CHAPTER 15

Section 15-2

15-1 1. The parameter of interest is median of pH.
2. $H_0 : \tilde{\mu} = 7.0$
3 $H_1 : \tilde{\mu} \neq 7.0$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 8$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 3$ is less than or equal to $\alpha = 0.05$.
7. Using the binomial distribution with $n = 10$ and $p = 0.5$, *P-value* = $2P(R^+ \geq 3 \mid p = 0.5) = 1$
8. Conclusion: we cannot reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the pH is 7.0

15-2 1. The parameter of interest is median titanium content.
2. $H_0 : \tilde{\mu} = 8.5$
3 $H_1 : \tilde{\mu} \neq 8.5$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 7$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 7$ is less than or equal to α=0.05.
7. Using the binomial distribution with n=10 and p=0.5, *P-value* = $2P(R^* \leq 7|p=0.5)=0.359$
8. Conclusion: we cannot reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the titanium content is 8.5.

15-3 1. Parameter of interest is the median impurity level.
2. $H_0 : \tilde{\mu} = 2.5$
3. $H_1 : \tilde{\mu} < 2.5$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 2$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 2$ is less than or equal to $\alpha = 0.05$.
7. Using the binomial distribution with $n = 22$ and $p = 0.5$, *P-value* = $P(R^+ \leq 2 \mid p = 0.5) = 0.0002$
8. Conclusion, reject H_0. The data supports the claim that the median is impurity level is less than 2.5.

15-4 1. The parameter of interest is median of pH.
2. $H_0 : \tilde{\mu} = 7.0$
3 $H_1 : \tilde{\mu} \neq 7.0$
4. α=0.05
5. The test statistic is $z_0 = \dfrac{r^* - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if $|Z_0|>1.96$ for α=0.05.
7. r*=8 and $z_0 = \dfrac{r^* - 0.5n}{0.5\sqrt{n}} = \dfrac{8 - 0.5(10)}{0.5\sqrt{10}} = 1.90$
8. Conclusion: we cannot reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the pH is 7.0

P-value = $2[1 - P(|Z_0| < 1.90)] = 2(0.0287) = 0.0574$

15-5 a)

1. Parameter of interest is the median compressive strength
2. $H_0 : \tilde{\mu} = 2250$
3. $H_1 : \tilde{\mu} > 2250$
4. $\alpha = 0.05$
5. The test statistic is the observed number of plus differences or $r^+ = 7$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 7$ is less than or equal to α=0.05.
7. Using the binomial distribution with $n = 12$ and $p = 0.5$, *P-value* = $P(R^+ \geq 7 \mid p = 0.5) = 0.3872$
8. Conclusion, cannot reject H_0. There is not enough evidence to conclude that the median compressive strength is greater than 2250.

b)

1. Parameter of interest is the median compressive strength
2. $H_0 : \tilde{\mu} = 2250$
3. $H_1 : \tilde{\mu} > 2250$
4. α=0.05
5. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$
7. Computation: $z_0 = \dfrac{7 - 0.5(12)}{0.5\sqrt{12}} = 0.577$
8. Conclusion, cannot reject H_0. The *P-value* = 1-Φ(0.58) = 1 - 0.7190 = 0.281. There is not enough evidence to conclude that the median compressive strength is greater than 2250.

15-6

1. Parameter of interest is the median margarine fat content
2. $H_0 : \tilde{\mu} = 17.0$
3. $H_1 : \tilde{\mu} \neq 17.0$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 3$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 7$ is less than or equal to α=0.05.
7. Using the binomial distribution with n=6 and p=0.5, *P-value* = $2^*P(R^* \geq 3|p=0.5,n=6)=1$.
8. Conclusion, cannot reject H_0. There is not enough evidence to conclude that the median fat content differs from 17.0.

15-7

1. Parameter of interest is the median titanium content
2. $H_0 : \tilde{\mu} = 8.5$
3. $H_1 : \tilde{\mu} \neq 8.5$
4. α=0.05
5. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$
7. Computation: $z_0 = \dfrac{7 - 0.5(20)}{0.5\sqrt{20}} = -1.34$
8. Conclusion, cannot reject H_0. There is not enough evidence to conclude that the median titanium content differs from 8.5. The *P-value* = 2*P($|Z|$ > 1.34) = 0.1802.

