CHAPTER 2

Section 2-1

2-1. Let "a", "b" denote a part above, below the specification
$S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

2-2. Let "e" denote a bit in error
Let "o" denote a bit not in error ("o" denotes okay)

$$S = \begin{Bmatrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{Bmatrix}$$

2-3. Let "a" denote an acceptable power supply
Let "f" ,"m","c" denote a supply with a functional, minor, or cosmetic error, respectively.
$S = \{a, f, m, c\}$

2-4. $S = \{0, 1, 2, \ldots\}$ = set of nonnegative integers

2-5. If only the number of tracks with errors is of interest, then $S = \{0, 1, 2, \ldots, 24\}$

2-6. A vector with three components can describe the three digits of the ammeter. Each digit can be 0,1,2,...,9. Then S is a sample space of 1000 possible three digit integers, $S = \{000, 001, \ldots, 999\}$

2-7. S is the sample space of 100 possible two digit integers.

2-8. Let an ordered pair of numbers, such as 43 denote the response on the first and second question. Then, S consists of the 25 ordered pairs $\{11, 12, \ldots, 55\}$

2-9. $S = \{0, 1, 2, \ldots,\}$ in ppb.

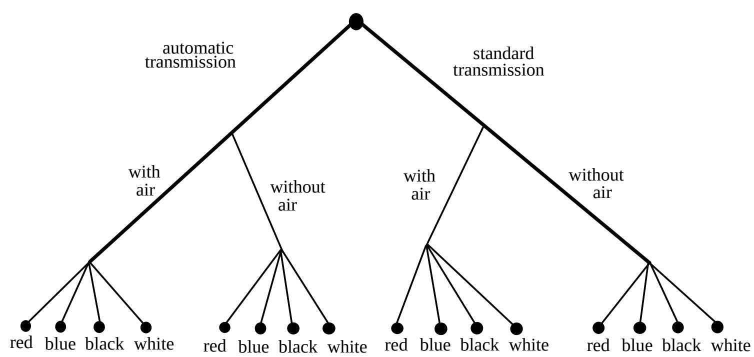
2-10. $S = \{0, 1, 2, \ldots,\}$ in milliseconds

2-11. $S = \{1.0, 1.1, 1.2, \ldots 14.0\}$

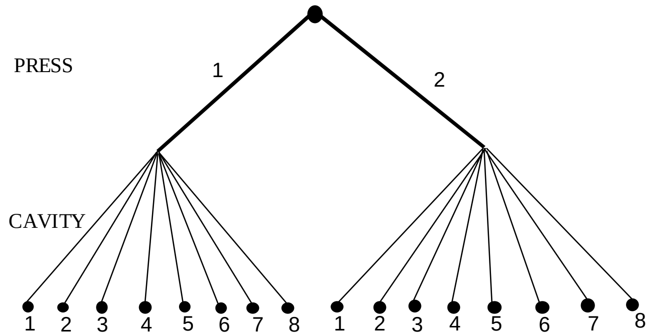
2-12. s = small, m = medium, l = large; S = {s, m, l, ss, sm, sl, ….}

2-13 $S = \{0, 1, 2, \ldots,\}$ in milliseconds.

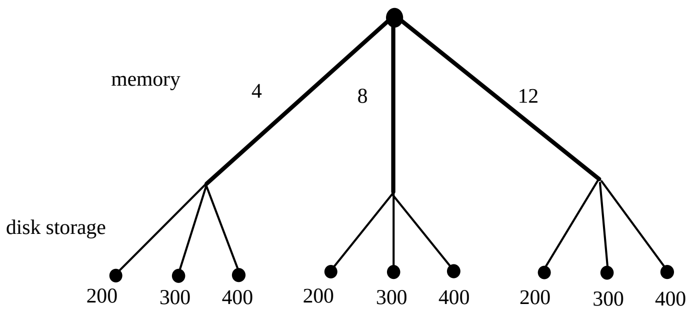
2-14.



2-15.



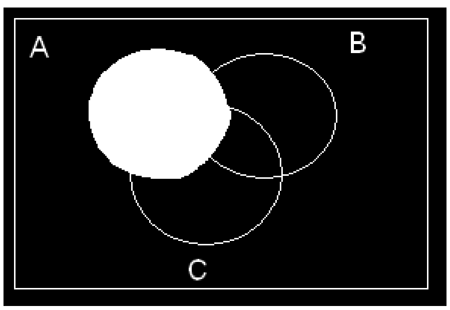
2-16.



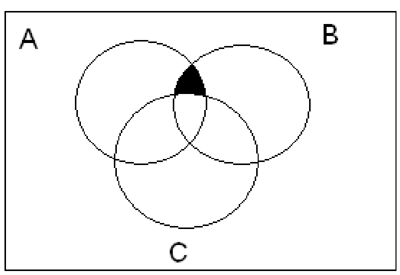
2-17. c = connect, b = busy, S = {c, bc, bbc, bbbc, bbbbc, …}

2-18. S = {s, fs, ffs, fffS, fffFS, fffFFS, fffFFFA}

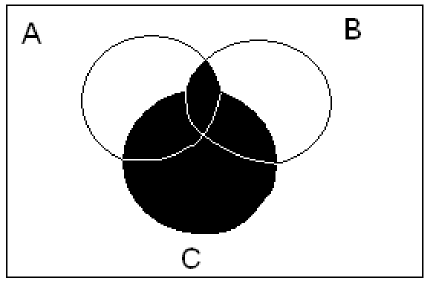
2-19 a.)



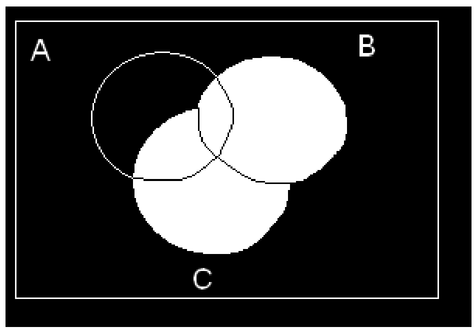
b.)



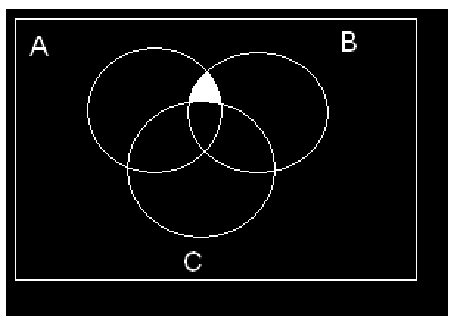
c.)



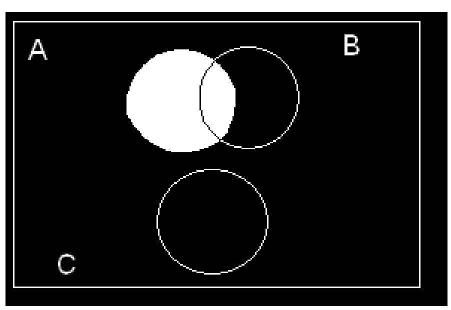
d.)



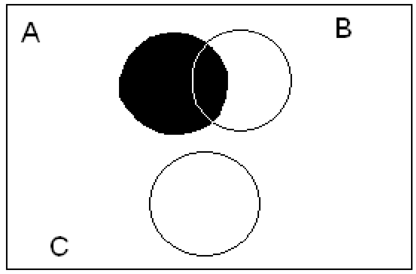
e.)



