

CHAPTER 4

Section 4-2

4-1. a) $P(1 < X) = \int_1^{\infty} e^{-x} dx = (-e^{-x}) \Big|_1^{\infty} = e^{-1} = 0.3679$

b) $P(1 < X < 2.5) = \int_1^{2.5} e^{-x} dx = (-e^{-x}) \Big|_1^{2.5} = e^{-1} - e^{-2.5} = 0.2858$

c) $P(X = 3) = \int_3^3 e^{-x} dx = 0$

d) $P(X < 4) = \int_0^4 e^{-x} dx = (-e^{-x}) \Big|_0^4 = 1 - e^{-4} = 0.9817$

e) $P(3 \leq X) = \int_3^{\infty} e^{-x} dx = (-e^{-x}) \Big|_3^{\infty} = e^{-3} = 0.0498$

4-2. a) $P(x < X) = \int_x^{\infty} e^{-x} dx = (-e^{-x}) \Big|_x^{\infty} = e^{-x} = 0.10.$

Then, $x = -\ln(0.10) = 2.3$

b) $P(X \leq x) = \int_0^x e^{-x} dx = (-e^{-x}) \Big|_0^x = 1 - e^{-x} = 0.10.$

Then, $x = -\ln(0.9) = 0.1054$

4-3 a) $P(X < 4) = \int_3^4 \frac{x}{8} dx = \frac{x^2}{16} \Big|_3^4 = \frac{4^2 - 3^2}{16} = 0.4375$, because $f_X(x) = 0$ for $x < 3$.

b) $P(X > 3.5) = \int_{3.5}^5 \frac{x}{8} dx = \frac{x^2}{16} \Big|_{3.5}^5 = \frac{5^2 - 3.5^2}{16} = 0.7969$ because $f_X(x) = 0$ for $x > 5$.

c) $P(4 < X < 5) = \int_4^5 \frac{x}{8} dx = \frac{x^2}{16} \Big|_4^5 = \frac{5^2 - 4^2}{16} = 0.5625$

d) $P(X < 4.5) = \int_3^{4.5} \frac{x}{8} dx = \frac{x^2}{16} \Big|_3^{4.5} = \frac{4.5^2 - 3^2}{16} = 0.7031$

e) $P(X > 4.5) + P(X < 3.5) = \int_{4.5}^5 \frac{x}{8} dx + \int_3^{3.5} \frac{x}{8} dx = \frac{x^2}{16} \Big|_{4.5}^5 + \frac{x^2}{16} \Big|_3^{3.5} = \frac{5^2 - 4.5^2}{16} + \frac{3.5^2 - 3^2}{16} = 0.5.$

4-4 a) $P(1 < X) = \int_4^{\infty} e^{-(x-4)} dx = -e^{-(x-4)} \Big|_4^{\infty} = 1$, because $f_X(x) = 0$ for $x < 4$. This can also be

obtained from the fact that $f_X(x)$ is a probability density function for $4 < x$.

b) $P(2 \leq X \leq 5) = \int_4^5 e^{-(x-4)} dx = -e^{-(x-4)} \Big|_4^5 = 1 - e^{-1} = 0.6321$

c) $P(5 < X) = 1 - P(X \leq 5)$. From part b., $P(X \leq 5) = 0.6321$. Therefore, $P(5 < X) = 0.3679$.

d) $P(8 < X < 12) = \int_8^{12} e^{-(x-4)} dx = -e^{-(x-4)} \Big|_8^{12} = e^{-4} - e^{-8} = 0.0180$

e) $P(X < x) = \int_4^x e^{-(x-4)} dx = -e^{-(x-4)} \Big|_4^x = 1 - e^{-(x-4)} = 0.90$.

Then, $x = 4 - \ln(0.10) = 6.303$

4-5 a) $P(0 < X) = 0.5$, by symmetry.

b) $P(0.5 < X) = \int_{0.5}^1 1.5x^2 dx = 0.5x^3 \Big|_{0.5}^1 = 0.5 - 0.0625 = 0.4375$

c) $P(-0.5 \leq X \leq 0.5) = \int_{-0.5}^{0.5} 1.5x^2 dx = 0.5x^3 \Big|_{-0.5}^{0.5} = 0.125$

d) $P(X < -2) = 0$

e) $P(X < 0 \text{ or } X > -0.5) = 1$

f) $P(x < X) = \int_x^1 1.5x^2 dx = 0.5x^3 \Big|_x^1 = 0.5 - 0.5x^3 = 0.05$

Then, $x = 0.9655$

4-6. a) $P(X > 3000) = \int_{3000}^{\infty} \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_{3000}^{\infty} = e^{-3} = 0.05$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233$

c) $P(X < 1000) = \int_0^{1000} \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_0^{1000} = 1 - e^{-1} = 0.6321$

d) $P(X < x) = \int_0^x \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_0^x = 1 - e^{-x/1000} = 0.10$.

Then, $e^{-x/1000} = 0.9$, and $x = -1000 \ln 0.9 = 105.36$.

$$4-7 \quad a) P(X > 50) = \int_{50}^{50.25} 2.0 dx = 2x \Big|_{50}^{50.25} = 0.5$$

$$b) P(X > x) = 0.90 = \int_x^{50.25} 2.0 dx = 2x \Big|_x^{50.25} = 100.5 - 2x$$

Then, $2x = 99.6$ and $x = 49.8$.

$$4-8. \quad a) P(X < 74.8) = \int_{74.6}^{74.8} 1.25 dx = 1.25x \Big|_{74.6}^{74.8} = 0.25$$

b) $P(X < 74.8 \text{ or } X > 75.2) = P(X < 74.8) + P(X > 75.2)$ because the two events are mutually exclusive. The result is $0.25 + 0.25 = 0.50$.

$$c) P(74.7 < X < 75.3) = \int_{74.7}^{75.3} 1.25 dx = 1.25x \Big|_{74.7}^{75.3} = 1.25(0.6) = 0.750$$

4-9 a) $P(X < 2.25 \text{ or } X > 2.75) = P(X < 2.25) + P(X > 2.75)$ because the two events are mutually exclusive. Then, $P(X < 2.25) = 0$ and

$$P(X > 2.75) = \int_{2.75}^{2.8} 2 dx = 2(0.05) = 0.10.$$

b) If the probability density function is centered at 2.55 meters, then $f_X(x) = 2$ for $2.3 < x < 2.8$ and all rods will meet specifications.

4-10. Because the integral $\int_{x_1}^{x_2} f(x) dx$ is not changed whether or not any of the endpoints x_1 and x_2 are included in the integral, all the probabilities listed are equal.

Section 4-3

4-11. a) $P(X < 2.8) = P(X \leq 2.8)$ because X is a continuous random variable. Then, $P(X < 2.8) = F(2.8) = 0.2(2.8) = 0.56$.

$$b) P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - 0.2(1.5) = 0.7$$

$$c) P(X < -2) = F_X(-2) = 0$$

$$d) P(X > 6) = 1 - F_X(6) = 0$$

4-12. a) $P(X < 1.8) = P(X \leq 1.8) = F_X(1.8)$ because X is a continuous random variable. Then,

$$F_X(1.8) = 0.25(1.8) + 0.5 = 0.95$$

$$b) P(X > -1.5) = 1 - P(X \leq -1.5) = 1 - .125 = 0.875$$

$$c) P(X < -2) = 0$$

$$d) P(-1 < X < 1) = P(-1 < X \leq 1) = F_X(1) - F_X(-1) = .75 - .25 = 0.50$$

4-13. Now, $f(x) = e^{-x}$ for $0 < x$ and $F_X(x) = \int_0^x e^{-x} dx = -e^{-x} \Big|_0^x = 1 - e^{-x}$

for $0 < x$. Then, $F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-x}, & x > 0 \end{cases}$

4-14. Now, $f(x) = x/8$ for $3 < x < 5$ and $F_X(x) = \int_3^x \frac{x}{8} dx = \frac{x^2}{16} \Big|_3^x = \frac{x^2 - 9}{16}$

for $0 < x$. Then, $F_X(x) = \begin{cases} 0, & x < 3 \\ \frac{x^2 - 9}{16}, & 3 \leq x < 5 \\ 1, & x \geq 5 \end{cases}$

4-15. Now, $f(x) = e^{-(x-4)}$ for $4 < x$ and

$$F_X(x) = \int_4^x e^{-(x-4)} dx = -e^{-(x-4)} \Big|_4^x = 1 - e^{-(x-4)}$$

for $4 < x$.

Then, $F_X(x) = \begin{cases} 0, & x \leq 4 \\ 1 - e^{-(x-4)}, & x > 4 \end{cases}$

4-16. Now, $f(x) = \frac{e^{-x/1000}}{1000}$ for $0 < x$ and

$$F_X(x) = 1/1000 \int_0^x e^{-x/1000} dx = -e^{-x/1000} \Big|_0^x = 1 - e^{-x/1000}$$

for $0 < x$.

Then, $F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-x/1000}, & x > 0 \end{cases}$

$$P(X > 3000) = 1 - P(X \leq 3000) = 1 - F(3000) = e^{-3000/1000} = 0.5$$

4-17. Now, $f(x) = 1.25$ for $74.6 < x < 75.4$ and $F(x) = \int_{74.6}^x 1.25 dx = 1.25x - 93.25$

for $74.6 < x < 75.4$. Then,

$$F(x) = \begin{cases} 0, & x < 74.6 \\ 1.25x - 93.25, & 74.6 \leq x < 75.4 \\ 1, & 75.4 \leq x \end{cases}$$

$P(X > 75) = 1 - P(X \leq 75) = 1 - F(75) = 1 - 0.5 = 0.5$ because X is a continuous random variable.

$$4-18 \quad f(x) = 2e^{-2x}, \quad x > 0$$

$$4-19. \quad f(x) = \begin{cases} 0.2, & 0 < x < 4 \\ 0.04, & 4 \leq x < 9 \end{cases}$$

$$4-20. \quad f_X(x) = \begin{cases} 0.25, & -2 < x < 1 \\ 0.5, & 1 \leq x < 1.5 \end{cases}$$

$$4-21. \quad F(x) = \int_0^x 0.5x dx = \left. \frac{0.5x^2}{2} \right|_0^x = 0.25x^2 \text{ for } 0 < x < 2. \text{ Then,}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.25x^2, & 0 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

Section 4-4

$$4-22. \quad E(X) = \int_0^4 0.25x dx = 0.25 \left. \frac{x^2}{2} \right|_0^4 = 2$$

$$V(X) = \int_0^4 0.25(x-2)^2 dx = 0.25 \left. \frac{(x-2)^3}{3} \right|_0^4 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$4-23. \quad E(X) = \int_0^4 0.125x^2 dx = 0.125 \left. \frac{x^3}{3} \right|_0^4 = 2.6667$$

$$V(X) = \int_0^4 0.125x(x - \frac{8}{3})^2 dx = 0.125 \int_0^4 (x^3 - \frac{16}{3}x^2 + \frac{64}{9}x) dx$$

$$= 0.125 \left(\frac{x^4}{4} - \frac{16}{3} \frac{x^3}{3} + \frac{64}{9} \cdot \frac{1}{2} x^2 \right) \Big|_0^4 = 0.88889$$

$$\begin{aligned}
4-24. \quad E(X) &= \int_{-1}^1 1.5x^3 dx = 1.5 \frac{x^4}{4} \Big|_{-1}^1 = 0 \\
V(X) &= \int_{-1}^1 1.5x^3 (x-0)^2 dx = 1.5 \int_{-1}^1 x^4 dx \\
&= 1.5 \frac{x^5}{5} \Big|_{-1}^1 = 0.6
\end{aligned}$$

$$\begin{aligned}
4-25. \quad E(X) &= \int_3^5 x \frac{x}{8} dx = \frac{x^3}{24} \Big|_3^5 = \frac{5^3 - 3^3}{24} = 4.083 \\
V(X) &= \int_3^5 (x - 4.083)^2 \frac{x}{8} dx = \int_3^5 \left(\frac{x^3}{8} - \frac{8.166x^2}{8} + \frac{16.6709x}{8} \right) dx \\
&= \frac{1}{8} \left(\frac{x^4}{4} - \frac{8.166x^3}{3} + \frac{16.6709x^2}{2} \right) \Big|_3^5 = 0.3264
\end{aligned}$$

$$\begin{aligned}
4-26. \quad E(X) &= \int_{49.75}^{50.25} 2x dx = x^2 \Big|_{49.75}^{50.25} = 50 \\
V(X) &= \int_{49.75}^{50.25} 2(x - 50)^2 dx = 2 \int_{49.75}^{50.25} (x^2 - 100x + 2500) dx \\
&= 2 \left(\frac{x^3}{3} - 100 \frac{x^2}{2} + 2500x \right) \Big|_{49.75}^{50.25} \\
&= 0.0208
\end{aligned}$$

$$\begin{aligned}
4-27. \quad \text{a.) } E(X) &= \int_{100}^{120} x \frac{600}{x^2} dx = 600 \ln x \Big|_{100}^{120} = 109.39 \\
V(X) &= \int_{100}^{120} (x - 109.39)^2 \frac{600}{x^2} dx = 600 \int_{100}^{120} \left(1 - \frac{2(109.39)}{x} + \frac{(109.39)^2}{x^2} \right) dx \\
&= 600(x - 218.78 \ln x - 109.39^2 x^{-1}) \Big|_{100}^{120} = 33.19
\end{aligned}$$

b.) Average cost per part = \$0.50 * 109.39 = \$54.70

$$4-28. \quad E(X) = \int_1^{\infty} x 2x^{-3} dx = -2x^{-1} \Big|_1^{\infty} = 2$$

