CHAPTER 14

<u>Section 14-3</u>

14-1 a) $H_0 : \tau_1 = \tau_2 = \tau_3 = 0$ H_1 : at least one $\tau_j \neq 0$

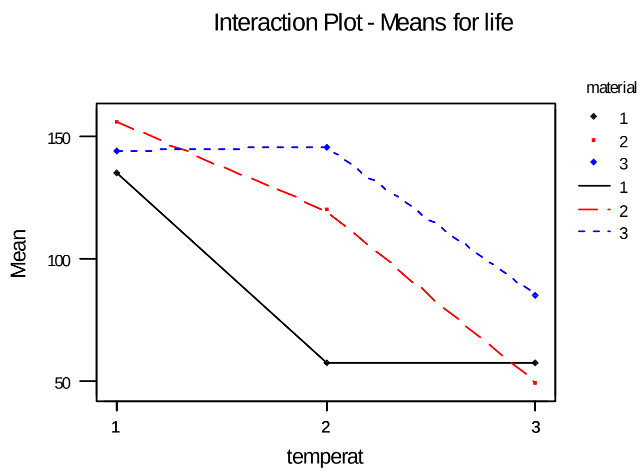
$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ H_1 : at least one $\beta_j \neq 0$

Analysis of Variance for life

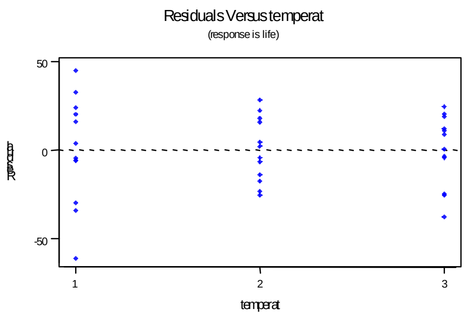
Source	DF	SS	MS	F	P
material	2	10683.7	5341.9	7.91	0.002
temperat	2	39118.7	19559.4	28.97	0.000
material*temperat	4	9613.8	2403.4	3.56	0.019
Error	27	18230.7	675.2		
Total	35	77647.0			

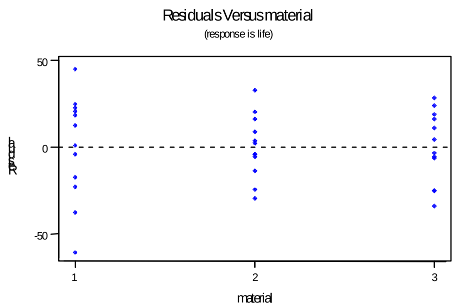
Main factors and interaction are all significant.

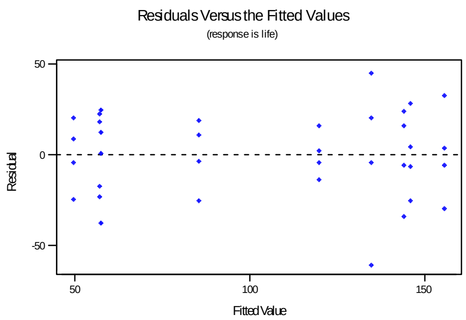
b) The mean life for material 2 is the highest at temperature level 1, in the middle at temperature level 2 and the lowest at temperature level 3. This shows that there is an interaction.

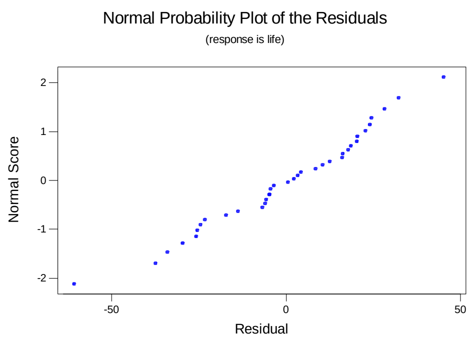


c) There appears to be slightly more variability at temperature 1 and material 1. The normal probability plot shows that the assumption of normality is reasonable.









14.2a) $H_0 : \tau_1 = \tau_2 = 0$

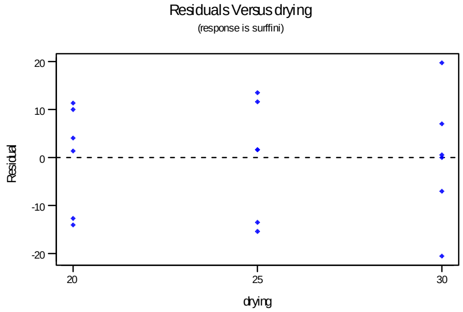
H_1 : at least one $\tau_j \neq 0$

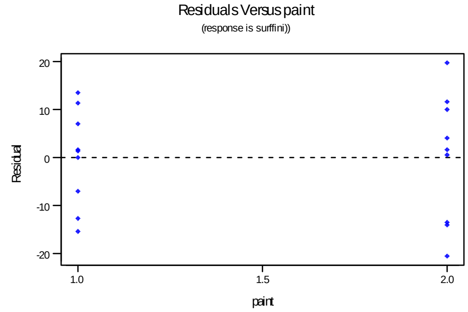
Analysis of variance for SURFACE FINISH

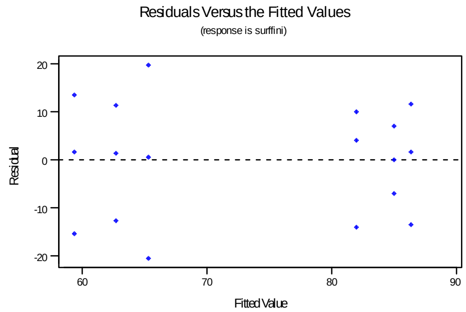
Source	DF	SS	MS	F	P
paint	1	355.6	355.6	1.90	0.193
drying	2	27.4	13.7	0.07	0.930
paint*drying	2	1878.8	939.4	5.03	0.026
Error	12	2242.7	186.9		
Total	17	4504.4			

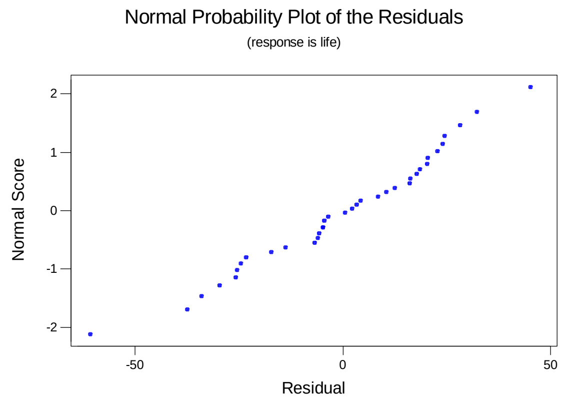
Only the interaction between the paint and drying time is significant.

b)The residual plots appear reasonable.









14-3 a) 1. $H_0: \tau_1 = \tau_2 = 0$

H_1: at least one $\tau_i \neq 0$

2. $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

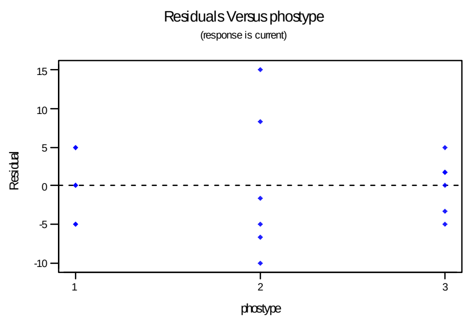
H_1: at least one $\beta_j \neq 0$

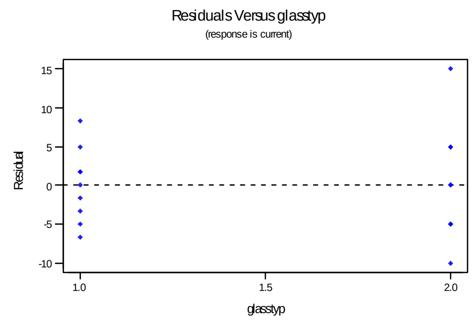
b) Analysis of Variance for current

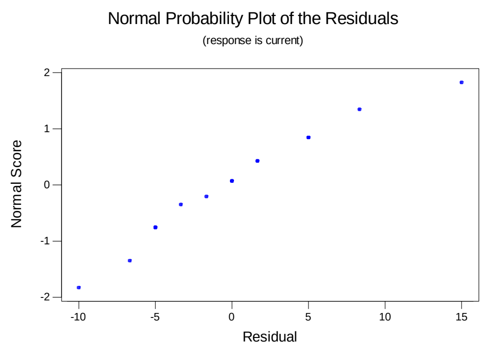
Source	DF	SS	MS	F	P
glasstyp	1	14450.0	14450.0	273.79	0.000
phostype	2	933.3	466.7	8.84	0.004
glasstyp*phostype	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

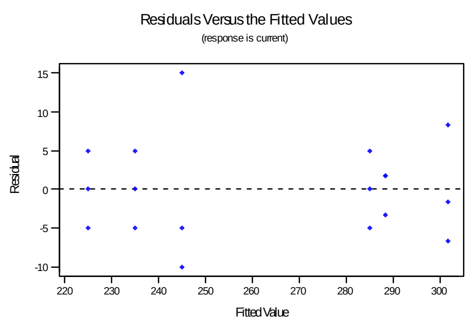
Main effects are significant, the interaction is not significant. Glass type 1 and phosphor type 2 lead to the highest mean current (brightness).

c) There appears to be more slightly variability at phosphor type 2 and glass type 2. The normal plot of the residuals shows that the assumption of normality is reasonable.









14-4 a) 1. $H_0 : \tau_1 = \tau_2 = 0$

H_1 : at least one $\tau_j \neq 0$

2. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

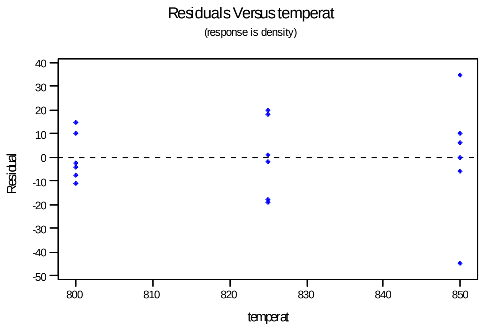
H_1 : at least one $\beta_j \neq 0$

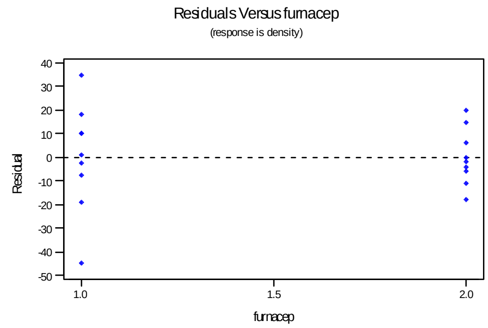
b)Analysis of Variance for DENSITY

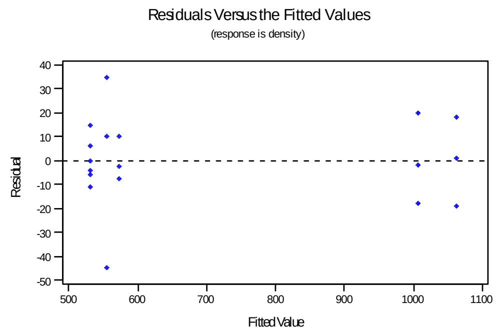
Source	DF	SS	MS	F	P
furnacep	1	7160	7160	16.00	0.002
temperat	2	945342	472671	1056.12	0.000
furnacep*temperat	2	818	409	0.91	0.427
Error	12	5371	448		
Total	17	958691			

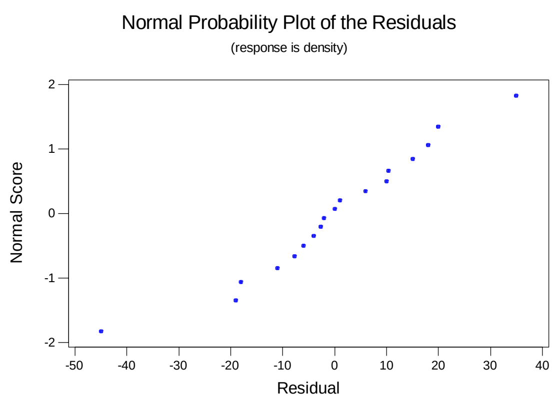
Reject H_0 for both main effects and conclude that both factors are significant.

c) There appears to be more variability at position 1 and temperature and the highest temperature level. There are two unusual points in the data.









14-5

```
Fisher's pairwise comparisons
Family error rate = 0.117
Individual error rate = 0.0500
Critical value = 2.131
Intervals for (column level mean) - (row level mean)
              800          825
 825        -518.4
            -445.0
 850         -27.9        453.8
              45.5        527.2
```

There are significant differences in the temperature levels 800 and 825, and 825 and 850. Therefore, temperature level 825 is different from the other two levels.

14-6 1. $H_0 : \tau_1 = \tau_2 = 0$

H_1 : at least one $\tau_j \neq 0$

2. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

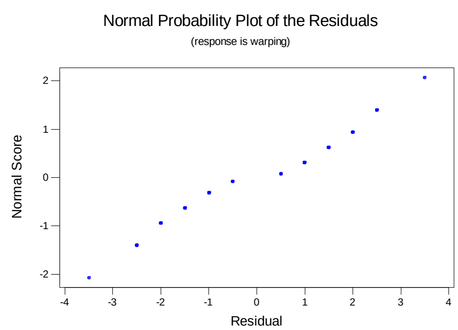
H_1 : at least one $\beta_j \neq 0$

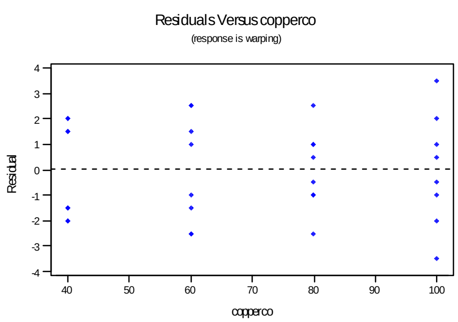
Analysis of Variance for warping

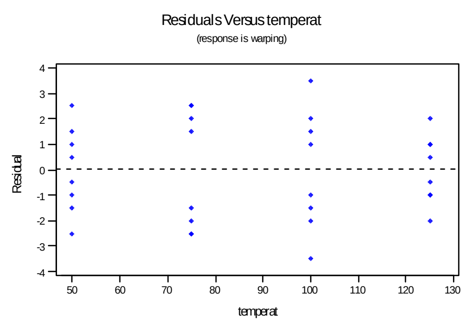
Source	DF	SS	MS	F	P
temp	3	156.09	52.03	7.67	0.002
copper	3	698.34	232.78	34.33	0.000
Interaction	9	113.78	12.64	1.86	0.133
Error	16	108.50	6.78		
Total	31	1076.72			

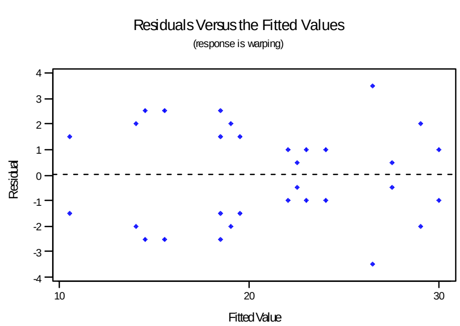
Reject H_0 for both factor effects and conclude that both temperature and copper content have an effect on the mean warping. The interaction is not significant.

b) The residuals for this experiment appear reasonable.

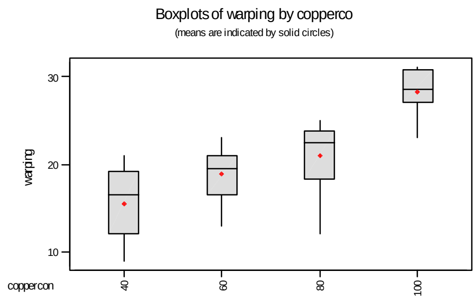








c.)




```
Fisher's pairwise comparisons
Family error rate = 0.195
Individual error rate = 0.0500
Critical value = 2.048
Intervals for (column level mean) - (row level mean)
                40          60          80
      60      -7.139
               0.389
      80      -9.264      -5.889
              -1.736       1.639
     100     -16.514     -13.139     -11.014
              -8.986      -5.611      -3.486
```

There are significant differences in the temperature levels 40 and 80, and 100 and all the other levels. This difference is apparent on the boxplot and using Fisher's LSD method. If low warping is desired, temperature level 40 would be most desirable.

d) No, because the factors do not interact.

14-7 The ratio $T = \dfrac{\bar{y}_{\cdot i \cdot} - \bar{y}_{\cdot j \cdot} - (\mu_i - \mu_j)}{\sqrt{2MS_E / n}}$ has a t distribution with *ab*(*n*-1) degrees of freedom

Therefore, the (1-α)% confidence interval on the difference in two treatment means is

$$\bar{y}_{\cdot i \cdot} - \bar{y}_{\cdot j \cdot} - t_{a/2, ab(n-1)} \sqrt{\frac{2MS_E}{n}} \le \mu_i - \mu_j \le \bar{y}_{\cdot i \cdot} - \bar{y}_{\cdot j \cdot} + t_{a/2, ab(n-1)} \sqrt{\frac{2MS_E}{n}}$$

For exercise 14-6, the mean warping at 80% copper concentration is 21.0 and the mean warping at 60% copper concentration is 18.88 $a = 4$, $b = 4$, $n = 2$ and $MS_E = 6.78$. The degrees of freedom are (4)(4)(1)=16

$$(21.0 - 18.88) - 2.120\sqrt{\frac{2(6.78)}{2}} \le \mu_3 - \mu_2 \le (21.0 - 18.88) + 2.120\sqrt{\frac{2(6.78)}{2}}$$
$$-3.40 \le \mu_3 - \mu_2 \le 7.64$$

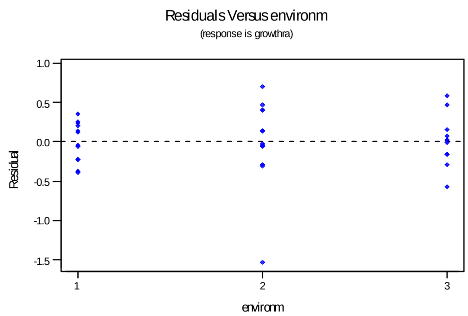
Therefore, there is no significant difference between the mean warping values at 80% and 60% copper concentration.

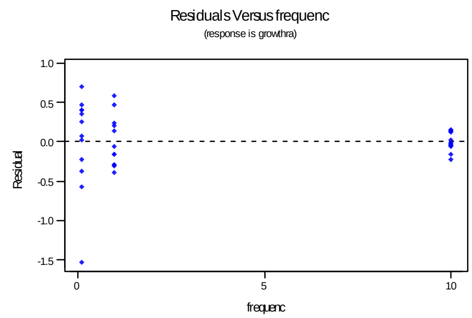
14-8. a) Analysis of Variance for Crack Growth

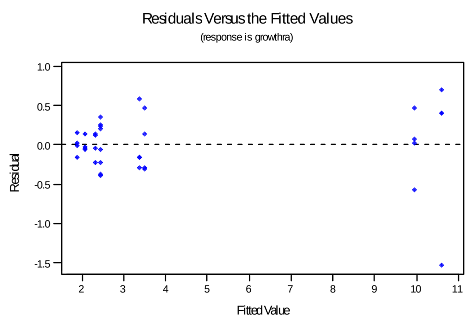
Source	DF	SS	MS	F	P
frequenc	2	209.893	104.946	522.40	0.000
environm	2	64.252	32.126	159.92	0.000
frequenc*environm	4	101.966	25.491	126.89	0.000
Error	27	5.424	0.201		
Total	35	381.535			

Both main factors and the interaction are significant.

b) There appear to be some problems with constant variance in the residual plots.







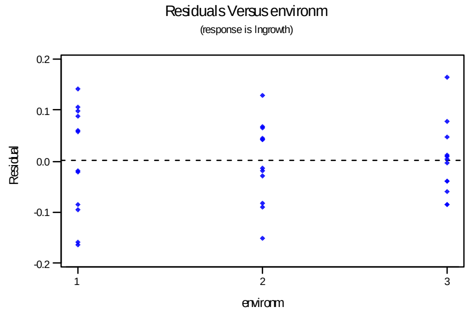
c)

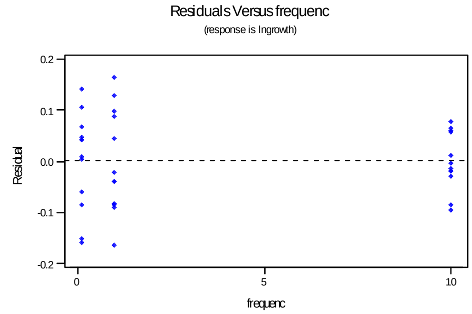
Analysis of Variance of Ln(Crack Growth)

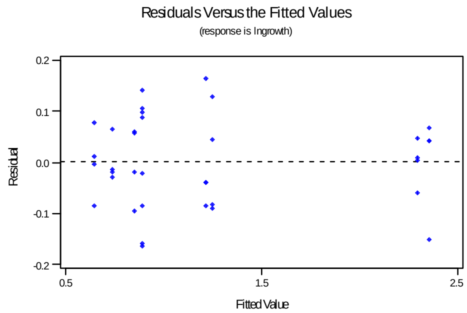
Source	DF	SS	MS	F	P
frequenc	2	7.5702	3.7851	404.09	0.000
environm	2	2.3576	1.1788	125.85	0.000
frequenc*environm	4	3.5284	0.8821	94.17	0.000
Error	27	0.2529	0.0094		
Total	35	13.7092			

The factors frequency, environment, and their interaction are all significant using the log of the data in the ANOVA

Residual plots on the log scale are improved. The variance appears to be more constant.







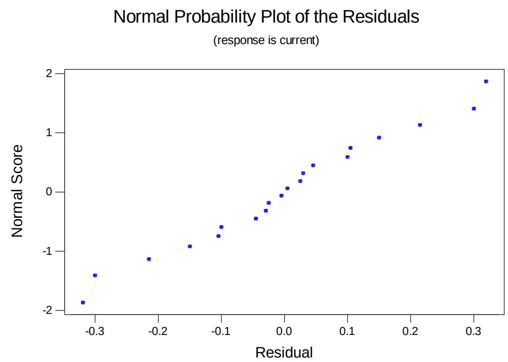
14-9 a) Analysis of Variance for current

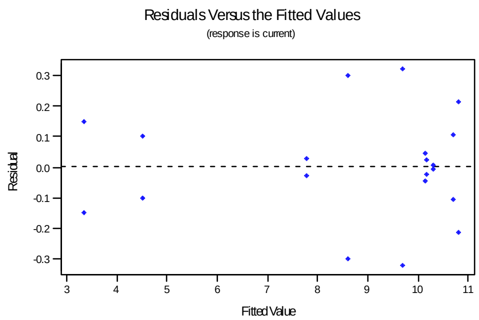
Source	DF	SS	MS	F	P
doping	1	1.442	1.442	25.23	0.000
anneal	4	124.238	31.059	543.52	0.000
doping*anneal	4	0.809	0.202	3.54	0.048
Error	10	0.571	0.057		
Total	19	127.060			

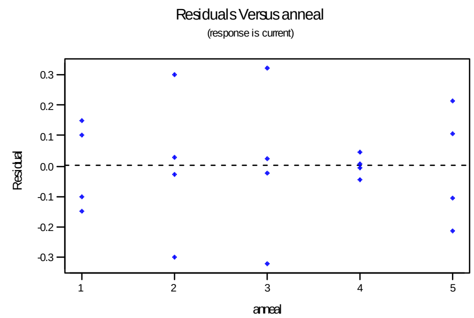
Both main factors are highly significant. The interaction is slightly significant at the 0.05 level of significance.