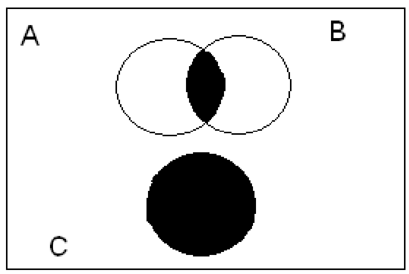
2.20 a.)



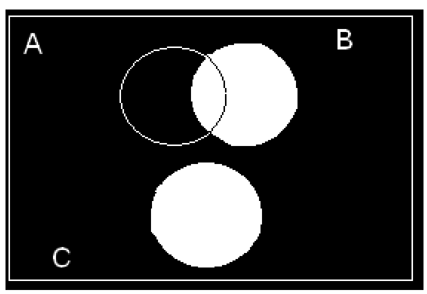
b.)



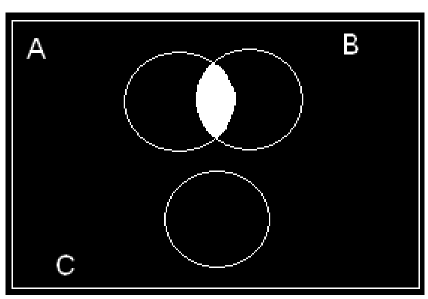
c.)



d.)

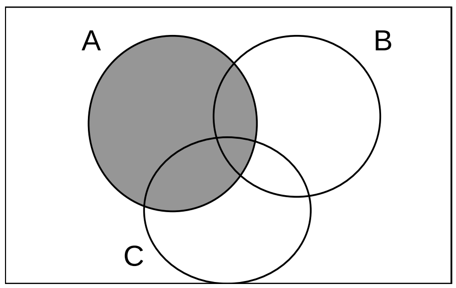


e.)

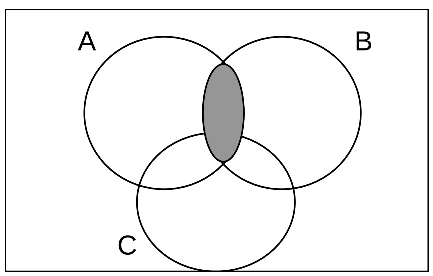


2-21. a) S = nonnegative integers from 0 to the largest integer that can be displayed by the scale.
Let X represent weight.
A is the event that $X > 11$ B is the event that $X \le 15$ C is the event that $8 \le X < 12$
S = {0, 1, 2, 3, …}
b) S
c) $11 < X \le 15$ or {12, 13, 14, 15}
d) $X \le 11$ or {0, 1, 2, …, 11}
e) S
f) $A \cup C$ would contain the values of X such that: $X \ge 8$
Thus $(A \cup C)'$ would contain the values of X such that: $X < 8$ or {0, 1, 2, …, 7}
g) $\varnothing$
h) B′ would contain the values of X such that $X > 15$. Therefore, $B' \cap C$ would be the empty set. They have no outcomes in common or $\varnothing$
i) $B \cap C$ is the event $8 \le X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \ge 8$ or {8, 9, 10, …}

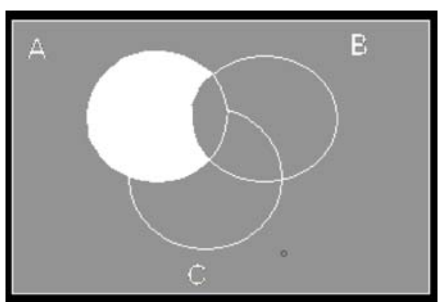
2-22. a)



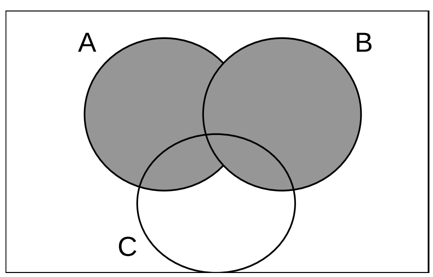
b)



c)



d.)



e.) If the events are mutually exclusive, then A∩B is equal to zero. Therefore, the process does not produce product parts with *X*=50 cm and *Y*=10 cm. The process would not be successful.

2-23. Let "d" denoted a distorted bit and let "o" denote a bit that is not distorted.

a) $S = \left\{ \begin{matrix} dddd, dodd, oddd, oodd, \\ dddo, dodo, oddo, oodo, \\ ddod, dood, odod, oood, \\ ddoo, dooo, odoo, oooo \end{matrix} \right\}$

b) No, for example $A_1 \cap A_2 = \{dddd, dddo, ddod, ddoo\}$

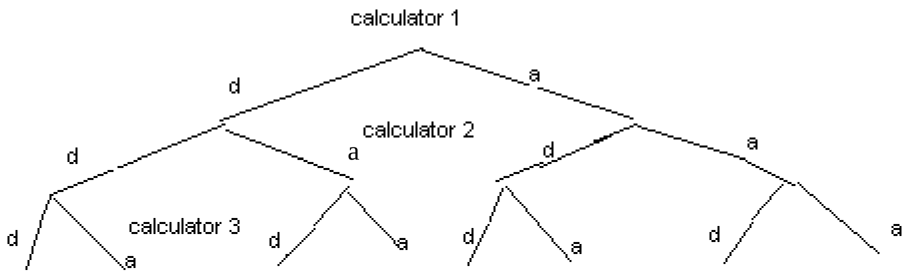
c) $A_1 = \left\{ \begin{matrix} dddd, dodd, \\ dddo, dodo \\ ddod, dood \\ ddoo, dooo \end{matrix} \right\}$

d) $A_1' = \left\{ \begin{matrix} oddd, oodd, \\ oddo, oodo, \\ odod, oood, \\ odoo, oooo \end{matrix} \right\}$

e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{dddd\}$

f) $(A_1 \cap A_2) \cup (A_3 \cap A_4) = \{dddd, dodd, dddo, oddd, ddod, oodd, ddoo\}$

2-24. Let "d" denote a defective calculator and let "a" denote an acceptable calculator



a) $S = \{ddd, add, dda, ada, dad, aad, daa, aaa\}$

b) $A = \{ddd, dda, dad, daa\}$

c) $B = \{ddd, dda, add, ada\}$

d) $A \cap B = \{ddd, dda\}$

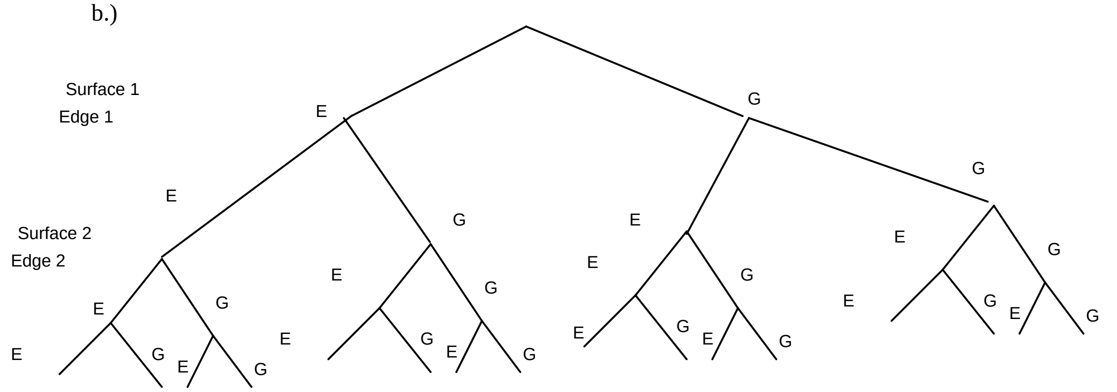
e) $B \cup C = \{ddd, dda, add, ada, dad, aad\}$

2-25. $2^{12} = 4096$

2-26. $A \cap B = 70$, $A' = 14$, $A \cup B = 95$

2-27. a.) $A' \cap B = 10$, $B' = 10$, $A \cup B = 92$

b.)



