PREFACE

This sixth edition of the text on the unit operations of chemical engineering has been extensively revised and updated, with much new material and considerable condensation of some sections. Its basic structure and general level of treatment, however, remain unchanged. It is an introductory text, written for undergraduate students in their junior or senior years who have completed the usual courses in mathematics, physics, chemistry, and an introduction to chemical engineering. An elementary knowledge of material and energy balances and of thermodynamic principles is assumed.

Separate chapters are devoted to each of the principal unit operations, grouped into four sections: fluid mechanics, heat transfer, mass transfer and equilibrium stages, and operations involving particulate solids. One-semester or one-quarter courses may be based on any of these sections or combinations of them. The order of the first 16 chapters has not been changed; later ones, dealing with mass transfer and operations involving solids, have been rearranged in a more logical order.

Nearly all equations have been written for SI units, and the Newton's law conversion factor g_c has been eliminated except in the few instances where it must be included. Symbols for dimensionless groups have been changed to Re for N_{Re}, for example, Pr for N_{Pr}, and so forth. Many new examples and problems have been added, some reflecting the importance of biochemical engineering processes. Material on handling, mixing, and grinding particulate solids has been greatly condensed and dealt with in a single chapter. The number of appendixes is reduced from 22 to 19.

Derivations of the differential equations for continuity and momentum balances, leading to the Navier-Stokes equation, have been added, as well as the differential forms of Fourier's law and Fick's law, emphasizing the analogies among momentum, heat, and mass transfer. The chapter on adsorption has been expanded to include new material on chromatography and ion exchange, and renamed "Fixed-bed Separations." Other new material has been added on viscoelastic fluids, laminar flow in annuli, drag coefficients, affinity laws for pumps, high-efficiency agitators and motionless mixers, plate-type heat exchangers, boiling by submerged tube bundles, cooling towers, aqueous phase extraction, cross-flow filtration, and many other topics.

Many of the problems at the ends of the chapters are new or revised. Most are expressed in SI units. Nearly all the problems can be solved with the aid of a pocket calculator; for a few, a computer solution is preferable.

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Solutions Manual

to accompany

Unit Operations of Chemical Engineering

Sixth Edition

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Chapter 1

1.1. (a) From Appendix 1,

$$1 \text{ ft-lb}_f = \frac{1}{0.73756} \text{ J} = \frac{1}{0.73756 \times 1055.06} \text{ Btu}$$

$$= \frac{1}{0.73756 \times 1055.06} \times 2.9307 \times 10^{-4} \text{ kWh}$$

$$= 3.76614 \times 10^{-7} \text{ kWh}$$

(b) 1 gal = 231 in.^{3} = 231 x 2.54^3/1000 = 3.7854 L

(c) 1 Btu/lb mol = 1055.06/0.45359237

= 2326 J/kg mol

1.2. 1 year = 365.25 days (close enough - exact value is 365.2422454 days). 1 day = 60 x 24 = 1,440 min

1 microcentury = 100 x 365.25 x 1440 x 10^{-6} = 52.596

say 53 minutes.

1.3. (a) Find z from Eq. (1.61) by multiplying by v/RT:

$$z = \frac{pv}{RT} = \frac{1 - c/vT^3}{v}\left(v + B_o - \frac{bB_o}{v}\right) - \frac{A_o}{vRT}\left(1 - \frac{a}{v}\right)$$

Clear of parentheses and collect terms in 1/v, $1/v^2$, and $1/v^3$:

$$z = 1 + \frac{1}{v}\left(B_o - \frac{c}{T^3} - \frac{A_o}{RT}\right) + \frac{1}{v^2}\left(- bB_o - \frac{cB_o}{T^3} + \frac{aA_o}{RT}\right) + \frac{1}{v^3}\left(\frac{cbB_o}{T^3}\right)$$

The virial coefficients are

$$B = B_o - A_o/RT - c/T^3$$

$$C = -bB_o + aA_o/RT - cB_o/T^3$$

$$D = cbB_o/T^3$$

(b) Since $1/v = \rho$, where ρ is the molar density, $z = 1 + \rho B + \rho^2 C + \rho^3 D$. Also, since 1 kg mol/m^3 = 1 g mol/l, the equations for z, B, C, and D calculated from the values of A_o, B_o, a, b, and c given in the problem statement can be used with SI units without change, provided the pressure is given in atmospheres.

For air, in SI units,

$$B = 0.04611 - 1.3012/0.08206T - 6.6x10^5/T^3$$

$$= 0.04611\ (1 - 343.89/T - 1.431 \times 10^7/T^3)$$

$$C = 0.01101 \times 0.04611 + 0.01931 \times 1.3012/0.08206T$$

$$- 0.04611 \times 6.6 \times 10^5/T^3$$

$$= 5.0767 \times 10^{-4}\ (1 + 603.13/T - 5.99 \times 10^7/T^3)$$

$$D = -0.01101 \times 0.04611 \times 6.6 \times 10^5/T^3 = -335/T^3$$

(c) At T = 300K

$$B = 0.04611 - 15.8567/300 - 6.600 \times 10^5/300^3$$

$$= -0.03119$$

$$C = 5.0767 \times 10^{-4} + 0.30615/300 - 3.04326 \times 10^4/300^3$$

$$= 4.0104 \times 10^{-4}$$

$$D = -335/300^3 = -1.241 \times 10^{-5}$$

When v = 0.200 m^3/kg mol, ρ = 5 kg mol/m^3, and

$$z = 1 - 5 \times 0.03119 + 5^2 \times 4.0104 \times 10^{-4} - 5^3 \times 1.251 \times 10^{-5}$$

$$= 1 - 0.15595 + 0.01003 - 0.00155 = 0.8525$$

1-4. <u>Basis: 1 minute; flows in g mol/min</u>

Molar volume at 15.6°C and 1 atm:

$$\frac{0.02241 \times (15.6 + 273.2)}{273.2} = 0.02369 \text{ mol/min}$$

Entering tower

Moles of gas: $28.32/0.02369 = 1195$ mol/min

Moles of NH_3: $0.25 \times 1195 = 299$ mol/min

Moles of air: $1195 - 299 = 896$ mol/min

Leaving tower

Moles of NH_3: $\dfrac{0.005 \times 896}{0.995} = 4.5$ mol/min

Per cent of NH_3 not absorbed: $\dfrac{4.5}{299} \times 100 = 1.5\%$

Vapor pressures of water (Appendix 8)

At 37.8°C: $0.949/14.696 = 0.0646$ atm

At 21.1°C: $0.363/14.696 = 0.0247$ atm

Partial pressure of water in entering gas: 0.0646 atm

Partial pressure of dry gas: $2.02 - 0.0646 = 1.9554$ atm

Water in inlet gas: $(0.0646/1.9554)1195 = 39.5$ mol/min

NH_3 + air leaving tower: $4.5 + 896 = 900.5$ mols/min

Water in exit stream: $900.5 \times \dfrac{0.0247}{2.02 - 0.0247} = 11.1$ mol/min

Water condensed: $39.5 - 11.1 = 28.4$ mol/min

NH_3 absorbed: $299 - 4.5 = 294.5$ mol/min

Since the leaving solution is 10% NH_3 by weight, the total water in this solution is $294.5 \times 17 \times 0.90/0.10 =$ 45058 g/min

Density of water: 998 kg/m^3

Volume flow of water: $60 \times 45058 \times 10^{-3}/998 = 2.71$ m^3/h

1-5. Basis: 1 hour

(a) Volume of entering gas: $1.5 \times 3600 \times \pi \times \dfrac{2^2}{4} =$ 16,965 ft^3/h

(b) Volume of one lb mol, entering conditions:

$359 \times (460 + 80)/492 = 394$ ft^3

Air entering: (16965/394) (0.75) = 32.29 mol/h

NH_3 entering: (16965/394) (0.25) = 10.76 mol/h

NH_3 leaving: (32.29) (1/99) = 0.33 mol/h

NH_3 absorbed = 10.43 mol/h

Absorption: 10.43 x 17 = 177 lb NH_3/h

1-6. Basis: 1 hour

NaOH fed: 25 x 0.1 = 2.5 tonnes

NaCl fed: 25 x 0.1 = 2.5 tonnes

Water fed: 25 x 0.8 = 20.0 tonnes

All the NaOH appears in the concentrated liquor

(c) Total mass of concentrated liquor:

$$2.5 \times \frac{100}{50} = 5 \text{ tonnes or } 5000 \text{ kg/h}$$

(a) Water in concentrated liquor: 0.48 x 5 = 2.4 tonnes

Water evaporated: 20 - 2.4 = 17.6 tonnes or 17,600 kg/h

(b) Salt in concentrated liquor: 0.02 x 5 = 0.1 tonne

Salt precipitated: 2.5 - 0.1 = 2.4 tonnes or 2,400 kg/h

1.7. Use Eq. (1.60) with $Z_b = Z_a$ and $W_s = 0$. Also, $H_b - H_a = c_p (T_b - T_a)$. From Eq. (1.60),

$$\frac{Q}{m} = c_p (T_b - T_a) + \frac{u_b^2 - u_a^2}{2g_c J}$$

where c_p = the specific heat at constant pressure

T_a, T_b = air temperatures at entrance and exit

u_a, u_b = air velocity at entrance and exit

Substituting the values from the problem statement gives

$$\frac{Q}{m} = 0.24\ (140-40) + \frac{75^2 - 50^2}{2 \times 32.174 \times 778}$$

$$= 24 + 0.06 = 24.06 \text{ Btu/lb}$$

1.8. Left-hand term: $[h] = \overline{H}\ \overline{L}^{-2}\ \overline{t}^{-1}\ \overline{T}^{-1}$

First right-hand term:

$[k\ D_p] = \overline{H}\ \overline{t}^{-1}\ \overline{L}^{-1}\ \overline{T}^{-1}\ \overline{L}^{-1} = \overline{H}\ \overline{L}^{-2}\ \overline{t}^{-1}\ \overline{T}^{-1}$

Second right-hand term:

$$\left[D_p^{-0.5} G^{0.5} \mu^{-0.17} c_p^{0.33} k^{0.67}\right] = (\overline{L}^{-0.5})(\overline{M}^{0.5}\ \overline{t}^{-0.5}\ \overline{L}^{-1})(\overline{M}^{-0.17}\ \overline{t}^{0.17}\ \overline{L}^{0.17})$$
$$\times\ (\overline{H}^{0.33}\ \overline{M}^{-0.33}\ \overline{T}^{-0.33})\ (\overline{H}^{0.67}\ \overline{t}^{-0.67}\ \overline{L}^{-0.67}\ \overline{T}^{-0.67})$$

Exponents of $\overline{H}$: $0.33 + 0.67 = 1$

Exponents of $\overline{L}$: $-0.5 - 1 + 0.17 - 0.67 = -2$

Exponents of $\overline{t}$: $-0.5 + 0.17 - 0.67 = -1$

Exponents of $\overline{T}$: $-0.33 - 0.67 = -1$

Exponents of $\overline{M}$: $0.5 - 0.17 - 0.33 = 0$

For the term: $\overline{H}\ \overline{L}^{-2}\ \overline{t}^{-1}\ \overline{T}^{-1}$. All terms have the same dimensions.

1.9. $[\Delta p] = [\overline{V}]^a [\rho]^b [\mu]^c [D]^d [D_o]^e$

Pressure has the units of force per unit area, say N/m^2. From Eq. (1.6) this becomes $kg.m/s^2.m^2 = kg/m.s^2$ or mass per unit length per time squared. Hence

$$\overline{M}\ \overline{L}^{-1}\ \overline{t}^{-2} = \overline{L}^a\ \overline{t}^{-a}\ \overline{M}^b\ \overline{L}^{-3b}\ \overline{M}^c\ \overline{L}^{-c}\ \overline{t}^{-c}\ \overline{L}^d\ \overline{L}^e$$

Exponents of $\overline{M}$: $1 = b + c$ (1)

Exponents of $\overline{L}$: $-1 = a - 3b - c + d + e$ (2)

Exponents of $\overline{t}$: $-2 = -a - c$ (3)

Retain exponents c and e. From Eq. (1),

$$b = 1 - c$$

From Eq. (3), $a = 2 - c$

From Eq. (2), $d = -1 - a + 3b + c - e$

$= -1 - 2 + c + 3 - 3c - e = 0 - c - e$

Hence $[\Delta p] = [\overline{V}]^{2-c} [\rho]^{1-c} [\mu]^{c} [D]^{-c-e} [D_o]^{e}$

Grouping by exponents,

$$\left(\frac{\Delta p}{\rho \overline{V}^2}\right) = \left(\frac{\mu}{D \overline{V} \rho}\right)^{c} \left(\frac{D_o}{D}\right)^{e}$$

The first group on the right is the reciprocal of the Reynolds number, Re. Any dimensionless group can be inverted without affecting its validity in the correlation.

Chapter 2

2-1. Use Eq. (2-10) noting that

$1\ g/cm^3 = 1000\ kg/m^3$. Quantities needed are:

$g = 9.80665\ m/s^2$ (Eq. (1-13)) $R_m = 0.2\ m$

$$p_a - p_b = 9.80665 \times 0.2\ (13.6 - 1.6)\ 1000$$

$$= 23536\ kg - m/m^2\text{-}s^2 = 23536\ N/m^2$$

2-2. Find a relation between temperature and elevation.