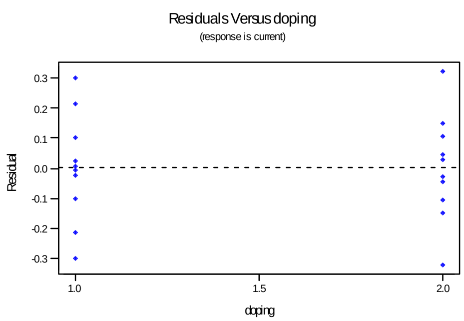
b) The interaction plot shows that there is a slight interaction because the lines are not parallel.

c) Analysis of the residual plots shows that all there is no problem with the model adequacy or the assumptions necessary to build the model.









14-10 Fisher's pairwise comparisons

```
Family error rate = 0.258
Individual error rate = 0.0500
Critical value = 2.131
Intervals for (column level mean) - (row level mean)
                 1           2           3           4
         2    -4.9187
              -3.6113
         3    -6.6662     -2.4012
              -5.3588     -1.0938
         4    -6.9487     -2.6837     -0.9362
              -5.6413     -1.3763      0.3712
         5    -7.4787     -3.2137     -1.4662     -1.1837
              -6.1713     -1.9063     -0.1588      0.1237
```

There are significant differences in anneal levels 1 and 2, 1 and 3, 1 and 4, 1 and 5, 2 and 3, 2 and 4, 2 and 5. Therefore levels 1 and 2 appear to be different from the other three.

Section 14-4

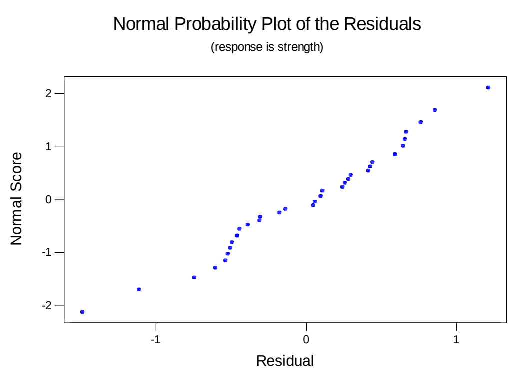
14-11 Parts a) and b)

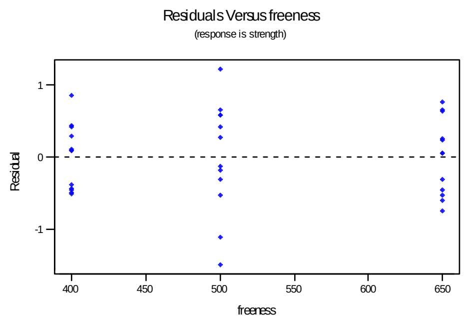
Analysis of Variance for strength

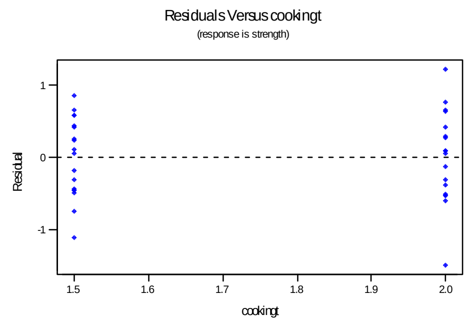
Source	DF	SS	MS	F	P
hardwood	2	8.3750	4.1875	7.64	0.003
cookingtime	1	17.3611	17.3611	31.66	0.000
freeness	2	21.8517	10.9258	19.92	0.000
hardwood*cookingtime	2	3.2039	1.6019	2.92	0.075
hardwood*freeness	4	6.5133	1.6283	2.97	0.042
cookingtime*freeness	2	1.0506	0.5253	0.96	0.399
Error	22	12.0644	0.5484		
Total	35	70.4200			

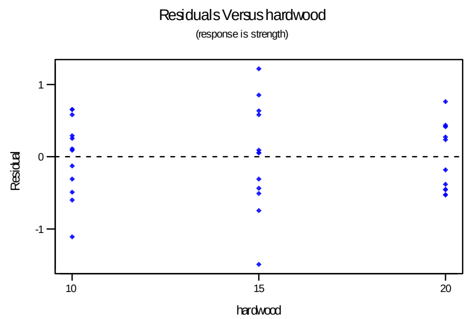
All main factors are significant. The interaction of hardwood*freeness is also significant.

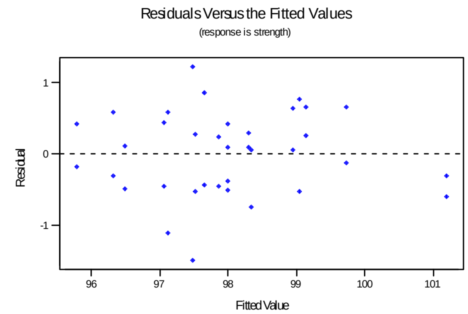
c) The residual plots do not indicate serious problems with normality or equality of variance.











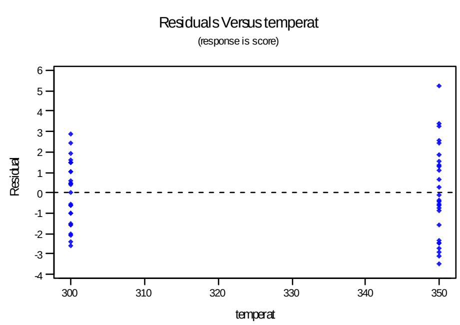
14-12 a) Analysis of Variance for dying score, using Adjusted SS for Tests

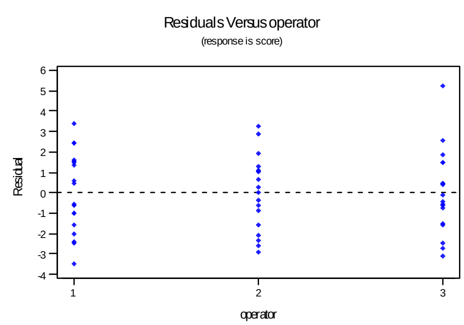
Source	DF	SS	MS	F	P
Time	2	396.778	198.389	39.85	0.000
Temp	1	73.500	73.500	14.77	0.000
Oper	2	256.333	128.167	25.75	0.000
Time*Temp	2	70.778	35.389	7.11	0.002
Time*Oper	4	300.222	75.056	15.08	0.000
Temp*Oper	2	14.111	7.056	1.42	0.254
Error	40	199.111	4.978		
Total	53	1310.833			

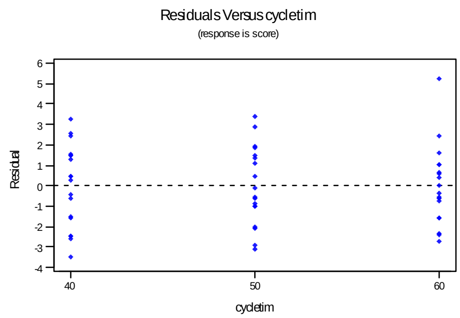
S = 2.23109 R-Sq = 84.81% R-Sq(adj) = 79.87%

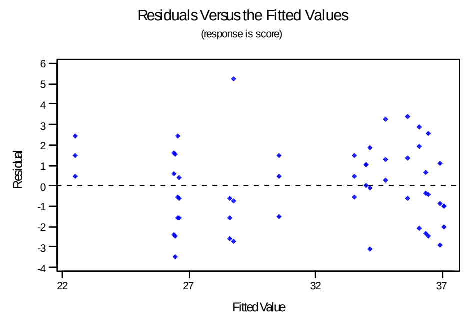
Only the operator*temperature interaction is *insignificant*.

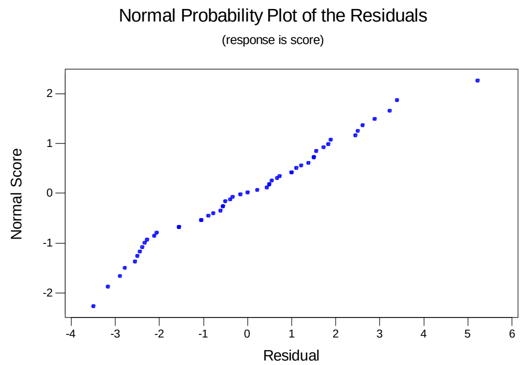
b)











The residuals are acceptable,

Section 14-7

14-13 a) Analysis of Variance for life (coded units)

Source	DF	SS	MS	F	P
A	1	1332	1332	0.54	0.483
B	1	28392	28392	11.53	0.009
C	1	20592	20592	8.36	0.020
A*B	1	506	506	0.21	0.662
A*C	1	56882	56882	23.10	0.001
B*C	1	2352	2352	0.96	0.357
A*B*C	1	4830	4830	1.96	0.199
Error	8	19700	2463		
Total	15	134588			

S = 49.6236   R-Sq = 85.36%   R-Sq(adj) = 72.56%

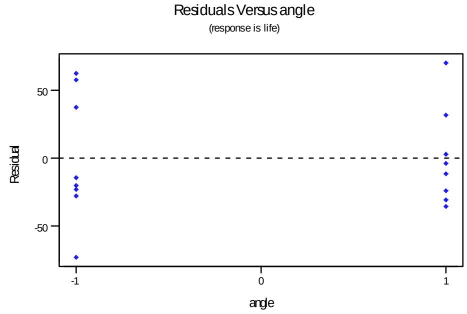
Hardness, angle and the speed-angle interaction are significant.

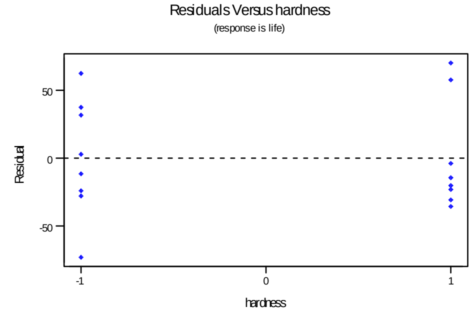
b) Estimated Effects and Coefficients for life (coded units)

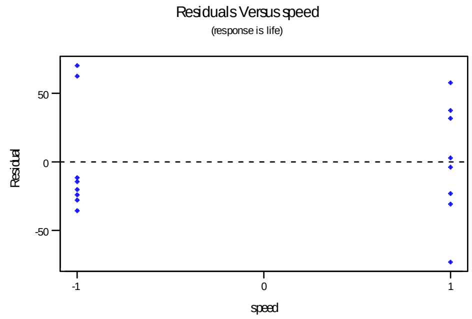
Term	Effect	Coef	SE Coef	T	P
Constant		413.13	12.41	33.30	0.000
speed	18.25	9.12	12.41	0.74	0.483
hardness	84.25	42.12	12.41	3.40	0.009
angle	71.75	35.87	12.41	2.89	0.020
speed*hardness	-11.25	-5.63	12.41	-0.45	0.662
speed*angle	-119.25	-59.62	12.41	-4.81	0.001
hardness*angle	-24.25	-12.12	12.41	-0.98	0.357
speed*hardness*angle	-34.75	-17.37	12.41	-1.40	0.199

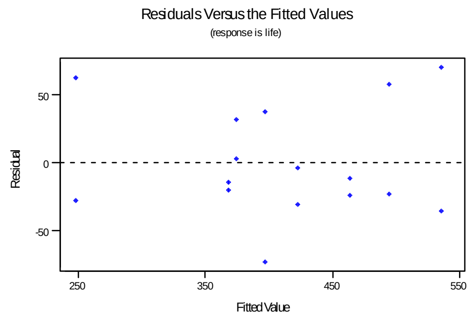
$$\hat{y} = 413.125 + 9.125x_1 + 45.12x_2 + 35.87x_3 - 59.62x_1x_3$$

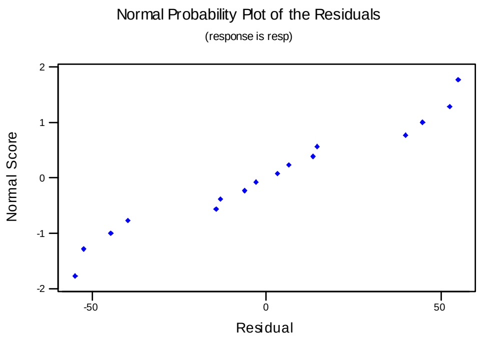
c) Analysis of the residuals shows that all assumptions are reasonable.











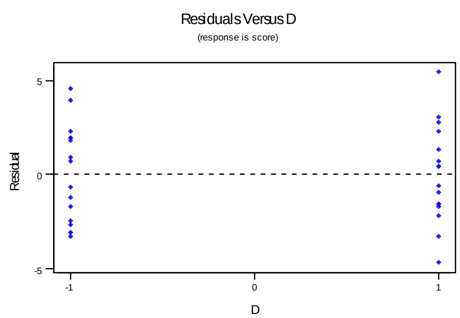
14-14

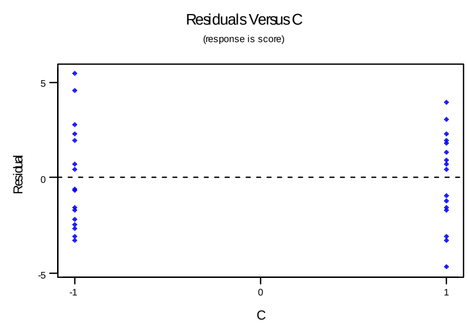
Term	Effect	Coef	SE Coef	T	P
Constant		175.250	0.5467	320.59	0.000
A	17.000	8.500	0.5467	15.55	0.000
B	-1.625	-0.812	0.5467	-1.49	0.157
C	10.875	5.438	0.5467	9.95	0.000
D	8.375	4.187	0.5467	7.66	0.000
A*B	-0.125	-0.063	0.5467	-0.11	0.910
A*C	-0.625	-0.313	0.5467	-0.57	0.575
A*D	9.125	4.562	0.5467	8.35	0.000
B*C	-0.250	-0.125	0.5467	-0.23	0.822
B*D	1.250	0.625	0.5467	1.14	0.270
C*D	-1.250	-0.625	0.5467	-1.14	0.270
A*B*C	0.750	0.375	0.5467	0.69	0.503
A*B*D	-0.500	-0.250	0.5467	-0.46	0.654
A*C*D	-0.000	-0.000	0.5467	-0.00	1.000
B*C*D	0.125	0.063	0.5467	0.11	0.910
A*B*C*D	-1.625	-0.812	0.5467	-1.49	0.157

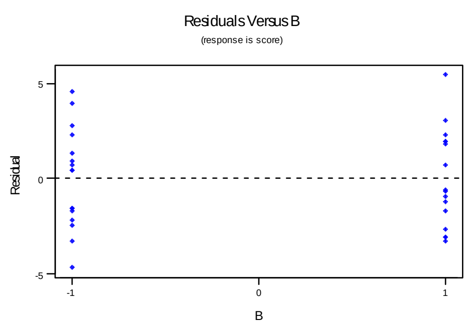
Factors A, C, and D are significant as well as the interaction AD.

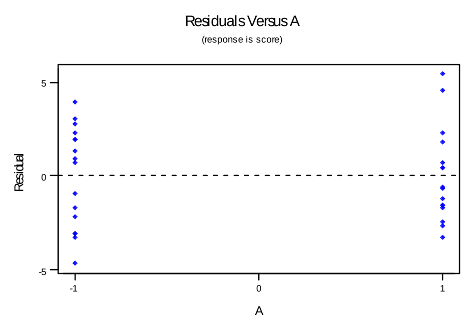
14-15 $\hat{y} = 175.25 + 8.5x_1 + 5.44x_3 + 4.19x_4 + 4.56x_1x_4$

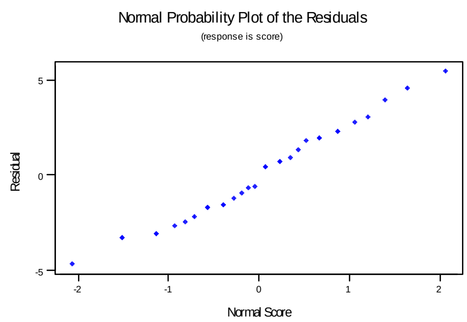
Analysis of the residuals shows that the model is adequate and the assumptions are reasonable.









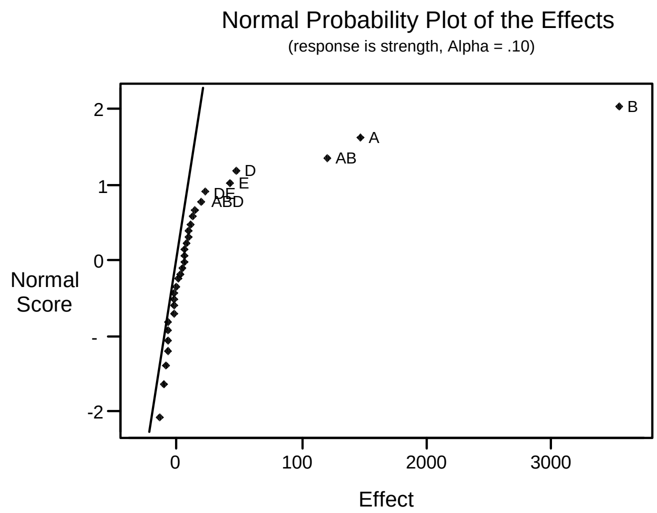


14-16 a)

Estimated Effects and Coefficients for strength

Term	Effect
A	1462.13
B	3537.87
C	-137.12
D	474.62
E	425.38
A*B	1199.62
A*C	124.62
A*D	62.87
A*E	62.12
B*C	-99.63
B*D	-12.88
B*E	-12.12
C*D	112.13
C*E	-62.13
D*E	224.62
A*B*C	-62.88
A*B*D	200.38
A*B*E	49.63
A*C*D	75.38
A*C*E	99.63
A*D*E	-87.12
B*C*D	99.62
B*C*E	-74.63
B*D*E	-62.88
C*D*E	37.13
A*B*C*D	-12.12
A*B*C*E	12.13
A*B*D*E	0.37
A*C*D*E	150.37
B*C*D*E	-25.38
A*B*C*D*E	62.88

b)



The effects that appear to be important are A, B, D, E, and the interactions AB, DE, and ABD.

c) To maximize strength, the variables A, B, D, and E should be increased. Variable C is not significant thus any level of C would be acceptable.

The regression equation is

$$\hat{y} = 2888.7 + 731.1x_1 + 1768.9x_2 + 237.3x_4 + 212.7x_5 + 599.8x_1x_2 + 112.3x_4x_5 + 100.2x_1x_2x_4$$

Predictor	Coef	StDev	T	P
Constant	2887.69	39.10	73.86	0.000
A	731.06	39.10	18.70	0.000
B	1768.94	39.10	45.24	0.000
D	237.31	39.10	6.07	0.000
E	212.69	39.10	5.44	0.000
AB	599.81	39.10	15.34	0.000
DE	112.31	39.10	2.87	0.008
ABD	100.19	39.10	2.56	0.017

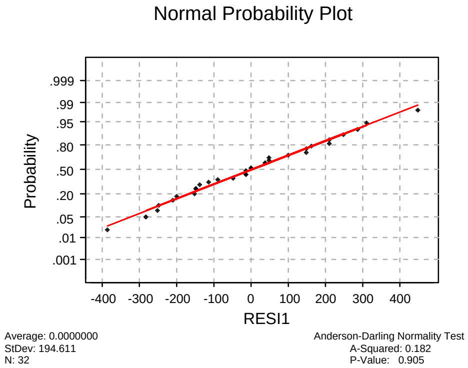
S = 221.2   R-Sq = 99.1%   R-Sq(adj) = 98.9%

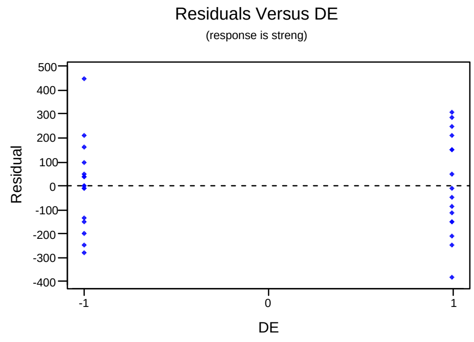
Analysis of Variance

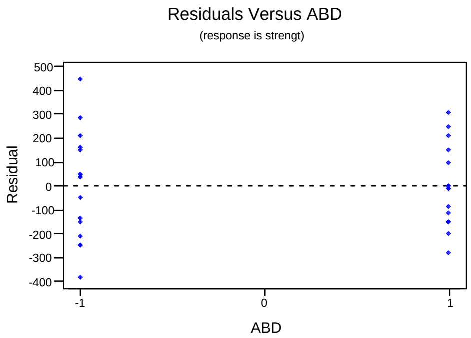
Source	DF	SS	MS	F	P
Regression	7	132722308	18960330	387.58	0.000
Error	24	1174077	48920		
Total	31	133896385			

Source	DF	Seq SS
A	1	17102476
B	1	100132476
D	1	1802151
E	1	1447551
AB	1	11512801
DE	1	403651
ABD	1	321201

d)







The normal probability plot of the residuals indicates the assumption of normality is reasonable. The model appears to be adequate.

