INVERSE MODELS

C. Wunsch, Massachusetts Institute of Technology, Cambridge, MA, USA

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Introduction

Inverse methods are formal procedures for making inferences about the ocean (or any other physical system) by using observations in combination with dynamical and kinematic models. As such, they are a part of the general methods of statistical inference done in the presence of known or assumed kinematic and dynamical constraints. Their first use in oceanography occurred in 1977 as a method for addressing the famous so-called level-of-no-motion problem in the oceanic general circulation, and the determination of the general circulation remains a major area of application. Subsequently, inverse methods became a central element of ocean acoustic tomography. More recently, they have begun to be applied widely in all areas of oceanography including biogeochemical problems.

In mathematical usage, 'inverse methods' often describe procedures directed at solving a variety of ill-posed problems, in the absence of observational noise. Although the terminology is much the same, noise-free observations exist only in textbooks, and this literature is useful, but tangential, to the oceanographic problem.

'Inverse methods' are often used in conjunction with, and thereby confused with, 'inverse models'. For historical reasons, and mathematical convention, one denotes many systems as being 'forward' or 'direct' problems or models. A simple example derives from a supposed theory that produces a rule for a variable, perhaps oceanic temperature, θ, as a function of z, in the form

$$\theta(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 = \sum_{i=0}^{3} a_i z^3 \qquad [1]$$

If the theory tells us the a_i, we can calculate θ for any value of z. The theory is often labeled a 'forward' or 'direct model' and, more generally, may be either an analytical or a very complex numerical one. Equation [1] is called a 'forward solution'. If, on the other hand, $\theta(z)$ were known, but one or more of the a_i were not, one would have an inverse model, also given by eqn [1], the label 'inverse' being employed only as a matter of convention – because there is a previously studied 'forward' version.

Another example comes from the classical advection/diffusion/decay equation for a concentration tracer, C,

$$w\frac{\partial C}{\partial z} - \kappa\frac{\partial^2 C}{\partial z^2} + \lambda C = q(z) \qquad [2]$$

where q is a source, w is the vertical advective velocity, κ is a diffusion coefficient and λ is a decay constant. All are known constants or functions of z. Given suitable boundary conditions, e.g., $C(z=0) = C_0, C(z \to \infty) = 0$, one has a well-understood, well-posed problem, in which eqn [2] and its boundary conditions are labeled as 'forward.' But an equally compelling, and commonplace problem is the following: Given $C(z)$ and λ, what are w and κ? This type of problem occupies much of the mathematical literature on inverse problems. The oceanographers' version is, however, more likely to be: Given observations,

$$y_i = C(z_i) + n_i \qquad [3]$$

where n_i are noise, at a finite number of discrete positions z_i, what are w, κ? Many variations of this problem are possible; for example, given noisy or uncertain observations or estimates of C, w, κ, λ what was the boundary condition C_0? In this context, eqn [2], along with any other available information, would now be described as the 'inverse model,' with the formal knowns and unknowns of the forward problem having in part, been interchanged.

Parameter determination from noisy observations goes back at least to Legendre and Gauss. The modern generalization, inverse theory, is often traced to the pioneering work of G. Backus and F. Gilbert in the solid-earth geophysics context. They initiated the study of model systems in which the number of formal unknowns greatly exceeds (perhaps infinitely so), the number of available data by exploiting the existence of an underlying differential or partial differential system. This form of inverse theory is rooted in functional analysis and is highly developed; see Parker (1994), or for oceanographic applications, Bennett (1992), in the Further Reading.

In oceanographic practice, even the simplest models are almost always reduced to some numerical,

discrete form, either by expansion into a finite set of modes or by finite differences or related methods. The polynomial form eqn [1] is already discrete, with four parameters. Many other forms are possible. For example eqn [2], when written in finite differences, becomes

$$w_i \frac{C_{i+1} - C_i}{z_{i+1} - z_i} - \kappa_i \frac{C_{i+1} - 2C_i + C_{i-1}}{z_{i+1} - 2z_i + z_{i-1}} + \lambda C_i = q_i, \quad 1 \le i \le M \qquad [4]$$

which is a set of M simultaneous linear equations in the finite discrete set w_i, κ_i, λ. Alternatively, if w, κ are constant, an analytic solution, subject to a fixed surface concentration $C(z = 0) = C_0$ is readily seen to be

$$C(z) = C_0 \exp\left\{ z \frac{w}{2\kappa} \left[1 + \left(1 + \frac{4\lambda\kappa}{w^2} \right)^{1/2} \right] \right\}, \quad z \le 0 \qquad [5]$$

which can be evaluated at $z = z_i$ to produce

$$C(z_i) = C_0 \exp\left\{ z_i \frac{w}{2\kappa} \left[1 + \left(1 + \frac{4\lambda\kappa}{w^2} \right)^{1/2} \right] \right\} \qquad [6]$$

