CHAPTER 9

Section 9-1

9-1 a) $H_0 : \mu = 25, H_1 : \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0 : \sigma > 10, H_1 : \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0 : \bar{x} = 50, H_1 : \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0 : p = 0.1, H_1 : p = 0.3$ No, the values in the null and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0 : s = 30, H_1 : s > 30$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-2 a) α = P(reject H_0 when H_0 is true)

$$= P(\bar{X} \le 11.5 \text{ when } \mu = 12) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{11.5-12}{0.5/\sqrt{4}}\right) = P(Z \le -2)$$

= 0.02275.

The probability of rejecting the null hypothesis when it is true is 0.02275.

b) β = P(accept H_0 when μ = 11.25) = $P(\bar{X} > 11.5 \text{ when } \mu = 11.25) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{4}}\right)$

P(Z > 1.0) = 1 − P(Z ≤ 1.0) = 1 − 0.84134 = 0.15866

The probability of accepting the null hypothesis when it is false is 0.15866.

9-3 a) $\alpha = P(\bar{X} \le 11.5 \mid \mu = 12) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{11.5-12}{0.5/\sqrt{16}}\right) = P(Z \le -4) = 0.$

The probability of rejecting the null, when the null is true, is approximately 0 with a sample size of 16.

b) $\beta = P(\bar{X} > 11.5 \mid \mu = 11.25) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{16}}\right) = P(Z > 2) = 1 - P(Z \le 2)$

= 1− 0.97725 = 0.02275.

The probability of accepting the null hypothesis when it is false is 0.02275.

9-4 Find the boundary of the critical region if α = 0.01:

$$0.01 = P\left(Z \le \frac{c-12}{0.5/\sqrt{4}}\right)$$

What Z value will give a probability of 0.01? Using Table 2 in the appendix, Z value is −2.33.

Thus, $\frac{c-12}{0.5/\sqrt{4}} = -2.33$, c = 11.4175

9-5. $P\left(Z \le \dfrac{c-12}{0.5/\sqrt{4}}\right) = 0.05$

What Z value will give a probability of 0.05? Using Table 2 in the appendix, Z value is −1.65.

Thus, $\dfrac{c-12}{0.5/\sqrt{4}} = -1.65$, c = 11.5875

9-6 a) $\alpha = P(\overline{X} \le 98.5) + P(\overline{X} > 101.5)$

$$= P\left(\frac{\overline{X}-100}{2/\sqrt{9}} \le \frac{98.5-100}{2/\sqrt{9}}\right) + P\left(\frac{\overline{X}-100}{2/\sqrt{9}} > \frac{101.5-100}{2/\sqrt{9}}\right)$$

= P(Z ≤ −2.25) + P(Z > 2.25)
= (P(Z ≤- 2.25)) + (1 − P(Z ≤ 2.25))
= 0.01222 + 1 − 0.98778
= 0.01222 + 0.01222 = 0.02444

b) $\beta = P(98.5 \le \overline{X} \le 101.5 \text{ when } \mu = 103)$

$$= P\left(\frac{98.5-103}{2/\sqrt{9}} \le \frac{\overline{X}-103}{2/\sqrt{9}} \le \frac{101.5-103}{2/\sqrt{9}}\right)$$

= P(−6.75 ≤ Z ≤ −2.25)
= P(Z ≤ −2.25) − P(Z ≤ −6.75)
= 0.01222 − 0 = 0.01222

c) $\beta = P(98.5 \le \overline{X} \le 101.5 \text{ when } \mu = 105)$

$$= P\left(\frac{98.5-105}{2/\sqrt{9}} \le \frac{\overline{X}-105}{2/\sqrt{9}} \le \frac{101.5-105}{2/\sqrt{9}}\right)$$

= P(−9.75≤ Z ≤ −5.25)
= P(Z ≤ −5.25) − P(Z ≤ −9.75)
= 0 − 0 = 0

The probability of accepting the null hypothesis when it is actually false is smaller in part (c) because the true mean, μ = 105, is further from the acceptance region. A larger difference exists.

9-7 Use *n* = 5, everything else held constant (from the values in exercise 9-6):

a) $P(\overline{X} \le 98.5) + P(\overline{X} > 101.5)$

$$= P\left(\frac{\overline{X}-100}{2/\sqrt{5}} \le \frac{98.5-100}{2/\sqrt{5}}\right) + P\left(\frac{\overline{X}-100}{2/\sqrt{5}} > \frac{101.5-100}{2/\sqrt{5}}\right)$$

= P(Z ≤ −1.68) + P(Z > 1.68)
= P(Z ≤ −1.68) + (1 − P(Z ≤ 1.68))
= 0.04648 + (1 − 0.95352)
= 0.09296

b) $\beta = P(98.5 \le \overline{X} \le 101.5 \text{ when } \mu = 103)$

$$= P\left(\frac{98.5-103}{2/\sqrt{5}} \le \frac{\overline{X}-103}{2/\sqrt{5}} \le \frac{101.5-103}{2/\sqrt{5}}\right)$$

= P(−5.03 ≤ Z ≤ −1.68)
= P(Z ≤ −1.68) − P(Z ≤ −5.03)
= 0.04648 − 0
= 0.04648

c) $\beta = P(98.5 \le \bar{x} \le 101.5 \text{ when } \mu = 105)$

$$= P\left(\frac{98.5-105}{2/\sqrt{5}} \le \frac{\bar{X}-105}{2/\sqrt{5}} \le \frac{101.5-105}{2/\sqrt{5}}\right)$$

$= P(-7.27 \le Z \le -3.91)$
$= P(Z \le -3.91) - P(Z \le -7.27)$
$= 0.00005 - 0$
$= 0.00005$

It is smaller, because it is not likely to accept the product when the true mean is as high as 105.

9-8 a) $\alpha = P(\bar{X} > 185 \text{ when } \mu = 175)$

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{10}} > \frac{185-175}{20/\sqrt{10}}\right)$$

$= P(Z > 1.58)$
$= 1 - P(Z \le 1.58)$
$= 1 - 0.94295$
$= 0.05705$

b) $\beta = P(\bar{X} \le 185 \text{ when } \mu = 195)$

$$= P\left(\frac{\bar{X}-195}{20/\sqrt{10}} \le \frac{185-195}{20/\sqrt{10}}\right)$$

$= P(Z \le -1.58)$
$= 0.05705.$

9-9 a) $z = \frac{190-175}{20/\sqrt{10}} = 2.37$, Note that z is large, therefore **reject** the null hypothesis and conclude that the mean foam height is greater than 175 mm.

b) $P(\bar{X} > 190 \text{ when } \mu = 175)$

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{10}} > \frac{190-175}{20/\sqrt{10}}\right)$$

$= P(Z > 2.37) = 1 - P(Z \le 2.37)$
$= 1 - 0.99111$
$= 0.00889.$

The probability that a value of at least 190 mm would be observed (if the true mean height is 175 mm) is only 0.00889. Thus, the sample value of $\bar{x}$ = 190 mm would be an unusual result.

9-10 Using n = 16:

a) $\alpha = P(\bar{X} > 185 \text{ when } \mu = 175)$

$$= P\left(\frac{\bar{X}-175}{20/\sqrt{16}} > \frac{185-175}{20/\sqrt{16}}\right)$$

$= P(Z > 2)$
$= 1 - P(Z \le 2)$
$= 1 - 0.97725$
$= 0.02275$

b) $\beta = P(\bar{X} \le 185 \text{ when } \mu = 195)$

$$= P\left(\frac{\bar{X} - 195}{20/\sqrt{16}} \le \frac{185 - 195}{20/\sqrt{16}}\right)$$

$= P(Z \le -2)$

$= 0.02275.$

9-11 a) $P(\bar{X} > c \mid \mu = 175) = 0.0571$

$$P\left(Z > \frac{c - 175}{20/\sqrt{16}}\right) = P(Z \ge 1.58)$$

Thus, $1.58 = \dfrac{c - 175}{20/\sqrt{16}}$, and $c = 182.9$

b) If the true mean foam height is 195 mm, then

$\beta = P(\bar{X} \le 182.9 \text{ when } \mu = 195)$

$$= P\left(Z \le \frac{182.9 - 195}{20/\sqrt{16}}\right)$$

$= P(Z \le -2.42)$

$= 0.00776$

c) For the same level of α, with the increased sample size, β is reduced.

9-12 a) $\alpha = P(\bar{X} \le 4.85 \text{ when } \mu = 5) + P(\bar{X} > 5.15 \text{ when } \mu = 5)$

$$= P\left(\frac{\bar{X} - 5}{0.25/\sqrt{8}} \le \frac{4.85 - 5}{0.25/\sqrt{8}}\right) + P\left(\frac{\bar{X} - 5}{0.25/\sqrt{8}} > \frac{5.15 - 5}{0.25/\sqrt{8}}\right)$$

$= P(Z \le -1.7) + P(Z > 1.7)$

$= P(Z \le -1.7) + (1 - P(Z \le 1.7)$

$= 0.04457 + (1 - 0.95543)$

$= 0.08914.$

b) Power $= 1 - \beta$

$\beta = P(4.85 \le \bar{X} \le 5.15 \text{ when } \mu = 5.1)$

$$= P\left(\frac{4.85 - 5.1}{0.25/\sqrt{8}} \le \frac{\bar{X} - 5.1}{0.25/\sqrt{8}} \le \frac{5.15 - 5.1}{0.25/\sqrt{8}}\right)$$

$= P(-2.83 \le Z \le 0.566)$

$= P(Z \le 0.566) - P(Z \le -2.83)$

$= 0.71566 - 0.00233$

$= 0.71333$

$1 - \beta = 0.2867.$

9-13 Using n = 16:

a) $\alpha = P(\overline{X} \le 4.85 \mid \mu = 5) + P(\overline{X} > 5.15 \mid \mu = 5)$

$$= P\left(\frac{\overline{X}-5}{0.25/\sqrt{16}} \le \frac{4.85-5}{0.25/\sqrt{16}}\right) + P\left(\frac{\overline{X}-5}{0.25/\sqrt{16}} > \frac{5.15-5}{0.25/\sqrt{16}}\right)$$

$= P(Z \le -2.4) + P(Z > 2.4)$
$= P(Z \le -2.4) + (1 - P(Z \le 2.4))$
$= 2(1 - P(Z \le 2.4))$
$= 2(1 - 0.99180)$
$= 2(0.0082)$
$= 0.0164.$

b) $\beta = P(4.85 \le \overline{X} \le 5.15 \mid \mu = 5.1)$

$$= P\left(\frac{4.85-5.1}{0.25/\sqrt{16}} \le \frac{\overline{X}-5.1}{0.25/\sqrt{16}} \le \frac{5.15-5.1}{0.25/\sqrt{16}}\right)$$

$= P(-4 \le Z \le 0.8) = P(Z \le 0.8) - P(Z \le -4)$
$= 0.78814 - 0$
$= 0.78814$
$1 - \beta = 0.21186$

9-14 Find the boundary of the critical region if $\alpha = 0.05$:

$$0.025 = P\left(Z \le \frac{c-5}{0.25/\sqrt{8}}\right)$$

What Z value will give a probability of 0.025? Using Table 2 in the appendix, Z value is −1.96.

Thus, $\frac{c-5}{0.25/\sqrt{8}} = -1.96$, c = 4.83 and

$$\frac{c-5}{0.25/\sqrt{8}} = 1.96, \quad c=5.17$$

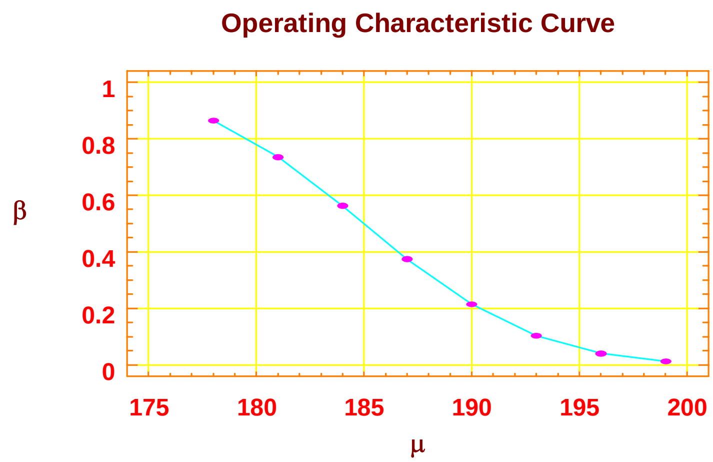
The acceptance region should be $(4.83 \le \overline{X} \le 5.17)$.

