begin{array}{r}\text{teaspoons}\rightarrow\\ \text{tablespoons}\rightarrow\end{array}\frac{3}{1}=\frac{18}{n}\begin{array}{l}\leftarrow\text{teaspoons}\\ \leftarrow\text{tablespoons}\end{array}$$

$$\begin{aligned}3\cdot n&=1\cdot 18\\ n&=\frac{18}{3}\\ n&=6\end{aligned}$$

6 tablespoons of granules are needed to treat 450 square feet of standing water.

33. Let n represent the estimated head-to-toe height of the Statue of Liberty.

$$\begin{array}{r}\text{height}\rightarrow\\ \text{arm length}\rightarrow\end{array}\frac{5\frac{1}{3}}{2}=\frac{n}{42}\begin{array}{l}\leftarrow\text{height}\\ \leftarrow\text{arm length}\end{array}$$

$$\begin{aligned}\left(5\frac{1}{3}\right)\cdot 42&=2\cdot n\\ \frac{16}{3}\cdot 42&=2\cdot n\\ 224&=2\cdot n\\ \frac{224}{2}&=n\\ 112&=n\end{aligned}$$

The head-to-toe height of the Statue of Liberty is estimated to be 112 feet.

$$112-111\frac{1}{12}=\frac{11}{12}$$

The difference is $\frac{11}{12}$ foot, or 11 inches.

37. Let n represent the estimated height of the Empire State Building.

$$\begin{array}{r}\text{height}\rightarrow\\ \text{stories}\rightarrow\end{array}\frac{850}{69}=\frac{n}{102}\begin{array}{l}\leftarrow\text{height}\\ \leftarrow\text{stories}\end{array}$$

$$\begin{aligned}850\cdot 102&=69\cdot n\\ 86{,}700&=69\cdot n\\ \frac{86{,}700}{69}&=n\\ 1257&\approx n\end{aligned}$$

The estimated height of the Empire State Building is 1257 feet.

41. Let n represent the number of SUV crossovers.

$$\begin{array}{r}\text{SUV crossovers}\rightarrow\\ \text{total vehicles}\rightarrow\end{array}\frac{6}{20}=\frac{n}{7}\begin{array}{l}\leftarrow\text{SUV crossovers}\\ \leftarrow\text{total vehicles}\end{array}$$

$$\begin{aligned}6\cdot 7&=20\cdot n\\ 42&=20\cdot n\\ \frac{42}{20}&=n\\ 2&\approx n\end{aligned}$$

Approximately 2 million of the vehicles sold were SUV crossovers.

45. **a.** Let n represent the gallons of oil.

$$\begin{array}{l} \text{gas} \rightarrow \\ \text{oil} \rightarrow \end{array} \frac{50}{1} = \frac{5}{n} \begin{array}{l} \leftarrow \text{gas} \\ \leftarrow \text{oil} \end{array}$$

$$50 \cdot n = 1 \cdot 5$$

$$n = \frac{5}{50}$$

$$n = 0.1$$

0.1 gallon of oil should be mixed with 5 gallons of gas.

b. Let n represent the number of fluid ounces.

$$\begin{array}{r} \text{gallons} \rightarrow \\ \text{fluid ounces} \rightarrow \end{array} \frac{1}{128} = \frac{0.1}{n} \begin{array}{l} \leftarrow \text{gallons} \\ \leftarrow \text{fluid ounces} \end{array}$$

$$1 \cdot n = 128 \cdot 0.1$$

$$n = 12.8$$

$$n \approx 13$$

Approximately 13 fluid ounces of oil should be mixed with 5 gallons of gas.

49. $\begin{array}{r} 5 \\ 3\overline{)15} \end{array}$

$15 = 3 \cdot 5$

53. $\begin{array}{r} 5 \\ 5\overline{)25} \\ 2\overline{)50} \\ 2\overline{)100} \\ 2\overline{)200} \end{array}$

$200 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 2^3 \cdot 5^2$

57. Let n represent the number of milliliters.

$$\begin{array}{l} \text{mg} \rightarrow \\ \text{ml} \rightarrow \end{array} \frac{15}{1} = \frac{12}{n} \begin{array}{l} \leftarrow \text{mg} \\ \leftarrow \text{ml} \end{array}$$

$$15 \cdot n = 1 \cdot 12$$

$$n = \frac{12}{15}$$

$$n = 0.8$$

0.8 milliliter should be administered.

61. Estimate 11 as 12 or 1 dozen. $8 \times 1.5 = 12$, so approximately 12 cups of milk are needed.

65. answers may vary

Exercise Set 5.4

1. $60 \text{ in.} = \dfrac{60 \text{ in.}}{1} \cdot \dfrac{1 \text{ ft}}{12 \text{ in.}} = \dfrac{60}{12} \text{ ft} = 5 \text{ ft}$

5. $$\begin{aligned} 42{,}240 \text{ ft} &= \frac{42{,}240 \text{ ft}}{1} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \\ &= \frac{42{,}240}{5280} \text{ mi} \\ &= 8 \text{ mi} \end{aligned}$$

9. $10 \text{ ft} = \dfrac{10 \text{ ft}}{1} \cdot \dfrac{1 \text{ yd}}{3 \text{ ft}} = \dfrac{10}{3} \text{ yd} = 3\dfrac{1}{3} \text{ yd}$

13. $$\begin{aligned} 162 \text{ in.} &= \frac{162 \text{ in.}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} \\ &= \frac{162}{36} \text{ yd} \\ &= 4.5 \text{ yd} \end{aligned}$$

17. $40 \text{ ft} = \dfrac{40 \text{ ft}}{1} \cdot \dfrac{1 \text{ yd}}{3 \text{ ft}} = \dfrac{40}{3} \text{ yd}$

$$\begin{array}{r} 13 \\ 3\overline{)\ 40} \\ \underline{-3} \\ 10 \\ \underline{-\ 9} \\ 1 \end{array}$$

$40 \text{ ft} = 13 \text{ yd } 1 \text{ ft}$

21. $10{,}000 \text{ ft} = \dfrac{10{,}000 \text{ ft}}{1} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft}} = \dfrac{10{,}000}{5280} \text{ mi}$

$$\begin{array}{r} 1 \\ 5280\overline{)\ 10{,}000} \\ \underline{-\ 5\,280} \\ 4\,720 \end{array}$$

$10{,}000 \text{ ft} = 1 \text{ mi } 4720 \text{ ft}$

25. $$\begin{aligned} 8 \text{ yd } 2 \text{ ft} &= \frac{8 \text{ yd}}{1} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} + 2 \text{ ft} \\ &= 24 \text{ ft} + 2 \text{ ft} \\ &= 26 \text{ ft} \end{aligned}$$

29. $$\begin{aligned} 3 \text{ ft } 10 \text{ in.} + 7 \text{ ft } 4 \text{ in.} &= 10 \text{ ft } 14 \text{ in.} \\ &= 10 \text{ ft} + 1 \text{ ft } 2 \text{ in.} \\ &= 11 \text{ ft } 2 \text{ in.} \end{aligned}$$

33. $$\begin{array}{r} 22 \text{ ft } 8 \text{ in.} \\ \underline{-16 \text{ ft } 3 \text{ in.}} \\ 6 \text{ ft } 5 \text{ in.} \end{array}$$

37. $28 \text{ ft } 8 \text{ in.} \div 2 = 14 \text{ ft } 4 \text{ in.}$

41. $60 \text{ m} = \dfrac{60 \text{ m}}{1} \cdot \dfrac{100 \text{ cm}}{1 \text{ m}} = 6000 \text{ cm}$

45. $500 \text{ m} = \dfrac{500 \text{ m}}{1} \cdot \dfrac{1 \text{ km}}{1000 \text{ m}} = \dfrac{500}{1000} \text{ km} = 0.5 \text{ km}$

49. $1500 \text{ cm} = \dfrac{1500 \text{ cm}}{1} \cdot \dfrac{1 \text{ m}}{100 \text{ cm}} = \dfrac{1500}{100} \text{ m} = 15 \text{ m}$

53. $7 \text{ km} = \dfrac{7 \text{ km}}{1} \cdot \dfrac{1000 \text{ m}}{1 \text{ km}} = 7000 \text{ m}$

57. $$\begin{aligned} 20.1 \text{ mm} &= \frac{20.1 \text{ mm}}{1} \cdot \frac{1 \text{ dm}}{100 \text{ mm}} \\ &= \frac{20.1}{100} \text{ dm} \\ &= 0.201 \text{ dm} \end{aligned}$$

61. $$\begin{array}{r} 8.6 \text{ m} \\ \underline{+\ 0.34 \text{ m}} \\ 8.94 \text{ m} \end{array}$$

65. $$\begin{array}{r} 24.8 \text{ mm} \\ \underline{-1.19 \text{ cm}} \\ \end{array} \quad \begin{array}{r} 24.8 \text{ mm} \\ \underline{-11.9 \text{ mm}} \\ 12.9 \text{ mm} \end{array} \quad \text{or} \quad \begin{array}{r} 2.48 \text{ cm} \\ \underline{-1.19 \text{ cm}} \\ 1.29 \text{ cm} \end{array}$$

69. $18.3 \text{ m} \times 3 = 54.9 \text{ m}$

73.

	Yards	Feet	Inches
Chrysler Building in New York City	$348\frac{2}{3}$	1046	12,552

77.

	Meters	**Millimeters**	**Kilometers**	**Centimeters**
Length of elephant	5	5000	0.005	500

81.

	Meters	**Millimeters**	**Kilometers**	**Centimeters**
Distance from London to Paris	342,000	342,000,000	342	34,200,000

85. $$\begin{array}{r} 6\text{ ft }10\text{ in.}\\ +\,3\text{ ft }\ \ 8\text{ in.}\\ \hline 9\text{ ft }18\text{ in.} \end{array} = 9\text{ ft} + 1\text{ ft }6\text{ in.} = 10\text{ ft }6\text{ in.}$$

The bamboo is 10 ft 6 in. tall.

89. $8\text{ ft }11\text{ in.} \div 21.5\text{ in.} = 107\text{ in.} \div 21.5\text{ in.} \approx 5.0$

Robert was 5.0 times as tall as Chandra.

93. $$\begin{array}{r} 80\text{ mm}\\ -5.33\text{ cm}\\ \hline \end{array} \qquad \begin{array}{r} 80.0\text{ mm}\\ -53.3\text{ mm}\\ \hline 26.7\text{ mm} \end{array}$$

The ice must be 26.7 mm thicker before skating is allowed.

97. $9\text{ ft }3\text{ in.} \div 3 = 3\text{ ft }1\text{ in.}$

Each piece is 3 ft 1 in. long.

101. $$\begin{array}{r} 3.35\\ 20\overline{)\ 67.00}\\ -60\\ \hline 7\,0\\ -6\,0\\ \hline 1\,00\\ -1\,00\\ \hline 0 \end{array}$$

Each piece will be 3.35 meters long.

105. $$\begin{array}{r} 44\\ 44\\ 65\\ +\,65\\ \hline 218 \end{array}$$

The perimeter of the sign is 218 feet.

$$218\text{ ft} = \frac{218\text{ ft}}{1} \cdot \frac{1\text{ yd}}{3\text{ ft}} = \frac{218}{3}\text{ yd} = 72\frac{2}{3}\text{ yd}$$

The perimeter of the sign is $72\frac{2}{3}$ yards.

109. $0.21 = \dfrac{21}{100}$

113. $$\begin{aligned} \frac{1}{4} &= \frac{1}{4} \cdot \frac{25}{25}\\ &= \frac{25}{100}\\ &= 0.25 \end{aligned}$$

117. Yes, glass for a drinking glass being 2 millimeters thick is reasonable.

121. 5 yd 2 in. is close to 5 yd and 7 yd 30 in. is close to 8 yd. Thus, 5 yd 2 in. + 7 yd 30 in. is estimated as 5 yd + 8 yd = 13 yd.

125. answers may vary

Exercise Set 5.5

1. $2\text{ lb} = \dfrac{2\text{ lb}}{1} \cdot \dfrac{16\text{ oz}}{1\text{ lb}} = 2 \cdot 16\text{ oz} = 32\text{ oz}$

5. $18{,}000\text{ lb} = \dfrac{18{,}000\text{ lb}}{1} \cdot \dfrac{1\text{ ton}}{2000\text{ lb}} = \dfrac{18{,}000}{2000}\text{ tons} = 9\text{ tons}$

9. $$\begin{aligned} 3500\text{ lb} &= \frac{3500\text{ lb}}{1} \cdot \frac{1\text{ ton}}{2000\text{ lb}}\\ &= \frac{3500}{2000}\text{ tons}\\ &= \frac{7}{4}\text{ tons}\\ &= 1\frac{3}{4}\text{ tons} \end{aligned}$$

13. $$\begin{aligned} 4.9\text{ tons} &= \frac{4.9\text{ tons}}{1} \cdot \frac{2000\text{ lb}}{1\text{ ton}}\\ &= 4.9 \cdot 2000\text{ lb}\\ &= 9800\text{ lb} \end{aligned}$$

17. $$\begin{aligned} 2950\text{ lb} &= \frac{2950\text{ lb}}{1} \cdot \frac{1\text{ ton}}{2000\text{ lb}}\\ &= \frac{2950}{2000}\text{ tons}\\ &= \frac{59}{40}\text{ tons}\\ &\approx 1.5\text{ tons} \end{aligned}$$

21. $$\begin{aligned} 5\frac{3}{4}\text{ lb} &= \frac{23}{4}\text{ lb} = \frac{\frac{23}{4}\text{ lb}}{1} \cdot \frac{16\text{ oz}}{1\text{ lb}}\\ &= \frac{23}{4} \cdot 16\text{ oz} = 23 \cdot 4\text{ oz} = 92\text{ oz} \end{aligned}$$

25. $89\text{ oz} = \dfrac{89\text{ oz}}{1} \cdot \dfrac{1\text{ lb}}{16\text{ oz}} = \dfrac{89}{16}\text{ lb}$

$$\begin{array}{r} 5\\ 16\overline{)\ 89}\\ -80\\ \hline 9 \end{array}$$

$89\text{ oz} = 5\text{ lb }9\text{ oz}$

29. $$\begin{aligned} &3\text{ tons }1820\text{ lb} + 4\text{ tons }\ 930\text{ lb}\\ &= 7\text{ tons }\ 2750\text{ lb}\\ &= 7\text{ tons} + 1\text{ ton }750\text{ lb}\\ &= 8\text{ tons }\ 750\text{ lb} \end{aligned}$$

33. $$\begin{array}{r} 12\text{ lb }4\text{ oz}\\ -\ \ 3\text{ lb }9\text{ oz}\\ \hline \end{array} \qquad \begin{array}{r} 11\text{ lb }20\text{ oz}\\ -\ 3\text{ lb }\ \ 9\text{ oz}\\ \hline 8\text{ lb }11\text{ oz} \end{array}$$

37. $$\begin{aligned} 6\text{ tons }\ 1500\text{ lb} \div 5 &= \frac{6}{5}\text{ tons }\ 300\text{ lb}\\ &= 1\frac{1}{5}\text{ tons }\ 300\text{ lb}\\ &= 1\text{ ton} + \frac{2000\text{ lb}}{5} + 300\text{ lb}\\ &= 1\text{ ton} + 400\text{ lb} + 300\text{ lb}\\ &= 1\text{ ton }\ 700\text{ lb} \end{aligned}$$

41. $4\text{ g} = \frac{4\text{ g}}{1}\cdot\frac{1000\text{ mg}}{1\text{ g}} = 4\cdot 1000\text{ mg} = 4000\text{ mg}$

45. $48\text{ mg} = \frac{48\text{ mg}}{1}\cdot\frac{1\text{ g}}{1000\text{ mg}} = \frac{48}{1000}\text{ g} = 0.048\text{ g}$

49. $$\begin{aligned} 15.14\text{ g} &= \frac{15.14\text{ g}}{1}\cdot\frac{1000\text{ mg}}{1\text{ g}}\\ &= 15.14\cdot 1000\text{mg}\\ &= 15{,}140\text{mg} \end{aligned}$$

53. $$\begin{aligned} 35\text{ hg} &= \frac{35\text{ hg}}{1}\cdot\frac{10{,}000\text{ cg}}{1\text{ hg}}\\ &= 35\cdot 10{,}000\text{ cg}\\ &= 350{,}000\text{ cg} \end{aligned}$$

57. $205\text{ mg} + 5.61\text{ g} = 0.205\text{ g} + 5.61\text{ g} = 5.815\text{ g}$

or

$$\begin{aligned} 205\text{ mg} + 5.61\text{ g} &= 205\text{ mg} + 5610\text{ mg}\\ &= 5815\text{ mg} \end{aligned}$$

61. $1.61\text{ kg} - 250\text{ g} = 1.61\text{ kg} - 0.250\text{ kg} = 1.36\text{ kg}$

or

$1.61\text{ kg} - 250\text{ g} = 1610\text{ g} - 250\text{ g} = 1360\text{ g}$

65. $17\text{ kg} \div 8 = \frac{17}{8}\text{ kg}$

$$\begin{array}{r} 2.125\\ 8\overline{)\;17.000}\\ -16\\ \hline 1\,0\\ -8\\ \hline 20\\ -16\\ \hline 40\\ -40\\ \hline 0 \end{array}$$

$17\text{ kg} \div 8 = 2.125\text{ kg}$

69.

Object	Tons	Pounds	Ounces
A 12-inch cube of osmium	$\frac{269}{400}$ or 0.6725	1345	21,520

73.

Object	Grams	Kilograms	Milligrams	Centigrams
A six-year-old boy	21,000	21	21,000,000	2,100,000

77. $$\begin{array}{r} 64\text{ lb }\ 8\text{ oz}\\ -28\text{ lb }10\text{ oz}\\ \hline \end{array}\qquad \begin{array}{r} 63\text{ lb }24\text{ oz}\\ -28\text{ lb }10\text{ oz}\\ \hline 35\text{ lb }14\text{ oz} \end{array}$$

Elizabeth's zucchini was 35 lb 14 oz lighter than the record weight.

81. $$\begin{aligned} 1\text{ lb }10\text{ oz} + 3\text{ lb }14\text{ oz} = 4\text{ lb }24\text{ oz} &= 4\text{ lb} + 1\text{ lb }8\text{ oz}\\ &= 5\text{ lb }\ 8\text{ oz} \end{aligned}$$

He has a total of 5 lb 8 oz of rice.

85. $3 \times 16 = 48$

3 cartons contain 48 boxes of fruit.

$3\text{ mg} \times 48 = 144\text{ mg}$

3 cartons contain 144 mg of preservatives.

89. $$\begin{aligned} 3\text{ lb }\ 4\text{ oz} \times 10 &= 30\text{ lb }\ 40\text{ oz}\\ &= 30\text{ lb} + 2\text{ lb }\ 8\text{ oz}\\ &= 32\text{ lb }\ 8\text{ oz} \end{aligned}$$

Each box weight 32 lb 8 oz.

$$\begin{aligned} 32\text{ lb }\ 8\text{ oz} \times 4 &= 128\text{ lb }\ 32\text{ oz}\\ &= 128\text{ lb} + 2\text{ lb}\\ &= 130\text{ lb} \end{aligned}$$

4 boxes of meat weight 130 lb.

