CHAPTER 3

Section 3-1

3-1. The range of X is $\{0,1,2,...,1000\}$

3-2 The range of X is $\{0,1,2,\ldots,50\}$

3-3. The range of X is $\{0,1,2,\ldots,99999\}$

3-4 The range of X is $\{0,1,2,3,4,5\}$

3-5. The range of X is $\{1,2,...,491\}$. Because 490 parts are conforming, a nonconforming part must be selected in 491 selections.

3-6 The range of X is $\{0,1,2,\ldots,100\}$. Although the range actually obtained from lots typically might not exceed 10%.

3-7. The range of X is conveniently modeled as all nonnegative integers. That is, the range of X is $\{0,1,2,\ldots\}$

3-8 The range of X is conveniently modeled as all nonnegative integers. That is, the range of X is $\{0,1,2,\ldots\}$

3-9. The range of X is $\{0,1,2,...,15\}$

3-10 The possible totals for two orders are 1/8 + 1/8 = 1/4, 1/8 + 1/4 = 3/8, 1/8 + 3/8 = 1/2, 1/4 + 1/4 = 1/2, 1/4 + 3/8 = 5/8, 3/8 + 3/8 = 6/8.
Therefore the range of X is $\left\{\frac{1}{4},\frac{3}{8},\frac{1}{2},\frac{5}{8},\frac{6}{8}\right\}$

3-11 The range of X is $\{0,1,2,\ldots,10000\}$

3-12 The range of X is $\{0,1,2,...,5000\}$

Section 3-2

3-13.

$$f_X(0) = P(X = 0) = 1/6 + 1/6 = 1/3$$
$$f_X(1.5) = P(X = 1.5) = 1/3$$
$$f_X(2) = 1/6$$
$$f_X(3) = 1/6$$

3-14 a) P(X=1.5) = 1/3
b) P(0.5< X < 2.7) = P(X=1.5) + P(X=2) = 1/6 + 1/3 = 1/2
c) P(X > 3) = 0
d) P (0 ≤ X < 2) = P(X=0) + P(X=1.5) = 1/3 + 1/3 = 2/3
e) P(X=0 or X=2) = 1/3 + 1/6 = 1/2

3-15. All probabilities are greater than or equal to zero and sum to one.

a) $P(X \le 2)=1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$
b) $P(X > -2) = 2/8 + 2/8 + 2/8 + 1/8 = 7/8$
c) $P(-1 \le X \le 1) = 2/8 + 2/8 + 2/8 =6/8 = 3/4$
d) $P(X \le -1$ or $X=2) = 1/8 + 2/8 +1/8 = 4/8 =1/2$

3-16 All probabilities are greater than or equal to zero and sum to one.
a) $P(X\le 1)=P(X=1)=0.5714$
b) $P(X>1)= 1-P(X=1)=1-0.5714=0.4286$
c) $P(2<X<6)=P(X=3)=0.1429$
d) $P(X\le 1$ or $X>1)= P(X=1)+ P(X=2)+P(X=3)=1$

3-17. Probabilities are nonnegative and sum to one.
a) $P(X = 4) = 9/25$
b) $P(X \le 1) = 1/25 + 3/25 = 4/25$
c) $P(2 \le X < 4) = 5/25 + 7/25 = 12/25$
d) $P(X > -10) = 1$

3-18 Probabilities are nonnegative and sum to one.
a) $P(X = 2) = 3/4(1/4)^2 = 3/64$
b) $P(X \le 2) = 3/4[1+1/4+(1/4)^2] = 63/64$
c) $P(X > 2) = 1 - P(X \le 2) = 1/64$
d) $P(X \ge 1) = 1 - P(X \le 0) = 1 - (3/4) = 1/4$

3-19. P(X = 10 million) = 0.3, P(X = 5 million) = 0.6, P(X = 1 million) = 0.1

3-20 P(X = 50 million) = 0.5, P(X = 25 million) = 0.3, P(X = 10 million) = 0.2

3-21. $P(X = 0) = 0.02^3 = 8 \times 10^{-6}$
$P(X = 1) = 3[0.98(0.02)(0.02)]=0.0012$
$P(X = 2) = 3[0.98(0.98)(0.02)]=0.0576$
$P(X = 3) = 0.98^3 = 0.9412$

3-22 X = number of wafers that pass
$P(X=0) = (0.2)^3 = 0.008$
$P(X=1) = 3(0.2)^2(0.8) = 0.096$
$P(X=2) = 3(0.2)(0.8)^2 = 0.384$
$P(X=3) = (0.8)^3 = 0.512$

3-23 P(X = 15 million) = 0.6, P(X = 5 million) = 0.3, P(X = -0.5 million) = 0.1

3-24 X = number of components that meet specifications
$P(X=0) = (0.05)(0.02) = 0.001$
$P(X=1) = (0.05)(0.98) + (0.95)(0.02) = 0.068$
$P(X=2) = (0.95)(0.98) = 0.931$

3-25. X = number of components that meet specifications
$P(X=0) = (0.05)(0.02)(0.01) = 0.00001$
$P(X=1) = (0.95)(0.02)(0.01) + (0.05)(0.98)(0.01)+(0.05)(0.02)(0.99) = 0.00167$
$P(X=2) = (0.95)(0.98)(0.01) + (0.95)(0.02)(0.99) + (0.05)(0.98)(0.99) = 0.07663$
$P(X=3) = (0.95)(0.98)(0.99) = 0.92169$

Section 3-3

3-26 $$F(x)=\begin{cases}0, & x<0\\ 1/3 & 0\le x<1.5\\ 2/3 & 1.5\le x<2\\ 5/6 & 2\le x<3\\ 1 & 3\le x\end{cases}$$ where
$$f_X(0)=P(X=0)=1/6+1/6=1/3$$
$$f_X(1.5)=P(X=1.5)=1/3$$
$$f_X(2)=1/6$$
$$f_X(3)=1/6$$

3-27.

$$F(x)=\begin{cases}0, & x<-2\\ 1/8 & -2\le x<-1\\ 3/8 & -1\le x<0\\ 5/8 & 0\le x<1\\ 7/8 & 1\le x<2\\ 1 & 2\le x\end{cases}$$ where
$$f_X(-2)=1/8$$
$$f_X(-1)=2/8$$
$$f_X(0)=2/8$$
$$f_X(1)=2/8$$
$$f_X(2)=1/8$$

a) P(X ≤ 1.25) = 7/8
b) P(X ≤ 2.2) = 1
c) P(-1.1 < X ≤ 1) = 7/8 – 1/8 = 3/4
d) P(X > 0) = 1 – P(X ≤ 0) = 1 – 5/8 = 3/8

3-28 $$F(x)=\begin{cases}0, & x<0\\ 1/25 & 0\le x<1\\ 4/25 & 1\le x<2\\ 9/25 & 2\le x<3\\ 16/25 & 3\le x<4\\ 1 & 4\le x\end{cases}$$ where
$$f_X(0)=1/25$$
$$f_X(1)=3/25$$
$$f_X(2)=5/25$$
$$f_X(3)=7/25$$
$$f_X(4)=9/25$$

a) P(X < 1.5) = 4/25
b) P(X ≤ 3) = 16/25
c) P(X > 2) = 1 – P(X ≤ 2) = 1 – 9/25 = 16/25
d) P(1 < X ≤ 2) = P(X ≤ 2) – P(X ≤ 1) = 9/25 – 4/25 = 5/25 = 1/5

3-29.

$$F(x)=\begin{cases}0, & x<1\\ 0.1, & 1\le x<5\\ 0.7, & 5\le x<10\\ 1, & 10\le x\end{cases}$$

where P(X = 10 million) = 0.3, P(X = 5 million) = 0.6, P(X = 1 million) = 0.1

3-30

$$F(x)=\begin{cases}0, & x<10\\ 0.2, & 10\le x<25\\ 0.5, & 25\le x<50\\ 1, & 50\le x\end{cases}$$

where P(X = 50 million) = 0.5, P(X = 25 million) = 0.3, P(X = 10 million) = 0.2

3-31.

$$F(x)=\begin{cases}0, & x<0\\ 0.008, & 0\le x<1\\ 0.104, & 1\le x<2\\ 0.488, & 2\le x<3\\ 1, & 3\le x\end{cases}\quad \text{where}\quad \begin{aligned}&f(0)=0.2^3=0.008,\\ &f(1)=3(0.2)(0.2)(0.8)=0.096,\\ &f(2)=3(0.2)(0.8)(0.8)=0.384,\\ &f(3)=(0.8)^3=0.512,\end{aligned}$$