9-15 Operating characteristic curve:

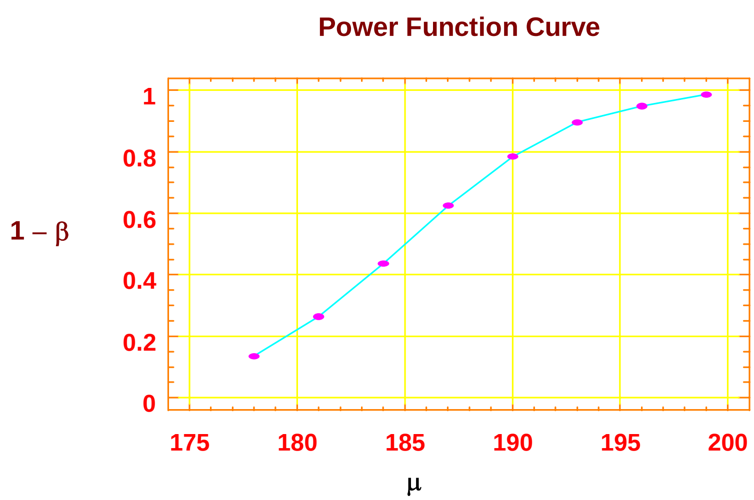
$\bar{x} = 185$

$$\beta = P\left(Z \le \frac{\bar{x}-\mu}{20/\sqrt{10}}\right) = P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right)$$

μ	$P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right) =$	β	$1-\beta$
178	$P(Z \le 1.11) =$	0.8665	0.1335
181	$P(Z \le 0.63) =$	0.7357	0.2643
184	$P(Z \le 0.16) =$	0.5636	0.4364
187	$P(Z \le -0.32) =$	0.3745	0.6255
190	$P(Z \le -0.79) =$	0.2148	0.7852
193	$P(Z \le -1.26) =$	0.1038	0.8962
196	$P(Z \le -1.74) =$	0.0409	0.9591
199	$P(Z \le -2.21) =$	0.0136	0.9864



9-16



9-17. The problem statement implies H_0: $p = 0.6$, H_1: $p > 0.6$ and defines an acceptance region as $\hat{p} \le \frac{400}{500} = 0.80$ and rejection region as $\hat{p} > 0.80$

a) $\alpha = P(\hat{p} > 0.80 \mid p = 0.60) = P\left(Z > \frac{0.80 - 0.60}{\sqrt{\frac{0.6(0.4)}{500}}}\right)$

$= P(Z > 9.13) = 1 - P(Z \le 9.13) \approx 0$

b) $\beta = P(\hat{p} \le 0.8 \text{ when } p = 0.75) = P(Z \le 2.58) = 0.99506$.

9-18 $X \sim \text{bin}(10, 0.3)$ Implicitly, H_0: $p = 0.3$ and H_1: $p < 0.3$
$n = 10$
Accept region: $\hat{p} > 0.1$
Reject region: $\hat{p} \le 0.1$
Use the normal approximation for parts a), b) and c):

a) When p =0.3 $\alpha =$ $P(\hat{p} \le 0.1) = P\left(Z \le \frac{0.1 - 0.3}{\sqrt{\frac{0.3(0.7)}{10}}}\right)$

$$= P(Z \le -1.38)$$
$$= 0.08379$$

b) When p = 0.2 $\beta =$ $P(\hat{p} > 0.1) = P\left(Z > \frac{0.1 - 0.2}{\sqrt{\frac{0.2(0.8)}{10}}}\right)$

$$= P(Z > -0.79)$$
$$= 1 - P(Z < -0.79)$$
$$= 0.78524$$

c) Power = 1 – β = 1 – 078524 = 0.21476

9-19 $X \sim \text{bin}(15, 0.4)$ H_0: $p = 0.4$ and H_1: $p \ne 0.4$

$p_1 = 4/15 = 0.267$ $p_2 = 8/15 = 0.533$

Accept Region: $0.267 \le \hat{p} \le 0.533$
Reject Region: $\hat{p} < 0.267$ or $\hat{p} > 0.533$

Use the normal approximation for parts a) and b)

a) When p = 0.4, $\alpha = P(\hat{p} < 0.267) + P(\hat{p} > 0.533)$

$$= P\left(Z < \frac{0.267 - 0.4}{\sqrt{\frac{0.4(0.6)}{15}}}\right) + P\left(Z > \frac{0.533 - 0.4}{\sqrt{\frac{0.4(0.6)}{15}}}\right)$$
$$= P(Z < -1.05) + P(Z > 1.05)$$
$$= P(Z < -1.05) + (1 - P(Z < 1.05))$$
$$= 0.14686 + 0.14686$$
$$= 0.29372$$

b) When p = 0.2,

$$\beta = P(0.267 \le \hat{p} \le 0.533) = P\left(\frac{0.267-0.2}{\sqrt{\frac{0.2(0.8)}{15}}} \le Z \le \frac{0.533-0.2}{\sqrt{\frac{0.2(0.8)}{15}}}\right)$$
$$= P(0.65 \le Z \le 3.22)$$
$$= P(Z \le 3.22) - P(Z < 0.65)$$
$$= 0.99936 - 0.74215$$
$$= 0.25721$$

Section 9-2

9-20 a) The exercise does not specify the direction of the one-sided hypothesis explicitly. If the goal is to demonstrate that the mean water temperature is greater than 100 degrees then the solution is as follows:

1) The parameter of interest is the true mean water temperature μ.

2) H_0: $\mu = 100$

3) H_1: $\mu > 100$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 98$, $\sigma = 2$

$$z_0 = \frac{98-100}{2/\sqrt{9}} = -3.0$$

8) Because -3.0 < 1.65 do not reject H_0 and conclude the water temperature is not significantly greater than 100 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(-3.0) = 1 - 0.00135 = 0.99865$

c) $\beta = \Phi\left(z_{0.05} + \dfrac{100-104}{2/\sqrt{9}}\right)$

$= \Phi(1.65 + -6)$

$= \Phi(-4.35)$

$\cong 0$

9-21 a) 1) The parameter of interest is the true mean yield μ.

2) H_0 : $\mu = 90$

3) H_1 : $\mu \ne 90$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\bar{x} = 90.48$, $\sigma = 3$

$$z_0 = \frac{90.48-90}{3/\sqrt{5}} = 0.36$$

8) Because $-1.96 < 0.36 < 1.96$ do not reject H_0 and conclude the yield is not significantly different from 90% at $\alpha = 0.05$.

b) P-value = $2[1 - \Phi(0.36)] = 2[1 - 0.64058] = 0.71884$

c) $n = \dfrac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.025} + z_{0.05})^2 3^2}{(85-90)^2} = \dfrac{(1.96 + 1.65)^2 9}{(-5)^2} = 4.69$

$n \cong 5$.

d) $\beta = \Phi\left(z_{0.025} + \dfrac{90-92}{3/\sqrt{5}}\right) - \Phi\left(-z_{0.025} + \dfrac{90-92}{3/\sqrt{5}}\right)$

$= \Phi(1.96 + -1.491) - \Phi(-1.96 + -1.491)$
$= \Phi(0.47) - \Phi(-3.45)$
$= \Phi(0.47) - (1 - \Phi(3.45))$
$= 0.68082 - (1 - 0.99972)$
$= 0.68054$.

e) Exercise should compare to the conclusion of part (a). For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

$\bar{x} - z_{0.025}\left(\dfrac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\dfrac{\sigma}{\sqrt{n}}\right)$

$90.48 - 1.96\left(\dfrac{3}{\sqrt{5}}\right) \le \mu \le 90.48 + 1.96\left(\dfrac{3}{\sqrt{5}}\right)$

$87.85 \le \mu \le 93.11$

With 95% confidence, we believe the true mean yield of the chemical process is between 87.85% and 93.11%. Because 90% is contained in the confidence interval, our decision in part (a) agrees with the confidence interval.

9-22 a) 1) The parameter of interest is the true mean crankshaft wear, μ.
2) $H_0 : \mu = 3$
3) $H_1 : \mu \ne 3$
4) $\alpha = 0.05$
5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
6)) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\bar{x} = 2.78$, $\sigma = 0.9$

$$z_0 = \frac{2.78 - 3}{0.9/\sqrt{15}} = -0.95$$

8) Because –0.95 > -1.96, do not reject the null hypothesis. There is not sufficient evidence to conclude that the mean crankshaft wear is differs from 3 at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.025} + \dfrac{3-3.25}{0.9/\sqrt{15}}\right) - \Phi\left(-z_{0.025} + \dfrac{3-3.25}{0.9/\sqrt{15}}\right)$

$= \Phi(1.96 + -1.08) - \Phi(-1.96 + -1.08)$
$= \Phi(0.88) - \Phi(-3.04)$
$= 0.81057 - (0.00118)$
$= 0.80939$

c) $n = \dfrac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.025} + z_{0.10})^2 \sigma^2}{(3.75-3)^2} = \dfrac{(1.96 + 1.29)^2 (0.9)^2}{(0.75)^2} = 15.21,$

$n \cong 16$

9-23 a) 1) The parameter of interest is the true mean melting point μ.

2) $H_0 : \mu = 155$

3) $H_1 : \mu \neq 155$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\overline{x} = 154.2$, $\sigma = 1.5$

$$z_0 = \frac{154.2 - 155}{1.5/\sqrt{10}} = -1.69$$

8) Because $-1.69 > -2.58$, do not reject the null hypothesis. There is not sufficient evidence to conclude that the mean melting point differs from155 °F at $\alpha = 0.01$.

b) P-value = 2*P(Z <- 1.69) =2* 0.045514 = 0.091028

c)
$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(155-150)\sqrt{10}}{1.5}\right) - \Phi\left(-2.58 - \frac{(155-150)\sqrt{10}}{1.5}\right)$$

$= \Phi(-7.96) - \Phi(-13.12) = 0 - 0 = 0$

d)
$$n = \frac{\left(z_{\alpha/2} + z_{\beta}\right)^2 \sigma^2}{\delta^2} = \frac{\left(z_{0.005} + z_{0.10}\right)^2 \sigma^2}{(150-155)^2} = \frac{(2.58+1.29)^2 (1.5)^2}{(5)^2} = 1.35,$$

$n \cong 2$.

9-24 a) 1) The parameter of interest is the true mean battery life in hours μ.

2) $H_0 : \mu = 40$

3) $H_1 : \mu > 40$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\overline{x} = 40.5$, $\sigma = 1.25$

$$z_0 = \frac{40.5 - 40}{1.25/\sqrt{10}} = 1.26$$

8) Because $1.26 < 1.65$ do not reject H_0 and conclude that battery life is not significantly greater than 40 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(1.26) = 1 - 0.8962 = 0.1038$

c) $\beta = \Phi\left(z_{0.05} + \dfrac{40-42}{1.25/\sqrt{10}}\right)$

$= \Phi(1.65 + -5.06)$

$= \Phi(-3.41)$

$\cong 0.000325$

d) $n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.10})^2 \sigma^2}{(40-44)^2} = \frac{(1.65+1.29)^2 (1.25)^2}{(4)^2} = 0.844,$

$n \cong 1$

e) 90% lower confidence interval

$$\bar{x} - z_{0.05}\sigma / \sqrt{n} \le \mu$$

$$40.5 - 1.65(1.25)/\sqrt{10} \le \mu$$

$$39.85 \le \mu$$

The lower bound of the 90 % confidence interval must be greater than 40 to have evidence that the true mean exceeds 40 hours.

9-25 a) 1) The parameter of interest is the true mean tensile strength μ.

2) $H_0 : \mu = 3500$

3) $H_1 : \mu \ne 3500$

4) $\alpha = 0.01$

5) $z_0 = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\bar{x} = 3250$, $\sigma = 60$

$$z_0 = \frac{3250 - 3500}{60/\sqrt{12}} = -14.43$$

8) Because −14.43 < −2.58, reject the null hypothesis and conclude the true mean tensile strength is significantly different from 3500 at $\alpha = 0.01$.

b) Smallest level of significance = P-value = $2[1 - \Phi(14.43)] \approx 2[1 - 1] = 0$

The smallest level of significance at which the null hypothesis is rejected is approximately 0.

c) $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 2.58\left(\frac{60}{\sqrt{12}}\right) \le \mu \le 3250 + 2.58\left(\frac{60}{\sqrt{12}}\right)$$

$3205.31 \le \mu \le 3294.69$

With 99% confidence, we believe that the true mean tensile strength is between 3205.31 psi and 3294.69 psi. We can test the hypotheses that the true mean tensile strength is not equal to 3500 by noting that the hypothesized value of 3500 is not within the confidence interval.