4-29. a) $E(X) = \int_5^{\infty} x 10e^{-10(x-5)} dx.$

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)} dx$, we obtain

$$E(X) = -xe^{-10(x-5)} \Big|_5^{\infty} + \int_5^{\infty} e^{-10(x-5)} dx = 5 - \frac{e^{-10(x-5)}}{10} \Big|_5^{\infty} = 5.1$$

Now, $V(X) = \int_5^{\infty} (x-5.1)^2 10e^{-10(x-5)} dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$$dv = 10e^{-10(x-5)}, \text{ we obtain } V(X) = -(x-5.1)^2 e^{-10(x-5)} \Big|_5^{\infty} + 2 \int_5^{\infty} (x-5.1) e^{-10(x-5)} dx. \text{ From}$$

the definition of $E(X)$ the integral above is recognized to equal 0.

$$\text{Therefore, } V(X) = (5-5.1)^2 = 0.01.$$

b) $P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)} dx = -e^{-10(x-5)} \Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$

4-30. a)

$$E(X) = \int_{1200}^{1210} x 0.1 dx = 0.05x^2 \Big|_{1200}^{1210} = 1205$$

$$V(X) = \int_{1200}^{1210} (x-1205)^2 0.1 dx = 0.1 \frac{(x-1205)^3}{3} \Big|_{1200}^{1210} = 8.333$$

$$\text{Therefore, } \sigma_x = \sqrt{V(X)} = 2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1195 < X < 1205) = P(1200 < X < 1205) = \int_{1200}^{1205} 0.1 dx = 0.1x \Big|_{1200}^{1205} = 0.5$$

Section 4-5

4-31. a) $E(X) = (5.5+1.5)/2 = 3.5,$

$$V(X) = \frac{(5.5-1.5)^2}{12} = 1.333, \text{ and } \sigma_x = \sqrt{1.333} = 1.155.$$

b) $P(X < 2.5) = \int_{1.5}^{2.5} 0.25 dx = 0.25x \Big|_{1.5}^{2.5} = 0.25$

4-32. a) $E(X) = (-1+1)/2 = 0$,

$$V(X) = \frac{(1 - (-1))^2}{12} = 1/3, \text{ and } \sigma_x = 0.577$$

b) $P(-x < X < x) = \int_{-x}^x \frac{1}{2} dt = 0.5t \Big|_{-x}^x = 0.5(2x) = x$

Therefore, x should equal 0.90.

4-33. a) $f(x) = 2.0$ for $49.75 < x < 50.25$.

$$E(X) = (50.25 + 49.75)/2 = 50.0,$$

$$V(X) = \frac{(50.25 - 49.75)^2}{12} = 0.0208, \text{ and } \sigma_x = 0.144.$$

b) $F(x) = \int_{49.75}^x 2.0 dx$ for $49.75 < x < 50.25$. Therefore,

$$F(x) = \begin{cases} 0, & x < 49.75 \\ 2x - 99.5, & 49.75 \leq x < 50.25 \\ 1, & 50.25 \leq x \end{cases}$$

c) $P(X < 50.1) = F(50.1) = 2(50.1) - 99.5 = 0.7$

4-34. a) The distribution of X is $f(x) = 10$ for $0.95 < x < 1.05$. Now,

$$F_X(x) = \begin{cases} 0, & x < 0.95 \\ 10x - 9.5, & 0.95 \leq x < 1.05 \\ 1, & 1.05 \leq x \end{cases}$$

b) $P(X > 1.02) = 1 - P(X \leq 1.02) = 1 - F_X(1.02) = 0.3$

c) If $P(X > x) = 0.90$, then $1 - F(X) = 0.90$ and $F(X) = 0.10$. Therefore, $10x - 9.5 = 0.10$ and $x = 0.96$.

d) $E(X) = (1.05 + 0.95)/2 = 1.00$ and $V(X) = \frac{(1.05 - 0.95)^2}{12} = 0.00083$

$$4-35 \quad E(X) = \frac{(1.5 + 2.2)}{2} = 1.85 \text{ min}$$

$$V(X) = \frac{(2.2 - 1.5)^2}{12} = 0.0408 \text{ min}^2$$

$$b) P(X < 2) = \int_{1.5}^2 \frac{1}{(2.2 - 1.5)} dx = \int_{1.5}^2 (1/0.7) dx = (1/0.7)x \Big|_{1.5}^2 = (1/0.7)(0.5) = 0.7143$$

$$c.) F(X) = \int_{1.5}^x \frac{1}{(2.2 - 1.5)} dx = \int_{1.5}^x (1/0.7) dx = (1/0.7)x \Big|_{1.5}^x \quad \text{for } 1.5 < x < 2.2. \text{ Therefore,}$$

$$F(x) = \begin{cases} 0, & x < 1.5 \\ (1/0.7)x - 2.14, & 1.5 \leq x < 2.2 \\ 1, & 2.2 \leq x \end{cases}$$

$$4-36 \quad f(x) = 0.04 \text{ for } 50 < x < 75$$

$$a) P(X > 70) = \int_{70}^{75} 0.04 dx = 0.2x \Big|_{70}^{75} = 0.2$$

$$b) P(X < 60) = \int_{50}^{60} 0.04 dx = 0.04x \Big|_{50}^{60} = 0.4$$

$$c) E(X) = \frac{75 + 50}{2} = 62.5 \text{ seconds}$$

$$V(X) = \frac{(75 - 50)^2}{12} = 52.0833 \text{ seconds}^2$$

$$4-37. \quad a) \text{ The distribution of } X \text{ is } f(x) = 100 \text{ for } 0.2050 < x < 0.2150. \text{ Therefore,}$$

$$F(x) = \begin{cases} 0, & x < 0.2050 \\ 100x - 20.50, & 0.2050 \leq x < 0.2150 \\ 1, & 0.2150 \leq x \end{cases}$$

$$b) P(X > 0.2125) = 1 - F(0.2125) = 1 - [100(0.2125) - 20.50] = 0.25$$

$$c) \text{ If } P(X > x) = 0.10, \text{ then } 1 - F(X) = 0.10 \text{ and } F(X) = 0.90.$$

$$\text{Therefore, } 100x - 20.50 = 0.90 \text{ and } x = 0.2140.$$

$$d) E(X) = (0.2050 + 0.2150)/2 = 0.2100 \mu\text{m} \text{ and}$$

$$V(X) = \frac{(0.2150 - 0.2050)^2}{12} = 8.33 \times 10^{-6} \mu\text{m}^2$$

4-38. a) $P(X > 35) = \int_{35}^{40} 0.1dx = 0.1x \Big|_{35}^{40} = 0.5$

b) $P(X > x) = 0.90$ and $P(X > x) = \int_x^{40} 0.1dt = 0.1(40 - x)$.

Now, $0.1(40 - x) = 0.90$ and $x = 31$

c) $E(X) = (30 + 40)/2 = 35$ and $V(X) = \frac{(40 - 30)^2}{12} = 8.33$

Section 4-6

- 4-39. a) $P(Z < 1.32) = 0.90658$
 b) $P(Z < 3.0) = 0.99865$
 c) $P(Z > 1.45) = 1 - 0.92647 = 0.07353$
 d) $P(Z > -2.15) = P(Z < 2.15) = 0.98422$
 e) $P(-2.34 < Z < 1.76) = P(Z < 1.76) - P(Z > 2.34) = 0.95116$
- 4-40. a) $P(-1 < Z < 1) = P(Z < 1) - P(Z > 1)$
 $= 0.84134 - (1 - 0.84134)$
 $= 0.68268$
 b) $P(-2 < Z < 2) = P(Z < 2) - [1 - P(Z < 2)]$
 $= 0.9545$
 c) $P(-3 < Z < 3) = P(Z < 3) - [1 - P(Z < 3)]$
 $= 0.9973$
 d) $P(Z > 3) = 1 - P(Z < 3)$
 $= 0.00135$
 e) $P(0 < Z < 1) = P(Z < 1) - P(Z < 0)$
 $= 0.84134 - 0.5$
 $= 0.34134$
- 4-41 a) $P(Z < 1.28) = 0.90$
 b) $P(Z < 0) = 0.5$
 c) If $P(Z > z) = 0.1$, then $P(Z < z) = 0.90$ and $z = 1.28$
 d) If $P(Z > z) = 0.9$, then $P(Z < z) = 0.10$ and $z = -1.28$
 e) $P(-1.24 < Z < z) = P(Z < z) - P(Z < -1.24)$
 $= P(Z < z) - 0.10749$.
 Therefore, $P(Z < z) = 0.8 + 0.10749 = 0.90749$ and $z = 1.33$
- 4-42. a) Because of the symmetry of the normal distribution, the area in each tail of the distribution must equal 0.025. Therefore the value in Table II that corresponds to 0.975 is 1.96. Thus, $z = 1.96$.
 b) Find the value in Table II corresponding to 0.995. $z = 2.58$.
 c) Find the value in Table II corresponding to 0.84. $z = 1.0$
 d) Find the value in Table II corresponding to 0.99865. $z = 3.0$.

$$4-43. \quad \begin{aligned} \text{a) } P(X < 13) &= P(Z < (13-10)/2) \\ &= P(Z < 1.5) \\ &= 0.93319 \end{aligned}$$

$$\begin{aligned} \text{b) } P(X > 9) &= 1 - P(X < 9) \\ &= 1 - P(Z < (9-10)/2) \\ &= 1 - P(Z < -0.5) \\ &= 0.69146. \end{aligned}$$

$$\begin{aligned} \text{c) } P(6 < X < 14) &= P\left(\frac{6-10}{2} < Z < \frac{14-10}{2}\right) \\ &= P(-2 < Z < 2) \\ &= P(Z < 2) - P(Z < -2)] \\ &= 0.9545. \end{aligned}$$

$$\begin{aligned} \text{d) } P(2 < X < 4) &= P\left(\frac{2-10}{2} < Z < \frac{4-10}{2}\right) \\ &= P(-4 < Z < -3) \\ &= P(Z < -3) - P(Z < -4) \\ &= 0.00132 \end{aligned}$$

$$\begin{aligned} \text{e) } P(-2 < X < 8) &= P(X < 8) - P(X < -2) \\ &= P\left(Z < \frac{8-10}{2}\right) - P\left(Z < \frac{-2-10}{2}\right) \\ &= P(Z < -1) - P(Z < -6) \\ &= 0.15866. \end{aligned}$$

$$4-44. \quad \text{a) } P(X > x) = P\left(Z > \frac{x-10}{2}\right) = 0.5. \text{ Therefore, } \frac{x-10}{2} = 0 \text{ and } x = 10.$$

$$\begin{aligned} \text{b) } P(X > x) &= P\left(Z > \frac{x-10}{2}\right) = 1 - P\left(Z < \frac{x-10}{2}\right) \\ &= 0.95. \end{aligned}$$

$$\text{Therefore, } P\left(Z < \frac{x-10}{2}\right) = 0.05 \text{ and } \frac{x-10}{2} = -1.64. \text{ Consequently, } x = 6.72.$$

$$\begin{aligned} \text{c) } P(x < X < 10) &= P\left(\frac{x-10}{2} < Z < 0\right) = P(Z < 0) - P\left(Z < \frac{x-10}{2}\right) \\ &= 0.5 - P\left(Z < \frac{x-10}{2}\right) = 0.2. \end{aligned}$$

$$\text{Therefore, } P\left(Z < \frac{x-10}{2}\right) = 0.3 \text{ and } \frac{x-10}{2} = -0.52. \text{ Consequently, } x = 8.96.$$

$$\begin{aligned} \text{d) } P(10 - x < X < 10 + x) &= P(-x/2 < Z < x/2) = 0.95. \\ \text{Therefore, } x/2 &= 1.96 \text{ and } x = 3.92 \end{aligned}$$

$$\begin{aligned} \text{e) } P(10 - x < X < 10 + x) &= P(-x/2 < Z < x/2) = 0.99. \\ \text{Therefore, } x/2 &= 2.58 \text{ and } x = 5.16 \end{aligned}$$

$$\begin{aligned}
 4-45. \quad a) P(X < 11) &= P\left(Z < \frac{11-5}{4}\right) \\
 &= P(Z < 1.5) \\
 &= 0.93319
 \end{aligned}$$

$$\begin{aligned}
 b) P(X > 0) &= P\left(Z > \frac{0-5}{4}\right) \\
 &= P(Z > -1.25) \\
 &= 1 - P(Z < -1.25) \\
 &= 0.89435
 \end{aligned}$$

$$\begin{aligned}
 c) P(3 < X < 7) &= P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right) \\
 &= P(-0.5 < Z < 0.5) \\
 &= P(Z < 0.5) - P(Z < -0.5) \\
 &= 0.38292
 \end{aligned}$$

$$\begin{aligned}
 d) P(-2 < X < 9) &= P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right) \\
 &= P(-1.75 < Z < 1) \\
 &= P(Z < 1) - P(Z < -1.75) \\
 &= 0.80128
 \end{aligned}$$

$$\begin{aligned}
 e) P(2 < X < 8) &= P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right) \\
 &= P(-0.75 < Z < 0.75) \\
 &= P(Z < 0.75) - P(Z < -0.75) \\
 &= 0.54674
 \end{aligned}$$

$$4-46. \quad a) P(X > x) = P\left(Z > \frac{x-5}{4}\right) = 0.5.$$

Therefore, $x = 5$.

$$b) P(X > x) = P\left(Z > \frac{x-5}{4}\right) = 0.95.$$

$$\text{Therefore, } P\left(Z < \frac{x-5}{4}\right) = 0.05$$

Therefore, $\frac{x-5}{4} = -1.64$, and $x = -1.56$.

$$c) P(x < X < 9) = P\left(\frac{x-5}{4} < Z < 1\right) = 0.2.$$

Therefore, $P(Z < 1) - P(Z < \frac{x-5}{4}) = 0.2$ where $P(Z < 1) = 0.84134$.