3-32

$$F(x)=\begin{cases}0, & x<-0.5\\ 0.1, & -0.5\le x<5\\ 0.4, & 5\le x<15\\ 1, & 15\le x\end{cases}$$

where P(X = 15 million) = 0.6, P(X = 5 million) = 0.3, P(X = -0.5 million) = 0.1

3-33. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: $f(1) = 0.5$, $f(3) = 0.5$
a) $P(X \le 3) = 1$
b) $P(X \le 2) = 0.5$
c) $P(1 \le X \le 2) = P(X=1) = 0.5$
d) $P(X>2) = 1 - P(X\le 2) = 0.5$

3-34 The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f(1) = 0.7, f(4) = 0.2, f(7) = 0.1
a) $P(X \le 4) = 0.9$
b) $P(X > 7) = 0$
c) $P(X \le 5) = 0.9$
d) $P(X>4) = 0.1$
e) $P(X\le 2) = 0.7$

3-35. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f(-10) = 0.25, f(30) = 0.5, f(50) = 0.25
a) $P(X\le 50) = 1$
b) $P(X\le 40) = 0.75$
c) $P(40 \le X \le 60) = P(X=50)=0.25$
d) $P(X<0) = 0.25$
e) $P(0\le X<10) = 0$
f) $P(-10<X<10) = 0$

3-36 The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f1/8) = 0.2, f(1/4) = 0.7, f(3/8) = 0.1
a) $P(X \le 1/18) = 0$
b) $P(X \le 1/4) = 0.9$
c) $P(X \le 5/16) = 0.9$
d) $P(X > 1/4) = 0.1$
e) $P(X \le 1/2) = 1$

Section 3-4

3-37 Mean and Variance

$$\mu = E(X) = 0f(0) + 1f(1) + 2f(2) + 3f(3) + 4f(4)$$
$$= 0(0.2) + 1(0.2) + 2(0.2) + 3(0.2) + 4(0.2) = 2$$
$$V(X) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) - \mu^2$$
$$= 0(0.2) + 1(0.2) + 4(0.2) + 9(0.2) + 16(0.2) - 2^2 = 2$$

3- 38 Mean and Variance

$$\mu = E(X) = 0f(0) + 1.5f(1.5) + 2f(2) + 3f(3)$$
$$= 0(1/3) + 1.5(1/3) + 2(1/6) + 3(1/6) = 1.333$$
$$V(X) = 0^2 f(0) + 1.5^2 f(1) + 2^2 f(2) + 3^2 f(3) - \mu^2$$
$$= 0(1/3) + 2.25(1/3) + 4(1/6) + 9(1/6) - 1.333^2 = 1.139$$

3-39 Determine E(X) and V(X) for random variable in exercise 3-15

$$\mu = E(X) = -2f(-2) - 1f(-1) + 0f(0) + 1f(1) + 2f(2)$$
$$= -2(1/8) - 1(2/8) + 0(2/8) + 1(2/8) + 2(1/8) = 0$$
$$V(X) = -2^2 f(-2) - 1^2 f(-1) + 0^2 f(0) + 1^2 f(1) + 2^2 f(2) - \mu^2$$
$$= 4(1/8) + 1(2/8) + 0(2/8) + 1(2/8) + 4(1/8) - 0^2 = 1.5$$

3-40 Determine E(X) and V(X) for random variable in exercise 3-15

$$\mu = E(X) = 0f(0) + 1f(1) + 2f(2) + 3f(3) + 4f(4)$$
$$= 0(0.04) + 1(0.12) + 2(0.2) + 3(0.28) + 4(0.36) = 2.8$$
$$V(X) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) - \mu^2$$
$$= 0(0.04) + 1(0.12) + 4(0.2) + 9(0.28) + 16(0.36) - 2.8^2 = 1.36$$

3-41. Mean and variance for exercise 3-19

$$\mu = E(X) = 10f(10) + 5f(5) + 1f(1)$$
$$= 10(0.3) + 5(0.6) + 1(0.1)$$
$$= 6.1 \text{ million}$$
$$V(X) = 10^2 f(10) + 5^2 f(5) + 1^2 f(1) - \mu^2$$
$$= 10^2(0.3) + 5^2(0.6) + 1^2(0.1) - 6.1^2$$
$$= 7.89 \text{ million}^2$$

3-42 Mean and variance for exercise 3-20

$$\mu = E(X) = 50f(50) + 25f(25) + 10f(10)$$
$$= 50(0.5) + 25(0.3) + 10(0.2)$$
$$= 34.5 \text{ million}$$
$$V(X) = 50^2 f(50) + 25^2 f(25) + 10^2 f(10) - \mu^2$$
$$= 50^2(0.5) + 25^2(0.3) + 10^2(0.2) - 34.5^2$$
$$= 267.25 \text{ million}^2$$

3-43. Mean and variance for random variable in exercise 3-22

$$\mu = E(X) = 0f(0) + 1f(1) + 2f(2) + 3f(3)$$
$$= 0(0.008) + 1(0.096) + 2(0.384) + 3(0.512) = 2.4$$

$$V(X) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) - \mu^2$$
$$= 0^2(0.008) + 1(0.096) + 4(0.384) + 9(0.512) - 2.4^2 = 0.48$$

3-44 Mean and variance for exercise 3-23

$$\mu = E(X) = 15f(15) + 5f(5) - 0.5f(5)$$
$$= 15(0.6) + 5(0.3) - 0.5(0.1)$$
$$= 10.45 \text{ million}$$

$$V(X) = 15^2 f(15) + 5^2 f(5) + (-0.5)^2 f(-0.5) - \mu^2$$
$$= 15^2(0.6) + 5^2(0.3) + (-0.5)^2(0.1) - 10.45^2$$
$$= 33.32 \text{ million}^2$$

3-45. Determine x where range is [0,1,2,3,x] and mean is 6.

$$\mu = E(X) = 6 = 0f(0) + 1f(1) + 2f(2) + 3f(3) + xf(x)$$
$$6 = 0(0.2) + 1(0.2) + 2(0.2) + 3(0.2) + x(0.2)$$
$$6 = 1.2 + 0.2x$$
$$4.8 = 0.2x$$
$$x = 24$$

Section 3-5

3-46 E(X) = (0+100)/2 = 50, V(X) = $[(100-0+1)^2-1]/12$ = 850

3-47. E(X) = (3+1)/2 = 2, V(X) = $[(3-1+1)^2 -1]/12$ = 0.667

3-48 $$E(X) = \frac{1}{8}\left(\frac{1}{3}\right) + \frac{1}{4}\left(\frac{1}{3}\right) + \frac{3}{8}\left(\frac{1}{3}\right) = \frac{1}{4},$$

$$V(X) = \left(\frac{1}{8}\right)^2\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)^2\left(\frac{1}{3}\right) + \left(\frac{3}{8}\right)^2\left(\frac{1}{3}\right) - \left(\frac{1}{4}\right)^2 = 0.0104$$

3-49. X=(1/100)Y, Y = 15, 16, 17, 18, 19.

E(X) = (1/100) E(Y) = $\frac{1}{100}\left(\frac{15+19}{2}\right) = 0.17$ mm

$$V(X) = \left(\frac{1}{100}\right)^2\left[\frac{(19-15+1)^2-1}{12}\right] = 0.0002 \text{ mm}^2$$

3-50 $E(X) = 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right) = 3$

in 100 codes the expected number of letters is 300

$$V(X) = (2)^2\left(\frac{1}{3}\right) + (3)^2\left(\frac{1}{3}\right) + (4)^2\left(\frac{1}{3}\right) - (3)^2 = \frac{2}{3}$$

in 100 codes the variance is 6666.67

3-51. X = 590 + 0.1Y, Y = 0, 1, 2, ..., 9

E(X) = $590 + 0.1\left(\frac{0+9}{2}\right) = 590.45$ mm,

$$V(X) = (0.1)^2\left[\frac{(9-0+1)^2-1}{12}\right] = 0.0825 \text{ mm}^2$$

3-52 The range of Y is 0, 5, 10, ..., 45, E(X) = (0+9)/2 = 4.5

E(Y) = 0(1/10)+5(1/10)+...+45(1/10)
= 5[0(0.1) +1(0.1)+ ... +9(0.1)]
= 5E(X)
= 5(4.5)
= 22.5

V(X) = 8.25, V(Y) = 5^2(8.25) = 206.25, σ_Y = 14.36

3-53 $E(cX) = \sum_x cxf(x) = c\sum_x xf(x) = cE(X)$,

$$V(cX) = \sum_x (cx - c\mu)^2 f(x) = c^2\sum_x (x-\mu)^2 f(x) = cV(X)$$

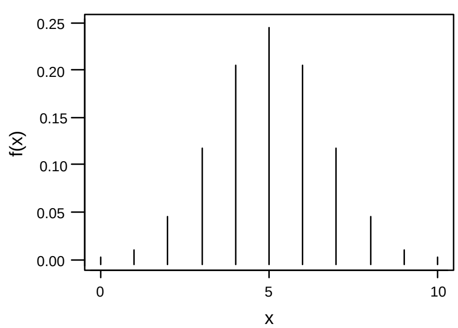
3-54 X is a discrete random variable. X is discrete because it is the number of fields out of 28 that has an error. However, X is not uniform because P(X=0) ≠ P(X=1).