9-26 $n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.20})^2 \sigma^2}{(3250-3500)^2} = \frac{(1.65+.84)^2 (60)^2}{(250)^2} = 0.357,$

$n \cong 1$

9-27 a) 1) The parameter of interest is the true mean speed μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu < 100$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_{0.05} = -1.65$

7) $\overline{x} = 102.2$, $\sigma = 4$

$$z_0 = \frac{102.2 - 100}{4/\sqrt{8}} = 1.56$$

8) Because $-1.65 < 1.56$ do not reject the null hypothesis. There is not sufficient evidence to conclude that true mean speed is less than 100 at $\alpha = 0.05$. The sample mean must be significantly less than 100 before the null hypothesis is rejected. In this exercise the sample mean is actually greater than 100 so it is known that the null hypothesis is not rejected before z_0 is computed.

b) $\beta = \Phi\left(-z_{0.05} - \dfrac{(95-100)\sqrt{8}}{4} \right)$ = $\Phi(-1.65 - -3.54) = \Phi(1.89) = 0.97062$

Power = $1-\beta = 1-0.97062 = 0.02938$

c) n = $\dfrac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.15})^2 \sigma^2}{(95-100)^2} = \dfrac{(1.65+1.03)^2 (4)^2}{(5)^2} = 4.597,$

$n \cong 5$

d) Typically this is answered with a one-sided, upper confidence limit bound. If the upper bound is less than the hypothesized value of 100 that is high confidence that the true mean is less than 100. The formula is

$$\mu \le \overline{x} + z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) = 102.2 + 1.65\left(\frac{4}{\sqrt{8}}\right) = 104.53$$

Because the upper limit of the CI is above 100, we cannot conclude that the true mean is below 100. This is an equivalent method to obtain the same conclusion as part (a).

9-28 a) 1) The parameter of interest is the true mean hole diameter μ.

2) $H_0 : \mu = 1.50$

3) $H_1 : \mu \neq 1.50$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\overline{x} = 1.4975$, $\sigma = 0.01$

$$z_0 = \frac{1.4975 - 1.50}{0.01/\sqrt{25}} = -1.25$$

8) Because $-2.58 < -1.25 < 2.58$, do not reject the null hypothesis. The true mean hole diameter is not significantly different from 1.5 in. at $\alpha = 0.01$.

b)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right) - \Phi\left(-2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right)$$

$= \Phi(5.08) - \Phi(-0.08) = 1 - .46812 = 0.53188$

power $= 1 - \beta = 0.46812$.

c) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_{\alpha/2} + z_{\beta})^2\sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2\sigma^2}{(1.495-1.50)^2} \cong \frac{(2.58+1.29)^2(0.01)^2}{(-0.005)^2} = 59.908,$$

$n \cong 60$.

d) For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$1.4975 - 2.58\left(\frac{0.01}{\sqrt{25}}\right) \le \mu \le 1.4975 + 2.58\left(\frac{0.01}{\sqrt{25}}\right)$$

$1.4923 \le \mu \le 1.5027$

The confidence interval contains the hypothesized value 1.5. Thus, this confidence interval does not provide evidence that the true mean differs from 1.5. In this manner a two-sided 99% confidence interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$. The conclusions necessarily must be the same.

9-29 a) 1) The parameter of interest is the true average battery life μ.

2) $H_0 : \mu = 4$

3) $H_1 : \mu > 4$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 4.05$, $\sigma = 0.2$

$$z_0 = \frac{4.05 - 4}{0.2/\sqrt{50}} = 1.77$$

8) Because 1.65 < 1.77, reject the null hypothesis. There is sufficient evidence to conclude that the true average battery life exceeds 4 hours at $\alpha = 0.05$.

b) $\beta = \Phi\left(z_{0.05} - \dfrac{(4.5-4)\sqrt{50}}{0.2}\right) = \Phi(1.65 - 17.68) = \Phi(-16.03) \approx 0$

Power $= 1 - \beta = 1 - 0 = 1$

c) $$n = \frac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.1})^2\sigma^2}{(4.5-4)^2} = \frac{(1.65+1.29)^2(0.2)^2}{(0.5)^2} = 1.38,$$

$n \cong 2$

d) $\bar{x} - z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu$

$$4.05 - 1.65\left(\frac{0.2}{\sqrt{50}}\right) \le \mu$$

$$4.003 \le \mu$$

Because the lower limit of the CI is just slightly above 4, we conclude that average life is greater than 4 hours at $\alpha = 0.05$.

Section 9-3

9-30 a) 1) The parameter of interest is the true mean interior temperature life μ.

2) $H_0 : \mu = 22.5$

3) $H_1 : \mu \ne 22.5$

4) $\alpha = 0.05$

5) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.776$

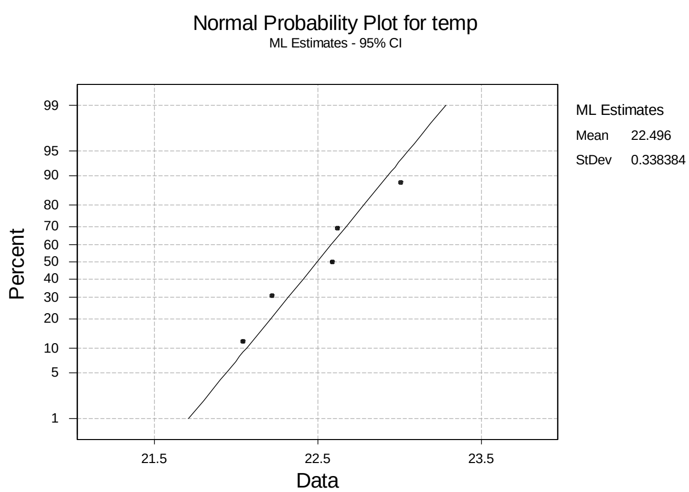
7) $\bar{x} = 22.496$, $s = 0.378$ $n=5$

$$t_0 = \frac{22.496 - 22.5}{0.378/\sqrt{5}} = -0.0237$$

8) Because $-0.0237 > -2.776$, we cannot reject the null hypothesis. There is not sufficient evidence to conclude that the true mean interior temperature is not equal to 22.5 °C at $\alpha = 0.05$.

$2*0.4 < P\text{-value} < 2*0.5$; $0.8 < P\text{-value} < 1.0$

b) The points on the normal probability plot fall along the line. Therefore, there is no evidence to conclude that the interior temperature data is not normally distributed.



c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|22.75 - 22.5|}{0.378} = 0.66$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 0.66$, and n = 5, $\beta \cong 0.8$ and power =1 − 0.8 = 0.2.

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|22.75 - 22.5|}{0.378} = 0.66$

Using the OC curve, Chart VI e) for $\alpha = 0.05$, $d = 0.66$, and $\beta \cong 0.1$ (power = 0.9), $n = 40$.

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right)$$

$$22.496 - 2.776\left(\frac{0.378}{\sqrt{5}}\right) \le \mu \le 22.496 + 2.776\left(\frac{0.378}{\sqrt{5}}\right)$$

$$22.027 \le \mu \le 22.965$$

We cannot conclude that the mean interior temperature differs from 22.5 because the value is included in the confidence interval.

9-31 a) 1) The parameter of interest is the true mean female body temperature μ.

2) $H_0 : \mu = 98.6$

3) $H_1 : \mu \ne 98.6$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.064$

7) $\bar{x} = 98.264$, s = 0.4821 n=25

$$t_0 = \frac{98.264 - 98.6}{0.4821/\sqrt{25}} = -3.48$$

8) Because −3.48 < −2.064, reject the null hypothesis. There is sufficient evidence to conclude that the true mean female body temperature is not equal to 98.6 °F at $\alpha = 0.05$.

2* 0.0005 < *P*-value < 2(0.001); 0.001 < *P*-value < 0.002

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|98 - 98.6|}{0.4821} = 1.24$

From the OC curve, Chart VI e) for $\alpha = 0.05$, d = 1.24, and n = 25, conclude $\beta \cong 0$ and power $\cong 1 - 0 = 1$.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|98.2 - 98.6|}{0.4821} = 0.83$

Using the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.83, $\beta \cong 0.1$ (power = 0.9), obtain $n = 20$.

d) 95% two sided confidence interval

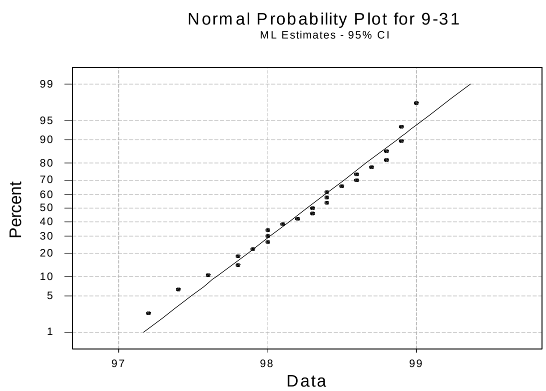
$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$98.264 - 2.064\left(\frac{0.4821}{\sqrt{25}}\right) \le \mu \le 98.264 + 2.064\left(\frac{0.4821}{\sqrt{25}}\right)$$

$$98.065 \le \mu \le 98.463$$

The mean female body temperature is significantly different from 98.6 because the hypothesized value of 98.6 is not included in the confidence interval.

e)



Data appear to be normally distributed**.**

9-32 a) 1) The parameter of interest is the true mean rainfall, μ.

2) $H_0 : \mu = 25$

3) $H_1 : \mu > 25$

4) $\alpha = 0.01$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

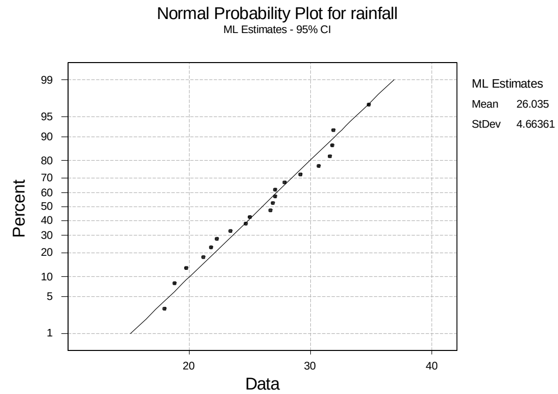
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,19} = 2.539$

7) $\bar{x} = 26.04$ $s = 4.78$ $n = 20$

$$t_0 = \frac{26.04 - 25}{4.78/\sqrt{20}} = 0.97$$

8) Because 0.97 < 2.539, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean rainfall is greater than 25 acre-feet at $\alpha = 0.01$. Also, $0.10 < P\text{-value} < 0.25$.

b) The data on the normal probability plot fall along the straight line. The normality assumption appears valid.



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|27 - 25|}{4.78} = 0.42$

From the OC curve, Chart VI h) for $\alpha = 0.01$, d = 0.42, n = 20, obtain $\beta \cong 0.7$ and power $\cong 1 - 0.7 = 0.3$

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|27.5 - 25|}{4.78} = 0.52$

From the OC curve, Chart VI h) for $\alpha = 0.01$, d = 0.42, $\beta \cong 0.1$ (power=0.9), obtain $n = 75$

e) 95% two sided confidence interval

$$\bar{x} - t_{0.01,19}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$26.04 - 2.539\left(\frac{4.78}{\sqrt{20}}\right) \le \mu$$

$$23.326 \le \mu$$

Because the lower limit of the CI is less than 25. There is not sufficient evidence to conclude that the true mean rainfall is greater than 25 acre-feet at $\alpha = 0.01$.

9-33 a) 1) The parameter of interest is the true mean sodium content, μ.

2) $H_0 : \mu = 130$
3) $H_1 : \mu \neq 130$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = 2.045$

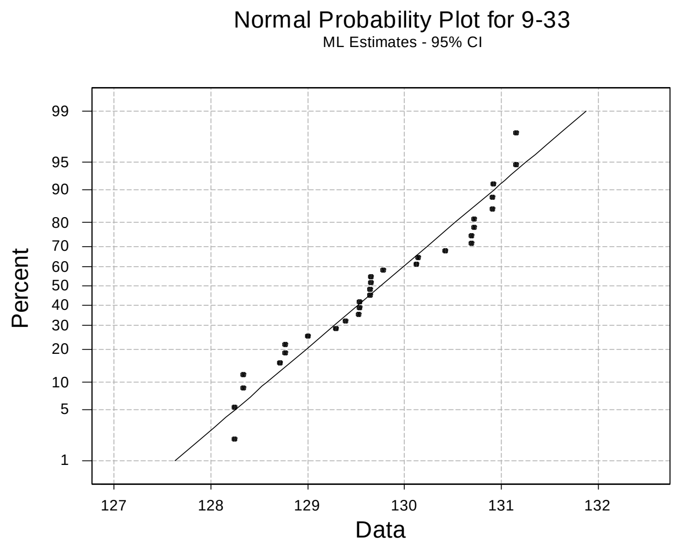
7) $\bar{x} = 129.753$, s = 0.929 n=30

$$t_0 = \frac{129.753 - 130}{0.929\sqrt{30}} = -1.456$$

8) Because 1.456 < 2.064, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean sodium content is different from 130mg at $\alpha = 0.05$.