14-17 a) Estimated Effects and Coefficients for charge

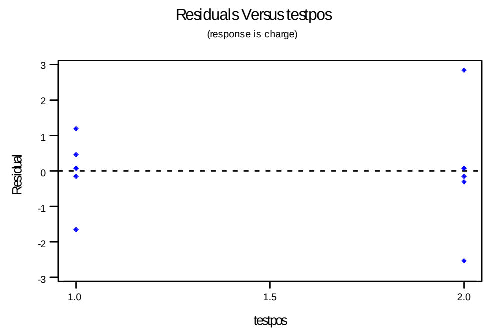
Term	Effect	Coef	StDev Coef	T	P
Constant		-1.000	0.4462	-2.24	0.055
cleanmet	-5.593	-2.797	0.4462	-6.27	0.000
testpos	-1.280	-0.640	0.4462	-1.43	0.189
cleanmet*testpos	-1.220	-0.610	0.4462	-1.37	0.209

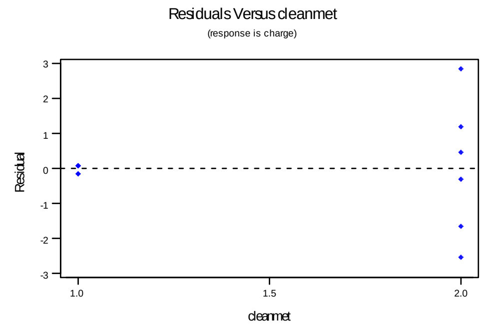
b) Analysis of Variance for charge

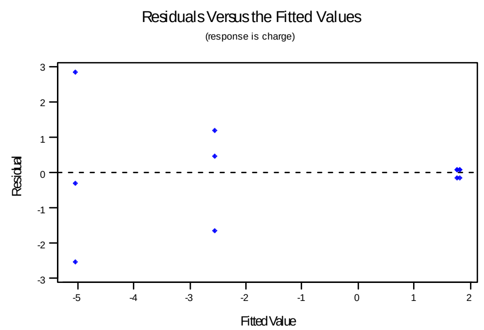
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	98.771	98.7713	49.386	20.67	0.001
2-Way Interactions	1	4.465	4.4652	4.465	1.87	0.209
Residual Error	8	19.110	19.1101	2.389		
Pure Error	8	19.110	19.1101	2.389		
Total	11	122.347				

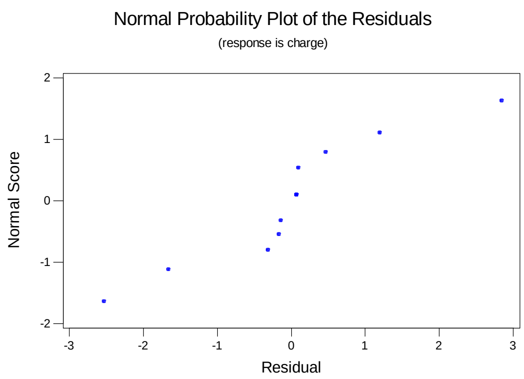
Cleaning Method is the only significant factor.

c) Analysis of the residuals shows that there is more variability at test position R and cleaning material SRD. In the case of the cleaning material, the difference in the variances is very large. The variation decreases with increased fitted values. The normal probability plot appears to have some variations from the straight line.

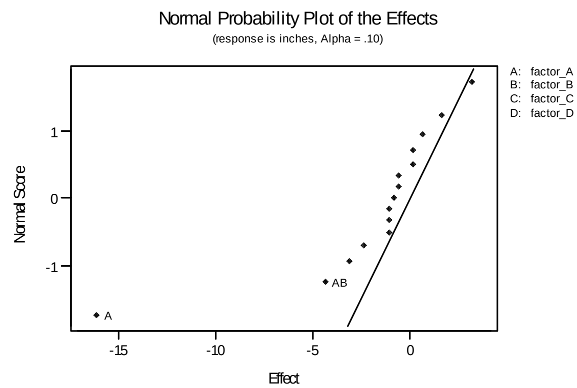








14-18 a)




```
Estimated Effects and Coefficients for inches
Term                               Effect      Coef
Constant                                     35.938
factor_A                          -16.125    -8.063
factor_B                            3.125     1.562
factor_C                           -1.125    -0.562
factor_D                           -1.125    -0.562
factor_A*factor_B                  -4.375    -2.187
factor_A*factor_C                  -0.625    -0.312
factor_A*factor_D                  -3.125    -1.562
factor_B*factor_C                   1.625     0.812
factor_B*factor_D                   0.125     0.063
factor_C*factor_D                  -0.625    -0.313
factor_A*factor_B*factor_C          0.625     0.313
factor_A*factor_B*factor_D         -2.375    -1.188
factor_A*factor_C*factor_D         -1.125    -0.563
factor_B*factor_C*factor_D         -0.875    -0.437
factor_A*factor_B*factor_C
*factor_D                           0.125     0.063
```

According to the normal probability plot, factors A, B, and AB appear to be significant.

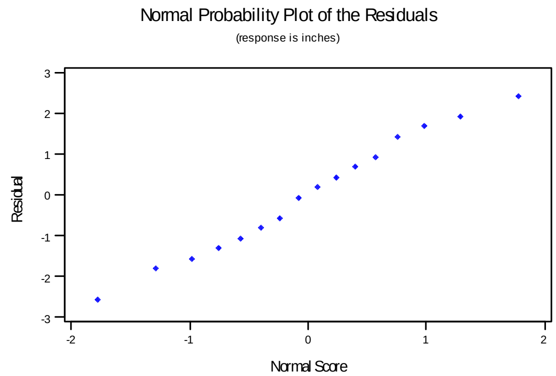
Parts b) and c)

Remove the three- and four-factor interactions to generate the following analysis:

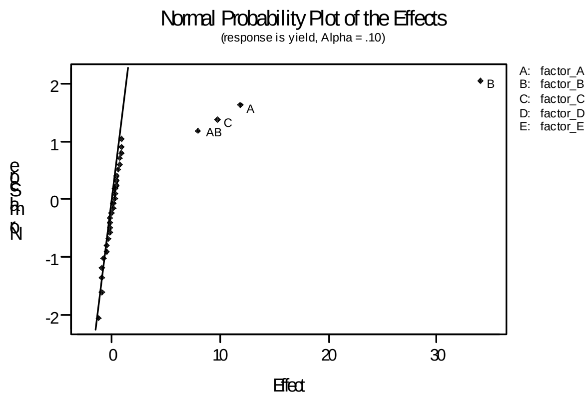
Term	Effect	Coef	StDev Coef	T	P
Constant		35.938	0.6355	56.55	0.000
factor_A	-16.125	-8.063	0.6355	-12.69	0.000
factor_B	3.125	1.562	0.6355	2.46	0.057
factor_C	-1.125	-0.562	0.6355	-0.89	0.417
factor_D	-1.125	-0.562	0.6355	-0.89	0.417
factor_A*factor_B	-4.375	-2.187	0.6355	-3.44	0.018
factor_A*factor_C	-0.625	-0.312	0.6355	-0.49	0.644
factor_A*factor_D	-3.125	-1.562	0.6355	-2.46	0.057
factor_B*factor_C	1.625	0.812	0.6355	1.28	0.257
factor_B*factor_D	0.125	0.063	0.6355	0.10	0.925
factor_C*factor_D	-0.625	-0.313	0.6355	-0.49	0.644

Analysis of Variance for resp, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
A	1	1040.06	1040.06	1040.06	131.03	0.000
B	1	39.06	39.06	39.06	4.92	0.047
A*B	1	76.56	76.56	76.56	9.65	0.009
Error	12	95.25	95.25	7.94		
Total	15	1250.94				



14-19 a) From the normal probability plot of the effects factors A, B, C, and the AB interaction appear to be significant.



b)

Analysis of Variance for yield (coded units)

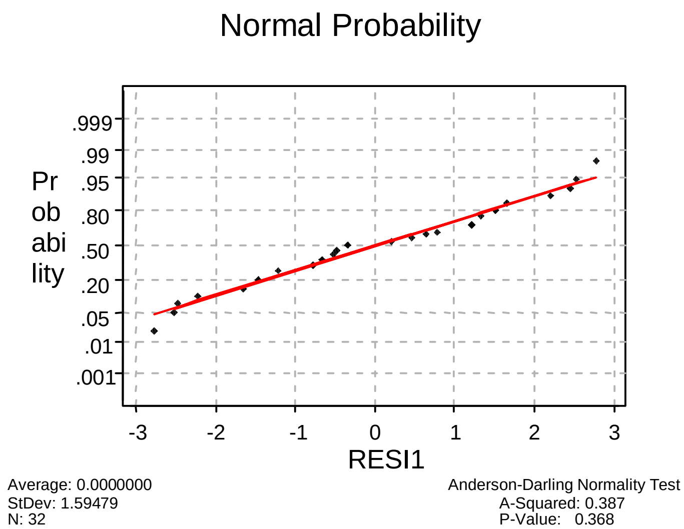
Term	Effect	Coef	StDev Coef	T	P
Constant		30.5312	0.2786	109.57	0.000
factor_A	11.8125	5.9063	0.2786	21.20	0.000
factor_B	33.9375	16.9687	0.2786	60.90	0.000
factor_C	9.6875	4.8437	0.2786	17.38	0.000
factor_D	-0.8125	-0.4063	0.2786	-1.46	0.164
factor_E	0.4375	0.2187	0.2786	0.79	0.444
factor_A*factor_B	7.9375	3.9687	0.2786	14.24	0.000
factor_A*factor_C	0.4375	0.2187	0.2786	0.79	0.444
factor_A*factor_D	-0.0625	-0.0313	0.2786	-0.11	0.912
factor_A*factor_E	0.9375	0.4687	0.2786	1.68	0.112
factor_B*factor_C	0.0625	0.0313	0.2786	0.11	0.912
factor_B*factor_D	-0.6875	-0.3437	0.2786	-1.23	0.235
factor_B*factor_E	0.5625	0.2813	0.2786	1.01	0.328
factor_C*factor_D	0.8125	0.4062	0.2786	1.46	0.164
factor_C*factor_E	0.3125	0.1563	0.2786	0.56	0.583
factor_D*factor_E	-1.1875	-0.5938	0.2786	-2.13	0.049

Analysis of Variance for yield

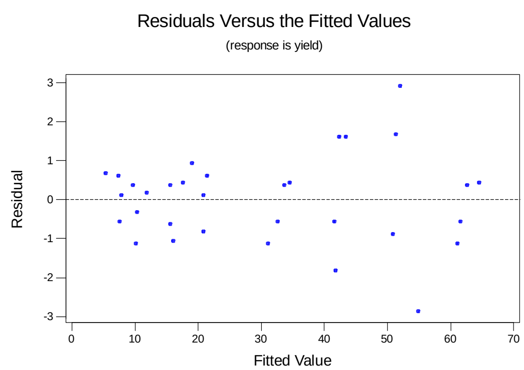
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	11087.9	11087.9	2217.58	892.61	0.000
2-Way Interactions	10	536.3	536.3	53.63	21.59	0.000
Residual Error	16	39.7	39.7	2.48		
Total	31	11664.0				

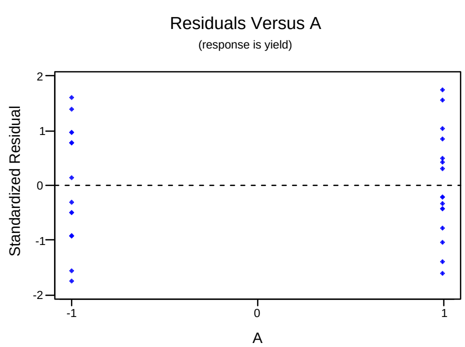
The analysis confirms our findings from part a)

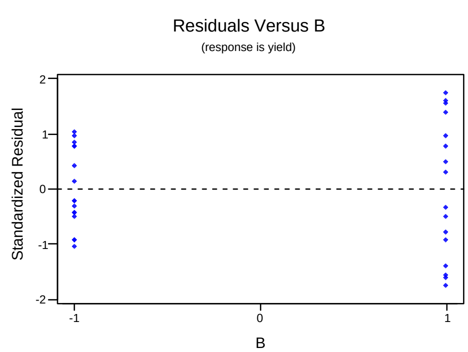
c) The normal probability plot of the residuals is satisfactory. However their variance appears to increase as the fitted value increases.

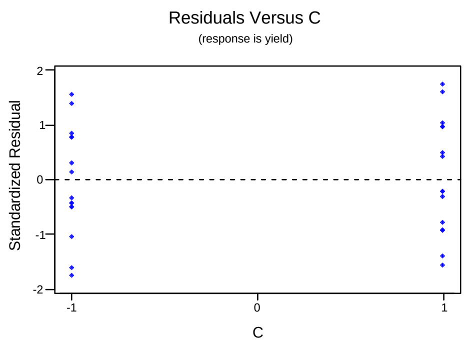


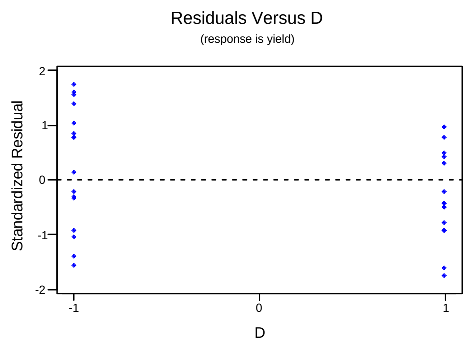
d) All plots support the constant variance assumption, although there is a very slight indication that variability is greater at the high level of factor B.

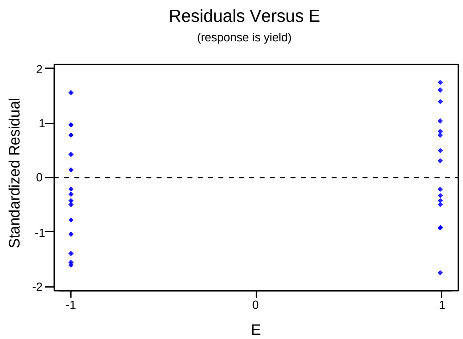




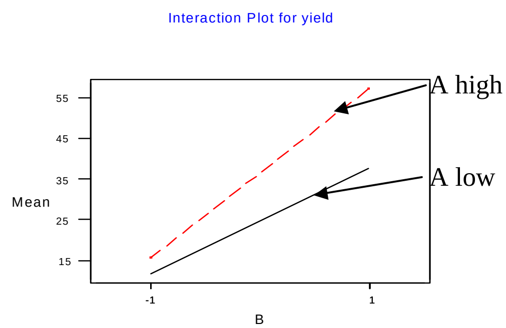




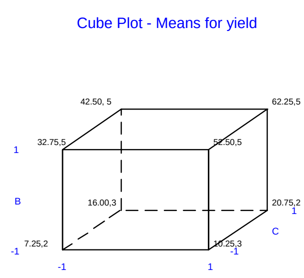




e) The AB interaction appears to be significant. The interaction plot from MINITAB indicates that a high level of A and of B increases the mean yield, while low levels of both factors would lead to a reduction in the mean yield.



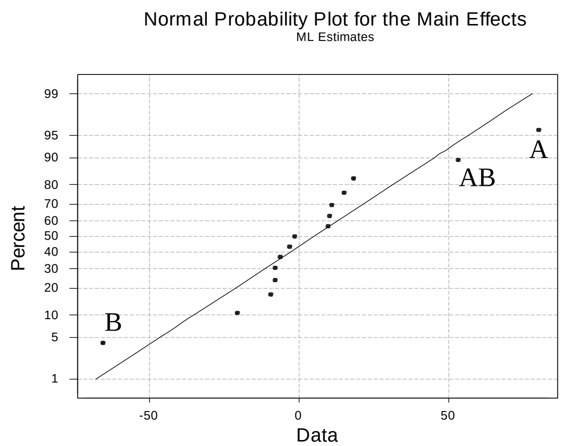
f) To increase yield and optimize the process, we would want to set A, B, and C at their high levels.
g) It is evident from the cube plot that we should run the process with all factors set at their high levels.



14-20 With only one replicate the full factorial cannot be analyzed without using the 3-way interaction for error.

```
Estimated Effects and Coefficients for resp (coded units)

Term          Effect      Coef     SE Coef       T      P
Constant                411.88       34.13   12.07  0.053
A              13.75      6.88       34.12    0.20  0.873
B             127.75     63.87       34.12    1.87  0.312
C              97.75     48.88       34.13    1.43  0.388
A*B           -21.25    -10.63       34.13   -0.31  0.808
A*C          -137.25    -68.63       34.13   -2.01  0.294
B*C           -52.25    -26.13       34.13   -0.77  0.584

Analysis of Variance for resp (coded units)

Source              DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects         3       52128      52128      17376   1.87  0.483
2-Way Interactions   3       44038      44038      14679   1.58  0.516
Residual Error       1        9316       9316       9316
Total                7      105483

The results show that none of the factors or interactions are
significant.
```

14-21 a)

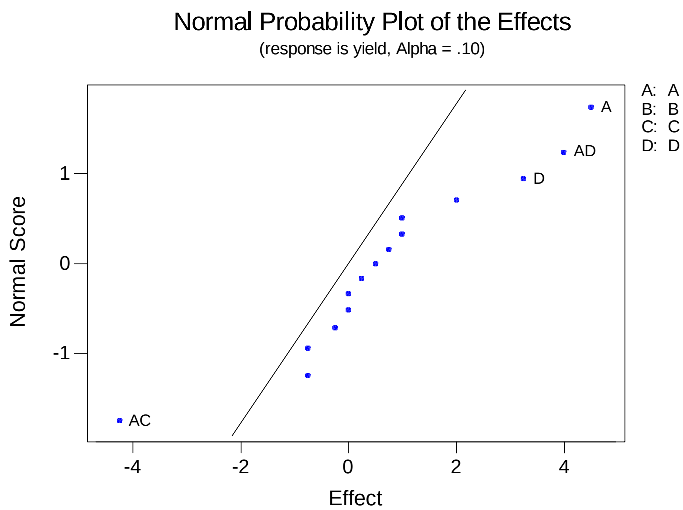


b) Based on the normal probability plot of the effects, factors A, B and AB are significant.

c) The estimated model is: $\hat{y} = 400 + 40.124x_1 - 32.75x_2 + 26.625x_1x_2$

14-22 a)

Term	Effect	Coef
Constant		17.375
A	4.500	2.250
B	0.500	0.250
C	2.000	1.000
D	3.250	1.625
A*B	-0.750	-0.375
A*C	-4.250	-2.125
A*D	4.000	2.000
B*C	0.250	0.125
B*D	0.000	0.000
C*D	0.000	0.000
A*B*C	1.000	0.500
A*B*D	0.750	0.375
A*C*D	-0.250	-0.125
B*C*D	-0.750	-0.375
A*B*C*D	1.000	0.500



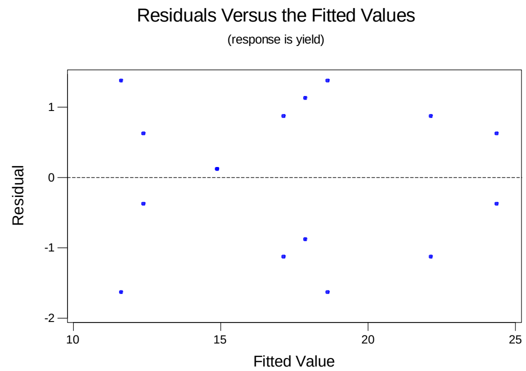
Factors A and D and interactions AC and AD are significant. Factor C should also be included in the model.

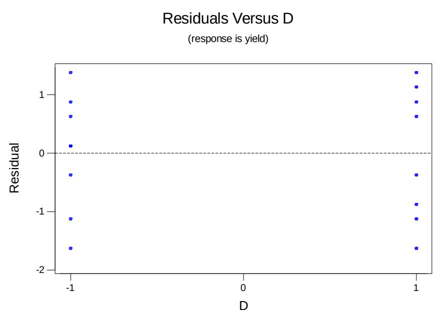
b) Analysis of Variance for yield, using Adjusted SS for Tests

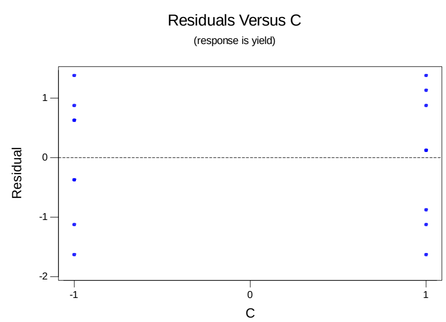
Source	DF	Seq SS	Adj SS	Adj MS	F	P
A	1	81.000	81.000	81.000	49.85	0.000
C	1	16.000	16.000	16.000	9.85	0.011
D	1	42.250	42.250	42.250	26.00	0.000
A*C	1	72.250	72.250	72.250	44.46	0.000
A*D	1	64.000	64.000	64.000	39.38	0.000
Error	10	16.250	16.250	1.625		
Total	15	291.750				

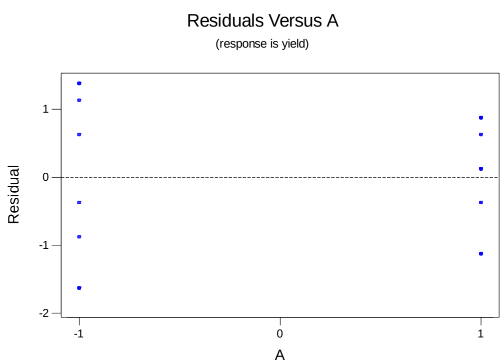
All of the factors and interactions in this table are significant at $\alpha = 0.05$.

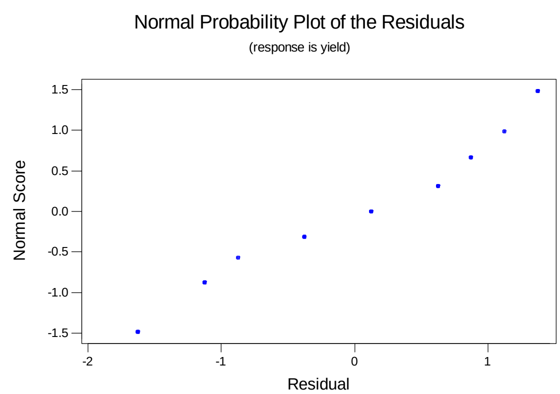
c) The analysis of the residuals shows that the assumptions of normality and constant variance are reasonable.









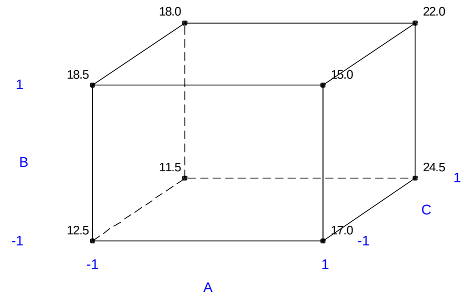


d) The regression equation is

$$\hat{y} = 17.4 + 2.25x_1 + 1.0x_3 + 1.62x_4 - 2.12x_1x_3 + 2.00x_1x_4$$

e) Yes, this design can be projected into a 2^3 design with 2 replicates by removing factor B.

Cube Plot (data means) for yield



The cube plot shows the means at the high and low of each level. It can also be used to identify the interactions.