Section 3-6

3-55. A binomial distribution is based on independent trials with two outcomes and a constant probability of success on each trial.

a) reasonable
b) independence assumption not reasonable
c) The probability that the second component fails depends on the failure time of the first component. The binomial distribution is not reasonable.
d) not independent trials with constant probability
e) probability of a correct answer not constant.
f) reasonable
g) probability of finding a defect not constant.
h) if the fills are independent with a constant probability of an underfill, then the binomial distribution for the number packages underfilled is reasonable.
i) because of the bursts, each trial (that consists of sending a bit) is not independent
j) not independent trials with constant probability

3-56



a.) $E(X) = np = 10(0.5) = 5$

b.) Values X=0 and X=10 are the least likely, the extreme values

3-57. a) $P(X = 5) = \binom{10}{5} 0.5^5 (0.5)^5 = 0.2461$

b) $P(X \le 2) = \binom{10}{0} 0.5^0 0.5^{10} + \binom{10}{1} 0.5^1 0.5^9 + \binom{10}{2} 0.5^2 0.5^8$

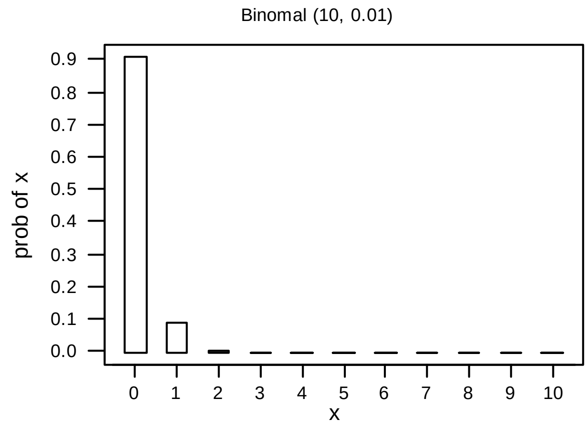
$= 0.5^{10} + 10(0.5)^{10} + 45(0.5)^{10} = 0.0547$

c) $P(X \ge 9) = \binom{10}{9} 0.5^9 (0.5)^1 + \binom{10}{10} 0.5^{10} (0.5)^0 = 0.0107$

d) $P(3 \le X < 5) = \binom{10}{3} 0.5^3 0.5^7 + \binom{10}{4} 0.5^4 0.5^6$

$= 120(0.5)^{10} + 210(0.5)^{10} = 0.3223$

3-58



P(X = 0) = 0.904, P(X = 1) = 0.091, P(X = 2) = 0.004, P(X = 3) = 0. P(X = 4) = 0 and so forth.
Distribution is skewed with $E(X) = np = 10(0.01) = 0.1$

a) The most-likely value of X is 0.

b) The least-likely value of X is 10.

3-59. a) $P(X = 5) = \binom{10}{5} 0.01^5 (0.99)^5 = 2.40 \times 10^{-8}$

$$b) P(X \le 2) = \binom{10}{0} 0.01^0 (0.99)^{10} + \binom{10}{1} 0.01^1 (0.99)^9 + \binom{10}{2} 0.01^2 (0.99)^8$$
$$= 0.9999$$

$$c) P(X \ge 9) = \binom{10}{9} 0.01^9 (0.99)^1 + \binom{10}{10} 0.01^{10} (0.99)^0 = 9.91 \times 10^{-18}$$

$$d) P(3 \le X < 5) = \binom{10}{3} 0.01^3 (0.99)^7 + \binom{10}{4} 0.01^4 (0.99)^6 = 1.138 \times 10^{-4}$$

3-60 n=3 and p=0.5

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.125 & 0 \le x < 1 \\ 0.5 & 1 \le x < 2 \\ 0.875 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases} \quad \text{where} \quad \begin{aligned} f(0) &= \left(\frac{1}{2}\right)^3 = \frac{1}{8} \\ f(1) &= 3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{3}{8} \\ f(2) &= 3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = \frac{3}{8} \\ f(3) &= \left(\frac{1}{4}\right)^3 = \frac{1}{8} \end{aligned}$$

3-61. n=3 and p=0.25

$$F(x)=\begin{cases} 0 & x<0 \\ 0.4219 & 0\le x<1 \\ 0.8438 & 1\le x<2 \\ 0.9844 & 2\le x<3 \\ 1 & 3\le x \end{cases} \quad \text{where} \quad \begin{aligned} f(0)&=\left(\frac{3}{4}\right)^3=\frac{27}{64} \\ f(1)&=3\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2=\frac{27}{64} \\ f(2)&=3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)=\frac{9}{64} \\ f(3)&=\left(\frac{1}{4}\right)^3=\frac{1}{64} \end{aligned}$$

3-62 Let X denote the number of defective circuits. Then, X has a binomial distribution with n = 40 and p = 0.01. Then, P(X = 0) = $\binom{40}{0}0.01^0 0.99^{40}=0.6690$.

3-63. a) $P(X=1)=\binom{1000}{1}0.001^1(0.999)^{999}=0.3681$

$$\text{b)}P(X\ge 1)=1-P(X=0)=1-\binom{1000}{0}0.001^0(0.999)^{999}=0.6319$$

$$\text{c)}P(X\le 2)=\binom{1000}{0}0.001^0(0.999)^{1000}+\binom{1000}{1}0.001^1(0.999)^{999}+\binom{1000}{2}0.001^2 0.999^{998}$$
$$=0.9198$$

$$\text{d)}E(X)=1000(0.001)=1$$
$$V(X)=1000(0.001)(0.999)=0.999$$

3-64 Let X denote the number of times the line is occupied. Then, X has a binomial distribution with n = 10 and p = 0.4

a.) P(X = 3) = $\binom{10}{3}0.4^3(0.6)^7=0.215$

b.) $P(X\ge 1)=1-P(X=0)=1-\binom{10}{0}0.4^0 0.6^{10}=0.994$

c.) $E(X)=10(0.4)=4$

3-65. a) n = 50, p = 5/50 = 0.1, since E(X) = 5 = np.

b) $P(X\le 2)=\binom{50}{0}0.1^0(0.9)^{50}+\binom{50}{1}0.1^1(0.9)^{49}+\binom{50}{2}0.1^2(0.9)^{48}=0.112$

c) $P(X\ge 49)=\binom{50}{49}0.1^{49}(0.9)^1+\binom{50}{50}0.1^{50}(0.9)^0=4.51\times 10^{-48}$

3-66 E(X) = 20 (0.01) = 0.2

V(X) = 20 (0.01) (0.99) = 0.198

$\mu_X + 3\sigma_X = 0.2 + 3\sqrt{0.198} = 1.53$

a)) X is binomial with n = 20 and p = 0.01

$$P(X > 1.53) = P(X \geq 2) = 1 - P(X \leq 1)$$
$$= 1 - \left[\binom{20}{0}0.01^0 0.99^{20} + \binom{20}{1}0.01^1 0.99^{19}\right] = 0.0169$$

b) X is binomial with n = 20 and p = 0.04

$$P(X > 1) = 1 - P(X \leq 1)$$
$$= 1 - \left[\binom{20}{0}0.04^0 0.96^{20} + \binom{20}{1}0.04^1 0.96^{19}\right] = 0.1897$$

c) Let Y denote the number of times X exceeds 1 in the next five samples. Then, Y is binomial with n = 5 and p = 0.190 from part b.

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - \left[\binom{5}{0}0.190^0 0.810^5\right] = 0.651$$

The probability is 0.651 that at least one sample from the next five will contain more than one defective.