2-28. $A' \cap B = 55$, $B' = 23$, $A \cup B = 85$

2-29. a) $A' = \{x \mid x \geq 72.5\}$

b) $B' = \{x \mid x \leq 52.5\}$

c) $A \cap B = \{x \mid 52.5 < x < 72.5\}$

d) $A \cup B = \{x \mid x > 0\}$

2.30 a) {*ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc*}

b) {*ab, ac, ad, ae, af, ag, bc, bd, be, bf, bg, cd, ce, cf, cg, ef, eg, fg, ba, ca, da, ea, fa, ga, cb, db, eb, fb, gb, dc, ec, fc, gc, fe, ge, gf*}

c) Let d = defective, g = good; S = {*gg, gd, dg, dd*}

d) Let d = defective, g = good; S = {*gd, dg, gg*}

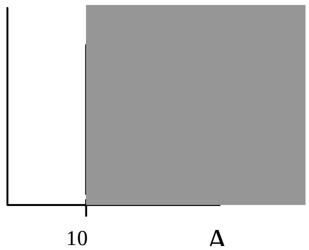
2.31 Let *g* denote a good board, *m* a board with minor defects, and *j* a board with major defects.

a.) S = {*gg, gm, gj, mg, mm, mj, jg, jm, jj*}

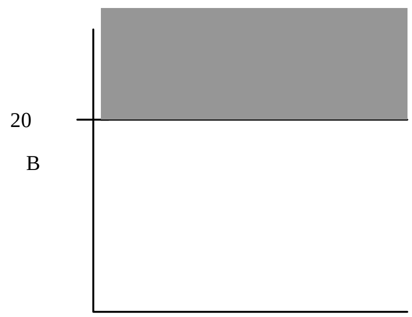
b) *S*={*gg,gm,gj,mg,mm,mj,jg,jm*}

2-32.a.) The sample space contains all points in the positive *X*-*Y* plane.

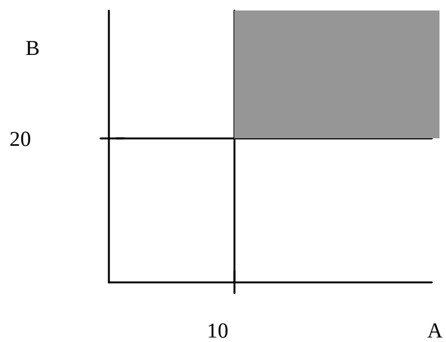
b)



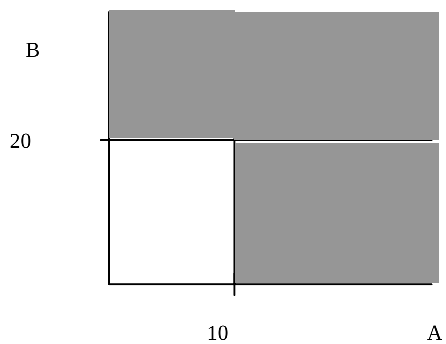
c)



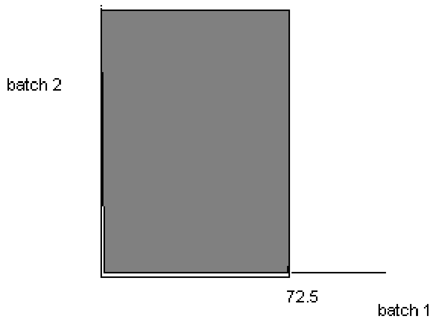
d)



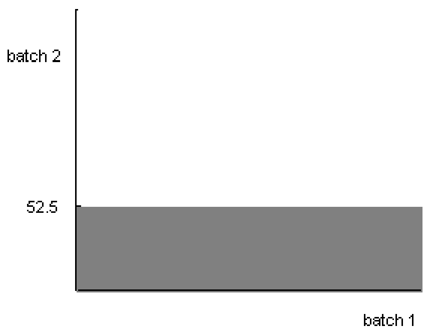
e)



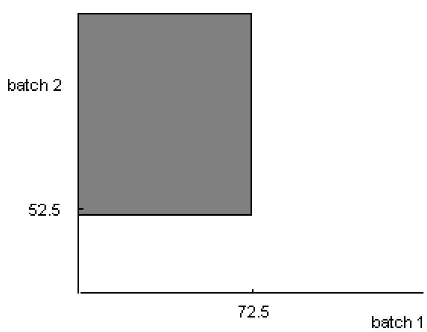
2-33 a)


batch 2
72.5
batch 1

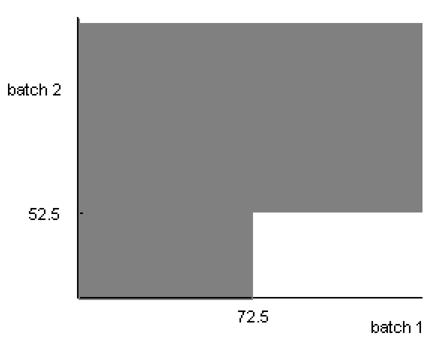
b)


batch 2
52.5
batch 1

c)


batch 2
52.5
72.5
batch 1

d)


batch 2
52.5
72.5
batch 1

Section 2-2

2-34. All outcomes are equally likely
a) P(A) = 2/5
b) P(B) = 3/5
c) P(A') = 3/5
d) $P(A \cup B) = 1$
e) $P(A \cap B) = P(\varnothing) = 0$

2-35. a) P(A) = 0.4
b) P(B) = 0.8
c) P(A') = 0.6
d) $P(A \cup B) = 1$
e) $P(A \cap B) = 0.2$

2-36. a) S = {1, 2, 3, 4, 5, 6}
b) 1/6
c) 2/6
d) 5/6

2-37. a) S = {1,2,3,4,5,6,7,8}
b) 2/8
c) 6/8

2-38. $\frac{x}{20} = 0.3, x = 6$

2-39. a) 0.5 + 0.2 = 0.7
b) 0.3 + 0.5 = 0.8

2-40. a) 1/10
b) 5/10

2-41. a) 0.25
b) 0.75

2-42. Total possible: 10^{16}, Only 10^8 valid, P(valid) = $10^8/10^{16} = 1/10^8$

2-43. 3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10);
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26); The probability your license plate is chosen is then $(1/10^3)*(1/26^3) = 5.7 \times 10^{-8}$

2-44. a) 5*5*4 = 100
b) (5*5)/100 = 25/100=1/4

2-45. a) P(A) = 86/100 = 0.86
b) P(B) = 79/100 = 0.79
c) P(A') = 14/100 = 0.14
d) $P(A \cap B) = 70/100 = 0.70$
e) $P(A \cup B) = (70+9+16)/100 = 0.95$
f) $P(A' \cup B) = (70+9+5)/100 = 0.84$

2-46. Let A = excellent surface finish; B = excellent length
a) P(A) = 82/100 = 0.82
b) P(B) = 90/100 = 0.90
c) P(A') = 1 – 0.82 = 0.18
d) $P(A \cap B) = 80/100 = 0.80$
e) $P(A \cup B) = 0.92$
f) $P(A' \cup B) = 0.98$