Section 14-8

14-23

BLOCK	A	B	C	y
1	-1	-1	-1	221
1	1	1	-1	552
1	1	-1	1	406
1	-1	1	1	605
2	1	-1	-1	325
2	-1	1	-1	354
2	-1	-1	1	440
2	1	1	1	392

Term	Effect	Coef
Constant		411.87
Block		34.12
factor_A	13.75	6.87
factor_B	127.75	63.87
factor_C	97.75	48.87
factor_A*factor_B	-21.25	-10.63
factor_A*factor_C	-137.25	-68.63
factor_B*factor_C	-52.25	-26.13

Term	Effect	Coef	StDev Coef	T	P
Constant		411.87	19.94	20.65	0.002
Block		34.12	19.94	1.71	0.229
factor_A	13.75	6.87	19.94	0.34	0.763
factor_B	127.75	63.87	19.94	3.20	0.085
factor_C	97.75	48.87	19.94	2.45	0.134
factor_A*factor_C	-137.25	-68.63	19.94	-3.44	0.075

Analysis of Variance for life

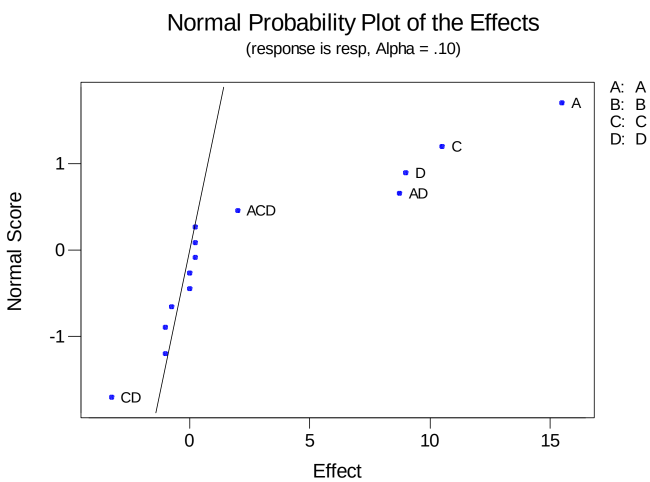
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Blocks	1	9316	9316	9316	2.93	0.229
Main Effects	3	52128	52128	17376	5.46	0.159
2-Way Interactions	1	37675	37675	37675	11.84	0.075
Residual Error	2	6363	6363	3182		
Total	7	105483				

In this model with blocking there are no significant factors.

14-24 Design with 2 blocks

Blocks	A	B	C	D	Rep I
1	-1	-1	-1	-1	159
1	1	1	-1	-1	166
1	1	-1	1	-1	179
1	-1	1	1	-1	173
1	1	-1	-1	1	187
1	-1	1	-1	1	163
1	-1	-1	1	1	168
1	1	1	1	1	194
2	1	-1	-1	-1	168
2	-1	1	-1	-1	158
2	-1	-1	1	-1	175
2	1	1	1	-1	179
2	-1	-1	-1	1	164
2	1	1	-1	1	185
2	1	-1	1	1	197
2	-1	1	1	1	170

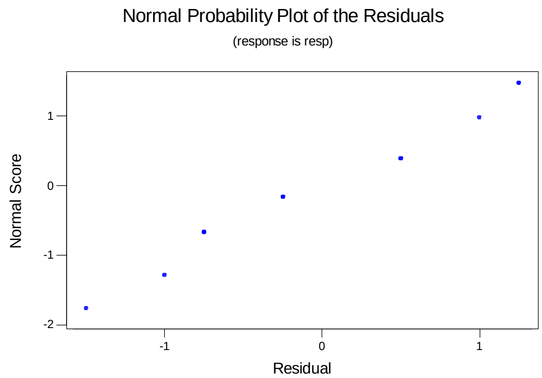
Term	Effect	Coef
Constant		174.063
Block		-0.438
A	15.625	7.813
B	-1.125	-0.563
C	10.625	5.312
D	8.875	4.437
A*B	-0.625	-0.313
A*C	0.125	0.062
A*D	8.875	4.437
B*C	0.375	0.188
B*D	0.125	0.063
C*D	-3.125	-1.562
A*B*C	-0.125	-0.062
A*B*D	-0.875	-0.437
A*C*D	1.875	0.937
B*C*D	0.125	0.063



Factors A, C, and D, and interactions AD, CD and ACD appear to be significant.

```
Estimated Effects and Coefficients for resp (coded units)

Term       Effect      Coef  SE Coef       T      P
Constant            174.063   0.2864  607.74  0.000
Block                 0.438   0.2864    1.53  0.165
A          15.625     7.812   0.2864   27.28  0.000
C          10.625     5.313   0.2864   18.55  0.000
D           8.875     4.438   0.2864   15.49  0.000
A*D         8.875     4.437   0.2864   15.49  0.000
C*D        -3.125    -1.563   0.2864   -5.46  0.001
A*C*D       1.875     0.937   0.2864    3.27  0.011

S = 1.14564   R-Sq = 99.51%   R-Sq(adj) = 99.07%
```

The main effects and interactions are all significant in a model that includes the factors listed above. The normal probability plot appears to support the assumption of normality.



14-25 Model with four blocks

BLOCK	A	B	C	D	var_1
1	-1	-1	-1	-1	166
1	1	1	-1	-1	197
1	1	-1	1	1	159
1	-1	1	1	1	170
2	-1	-1	1	-1	175
2	1	-1	-1	1	163
2	-1	1	-1	1	179
2	1	1	1	-1	187
3	-1	-1	-1	1	179
3	1	-1	1	-1	185
3	-1	1	1	-1	173
3	1	1	-1	1	164
4	1	-1	-1	-1	158
4	-1	1	-1	-1	168
4	-1	-1	1	1	168
4	1	1	1	1	194

```
Term          Effect       Coef      SE Coef       T       P
Constant                 174.063      0.6760  257.47   0.000
Block   1                 -1.063      1.1709   -0.91   0.460
        2                  1.937      1.1709    1.65   0.240
        3                  1.188      1.1709    1.01   0.417
A             15.625       7.812      0.6760   11.56   0.007
B             -1.125      -0.563      0.6760   -0.83   0.493
```

```
C            10.625     5.312      0.6760    7.86  0.016
D             8.875     4.438      0.6760    6.56  0.022
A*B          -0.625    -0.312      0.6760   -0.46  0.689
A*C           0.125     0.062      0.6760    0.09  0.935
A*D           8.875     4.438      0.6760    6.56  0.022
B*D           0.125     0.063      0.6760    0.09  0.935
B*C*D         0.125     0.062      0.6760    0.09  0.935
A*B*C*D      -0.875    -0.437      0.6760   -0.65  0.584

Alias Structure

I
Blocks = C*D + A*B*C + A*B*D
A
B
C
D
A*B
A*C
A*D
B*D
B*C*D
A*B*C*D
```

Effects A, C, D, and the AD interaction are significant in the model with 4 blocks. There is no information about interaction CD as it is confounded with block effects.

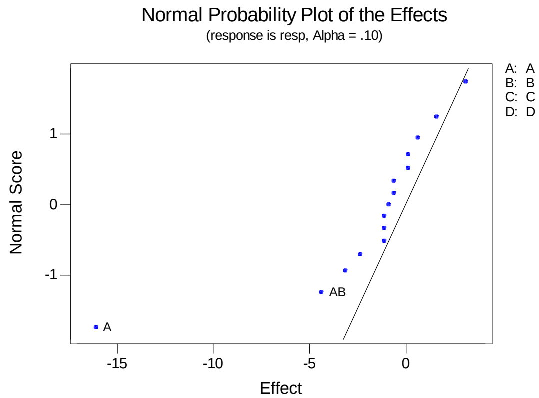
14-26 2^5 Design in 2 Blocks with ABCDE confounded with blocks.

Run	Block	A	B	C	D	E
1	1	-	-	-	-	-
2	1	+	+	-	-	-
3	1	+	-	+	-	-
4	1	-	+	+	-	-
5	1	+	-	-	+	-
6	1	-	+	-	+	-
7	1	-	-	+	+	-
8	1	+	+	+	+	-
9	1	+	-	-	-	+
10	1	-	+	-	-	+
11	1	-	-	+	-	+
12	1	+	+	+	-	+
13	1	-	-	-	+	+
14	1	+	+	-	+	+
15	1	+	-	+	+	+
16	1	-	+	+	+	+
17	2	+	-	-	-	-
18	2	-	+	-	-	-
19	2	-	-	+	-	-
20	2	+	+	+	-	-
21	2	-	-	-	+	-
22	2	+	+	-	+	-
23	2	+	-	+	+	-
24	2	-	+	+	+	-
25	2	-	-	-	-	+
26	2	+	+	-	-	+
27	2	+	-	+	-	+
28	2	-	+	+	-	+
29	2	+	-	-	+	+
30	2	-	+	-	+	+
31	2	-	-	+	+	+
32	2	+	+	+	+	+

14-27 2^5 Design in 4 Blocks.

Run	Block	A	B	C	D	E
1	1	-	-	-	-	-
2	1	+	+	-	-	-
3	1	+	-	+	+	-
4	1	-	+	+	+	-
5	1	+	-	+	-	+
6	1	-	+	+	-	+
7	1	-	-	-	+	+
8	1	+	+	-	+	+
9	2	+	-	-	-	-
10	2	-	+	-	-	-
11	2	-	-	+	+	-
12	2	+	+	+	+	-
13	2	-	-	+	-	+
14	2	+	+	+	-	+
15	2	+	-	-	+	+
16	2	-	+	-	+	+
17	3	+	-	+	-	-
18	3	-	+	+	-	-
19	3	-	-	-	+	-
20	3	+	+	-	+	-
21	3	-	-	-	-	+
22	3	+	+	-	-	+
23	3	+	-	+	+	+
24	3	-	+	+	+	+
25	4	-	-	+	-	-
26	4	+	+	+	-	-
27	4	+	-	-	+	-
28	4	-	+	-	+	-
29	4	+	-	-	-	+
30	4	-	+	-	-	+
31	4	-	-	+	+	+
32	4	+	+	+	+	+

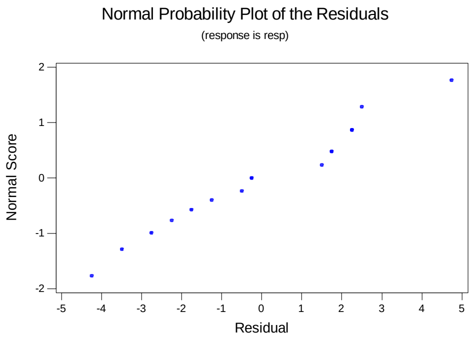
14-28

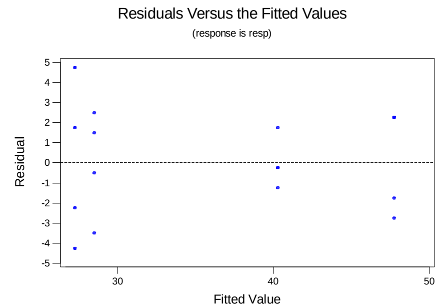


Factor A and interaction AB are significant. Factor B should also be included in the modelto make the model hierarchical..

```
Term          Effect      Coef
Constant                35.938
BLOCK                   -0.063
A            -16.125    -8.062
B              3.125     1.562
C             -1.125    -0.562
D             -1.125    -0.562
A*B           -4.375    -2.188
A*C           -0.625    -0.313
A*D           -3.125    -1.563
B*C            1.625     0.813
B*D            0.125     0.063
C*D           -0.625    -0.312
A*B*C          0.625     0.312
A*B*D         -2.375    -1.187
A*C*D         -1.125    -0.562
B*C*D         -0.875    -0.438

Estimated Effects and Coefficients for resp (coded units)

Term          Effect      Coef     SE Coef       T      P
Constant                35.938      0.7043   51.02  0.000
A            -16.125    -8.062      0.7043  -11.45  0.000
B              3.125     1.562      0.7043    2.22  0.047
A*B           -4.375    -2.188      0.7043   -3.11  0.009

Analysis of Variance for resp (coded units)

Source                 DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects            2     1079.12    1079.12    539.562  67.98  0.000
2-Way Interactions      1       76.56      76.56     76.563   9.65  0.009
Residual Error         12       95.25      95.25      7.938
  Pure Error           12       95.25      95.25      7.938
Total                  15     1250.94
```

Effects A, B, and the AB interaction are significant at $\alpha = 0.05$. The residual analysis shows some slight differences in variability in the data.





14-29 a) Estimated Effects and Coefficients for y

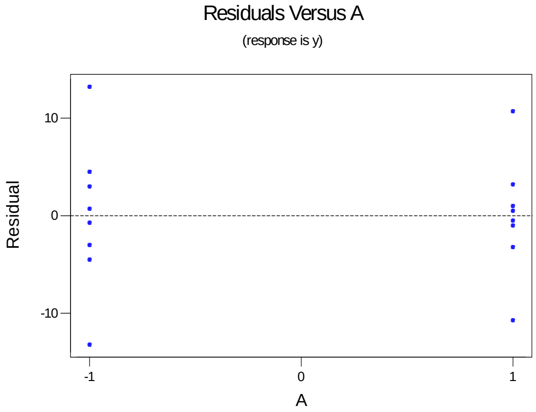
Term	Effect	Coef	StDev Coef	T	P
Constant		56.37	2.633	21.41	0.000
Block 1		15.63	4.560	3.43	0.014
2		-3.38	4.560	-0.74	0.487
3		-10.88	4.560	-2.38	0.054
A	-45.25	-22.62	2.633	-8.59	0.000
B	-1.50	-0.75	2.633	-0.28	0.785
C	14.50	7.25	2.633	2.75	0.033
A*B	19.00	9.50	2.633	3.61	0.011
A*C	-14.50	-7.25	2.633	-2.75	0.033
B*C	-9.25	-4.63	2.633	-1.76	0.130

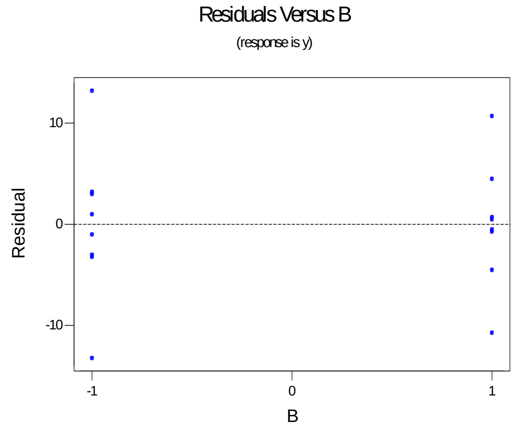
Analysis of Variance for y

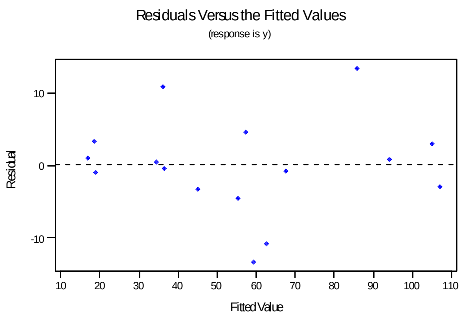
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Blocks	3	1502.8	1502.8	500.9	4.52	0.055
Main Effects	3	9040.2	9040.2	3013.4	27.17	0.001
2-Way Interactions	3	2627.2	2627.2	875.7	7.90	0.017
Residual Error	6	665.5	665.5	110.9		
Total	15	13835.7				

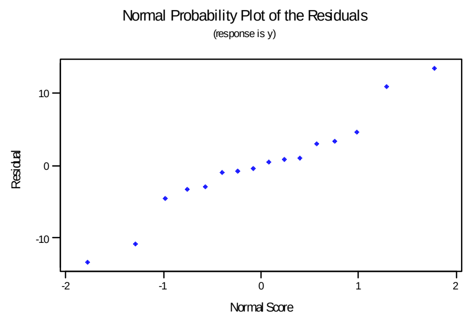
Factors A, C, AB, and AC are significant.

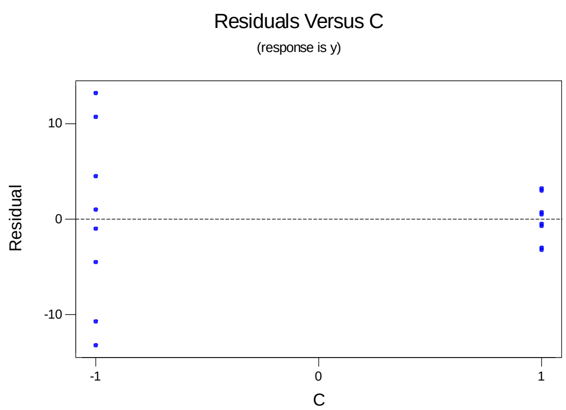
b) There is more variability on the response associated with the low setting of factor C.











c) Some of the information from the experiment is lost because the design is run in 4 blocks. This causes us to lose information on the ABC interaction even though we have replicated the experiment twice. If it is possible to run the experiment in only 2 blocks, there would be information on all interactions.

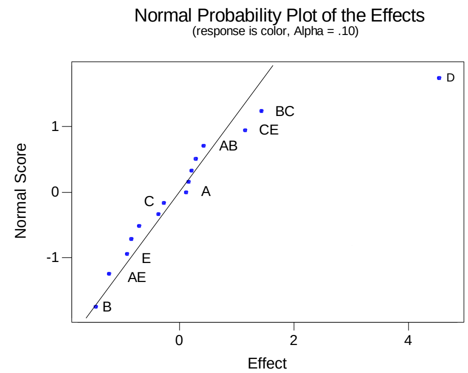
d) To have data on all interactions, we could run the experiment so that each replicate is a block (therefore only 2 blocks).

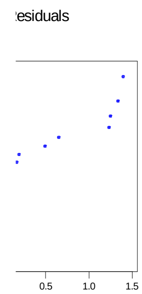
14-30 2^6 in 4 blocks.

Run	Block	A	B	C	D	E	F
1	1	-	-	-	+	+	-
2	1	+	-	+	+	-	-
3	1	-	+	-	-	+	+
4	1	-	+	+	+	-	-
5	1	+	+	-	+	+	-
6	1	-	-	+	+	+	+
7	1	+	-	+	-	+	-
8	1	+	+	+	-	-	+
9	1	-	-	+	-	-	+
10	1	+	+	-	-	-	-
11	1	-	+	+	-	+	-
12	1	+	-	-	-	+	+
13	1	-	-	-	-	-	-
14	1	-	+	-	+	-	+
15	1	+	+	+	+	+	+
16	1	+	-	-	+	-	+
17	2	-	+	+	-	+	+
18	2	+	+	+	-	-	-
19	2	-	-	-	-	-	+
20	2	-	+	+	+	-	+
21	2	+	+	-	-	-	+
22	2	+	-	-	+	-	-
23	2	-	-	-	+	+	+
24	2	-	-	+	+	+	-
25	2	-	-	+	-	-	-
26	2	+	-	+	-	+	+
27	2	-	+	-	+	-	-
28	2	+	-	+	+	-	+
29	2	+	-	-	-	+	-
30	2	+	+	+	+	+	-
31	2	-	+	-	-	+	-
32	2	+	+	-	+	+	+
33	3	+	+	+	+	-	+
34	3	-	-	-	-	+	-
35	3	-	+	+	-	-	-
36	3	-	-	+	-	+	+
37	3	-	+	-	-	-	+
38	3	-	-	+	+	-	+
39	3	+	-	-	+	+	+
40	3	+	-	+	+	+	-
41	3	+	-	+	-	-	-
42	3	+	+	-	-	+	-
43	3	-	+	+	+	+	-
44	3	-	+	-	+	+	+
45	3	+	-	-	-	-	+
46	3	+	+	-	+	-	-
47	3	-	-	-	+	-	-
48	3	+	+	+	-	+	+
49	4	+	-	+	+	+	+
50	4	-	-	-	-	+	+
51	4	+	-	-	-	-	-
52	4	-	-	+	+	-	-
53	4	-	+	+	+	+	+
54	4	-	+	-	-	-	-
55	4	+	-	+	-	-	+
56	4	+	+	-	+	-	+
57	4	-	+	-	+	+	-
58	4	+	+	+	-	+	-
59	4	+	+	-	-	+	+
60	4	-	-	+	-	+	-
61	4	-	+	+	-	-	+
62	4	+	-	-	+	+	-
63	4	-	-	-	+	-	+
64	4	+	+	+	+	-	-

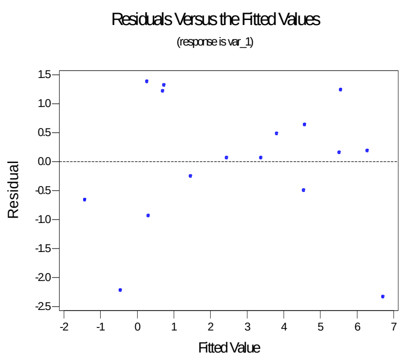
Section 14-9

14-31 a) Several factors and interactions are significant.

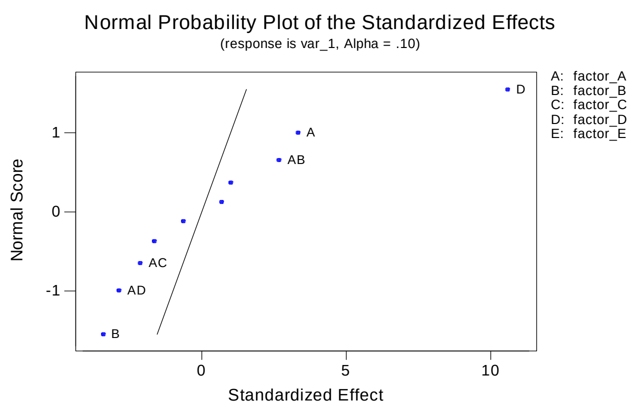




b) There are no serious problems with the residual plots. The normal probability plot has a little bit of curvature and there is a little more variability at the lower and higher ends of the fitted values.



c) Normal probability plot shows that we can collapse using only factors A, B, and D



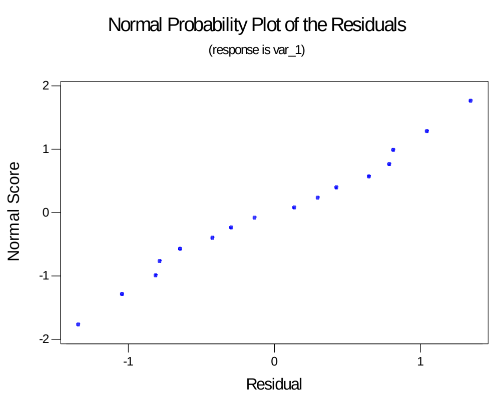
Estimated Effects and Coefficients for var_1

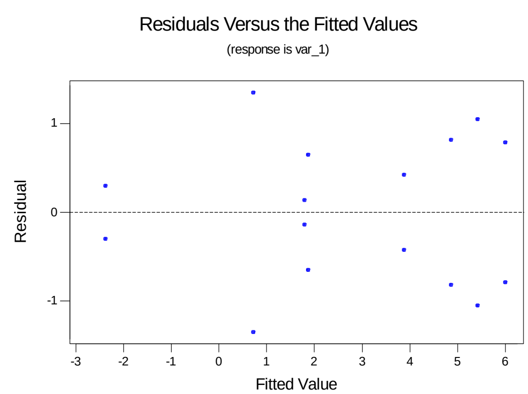
Term	Effect	Coef	StDev Coef	T	P
Constant		2.7700	0.2762	10.03	0.000
factor_A	1.4350	0.7175	0.2762	2.60	0.032
factor_B	-1.4650	-0.7325	0.2762	-2.65	0.029
factor_D	4.5450	2.2725	0.2762	8.23	0.000
factor_A*factor_B	1.1500	0.5750	0.2762	2.08	0.071
factor_A*factor_D	-1.2300	-0.6150	0.2762	-2.23	0.057
factor_B*factor_D	0.1200	0.0600	0.2762	0.22	0.833
factor_A*factor_B*factor_D	-0.3650	-0.1825	0.2762	-0.66	0.527

Analysis of Variance for var_1

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	99.450	99.4499	33.1500	27.15	0.000
2-Way Interactions	3	11.399	11.3992	3.7997	3.11	0.088
3-Way Interactions	1	0.533	0.5329	0.5329	0.44	0.527
Residual Error	8	9.767	9.7668	1.2208		
Pure Error	8	9.767	9.7668	1.2208		
Total	15	121.149				

Factors A, B, D, AB and AD are significant.