Thus $P(Z < \frac{x-5}{4}) = 0.64134$. Consequently, $\frac{x-5}{4} = 0.36$ and $x = 6.44$.

$$d) P(3 < X < x) = P\left(\frac{3-5}{4} < Z < \frac{x-5}{4}\right) = 0.95.$$

$$\text{Therefore, } P\left(Z < \frac{x-5}{4}\right) - P(Z < -0.5) = 0.95 \text{ and } P\left(Z < \frac{x-5}{4}\right) - 0.30854 = 0.95.$$

Consequently,

$$P\left(Z < \frac{x-5}{4}\right) = 1.25854. \text{ Because a probability can not be greater than one, there is no}$$

solution for x . In fact, $P(3 < X) = P(-0.5 < Z) = 0.69146$. Therefore, even if x is set to infinity the probability requested cannot equal 0.95.

e) This part of the exercise was changed in Printing 3 from $P(-x < X < x)$ to $P(-x < X - 5 < x)$.

$$\begin{aligned} P(-x < X - 5 < x) &= P(5 - x < X < 5 + x) = P\left(\frac{5-x-5}{4} < Z < \frac{5+x-5}{4}\right) \\ &= P\left(\frac{-x}{4} < Z < \frac{x}{4}\right) = 0.99 \end{aligned}$$

Therefore, $x/4 = 2.58$ and $x = 10.32$.

$$\begin{aligned} 4-47. \quad a) P(X < 6250) &= P\left(Z < \frac{6250-6000}{100}\right) \\ &= P(Z < 2.5) \\ &= 0.99379 \end{aligned}$$

$$\begin{aligned} b) P(5800 < X < 5900) &= P\left(\frac{5800-6000}{100} < Z < \frac{5900-6000}{100}\right) \\ &= P(-2 < Z < -1) \\ &= P(Z < -1) - P(Z < -2) \\ &= 0.13591 \end{aligned}$$

$$c) P(X > x) = P\left(Z > \frac{x-6000}{100}\right) = 0.95.$$

Therefore, $\frac{x-6000}{100} = -1.65$ and $x = 5835$.

$$\begin{aligned} 4-48. \quad a) P(X < 40) &= P\left(Z < \frac{40-35}{2}\right) \\ &= P(Z < 2.5) \\ &= 0.99379 \end{aligned}$$

$$\begin{aligned} b) P(X < 30) &= P\left(Z < \frac{30-35}{2}\right) \\ &= P(Z < -2.5) \\ &= 0.00621 \end{aligned}$$

0.621% are scrapped

$$\begin{aligned}
 4-49. \quad a) P(X > 0.62) &= P\left(Z > \frac{0.62 - 0.5}{0.05}\right) \\
 &= P(Z > 2.4) \\
 &= 1 - P(Z < 2.4) \\
 &= 0.0082
 \end{aligned}$$

$$\begin{aligned}
 b) P(0.47 < X < 0.63) &= P\left(\frac{0.47 - 0.5}{0.05} < Z < \frac{0.63 - 0.5}{0.05}\right) \\
 &= P(-0.6 < Z < 2.6) \\
 &= P(Z < 2.6) - P(Z < -0.6) \\
 &= 0.99534 - 0.27425 \\
 &= 0.72109
 \end{aligned}$$

$$c) P(X < x) = P\left(Z < \frac{x - 0.5}{0.05}\right) = 0.90.$$

$$\text{Therefore, } \frac{x - 0.5}{0.05} = 1.28 \text{ and } x = 0.564.$$

$$4-50. \quad a) P(X < 12) = P\left(Z < \frac{12 - 12.4}{0.1}\right) = P(Z < -4) \cong 0$$

$$b) P(X < 12.1) = P\left(Z < \frac{12.1 - 12.4}{0.1}\right) = P(Z < -3) = 0.00135$$

and

$$P(X > 12.6) = P\left(Z > \frac{12.6 - 12.4}{0.1}\right) = P(Z > 2) = 0.02275.$$

Therefore, the proportion of cans scrapped is $0.00135 + 0.02275 = 0.0241$, or 2.41%

$$c) P(12.4 - x < X < 12.4 + x) = 0.99.$$

$$\text{Therefore, } P\left(-\frac{x}{0.1} < Z < \frac{x}{0.1}\right) = 0.99$$

$$\text{Consequently, } P\left(Z < \frac{x}{0.1}\right) = 0.995 \text{ and } x = 0.1(2.58) = 0.258.$$

The limits are (12.142, 12.658).

$$4-51. \quad a) P(X < 45) = P\left(Z < \frac{45 - 60}{5}\right) = P(Z < -3) = 0.00135$$

$$\begin{aligned}
 b) P(X > 65) &= P\left(Z > \frac{65 - 60}{5}\right) = P(Z > 1) = 1 - P(Z < 1) \\
 &= 1 - 0.841345 = 0.158655
 \end{aligned}$$

$$c) P(X < x) = P\left(Z < \frac{x - 60}{5}\right) = 0.99.$$

$$\text{Therefore, } \frac{x - 60}{5} = 2.33 \text{ and } x = 72$$

4-52. a) If $P(X > 12) = 0.999$, then $P\left(Z > \frac{12 - \mu}{0.1}\right) = 0.999$.

Therefore, $\frac{12 - \mu}{0.1} = -3.09$ and $\mu = 12.309$.

b) If $P(X > 12) = 0.999$, then $P\left(Z > \frac{12 - \mu}{0.05}\right) = 0.999$.

Therefore, $\frac{12 - \mu}{0.05} = -3.09$ and $\mu = 12.1545$.

4-53. a) $P(X > 0.5) = P\left(Z > \frac{0.5 - 0.4}{0.05}\right)$
 $= P(Z > 2)$
 $= 1 - 0.97725$
 $= 0.02275$

b) $P(0.4 < X < 0.5) = P\left(\frac{0.4 - 0.4}{0.05} < Z < \frac{0.5 - 0.4}{0.05}\right)$
 $= P(0 < Z < 2)$
 $= P(Z < 2) - P(Z < 0)$
 $= 0.47725$

c) $P(X > x) = 0.90$, then $P\left(Z > \frac{x - 0.4}{0.05}\right) = 0.90$.

Therefore, $\frac{x - 0.4}{0.05} = -1.28$ and $x = 0.336$.

4-54 a) $P(X > 70) = P\left(Z > \frac{70 - 60}{4}\right)$
 $= 1 - P(Z < 2.5)$
 $= 1 - 0.99379 = 0.00621$

b) $P(X < 58) = P\left(Z < \frac{58 - 60}{4}\right)$
 $= P(Z < -0.5)$
 $= 0.308538$

c) $1,000,000 \text{ bytes} * 8 \text{ bits/byte} = 8,000,000 \text{ bits}$

$$\frac{8,000,000 \text{ bits}}{60,000 \text{ bits/sec}} = 133.33 \text{ seconds}$$

$$a) P(X > 90.3) + P(X < 89.7)$$

$$\begin{aligned}
 &= P\left(Z > \frac{90.3 - 90.2}{0.1}\right) + P\left(Z < \frac{89.7 - 90.2}{0.1}\right) \\
 &= P(Z > 1) + P(Z < -5) \\
 &= 1 - P(Z < 1) + P(Z < -5) \\
 &= 1 - 0.84134 + 0 \\
 &= 0.15866.
 \end{aligned}$$

Therefore, the answer is 0.15866.

b) The process mean should be set at the center of the specifications; that is, at $\frac{90.3 + 89.7}{2} = 90.0$.

$$\begin{aligned}
 c) P(89.7 < X < 90.3) &= P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right) \\
 &= P(-3 < Z < 3) = 0.9973.
 \end{aligned}$$

The yield is $100 \times 0.9973 = 99.73\%$

$$\begin{aligned}
 4-56. \quad a) P(89.7 < X < 90.3) &= P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right) \\
 &= P(-3 < Z < 3) \\
 &= 0.9973. \\
 P(X=10) &= (0.9973)^{10} = 0.9733
 \end{aligned}$$

b) Let Y represent the number of cases out of the sample of 10 that are between 89.7 and 90.3 ml. Then Y follows a binomial distribution with $n=10$ and $p=0.9973$. Thus, $E(Y) = 9.973$ or 10.

$$\begin{aligned}
 4-57. \quad a) P(50 < X < 80) &= P\left(\frac{50 - 100}{20} < Z < \frac{80 - 100}{20}\right) \\
 &= P(-2.5 < Z < -1) \\
 &= P(Z < -1) - P(Z < -2.5) \\
 &= 0.15245.
 \end{aligned}$$

$$b) P(X > x) = 0.10. \text{ Therefore, } P\left(Z > \frac{x - 100}{20}\right) = 0.10 \text{ and } \frac{x - 100}{20} = 1.28.$$

Therefore, x

= 126. hours

$$4-58. \quad a) P(X < 5000) = P\left(Z < \frac{5000 - 7000}{600}\right) \\ = P(Z < -3.33) = 0.00043.$$

$$b) P(X > x) = 0.95. \text{ Therefore, } P\left(Z > \frac{x - 7000}{600}\right) = 0.95 \text{ and } \frac{x - 7000}{600} = -1.64. \text{ Consequently, } x = 6016.$$

$$c) P(X > 7000) = P\left(Z > \frac{7000 - 7000}{600}\right) = P(Z > 0) = 0.5$$

$$P(\text{three lasers operating after 7000 hours}) = (1/2)^3 = 1/8$$

$$4-59. \quad a) P(X > 0.0026) = P\left(Z > \frac{0.0026 - 0.002}{0.0004}\right) \\ = P(Z > 1.5) \\ = 1 - P(Z < 1.5) \\ = 0.06681.$$

$$b) P(0.0014 < X < 0.0026) = P\left(\frac{0.0014 - 0.002}{0.0004} < Z < \frac{0.0026 - 0.002}{0.0004}\right) \\ = P(-1.5 < Z < 1.5) \\ = 0.86638.$$

$$c) P(0.0014 < X < 0.0026) = P\left(\frac{0.0014 - 0.002}{\sigma} < Z < \frac{0.0026 - 0.002}{\sigma}\right) \\ = P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right).$$

$$\text{Therefore, } P\left(Z < \frac{0.0006}{\sigma}\right) = 0.9975. \text{ Therefore, } \frac{0.0006}{\sigma} = 2.81 \text{ and } \sigma = 0.000214.$$

$$4-60. \quad a) P(X > 13) = P\left(Z > \frac{13 - 12}{0.5}\right) \\ = P(Z > 2) \\ = 0.02275$$

$$b) \text{ If } P(X < 13) = 0.999, \text{ then } P\left(Z < \frac{13 - 12}{\sigma}\right) = 0.999.$$

$$\text{Therefore, } 1/\sigma = 3.09 \text{ and } \sigma = 1/3.09 = 0.324.$$

$$c) \text{ If } P(X < 13) = 0.999, \text{ then } P\left(Z < \frac{13 - \mu}{0.5}\right) = 0.999.$$

$$\text{Therefore, } \frac{13 - \mu}{0.5} = 3.09 \text{ and } \mu = 11.455$$

Section 4-7

4-61. a) $E(X) = 200(0.4) = 80$, $V(X) = 200(0.4)(0.6) = 48$ and $\sigma_X = \sqrt{48}$.

Then, $P(X \leq 70) \cong P\left(Z \leq \frac{70-80}{\sqrt{48}}\right) = P(Z \leq -1.44) = 0.074934$

b)

$$P(70 < X \leq 90) \cong P\left(\frac{70-80}{\sqrt{48}} < Z \leq \frac{90-80}{\sqrt{48}}\right) = P(-1.44 < Z \leq 1.44) \\ = 0.925066 - 0.074934 = 0.85013$$

4-62. a)

$$P(X < 4) = \binom{100}{0} 0.1^0 0.9^{100} + \binom{100}{1} 0.1^1 0.9^{99} \\ + \binom{100}{2} 0.1^2 0.9^{98} + \binom{100}{3} 0.1^3 0.9^{97} = 0.0078$$

b) $E(X) = 10$, $V(X) = 100(0.1)(0.9) = 9$ and $\sigma_X = 3$.