Because eqns [4] and [6] are algebraic equations in w, κ, λ the problem of determining them has been reduced from that of an infinite-dimensional Hilbert or Banach space to that of an ordinary finite-dimensional vector space. No matter how great the value of M, the corresponding mathematical simplifications render the methods of inverse theory much more transparent in this case. Reduction of the functional analysis methods of Backus and Gilbert to that of finite-dimensional spaces appears to begin with Wiggins, who used the singular value decomposition of Eckart and Young. In practice, almost all real inverse problems are solved on computers; they are thus automatically discrete and of finite dimension and this mathematical representation is the most useful one. Note that eqn [6] is nonlinear in the parameters, showing that the same problem can be rendered linear depending upon exactly how it is formulated.

Example

Assuming then, that observations always have a noise component, we can proceed to estimate whatever is known. For the simple power law of eqn [1], let us suppose that the 'truth' is

$$\theta(z) = 30 - 0.0005z^2, \quad -100 \le z \le 0 \qquad [7]$$

but the theory says that it could actually be of the general form (1) so that the correct answer would be $a_0 = 30$, $a_2 = 5 \times 10^{-4}$, $a_i = 0$, otherwise. Defining $y_i = C_i + n_i$, the problem has become one of finding an estimate, $\tilde{\mathbf{x}}$, satisfying,

$$\mathbf{E}\mathbf{x} + \mathbf{n} = \mathbf{y} \qquad [8]$$

Here the matrix vector notation $\mathbf{x} = [a_i]$ (called the 'state vector'), $\mathbf{n} = [n_i]$, $\mathbf{y} = [y_i]$ is being used, and $\mathbf{E}$ is the coefficient matrix. As stated, this is now a problem in polynomial regression theory, and much of inverse theory overlaps that branch of statistics.

The tracer problem (eqn [2]) can be reduced to this same form. Suppose that w_i, κ_i, are believed constant, independent of i, and that we have noisy measurements ξ_i [3] of C_i. Then one can readily make the substitution $C_i \to \xi_i$ in eqn [4], producing a set of simultaneous equations for w, κ_i, where the coefficient matrix $\mathbf{E}$ has elements depending upon

$$\frac{C_{i+1} - C_i}{z_{i+1} - z_i} \sim \frac{\xi_{i+1} - \xi_i}{z_{i+1} - z_i} \qquad [9]$$

etc. and $\mathbf{y}$ now involves $\lambda\xi_i$, q_i, etc. $\mathbf{n}$ is a noise-vector representing the errors introduced by the observational noise, and any misrepresentation of the true full physics by eqn [2]. In practice, the structure of $\mathbf{n}$ can be a complicated function of the structure of $\mathbf{E}$, because equations such as [9] render the problem, in a rigorous sense, nonlinear, too. This nonlinearity is often ignored and there are many inverse problems where $\mathbf{E}$ is known exactly. Equation [8] can now be regarded as the 'model' instead of, or in addition to, eqn [2]. The state vector consists of the two unknowns $\mathbf{x}^{\mathrm{T}} = [w, \kappa_i]^{\mathrm{T}}$ (superscript T denotes a transpose; all vectors here are column vectors). The form of eqns [8] obviously generalizes to any number of unknown elements, N, in $\mathbf{x}$.