The normal probability plot looks good. The reduced model ignores factor C and it is two replicates of a full factorial experiment in factors A, B, and D. There are 8 unique test points with two replicates at each. The model shown has 8 coefficients so that the fitted value is the mean of the replicates at each of the 8 unique test points. Therefore, at each unique test point there are equal positive and negative residuals. Consequently, the plot of residuals versus fitted values has a symmetry about zero.

14-32 a) Design Generators: D = ABC

Alias Structure
I + ABCD
A + BCD
B + ACD
C + ABD
D + ABC
AB + CD
AC + BD
AD + BC

b)

Term	Effect	Coef
Constant		70.750
A	19.000	9.500
B	1.500	0.750
C	14.000	7.000
D	16.500	8.250
A*B	-1.000	-0.500
A*C	-18.500	-9.250
A*D	19.000	9.500

A, C, D, AC, and AD have large estimated effects.

c) Estimated Effects and Coefficients for rate

Term	Effect	Coef	StDev Coef	T	P
Constant		70.750	0.6374	111.00	0.000
A	19.000	9.500	0.6374	14.90	0.004
C	14.000	7.000	0.6374	10.98	0.008
D	16.500	8.250	0.6374	12.94	0.006
A*C	-18.500	-9.250	0.6374	-14.51	0.005
A*D	19.000	9.500	0.6374	14.90	0.004

Analysis of Variance for rate

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	1658.50	1658.50	552.833	170.10	0.006
2-Way Interactions	2	1406.50	1406.50	703.250	216.38	0.005
Residual Error	2	6.50	6.50	3.250		
Total	7	3071.50				

A, C, D, AC, and AD are significant. This appears to be the appropriate model for the data.

14-33 a.)The design generators are I = ACE and I = BDE. This is verified by looking at the following table. The contrast for E is calculated using E = AC and the contrast for D is calculated using D = BE.

A	B	C	D	E	reponse
-1	-1	-1	-1	1	23.2
1	1	-1	-1	-1	15.5
1	-1	-1	1	-1	16.9
-1	1	1	-1	-1	16.2
-1	-1	1	1	-1	23.8
1	-1	1	-1	1	23.4
-1	1	-1	1	1	16.8
1	1	1	1	1	18.1

b.) Design Generator: D=BE, E=AC
Defining Relation: I=ACE=BDE=ABCDE

Aliases
A=CE=BCDE=ABDE
B=DE=ACDE=ABCE
C=AE=ABDE=BCDE
D=BE=ABCE=ACDE
E=AC=BD=ABCD

c.) Estimated Effects and Coefficients for reponse (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		19.238	0.7871	24.44	0.002
A	-1.525	-0.762	0.7871	-0.97	0.435
B	-5.175	-2.587	0.7871	-3.29	0.081
C	2.275	1.138	0.7871	1.45	0.285
D	-0.675	-0.337	0.7871	-0.43	0.710
E	2.275	1.137	0.7871	1.45	0.285

d) Estimated Effects and Coefficients for reponse (coded units)

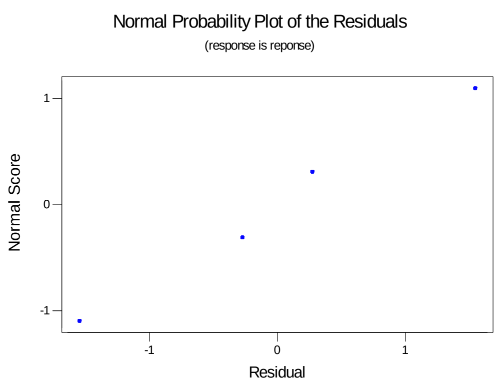
Term	Effect	Coef	SE Coef	T	P
Constant		19.238	1.138	16.91	0.038
A	-1.525	-0.762	1.138	-0.67	0.624
B	-5.175	-2.587	1.138	-2.27	0.264
C	2.275	1.138	1.138	1.00	0.500
D	-0.675	-0.337	1.138	-0.30	0.816
A*B	1.825	0.913	1.138	0.80	0.570
A*D	-1.275	-0.638	1.138	-0.56	0.675

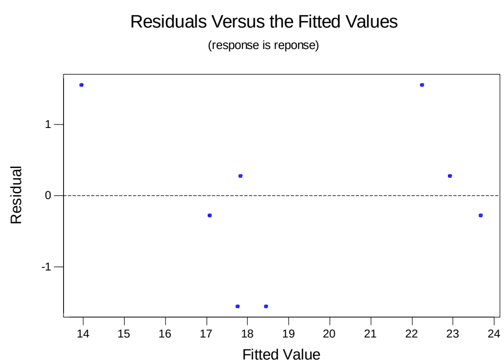
Analysis of Variance for reponse (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	69.475	69.475	17.369	1.68	0.517
2-Way Interactions	2	9.913	9.913	4.956	0.48	0.715
Residual Error	1	10.351	10.351	10.351		
Total	7	89.739				

Interactions AD and AB are not significant in the model, and therefore may be used as error.

e) The normal probability plot and the plot of the residuals versus fitted values are satisfactory.



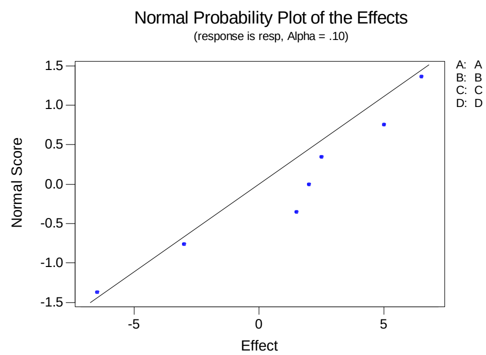


14-34 Because factors A, B, C, and E form a word in the complete defining relation, it can be verified that the resulting design is two replicates of a 2^{4-1} fractional factorial.

14-35 Because factors A, B, C, and E form a word in the complete defining relation, it can be verified that the resulting design is two replicates of a 2^{4-1} fractional factorial. This is different than the design that results when C and E are dropped from the 2^{6-2} in Table 14-28 which results in a full factorial because, the factors ABDF do not form a word in the complete defining relation.

14-36 `Estimated Effects and Coefficients for resp (coded units)`

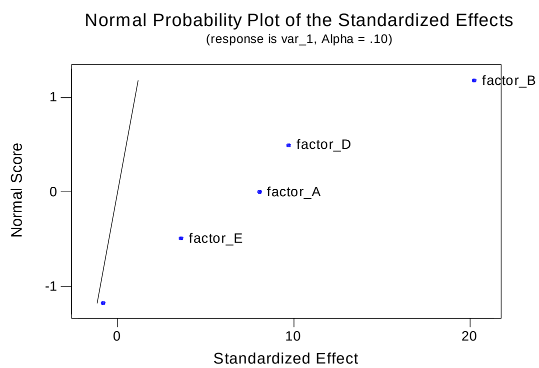
```
Estimated Effects and Coefficients for yield (coded units)

Term         Effect    Coef
Constant              17.875
A             3.750    1.875
B             0.250    0.125
C             2.750    1.375
D             4.250    2.125
A*B          -0.750   -0.375
A*C          -4.250   -2.125
A*D           4.250    2.125
```



None of the factors appear to be significant in the 2^{4-1} design.

14-37 Generators D = AB, E = AC for 2^{5-2}, Resolution III

A	B	C	D	E	var_1
-1	-1	-1	1	1	1900
1	-1	-1	-1	-1	900
-1	1	-1	-1	1	3500
1	1	-1	1	-1	6100
-1	-1	1	1	-1	800
1	-1	1	-1	1	1200
-1	1	1	-1	-1	3000
1	1	1	1	1	6800



The normal probability plot and table below show that factors A, B, and D are significant.

```
Estimated Effects and Coefficients for var_1 (coded units)

Term          Effect      Coef     SE Coef       T      P
Constant                3025.00      90.14   33.56  0.001
factor_A    1450.00    725.00      90.14    8.04  0.015
factor_B    3650.00   1825.00      90.14   20.25  0.002
factor_C    -150.00    -75.00      90.14   -0.83  0.493
factor_D    1750.00    875.00      90.14    9.71  0.010
factor_E     650.00    325.00      90.14    3.61  0.069

Analysis of Variance for var_1 (coded units)
Source               DF      Seq SS     Adj SS     Adj MS      F       P
Main Effects          5    37865000   37865000    7573000 116.51  0.009
Residual Error        2      130000     130000      65000
Total                 7    37995000
```

Factors A, B and D are significant. In these factors, the design is a 2^2 with two replicates.

14-38 2^{8-4} fractional factorial

```
Design Generators:  E = BCD  F = ACD  G = ABC  H = ABD

Alias Structure (up to order 4)
I + ABCG + ABDH + ABEF + ACDF + ACEH + ADEG + AFGH + BCDE + BCFH + BDFG +
BEGH+ CDGH + CEFG + DEFH

A + BCG + BDH + BEF + CDF + CEH + DEG + FGH
B + ACG + ADH + AEF + CDE + CFH + DFG + EGH
C + ABG + ADF + AEH + BDE + BFH + DGH + EFG
D + ABH + ACF + AEG + BCE + BFG + CGH + EFH
E + ABF + ACH + ADG + BCD + BGH + CFG + DFH
F + ABE + ACD + AGH + BCH + BDG + CEG + DEH
G + ABC + ADE + AFH + BDF + BEH + CDH + CEF
H + ABD + ACE + AFG + BCF + BEG + CDG + DEF
AB + CG + DH + EF + ACDE + ACFH + ADFG + AEGH + BCDF + BCEH + BDEG + BFGH
AC + BG + DF + EH + ABDE + ABFH + ADGH + AEFG + BCDH + BCEF + CDEG + CFGH
AD + BH + CF + EG + ABCE + ABFG + ACGH + AEFH + BCDG + BDEF + CDEH + DFGH
AE + BF + CH + DG + ABCD + ABGH + ACFG + ADFH + BCEG + BDEH + CDEF + EFGH
AF + BE + CD + GH + ABCH + ABDG + ACEG + ADEH + BCFG + BDFH + CEFH + DEFG
AG + BC + DE + FH + ABDF + ABEH + ACDH + ACEF + BDGH + BEFG + CDFG + CEGH
AH + BD + CE + FG + ABCF + ABEG + ACDG + ADEF + BCGH + BEFH + CDFH + DEGH
Design
```

run no.	A	B	C	D	E	F	G	H
1	-1	-1	-1	-1	-1	-1	-1	-1
2	1	-1	-1	-1	-1	1	1	1
3	-1	1	-1	-1	1	-1	1	1
4	1	1	-1	-1	1	1	-1	-1
5	-1	-1	1	-1	1	1	1	-1
6	1	-1	1	-1	1	-1	-1	1
7	-1	1	1	-1	-1	1	-1	1
8	1	1	1	-1	-1	-1	1	-1
9	-1	-1	-1	1	1	1	-1	1
10	1	-1	-1	1	1	-1	1	-1
11	-1	1	-1	1	-1	1	1	-1
12	1	1	-1	1	-1	-1	-1	1
13	-1	-1	1	1	-1	-1	1	1
14	1	-1	1	1	-1	1	-1	-1
15	-1	1	1	1	1	-1	-1	-1
16	1	1	1	1	1	1	1	1

14-39 2^{6-3} fractional factorial
Design Generators: D=AB, E=AC, F=BC
Defining Relation: I=ABD=ACE=BCF=BCDE=ACDF=ABEF

A=BD=CE	AB=EF
B=AD=CF	AC=DF
C=AE=BF	AD=CF
D=AB	AE=BF
E=AC	AF=BE=CD
F=BC	BC=DE

14-40 a) Design 2^{5-1}

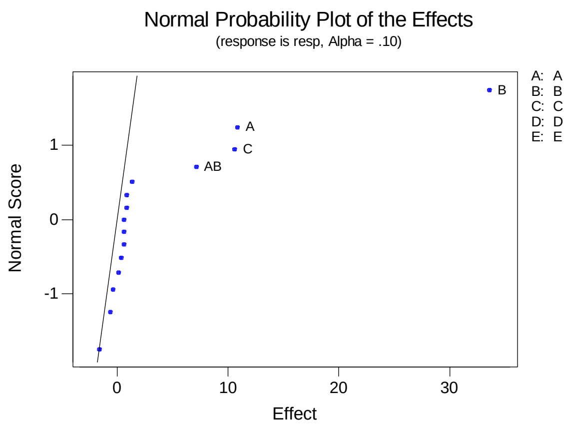
Run No.	A	B	C	D	E	resp
1	-1	-1	-1	-1	1	8
2	1	-1	-1	-1	-1	9
3	-1	1	-1	-1	-1	34
4	1	1	-1	-1	1	52
5	-1	-1	1	-1	-1	16
6	1	-1	1	-1	1	22
7	-1	1	1	-1	1	45
8	1	1	1	-1	-1	60
9	-1	-1	-1	1	-1	8
10	1	-1	-1	1	1	10
11	-1	1	-1	1	1	30
12	1	1	-1	1	-1	50
13	-1	-1	1	1	1	15
14	1	-1	1	1	-1	21
15	-1	1	1	1	-1	44
16	1	1	1	1	1	63

b)
```
Design Generators:  E = ABCD

Alias Structure

I + ABCDE

A + BCDE
B + ACDE
C + ABDE
D + ABCE
E + ABCD
AB + CDE
AC + BDE
AD + BCE
AE + BCD
BC + ADE
BD + ACE
BE + ACD
CD + ABE
CE + ABD
DE + ABC
```

c)

Term	Effect	Coef
Constant		30.4375
factor_A	10.8750	5.4375
factor_B	33.6250	16.8125
factor_C	10.6250	5.3125
factor_D	-0.6250	-0.3125
factor_E	0.3750	0.1875
factor_A*factor_B	7.1250	3.5625
factor_A*factor_C	0.6250	0.3125
factor_A*factor_D	0.8750	0.4375
factor_A*factor_E	1.3750	0.6875
factor_B*factor_C	0.8750	0.4375
factor_B*factor_D	-0.3750	-0.1875
factor_B*factor_E	0.1250	0.0625
factor_C*factor_D	0.6250	0.3125
factor_C*factor_E	0.6250	0.3125
factor_D*factor_E	-1.6250	-0.8125

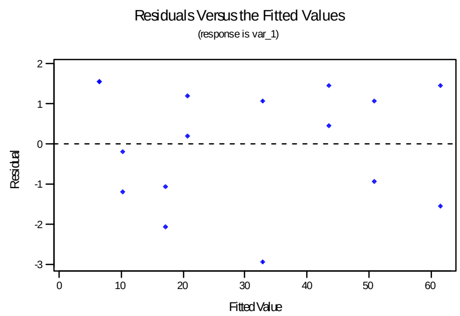
d) Factors A, B, and C and interaction AB are significant

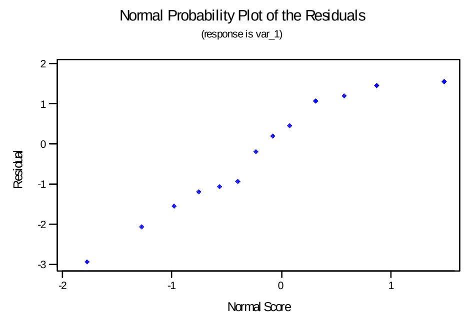


e)

Term	Effect	Coef	StDev Coef	T	P
Constant		30.438	0.4243	71.73	0.000
factor_A	10.875	5.438	0.4243	12.81	0.000
factor_B	33.625	16.812	0.4243	39.62	0.000
factor_C	10.625	5.313	0.4243	12.52	0.000
factor_A*factor_B	7.125	3.562	0.4243	8.40	0.000

f)





g) The main factors A, B, C, and the interaction AB are all significant.

Supplemental Exercises

14-41 a) Estimated Effects and Coefficients for var_1 (coded units)

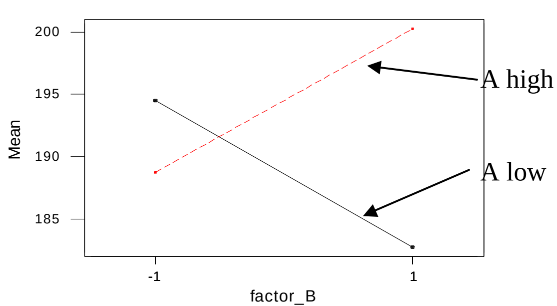
Term	Effect	Coef	SE Coef	T	P
Constant		191.563	1.158	165.49	0.000
factor_A (PH)	5.875	2.937	1.158	2.54	0.026
factor_B (CC)	-0.125	-0.062	1.158	-0.05	0.958
factor_A*factor_B	11.625	5.812	1.158	5.02	0.000

Analysis of Variance for var_1 (coded units)

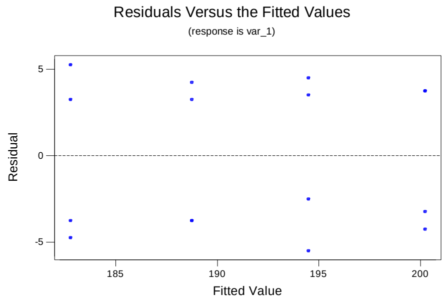
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	138.125	138.125	69.06	3.22	0.076
2-Way Interactions	1	540.562	540.562	540.56	25.22	0.000
Residual Error	12	257.250	257.250	21.44		
Pure Error	12	257.250	257.250	21.44		
Total	15	935.938				

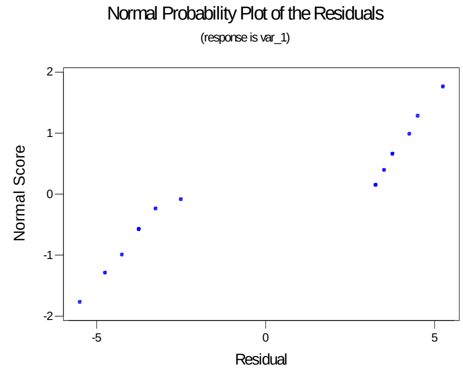
The main effect of pH and the interaction of pH and Catalyst Concentration (CC) are significant at the 0.05 level of significance. The model used is viscosity = $191.563 + 2.937x_1 - 0.062x_2 + 5.812x_{12}$

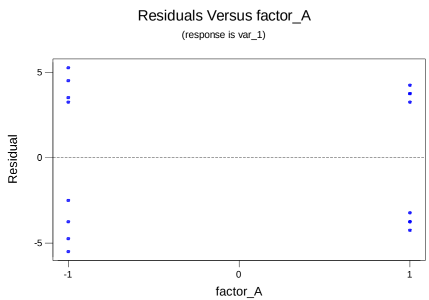
b) The interaction plot shows that there is a strong interaction. When Factor A is at its low level, the mean response is large at the low level of B and is small at the high level of B. However, when A is at its high level, the results are opposite.

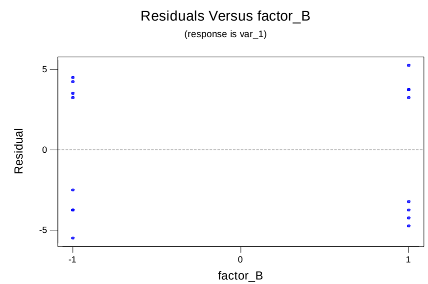


c) The plots of the residuals show that the equality of variance assumption is reasonable. However, there is a large gap in the middle of the normal probability plot. Sometimes, this can indicate that there is another variable that has an effect on the response but which is not included in the experiment. For example, in this experiment, note that the replicates in each cell have two pairs of values that are very similar, but there is a rather large difference in the mean values of the two pairs. (Cell 1 has 189 and 192 as one pair and 198 and 199 as the other.)







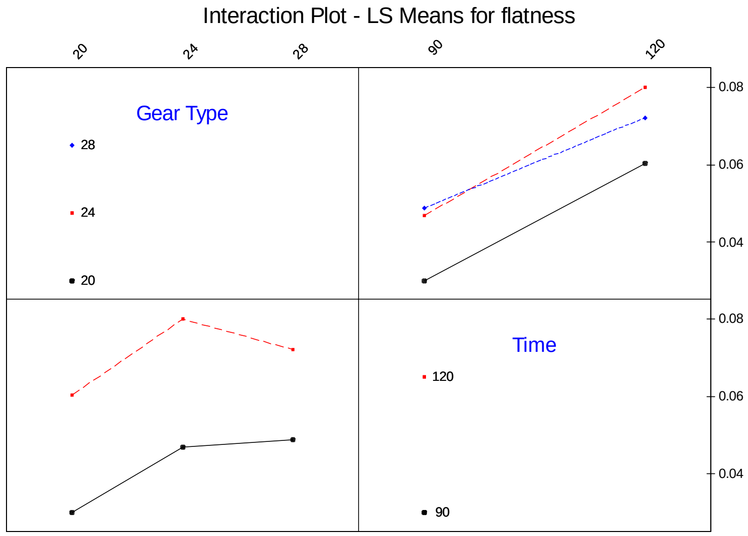


14-42 a)

```
Factor      Type Levels Values
Gear Typ  fixed       3 20 24 28
Time      fixed       2  90 120

Analysis of Variance for flatness, using Adjusted SS for Tests

Source          DF     Seq SS     Adj SS     Adj MS       F      P
Gear Typ         2  0.0007591  0.0007591  0.0003796    3.90  0.082
Time             1  0.0024941  0.0024941  0.0024941   25.66  0.002
Gear Typ*Time    2  0.0000505  0.0000505  0.0000253    0.26  0.779
Error            6  0.0005833  0.0005833  0.0000972
Total           11  0.0038870

Term              Coef   SE Coef        T      P
Constant      0.056500  0.002846    19.85  0.000
Gear Typ
20           -0.011125  0.004025    -2.76  0.033
24            0.007000  0.004025     1.74  0.133
Time
 90          -0.014417  0.002846    -5.07  0.002
Gear Typ*Time
20        90 -0.000708  0.004025    -0.18  0.866
24        90 -0.002083  0.004025    -0.52  0.623
```

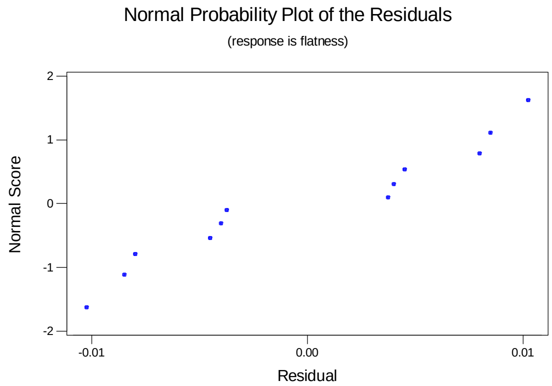
There is weak evidence that flatness distortion is different for the different gear types (p = 0.082). Gear type is significant at α = 0.1, but not at α = 0.05. Also, the gear type 20 coefficient has a p-value = 0.033. Heat-treating time affects the flatness distortion (p = 0.002). There is no evidence that factors interact (p = 0.779).