From Table IV the t_0 value is between the values of 0.05 and 0.1 with 29 degrees of freedom. Therefore, 2*0.05 < *P*-value = 2* 0.1. That is, 0.1 < *P*-value < 0.2.

b)



The assumption of normality appears to be reasonable.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|130.5 - 130|}{0.929} = 0.538$

From the OC curve, Chart VI e) for $\alpha = 0.05$, d = 0.53, n = 30, obtain $\beta \cong 0.2$ and power $\cong 1 - 0.20 = 0.80$

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|130.1 - 130|}{0.929} = 0.11$

From the OC curve, Chart VI e) for $\alpha = 0.05$, d = 0.11, $\beta \cong 0.25$ (power = 0.75), obtain $n = 100$

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,29}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,29}\left(\frac{s}{\sqrt{n}}\right)$$

$$129.753 - 2.045\left(\frac{0.929}{\sqrt{30}}\right) \le \mu \le 129.753 + 2.045\left(\frac{0.929}{\sqrt{30}}\right)$$

$$129.406 \le \mu \le 130.100$$

Because the hypothesized value 130 is in the confidence interval there is not sufficient evidence to conclude that the mean sodium content differs from 130

9-34 a)1) The parameter of interest is the true mean tire life, μ.

2) $H_0 : \mu = 60000$

3) $H_1 : \mu > 60000$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,15} = 1.753$

7) $n = 16 \quad \bar{x} = 60{,}139.7 \quad s = 3645.94$

$$t_0 = \frac{60139.7 - 60000}{3645.94/\sqrt{16}} = 0.15$$

8) Because 0.15 < 1.753., do not reject the null hypothesis. There is not sufficient evidence to indicate that the true mean tire life is greater than 60,000 kilometers at $\alpha = 0.05$. Also, P-value > 0.40.

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|61000 - 60000|}{3645.94} = 0.27$

From the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.27, $\beta = 0.1$ (power=0.9), obtain $n = 4$. Yes, the sample size of 16 was sufficient.

9-35 In order to use t statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean impact strength μ.

2) $H_0 : \mu = 1.0$

3) $H_1 : \mu > 1.0$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,19} = 1.729$

7) $\bar{x} = 1.25 \quad s = 0.25 \quad n = 20$

$$t_0 = \frac{1.25 - 1.0}{0.25/\sqrt{20}} = 4.47$$

8) Because 1.729 < 4.47, reject the null hypothesis. There is sufficient evidence to conclude that the true mean impact strength is greater than 1.0 ft-lb/in at $\alpha = 0.05$. Also, P-value < 0.0005.

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9-36 In order to use t statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean current μ.

2) $H_0 : \mu = 300$

3) $H_1 : \mu > 300$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,9} = 1.833$

7) $n = 10 \quad \bar{x} = 317.2 \quad s = 15.7$

$$t_0 = \frac{317.2 - 300}{15.7/\sqrt{10}} = 3.46$$

8) Because 3.46 > 1.833, reject the null hypothesis. There is sufficient evidence to conclude that the true mean current is greater than 300 microamps at $\alpha = 0.05$. Also. 0.0025 < P-value < 0.005.

9-37 In order to use t statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean coefficient of restitution μ.

2) $H_0 : \mu = 0.635$
3) $H_1 : \mu > 0.635$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,39} = 1.685$

7) $\overline{x} = 0.624$ $s = 0.013$ $n = 40$

$$t_0 = \frac{0.624 - 0.635}{0.013/\sqrt{40}} = -5.35$$

8) Because –5.25 < 1.685, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean coefficient of restitution is greater than 0.635 at $\alpha = 0.05$. In this exercise the sample mean is less than the hypothesized value of 0.635 so that it is known that the null hypothesis is not rejected before t_0 is computed.

b) The *P*-value > 0.4 from Table IV.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.64 - 0.635|}{0.013} = 0.38$
From the OC curve, Chart VI g) with $\alpha = 0.05$, d = 0.38, n = 40, obtain $\beta \cong 0.25$ and power $\cong 1 - 0.25 = 0.75$

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|0.64 - 0.635|}{0.013} = 0.38$
From the OC curve, Chart VI g) for $\alpha = 0.05$, d = 0.38, $\beta \cong 0.25$ (power = 0.75), obtain *n* = 40

9-38 a) In order to use *t* statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.
1) The parameter of interest is the true mean oxygen concentration μ.
2) $H_0 : \mu = 4$
3) $H_1 : \mu \neq 4$
4) $\alpha = 0.01$
5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{a/2, n-1} = t_{0.005, 19} = 2.861$
7) $\overline{x} = 3.265$, $s = 2.127$, $n = 20$

$$t_0 = \frac{3.265 - 4}{2.127/\sqrt{20}} = -1.55$$

8) Because -2.861<-1.55 < 2.861, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean oxygen differs from 4 at $\alpha = 0.01$.

b) The *P*-value: 2*0.05 < *P*-value < 2*0.10. That is, 0.10< *P*-value<0.20
c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|3 - 4|}{2.127} = 0.47$

From the OC curve, Chart VI f) for $\alpha = 0.01$, $d = 0.47$, $n = 20$, obtain $\beta \cong 0.70$ and power $\cong 1 - 0.70 = 0.30$

d.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|2.5 - 4|}{2.127} = 0.71$
From the OC curve, Chart VI f) for $\alpha = 0.01$, $d = 0.71$, $\beta \cong 0.10$ (power = 0.90), obtain *n* = 40

9-39 a) In order to use t statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean cigar tar content μ.

2) $H_0 : \mu = 1.5$

3) $H_1 : \mu > 1.5$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,29} = 1.699$

7) $\overline{x} = 1.529$ $s = 0.0566$ $n = 30$

$$t_0 = \frac{1.529 - 1.5}{0.0566/\sqrt{30}} = 2.806$$

8) Because 2.806 > 1.699, reject the null hypothesis. There is sufficient evidence to conclude that the true mean tar content is greater than 1.5 at $\alpha = 0.05$.

b) From table IV the t_0 value is between the values of 0.0025 and 0.005 with 29 degrees of freedom. Therefore, 0.0025< P-value < 0.005. Minitab software gives P-value = 0.004.

c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|1.6 - 1.5|}{0.0566} = 1.77$

From the OC curve, Chart VI g) for $\alpha = 0.05$, $d = 1.77$, $n = 30$, obtain $\beta \cong 0$ and power $\cong 1 - 0 = 1$

e.) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|1.6 - 1.5|}{0.0566} = 1.77$

From the OC curve, Chart VI g) for $\alpha = 0.05$, $d = 1.77$, $\beta \cong 0.20$ (power = 0.80), obtain n = 4

9-40 a)

1) The parameter of interest is the true mean height of female engineering students μ.

2) $H_0 : \mu = 65$

3) $H_1 : \mu \neq 65$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{0.025,59} = 2.0281$

7) $\overline{x} = 65.811$ inches $s = 2.106$ inches $n = 37$

$$t_0 = \frac{65.811 - 65}{2.11/\sqrt{37}} = 2.34$$

8) Because 2.0281 < 2.34, reject the null hypothesis. There is sufficient evidence to conclude that the true mean height of female engineering students is differs from 65 at $\alpha = 0.05$.

b) P-value: 0.02 < P-value < 0.05

c) $d = \dfrac{|62 - 65|}{2.11} = 1.42$

From the OC Chart VI e) for $\alpha = 0.05$, $d = 1.42$, $n = 37$, obtain $\beta \cong 0$ and power $\cong 1$

d) $d = \dfrac{|64 - 65|}{2.11} = 0.47$

From the OC Chart VI e) for $\alpha = 0.05$, $d = 0.47$, $\beta \cong 0.2$ (power = 0.8), obtain $n = 40$

9-41 a) In order to use *t* statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean concentration of suspended solids μ.
2) $H_0 : \mu = 55$
3) $H_1 : \mu \neq 55$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{0.025,59} = 2.000$

7) $\bar{x} = 59.87$ $s = 12.50$ $n = 60$

$$t_0 = \frac{59.87 - 55}{12.50/\sqrt{60}} = 3.018$$

8) Because 3.018 > 2.000, reject the null hypothesis. There is sufficient evidence to conclude that the true mean concentration of suspended solids differs from 55 at $\alpha = 0.05$.

b) From Table IV the t_0 value is between the values of 0.001 and 0.0025 with 59 degrees of freedom. Therefore, 2*0.001 < *P*-value = 2* 0.0025. That is, 0.002 < *P*-value<0.005. Minitab software gives a *P*-value of 0.0038.

c) $d = \dfrac{|50 - 55|}{12.50} = 0.4$

From the OC Chart VI e) for $\alpha = 0.05$, $d = 0.4$, $n = 60$, obtain $\beta \cong 0.2$ and power $\cong 1 - 0.2 = 0.8$

d) $d = \dfrac{|50 - 55|}{12.50} = 0.4$

From the OC Chart VI e) for $\alpha = 0.05$, $d = 0.4$, $\beta \cong 0.10$ (power=0.90), obtain $n = 75$

9-42 a) In order to use *t* statistics in hypothesis testing, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean distance, μ.
2) $H_0 : \mu = 280$
3) $H_1 : \mu > 280$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,99} = 1.6604$

7) $\bar{x} = 260.3$ $s = 13.41$ $n = 100$

$$t_0 = \frac{260.3 - 280}{13.41/\sqrt{100}} = -14.69$$

8) Because –14.69 < 1.6604, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean distance is greater than 280 at $\alpha = 0.05$. In this exercise the sample mean is less than 280 so it is known that the null hypothesis is not rejected before t_0 is computed.

b) From table IV the t_0 value is above the value 0.005. Therefore, the *P*-value is greater than 0.995. This occurs because the sample mean is less than the hypothesized value of 280 but the alternative hypothesis is greater than 280.

c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|290 - 280|}{13.41} = 0.75$

From the OC curve, Chart VI g) for $\alpha = 0.05$, $d = 0.75$, $n = 100$, obtain $\beta \cong 0$ and power $\cong 1 - 0 = 1$

d) (Labeled incorrectly as part (e)) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|290 - 280|}{13.41} = 0.75$

From the OC curve, Chart VI g) for $\alpha = 0.05$, $d = 0.75$, $\beta \cong 0.20$ (power = 0.80), obtain $n = 15$

Section 9-4

9-43 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true standard deviation of the diameter, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) $H_0 : \sigma^2 = 0.0001$

3) $H_1 : \sigma^2 > 0.0001$

4) $\alpha = 0.01$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 > \chi^2_{\alpha,n-1}$ where $\chi^2_{0.01,14} = 29.14$

7) n = 15, s = 0.008

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.008)^2}{0.0001} = 8.96$$

8) Because 8.96 < 29.14 do not reject H_0 and conclude there is insufficient evidence to indicate the true standard deviation of the diameter exceeds 0.01 at $\alpha = 0.01$.

b) *P*-value = $P(\chi^2 > 8.96)$ for 14 degrees of freedom. That is, 0.5 < *P*-value < 0.9

c) $\lambda = \dfrac{\sigma}{\sigma_0} = \dfrac{0.0125}{0.01} = 1.25$

From Chart VI l) for β=0.2 (power = 0.8), $\lambda = 1.25$, obtain $n = 100$

9-44 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true variance of sugar content σ^2.

2) $H_0 : \sigma^2 = 18$

3) $H_1 : \sigma^2 \neq 18$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.975,9} = 2.70$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.025,9} = 19.02$

7) n = 10, s = 4.8

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(4.8)^2}{18} = 11.52$$

8) Because 11.52 < 19.02 do not reject H_0. There is not sufficient evidence to conclude that the true variance of sugar content differs from 18 at $\alpha = 0.01$.

b) From Table III the χ_0^2 is between 0.50 and 0.10. Therefore, 0.2 < *P*-value < 1

c)
$$\frac{9(4.8)^2}{19.02} \le \sigma^2 \le \frac{9(4.8)^2}{2.70}$$
$$10.90 \le \sigma^2 \le 76.80$$

The 95% confidence interval includes the value hypothesized of 18. Therefore, there is not sufficient evidence to conclude that the variance differs from 18.