Then, $P(X < 4) \cong P(Z < \frac{4-10}{3}) = P(Z < -2) = 0.023$

c) $P(8 < X < 12) \cong P(\frac{8-10}{3} < Z < \frac{12-10}{3}) = P(-0.67 < Z < 0.67) = 0.497$

4-63. Let X denote the number of defective chips in the lot.

Then, $E(X) = 1000(0.02) = 20$, $V(X) = 1000(0.02)(0.98) = 19.6$.

a) $P(X > 25) \cong P\left(Z > \frac{25-20}{\sqrt{19.6}}\right) = P(Z > 1.13) = 1 - P(Z \leq 1.13) = 0.129$

b) $P(20 < X < 30) \cong P(0 < Z < \frac{10}{\sqrt{19.6}}) = P(0 < Z < 2.26) \\ = P(Z \leq 2.26) - P(Z < 0) = 0.98809 - 0.5 = 0.488$

4-64. Let X denote the number of defective electrical connectors in the sample.

Then, $E(X) = 25(100/1000) = 2.5$, $V(X) = 25(0.1)(0.9) = 2.25$.

a) $P(X=0) = 0.9^{25} = 0.0718$

b) $P(X \leq 0) \cong P(Z < \frac{0-2.5}{\sqrt{2.25}}) = P(Z < -1.67) = 0.047$

The approximation is smaller than the binomial. It is not satisfactory since $np < 5$.

c) Then, $E(X) = 25(100/500) = 5$, $V(X) = 25(0.2)(0.8) = 4$.

$P(X=0) = 0.8^{25} = 0.00377$

$P(X \leq 0) \cong P(Z < \frac{0-5}{\sqrt{4}}) = P(Z < -2.5) = 0.006$

Normal approximation is now closer to the binomial; however, it is still not satisfactory since $np = 5$ is not > 5 .

- 4-65. Let X denote the number of original components that fail during the useful life of the product. Then, X is a binomial random variable with $p = 0.001$ and $n = 5000$. Also, $E(X) = 5000(0.001) = 5$ and $V(X) = 5000(0.001)(0.999) = 4.995$.

$$P(X \geq 10) \cong P\left(Z \geq \frac{10-5}{\sqrt{4.995}}\right) = P(Z \geq 2.24) = 1 - P(Z < 2.24) = 1 - 0.987 = 0.013.$$

- 4-66. Let X denote the number of particles in 10 cm^2 of dust. Then, X is a Poisson random variable with $\lambda = 10(1000) = 10,000$. Also, $E(X) = \lambda = 10^4$ and $V(X) = \lambda = 10^4$

$$P(X > 10,000) \cong P\left(Z > \frac{10,000-10,000}{\sqrt{10,000}}\right) = P(Z > 0) = 0.5$$

- 4-67 Let X denote the number of errors on a web site. Then, X is a binomial random variable with $p = 0.05$ and $n = 100$. Also, $E(X) = 100(0.05) = 5$ and $V(X) = 100(0.05)(0.95) = 4.75$

$$P(X \geq 1) \cong P\left(Z \geq \frac{1-5}{\sqrt{4.75}}\right) = P(Z \geq -1.84) = 1 - P(Z < -1.84) = 1 - 0.03288 = 0.96712$$

- 4-68. Let X denote the number of hits to a web site. Then, X is a Poisson random variable with a mean of $\lambda = 10,000$ hits per day. Also, $E(X) = \lambda = 10,000 = V(X)$.

a)

$$P(X \geq 20,000) \cong P\left(Z \geq \frac{20,000-10,000}{\sqrt{10,000}}\right) = P(Z \geq 100) \\ = 1 - P(Z < 100) \approx 1 - 1 = 0$$

b.) $P(X < 9,900) \cong P\left(Z < \frac{9,900-10,000}{\sqrt{10,000}}\right) = P(Z < -1) = 0.1587$

c.) If $P(X > x) = 0.01$, then $P\left(Z > \frac{x-10,000}{\sqrt{10,000}}\right) = 0.01$.

Therefore, $\frac{x-10,000}{100} = 2.33$ and $x = 10,233$

- 4-69 Let X denote the number of hits to a web site. Then, X is a Poisson random variable with a of mean $10,000$ per day. $E(X) = \lambda = 10,000$ and $V(X) = 10,000$

a)

$$P(X \geq 10,200) \cong P\left(Z \geq \frac{10,200-10,000}{\sqrt{10,000}}\right) = P(Z \geq 2) = 1 - P(Z < 2) \\ = 1 - 0.9772 = 0.0228$$

Expected value of hits days with more than 10,200 hits per day is $(0.0228) \times 365 = 8.32$ days per year

- 4-69 b.) Let Y denote the number of days per year with over 10,200 hits to a web site.
Then, Y is a binomial random variable with $n=365$ and $p=0.0228$.
 $E(Y) = 8.32$ and $V(Y) = 365(0.0228)(0.9772)=8.13$

$$P(Y > 15) \cong P\left(Z \geq \frac{15 - 8.32}{\sqrt{8.13}}\right) = P(Z \geq 2.34) = 1 - P(Z < 2.34) \\ = 1 - 0.9904 = 0.0096$$

- 4-70 $E(X) = 1000(0.2) = 200$ and $V(X) = 1000(0.2)(0.8) = 160$

a) $P(X \geq 225) \cong 1 - P\left(Z \leq \frac{225 - 200}{\sqrt{160}}\right) = 1 - P(Z \leq 1.98) = 1 - 0.97615 = 0.02385$

b) $P(175 \leq X \leq 225) \cong P\left(\frac{175 - 200}{\sqrt{160}} \leq Z \leq \frac{225 - 200}{\sqrt{160}}\right) = P(-1.98 \leq Z \leq 1.98) \\ = 0.97615 - 0.02385 = 0.9523$

c) If $P(X > x) = 0.01$, then $P\left(Z > \frac{x - 200}{\sqrt{160}}\right) = 0.01$.

Therefore, $\frac{x - 200}{\sqrt{160}} = 2.33$ and $x = 229.5$

- 4-71 X is the number of minor errors on a test pattern of 1000 pages of text. X is a Poisson random variable with a mean of 0.4 per page

- a) The numbers of errors per page are random variables. The assumption that the occurrence of an event in a subinterval in a Poisson process is independent of events in other subintervals implies that the numbers of events in disjoint intervals are independent. The pages are disjoint intervals and the consequently the error counts per page are independent.

b) $P(X = 0) = \frac{e^{-0.4} 0.4^0}{0!} = 0.670$

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.670 = 0.330$$

The mean number of pages with one or more errors is $1000(0.330)=330$ pages

- c) Let Y be the number of pages with errors.

$$P(Y > 350) \cong P\left(Z \geq \frac{350 - 330}{\sqrt{1000(0.330)(0.670)}}\right) = P(Z \geq 1.35) = 1 - P(Z < 1.35) \\ = 1 - 0.9115 = 0.0885$$

Section 4-9

4-72. a) $P(X \leq 0) = \int_0^0 \lambda e^{-\lambda x} dx = 0$

b) $P(X \geq 2) = \int_2^{\infty} 2e^{-2x} dx = -e^{-2x} \Big|_2^{\infty} = e^{-4} = 0.0183$

c) $P(X \leq 1) = \int_0^1 2e^{-2x} dx = -e^{-2x} \Big|_0^1 = 1 - e^{-2} = 0.8647$

d) $P(1 < X < 2) = \int_1^2 2e^{-2x} dx = -e^{-2x} \Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

e) $P(X \leq x) = \int_0^x 2e^{-2t} dt = -e^{-2t} \Big|_0^x = 1 - e^{-2x} = 0.05$ and $x = 0.0256$

4-73. If $E(X) = 10$, then $\lambda = 0.1$.

a) $P(X > 10) = \int_{10}^{\infty} 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_{10}^{\infty} = e^{-1} = 0.3679$

b) $P(X > 20) = -e^{-0.1x} \Big|_{20}^{\infty} = e^{-2} = 0.1353$

c) $P(X > 30) = -e^{-0.1x} \Big|_{30}^{\infty} = e^{-3} = 0.0498$

d) $P(X < x) = \int_0^x 0.1e^{-0.1t} dt = -e^{-0.1t} \Big|_0^x = 1 - e^{-0.1x} = 0.95$ and $x = 29.96$.

4-74. Let X denote the time until the first count. Then, X is an exponential random variable with $\lambda = 2$ counts per minute.

a) $P(X > 0.5) = \int_{0.5}^{\infty} 2e^{-2x} dx = -e^{-2x} \Big|_{0.5}^{\infty} = e^{-1} = 0.3679$

b) $P(X < \frac{10}{60}) = \int_0^{1/6} 2e^{-2x} dx = -e^{-2x} \Big|_0^{1/6} = 1 - e^{-1/3} = 0.2835$

c) $P(1 < X < 2) = -e^{-2x} \Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

4-75. a) $E(X) = 1/\lambda = 1/3 = 0.333$ minutes

b) $V(X) = 1/\lambda^2 = 1/3^2 = 0.111$, $\sigma = 0.3333$

c) $P(X < x) = \int_0^x 3e^{-3t} dt = -e^{-3t} \Big|_0^x = 1 - e^{-3x} = 0.95$, $x = 0.9986$

- 4-76. The time to failure (in hours) for a laser in a cytometry machine is modeled by an exponential distribution with 0.00004.

$$a) P(X > 20,000) = \int_{20000}^{\infty} 0.00004e^{-0.00004x} dx = -e^{-0.00004x} \Big|_{20000}^{\infty} = e^{-0.8} = 0.4493$$

$$b) P(X < 30,000) = \int_{30000}^{\infty} 0.00004e^{-0.00004x} dx = -e^{-0.00004x} \Big|_0^{30000} = 1 - e^{-1.2} = 0.6988$$

c)

$$\begin{aligned} P(20,000 < X < 30,000) &= \int_{20000}^{30000} 0.00004e^{-0.00004x} dx \\ &= -e^{-0.00004x} \Big|_{20000}^{30000} = e^{-0.8} - e^{-1.2} = 0.1481 \end{aligned}$$

- 4-77. Let X denote the time until the first call. Then, X is exponential and calls/minute.

$$\lambda = \frac{1}{E(X)} = \frac{1}{15}$$

$$a) P(X > 30) = \int_{30}^{\infty} \frac{1}{15} e^{-\frac{x}{15}} dx = -e^{-\frac{x}{15}} \Big|_{30}^{\infty} = e^{-2} = 0.1353$$

b) The probability of at least one call in a 10-minute interval equals one minus the probability of zero calls in

a 10-minute interval and that is $P(X > 10)$.

$$P(X > 10) = -e^{-\frac{x}{15}} \Big|_{10}^{\infty} = e^{-2/3} = 0.5134.$$

Therefore, the answer is $1 - 0.5134 = 0.4866$. Alternatively, the requested probability is equal to $P(X < 10) = 0.4866$.

$$c) P(5 < X < 10) = -e^{-\frac{x}{15}} \Big|_5^{10} = e^{-1/3} - e^{-2/3} = 0.2031$$

$$d) P(X < x) = 0.90 \text{ and } P(X < x) = -e^{-\frac{x}{15}} \Big|_0^x = 1 - e^{-x/15} = 0.90. \text{ Therefore, } x = 34.54 \text{ minutes.}$$

- 4-78. Let X be the life of regulator. Then, X is an exponential random variable with $\lambda = 1/E(X) = 1/6$

a) Because the Poisson process from which the exponential distribution is derived is memoryless, this probability is

$$P(X < 6) = \int_0^6 \frac{1}{6} e^{-x/6} dx = -e^{-x/6} \Big|_0^6 = 1 - e^{-1} = 0.6321$$

b) Because the failure times are memoryless, the mean time until the next failure is $E(X) = 6$ years.

- 4-79. Let X denote the time to failure (in hours) of fans in a personal computer. Then, X is an exponential random variable and $\lambda = 1/E(X) = 0.0003$.

$$\text{a) } P(X > 10,000) = \int_{10,000}^{\infty} 0.0003e^{-x \cdot 0.0003} dx = -e^{-x \cdot 0.0003} \Big|_{10,000}^{\infty} = e^{-3} = 0.0498$$

$$\text{b) } P(X < 7,000) = \int_0^{7,000} 0.0003e^{-x \cdot 0.0003} dx = -e^{-x \cdot 0.0003} \Big|_0^{7,000} = 1 - e^{-2.1} = 0.8775$$

- 4-80. Let X denote the time until a message is received. Then, X is an exponential random variable and $\lambda = 1/E(X) = 1/2$.

$$\text{a) } P(X > 2) = \int_2^{\infty} \frac{1}{2} e^{-x/2} dx = -e^{-x/2} \Big|_2^{\infty} = e^{-1} = 0.3679$$

b) The same as part a.

c) $E(X) = 2$ hours.