From the given conditions:

$$\frac{dT}{dZ} = -\frac{5}{1000} = -0.005$$

Integrating between limits, setting T = 288K at Z = 0 gives

$$T = 288 - 0.005Z$$

Substituting for T in Eq. (2-[illegible]) and integrating gives

$$\int_{p_a}^{p_b} \frac{dp}{p} = -\frac{gM}{R} \int_{Z_a}^{Z_b} \frac{dZ}{288 - 0.005Z}$$

Since $Z_a = 0$

$$\ln \frac{p_a}{p_b} = \frac{gM}{0.005R} \ln \frac{288}{288 - 0.005Z_b}$$

Given: $p_a/p_b = 760/380 = 2 \quad g = 9.80665\ m/s^2$

$M = 29$ kg/kg-mol $R = 8314.3$ J-K/kg mol (Table 1.2)

$$= 8314.3 \frac{(kg - m^2/s^2)K}{kg\ mol}$$

Note also that the coefficient 0.005 has the units K/m.

Substituting gives

$$\ln 2 = \frac{9.80665 \times 29}{0.005 \times 8314.3} \ln \frac{288}{288 - 0.005Z_b}$$

from which $Z_b = 5550$ m.

2-3. Quantities needed for substitution in Eq. (2.4) are:

$p_a = 380$ mm Hg $= 0.5$ atm $= 0.50663 \times 10^5\ N/m^2$

$p_b = 760$ mm Hg $= 1.0$ atm $= 1.01325 \times 10^5\ N/m^2$

(see Appendix 1)

Average pressure: $\bar{p} = 0.75$ atm

Density of air: $= \frac{29}{0.02241} \times \frac{0.75}{1} = 970.5\ g/m^3$ or

$0.9705\ kg/m^3$

$g = 9.80665\ m/s^2 \qquad g_c = 1\ kg\text{-}m/N\text{-}s^2$

Hence $\frac{(1.01325 - 0.50663) \times 10^5}{0.9705} = \frac{9.80665}{1}(Z_a - Z_b)$

$Z_a - Z_b = 5323$ m

Error: $5550 - 5323 = 227$ m

or $(227/5550) \times 100 = 4.1\%$

2-4. (a) From Eq. (2-14),

$$Z_{A2} = Z_{A1}(1 - \rho_B/\rho_A) + Z_T\,\rho_B/\rho_A$$

Given $\rho_B/\rho_A = 1020/1109 = 0.9197$

$Z_{A1} = 0.6 \qquad Z_T = 1$

$Z_{A2} = 0.6(1 - 0.9197) + 1(0.9197) = 0.9679$ m

(b) Differentiating the equation in part (a),

$$dZ_{A2} = (1 - \rho_B/\rho_A)dZ_{A1}$$

For finite increments, therefore,

$$\Delta Z_{A1} = \frac{\Delta Z_{A2}}{1 - \rho_B/\rho_A} = \frac{50}{1 - 0.9197}$$

$$= 623 \text{ mm or } 0.623 \text{ m}$$

2-5. Use Eq. (2-15). Quantities needed are:

μ at 35°C (from Appendix 6) = 0.72 cP

ρ_A = 1109 kg/m^3 ρ_B = 1020 kg/m^3

Thus:

$$t = \frac{100 \times 0.72}{1109 - 1020} = 0.809 \text{ h}$$

Volume of chlorobenzene:

(0.809 × 1600)/1109 = 1.167 m^3

Volume of wash:

(0.809 × 2100)/1020 = 1.666 m^3

Total volume: 1.167 + 1.666 = 2.833 m^3

Volume of separator, allowing for 10% vapor space: 2.833 × 1.10 = 3.12 m^3

2.6. Use Eq. (2.8). Quantities needed are:

$\omega = \pi n$ = π x 4000/60 = 209.4 rad/s

ρ = 1,022 kg/m^2

r_2 = 0.125 m r_1 = 0.075 m

Gauge pressure:

$$p_2 - p_1 = (209.4^2 \times 1022(0.125^2 - 0.075^2))/2$$

$$= 224{,}065 \text{ N/m}^2 \text{ or } 2.241 \text{ bar}$$

2-7. Refer to Fig. 2-6. Solve Eq. (2-17) for r_A, given $r_B = 0.04$ m, $r_2 = 0.150/2 = 0.075$ m. Find r_1. To give equal volumes of the two liquids, $\pi(r_1^2 - r_B^2) = \pi(r_2^2 - r_1^2)$

$$r_1^2 - 0.04^2 = 0.075^2 - r_1^2$$

$$r_1 = 0.060 \text{ m}$$

From Eq. (2-17),

$$r_A^2 = r_1^2(1 - \rho_B/\rho_A) + (\rho_B/\rho_A) r_B^2$$

$$\rho_B/\rho_A = 1020/1109 = 0.9197$$

$$r_A^2 = 0.06^2(1 - 0.9197) + (0.9197 \times 0.04^2)$$

$$= 0.00176$$

$r_A = 0.042$ m or 42 mm

2.8. Densities are: For water, $\rho_A = 62.23$ lb/ft³.

For air: $$\rho_B = \frac{29 \times 74.59 \times 273.15}{359 \times 14.59 \times 299.15} = 0.377 \text{ lb/ft}^3$$

(a) From Eq. (2.10): $p_a - p_b = g/g_c \times R_m(\rho_A - \rho_B)$

$$\Delta p = 1 \times \frac{10}{12}(62.23 - 0.377) = 51.54 \text{ lb}_f/\text{ft}^2$$

$$= 51.54/144 = 0.358 \text{ lb}_f/\text{in.}^2$$

or $0.358/14.59 = 0.0245$ atm.

(b) Neglecting r_B gives $$\Delta p = \frac{10}{12}(62.23) = 51.86 \text{ lb}_f/\text{ft}^2$$

Error: $$\frac{51.86 - 51.54}{51.54} = 0.0062 \text{ or } 0.62\%$$

2.9. Use Eq. (2.4). Set $Z_a = 0$. Since the pressure inside the submarine and at the surface is atmospheric, set $p_a = 0$. Then $p_b = \rho\ g\ Z_b$.

$$\rho = 1028\ \text{kg/m}^3 \quad g = 9.80665\ \text{m/s}^2 \quad Z_b = 3000\ \text{m}$$

$$p_b = 1028 \times 9.80665 \times 3000 = 30{,}243{,}700\ \text{N/m}^2$$

Area of window: $A = pD^2/4 = \pi \times 0.15^2/4 = 0.01767\ \text{m}^2$

Force on window: $F = 0.01767 \times 30243700 = 534400$ N

2.10. Let sand be substance *A* and water be substance *B*. From Perry's *Chemical Engineers' Handook*, 7th ed., p. **2**-120

$\rho_A = 143\ \text{lb/ft}^3$ or $143 \times 16.018 = 2291\ \text{kg/m}^3$

$\rho_B = 996\ \text{kg/m}^3$. The density of coal is not needed for the solution. Let x be the volume fraction of sand in the suspension. Basis: 1 m^3.

$$2291x + 996(1 - x) = 1500 \times 1$$

From this, $x = 0.389$

Mass fraction: Let y be the mass fraction of sand. Basis: 1500 kg.

$$\frac{y}{2291} + \frac{(1-y)}{996} = \frac{1}{1500}$$

$$\frac{1500y}{2291} + \frac{1500(1-y)}{996} = 0.6547y + 1.506(1-y) = 1$$

From this, $y = 0.594$

Alternative solution. Mass of 0.389 m^3: 0.389 x 2291 = 891.2 kg Mass fraction: 891.2/1500 = 0.594.

Chapter 3

3-1. (a) $D = 0.1$ m $\bar{V} = 2$ m/s

For water at 10°C, from Appendix 6:

$\mu = 1.310$ cP $= 1.31 \times 10^{-3}$ Pa-s

$\rho = 62.42 \times 16.018 = 999.8$ kg/m^3

$Re = (0.1 \times 2 \times 999.8)/(1.31 \times 10^{-3})$

$= 1.53 \times 10^5$

Flow is turbulent.

(b) $T = 460 + 180 = 640$°R $p = 2$ atm

$\rho = (29 \times 492 \times 2)/(359 \times 640)$

$= 0.124$ lb/ft^3

$D = 1$ ft $\bar{V} = 50$ ft/s

$\mu = 0.02$ cP (Appendix 8)

$Re = (1 \times 50 \times 0.124)/(0.02 \times 6.72 \times 10^{-4})$

$= 4.61 \times 10^5$

Flow is turbulent.

(c) $D = 2/12 = 0.1667$ ft $\bar{V} = 5$ ft/s

$\rho = 0.78 \times 62.37 = 48.6$ lb/ft^3 $\mu = 20$ cP

$Re = (0.1667 \times 5 \times 48.6)/(20 \times 6.72 \times 10^{-4})$

$= 3{,}014$

Flow is marginally turbulent.

(d) $D = 0.015$ m $\bar{V} = 0.2$ m/s

$\rho = 900$ kg/m³ $\mu = 1$ Pa-s

$$Re = (0.015 \times 0.2 \times 900)/1 = 2.7$$

Flow is laminar.

3-2 Reynolds number based on x is given by Eq. (3-21).

Reynolds number based on Z_x is

$$Re,Z = \frac{Z_x u_\infty \rho}{\mu}$$

Substituting from the equation in problem statement gives

$$Re,Z = 5.5\left(\frac{\mu x}{u_\infty \rho}\right)^{1/2} \frac{u_\infty \rho}{\mu}$$

$$= 5.5\left(\frac{x u_\infty \rho}{\mu}\right)^{1/2} = 5.5\ N_{Re,x}^{1/2}$$

Since Re_x ranges from 10^5 to 3×10^6 (see text),

$$Re,Z = 5.5 \times 10^{2.5} \text{ to } 5.5 \times (3 \times 10^6)^{1/2}$$

$$= 1.74 \times 10^3 \text{ to } 9.5 \times 10^3$$

For laminar flow in a pipe, Re is typically less than 3×10^3 (see text).

3-3. From Appendix 8:

Temperature °C	K	Viscosity, cP CO	He	Value of n CO	He
0	273	0.0169	0.0176		
				0.70	0.72
300	573	.0285	.0300		
				0.68	0.68
600	873	.0380	.0400		

Sample calculation for CO, 0 to 300°C:

$$\mu_{573}/\mu_{273} = 0.02\quad/0.016\quad = (573/273)^n$$

$$n = \ln(0.0285/0.0169)/\ln(573/273) = 0.70$$

3-4. For glycerol, μ = 98 cP = 0.098 Pa.s (Appendix 9)

(a) $\bar{V}$ = 0.3 m/s D = 0.015 m ρ = 1240 kg/m^3

Re = (0.015 x 0.3 x 1240/0.098)

= 56.9

From Eq. (3-22):

x_t = 0.05 x 56.9 x 0.015

= 0.043 m or 43 mm.

(b) For n-propyl alcohol, μ = 1.9 cP (Appendix 9)

ρ = 50 lb/ft^3 $\bar{V}$ = 7 ft/s

D = 3/12 = 0.25 ft

Re = (0.25 x 7 x 50)/(1.9 x 6.72 x 10^{-4})

= 6.8 x 10^4

This is turbulent flow. Estimate x_t to be 45 pipe diameters or 45 x 0.25 $\approx$ 11 ft.

3-5. (a) Estimated diameter of exhaust pipe:

50 mm or 0.05 m

Cross-sectional area:

π x $0.05^2/4$ = 0.002 m^2

Approximate composition of exhaust gas:

15% CO_2, 10% H_2O, 75% N_2

Molecular weight: (0.15 x 44) + (0.10 X 18) +

(0.75 x 28) = 29.4

Estimated temperature: 300°C

Estimated pressure: 1.1 atm

Approximate density:

$$\rho = pM/RT = (1.1 \times 29.4)/(0.082056 \times 573)$$
$$= 0.688 \text{ kg/m}^3$$

Volume flow. In a 4-cycle engine, every second stroke is a compression stroke during which no gas is discharged. Hence the volume flow is $(2 \times 3000)/(2 \times 60) = 50$ l/s or 0.05 m^3/s.