9-45 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the standard deviation of tire life σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = 40{,}000$
3) $H_1: \sigma^2 > 40{,}000$
4) $\alpha = 0.05$
5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$ where $\chi_{0.05,15}^2 = 25.00$
7) $n = 16$, $s^2 = (3645.94)^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{15(3645.94)^2}{40000} = 4984.83$$

8) Because 25.00 < 4984.83 reject H_0. There is sufficient evidence to conclude that the true standard deviation of tire life exceeds 200 km at $\alpha = 0.05$.

b) P-value = $P(\chi^2 > 4984.83)$ for 15 degrees of freedom. From Table III, P-value < 0.005

9-46 a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true standard deviation of Izod impact strength, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) $H_0: \sigma^2 = (0.10)^2$
3) $H_1: \sigma^2 \neq (0.10)^2$
4) $\alpha = 0.01$
5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha/2,n-1}^2$ where $\chi_{0.995,19}^2 = 6.84$ or $\chi_0^2 > \chi_{\alpha,2,n-1}^2$ where $\chi_{0.005,19}^2 = 38.58$
7) $n = 20$, $s = 0.25$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.25)^2}{(0.10)^2} = 118.75$$

8) Because 118.75 > 38.58 reject H_0. There is sufficient evidence to conclude that the true standard deviation of Izod impact strength is significantly different from 0.10 at $\alpha = 0.01$.

b) From Table III, the P-value < 2*0.005 = 0.01. The multiplier of "2" is used because this is a two-sided test.

c) 99% confidence interval for σ:

For $\alpha = 0.01$ and n = 20, $\chi_{\alpha/2,n-1}^2 = \chi_{0.995,19}^2 = 6.84$ and $\chi_{1-\alpha/2,n-1}^2 = \chi_{0.005,19}^2 = 38.58$

$$\frac{19(0.25)^2}{38.58} \le \sigma^2 \le \frac{19(0.25)^2}{6.84}$$

$$0.03078 \le \sigma^2 \le 0.1736$$

Because 0.01 falls below the lower confidence bound there is sufficient evidence to conclude that the population standard deviation differs from 0.01 at significance level of 0.01.

9-47. a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the data are a random sample from a normal distribution.

1) The parameter of interest is the true standard deviation of titanium percentage σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = (0.25)^2$

3) H_1 : $\sigma^2 \neq (0.25)^2$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.975,50} = 32.36$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.025,50} = 71.42$

7) $n = 51$, $s = 0.37$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{50(0.37)^2}{(0.25)^2} = 109.52$$

8) Because 109.52 > 71.42 reject H_0. There is sufficient evidence to conclude that the true standard deviation of titanium percentage differs from 0.25 at $\alpha = 0.05$.

b) 95% confidence interval for σ:

For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{71.42} \le \sigma^2 \le \frac{50(0.37)^2}{32.36}$$

$$0.096 \le \sigma^2 \le 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

Because 0.25 falls below the lower confidence bound there is sufficient evidence to conclude that the population standard deviation is not equal to 0.25.

9-48 $\lambda = \dfrac{0.012}{0.008} = 1.5$

From the OC Chart VI l) for $\alpha = 0.01$, $n = 15$, obtain $\beta = 0.60$

9-49 $\lambda = \sqrt{\dfrac{40}{18}} = 1.49$

From the OC Chart VI i), for $\alpha = 0.05$, $\beta = 0.10$, obtain $n = 30$

Section 9-5

9-50 a) 1) The parameter of interest is the true proportion of engine crankshaft bearings exhibiting surface roughness.

2) $H_0 : p = 0.10$

3) $H_1 : p > 0.10$

4) $\alpha = 0.05$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach yields the same conclusion

6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.05} = 1.65$

7) $x = 10$ $n = 85$ $\hat{p} = \frac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{10 - 85(0.10)}{\sqrt{85(0.10)(0.90)}} = 0.54$$

8) Because $0.53 < 1.65$ do not reject the null hypothesis. There is not sufficient evidence to conclude that the true proportion of crankshaft bearings exhibiting surface roughness is greater than 0.10, at $\alpha = 0.05$.

9-51 $p = 0.15$, $p_0 = 0.10$, $n = 85$, and $z_{\alpha/2} = 1.96$

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

$$= \Phi\left(\frac{0.10 - 0.15 + 1.96\sqrt{0.10(1-0.10)/85}}{\sqrt{0.15(1-0.15)/85}}\right)$$

$$= \Phi(0.95) = 0.5378$$

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} - z_\beta\sqrt{p(1-p)}}{p - p_0}\right)^2$$

$$= \left(\frac{1.65\sqrt{0.10(1-0.10)} - 1.28\sqrt{0.15(1-0.15)}}{0.15 - 0.10}\right)^2$$

$$= (19.04)^2 = 362.56 \cong 363$$

9-52 a) Using the information from Exercise 8-48, test

1) The parameter of interest is the true fraction defective integrated circuits

2) $H_0 : p = 0.05$

3) $H_1 : p \neq 0.05$

4) $\alpha = 0.05$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach yields the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) $x = 13$ $n = 300$ $\hat{p} = \frac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{13 - 300(0.05)}{\sqrt{300(0.05)(0.95)}} = -0.53$$

8) Because −1.96 < −0.53 < 1.96 do not reject the null hypothesis. There is not sufficient evidence to conclude that the true fraction of defective integrated circuits differs from 0.05, at $\alpha = 0.05$.

b) The *P*-value is $2(1 - \Phi(0.53)) = 2(1 - 0.70194) = 0.5961$

9-53. a) Using the information from Exercise 8-48, test
1) The parameter of interest is the true fraction defective integrated circuits
2) $H_0 : p = 0.05$
3) $H_1 : p < 0.05$
4) $\alpha = 0.05$
5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.05} = -1.65$
7) $x = 13$ $n = 300$ $\hat{p} = \frac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{13 - 300(0.05)}{\sqrt{300(0.05)(0.95)}} = -0.53$$

8) Because −1.65 < −0.53, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true fraction of defective integrated circuits is less than 0.05 at $\alpha = 0.05$.

b) *P*-value = $1 - \Phi(0.53) = 0.29806$

9-54 a) 1) The parameter of interest is the true proportion of engineering students planning graduate studies
2) $H_0 : p = 0.50$
3) $H_1 : p \neq 0.50$
4) $\alpha = 0.05$
5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$
7) $x = 117$ $n = 484$ $\hat{p} = \frac{117}{484} = 0.2417$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{117 - 484(0.5)}{\sqrt{484(0.5)(0.5)}} = -11.36$$

8) Because −11.36 > −1.65, reject the null hypothesis. There is sufficient evidence to conclude that the true proportion of engineering students planning graduate studies differs from 0.5 at $\alpha = 0.05$.

b) *P*-value $= 2[1 - \Phi(11.36)] \cong 0$

c) $\hat{p} = \frac{117}{484} = 0.242$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.242 - 1.96\sqrt{\frac{0.242(0.758)}{484}} \le p \le 0.242 + 1.96\sqrt{\frac{0.242(0.758)}{484}}$$

$$0.204 \le p \le 0.280$$

Because the 95% confidence interval does not contain the value 0.5 there is sufficient evidence to conclude that the true fraction of engineering students planning graduate studies differs from 0.5, at $\alpha = 0.05$.

9-55. a) 1) The parameter of interest is the true percentage of polished lenses that contain surface defects p.
2) $H_0 : p = 0.02$
3) $H_1 : p < 0.02$
4) $\alpha = 0.05$
5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.05} = -1.65$
7) $x = 6$ $n = 250$ $\hat{p} = \frac{6}{250} = 0.024$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.024 - 0.02}{\sqrt{\frac{0.02(1-0.02)}{250}}} = 0.452$$

8) Because $-1.65 < 0.452$ do not reject the null hypothesis. There is not sufficient evidence to conclude that the true fraction of lens with surface defects is less than 0.02 at $\alpha = 0.05$.

b) P-value = $\Phi(0.452) = 0.674$

9-56. a) 1) The parameter of interest is the true percentage of football helmets that contain flaws p
2) $H_0 : p = 0.1$
3) $H_1 : p > 0.1$
4) $\alpha = 0.01$
5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach yields the same conclusion
6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.01} = 2.33$
7) $x = 16$ $n = 200$ $\hat{p} = \frac{16}{200} = 0.08$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.08 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{200}}} = -0.94$$

8) Because $-0.94 < 2.33$ do not reject the null hypothesis. There is not sufficient evidence to conclude that the proportion of football helmets with flaws exceeds 10% at $\alpha = 0.01$.

b) P-value = $\Phi(0.94) = 0.826$

9-57. The problem statement implies that H_0: $p = 0.6$, H_1: $p > 0.6$ and defines an acceptance region as $\hat{p} \le \frac{315}{500} = 0.63$ and a rejection region as $\hat{p} > 0.63$

a) The probability of a type 1 error is

$$\alpha = P(\hat{p} \ge 0.63 \mid p = 0.6) = P\left(Z \ge \frac{0.63 - 0.6}{\sqrt{\frac{0.6(0.4)}{500}}} \right) = P(Z \ge 1.37) = 1 - P(Z < 1.37) = 0.08535$$

b) $\beta = P(\hat{P} \le 0.63 \mid p = 0.75) = P(Z \le -6.196) = 0$

9-58 1) The parameter of interest is the true proportion of batteries that fail before 48 hours p.

2) $H_0 : p = 0.002$

3) $H_1 : p < 0.002$

4) $\alpha = 0.01$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach yields the same conclusion

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_\alpha = z_{0.01} = 2.33$

7) $x = 15$ $n = 5000$ $\hat{p} = \frac{15}{5000} = 0.003$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.003 - 0.002}{\sqrt{\frac{0.002(0.998)}{5000}}} = 1.58$$

8) Because -2.33 < 1.58 do not reject the null hypothesis. There is not sufficient evidence to conclude that the proportion of proportion of cell phone batteries that fail is less than 0.2% at α=0.01.

Section 9-7

9-59. Expected Frequency is obtained from the Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ where } \lambda = [0(24) + 1(30) + 2(31) + 3(11) + 4(4)]/100 = 1.41$$

Value	0	1	2	3	4
Observed Frequency	24	30	31	11	4
Expected Frequency	30.12	36.14	21.69	8.67	2.60

Because value 4 has an expected frequency less than 3, combine this category with the previous category:

Value	0	1	2	3-4
Observed Frequency	24	30	31	15
Expected Frequency	30.12	36.14	21.69	11.27

The degrees of freedom are $k - p - 1 = 4 - 0 - 1 = 3$

a) 1) The variable of interest is the form of the distribution for *X*.
2) H_0: The form of the distribution is Poisson with $\lambda = 1.2$
3) H_1: Not H_0
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$

7) $$\chi_0^2 = \frac{(24-30.12)^2}{30.12} + \frac{(30-36.14)^2}{36.14} + \frac{(31-21.69)^2}{21.69} + \frac{(15-11.27)^2}{11.27} = 7.52$$

8) Because 7.52 < 7.81 do not reject H_0. There is not sufficient evidence to conclude that the distribution differs from a Poisson distribution at $\alpha = 0.05$.

b) From Table III, the *P*-value is between 0.05 and 0.1. The *P*-value = 0.057 from Minitab software.

9-60. Expected Frequency is obtained from the Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ where } \lambda = [1(1) + 2(11) + \ldots + 7(10) + 8(9)]/75 = 4.907$$

Estimated mean = 4.907

Value	1	2	3	4	5	6	7	8
Observed Frequency	1	11	8	13	11	12	10	9
Expected Frequency	2.7214	6.6770	10.9213	13.3977	13.1485	10.7533	7.5381	4.6237

Because the first category has an expected frequency less than 3, combine it with the next category:

Value	1-2	3	4	5	6	7	8
Observed Frequency	12	8	13	11	12	10	9
Expected Frequency	9.3984	10.9213	13.3977	13.1485	10.7533	7.5381	4.6237

The degrees of freedom are $k - p - 1 = 7 - 1 - 1 = 5$

a) 1) The variable of interest is the form of the distribution for the number of flaws.

2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.01$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.01,5}^2 = 15.09$
7)

$$\chi_0^2 = \frac{(12 - 9.3984)^2}{9.3984} + \cdots + \frac{(9 - 4.6237)^2}{4.6237} = 6.955$$

8) Because 6.955 < 15.09 do not reject H_0. There is not sufficient evidence to conclude that the distribution differs from a Poisson distribution at $\alpha = 0.05$.

b) *P*-value = 0.224 from Minitab software.

9-61. Estimated mean = 10.131

Value	5	6	8	9	10	11	12	13	14	15
Rel. Freq	0.067	0.067	0.100	0.133	0.200	0.133	0.133	0.067	0.033	0.067
Observed (Days)	2	2	3	4	6	4	4	2	1	2
Expected (Days)	1.0626	1.7942	3.2884	3.7016	3.7501	3.4538	2.9159	2.2724	1.6444	1.1106

Because there are several cells with expected frequencies less than 3, the revised table would be:

Value	5-8	9	10	11	12-15
Observed (Days)	7	4	6	4	9
Expected (Days)	6.1452	3.7016	3.7501	3.4538	7.9433

The degrees of freedom are $k - p - 1 = 5 - 1 - 1 = 3$

a) 1) The variable of interest is the form of the distribution for the number of calls arriving to a switchboard from noon to 1 pm during business days.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$
7)

$$\chi_0^2 = \frac{(7 - 6.1452)^2}{6.1452} + \frac{(4 - 3.7016)^2}{3.7016} + \frac{(6 - 3.7501)^2}{3.7501} + \frac{(4 - 3.4538)^2}{3.4538} + \frac{(9 - 7.9433)^2}{7.9433} = 1.72$$

8) Because 1.72 < 7.81 do not reject H_0. There is not sufficient evidence to conclude that the distribution differs from a Poisson distribution at $\alpha = 0.05$.

b) From Table III the *P*-value is between 0.9 and 0.5. The *P*-value = 0.6325 from Minitab software.