2-47. a) P(A) = 30/100 = 0.30
b) P(B) = 77/100 = 0.77
c) P(A') = 1 – 0.30 = 0.70
d) $P(A \cap B)$ = 22/100 = 0.22
e) $P(A \cup B)$ = 85/100 = 0.85
f) $P(A' \cup B)$ =92/100 = 0.92

2-48. a) Because E and E' are mutually exclusive events and $E \cup E' = S$
$1 = P(S) = P(E \cup E') = P(E) + P(E')$. Therefore, P(E') = 1 - P(E)
b) Because S and $\varnothing$ are mutually exclusive events with $S = S \cup \varnothing$
$P(S) = P(S) + P(\varnothing)$. Therefore, $P(\varnothing) = 0$
c) Now, $B = A \cup (A' \cap B)$ and the events A and $A' \cap B$ are mutually exclusive. Therefore,
$P(B) = P(A) + P(A' \cap B)$. Because $P(A' \cap B) \geq 0$, $P(B) \geq P(A)$.

<u>Section 2-3</u>

2-49. a) P(A') = 1- P(A) = 0.7
b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 0.3+0.2 - 0.1 = 0.4
c) $P(A' \cap B) + P(A \cap B) = P(B)$. Therefore, $P(A' \cap B)$ = 0.2 - 0.1 = 0.1
d) $P(A) = P(A \cap B) + P(A \cap B')$. Therefore, $P(A \cap B')$ = 0.3 - 0.1 = 0.2
e) $P((A \cup B)') = 1 - P(A \cup B)$ = 1 - 0.4 = 0.6
f) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$ = 0.7 + 0.2 - 0.1 = 0.8

2-50. a) $P(A \cup B \cup C)$ = P(A) + P(B) + P(C), because the events are mutually exclusive. Therefore,
$P(A \cup B \cup C)$ = 0.2+0.3+0.4 = 0.9
b) $P(A \cap B \cap C) = 0$, because $A \cap B \cap C = \varnothing$
c) $P(A \cap B) = 0$, because $A \cap B = \varnothing$
d) $P((A \cup B) \cap C) = 0$, because $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = \varnothing$
e) $P(A' \cap B' \cap C')$ =1-[P(A) + P(B) + P(C)] = 1-(0.2+0.3+0.4) = 0.1

2-51. If A,B,C are mutually exclusive, then $P(A \cup B \cup C)$ = P(A) + P(B) + P(C) = 0.3 + 0.4 + 0.5 = 1.2, which greater than 1. Therefore, P(A), P(B),and P(C) cannot equal the given values.

2-52. a) 70/100 = 0.70
b) (79+86-70)/100 = 0.95
c) No, $P(A \cap B) \neq 0$

2-53. a) 350/370
b) $\frac{345+5+12}{370} = \frac{362}{370}$
c) $\frac{345+5+8}{370} = \frac{358}{370}$
d) 345/370

2-54. a) 170/190 = 17/19
b) 7/190

2-55. a) P(unsatisfactory) = (5+10-2)/130 = 13/130
b) P(both criteria satisfactory) = 117/130 = 0.90, No

2-56. a) (207+350+357-201-204-345+200)/370 = 0.9838
b) 366/370 = 0.989
c) (200+163)/370 = 363/370 = 0.981
d) (201+163)/370 = 364/370 = 0.984

<u>Section 2-4</u>

2-57. a) P(A) = 86/100 b) P(B) = 79/100

c) P(A|B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{70/100}{79/100} = \dfrac{70}{79}$

d) P(B|A) = $\dfrac{P(A \cap B)}{P(A)} = \dfrac{70/100}{86/100} = \dfrac{70}{86}$

2-58.a) 0.82

b) 0.90

c) 8/9 = 0.889

d) 80/82 = 0.9756

e) 80/82 = 0.9756

f) 2/10 = 0.20

2-59. a) 345/357 b) 5/13

2-60. a) 12/100 b) 12/28 c) 34/122

2-61.

a) P(A) = 0.05 + 0.10 = 0.15

b) P(A|B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{0.04+0.07}{0.72}$ = 0.153

c) P(B) = 0.72

d) P(B|A) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{0.04+0.07}{0.15}$ = 0.733

e) P(A ∩ B) = 0.04 +0.07 = 0.11

f) P(A ∪ B) = 0.15 + 0.72 – 0.11 = 0.76

2-62. a) 20/100

b) 19/99

c) (20/100)(19/99) = 0.038

d) If the chips are replaced, the probability would be (20/100) = 0.2

2-63. a) P(A) = 15/40

b) P(B|A) = 14/39

c) P(A ∩ B) = P(A) P(B/A) = (15/40) (14/39) = 0.135

d) P(A ∪ B) = 1 – P(A' and B') = $1-\left(\dfrac{25}{40}\right)\left(\dfrac{24}{39}\right)=0.615$

2-64. A = first is local, B = second is local, C = third is local

a) P(A ∩ B ∩ C) = (15/40)(14/39)(13/38) = 0.046

b) P(A ∩ B ∩ C') = (15/40)(14/39)(25/39) = 0.089

2-65. a) 4/499 = 0.0080

b) (5/500)(4/499) = 0.000080

c) (495/500)(494/499) = 0.98

2-66. a) 3/498 = 0.0060

b) 4/498 = 0.0080

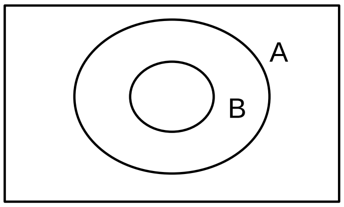
c) $\left(\dfrac{5}{500}\right)\left(\dfrac{4}{499}\right)\left(\dfrac{3}{498}\right) = 4.82x10^{-7}$

2-67. a) P(gas leak) = (55 + 32)/107 = 0.813

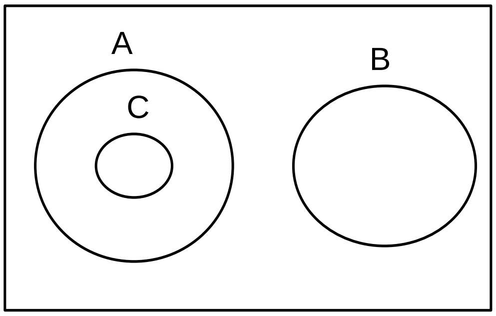
b) P(electric failure|gas leak) = (55/107)/(87/102) = 0.632

c) P(gas leak| electric failure) = (55/107)/(72/107) = 0.764

2-68. No, if $B \subset A$, then P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$



2-69.



<u>Section 2-5</u>

2-70. a) $P(A \cap B) = P(A|B)P(B) = (0.4)(0.5) = 0.20$

b) $P(A' \cap B) = P(A'|B)P(B) = (0.6)(0.5) = 0.30$

2-71.