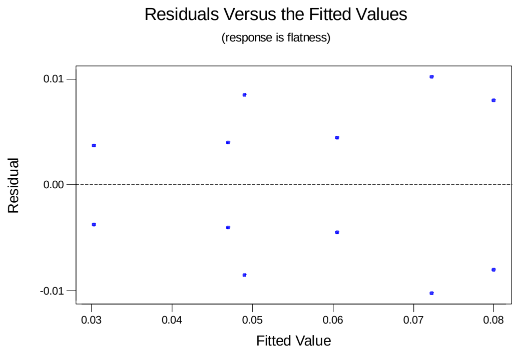
b)

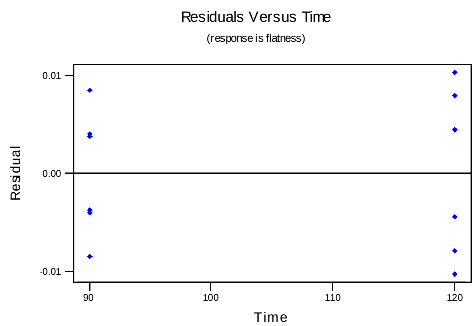


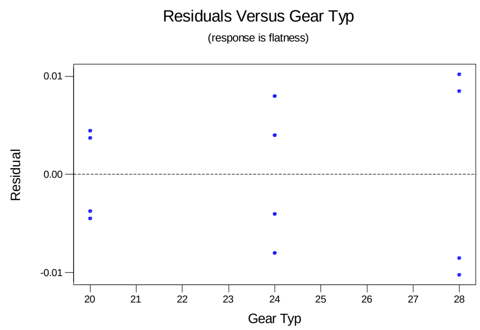
The interaction plot for the effects indicates that there is no interaction between gear type and time. The interaction plot does indicate there may be some significant difference between the low and high levels of time. There is also a difference between one of the gear types and the other two.

c) The model used is $\hat{y} = 0.0565 = 0.0111x_1 - 0.01144x_2$









The residual plots are adequate. There does not appear to be any serious departure from normality or violation of the assumption of constant variance.

14-43 a)

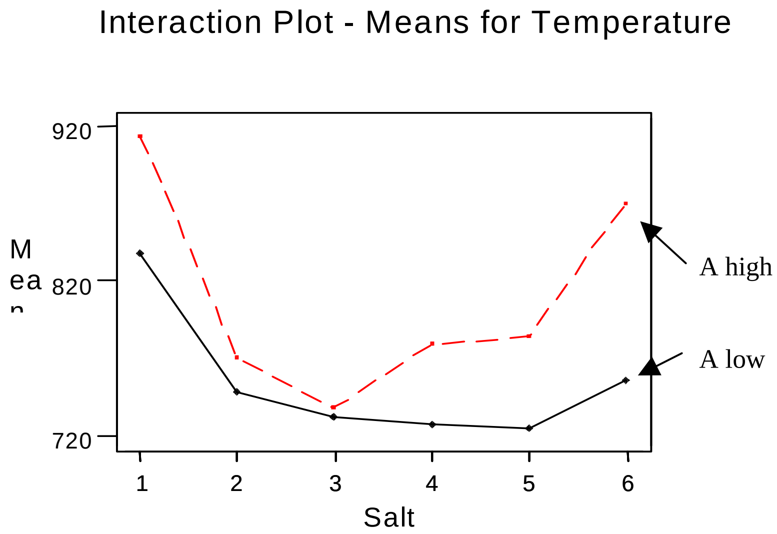
Factor	Type	Levels	Values					
Level	fixed	2	1	2				
Salt	fixed	6	1	2	3	4	5	6

Analysis of Variance for temperat

Source	DF	SS	MS	F	P
level	1	27390	27390	63.24	0.000
salt	5	86087	17217	39.75	0.000
Interaction	5	11459	2292	5.29	0.002
Error	24	10395	433		
Total	35	135332			

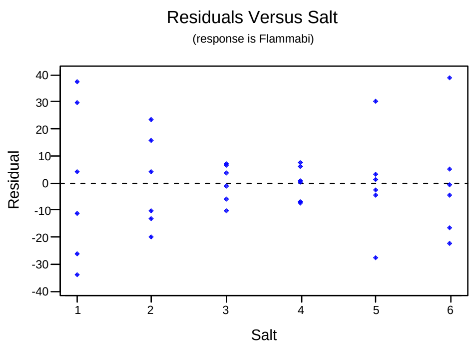
There is a significant difference between the application levels, the salts, and there is a significant interaction between the two factors.

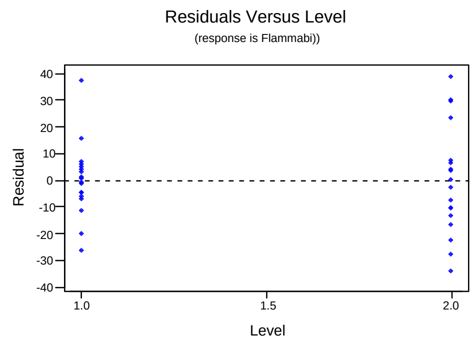
b)

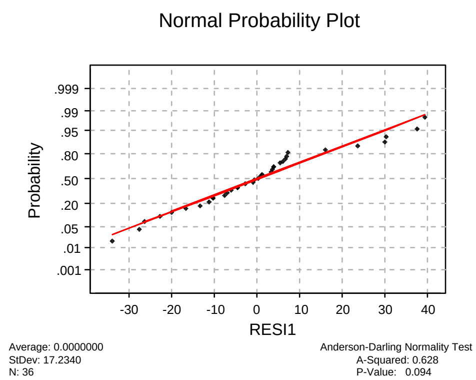


From the interaction plot, we see that the untreated salt 1 has a higher flammability average than any of the other five levels. The remaining five levels ($MgCl_2$, NaCl, $CaCO_3$, $CaCl_2$, Na_2CO_3) also seem to differ. Overall, application level 1 increases the flammability average. Also, the difference between application level varies with the salt type and this indicates a significant interaction.

c)







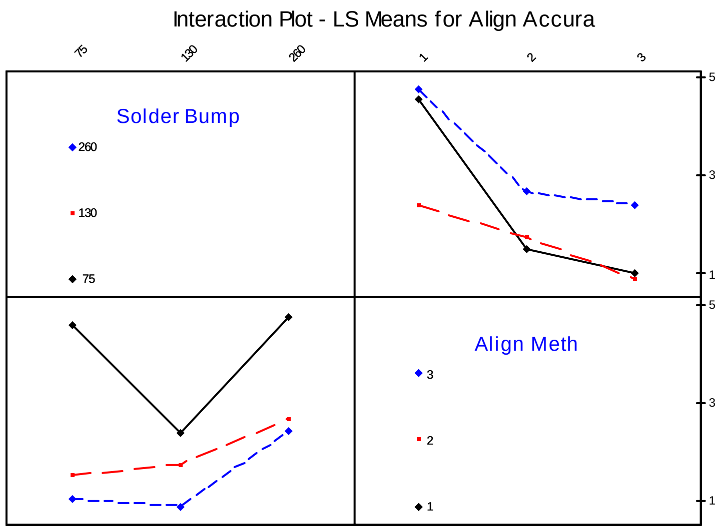
The residual plots do not indicate major problems with the assumptions. There is some concern with the constant variability assumption in the plot of residuals versus salts.

14-44 a)

```
Factor      Type Levels Values
Solder B   fixed      3  75 130 260
Align Me   fixed      3 1 2 3

Analysis of Variance for Align Ac, using Adjusted SS for Tests

Source              DF     Seq SS     Adj SS     Adj MS       F       P
Solder B             2     7.7757     7.7757     3.8879  297.92   0.000
Align Me             2    20.1241    20.1241    10.0621  771.04   0.000
Solder B*Align Me    4     3.5001     3.5001     0.8750   67.05   0.000
Error                9     0.1174     0.1174     0.0130
Total               17    31.5174

Term                    Coef    SE Coef        T       P
Constant             2.43611    0.02693    90.47   0.000
Solder B
 75                 -0.07278    0.03808    -1.91   0.088
130                 -0.76611    0.03808   -20.12   0.000
Align Me
1                    1.46389    0.03808    38.44   0.000
2                   -0.46778    0.03808   -12.28   0.000
Solder B*Align Me
 75       1          0.73778    0.05385    13.70   0.000
 75       2         -0.39556    0.05385    -7.35   0.000
130       1         -0.74889    0.05385   -13.91   0.000
130       2          0.53778    0.05385     9.99   0.000
```

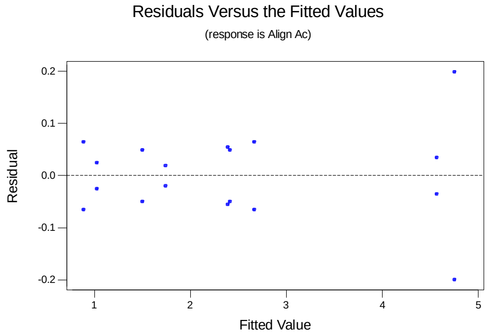
he analysis indicates that both solder size and alignment method significantly affect alignment accuracy. The interaction between solder size and alignment method is also significant in affecting alignment accuracy.

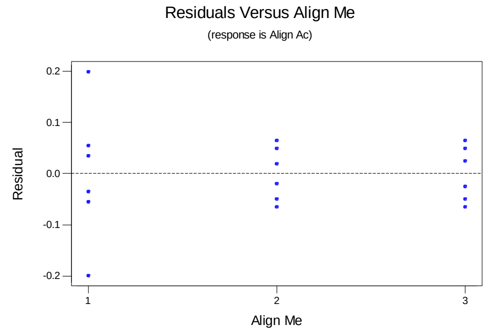
b) The lines for factor A intersect at the lower level of alignment.

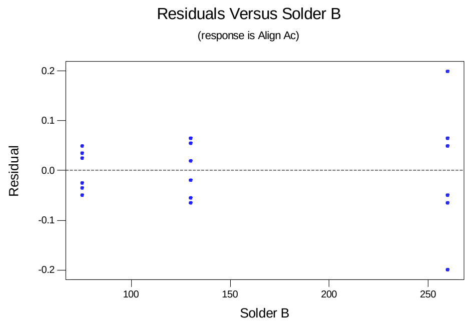


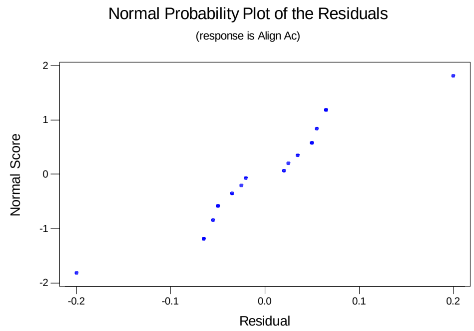
Because the smaller value is preferred, then, to improve alignment accuracy, it seems that solder size should be set at its middle level (130µm) while the alignment method used should be method 3. There is not much difference between bump size of 130 µm and 75 µm when method 3 is used.

c)







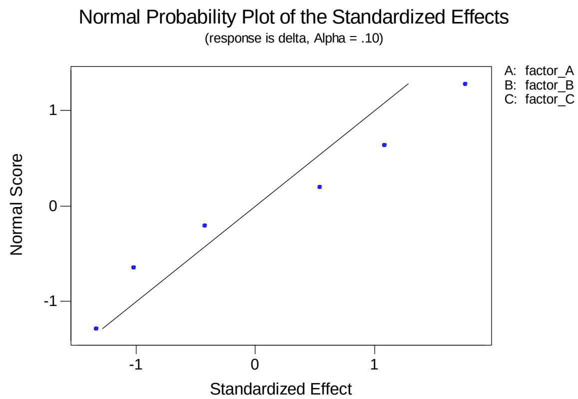


The normal probability plot does not suggest a departure from normality. The assumption of constant variance may be of concern. It appears that the variability is lower for the high level of the factor A.

14-45 a) Estimated Effects

Term	Effect
A	-2.74
B	-6.66
C	3.49
A*B	-8.71
A*C	7.04
B*C	11.46
A*B*C	-6.49

b) None of the factor effects are significant

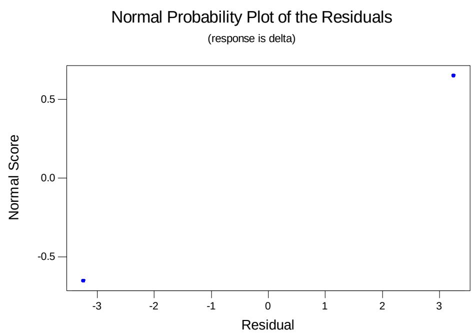


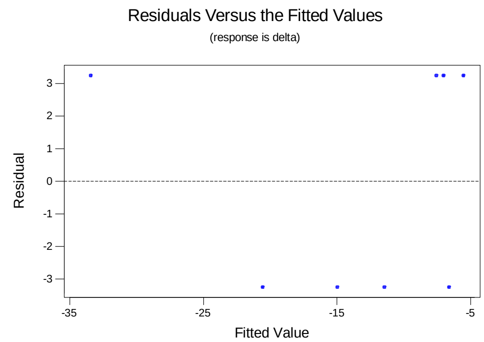
c) Analysis of Variance for delta (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	128.07	128.07	42.69	0.51	0.746
2-Way Interactions	3	512.99	512.99	171.00	2.02	0.467
Residual Error	1	84.50	84.50	84.50		
Total	7	725.56				

Based on the analysis of this data, a significant model cannot be built. It appears that more data should be collected or that maybe a factor is missing.

d.) The residual plots shows there are serious problems with the data.



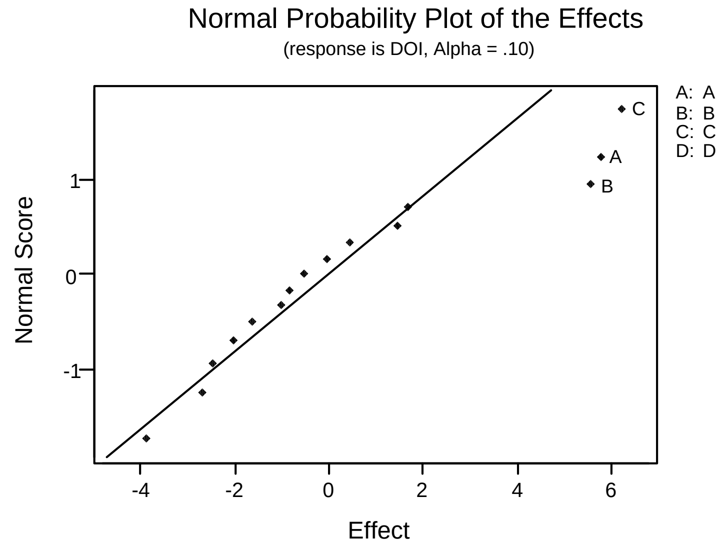


14-46 a)

Estimated Effects and Coefficients for DOI

Term	Effect	Coef
Constant		72.894
A	5.763	2.881
B	5.563	2.781
C	6.238	3.119
D	-3.862	-1.931
A*B	-2.013	-1.006
A*C	-0.837	-0.419
A*D	-2.488	-1.244
B*C	-0.537	-0.269
B*D	1.462	0.731
C*D	-0.063	-0.031
A*B*C	0.437	0.219
A*B*D	-1.013	-0.506
A*C*D	1.663	0.831
B*C*D	-2.687	-1.344
A*B*C*D	-1.612	-0.806

b)



Based on the normal probability plot it appears that factors A, B, and C are significant.

c) Run an analysis using the main factors A, B, and C and interactions among these variables to see if any are significant.

Predictor	Coef	StDev	T	P
Constant	72.894	1.073	67.92	0.000
A	2.881	1.073	2.68	0.028
B	2.781	1.073	2.59	0.032
C	3.119	1.073	2.91	0.020
AB	-1.006	1.073	-0.94	0.376
AC	-0.419	1.073	-0.39	0.707
BC	-0.269	1.073	-0.25	0.809
ABC	0.219	1.073	0.20	0.844

```
S = 4.293       R-Sq = 74.6%     R-Sq(adj) = 52.4%
```

Based on this analysis, only the main effects A, B, and C are significant. Run a regression analysis using these important factors.

d)

```
The regression equation is
DOI = 72.9 + 2.88 A + 2.78 B + 3.12 C
```

Predictor	Coef	StDev	T	P
Constant	72.8937	0.9365	77.84	0.000
A	2.8813	0.9365	3.08	0.010
B	2.7813	0.9365	2.97	0.012
C	3.1188	0.9365	3.33	0.006

```
S = 3.746       R-Sq = 71.0%     R-Sq(adj) = 63.7%
```

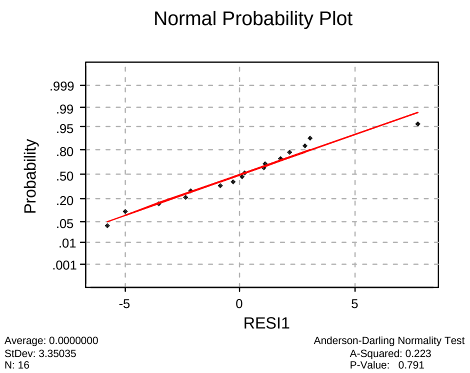
Analysis of Variance

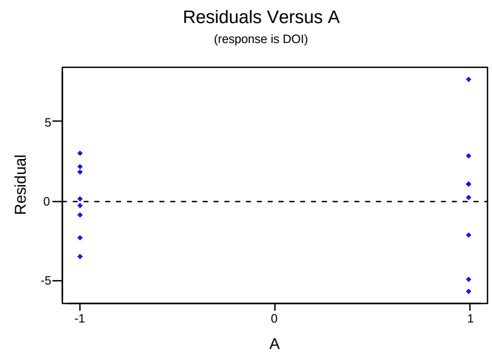
Source	DF	SS	MS	F	P
Regression	3	412.22	137.41	9.79	0.002
Error	12	168.37	14.03		
Total	15	580.59			

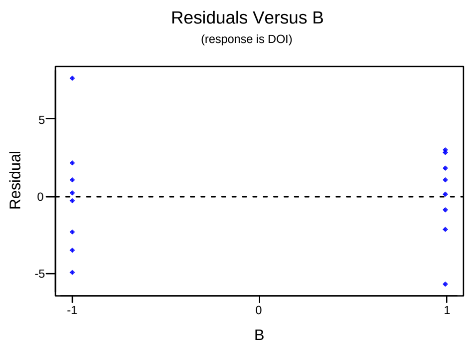
Source	DF	Seq SS
A	1	132.83
B	1	123.77
C	1	155.63

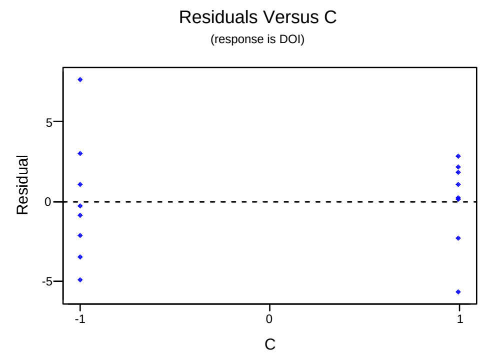
$$\hat{y} = 72.9 + 2.88x_1 + 2.78x_2 + 3.12x_3$$

e)







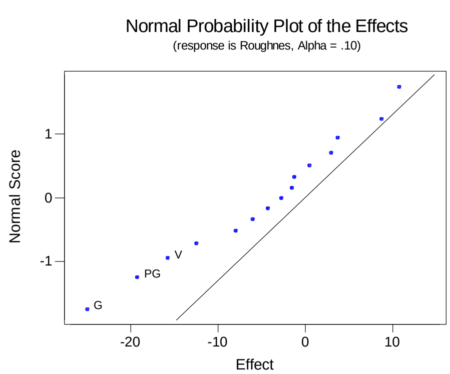


The residual plots appear to be adequate.

14-47 a)

Term	Effect
V	-15.75
F	8.75
P	10.75
G	-25.00
V*F	3.00
V*P	-8.00
V*G	-2.75
F*P	-6.00
F*G	3.75
P*G	-19.25
V*F*P	-1.25
V*F*G	0.50
V*P*G	-1.50
F*P*G	-12.50
V*F*P*G	-4.25

b)



According to the probability plot, factors V, P, and G and, PG are possibly significant.

```
Estimated Effects and Coefficients for roughnes (coded units)

Term         Effect      Coef     SE Coef       T      P
Constant                102.75      2.986   34.41  0.000
V            -15.75      -7.87      2.986   -2.64  0.046
F              8.75       4.37      2.986    1.46  0.203
P             10.75       5.37      2.986    1.80  0.132
G            -25.00     -12.50      2.986   -4.19  0.009
V*F            3.00       1.50      2.986    0.50  0.637
V*P           -8.00      -4.00      2.986   -1.34  0.238
V*G           -2.75      -1.38      2.986   -0.46  0.665
F*P           -6.00      -3.00      2.986   -1.00  0.361
F*G            3.75       1.88      2.986    0.63  0.558
P*G          -19.25      -9.63      2.986   -3.22  0.023

Analysis of Variance for roughness (coded units)

Analysis of Variance for Roughness (coded units)

Source               DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects          4      4260.7     4260.7     1065.2   7.46  0.024
2-Way Interactions    6      2004.7     2004.7      334.1   2.34  0.184
Residual Error        5       713.5      713.5      142.7
Total                15      6979.0
```

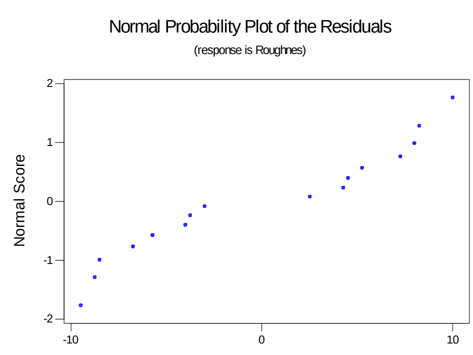
$$\hat{y} = 102.75 - 7.87x_1 + 5.37x_3 - 12.50x_4 - 9.63x_{34}$$

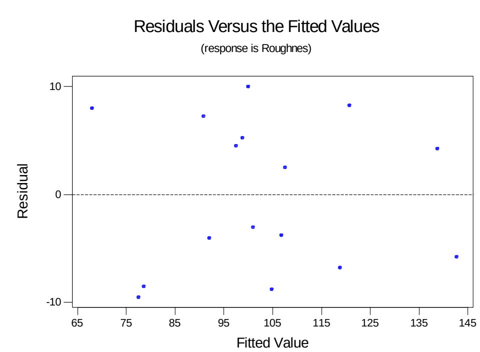
c) From the analysis, we see that water jet pressure (P), abrasive grain size (G), and jet traverse speed (V) are significant along with the interaction of water jet pressure and abrasive grain size. The model without the interaction is a reasonable model. With the interaction, there is a problem with collinearity. Without the interaction, Factor P is likely not required in the mode.