93. $\frac{4}{25} = \frac{4}{25}\cdot\frac{4}{4} = \frac{16}{100} = 0.16$

97. No, a pill containing 2 kg of medication is not reasonable.

101. No, a professor weighing less than 150 g is not reasonable.

105. True, 1 kilogram is 1000 grams.

Exercise Set 5.6

1. $32\text{ fl oz} = \frac{32\text{ fl oz}}{1}\cdot\frac{1\text{ c}}{8\text{ fl oz}} = \frac{32}{8}\text{ c} = 4\text{ c}$

5. $14\text{ qt} = \frac{14\text{ qt}}{1}\cdot\frac{1\text{ gal}}{4\text{ qt}} = \frac{14}{4}\text{ gal} = 3\frac{1}{2}\text{ gal}$

9. $2\text{ qt} = \frac{2\text{ qt}}{1}\cdot\frac{2\text{ pt}}{1\text{ qt}}\cdot\frac{2\text{ c}}{1\text{ pt}} = 2\cdot 2\cdot 2\text{ c} = 8\text{ c}$

13. $42\text{ cups} = \frac{42\text{ cups}}{1}\cdot\frac{1\text{ pt}}{2\text{ cups}}\cdot\frac{1\text{ qt}}{2\text{ pt}} = \frac{42}{4}\text{ qt} = 10\frac{1}{2}\text{ qt}$

17. $$\begin{aligned} 5\text{ gal }3\text{ qt} &= \frac{5\text{ gal}}{1}\cdot\frac{4\text{ qt}}{1\text{ gal}} + 3\text{ qt}\\ &= 5\cdot 4\text{ qt} + 3\text{ qt}\\ &= 20\text{ qt} + 3\text{ qt}\\ &= 23\text{ qt} \end{aligned}$$

21. $$\begin{aligned} 58\text{ qt} &= 56\text{ qt} + 2\text{ qt}\\ &= \frac{56\text{ qt}}{1}\cdot\frac{1\text{ gal}}{4\text{ qt}} + 2\text{ qt}\\ &= \frac{56}{4}\text{ gal} + 2\text{ qt}\\ &= 14\text{ gal }2\text{ qt} \end{aligned}$$

25. $$\begin{aligned} 2\tfrac{3}{4}\text{ gal} &= \frac{11}{4}\text{ gal}\\ &= \frac{\frac{11}{4}\text{ gal}}{1}\cdot\frac{4\text{ qt}}{1\text{ gal}}\cdot\frac{2\text{ pt}}{1\text{ qt}}\\ &= \frac{11}{4}\cdot 4\cdot 2\text{ pt}\\ &= 22\text{ pt} \end{aligned}$$

29. $$\begin{aligned} 1\text{ c }5\text{ fl oz} + 2\text{ c }7\text{ fl oz} &= 3\text{ c }12\text{ fl oz}\\ &= 3\text{ c} + 1\text{ c }\ 4\text{ fl oz}\\ &= 4\text{ c }4\text{ fl oz} \end{aligned}$$

33. $$\begin{array}{r} 3\text{ gal }1\text{ qt}\\ -\qquad 1\text{ qt }1\text{ pt}\\ \hline \end{array}\qquad \begin{array}{r} 2\text{ gal }5\text{ qt}\\ -\qquad 1\text{ qt }1\text{ pt}\\ \hline \end{array}\qquad \begin{array}{r} 2\text{ gal }4\text{ qt }2\text{ pt}\\ -\qquad 1\text{ qt }1\text{ pt}\\ \hline 2\text{ gal }3\text{ qt }1\text{ pt} \end{array}$$

37. $$\begin{aligned} 9\text{ gal }2\text{ qt} \div 2 &= (8\text{ gal }4\text{ qt} + 2\text{ qt}) \div 2\\ &= 8\text{ gal }6\text{ qt} \div 2\\ &= 4\text{ gal }3\text{ qt} \end{aligned}$$

41. $0.16\text{ L} = \frac{0.16\text{ L}}{1}\cdot\frac{1\text{ kl}}{1000\text{ L}} = \frac{0.16}{1000}\text{ kl} = 0.00016\text{ kl}$

45. $3.2\text{ L} = \frac{3.2\text{ L}}{1}\cdot\frac{100\text{ cl}}{1\text{ L}} = 3.2\cdot 100\text{ cl} = 320\text{ cl}$

49. $64\text{ ml} = \frac{64\text{ ml}}{1}\cdot\frac{1\text{ L}}{1000\text{ ml}} = \frac{64}{1000}\text{ L} = 0.064\text{ L}$

53. $3.6\text{ L} = \frac{3.6\text{ L}}{1}\cdot\frac{1000\text{ ml}}{1\text{ L}} = 3.6\cdot 1000\text{ ml} = 3600\text{ ml}$

57. $2700\text{ ml} + 1.8\text{ L} = 2700\text{ ml} + 1800\text{ ml} = 4500\text{ ml}$
or
$2700\text{ ml} + 1.8\text{ L} = 2.7\text{ L} + 1.8\text{ L} = 4.5\text{ L}$

61. $17{,}500\text{ ml} - 0.9\text{ L} = 17{,}500\text{ ml} - 900\text{ ml} = 16{,}600\text{ ml}$
or
$17{,}500\text{ ml} - 0.9\text{ L} = 17.5\text{ L} - 0.9\text{ L} = 16.6\text{ L}$

65. $$\begin{aligned} 81.2\text{ L} \div 0.5 &= 81.2\text{ L} \div \frac{1}{2} \\ &= 81.2\text{ L} \cdot 2 \\ &= 162.4\text{ L} \end{aligned}$$

69.

Capacity	Cups	Gallons	Quarts	Pints
Your kidneys filter about this amount of blood every minute	4	$\frac{1}{4}$	1	2

73. $354\text{ ml} + 18.6\text{ L} = 0.354\text{ L} + 18.6\text{ L} = 18.954\text{ L}$
The total amount of gasoline in the tank is 18.954 liters.

77. $$\begin{aligned} 5\text{ pt }1\text{ c} + 2\text{ pt }1\text{ c} &= 7\text{ pt }2\text{ c} \\ &= 7\text{ pt} + 1\text{ pt} \\ &= 8\text{ pt} \\ &= \frac{8\text{ pt}}{1} \cdot \frac{1\text{ qt}}{2\text{ pt}} \\ &= \frac{8}{2}\text{ qt} \\ &= \frac{4\text{ qt}}{1} \cdot \frac{1\text{ gal}}{4\text{ qt}} \\ &= \frac{4}{4}\text{ gal} \\ &= 1\text{ gal} \end{aligned}$$
Yes, the liquid can be poured into the container without causing it to overflow.

81. $\frac{20}{25} = \frac{4 \cdot 5}{5 \cdot 5} = \frac{4}{5}$

85. $\frac{72}{80} = \frac{8 \cdot 9}{8 \cdot 10} = \frac{9}{10}$

89. No, a bathtub filled with 3000 ml of water is not reasonable.

93. answers may vary

97. B indicates 1.5 cc.

101. B indicates 54 u or 0.54 cc.

Exercise Set 5.7

1. $756\text{ ml} \approx \frac{756\text{ ml}}{1} \cdot \frac{1\text{ fl oz}}{29.57\text{ ml}} \approx 25.57\text{ fl oz}$

5. $1000\text{ g} \approx \frac{1000\text{ g}}{1} \cdot \frac{0.04\text{ oz}}{1\text{ g}} \approx 40\text{ oz}$

9. $14.5\text{ L} \approx \frac{14.5\text{ L}}{1} \cdot \frac{0.26\text{ gal}}{1\text{ L}} \approx 3.77\text{ gal}$

13.

	Meters	Yards	Centimeters	Feet	Inches
The height of a woman	1.5	$1\frac{2}{3}$	150	5	60

17. $10\text{ cm} = \frac{10\text{ cm}}{1} \cdot \frac{1\text{ in.}}{2.54\text{ cm}} \approx 3.94\text{ in.}$
The balance beam is approximately 3.94 inches wide.

21. $200\text{ mg} = 0.2\text{ g} \approx \frac{0.2\text{ g}}{1} \cdot \frac{0.04\text{ oz}}{1\text{ g}} \approx 0.008\text{ oz}$

25. $$\begin{aligned} 16\text{ billion km} &\approx \frac{16\text{ billion km}}{1} \cdot \frac{0.62\text{ mi}}{1\text{ km}} \\ &\approx 9.92\text{ billion mi} \end{aligned}$$

29. $$\begin{aligned} 4500\text{ km} &\approx \frac{4500\text{ km}}{1} \cdot \frac{0.62\text{ mi}}{1\text{ km}} \\ &\approx 2790\text{ mi} \end{aligned}$$

33. $1.5\text{ lb} - 1.25\text{ lb} = 0.25\text{ lb}$
$0.25\text{ lb} \approx \frac{0.25\text{ lb}}{1} \cdot \frac{0.45\text{ kg}}{1\text{ lb}} \cdot \frac{1000\text{ g}}{1\text{ kg}} \approx 112.5\text{ g}$
The difference is approximately 112.5 g.

37. $8\text{ m} \approx \frac{8\text{ m}}{1} \cdot \frac{3.28\text{ ft}}{1\text{ m}} \approx 26.24\text{ ft}$
The base diameter is approximately 26.24 ft.

41. One dose every 4 hours results in $\frac{24}{4} = 6$ doses per day and $6 \times 7 = 42$ doses per week.
$5\text{ ml} \times 42 = 210\text{ ml}$
$210\text{ ml} \approx \frac{210\text{ ml}}{1} \cdot \frac{1\text{ fl oz}}{29.57\text{ ml}} \approx 7.1\text{ fl oz}$
8 fluid ounces of medicine should be purchased.

45. A liter has greater capacity than a quart; **b**.

49. An $8\frac{1}{2}$-ounce glass of water has a capacity of about $250\text{ ml}\left(\frac{1}{4}\text{ L}\right)$; **d**.

53. $6 \cdot 4 + 5 \div 1 = 24 + 5 = 29$

57. $$\begin{aligned} 3 + 5(19 - 17) - 8 &= 3 + 5(2) - 8 \\ &= 3 + 10 - 8 \\ &= 13 - 8 \\ &= 5 \end{aligned}$$

61. $\text{BSA} = \sqrt{\frac{90 \times 182}{3600}} \approx 2.13$
The BSA is approximately 2.13 sq m.

65. $60\text{ in.} = \frac{60\text{ in.}}{1} \cdot \frac{2.54\text{ cm}}{1\text{ in.}} = 152.4\text{ cm}$
$150\text{ lb} \approx \frac{150\text{ lb}}{1} \cdot \frac{0.45\text{ kg}}{1\text{ lb}} \approx 67.5\text{ kg}$
$\text{BSA} \approx \sqrt{\frac{67.5 \times 152.4}{3600}} \approx 1.690$
The BSA is approximately 1.69 sq m.

69. $20\text{ m} \times 40\text{ m} = 800\text{ sq m}$
$20\text{ m} \approx \frac{20\text{ m}}{1} \cdot \frac{3.28\text{ ft}}{1\text{ m}} \approx 65.6\text{ ft}$
$40\text{ m} \approx \frac{40\text{ m}}{1} \cdot \frac{3.28\text{ ft}}{1\text{ m}} \approx 131.2\text{ ft}$
$20\text{ m} \times 40\text{ m} \approx 65.6\text{ ft} \times 131.2\text{ ft} \approx 8606.72\text{ sq ft}$
The area is 800 sq m or approximately 8606.72 sq ft.

Chapter 5 Test

1. $\frac{\$75}{\$10} = \frac{75}{10} = \frac{5 \cdot 15}{5 \cdot 2} = \frac{15}{2}$

5. $\frac{591\text{ ft}}{186\text{ ft}} = \frac{591}{186} = \frac{3 \cdot 197}{3 \cdot 62} = \frac{197}{62}$

9.
$$\begin{array}{r} 0.148 \approx 0.15 \\ 8\overline{)\,1.190} \\ -8 \\ \hline 39 \\ -32 \\ \hline 70 \\ -64 \\ \hline 6 \end{array}$$

The 8-oz size costs approximately \$0.15/ounce.

$$\begin{array}{r} 0.157 \approx 0.16 \\ 12\overline{)\,1.890} \\ -1\,2 \\ \hline 69 \\ -60 \\ \hline 90 \\ -84 \\ \hline 6 \end{array}$$

The 12-ounce size costs approximately \$0.16/ounce. The 8-oz size is the better buy.

13.
$$\begin{aligned} \frac{-1.5}{5} &= \frac{2.4}{n} \\ -1.5 \cdot n &= 5 \cdot 2.4 \\ -1.5 \cdot n &= 12 \\ n &= \frac{12}{-1.5} \\ n &= -8 \end{aligned}$$

17. Let n represent the standard dose.

$$\begin{aligned} \text{dose} \rightarrow \frac{10}{15} &= \frac{n}{80} \leftarrow \text{dose} \\[-2pt] \text{pounds} \rightarrow \phantom{\frac{1}{1}} & \phantom{\frac{1}{1}} \leftarrow \text{pounds} \\ 10 \cdot 80 &= 15 \cdot n \\ 800 &= 15 \cdot n \\ \frac{800}{15} &= n \\ \frac{160}{3} &= n \\ 53\frac{1}{3} &= n \end{aligned}$$

The standard dose for a dog weighing 80 pounds is $53\frac{1}{3}$ grams.

21. $40 \text{ mg} = \dfrac{40 \text{ mg}}{1} \cdot \dfrac{1 \text{ g}}{1000 \text{ mg}} = \dfrac{40}{1000} \text{ g} = 0.04 \text{ g}$

25. 5 gal 2 qt ÷ 2
= 22 qt ÷ 2
= 11 qt
= 2 gal 3 qt

29. 88 m + 340 cm = 88 m + 3.40 m = 91.4 m
The span is 91.4 meters

33. $5 \text{ km} \approx \dfrac{5 \text{ km}}{1} \cdot \dfrac{0.62 \text{ mi}}{1 \text{ km}} \approx 3.1 \text{ mi}$
5 km is about 3.1 mi.

Chapter 6

Exercise Set 6.1

1. $96 \text{ out of } 100 = \dfrac{96}{100} = 96\%$
96% of college students use the Internet.

5. The largest sector corresponds to football, so football was preferred by most adults.
$\dfrac{37}{100} = 37\%$
37% percent of adults prefer football.

9. $41\% = 41(0.01) = 0.\underset{\curvearrowleft}{41}$

13. $100\% = 100(0.01) = 1.\underset{\curvearrowleft}{00}$ or 1

17. $2.8\% = 2.8(0.01) = 0.\underset{\curvearrowleft}{02}8$

21. $300\% = 300(0.01) = 3.\underset{\curvearrowleft}{00}$ or 3

25. $12\% = 12 \cdot \dfrac{1}{100} = \dfrac{12}{100} = \dfrac{3 \cdot 4}{4 \cdot 25} = \dfrac{3}{25}$

29. $4.5\% = 4.5 \cdot \dfrac{1}{100} = \dfrac{4.5}{100} = \dfrac{4.5 \cdot 10}{100 \cdot 10} = \dfrac{45}{1000} = \dfrac{5 \cdot 9}{5 \cdot 200} = \dfrac{9}{200}$

33. $6.25\% = 6.25 \cdot \dfrac{1}{100} = \dfrac{6.25}{100} = \dfrac{6.25 \cdot 100}{100 \cdot 100} = \dfrac{625}{10{,}000}$
$= \dfrac{1 \cdot 625}{16 \cdot 625} = \dfrac{1}{16}$

37. $22\frac{3}{8}\% = 22\frac{3}{8} \cdot \dfrac{1}{100} = \dfrac{179}{8} \cdot \dfrac{1}{100} = \dfrac{179}{800}$

41. $0.22 = 0.22(100\%) = 22\%$

45. $0.056 = 0.056(100\%) = 5.6\%$

49. $3 = 3(100\%) = 300\%$

53. $\dfrac{7}{10} = \dfrac{7}{10} \cdot 100\% = \dfrac{7}{10} \cdot \dfrac{100}{1}\% = \dfrac{700}{10}\% = 70\%$

57. $\dfrac{17}{50} = \dfrac{17}{50} \cdot 100\% = \dfrac{17}{50} \cdot \dfrac{100}{1}\% = \dfrac{1700}{50}\% = 34\%$

61. $\dfrac{7}{9} = \dfrac{7}{9} \cdot 100\% = \dfrac{7}{9} \cdot \dfrac{100}{1}\% = \dfrac{700}{9}\% = 77\frac{7}{9}\%$

65. $1\frac{9}{10} = 1\frac{9}{10} \cdot 100\% = \dfrac{19}{10} \cdot \dfrac{100}{1}\% = \dfrac{1900}{10}\% = 190\%$

69. $\dfrac{4}{15} = \dfrac{4}{15} \cdot 100\% = \dfrac{4}{15} \cdot \dfrac{100}{1}\% = \dfrac{400}{15}\% = \dfrac{80}{3}\%$

$$\begin{array}{r} 26.666 \\ 3\overline{)\,80.000} \\ -6 \\ \hline 20 \\ -18 \\ \hline 2\,0 \\ -1\,8 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

$\dfrac{4}{15} \approx 26.67\%$

73.

Percent	Decimal	Fraction
40%	$40\% = 0.40 = 0.4$	$40\% = \frac{40}{100} = \frac{2}{5}$
$0.235 = 0.235(100\%)$ $= 23.5\%$	0.235	$0.235 = \frac{235}{1000}$ $= \frac{47}{200}$
$\frac{4}{5} = \frac{4}{5} \cdot 100\%$ $= \frac{400}{5}\%$ $= 80\%$	$\frac{4}{5} = \frac{80}{100} = 80\%$	$\frac{4}{5}$
$33\frac{1}{3}\%$	$33\frac{1}{3}\% = \frac{100}{3} \cdot 0.01$ $= 33.\overline{3} \cdot 0.01$ $= 0.33\overline{3}$	$33\frac{1}{3}\% = 33\frac{1}{3} \cdot \frac{1}{100}$ $= \frac{100}{3} \cdot \frac{1}{100}$ $= \frac{1}{3}$
$\frac{7}{8} = \frac{7}{8} \cdot 100\%$ $= \frac{700}{8}\%$ $= 87.5\%$	$\frac{7}{8} = \frac{7 \cdot 125}{8 \cdot 125}$ $= \frac{875}{1000}$ $= 0.875$	$\frac{7}{8}$
7.5%	$7.5\% = 7.5(0.01)$ $= 0.075$	$7.5\% = \frac{7.5}{100}$ $= \frac{75}{1000}$ $= \frac{3}{40}$

77. $66\% = 66(0.01) = 0.66$

$66\% = 66 \cdot \frac{1}{100} = \frac{66}{100} = \frac{33}{50}$

81. $\frac{12}{25} = \frac{12}{25} \cdot 100\% = \frac{12}{25} \cdot \frac{100}{1}\% = \frac{1200}{25}\% = 48\%$

85. $\frac{9}{50} = \frac{9}{50} \cdot 100\% = \frac{9}{50} \cdot \frac{100}{1}\% = \frac{900}{50}\% = 18\%$

89. $16.2\% = 16.2(0.01) = 0.162$

$16.2\% = \frac{16.2}{1} \cdot \frac{1}{100} = \frac{16.2}{100} = \frac{162}{1000} = \frac{81}{500}$

93. $3 \cdot n = 45$

$n = \frac{45}{3}$

$n = 15$

97. $0.7682 = 0.7682(100\%) = 76.82\% \approx 77\%$

101. **a.** 52.8647% rounded to the nearest tenth of a percent is 52.9%.

b. 52.8647% rounded to the nearest hundredth of a percent is 52.86%.

105. If the percentages of the four blood types are added, the total will be 100%, since the blood types cover the whole population.

$45\% + 40\% + 11\% = (45 + 40 + 11)\% = 96\%$

$100\% - 96\% = (100 - 96)\% = 4\%$

4% of the U.S. population has the blood type AB.