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)P(B) + P(A|B')P(B') \\ &= (0.2)(0.8) + (0.3)(0.2) \\ &= 0.16 + 0.06 = 0.22 \end{aligned}$$

2-72. Let F denote the event that a connector fails.
Let W denote the event that a connector is wet.

$$\begin{aligned} P(F) &= P(F|W)P(W) + P(F|W')P(W') \\ &= (0.05)(0.10) + (0.01)(0.90) = 0.014 \end{aligned}$$

2-73. Let F denote the event that a roll contains a flaw.
Let C denote the event that a roll is cotton.

$$\begin{aligned} P(F) &= P(F|C)P(C) + P(F|C')P(C') \\ &= (0.02)(0.70) + (0.03)(0.30) = 0.023 \end{aligned}$$

2-74. a) P(A) = 0.03
b) P(A') = 0.97
c) P(B|A) = 0.40
d) P(B|A') = 0.05
e) $P(A \cap B) = P(B|A)P(A) = (0.40)(0.03) = 0.012$
f) $P(A \cap B') = P(B'|A)P(A) = (0.60)(0.03) = 0.018$
g) $P(B) = P(B|A)P(A) + P(B|A')P(A') = (0.40)(0.03) + (0.05)(0.97) = 0.0605$

2-75. Let R denote the event that a product exhibits surface roughness. Let N,A, and W denote the events that the blades are new, average, and worn, respectively. Then,

P(R)= P(R|N)P(N) + P(R|A)P(A) + P(R|W)P(W)
= (0.01)(0.25) + (0.03) (0.60) + (0.05)(0.15)
= 0.028

2-76. Let B denote the event that a glass breaks.
Let L denote the event that large packaging is used.
P(B)= P(B|L)P(L) + P(B|L')P(L')
= 0.01(0.60) + 0.02(0.40) = 0.014

2-77. Let U denote the event that the user has improperly followed installation instructions.
Let C denote the event that the incoming call is a complaint.
Let P denote the event that the incoming call is a request to purchase more products.
Let R denote the event that the incoming call is a request for information.
a) P(U|C)P(C) = (0.75)(0.03) = 0.0225
b) P(P|R)P(R) = (0.50)(0.25) = 0.125

2-78. a) (0.88)(0.27) = 0.2376
b) (0.12)(0.13+0.52) = 0.0.078

2-79. Let A denote a event that the first part selected has excessive shrinkage.
Let B denote the event that the second part selected has excessive shrinkage.
a) P(B)= P(B|A)P(A) + P(B|A ')P(A')
= (4/24)(5/25) + (5/24)(20/25) = 0.20
b) Let C denote the event that the third part selected has excessive shrinkage.

$$P(C) = P(C|A\cap B)P(A\cap B) + P(C|A\cap B')P(A\cap B')$$
$$+P(C|A'\cap B)P(A'\cap B) + P(C|A'\cap B')P(A'\cap B')$$
$$= \frac{3}{23}\left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{20}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{5}{24}\right)\left(\frac{20}{25}\right) + \frac{5}{23}\left(\frac{19}{24}\right)\left(\frac{20}{25}\right)$$
$$= 0.20$$

2-80. Let A and B denote the events that the first and second chips selected are defective, respectively.
a) P(B) = P(B|A)P(A) + P(B|A')P(A') = (19/99)(20/100) + (20/99)(80/100) = 0.2
b) Let C denote the event that the third chip selected is defective.

$$P(A\cap B\cap C) = P(C|A\cap B)P(A\cap B) = P(C|A\cap B)P(B|A)P(A)$$
$$= \frac{18}{98}\left(\frac{19}{99}\right)\left(\frac{20}{100}\right)$$
$$= 0.00705$$

Section 2-6

2-81. Because P(A|B) $\neq$ P(A), the events are not independent.

2-82. P(A') = 1 - P(A) = 0.7 and P(A'|B) = 1 - P(A|B) = 0.7
Therefore, A' and B are independent events.

2-83. P($A\cap B$) = 70/100, P(A) = 86/100, P(B) = 77/100.
Then, P($A\cap B$) $\neq$ P(A)P(B), so A and B are not independent.

2-84. $P(A \cap B) = 80/100$, $P(A) = 82/100$, $P(B) = 90/100$.
Then, $P(A \cap B) \neq P(A)P(B)$, so A and B are <u>not</u> independent.

2-85. a) $P(A \cap B) = 22/100$, $P(A) = 30/100$, $P(B) = 77/100$, Then $P(A \cap B) \neq P(A)P(B)$, therefore, A and B are not independent.
b) $P(B|A) = P(A \cap B)/P(A) = (22/100)/(30/100) = 0.733$

2-86. If A and B are mutually exclusive, then $P(A \cap B) = 0$ and $P(A)P(B) = 0.04$.
Therefore, A and B are <u>not</u> independent.

2-87. It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the ith sample contains high levels of contamination.

a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = P(H_1')P(H_2')P(H_3')P(H_4')P(H_5')$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^5 = 0.59$

b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4' \cap H_5')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4' \cap H_5')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4' \cap H_5')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4 \cap H_5')$

$A_5 = (H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^4(0.1) = 0.0656$. Therefore, the answer is 5(0.0656) = 0.328.

c) Let B denote the event that no sample contains high levels of contamination. The requested probability is P(B') = 1 - P(B). From part (a), P(B') = 1 - 0.59 = 0.41.

2-88. Let A_i denote the event that the ith bit is a one.

a) By independence $P(A_1 \cap A_2 \cap \ldots \cap A_{10}) = P(A_1)P(A_2)\ldots P(A_{10}) = (\frac{1}{2})^{10} = 0.000976$

b) By independence, $P(A_1' \cap A_2' \cap \ldots \cap A_{10}') = P(A_1')P(A_2')\ldots P(A_{10}^c) = (\frac{1}{2})^{10} = 0.000976$

c) The probability of the following sequence is

$P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5' \cap A_6 \cap A_7 \cap A_8 \cap A_9 \cap A_{10}) = (\frac{1}{2})^{10}$, by independence. The number of sequences consisting of five "1"'s, and five "0"'s is $\binom{10}{5} = \frac{10!}{5!5!} = 252$. The answer is $252\left(\frac{1}{2}\right)^{10} = 0.246$

2-89. Let A denote the event that a sample is produced in cavity one of the mold.

a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup \ldots \cup B_8) = P(B_1) + P(B_2) + \ldots + P(B_8)$

From part a., $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4(\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4(\frac{7}{8}) = 0.00107$.

2-90. Let A denote the upper devices function. Let B denote the lower devices function.
$P(A) = (0.9)(0.8)(0.7) = 0.504$
$P(B) = (0.95)(0.95)(0.95) = 0.8574$
$P(A\cap B) = (0.504)(0.8574) = 0.4321$
Therefore, the probability that the circuit operates = $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.9293$

2-91. $[1-(0.1)(0.05)][1-(0.1)(0.05)][1-(0.2)(0.1)] = 0.9702$

2-92. Let A_i denote the event that the ith readback is successful. By independence,
$P(A_1' \cap A_2' \cap A_3') = P(A_1')P(A_2')P(A_3') = (0.02)^3 = 0.000008.$

2-93. a) $P(B|A) = 4/499$ and

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = (4/499)(5/500) + (5/499)(495/500) = 5/500$$

Therefore, A and B are <u>not</u> independent.
b) A and B are independent.