9-62 Use the binomial distribution to get the expected frequencies with the mean = np = 6(0.25) = 1.5

Value	0	1	2	3	4
Observed	4	21	10	13	2
Expected	8.8989	17.7979	14.8315	6.5918	1.6479

The expected frequency for value 4 is less than 3. Combine this cell with value 3:

Value	0	1	2	3-4
Observed	4	21	10	15
Expected	8.8989	17.7979	14.8315	8.2397

The degrees of freedom are $k - p - 1 = 4 - 0 - 1 = 3$

a) 1) The variable of interest is the form of the distribution for the random variable X.
2) H_0: The form of the distribution is binomial with $n = 6$ and $p = 0.25$
3) H_1: The form of the distribution is not binomial with $n = 6$ and $p = 0.25$
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$
7)

$$\chi_0^2 = \frac{(4-8.8989)^2}{8.8989} + \cdots + \frac{(15-8.2397)^2}{8.2397} = 10.39$$

8) Because 7.81 < 10.39 reject H_0. There is sufficient evidence to conclude that the distribution is not binomial with n = 6 and p = 0.25 at $\alpha = 0.05$.

b) *P*-value = 0.0155 from Minitab software.

9-63 The value of *p* must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(39)+1(23)+2(12)+3(1)}{75} = 0.6667$$

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{0.6667}{24} = 0.02778$$

From the binomial distribution with $n = 24$ and $p = 0.02778$, the probability of a carton with no underfilled bottles is $(1 - 0.02778)^{24} = 0.5086$.

The expected number of cartons with no underfilled bottles is 75(0.5086) = 38.1426.

Value	0	1	2	3
Observed	39	23	12	1
Expected	38.1426	26.1571	8.5952	1.8010

Because value 3 has an expected frequency less than 3, combine this cell with that of value 2:

Value	0	1	2-3
Observed	39	23	13
Expected	38.1426	26.1571	10.3962

The degrees of freedom are $k - p - 1 = 3 - 1 - 1 = 1$

a) 1) The variable of interest is the form of the distribution for the number of underfilled bottles, *X*.
2) H_0: The form of the distribution is binomial

3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,1}^2 = 3.84$

7) $$\chi_0^2 = \frac{(39 - 38.1426)^2}{38.1426} + \frac{(23 - 26.1571)^2}{26.1571} + \frac{(13 - 10.3962)^2}{10.39} = 1.053$$

8) Because 1.053 < 3.84 do not reject H_0. There is not sufficient evidence to conclude that the distribution of the number of underfilled cartons differs from a binomial at $\alpha = 0.05$.

b) From Table III the *P*-value is between 0.1 and 0.5. The *P*-value = 0.3048 from Minitab software.

9-64 a) 1) The variable of interest is the form of the distribution for the number of cars passing through the intersection.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,24}^2 = 36.42$

7) Estimated mean = 49.6741. Use a Poisson distribution with $\lambda = 49.674$.

$\chi_0^2 = 769.57$

The degrees of freedom are $k - p - 1 = 26 - 1 - 1 = 24$

8) Because 36.42 << 769.57, reject H_0. Conclude that the distribution is not Poisson at $\alpha = 0.05$.

b) *P*-value ≈ 0 (found using Minitab)

Section 9-8

9-65. 1. The variable of interest is breakdowns among shifts.
2. H_0: Breakdowns are independent of shift.
3. H_1: Breakdowns are not independent of shift.
4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.05,6} = 12.592$
7. The calculated test statistic is $\chi^2_0 = 11.65$
8. Because 11.65 < 12.592, do not reject H_0. Conclude that the data provide insufficient evidence to claim that machine breakdowns and shift are dependent at $\alpha = 0.05$.

P-value = 0.070 from Minitab software.

9-66 1. The variable of interest is calls by surgical-medical patients.
2. H_0:Calls by surgical-medical patients are independent of Medicare status.
3. H_1:Calls by surgical-medical patients are not independent of Medicare status.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.01,1} = 6.637$
7. The calculated test statistic is $\chi^2_0 = 0.033$
8. Because 0.033 < 6.637, do not reject H_0. The evidence is not sufficient to claim that surgical-medical patients and Medicare status are dependent at $\alpha = 0.01$.

P-value = 0.85 from Minitab software.

9-67 1. The variable of interest is statistics grades and OR grades.
2. H_0: Statistics grades are independent of OR grades.
3. H_1: Statistics and OR grades are not independent.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{.01,9} = 21.665$
7. The calculated critical value is $\chi^2_0 = 25.55$
8. Because 21.665 < 25.55, reject H_0. Conclude that the grades are not independent at $\alpha = 0.01$.

P-value = 0.002 from Minitab software.

9-68 1. The variable of interest is characteristic among deflections and ranges.
2. H_0: Deflection and range are independent.
3. H_1: Deflection and range are not independent.

4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{0.05,4} = 9.488$
7. The calculated test statistic is $\chi^2_0 = 2.46$
8. Because 2.46 < 9.488, do not reject H_0. There is not sufficient evidence to conclude that deflection and range are dependent at $\alpha = 0.05$.

P-value = 0.652 from Minitab software.

9-69 1. The variable of interest is failures of an electronic component.
2. H_0: Type of failure is independent of mounting position.
3. H_1: Type of failure is not independent of mounting position.
4. $\alpha = 0.01$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{0.01,3} = 11.344$
7. The calculated test statistic is $\chi^2_0 = 10.71$
8. Because 10.71 < 11.344, do not reject H_0. There is not sufficient evidence to conclude that the type of failure depends on the mounting position at $\alpha = 0.01$.

P-value = 0.013 from Minitab software.

9-70 1. The variable of interest is opinion on core curriculum change.
2. H_0: Opinion of the change is independent of the class standing.
3. H_1: Opinion of the change is not independent of the class standing.
4. $\alpha = 0.05$
5. The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

6. The critical value is $\chi^2_{0.05,3} = 7.815$
7. The calculated test statistic is $\chi^2_{.0} = 26.97$.
8. Because 7.815 < 26.97, reject H_0. There is sufficient evidence to conclude that opinions on the change are not independent of class standing at $\alpha = 0.05$.

P-value ≈ 0

Supplemental Exercises

9-71 Sample Mean = $\hat{p}$ Sample Variance = $\frac{\hat{p}(1-\hat{p})}{n}$

	Sample Size, n	Sampling Distribution	Sample Mean	Sample Variance
a.	50	Normal	p	$\frac{p(1-p)}{50}$
b.	80	Normal	p	$\frac{p(1-p)}{80}$
c.	100	Normal	p	$\frac{p(1-p)}{100}$

d) As the sample size increases, the variance of the sampling distribution decreases.

9-72

	n	Test statistic	P-value	conclusion
a.	50	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/50}} = -0.12$	0.4522	Do not reject H_0
b.	100	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/100}} = -0.17$	0.4325	Do not reject H_0
c.	500	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/500}} = -0.37$	0.3557	Do not reject H_0
d.	1000	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/1000}} = -0.53$	0.2981	Do not reject H_0

e. The *P*-value decreases as the sample size increases.

9-73. $\sigma = 12$, $\delta = 205 - 200 = 5$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$,

a) $n = 20$: $\beta = \Phi\left(1.96 - \frac{5\sqrt{20}}{12}\right) = \Phi(0.163) = 0.564$

b) $n = 50$: $\beta = \Phi\left(1.96 - \frac{5\sqrt{50}}{12}\right) = \Phi(-0.986) = 1 - \Phi(0.986) = 1 - 0.839 = 0.161$

c) $n = 100$: $\beta = \Phi\left(1.96 - \frac{5\sqrt{100}}{12}\right) = \Phi(-2.207) = 1 - \Phi(2.207) = 1 - 0.986 = 0.014$

d) The probability of a Type II error, β, decreases as the sample size increases because the variance of the sample mean decreases. Consequently, the probability of observing a sample mean in the acceptance region when the true mean is 205 ml/h decreases with larger *n*.

9-74 $\sigma = 14$, $\delta = 205 - 200 = 5$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$,

a) n = 20: $\beta = \Phi\left(1.96 - \frac{5\sqrt{20}}{14}\right) = \Phi(0.362) = 0.641$

b) n = 50: $\beta = \Phi\left(1.96 - \frac{5\sqrt{50}}{14}\right) = \Phi(-0.565) = 1 - \Phi(0.565) = 1 - 0.716 = 0.284$

c) n = 100: $\beta = \Phi\left(1.96 - \frac{5\sqrt{100}}{14}\right) = \Phi(-1.611) = 1 - \Phi(1.611) = 1 - 0.946 = 0.054$

d) The probability of a Type II error increases with an increase in the standard deviation.

9-75 $\sigma = 8$, $\delta = 200 - 204 = -4$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$.

a) n = 20: $\beta = \Phi\left(1.65 - \frac{4\sqrt{20}}{8}\right) = \Phi(-0.59) = 1 - \Phi(0.59) = 1 - 0.722 = 0.278$

Therefore, power = $1 - \beta = 0.722$

b) n = 50: $\beta = \Phi\left(1.65 - \frac{4\sqrt{50}}{8}\right) = \Phi(-2.89) = 1 - \Phi(2.89) = 1 - 0.971 = 0.029$

Therefore, power = $1 - \beta = 0.971$

c) n = 100: $\beta = \Phi\left(1.65 - \frac{4\sqrt{100}}{8}\right) = \Phi(-3.35) = 1 - \Phi(3.35) = 1 - 0.9996 = 0.0004$

Therefore, power = $1 - \beta = 0.9996$

d) As sample size increases, and all other values are held constant, the power increases because the variance of the sample mean decreases. Consequently, the probability of observing a sample mean in the acceptance region when the true mean is 204 decreases with larger *n*.

9-76 $\alpha = 0.01$

a) n = 25 $\beta = \Phi\left(z_{0.01} + \frac{85 - 86}{16/\sqrt{25}}\right) = \Phi(2.33 - 0.31) = \Phi(2.02) = 0.9783$

n = 100 $\beta = \Phi\left(z_{0.01} + \frac{85 - 86}{16/\sqrt{100}}\right) = \Phi(1.71) = 0.9564$

n = 400 $\beta = \Phi\left(z_{0.01} + \frac{85 - 86}{16/\sqrt{400}}\right) = \Phi(2.33 - 1.25) = \Phi(1.08) = 0.8599$

n = 2500 $\beta = \Phi\left(z_{0.01} + \frac{85 - 86}{16/\sqrt{2500}}\right) = \Phi(2.33 - 3.13) = \Phi(-0.80) = 0.2119$

b) n = 25 $z_0 = \frac{86 - 85}{16/\sqrt{25}} = 0.31$ *P*-value: $1 - \Phi(0.31) = 1 - 0.6217 = 0.3783$

n = 100 $z_0 = \frac{86 - 85}{16/\sqrt{100}} = 0.63$ *P*-value: $1 - \Phi(0.63) = 1 - 0.7357 = 0.2643$

$n = 400$ $z_0 = \dfrac{86-85}{16/\sqrt{400}} = 1.25$ *P*-value: $1-\Phi(1.25) = 1-0.8944 = 0.1056$

$n = 2500$ $z_0 = \dfrac{86-85}{16/\sqrt{2500}} = 3.13$ *P*-value: $1-\Phi(3.13) = 1-0.9991 = 0.0009$

The data would be statistically significant when n = 2500 at α=0.01

9-77 a) Rejecting a null hypothesis provides a *stronger conclusion* than failing to reject a null hypothesis. Therefore, the alternative hypothesis should state the claim that is to be proved.

Assume that the data are a random sample from a normal distribution.

b) 1) the parameter of interest is the mean weld strength μ.
2) $H_0 : \mu = 150$
3) $H_1 : \mu > 150$
4) Not given
5) The test statistic is:

$$t_0 = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$

6) Because no critical value is given, calculate the *P*-value

7) $\bar{x} = 153.7$, s= 11.3, n=20

$$t_0 = \frac{153.7-150}{11.3\sqrt{20}} = 1.46$$

P-value = P(t > 1.46) and 0.05 < *P*-value < 0.10

8) There is some modest evidence to support the claim that the weld strength exceeds 150 psi. If α = 0.01 or 0.05, we would not reject the null hypothesis, thus the claim would not be supported. If we used α = 0.10, we would reject the null in favor of the alternative and conclude that there is sufficient evidence that the weld strength exceeds 150 psi at α = 0.10.