109. $30\% = 30(0.01) = 0.30$

113. 4 of the 5 equal parts are shaded, so $\frac{4}{5}$ of the figure is shaded.

$\frac{4}{5} = \frac{4}{5} \cdot 100\% = \frac{4}{5} \cdot \frac{100}{1}\% = \frac{400}{5}\% = 80\%$

80% of the figure is shaded.

117. answers may vary

121.

$$\begin{array}{r} 1.1548 \\ 736\overline{)\,850.0000} \\ -736 \\ \hline 114\,0 \\ -73\,6 \\ \hline 40\,40 \\ -36\,80 \\ \hline 3\,600 \\ -2\,944 \\ \hline 6560 \\ -5888 \\ \hline 672 \end{array}$$

$\frac{850}{736} \approx 1.155 \quad \text{or} \quad 115.5\%$

Exercise Set 6.2

1.
$$\begin{array}{ccccc} 18\% & \text{of} & 81 & \text{is} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 18\% & \cdot & 81 & = & n \end{array}$$

5.
$$\begin{array}{ccccc} 0.6 & \text{is} & 40\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.6 & = & 40\% & \cdot & n \end{array}$$

9.
$$\begin{array}{ccccc} \underbrace{\text{What number}} & \text{is} & 9\% & \text{of} & 43? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & = & 9\% & \cdot & 43 \end{array}$$

13. $10\% \cdot 35 = n$

$0.10 \cdot 35 = n$

$3.5 = n$

10% of 35 is 3.5.

17. $1.2 = 12\% \cdot n$

$1.2 = 0.12 \cdot n$

$\frac{1.2}{0.12} = n$

$10 = n$

1.2 is 12% of 10.

21. $n \cdot 80 = 88$

$n = \frac{88}{80}$

$n = 1.1$

$n = 110\%$

88 is 110% of 80.

25. $0.1 = 10\% \cdot n$

$0.1 = 0.10 \cdot n$

$\frac{0.1}{0.1} = n$

$1 = n$

0.1 is 10% of 1.

29. $82.5 = 16\frac{1}{2}\% \cdot n$

$82.5 = 0.165 \cdot n$

$\frac{82.5}{0.165} = n$

$500 = n$

82.5 is $16\frac{1}{2}\%$ of 500.

33. $n = 42\% \cdot 60$

$n = 0.42 \cdot 60$

$n = 25.2$

25.2 is 42% of 60.

37. $$\begin{aligned} 120\% \cdot n &= 42 \\ 1.20 \cdot n &= 42 \\ n &= \frac{42}{1.2} \\ n &= 35 \end{aligned}$$
120% of 35 is 42.

41. $$\begin{aligned} n \cdot 600 &= 3 \\ n &= \frac{3}{600} \\ n &= 0.005 \\ n &= 0.5\% \end{aligned}$$
0.5% of 600 is 3.

45. $$\begin{aligned} 1575 &= n \cdot 2500 \\ \frac{1575}{2500} &= n \\ 0.63 &= n \\ 63\% &= n \end{aligned}$$
1575 is 63% of 2500.

49. $$\begin{aligned} \frac{27}{n} &= \frac{9}{10} \\ 27 \cdot 10 &= 9 \cdot n \\ 270 &= 9 \cdot n \\ \frac{270}{9} &= n \\ 30 &= n \end{aligned}$$

53. $\dfrac{17}{12} = \dfrac{n}{20}$

57. In the equation $5 \cdot n = 32$, the step that should be taken to find the value of n is to divide by 5, obtaining $n = \dfrac{32}{5}$, which is choice **c**.

61. $20\% \cdot n = 18.6$ in words is "twenty percent of some number is eighteen and six tenths."

65. Since 85 is less than 120, the percent is less than 100%; **c**.

69. Since 100% is 1, 100% of 45 is equal to 45; **a**.

73. answers may vary

77. $$\begin{aligned} 22{,}113 &= 180\% \cdot n \\ 22{,}113 &= 1.80 \cdot n \\ \frac{22{,}113}{1.8} &= n \\ 12{,}285 &= n \end{aligned}$$
22,113 is 180% of 12,285.

Exercise Set 6.3

1. $$\underset{\text{percent}}{\underset{\downarrow}{98\%}} \quad \text{of} \quad \underset{\text{base}}{\underset{\downarrow}{45}} \quad \text{is} \quad \underset{\text{amount} = a}{\underset{\downarrow}{\underbrace{\text{what number}}}}?$$
$$\frac{a}{45} = \frac{98}{100}$$

5. $$\underset{\text{amount}}{\underset{\downarrow}{14.3}} \quad \text{is} \quad \underset{\text{percent}}{\underset{\downarrow}{26\%}} \quad \text{of} \quad \underset{\text{base} = b}{\underset{\downarrow}{\underbrace{\text{what number}}}}\ ?$$
$$\frac{14.3}{b} = \frac{26}{100}$$

9. $$\underset{\text{percent} = p}{\underset{\downarrow}{\underbrace{\text{What percent}}}} \quad \text{of} \quad \underset{\text{base}}{\underset{\downarrow}{400}} \quad \text{is} \quad \underset{\text{amount}}{\underset{\downarrow}{70?}}$$
$$\frac{70}{400} = \frac{p}{100}$$

13. $\dfrac{a}{65} = \dfrac{40}{100}$ or $\dfrac{a}{65} = \dfrac{2}{5}$
$$\begin{aligned} a \cdot 5 &= 65 \cdot 2 \\ 5 \cdot a &= 130 \\ a &= \frac{130}{5} \\ a &= 26 \end{aligned}$$
40% of 65 is 26.

17. $\dfrac{90}{b} = \dfrac{15}{100}$ or $\dfrac{90}{b} = \dfrac{3}{20}$
$$\begin{aligned} 90 \cdot 20 &= b \cdot 3 \\ 1800 &= b \cdot 3 \\ \frac{1800}{3} &= b \\ 600 &= b \end{aligned}$$
15% of 600 is 90.

21. $\dfrac{42}{35} = \dfrac{p}{100}$ or $\dfrac{6}{5} = \dfrac{p}{100}$
$$\begin{aligned} 6 \cdot 100 &= 5 \cdot p \\ 600 &= 5 \cdot p \\ \frac{600}{5} &= p \\ 120 &= p \end{aligned}$$
42 is 120% of 35.

25. $\dfrac{3.7}{b} = \dfrac{10}{100}$ or $\dfrac{3.7}{b} = \dfrac{1}{10}$
$$\begin{aligned} 3.7 \cdot 10 &= b \cdot 1 \\ 37 &= b \end{aligned}$$
3.7 is 10% of 37.

29. $\dfrac{160}{b} = \dfrac{16}{100}$ or $\dfrac{160}{b} = \dfrac{4}{25}$
$$\begin{aligned} 160 \cdot 25 &= b \cdot 4 \\ 4000 &= b \cdot 4 \\ \frac{4000}{4} &= b \\ 1000 &= b \end{aligned}$$
160 is 16% of 1000.

33. $\dfrac{a}{62} = \dfrac{89}{100}$
$$\begin{aligned} a \cdot 100 &= 62 \cdot 89 \\ a \cdot 100 &= 5518 \\ a &= \frac{5518}{100} \\ a &= 55.18 \end{aligned}$$
55.18 is 89% of 62.

37. $\dfrac{105}{b} = \dfrac{140}{100}$ or $\dfrac{105}{b} = \dfrac{7}{5}$
$$\begin{aligned} 105 \cdot 5 &= b \cdot 7 \\ 525 &= b \cdot 7 \\ \frac{525}{7} &= b \\ 75 &= b \end{aligned}$$
140% of 75 is 105.

41. $\dfrac{4}{800} = \dfrac{p}{100}$ or $\dfrac{1}{200} = \dfrac{p}{100}$
$$\begin{aligned} 1 \cdot 100 &= 200 \cdot p \\ 100 &= 200 \cdot p \\ \frac{100}{200} &= p \\ 0.5 &= p \end{aligned}$$
0.5% of 800 is 4.

45. $\dfrac{a}{48} = \dfrac{20}{100}$ or $\dfrac{a}{48} = \dfrac{1}{5}$
$$\begin{aligned} a \cdot 5 &= 48 \cdot 1 \\ a \cdot 5 &= 48 \\ a &= \frac{48}{5} \\ a &= 9.6 \end{aligned}$$
20% of 48 is 9.6.

49. $-\frac{11}{16}+\left(-\frac{3}{16}\right)=\frac{-11+(-3)}{16}=\frac{-14}{16}=-\frac{7\cdot 2}{8\cdot 2}=-\frac{7}{8}$

53.
$$\begin{array}{r} 0.\overset{1}{4}1 \\ \underline{+0.29} \\ 0.70 = 0.7 \end{array}$$

57. answers may vary

61.
$$\begin{aligned} \frac{p}{100} &= \frac{13}{52} \\ \frac{25}{100} &\overset{?}{=} \frac{13}{52} \\ \frac{1}{4} &= \frac{1}{4} \quad \text{True} \end{aligned}$$

Yes, the percent is 25.

65.
$$\begin{aligned} \frac{a}{53{,}862} &= \frac{22.3}{100} \\ a\cdot 100 &= 22.3\cdot 53{,}862 \\ a\cdot 100 &= 1{,}201{,}122.6 \\ a &= \frac{1{,}201{,}122.6}{100} \\ a &\approx 12{,}011.2 \end{aligned}$$

Exercise Set 6.4

1. 24 is 1.5% of what number?

Method 1:
$$\begin{aligned} 24 &= 1.5\%\cdot n \\ 24 &= 0.015\cdot n \\ \frac{24}{0.015} &= n \\ 1600 &= n \end{aligned}$$

1600 bolts were inspected.

Method 2:
$$\begin{aligned} \frac{24}{b} &= \frac{1.5}{100} \\ 24\cdot 100 &= 1.5\cdot b \\ 2400 &= 1.5\cdot b \\ \frac{2400}{1.5} &= b \\ 1600 &= b \end{aligned}$$

1600 bolts were inspected.

5. 378 is what percent of 2700?

Method 1:
$$\begin{aligned} 378 &= n\cdot 2700 \\ \frac{378}{2700} &= n \\ 0.14 &= n \\ 14\% &= n \end{aligned}$$

The student spent 14% of last semester's college costs on books.

Method 2:
$$\begin{aligned} \frac{378}{2700} &= \frac{p}{100} \\ 378\cdot 100 &= 2700\cdot p \\ 37{,}800 &= 2700\cdot p \\ \frac{37{,}800}{2700} &= p \\ 14 &= p \end{aligned}$$

The student spent 14% of last semester's college costs on books.

9. 14,250 is what percent of 36,900?

Method 1:
$$\begin{aligned} 14{,}250 &= n\cdot 36{,}900 \\ \frac{14{,}250}{36{,}900} &= n \\ 0.386 &\approx n \\ 38.6\% &\approx n \end{aligned}$$

38.6% of McDonald's restaurants were in the United States in 2016.

Method 2:
$$\begin{aligned} \frac{14{,}250}{36{,}900} &= \frac{p}{100} \\ 14{,}250\cdot 100 &= 36{,}900\cdot p \\ 1{,}425{,}000 &= 36{,}900\cdot p \\ \frac{1{,}425{,}000}{36{,}900} &= p \\ 38.6 &\approx p \end{aligned}$$

38.6% of McDonald's restaurants were in the United States in 2016.

13. What number is 43% of 41,900?

Method 1:
$$\begin{aligned} n &= 43\%\cdot 41{,}900 \\ n &= 0.43\cdot 41{,}900 \\ n &= 18{,}017 \end{aligned}$$

The number people employed as occupational therapy assistants is expected to be 41,900 + 18,017 = 59,917.

Method 2:
$$\begin{aligned} \frac{a}{41{,}900} &= \frac{43}{100} \\ a\cdot 100 &= 41{,}900\cdot 43 \\ 100\cdot a &= 1{,}801{,}700 \\ a &= \frac{1{,}801{,}700}{100} \\ a &= 18{,}017 \end{aligned}$$

The number of people employed as occupational therapy assistants is expected to be 41,900 + 18,017 = 59,917.

17. 38 is what percent of 131?

Method 1:
$$\begin{aligned} 38 &= n\cdot 131 \\ \frac{38}{131} &= n \\ 0.29 &\approx n \\ 29\% &\approx n \end{aligned}$$

29% of the ski runs at Keystone ski area are rated intermediate.

Method 2:
$$\begin{aligned} \frac{38}{131} &= \frac{p}{100} \\ 38\cdot 100 &= 131\cdot p \\ 3800 &= 131\cdot p \\ \frac{3800}{131} &= p \\ 29 &\approx p \end{aligned}$$

29% of the ski runs at Keystone ski area are rated intermediate.

21. 10 is what percent of 80?

Method 1:
$$\begin{aligned} 10 &= n\cdot 80 \\ \frac{10}{80} &= n \\ 0.125 &= n \\ 12.5\% &= n \end{aligned}$$

12.5% of the total calories come from fat.

Method 2:

$$\begin{aligned} \frac{10}{80} &= \frac{p}{100} \\ 10 \cdot 100 &= 80 \cdot p \\ 1000 &= 80 \cdot p \\ \frac{1000}{80} &= p \\ 12.5 &= p \end{aligned}$$

12.5% of the total calories come from fat.

25. 26,250 is 15% of what number?

Method 1:

$$\begin{aligned} 26{,}250 &= 15\% \cdot n \\ 26{,}250 &= 0.15 \cdot n \\ \frac{26{,}250}{0.15} &= n \\ 175{,}000 &= n \end{aligned}$$

The price of the home was $175,000.

Method 2:

$$\begin{aligned} \frac{26{,}250}{b} &= \frac{15}{100} \\ 26{,}250 \cdot 100 &= b \cdot 15 \\ 2{,}625{,}000 &= b \cdot 15 \\ \frac{2{,}625{,}000}{15} &= b \\ 175{,}000 &= b \end{aligned}$$

The price of the home was $175,000.

29. What number is 4.5% of 19,286?

Method 1:

$$\begin{aligned} n &= 4.5\% \cdot 19{,}286 \\ n &= 0.045 \cdot 19{,}286 \\ n &= 867.87 \end{aligned}$$

The price of the car will increase by $867.87. The new price of that model will be $19,286 + $867.87 = $20,153.87.

Method 2:

$$\begin{aligned} \frac{a}{19{,}286} &= \frac{4.5}{100} \\ a \cdot 100 &= 4.5 \cdot 19{,}286 \\ a \cdot 100 &= 86{,}787 \\ a &= \frac{86{,}787}{100} \\ a &= 867.87 \end{aligned}$$

The price of the car will increase by $867.87. The new price of that model will be $19,286 + $867.87 = $20,153.87.

33. 11.6% of 8994 is what number?

Method 1:

$$\begin{aligned} 11.6\%(8994) &= n \\ 0.116(8994) &= n \\ 1043.304 &= n \\ 1043 &\approx n \end{aligned}$$

The increase in tuition was $1043. The tuition for the 2016–2017 school year was $8994 + $1043 = $10,037.

Method 2:

$$\begin{aligned} \frac{a}{8994} &= \frac{11.6}{100} \\ a \cdot 100 &= 11.6 \cdot 8994 \\ a \cdot 100 &= 104{,}330.4 \\ a &= \frac{104{,}330.4}{100} \\ a &= 1043.304 \\ a &\approx 1043 \end{aligned}$$

The increase in tuition was $1043. The tuition for the 2016–2017 school year was $8994 + $1043 = $10,037.

37.

Original Amount	New Amount	Amount of Increase	Percent of Increase
50	80	80 − 50 = 30	$\frac{30}{50} = 0.6 = 60\%$

41.

Original Amount	New Amount	Amount of Decrease	Percent of Decrease
8	6	8 − 6 = 2	$\frac{2}{8} = 0.25 = 25\%$

45.
$$\begin{aligned} \text{percent of decrease} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{150 - 84}{150} \\ &= \frac{66}{150} \\ &= 0.44 \end{aligned}$$

The decrease in calories per cup is 44%.

49.
$$\begin{aligned} \text{percent of increase} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{421 - 174}{174} \\ &= \frac{247}{174} \\ &\approx 1.420 \end{aligned}$$

The increase in the size of privately owned farms in the United States was about 142.0%.

53.
$$\begin{aligned} \text{percent of increase} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{4151 \text{ thousand} - 3782 \text{ thousand}}{3782 \text{ thousand}} \\ &= \frac{369 \text{ thousand}}{3782 \text{ thousand}} \\ &= \frac{369}{3782} \\ &\approx 0.098 \end{aligned}$$

The increase in the number of elementary and secondary teachers in the United State is expected to be 9.8%

57.
$$\begin{aligned} \text{percent of increase} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{8.87 - 6.88}{6.88} \\ &= \frac{1.99}{6.88} \\ &\approx 0.289 \end{aligned}$$

The increase in the U.S. movie theater ticket price was 28.9%.

61.
$$\begin{array}{r} 0.12 \\ \times\, 38 \\ \hline 96 \\ 360 \\ \hline 4.56 \end{array}$$

65.
$$\begin{array}{r} 78.00 \\ -19.46 \\ \hline 58.54 \end{array}$$

69.
$$\begin{aligned} \text{percent of increase} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{180 - 150}{150} \\ &= \frac{30}{150} \\ &= 0.20 \end{aligned}$$

The percent of increase is 20%.

Exercise Set 6.5

1. $\text{sales tax} = 5\% \cdot \$150 = 0.05 \cdot \$150 = \$7.50$
The sales tax is \$7.50.

5. $\$335.30 = r \cdot \$4790$
$$\frac{335.30}{4790} = r$$
$$0.07 = r$$
The sales tax rate is 7%.

9. $\text{sales tax} = 6.5\% \cdot \$1800 = 0.065 \cdot \$1800 = \$117$
$\text{total price} = \$1800 + \$117 = \$1917$
The sales tax is \$117 and the total price of the bracelet is \$1917.

13. $\$98.70 = r \cdot \$1645$
$$\frac{98.70}{1645} = r$$
$$0.06 = r$$
The sales tax rate is 6%.

17. $\text{commission} = 4\% \cdot \$1{,}329{,}401 = 0.04 \cdot \$1{,}329{,}401$
$= \$53{,}176.04$
Her commission was \$53,176.04.

21. $\text{commission} = 1.5\% \cdot \$325{,}900 = 0.015 \cdot \$325{,}900$
$= \$4888.50$
His commission will be \$4888.50.

	Original Price	Discount Rate	Amount of Discount	Sale Price
25.	\$89	10%	$10\% \cdot \$89 = \$8.90$	$\$89 - \$8.90 = \$80.10$
29.	\$410	35%	$35\% \cdot \$410 = \$143.50$	$\$410 - \$143.50 = \$266.50$

33. $\text{discount} = 15\% \cdot \$300 = 0.15 \cdot \$300 = \$45$
$\text{sale price} = \$300 - \$45 = \$255$
The discount is \$45 and the sale price is \$255.

37.

Purchase Price	Tax Rate	Sales Tax	Total Price
\$56	5.5%	$5.5\% \cdot \$56 = \$3.08$	$\$56 + \$3.08 = \$59.08$

41.

Sale	Commission Rate	Commission
\$17,900	$\frac{\$1432}{\$17{,}900} = 0.08 = 8\%$	\$1432

45. $400 \cdot \frac{3}{100} \cdot 11 = 12 \cdot 11 = 132$

49. Round \$68 to \$70 and 9.5% to 10%.
$10\% \cdot \$70 = 0.10 \cdot \$70 = \$7$
$\$70 + \$7 = \$77$
The best estimate of the total price is \$77; **d**.