- 4-81. Let X denote the time until the arrival of a taxi. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.1$ arrivals/minute.

$$\text{a) } P(X > 60) = \int_{60}^{\infty} 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_{60}^{\infty} = e^{-6} = 0.0025$$

$$\text{b) } P(X < 10) = \int_0^{10} 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_0^{10} = 1 - e^{-1} = 0.6321$$

- 4-82. a) $P(X > x) = \int_x^{\infty} 0.1e^{-0.1t} dt = -e^{-0.1t} \Big|_x^{\infty} = e^{-0.1x} = 0.1$ and $x = 23.03$ minutes.

b) $P(X < x) = 0.9$ implies that $P(X > x) = 0.1$. Therefore, this answer is the same as part a.

$$\text{c) } P(X < x) = -e^{-0.1t} \Big|_0^x = 1 - e^{-0.1x} = 0.5 \text{ and } x = 6.93 \text{ minutes.}$$

- 4-83. Let X denote the distance between major cracks. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.2$ cracks/mile.

$$\text{a) } P(X > 10) = \int_{10}^{\infty} 0.2e^{-0.2x} dx = -e^{-0.2x} \Big|_{10}^{\infty} = e^{-2} = 0.1353$$

b) Let Y denote the number of cracks in 10 miles of highway. Because the distance between cracks is exponential, Y is a Poisson random variable with $\lambda = 10(0.2) = 2$ cracks per 10 miles.

$$P(Y = 2) = \frac{e^{-2} 2^2}{2!} = 0.2707$$

c) $\sigma_X = 1/\lambda = 5$ miles.

4-84. a) $P(12 < X < 15) = \int_{12}^{15} 0.2e^{-0.2x} dx = -e^{-0.2x} \Big|_{12}^{15} = e^{-2.4} - e^{-3} = 0.0409$

b) $P(X > 5) = -e^{-0.2x} \Big|_5^{\infty} = e^{-1} = 0.3679$. By independence of the intervals in a Poisson process,

the answer is $0.3679^2 = 0.1353$. Alternatively, the answer is $P(X > 10) = e^{-2} = 0.1353$. The probability does depend on whether or not the lengths of highway are consecutive.

c) By the memoryless property, this answer is $P(X > 10) = 0.1353$ from part b.

4-85. Let X denote the lifetime of an assembly. Then, X is an exponential random variable with $\lambda = 1/E(X) = 1/400$ failures per hour.

a) $P(X < 100) = \int_0^{100} \frac{1}{400} e^{-x/400} dx = -e^{-x/400} \Big|_0^{100} = 1 - e^{-0.25} = 0.2212$

b) $P(X > 500) = -e^{-x/400} \Big|_{500}^{\infty} = e^{-5/4} = 0.2865$

c) From the memoryless property of the exponential, this answer is the same as part a., $P(X < 100) = 0.2212$.

4-86. a) Let U denote the number of assemblies out of 10 that fail before 100 hours. By the memoryless property of a Poisson process, U has a binomial distribution with $n = 10$ and $p = 0.2212$ (from Exercise 4-85a). Then,

$$P(U \geq 1) = 1 - P(U = 0) = 1 - \binom{10}{0} 0.2212^0 (1 - 0.2212)^{10} = 0.9179$$

b) Let V denote the number of assemblies out of 10 that fail before 800 hours. Then, V is a binomial random variable with $n = 10$ and $p = P(X < 800)$, where X denotes the lifetime of an assembly.

$$\text{Now, } P(X < 800) = \int_0^{800} \frac{1}{400} e^{-x/400} dx = -e^{-x/400} \Big|_0^{800} = 1 - e^{-2} = 0.8647.$$

$$\text{Therefore, } P(V = 10) = \binom{10}{10} 0.8647^{10} (1 - 0.8647)^0 = 0.2337.$$

4-87. Let X denote the number of calls in 3 hours. Because the time between calls is an exponential random variable, the number of calls in 3 hours is a Poisson random variable. Now, the mean time between calls is 0.5 hours and $\lambda = 1/0.5 = 2$ calls per hour = 6 calls in 3 hours.

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \left[\frac{e^{-6} 6^0}{0!} + \frac{e^{-6} 6^1}{1!} + \frac{e^{-6} 6^2}{2!} + \frac{e^{-6} 6^3}{3!} \right] = 0.8488$$

4-88. Let Y denote the number of arrivals in one hour. If the time between arrivals is exponential, then the count of arrivals is a Poisson random variable and $\lambda = 1$ arrival per hour.

$$P(Y > 3) = 1 - P(Y \leq 3) = 1 - \left[\frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!} + \frac{e^{-1} 1^2}{2!} + \frac{e^{-1} 1^3}{3!} \right] = 0.01899$$

- 4-89. a) From Exercise 4-88, $P(Y > 3) = 0.01899$. Let W denote the number of one-hour intervals out of 30 that contain more than 3 arrivals. By the memoryless property of a Poisson process, W is a binomial random variable with $n = 30$ and $p = 0.01899$.

$$P(W = 0) = \binom{30}{0} 0.01899^0 (1 - 0.01899)^{30} = 0.5626$$

- b) Let X denote the time between arrivals. Then, X is an exponential random variable with $\lambda = 1$ arrivals per hour. $P(X > x) = 0.1$ and $P(X > x) = \int_x^{\infty} 1e^{-1t} dt = -e^{-1t} \Big|_x^{\infty} = e^{-1x} = 0.1$. Therefore, $x = 2.3$ hours.

- 4-90. Let X denote the number of calls in 30 minutes. Because the time between calls is an exponential random variable, X is a Poisson random variable with $\lambda = 1/E(X) = 0.1$ calls per minute = 3 calls per 30 minutes.

$$a) P(X > 3) = 1 - P(X \leq 3) = 1 - \left[\frac{e^{-3} 3^0}{0!} + \frac{e^{-3} 3^1}{1!} + \frac{e^{-3} 3^2}{2!} + \frac{e^{-3} 3^3}{3!} \right] = 0.3528$$

$$b) P(X = 0) = \frac{e^{-3} 3^0}{0!} = 0.04979$$

- c) Let Y denote the time between calls in minutes. Then, $P(Y \geq x) = 0.01$ and

$$P(Y \geq x) = \int_x^{\infty} 0.1e^{-0.1t} dt = -e^{-0.1t} \Big|_x^{\infty} = e^{-0.1x}. \text{ Therefore, } e^{-0.1x} = 0.01 \text{ and } x = 46.05 \text{ minutes.}$$

- 4-91. a) From Exercise 4-90, $P(Y > 120) = \int_{120}^{\infty} 0.1e^{-0.1y} dy = -e^{-0.1y} \Big|_{120}^{\infty} = e^{-12} = 6.14 \times 10^{-6}$.

- b) Because the calls are a Poisson process, the number of calls in disjoint intervals are independent. From Exercise 4-90 part b., the probability of no calls in one-half hour is $e^{-3} = 0.04979$. Therefore, the answer is $[e^{-3}]^4 = e^{-12} = 6.14 \times 10^{-6}$. Alternatively, the answer is the probability of no calls in two hours. From part a. of this exercise, this is e^{-12} .

- c) Because a Poisson process is memoryless, probabilities do not depend on whether or not intervals are consecutive. Therefore, parts a. and b. have the same answer.

- 4-92. a) $P(X > \theta) = \int_{\theta}^{\infty} \frac{1}{\theta} e^{-x/\theta} dx = -e^{-x/\theta} \Big|_{\theta}^{\infty} = e^{-1} = 0.3679$

$$b) P(X > 2\theta) = -e^{-x/\theta} \Big|_{2\theta}^{\infty} = e^{-2} = 0.1353$$

$$c) P(X > 3\theta) = -e^{-x/\theta} \Big|_{3\theta}^{\infty} = e^{-3} = 0.0498$$

- d) The results do not depend on θ .

4-93. X is an exponential random variable with $\lambda = 0.2$ flaws per meter.

a) $E(X) = 1/\lambda = 5$ meters.

b) $P(X > 10) = \int_{10}^{\infty} 0.2e^{-0.2x} dx = -e^{-0.2x} \Big|_{10}^{\infty} = e^{-2} = 0.1353$

c) No, see Exercise 4-91 part c.

d) $P(X < x) = 0.90$. Then, $P(X < x) = -e^{-0.2x} \Big|_0^x = 1 - e^{-0.2x}$. Therefore, $1 - e^{-0.2x} = 0.9$ and $x = 11.51$.

4-94. $P(X > 8) = \int_8^{\infty} 0.2e^{-0.2x} dx = -e^{-8/5} = 0.2019$

The distance between successive flaws is either less than 8 meters or not. The distances are independent and $P(X > 8) = 0.2019$. Let Y denote the number of flaws until the distance exceeds 8 meters. Then, Y is a geometric random variable with $p = 0.2019$.

a) $P(Y = 5) = (1 - 0.2019)^4 0.2019 = 0.0819$.

b) $E(Y) = 1/0.2019 = 4.95$.

4-95. $E(X) = \int_0^{\infty} x\lambda e^{-\lambda x} dx$. Use integration by parts with $u = x$ and $dv = \lambda e^{-\lambda x}$.

Then, $E(X) = -xe^{-\lambda x} \Big|_0^{\infty} + \int_0^{\infty} e^{-\lambda x} dx = \frac{-e^{-\lambda x}}{\lambda} \Big|_0^{\infty} = 1/\lambda$

$V(X) = \int_0^{\infty} (x - \frac{1}{\lambda})^2 \lambda e^{-\lambda x} dx$. Use integration by parts with $u = (x - \frac{1}{\lambda})^2$ and

$dv = \lambda e^{-\lambda x}$. Then,

$V(X) = -(x - \frac{1}{\lambda})^2 e^{-\lambda x} \Big|_0^{\infty} + 2 \int_0^{\infty} (x - \frac{1}{\lambda}) e^{-\lambda x} dx = (\frac{1}{\lambda})^2 + \frac{2}{\lambda} \int_0^{\infty} (x - \frac{1}{\lambda}) \lambda e^{-\lambda x} dx$

The last integral is seen to be zero from the definition of $E(X)$. Therefore, $V(X) = (\frac{1}{\lambda})^2$.

Section 4-10

4-96 a) The time until the tenth call is an Erlang random variable with $\lambda = 5$ calls per minute and $r = 10$.

b) $E(X) = 10/5 = 2$ minutes. $V(X) = 10/25 = 0.4$ minutes.

c) Because a Poisson process is memoryless, the mean time is $1/5 = 0.2$ minutes or 12 seconds

- 4-97. Let Y denote the number of calls in one minute. Then, Y is a Poisson random variable with $\lambda = \frac{45-65}{5}$ calls per minute.

$$a) P(Y = 4) = \frac{e^{-5} 5^4}{4!} = 0.1755$$

$$b) P(Y > 2) = 1 - P(Y \leq 2) = 1 - \frac{e^{-5} 5^0}{0!} - \frac{e^{-5} 5^1}{1!} - \frac{e^{-5} 5^2}{2!} = 0.8754.$$

Let W denote the number of one minute intervals out of 10 that contain more than 2 calls. Because the

calls are a Poisson process, W is a binomial random variable with $n = 10$ and $p = 0.8754$.

$$\text{Therefore, } P(W = 10) = \binom{10}{10} 0.8754^{10} (1 - 0.8754)^0 = 0.2643.$$

- 4-98. Let X denote the pounds of material to obtain 15 particles. Then, X has an Erlang distribution with $r = 15$ and $\lambda = 0.01$.

$$a) E(X) = \frac{r}{\lambda} = \frac{15}{0.01} = 1500 \text{ pounds.}$$

$$b) V(X) = \frac{15}{0.01^2} = 150,000 \text{ and } \sigma_X = \sqrt{150,000} = 387.3 \text{ pounds.}$$

- 4-99 Let X denote the time between failures of a laser. X is exponential with a mean of 25,000.

a.) Expected time until the second failure $E(X) = r / \lambda = 2 / 0.00004 = 50,000$ hours

b.) N=no of failures in 50000 hours

$$E(N) = \frac{50000}{25000} = 2$$

$$P(N \leq 2) = \sum_{k=0}^2 \frac{e^{-2} (2)^k}{k!} = 0.6767$$

- 4-100 Let X denote the time until 5 messages arrive at a node. Then, X has an Erlang distribution with $r = 5$ and $\lambda = 30$ messages per minute.

a) $E(X) = 5/30 = 1/6$ minute = 10 seconds.

b) $V(X) = \frac{5}{30^2} = 1/180$ minute² = 1/3 second and $\sigma_X = 0.0745$ minute = 4.472 seconds.

c) Let Y denote the number of messages that arrive in 10 seconds. Then, Y is a Poisson random variable with $\lambda = 30$ messages per minute = 5 messages per 10 seconds.

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - \left[\frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!} + \frac{e^{-5} 5^3}{3!} + \frac{e^{-5} 5^4}{4!} \right]$$

$$= 0.5595$$

d) Let Y denote the number of messages that arrive in 5 seconds. Then, Y is a Poisson random variable with

$\lambda = 2.5$ messages per 5 seconds.