15-8 1. Parameter of interest is the median impurity level

2. $H_0 : \tilde{\mu} = 2.5$

3. $H_1 : \tilde{\mu} < 2.5$

4. α=0.05

5. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$

6. We reject H_0 if the $Z_0 < Z_{0.05} = -1.65$

7. Computation: $z_0 = \dfrac{2 - 0.5(22)}{0.5\sqrt{22}} = -3.84$

8. Conclusion, reject H_0 and conclude that the median impurity level is less than 2.5. The *P-value* = P(Z<-3.84) = 0.000062

15-9 1. Parameters of interest are the median hardness readings for the two tips

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.05

5. The test statistic is $r = min(r^+, r^-)$.

6. Because $\alpha = 0.05$ and $n = 8$, Appendix A, Table VII gives the critical value of $r^*_{0.05} = 0$. We reject H_0 in favor of H_1 if $r \leq 0$.

7. $r^+ = 6$ and $r^- = 2$ and $r = min(6,2) = 2$

8. Conclusion, cannot reject H_0. There is not a significant difference in the tips.

15-10 1. Parameters of interest are the median drying times for the two formulations

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.01

5. The test statistic is $r = min(r^+, r^-)$.

6. Because $\alpha = 0.01$ and $n = 20$, Appendix A, Table VII gives the critical value of $r^*_{0.01} = 3$. We reject H_0 in favor of H_1 if $r \leq 3$.

7. There are two ties that are ignored so that $r^+ = 3$ and $r^- = 15$ and $r = min(3,15) = 3$

8. Conclusion, reject H_0. There is a significant difference in the drying times of the two formulations at α = 0.01.

15-11 1. Parameters of interest are the median drying times of the two formulations

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.01

5. Test statistic $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$

6. We reject H_0 if the $|Z_0| > Z_{0.005} = 2.58$

7. Computation: $z_0 = \dfrac{3 - 0.5(18)}{0.5\sqrt{18}} = -2.83$

8. Conclusion, reject H_0 and conclude that there is a significant difference between the drying times for the two formulations at α = 0.01.

The *P-value* = 2[1-P($|Z_0| < 2.83$) = 0.005.

15-12 1. Parameters of interest are the median caliper measurements

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.05

5. The test statistic is $r = min(r^+, r^-)$.

6. Because α =0.05 and n = 12, Appendix A, Table VII gives the critical value of $r^*_{0.05}$ = 2. We reject H_0 in favor of H_1 if $r \leq 2$.

7. There are four ties that are ignored so that $r^+ = 6$ and $r^- = 2$ and $r = min(6,2) = 2$

8. Conclusion, reject H_0. There is a significant difference in the median measurements of the two calipers at $\alpha = 0.05$.

15-13 1. Parameters of interest are the median blood cholesterol levels

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.05

5. The test statistic $r = min(r^+, r^-)$.

6. Because α = 0.05 and n = 15, Appendix A, Table VII gives the critical value of $r^*_{0.05}$ = 3. We reject H_0 in favor of H_1 if $r \leq 3$.

7. $r^+ = 14$ and $r^- = 1$ and $r = min(14,1) = 1$

8. Conclusion, reject H_0. There a significant difference in the blood cholesterol levels at $\alpha = 0.05$.

15-14 1. Parameters of interest are the median caliper measurements

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.05

5. Test statistic $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$

6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$

7. Computation: $z_0 = \dfrac{6 - 0.5(8)}{0.5\sqrt{8}} = 1.41$

8. Conclusion, do not reject H_0. There is not a significant difference in the median measurements of the two calipers at $\alpha = 0.05$.

The *P-value* = 2[1-P($|Z_0| < 1.41$) = 0.159.

15-15 1. Parameter of interest is the difference in blood cholesterol levels

2. $H_0 : \tilde{\mu}_D = 0$

3. $H_1 : \tilde{\mu}_D \neq 0$

4. α=0.05

5. Test statistic $z_0 = \dfrac{r^+ - 0.5n}{0.5n}$

6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$

7. Computation: $z_0 = \dfrac{14 - 0.5(15)}{0.5\sqrt{15}} = 3.36$

8. Conclusion, reject H_0. There is a difference between the blood cholesterol levels.

The *P-value* = 2*P ($|Z| > 3.36$) = 0.0008.