9-78 a) α = 0.05

$n = 100$ $$\beta = \Phi\left(\frac{0.5-0.6+z_{0.05}\sqrt{0.5(0.5)/100}}{\sqrt{0.6(0.4)/100}}\right) = \Phi(-0.36) = 0.3594$$

$$Power = 1-\beta = 1-0.3594 = 0.6406$$

$n = 150$ $$\beta = \Phi\left(\frac{0.5-0.6+z_{0.05}\sqrt{0.5(0.5)/150}}{\sqrt{0.6(0.4)/150}}\right) = \Phi(-0.82) = 0.2061$$

$$Power = 1-\beta = 1-0.206 = 0.7881$$

$n = 300$ $$\beta = \Phi\left(\frac{0.5-0.6+z_{0.05}\sqrt{0.5(0.5)/300}}{\sqrt{0.6(0.4)/300}}\right) = \Phi(-1.85) = 0.0322$$

$$Power = 1-\beta = 1-0.0322 = 0.9678$$

The power increases as the sample size increases.

b) α = 0.01

$n = 100$ $\beta = \Phi\left(\dfrac{0.5 - 0.6 + z_{0.01}\sqrt{0.5(0.5)/100}}{\sqrt{0.6(0.4)/100}}\right) = \Phi(0.34) = 0.6331$

$Power = 1 - \beta = 1 - 0.6331 = 0.3669$

$n = 150$ $\beta = \Phi\left(\dfrac{0.5 - 0.6 + z_{0.01}\sqrt{0.5(0.5)/150}}{\sqrt{0.6(0.4)/150}}\right) = \Phi(-0.12) = 0.4522$

$Power = 1 - \beta = 1 - 0.4522 = 0.5478$

$n = 300$ $\beta = \Phi\left(\dfrac{0.5 - 0.6 + z_{0.01}\sqrt{0.5(0.5)/300}}{\sqrt{0.6(0.4)/300}}\right) = \Phi(-1.16) = 0.1230$

$Power = 1 - \beta = 1 - 0.1230 = 0.8770$

Decreasing the value of α lowers the power of the test for the same sample size.

c) $\alpha = 0.05$

$n = 100$ $\beta = \Phi\left(\dfrac{0.5 - 0.8 + z_{0.01}\sqrt{0.5(0.5)/100}}{\sqrt{0.8(0.2)/100}}\right) = \Phi(-5.44) \cong 0.0$

$Power = 1 - \beta = 1 - 0 \cong 1$

The true value of p has a large effect on the power. The further p is away from p_0 the larger the power of the test.

d)

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p(1-p)}}{p - p_0}\right)^2$$

$$= \left(\frac{2.58\sqrt{0.5(1-0.50)} + 1.65\sqrt{0.6(1-0.6)}}{0.6 - 0.5}\right)^2 = (4.82)^2 = 23.2 \cong 24$$

$$n = \left(\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p(1-p)}}{p - p_0}\right)^2$$

$$= \left(\frac{2.58\sqrt{0.5(1-0.50)} + 1.65\sqrt{0.8(1-0.8)}}{0.8 - 0.5}\right)^2 = (6.5)^2 = 42.25 \cong 43$$

The true value of p has a large effect on the power. The further p is away from p_0 the smaller the sample size that is required to achieve a specified power.

9-79 a) 1) The parameter of interest is the standard deviation σ

2) $H_0 : \sigma^2 = 400$

3) $H_1 : \sigma^2 < 400$

4) Not given

5) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Because no critical value is given, we will calculate the *P*-value

7) $n = 10$, $s = 15.7$

$$\chi_0^2 = \frac{9(15.7)^2}{400} = 5.546$$

P-value = $P(\chi^2 < 5.546)$; From Table III, $0.1 < P\text{-value} < 0.5$.

8) The *P*-value is greater than a reasonable significance level α. Therefore we do not reject the null hypothesis. There is not sufficient evidence to support the claim that the standard deviation is less than 20 microamps.

b) 7) $n = 51$, $s = 20$

$$\chi_0^2 = \frac{50(15.7)^2}{400} = 30.81$$

P-value = $P(\chi^2 < 30.81)$; From Table III, $0.01 < P\text{-value} < 0.025$

8) The *P*-value is less than 0.05. Therefore, reject the null hypothesis. There is sufficient evidence to conclude that the standard deviation is less than 20 microamps at $\alpha = 0.05$.

c) Increasing the sample size increases the test statistic χ_0^2 and therefore decreases the *P*-value, providing more evidence against the null hypothesis.

9-80 a) 1) The parameter of interest is the variance of fatty acid measurements σ^2
2) $H_0 : \sigma^2 = 1.0$
3) $H_1 : \sigma^2 \neq 1.0$
4) $\alpha = 0.01$
5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) $\chi^2_{0.995,5} = 0.41$ and $\chi^2_{0.005,5} = 16.75$, reject H_0 if $\chi_0^2 < 0.41$ or if $\chi_0^2 > 16.75$

7) $n = 6$, $s = 0.319$

$$\chi_0^2 = \frac{5(0.319)^2}{1^2} = 0.509$$

P-value = $P(\chi^2 < 0.509)$; From Table III, $0.01 < P\text{-value} < 0.02$

8) Because $0.41 < 0.509$, do not reject the null hypothesis. There is not sufficient evidence to conclude that the variance differs from 1.0 at $\alpha = 0.01$. Alternatively, because the *P*-value is greater 0.01we do not reject the null hypothesis.

b) 1) The parameter of interest is the variance of fatty acid measurements σ^2 (now $n = 51$)
2) $H_0 : \sigma^2 = 1.0$
3) $H_1 : \sigma^2 \neq 1.0$
4) α=0.01
5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) $\chi^2_{0.995,50} = 27.99$ and $\chi^2_{0.005,5} = 79.49$, reject H_0 if $\chi_0^2 < 27.99$ or if $\chi_0^2 > 79.49$

7) $n = 51$, $s = 0.319$

$$\chi_0^2 = \frac{50(0.319)^2}{1^2} = 5.09$$

P-value = $P(\chi^2 < 5.09)$; From Table III, *P*-value < 0.01

8) Because 5.09 < 27.99, reject the null hypothesis. There is not sufficient evidence to conclude that the variance differs from 1.0. Alternatively, the *P*-value is smaller than a typical significance level, α. Therefore we do reject the null hypothesis.

c) The sample size changes the conclusion. With a small sample size, the null hypothesis is not rejected at $\alpha = 0.01$. A larger sample size provides strong evidence that the variance differs from 1.

9-81. Assume the data are a random sample from a normal distribution.

a) 1) The parameter of interest is the standard deviation σ.

2) $H_0 : \sigma^2 = (0.00002)^2$

3) $H_1 : \sigma^2 < (0.00002)^2$

4) $\alpha = 0.01$

5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) $\chi^2_{0.99,7} = 1.24$ reject H_0 if $\chi_0^2 < 1.24$

7) $s = 0.00001$

$$\chi_0^2 = \frac{7(0.00001)^2}{(0.00002)^2} = 1.75$$

Because 1.24 < 1.75, do not reject the null hypothesis. There is not sufficient evidence to conclude that the standard deviation is less than 0.00002 mm.

b) Although the sample standard deviation is less than the hypothesized value of 0.00002, it is *not significantly less* (when $\alpha = 0.01$) than 0.00002. The true standard deviation could be larger than 0.00002 yet the value of 0.00001 could have occurred as a result of sampling variation.

9-82 Assume the data are a random sample from a normal distribution..

1) The parameter of interest is the standard deviation of the concentration, σ.

2) $H_0 : \sigma^2 = 4^2$

3) $H_1 : \sigma^2 < 4^2$

4) not given

5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) based on the *P*-value

7) $s = 0.004$ and $n = 10$

$$\chi_0^2 = \frac{9(0.004)^2}{(4)^2} = 0.000009$$

P-value = $P(\chi^2 < 0.00009)$; From Table III, P-value ≈ 0.

The *P*-value is approximately 0, therefore we reject the null hypothesis. There is sufficient evidence to conclude that the standard deviation of the concentration is less than 4 grams per liter at reasonable significance levels.

9-83. Create a table for the number of nonconforming coil springs (value) and the observed number of times the number appeared. One possible table is:

Value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

Obs	0	0	0	1	4	3	4	6	4	3	0	3	3	2	1	1	0	2	1	2

The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(0) + 1(0) + 2(0) + \cdots + 19(2)}{40} = 9.325$$

$$\hat{p}_{sample} = \frac{sample \quad mean}{n} = \frac{9.325}{50} = 0.1865$$

The probability of each value is calculated from the binomial distribution with $n = 50$, $p = 0.1865$. The expected value is the probability of a value times 40.

Value	Observed	Expected
0	0	0.00132
1	0	0.01511
2	0	0.08486
3	1	0.31128
4	4	0.83853
5	3	1.76859
6	4	3.04094
7	6	4.38212
8	4	5.39988
9	3	5.77713
10	0	5.43022
11	3	4.52692
12	3	3.37296
13	2	2.26033
14	1	1.36952
15	1	0.75353
16	0	0.37789
17	2	0.17327
18	1	0.07283
19	2	0.02812

Because several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	Observed	Expected
0-5	8	3.01969
6	4	3.04094
7	6	4.38212
8	4	5.39988
9	3	5.77713
10	0	5.43022
11	3	4.52695
12	3	3.37296
≥13	9	5.03549

The degrees of freedom are $k - p - 1 = 9 - 1 - 1 = 7$

a) 1) The variable of interest is the form of the distribution for the number of nonconforming coil springs.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_0^2 > \chi_{0.05,7}^2 = 14.07$

7)

$$\chi_0^2 = \frac{(8-3.01969)^2}{3.01969} + \frac{(4-3.04094)^2}{3.04094} + \cdots + \frac{(9-5.03549)^2}{5.03549} = 19.919$$

8) Because 19.919 > 14.07 reject H_0. There is sufficient evidence to conclude that the distribution of nonconforming springs is not binomial at $\alpha = 0.05$.

b) *P*-value = 0.0057 from Minitab software

9-84 Create a table for the number of errors in a string of 1000 bits (value) and the observed number of times the number appeared. One possible table is:

Value	0	1	2	3	4	5
Obs	3	7	4	5	1	0

The value of *p* must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(3)+1(7)+2(4)+3(5)+4(1)+5(0)}{20} = 1.7$$

$$\hat{p}_{sample} = \frac{sample\ \ mean}{n} = \frac{1.7}{1000} = 0.0017$$

Value	0	1	2	3	4	5
Observed	3	7	4	5	1	0
Expected	3.64839	6.21282	5.28460	2.99371	1.27067	0.43103

Because several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	0	1	2	≥3
Observed	3	7	4	6
Expected	3.64839	6.21282	5.28460	4.69541

The degrees of freedom are $k - p - 1 = 4 - 1 - 1 = 2$

a) 1) The variable of interest is the form of the distribution for the number of errors in a string of 1000 bits.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_0^2 > \chi_{0.05,2}^2 = 5.99$

7)

$$\chi_0^2 = \frac{(3-3.64839)^2}{3.64839} + \cdots + \frac{(6-4.69541)^2}{4.69541} = 0.88971$$

8) Because 0.88971 < 5.99, do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of errors is binomial at $\alpha = 0.05$.

b) *P*-value = 0.6409 from Minitab software

9-85 Divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0, 0.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is p = 1/8 = 0.125. Therefore, the expected cell frequencies are E = *np* = (100) (0.125) = 12.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
x ≤ 5332.5	1	12.5
5332.5< x ≤ 5357.5	3	12.5
5357.5< x ≤ 5382.5	7	12.5
5382.5< x ≤ 5407.5	23	12.5
5407.5< x ≤ 5432.5	30	12.5
5432.5< x ≤ 5457.5	20	12.5
5457.5< x ≤ 5482.5	14	12.5
x ≥ 5482.5	2	12.5

The test statistic is:

$$\chi_0^2 = \frac{(1 - 12.5)^2}{12.5} + \frac{(3 - 12.5)^2}{12.5} + \cdots + \frac{(14 - 12.5)^2}{12.5} + \frac{(2 - 12.5)^2}{12.5} = 67.04$$

and reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Because $\chi_o^2 > \chi^2_{0.05,5}$, reject the hypothesis that the data are normally distributed

9-86 a) Assume the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean concentration of suspended solids μ.