Gas velocity: $\bar{V} = 0.05/0.002 = 25$ m/s

Viscosity of nitrogen at 300°C (Appendix 8):

$\mu = 0.029$ cP

Reynolds number:

$$Re = (0.05 \times 25 \times 0.688)/(0.029 \times 10^{-3})$$
$$= 29{,}700$$

(b) Assume the cross-sectional area of the channels is 4 times that of the exhaust pipe. The linear velocity in each channel is then 25/4 = 6.25 m/s. To ensure laminar flow, set the Reynolds number at 1500. The channel diameter D_c would then be:

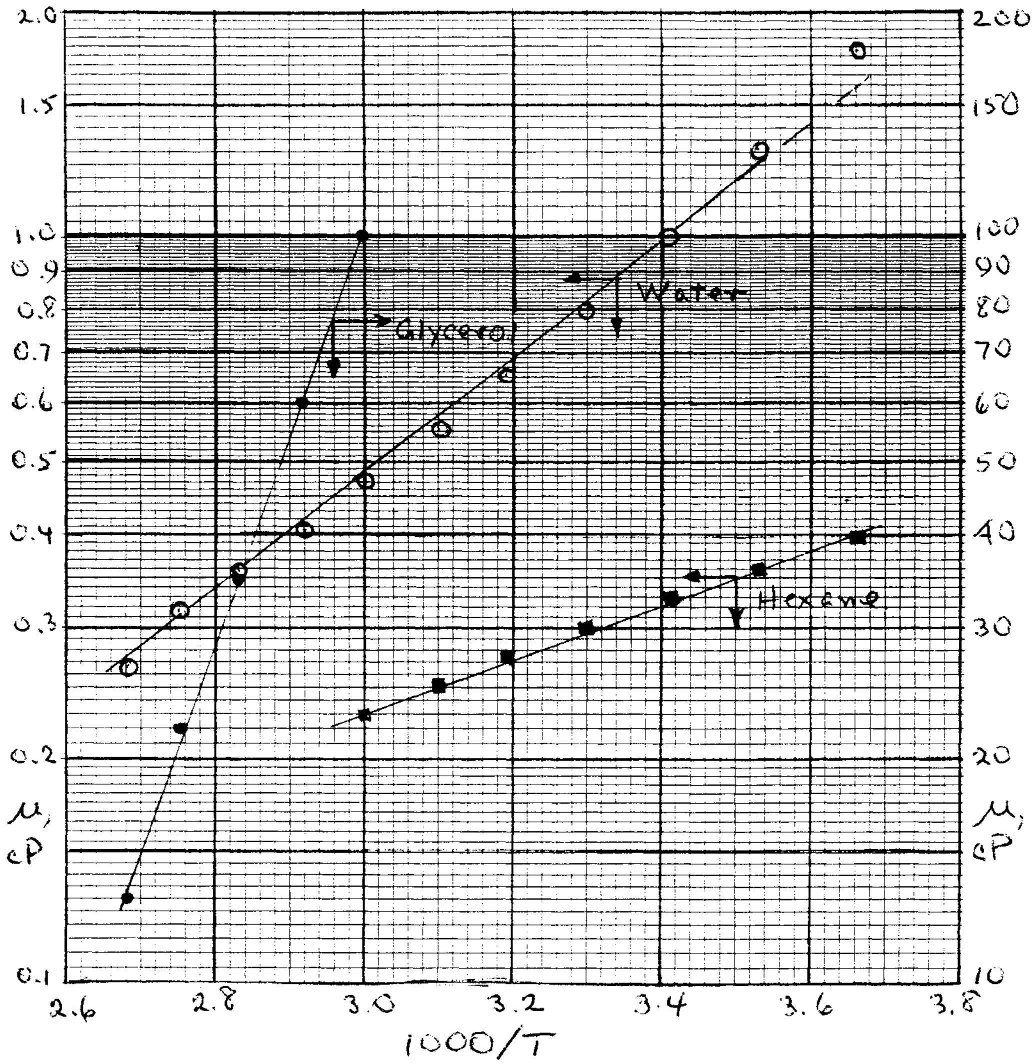
$$D_c = (1500 \times 0.029 \times 10^{-3})/(6.25 \times 0.688)$$
$$= 0.01 \text{ m or } 10 \text{ mm}$$

The actual channels are much smaller than tnis.

3-6. Find viscosities from Appendix 6 for water and from Appendix 9 for hexane and glycerol.

Temperature °C	K	1/T	Viscosity, cP Water	Hexane	Glycerol
0	273	0.00366	1.794	0.40	
10	283	.00353	1.310	.36	
20	293	.00341	1.000	.33	
30	303	.00330	.800	.30	
40	313	.00319	.651	.275	
50	323	.00310	.554	.25	
60	333	.00300	.470	.23	100
70	343	.00292	.404		60
80	353	.00283	.357		35
90	363	.00275	.317		22
100	373	.00268	.288		13

The linear semi-log graphs show that for hexane and glycerol, over the temperature ranges used, the viscosity closely follows an Arrhenius relationship. (Note: the normal boiling point of n-hexane is 69°C.) For water the graph is almost linear between 10 and 100°C, but not at temperatures below 10°C. (The variation of density with temperature is also anomalous for water between 0 and 10°C.)

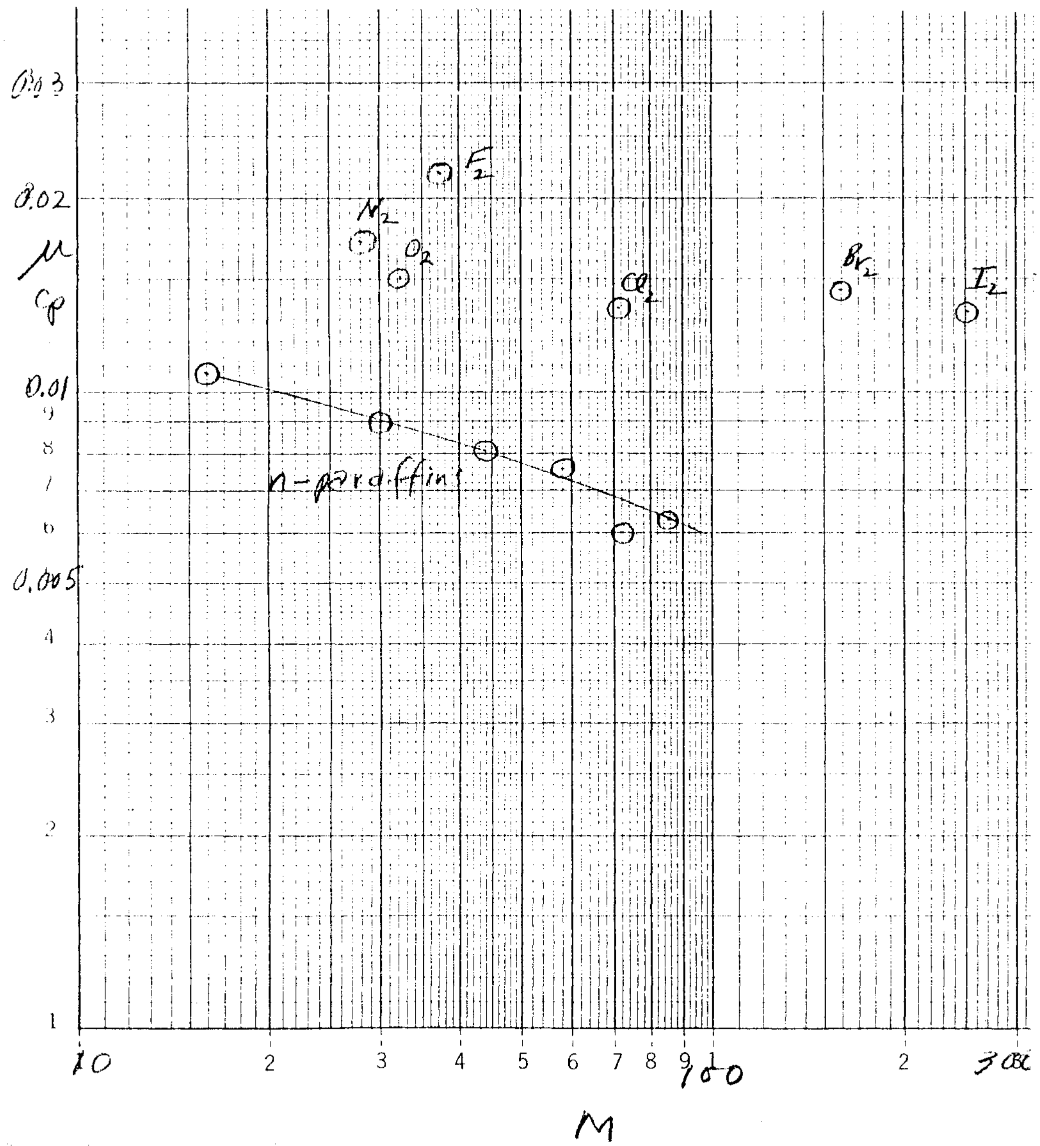


3.7 From Appendix 8 nomograph the viscosities are given in the table on the next page.

The viscosities of the halogens are 2 to 2.5 times those of the paraffins. They show no definite trend with increasing molecular weight. For rigid non-interacting spheres, the viscosity is predicted to vary with $(MT)^{1/2}/\sigma^2$, where σ is the hard sphere diameter. If σ is proportional to $M^{1/3}$, μ should vary with $M^{-1/6}$.

Formula	Gas	*M*	μ at 20°C, cP
F_2	Fluorine	38.00	0.022
Cl_2	Chlorine	70.91	0.0135
Br_2	Bromine	159.83	0.0145
I_2	Iodine	253.82	0.0135
CH_4	Methane	16.04	0.0107
C_2H_6	Ethane	30.07	0.0090
C_3H_8	Propane	44.09	0.0081
*C_4H_{10}	*n*-Butane	58.12	0.0076
C_5H_{12}	*n*-Pentane	72.15	0.0060
C_6H_{14}	*n*-Hexane	86.17	0.0063

*From Perry, *Chemical Engineers' Handbook*, 7th ed. p. 2-320.



The viscosity of hydrocarbon gases decreases gradually with increasing molecular weight. The data for pentane are out of line and may be in error. The slope of the line is about -0.3, somewhat greater than predicted for spherical molecules.

3.8. Cross-sectional area of pipe (App. 3): 0. 00211 ft^2

Air velocity: $\bar{V} = 4/(60 \times 0.00211) = 31.6$ ft/s

Inside dia. of pipe (App. 3): D = 0.493 in.

Density of air: $\rho = \dfrac{29 \times 273 \times 5}{359 \times 303} = 0.364$ lb/ft^3

Viscosity of air at 30°C (App. 8): 0.018 cP

$$\text{Re} = \frac{0.493 \times 31.6 \times 0.364}{12 \times 0.018 \times 6.72 \times 10^{-4}} = 39100$$

Flow is turbulent.

3.9. The density of crude oil varies, depending on the source. Assume ρ = 900 kg/m^3.

D= 1 m. $\bar{V} = 1.5$ m/s. Set the Reynolds number at 2100. Then $\mu = \dfrac{D\bar{V}\rho}{2100} = \dfrac{1 \times 1.5 \times 900}{2100} = 0.643$ Pa-s or 643 cP.

3.10. (a) From Table 3.3, for paper pulp, n'= 0.575, K'= 20,020 g/m-s^{2-n} or 20.02 kg/m-s^{2-n}. At du/dy = 10 s^{-1}, from Eq. (3.7),

$$t_v = 20.02 \times 10^{0.575} = 75.24$$

Apparent viscosity: μ_{app} = 75.24/10 = 7.524 Pa-s or 7,524 cP

At du/dy = 1000 s^{-1}, t_v= 20.02 x $1000^{0.575}$ = 1062

μ_{app} = 1062/1000 = 1.062 Pa-s or 1062 cP.

(*b*) For the clay suspension, from Table 3.3, n'= 0.185, K'= 1590 g/m-s^{2-n} or 1.59 kg/m-s^{2-n}

At du/dy = 10 s^{-1}, from Eq. (3.7),

τ_v= 1.59 x $10^{0.185}$ = 2.43

μ_{app} = 2.43/10 = 0.243 Pa-s or 243 cP

At du/dy = 1000 s^{-1}, τ_v= 1.59 x $1000^{0.185}$ = 3.59

μ_{app} = 3.59/1000 = 0.00359 Pa-s or 3.59 cP

Chapter 4

4-1. (a) From Eq. (4-9)

$$\bar{V} = \frac{1}{S}\int_S u\,dS$$

Since $S = \pi r_w^2$ and $dS = 2\pi r dr$, where r is the radial distance from the pipe axis and r_w the radius of the pipe

$$\bar{V} = \frac{2}{r_w^2}\int_0^{r_w} ur\,dr$$

By integration of a plot of ur versus r (see table below)

$$\int_0^{r_w} ur\,dr = 567.8 \times 10^{-6}\ m^3/s$$

$$\bar{V} = \frac{2 \times 567.8 \times 10^{-6}}{(37.5 \times 10^{-3})^2} = 0.808\ m/s$$

(b) From Eq. (4-61):

$$\alpha = \frac{\int_S u^3 dS}{\bar{V}^3 S} = \frac{2}{\bar{V}^3 r_w^2}\int_0^{r_w} u^3 r\,dr$$

By integration of a plot of u^3r versus r,

$$\int_0^{r_w} u^3 r\,dr = 430.7 \times 10^{-6}\ m^5/s^3$$

$$\alpha = \frac{2 \times 430.7 \times 10^{-6}}{0.808^3 \times (37.5 \times 10^{-3})^2} = 1.16$$

(c) From Eq. (4-41), since $\bar{V}$ is constant,

$$\beta = \frac{1}{S\,\bar{V}^2}\int_S u^2 dS = \frac{2}{\bar{V}^2 r_w^{\ 2}}\int_0^{r_w} u^2 r\,dr$$

By integration of a plot of u^2r versus r,

$$\int_0^{r_w} u^2 r\,dr = 486.3 \times 10^{-6}\ m^4/s^2$$

$$\beta = \frac{2 \times 486.3 \times 10^{-6}}{0.808^2 \times (37.5 \times 10^{-3})^2} = 1.059$$