53.

Bill Amount	10%	15%	20%
$\$72.17 \approx \$72.00$	\$7.20	$\$7.20 + \frac{1}{2}(\$7.20) = \$7.20 + \$3.60 = \$10.80$	$2(\$7.20) = \$14.40$

57. $7.5\% \cdot \$24{,}966 = 0.075 \cdot \$24{,}966 = \$1872.45$
$\$24{,}966 + \$1872.45 = \$26{,}838.45$
The total price of the necklace is \$26,838.45.

Exercise Set 6.6

1.
$$\begin{aligned}\text{simple interest} &= \text{principal} \cdot \text{rate} \cdot \text{time}\\ &= (\$200)(8\%)(2)\\ &= (\$200)(0.08)(2) = \$32\end{aligned}$$

5.
$$\begin{aligned}\text{simple interest} &= \text{principal} \cdot \text{rate} \cdot \text{time}\\ &= (\$5000)(10\%)\left(1\frac{1}{2}\right)\\ &= (\$5000)(0.10)(1.5) = \$750\end{aligned}$$

9.
$$\begin{aligned}\text{simple interest} &= \text{principal} \cdot \text{rate} \cdot \text{time}\\ &= (\$2500)(16\%)\left(\frac{21}{12}\right)\\ &= (\$2500)(0.16)(1.75) = \$700\end{aligned}$$

13.
$$\begin{aligned}\text{simple interest} &= \text{principal} \cdot \text{rate} \cdot \text{time}\\ &= \$5000(9\%)\left(\frac{15}{12}\right)\\ &= \$5000(0.09)(1.25) = \$562.5\end{aligned}$$
$\text{Total} = \$5000 + \$562.50 = \$5562.50$

17.
$$\begin{aligned}A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t}\\ &= 6150\left(1 + \frac{0.14}{2}\right)^{2 \cdot 15}\\ &= 6150(1.07)^{30}\\ &\approx 46{,}815.37\end{aligned}$$
The total amount is \$46,815.37.

21.
$$\begin{aligned}A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t}\\ &= 10{,}000\left(1 + \frac{0.09}{2}\right)^{2 \cdot 20}\\ &= 10{,}000(1.045)^{40}\\ &\approx 58{,}163.65\end{aligned}$$
The total amount is \$58,163.65.

25.
$$\begin{aligned}A &= P\left(1 + \frac{r}{n}\right)^{n \cdot t}\\ &= 2000\left(1 + \frac{0.08}{1}\right)^{1 \cdot 5}\\ &= 2000(1.08)^{5}\\ &\approx 2938.66\end{aligned}$$
The total amount is \$2938.66.

29.
$$\begin{aligned}\text{monthly payment} &= \frac{\text{principal} + \text{interest}}{\text{number of payments}}\\ &= \frac{\$1500 + \$61.88}{6}\\ &= \frac{\$1561.88}{6}\\ &\approx \$260.31\end{aligned}$$
The monthly payment is \$260.31.

33. $\text{perimeter} = 10 + 6 + 10 + 6 = 32$
The perimeter is 32 yards.

37. answers may vary

Chapter 6 Test

1. $85\% = 85(0.01) = 0.85$

5. $6.1 = 6.1(100\%) = 610\%$

9. $0.2\% = \frac{0.2}{100} = \frac{2}{1000} = \frac{1}{500}$

13. What number is 42% of 80?

Method 1:

$n = 42\% \cdot 80$

$n = 0.42 \cdot 80$

$n = 33.6$

33.6 is 42% of 80.

Method 2:

$$\frac{a}{80} = \frac{42}{100}$$
$$a \cdot 100 = 80 \cdot 42$$
$$a \cdot 100 = 3360$$
$$a = \frac{3360}{100}$$
$$a = 33.60$$

33.6 is 42% of 80.

17. 20% of what number is 11,350?

Method 1:

$$20\% \cdot n = 11{,}350$$
$$0.20 \cdot n = 11{,}350$$
$$n = \frac{11{,}350}{0.20}$$
$$n = 56{,}750$$

The total value of his potential crop was $56,750.

Method 2:

$$\frac{11{,}350}{b} = \frac{20}{100}$$
$$\frac{11{,}350}{b} = \frac{1}{5}$$
$$11{,}350 \cdot 5 = b \cdot 1$$
$$56{,}750 = b$$

The total value of his potential crop was $56,750.

21. commission $= 4\% \cdot \$9875 = 0.04 \cdot \$9875 = \$395$

His commission is $395.

25.
$$\begin{aligned} \text{simple interest} &= \text{principal} \cdot \text{rate} \cdot \text{time} \\ &= (\$400)(13.5\%)\left(\frac{6}{12}\right) \\ &= (\$400)(0.135)(0.5) \\ &= \$27.00 \end{aligned}$$

Total amount due the bank $= \$400 + \$27 = \$427$.

Chapter 7

Exercise Set 7.1

1. Kansas has the greatest number of wheat icons, so the greatest acreage of wheat was planted in the state of Kansas.

5. The states with fewer than 3 wheat icons are South Dakota, Colorado, and Washington, so the states of South Dakota, Colorado, and Washington plant less than 3 million acres in wheat.

9. The year 2013 has 4 flames and each flame represents 12,000 wildfires, so there were approximately $4(12{,}000) = 48{,}000$ wildfires in 2013.

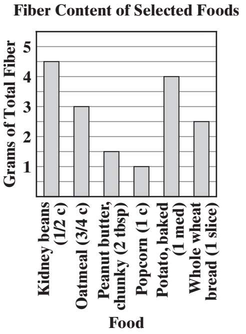
13. 2012 has 5.5 flames and 2013 has 4 flames, which is 1.5 less. Thus, the decrease in the number of wildfires from 2012 to 2013 was about $1.5(12{,}000) = 18{,}000$.

17. The longest bar corresponds to September, so the month in which most hurricanes made landfall is September.

21. Three of the 54 hurricanes that made landfall in October did so in 2005. The fraction is $\frac{3}{54} = \frac{1}{18}$.

25. The only bar corresponding to a city in the United States is the bar for New York City. The population is approximately 21.4 million or 21,400,000.

29.



33. The height of the bar for 100–149 miles per week is 15, so 15 of the adults drive 100–149 miles per week.

37. 15 of the adults drive 100–149 miles per week and 9 of the adults drive 150–199 miles per week, so $15 + 9 = 24$ of the adults drive 100–199 miles per week.

41. 9 of the 100 adults surveyed drive 150–199 miles per week, so the ratio is $\frac{9}{100}$.

45. According to the histogram, the population of 20- to 44-year-olds is projected to be 109 million in 2020.

49. answers may vary

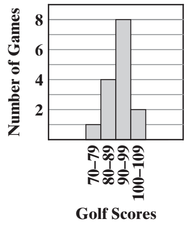
53.

Class Interval (Scores)	Tally	Class Frequency (Number of Games)
90–99	𝍸 |||	8

57.

Class Interval (Account Balances)	Tally	Class Frequency (Number of People)
$200–$299	𝍸 |	6

61.



65. The highest point on the graph corresponds to 2009, so the average number of goals per game was the highest in 2009.

69. The dots for 2007 and 2013 are below the 8-level, so the average number of goals per game was less than 8 in 2007 and 2013.

73. 10% of 62 is $0.10 \cdot 62 = 6.2$

77. $\frac{17}{50} = \frac{17}{50} \cdot 100\% = \frac{17 \cdot 2 \cdot 50}{50}\% = 34\%$

81. The lowest point on the graph of low temperatures corresponds to Sunday. The low temperature on Sunday was 68°F.

85. answers may vary

Exercise Set 7.2

1. The largest sector corresponds to the category "parent or guardian's home," so most of the students live in a parent or guardian's home.

5. 180 of the students live in campus housing while 320 live in a parent or guardian's home.

$\frac{180}{320} = \frac{9}{16}$

The ratio is $\frac{9}{16}$.

9. $30\% + 7\% = 37\%$

37% of the land on Earth is accounted for by Europe and Asia.

13. Australia accounts for 5% of the land on Earth.

$5\% \text{ of } 57{,}000{,}000 = 0.05 \cdot 57{,}000{,}000$

$= 2{,}850{,}000$

Australia is 2,850,000 square miles.

17. The second-largest sector corresponds to nonfiction, so the second-largest category of books is nonfiction.

21. Children's fiction accounts for 22% of the books.

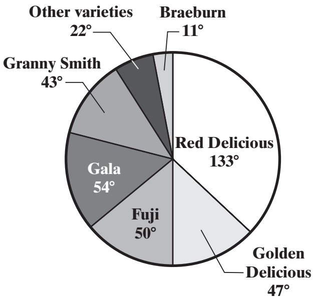
$22\% \text{ of } 125{,}600 = 0.22 \cdot 125{,}600$

$= 27{,}632$

The library has 27,632 children's fiction books.

25.

Type of Apple	Percent	Degrees in Sector
Red Delicious	37%	$37\% \text{ of } 360° = 0.37(360°) \approx 133°$
Golden Delicious	13%	$13\% \text{ of } 360° = 0.13(360°) \approx 47°$
Fuji	14%	$14\% \text{ of } 360° = 0.14(360°) \approx 50°$
Gala	15%	$15\% \text{ of } 360° = 0.15(360°) = 54°$
Granny Smith	12%	$12\% \text{ of } 360° = 0.12(360°) \approx 43°$
Other varieties	6%	$6\% \text{ of } 360° = 0.06(360°) \approx 22°$
Braeburn	3%	$3\% \text{ of } 360° = 0.03(360°) \approx 11°$



29. $20 = 2 \times 10 = 2 \times 2 \times 5 = 2^2 \times 5$

33. $85 = 5 \times 17$

37. Pacific Ocean:

$49\% \cdot 264{,}489{,}800 = 0.49 \cdot 264{,}489{,}800$

$= 129{,}600{,}002$ square kilometers

41. $24\% \cdot 2800 = 0.24 \cdot 2800 = 672$

672 respondents said that they spend \$0 online each month.

45.
$$\frac{\text{number of respondents who spend \$0}}{\text{number of respondents who spend \$1–\$100}} = \frac{672}{1736} = \frac{12 \cdot 56}{31 \cdot 56} = \frac{12}{31}$$

49. answers may vary

Exercise Set 7.3

1. Mean: $\frac{15 + 23 + 24 + 18 + 25}{5} = \frac{105}{5} = 21$

Median: Write the numbers in order: 15, 18, 23, 24, 25

The middle number is 23.

Mode: There is no mode, since each number occurs once.

5. Mean:

$$\frac{0.5 + 0.2 + 0.2 + 0.6 + 0.3 + 1.3 + 0.8 + 0.1 + 0.5}{9} = \frac{4.5}{9} = 0.5$$

Median: Write the numbers in order:

0.1, 0.2, 0.2, 0.3, 0.5, 0.5, 0.6, 0.8, 1.3

The middle number is 0.5.

Mode: Since 0.2 and 0.5 occur twice, there are two modes, 0.2 and 0.5.

9. Mean:

$$\frac{2717 + 2073 + 1972 + 1965 + 1819}{5} = \frac{10{,}546}{5} = 2109.2$$

The mean height of the five tallest buildings is 2109.2 feet.

13. answers may vary

17. $\text{GPA} = \frac{4 \cdot 3 + 4 \cdot 3 + 4 \cdot 4 + 3 \cdot 3 + 2 \cdot 1}{3 + 3 + 4 + 3 + 1} = \frac{51}{14} \approx 3.64$

21. Mode: 6.9 since this number appears twice.

25. Mean: $\frac{\text{sum of 15 pulse rates}}{15} = \frac{1095}{15} = 73$

29. There are 9 rates lower than the mean. They are 66, 68, 71, 64, 71, 70, 65, 70, and 72.

33. $\text{range} = 20 - 8 = 12$

37. $\text{range} = 11 - 9 = 2$

41. a. $\text{mean} = \frac{1 \cdot 5 + 1 \cdot 6 + 2 \cdot 7 + 5 \cdot 8 + 6 \cdot 9 + 2 \cdot 10}{1 + 1 + 2 + 5 + 6 + 2} = \frac{139}{17} \approx 8.2$

b. median: $\frac{n + 1}{2} = \frac{17 + 1}{2} = 9$. Thus the median is the data item in the 9th position.

median = 8

c. mode: 9 occurs most often.

45. a. $\text{mean:} = \frac{2 \cdot 5 + 3 \cdot 10 + 6 \cdot 15 + 5 \cdot 20 + 1 \cdot 25}{2 + 3 + 6 + 5 + 1} = \frac{255}{17} = 15$

b. median: $\frac{n + 1}{2} = \frac{17 + 1}{2} = 9$. Thus the median is the data item in the 9th position.

median = 15

c. mode: 15 occurs most often.

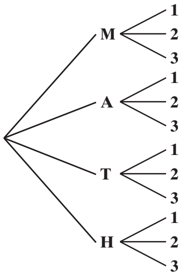
49. $\frac{12}{20} = \frac{3 \cdot 4}{4 \cdot 5} = \frac{3}{5}$

53. $\frac{35}{100} = \frac{5 \cdot 7}{5 \cdot 20} = \frac{7}{20}$

57. yes; answers may vary

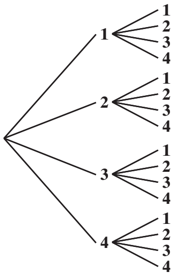
Exercise Set 7.4

1.



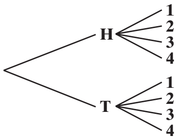
12 outcomes

5.



16 outcomes

9.



8 outcomes

13. A 1 or a 6 are two of the six possible outcomes. The probability is $\frac{2}{6} = \frac{1}{3}$.

17. Four of the six possible outcomes are numbers greater than 2. The probability is $\frac{4}{6} = \frac{2}{3}$.

21. A 1, a 2, or a 3 are three of three possible outcomes. The probability is $\frac{3}{3} = 1$.

25. One of the seven marbles is red. The probability is $\frac{1}{7}$.

29. Four of the seven marbles are either green or red. The probability is $\frac{4}{7}$.

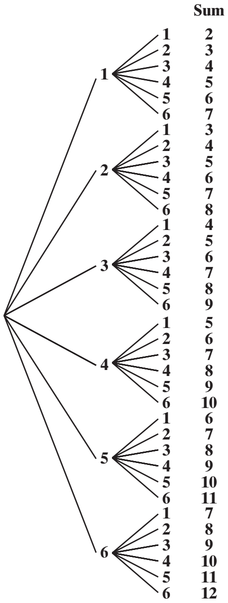
33. The blood pressure did not change for 10 of the 200 people. The probability is $\frac{10}{200} = \frac{1}{20}$.

37. $\frac{1}{2} \cdot \frac{1}{3} = \frac{1 \cdot 1}{2 \cdot 3} = \frac{1}{6}$

41. One of the 52 cards is the king of hearts. The probability is $\frac{1}{52}$.

45. Thirteen of the 52 cards are hearts. The probability is $\frac{13}{52} = \frac{1}{4}$.

49.



Five of the 36 sums are 6. The probability is $\frac{5}{36}$.

53. answers may vary

Chapter 7 Test

1. There are $4\frac{1}{2}$ dollar symbols for the second week. Each dollar symbol corresponds to \$50.

$4\frac{1}{2} \cdot \$50 = \frac{9}{2} \cdot \$50 = \frac{\$450}{2} = \$225$

\$225 was collected during the second week.

5. The shortest bar corresponds to February. The normal monthly precipitation in February in Chicago is 3 centimeters.

9. The line is above the 3 level for 2008 and 2011. Thus, the inflation rate was greater than 3% in 2008 and 2011

13. People under 18 are expected to account for 22% of the U.S. population in 2020.

$$\begin{aligned} 22\% \text{ of } 335 \text{ million} &= 0.22 \times 335 \text{ million} \\ &= 73.7 \text{ million} \end{aligned}$$

74 million people are projected to be under 18 in 2020.

17.

Class Intervals (Scores)	Tally	Class Frequency (Number of Students)
40–49	|	1
50–59	|||	3
60–69	||||	4
70–79	𝍸	5
80–89	𝍸 |||	8
90–99	||||	4

21.

Grade	Point Value	Credit Hours	$\left(\text{Point Value}\right) \cdot \left(\text{Credit Hours}\right)$
A	4	3	12
B	3	3	9
C	2	3	6
B	3	4	12
A	4	1	4
	Totals	14	43

$\frac{43}{14} \approx 3.07$

The grade point average is about 3.07.

25. a. mean $= \frac{3 \cdot 15 + 6 \cdot 20 + 7 \cdot 25 + 3 \cdot 30 + 8 \cdot 35}{3 + 6 + 7 + 3 + 8}$

$= \frac{710}{27} \approx 26.3$

b. median: $\frac{n+1}{2} = \frac{27+1}{2} = 14$. Thus, the median is the data item in the 14th position.

median $= 25$

c. mode: 35 occurs most often

d. range $= 35 - 15 = 20$

29. A 3 or a 4 are two of the ten possible outcomes. The probability is $\frac{2}{10} = \frac{1}{5}$.

Chapter 8

Exercise Set 8.1

1. $$\begin{aligned} 3 + 2z &= 3 + 2(-3) \\ &= 3 + (-6) \\ &= -3 \end{aligned}$$

5. $$\begin{aligned} z - x + y &= -3 - (-2) + 5 \\ &= -3 + 2 + 5 \\ &= -1 + 5 \\ &= 4 \end{aligned}$$

9. $$\begin{aligned} 8 - (5y - 7) &= 8 - (5 \cdot 5 - 7) \\ &= 8 - (25 - 7) \\ &= 8 - 18 \\ &= -10 \end{aligned}$$

13. $$\frac{6xy}{4} = \frac{6(-2)(5)}{4} = \frac{-12(5)}{4} = \frac{-60}{4} = -15$$

17. $$\begin{aligned} \frac{x + 2y}{2z} &= \frac{-2 + 2 \cdot 5}{2(-3)} \\ &= \frac{-2 + 10}{-6} \\ &= \frac{8}{-6} \\ &= -\frac{4}{3} \text{ or } -1\frac{1}{3} \end{aligned}$$

21. $$\begin{aligned} \frac{xz}{y} + \frac{3}{10} &= \frac{-2(-3)}{5} + \frac{3}{10} \\ &= \frac{6}{5} + \frac{3}{10} \\ &= \frac{12}{10} + \frac{3}{10} \\ &= \frac{15}{10} \\ &= \frac{3}{2} \text{ or } 1\frac{1}{2} \end{aligned}$$

25. $3x + 5x = (3 + 5)x = 8x$

29. $$\begin{aligned} 4c + c - 7c &= (4 + 1 - 7)c \\ &= (5 - 7)c \\ &= -2c \end{aligned}$$

33. $$\begin{aligned} 4a + 3a + 6a - 8 &= (4 + 3 + 6)a - 8 \\ &= (7 + 6)a - 8 \\ &= 13a - 8 \end{aligned}$$

37. $$\begin{aligned} 3x + 7 - x - 14 &= 3x - x + 7 - 14 \\ &= (3 - 1)x + (7 - 14) \\ &= 2x + (-7) \\ &= 2x - 7 \end{aligned}$$

41. $$\begin{aligned} \frac{5}{6} - \frac{7}{12}x - \frac{1}{3} - \frac{3}{10}x &= \frac{5}{6} - \frac{1}{3} - \frac{7}{12}x - \frac{3}{10}x \\ &= \left(\frac{5}{6} - \frac{1}{3}\right) + \left(-\frac{7}{12} - \frac{3}{10}\right)x \\ &= \left(\frac{5}{6} - \frac{2}{6}\right) + \left(-\frac{35}{60} - \frac{18}{60}\right)x \\ &= \frac{3}{6} + \left(-\frac{53}{60}\right)x \\ &= \frac{1}{2} - \frac{53}{60}x \end{aligned}$$

45. $6(5x) = (6 \cdot 5)x = 30x$

49. $-0.6(7a) = (-0.6 \cdot 7)a = -4.2a$

53. $2(y + 2) = 2 \cdot y + 2 \cdot 2 = 2y + 4$

57. $$\begin{aligned} -4(3x + 7) &= -4 \cdot 3x + (-4) \cdot 7 \\ &= (-4 \cdot 3)x + (-28) \\ &= -12x - 28 \end{aligned}$$

61. $$\begin{aligned} \frac{1}{2}(-8x - 3) &= \frac{1}{2}(-8x) - \frac{1}{2}(3) \\ &= \left(\frac{1}{2} \cdot -8\right)x - \frac{3}{2} \\ &= -4x - \frac{3}{2} \end{aligned}$$

65. $$\begin{aligned} 4(6n - 5) + 3n &= 4 \cdot 6n - 4 \cdot 5 + 3n \\ &= 24n - 20 + 3n \\ &= 24n + 3n - 20 \\ &= (24 + 3)n - 20 \\ &= 27n - 20 \end{aligned}$$

69. $$\begin{aligned} &-2(3x + 1) - 5(x - 2) \\ &= -2 \cdot 3x + (-2) \cdot 1 - 5 \cdot x - (-5) \cdot 2 \\ &= (-2 \cdot 3)x + (-2) - 5x - (-10) \\ &= -6x - 2 - 5x + 10 \\ &= (-6 - 5)x + (-2 + 10) \\ &= -11x + 8 \end{aligned}$$

73. $$\begin{aligned} &2a + 2a + 6 + 5a + 6 + 2a \\ &= 2a + 2a + 5a + 2a + 6 + 6 \\ &= (2 + 2 + 5 + 2)a + 6 + 6 \\ &= 11a + 12 \end{aligned}$$

The perimeter is $(11a + 12)$ feet.