15-16 a) $f(x) = \lambda e^{-\lambda x}$ for x > 0. Because 3.5 is the median $\int_0^{3.5} \lambda e^{-\lambda x} = 0.5$. Solving for λ, we find λ = 0.198 and E(X) = 1/λ = 5.05

b) Because $r^* = 3 > 1$, do not reject H_0

c) β = P(Type II error)=P($R^* > 1$| $\tilde{\mu}$ =4.5)

For $\tilde{\mu}$ = 4.5, E(X) = 6.49 and P(X < 3.5| $\tilde{\mu}$ =4.5) = $\int_0^{3.5} 0.154e^{-0.154x}$ = 0.4167. Therefore, with

n =10, *p* = 0.4167, β = P(Type II error)=P($R^* > 1$| $\tilde{\mu}$ =4.5) = 0.963

15-17 a) α = P(Z > 1.96) = 0.025

b) $\beta = P\left(\frac{\overline{X}}{\sigma/\sqrt{n}} = 1.96 \mid \mu = 1\right) = P(Z < -1.20) = 0.115$

c) The sign test that rejects if $R^- \leq 1$ has α = 0.011 based on the binomial distribution.

d) $\beta = P(R^- \geq 2 \mid \mu = 1) = P(Z > 1) = 0.1587$. Therefore, R^- has a binomial distribution with *p* = 0.1587 and *n* = 10 when μ = 1. Then β = 0.487. The value of β is greater for the sign test than for the normal test because the Z-test was designed for the normal distribution.

15-18 *P*-value = 2P($R^- \geq 6$ | p = 0.5) = 0.289. The *P*-value is greater than 0.05, therefore we cannot reject H_0 and conclude that there is no significant difference between median hardness readings for the two tips. This result agrees with the result found in Exercise 15-9.

15-19 *P*-value = 2P($R^- \leq 3$ | p = 0.5) = 0.0075. The exact *P*-value computed here agrees with the normal approximation in Exercise 15-11 in the sense that both calculations would lead to the rejection of H_0.

Section 15-3

Note to Instructors: When an observation equals the hypothesized mean the observation is dropped and not considered in the analysis.

15-20
1. Parameter of interest is the mean pH
2. $H_0 : \mu = 7.0$
3. $H_1 : \mu \neq 7.0$
4. α = 0.05
5. The test statistic is $w = min(w^+, w^-)$.
6. We reject H_0 if $w \leq w^*_{0.05}$ = 8, because α = 0.05 and n = 10, Appendix A, Table VIII gives the critical value.
7. w^+ = 50.5 and w^- = 4.5 and $w = min(50.5, 4.5) = 4.5$
8. Conclusion, because 4.5 < 8, we reject H_0 and conclude that the mean pH is not equal to 7.

15-21
1. Parameter of interest is the mean titanium content
2. $H_0 : \mu = 8.5$
3. $H_1 : \mu \neq 8.5$
4. α=0.05
5. The test statistic is $w = min(w^+, w^-)$.
6. We reject H_0 if $w \leq w^*_{0.05}$ = 52, because α = 0.05 and n = 20, Appendix A, Table VIII gives the critical value.
7. w^+ = 80.5 and w^- = 109.5 and $w = min(80.5,109.5) = 80.5$

8. Conclusion, because 80.5 > 52, we cannot reject H_0. The mean titanium content is not significantly different from 8.5 at $\alpha = 0.05$.

15-22 1. Parameter of interest is the mean titanium content

2. $H_0 : \mu = 8.5$

3. $H_1 : \mu \neq 8.5$

4. α=0.05

5. The test statistic is $Z_0 = \dfrac{W^+ - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}}$

6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$, at $\alpha = 0.05$

7. $w^+ = 80.5$ and $w^- = 109.5$ and

$$Z_0 = \frac{80 - 19(20)/4}{\sqrt{19(20)(39)/24}} = -0.58$$

8. Conclusion, because 0.58 < 1.96, we cannot reject H_0. The mean titanium content is not significantly different from 8.5 at α=0.05.

15-23 1. Parameter of interest is the mean impurity level

2. $H_0 : \mu = 2.5$

3. $H_1 : \mu < 2.5$

4. $\alpha = 0.05$

5. The test statistic is $w = min(w^+, w^-)$.

6. We reject H_0 if $w \leq w^*_{0.05} = 52$ because $\alpha = 0.05$ and $n = 20$ Appendix A, Table VIII gives the critical value.

7. $w^+ = 205$ and $w^- = 5$ and $w = min(205, 5) = 5$

8. Conclusion, because 5 < 65, we reject H_0 and conclude that the mean impurity level is less than 2.5 ppm.

15-24 1. Parameter of interest is the difference in mean hardness readings for the two tips

2. $H_0 : \mu_D = 0$

3. $H_1 : \mu_D \neq 0$

4. $\alpha = 0.05$

5. The test statistic is $w = min(w^+, w^-)$.

6 We reject H_0 if $w \leq w^*_{0.05} = 3$, because $\alpha = 0.05$ and $n = 8$ Appendix A, Table VIII gives the critical value.