A common method for dealing with equations of this type is to seek a least-squares solution that renders the noise vector $\mathbf{n}$ as small as possible:

$$J = \mathbf{n}^{\mathrm{T}}\mathbf{n} = (\mathbf{y} - \mathbf{E}\mathbf{x})^{\mathrm{T}}(\mathbf{y} - \mathbf{E}\mathbf{x}) \qquad [10]$$

By taking the derivatives of the 'objective' or 'cost' function J with respect to the elements of $\mathbf{x}$ and setting them to zero, one obtains the 'normal equations.' Here, one finds, using this matrix notation, that the so-called normal equations are

$$\mathbf{E}^{\mathrm{T}}\mathbf{E}\mathbf{x} = \mathbf{E}^{\mathrm{T}}\mathbf{y} \qquad [11]$$

whose solution is

$$\tilde{\mathbf{x}} = (\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{y} \qquad [12]$$

and which permits an estimate of the noise unknowns,

$$\tilde{\mathbf{n}} = \mathbf{y} - \mathbf{E}\tilde{\mathbf{x}} = \mathbf{y} - \mathbf{E}(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{y} \qquad [13]$$

The forward model [1] was used to generate 'data' at seven observation points, and these values were then corrupted with noise having a standard deviation of 1. The least-squares solution produces a state vector $\tilde{\mathbf{x}}^{\mathrm{T}} = [31,\ 0.02,\ 0.002,\ 6\times10^{-5}, 4\times10^{-7}]$, and the corresponding estimated $\tilde{\theta}$ is shown in **Figure 1**. It passes near the data points but is clearly not 'correct' in the sense of reproducing the known true values.

Although a very easy-to-use and common parameter-determining procedure, least-squares in this form is not an inverse method (statisticians call it 'curve-fitting'). The reasons for seeking a more powerful method are easy to see. At least as written, one can raise a number of questions about the solution:

1. Why should the smallest mean-square noise, $\mathbf{n}^{\mathrm{T}}\mathbf{n}$, be regarded as the correct solution to choose?
2. What would happen if the inverse of $\mathbf{E}^{\mathrm{T}}\mathbf{E}$ failed to exist? ($\mathbf{E}$ corresponding to eqn [7] is a Vandermonde matrix, and known to be very badly conditioned.)
3. Suppose one knew that some of the noise values were likely to be much greater than others: Could that information be used?
4. Suppose one had a reasonable idea of the magnitude of the a_i, i.e., of $\mathbf{x}$: Cannot that information be used?
5. Just how reliable is the solution given the noisiness of the data, and the particular structure of the observations?
6. Are some observations more important than others in determining the solution?
7. Could one require $\kappa > 0$ in the advection–diffusion problem?
8. In general, there are M eqns in [8] and N elements of $\mathbf{x}$. When $M > N$, the system appears to be comfortably overdetermined. But that comfort disappears when one recognizes that N noise unknowns have also been determined in eqn [13], producing a total of $M + N$ previously unknown values. Is it still sensible to term the problem 'overdetermined.'
9. Problems such as the advection-diffusion one involve a quantity C that is necessarily positive and that often renders the noise statistics highly non-Gaussian. How can one understand how that affects the best solution?

These and other issues lead one to find other means to make an estimate of $\mathbf{x}$. It is possible to modify the least-squares procedure so that it will produce solutions obtained by other methods; this correspondence has led, in the literature, to serious confusion about what is going on.

Two More Solution Methods

There are a number of methods available for estimating the elements of $\mathbf{x}$, $\mathbf{n}$ that produce useful answers to some or all of the questions listed above. We will briefly describe two such methods, leaving to the references listed in Further Reading (see Menke, 1989; Tarantola, 1987; Munk *et al.*, 1995;

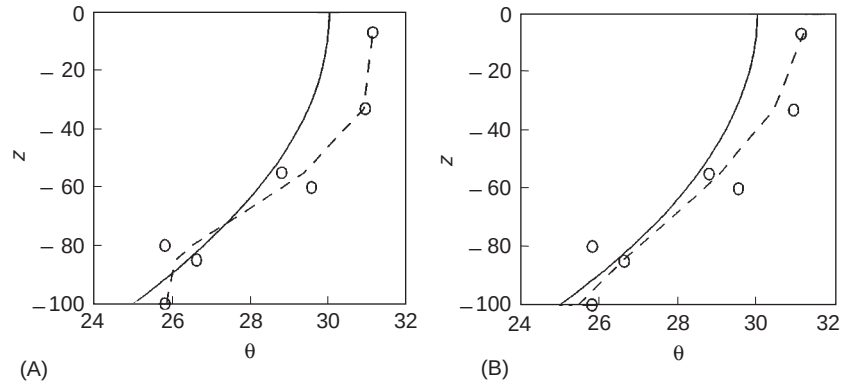

Figure 1 (A) A hypothetical temperature, θ, profile with depth, $-z$, drawn as a solid line. 'Observations' of that profile at discrete depths contaminated with unit noise variance errors are shown as open circles. Dashed line is the ordinary least-squares fit. (B) Same as in (A), except that the dashed line is the Gauss–Markov solution computed using *a priori* knowledge of the statistics of the noise and the solution itself. The fit is less structured, and a specific error estimate is available at every point, showing the true line to lie within the estimated uncertainty.