Table for Solution of Problem 4-1

u	r $\times 10^2$	ur $\times 10^2$	u^2r $\times 10^2$	u^3r $\times 10^2$
1.042	0	0	0	0
1.033	0.375	0.387	0.400	0.413
1.019	.750	.764	.779	.794
0.996	1.125	1.121	1.117	1.113
.978	1.500	1.467	1.435	1.403
.955	1.875	1.791	1.710	1.633
.919	2.250	2.068	1.900	1.746
.864	2.625	2.268	1.960	1.693
.809	3.000	2.427	1.963	1.588
.699	3.375	2.359	1.650	1.153
.507	3.5625	1.806	.916	.464
0	3.750	0	0	0

4.2. (*a*) For none of the oil to be flowing downward, dv/dx must equal or be greater than zero at $x = 0$. From Eq. (4.33), neglecting dp/dy in comparison with ρg,

$$\frac{dv}{dx} = C_1 = \frac{v_0}{B} - \frac{B\rho g}{2\mu}$$

B = 1 mm = 0.001 m $\mu = 20$ mPa-s or 0.5 kg/m-s

Set $dv/dx = C_1 = 0$. Then

$$0 = \frac{v_0}{0.001} - \frac{0.001 \times 900 \times 9.80665}{2 \times 0.05} = 1000 v_0 - 88.26$$

$v_0 = 0.08826$ m/s or 88.26 mm/s

(*b*) At $x = 0.5B = 0.0005$ m, from Eq. (4.35),

$$v = -\left(\frac{900 \times 9.80665}{2 \times 0.05}\right)\left((0.001 \times 0.0005) - 0.0005^2\right) + (0.08826 \times 0.5)$$

$= 0.02206$ m/s or 22.06 m/s

(*c*) At the stationary plate, $dv/dx = 0$. Also, $C_1 = 0$. At the moving plate, $x = 0.001$ and from Eq. (4.33),

$$\frac{dv}{dx} = \frac{x\rho g}{2\mu} = \frac{0.001 \times 900 \times 9.80665}{2 \times 0.05} = 88.26 \ \text{s}^{-1}$$

Halfway between the plates, $x = 0.0005$ m, and $dv/dx = 88.26/2 = 44.13 \ \text{s}^{-1}$

4-3. (a) As in Example 4-4, substitution in Eq. (4-58) gives $u_b^2/2 = gZ$, and therefore $u_b = \sqrt{2gZ}$

Initially, Z = 25 ft and

$$u_b = \sqrt{2 \times 32.17 \times 25} = 40.1 \text{ ft/s}$$

Cross-sectional area of Schedule 40 4-in. outlet, (from Appendix 5):

$$S = 0.0884 \text{ ft}^2$$

Volume flow: $q = u_b S = 40.1 \times 0.0884$

$$= 3.54 \text{ ft}^3\text{/s or } 1590 \text{ gal/min}$$

(b) Volume of liquid in tank at any time:

$$V = (\pi \times 30^2 \times Z)/4 = 706.9Z$$

Volume flow: $q = -(dV/dt) = -706.9(dZ/dt)$

Also, $q = Su_b = S\sqrt{2gZ} = 0.0884\sqrt{2 \times 32.17Z}$

$$= 0.709Z^{1/2}$$

Hence $-706.9(dZ/dt) = 0.709Z^{1/2}$

Initial height: $Z_1 = 25$ ft

Final height: $Z_2 = 0$

Integrating between limits:

$$\int_{25}^{0} Z^{-1/2} dZ = (0.709/706.9)\int_{0}^{t_T} dt$$

$$= 0.001003\int_{0}^{t_T} dt$$

$$t_T = (2 \times \sqrt{25})/0.001003 = 9970 \text{ s or } 2.77 \text{ h}$$

(c) Initial volume: $V_1 = 706.9 \times 25 = 17673 \text{ ft}^3$

Average flow rate: $q = 17673/9970 = 1.77 \text{ ft}^3\text{/s}$

This is exactly half the initial flow rate.

4.4. Flow rate: $\dot{m} = (9 \times 998)/3600 = 2.495$ kg/s

Cross-sectional area of pipe:

$$S = (\pi \times 0.05^2)/4 = 0.00196 \text{ m}^2$$

$$\bar{V}_b = 9/(3600 \times 0.00196) = 1.276 \text{ m/s}$$

$$\bar{V}_a = 0 \qquad Z_b = 5 \text{ m}$$

$$\alpha_a = \alpha_b = 1 \qquad p_a = p_b$$

$$h_f = 2.5 \text{ J/kg} = 2.5 \text{ m}^2/\text{s}^2 \text{ (see Eq. (1-8))}$$

From Eq. (4-32),

$$\eta W_b = g(Z_b - Z_a) + (\bar{V}_b^2 - \bar{V}_a^2)/2 + h_f$$

$$= 9.81(5 - Z_a) + (1.276^2/2) + 2.5$$

$$= 52.344 - 9.81 Z_a \text{ J/kg}$$

Given: Power P = 0.1 kW = 100 J/s

$$\eta W_b = P/\dot{m} = 100/2.495 = 40.08 \text{ J/kg}$$

So: $52.344 - 9.81 Z_a = 40.08$

$$Z_a = (52.344 - 40.08)/9.81 = 1.25 \text{ m}$$

4.5. Assume elbow is in the x-z plane with the water entering in the positive x direction and leaving in the positive z direction. The forces in each plane are found by combining Eqs. (4.42) and (4.43), as in Example 4-5, to give (since $F_g = 0$),

In x direction:

$$\dot{m}(\beta_b \bar{V}_{b,x} - \beta_a \bar{V}_{a,x}) = p_a S_{a,x} - p_b S_{b,x} + F_{w,x}$$

In z direction:

$$\dot{m}(\beta_b \bar{V}_{b,z} - \beta_a \bar{V}_{a,z}) = p_a S_{a,z} - p_b S_{b,z} + F_{w,z}$$

The quantities needed are:

$$\dot{m} = \frac{6\ \pi\ (0.1)^2}{4} \times 998 = 47.03\ \text{kg/s}$$

$\beta_a = \beta_b = 1$ (by assumption)

$\bar{V}_a = \bar{V}_{a,x} = 6$ m/s $\quad \bar{V}_{a,z} = 0$

$\bar{V}_b = \bar{V}_{b,z} = 6$ m/s $\quad \bar{V}_{b,x} = 0$

$p_a = p_b = 70{,}000\ \text{N/m}^2$

$S_a = S_b = S_{a,x} = \pi \times 0.1^2/4 = 0.00785\ \text{m}^2$

$S_{a,z} = 0 \qquad S_{b,x} = 0$

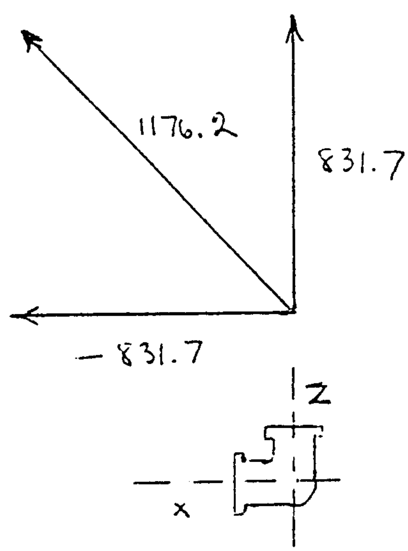
Substituting gives

$$F_{w,x} = 47.03\ (0 - 6) - (70{,}000 \times 0.00785) + 0$$
$$= -282.2 - 549.5 = -831.7\ \text{N}$$
$$F_{w,z} = 47.03\ (6 - 0) - 0 + (70{,}000 \times 0.00785)$$
$$= 282.2 + 549.5 = 831.7\ \text{N}$$

Total force (see diagram):

$F = 831.7\ \sqrt{2} = 1176.2$ N

acting in the direction indicated.



4.6. Use Eq. (4.50). Here $\beta = 0^0$ and $\cos \beta = 1$. Since

ρ = 998 kg/m^3,

$$\dot{m} = 0.15 \times 998/60 = 2.495 \text{ kg/s}$$

$$b = \pi \times 2.5 = 7.854 \text{ m}$$

μ at 40°C = 0.656 cP (Appendix 6)

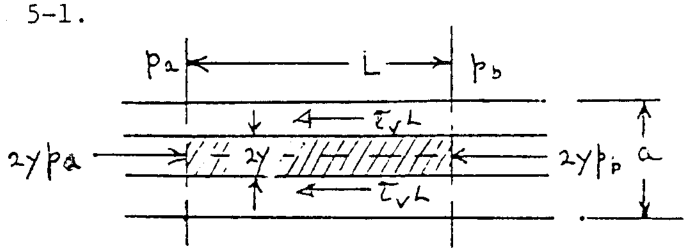
From Eq. (4.50),

$$\delta = \left(\frac{3 \times 0.656 \times 10^{-3} \times 2.495}{998^2 \times 9.80665 \times 7.854} \right)^{1/3}$$

$$P = 4.0 \times 10^{-4} \text{ m or } 0.4 \text{ mm}$$

Chapter 5

5-1.



Basis: Breadth = 1 ft

Consider slice of thickness 2y in center channel

Force balance: $2yp_a - 2yp_b = 2\tau_v L$

$$\frac{p_a - p_b}{L} = \frac{\tau_v}{y}$$

From the definition of μ, Eq. (3.3),

$$\tau_v = -\mu \, du/dy$$

Substituting:

$$\frac{(p_a - p_b)}{L\mu} \int_{a/2}^{y} y dy = - \int_{o}^{u} du$$

$$u = \frac{(p_a - p_b)}{2L\mu} \left(\frac{a^2}{4} - y^2 \right)$$

$$\bar{V} = \frac{1}{a/2}\int_o^{a/2} u\,dy = \frac{(p_a - p_b)\, g_c a^2}{12L\mu}$$

$$p_a - p_b = \frac{12\mu\bar{V}L}{a^2}$$

5-2. From Eq. (5.31), $\bar{V} = u^*/\sqrt{f/2}$. Hence

$$Re = D\bar{V}\rho/\mu = \frac{Du^*\rho}{\mu\sqrt{f/2}} \text{ and}$$

$$u^*\rho/\mu = Re\sqrt{f/2}/D$$

From Eq. (5.33), $y = y^+ D/Re\ \sqrt{f/2}$

Here $y^+ = 5$, $D = 60$ mm, $Re = 5 \times 10^4$

From Fig. 5.10 for smooth tubing, $f = 0.0051$, and

$$y = (5 \times 60)/(5 \times 10^4)\sqrt{0.0051/2} = 0.119 \text{ mm}$$

Cross-sect. area: $S = \pi \times 60^2/4 = 2827.43$ mm²

Diameter of turbulent core:

$$D_i = 60 - (2 \times 0.119) = 59.762 \text{ mm}$$

Area of core $= \pi \times 59.762^2/4 = 2805.05$ mm²

Area of viscous layer:

$$2827.43 - 2805.05 = 22.38 \text{ mm}^2$$

% of total area: $22.38 \times 100/2827.43 = 0.79$ %

Volume flow, q_v, in viscous layer:

$$q_v = \int u\, dS = \int u(2\pi r\, dy) = 2\pi\int u((D/2) - y)dy$$

Since $u^+ = y^+$, $u/\bar{V}\sqrt{f/2} = y\, Re\sqrt{f/2}/D$ and

$$u = y\bar{V}\, Re\, f/2D. \quad \text{Hence}$$

$$q_v = \frac{\pi V Re\, f}{D}\left(\int_0^{0.119}\frac{Dy}{2}\,dy - \int_0^{0.119} y^2\,dy\right)$$

$$q_v = \frac{\pi \bar{V} \times 5 \times 10^4 \times 0.0051}{60} \left(\frac{60 \times 0.119^2}{4} - \frac{0.119^3}{3} \right)$$

$$= 2.828\bar{V}$$

Total flow q_T is $\bar{V}S$ or $2827.43\bar{V}$

Fraction of flow in viscous layer:

$$q_v / q_T = 2.828\bar{V}/2827.43\bar{V} = 0.001 \text{ or } 0.1\%$$

5-3. From the solution to Problem 5-1:

$$\frac{u}{\bar{V}} = \frac{\frac{(p_a - p_b)}{2L\mu}\left(\frac{a^2}{4} - y^2\right)}{\frac{(p_a - p_b)}{12L\mu}(a^2)}$$

$$= 1.5 - 6\ (y^2/a^2)$$

$u = u_{max}$ when $y = 0$. Hence $u_{max}/\bar{V} = 1.5$.