3-67. Let X denote the passengers with tickets that do not show up for the flight. Then, X is binomial with n = 125 and p = 0.1.

$$a) P(X \geq 5) = 1 - P(X \leq 4)$$
$$= 1 - \left[\begin{matrix}\binom{125}{0}0.1^0(0.9)^{125} + \binom{125}{1}0.1^1(0.9)^{124} + \binom{125}{2}0.1^2(0.9)^{123} \\ + \binom{125}{3}0.1^3(0.9)^{122} + \binom{125}{4}0.1^4(0.9)^{121}\end{matrix}\right]$$
$$= 0.9961$$

$$b) P(X > 5) = 1 - P(X \leq 5) = 0.9886$$

3-68 Let X denote the number of defective components among those stocked.

$$a) \quad P(X = 0) = \binom{100}{0}0.02^0 0.98^{100} = 0.133$$

$$b) \quad P(X \leq 2) = \binom{102}{0}0.02^0 0.98^{102} + \binom{102}{1}0.02^1 0.98^{101} + \binom{102}{2}0.02^2 0.98^{100} = 0.666$$

$$c) \quad P(X \leq 5) = 0.981$$

3-69. Let X denote the number of questions answered correctly. Then, X is binomial with n = 25 and p = 0.25.

$$a) P(X \geq 20) = \binom{25}{20} 0.25^{20}(0.75)^{5} + \binom{25}{21} 0.25^{21}(0.75)^{4} + \binom{25}{22} 0.25^{22}(0.75)^{3}$$

$$+ \binom{25}{23} 0.25^{23}(0.75)^{2} + \binom{25}{24} 0.25^{24}(0.75)^{1} + \binom{25}{25} 0.25^{25}(0.75)^{0} = 9.677 \times 10^{-10}$$

$$b) P(X < 5) = \binom{25}{0} 0.25^{0}(0.75)^{25} + \binom{25}{1} 0.25^{1}(0.75)^{24} + \binom{25}{2} 0.25^{2}(0.75)^{23}$$

$$+ \binom{25}{3} 0.25^{3}(0.75)^{22} + \binom{25}{4} 0.25^{4}(0.75)^{21} = 0.2137$$

3-70 Let X denote the number of mornings the light is green.

$$a) \quad P(X = 1) = \binom{5}{1} 0.2^{1} 0.8^{4} = 0.410$$

$$b) \quad P(X = 4) = \binom{20}{4} 0.2^{4} 0.8^{16} = 0.218$$

$$c) \quad P(X > 4) = 1 - P(X \leq 4) = 1 - 0.630 = 0.370$$

Section 3-7

3-71. a) $P(X = 1) = (1 - 0.5)^{0} 0.5 = 0.5$

b) $P(X = 4) = (1 - 0.5)^{3} 0.5 = 0.5^{4} = 0.0625$

c) $P(X = 8) = (1 - 0.5)^{7} 0.5 = 0.5^{8} = 0.0039$

d) $P(X \leq 2) = P(X = 1) + P(X = 2) = (1 - 0.5)^{0} 0.5 + (1 - 0.5)^{1} 0.5$

$= 0.5 + 0.5^{2} = 0.75$

e.) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.75 = 0.25$

3-72 E(X) = 2.5 = 1/p giving p = 0.4

a) $P(X = 1) = (1 - 0.4)^{0} 0.4 = 0.4$

b) $P(X = 4) = (1 - 0.4)^{3} 0.4 = 0.0864$

c) $P(X = 5) = (1 - 0.5)^{4} 0.5 = 0.05184$

d) $P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3)$

$= (1 - 0.4)^{0} 0.4 + (1 - 0.4)^{1} 0.4 + (1 - 0.4)^{2} 0.4 = 0.7840$

e) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.7840 = 0.2160$

3-73. Let X denote the number of trials to obtain the first successful alignment. Then X is a geometric random variable with p = 0.8

a) $P(X=4)=(1-0.8)^3 0.8=0.2^3 0.8=0.0064$

b) $P(X\le 4)=P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$$=(1-0.8)^0 0.8+(1-0.8)^1 0.8+(1-0.8)^2 0.8+(1-0.8)^3 0.8$$

$$=0.8+0.2(0.8)+0.2^2(0.8)+0.2^3 0.8=0.9984$$

c) $P(X\ge 4)=1-P(X\le 3)=1-[P(X=1)+P(X=2)+P(X=3)]$

$$=1-[(1-0.8)^0 0.8+(1-0.8)^1 0.8+(1-0.8)^2 0.8]$$

$$=1-[0.8+0.2(0.8)+0.2^2(0.8)]=1-0.992=0.008$$

3-74 Let X denote the number of people who carry the gene. Then X is a negative binomial random variable with r=2 and p = 0.1

a) $P(X\ge 4)=1-P(X<4)=1-[P(X=2)+P(X=3)]$

$$=1-\left[\binom{1}{1}(1-0.1)^0 0.1^2+\binom{2}{1}(1-0.1)^1 0.1^2\right]=1-(0.01+0.018)=0.972$$

b) $E(X)=r/p=2/0.1=20$

3-75. Let X denote the number of calls needed to obtain a connection. Then, X is a geometric random variable with p = 0.02

a) $P(X=10)=(1-0.02)^9 0.02=0.98^9 0.02=0.0167$

b) $P(X>5)=1-P(X\le 4)=1-[P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)]$

$$=1-[0.02+0.98(0.02)+0.98^2(0.02)+0.98^3(0.02)+0.98^3(0.02)+0.98^4(0.02)]$$

$$=1-0.0961=0.9039$$

May also use the fact that P(X > 5) is the probability of no connections in 5 trials. That is,

$$P(X>5)=\binom{5}{0}0.02^0 0.98^5=0.9039$$

c) E(X) = 1/0.02 = 50

3-76 Let X denote the number of mornings needed to obtain a green light. Then X is a geometric random variable with p = 0.20.

a) $P(X = 4) = (1-0.2)^3 0.2 = 0.1024$

b) By independence, $(0.8)^{10} = 0.1074$. (Also, P(X > 10) = 0.1074)

3-77 p = 0.005 , r = 8

a.) $P(X=8)=0.005^8=3.91x10^{-19}$

b). $\mu=E(X)=\dfrac{1}{0.005}=200$ days

c) Mean number of days until all 8 computers fail. Now we use $p=3.91x10^{-19}$

$$\mu=E(Y)=\frac{1}{3.91x10^{-91}}=2.56x10^{18}$$ days or $7.01\ x10^{15}$ years

3-78 Let Y denote the number of samples needed to exceed 1 in Exercise 3-66. Then Y has a geometric distribution with p = 0.0169.

a) $P(Y = 10) = (1 - 0.0169)^9(0.0169) = 0.0145$

b) Y is a geometric random variable with p = 0.1897 from Exercise 3-66.

$P(Y = 10) = (1 - 0.1897)^9(0.1897) = 0.0286$

c) $E(Y) = 1/0.1897 = 5.27$

3-79. Let X denote the number of trials to obtain the first success.

a) E(X) = 1/0.2 = 5

b) Because of the lack of memory property, the expected value is still 5.

3-80 Negative binomial random variable: f(x; p, r) = $\binom{x-1}{r-1}(1-p)^{x-r}p^r$.

When r = 1, this reduces to f(x; p, r) = $(1-p)^{x-1}p$, which is the pdf of a geometric random variable.
Also, E(X) = r/p and V(X) = $[r(1-p)]/p^2$ reduce to E(X) = 1/p and V(X) = $(1-p)/p^2$, respectively.

3-81. a) E(X) = 4/0.2 = 20

b) P(X=20) = $\binom{19}{3}(0.80)^{16}0.2^4 = 0.0436$

c) P(X=19) = $\binom{18}{3}(0.80)^{15}0.2^4 = 0.0459$

d) P(X=21) = $\binom{20}{3}(0.80)^{17}0.2^4 = 0.0411$

e) The most likely value for X should be near μ_X. By trying several cases, the most likely value is x = 19.

3-82 Let X denote the number of attempts needed to obtain a calibration that conforms to specifications. Then, X is geometric with p = 0.6.

$P(X \le 3) = P(X=1) + P(X=2) + P(X=3) = 0.6 + 0.4(0.6) + 0.4^2(0.6) = 0.936$.