Wunsch, 1996) both the details and discussion of other approaches.

Gauss–Markov Method

We postulate some *a priori* knowledge of the elements of $\mathbf{x}$, $\mathbf{n}$ in a common, statistical, form. In particular, we assume that $\langle \mathbf{x} \rangle = \langle \mathbf{n} \rangle = 0$, where the brackets denote the expected value, and that there is some knowledge of the second moments, $\mathbf{S} = \langle \mathbf{x}\mathbf{x}^{\mathrm{T}} \rangle, \mathbf{R} = \langle \mathbf{n}\mathbf{n}^{\mathrm{T}} \rangle$.

The so-called Gauss–Markov method (sometimes known as the 'stochastic inverse') produces a solution that minimizes the variance about the true value, i.e., it is a statistical method that minimizes not the sum of squares but, individually, all terms of the form $\langle (\tilde{\mathbf{x}}_i - x_i)^2 \rangle$ where x_i is the true value, and $\tilde{x}_i$ is the estimate made. The solution is

$$\tilde{\mathbf{x}} = \mathbf{S}\mathbf{E}^{\mathrm{T}}(\mathbf{E}\mathbf{S}\mathbf{E}^{\mathrm{T}} + \mathbf{R})^{-1}\mathbf{y} \qquad [14]$$

with solution uncertainty (error estimate)

$$\mathbf{P} = \langle (\tilde{\mathbf{x}} - \mathbf{x})(\tilde{\mathbf{x}} - \mathbf{x})^{\mathrm{T}} \rangle = \mathbf{S} - \mathbf{S}\mathbf{E}^{\mathrm{T}}(\mathbf{E}\mathbf{S}\mathbf{E}^{\mathrm{T}} + \mathbf{R})^{-1}\mathbf{E}\mathbf{S} \qquad [15]$$

All of the available information has been used. A solution for the simple example is shown in **Figure 1**, where $\mathbf{S} = \mathrm{diag}\,([100, 1\times10^{-10}, 1\times10^{-6}, 1\times10^{-10}, 1\times10^{-10}])$, $\mathbf{R} = \{\delta_{ij}\}$. One obtains,

$$\tilde{\mathbf{x}}^{\mathrm{T}} = [31 \pm 0.8, -1.4\times10^{-10} \pm 1\times10^{-5}, -6.1\times10^{-4} \pm 5.1\times10^{-4}, 4.5\times10^{-6} \pm 9.1\times10^{6}, 4.9\times10^{-8} \pm 6.9\times10^{-8}] \qquad [16]$$

that is, the correct answer now lies within two standard errors and, although it is not displayed here, the $\mathbf{P}$ matrix provides a full statement of the extent to which the noise elements in $\tilde{\mathbf{x}}$ are correlated with each other (which can be of the utmost importance in many problems).

Least-Squares by Singular Value Decomposition

As noted, least-squares can be modified so that it is more fully capable of producing answers to the questions put above. There are at least two ways to do this. The more interesting one is that based upon the singular value decomposition (SVD) and the so-called Cholesky decomposition of the covariance matrices, $\mathbf{S} = \mathbf{S}^{\mathrm{T}/2}\mathbf{S}^{1/2}$, $\mathbf{R} = \mathbf{R}^{\mathrm{T}/2}\mathbf{R}^{1/2}$.

One takes the original eqn [18] and employs the matrices $\mathbf{S}$, $\mathbf{R}$ to rotate and stretch $\mathbf{E}$, $\mathbf{n}$, $\mathbf{y}$, $\mathbf{x}$ into new vector spaces in which both observations and solution have uncorrelated structures:

$$\mathbf{R}^{-\mathrm{T}/2}\mathbf{E}\mathbf{S}^{\mathrm{T}/2}\mathbf{S}^{-\mathrm{T}/2}\mathbf{x} + \mathbf{R}^{-\mathrm{T}/2}\mathbf{n} = \mathbf{R}^{-\mathrm{T}/2}\mathbf{y} \qquad [17]$$

or

$$\mathbf{E}'\mathbf{x}' + \mathbf{n}' = \mathbf{y}' \qquad [18]$$

where

$$\mathbf{E}' = \mathbf{R}^{-\mathrm{T}/2}\mathbf{E}\mathbf{S}^{\mathrm{T}/2}, \quad \mathbf{x}' = \mathbf{S}^{-\mathrm{T}/2}\mathbf{x}, \quad \mathbf{n}' = \mathbf{R}^{-\mathrm{T}/2}\mathbf{n}, \quad \mathbf{y}' = \mathbf{R}^{-\mathrm{T}/2}\mathbf{y} \qquad [19]$$