<u>Section 2-7</u>

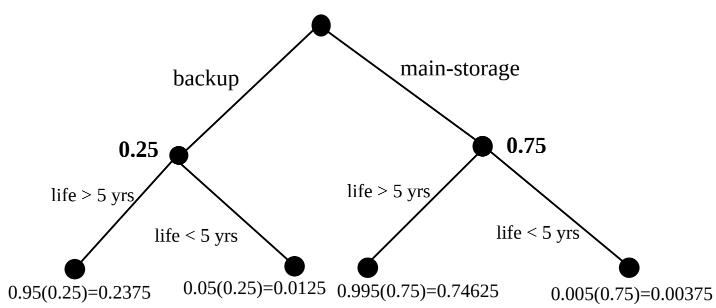
2-94. Because, $P(A|B)P(B) = P(A\cap B) = P(B|A)P(A)$,

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.7(0.2)}{0.5} = 0.28$$

2-95. Let F denote a fraudulent user and let T denote a user that originates calls from two or more metropolitan areas in a day. Then,

$$P(F|T) = \frac{P(T|F)P(F)}{P(T|F)P(F) + P(T|F')P(F')} = \frac{0.30(0.0001)}{0.30(0.0001) + 0.01(.9999)} = 0.003$$

2-96.



a) $P(B) = 0.25$
b) $P(A|B) = 0.95$
c) $P(A|B') = 0.995$
d) $P(A\cap B) = P(A|B)P(B) = 0.95(0.25) = 0.2375$
e) $P(A\cap B') = P(A|B')P(B') = 0.995(0.75) = 0.74625$
f) $P(A) = P(A\cap B) + P(A\cap B') = 0.95(0.25) + 0.995(0.75) = 0.98375$
g) $0.95(0.25) + 0.995(0.75) = 0.98375.$
h)

$$P(B|A') = \frac{P(A'|B)P(B)}{P(A'|B)P(B) + P(A'|B')P(B')} = \frac{0.05(0.25)}{0.05(0.25) + 0.005(0.75)} = 0.769$$

2-97. Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.

a)

$$\begin{aligned} P(G) &= P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P) \\ &= 0.95(0.40) + 0.60(0.35) + 0.10(0.25) \\ &= 0.615 \end{aligned}$$

b) Using the result from part a.,

$$P(H|G) = \frac{P(G|H)P(H)}{P(G)} = \frac{0.95(0.40)}{0.615} = 0.618$$

c) $P(H|G') = \frac{P(G'|H)P(H)}{P(G')} = \frac{0.05(0.40)}{1-0.615} = 0.052$

2-98. a) P(D)=P(D|G)P(G)+P(D|G')P(G')=(.005)(.991)+(.99)(.009)=0.013865

b) $P(G|D')=P(G\cap D')/P(D')=P(D'|G)P(G)/P(D')=(.995)(.991)/(1-.013865)=0.9999$

2-99. a) P(S) = 0.997(0.60) + 0.9995(0.27) + 0.897(0.13) = 0.9847

b) P(Ch|S) =(0.13)(0.897)/0.9847 = 0.1184

Section 2-8

2-100. Continuous: a, c, d, f, h, i; Discrete: b, e, and g

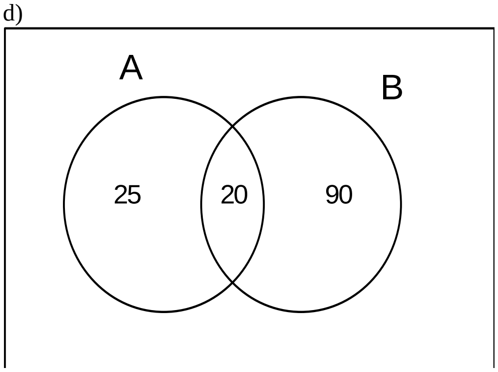
Supplemental Exercises

2-101. Let D_i denote the event that the primary failure mode is type i and let A denote the event that a board passes the test.

The sample space is S = $\{A, A'D_1, A'D_2, A'D_3, A'D_4, A'D_5\}$.

2-102. a) 20/200 b) 135/200 c) 65/200

d)



2-103. a) $P(A) = 19/100 = 0.19$
b) $P(A \cap B) = 15/100 = 0.15$
c) $P(A \cup B) = (19 + 95 - 15)/100 = 0.99$
d) $P(A' \cap B) = 80/100 = 0.80$
e) $P(A|B) = P(A \cap B)/P(B) = 0.158$

2-104. Let A_i denote the event that the *i*th order is shipped on time.

a) By independence, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) = (0.95)^3 = 0.857$

b) Let

$$B_1 = A_1' \cap A_2 \cap A_3$$
$$B_2 = A_1 \cap A_2' \cap A_3$$
$$B_3 = A_1 \cap A_2 \cap A_3'$$

Then, because the B's are mutually exclusive,

$$\begin{aligned} P(B_1 \cup B_2 \cup B_3) &= P(B_1) + P(B_2) + P(B_3) \\ &= 3(0.95)^2(0.05) \\ &= 0.135 \end{aligned}$$

c) Let

$$B_1 = A_1' \cap A_2' \cap A_3$$
$$B_2 = A_1' \cap A_2 \cap A_3'$$
$$B_3 = A_1 \cap A_2' \cap A_3'$$
$$B_4 = A_1' \cap A_2' \cap A_3'$$

Because the B's are mutually exclusive,

$$\begin{aligned} P(B_1 \cup B_2 \cup B_3 \cup B_4) &= P(B_1) + P(B_2) + P(B_3) + P(B_4) \\ &= 3(0.05)^2(0.95) + (0.05)^3 \\ &= 0.00725 \end{aligned}$$

2-105. a) No, $P(E_1 \cap E_2 \cap E_3) \neq 0$
b) No, $E_1' \cap E_2'$ is not $\varnothing$
c) $P(E_1' \cup E_2' \cup E_3') = P(E_1') + P(E_2') + P(E_3') - P(E_1' \cap E_2') - P(E_1' \cap E_3') - P(E_2' \cap E_3')$
$+ P(E_1' \cap E_2' \cap E_3')$
$= 40/240$
d) $P(E_1 \cap E_2 \cap E_3) = 200/240$
e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) - P(E_1 \cap E_3) = 234/240$
f) $P(E_1 \cup E_2 \cup E_3) = 1 - P(E_1' \cap E_2' \cap E_3') = 1 - 0 = 1$

2-106. $(0.20)(0.30) + (0.7)(0.9) = 0.69$

2-107. Let A_i denote the event that the *i*th bolt selected is not torqued to the proper limit.

a) Then,

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_4 | A_1 \cap A_2 \cap A_3) P(A_1 \cap A_2 \cap A_3)$$
$$= P(A_4 | A_1 \cap A_2 \cap A_3) P(A_3 | A_1 \cap A_2) P(A_2 | A_1) P(A_1)$$
$$= \left(\frac{12}{17}\right)\left(\frac{13}{18}\right)\left(\frac{14}{19}\right)\left(\frac{15}{20}\right) = 0.282$$

b) Let B denote the event that at least one of the selected bolts are not properly torqued. Thus, B' is the event that all bolts are properly torqued. Then,

$P(B) = 1 - P(B') = 1 - \left(\frac{15}{20}\right)\left(\frac{14}{19}\right)\left(\frac{13}{18}\right)\left(\frac{12}{17}\right) = 0.718$

2-108. Let A,B denote the event that the first, second portion of the circuit operates. Then, P(A) = (0.99)(0.99)+0.9-(0.99)(0.99)(0.9) = 0.998

P(B) = 0.9+0.9-(0.9)(0.9) = 0.99 and

$P(A \cap B) = P(A)\,P(B) = (0.998)\,(0.99) = 0.988$

2-109. A_1 = by telephone, A_2 = website; $P(A_1) = 0.92$, $P(A_2) = 0.95$;

By independence $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.92 + 0.95 - 0.92(0.95) = 0.996$