5-4. $\mu = 0.025$ kg/m-s

$a = 0.006$ m $\quad L = 3$ m

$\bar{V} = 100/(0.006 \times 3600) = 4.630$ m/s

$$p_a - p_b = \frac{12 \times 0.025 \times 4.360 \times 3}{0.006^2}$$

$$= 109{,}000 \text{ N/m}^2$$

Flow rate, $q = 100/3600 = 0.02778$ m^3/s

Power, $P = 109{,}000 \times 0.02778$

$= 3028$ N-m/s $= 3028$ W

5.5. Assume all the energy supplied to the oil is transformed into heat. Volume flow rate (from Problem 5.4): $q = 0.02778$ m^3/s Mass flow rate:

$$\dot{m} = 0.02778 \times 820 = 22.78 \text{ kg/s}$$

Rate of heat generation:

$Q/t = W/t = 3028$ W = 3028 J/s

$Q = 3028$ J From Appendix 1,

$c_p = 0.5 \times 1000 \times 4.184 = 2092$ J/kg-°C

Temperature rise: $\Delta T = Q/c_p\dot{m} = 3028/(2092 \times 22.78)$

$= 0.064°C$

5.6. Substitute $\bar{V} = 4\dot{m}/\pi D^2\rho$ in Eq. (5.8) and solve for D^5 in terms of f. Quantities needed are:

$\rho = 2.6 \times 62.3 = 162$ lb/ft^3

$\mu = 2 \times 2.42 = 4.84$ lb/ft-h $\qquad \dot{m} = 7000$ lb/h

$\Delta L = 1.73 \times 5280 = 9134$ ft

$g_c = 4.17 \times 10^8$ ft-lb /lb-h^2

$\Delta p_s = 0.183 \times 144 = 26.35$ lb /ft^2

$$D^5 = \frac{32 \times 9134 \times 7000^2 f}{\pi^2 \times 162 \times 4.17 \times 10^8 \times 26.35} = 0.815f$$

From Eq. (4.51), $Re = 4\dot{m}/L_p\mu = 4\dot{m}/\pi D\mu =$

$= (4 \times 7000)/(\pi \times 4.84D) = 1841/D.$

By trial from Fig. 5.10 D = 0.375 ft or 4.5 in.

$Re = 1841/0.375 = 4909 \qquad f = 0.0092$

Check: $D^5 = 0.375^5 = 0.00741$

$0.815f = 0.815 \times 0.0092 = 0.00750$ (close enough)

Thus D = 0.375 ft. Since Re = 4909, flow is turbulent, as assumed.

NOTE: Eq. (4.51) applies to layer flow and also to flow when the liquid fills the pipe.

5.7 <u>Basis of solution:</u> Pressure drop in by-pass is the same as pressure drop in bypassed section. Let subscript 1 refer to the bypass; subscript 4 refer to bypassed section. From Eq. (5.72), since $Z_a = Z_b$ and fittings losses and contraction and expansion losses are neglected,

$$\frac{p_a - p_b}{\rho} = \left(4 f_4 \frac{L_4}{D_4}\right)\frac{\overline{V_4}^2}{2g_c} = \left(4 f_1 \frac{L_1}{D_1}\right)\frac{\overline{V_1^2}}{2g_c}$$

from which:

$$f_1 \overline{V_1}^2 = \frac{D_1 L_4}{D_4 L_1} f_4 \overline{V_4^2}$$

Given L_1 = 22 ft; L_4 = 16ft; D_1 = 0.1667 ft
D_4 = 0.6667 ft

$f_1 \overline{V}_1^2$ = ((0.1667 x 16) / (0.6667 x 22)) $f_4 \overline{V}_4^2$

= 0.182$f_4 \overline{V}_4^2$

A second relation between $\overline{V}_1$ and $\overline{V}_4$ is found from continuity:

$$\dot{m}_1 + \dot{m}_4 = \dot{m}_0$$

where $\dot{m}_0$ = total mass flow rate in upstream section.

Since $\dot{m} = \rho \overline{V} S = \frac{\pi \rho \overline{V} D^2}{4}$, $\overline{V}_1 D_1^2 - \overline{V}_4 D_4^2 = \overline{V}_0 D_4^2$

$\overline{V}_1 = (\overline{V}_0 D_4^2 - \overline{V}_4 D_4^2) \ / \ D_1^2$

Given $\overline{V}_0 = 6$ ft/s, $\overline{V}_1 = (6 - \overline{V}_4)(D_4^2)/(D_1^2)$

$= (6 - \overline{V}_4)(0.6667^2/0.1667^2)$

$= 96 - 16\overline{V}_4$ (B)

Assume values of $\overline{V}_1$; find $\overline{V}_4$; calculate the corresponding Reynolds numbers; from Fig. (5.10) find friction factors, using appropriate roughness parameters. For steel pipe, k = 0.00015ft. For a 2-inch pipe, assuming a viscosity of 1 cP and a density of 62.3 lb/ft^3,

$Re_1 = D_1 V_1 \rho/\mu$

$= 2 \times 62.3\overline{V}_1/12 \times 6.72 \times 10^{-4} = 1.55 \times 10^4 \overline{V}_1$

$k_1/D_1 = 0.00015 \times 12/2 = 0.0009$

For 8-inch pipe,

$$Re_4 = \frac{8 \times 62.3\overline{V}_4}{12 \times 6.72 \times 10} = 6.18 \times 10^4 \overline{V}_4$$

$k_4/D_4 = 0.00015 \times 12/8 = 0.00023$

From Fig (5.10), by trial, the following quantities are found to satisfy Equation (A) and (B):

Bypass	Bypassed section
$\overline{V}_1 = 2.02$ ft/s	$\overline{V}_4 = 5.87$ ft/s
$Re_1 = 31{,}310$	$Re_4 = 363{,}000$
$f_1 = 0.0063$	$f_4 = 0.0041$
$f_1\overline{V}_1^2 = 0.0257$	0.0182 $f_4\overline{V}_4^2 = 0.0257$

Mass flow rate through bypass:

$$\dot{m}_1 = 2.02 \times 3600 \times (\pi/4) * 0.1667^2 \times 62.3$$
$$= 9{,}888 \text{ lb/h}$$

Total flow rate:

$$\dot{m}_0 = 6 \times 3600 \times (\pi/4) \times 0.6667^2 \times 62.3$$
$$= 469{,}800 \text{ lb/h}$$

Fraction of total flow through bypass:

$$\dot{m}_1 / \dot{m}_0 = 9{,}888 / 469{,}800 = 0.021 \text{ or } 2.1\%$$

5.8 $\overline{V} = 15$ft/s D = 2ft

Assume $\mu = 1$ cP, $\rho = 52.3$ lb/ft^3

$$Re = \frac{2 \times 15 \times 62.3}{6.72 \times 10^{-4}} = 2.78 \times 10^6$$

Let subscript 1 refer to the original pipe, 2 to lined pipe.

For k/D = 0.0003/2 = 0.00015, from Fig. 5.10:

$f_1 = 0.0032$

For smooth pipe, $f_2 = 0.0023$. From Eq. (5.7), since $D_2 = 1.9$ft:

$$\frac{\Delta p_2}{\Delta p_1} = \frac{4 f_2 \overline{V}_2{}^2 \Delta L \rho / 2D_2}{4 f_1 \overline{V}_1{}^5 \Delta L \rho / 2D_1}$$

$= \quad f_2\ \overline{V}_2{}^2\ D_1/f_1\ \overline{V}_1{}^2 D_2$

Since for constant volumetric flow rate $\overline{V}_1 / \overline{V}_2 = D_1^2 / D_2^2$,

$$\frac{\Delta p_2}{\Delta p_1} = \frac{f_2 D_1{}^5}{f_1 D_2{}^5} = \frac{0.0023x2^5}{0.0032x1.9^5} = 0.929$$

For a constant pressure drop $\Delta p_2 / \Delta p_1 = 1$. Hence

$$\overline{V_2}^2 = \frac{\Delta p_2 f_1 D_2 \overline{V_1^2}}{\Delta p_1 f_2 D_1} = \frac{0.0032x1.9\overline{V_1^2}}{0.0023x2} = 1.322\overline{V_1}^2$$

Ratio of volumetric flows is, since $S_2/S_1 = (D_2/D_1)^2$

$$\frac{q_2}{q_1} = \frac{\overline{V_2} D_2{}^2}{\overline{V_1} D_1{}^2} = (1.322)^{1/2} x (1.9/2)^2 = 1.038$$

5.9 Cross-sectional area of pipe: 0.165 ft^2 (Perry, 7th ed., p.10-73)

$D = 5.501/12 = 0.458$ ft (Perry, 7th Ed., p.10-73)

$\rho = 62.37\ lb/ft^3$ $\quad\mu = 1.129$ cP (Appendix 6)

$$\mathrm{Re} = \frac{0.458x12x62.37}{1.129x6.72x10^{-4}} = 4.52x10^5$$

$k/D = 0.00015/0.458 = 0.00033$

$f = 0.0041$ (Fig. 5.10) $\quad \Delta L = 4500$ ft

From Eq. (5-7),

h_{fs} $= 0.0041 \times 2 \times 4500 \times 12^2 / (32.174 \times 0.458)$

$= 361$ ft-lb_f / lb

Static lift = 3000 ft. Neglect velocity head.

Total work: W = 3000 + 361 = 3361 ft-lb_f / lb

$Q = 0.165 \times 12 = 1.98\ ft^3/s$

$P = 3361 \times 1.98 \times 62.37/550 = 733$ hp (theoret.)

Cost: $755 \times 0.746 \times 0.04/0.70 = \$32.18/h$

5.10 Total volumetric flow, since $1m^3 = 264.17$ gal:

$$q = 2000 / (264.17 \times 24 \times 3600)$$

$$= 8.763 \times 10^{-5}\ m^3/s$$

Cross sectional area of one tube:

$$S = \pi \times (42 \times 10^{-6})^2/4 = 1.385 \times 10^{-9}\ m^2$$

Average velocity in tubes:

$$\overline{V} = \frac{8.763 \times 10^{-5}}{9 \times 10^5 \times 1.385 \times 10^{-9}} = 0.0703\ m/s$$

From Eq. (5.20), since $\Delta L = 3/3.208 = 0.914$ m

and $\mu = 1$ cP or 1×10^{-3} Pa-s:

$$\Delta p = 32\ \Delta L \overline{V} \mu\ /\ (D^2)$$

$$= \frac{32 \times 0.914 \times 0.703 \times 1 \times 10^{-3}}{(42 \times 10^{-6})^2}$$

$$= 1.17 \times 10^6\ N/m^2\ (169\ lb_f/in^2)$$

5.11 Assume the average pressure = 1.6 atm and the pressure drop in the line is small. Neglect the temperature rise on compression, so T = 200°C.

$\dot{m} = 4 \times 60 = 240$ lb/h   M = 70.91

$\rho = (70.91 \times 273 \times 1.6) / (359 \times 473)$

0.182 lb/ft^3

$q = 240 / (0.182 \times 3600) = 0.366$ ft^3/s

The gas velocity should be between 20 and 50 ft/s. Try 1-1/2 inch pipe, Sch. 40. From Appendix 3, S = 0.01414 ft^2. Velocity is:

$\overline{V} = 0.366/0.01414 = 25.9$ ft/s   (OK)

$D = 1.61/12 = 0.134$ ft

$\mu = 0.021$ cP (Appendix 8)

$$\mathrm{Re} = \frac{0.134x25.9x0.182}{0.021x6.72x10^{-4}} = 4.5x10^4$$

$k/d = 0.00015 / 0.134 = 0.0011$

$f = 0.0061$ (Fig. 5.10)

From Table 5-1:

$K_f = (2 \times 10) + (6 \times 0.9) = 25.4$

Use Eq. (5.73). Assume $Z_a = Z_b$; $K_e = 0$

$P_a - P_b = \rho(4fL/D + K_f)\ \overline{V}^2/2gc$

$$= 0.182\left(\frac{4x0.0061x80}{0.134} + 25.4\right)\frac{25.9^2}{2x32.174}$$

= 75.9 lb$_f$/ft^2 or 75.9 /(144 x 14.7)= 0.036 atm. This is very small and so the assumed average pressure of 1.6 atm is satisfactory.

5.12 Equivalent diameter:

$D_e = 4 \times 15 \times 20/((2 \times 15)+(2 \times 20)) = 17.14$ ft

$\bar{V}$ = 50 ft/s $\qquad \Delta L$ = 250 ft

ρ = (29 x 492)/(359 x 640) =0.062 lb/ft^3

At 180°F, μ = 0.02 cP (Appendix 8)

$$Re = \frac{17.14x50x0.0062}{0.02x6.72x10^{-4}} = 3.95x10^6$$

For such a large duct, k/D≈0. From Fig. 5.10,

f = 0.0023

From Table 5-1, K_f = 2 x 0.9 = 1.8.

From Eq. (5.72),

$$h_f = \left(\frac{4nx0.0023x250}{17.14} + 1.8\right)\frac{50^2}{2x32.17}$$

= 75.1 ft-lb_f/lb

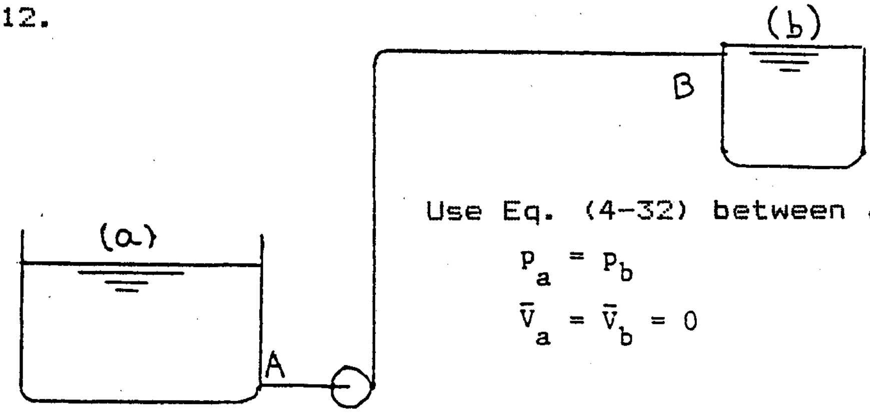
$P_a - P_b = \rho h_f$ = 0.062 x 75.1 = 4.65 lb_f/in^2

or 0.032 lb_f/in^2

$\dot{m}$ = $\rho\bar{V}$S = 0.062 x 50 x 15 x 20 = 930 lb/s

P = $\dot{m}h_f$ = 930 x 75.1/550 = 127 hp (94.8 kW)

5-12.