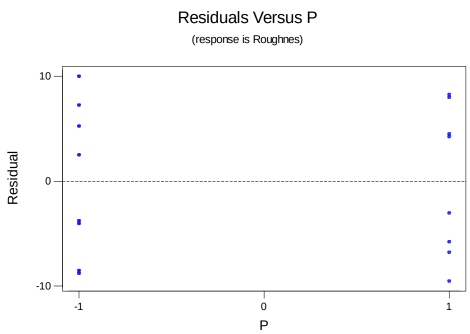
```
Analysis of Variance for Roughnes, using Adjusted SS for Tests

Source     DF     Seq SS     Adj SS     Adj MS      F      P
V           1      992.2      992.2      992.2   3.74  0.077
P           1      306.2      306.2      306.2   1.16  0.304
G           1     2500.0     2500.0     2500.0   9.43  0.010
Error      12     3180.5     3180.5      265.0
Total      15     6979.0
```

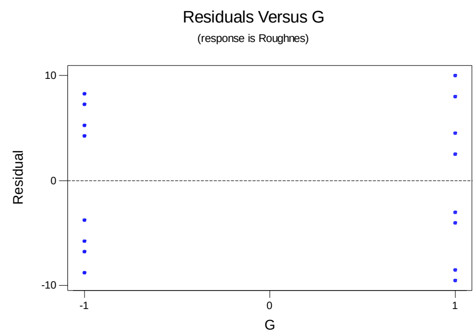
d) To minimize, abrasive grain size should be at the higher level with jet traverse speed at the lower level.

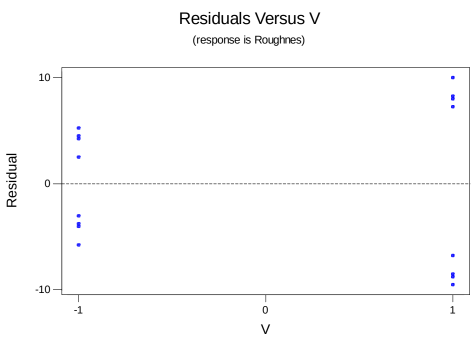
e) The residual plots appear to indicate the assumption of constant variance may not be met. The assumption of normality appears reasonable.

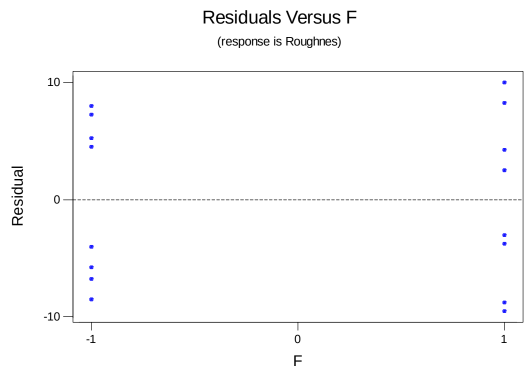










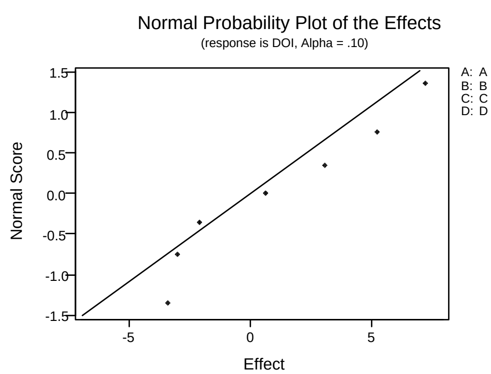


14-48 One possible 2^{4-1} design is I =ABCD

A	B	C	D	DOI
-1	-1	-1	-1	63.8
1	-1	-1	1	64.9
-1	1	-1	1	72.7
1	1	-1	-1	76.5
-1	-1	1	1	68.0
1	-1	1	-1	77.2
-1	1	1	-1	77.7
1	1	1	1	75.9

Estimated Effects

Term	Effect
A	3.075
B	7.225
C	5.225
D	-3.425
AB	-2.075
AC	0.625
AD	-3.025



It may be useful to run analysis with the main effects only to see which main effect is significant.

Term	Effect	Coef	StDev Coef	T	P
Constant		72.088	1.074	67.11	0.000
A	3.075	1.538	1.074	1.43	0.248
B	7.225	3.612	1.074	3.36	0.044
C	5.225	2.612	1.074	2.43	0.093
D	-3.425	-1.712	1.074	-1.59	0.209

Analysis of Variance for DOI

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	201.38	201.38	50.344	5.45	0.097
Residual Error	3	27.69	27.69	9.231		
Total	7	229.07				

Based on this analysis, only factor B appears to be significant at the 0.05 level of significance. Comparing this result to that of problem 14-46, we see that the Factors A and C were not revealed as significant in the smaller design.

14-49

V	F	P	G	Surface Roughness
-1	-1	-1	-1	129
1	-1	-1	1	98
-1	1	-1	1	110
1	1	-1	-1	103
-1	-1	1	1	76
1	-1	1	-1	137
-1	1	1	-1	98
1	1	1	1	88

Estimated Effects and Coefficients for roughness (coded units)

Estimated Effects and Coefficients for surf (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		104.88	7.950	13.19	0.006
V	3.25	1.63	7.950	0.20	0.857
F	-10.25	-5.12	7.950	-0.64	0.585
P	-10.25	-5.12	7.950	-0.64	0.585
G	-23.75	-11.88	7.950	-1.49	0.274
P*G	-11.75	-5.87	7.950	-0.74	0.537

Alias Structure

I + V*F*P*G
V + F*P*G
F + V*P*G
P + V*F*G
G + V*F*P
V*F + P*G

None of the effects are significant for the fractional factorial model. The fractional factorial does not provide enough information to detect significant effects in this experiment.

14-50 ABCDEFGH 2^{8-4} Resolution IV

```
Design Generators:  E = BCD  F = ACD  G = ABC  H = ABD

Alias Structure (up to order 4)

I + ABCG + ABDH + ABEF + ACDF + ACEH + ADEG + AFGH + BCDE + BCFH + BDFG +
BEGH+ CDGH + CEFG + DEFH

A + BCG + BDH + BEF + CDF + CEH + DEG + FGH
B + ACG + ADH + AEF + CDE + CFH + DFG + EGH
C + ABG + ADF + AEH + BDE + BFH + DGH + EFG
D + ABH + ACF + AEG + BCE + BFG + CGH + EFH
E + ABF + ACH + ADG + BCD + BGH + CFG + DFH
F + ABE + ACD + AGH + BCH + BDG + CEG + DEH
G + ABC + ADE + AFH + BDF + BEH + CDH + CEF
H + ABD + ACE + AFG + BCF + BEG + CDG + DEF
AB + CG + DH + EF + ACDE + ACFH + ADFG + AEGH + BCDF + BCEH + BDEG + BFGH
AC + BG + DF + EH + ABDE + ABFH + ADGH + AEFG + BCDH + BCEF + CDEG + CFGH
AD + BH + CF + EG + ABCE + ABFG + ACGH + AEFH + BCDG + BDEF + CDEH + DFGH
AE + BF + CH + DG + ABCD + ABGH + ACFG + ADFH + BCEG + BDEH + CDEF + EFGH
AF + BE + CD + GH + ABCH + ABDG + ACEG + ADEH + BCFG + BDFH + CEFH + DEFG
AG + BC + DE + FH + ABDF + ABEH + ACDH + ACEF + BDGH + BEFG + CDFG + CEGH
AH + BD + CE + FG + ABCF + ABEG + ACDG + ADEF + BCGH + BEFH + CDFH + DEGH
```

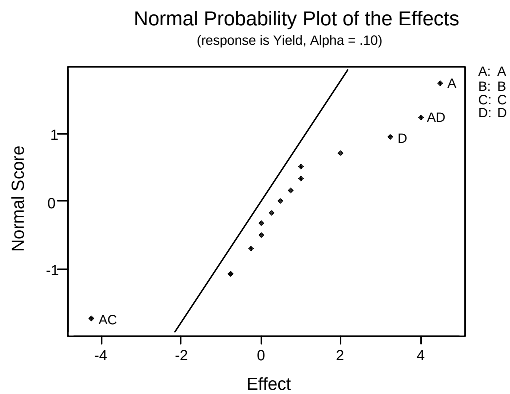
14-51 ABCDE 2^{5-2} 8 runs

```
Design Generators:  D = AB  E = AC

Alias Structure  I + ABD + ACE + BCDE

A + BD + CE + ABCDE
B + AD + CDE + ABCE
C + AE + BDE + ABCD
D + AB + BCE + ACDE
E + AC + BCD + ABDE
BC + DE + ABE + ACD
BE + CD + ABC + ADE
```

Design

StdOrder	A	B	C	D	E
1	-1	-1	-1	1	1
2	1	-1	-1	-1	-1
3	-1	1	-1	-1	1
4	1	1	-1	1	-1
5	-1	-1	1	1	-1
6	1	-1	1	-1	1
7	-1	1	1	-1	-1
8	1	1	1	1	1

14-52 a)



The factors that appear to have large effects are A, D, AC, and AD.

b) The regression analysis is

Estimated Effects and Coefficients for Yield

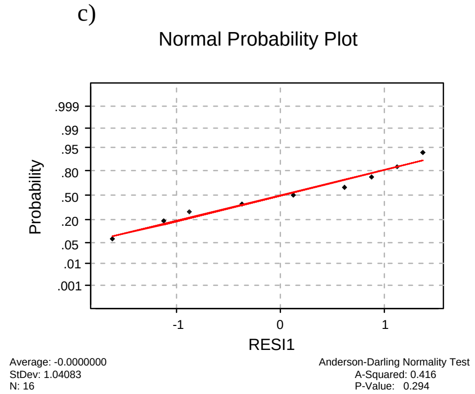
Term	Effect	Coef	StDev Coef	T	P
Constant		17.375	0.3187	54.52	0.000
A	4.500	2.250	0.3187	7.06	0.000
C	2.000	1.000	0.3187	3.14	0.011
D	3.250	1.625	0.3187	5.10	0.000
A*C	-4.250	-2.125	0.3187	-6.67	0.000
A*D	4.000	2.000	0.3187	6.28	0.000

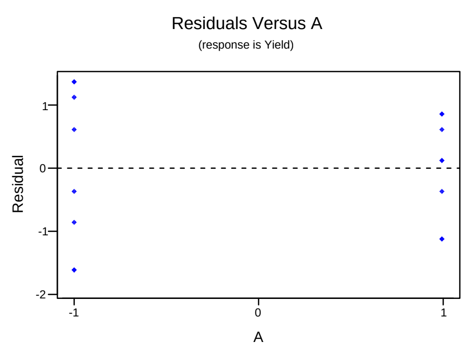
Analysis of Variance for Yield

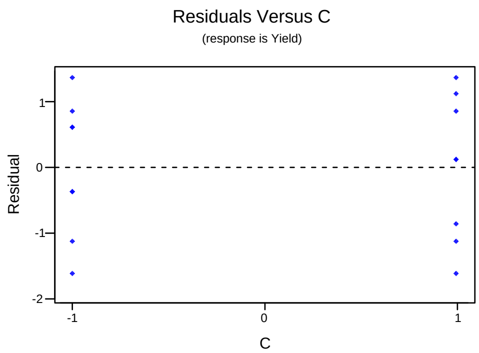
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	139.250	139.250	46.4167	28.56	0.000
2-Way Interactions	2	136.250	136.250	68.1250	41.92	0.000
Residual Error	10	16.250	16.250	1.6250		
Lack of Fit	2	0.250	0.250	0.1250	0.06	0.940
Pure Error	8	16.000	16.000	2.0000		
Total	15	291.750				

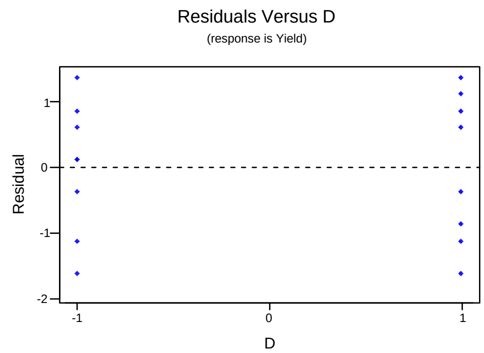
The conclusions are that factors A, C, and D are significant along with the interactions AC and AD.

c)



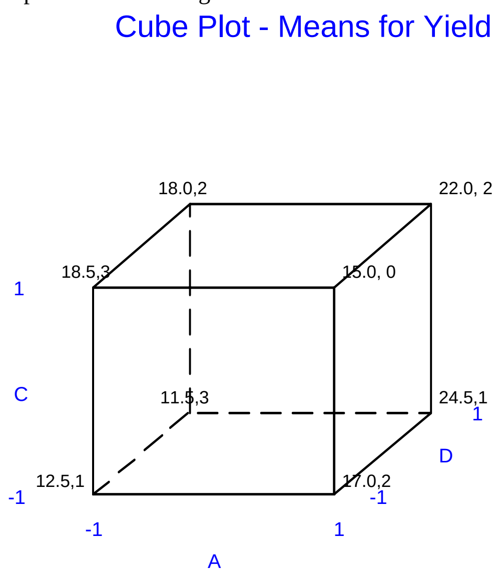






The residual plots appear to be adequate.

d) The design can be collapsed into a 2^3 design as shown



The data shown on the plot are the mean and range of the response for each location. It is evident that the mean yield is largest for high levels of A and D and low level of C.

14-53 The generator for this fraction was I = ABCD

```
I = A*B*C*D
A = B*C*D
B = A*C*D
C = A*B*D
D = A*B*C
E = A*B*C*D*E
A*B = C*D
A*C = B*D
A*D = B*C
A*E = B*C*D*E
B*E = A*C*D*E
C*E = A*B*D*E
D*E = A*B*C*E
A*B*E = C*D*E
A*C*E = B*D*E
A*D*E = B*C*E
```

b) Estimated Effects and Coefficients for freeheig

Term	Effect	Coef	StDev Coef	T	P
Constant		7.6400	0.01901	401.97	0.000
A	0.2133	0.1067	0.01901	5.61	0.000
B	-0.1925	-0.0963	0.01901	-5.06	0.000
C	-0.0783	-0.0392	0.01901	-2.06	0.048
D	0.0625	0.0313	0.01901	1.64	0.110
E	-0.2100	-0.1050	0.01901	-5.52	0.000
A*B	-0.0008	-0.0004	0.01901	-0.02	0.983
A*C	0.0300	0.0150	0.01901	0.79	0.436
A*D	0.0058	0.0029	0.01901	0.15	0.879
A*E	0.0350	0.0175	0.01901	0.92	0.364
B*E	0.1242	0.0621	0.01901	3.27	0.003
C*E	-0.0617	-0.0308	0.01901	-1.62	0.115
D*E	0.0108	0.0054	0.01901	0.28	0.777
A*B*E	0.0308	0.0154	0.01901	0.81	0.423
A*C*E	0.0483	0.0242	0.01901	1.27	0.213
A*D*E	-0.0308	-0.0154	0.01901	-0.81	0.423

Analysis of Variance for freeheig

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	1.64052	1.64052	0.32810	18.92	0.000
2-Way Interactions	7	0.25797	0.25797	0.03685	2.13	0.069
3-Way Interactions	3	0.05085	0.05085	0.01695	0.98	0.416
Residual Error	32	0.55487	0.55487	0.01734		
Pure Error	32	0.55487	0.55487	0.01734		
Total	47	2.50420				

Based on the analysis, factors A, B, C, and E are significant. The interaction of BE is also significant.

c)

A	B	C	D	E	Range
-1	-1	-1	-1	-1	0.03
1	-1	-1	1	-1	0.30
-1	1	-1	1	-1	0.06
1	1	-1	-1	-1	0.19
-1	-1	1	1	-1	0.46
1	-1	1	-1	-1	0.40
-1	1	1	-1	-1	0.12
1	1	1	1	-1	0.25
-1	-1	-1	-1	1	0.06
1	-1	-1	1	1	0.44
-1	1	-1	1	1	0.06
1	1	-1	-1	1	0.19
-1	-1	1	1	1	0.12
1	-1	1	-1	1	0.13
-1	1	1	-1	1	0.07
1	1	1	1	1	0.31

Estimated Effects and Coefficients for Range

Term	Effect	Coef	StDev Coef	T	P
Constant		0.19938	0.02714	7.35	0.000
A	0.15375	0.07688	0.02714	2.83	0.018
B	-0.08625	-0.04313	0.02714	-1.59	0.143
C	0.06625	0.03312	0.02714	1.22	0.250
D	0.10125	0.05062	0.02714	1.87	0.092
E	-0.05375	-0.02687	0.02714	-0.99	0.345

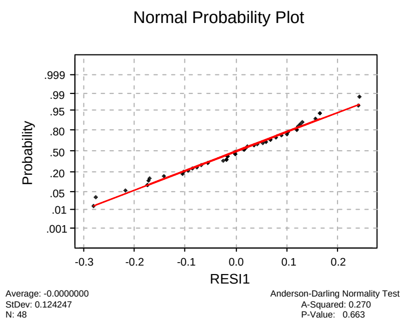
Analysis of Variance for Range

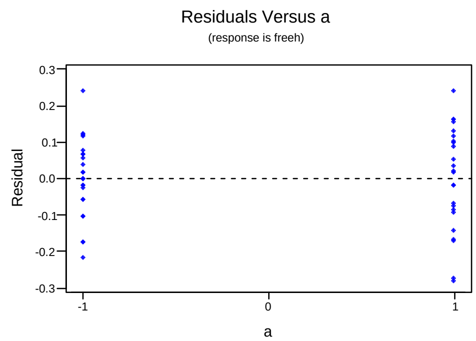
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	0.1944	0.1944	0.03889	3.30	0.051
Residual Error	10	0.1179	0.1179	0.01179		
Total	15	0.3123				

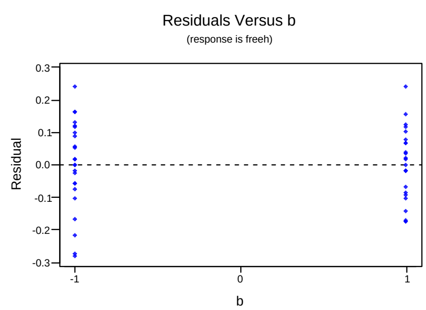
From the analysis factor A is significant for variability in free height.

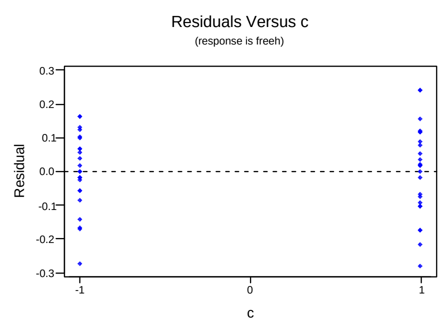
Using the model $\hat{y} = 0.19938 + 0.07688x_1$

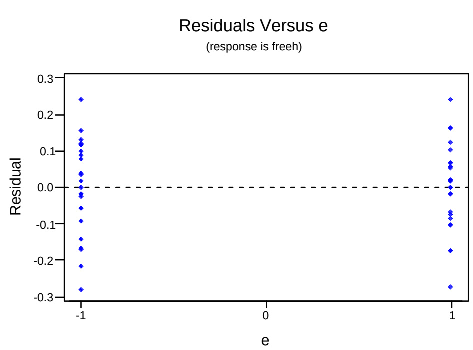
d)











The residual plots appear to be adequate.

14-54 a) The design used is a 2^2 full factorial with 2 replicates.

b) Factors x_1 and x_2 are significant. The interaction between x_1 and x_2 is not significant

```
Term          Effect       Coef     SE Coef       T      P
Constant                13.5800      0.1241  109.42  0.000
x1            0.7950     0.3975      0.1241    3.20  0.033
x2           -1.1600    -0.5800      0.1241   -4.67  0.009
x1*x2         0.0850     0.0425      0.1241    0.34  0.749

Analysis of Variance for y (coded units)
Source                DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects           2     3.95525    3.95525    1.97763  16.05  0.012
2-Way Interactions     1     0.01445    0.01445    0.01445   0.12  0.749
Residual Error         4     0.49290    0.49290    0.12323
  Pure Error           4     0.49290    0.49290    0.12322
Total                  7     4.46260
```

Mind Expanding Exercises

14-55 The ABCD interaction is

$$\frac{1}{8}\left[(1)+ab+ac+bc+ad+bd+cd+abcd\right]-\left[a+b+c+d+abc+abd+acd+bcd\right]$$

If *ab* is missing, then ABCD interaction will be zero when
[550 + ab + 642 + 601 + 749 + 1052 + 1075 + 729] - [669 + 604 + 633 + 1037 + 635 + 868 + 860 + 1063]=0
Therefore, $ab + 5398 - 6369 = 0$ or $ab = 971$. After estimating *ab*, only the A and AD effects appear significant.

14-56 Two three-factor interactions could be used to generate the blocks such as ABC and ACD. This would confound these effects and ABC • ACD = BD with blocks. Therefore, only one two-factor and no main effects are confounded with blocks.

14-57

	A	B	AB	block
(1)	-	-	+	1
a	+	-	-	2
b	-	+	-	2
ab	+	+	+	1

The block effect is estimated by $\frac{a+b}{2}-\frac{(1)-ab}{2}$ which is the same as the estimate of the effect of AB.

14-58 a) A different effect can be confounded in each replicate as follows.

Replicate 1		Replicate 2		Replicate 3		Replicate 4	
ABC confounded		AB confounded		BC confounded		AC confounded	
(1)	a	(1)	a	(1)	b	(1)	a
ab	b	c	b	a	c	b	c
ac	c	ab	ac	bc	ab	ac	ab
bc	abc	abc	bc	abc	ac	abc	bc

b)

Source of Variation	Degrees of freedom
Replicates	3
Blocks with replicates	4
[or ABC (rep. 1) + AB (rep. 2) + BC (rep. 3) + AC (rep. 4)]	
A	1
B	1
C	1
AB (from replicates 1, 3, and 4)	1
AC (from replicates 1, 2, and 3)	1
BC (from replicates 1, 2, and 4)	1
ABC (from replicates 2, 3, and 4)	1
Error (by subtraction)	17
Total	31

In calculating an interaction sum of squares, only data from the replicates in which the interaction is un-confounded are used.

14-59

A	B	C	D	E = ABCD	AB = CDE	block
-	-	-	-	+	+	1
+	-	-	-	-	-	2
-	+	-	-	-	-	2
+	+	-	-	+	+	1
-	-	+	-	-	+	1
+	-	+	-	+	-	2
-	+	+	-	+	-	2
+	+	+	-	-	+	1
-	-	-	+	-	+	1
+	-	-	+	+	-	2
-	+	-	+	+	-	2
+	+	-	+	-	+	1
-	-	+	+	+	+	1
+	-	+	+	-	-	2
-	+	+	+	-	-	2
+	+	+	+	+	+	1

This uses AB = CDE as the effect to confound with blocks.