7. $w^+ = 24.5$ and $w^- = 11.5$ and $w = min(24.5, 11.5) = 11.5$

8. Conclusion, because 11.5 > 3 cannot reject H_0. There is not a significant difference in the tips.

15-25 1. Parameter of interest is the difference in mean drying times

2. $H_0 : \mu_D = 0$

3. $H_1 : \mu_D \neq 0$

4. α=0.05

5. The test statistic is $w = min(w^+, w^-)$.

6. We reject H_0 if $w \leq w^*_{0.05} = 27$ because $\alpha = 0.01$ and $n = 18$ Appendix A, Table VIII gives the critical value.

7. $w^+ = 144$ and $w^- = 27$ and $w = min(144, 27) = 27$

8. Conclusion, because 27 = 27, we reject H_0.

15-26 1. Parameter of interest is the difference in the mean caliper measurements

2. $H_0 : \mu_D = 0$

3. $H_1 : \mu_D \neq 0$

4. α=0.05

5. The test statistic is $w = min(w^+, w^-)$.

6. We reject H_0 if $w \le w^*_{0.05} = 3$ because $\alpha = 0.05$ and $n = 8$ Appendix A, Table VIII gives the critical value.
7. $w^+ = 21.5$ and $w^- = 14.5$ and $w = min(21.5,14.5) = 14.5$
8. Conclusion, do not reject H_0 because 14.5 > 13. There is a not a significant difference in the mean measurements of the two calipers at $\alpha = 0.05$.

15-27 1. Parameter of interest is the difference in mean blood cholesterol levels
2. $H_0 : \mu_D = 0$
3. $H_1 : \mu_D \neq 0$
4. $\alpha=0.05$
5. The test statistic is $w = min(w^+, w^-)$.
6. We reject H_0 if $w \le w^*_{0.05} = 25$, because $\alpha=0.05$ and n=15, Appendix A, Table VIII gives the critical value.
7. $w^+ = 119$ and $w^- = 1$ and $w = min(119, 1) = 1$
8. Conclusion, because 1 < 25, we reject H_0 and conclude that the difference in the mean blood cholesterol levels is significant at $\alpha=0.05$.

Section 15-4

15-28 1. The parameters of interest are the mean current (note: set circuit 1 equal to sample 2 so that Table IX can be used. Therefore μ_1=mean of circuit 2 and μ_2=mean of circuit 1)
2. $H_0 : \mu_1 = \mu_2$
3. $H_1 : \mu_1 > \mu_2$
4. $\alpha = 0.025$
5. The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$
6. We reject H_0 if $w_2 \le w^*_{0.025} = 51$, because $\alpha=0.025$ and $n_1=8$ and $n_2=9$, Appendix A, Table IX gives the critical value.
7. $w_1 = 78$and $w_2 = 75$ and because 75 is less than 51, do not reject H_0
8. Conclusion, do not reject H_0. There is not enough evidence to conclude that the mean of circuit 1 exceeds the mean of circuit 2.

15-29 1. The parameters of interest are the mean flight delays
2. $H_0 : \mu_1 = \mu_2$
3. $H_1 : \mu_1 \neq \mu_2$
4. $\alpha=0.01$
5. The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$
6. We reject H_0 if $w \le w^*_{0.01} = 23$, because $\alpha=0.01$ and $n_1=6$ and $n_2=6$, Appendix A, Table IX gives the critical value.
7. $w_1 = 40$ and $w_2 = 38$ and because 40 and 38 are greater than 23, we cannot reject H_0
8. Conclusion, do not reject H_0. There is no significant difference in the flight delays at $\alpha = 0.01$.

15-30 1. The parameters of interest are the mean heat gains for heating units
2. $H_0 : \mu_1 = \mu_2$
3. $H_1 : \mu_1 \neq \mu_2$
4. $\alpha = 0.05$
5. The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

6. We reject H_0 if $w \le w^*_{0.01} = 78$, because $\alpha = 0.01$ and $n_1 = 10$ and $n_2 = 10$, Appendix A, Table IX gives the critical value.
7. $w_1 = 77$ and $w_2 = 133$ and because 77 is less than 78, we can reject H_0
8. Conclusion, reject H_0 and conclude that there is a significant difference in the heating units at $\alpha = 0.05$.