One then computes the singular value decomposition

$$\mathbf{E}' = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^{\mathrm{T}} \qquad [20]$$

where $\mathbf{\Lambda}$ is a diagonal matrix, and $\mathbf{U}$, $\mathbf{V}$ are orthogonal matrices. Let there be K nonzero values on the diagonal of $\mathbf{\Lambda}$, and let the columns of $\mathbf{U}$, $\mathbf{V}$ be denoted $\mathbf{u}_i$, $\mathbf{v}_i$ respectively. Then it can be shown in straightforward fashion that the $\mathbf{u}_i^{\mathrm{T}}\mathbf{u}_j = \delta_{ij}, \mathbf{v}_i^{\mathrm{T}}\mathbf{v}_j = \delta_{ij}$ and that the solution to eqn. [18] is

$$\mathbf{x}' = \sum_{k=1}^{K} \mathbf{v}_k \frac{\mathbf{u}_k^{\mathrm{T}}\mathbf{y}'}{\lambda_k} + \sum_{k=K+1}^{N} \alpha_k \mathbf{v}_k \qquad [21]$$

where the α_k are completely arbitrary. The physical solution is obtained from $\tilde{\mathbf{x}} = \mathbf{S}^{\mathrm{T}/2}\tilde{\mathbf{x}}'$. A major advantage of this solution (and the uncertainly matrix, $\mathbf{P}$, can also be computed for it), is that it explicitly produces the solution in orthonormal structures, $\mathbf{v}_i$, in terms of orthonormal structures of the observations $\mathbf{u}_i$, and separates these elements from the second sum on the right of equations [21], which defines the so-called null space of the problem. The null space represents the structures possibly present in the true solution, about which the equations [18] carry no information. A null space is present too, in the Gauss–Markov solution, but the corresponding null space vectors are given coefficients α_k so as best to reproduce the *a priori* values of $\mathbf{S}$. The SVD leads naturally to a discussion of what is called 'resolution' both of data and of the solution, and provides complete information about the solution. These issues, with examples, are discussed in Wunsch (1996) (see Further Reading).

A Hydrographic Example

The first use of inverse methods in oceanography was the hydrographic problem. In this classical problem, oceanographers are able to determine the density field, $\rho(z)$ at two nearby 'hydrographic

stations' where temperature and salinity were measured by a ship as functions of depth. By computing the horizontal differences in density as a function of depth, and invoking geostrophic balance, an estimate could be made of the velocity field, up to an unknown constant (with depth) of integration. Although mathematically trivial, the inability to determine the integration constant between pairs of stations plagued oceanography for 100 years, and led to the employment of the *ad hoc* assumption that the flow field vanished at some specific depth z_0. This depth became known as the 'level-of-no-motion.' But by employing physical requirements such as mass and salt conservation (or any other constraint involving the absolute velocity), it is straightforward to reduce the problem to one of deducing the actual flow field at the level-of-no-motion (and which is thus better called the 'reference-level'). These constraints are readily written in the form of eqn [8], and solved with error estimates, etc.

An example of the practical application of this method is shown in **Figure 2**. In this calculation, constraints were written for mass and salt conservation in a triangular region bounded by the US coastline, and a pair of hydrographic sections in which the Gulf Stream flowed into the region across one section and out again in the other. Direct velocity measurements obtained from the ship permitted additional equations to be written for the unknown flow velocity at the reference level. The resulting estimated absolute flow at the reference level is shown, with error estimates, in the top panels. The lower panels show the total estimated absolute velocity. In general, any available information can be used to estimate the unknowns of the system as long as one has a plausible estimate of the error contained in the resulting constraint. For the hydrographic problem in particular, a number of variations on the constraints have been proposed, under the labels of 'beta-spiral,' 'Bernoulli-method,' etc., which we must leave to the references listed in Further Reading.