77. $$\begin{aligned} \text{Area} &= (\text{length}) \cdot (\text{width}) \\ &= (4y) \cdot (9) \\ &= (4 \cdot 9)y \\ &= 36y \end{aligned}$$

The area is $36y$ square inches.

81. $$\begin{aligned} \text{Perimeter} &= 2 \cdot (\text{length}) + 2 \cdot (\text{width}) \\ &= 2 \cdot (20) + 2 \cdot (3y + 1) \\ &= 40 + 2 \cdot (3y) + 2 \cdot (1) \\ &= 40 + 6y + 2 \\ &= 6y + 42 \end{aligned}$$

$$\begin{aligned} \text{Area} &= (\text{length}) \cdot (\text{width}) \\ &= (20) \cdot (3y + 1) \\ &= 20 \cdot 3y + 20 \cdot 1 \\ &= (20 \cdot 3)y + 20 \\ &= 60y + 20 \end{aligned}$$

The perimeter is $(6y + 42)$ miles and the area is $(60y + 20)$ square miles.

85. $$\begin{aligned} \text{Perimeter} &= 2 \cdot (\text{length}) + 2 \cdot (\text{width}) \\ &= 2 \cdot (18) + 2 \cdot (14) \\ &= 36 + 28 \\ &= 64 \end{aligned}$$

The perimeter is 64 feet.

89. $$\begin{aligned} I &= PRT \\ &= (3000) \cdot (0.06) \cdot (2) \\ &= 360 \end{aligned}$$

The account will earn \$360 in interest.

93. $$\begin{aligned} F &= \frac{9}{5}C + 32 \\ &= \frac{9}{5}(-5) + 32 \\ &= -9 + 32 \\ &= 23 \end{aligned}$$

The low temperature is 23°F.

97. Let $l = 7.6$, $w = 3$, and $h = 4x$.
$V = lwh = 7.6 \cdot 3 \cdot 4x = 91.2x$.
The volume is $91.2x$ cubic inches.

101. $-4-(-12)=-4+12=8$

105. The given result is incorrect.

$$\begin{aligned}5(3x-2)&=5\cdot 3x-5\cdot 2\\&=15x-10\end{aligned}$$

109. The parentheses have been removed by the distributive property.

113. answers may vary

117. Let $d=173$.

$$\begin{aligned}A&=0.7854d^2\\&=0.7854(173)^2\\&=0.7854\cdot 29{,}929\\&\approx 23{,}506.2\end{aligned}$$

The trunk area is approximately 23,506.2 square inches.

Exercise Set 8.2

1.
$$\begin{aligned}x-8&=2\\10-8&\overset{?}{=}2\\2&\overset{?}{=}2\quad\text{True}\end{aligned}$$

Yes, 10 is a solution of the equation.

5.
$$\begin{aligned}-9f&=64-f\\-9(-8)&\overset{?}{=}64-(-8)\\72&\overset{?}{=}64+8\\72&\overset{?}{=}72\quad\text{True}\end{aligned}$$

Yes, -8 is a solution of the equation.

9.
$$\begin{aligned}a+5&=23\\a+5-5&=23-5\\a&=18\end{aligned}$$

Check:
$$\begin{aligned}a+5&=23\\18+5&\overset{?}{=}23\\23&\overset{?}{=}23\quad\text{True}\end{aligned}$$

The solution of the equation is 18.

13.
$$\begin{aligned}7&=y-2\\7+2&=y-2+2\\9&=y\end{aligned}$$

Check:
$$\begin{aligned}7&=y-2\\7&\overset{?}{=}9-2\\7&\overset{?}{=}7\quad\text{True}\end{aligned}$$

The solution of the equation is 9.

17.
$$\begin{aligned}x+\frac{1}{2}&=\frac{7}{2}\\x+\frac{1}{2}-\frac{1}{2}&=\frac{7}{2}-\frac{1}{2}\\x&=\frac{6}{2}\\x&=3\end{aligned}$$

Check:
$$\begin{aligned}x+\frac{1}{2}&=\frac{7}{2}\\3+\frac{1}{2}&\overset{?}{=}\frac{7}{2}\\\frac{6}{2}+\frac{1}{2}&\overset{?}{=}\frac{7}{2}\\\frac{7}{2}&\overset{?}{=}\frac{7}{2}\quad\text{True}\end{aligned}$$

The solution of the equation is 3.

21.
$$\begin{aligned}x-3&=-1+4\\x-3&=3\\x-3+3&=3+3\\x&=6\end{aligned}$$

Check:
$$\begin{aligned}x-3&=-1+4\\6-3&\overset{?}{=}-1+4\\3&\overset{?}{=}3\quad\text{True}\end{aligned}$$

The solution of the equation is 6.

25.
$$\begin{aligned}x-0.6&=4.7\\x-0.6+0.6&=4.7+0.6\\x&=5.3\end{aligned}$$

Check:
$$\begin{aligned}x-0.6&=4.7\\5.3-0.6&\overset{?}{=}4.7\\4.7&\overset{?}{=}4.7\quad\text{True}\end{aligned}$$

The solution of the equation is 5.3.

29.
$$\begin{aligned}y+2.3&=-9.2-8.6\\y+2.3&=-17.8\\y+2.3-2.3&=-17.8-2.3\\y&=-20.1\end{aligned}$$

Check:
$$\begin{aligned}y+2.3&=-9.2-8.6\\-20.1+2.3&\overset{?}{=}-9.2-8.6\\-17.8&\overset{?}{=}-17.8\quad\text{True}\end{aligned}$$

The solution of the equation is -20.1.

33.
$$\begin{aligned}5+(-12)&=5x-7-4x\\5+(-12)&=5x-4x-7\\-7&=x-7\\-7+7&=x-7+7\\0&=x\end{aligned}$$

Check:
$$\begin{aligned}5+(-12)&=5x-7-4x\\5+(-12)&\overset{?}{=}5(0)-7-4(0)\\5+(-12)&\overset{?}{=}0-7-0\\-7&\overset{?}{=}-7\quad\text{True}\end{aligned}$$

The solution of the equation is 0.

37.
$$\begin{aligned}2(5x-3)&=11x\\2\cdot 5x-2\cdot 3&=11x\\10x-6&=11x\\10x-10x-6&=11x-10x\\-6&=x\end{aligned}$$

41.
$$\begin{aligned}21y&=5(4y-6)\\21y&=5\cdot 4y-5\cdot 6\\21y&=20y-30\\21y-20y&=20y-20y-30\\y&=-30\end{aligned}$$

45. $\frac{-7}{-7}=1$ because $1\cdot -7=-7$.

49. $-\frac{2}{3}\cdot-\frac{3}{2}=\frac{2\cdot 3}{3\cdot 2}=\frac{6}{6}=1$

53. To solve $-\frac{1}{7}=-\frac{4}{5}+x$, $\frac{4}{5}$ should be added to both sides of the equation.

57.
$$\begin{aligned}x-76{,}862&=86{,}102\\x-76{,}862+76{,}862&=86{,}102+76{,}862\\x&=162{,}964\end{aligned}$$

61. Use $I=R-E$, where $I=6{,}742{,}000{,}000$ and $E=11{,}944{,}000{,}000$.

$$\begin{aligned}I&=R-E\\6{,}742{,}000{,}000&=R-11{,}944{,}000{,}000\\6{,}742{,}000{,}000+11{,}944{,}000{,}000&=R-11{,}944{,}000{,}000\\&\quad+11{,}944{,}000{,}000\\18{,}686{,}000{,}000&=R\end{aligned}$$

Kohl's total revenues for the year were $18,686,000,000.

Exercise Set 8.3

1.
$$\begin{aligned} 5x &= 20 \\ \frac{5x}{5} &= \frac{20}{5} \\ x &= 4 \end{aligned}$$

5.
$$\begin{aligned} 0.4y &= -12 \\ \frac{0.4y}{0.4} &= \frac{-12}{0.4} \\ y &= -30 \end{aligned}$$

9.
$$\begin{aligned} -0.3x &= -15 \\ \frac{-0.3x}{-0.3} &= \frac{-15}{-0.3} \\ x &= 50 \end{aligned}$$

13.
$$\begin{aligned} \frac{1}{6}y &= -5 \\ \frac{6}{1}\cdot\frac{1}{6}y &= \frac{6}{1}\cdot -5 \\ y &= -30 \end{aligned}$$

17.
$$\begin{aligned} -\frac{2}{9}z &= \frac{4}{27} \\ -\frac{9}{2}\cdot -\frac{2}{9}z &= -\frac{9}{2}\cdot\frac{4}{27} \\ z &= -\frac{2}{3} \end{aligned}$$

21.
$$\begin{aligned} 16 &= 10t - 8t \\ 16 &= 2t \\ \frac{16}{2} &= \frac{2t}{2} \\ 8 &= t \end{aligned}$$

25.
$$\begin{aligned} 4 - 10 &= -3z \\ -6 &= -3z \\ \frac{-6}{-3} &= \frac{-3z}{-3} \\ 2 &= z \end{aligned}$$

29.
$$\begin{aligned} 0.4 &= -8z \\ \frac{0.4}{-8} &= \frac{-8z}{-8} \\ -0.05 &= z \end{aligned}$$

33.
$$\begin{aligned} -\frac{3}{5}x &= -\frac{6}{15} \\ -\frac{5}{3}\cdot -\frac{3}{5}x &= -\frac{5}{3}\cdot -\frac{6}{15} \\ x &= \frac{2}{3} \end{aligned}$$

37.
$$\begin{aligned} 5 - 5 &= 2x + 7x \\ 0 &= 9x \\ \frac{0}{9} &= \frac{9x}{9} \\ 0 &= x \end{aligned}$$

41.
$$\begin{aligned} -3x - 3x &= 50 - 2 \\ -6x &= 48 \\ \frac{-6x}{-6} &= \frac{48}{-6} \\ x &= -8 \end{aligned}$$

45.
$$\begin{aligned} \frac{1}{4}x - \frac{5}{8}x &= 20 - 47 \\ \frac{2}{8}x - \frac{5}{8}x &= -27 \\ -\frac{3}{8}x &= -27 \\ -\frac{8}{3}\cdot -\frac{3}{8}x &= -\frac{8}{3}\cdot -27 \\ x &= 72 \end{aligned}$$

49.
$$\begin{aligned} \frac{x}{-4} &= 1 - (-6) \\ \frac{x}{-4} &= 1 + 6 \\ \frac{x}{-4} &= 7 \\ -4\cdot\frac{x}{-4} &= -4\cdot 7 \\ x &= -28 \end{aligned}$$

53. $\frac{x-3}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$

57. The longest bar corresponds to 2015, so the number of acres burned by wildfires was greatest in 2015.

61. Addition should be used to solve the equation $12 = x - 5$. Specifically, 5 should be added to both sides of the equation.

65. answers may vary

69. Use $d = r\cdot t$, where $d = 294$ and $t = 5$.
$$\begin{aligned} d &= r\cdot t \\ 294 &= 5\cdot r \\ \frac{294}{5} &= \frac{5r}{5} \\ \frac{294}{5} &= r \end{aligned}$$
The driver should drive $\frac{294}{5} = 58.8$ miles per hour to make the trip in 5 hours.

73.
$$\begin{aligned} \frac{y}{72} &= -86 - (-1029) \\ \frac{y}{72} &= -86 + 1029 \\ \frac{y}{72} &= 943 \\ 72\cdot\frac{y}{72} &= 72\cdot 943 \\ y &= 67{,}896 \end{aligned}$$

Exercise Set 8.4

1.
$$\begin{aligned} 2x - 6 &= 0 \\ 2x - 6 + 6 &= 0 + 6 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

5.
$$\begin{aligned} 6 - n &= 10 \\ 6 - 6 - n &= 10 - 6 \\ -n &= 4 \\ \frac{-n}{-1} &= \frac{4}{-1} \\ n &= -4 \end{aligned}$$

9.
$$\begin{aligned} 1.7 &= 2y + 9.5 \\ 1.7 - 9.5 &= 2y + 9.5 - 9.5 \\ -7.8 &= 2y \\ \frac{-7.8}{2} &= \frac{2y}{2} \\ -3.9 &= y \end{aligned}$$

13.
$$\begin{aligned} 3x - 7 &= 4x + 5 \\ 3x - 7 - 5 &= 4x + 5 - 5 \\ 3x - 12 &= 4x \\ 3x - 3x - 12 &= 4x - 3x \\ -12 &= x \end{aligned}$$

17.
$$\begin{aligned} 9 - 3x &= 14 + 2x \\ 9 - 14 - 3x &= 14 - 14 + 2x \\ -5 - 3x &= 2x \\ -5 - 3x + 3x &= 2x + 3x \\ -5 &= 5x \\ \frac{-5}{5} &= \frac{5x}{5} \\ -1 &= x \end{aligned}$$

21.
$$\begin{aligned} x + 20 + 2x &= -10 - 2x - 15 \\ x + 2x + 20 &= -10 - 15 - 2x \\ 3x + 20 &= -25 - 2x \\ 3x + 2x + 20 &= -25 - 2x + 2x \\ 5x + 20 &= -25 \\ 5x + 20 - 20 &= -25 - 20 \\ 5x &= -45 \\ \frac{5x}{5} &= \frac{-45}{5} \\ x &= -9 \end{aligned}$$

25.
$$\begin{aligned} -2(y + 4) &= 2 \\ -2y - 8 &= 2 \\ -2y - 8 + 8 &= 2 + 8 \\ -2y &= 10 \\ \frac{-2y}{-2} &= \frac{10}{-2} \\ y &= -5 \end{aligned}$$

29.
$$\begin{aligned} 35 - 17 &= 3(x - 2) \\ 18 &= 3x - 6 \\ 18 + 6 &= 3x - 6 + 6 \\ 24 &= 3x \\ \frac{24}{3} &= \frac{3x}{3} \\ 8 &= x \end{aligned}$$

33.
$$\begin{aligned} 2t - 1 &= 3(t + 7) \\ 2t - 1 &= 3t + 21 \\ 2t - 1 - 21 &= 3t + 21 - 21 \\ 2t - 22 &= 3t \\ 2t - 2t - 22 &= 3t - 2t \\ -22 &= t \end{aligned}$$

37.
$$\begin{aligned} -4x &= 44 \\ \frac{-4x}{-4} &= \frac{44}{-4} \\ x &= -11 \end{aligned}$$

41.
$$\begin{aligned} 8 - b &= 13 \\ 8 - 8 - b &= 13 - 8 \\ -b &= 5 \\ \frac{-b}{-1} &= \frac{5}{-1} \\ b &= -5 \end{aligned}$$

45.
$$\begin{aligned} 2x - 1 &= -7 \\ 2x - 1 + 1 &= -7 + 1 \\ 2x &= -6 \\ \frac{2x}{2} &= \frac{-6}{2} \\ x &= -3 \end{aligned}$$

49.
$$\begin{aligned} 9a + 29 &= -7 \\ 9a + 29 - 29 &= -7 - 29 \\ 9a &= -36 \\ \frac{9a}{9} &= \frac{-36}{9} \\ a &= -4 \end{aligned}$$

53.
$$\begin{aligned} 11(x - 2) &= 22 \\ 11x - 22 &= 22 \\ 11x - 22 + 22 &= 22 + 22 \\ 11x &= 44 \\ \frac{11x}{11} &= \frac{44}{11} \\ x &= 4 \end{aligned}$$

57.
$$\begin{aligned} 3(x - 5) &= -7 - 11 \\ 3x - 15 &= -18 \\ 3x - 15 + 15 &= -18 + 15 \\ 3x &= -3 \\ \frac{3x}{3} &= \frac{-3}{3} \\ x &= -1 \end{aligned}$$

61.
$$\begin{aligned} 4x + 3 &= 2x + 11 \\ 4x + 3 - 3 &= 2x + 11 - 3 \\ 4x &= 2x + 8 \\ 4x - 2x &= 2x - 2x + 8 \\ 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

65.
$$\begin{aligned} \frac{3}{8}x + 14 &= \frac{5}{8}x - 2 \\ \frac{3}{8}x + 14 + 2 &= \frac{5}{8}x - 2 + 2 \\ \frac{3}{8}x + 16 &= \frac{5}{8}x \\ \frac{3}{8}x - \frac{3}{8}x + 16 &= \frac{5}{8}x - \frac{3}{8}x \\ 16 &= \frac{2}{8}x \\ 16 &= \frac{1}{4}x \\ 4 \cdot 16 &= 4 \cdot \frac{1}{4}x \\ 64 &= x \end{aligned}$$

69.
$$\begin{aligned} \frac{5}{8}a &= \frac{1}{8}a + \frac{3}{4} \\ \frac{5}{8}a - \frac{1}{8}a &= \frac{1}{8}a - \frac{1}{8}a + \frac{3}{4} \\ \frac{4}{8}a &= \frac{3}{4} \\ \frac{1}{2}a &= \frac{3}{4} \\ 2 \cdot \frac{1}{2}a &= 2 \cdot \frac{3}{4} \\ a &= \frac{3}{2} \end{aligned}$$

73.
$$\begin{aligned} 3 + 2(2n - 5) &= 1 \\ 3 + 4n - 10 &= 1 \\ 4n - 7 &= 1 \end{aligned}$$

$$\begin{aligned} 4n - 7 + 7 &= 1 + 7 \\ 4n &= 8 \\ \frac{4n}{4} &= \frac{8}{4} \\ n &= 2 \end{aligned}$$

77.
$$\begin{aligned} -20 - (-50) &= \frac{x}{9} \\ -20 + 50 &= \frac{x}{9} \\ 30 &= \frac{x}{9} \\ 9 \cdot 30 &= 9 \cdot \frac{x}{9} \\ 270 &= x \end{aligned}$$

81.
$$\begin{aligned} 3(5c - 1) - 2 &= 13c + 3 \\ 15c - 3 - 2 &= 13c + 3 \\ 15c - 5 &= 13c + 3 \\ 15c - 5 + 5 &= 13c + 3 + 5 \\ 15c &= 13c + 8 \\ 15c - 13c &= 13c - 13c + 8 \\ 2c &= 8 \\ \frac{2c}{2} &= \frac{8}{2} \\ c &= 4 \end{aligned}$$

85. "The sum of -42 and 16 is -26" translates to $-42 + 16 = -26$.