2) $H_0 : \mu = 50$

3) $H_1 : \mu < 50$

4) α = 0.05

5) Because n>>30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 59.87$ $s = 12.50$ $n = 60$

$$z_0 = \frac{59.87 - 50}{12.50/\sqrt{60}} = 6.12$$

8) Because –1.65 < 6.12, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean concentration of suspended solids is less than 50 ppm at α = 0.05.

b) The *P*-value = $\Phi(6.12) \cong 1$.

c) We divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0, 0.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is *p* = 1/8 = 0.125 so the expected cell frequencies are E = *np* = (60) (0.125) = 7.5. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
x ≤ 45.50	9	7.5
45.50< x ≤ 51.43	5	7.5
51.43< x ≤ 55.87	7	7.5
55.87< x ≤ 59.87	11	7.5
59.87< x ≤ 63.87	4	7.5
63.87< x ≤ 68.31	9	7.5
68.31< x ≤ 74.24	8	7.5
x ≥ 74.24	7	7.5

The test statistic is:

$$\chi^2_o = \frac{(9-7.5)^2}{7.5} + \frac{(5-7.5)^2}{7.5} + \cdots + \frac{(8-7.5)^2}{7.5} + \frac{(7-7.5)^2}{7.5} = 4.8$$

and we reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Because 4.8 < 11.07, do not reject the hypothesis that the data are normally distributed.

9-87 a) Assume the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean overall distance for this brand of golf ball μ

2) $H_0 : \mu = 270$

3) $H_1 : \mu < 270$

4) $\alpha = 0.05$

5) Because n >> 30 we can use the normal distribution

$$z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 260.30$ $s = 13.41$ $n = 100$

$$z_0 = \frac{260.30 - 270.0}{13.41/\sqrt{100}} = -7.23$$

8) Because –7.23 < –1.65, reject the null hypothesis. There is sufficient evidence to conclude that the true mean distance is less than 270 yds at $\alpha = 0.05$.

b) The *P*-value ≅ 0

c) We divide the real line under a standard normal distribution into eight intervals with equal probability. These intervals are [0, 0.32), [0.32, 0.675), [0.675, 1.15), [1.15, ∞) and their negative counterparts. The probability for each interval is $p = 1/8 = 0.125$ so the expected cell frequencies are $E = np = (100)(0.125) = 12.5$. The table of ranges and their corresponding frequencies is completed as follows.

Interval	Obs. Frequency.	Exp. Frequency.
x ≤ 244.88	16	12.5
244.88< x ≤ 251.25	6	12.5
251.25< x ≤ 256.01	17	12.5
256.01< x ≤ 260.30	9	12.5
260.30< x ≤ 264.59	13	12.5
264.59< x ≤ 269.35	8	12.5
269.35< x ≤ 275.72	19	12.5
x ≥ 275.72	12	12.5

The test statistic is:

$$\chi^2_o = \frac{(16-12.5)^2}{12.5} + \frac{(6-12.5)^2}{12.5} + \cdots + \frac{(19-12.5)^2}{12.5} + \frac{(12-12.5)^2}{12.5} = 12$$

and we would reject if this value exceeds $\chi^2_{0.05,5} = 11.07$. Because 11.07 < 12, reject the hypothesis that the data are normally distributed.

9-88 a) Assume the data are a random sample from a normal distribution.

1) The parameter of interest is the true mean coefficient of restitution μ.

2) $H_0 : \mu = 0.635$

3) $H_1 : \mu > 0.635$

4) $\alpha = 0.01$

5) Because $n > 30$ we can use the normal distribution

$$z_0 = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05}$ =2.33

7) $\overline{x}$ = 0.324 s = 0.0131 n = 40

$$z_0 = \frac{0.624 - 0.635}{0.0131/\sqrt{40}} = -5.31$$

8) Because –5.31< 2.33, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean coefficient of restitution is greater than 0.635 at α = 0.01.

For this exercise the alternative hypothesis is that the true mean is greater than 0.635 yet sample mean is less than the hypothesized value of 0.635. Therefore, it is known that the null hypothesis is not rejected before the *z* statistic is computed.

b) The P-value is $\Phi(5.31) \cong 1$

c) If the lower bound of the CI was above the value 0.635 then we could conclude that the mean coefficient of restitution was greater than 0.635.

9-89 a) Assume the data are a random sample from a normal distribution. Use the t-test to test the hypothesis that the true mean is 2.5 mg/L.
1) State the parameter of interest: The parameter of interest is the true mean dissolved oxygen level, μ.
2) State the null hypothesis $H_0 : \mu = 2.5$
3) State the alternative hypothesis $H_1 : \mu \neq 2.5$
4) Give the significance level $\alpha = 0.05$
5) Give the statistic

$$t_0 = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $|t_0| < t_{\alpha/2,n-1}$
7) Compute the sample statistic $\overline{x}$ = 3.265, *s* = 2.127, *n* = 20 and calculate the t-statistic

$$t_0 = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

8) Draw your conclusion and find the P-value.

b) Assume the data are a random sample from a normal distribution.
1) The parameter of interest is the true mean dissolved oxygen level, μ.
2) $H_0 : \mu = 2.5$
3) $H_1 : \mu \neq 2.5$
4) $\alpha = 0.05$
5) Test statistic

$$t_0 = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

6) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $t_{\alpha/2,n-1} = t_{0.025,19}$ =2.093
7) $\overline{x}$ = 3.265 *s* =2.127 *n* = 20

$$t_0 = \frac{3.265 - 2.5}{2.127/\sqrt{20}} = 1.608$$

8) Because 1.608 < 2.093, do not reject the null hypotheses. There is not sufficient evidence to conclude that the true mean differs from 2.5 mg/L.

c) From Table IV, 1.608 is between 0.05 and 0.1 for 19 degrees of freedom. Therefore, the *P*-value is between 2(0.05) = 0.1 and 2(0.1) = 0.2. The *P*-value = 0.124 from Minitab software.

d.) The confidence interval found in exercise 8-81 b) agrees with the hypothesis test above. The value of 2.5 is within the 95% confidence limits. The confidence interval is quite wide due to the large sample standard deviation.

$$\bar{x} - t_{0.025,19}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{0.025,19}\frac{s}{\sqrt{n}}$$

$$3.265 - 2.093\frac{2.127}{\sqrt{20}} \le \mu \le 3.265 + 2.093\frac{2.127}{\sqrt{20}}$$

$$2.270 \le \mu \le 4.260$$

9-90 a) Assume the data are a random sample from a normal distribution.

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|73 - 75|}{1} = 2$$

From the OC Chart g) for $\alpha = 0.05$, $d = 2$, n = 10, obtain $\beta \cong 0.0$ and power $\cong 1 - 0.0 = 1$

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72 - 75|}{1} = 3$$

From the OC Chart g) for $\alpha = 0.05$, $d = 3$, n = 10, obtain $\beta \cong 0.0$ and power $\cong 1 - 0.0 = 1$

b) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72 - 75|}{1} = 3$

From the OC Chart VI g) for $\alpha = 0.05$, $d = 3$, $\beta = 0.1$ (power = 0.9), obtain $n = 3$.

c) For $\sigma = 2$

Repeat of part (a)

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|73 - 75|}{2} = 1$$

From the OC Chart VI g) for $\alpha = 0.05$, $d = 1$, n = 10, obtain $\beta \cong 0.10$ and power $\cong 1 - 0.10 = 0.90$

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72 - 75|}{2} = 1.5$$

From the OC Chart VI g) for $\alpha = 0.05$, $d = 1.5$, n = 10, obtain $\beta \cong 0.0$ and power $\cong 1 - 0.0 = 1$

Repeat of part (b)

$$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|72 - 75|}{2} = 1.5$$

From the OC Chart VI g) for $\alpha = 0.05$, $d = 3$, $\beta \cong 0.1$ (power = 0.9), obtain $n = 7$

Increasing the standard deviation lowers the power of the test and increases the sample size required to obtain a certain power.

Mind Expanding Exercises

9-91 The parameter of interest is the true mean μ

H_0: $\mu = \mu_0$

H_1: $\mu \neq \mu_0$

a) Reject H_0 if $z_0 < -z_{\alpha-\varepsilon}$ or $z_0 > z_\varepsilon$

$$P(\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} < -\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} = \mid \mu=\mu_0) + P(\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} > \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} \mid \mu=\mu_0) \text{ P}$$

$$P(z_0 < -z_{\alpha-\varepsilon}) + P(z_0 > z_\varepsilon) = \Phi(-z_{\alpha-\varepsilon}) + 1 - \Phi(z_\varepsilon)$$

$$= ((\alpha-\varepsilon)) + (1-(1-\varepsilon)) = \alpha$$

b) $\beta = P(z_\varepsilon \le \overline{X} \le z_\varepsilon$ when $\mu_1 = \mu_0 + d$)

or $\beta = P(-z_{\alpha-\varepsilon} < Z_0 < z_\varepsilon | \mu_1 = \mu_0 + \delta)$

$$\beta = P(-z_{\alpha-\varepsilon} < \frac{\bar{x}-\mu_0}{\sqrt{\sigma^2/n}} < z_\varepsilon | \mu_1 = \mu_0 + \delta)$$

$$= P(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}} < Z < z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}})$$

$$= \Phi(z_\varepsilon - \frac{\delta}{\sqrt{\sigma^2/n}}) - \Phi(-z_{\alpha-\varepsilon} - \frac{\delta}{\sqrt{\sigma^2/n}})$$

9-92

$$\beta = P(-X^2_{1-\alpha/2,n-1} < X_0^2 < X^2_{\alpha/2,n-1} \mid \sigma^2 = \sigma^2{}_1)$$

$$= P\left(\sigma_1\sqrt{\frac{X^2_{1-\alpha/2,n-1}}{n-1}} < s < \sigma_1\sqrt{\frac{X^2_{\alpha/2,n-1}}{n-1}}\right)$$

]

9-93 1) The parameter of interest is the true mean number of open circuits λ

2) $H_0 : \lambda = 2$

3) $H_1 : \lambda > 2$

4) $\alpha = 0.05$

5) Because n > 30 we can use the normal distribution

$$z_0 = \frac{\overline{X}-\lambda}{\sqrt{\lambda/n}}$$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x}$= 1038/500=2.076 $n = 500$

$$z_0 = \frac{2.076-2}{2/\sqrt{500}} = 0.85$$

8) Because 0.85< 1.65, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true mean number of open circuits is greater than 2 at $\alpha = 0.01$

9-94 1) The parameter of interest is the true standard deviation of the golf ball distance σ

2) $H_0 : \sigma = 10$

3) $H_1 : \sigma < 10$

4) α=0.05

5) Exercise states that the normal distribution approximation should be used

$$z_0 = \frac{S-\sigma_0}{\sqrt{\sigma_0^2/(2n)}}$$

6) Reject H_0 if $z_0 < z_\alpha$ where $z_{0.05} = -1.65$
7) $s = 13.41$ n = 100

$$z_0 = \frac{13.41 - 10}{\sqrt{10^2/(200)}} = 4.82$$

8) Because 4.82 > -1.65, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true standard deviation is less than 10 at $\alpha = 0.05$

9-95 95% percentile $\theta = \mu + 1.645\sigma$

95% percentile estimator is $\hat{\theta} = \overline{X} + 1.645S$

$$S.E.(\hat{\theta}) = \sqrt{V\left(\overline{X} + 1.645S\right)} = \sqrt{V\left(\overline{X}\right) + 1.645^2 V(S)} = \sqrt{\frac{\sigma^2}{n} + 2.706\frac{\sigma^2}{2n}} = \sigma\sqrt{\frac{2.353}{n}}$$

The standard error of the estimator is estimated by $1.534s/\sqrt{n}$.

9-96 1) The parameter of interest is the true 95th percentile golf ball distance θ
2) $H_0 : \theta = 285$
3) $H_1 : \theta > 285$
4) $\alpha = 0.05$
5) Exercise states that a normal distribution approximation should be used and the results from the previous exercise are used to obtain the test statistic

$$z_0 = \frac{\hat{\theta} - \theta_0}{\sqrt{2.353\sigma^2 / n}}$$

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$
7) $\hat{\theta} = 282.36$ and $n = 100$

$$z_0 = \frac{282.36 - 285}{\sqrt{2.353(179.78)/100}} = -1.28$$

8) Because -1.28 < 1.65, do not reject the null hypothesis. There is not sufficient evidence to conclude that the true 95th percentile is greater than 285 at $\alpha = 0.05$

9-97 1) The parameter of interest is the parameter λ of an exponential distribution
2) $H_0 : \lambda = \lambda_0$
3) $H_1 : \lambda \neq \lambda_0$
4) $\alpha = 0.05$
5) Test statistic

$$\chi_0^2 = 2\lambda_0 \sum_{i=1}^{n} X_i$$

6) Reject H_0 if $\chi_0^2 > \chi^2_{a/2,2n}$ or $\chi_0^2 < \chi^2_{1-a/2,2n}$

7) Compute $2\lambda_0 \sum_{i=1}^{n} X_i$

8) Make conclusions

Alternative hypotheses
1) $H_0 : \lambda = \lambda_0$
$H_1 : \lambda > \lambda_0$ Reject H_0 if $\chi_0^2 > \chi^2_{a,2n}$
2) $H_0 : \lambda = \lambda_0$
$H_1 : \lambda < \lambda_0$ Reject H_0 if $\chi_0^2 < \chi^2_{a,2n}$