3-83. Let X denote the number of fills needed to detect three underweight packages. Then X is a negative binomial random variable with p = 0.001 and r = 3.

a) E(X) = 3/0.001 = 3000

b) V(X) = $[3(0.999)/0.001^2]$ = 2997000. Therefore, σ_X = 1731.18

3-84 Let X denote the number of transactions until all computers have failed. Then, X is negative binomial random variable with p = 10^{-8} and r = 3.

a) E(X) = 3 x 10^8

b) V(X) = $[3(1-10^{-8}0]/(10^{-16})$ = 3.0 x 10^{16}

3-85 Let X denote a geometric random variable with parameter p. Let q = 1-p.

$$E(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$$

$$= p\sum_{x=1}^{\infty} xq^{x-1} = p\frac{d}{dq}\left[\sum_{x=0}^{\infty} q^x\right] = p\frac{d}{dq}\left[\frac{1}{1-q}\right]$$

$$= \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$V(X) = \sum_{x=1}^{\infty} (x - \tfrac{1}{p})^2 (1-p)^{x-1} p = \sum_{x=1}^{\infty} \left(px^2 - 2x + \tfrac{1}{p}\right)(1-p)^{x-1}$$

$$= p\sum_{x=1}^{\infty} x^2 q^{x-1} - 2\sum_{x=1}^{\infty} xq^{x-1} + \tfrac{1}{p}\sum_{x=1}^{\infty} q^{x-1}$$

$$= p\sum_{x=1}^{\infty} x^2 q^{x-1} - \tfrac{2}{p^2} + \tfrac{1}{p^2}$$

$$= p\sum_{x=1}^{\infty} x^2 q^{x-1} - \tfrac{1}{p^2}$$

$$= p\tfrac{d}{dq}\left[q + 2q^2 + 3q^3 + \ldots\right] - \tfrac{1}{p^2}$$

$$= p\tfrac{d}{dq}\left[q(1 + 2q + 3q^2 + \ldots)\right] - \tfrac{1}{p^2}$$

$$= p\tfrac{d}{dq}\left[\tfrac{q}{(1-q)^2}\right] - \tfrac{1}{p^2} = 2pq(1-q)^{-3} + p(1-q)^{-2} - \tfrac{1}{p^2}$$

$$= \frac{\left[2(1-p) + p - 1\right]}{p^2} = \frac{(1-p)}{p^2} = \frac{q}{p^2}$$

Section 3-8

3-86 X has a hypergeometric distribution N=100, n=4, K=20

a.) $P(X = 1) = \dfrac{\binom{20}{1}\binom{80}{3}}{\binom{100}{4}} = \dfrac{20(82160)}{3921225} = 0.4191$

b.) $P(X = 6) = 0$, the sample size is only 4

c.) $P(X = 4) = \dfrac{\binom{20}{4}\binom{80}{0}}{\binom{100}{4}} = \dfrac{4845(1)}{3921225} = 0.001236$

d.) $E(X) = np = n\dfrac{K}{N} = 4\left(\dfrac{20}{100}\right) = 0.8$

$$V(X) = np(1-p)\left(\frac{N-n}{N-1}\right) = 4(0.2)(0.8)\left(\frac{96}{99}\right) = 0.6206$$

3-87. a) $P(X=1)=\frac{\binom{4}{1}\binom{16}{3}}{\binom{20}{4}}=\frac{(4\times16\times15\times14)/6}{(20\times19\times18\times17)/24}=0.4623$

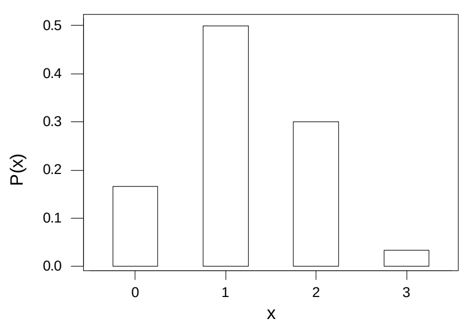
b) $P(X=4)=\frac{\binom{4}{4}\binom{16}{0}}{\binom{20}{4}}=\frac{1}{(20\times19\times18\times17)/24}=0.00021$

c)

$$P(X\le2)=P(X=0)+P(X=1)+P(X=2)$$
$$=\frac{\binom{4}{0}\binom{16}{4}}{\binom{20}{4}}+\frac{\binom{4}{1}\binom{16}{3}}{\binom{20}{4}}+\frac{\binom{4}{2}\binom{16}{2}}{\binom{20}{4}}$$
$$=\frac{\left(\frac{16\times15\times14\times13}{24}+\frac{4\times16\times15\times14}{6}+\frac{6\times16\times15}{2}\right)}{\left(\frac{20\times19\times18\times17}{24}\right)}=0.9866$$

d) E(X) = 4(4/20) = 0.8
V(X) = 4(0.2)(0.8)(16/19) = 0.539

3-88 N=10, n=3 and K=4



3-89.

$$F(x) = \begin{cases} 0, & x < 0 \\ 1/6, & 0 \le x < 1 \\ 2/3, & 1 \le x < 2 \\ 29/30, & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$ where

$$f(0) = \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = 0.1667 \quad , \quad f(1) = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = 0.5,$$

$$f(2) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = 0.3, \quad f(3) = \frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = 0.0333$$

3-90 Let X denote the number of unacceptable washers in the sample of 10.

$$a.)\quad P(X=0) = \frac{\binom{5}{0}\binom{70}{10}}{\binom{75}{10}} = \frac{\frac{70!}{10!60!}}{\frac{75!}{10!65!}} = \frac{65\times 64\times 63\times 62\times 61}{75\times 74\times 73\times 72\times 71} = 0.4786$$

$$b.)\quad P(X \ge 1) = 1 - P(X=0) = 0.5214$$

$$c.)\quad P(X=1) = \frac{\binom{5}{1}\binom{70}{9}}{\binom{75}{10}} = \frac{\frac{5!70!}{9!61!}}{\frac{75!}{10!65!}} = \frac{5\times 65\times 64\times 63\times 62\times 10}{75\times 74\times 73\times 72\times 71} = 0.3923$$

$$d.)\quad E(X) = 10(5/75) = 2/3$$

3-91. Let X denote the number of men who carry the marker on the male chromosome for an increased risk for high blood pressure. N=800, K=240 n=10

a) n=10

$$P(X=1) = \frac{\binom{240}{1}\binom{560}{9}}{\binom{800}{10}} = \frac{\left(\frac{240!}{1!239!}\right)\left(\frac{560!}{9!551!}\right)}{\frac{800!}{10!790!}} = 0.1201$$

b) n=10

$$P(X>1) = 1 - P(X \le 1) = 1 - [P(X=0) + P(X=1)]$$

$$P(X=0) = \frac{\binom{240}{0}\binom{560}{10}}{\binom{800}{10}} = \frac{\left(\frac{240!}{0!240!}\right)\left(\frac{560!}{10!550!}\right)}{\frac{800!}{10!790!}} = 0.0276$$

$$P(X>1) = 1 - P(X \le 1) = 1 - [0.0276 + 0.1201] = 0.8523$$

3-92 . Let X denote the number of cards in the sample that are defective.

a)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \frac{\binom{20}{0}\binom{120}{20}}{\binom{140}{20}} = \frac{\frac{120!}{20!100!}}{\frac{140!}{20!120!}} = 0.0356$$

$$P(X \geq 1) = 1 - 0.0356 = 0.9644$$

b)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \frac{\binom{5}{0}\binom{135}{20}}{\binom{140}{20}} = \frac{\frac{135!}{20!115!}}{\frac{140!}{20!120!}} = \frac{135!120!}{115!140!} = 0.4571$$

$$P(X \geq 1) = 1 - 0.4571 = 0.5429$$

3-93. Let X denote the number of blades in the sample that are dull.

a)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \frac{\binom{10}{0}\binom{38}{5}}{\binom{48}{5}} = \frac{\frac{38!}{5!33!}}{\frac{48!}{5!43!}} = \frac{38!43!}{48!33!} = 0.2931$$

$$P(X \geq 1) = 1 - P(X = 0) = 0.7069$$

b) Let Y denote the number of days needed to replace the assembly.

P(Y = 3) = $0.2931^2(0.7069) = 0.0607$

c) On the first day, $P(X = 0) = \frac{\binom{2}{0}\binom{46}{5}}{\binom{48}{5}} = \frac{\frac{46!}{5!41!}}{\frac{48!}{5!43!}} = \frac{46!43!}{48!41!} = 0.8005$

On the second day, $P(X = 0) = \frac{\binom{6}{0}\binom{42}{5}}{\binom{48}{5}} = \frac{\frac{42!}{5!37!}}{\frac{48!}{5!43!}} = \frac{42!43!}{48!37!} = 0.4968$

On the third day, P(X = 0) = 0.2931 from part a. Therefore,
P(Y = 3) = 0.8005(0.4968)(1-0.2931) = 0.2811.

3-94 Let X denote the count of the numbers in the state's sample that match those in the player's sample. Then, X has a hypergeometric distribution with N = 40, n = 6, and K = 6.

a) $P(X = 6) = \frac{\binom{6}{6}\binom{34}{0}}{\binom{40}{6}} = \left(\frac{40!}{6!34!}\right)^{-1} = 2.61 \times 10^{-7}$

b) $P(X = 5) = \frac{\binom{6}{5}\binom{34}{1}}{\binom{40}{6}} = \frac{6 \times 34}{\binom{40}{6}} = 5.31 \times 10^{-5}$

c) $P(X = 4) = \frac{\binom{6}{4}\binom{34}{2}}{\binom{40}{6}} = 0.00219$

d) Let Y denote the number of weeks needed to match all six numbers. Then, Y has a geometric distribution with p = $\frac{1}{3{,}838{,}380}$ and E(Y) = 1/p = $3{,}838{,}380$ weeks. This is more than 738 centuries!