89. "Three times the difference of -14 and 2 amounts to -48" translates to $3(-14 - 2) = -48$.

93. From the label of the 2014 bar, 126 million or 126,000,000 returns were filed electronically in 2014.

97. The first step in solving $2x - 5 = -7$ is to add 5 to both sides, which is choice **b**.

101. The error is in the second line.
$$\begin{aligned} 2(3x - 5) &= 5x - 7 \\ 6x - 10 &= 5x - 7 \\ 6x - 10 + 10 &= 5x - 7 + 10 \\ 6x &= 5x + 3 \\ 6x - 5x &= 5x + 3 - 5x \\ x &= 3 \end{aligned}$$

105.
$$\begin{aligned} 2^3(x + 4) &= 3^2(x + 4) \\ 8(x + 4) &= 9(x + 4) \\ 8x + 32 &= 9x + 36 \\ 8x + 32 - 36 &= 9x + 36 - 36 \\ 8x - 4 &= 9x \\ 8x - 4 - 8x &= 9x - 8x \\ -4 &= x \end{aligned}$$

109. Use $C = \frac{5}{9}(F - 32)$ with $C = 50.7$.
$$\begin{aligned} C &= \frac{5}{9}(F - 32) \\ 50.7 &= \frac{5}{9}(F - 32) \\ \frac{9}{5}(50.7) &= \frac{9}{5} \cdot \frac{5}{9}(F - 32) \\ 91.26 &= F - 32 \\ 91.26 + 32 &= F - 32 + 32 \\ 123.26 &= F \end{aligned}$$
The temperature was 123.26°F.

Exercise Set 8.5

1. "The sum of a number and five" is $x + 5$.

5. "Twenty decreased by a number" is $20 - x$.

9. "A number divided by 2" is $x \div 2$ or $\frac{x}{2}$.

13. "A number added to -5 is -7" is $-5 + x = -7$.

17. "A number subtracted from -20 amounts to 104" is $-20 - x = 104$.

21. "A number subtracted from 11" is $11 - x$.

25. "Fifty decreased by eight times a number" is $50 - 8x$.

29. "Three times a number, added to 9, is 33" is $9 + 3x = 33$.
$$\begin{aligned} 9 + 3x &= 33 \\ 9 + 3x - 9 &= 33 - 9 \\ 3x &= 24 \\ \frac{3x}{3} &= \frac{24}{3} \\ x &= 8 \end{aligned}$$
The number is 8.

33. "The difference of a number and 3 is equal to the quotient of 10 and 5" is $x - 3 = \frac{10}{5}$.
$$\begin{aligned} x - 3 &= \frac{10}{5} \\ x - 3 &= 2 \\ x - 3 + 3 &= 2 + 3 \\ x &= 5 \end{aligned}$$
The number is 5.

37. "40 subtracted from five times a number is 8 more than the number" is $5x - 40 = x + 8$.
$$\begin{aligned} 5x - 40 &= x + 8 \\ 5x - 40 + 40 &= x + 8 + 40 \\ 5x &= x + 48 \\ 5x - x &= x - x + 48 \\ 4x &= 48 \\ \frac{4x}{4} &= \frac{48}{4} \\ x &= 12 \end{aligned}$$
The number is 12.

41. "The product of 4 and a number is the same as 30 less twice that same number" is $4x = 30 - 2x$.
$$\begin{aligned} 4x &= 30 - 2x \\ 4x + 2x &= 30 - 2x + 2x \\ 6x &= 30 \\ \frac{6x}{6} &= \frac{30}{6} \\ x &= 5 \end{aligned}$$
The number is 5.

45. Let x be the fastest speed for the pheasant. Since the top speed of the falcon is five times the top speed of the pheasant, the top speed of the falcon is $5x$. Since the total of the speeds is 222 miles per hour, the sum of x and $5x$ is 222.
$$\begin{aligned} x + 5x &= 222 \\ 6x &= 222 \\ \frac{6x}{6} &= \frac{222}{6} \\ x &= 37 \end{aligned}$$
The top speed of the pheasant is 37 miles per hour and the top speed of the falcon is $5 \cdot 37 = 185$ miles per hour.

49. Let x be the cost of *Kinect Adventures*. Since the cost of *Grand Theft Auto V* is \$18 more than the cost of *Kinect Adventures*, the cost of *Grand Theft Auto V* is $x + 18$. Since the total cost is \$42, the sum of x and $x + 18$ is 42.

$$\begin{aligned} x + x + 18 &= 42 \\ 2x + 18 &= 42 \\ 2x + 18 - 18 &= 42 - 18 \\ 2x &= 24 \\ \frac{2x}{2} &= \frac{24}{2} \\ x &= 12 \end{aligned}$$

The cost of *Kinect Adventures* is \$12 and the cost of *Grand Theft Auto V* is \$12 + \$18 = \$30.

53. Let x be the capacity of Beaver Stadium. Since the capacity of Michigan Stadium is 3329 more than the capacity of Beaver Stadium, the capacity of Michigan Stadium is $x + 3329$. Since the total capacity of the two stadiums is 216,473, the sum of x and $x + 3329$ is 216,473.

$$\begin{aligned} x + x + 3329 &= 216{,}473 \\ 2x + 3329 &= 216{,}473 \\ 2x + 3329 - 3329 &= 216{,}473 - 3329 \\ 2x &= 213{,}144 \\ \frac{2x}{2} &= \frac{213{,}144}{2} \\ x &= 106{,}572 \end{aligned}$$

The capacity of Beaver Stadium is 106,572 and the capacity of Michigan Stadium is $106{,}572 + 3329 = 109{,}901$.

57. Let x be the number of points scored by the Gonzaga Bulldogs. Since the North Carolina Tar Heels scored 6 points more than the Gonzaga Bulldogs, the North Carolina Tar Heels scored $x + 6$ points. Since the total number of points scored by the two teams was 136 points, the sum of x and $x + 6$ is 136.

$$\begin{aligned} x + x + 6 &= 136 \\ 2x + 6 &= 136 \\ 2x + 6 - 6 &= 136 - 6 \\ 2x &= 130 \\ \frac{2x}{2} &= \frac{130}{2} \\ x &= 65 \end{aligned}$$

$x + 6 = 65 + 6 = 71$

The 2017 Champion North Carolina Tar Heels scored 71 points.

61. Let x be the speed of a lion. Since the speed of a cheetah is 25 mph faster, its speed is $x + 25$. Since the total of their speeds is 123 mph, the sum of x and $x + 25$ is 123.

$$\begin{aligned} x + x + 25 &= 123 \\ 2x + 25 &= 123 \\ 2x + 25 - 25 &= 123 - 25 \\ 2x &= 98 \\ \frac{2x}{2} &= \frac{98}{2} \\ x &= 49 \end{aligned}$$

The speed of a lion is 49 mph and the speed of a cheetah is $(49 + 25)$ mph or 74 mph.

65. Let x be the number of Internet users, in millions, in Europe. Since Asia had 1319 million more Internet users than Europe, the number of Internet users in Asia was $x + 1319$. Since the total number of Internet users for both continents was 2727 million, the sum of x and $x + 1319$ is 2727.

$$\begin{aligned} x + x + 1319 &= 2727 \\ 2x + 1319 &= 2727 \\ 2x + 1319 - 1319 &= 2727 - 1319 \\ 2x &= 1408 \\ \frac{2x}{2} &= \frac{1408}{2} \\ x &= 704 \end{aligned}$$

Europe had 704 million Internet users and Asia had 704 million + 1319 million = 2023 million Internet users.

69. 1026 rounded to the nearest hundred is 1000.

73. yes; answers may vary

77. Use $P = C + M$, where $P = 999$ and $M = 450$.

$$\begin{aligned} P &= C + M \\ 999 &= C + 450 \\ 999 - 450 &= C + 450 - 450 \\ 549 &= C \end{aligned}$$

The wholesale cost is \$549.

Chapter 8 Test

1.
$$\begin{aligned} \frac{3x - 5}{2y} &= \frac{3 \cdot 7 - 5}{2(-8)} \\ &= \frac{21 - 5}{-16} \\ &= \frac{16}{-16} \\ &= -1 \end{aligned}$$

5.
$$\begin{aligned} \text{Area} &= \text{length} \cdot \text{width} \\ A &= 4 \cdot (3x - 1) \\ &= 4 \cdot 3x - 4 \cdot 1 \\ &= 12x - 4 \end{aligned}$$

The area is $4(3x - 1)$ square meters or $(12x - 4)$ square meters.

9.
$$\begin{aligned} -\frac{5}{8}x &= -25 \\ -\frac{8}{5} \cdot -\frac{5}{8}x &= -\frac{8}{5} \cdot -25 \\ x &= 40 \end{aligned}$$

13.
$$\begin{aligned} -4x + 7 &= 15 \\ -4x + 7 - 7 &= 15 - 7 \\ -4x &= 8 \\ \frac{-4x}{-4} &= \frac{8}{-4} \\ x &= -2 \end{aligned}$$

17.
$$\begin{aligned} 10y - 1 &= 7y + 21 \\ 10y - 1 + 1 &= 7y + 21 + 1 \\ 10y &= 7y + 22 \\ 10y - 7y &= 7y - 7y + 22 \\ 3y &= 22 \\ \frac{3y}{3} &= \frac{22}{3} \\ y &= \frac{22}{3} \end{aligned}$$

21. Use $A = \frac{1}{2}bh$, where $b = 5$ and $h = 12$.

$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 5 \cdot 12 = 30$$

The area is 30 square feet.

25. Let x be the number of women runners entered. Then the number of men runners entered is $x + 112$, and the sum of these two quantities is 600.

$$\begin{aligned} x + x + 112 &= 600 \\ 2x + 112 &= 600 \\ 2x + 112 - 112 &= 600 - 112 \\ 2x &= 488 \\ \frac{2x}{2} &= \frac{488}{2} \\ x &= 244 \end{aligned}$$

There were 244 women runners entered.

Chapter 9

Exercise Set 9.1

1. The figure extends indefinitely in two directions. It is line CD, line l, or $\overleftrightarrow{CD}$.

5. The figure has two rays with a common endpoint. It is an angle, which can be named $\angle GHI$, $\angle IHG$, or $\angle H$.

9. Two other ways to name $\angle x$ are $\angle CPR$ and $\angle RPC$.

13. $\angle S$ is a straight angle.

17. $\angle Q$ measures between 90° and 180°. It is an obtuse angle.

21. The complement of an angle that measures 23° is an angle that measures $90° - 23° = 67°$.

25. The complement of an angle that measures 58° is an angle that measures $90° - 58° = 32°$.

29. $52° + 38° = 90°$, so $\angle PNQ$ and $\angle QNR$ are complementary. $60° + 30° = 90°$, so $\angle MNP$ and $\angle RNO$ are complementary.

33. $m\angle x = 74° - 47° = 27°$

37. $\angle x$ and the angle marked 150° are supplementary, so $m\angle x = 180° - 150° = 30°$. $\angle y$ and the angle marked 150° are vertical angles, so $m\angle y = 150°$. $\angle z$ and $\angle x$ are vertical angles so $m\angle z = m\angle x = 30°$.

41. $\angle x$ and the angle marked 80° are supplementary, so $m\angle x = 180° - 80° = 100°$. $\angle y$ and the angle marked 80° are alternate interior angles, so $m\angle y = 80°$. $\angle x$ and $\angle z$ are corresponding angles, so $m\angle z = m\angle x = 100°$.

45. $\angle x$ can also be named $\angle ABC$ or $\angle CBA$.

49. $m\angle ABC = 15°$

53.
$$\begin{aligned} m\angle DBA &= m\angle DBC + m\angle CBA \\ &= 50° + 15° \\ &= 65° \end{aligned}$$

57. $\frac{7}{8} + \frac{1}{4} = \frac{7}{8} + \frac{2}{8} = \frac{9}{8}$ or $1\frac{1}{8}$

61.
$$\begin{aligned} 3\frac{1}{3} - 2\frac{1}{2} &= \frac{10}{3} - \frac{5}{2} \\ &= \frac{20}{6} - \frac{15}{6} \\ &= \frac{5}{6} \end{aligned}$$

65. Since there are 360° in a full revolution, there are 360° around the earth at the equator.

69. The complement of an angle that measures 45° is an angle with measure $90° - 45° = 45°$.

73. True; because 120° is less than 180°, it is possible to find the supplement of a 120° angle.

77. no; answers may vary

Exercise Set 9.2

1. The figure has five sides, so it is a pentagon.

5. The figure has four sides, so it is a quadrilateral.

9. All three sides of the triangle have the same length, therefore the triangle is equilateral.

13. Two sides of the triangle have the same length, therefore the triangle is isosceles.

17. $m\angle x = 180° - 95° - 72° = 13°$

21. Twice the radius of a circle is its diameter.

25. A quadrilateral with opposite sides parallel is a parallelogram.

29. $d = 2 \cdot r = 2 \cdot 7 \text{ m} = 14 \text{ m}$

33. $d = 2 \cdot r = 2 \cdot 20.3 \text{ cm} = 40.6 \text{ cm}$

37. The solid is a cylinder.

41. The solid is a cone.

45. The object has the shape of a rectangular solid.

49. The object has the shape of a pyramid.

53. $r = \frac{1}{2}d = \frac{1}{2} \cdot 26 \text{ miles} = 13 \text{ miles}$

57. $2(18) + 2(36) = 36 + 72 = 108$

61. True; since all four sides of a square are equal in length and a square is a parallelogram, a square is also a rhombus.

65. False; a pentagon has five sides, so it cannot be a quadrilateral, which has four sides.

69. answers may vary

Exercise Set 9.3

1.
$$\begin{aligned} P &= 2 \cdot l + 2 \cdot w \\ &= 2 \cdot 17 \text{ ft} + 2 \cdot 15 \text{ ft} \\ &= 34 \text{ ft} + 30 \text{ ft} \\ &= 64 \text{ ft} \end{aligned}$$

The perimeter is 64 feet.

5.
$$\begin{aligned} P &= a + b + c \\ &= 5 \text{ in.} + 7 \text{ in.} + 9 \text{ in.} \\ &= 21 \text{ in.} \end{aligned}$$

The perimeter is 21 inches.

9. All sides of a regular triangle have the same length.

$P = a + b + c = 14 \text{ in.} + 14 \text{ in.} + 14 \text{ in.} = 42 \text{ in.}$

The perimeter is 42 inches.

13. Sum the lengths of the sides.

$$\begin{aligned} P &= 5 \text{ ft} + 3 \text{ ft} + 2 \text{ ft} + 7 \text{ ft} + 4 \text{ ft} \\ &= 21 \text{ ft} \end{aligned}$$

The perimeter is 21 feet.

17.
$$\begin{aligned} P &= 2 \cdot l + 2 \cdot w \\ &= 2 \cdot 120 \text{ yd} + 2 \cdot 53 \text{ yd} \\ &= 240 \text{ yd} + 106 \text{ yd} \\ &= 346 \text{ yd} \end{aligned}$$

The perimeter of the football field is 346 yards.

21. The amount of stripping needed is 22 feet.
22 feet · \$2.50 per foot = \$55
The total cost of the stripping is \$55.

25. $P = 4 \cdot s = 4 \cdot 7 \text{ in.} = 28 \text{ in.}$
The perimeter is 28 inches.

29. The unmarked vertical side must have length
28 m − 20 m = 8 m.
The unmarked horizontal side must have length
20 m − 17 m = 3 m.
Sum the lengths of the sides.
$$\begin{aligned} P &= 17 \text{ m} + 8 \text{ m} + 3 \text{ m} + 20 \text{ m} + 20 \text{ m} + 28 \text{ m} \\ &= 96 \text{ m} \end{aligned}$$
The perimeter is 96 meters.

33. The unmarked vertical side must have length
5 cm + 14 cm = 19 cm.
The unmarked horizontal side must have length
18 cm − 9 cm = 9 cm.
Sum the lengths of the sides.
$$\begin{aligned} P &= 18 \text{ cm} + 19 \text{ cm} + 9 \text{ cm} + 14 \text{ cm} + 9 \text{ cm} + 5 \text{ cm} \\ &= 74 \text{ cm} \end{aligned}$$
The perimeter is 74 centimeters.

37.
$$\begin{aligned} C &= 2 \cdot \pi \cdot r \\ &= 2 \cdot \pi \cdot 8 \text{ mi} \\ &= 16\pi \text{ mi} \\ &\approx 50.24 \text{ mi} \end{aligned}$$
The circumference is exactly 16π miles, which is approximately 50.24 miles.

41. $C = \pi \cdot d = \pi \cdot 150 \text{ ft} = 150\pi \text{ ft} \approx 471 \text{ ft}$
The circumference of the barn is 150π feet, which is approximately 471 feet.

45. Sum the lengths of the sides.
$$\begin{aligned} P &= 9 \text{ mi} + 6 \text{ mi} + 11 \text{ mi} + 4.7 \text{ mi} \\ &= 30.7 \text{ mi} \end{aligned}$$
The perimeter is 30.7 miles.

49. The sides of a regular pentagon all have the same length.
Sum the lengths of the sides.
$$\begin{aligned} P &= 8 \text{ mm} + 8 \text{ mm} + 8 \text{ mm} + 8 \text{ mm} + 8 \text{ mm} \\ &= 40 \text{ mm} \end{aligned}$$
The perimeter is 40 millimeters.

53. $5 + 6 \cdot 3 = 5 + 18 = 23$

57. $72 \div (2 \cdot 6) = 72 \div 12 = 6$

61. a. The first age category that 8-year-old children fit into is "Under 9." Thus the minimum width is 30 yards, and the minimum length is 40 yards.

b.
$$\begin{aligned} P &= 2 \cdot l + 2 \cdot w \\ &= 2 \cdot 40 \text{ yd} + 2 \cdot 30 \text{ yd} \\ &= 80 \text{ yd} + 60 \text{ yd} \\ &= 140 \text{ yd} \end{aligned}$$
The perimeter of the field is 140 yards.

65. a. Smaller circle:
$$\begin{aligned} C &= 2 \cdot \pi \cdot r \\ &= 2 \cdot \pi \cdot 10 \text{ m} \\ &= 20\pi \text{ m} \\ &\approx 62.8 \text{ m} \end{aligned}$$
Larger circle:
$$\begin{aligned} C &= 2 \cdot \pi \cdot r \\ &= 2 \cdot \pi \cdot 20 \text{ m} \\ &= 40\pi \text{ m} \\ &\approx 125.6 \text{ m} \end{aligned}$$

b. Yes, when the radius of a circle is doubled, the circumference is also doubled.

69. Each of the three linear sides has length 6 meters. The length of the curved side is half of the circumference of a circle with diameter 6 meters, or
$$\frac{1}{2}\pi \cdot d = \frac{1}{2}\pi \cdot 6 \text{ meters} = 3\pi \text{ meters} \approx 9.4 \text{ meters}$$
$6 \text{ m} + 6 \text{ m} + 6 \text{ m} + 9.4 \text{ m} = 27.4 \text{ m}$
The perimeter is 27.4 meters.