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - 0.8912 = 0.1088$$

4-101. Let X denote the number of bits until five errors occur. Then, X has an Erlang distribution with $r = 5$ and $\lambda = 10^{-5}$ error per bit.

a) $E(X) = \frac{r}{\lambda} = 5 \times 10^5$ bits.

b) $V(X) = \frac{r}{\lambda^2} = 5 \times 10^{10}$ and $\sigma_X = \sqrt{5 \times 10^{10}} = 223607$ bits.

c) Let Y denote the number of errors in 10^5 bits. Then, Y is a Poisson random variable with $\lambda = 1/10^5 = 10^{-5}$ error per bit = 1 error per 10^5 bits.

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \left[\frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!} + \frac{e^{-1} 1^2}{2!} \right] = 0.0803$$

4-102 $\lambda = 1/20 = 0.05$ $r = 100$

a) $E(X) = r/\lambda = 100/0.05 = 2000$ minutes

b) 4 min - 2.5 min = 1.5 min

c) Let Y be the number of calls before 15 seconds $\lambda = 0.25 \times 20 = 5$

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \left[\frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!} \right] = 1 - 0.1247 = 0.8753$$

4-103. a) Let X denote the number of customers that arrive in 10 minutes. Then, X is a Poisson random variable with $\lambda = 0.2$ arrivals per minute = 2 arrivals per 10 minutes.

$$P(X > 3) = 1 - P(X \leq 3) = 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} 2^3}{3!} \right] = 0.1429$$

b) Let Y denote the number of customers that arrive in 15 minutes. Then, Y is a Poisson random variable with $\lambda = 3$ arrivals per 15 minutes.

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - \left[\frac{e^{-3} 3^0}{0!} + \frac{e^{-3} 3^1}{1!} + \frac{e^{-3} 3^2}{2!} + \frac{e^{-3} 3^3}{3!} + \frac{e^{-3} 3^4}{4!} \right] = 0.1847$$

4-104. Let X denote the time in days until the fourth problem. Then, X has an Erlang distribution with $r = 4$ and $\lambda = 1/30$ problem per day.

a) $E(X) = \frac{4}{30^{-1}} = 120$ days.

b) Let Y denote the number of problems in 120 days. Then, Y is a Poisson random variable with $\lambda = 4$ problems per 120 days.

$$P(Y < 4) = \left[\frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!} + \frac{e^{-4} 4^2}{2!} + \frac{e^{-4} 4^3}{3!} \right] = 0.4335$$

4-105. a) $\Gamma(6) = 5! = 120$

b) $\Gamma(\frac{5}{2}) = \frac{3}{2} \Gamma(\frac{3}{2}) = \frac{3}{2} \cdot \frac{1}{2} \Gamma(\frac{1}{2}) = \frac{3}{4} \pi^{1/2} = 1.32934$

c) $\Gamma(\frac{9}{2}) = \frac{7}{2} \Gamma(\frac{7}{2}) = \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \Gamma(\frac{1}{2}) = \frac{105}{16} \pi^{1/2} = 11.6317$

4-106 $\Gamma(r) = \int_0^{\infty} x^{r-1} e^{-x} dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$$\Gamma(r) = -x^{r-1} e^{-x} \Big|_0^{\infty} + (r-1) \int_0^{\infty} x^{r-2} e^{-x} dx = (r-1) \Gamma(r-1).$$

4-107 $\int_0^{\infty} f(x; \lambda, r) dx = \int_0^{\infty} \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)} dx$. Let $y = \lambda x$, then the integral is $\int_0^{\infty} \frac{\lambda y^{r-1} e^{-y}}{\Gamma(r)} \frac{dy}{\lambda}$. From the definition of $\Gamma(r)$, this integral is recognized to equal 1.

4-108. If X is a chi-square random variable, then X is a special case of a gamma random variable. Now,

$$E(X) = \frac{r}{\lambda} = \frac{(7/2)}{(1/2)} = 7 \text{ and } V(X) = \frac{r}{\lambda^2} = \frac{(7/2)}{(1/2)^2} = 14.$$

Section 4-11

4-109. $\beta=0.2$ and $\delta=100$ hours

$$E(X) = 100\Gamma(1 + \frac{1}{0.2}) = 100 \times 5! = 12,000$$

$$V(X) = 100^2 \Gamma(1 + \frac{2}{0.2}) - 100^2 [\Gamma(1 + \frac{1}{0.2})]^2 = 3.61 \times 10^{10}$$

4-110. a) $P(X < 10000) = F_X(10000) = 1 - e^{-100^{0.2}} = 1 - e^{-2.512} = 0.9189$

b) $P(X > 5000) = 1 - F_X(5000) = e^{-50^{0.2}} = 0.1123$

4-111. Let X denote lifetime of a bearing. $\beta=2$ and $\delta=10000$ hours

a) $P(X > 8000) = 1 - F_X(8000) = e^{-\left(\frac{8000}{10000}\right)^2} = e^{-0.8^2} = 0.5273$

b)

$$\begin{aligned} E(X) &= 10000\Gamma(1 + \frac{1}{2}) = 10000\Gamma(1.5) \\ &= 10000(0.5)\Gamma(0.5) = 5000\sqrt{\pi} = 8862.3 \\ &= 8862.3 \text{ hours} \end{aligned}$$

c) Let Y denote the number of bearings out of 10 that last at least 8000 hours. Then, Y is a binomial random variable with $n = 10$ and $p = 0.5273$.

$$P(Y = 10) = \binom{10}{10} 0.5273^{10} (1 - 0.5273)^0 = 0.00166.$$

4-112 a) $E(X) = \delta\Gamma(1 + \frac{1}{\beta}) = 900\Gamma(1 + 1/3) = 900\Gamma(4/3) = 900(0.89298) = 803.68 \text{ hours}$

b)

$$\begin{aligned} V(X) &= \delta^2 \Gamma(1 + \frac{2}{\beta}) - \delta^2 \left[\Gamma(1 + \frac{2}{\beta}) \right]^2 = 900^2 \Gamma(1 + \frac{2}{3}) - 900^2 \left[\Gamma(1 + \frac{1}{3}) \right]^2 \\ &= 900^2 (0.90274) - 900^2 (0.89298)^2 = 85319.64 \text{ hours}^2 \end{aligned}$$

c) $P(X < 500) = F_X(500) = 1 - e^{-\left(\frac{500}{900}\right)^3} = 0.1576$

4-113. Let X denote the lifetime.

a) $E(X) = \delta\Gamma(1 + \frac{1}{0.5}) = \delta\Gamma(3) = 2\delta = 600$. Then $\delta = 300$. Now,

$$P(X > 500) = e^{-\left(\frac{500}{300}\right)^{0.5}} = 0.2750$$

$$b) P(X < 400) = 1 - e^{-\left(\frac{400}{300}\right)^{0.5}} = 0.6848$$

4-114 Let X denote the lifetime

a) $E(X) = 700\Gamma(1 + \frac{1}{2}) = 620.4$

b)

$$\begin{aligned} V(X) &= 700^2\Gamma(2) - 700^2[\Gamma(1.5)]^2 \\ &= 700^2(1) - 700^2(0.25\pi) = 105,154.9 \end{aligned}$$

$$c) P(X > 620.4) = e^{-\left(\frac{620.4}{700}\right)^2} = 0.4559$$

4-115 a) $\beta = 2, \delta = 500$

$$\begin{aligned} E(X) &= 500\Gamma(1 + \frac{1}{2}) = 500\Gamma(1.5) \\ &= 500(0.5)\Gamma(0.5) = 250\sqrt{\pi} = 443.11 \\ &= 443.11 \text{ hours} \end{aligned}$$

$$\begin{aligned} b) V(X) &= 500^2\Gamma(2) - 500^2[\Gamma(1.5)]^2 \\ &= 500^2\Gamma(2) - 500^2[\Gamma(1.5)]^2 = 53650.5 \end{aligned}$$

$$c) P(X < 250) = F(250) = 1 - e^{-\left(\frac{250}{500}\right)^2} = 1 - 0.7788 = 0.2212$$

4-116 If X is a Weibull random variable with $\beta=1$ and $\delta=1000$, the distribution of X is the exponential distribution with $\lambda=.001$.

$$\begin{aligned} f(x) &= \left(\frac{1}{1000}\right)\left(\frac{x}{1000}\right)^0 e^{-\left(\frac{x}{1000}\right)^1} \text{ for } x > 0 \\ &= 0.001e^{-0.001x} \text{ for } x > 0 \end{aligned}$$

The mean of X is $E(X) = 1/\lambda = 1000$.

Section 4-11

4-117 X is a lognormal distribution with $\theta=5$ and $\omega^2=9$

a.)

$$\begin{aligned} P(X < 13300) &= P(e^W < 13300) = P(W < \ln(13300)) = \Phi\left(\frac{\ln(13300) - 5}{3}\right) \\ &= \Phi(1.50) = 0.9332 \end{aligned}$$

b.) Find the value for which $P(X \leq x) = 0.95$

$$P(X \leq x) = P(e^W \leq x) = P(W \leq \ln(x)) = \Phi\left(\frac{\ln(x) - 5}{3}\right) = 0.95$$

$$\frac{\ln(x) - 5}{3} = 1.65 \quad x = e^{1.65(3) + 5} = 20952.2$$

$$c.) \mu = E(X) = e^{\theta + \omega^2 / 2} = e^{5 + 9 / 2} = e^{9.5} = 13359.7$$

$$V(X) = e^{2\theta + \omega^2} (e^{\omega^2} - 1) = e^{10 + 9} (e^9 - 1) = e^{19} (e^9 - 1) = 1.45 \times 10^{12}$$

4-118 a.) X is a lognormal distribution with $\theta=-2$ and $\omega^2=9$

$$\begin{aligned} P(500 < X < 1000) &= P(500 < e^W < 1000) = P(\ln(500) < W < \ln(1000)) \\ &= \Phi\left(\frac{\ln(1000) + 2}{3}\right) - \Phi\left(\frac{\ln(500) + 2}{3}\right) = \Phi(2.97) - \Phi(2.74) = 0.0016 \end{aligned}$$

$$b.) P(X < x) = P(e^W \leq x) = P(W \leq \ln(x)) = \Phi\left(\frac{\ln(x) + 2}{3}\right) = 0.1$$

$$\frac{\ln(x) + 2}{3} = -1.28 \quad x = e^{-1.28(3) - 2} = 0.0029$$

$$c.) \mu = E(X) = e^{\theta + \omega^2 / 2} = e^{-2 + 9 / 2} = e^{2.5} = 12.1825$$

$$V(X) = e^{2\theta + \omega^2} (e^{\omega^2} - 1) = e^{-4 + 9} (e^9 - 1) = e^5 (e^9 - 1) = 1,202,455.87$$

4-119 a.) X is a lognormal distribution with $\theta=2$ and $\omega^2=4$

$$P(X < 500) = P(e^W < 500) = P(W < \ln(500)) = \Phi\left(\frac{\ln(500) - 2}{2}\right) \\ = \Phi(2.11) = 0.9826$$

b.)

$$P(X < 15000 | X > 1000) = \frac{P(1000 < X < 15000)}{P(X > 1000)} \\ = \frac{\left[\Phi\left(\frac{\ln(15000) - 2}{2}\right) - \Phi\left(\frac{\ln(1000) - 2}{2}\right) \right]}{\left[1 - \Phi\left(\frac{\ln(1000) - 2}{2}\right) \right]} \\ = \frac{\Phi(2.66) - \Phi(2.45)}{(1 - \Phi(2.45))} = \frac{0.9961 - 0.9929}{(1 - 0.9929)} = 0.0032 / 0.007 = 0.45$$

c.) The product has degraded over the first 1000 hours, so the probability of it lasting another 500 hours is very low.