3-95. a) For Exercise 3-86, the finite population correction is 96/99.
For Exercise 3-87, the finite population correction is 16/19.
Because the finite population correction for Exercise 3-86 is closer to one, the binomial approximation to the distribution of X should be better in Exercise 3-86.

b) Assuming X has a binomial distribution with n = 4 and p = 0.2,

$$P(X = 1) = \binom{4}{1} 0.2^1 0.8^3 = 0.4096$$

$$P(X = 4) = \binom{4}{4} 0.2^4 0.8^0 = 0.0016$$

The results from the binomial approximation are close to the probabilities obtained in Exercise 3-86.

c) Assume X has a binomial distribution with n = 4 and p = 0.2. Consequently, P(X = 1) and P(X = 4) are the same as computed in part b. of this exercise. This binomial approximation is not as close to the true answer as the results obtained in part b. of this exercise.

3-96 a.) From Exercise 3-92, X is approximately binomial with n = 20 and p = 20/140 = 1/7.

$$P(X \geq 1) = 1 - P(X = 0) = \binom{20}{0}\left(\tfrac{1}{7}\right)^0 \left(\tfrac{6}{7}\right)^{20} = 1 - 0.0458 = 0.9542$$

finite population correction is 120/139=0.8633

b) From Exercise 3-92, X is approximately binomial with n = 20 and p = 5/140 =1/28

$$P(X \geq 1) = 1 - P(X = 0) = \binom{20}{0}\left(\tfrac{1}{28}\right)^0 \left(\tfrac{27}{28}\right)^{20} = 1 - 0.4832 = 0.5168$$

finite population correction is 120/139=0.8633

Section 3-9

3-97. a) $P(X = 0) = \dfrac{e^{-4} 4^0}{0!} = e^{-4} = 0.0183$

b) $P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= e^{-4} + \frac{e^{-4} 4^1}{1!} + \frac{e^{-4} 4^2}{2!}$$

$$= 0.2381$$

c) $P(X = 4) = \dfrac{e^{-4} 4^4}{4!} = 0.1954$

d) $P(X = 8) = \dfrac{e^{-4} 4^8}{8!} = 0.0298$

3-98 a) $P(X = 0) = e^{-0.4} = 0.6703$

b) $P(X \le 2) = e^{-0.4} + \dfrac{e^{-0.4}(0.4)}{1!} + \dfrac{e^{-0.4}(0.4)^2}{2!} = 0.9921$

c) $P(X = 4) = \dfrac{e^{-0.4}(0.4)^4}{4!} = 0.000715$

d) $P(X = 8) = \dfrac{e^{-0.4}(0.4)^8}{8!} = 1.09 \times 10^{-8}$

3-99. $P(X = 0) = e^{-\lambda} = 0.05$. Therefore, $\lambda = -\ln(0.05) = 2.996$.
Consequently, E(X) = V(X) = 2.996.

3-100 a) Let X denote the number of calls in one hour. Then, X is a Poisson random variable with $\lambda = 10$.

$$P(X = 5) = \frac{e^{-10}10^5}{5!} = 0.0378 .$$

b) $P(X \le 3) = e^{-10} + \frac{e^{-10}10}{1!} + \frac{e^{-10}10^2}{2!} + \frac{e^{-10}10^3}{3!} = 0.0103$

c) Let Y denote the number of calls in two hours. Then, Y is a Poisson random variable with $\lambda = 20$. $P(Y = 15) = \frac{e^{-20}20^{15}}{15!} = 0.0516$

d) Let W denote the number of calls in 30 minutes. Then W is a Poisson random variable with $\lambda = 5$. $P(W = 5) = \frac{e^{-5}5^5}{5!} = 0.1755$

3-101. a) Let X denote the number of flaws in one square meter of cloth. Then, X is a Poisson random variable with $\lambda = 0.1$. $P(X = 2) = \frac{e^{-0.1}(0.1)^2}{2!} = 0.0045$

b) Let Y denote the number of flaws in 10 square meters of cloth. Then, Y is a Poisson random variable with $\lambda = 1$. $P(Y = 1) = \frac{e^{-1}1^1}{1!} = e^{-1} = 0.3679$

c) Let W denote the number of flaws in 20 square meters of cloth. Then, W is a Poisson random variable with $\lambda = 2$. $P(W = 0) = e^{-2} = 0.1353$

d) $P(Y \ge 2) = 1 - P(Y \le 1) = 1 - P(Y = 0) - P(Y = 1)$

$$= 1 - e^{-1} - e^{-1}$$

$$= 0.2642$$

3-102 a) $E(X) = \lambda = 0.2$ errors per test area

b.) $P(X \le 2) = e^{-0.2} + \frac{e^{-0.2}0.2}{1!} + \frac{e^{-0.2}(0.2)^2}{2!} = 0.9989$

99.89% of test areas

3-103. a) Let X denote the number of cracks in 5 miles of highway. Then, X is a Poisson random variable with $\lambda = 10$. $P(X = 0) = e^{-10} = 4.54 \times 10^{-5}$

b) Let Y denote the number of cracks in a half mile of highway. Then, Y is a Poisson random variable with $\lambda = 1$. $P(Y \ge 1) = 1 - P(Y = 0) = 1 - e^{-1} = 0.6321$

c) The assumptions of a Poisson process require that the probability of a count is constant for all intervals. If the probability of a count depends on traffic load and the load varies, then the assumptions of a Poisson process are not valid. Separate Poisson random variables might be appropriate for the heavy and light load sections of the highway.

3-104 a.) $E(X) = \lambda = 0.01$ failures per 100 samples. Let Y= the number of failures per day

$E(Y) = E(5X) = 5E(X) = 5\lambda = 0.05$ failures per day.

b.)Let W= the number of failures in 500 participants, now λ=0.05 and $P(W = 0) = e^{-0.05} = 0.9512$

3-105. a) Let X denote the number of flaws in 10 square feet of plastic panel. Then, X is a Poisson random variable with $\lambda = 0.5$. $P(X = 0) = e^{-0.5} = 0.6065$

b) Let Y denote the number of cars with no flaws,

$$P(Y = 10) = \binom{10}{10}(0.6065)^{10}(0.3935)^{0} = 0.0067$$

c) Let W denote the number of cars with surface flaws. Because the number of flaws has a Poisson distribution, the occurrences of surface flaws in cars are independent events with constant probability. From part a., the probability a car contains surface flaws is 1–0.6065 = 0.3935. Consequently, W is binomial with n = 10 and p = 0.3935.

$$P(W = 0) = \binom{10}{0}(0.3935)^{0}(0.6065)^{10} = 0.0067$$

$$P(W = 1) = \binom{10}{1}(0.3935)^{1}(0.6065)^{9} = 0.0437$$

$$P(W \le 1) = 0.0067 + 0.0437 = 0.0504$$

3-106 a) Let X denote the failures in 8 hours. Then, X has a Poisson distribution with $\lambda = 0.16$.

$P(X = 0) = e^{-0.16} = 0.8521$

b) Let Y denote the number of failure in 24 hours. Then, Y has a Poisson distribution with $\lambda = 0.48$. $P(Y \ge 1) = 1 - P(Y = 0) = 1 - e^{-48} = 0.3812$

Supplemental Exercises

3-107. Let X denote the number of totes in the sample that do not conform to purity requirements. Then, X has a hypergeometric distribution with N = 15, n = 3, and K = 2.

$$P(X \ge 1) = 1 - P(X = 0) = 1 - \frac{\binom{2}{0}\binom{13}{3}}{\binom{15}{3}} = 1 - \frac{13!12!}{10!15!} = 0.3714$$

3-108 Let X denote the number of calls that are answered in 30 seconds or less. Then, X is a binomial random variable with p = 0.75.

a) P(X = 9) = $\binom{10}{9}(0.75)^{9}(0.25)^{1} = 0.1877$

b) P(X ≥ 16) = P(X=16) +P(X=17) + P(X=18) + P(X=19) + P(X=20)

$$= \binom{20}{16}(0.75)^{16}(0.25)^{4} + \binom{20}{17}(0.75)^{17}(0.25)^{3} + \binom{20}{18}(0.75)^{18}(0.25)^{2}$$

$$+ \binom{20}{19}(0.75)^{19}(0.25)^{1} + \binom{20}{20}(0.75)^{20}(0.25)^{0} = 0.4148$$

c) E(X) = 20(0.75) = 15

3-109. Let Y denote the number of calls needed to obtain an answer in less than 30 seconds.

a) $P(Y=4)=(1-0.75)^3 0.75 = 0.25^3 0.75 = 0.0117$

b) E(Y) = 1/p = 1/0.75 = 4/3

3-110 Let W denote the number of calls needed to obtain two answers in less than 30 seconds. Then, W has a negative binomial distribution with p = 0.75.