73.
$$\begin{aligned} P &= 2 \cdot l + 2 \cdot w \\ 31 \text{ ft} &= 2 \cdot 9 \text{ ft} + 2 \cdot w \\ 31 \text{ ft} &= 18 \text{ ft} + 2 \cdot w \\ 13 \text{ ft} &= 2 \cdot w \\ 6.5 \text{ ft} &= w \end{aligned}$$
The width is 6.5 feet.

Exercise Set 9.4

1. $A = l \cdot w = 3.5 \text{ m} \cdot 2 \text{ m} = 7 \text{ sq m}$

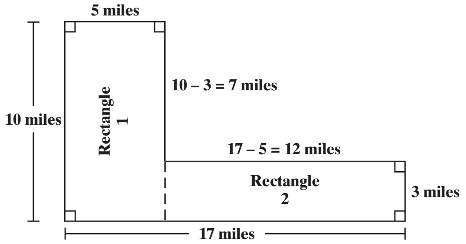
5. $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 6 \text{ yd} \cdot 5 \text{ yd} = 15 \text{ sq yd}$

9. $A = s^2 = (4.2 \text{ ft})^2 = 17.64 \text{ sq ft}$

13.
$$\begin{aligned} A &= \frac{1}{2}(b + B) \cdot h \\ &= \frac{1}{2}(7 \text{ yd} + 4 \text{ yd}) \cdot 4 \text{ yd} \\ &= \frac{1}{2}(11 \text{ yd}) \cdot 4 \text{ yd} \\ &= 22 \text{ sq yd} \end{aligned}$$

17.
$$\begin{aligned} A &= b \cdot h \\ &= 5 \text{ in.} \cdot 4\frac{1}{2} \text{ in.} \\ &= 5 \text{ in.} \cdot \frac{9}{2} \text{ in.} \\ &= \frac{45}{2} \text{ sq in.} \\ &= 22\frac{1}{2} \text{ sq in.} \end{aligned}$$

21.



Rectangle 1: $A = l \cdot w = 10 \text{ mi} \cdot 5 \text{ mi} = 50 \text{ sq mi}$
Rectangle 2: $A = l \cdot w = 12 \text{ mi} \cdot 3 \text{ mi} = 36 \text{ sq mi}$
The area of the figure is 50 sq mi + 36 sq mi = 86 sq mi.

25. $A = \pi r^2 = \pi(6 \text{ in.})^2 = 36\pi \text{ sq in.} \approx 113\frac{1}{7} \text{ sq in.}$

29. $A = l \cdot w = 505 \text{ ft} \cdot 255 \text{ ft} = 128{,}775 \text{ sq ft}$
The area of the flag is 128,775 square feet.

33. $A = l \cdot w = 16 \text{ in.} \cdot 8 \text{ in.} = 128 \text{ sq in.}$

$128 \text{ sq in.} \cdot \dfrac{1 \text{ sq ft}}{144 \text{ sq in.}} = \dfrac{128}{144} \text{ sq ft} = \dfrac{8}{9} \text{ sq ft}$

The side has area 128 square inches, which is $\frac{8}{9}$ square foot.

37. $A = l \cdot w = 7 \text{ ft} \cdot 6 \text{ ft} = 42 \text{ sq ft}$

$4 \cdot 42 \text{ sq ft} = 168 \text{ sq ft}$

Four panels have an area of 168 square feet.

41. **a.** $A = \frac{1}{2}(b + B) \cdot h$

$= \frac{1}{2}(25 \text{ ft} + 36 \text{ ft}) \cdot 12\frac{1}{2} \text{ ft}$

$= \frac{1}{2} \cdot 61 \text{ ft} \cdot 12\frac{1}{2} \text{ ft}$

$= 381\frac{1}{4} \text{ sq ft}$

To the nearest square foot, the area is 381 square feet.

b. Since each square covers 100 square feet, 4 squares of shingles need to be purchased.

45. Sum the lengths of the sides.

$3 \text{ ft} + 3\frac{1}{2} \text{ ft} + 6 \text{ ft} + 8\frac{1}{2} \text{ ft} + 4 \text{ ft} = 25 \text{ ft}$

The perimeter is 25 feet.

49. A fence goes around the edge of a yard, thus the situation involves perimeter.

53. Paint covers the surface of a wall, thus the situation involves area.

57. Note that the dimensions given are the diameters of the pizzas.

12-inch pizza:

$r = \frac{1}{2} \cdot d = \frac{1}{2} \cdot 12 \text{ in.} = 6 \text{ in.}$

$A = \pi \cdot r^2 = \pi(6 \text{ in.})^2 = 36\pi \text{ sq in.}$

Price per square inch $= \dfrac{\$10}{36\pi \text{ sq in.}} \approx \$0.0884$

8-inch pizzas:

$r = \frac{1}{2} \cdot d = \frac{1}{2} \cdot 8 \text{ in.} = 4 \text{ in.}$

$A = \pi \cdot r^2 = \pi(4 \text{ in.})^2 = 16\pi \text{ sq in.}$

$2 \cdot A = 2 \cdot 16\pi \text{ sq in.} = 32\pi \text{ sq in.}$

Price per square inch: $\dfrac{\$9}{32\pi \text{ sq in.}} \approx \$0.0895$

Since the price per square inch for the 12-inch pizza is less, the 12-inch pizza is the better deal.

61. The area of the shaded region is the area of the square minus the area of the circle.

Square: $A = s^2 = (6 \text{ in.})^2 = 36 \text{ sq in.}$

Circle: $r = \frac{1}{2} \cdot d = \frac{1}{2}(6 \text{ in.}) = 3 \text{ in.}$

$A = \pi \cdot r^2 = \pi(3 \text{ in.})^2 = 9\pi \text{ sq in.} \approx 28.26 \text{ sq in.}$

$36 \text{ sq in.} - 28.26 \text{ sq in.} = 7.74 \text{ sq in.}$

The shaded region has area of approximately 7.74 square inches.

65. The skating area is a rectangle with a half circle on each end. Rectangle: $A = l \cdot w = 22 \text{ m} \cdot 10 \text{ m} = 220 \text{ sq m}$

Half circles: $A = 2 \cdot \frac{1}{2} \cdot \pi \cdot r^2$

$= \pi(5 \text{ m})^2$

$= 25\pi \text{ sq m} \approx 78.5 \text{ sq m}$

$220 \text{ sq m} + 78.5 \text{ sq m} = 298.5 \text{ sq m}$

The skating surface has area of 298.5 square meters.

69. no; answers may vary

Exercise Set 9.5

1. $V = l \cdot w \cdot h = 6 \text{ in.} \cdot 4 \text{ in.} \cdot 3 \text{ in.} = 72 \text{ cu in.};$

$SA = 2lh + 2wh + 2lw$

$= 2(6 \text{ in.})(3 \text{ in.}) + 2(4 \text{ in.})(3 \text{ in.}) + 2(6 \text{ in.})(4 \text{ in.})$

$= 36 \text{ sq in.} + 24 \text{ sq in.} + 48 \text{ sq in.}$

$= 108 \text{ sq in.}$

5. $V = \frac{1}{3}\pi \cdot r^2 \cdot h = \frac{1}{3}\pi \cdot (2 \text{ yd})^2 \cdot 3 \text{ yd} = 4\pi \text{ cu yd} \approx 12\frac{4}{7} \text{ cu yd};$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

$= \pi(2 \text{ yd})\sqrt{(2 \text{ yd})^2 + (3 \text{ yd})^2} + \pi(2 \text{ yd})^2$

$= 2\pi \text{ yd } \sqrt{4 \text{ sq yd} + 9 \text{ sq yd}} + \pi(4 \text{ sq yd})$

$= 2\pi \text{ yd } \sqrt{13 \text{ sq yd}} + 4\pi \text{ sq yd}$

$= 2\pi \text{ yd } \sqrt{13} \text{ yd} + 4\pi \text{ sq yd}$

$= 2\sqrt{13}\,\pi \text{ sq yd} + 4\pi \text{ sq yd}$

$= (2\sqrt{13}\pi + 4\pi) \text{ sq yd} \approx 35.23 \text{ sq yd}$

9. $V = \pi \cdot r^2 \cdot h$

$= \pi(1 \text{ in.})^2 \cdot 9 \text{ in.}$

$= 9\pi \text{ cu in.}$

$\approx 28\frac{2}{7} \text{ cu in.}$

13. $V = s^3 = \left(1\frac{1}{3} \text{ in.}\right)^3$

$= \left(\frac{4}{3} \text{ in.}\right)^3$

$= \frac{64}{27} \text{ cu in.}$

$= 2\frac{10}{27} \text{ cu in.}$

17. $V = \frac{1}{3}s^2h$

$= \frac{1}{3}(12 \text{ cm})^2(20 \text{ cm})$

$= \frac{1}{3} \cdot 144 \cdot 20 \text{ cu cm}$

$= 960 \text{ cu cm}$

21. $V = l \cdot w \cdot h$

$= (2 \text{ ft}) \cdot \left(2\frac{1}{2} \text{ ft}\right) \cdot \left(1\frac{1}{2} \text{ ft}\right)$

$= 2 \cdot \frac{5}{2} \cdot \frac{3}{2} \text{ cu ft}$

$= \frac{15}{2} \text{ cu ft}$

$= 7\frac{1}{2} \text{ cu ft}$

25. $r = \frac{1}{2} \cdot d = \frac{1}{2} \cdot 3 \text{ m} = 1.5 \text{ m}$

$V = \frac{4}{3}\pi r^3$

$$\begin{aligned} &= \frac{4}{3}\pi(1.5\text{ m})^3 \\ &= \frac{4}{3}\pi\cdot 3.375\text{ cu m} \\ &= \pi\cdot 4.5\text{ cu m} \\ &\approx 14.13\text{ cu m} \end{aligned}$$

The exact volume is 4.5π cubic meters, which is about 14.13 cubic meters.

29. $r = \frac{1}{2}d = \frac{1}{2}(12\text{ ft}) = 6\text{ ft}$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(6\text{ ft})^3 \\ &= \frac{4}{3}\pi\cdot 216\text{ cu ft} \\ &= 288\pi\text{ cu ft} \end{aligned}$$

33. $V = lwh = (2\text{ in.})(2\text{ in.})(2.2\text{ in.}) = 8.8\text{ cu in.}$
The volume of the Space Cube is 8.8 cubic inches.

37. $5^2 = 5\cdot 5 = 25$

41. $1^2 + 2^2 = 1\cdot 1 + 2\cdot 2 = 1 + 4 = 5$

45. $r = \frac{1}{2}\cdot d = \frac{1}{2}\cdot 20\text{ m} = 10\text{ m}$

$$\begin{aligned} V &= \frac{1}{2}\cdot\frac{4}{3}\pi r^3 \\ &= \frac{2}{3}\pi(10\text{ m})^3 \\ &= \frac{2}{3}\pi\cdot 1000\text{ cu m} \\ &= \frac{2000}{3}\pi\text{ cu m} \\ &\approx \frac{2000}{3}\cdot 3.14\text{ cu m} \\ &\approx 2093.33\text{ cu m} \end{aligned}$$

The volume of the dome is about 2093.33 cubic meters.

49. Kennel **(a)**:

$$\begin{aligned} 2\frac{1}{12}\text{ ft}\cdot 1\frac{8}{12}\text{ ft}\cdot 1\frac{7}{12}\text{ ft} &= \left(\frac{25}{12}\cdot\frac{20}{12}\cdot\frac{19}{12}\right)\text{ cu ft} \\ &= \frac{9500}{1728}\text{ cu ft} \\ &= \frac{2375}{432}\text{ cu ft} \\ &\approx 5.5\text{ cu ft} \end{aligned}$$

Kennel **(b)**:

$$\begin{aligned} 1\frac{1}{12}\text{ ft}\cdot 2\text{ ft}\cdot 2\frac{8}{12}\text{ ft} &= \left(\frac{13}{12}\cdot\frac{2}{1}\cdot\frac{32}{12}\right)\text{ cu ft} \\ &= \frac{832}{144}\text{ cu ft} \\ &= \frac{52}{9}\text{ cu ft} \\ &\approx 5.8\text{ cu ft} \end{aligned}$$

Kennel **(b)** is larger.

Exercise Set 9.6

1. The triangles are congruent by Side-Side-Side.

5. The triangles are congruent by Angle-Side-Angle.

9. $\frac{22}{11} = \frac{14}{7} = \frac{12}{6} = \frac{2}{1}$

The ratio of corresponding sides is $\frac{2}{1}$.

13.
$$\begin{aligned} \frac{n}{3} &= \frac{9}{6} \\ 6\cdot n &= 3\cdot 9 \\ 6\cdot n &= 27 \\ n &= \frac{27}{6} \\ n &= 4.5 \end{aligned}$$

17.
$$\begin{aligned} \frac{n}{3.75} &= \frac{12}{9} \\ 9\cdot n &= 12\cdot 3.75 \\ 9\cdot n &= 45 \\ n &= \frac{45}{9} \\ n &= 5 \end{aligned}$$

21.
$$\begin{aligned} \frac{n}{3.25} &= \frac{17.5}{3.25} \\ 3.25\cdot n &= 17.5\cdot 3.25 \\ 3.25\cdot n &= 56.875 \\ n &= \frac{56.875}{3.25} \\ n &= 17.5 \end{aligned}$$

25.
$$\begin{aligned} \frac{n}{60} &= \frac{15}{32} \\ 32\cdot n &= 60\cdot 15 \\ 32\cdot n &= 900 \\ n &= \frac{900}{32} \\ n &= 28.125 \end{aligned}$$

29.
$$\begin{aligned} \frac{n}{13} &= \frac{80}{2} \\ 2\cdot n &= 80\cdot 13 \\ 2\cdot n &= 1040 \\ n &= \frac{1040}{2} \\ n &= 520 \end{aligned}$$

The observation deck is 520 feet high.

33.
$$\begin{aligned} \frac{n}{5} &= \frac{48}{4} \\ \frac{n}{5} &= \frac{12}{1} \\ 1\cdot n &= 12\cdot 5 \\ n &= 60 \end{aligned}$$

The tree is 60 feet tall.

37.
$$\begin{aligned} \frac{n}{55} &= \frac{19}{20} \\ 20\cdot n &= 55\cdot 19 \\ 20\cdot n &= 1045 \\ n &= \frac{1045}{20} \\ n &= 52.25 \end{aligned}$$

Pete would place 52 neon tetras in a 55-gallon tank.

41.
$$\begin{array}{r} \overset{1}{3}.60 \\ +\ 0.41 \\ \hline 4.01 \end{array}$$

45.
$$\begin{aligned} \frac{n}{7\text{ in.}} &= \frac{5\text{ in.}}{9\text{ in.}} \\ \frac{n}{7} &= \frac{5}{9} \\ 7\cdot 5 &= 9\cdot n \\ 35 &= 9\cdot n \\ \frac{35}{9} &= n \\ n &= \frac{35}{9}\text{ or }3\frac{8}{9}\text{ in.} \end{aligned}$$

The new width is $3\frac{8}{9}$ in. Since the print area is now $3\frac{8}{9}$ in. by 5 in., it will not fit on a 3-by-5 inch index card. (Lengths are same but $3\frac{8}{9} > 3$.)

49. answers may vary

Chapter 9 Test

1. The complement of an angle that measures 78° is an angle that measures $90° - 78° = 12°$.

5. $\angle x$ and the angle marked 73° are vertical angles, so $m\angle x = 73°$. $\angle x$ and $\angle y$ are alternate interior angles, so $m\angle y = m\angle x = 73°$. $\angle x$ and $\angle z$ are corresponding angles, so $m\angle z = m\angle x = 73°$.

9. Circumference:

$$\begin{aligned} C &= 2 \cdot \pi \cdot r \\ &= 2 \cdot \pi \cdot 9 \text{ in.} \\ &= 18\pi \text{ in.} \\ &\approx 56.52 \text{ in.} \end{aligned}$$

Area:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(9 \text{ in.})^2 \\ &= 81\pi \text{ sq in.} \\ &\approx 254.34 \text{ sq in.} \end{aligned}$$

13. $V = l \cdot w \cdot h = 5 \text{ ft} \cdot 3 \text{ ft} \cdot 2 \text{ ft} = 30 \text{ cu ft}$;

$$\begin{aligned} SA &= 2lh + 2wh + 2lw \\ &= 2(5 \text{ ft})(2 \text{ ft}) + 2(3 \text{ ft})(2 \text{ ft}) + 2(5 \text{ ft})(3 \text{ ft}) \\ &= 20 \text{ sq ft} + 12 \text{ sq ft} + 30 \text{ sq ft} = 62 \text{ sq ft} \end{aligned}$$

17. First find the area of the lawn.

$A = l \cdot w = 123.8 \text{ ft} \cdot 80 \text{ ft} = 9904 \text{ sq ft}$

0.02 ounce per square foot is $\frac{0.02 \text{ oz}}{1 \text{ sq ft}}$.

$$\frac{0.02 \text{ oz}}{1 \text{ sq ft}} \cdot 9904 \text{ sq ft} = 198.08 \text{ oz}$$

The area of Vivian's lawn is 9904 sq ft and she needs to purchase 198.08 ounces of insecticide.