15-31 1. The parameters of interest are the mean image brightness of the two tubes
2. $H_0 : \mu_1 = \mu_2$
3. $H_1 : \mu_1 > \mu_2$
4. $\alpha = 0.025$
5. The test statistic is $z_0 = \dfrac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$
6. We reject H_0 if $Z_0 > Z_{0.025} = 1.96$
7. $w_1 = 78$, $\mu_{w_1} = 72$ and $\sigma^2_{w_1} = 108$

$$z_0 = \frac{78 - 72}{10.39} = 0.58$$

Because $Z_0 < 1.96$, cannot reject H_0
8. Conclusion, fail to reject H_0. There is not a significant difference in the heat gain for the heating units at $\alpha = 0.05$. P-value $= 2[1 - P(Z < 0.58)] = 0.5619$

15-32 1. The parameters of interest are the mean heat gain for heating units
2. $H_0 : \mu_1 = \mu_2$
3. $H_1 : \mu_1 \neq \mu_2$
4. $\alpha = 0.05$
5. The test statistic is $z_0 = \dfrac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$
6. We reject H_0 if $|Z_0| > Z_{0.025} = 1.96$
7. $w_1 = 77$, $\mu_{w_1} = 105$ and $\sigma^2_{w_1} = 175$

$$z_0 = \frac{77 - 105}{13.23} = -2.12$$

Because $|Z_0| > 1.96$, reject H_0
8. Conclusion, reject H_0 and conclude that there is a difference in the heat gain for the heating units at $\alpha=0.05$. P-value $= 2[1 - P(Z < 2.19)] = 0.034$

15-33 1. The parameters of interest are the mean etch rates
2. $H_0 : \mu_1 = \mu_2$
3. $H_1 : \mu_1 \neq \mu_2$
4. $\alpha = 0.05$
5. The test statistic is $w_2 = \dfrac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$
6. We reject H_0 if $w \le w^*_{0.05} = 78$, because $\alpha = 0.05$ and $n_1 = 10$ and $n_2 = 10$, Appendix A, Table IX gives the critical value.
7. $w_1 = 73$ and $w_2 = 137$ and because 73 is less than 78, we reject H_0
8. Conclusion, reject H_0 and conclude that there is a significant difference in the mean etch rate at $\alpha = 0.05$.

15-34 1. The parameters of interest are the mean temperatures

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α = 0.05

5. The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

6. We reject H_0 if $w \leq w^*_{0.05} = 185$, because α = 0.05 and $n_1 = 15$ and $n_2 = 15$, Appendix A, Table IX gives the critical value.

7. $w_1 = 258$ and $w_2 = 207$ and because both 258 and 207 are greater than 185, we cannot reject H_0

8. Conclusion, do not reject H_0. There is not significant difference in the pipe deflection temperature at α = 0.05.

15-35. 1. The parameters of interest are the mean etch rates

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α=0.05

5. The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

6. We reject H_0 if $|Z_0| > Z_{0.025} = 1.96$

7. $w_1 = 73$, $\mu_{w_1} = 105$ and $\sigma^2_{w_1} = 175$

$$z_0 = \frac{73 - 105}{13.23} = -2.42$$

Because $|Z_0| > 1.96$, reject H_0

8. Conclusion, reject H_0 and conclude that there is a significant difference between the mean etch rates. *P*-value = 0.0155

15-36 1. The parameters of interest are the mean temperatures

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α = 0.05

5. The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

6. We reject H_0 if $|Z_0| > Z_{0.025} = 1.96$

7. $w_1 = 55$, $\mu_{w_1} = 232.5$ and $\sigma^2_{w_1} = 581.25$

$$z_0 = \frac{258 - 232.5}{24.11} = 1.06$$

Because $|Z_0| < 1.96$, do not reject H_0

8. Conclusion, do not reject H_0. There is not a significant difference in the pipe deflection temperatures at α = 0.05. *P*-value =2[1 - P(Z < 1.06)] = 0.2891

Section 15-5

15-37 Kruskal-Wallis Test on strength

mixingte	N	Median	Ave Rank	Z
1	4	2945	9.6	0.55
2	4	3075	12.9	2.12
3	4	2942	9.0	0.24
4	4	2650	2.5	-2.91
Overall	16		8.5	

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H = 10.00  DF = 3  P = 0.019
H = 10.03  DF = 3  P = 0.018 (adjusted for ties)
* NOTE * One or more small samples
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Reject H_0 and conclude that the mixing technique has an effect on the strength.