a) P(W=6) = $\binom{5}{1}(0.25)^4(0.75)^2 = 0.0110$

b) E(W) = r/p = 2/0.75 = 8/3

3-111. a) Let X denote the number of messages sent in one hour. $P(X=5)=\frac{e^{-5}5^5}{5!}=0.1755$

b) Let Y denote the number of messages sent in 1.5 hours. Then, Y is a Poisson random variable with λ =7.5. $P(Y=10)=\frac{e^{-7.5}(7.5)^{10}}{10!}=0.0858$

c) Let W denote the number of messages sent in one-half hour. Then, W is a Poisson random variable with λ = 2.5. $P(W<2)=P(W=0)+P(W=1)=0.2873$

3-112 X is a negative binomial with r=4 and p=0.0001

$E(X)=r/p=4/0.0001=40000$ requests

3-113. X ~ Poisson(λ = 0.01), X ~ Poisson(λ = 1)

$$P(Y\le 3)=e^{-1}+\frac{e^{-1}(1)^1}{1!}+\frac{e^{-1}(1)^2}{2!}+\frac{e^{-1}(1)^3}{3!}=0.9810$$

3-114 Let X denote the number of individuals that recover in one week. Assume the individuals are independent. Then, X is a binomial random variable with n = 20 and p = 0.1. P(X ≥ 4) = 1 – P(X ≤ 3) = 1 – 0.8670 = 0.1330.

3-115 a.) P(X=1) = 0 , P(X=2) = 0.0025, P(X=3) = 0.01, P(X=4) = 0.03, P(X=5) = 0.065
P(X=6) = 0.13, P(X=7) = 0.18, P(X=8) = 0.2225, P(X=9) = 0.2, P(X=10) = 0.16

b.) P(X=1) = 0.0025, P(X=1.5) = 0.01, P(X=2) = 0.03, P(X=2.5) = 0.065, P(X=3) = 0.13
P(X=3.5) = 0.18, P(X=4) = 0.2225, P(X=4.5) = 0.2, P(X=5) = 0.16

3-116 Let X denote the number of assemblies needed to obtain 5 defectives. Then, X is a negative binomial random variable with p = 0.01 and r=5.

a) E(X) = r/p = 500.

b) V(X) =(5* 0.99/0.01^2 = 49500 and σ_X = 222.49

3-117. If n assemblies are checked, then let X denote the number of defective assemblies. If P(X ≥ 1) ≥ 0.95, then P(X=0) ≤ 0.05. Now,

P(X=0) = $\binom{n}{0}(0.01)^0(0.99)^n = 99^n$ and $0.99^n \le 0.05$. Therefore,

$$n(\ln(0.99))\le \ln(0.05)$$

$$n\ge \frac{\ln(0.05)}{\ln(0.95)}=298.07$$

This would require n = 299.

3-118 Require f(1) + f(2) + f(3) + f(4) = 1. Therefore, $c(1+2+3+4) = 1$. Therefore, $c = 0.1$.

3-119. Let X denote the number of products that fail during the warranty period. Assume the units are independent. Then, X is a binomial random variable with n = 500 and p = 0.02.

a) P(X = 0) = $\binom{500}{0}(0.02)^0(0.98)^{500} = 4.1 \times 10^{-5}$

b) E(X) = 500(0.02) = 10

c) P(X >2) = 1 − P(X ≤ 2) = 0.9995

3-120
$$f_X(0) = (0.1)(0.7) + (0.3)(0.3) = 0.16$$
$$f_X(1) = (0.1)(0.7) + (0.4)(0.3) = 0.19$$
$$f_X(2) = (0.2)(0.7) + (0.2)(0.3) = 0.20$$
$$f_X(3) = (0.4)(0.7) + (0.1)(0.3) = 0.31$$
$$f_X(4) = (0.2)(0.7) + (0)(0.3) = 0.14$$

3-121. a) P(X ≤ 3) = 0.2 + 0.4 = 0.6

b) P(X > 2.5) = 0.4 + 0.3 + 0.1 = 0.8

c) P(2.7 < X < 5.1) = 0.4 + 0.3 = 0.7

d) E(X) = 2(0.2) + 3(0.4) + 5(0.3) + 8(0.1) = 3.9

e) $V(X) = 2^2(0.2) + 3^2(0.4) + 5^2(0.3) + 8^2(0.1) - (3.9)^2 = 3.09$

3-122

x	2	5.7	6.5	8.5
f(x)	0.2	0.3	0.3	0.2

3-123. Let X denote the number of bolts in the sample from supplier 1 and let Y denote the number of bolts in the sample from supplier 2. Then, x is a hypergeometric random variable with N = 100, n = 4, and K = 30. Also, Y is a hypergeometric random variable with N = 100, n = 4, and K = 70.

a) P(X=4 or Y=4) = P(X = 4) + P(Y = 4)
$$= \frac{\binom{30}{4}\binom{70}{0}}{\binom{100}{4}} + \frac{\binom{30}{0}\binom{70}{4}}{\binom{100}{4}}$$
$$= 0.2408$$

b) P[(X=3 and Y=1) or (Y=3 and X = 1)]= $= \dfrac{\binom{30}{3}\binom{70}{1} + \binom{30}{1}\binom{70}{3}}{\binom{100}{4}} = 0.4913$

3-124 Let X denote the number of errors in a sector. Then, X is a Poisson random variable with λ = 0.32768.

a) $P(X>1) = 1 - P(X\le1) = 1 - e^{-0.32768} - e^{-0.32768}(0.32768) = 0.0433$

b) Let Y denote the number of sectors until an error is found. Then, Y is a geometric random variable and
$P = P(X \ge 1) = 1 - P(X=0) = 1 - e^{-0.32768} = 0.2794$
E(Y) = 1/p = 3.58

3-125. Let X denote the number of orders placed in a week in a city of 800,000 people. Then X is a Poisson random variable with $\lambda = 0.25(8) = 2$.

a) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - [e^{-2} + e^{-2}(2) + (e^{-2}2^2)/2!] = 1 - 0.6767 = 0.3233$.

b) Let Y denote the number of orders in 2 weeks. Then, Y is a Poisson random variable with $\lambda = 4$, and $P(Y>2) = 1 - P(Y \leq 2) = e^{-4} + (e^{-4}4^1)/1! + (e^{-4}4^2)/2! = 1 - [0.01832+0.07326+0.1465] = 0.7619$.

3-126 a.) hypergeometric random variable with N = 500, n = 5, and K = 125

$$f_X(0) = \frac{\binom{125}{0}\binom{375}{5}}{\binom{500}{5}} = \frac{6.0164E10}{2.5524E11} = 0.2357$$

$$f_X(1) = \frac{\binom{125}{1}\binom{375}{4}}{\binom{500}{5}} = \frac{125(8.10855E8)}{2.5525E11} = 0.3971$$

$$f_X(2) = \frac{\binom{125}{2}\binom{375}{3}}{\binom{500}{5}} = \frac{7750(8718875)}{2.5524E11} = 0.2647$$

$$f_X(3) = \frac{\binom{125}{3}\binom{375}{2}}{\binom{500}{5}} = \frac{317750(70125)}{2.5524E11} = 0.0873$$

$$f_X(4) = \frac{\binom{125}{4}\binom{375}{1}}{\binom{500}{5}} = \frac{9691375(375)}{2.5524E11} = 0.01424$$

$$f_X(5) = \frac{\binom{125}{5}\binom{375}{0}}{\binom{500}{5}} = \frac{2.3453E8}{2.5524E11} = 0.00092$$

b.)

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0.0546	0.1866	0.2837	0.2528	0.1463	0.0574	0.0155	0.0028	0.0003	0.0000	0.0000

3-127. Let X denote the number of totes in the sample that exceed the moisture content. Then X is a binomial random variable with n = 30. We are to determine p.

If $P(X \geq 1) = 0.9$, then $P(X = 0) = 0.1$. Then $\binom{30}{0}(p)^0(1-p)^{30} = 0.1$, giving $30\ln(1-p)=\ln(0.1)$, which results in p = 0.0739.