Use Eq. (4-32) between a and b

$$p_a = p_b$$

$$\bar{V}_a = \bar{V}_b = 0$$

$0.60\ W_p = 200 + h_f$

Friction: Contraction at A, friction in pipe, expansion at B.

Pipe: D = 4.026 /12 = 0.336 ft (Appendix 3)

Cross-sect. area = 0.0884 ft^2 (Appendix 3)

$\bar{V} = 400/(7.48 \times 60 \times 0.0884\ ft^2) = 10.1$ ft/s

K/D = 0.00015/0.336 = 0.0004

$$Re = \frac{0.336 \times 10.1 \times 1.18 \times 62.3}{1.2 \times 6.72 \times 10^{-4}} = 3.1 \times 10^5$$

f = 0.0045 (Fig. 5.10)

K_f for fittings:	2 gate valves	2 x 0.2 = 0.4
(Table 5-1)	4 elbows	4 x 0.9 = 3.6
	4 tees	4 x 1.8 = 7.2
		K_f = 11.2

Contraction loss: $K_c = 0.4$ [Eq. (5.69)]

Expansion loss: $K_e = 1.0$ [Eq. (5.67)]

From Eq. (5.72),

$$h_f = \left(\frac{4 \times 0.0045 \times 600}{0.0036} + 0.4 + 1.0 + 11.2 \right) \times \frac{10^2}{2 \times 32.174} = 69.5 \text{ ft-lb}_f / \text{lb}$$

From Eq. (4.65),

$W = (\Delta Z + h_f)/\eta = (150 + 69.5)/0.60$

$= 366 \text{ ft-lb}_f/\text{lb}$

$\dot{m} = 400 \times 8.33 \times 1.18/60 = 65.5 \text{ lb/s}$

$P = 366 \times 65.5/550 = 43.6 \text{ hp}$

Cost $= (43.6 \times 400)/300 = \$58.13$/day.

5.14. Mass flow: $\dot{m} = 600 \times 8.33 \times 60 = 300{,}000 \text{lb/h}$

From Eq. (4.51), with $L_p = \pi \times 6.6065 / 12 =$ 1.588 ft and $\mu = 1$cP:

$$Re = \frac{4x300{,}000}{1.488x1x2.42} = 312{,}300$$

$k/D = 0.00015/0.5 = 0.0003$

From Fig. 5.10, $f = 0.0043$

From Appendix 3, $\bar{V} = 300{,}000/45{,}000 = 6.67 \text{ft/s}$

$\Delta L = 2500$ ft $\qquad \rho = 62.3 \text{ lb/ft}^3$

From Eq. (5-7),

$$\Delta P_s = \frac{4x0.0043x2500x6.67^2 x62.3}{(6.065/12)x2x32.174}$$

$= 3665 \text{ lb}_f/\text{ft}^2$ or $25.4 \text{ lb}_f/\text{in}^2$

$u_t = 1.75\ (9.80665 \times 60 \times 10^{-4} \times 0.3)^{1/2}$

$= 0.233$ m/s

$t = 2/0.233 = 8.58$s

Case 4: $u_t = \sqrt{\dfrac{4g(s-1)D_P}{3C_D}} = \sqrt{\dfrac{4x9.80665x6.5x2.5x10^{-4}}{3C_D}}$

$= 0.146/C_D^{1/2}$

$Re = \dfrac{2.5x10^{-4}xu_t x998}{1.05x10^{-3}} = 237.6u_t$

By trial from Fig. 7-6, $u_t = 0.092$ m/s

$Re = 237.6 \times 0.092 = 21.9$; $C_D = 2.5$

Check

$u_t = 0.146/2.5^{1/2} = 0.092$ m/s

$t = 2/0.092 = 21.7$s

Case 2a

$u_t = \sqrt{\dfrac{4x9.80665x1.65x2.5x10^{-4}}{3C_D}} = 0.0734/C_D^{1/2}$

$Re = 237.6\ u_t$ (as in Case 1a)

By trial, $V = 0.033$ m/s $N_{Re} = 7.8$

From Fig. 7-6, $C_D = 5.0$

$u_t = 0.0734/5.0^{1/2} = 0.033$ m/s

$t = 2/0.033 = 60.6$s

7.8. Rate of rotation near wall:

$N = 16/(\pi/0.6) = 8.49$ rev/s

$= 2 \times \pi \times 8.49 = 53.3$ rad/s

Centrifugal acceleration :

$r\omega^2 = 0.3 \times 53.3^2 = 852$ m/s^2

$\rho_p =$ for sand is about $2.2 \times 998 = 2196$ kg/m^3

423) = 0.836 kg/m^3

= 0.023 x 10^{-3} kg/m-s

from Eq. (7.45), for

3

= 2.87

e range and u_t = must be

7.6. From Eq. (7.37)

20x10^{-6}

u_t/0.023 x 10^{-3}

al, u_t = 1.6 m/s,

.0.

= 5.74

.72/C_D) = 10.92/$\sqrt{C_D}$

= 1.454 u_t

g. 7.6, u_t = 4.61 m/s

.61 = 6.70

= 10.92/5.6 = 4.61 m/s

for D_P = 20μm:

665/852)$^{1/3}$ = 0.65

77

55

kg/m s

an

la

From Eqs. (7.35) and (7.36):

$m = \pi \times 0.006^3 \times 1330/6 = 1.50 \times 10^{-4}$g

$A_p = \pi \times 0.006^2/4 = 2.83 \times 10^{-5}$ m^2

Substituting in Eq. (7.30):

$$\frac{du}{dt} = \frac{9.80665(1330-1.206)}{1330} - \frac{0.44x1.206x2.83x10^{-5}u^2}{2x1.50x10^{-4}}$$

$$= 9.798 - 0.0501\ u^2$$

Hence

$$\frac{du}{9.798-0.0501u^2} = dt$$

and

$$\int_0^u \frac{udu}{9.798-0.0501u^2} = \int udt = \int_0^x dx = x$$

since u = dx/dt and udt = dx.

Let 9.798 $-0.0501u^2$ = V. Then dV = -0.1002u du and u du = -9.98 dV. Integration limits are u = 0 and u = 0.99 x 14.1 = 13.959; limiting values of V are 9.798 and 9.798 - 9.76217 = 0.03583.

$$\int_{9.798}^{0.03583} 9.98dV/V = 9.98\ln(9.798/0.03583) = 56.0m$$

The distance to reach 99% of u_t is 56m, which is greater than the height of the tower.

7.10 The pressure drop across an individual particle equals the effective mass divided by the projected area.

$$\Delta p = \frac{mg}{A_p g_c}\left(\frac{\rho_p - \rho}{\rho_p}\right)$$

Since m = $\pi D_p^3 \rho_p/6$ and $A_p = \pi D_p^2/4$,

$$\Delta p = \frac{2D_p(\rho_p - \rho)g}{3\ g_c}$$

Here L = D_p, so

$$\frac{\Delta p}{L} = 2x(1.5-1)\frac{998x9.80665}{3}$$

$$= 3262\ (N/m^2)/m$$

From Eq. (5.20) for laminar flow, with D = 10^{-5}m, and μ = 10^{-3} kg/m-s:

$$\overline{V}_{pore} = \frac{3262x(10^{-5})^2}{32x10^{-3}} = 1.0x10^{-5}\, m/s$$

The superficial velocity $\overline{V}_0$ is 2 $\overline{V}_{0M}$. From Eq. (7.51), with ϵ taken as 0.40, Φ_s = 1; and D_p = 10^{-3}m:

$$1.406 \times 10^6\ \overline{V}_{0M} + 2.73 \times 10^7\ \overline{V}_{0M}^2 = 4920$$

$$\overline{V}_{0M} = 3.27 \times 10^{-3}\ m/s$$

$$\overline{V}_0 = 2 \times 3.27 \times 10^{-3} = 6.54 \times 10^{-3 m/s}$$

$$\frac{\overline{V}}{\overline{V}_{pore}} = \frac{6.54x10^{-3}}{1x10^{-5}} = 654$$

Even at incipient fluidization, the cross-sectional area for flow in the pores is 0.40 x (1-0.4)= 0.24 of the total area and that in the channels is 0.4 of the total area, and the ratio of the flow through the channels between particles to that in the pores is

$$\frac{q_0}{q_{pore}} = \frac{\bar{V}_0 x0.4}{\bar{V}_{pore} x0.24} = 1090$$

Hence the flow through the pores is less than 0.1% of that in the channels.

7.11 Use Eq. (7.51). Assume ϵ_M = 0.45. D_P = 0.005m. For air,

$$\rho = \frac{29}{22.4} x \frac{273}{353} = 1.00 kg/m^3$$

μ = 0.02 x 10^{-3} kg/m-s

$$\frac{150x0.02x10^{-3}(1-0.45)\bar{V}_{oM}}{0.86^2 x0.005^2 x0.45^3} + \frac{1.75x1.00\bar{V}_{oM}^{\ 2}}{0.86x0.005x0.45^3}$$

= 9.80665(960-1.00)

979.27$\bar{V}_{oM}$ + 4466$\bar{V}_{oM}^2$ = 9404.5

$\bar{V}_{OM}$ = 1.56 m/s

G = $\dot{m}$/S = $\rho\bar{V}_{oM}$

$$= \frac{45000}{3600S} = 1.00x1.56$$

S = 8.01 m^2 = $\pi D^2/4$

D = 3.19m.

7.12. (a) From Appendix 5, 20 mesh = 0.833 mm, 50 mesh = 0.287 mm.

$$\overline{D}_p = 0.56\, mm = 5.6 \times 10^{-4}\ m$$

For spherical beads, Φ_s = 1.0. Assume ε = 0.35.

At 10 gal/min-ft^2,

$$\overline{V} = \frac{10}{264.17\ gal/m^3} \times \frac{1}{60} \times \frac{1}{0.0929\ m^2/ft^2} = 6.79 \times 10^{-3}\ \text{m/s}$$

Assume μ = 1 cP = 10^{-3} Pa.s $\quad \rho$ = 1000 kg/m^3

From Eq. (7.22),

$$\frac{\Delta p}{L} = \frac{150 \overline{V}_0\, \mu (1-\varepsilon)^2}{(\Phi_s D_p)^2} + \frac{1.75\ \overline{V}_0^2\ \rho}{\Phi_s D}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$$

$$= \frac{150(6.79 \times 10^{-3})(10^{-3})(0.65)^2}{(5.6 \times 10^{-4})^2\ (0.35)^3} + \frac{1.75(6.79 \times 10^{-3})^2 (10^3)(0.65)}{5.6 \times 10^{-4} \times 0.35^3}$$

= 3.42 x 10^4 Pa/m or 1.51 (lb$_f$/in^2.)/ft

(b) Repeat the calculation for other values of ε, using ratios.

ε	$(1-\varepsilon)^2/\varepsilon^3$	$(1-\varepsilon)/\varepsilon^2$	Term 1	Term 2	Total	$\Delta p/L$
0.35	9.85	15.17	3.20 x 104	2.18 x 104	3.42 x 104	1.51
0.40	5.62	9.38	1.83 x 104	1.35 x 104	1.97 x 104	0.87
0.41	5.05	8.96	1.64 x 104	1.23 x 104	1.76 x 104	0.78

A change in ε from 0.35 to 0.41 would be needed for agreement with the published pressure drop if the average particle size is 0.56 mm. With a wide range of sizes, the average size might be significantly larger than the arithmetic mean based on mesh sizes.

If $\overline{D}_p$ = 0.65 mm instead of 0.56 mm:

$$\frac{\Delta p}{L} = 3.2\times10^4\left(\frac{0.56}{0.65}\right)^2 + 2.18\times10^{-3}\left(\frac{0.56}{0.65}\right) = 2.56\times10^4$$

Predicted Δp/L = (2.56/3.42) x 1.51 = 1.13 $(lb_f/in^2.)/ft$

If $\overline{D}_p$ = 0.70 mm (unlikely, since the largest particles are only 0.83 mm):

$$\frac{\Delta p}{L} = 3.2\times10^4\left(\frac{0.56}{0.70}\right)^2 + 2.18\times10^{-3}\left(\frac{0.56}{0.70}\right) = 2.22\times10^4$$

Predicted Δp/L = (2.22/3.42) x 1.51 = 0.98 $(lb_f/in^2.)/ft$

A slightly larger void fraction is the most likely reason for the difference in the predicted and reported pressure drops.