4-120 X is a lognormal distribution with $\theta=0.5$ and $\omega^2=1$

a)

$$P(X > 10) = P(e^W > 10) = P(W > \ln(10)) = 1 - \Phi\left(\frac{\ln(10) - 0.5}{1}\right) \\ = 1 - \Phi(1.80) = 1 - 0.96407 = 0.03593$$

$$b.) \quad P(X \leq x) = P(e^W \leq x) = P(W \leq \ln(x)) = \Phi\left(\frac{\ln(x) - 0.5}{1}\right) = 0.50$$

$$\frac{\ln(x) - 0.5}{1} = 0 \quad x = e^{0(1) + 0.5} = 1.65 \text{ seconds}$$

$$c.) \quad \mu = E(X) = e^{\theta + \omega^2/2} = e^{0.5 + 1/2} = e^1 = 2.7183$$

$$V(X) = e^{2\theta + \omega^2} (e^{\omega^2} - 1) = e^{1+1} (e^1 - 1) = e^2 (e^1 - 1) = 12.6965$$

4-121 Find the values of θ and ω^2 given that $E(X) = 100$ and $V(X) = 85,000$

$$100 = e^{\theta + \omega^2 / 2} \quad 85000 = e^{2\theta + \omega^2} (e^{\omega^2} - 1)$$

$$\text{let } x = e^\theta \text{ and } y = e^{\omega^2} \text{ then (1) } 100 = x\sqrt{y} \text{ and (2) } 85000 = x^2 y(y-1) = x^2 y^2 - x^2 y$$

$$\text{Square (1) } 10000 = x^2 y \text{ and substitute into (2)}$$

$$85000 = 10000 (y - 1)$$

$$y = 9.5$$

$$\text{Substitute } y \text{ into (1) and solve for } x \quad x = \frac{100}{\sqrt{9.5}} = 32.444$$

$$\theta = \ln(32.444) = 3.48 \text{ and } \omega^2 = \ln(9.5) = 2.25$$

4-122 a.) Find the values of θ and ω^2 given that $E(X) = 10000$ and $\sigma = 20,000$

$$10000 = e^{\theta + \omega^2 / 2} \quad 20000^2 = e^{2\theta + \omega^2} (e^{\omega^2} - 1)$$

$$\text{let } x = e^\theta \text{ and } y = e^{\omega^2} \text{ then (1) } 10000 = x\sqrt{y} \text{ and}$$

$$(2) 20000^2 = x^2 y(y-1) = x^2 y^2 - x^2 y$$

$$\text{Square (1) } 10000^2 = x^2 y \text{ and substitute into (2)}$$

$$20000^2 = 10000^2 (y - 1)$$

$$y = 5$$

$$\text{Substitute } y \text{ into (1) and solve for } x \quad x = \frac{10000}{\sqrt{5}} = 4472.1360$$

$$\theta = \ln(4472.1360) = 8.4056 \text{ and } \omega^2 = \ln(5) = 1.6094$$

b.)

$$P(X > 10000) = P(e^W > 10000) = P(W > \ln(10000)) = 1 - \Phi\left(\frac{\ln(10000) - 8.4056}{1.2686}\right)$$

$$= 1 - \Phi(0.63) = 1 - 0.7357 = 0.2643$$

$$\text{c.) } P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\frac{\ln(x) - 8.4056}{1.2686}\right) = 0.1$$

$$\frac{\ln(x) - 8.4056}{1.2686} = -1.28 \quad x = e^{-1.280 (1.2686) + 8.4056} = 881.65 \text{ hours}$$

4-123 Let $X \sim N(\mu, \sigma^2)$, then $Y = e^X$ follows a lognormal distribution with mean μ and variance σ^2 . By definition, $F_Y(y) = P(Y \leq y) = P(e^X < y) = P(X < \log y) = F_X(\log y) = \Phi\left(\frac{\log y - \mu}{\sigma}\right)$.

Since $Y = e^X$ and $X \sim N(\mu, \sigma^2)$, we can show that $f_Y(Y) = \frac{1}{y} f_X(\log y)$

$$\text{Finally, } f_Y(y) = \frac{\partial F_Y(y)}{\partial y} = \frac{\partial F_X(\log y)}{\partial y} = \frac{1}{y} f_X(\log y) = \frac{1}{y} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\log y - \mu}{2\sigma}\right)^2}.$$

Supplemental Exercises

$$4-124 \quad a) P(X < 2.5) = \int_2^{2.5} (0.5x - 1) dx = \left(0.5 \frac{x^2}{2} - x \right) \Big|_2^{2.5} = 0.0625$$

$$b) P(X > 3) = \int_3^4 (0.5x - 1) dx = 0.5 \frac{x^2}{2} - x \Big|_3^4 = 0.75$$

$$c) P(2.5 < X < 3.5) = \int_{2.5}^{3.5} (0.5x - 1) dx = 0.5 \frac{x^2}{2} - x \Big|_{2.5}^{3.5} = 0.5$$

$$4-125 \quad F(x) = \int_2^x (0.5t - 1) dt = 0.5 \frac{t^2}{2} - t \Big|_2^x = \frac{x^2}{4} - x + 1. \text{ Then,}$$

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{x^2}{4} - x + 1, & 2 \leq x < 4 \\ 1, & 4 \leq x \end{cases}$$

$$4-126 \quad E(X) = \int_2^4 x(0.5x - 1) dx = 0.5 \frac{x^3}{3} - \frac{x^2}{2} \Big|_2^4 = \frac{32}{3} - 8 - \left(\frac{4}{3} - 2 \right) = \frac{10}{3}$$

$$\begin{aligned} V(X) &= \int_2^4 \left(x - \frac{10}{3}\right)^2 (0.5x - 1) dx = \int_2^4 \left(x^2 - \frac{20}{3}x + \frac{100}{9}\right)(0.5x - 1) dx \\ &= \int_2^4 \left(0.5x^3 - \frac{13}{3}x^2 + \frac{110}{9}x - \frac{100}{9}\right) dx = \frac{x^4}{8} - \frac{13}{9}x^3 + \frac{55}{9}x^2 - \frac{100}{9}x \Big|_2^4 \\ &= 0.2222 \end{aligned}$$

4-127. Let X denote the time between calls. Then, $\lambda = 1/E(X) = 0.1$ calls per minute.

$$\text{a) } P(X < 5) = \int_0^5 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_0^5 = 1 - e^{-0.5} = 0.3935$$

$$\text{b) } P(5 < X < 15) = -e^{-0.1x} \Big|_5^{15} = e^{-0.5} - e^{-1.5} = 0.3834$$

$$\text{c) } P(X < x) = 0.9. \text{ Then, } P(X < x) = \int_0^x 0.1e^{-0.1t} dt = 1 - e^{-0.1x} = 0.9. \text{ Now, } x = 23.03 \text{ minutes.}$$

4-128 a) This answer is the same as part a. of Exercise 4-127.

$$P(X < 5) = \int_0^5 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_0^5 = 1 - e^{-0.5} = 0.3935$$

b) This is the probability that there are no calls over a period of 5 minutes. Because a Poisson process is memoryless, it does not matter whether or not the intervals are consecutive.

$$P(X > 5) = \int_5^{\infty} 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_5^{\infty} = e^{-0.5} = 0.6065$$

4-129. a) Let Y denote the number of calls in 30 minutes. Then, Y is a Poisson random variable with

$$\lambda = 3. \quad P(Y \leq 2) = \frac{e^{-3} 3^0}{0!} + \frac{e^{-3} 3^1}{1!} + \frac{e^{-3} 3^2}{2!} = 0.423.$$

b) Let W denote the time until the fifth call. Then, W has an Erlang distribution with $P\left(Z < \frac{x-60}{5}\right)$ and $r = 5$.

$$E(W) = 5/0.1 = 50 \text{ minutes.}$$

4-130 Let X denote the lifetime. Then $\lambda = 1/E(X) = 1/6$.

$$P(X < 3) = \int_0^3 \frac{1}{6} e^{-x/6} dx = -e^{-x/6} \Big|_0^3 = 1 - e^{-0.5} = 0.3935.$$

4-131. Let W denote the number of CPUs that fail within the next three years. Then, W is a binomial random variable with $n = 10$ and $p = 0.3935$ (from Exercise 4-130). Then,

$$P(W \geq 1) = 1 - P(W = 0) = 1 - \binom{10}{0} 0.3935^0 (1 - 0.3935)^{10} = 0.9933.$$

4-132 X is a lognormal distribution with $\theta=0$ and $\omega^2=4$

a.)

$$\begin{aligned} P(10 < X < 50) &= P(10 < e^W < 50) = P(\ln(10) < W < \ln(50)) \\ &= \Phi\left(\frac{\ln(50)-0}{2}\right) - \Phi\left(\frac{\ln(10)-0}{2}\right) \\ &= \Phi(1.96) - \Phi(1.15) = 0.975002 - 0.874928 = 0.10007 \end{aligned}$$

$$\text{b.) } P(X < x) = P(e^W < x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-0}{2}\right) = 0.05$$

$$\frac{\ln(x)-0}{2} = -1.64 \quad x = e^{-1.64(2)} = 0.0376$$

$$\text{c.) } \mu = E(X) = e^{\theta+\omega^2/2} = e^{0+4/2} = e^2 = 7.389$$

$$V(X) = e^{2\theta+\omega^2}(e^{\omega^2} - 1) = e^{0+4}(e^4 - 1) = e^4(e^4 - 1) = 2926.40$$

4-133 a.) Find the values of θ and ω^2 given that $E(X) = 50$ and $V(X) = 4000$

$$50 = e^{\theta+\omega^2/2} \quad 4000 = e^{2\theta+\omega^2}(e^{\omega^2} - 1)$$

$$\text{let } x = e^\theta \text{ and } y = e^{\omega^2} \text{ then (1) } 50 = x\sqrt{y} \text{ and (2) } 4000 = x^2y(y-1) = x^2y^2 - x^2y$$

$$\text{Square (1) for } x \quad x = \frac{50}{\sqrt{y}} \text{ and substitute into (2)}$$

$$4000 = \left(\frac{50}{\sqrt{y}}\right)^2 y^2 - \left(\frac{50}{\sqrt{y}}\right)^2 y = 2500y - 2500$$

$$y = 2.6$$

$$\text{substitute } y \text{ back in to (1) and solve for } x \quad x = \frac{50}{\sqrt{2.6}} = 31$$

$$\theta = \ln(31) = 3.43 \text{ and } \omega^2 = \ln(2.6) = 0.96$$

b.)

$$\begin{aligned} P(X < 150) &= P(e^W < 150) = P(W < \ln(150)) = \Phi\left(\frac{\ln(150)-3.43}{0.98}\right) \\ &= \Phi(1.61) = 0.946301 \end{aligned}$$

4-134 Let X denote the number of fibers visible in a grid cell. Then, X has a Poisson distribution and $\lambda = 100 \text{ fibers per cm}^2 = 80,000 \text{ fibers per sample} = 0.5 \text{ fibers per grid cell}$.

a) $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{e^{-0.5} 0.5^0}{0!} = 0.3935$.

b) Let W denote the number of grid cells examined until 10 contain fibers. If the number of fibers have a Poisson distribution, then the number of fibers in each grid cell are independent. Therefore, W has a negative binomial distribution with $p = 0.3935$. Consequently, $E(W) = 10/0.3935 = 25.41$ cells.

c) $V(W) = \frac{10(1 - 0.3935)}{0.3935^2}$. Therefore, $\sigma_W = 6.25$ cells.

4-135. Let X denote the height of a plant.

a) $P(X > 2.25) = P\left(Z > \frac{2.25 - 2.5}{0.5}\right) = P(Z > -0.5) = 1 - P(Z \leq -0.5) = 0.6915$

b) $P(2.0 < X < 3.0) = P\left(\frac{2.0 - 2.5}{0.5} < Z < \frac{3.0 - 2.5}{0.5}\right) = P(-1 < Z < 1) = 0.683$

c.) $P(X > x) = 0.90 = P\left(Z > \frac{x - 2.5}{0.5}\right) = 0.90$ and $\frac{x - 2.5}{0.5} = -1.28$.

Therefore, $x = 1.86$.

4-136 a) $P(X > 3.5)$ from part a. of Exercise 4-135 is 0.023.

b) Yes, because the probability of a plant growing to a height of 3.5 centimeters or more without irrigation is low.

4-137. Let X denote the thickness.

a) $P(X > 5.5) = P\left(Z > \frac{5.5 - 5}{0.2}\right) = P(Z > 2.5) = 0.0062$

b) $P(4.5 < X < 5.5) = P\left(\frac{4.5 - 5}{0.2} < Z < \frac{5.5 - 5}{0.2}\right) = P(-2.5 < Z < 2.5) = 0.9876$

Therefore, the proportion that do not meet specifications is

$1 - P(4.5 < X < 5.5) = 0.012$.

c) If $P(X < x) = 0.95$, then $P\left(Z > \frac{x - 5}{0.2}\right) = 0.95$. Therefore, $\frac{x - 5}{0.2} = 1.65$ and $x = 5.33$.

4-138 Let X denote the dot diameter. If $P(0.0014 < X < 0.0026) = 0.9973$, then

$P\left(\frac{0.0014 - 0.002}{\sigma} < Z < \frac{0.0026 - 0.002}{\sigma}\right) = P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right) = 0.9973$.

Therefore, $\frac{0.0006}{\sigma} = 3$ and $\sigma = 0.0002$.

4-139. If $P(0.002-x < X < 0.002+x)$, then $P(-x/0.0004 < Z < x/0.0004) = 0.9973$. Therefore, $x/0.0004 = 3$ and $x = 0.0012$. The specifications are from 0.0008 to 0.0032.

4-140 Let X denote the life.

a) $P(X < 5800) = P(Z < \frac{5800-7000}{600}) = P(Z < -2) = 1 - P(Z \leq 2) = 0.023$

d) If $P(X > x) = 0.9$, then $P(Z < \frac{x-7000}{600}) = -1.28$. Consequently, $\frac{x-7000}{600} = -1.28$ and $x = 6232$ hours.

4-141. If $P(X > 10,000) = 0.99$, then $P(Z > \frac{10,000-\mu}{600}) = 0.99$. Therefore, $\frac{10,000-\mu}{600} = -2.33$ and $\mu = 11,398$.

4-142 The probability a product lasts more than 10000 hours is $[P(X > 10000)]^3$, by independence. If $[P(X > 10000)]^3 = 0.99$, then $P(X > 10000) = 0.9967$.

Then, $P(X > 10000) = P(Z > \frac{10000-\mu}{600}) = 0.9967$. Therefore, $\frac{10000-\mu}{600} = -2.72$ and $\mu = 11,632$ hours.