3-128 Let t denote an interval of time in hours and let X denote the number of messages that arrive in time t. Then, X is a Poisson random variable with $\lambda = 10t$.
Then, $P(X=0) = 0.9$ and $e^{-10t} = 0.9$, resulting in t = 0.0105 hours = 0.63 seconds

3-129. a) Let X denote the number of flaws in 50 panels. Then, X is a Poisson random variable with $\lambda = 50(0.02) = 1$. $P(X = 0) = e^{-1} = 0.3679$.

b) Let Y denote the number of flaws in one panel, then $P(Y \geq 1) = 1 - P(Y=0) = 1 - e^{-0.02} = 0.0198$. Let W denote the number of panels that need to be inspected before a flaw is found. Then W is a geometric random variable with p = 0.0198 and $E(W) = 1/0.0198 = 50.51$ panels.

c) $P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-0.02} = 0.0198$

Let V denote the number of panels with 1 or more flaws. Then V is a binomial random variable with n=50 and p=0.0198

$$P(V \leq 2) = \binom{50}{0}0.0198^0(.9802)^{50} + \binom{50}{1}0.0198^1(0.9802)^{49}$$
$$+ \binom{50}{2}0.0198^2(0.9802)^{48} = 0.9234$$

Mind Expanding Exercises

3-130. Let X follow a hypergeometric distribution with parameters K, n, and N.

To solve this problem, we can find the general expectation:

$$E(X^k) = \sum_{i=0}^{n} i^k P(X = i) = \sum_{i=0}^{n} i^k \frac{\binom{K}{i}\binom{N-K}{n-i}}{\binom{N}{n}}$$

Using the relationships

$$i\binom{K}{i} = K\binom{K-1}{i-1} \quad \text{and} \quad n\binom{N}{n} = N\binom{N-1}{n-1}$$

we can substitute into $E(X^K)$:

$$E(X^k) = \sum_{i=0}^{n} i^k P(X = i)$$

$$= \sum_{i=0}^{n} i^k \frac{\binom{K}{i}\binom{N-K}{n-i}}{\binom{N}{n}}$$

$$= n\sum_{i=0}^{n} i^{k-1} \frac{K\binom{K-1}{i-1}\binom{N-K}{n-i}}{N\binom{N-1}{n-1}}$$

$$= \frac{nK}{N}\sum_{j=0}^{n-1} (j+1)^{k-1} \frac{\binom{K-1}{j}\binom{N-K}{n-1-j}}{\binom{N-1}{n-1}}$$

$$= \frac{nK}{N} E[(Z+1)^{k-1}]$$

Now, Z is also a hypergeometric random variable with parameters n – 1, N – 1, and K – 1.

To find the mean of X, E(X), set k = 1:

$$E(X) = = \frac{nK}{N} E[(Z+1)^{1-1}] = \frac{nK}{N}$$

If we let p = K/N, then E(X) = np. In order to find the variance of X using the formula V(X) = $E(X^2) - [E(X)\}^2$, the $E(X^2)$ must be found. Substituting k = 2 into $E(X^k)$ we get

$$E(X^2) = \frac{nK}{N} E[(Z+1)^{2-1}] = \frac{nK}{N} E(Z+1)$$

$$= \frac{nK}{N}\left[E(Z) + E(1)\right] = \frac{nK}{N}\left[\frac{(n-1)(K-1)}{N-1} + 1\right]$$

Therefore, V(X) = $\frac{nK}{N}\left[\frac{(n-1)(K-1)}{N-1} + 1\right] - \left(\frac{nK}{N}\right)^2 = \frac{nK}{N}\left[\frac{(n-1)(K-1)}{N-1} + 1 - \frac{nK}{N}\right]$

If we let p = K/N, the variance reduces to V(X) = $\left(\frac{N-n}{N-1}\right) np(1-p)$

3-131. Show that $\sum_{i=1}^{\infty} (1-p)^{i-1} p = 1$ using an infinite sum.

To begin, $\sum_{i=1}^{\infty} (1-p)^{i-1} p = p\sum_{i=1}^{\infty} (1-p)^{i-1}$, by definition of an infinite sum this can be rewritten as

$$p\sum_{i=1}^{\infty} (1-p)^{i-1} = \frac{p}{1-(1-p)} = \frac{p}{p} = 1$$

3-132

$$E(X) = [(a + (a+1) + ... + b](b - a + 1)$$

$$= \left[\sum_{i=1}^{b} i - \sum_{i=1}^{a-1} i\right] \Big/ (b-a+1) = \left[\frac{b(b+1)}{2} - \frac{(a-1)a}{2}\right] \Big/ (b-a+1)$$

$$= \left[\frac{(b^2 - a^2 + b + a)}{2}\right] \Big/ (b-a+1) = \left[\frac{(b+a)(b-a+1)}{2}\right] \Big/ (b-a+1)$$

$$= \frac{(b+a)}{2}$$

$$V(X) = \frac{\sum_{i=a}^{b}\left[i - \frac{b+a}{2}\right]^2}{b+a-1} = \frac{\left[\sum_{i=a}^{b} i^2 - (b+a)\sum_{i=a}^{b} i + \frac{(b-a+1)(b+a)^2}{4}\right]}{b+a-1}$$

$$= \frac{\frac{b(b+1)(2b+1)}{6} - \frac{(a-1)a(2a-1)}{6} - (b+a)\left[\frac{b(b+1)-(a-1)a}{2}\right] + \frac{(b-a+1)(b+a)^2}{4}}{b-a+1}$$

$$= \frac{(b-a+1)^2 - 1}{12}$$

3-133 Let X denote the number of nonconforming products in the sample. Then, X is approximately binomial with p = 0.01 and n is to be determined.

If $P(X \geq 1) \geq 0.90$, then $P(X = 0) \leq 0.10$.

Now, P(X = 0) = $\binom{n}{0} p^0 (1-p)^n = (1-p)^n$. Consequently, $(1-p)^n \leq 0.10$, and

$n \leq \frac{\ln 0.10}{\ln(1-p)} = 229.11$. Therefore, n = 230 is required

3-134 If the lot size is small, 10% of the lot might be insufficient to detect nonconforming product. For example, if the lot size is 10, then a sample of size one has a probability of only 0.2 of detecting a nonconforming product in a lot that is 20% nonconforming.

If the lot size is large, 10% of the lot might be a larger sample size than is practical or necessary. For example, if the lot size is 5000, then a sample of 500 is required. Furthermore, the binomial approximation to the hypergeometric distribution can be used to show the following. If 5% of the lot of size 5000 is nonconforming, then the probability of zero nonconforming product in the sample is approximately 7×10^{-12}. Using a sample of 100, the same probability is still only 0.0059. The sample of size 500 might be much larger than is needed.

3-135 Let X denote the number of panels with flaws. Then, X is a binomial random variable with n =100 and p is the probability of one or more flaws in a panel. That is, p = $1-e^{-0.1}$ = 0.095.

$$P(X<5)=P(X\le 4)=P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$$

$$=\binom{100}{0}p^0(1-p)^{100}+\binom{100}{1}p^1(1-p)^{99}+\binom{100}{2}p^2(1-p)^{98}$$

$$+\binom{100}{3}p^3(1-p)^{97}+\binom{100}{4}p^4(1-p)^{96}$$

$$=0.034$$

3-136 Let X denote the number of rolls produced.

Revenue at each demand				
	0	1000	2000	3000
$0 \le x \le 1000$	0.05x	0.3x	0.3x	0.3x
mean profit = 0.05x(0.3) + 0.3x(0.7) - 0.1x				
$1000 \le x \le 2000$	0.05x	0.3(1000) + 0.05(x-1000)	0.3x	0.3x
mean profit = 0.05x(0.3) + [0.3(1000) + 0.05(x-1000)](0.2) + 0.3x(0.5) - 0.1x				
$2000 \le x \le 3000$	0.05x	0.3(1000) + 0.05(x-1000)	0.3(2000) + 0.05(x-2000)	0.3x
mean profit = 0.05x(0.3) + [0.3(1000)+0.05(x-1000)](0.2) + [0.3(2000) + 0.05(x-2000)](0.3) + 0.3x(0.2) - 0.1x				
$3000 \le x$	0.05x	0.3(1000) + 0.05(x-1000)	0.3(2000) + 0.05(x-2000)	0.3(3000)+ 0.05(x-3000)
mean profit = 0.05x(0.3) + [0.3(1000)+0.05(x-1000)](0.2) + [0.3(2000)+0.05(x-2000)]0.3 + [0.3(3000)+0.05(x-3000)]0.2 - 0.1x				

	Profit	Max. profit
$0 \le x \le 1000$	0.125 x	\$ 125 at x = 1000
$1000 \le x \le 2000$	0.075 x + 50	\$ 200 at x = 2000
$2000 \le x \le 3000$	200	\$200 at x = 3000
$3000 \le x$	-0.05 x + 350	\$200 at x = 3000

The bakery can make anywhere from 2000 to 3000 and earn the same profit.

3-137 Let X denote the number of acceptable components. Then, X has a binomial distribution with p = 0.98 and n is to be determined such that $P(X \ge 100) \ge 0.95$.

n	$P(X \ge 100)$
102	0.666
103	0.848
104	0.942
105	0.981

Therefore, 105 components are needed.