15-38 Kruskal-Wallis Test on strength

methods	N	Median	Ave Rank	Z
1	5	550.0	8.7	0.43
2	5	553.0	10.7	1.65
3	5	528.0	4.6	-2.08
Overall	15		8.0	

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H = 4.83  DF = 2  P = 0.089
H = 4.84  DF = 2  P = 0.089 (adjusted for ties)
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Do not reject H_0. The conditioning method does not have an effect on the breaking strength at $\alpha = 0.05$.

15-39 Kruskal-Wallis Test on angle

manufact	N	Median	Ave Rank	Z
1	5	39.00	7.6	-1.27
2	5	44.00	12.4	0.83
3	5	48.00	15.8	2.31
4	5	30.00	6.2	-1.88
Overall	20		10.5	

```
H = 8.37  DF = 3  P = 0.039
```

Do not reject H_0. There is not a significant difference between the manufacturers at $\alpha = 0.01$.

15-40 Kruskal-Wallis Test on uniformity

flow rate	N	Median	Ave Rank	Z
125	6	3.100	5.8	-2.06
160	6	4.400	13.0	1.97
250	6	3.800	9.7	0.09
Overall	18		9.5	

```
H = 5.42  DF = 2  P = 0.067
H = 5.44  DF = 2  P = 0.066 (adjusted for ties)
```

Do not reject H_0. There is not a significant difference between the flow rates on the uniformity at $\alpha = 0.05$.

15-41 *P*-value = 0.018 (use the chi-square distribution)

15-42 *P*-value = 0.066 (use the chi-square distribution)

Supplemental Exercises

15-43 1. Parameter of interest is median surface finish
2. $H_0 : \tilde{\mu} = 10.0$
3 $H_1 : \tilde{\mu} \neq 10.0$
4. $\alpha = 0.05$
5. The test statistic is the observed number of plus differences or $r^+ = 5$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 5$ is less than or equal to $\alpha = 0.05$.
7. Using the binomial distribution with $n = 10$ and $p = 0.5$, *P-value* = $2P(R^* \geq 5 | p = 0.5) = 1.0$
8. Conclusion, we cannot reject H_0. We cannot reject the claim that the median is 10 μin.

15-44 1. Parameter of interest is median of surface finish
2. $H_0 : \tilde{\mu} = 10.0$
3 $H_1 : \tilde{\mu} \neq 10.0$
4. α=0.05
5. Test statistic is $z_0 = \frac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$
7. Computation: $z_0 = \frac{5 - 0.5(10)}{0.5\sqrt{10}} = 0$
8. Conclusion, cannot reject H_0. There is not sufficient evidence that the median surface finish differs from 10 microinches.

The *P-value* = $2[1-\Phi(0)] = 1$

15-45 1. The parameter of interest is the median fluoride emissions.
2. $H_0 : \tilde{\mu} = 6$
3. $H_1 : \tilde{\mu} < 6$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 4$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 4$ is less than or equal to $\alpha = 0.05$.
7. Using the binomial distribution with $n = 15$ and $p = 0.5$, *P-value* = $P(R^+ \leq 4 | p = 0.5) = 0.1334$
8. Conclusion, do not reject H_0. The data does not support the claim that the median fluoride impurity level is less than 6.

Using Minitab (Sign Rank Test)

```
Sign test of median = 6.000 versus  <  6.000
                N  Below  Equal  Above        P     Median
y              15      9      2      4   0.1334      4.000
```

Do not reject H_0

15-46 1. Parameter of interest is median fluoride impurity level
2. $H_0 : \tilde{\mu} = 6.0$
3 $H_1 : \tilde{\mu} < 6.0$
4. α=0.05
5. Test statistic is $z_0 = \frac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $Z_0 < -Z_{0.05} = -1.65$
7. Computation: $z_0 = \frac{4 - 0.5(13)}{0.5\sqrt{13}} = -1.39$
8. Conclusion, do not reject H_0. There is not enough evidence to conclude that the median fluoride impurity level is less than 6.0 ppm. The *P-value* = $2[1-\Phi(1.39)] = 0.1645$.