4-143 X is an exponential distribution with $E(X) = 7000$ hours

a.) $P(X < 5800) = \int_0^{5800} \frac{1}{7000} e^{-\frac{x}{7000}} dx = 1 - e^{-\left(\frac{5800}{7000}\right)} = 0.5633$

b.) $P(X > x) = \int_x^{\infty} \frac{1}{7000} e^{-\frac{x}{7000}} dx = 0.9$ Therefore, $e^{-\frac{x}{7000}} = 0.9$ and $x = -7000 \ln(0.9) = 737.5$ hours

4-144 Find the values of θ and ω^2 given that $E(X) = 7000$ and $\sigma = 600$

$$7000 = e^{\theta + \omega^2 / 2} \quad 600^2 = e^{2\theta + \omega^2} (e^{\omega^2} - 1)$$

let $x = e^\theta$ and $y = e^{\omega^2}$ then (1) $7000 = x\sqrt{y}$ and

$$(2) 600^2 = x^2 y (y - 1) = x^2 y^2 - x^2 y$$

Square (1) $7000^2 = x^2 y$ and substitute into (2)

$$600^2 = 7000^2 (y - 1)$$

$$y = 1.0073$$

Substitute y into (1) and solve for x $x = \frac{7000}{\sqrt{1.0073}} = 6974.6$

$$\theta = \ln(6974.6) = 8.850 \quad \text{and} \quad \omega^2 = \ln(1.0073) = 0.0073$$

a)

$$\begin{aligned} P(X < 5800) &= P(e^W < 5800) = P(W < \ln(5800)) = \Phi\left(\frac{\ln(5800) - 8.85}{0.0854}\right) \\ &= \Phi(-2.16) = 0.015 \end{aligned}$$

$$\text{b) } P(X > x) = P(e^W > x) = P(W > \ln(x)) = 1 - \Phi\left(\frac{\ln(x) - 8.85}{0.0854}\right) = 0.9$$

$$\frac{\ln(x) - 8.85}{0.0854} = -1.28 \quad x = e^{-1.28(0.0854) + 8.85} = 6252.20 \text{ hours}$$

4-145. a) Using the normal approximation to the binomial with $n = 50 \cdot 36 \cdot 36 = 64,800$, and $p = 0.0001$ we have: $E(X) = 64800(0.0001) = 6.48$

$$\begin{aligned} P(X \geq 1) &\cong P\left(\frac{X - np}{\sqrt{np(1-p)}} \geq \frac{1 - 6.48}{\sqrt{64800(0.0001)(0.9999)}}\right) \\ &= P(Z > -2.15) = 1 - 0.01578 = 0.98422 \end{aligned}$$

b)

$$\begin{aligned} P(X \geq 4) &\cong P\left(\frac{X - np}{\sqrt{np(1-p)}} \geq \frac{4 - 6.48}{\sqrt{64800(0.0001)(0.9999)}}\right) \\ &= P(Z \geq -0.97) = 1 - 0.166023 = 0.8340 \end{aligned}$$

- 4-146 Using the normal approximation to the binomial with X being the number of people who will be seated.

$X \sim \text{Bin}(200, 0.9)$.

$$\text{a) } P(X \leq 185) = P\left(\frac{X - np}{\sqrt{np(1-p)}} \geq \frac{185 - 180}{\sqrt{200(0.9)(0.1)}}\right) = P(Z \leq 1.18) = 0.8810$$

b)

$$P(X < 185)$$

$$= P(X \leq 184) = P\left(\frac{X - np}{\sqrt{np(1-p)}} \geq \frac{184 - 180}{\sqrt{200(0.9)(0.1)}}\right) = P(Z \leq 0.94) = 0.8264$$

c) $P(X \leq 185) \cong 0.95$, Successively trying various values of n: The number of reservations taken could be reduced to about 198.

n	Z_0	Probability $P(Z < Z_0)$
190	3.39	0.99965
195	2.27	0.988396
198	1.61	0.946301

Mind-Expanding Exercises

- 4-147. a) $P(X > x)$ implies that there are $r - 1$ or less counts in an interval of length x . Let Y denote the number of counts in an interval of length x . Then, Y is a Poisson random variable with parameter

$$\lambda x. \text{ Then, } P(X > x) = P(Y \leq r - 1) = \sum_{i=0}^{r-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!}.$$

$$\text{b) } P(X \leq x) = 1 - \sum_{i=0}^{r-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!}$$

$$\text{c) } f_X(x) = \frac{d}{dx} F_X(x) = \lambda e^{-\lambda x} \sum_{i=0}^{r-1} \frac{(\lambda x)^i}{i!} - e^{-\lambda x} \sum_{i=0}^{r-1} \lambda i \frac{(\lambda x)^i}{i!} = \lambda e^{-\lambda x} \frac{(\lambda x)^{r-1}}{(r-1)!}$$

- 4-148. Let X denote the diameter of the maximum diameter bearing. Then, $P(X > 1.6) = 1 - P(X \leq 1.6)$.

Also, $X \leq 1.6$ if and only if all the diameters are less than 1.6. Let Y denote the diameter of a bearing. Then, by independence

$$P(X \leq 1.6) = [P(Y \leq 1.6)]^{10} = \left[P\left(Z \leq \frac{1.6-1.5}{0.025}\right)\right]^{10} = 0.999967^{10} = 0.99967$$

Then, $P(X > 1.575) = 0.0033$.

4-149. a) Quality loss = $Ek(X - m)^2 = kE(X - m)^2 = k\sigma^2$, by the definition of the variance.

b)

$$\begin{aligned}\text{Quality loss} &= Ek(X - m)^2 = kE(X - \mu + \mu - m)^2 \\ &= kE[(X - \mu)^2 + (\mu - m)^2 + 2(\mu - m)(X - \mu)] \\ &= kE(X - \mu)^2 + k(\mu - m)^2 + 2k(\mu - m)E(X - \mu).\end{aligned}$$

The last term equals zero by the definition of the mean.

Therefore, quality loss = $k\sigma^2 + k(\mu - m)^2$.

4-150. Let X denote the event that an amplifier fails before 60,000 hours. Let A denote the event that an amplifier mean is 20,000 hours. Then A' is the event that the mean of an amplifier is 50,000 hours. Now, $P(E) = P(E|A)P(A) + P(E|A')P(A')$ and

$$\begin{aligned}P(E | A) &= \int_0^{60,000} \frac{1}{20,000} e^{-x/20,000} dx = -e^{-x/20,000} \Big|_0^{60,000} = 1 - e^{-3} = 0.9502 \\ P(E | A') &= -e^{-x/50,000} \Big|_0^{60,000} = 1 - e^{-6/5} = 0.6988.\end{aligned}$$

Therefore, $P(E) = 0.9502(0.10) + 0.6988(0.90) = 0.7239$

4-151. $P(X < t_1 + t_2 | X > t_1) = \frac{P(t_1 < X < t_1 + t_2)}{P(X > t_1)}$ from the definition of conditional probability. Now,

$$P(t_1 < X < t_1 + t_2) = \int_{t_1}^{t_1 + t_2} \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big|_{t_1}^{t_1 + t_2} = e^{-\lambda t_1} - e^{-\lambda(t_1 + t_2)}$$

$$P(X > t_1) = -e^{-\lambda x} \Big|_{t_1}^{\infty} = e^{-\lambda t_1}$$

Therefore, $P(X < t_1 + t_2 | X > t_1) = \frac{e^{-\lambda t_1}(1 - e^{-\lambda t_2})}{e^{-\lambda t_1}} = 1 - e^{-\lambda t_2} = P(X < t_2)$

4-152. a)

$$1 - P(\mu_0 - 6\sigma < X < \mu_0 + 6\sigma) = 1 - P(-6 < Z < 6) \\ = 1.97 \times 10^{-9} = 0.00197 \text{ ppm}$$

b)

$$1 - P(\mu_0 - 6\sigma < X < \mu_0 + 6\sigma) = 1 - P(-7.5 < \frac{X - (\mu_0 + 1.5\sigma)}{\sigma} < 4.5) \\ = 3.4 \times 10^{-6} = 3.4 \text{ ppm}$$

c)

$$1 - P(\mu_0 - 3\sigma < X < \mu_0 + 3\sigma) = 1 - P(-3 < Z < 3) \\ = .0027 = 2,700 \text{ ppm}$$

d)

$$1 - P(\mu_0 - 3\sigma < X < \mu_0 + 3\sigma) = 1 - P(-4.5 < \frac{X - (\mu_0 + 1.5\sigma)}{\sigma} < 1.5) \\ = 0.0668106 = 66,810.6 \text{ ppm}$$

e) If the process is centered six standard deviations away from the specification limits and the process shifts, there will be significantly less product loss. If the process is centered only three standard deviations away from the specifications and the process shifts, there could be a great loss of product.

Section 4-8 on CD

S4-1. $E(X) = 50(0.1) = 5$ and $V(X) = 50(0.1)(0.9) = 4.5$

a) $P(X \leq 2) = P(X \leq 2.5) \cong P(Z \leq \frac{2.5-5}{\sqrt{4.5}}) = P(Z \leq -1.18) = 0.119$

b) $P(X \leq 2) \cong P(Z \leq \frac{2-2.5}{\sqrt{4.5}}) = P(Z \leq -0.24) = 0.4052$

c) $P(X \leq 2) = \binom{50}{0} 0.1^0 0.9^{50} + \binom{50}{1} 0.1^1 0.9^{49} + \binom{50}{2} 0.1^2 0.9^{48} = 0.118$

The probability computed using the continuity correction is closer.

d) $P(X < 10) = P(X \leq 9.5) \cong P\left(Z \leq \frac{9.5-5}{\sqrt{4.5}}\right) = P(Z \leq 2.12) = 0.983$

S4-2. $E(X) = 50(0.1) = 5$ and $V(X) = 50(0.1)(0.9) = 4.5$

a) $P(X \geq 2) = P(X \geq 1.5) \cong P(Z \leq \frac{1.5-5}{\sqrt{4.5}}) = P(Z \geq -1.65) = 0.951$

b) $P(X \geq 2) \cong P(Z \geq \frac{2-5}{\sqrt{4.5}}) = P(Z \geq -1.414) = 0.921$

c) $P(X \geq 2) = 1 - P(X < 2) = 1 - \left(\binom{50}{0} 0.1^0 0.9^{50} + \binom{50}{1} 0.1^1 0.9^{49}\right) = 0.966$

The probability computed using the continuity correction is closer.

d) $P(X > 6) = P(X \geq 7) = P(X \geq 6.5) \cong P(Z \geq 0.707) = 0.24$

S4-3. $E(X) = 50(0.1) = 5$ and $V(X) = 50(0.1)(0.9) = 4.5$

a)

$$\begin{aligned} P(2 \leq X \leq 5) &= P(1.5 \leq X \leq 5.5) \cong P\left(\frac{1.5-5}{\sqrt{4.5}} \leq Z \leq \frac{5.5-5}{\sqrt{4.5}}\right) \\ &= P(-1.65 \leq Z \leq 0.236) = P(Z \leq 0.24) - P(Z \leq -1.65) \\ &= 0.5948 - (1 - 0.95053) = 0.5453 \end{aligned}$$

b)

$$\begin{aligned} P(2 \leq X \leq 5) &\cong P\left(\frac{2-5}{\sqrt{4.5}} \leq Z \leq \frac{5-5}{\sqrt{4.5}}\right) = P(-1.414 \leq Z \leq 0) \\ &= 0.5 - P(Z \leq -1.414) \\ &= 0.5 - (1 - 0.921) = 0.421 \end{aligned}$$

The exact probability is 0.582

S4-4. $E(X) = 50(0.1) = 5$ and $V(X) = 50(0.1)(0.9) = 4.5$

a)

$$\begin{aligned} P(X = 10) &= P(9.5 \leq X \leq 10.5) \cong P\left(\frac{9.5-5}{\sqrt{4.5}} \leq Z \leq \frac{10.5-5}{\sqrt{4.5}}\right) \\ &= P(2.121 \leq Z \leq 2.593) = 0.012 \end{aligned}$$

b)

$$\begin{aligned} P(X = 5) &= P(4.5 \leq X \leq 5.5) \cong P\left(\frac{4.5-5}{\sqrt{4.5}} \leq Z \leq \frac{5.5-5}{\sqrt{4.5}}\right) \\ &= P(-0.24 \leq Z \leq 0.24) = 0.1897 \end{aligned}$$

S4-5 Let X be the number of chips in the lot that are defective. Then $E(X)=1000(0.02)=20$ and $V(X)=1000(0.02)(0.98)=19.6$

a) $P(20 \leq X \leq 30)=P(19.5 \leq X \leq 30.5) =$

$$P\left(\frac{19.5-20}{\sqrt{19.6}} \leq Z \leq \frac{30.5-20}{\sqrt{19.6}}\right) = P(-.11 \leq Z \leq 2.37) = 0.9911 - 0.4562 = 0.5349$$

b) $P(X=20)=P(19.5 \leq X \leq 20.5) =P(-0.11 \leq Z \leq 0.11)=0.5438-0.4562=0.0876.$

d) The answer should be close to the mean. Substituting values close to the mean, we find $x=20$ gives the maximum probability.