2-110. P(Possess) = 0.95(0.99) +(0.05)(0.90) = 0.9855

2-111. Let D denote the event that a container is incorrectly filled and let H denote the event that a container is filled under high-speed operation. Then,

a) $P(D) = P(D|H)P(H) + P(D|H')P(H') = 0.01(0.30) + 0.001(0.70) = 0.0037$

b) $P(H|D) = \frac{P(D|H)P(H)}{P(D)} = \frac{0.01(0.30)}{0.0037} = 0.8108$

2-112. a) $P(E' \cap T' \cap D') = (0.995)(0.99)(0.999) = 0.984$

b) $P(E \cup D) = P(E) + P(D) - P(E \cap D) = 0.005995$

2-113. D = defective copy

a) $P(D = 1) = \left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{72}{73}\right) + \left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{72}{73}\right) + \left(\frac{73}{75}\right)\left(\frac{72}{74}\right)\left(\frac{2}{73}\right) = 0.0778$

b) $P(D = 2) = \left(\frac{2}{75}\right)\left(\frac{1}{74}\right)\left(\frac{73}{73}\right) + \left(\frac{2}{75}\right)\left(\frac{73}{74}\right)\left(\frac{1}{73}\right) + \left(\frac{73}{75}\right)\left(\frac{2}{74}\right)\left(\frac{1}{73}\right) = 0.00108$

c) Let A represent the event that the two items NOT inspected are not defective. Then, P(A)=(73/75)(72/74)=0.947.

2-114. The tool fails if any component fails. Let F denote the event that the tool fails. Then, $P(F') = 0.99^{10}$ by independence and $P(F) = 1 - 0.99^{10} = 0.0956$

2-115. a) (0.3)(0.99)(0.985) + (0.7)(0.98)(0.997) = 0.9764

b) $P(route1|E) = \frac{P(E|route1)P(route1)}{P(E)} = \frac{0.02485(0.30)}{1-0.9764} = 0.3159$

2-116. a) By independence, $0.15^5 = 7.59 \times 10^{-5}$

b) Let A_i denote the events that the machine is idle at the time of your ith request. Using independence, the requested probability is

c) As in part b, the probability of 3 of the events is

$$P(A_1 A_2 A_3 A_4' A_5' \quad or \quad A_1 A_2 A_3' A_4 A_5' \quad or \quad A_1 A_2 A_3' A_4' A_5 \quad or \quad A_1 A_2' A_3 A_4 A_5' \quad or \quad A_1 A_2' A_3 A_4' A_5 \; or$$

$$A_1 A_2' A_3' A_4 A_5 \quad or \quad A_1' A_2 A_3 A_4 A_5' \quad or \quad A_1' A_2 A_3 A_4' A_5 \quad or \quad A_1' A_2 A_3' A_4 A_5 \quad or \quad A_1' A_2' A_3 A_4 A_5)$$

$$= 10(0.15^3)(0.85^2)$$

$$= 0.0244$$

For the probability of at least 3, add answer parts a) and b) to the above to obtain the requested probability. Therefore, the answer is 0.0000759 + 0.0022 + 0.0244 = 0.0267

2-117. Let A_i denote the event that the ith washer selected is thicker than target.

a) $\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{8}\right) = 0.207$

b) 30/48 = 0.625

c) The requested probability can be written in terms of whether or not the first and second washer selected are thicker than the target. That is,

2-118. a) If n washers are selected, then the probability they are all less than the target is $\frac{20}{50} \cdot \frac{19}{49} \cdot \ldots \frac{20-n+1}{50-n+1}$.

n	probability all selected washers are less than target
1	20/50 = 0.4
2	(20/50)(19/49) = 0.155
3	(20/50)(19/49)(18/48) = 0.058

Therefore, the answer is n = 3

b) Then event E that one or more washers is thicker than target is the complement of the event that all are less than target. Therefore, P(E) equals one minus the probability in part a. Therefore, n = 3.

2-119.

$$a)\quad P(A\cup B)=\frac{112+68+246}{940}=0.453$$

$$b)\quad P(A\cap B)=\frac{246}{940}=0.262$$

$$c)\quad P(A'\cup B)=\frac{514+68+246}{940}=0.881$$

$$d)\quad P(A'\cap B')=\frac{514}{940}=0.547$$

. e) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{246/940}{314/940}=0.783$

f) $P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{246/940}{358/940}=0.687$

2-120. Let E denote a read error and let S,O,P denote skewed, off-center, and proper alignments, respectively. Then,

a) P(E) = P(E|S) P(S) + P(E|O) P (O) + P(E|P) P(P)
= 0.01(0.10) + 0.02(0.05) + 0.001(0.85)
= 0.00285

b) $P(S|E)=\frac{P(E|S)P(S)}{P(E)}=\frac{0.01(0.10)}{0.00285}=0.351$

2-121. Let A_i denote the event that the ith row operates. Then,

$P(A_1)=0.98, P(A_2)=(0.99)(0.99)=0.9801, P(A_3)=0.9801,\quad P(A_4)=0.98.$

The probability the circuit does not operate is

$P(A_1')P(A_2')P(A_3')P(A_4')=(0.02)(0.0199)(0.0199)(0.02)=1.58\times10^{-7}$

2-122. a) (0.4)(0.1) + (0.3)(0.1) +(0.2)(0.2) + (0.4)(0.1) = 0.15
b) P(4 or more|provided) = (0.4)(0.1)/0.15 = 0.267

Mind-Expanding Exercises

2-123. Let E denote a read error and let S, O, B, P denote skewed, off-center, both, and proper alignments, respectively.
P(E) = P(E|S)P(S) + P(E|O)P(O) + P(E|B)P(B) + P(E|P)P(P)
= 0.01(0.10) + 0.02(0.05) + 0.06(0.01) + 0.001(0.84) = 0.00344

2-124. Let n denote the number of washers selected.

a) The probability that all are less than the target is 0.4^n , by independence.

n	0.4^n
1	0.4
2	0.16
3	0.064

Therefore, n = 3

b) The requested probability is the complement of the probability requested in part a. Therefore, n = 3

2-125. Let x denote the number of kits produced.

Revenue at each demand

	0	50	100	200
$0 \le x \le 50$	-5x	100x	100x	100x
Mean profit = 100x(0.95)-5x(0.05)-20x				
$50 \le x \le 100$	-5x	100(50)-5(x-50)	100x	100x
Mean profit = [100(50)-5(x-50)](0.4) + 100x(0.55)-5x(0.05)-20x				
$100 \le x \le 200$	-5x	100(50)-5(x-50)	100(100)-5(x-100)	100x
Mean profit = [100(50)-5(x-50)](0.4) + [100(100)-5(x-100)](0.3) + 100x(0.25) - 5x(0.05) - 20x				

	Mean Profit	Maximum Profit
$0 \le x \le 50$	74.75 x	\$ 3737.50 at x=50
$50 \le x \le 100$	32.75 x + 2100	\$ 5375 at x=100
$100 \le x \le 200$	1.25 x + 5250	\$ 5500 at x=200

Therefore, profit is maximized at 200 kits. However, the difference in profit over 100 kits is small.