7.13. Liquid velocity:

$$\overline{V} = \frac{10\,gal/\min\text{-}ft^2}{264.17\,gal/m^3}\times\frac{1}{60}\times\frac{1}{0.0929\,m^2/ft^2} = 2.72\times10^{-4}\ \text{m/s}$$

or 0.0272 cm/s

At this low velocity only the first term of the Ergun equation is important, and Eq. (7.52) for the minimum fluidization velocity can be used.

$$\overline{V}_{0M} = \frac{g(\rho_p-\rho)}{150\mu}\frac{\varepsilon_M^3}{1-\varepsilon}\Phi_s^2 D_p^2$$

Assume T = 20°C. μ = 1 cP = 10^{-3} Pa.s ρ = 1 g/cm^3

For 20- to 50-mesh beads, as in Problem 17.12,

$\overline{D}_p = 0.56\,mm = 5.6\times10^{-4}\,m$, Φ_s = 1.0.

Assume ε = 0.35. The particle density for Dowex 50-WX8 resin is 1.24 g/cm^3 for the H-form resin, but it may be different for other forms such as NaR (Ref. 10, Chap 17).

Use 1.24 - 1.0 = 0.24 g/cm^3 or 240 kg/m^3 for $\Delta\rho$.

$$\bar{V}_{0M} = \frac{9.8\ m/s^2 \times 240\ kg/m^3 \times 0.35^3 \times (0.56 \times 10^{-3})^2}{150 \times 10^{-3}\ Pa \cdot s \times 0.65}$$

$$= 3.24 \times 10^{-4} \text{ m/s or } 0.0324 \text{ cm/s}$$

When $\bar{V}$ = 0.0324, the flow is 0.476 gal/min-ft^2, quite close to the reported value of 0.4 gal/min-ft^2 for the start of expansion.

When the flow is 6 gal/min-ft^2, or 15 times the minimum for fluidization, the bed expansion should follow Eq. (7.57) for particulate fluidization.

$$\frac{\varepsilon^3}{1-\varepsilon} = \frac{150\ \bar{V}_0\ \mu}{g(\rho_p - \rho)\ \Phi_s^2\ D_p^2} = \beta \times \text{ (flowrate)}$$

For ε = 0.35, $\beta = \left(\frac{0.35^3}{0.65}\right)\left(\frac{1}{0.4\ gal/\min\text{-}ft^2}\right) = 0.165$

At a flow rate of 6 gal/min-ft^2:

$$\frac{\varepsilon^3}{1-\varepsilon} = 0.165 \times 6 = 0.99$$

Solving by trial, ε = 0.68. From Eq. (7.58),

$$\frac{L}{L_M} = \frac{1-\varepsilon_M}{1-\varepsilon} = \frac{1-0.35}{1-0.68} = 2.03$$

The predicted bed expansion is 100%, compared to the reported value of 45%.

Chapter 8

8.1. (a) Assume the average gas velocity is 20 m/s. (See p. 189). The actual volume flow is

$$q = \frac{10{,}000x1x293}{3600x3x273} = 0.994 m^3/s$$

Cross-sectional area required:

$S = 0.994/20 = 0.0497 m^2$ ($0.535\ ft^2$)

Using Schedule 40 steel pipe, the closest standard pipe size, from Appendix 3, is 10 in. (255mm ID).

(b) Mass flow rates:

p-Nitrophenol		1000 kg/h
Water:	1000 x 0.55/0.45 =	1222

Total		2222 kg/h

Volume flow rates:

p-Nitrophenol:	
1000/(3600 x 1475) =	$1.883 \times 10^{-4}\ m^3/s$
Water:	
1222/(3600 x 998) =	$3.401 \times 10^{-4}\ m^3/s$

Total	$5.284 \times 10^{-4}\ m^3/s$

Assume a slurry velocity of 2 m/s. The cross-sectional area required is $(5.284 \times 10^{-4})/2 = 2.642 \times 10^{-4}\ m^2$ ($0.00284\ ft^2$). The closest pipe size, from Appendix 3, is ¾-in. Schedule 80 steel pipe (18.8 mm ID).

Note: These are preliminary estimates only, suitable for high-spot evaluations. The optimum pipe size for the final installation should be checked by careful evaluation of all pertinent economic factors, using Eq. (8.1).

8.2. Use Eq. (8.9a). The allowable NPSH for the pump is 2.5m. Also $p_a' = p_v = 1.1$ atm, so $p_a' - p_v = 0$. Friction loss h_{fs} equals the frictional pressure drop Δp_f divided by density ρ. Δp_f is given as 7000 N/m^2; hence

$$h_{fs} = 7000/866 = 8.083 \text{ J/kg}$$

Substituting Eq. (8.9a) gives

$$Z_a = -2.5 - \frac{0 + 8.083}{9.80665}$$

$$= -3.32 \text{ m}$$

Thus the liquid level in the reboiler must be at least 3.32m above the pump suction.

8.3. Use Eq. (4.65). Assume the pump elevates the toluene 10m from the pump inlet. Referring to Fig. 4.9, Z_a then equals 3.32m and Z_b = 10m. $\bar{V}_b =$ 2 m/s. Assume that $\alpha_a = \alpha_b = 1.0$. Other quantities needed are:

$\bar{V}_a = 0$ $P_a = 1.1$ atm $P_b = 1.0$atm

$h_f = 35000/866 = 40.42$ J/kg

Note that 1 atm = 101325 N/m^2.

Substituting in Eq. (4.65) gives

$$nW_p = 9.80665(10-3.32) - \frac{(1.1-1.0)101325}{866} + 40.42 + \frac{2^2+0^2}{2}$$

$$= 96.23 \text{ J/kg}$$

The theoretical power, assuming $\eta = 1$, is

$$P = W_p \dot{m} = 96.23 \times 10{,}000/3600 = 267.3 \text{ W.}$$

If $\eta = 0.6$, P would be 267.3/0.6 = 445.5 W.

8.4. $\sqrt[4]{4014.7/14.7} = \sqrt[4]{273.1} = 4.065$ (See Example 8.3)

Use four stages.

Theoretical shaft work [Eq. (8.27)]:

$1-1/\gamma = 1-1/1.40 = 0.286$

p_a (first stage) $= 14.7 \times 144$

$= 2117 \text{ lb}_f/\text{ft}^2$

$\rho_a = \frac{29}{359} x \frac{492}{530} = 0.0750 \text{ lb/ft}^3$ (first stage)

Assuming gas is cooled to 70°F by each intercooler, $p_a \rho_a$ is the same for each stage. Then

$$W_{pr} = \frac{4x2117}{0.286x0.0750}[4.065^{0.286} - 1]$$

$$= 194{,}800 \text{ ft-lb}_f/\text{lb}$$

Mass of one standard cubic ft $= \frac{29}{359} x \frac{492}{520} = 0.0764$ lb

The theoretical shaft work per standard cubic foot:

$W_{pr}\rho_B = 194{,}800 \times 0.0764 = 14882 \text{ ft-lb}_f/\text{std. ft}^3$

Brake horsepower:

$$P_B = \frac{14882x125}{33000x0.85} = 66.3 \text{ hp}$$

8.5 From Eq. (8.24)

$$T_1 = 530 \text{ x } 4.065^{0.286} = 792°R \text{ or } 332°F$$

8.6 The mass flow rate is proportional to the square root of the pressure drop, provided C_0 is constant. Approximately, therefore, since Re_0 = 376,000 for a manometer reading of 1.25m (Example 8.4), Re_0 for the smaller reading is approximately

$$Re_0 = 376{,}000 \text{ x } (45/1250)^{0.5} = 71{,}300$$

This is large enough so that C_0 may be taken as 0.61. Since the flow rate in Example 8.4 was 50 m^3/h, it is now:

$$50 \text{ x } (45/1250)^{0.5} = 9.50 \text{ m}^3/\text{h}$$

8.7 Assume Eq. (8.42) applies. Needed data are

$$\rho_a = \frac{0.60x29}{359}x\frac{20}{14.69}x\frac{492}{560} = 0.058 \text{ lb/ft}^3$$

$$p_a - p_b = \frac{46.3}{12}x62.37 = 240.6 lb_f/ft^2$$

$$S_0 = \frac{\pi x2.00^2}{4x144} = 0.0218 \text{ ft}^2 \qquad \gamma = 1.30$$

$p_a = 20 \text{ x } 144 = 2880 \text{ lb}_f/\text{ft}^2$ $\quad$ $p_b = 2880 - 240.6$
$= 2639.4 \text{lb}_f/\text{ft}^2$

$$\frac{p_b}{p_a} = \frac{2639.4}{2880} = 0.9165 \qquad \beta = 2.00/6.065 = 0.330$$

From Eq. (8.44)

$$Y = 1 - \frac{0.41 + 0.35x0.330^4}{1.30}(1 - 0.9165) = 0.973$$

From Eq. (8.42):

$$\dot{m} = 0.61 \times 0.973 \times 0.0218\sqrt{2x32.174x240.6x0.058}$$

$= 0.388$ lb/s

Density at base conditions:

$$\rho_B = 0.058 \times \frac{14.4}{20}x\frac{560}{520} = 0.045 lb/ft^3$$

Volume flow:

$$q = \frac{0.388x60}{0.045} = 517 \ ft^3/min$$

Check on Reynolds number, from Eq. (8.40):

$$Re_0 = \frac{4x0.388}{\pi x2/12x0.011x6.72x10^{-4}} = 401,000$$

Hence C_0 may be taken as 0.61 and Eq. (8.44) applies.

8.8. Use Eq. (8.37). Assume $C_v = 0.98$. $R_m = 0.5$m

$$S_b = \frac{0.020^2\pi}{4} = 0.000314 \ m^2$$

$\beta = 20/75 = 0.267$ $\rho = 999$ kg/m^3

From Eq. (2-10):

$$(p_a - p_b) = 9.80665 \times 0.5(13.6-1) \times 999$$
$$= 61720 \ N/m^2$$

From Eq. (8.37):

$$q = \frac{0.98x0.000314}{\sqrt{1-0.267^4}}\sqrt{\frac{2x61720}{999}}$$

$= 0.00343\ m^3/s$ or $12.35\ m^3/h$

Power loss: 0.12 x 61720 x 0.000343 = 25.4 W

8.9. (a) Slurry density: volume of 1000kg of slurry

Volume = (850/998) + (150/1250) = 0.9717 m^3

$\rho = 1000/0.9717 = 1029\ kg/m^3$

For orifice area use Table 1-19a, Perry ,6th ed., p. 1-26. For h/D = 1/3 or h/R = 0.667, $S_0 = 0.9158R^2$. For 3-in. pipe,

$$S_0 = 0.9158 \text{ x } \left(\frac{3.068}{2x12}\right)^2 = 1.497 \text{ x } 10^{-2} \text{ ft}$$

At 150 gal/min,

$$u_0 = \frac{150}{60}x0.13368\frac{ft^3}{gal}x\frac{1}{0.01497} = 22.32 ft/s$$

$u_0 = 22.32 \text{ x } 0.3048 = 6.803\ m/s$

$C_0 = 0.80$. From Eq. (8.39), in fps units,

$$(p_a - p_b) = \frac{u_0^{\,2}(1-\beta^4)}{C_0^{\,2}}\frac{\rho}{2g_c}$$

Since the orifice is not circular, β^2 must be defined as the ratio of upstream velocity to orifice velocity as in Eq. (8.34).

$$\beta^2 = V_a/V_b = V_a/u_0 = \frac{\text{Orifice area}}{\text{Pipe area}}$$

Pipe area (Appendix 3) = 0.05130 ft^2

$\beta^2 = 0.01497/0.05130 = 0.2918$

$$p_a - p_b = \frac{6.803^2\left(1-0.2918^2\right)}{0.8^2} x \frac{1029}{2}$$

= 34039 Pa or 4.93 $lb_f/in.^2$

(b) Since $u_0 = B\Delta p^{1/2}$, where B is a constant,

$du_0/u_0 = 1/2(d\Delta p/\Delta p)$

At maximum flow of 150 gal/min,

$du_0/u_0 = ½ \times (0.05/4.93) = 0.005$

0.5% accuracy

At minimum flow of 30 gal/min,

$\Delta p = 4.93 \times (30/150)^2 = 0.1972\ lb_f/in.^2$

$du_0/u_0 = ½ \times (0.05/0.1972) = 0.127$

or 12.7% accuracy

8.10 From appendix 14, find the average molar heat capacity of the gas.

Gas	Vol. %	Mol. Wt.	C_p at 150°C
N_2	76	28	7.01
O_2	3	32	7.25
CO_2	14	44	10.08
H_2O	7	18	8.21

Average heat capacity:

$\bar{C}_p =$ 0.76(7.01) + 0.03(7.25) + 0.14(10.08) + 0.07(8.21) = 7.53 Btu/lb mol$^{-°F}$

= 4.168 x 7.53 = 31.39 w/g mol-°C

Duct area = 1.2 x 2 = 2.4m^2. Let F be the molar flow rate. Then, since T = 150 + 273 = 423 K,

F = 2.4 x 12 x 1000 x 273/(22.4 x 423)= 829.8g mol/s

Temperature change:

$T_b - T_a = q/FC_p$

$$= 5000x\frac{1}{31.39x829.8} = 0.192^{\circ}C$$

If each temperature measurement is ± 0.01°C, the maximum difference is ± 0.02°C, which is about 10% of the expected reading.