15-47 1. Parameter of interest is the difference between the impurity levels.
2. $H_0 : \tilde{\mu}_D = 0$
3. $H_1 : \tilde{\mu}_D \neq 0$
4. $\alpha = 0.05$
5. The test statistic $r = min(r^+, r^-)$.
6. Because $\alpha = 0.05$ and $n = 7$, Appendix A, Table VII gives the critical value of $r^*_{0.05} = 0$. We reject H_0 in favor of H_1 if $r \leq 10$.
7. $r^+ = 1$ and $r^- = 6$ and $r = min(1,6) = 1$
8. Conclusion, cannot reject H_0. There is not a significant difference in the impurity levels.

15-48 1. Parameter of interest is mean surface finish.
2. $H_0 : \mu = 10.0$
3 $H_1 : \mu \neq 10.0$
4. α=0.05
5. The test statistic is $w = min(w^+, w^-)$.
6. We reject H_0 if $w \leq w^*_{0.05} = 8$, because $\alpha = 0.05$ and $n = 10$, Appendix A, Table VIII gives the critical value.
7. $w^+ = 28.5$ and $w^- = 26.5$ and $w = min(28.5, 26.5) = 26.5$
8. Conclusion, we cannot reject H_0. We cannot reject the claim that the mean surface finish is 10 μin.

15-49 1. Parameter of interest is mean fluoride impurity level.
2. $H_0 : \mu = 6.0$
3 $H_1 : \mu < 6.0$
4. $\alpha = 0.05$
5. The test statistic is $w = min(w^+, w^-)$.
6. We reject H_0 if $w \leq w^*_{0.05} = 21$, because $\alpha = 0.05$ and $n = 13$, Appendix A, Table VIII gives the critical value.
7. $w^+ = 19$ and $w^- = 72$ and $w = min(19,72) = 19$
8. Conclusion, reject H_0. We conclude that the mean fluoride impurity level is less than 6.0 ppm.

```
Using Minitab Wilcoxon signed-rank t test
Test of median = 6.000 versus median < 6.000
                N for   Wilcoxon           Estimated
             N   Test  Statistic        P     Median
y           15     13       19.0    0.035      5.000
```
Reject H_0

The Wilcoxon signed-rank test applies to symmetric, continuous distributions. The test in this exercise applies to the mean of the distribution while the sign test applied to the median.

15-50 1. Parameters of interest are the mean weights
2. $H_0 : \mu_D = 0$
3. $H_1 : \mu_D \neq 0$
4. α=0.05
5. The test statistic is $w = min(w^+, w^-)$.
6. We reject H_0 if $w \leq w^*_{0.05} = 8$ because $\alpha = 0.05$ and n = 10 Appendix A, Table VIII gives the critical value.
7. $w^+ = 55$ and $w^- = 0$ and $w = min(55, 0) = 0$
8. Conclusion, because 0 < 8, we reject H_0 and conclude that there is a difference in the weights due to the diet modification experiment.

15-51 1. The parameters of interest are the mean mileages for a Volkswagen and Mercedes.

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α = 0.01

5. The test statistic is $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

6. We reject H_0 if $w \leq w^*_{0.01} = 71$, because α = 0.01 and $n_1 = 10$ and $n_2 = 10$, Appendix A, Table IX gives the critical value.

7. $w_1 = 55$ and $w_2 = 155$ and because 55 is less than 71, we reject H_0

8. Conclusion, reject H_0 and conclude that there is a difference in the mean mileages of the two different vehicles.

15-52 1. The parameters of interest are the mean mileages for a Volkswagen and Mercedes

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α = 0.01

5. The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

6. We reject H_0 if $|Z_0| > Z_{0.005} = 2.58$

7. $w_1 = 55$, $\mu_{w_1} = 105$ and $\sigma^2_{w_1} = 175$

$$z_0 = \frac{55 - 105}{13.23} = -3.78$$

Because $|Z_0| > 2.58$, reject H_0

8. Conclusion, reject H_0 and conclude that there is a significant difference in the mean mileages at α = 0.01.
P-value $= 2[1 - P(Z < 3.78)] = 0.00016$

15-53 1. The parameters of interest are the in mean fill volumes

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α=0.05

5. The test statistic $w_2 = \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - w_1$

6. We reject if H_0 $w \leq w^*_{0.05} = 78$, because α = 0.05 and $n_1 = 10$ and $n_2 = 10$, Appendix A, Table IX gives the critical value.