Appendices

Appendix A

Exercise Set A.1

1. $\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$

5. $\begin{array}{r} 3 \\ +9 \\ \hline 12 \end{array}$

9. $\begin{array}{r} 9 \\ +5 \\ \hline 14 \end{array}$

13. $\begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array}$

17. $\begin{array}{r} 2 \\ +9 \\ \hline 11 \end{array}$

21. $\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$

25. $\begin{array}{r} 9 \\ +8 \\ \hline 17 \end{array}$

29. $\begin{array}{r} 7 \\ +5 \\ \hline 12 \end{array}$

33. $\begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$

37. $\begin{array}{r} 7 \\ +1 \\ \hline 8 \end{array}$

41. $\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array}$

45. $\begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array}$

49. $\begin{array}{r} 2 \\ +5 \\ \hline 7 \end{array}$

53. $\begin{array}{r} 8 \\ +3 \\ \hline 11 \end{array}$

57. $\begin{array}{r} 0 \\ +5 \\ \hline 5 \end{array}$

61. $\begin{array}{r} 6 \\ +7 \\ \hline 13 \end{array}$

65. $\begin{array}{r} 0 \\ +7 \\ \hline 7 \end{array}$

69. $\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$

73. $\begin{array}{r} 7 \\ +9 \\ \hline 16 \end{array}$

77. $\begin{array}{r} 1 \\ +0 \\ \hline 1 \end{array}$

81. $\begin{array}{r} 2 \\ +8 \\ \hline 10 \end{array}$

85. $\begin{array}{r} 5 \\ +0 \\ \hline 5 \end{array}$

89. $\begin{array}{r} 9 \\ +2 \\ \hline 11 \end{array}$

93. $\begin{array}{r} 2 \\ +7 \\ \hline 9 \end{array}$

97. $\begin{array}{r} 6 \\ +8 \\ \hline 14 \end{array}$

Exercise Set A.2

1. $\begin{array}{r} 1 \\ \times 1 \\ \hline 1 \end{array}$

5. $\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \end{array}$

9. $\begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array}$

13. $\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \end{array}$

17. $\begin{array}{r} 4 \\ \times 6 \\ \hline 24 \end{array}$

21. $\begin{array}{r} 5 \\ \times 8 \\ \hline 40 \end{array}$

25. $\begin{array}{r} 9 \\ \times 6 \\ \hline 54 \end{array}$

29. $\begin{array}{r} 6 \\ \times 9 \\ \hline 54 \end{array}$

33. $\begin{array}{r} 4 \\ \times 9 \\ \hline 36 \end{array}$

37. $\begin{array}{r} 9 \\ \times 4 \\ \hline 36 \end{array}$

41. $\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$

45. $\begin{array}{r} 9 \\ \times 0 \\ \hline 0 \end{array}$

49. $\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$

53. $\begin{array}{r} 1 \\ \times 0 \\ \hline 0 \end{array}$

57. $\begin{array}{r} 0 \\ \times 6 \\ \hline 0 \end{array}$

61. $\begin{array}{r} 9 \\ \times 2 \\ \hline 18 \end{array}$

65. $\begin{array}{r} 6 \\ \times 6 \\ \hline 36 \end{array}$

69. $\begin{array}{r} 7 \\ \times 5 \\ \hline 35 \end{array}$

73. $\begin{array}{r} 8 \\ \times 5 \\ \hline 40 \end{array}$

77. $\begin{array}{r} 9 \\ \times 1 \\ \hline 9 \end{array}$

81. $\begin{array}{r} 1 \\ \times 5 \\ \hline 5 \end{array}$

85. $\begin{array}{r} 3 \\ \times 7 \\ \hline 21 \end{array}$

89. $\begin{array}{r} 8 \\ \times 2 \\ \hline 16 \end{array}$

93. $\begin{array}{r} 4 \\ \times 1 \\ \hline 4 \end{array}$

97. $\begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array}$

Appendix B

Exercise Set B.1

1. $\begin{aligned}(2x + 3) + (-7x - 27) &= (2x - 7x) + (3 - 27)\\ &= -5x + (-24)\\ &= -5x - 24\end{aligned}$

5. $\begin{aligned}&(12y - 20) + (9y^2 + 13y - 20)\\ &= 9y^2 + (12y + 13y) + (-20 - 20)\\ &= 9y^2 + 25y - 40\end{aligned}$

9. $\begin{aligned}(5a - 6) - (a + 2) &= (5a - 6) + (-a - 2)\\ &= (5a - a) + (-6 - 2)\\ &= 4a - 8\end{aligned}$

13. $\begin{aligned}&(10y^2 - 7) - (20y^3 - 2y^2 - 3)\\ &= (10y^2 - 7) + (-20y^3 + 2y^2 + 3)\\ &= -20y^3 + (10y^2 + 2y^2) + (-7 + 3)\\ &= -20y^3 + 12y^2 - 4\end{aligned}$

17. $\begin{array}{r} 13y^2 - 6y - 14\\ \underline{-(5y^2 + 4y - 6)}\end{array} \qquad \begin{array}{r} 13y^2 - 6y - 14\\ \underline{+\ -5y^2 - 4y + 6}\\ 8y^2 - 10y - 8\end{array}$

21. $\begin{aligned}(4y + 4) - (3y + 8) &= (4y + 4) + (-3y - 8)\\ &= (4y - 3y) + (4 - 8)\\ &= y - 4\end{aligned}$

25. $\begin{aligned}(10x + 4.5) + (-x - 8.6) &= (10x - x) + (4.5 - 8.6)\\ &= 9x - 4.1\end{aligned}$

29. $\begin{aligned}&(21y - 4.6) - (36y - 8.2)\\ &= (21y - 4.6) + (-36y + 8.2)\\ &= (21y - 36y) + (-4.6 + 8.2)\\ &= -15y + 3.6\end{aligned}$

33. $\begin{aligned}&(b^3 - 2b^2 + 10b + 11) + (b^2 - 3b - 12)\\ &= b^3 + (-2b^2 + b^2) + (10b - 3b) + (11 - 12)\\ &= b^3 - b^2 + 7b - 1\end{aligned}$

37. $\begin{aligned}&\left(-3z + \frac{6}{7}\right) - \left(3z - \frac{3}{7}\right)\\ &= \left(-3z + \frac{6}{7}\right) + \left(-3z + \frac{3}{7}\right)\\ &= (-3z - 3z) + \left(\frac{6}{7} + \frac{3}{7}\right)\\ &= -6z + \frac{9}{7}\end{aligned}$

41. Replace x with 2.
$\begin{aligned}x^2 - 6x + 3 &= (2)^2 - 6(2) + 3\\ &= 4 - 6(2) + 3\\ &= 4 - 12 + 3\\ &= -5\end{aligned}$

45. Replace x with 5.
$2x + 10 = 2(5) + 10 = 10 + 10 = 20$

49. Replace x with 5.
$\begin{aligned}2x^2 + 4x - 20 &= 2(5)^2 + 4(5) - 20\\ &= 2(25) + 4(5) - 20\\ &= 50 + 20 - 20\\ &= 50\end{aligned}$

53. Evaluate $3000 + 20x$ for $x = 10$.
$3000 + 20x = 3000 + 20(10) = 3000 + 200 = 3200$
The total cost to manufacture 10 file cabinets is \$3200.

57. $\begin{aligned}&(2x + 1) + (x + 11) + (5x - 10)\\ &= (2x + x + 5x) + (1 + 11 - 10)\\ &= 8x + 2\end{aligned}$
The perimeter is $(8x + 2)$ inches.

61. $\begin{array}{r} 3x^2 + \underline{\ }x - \underline{\ }\\ \underline{+\ \underline{\ }x^2 - 6x + 2}\\ 5x^2 + 14x - 4\end{array}$

$\left[\begin{array}{l}(3 + \underline{\quad})x^2 = 5x\\ (3 + \underline{2})x^2 = 5x^2\end{array}\right]$

$\left[\begin{array}{l}(\underline{\quad} - 6)x = 14x\\ (\underline{20} - 6)x = 14x\end{array}\right]$

$\left[\begin{array}{l}(\underline{\quad} + 2) = -4\\ (-\underline{6} + 2) = -4\end{array}\right]$

$\begin{array}{r} 3x^2 + \underline{20}x - \underline{6}\\ \underline{+2x^2 - 6x + 2}\\ 5x^2 + 14x - 4\end{array}$

65. When $t = 8$ seconds:
$\begin{aligned}1053 - 16t^2 &= 1053 - 16(8)^2\\ &= 1053 - 16 \cdot 64\\ &= 1053 - 1024\\ &= 29\end{aligned}$
The height of the object above the river after 8 seconds is 29 feet.
When $t = 9$ seconds:
$\begin{aligned}1053 - 16t^2 &= 1053 - 16(9)^2\\ &= 1053 - 16 \cdot 81\\ &= 1053 - 1296\\ &= -243\end{aligned}$
The height of the object above the river after 9 seconds is -243 feet. answers may vary

Exercise Set B.2

1. $x^5 \cdot x^9 = x^{5+9} = x^{14}$

5. $3z^3 \cdot 5z^2 = (3 \cdot 5)(z^3 \cdot z^2) = 15z^5$

9. $\begin{aligned}(-5x^2y^3)(-5x^4y) &= (-5)(-5)(x^2 \cdot x^4)(y^3 \cdot y)\\ &= 25x^6y^4\end{aligned}$

13. $2x \cdot 3x \cdot 7x = (2 \cdot 3 \cdot 7)(x \cdot x \cdot x) = 42x^3$

17. $(x^5)^3 = x^{5 \cdot 3} = x^{15}$

21. $\begin{aligned}(b^7)^6(b^2)^{10} &= b^{7 \cdot 6} \cdot b^{2 \cdot 10}\\ &= b^{42} \cdot b^{20}\\ &= b^{42+20}\\ &= b^{62}\end{aligned}$

25. $(a^{11}b^8)^3 = a^{11 \cdot 3}b^{8 \cdot 3} = a^{33}b^{24}$

29. $\begin{aligned}(-3y)(2y^7)^3 &= (-3y) \cdot 2^3(y^7)^3\\ &= (-3y) \cdot 8y^{21}\\ &= (-3)(8)(y^1 \cdot y^{21})\\ &= -24y^{22}\end{aligned}$

33. $(4x^6)^2 = 4^2(x^6)^2 = 16x^{12}$
The area is $16x^2$ square inches.

37. $\begin{aligned}&(14a^7b^6)^3(9a^6b^3)^4\\ &= (14)^3(a^7)^3(b^6)^3(9)^4(a^6)^4(b^3)^4\\ &= 2744a^{21}b^{18} \cdot 6561a^{24}b^{12}\\ &= (2744 \cdot 6561)(a^{21} \cdot a^{24})(b^{18} \cdot b^{12})\\ &= 18{,}003{,}384a^{45}b^{30}\end{aligned}$

41. $\begin{aligned}(a^{20}b^{10}c^5)^5(a^9b^{12})^3 &= (a^{20})^5(b^{10})^5(c^5)^5 \cdot (a^9)^3(b^{12})^3\\ &= a^{100}b^{50}c^{25} \cdot a^{27}b^{36}\\ &= a^{127}b^{86}c^{25}\end{aligned}$

Exercise Set B.3

1. $\begin{aligned}3x(9x^2 - 3) &= 3x \cdot 9x^2 + 3x \cdot (-3)\\ &= (3 \cdot 9)(x \cdot x^2) + (3)(-3)(x)\\ &= 27x^3 + (-9x)\\ &= 27x^3 - 9x\end{aligned}$

5. $7x^2(6x^2 - 5x + 7)$
$= (7x^2)(6x^2) + (7x^2)(-5x) + (7x^2)(7)$
$= (7 \cdot 6)(x^2 \cdot x^2) + (7)(-5)(x^2 \cdot x) + (7 \cdot 7)x^2$
$= 42x^4 - 35x^3 + 49x^2$

9. $(2x - 6)(x + 4) = 2x(x + 4) - 6(x + 4)$
$= 2x \cdot x + 2x \cdot 4 - 6 \cdot x - 6 \cdot 4$
$= 2x^2 + 8x - 6x - 24$
$= 2x^2 + 2x - 24$

13. $(a + 6)(a^2 - 6a + 3)$
$= a(a^2 - 6a + 3) + 6(a^2 - 6a + 3)$
$= a \cdot a^2 + a(-6a) + a \cdot 3 + 6 \cdot a^2 + 6(-6a) + 6 \cdot 3$
$= a^3 - 6a^2 + 3a + 6a^2 - 36a + 18$
$= a^3 - 33a + 18$

17. $(x^3 + 2x + x^2)(3x + 1 + x^2)$
$= x^3(3x + 1 + x^2) + 2x(3x + 1 + x^2)$
$+ x^2(3x + 1 + x^2)$
$= x^3 \cdot 3x + x^3 \cdot 1 + x^3 \cdot x^2 + 2x \cdot 3x + 2x \cdot 1$
$+ 2x \cdot x^2 + x^2 \cdot 3x + x^2 \cdot 1 + x^2 \cdot x^2$
$= 3x^4 + x^3 + x^5 + 6x^2 + 2x + 2x^3$
$+ 3x^3 + x^2 + x^4$
$= x^5 + 4x^4 + 6x^3 + 7x^2 + 2x$

21. $-2y^2(3y + y^2 - 6)$
$= -2y^2 \cdot 3y + (-2y^2) \cdot y^2 + (-2y^2)(-6)$
$= -6y^3 - 2y^4 + 12y^2$

25. $(2a + 3)(2a - 3)$
$= 2a(2a - 3) + 3(2a - 3)$
$= 2a \cdot 2a + 2a(-3) + 3 \cdot 2a + 3(-3)$
$= 4a^2 - 6a + 6a - 9$
$= 4a^2 - 9$

29. $\left(b + \frac{3}{5}\right)\left(b + \frac{4}{5}\right) = b\left(b + \frac{4}{5}\right) + \frac{3}{5}\left(b + \frac{4}{5}\right)$
$= b^2 + \frac{4}{5}b + \frac{3}{5}b + \frac{3}{5} \cdot \frac{4}{5}$
$= b^2 + \frac{7}{5}b + \frac{12}{25}$

33. $(7x + 5)^2 = (7x + 5)(7x + 5)$
$= 7x(7x + 5) + 5(7x + 5)$
$= 49x^2 + 35x + 35x + 25$
$= 49x^2 + 70x + 25$

37. $(2x^2 - 3)(4x^3 + 2x - 3)$
$= 2x^2(4x^3 + 2x - 3) - 3(4x^3 + 2x - 3)$
$= 8x^5 + 4x^3 - 6x^2 - 12x^3 - 6x + 9$
$= 8x^5 - 8x^3 - 6x^2 - 6x + 9$

41.
$$\begin{array}{rrrrrr}
 & & & 2z^2 - & z & + 1 \\
 & \times & & 5z^2 + & z & - 2 \\
\hline
 & & & -4z^2 + & 2z & - 2 \\
 & & 2z^3 - & z^2 + & z & \\
10z^4 & - & 5z^3 + & 5z^2 & & \\
\hline
10z^4 & - & 3z^3 & & + 3z & - 2
\end{array}$$

45. larger area − smaller area
$= (x^2 - 1)(x^2 - 1) - (x)(x)$
$= x^2(x^2 - 1) - 1(x^2 - 1) - (x)(x)$
$= x^4 - x^2 - x^2 + 1 - x^2$
$= x^4 - 3x^2 + 1$

The area of the shaded figure is $(x^4 - 3x^2 + 1)$ square meters.

Appendix C

Exercise Set

1. This is a general conclusion based on specific observations, which is inductive reasoning.

5. This is a general conclusion based on specific observations, which is inductive reasoning.

9. A, Z, B, Y, C

These are letters of the alphabet in forward order in positions 1 and 3; the second and fourth letters are the letters of the alphabet in reverse order. So, the fifth letter must be the letter that comes directly after B, namely C.

13. O, T, T, F, F, S, S

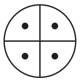
These are the first letters of the first seven counting numbers: One, Two, Three, Four, Five, Six, Seven, so the next letter should be E (Eight).

17. 2, 7, 4, 8, 6

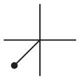
The first, third, and fifth numbers are even numbers in increasing order. The second and fourth numbers are counting numbers beginning at 7. So, the next number, the sixth, should be the next counting number after 8, which is 9.

21. 1, 11, 38, 84, 151
↘↙↘↙↘↙↘↙
10 27 46 67
↘↙↘↙↘↙
17 19 21
↘↙↘↙
2 2
21 + 2 = 23
67 + 23 = 90
151 + 90 = 241
The next number should be 241.

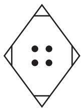
25. The first figure is a circle divided in half and the second figure is the same as the first figure with a dot added in each sector of the circle. The third figure is a circle divided in four, so the fourth figure should be a circle divided into fourths with a dot added in each sector of the circle.



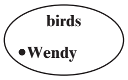
29. The figures are plus signs, each with an added segment from the middle of the plus sign diagonally out. The added segments start in the lower right and rotate counterclockwise to the upper right, then to the upper left. The segments alternate between having an open dot and a solid dot at the 'outer' end. The fourth figure should be a plus sign with the added segment extending to the lower left, ending with a solid dot.



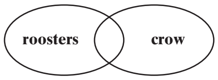
33. The figures are diamonds with increasing numbers of dots and with segments in the same number of corners as there are dots in the figure. The fourth figure should be a diamond with four dots and with segments in all four corners.



37.



41.



45. Since Sal finished ahead of Mark, who finished ahead of Bob, and Joey finished between Mark and Bob, the order of finishing was Sal, Mark, Joey, Bob, and Joey finished third.

49. From statement **a**, we can conclude that either Donna or Maria is the electrical engineer. From statement **b**, we can conclude that Donna is not the electrical engineer, thus Maria is. This means that Maria is not the civil engineer. From statement **c**, Marty is not the civil engineer. From statement **d**, Aaron is not the civil engineer. So Donna must be the civil engineer.

53. 10 people said they drink Coke and Pepsi. A total of 65 people said they drink Coke. This means that $65 - 10 = 55$ is the number of people that drink only Coke. Since a total of 40 people said they drink Pepsi, $40 - 10 = 30$ people only drink Pepsi. This means that $55 + 10 + 30 = 95$ people drink Coke or Pepsi. Since 100 people were polled, $100 - 95 = 5$ people drink neither Coke nor Pepsi.

Practice Final Exam

1.
$$\begin{array}{r} 600 \\ \underline{-487} \\ 113 \end{array}$$

5.
$$\begin{aligned}\frac{64 \div 8 \cdot 2}{(\sqrt{9} - \sqrt{4})^2 + 1} &= \frac{8 \cdot 2}{(3 - 2)^2 + 1} = \frac{16}{1^2 + 1} \\ &= \frac{16}{1 + 1} \\ &= \frac{16}{2} \\ &= 8\end{aligned}$$

9.
$$\begin{aligned}3\frac{1}{3} \cdot 6\frac{3}{4} = \frac{10}{3} \cdot \frac{27}{4} = \frac{10 \cdot 27}{3 \cdot 4} &= \frac{2 \cdot 5 \cdot 3 \cdot 9}{3 \cdot 2 \cdot 2} \\ &= \frac{5 \cdot 9}{2} \\ &= \frac{45}{2} \\ &= 22\frac{1}{2}\end{aligned}$$

13. $-7 - (-19) = -7 + 19 = 12$

17. To round 34.8923 to the nearest tenth, observe that the digit in the hundredths place is 9. Since the digit is at least 5, we add 1 to the digit in the tenths place and drop all digits to the right of the tenths place. The number 34.8923 rounded to the nearest tenth is 34.9.

21. $\frac{3}{8} = \frac{3}{8}(100\%) = \frac{300}{8}\% = 37.5\%$

$$\begin{array}{r} 37.5 \\ 8\overline{)300.0} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 4\,0 \\ \underline{-4\,0} \\ 0 \end{array}$$

25. The rate of 9 inches in 30 days is $\frac{9\,\text{inches}}{30\,\text{days}} = \frac{3\,\text{inches}}{10\,\text{days}}$.

29.
$$\begin{array}{r} 20.0 \\ \underline{-8.6} \\ 11.4 \end{array}$$

33. 0.6% of what number is 7.5?

Method 1:
$$\begin{aligned}0.6\% \cdot n &= 7.5 \\ 0.006 \cdot n &= 7.5 \\ n &= \frac{7.5}{0.006} \\ n &= 1250\end{aligned}$$
0.6% of 1250 is 7.5.

Method 2:
$$\begin{aligned}\frac{7.5}{b} &= \frac{0.6}{100} \\ 7.5 \cdot 100 &= 0.6 \cdot b \\ 750 &= 0.6 \cdot b \\ \frac{750}{0.6} &= b \\ 1250 &= b\end{aligned}$$
0.6% of 1250 is 7.5.

37. $40 \text{ mg} = \frac{40 \text{ mg}}{1} \cdot \frac{1 \text{ g}}{1000 \text{ mg}} = \frac{40}{1000} \text{ g} = 0.04 \text{ g}$

41. 5 of the students are 5′0″ − 5′3″ and 6 of the students are 5′3″ − 5′7″, so $5 + 6 = 11$ of the students are 5′7″ or shorter.

45.
$$\begin{aligned}5x + 12 - 4x - 14 &= 22 \\ 5x - 4x + 12 - 14 &= 22 \\ x - 2 &= 22 \\ x - 2 + 2 &= 22 + 2 \\ x &= 24\end{aligned}$$

49. $\angle x$ and the angle marked 62° are supplementary, so $m\angle x = 180° - 62° = 118°$.
$\angle y$ and the angle marked 62° are vertical angles, so $m\angle y = 62°$.
$\angle x$ and $\angle z$ are vertical angles, so $m\angle z = m\angle x = 118°$.

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