Extensions

Like eqn [6], many inverse problems are nonlinear. Tarantola (1987) provides some general background and specific oceanographic applications may be seen

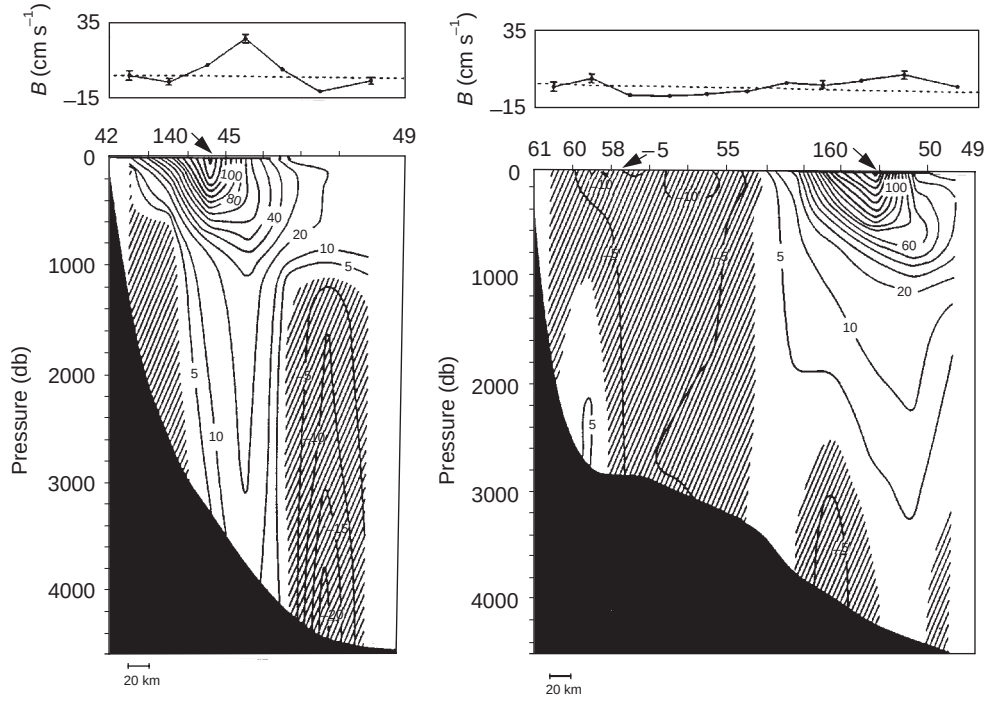

Figure 2 Example of the inversion for the reference level velocity in a triangular region bounded on two sides by hydrographic sections crossing the Gulf Stream, and on the third side by the US coastline. Velocity contours are in centimeters. They are the sum of the so-called thermal wind, which involves setting the velocity to zero at a reference depth. The actual flow at that depth (the 'reference level velocity') is shown in the top panels as estimated, with uncertainties, from the singular value decomposition solution, with $K = 30$. (From Joyce *et al.*, 1986.) The 'columnar' structure, which is so apparent here, first appeared in inverse solutions, and was greeted with disbelief by those who 'knew' that oceanic flows were 'layered' in form. Acceptance of this type of structure is now commonplace.

in Mercier (1989), Mercier *et al.* (1991), and Wunsch (1994). The use of inequality constraints leads to the general subject of mathematical programming, a part of the wide subject of optimization theory (see Arthnari and Dodge, 1981).

The Gauss–Markov solution method and the SVD version of least-squares have a ready interpretation, as minimum variance estimates of the true field. If the fields are all normally distributed, the solutions are also maximum-likelihood estimates, a methodology that is readily extended to non-Gaussian fields.

Time-dependent Problems

As originally formulated by Backus and Gilbert, and as exploited in most of the oceanographic literature to date, the problems have been essentially static, with time evolution not accounted for. If one has a system that evolves in time, the observations and state vector, $\mathbf{x}$, will also be time evolving. If one simply writes down the relationship between data and a model in which the two are connected by a set of equations, linear or nonlinear, one sees immediately that, mathematically, the problem is identical to that posed by eqn [8], in which the time of the observation or of the model calculation is just a bookkeeping index. The difficulty that arises is purely a practical one: the potentially extremely large growth in the number of equations that must be dealt with over long time spans. With a sufficiently large and fast computer, the most efficient way to solve such inverse problems would be the straightforward application of the same methods developed for static problems: Write out all of the equations explicitly and solve them all at once. In oceanographic practice however, one rapidly outstrips the largest available computers and the static methods become impractical if used naively.