7. $w_1 = 58$ and $w_2 = 152$ and because 58 is less than 78, we reject H_0

8. Conclusion, reject H_0 and conclude that there is a significant difference in the mean fill volumes at α = 0.05.

15-54 1. The parameters of interest are the mean mileages for a Volkswagen and Mercedes

2. $H_0 : \mu_1 = \mu_2$

3. $H_1 : \mu_1 \neq \mu_2$

4. α=0.05

5. The test statistic is $z_0 = \frac{W_1 - \mu_{w_1}}{\sigma_{w_1}}$

6. We reject H_0 if $|Z_0| > Z_{0.025} = 1.96$

7. $w_1 = 58$, $\mu_{w_1} = 105$ and $\sigma^2_{w_1} = 175$

$$z_0 = \frac{58 - 105}{13.23} = -3.55$$

Because $|Z_0| > 1.96$, reject H_0

8. Conclusion, reject H_0 and conclude that there is a significant difference in the mean mileages at α = 0.05.

P-value =2[1 - P(Z < 3.55)] ≅ 0.000385

15-55.

```
Kruskal-Wallis Test on RESISTAN
ALLOY        N    Median   Ave Rank         Z
1           10     98.00        5.7     -4.31
2           10    102.50       15.3     -0.09
3           10    138.50       25.5      4.40
Overall     30                 15.5

H = 25.30  DF = 2  P = 0.000
H = 25.45  DF = 2  P = 0.000 (adjusted for ties)
```

Reject H_0, P-value ≅ 0

15-56

```
Kruskal-Wallis Test on TIME

time         N    Median    Ave Rank         Z
 0           9    1106.0        24.8      2.06
 7           9     672.2        19.7      0.38
14           9     646.2        20.9      0.79
21           9     377.6         8.7     -3.23
Overall     36                  18.5

H = 11.61  DF = 3  P = 0.009
```

Reject H_0 at α = 0.05

15-57.

```
Kruskal-Wallis Test on VOLUME
TEMPERAT     N    Median   Ave Rank         Z
70           5      1245       12.4      2.69
75           5      1220        7.9     -0.06
80           5      1170        3.7     -2.63
Overall     15                  8.0
H = 9.46  DF = 2  P = 0.009
H = 9.57  DF = 2  P = 0.008 (adjusted for ties)
```

Reject H_0 at α = 0.05

Mind-Expanding Exercises

15-58 Under the null hypothesis each rank has probability of 0.5 of being either positive or negative. Define the random variable X_i as

$$X_i = \begin{cases} 1 & rank \quad is \quad positive \\ 0 & rank \quad is \quad negative \end{cases}$$

Then,

$$R^+ = \sum_{i=1}^{n} x_i i \text{ and } E(R^+) = \sum_{i=1}^{n} E(x_i) i = \frac{1}{2}\sum i = \frac{n(n+1)}{4} \quad \text{since} \quad E(x_i) = \frac{1}{2}$$

$$\text{where } V(R^+) = \sum V(x_i) i^2 \text{ by independence}$$

$$V(X_i) = 1^2 \frac{1}{2} + 0^2 \frac{1}{2} - [E(X_i)]^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\text{Then, } V(R^+) = \frac{n(n+1)(2n+1)}{24}$$

15-59 a) $p = P(X_{i+5} > X_i) = \frac{4}{5}$ There appears to be an upward trend.

b) V is the number of values for which $X_{i+5} > X_i$. The probability distribution of V is binomial with $n = 5$ and $p = 0.5$.

c) $V = 4$

1. The parameter of interest is number of values of i for which $X_{i+5} > X_i$.
2. H_0 : there is no trend
3 H_1 : there is an upward trend
4. $\alpha = 0.05$
5. The test statistic is the observed number of values where $X_{i+5} > X_i$ or $V = 4$.
6. We reject H_0 if the *P-value* corresponding to $V = 4$ is less than or equal to $\alpha = 0.05$.
7. Using the binomial distribution with $n = 5$ and $p = 0.5$, *P-value* = $P(V \geq 4| p = 0.5) = 01875$
8. Conclusion: do not reject H_0. There is not enough evidence to suspect an upward trend in the wheel opening dimensions at α=0.05.

15-60 a) 32 sequences are possible

b) Because each sequence has probability 1/32 under H_0, the distribution of W^* is obtained by counting the sequences that result in each value of W^*

w^*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Prob*1/32	1	1	1	2	2	3	3	3	3	3	3	2	2	1	1	1

c) $P(W^* > 13) = 2/32$

d) By enumerating the possible sequences, the probability that W^* exceeds any value can be calculated under the null hypothesis as in part (c). This approach can be used to determine the critical values for the test.