14-60 The generators are F = ABCD and G = ABDE. The complete defining relation is
I = ABCDF = ABDEG = CEFG.
The design can be constructed in four blocks by confounding ACE = AFG and BCE = BFG with blocks. This also confounds AB = CDF = DEG with blocks.
Yes, a two-factor interaction is confounded with blocks. The best blocking scheme confounds only one two-factor interaction with blocks.

14-61 The generators are E = ABC, F = BCD, and G = ACD. The complete defining relation is I = ABCE = BCDF = ADEF = ACDG = BDEG = ABFG = CEFG. The alias set ABD = CDE = ACF = BEF = BCG = AEG = DFG can be used to construct the blocks. Then, only three-factor interactions are confounded with blocks.

14-62 a)

A	B	C	D = AB	E = AC	F = BC	G = ABC
-	-	-	+	+	+	-
+	-	-	-	-	+	+
-	+	-	-	+	-	+
+	+	-	+	-	-	-
-	-	+	+	-	-	+
+	-	+	-	+	-	-
-	+	+	-	-	+	-
+	+	+	+	+	+	+

The complete defining relation is I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF = ABCG = CDG = BEG = ADEG = AFG = BDFG = CEFG = ABCDEFG.
The alias structure follows (including only one- and two-factor effects).
A = BD = CE = FG
B = AD = CF = EG
C = AE = BF = DG
D = AB = EF = CG
E = AC = DF = BG
F = BC = DE = AG
G = CD = BE = AF

b) The complete defining relation is I = -ABD = -ACE = BCDE = -BCF = ACDF = ABEF = -DEF = ABCG = -CDG = -BEG = ADEG = -AFG = BDFG = CEFG = -ABCDEFG.
The aliases (up to two-factor effects) are:
A = -BD = -CE = -FG
B = -AD = -CF = -EG
C = -AE = -BF = -DG
D = -AB = -EF = -CG
E = -AC = -DF = -BG
F = -BC = -DE = -AG
G = -CD = -BE = -AF

c) All the main effects can be estimated. Use the average response from the alias set that contains the main effect in each fraction. The two-factor effects cancel when this average is computed.

Exercises are based on Section 14-6 on CD

14-63 a)
```
Analysis of Variance for Force
Source          DF        SS          MS        F       P
Primer           2     4.5811      2.2906    19.00   0.050
Appl             1     4.9089      4.9089    40.72   0.024
Primer*Appl      2     0.2411      0.1206     1.47   0.269
Error           12     0.9867      0.0822
Total           17    10.7178
```

b) $\hat{\sigma}^2 = 0.082$

$$\hat{\sigma}^2_{\tau\beta} = \frac{0.1206 - 0.0822}{3} = 0.013$$

$$\hat{\sigma}^2_{\beta} = \frac{4.9089 - 0.1206}{9} = 0.532$$

$$\hat{\sigma}^2_{\tau} = \frac{2.2906 - 0.1206}{6} = 0.362$$

14-64

Source	DF	SS	MS	F	P
Machine	3	24.333	8.111	1.36	0.340
Operator	2	26.333	13.167	2.21	0.190
Machine*Operator	6	35.667	5.944	1.59	0.234
Error	12	45.000	3.750		
Total	23	131.333			

With operators random, there is no significant difference among operators or machines.

14-65 $H_0 : \tau\beta = 0$

$H_1 : \tau\beta \neq 0$

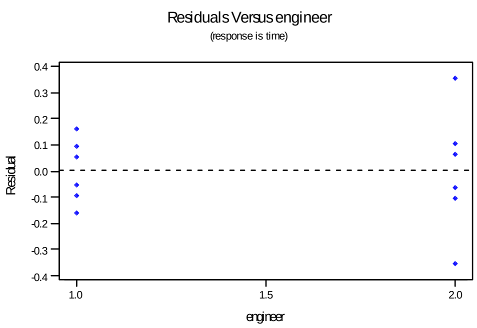
b)

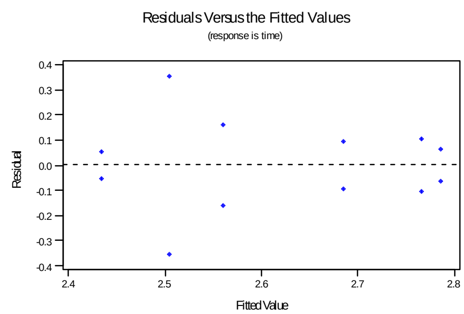
Source	DF	SS	MS	F	P
operator	2	0.01005	0.00503	0.07	0.937
engineer	1	0.04688	0.04688	0.62	0.512
operator*engineer	2	0.15005	0.07503	1.26	0.350
Error	6	0.35785	0.05964		
Total	11	0.56483			

Source	Variance component	Error term	Expected Mean Square (using unrestricted model)
1 operator	-0.01750	3	(4) + 2(3) + 4(1)
2 engineer		3	(4) + 2(3) + Q[2]
3 operator*engineer	0.00769	4	(4) + 2(3)
4 Error	0.05964		(4)

Because the significance level for operator*engineer is 0.350, we cannot reject H_0.

c) Modest concern with the constant variance assumption.





d) $\hat{\sigma}^2 = 0.0596$

$$\hat{\sigma}^2_{\tau\beta} = \frac{0.0750 - 0.0596}{2} = 0.0077$$

$$\hat{\sigma}^2_{\beta} = \frac{0.0050 - 0.0596}{4} = -0.0137 \rightarrow 0$$

14-66 a) $H_0: \tau_1 = \tau_2 = 0$ $\quad H_1: not\ all\ \tau_i = 0$

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$ $\quad H_1:$ not all $\beta_i = 0$

$H_0: \tau\beta_{12} = \tau\beta_{13} = \tau\beta_{23} = 0$ $\quad H_1:$ not all $\tau\beta_{ij} = 0$

b)

Source	DF	SS	MS	F	P
furnacep	1	7160	7160	17.98	0.002
temperat	2	945342	472671	1155.52	0.001
furnacep*temperat	2	818	409	0.91	0.427
Error	12	5371	448		
Total	17	958691			

Source	Variance component	Error term	Expected Mean Square (using unrestricted model)
1 furnacep	750.11	3	(4) + 3(3) + 9(1)
2 temperat		3	(4) + 3(3) + Q[2]
3 furnacep*temperat	-12.83	4	(4) + 3(3)
4 Error	447.56		(4)

The first and second null hypotheses can be rejected at α = 0.05 from the ANOVA table. The third null hypothesis cannot be rejected.

c) $\hat{\sigma}^2 = 447.56$

$$\hat{\sigma}^2_{\tau\beta} = \frac{409.056 - 447.556}{3} = -12.83 \to 0$$

$$\hat{\sigma}^2_{\tau} = \frac{7160.06 - 447.556}{9} = 745.83$$

14-67

Source	DF	SS	MS	F	P
operator	2	0.01005	0.00503	0.07	0.937
engineer	1	0.04688	0.04688	0.62	0.512
operator*engineer	2	0.15005	0.07503	1.26	0.350
Error	6	0.35785	0.05964		
Total	11	0.56483			

Source	Variance component	Error term	Expected Mean Square (using unrestricted model)
1 operator	-0.01750	3	(4) + 2(3) + 4(1)
2 engineer	-0.00469	3	(4) + 2(3) + 6(2)
3 operator*engineer	0.00769	4	(4) + 2(3)
4 Error	0.05964		(4)

The ANOVA table is shown above. The conclusions do not differ from Exercise 14-65. No effects are significant.

<u>Exercises are based on Section 14-7.4 on CD</u>

14-68 a)

Term	Effect	Coef	SE Coef	T	P
Constant		30.5313	0.5062	60.31	0.000
A	11.8125	5.9063	0.5062	11.67	0.000
B	33.9375	16.9687	0.5062	33.52	0.000
C	9.6875	4.8438	0.5062	9.57	0.001
D	-0.8125	-0.4062	0.5062	-0.80	0.467
E	0.4375	0.2188	0.5062	0.43	0.688
A*B	7.9375	3.9687	0.5062	7.84	0.001
A*C	0.4375	0.2187	0.5062	0.43	0.688
A*D	-0.0625	-0.0313	0.5062	-0.06	0.954
A*E	0.9375	0.4688	0.5062	0.93	0.407
B*C	0.0625	0.0313	0.5062	0.06	0.954
B*D	-0.6875	-0.3437	0.5062	-0.68	0.534
B*E	0.5625	0.2813	0.5062	0.56	0.608

C*D	0.8125	0.4062	0.5062	0.80	0.467
C*E	0.3125	0.1563	0.5062	0.31	0.773
D*E	-1.1875	-0.5938	0.5062	-1.17	0.306
A*B*C	-0.4375	-0.2187	0.5062	-0.43	0.688
A*B*D	0.3125	0.1563	0.5062	0.31	0.773
A*B*E	-0.1875	-0.0937	0.5062	-0.19	0.862
A*C*D	-0.4375	-0.2187	0.5062	-0.43	0.688
A*C*E	0.3125	0.1562	0.5062	0.31	0.773
A*D*E	0.8125	0.4062	0.5062	0.80	0.467
B*C*D	0.4375	0.2187	0.5062	0.43	0.688
B*C*E	0.9375	0.4688	0.5062	0.93	0.407
B*D*E	0.1875	0.0938	0.5062	0.19	0.862
C*D*E	-0.8125	-0.4063	0.5062	-0.80	0.467
A*B*C*D	-0.0625	-0.0312	0.5062	-0.06	0.954
A*B*C*E	0.1875	0.0938	0.5062	0.19	0.862
A*B*D*E	0.9375	0.4688	0.5062	0.93	0.407
A*C*D*E	-0.3125	-0.1562	0.5062	-0.31	0.773
B*C*D*E	-0.9375	-0.4688	0.5062	-0.93	0.407
A*B*C*D*E	-0.1875	-0.0938	0.5062	-0.19	0.862
Ct Pt		12.6688	1.3770	9.20	0.001

S = 2.86356   R-Sq = 99.74%   R-Sq(adj) = 97.62%

Analysis of Variance for y (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	11087.9	11087.9	2217.58	270.44	0.000
2-Way Interactions	10	536.3	536.3	53.63	6.54	0.043
3-Way Interactions	10	24.3	24.3	2.43	0.30	0.946
4-Way Interactions	5	15.2	15.2	3.03	0.37	0.848
5-Way Interactions	1	0.3	0.3	0.28	0.03	0.862
Curvature	1	694.0	694.0	694.04	84.64	0.001
Residual Error	4	32.8	32.8	8.20		
Pure Error	4	32.8	32.8	8.20		
Total	36	12390.8				

b) The hand calculation of $\hat{\sigma}_c^2$ is

$$\hat{\sigma}_c^2 = \sum_{\substack{center\\pts}} \frac{(y_i - \bar{y}_c)^2}{(n_c - 1)} = \frac{32.8}{4} = 8.2 \text{ where } \bar{y}_c = 43.2$$

If all 3-factor and higher interactions are removed from the model the following ANOVA results:

Analysis of Variance for y (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	11087.9	11087.9	2217.58	628.62	0.000
2-Way Interactions	9	534.8	534.8	59.42	16.84	0.000
Curvature	1	694.0	694.0	694.04	196.74	0.000
Residual Error	21	74.1	74.1	3.53		
Lack of Fit	17	41.3	41.3	2.43	0.30	0.967
Pure Error	4	32.8	32.8	8.20		
Total	36	12390.8				

The estimate of σ^2 from these higher-order effects is 2.43. This is a pooled estimate obtained from the sum of squares divided by the degrees of freedom of all effects removed from the model. The estimate of 8.2 is based on replicate tests. The ratio of F = 2.43/8.2 =0.30 tests the lack of fit of the model that excludes 3-factor and higher interactions. Because this ratio is less than one it is not significant. However, the test for curvature in the ANOVA is 694.04/3.53 = 196.74 and the curvature effect is significant. Therefore, the linear model of only factorial effects is not adequate for this data.

14-69 a)

Term	Effect	Coef	SE Coef	T	P
Constant		411.88	5.470	75.30	0.000
A	13.75	6.87	5.470	1.26	0.298
B	127.75	63.87	5.470	11.68	0.001
C	97.75	48.88	5.470	8.94	0.003
A*B	-21.25	-10.62	5.470	-1.94	0.147
A*C	-137.25	-68.63	5.470	-12.55	0.001
B*C	-52.25	-26.13	5.470	-4.78	0.017
A*B*C	-68.25	-34.13	5.470	-6.24	0.008
Ct Pt		8.13	9.474	0.86	0.454

```
S = 15.4704   R-Sq = 99.33%   R-Sq(adj) = 97.53%
```

```
Analysis of Variance for resp (coded units)
```

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	52128	52128.4	17376.1	72.60	0.003
2-Way Interactions	3	44038	44038.4	14679.5	61.33	0.003
3-Way Interactions	1	9316	9316.1	9316.1	38.93	0.008
Curvature	1	176	176.0	176.0	0.74	0.454
Residual Error	3	718	718.0	239.3		
Pure Error	3	718	718.0	239.3		
Total	11	106377				

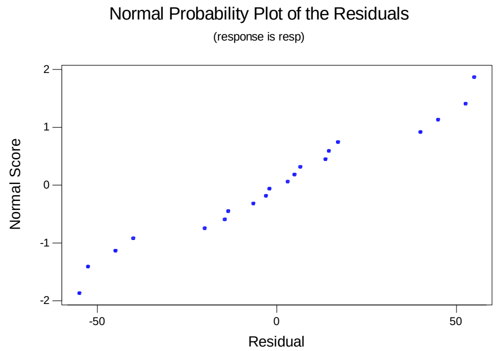
b) The hand calculation of $\hat{\sigma}_c^2$ is

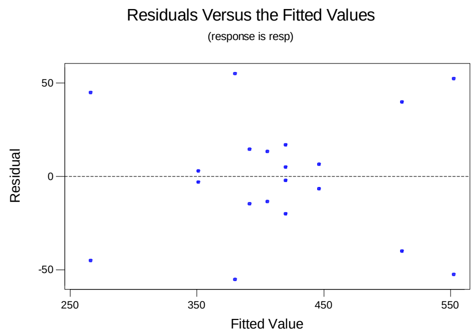
$$\hat{\sigma}_c^2 = \sum_{\substack{center\\pts}} \frac{(y_i - \bar{y}_c)^2}{(n_c - 1)} = \frac{718}{3} = 239.3 \quad \text{where } \bar{y}_c = 420$$

Because the 3-factor interaction is significant all the effects are kept in the model. The curvature test is not significant. There is no significant lack of fit. That is, the hypothesis that the full factorial model fits is not rejected.

c) See part (a). Factors B and C, and interactions AC, BC, and ABC are significant.

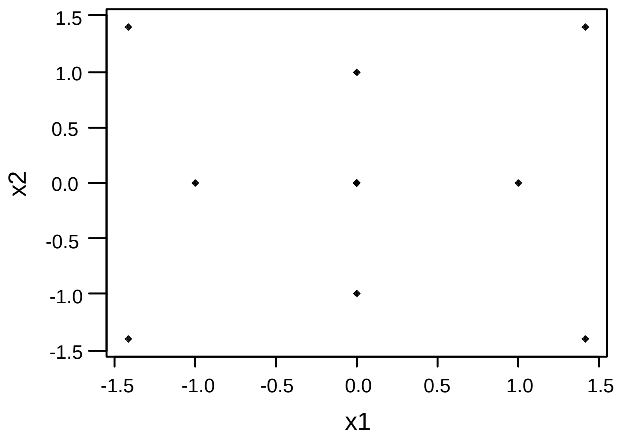
d) The assumptions of normality and constant variance seem reasonable although there are some slight abnormalities on the residual plots.





Exercises are based on Section 14-10 on CD

14-70 a)

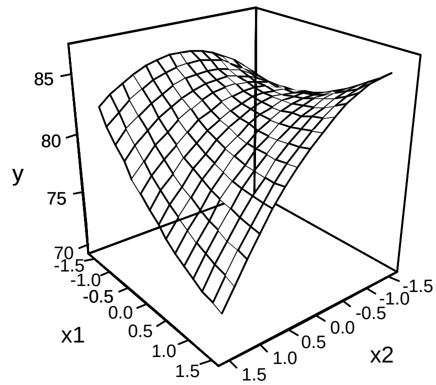


b)

```
Estimated Regression Coefficients for y
Term             Coef       StDev          T      P
Constant       82.024      0.5622    145.905  0.000
x1             -1.115      0.4397     -2.536  0.044
x2             -2.408      0.4397     -5.475  0.002
x1*x1           0.861      0.7343      1.172  0.286
x2*x2          -1.590      0.7342     -2.165  0.074
x1*x2          -1.801      0.3477     -5.178  0.002
S = 1.390       R-Sq = 92.0%     R-Sq(adj) = 85.3%

Analysis of Variance for y
Source          DF    Seq SS    Adj SS    Adj MS      F      P
Regression       5   132.837   132.837   26.5674  13.74  0.003
  Linear         2    70.393    70.391   35.1957  18.21  0.003
  Square         2    10.602    10.610    5.3048   2.74  0.142
  Interaction    1    51.842    51.842   51.8425  26.82  0.002
Residual Error   6    11.600    11.600    1.9333
  Lack-of-Fit    3    10.052    10.052    3.3507   6.50  0.079
  Pure Error     3     1.548     1.548    0.5158
Total           11   144.437
```

The second order model appears to be significant for the interaction term (p = 0.002). However, the square terms are not significant (p = 0.142).

c)



There appears to be a saddle point in the experimental region. The yield increases as x_1 is decreased and x_2 is near the zero level.

14-71 Move 1.5 units in the direction of x_1 for every –0.8 unit in the direction of x_2. Thus, the path of steepest ascent passes through the point (0, 0) and has a slope -0.8/1.5 = -0.533.

14-72 a) $10 + 5x_1 + 2x_2 > 12$ $\quad\quad$ $23 + 3x_1 + 4x_2 < 27.50$

$x_2 > -\frac{5}{2}x_1 + 1$ $\quad\quad$ $x_2 < -0.75x_1 + 1.125$

The feasible region is between these two lines, which can be shown graphically on the x_1-x_2 plane.

b) Operating the process with $x_1 = 1.5$ and $x_2 = -1.5$ results in y_1 and y_2 comfortably within the feasible region.

14-73. a) A central composite design has been used but it is not rotatable.

b)

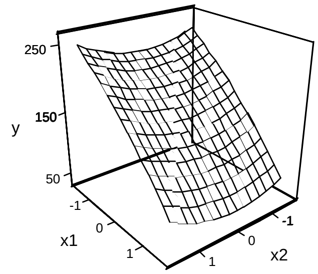
Term	Coef	StDev	T	P
Constant	150.04	7.821	19.184	0.000
x1	-58.47	5.384	-10.861	0.000
x2	3.35	5.384	0.623	0.556
x1*x1	-6.53	5.693	-1.147	0.295
x2*x2	10.58	5.693	1.859	0.112
x1*x2	0.50	7.848	0.064	0.951

S = 15.70 R-Sq = 95.4% R-Sq(adj) = 91.6%

Analysis of Variance for y

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	5	30688.7	30688.7	6137.7	24.91	0.001
Linear	2	29155.4	29155.4	14577.7	59.17	0.000
Square	2	1532.3	1532.3	766.1	3.11	0.118
Interaction	1	1.0	1.0	1.0	0.00	0.951
Residual Error	6	1478.2	1478.2	246.4		
Lack-of-Fit	3	4.2	4.2	1.4	0.00	1.000
Pure Error	3	1474.0	1474.0	491.3		
Total	11	32166.9				

The linear terms appear to be significant (p = 0.001) while both the square terms and interaction terms are insignificant (p = 0.118 and p = 0.951, respectively). Because x_1 is the only significant factor, to minimize ash increase the value of x_1.



14-74. a) Response Surface Regression

```
Estimated Regression Coefficients for y

Term        Coef  SE Coef       T      P
Constant  327.62    38.76   8.453  0.000
x3        131.47    17.94   7.328  0.000
x2        109.43    17.94   6.099  0.000
x1        177.00    17.94   9.866  0.000
x3*x3     -29.06    31.08  -0.935  0.363
x2*x2     -22.38    31.08  -0.720  0.481
x1*x1      32.01    31.08   1.030  0.317
x3*x2      43.58    21.97   1.983  0.064
x3*x1      75.47    21.97   3.435  0.003
x2*x1      66.03    21.97   3.005  0.008

S = 76.12   R-Sq = 92.7%   R-Sq(adj) = 88.8%

Analysis of Variance for y

Source          DF   Seq SS   Adj SS  Adj MS      F      P
Regression       9  1248237  1248237  138693  23.94  0.000
  Linear         3  1090558  1090558  363519  62.74  0.000
  Square         3    14219    14219    4740   0.82  0.502
  Interaction    3   143461   143461   47820   8.25  0.001
Residual Error  17    98498    98498    5794
Total           26  1346735
Reduced model:

Term        Coef  SE Coef       T      P
Constant  314.67    15.46  20.354  0.000
x3        131.47    18.93   6.943  0.000
x2        109.43    18.93   5.779  0.000
x1        177.00    18.93   9.348  0.000
x3*x1      75.47    23.19   3.255  0.004
x2*x1      66.03    23.19   2.847  0.010

S = 80.33   R-Sq = 89.9%   R-Sq(adj) = 87.5%
```

The quadratic model for y_1 is

$$y_1 = 314.67 + 177.00\, x_1 + 109.43\, x_2 + 131.47\, x_3 + 66.03\, x_1 x_2 + 75.47\, x_1 x_3$$

b) Response Surface Regression

```
Estimated Regression Coefficients for y2

Term       Coef  SE Coef      T      P
```

```
Constant  48.00    7.808  6.147  0.000
x3        29.19    9.563  3.052  0.006
x2        15.32    9.563  1.602  0.123
x1        11.53    9.563  1.205  0.240

S = 40.57   R-Sq = 36.7%   R-Sq(adj) = 28.4%

Analysis of Variance for y2

Source          DF   Seq SS   Adj SS   Adj MS     F      P
Regression       3  21957.3  21957.3  7319.09  4.45  0.013
  Linear         3  21957.3  21957.3  7319.09  4.45  0.013
Residual Error  23  37863.6  37863.6  1646.24
Total           26  59820.9
```

The linear model for y_2 is given by

$$y_2 = 48.00 + 29.19 x_3$$

c) The equations for y_1 and y_2 are used to determine values for the x's. Given values for x_1 and x_2 a value for x_3 can be solved to set y_1 to a target. Each x_i should range from -1 to 1 to stay within the experimental region for the models. The standard deviation is minimized with the smallest feasible value for x_3. When $x_3 = -1$ at least one of x_1 and x_2 must exceed 1 in order to set $y_1 = 500$. Therefore, x_3 is greater than -1. To keep a solution within the feasible region, we set $x_1 = 1$ and $x_2 = 1$. With these values the value for x_3 that sets $y_1 = 500$ is $x_3 = -0.808$ and this minimizes the standard deviation.