Fortunately, time-evolving oceanographic models have a very special algebraic structure, which permits one to solve the corresponding normal equations [11] by methods not requiring storage of everything in the computer at one time. Numerous such special methods exist, and go by names such as sequential estimators (Kalman filter and various smoothers), adjoint equations (Pontryagin principle or method of Lagrange multipliers), Monte Carlo methods, etc. The generic terminology that has come into use is 'data assimilation' borrowed from meteorological forecasting terminology. Such methods are highly developed, if often abused or misunderstood, and require a separate discussion. What is important to know, however, is that they are simply algorithmically efficient solutions to the inverse problem as described here.

Common Misconceptions and Difficulties

Some chronic misunderstandings and difficulties arise. The most pernicious of these is the attempt to use inverse models, which are physically inconsistent with the known forward model or physics. This blunder corresponds, for example, in the simplest model above, to imposing a linear relationship (model) between θ and z, when it is clear or suspected that higher powers of z are likely to be present. Some writers have gone so far as to show that such an inversion does not reproduce a known forward solution, and then declared inverse methods to be failures. Another blunder is to confuse the inability to resolve or determine a parameter of interest, when the data are inadequate for the purpose, with a methodological or model failure. Inverse methods are very powerful tools. Like any powerful tool (a chain saw, for example), when properly used they are useful and even essential; when improperly used they are a grave danger to the user.

See also

Data Assimilation in Models. Elemental Distribution: Overview. General Circulation Models. Tomography. Tracer Release Experiments.

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IR RADIOMETERS

C. J. Donlon, Space Applications Institute, Ispra, Italy

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Introduction

Measurements of sea surface temperature (SST) are most important for the investigation of the processes underlying heat and gas exchange across the air–sea interface, the surface energy balance, and the general circulation of both the atmosphere and the oceans. Complementing traditional subsurface contact temperature measurements, there is a wide variety of infrared radiometers, spectroradiometers, and thermal imaging systems that can be used to determine the SST by measuring thermal emissions from the sea surface. However, the SST determined from thermal emission can be significantly different from the subsurface temperature ($> \pm 1$ K) because the heat flux passing through the air–sea interface typically results in a strong temperature gradient. Radiometer systems deployed on satellite platforms provide daily global maps of SSST (sea surface temperature) at high spatial resolution (~ 1 km) whereas those deployed from ships and aircraft provide data at small spatial scales of centimeters to meters. In particular, the development of satellite radiometer systems providing a truly synoptic view of surface ocean thermal features has been pivotal in the description and understanding of the global oceans.

This article reviews the infrared properties of water and some of the instruments developed to measure thermal emission from the sea surface. It focuses on *in situ* radiometers although the general principles described are applicable to satellite sensors treated elsewhere in this volume.

Infrared Measurement Theory

Infrared (IR) radiation is heat energy that is emitted from all objects that have a temperature above 0 K (-273.16°C). It includes all wavelengths of the electromagnetic spectrum between 0.75 μm and ~ 100 μm (**Figure 1**) and has the same optical properties as visible light, being capable of reflection, refraction, and forming interference patterns.

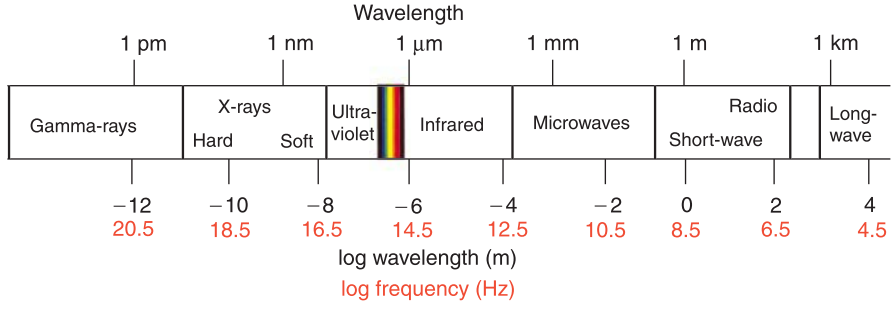

Figure 1 Schematic diagram of the electromagnetic spectrum showing the location and interval of the infrared waveband.