2-126. Let E denote the probability that none of the bolts are identified as incorrectly torqued. The requested probability is P(E'). Let X denote the number of bolts in the sample that are incorrect. Then,
P(E) = P(E|X=0)P(X=0) + P(E|X=1) P(X=1) + P(E|X=2) P(X=2) + P(E|X=3) P(X=3) + P(E|X=4)P(X=4)
and P(X=0) = (15/20)(14/19)(13/18)(12/17) = 0.2817. The remaining probability for x can be determined from the counting methods in Appendix B-1. Then,

$$P(X=1)=\frac{\binom{5}{1}\binom{15}{3}}{\binom{20}{4}}=\frac{\left(\frac{5!}{4!1!}\right)\left(\frac{15!}{3!12!}\right)}{\left(\frac{20!}{4!16!}\right)}=\frac{5!15!4!16!}{4!3!12!20!}=0.4696$$

$$P(X=2)=\frac{\binom{5}{2}\binom{15}{2}}{\binom{20}{4}}=\frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{2!13!}\right)}{\left(\frac{20!}{4!16!}\right)}=0.2167$$

$$P(X=3)=\frac{\binom{5}{3}\binom{15}{1}}{\binom{20}{4}}=\frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{1!14!}\right)}{\left(\frac{20!}{4!16!}\right)}=0.0309$$

P(X=4) = (5/20)(4/19)(3/18)(2/17) = 0.0010 and P(E|X=0) = 1, P(E|X=1) = 0.05, P(E|X=2) = $0.05^2 = 0.0025$, P(E|X=3) = $0.05^3 = 125 \times 10^{-4}$, P(E|X=4) = $0.05^4 = 6.25 \times 10^{-6}$. Then,

$$\begin{aligned}P(E) &= 1(0.2817)+0.05(0.4696)+0.0025(0.2167)+125\times10^{-4}(0.0309)\\ &\quad +6.25\times10^{-6}(0.0010)\\ &= 0.306\end{aligned}$$

and P(E') = 0.694

2-127.

$$\begin{aligned}P(A'\cap B') &= 1-P([A'\cap B']') = 1-P(A\cup B)\\ &= 1-[P(A)+P(B)-P(A\cap B)]\\ &= 1-P(A)-P(B)+P(A)P(B)\\ &= [1-P(A)][1-P(B)]\\ &= P(A')P(B')\end{aligned}$$

2-128. The total sample size is ka + a + kb + b = (k + 1)a + (k +1)b.

$P(A) = \frac{k(a+b)}{(k+1)a + (k+1)b}$, $P(B) = \frac{ka + a}{(k+1)a + (k+1)b}$

and

$P(A \cap B) = \frac{ka}{(k+1)a + (k+1)b} = \frac{ka}{(k+1)(a+b)}$

Then,

$P(A)P(B) = \frac{k(a+b)(ka+a)}{[(k+1)a + (k+1)b]^2} = \frac{k(a+b)(k+1)a}{(k+1)^2(a+b)^2} = \frac{ka}{(k+1)(a+b)} = P(A \cap B)$

<u>Section 2-1.4 on CD</u>

S2-1. From the multiplication rule, the answer is $5 \times 3 \times 4 \times 2 = 120$

S2-2. From the multiplication rule, $3 \times 4 \times 3 = 36$

S2-3. From the multiplication rule, $3 \times 4 \times 3 \times 4 = 144$

S2-4. From equation S2-1, the answer is 10! = 3628800

S2-5. From the multiplication rule and equation S2-1, the answer is 5!5! = 14400

S2-6. From equation S2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

S2-7. a) From equation S2-4, the number of samples of size five is $\binom{140}{5} = \frac{140!}{5!135!} = 416965528$

b) There are 10 ways of selecting one nonconforming chip and there are $\binom{130}{4} = \frac{130!}{4!126!} = 11358880$ ways of selecting four conforming chips. Therefore, the number of samples that contain exactly one nonconforming chip is $10 \times \binom{130}{4} = 113588800$

c) The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{140}{5}$ minus the number of samples that contain no nonconforming chips $\binom{130}{5}$.

That is $\binom{140}{5}$ - $\binom{130}{5} = \frac{140!}{5!135!} - \frac{130!}{5!125!} = 130721752$

S2-8. a) If the chips are of different types, then every arrangement of 5 locations selected from the 12 results in a different layout. Therefore, $P_5^{12} = \frac{12!}{7!} = 95040$ layouts are possible.

b) If the chips are of the same type, then every subset of 5 locations chosen from the 12 results in a different layout. Therefore, $\binom{12}{5} = \frac{12!}{5!7!} = 792$ layouts are possible.

S2-9. a) $\frac{7!}{2!5!} = 21$ sequences are possible.

b) $\frac{7!}{1!1!1!1!1!2!} = 2520$ sequences are possible.

c) 6! = 720 sequences are possible.

S2-10. a) Every arrangement of 7 locations selected from the 12 comprises a different design.

$P_7^{12} = \frac{12!}{5!} = 3991680$ designs are possible.

b) Every subset of 7 locations selected from the 12 comprises a new design. $\frac{12!}{5!7!} = 792$ designs are possible.

c) First the three locations for the first component are selected in $\binom{12}{3} = \frac{12!}{3!9!} = 220$ ways. Then, the four locations for the second component are selected from the nine remaining locations in $\binom{9}{4} = \frac{9!}{4!5!} = 126$ ways. From the multiplication rule, the number of designs is $220 \times 126 = 27720$

S2-11. a) From the multiplication rule, $10^3 = 1000$ prefixes are possible

b) From the multiplication rule, $8 \times 2 \times 10 = 160$ are possible

c) Every arrangement of three digits selected from the 10 digits results in a possible prefix.

$P_3^{10} = \frac{10!}{7!} = 720$ prefixes are possible.

S2-12. a) From the multiplication rule, $2^8 = 256$ bytes are possible

b) From the multiplication rule, $2^7 = 128$ bytes are possible

S2-13. a) The total number of samples possible is $\binom{24}{4} = \frac{24!}{4!20!} = 10626.$ The number of samples in which exactly one tank has high viscosity is $\binom{6}{1}\binom{18}{3} = \frac{6!}{1!5!} \times \frac{18!}{3!15!} = 4896$. Therefore, the probability is

$$\frac{4896}{10626} = 0.461$$

b) The number of samples that contain no tank with high viscosity is $\binom{18}{4} = \frac{18!}{4!14!} = 3060.$ Therefore, the requested probability is $1 - \frac{3060}{10626} = 0.712$.

c) The number of samples that meet the requirements is $\binom{6}{1}\binom{4}{1}\binom{14}{2} = \frac{6!}{1!5!} \times \frac{4!}{1!3!} \times \frac{14!}{2!12!} = 2184$.

Therefore, the probability is $\frac{2184}{10626} = 0.206$

S2-14. a) The total number of samples is $\binom{12}{3} = \frac{12!}{3!\,9!} = 220$. The number of samples that result in one nonconforming part is $\binom{2}{1}\binom{10}{2} = \frac{2!}{1!1!} \times \frac{10!}{2!8!} = 90.$ Therefore, the requested probability is 90/220 = 0.409.

b) The number of samples with no nonconforming part is $\binom{10}{3} = \frac{10!}{3!7!} = 120.$ The probability of at least one nonconforming part is $1 - \frac{120}{220} = 0.455$.

S2-15. a) The probability that both parts are defective is $\frac{5}{50} \times \frac{4}{49} = 0.0082$

b) The total number of samples is $\binom{50}{2} = \frac{50!}{2!48!} = \frac{50 \times 49}{2}$. The number of samples with two defective parts is $\binom{5}{2} = \frac{5!}{2!3!} = \frac{5 \times 4}{2}$. Therefore, the probability is $\frac{\frac{5\times4}{2}}{\frac{50\times49}{2}} = \frac{5 \times 4}{50 \times 49} = 0.0082$.