(b) A gas with 12% CO_2 would have a lower heat capacity. Assuming the same content of O_2 and H_2) and with 78% N_2,

$\bar{C}_p$ = 0.78(7.01) + (0.03(7.25) + 0.12(10.08) + 0.07(8.21) = 7.47 Btu/lb $mol^{-°F}$ or 31.13 J/g $mol^{-°C}$. This is 1% less than the base value, so the flow rate determined from the normal calibration would be 1 percent lower than the correct value.

Chapter 9

9.1. D_a = 0.36 m ρ = 800 kg/m^3

n = 800/60 = 13.33 r/s μ = 1 Pa-s or 1 kg/m-s

N_{Re} = 13.33 x 0.36^2 x 800/1 = 1382

N_{Fr} = 13.33^2 x 0.36/9.80665 = 6.52

From Fig. 9.14, for unbaffled tanks, N_P = 0.73

P = 0.73 x 13.33^3 x 0.36^5 x 800 = 8364 W

The motor is not adequate.

9.2. Motor delivers 8 kw. Viscosity = 0.1 kg/m-s.

From Problem 9.1, $n = 13.33\ s^{-1}$ for $\mu = 1$ kg/m-s

Guess $n = 14\ s^{-1}$

$Re = 0.36^2 \times 800 \times 14/0.1 = 14518$

$N_P = 0.58$ (extrapolated from Fig. 14)

$P = 0.58 \times 14^3 \times 0.36^5 \times 800 = 7699$ W

Close enough, n = 14 rev/s.

9.3. $Re = 15 \times 0.36^2 \times 800/1 = 1555$

From Fig. 9.14, $N_p = 0.9$. By Eq. (9.18),

$P = 0.9 \times 15^3 \times 0.36^5 \times 800 = 14690$ W

An 8-kW motor would not be adequate.

9.4. $D_a = 0.4$m

Volume $= (\pi/4) \times 1.2^2 \times 1.2 = 1.357\ m^3$

$\mu = 15\ P = 1.5$ kg/m-s $\qquad P = 1.357$ kW

From Eqs. (9.18) and (9.24)

$N_p = 1357/(950 \times 0.4^5 n^3) = 139.5/n^3$

$K' = 1.5/10^{(0.75 - 1)} = 2.667\ kg/m\text{-}s^{1.25}$

Note that K′ must be expressed in the correct units, whereas exponent n′ is dimensionless.

From Eq. (9.27),

$$Re_n = \frac{n^{1.25} x 0.4^2 x 950}{11^{-0.25} x 2.667} = 103.9n^{1.25}$$

By trial, using Fig. 9.15, n = 3.38 r/s.

$Re_n = 475$, $\qquad N_P = 139.5/3.38^3 = 3.61$

N_P from Fig. 9.15 is 3.6. Close enough.

9.5. Find the Reynolds number.

$$Re = D_a^2 n\rho/\mu$$

$$= \frac{1.5^2 x 75 x 65}{60 x 3 x 6.72 x 10^{-4}} = 9.07 \times 10^4$$

Flow is turbulent; N_P is constant (Fig. 9.13). For $D_a = D_t/4 = 4.5 \times 12/4 = 13.5$ in., since N_P and p are constant:

$$n = 75(18/13.5)^{5/3} = 121.1.\ \text{r/min or } 2.018\ \text{r/s}$$

$$Re = \frac{(13.5/12)^2 x 2.018 x 65}{3 x 6.72 x 10^{-4}} = 8.23 x 10^4$$

Mixing time. Since f_t is constant (Fig. 9.17),

$$t_T = 29 \times \left(\frac{75}{121.1}\right)^{2/3}\left(\frac{18}{13.5}\right)^{11/6} = 35.7\text{s}$$

For $D_a = 4.5 \times 12/2 = 27$ in.:

$$n = 75\ (18/27)^{5/3} = 38.2\ \text{r/min}$$

$$Re = \left(\frac{27}{12}\right)^2 \frac{38.2 x 65}{60 x 3 x 6.72 x 10^{-4}}$$

$= 1.04 \times 10^5$. $N_P = 6.3$; $f_t = 6$

Mixing time (since f_t is constant):

$$t_T = 29 \times \left(\frac{75}{38.2}\right)^{2/3}\left(\frac{18}{27}\right)^{11/6} = 21.6\ \text{s}$$

9.6.(a) Linear scale factor is $500^{1/3} = 7.937$

For the pilot plant reactor,

D_t and H are both $2/7.937 = 0.252$ m or 252 mm

$D_a = 0.6/7.937 = 0.076$ m or 76 mm

(b) $\rho = 61.04 \times 16.0185 = 977.8\ \text{kg/m}^3$ (Appendix 6)

$\mu = 0.408$ cP

$$\text{Re} = \frac{0.076^2 x(330/60)x977.8}{0.408x10^{-3}} = 7.61x10^4$$

Since Re > 10^4, $N_P = K_T$ = constant

Let subscript 1 refer to pilot-plant vessel, subscript 2 to large vessel.

$$\frac{P_1}{D_{a1}{}^3 D_{a1}{}^2 n_1{}^3 \rho} = \frac{P_2}{D_{a2}{}^3 D_{a2}{}^2 n_2{}^3 \rho}$$

Since $P_1/D_{a1}{}^3$ is to be set equal to $P_2/D_{a2}{}^3$ (power per unit volume is constant).

$$n_2 = n_1\left(\frac{D_{a1}}{D_{a2}}\right)^{2/3} = \frac{330}{7.937^{2/3}} = 82.9 \text{ r/min}$$

From Fig. 9.16, nt_T is constant at these Reynolds numbers:

$$\frac{t_{T2}}{t_{T1}} = \frac{n_1}{n_2} = \frac{330}{82.9} = 3.98$$

(c) From Fig. 9-16, nt_T is constant and n2 = n1 = 330 r/min

For this: Power per unit volume is

$$\frac{P_2/D_{a2}{}^3}{P_1/D_{a1}{}^3} = \left(\frac{D_{a2}}{D_{a1}}\right)^2 = 7.937^2 = 63$$

(d) Since $D_a{}^2 n\rho/\mu$ is constant

$$n_2 = n_1\left(\frac{D_{a1}}{D_{a2}}\right)^2 = \frac{330}{7.937^2} = 5.24 r/\min$$

N_P is constant; hence

$$\frac{P_2/D^3{}_{a2}}{P_1/D^3{}_{a1}} = \left(\frac{D_{a2}}{D_{a1}}\right)^2\left(\frac{n_2}{n_1}\right)^3 = 7.937^2 x\left(\frac{5.24}{330}\right)^3 = \frac{1}{3965}$$

Blending time. Since nt_T is constant at high Reynolds numbers

$$\frac{t_{T2}}{t_{T1}} = \frac{n_1}{n_2} = 330/5.24 = 63$$

(e) Ratios (large vessel/small vessel)

Part	Speed	Power per unit volume	Blending Times
b	1/3.98	1	3.98
c	1.00	63	1
d	1/63	1/3965	63

Choose (b), keeping power per unit volume constant. This gives reasonable speed, reasonable power load, and modest increase in blending time.

9.7 n = 400/60 = 6.67 r/s

ρ = 60 lb/ft^3, μ = 5 cP. Then

$$\text{Re} = \frac{6.67x1^2x60}{5x6.72x10^{-4}} = 1.19 \times 10^5$$

Turbulent flow, so from Eq. (*9.31), omitting the Fr term, $nt_T(D_a/D_t)^2(D_t/H)^{1/2}$ is constant.

(a) Assume (D_t/H) is constant; given that t_T is constant. If subscript 1 refers to the original vessel, 2 to the new vessel:

$(D_a/D_t)_1 = 1/3$ $\qquad$ $(D_a/D_t)_2 = 3/7$

Hence $\dfrac{n_1t_T}{n_2t_T} = \dfrac{n_1}{n_2} = \left(\dfrac{3/7}{1/3}\right)^2 = 1.653$

$n_2 = 6.67/1.653 = 4.03$ r/s or 242 r/min

(b) At $N_{Re} = 10^5$, $N_P = Pg_c/n^3D_a^5\rho$ is constant. Since g_c and ρ are constant

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^3\left(\frac{D_{a2}}{D_{a1}}\right)^5 = \left(\frac{4.03}{6.67}\right)^3\left(\frac{3}{1}\right)^5 = 53.6$$

$V_2/V_1 = (7/3)^3 = 12.7$

$$\frac{P_2/V_2}{P_1/V_1} = \frac{53.6}{12.7} = 4.22$$

9.8. Use Fig. 9.21. First find $\bar{V}_s$.

$P = 60 + 14.7 = 74.7\ lb_f/in.^2$ abs

$T = (90 \times 1.8) + 32 = 194°F$ or $654°R$

$S = \pi \times 9^2/4 = 63.62\ ft^2$

$$q = \frac{1800x654x14.7}{520x74.7x60} = 7.425\ ft^3/s$$

$\bar{V}_s = 7.425/63.622 = 0.117$ ft/s or 35.6 mm/s

From Fig. 9.21, $P_g/P_o = 0.33$

Tank volume $= 63.62 \times 12 = 763.4\ ft^3$ or 5710 gal

$P_g = 5 \times 5710/1000 = 28.6$ hp

$P_o = 28.6/0.33 = 86.5$ hp

Assume that flow is turbulent and $N_P = K_T$

From Table 9.2, $K_T = N_P = 5.75$.

For 1 wt% catalyst

$\rho = 62.3/((0.99/0.84) + (0.01/4))$

$= 52.7\ lb/ft^3$

Since $N_P = 5.75$,

$P_o = (5.75 \times 3^5 \times 52.7n^3)/32.174$

$= 2289n^3 = 86.5 \times 550\ lb_f\text{-}ft/s$

$n = (86.5 \times 550/2289)^{1/3}$

$= 2.75$ r/s or 165 r/min

(b) Gas flow on: $P_g = 28.6$ hp

Gas flow off: $P_o = 86.5$ hp

9.9. Use Eq. (9.35) for n_c.

$D_a = 3$ft. S (from Table 9.4) = 7.5.

$$\nu = \frac{1.6x6.72x10^{-4}}{0.84x62.3} = 2.05x10^{-5} \text{ ft}^2/\text{s}$$

$D_P = 50 \times 10^{-6} \times 3.2808 = 1.64 \times 10^{-4}$ ft

$g = 32.2$ ft/s^2 $\Delta\rho/\rho = (4-0.84)/0.84 = 3.76$

For 1 wt% catalyst, $B = 100 \times 0.01/0.99 = 1.01$

From Eq.(9.35),

$$n_c = \frac{7.5(2.05x10^{-5})^{0.1}(1.64x10^{-4})^{0.2}}{3^{0.85}}x(32.2x3.76)^{0.45}x1.01^{0.13}$$

$= 1.52$ r/s or 91 r/min

As in Problem 9.8, $N_P = 5.75$. Thus

$$P = \frac{5.75x1.52^3x3^5x52.7}{32.174}$$

$= 8037$ lb_f-ft/s or 14.6 hp

This is 14.6/5.710 = 2.56 hp/1000 gal. From Fig. 9.19, for a 12-ft vessel P/V = 2.2 to 3.1 hp/1000 gal.

9.10 (a) Estimates of n_c can be obtained directly from Eq. (9.35) and Table 9.3 or from Fig. 9.20, using Eq. (9.35) to correct for changed conditions as in Example 9.4.

D_a = 20/3 = 6.667 ft. Assume μ = 1 cP and ρ = 62.3 lb/ft^3. Then

$\nu = 6.72 \times 10^{-4}/62.3 = 1.08 \times 10^{-5}$ ft^2/s

Find D from Appendix 5:

20 mesh = 0.0328 in. 28 mesh = 0.0232 in.

D_P = (0.0328 + 0.0232)/(2 x 12) = 0.00233 ft

ρ for limestone (Perry, 7th ed., p. 2-119)

= 155 lb/ft^3

$\Delta\rho/\rho$ = (155 - 62.3)/62.3 = 1.488

B = 100 x 15/85 = 17.65

From Table 9.3, use S = 6.5, assuming the 45° turbine is more like a propeller than a standard turbine. From Eq. (9.35),

$$n_c = \frac{6.5(1.08x10^{-5})^{0.1}(0.00233)^{0.2}}{6.667^{0.85}} \times (32.2 \times 1.488)^{0.45} \times 17.65^{0.13}$$

= 1.02 r/s or 61 r/min.

Alternate solution The data of Buurman in Fig. 9.20 shows P/V is about 2.1 hp/1000 gal for 200-μm sand with B = 11.1. D_p for limestone is 0.00233 x 304,800 = 710 μm.

Predicted change in n_c

$(710/200)^{0.2}(1.49/1.59)^{0.45}(17.65/11.1)^{0.13} = 1.329$

Corrected P/V = 2.1 x 1.329 = 4.93 hp/1000 gal

ρ_m = 1/((0.85/62.3) + (0.15/155)) = 68.4 lb/ft^3

From Table 9.1, K_T = 2.0 for 6-blade 45° turbine

V = 20 x (π/4)20^2 = 6283 ft^3 or 47,000 gal

$P = 4.93 \times 47 = 232$ hp

$$n_c^3 = \frac{232x32.174x550}{2.0x6.67^5 x68.4} = 2.27$$

$n_c = 1.31$ r/s or 78.6 r/min

The two methods differ by 30% in the predicted critical stirrer speed.

(b) $D_a = 0.4 \times 20 = 8$ ft. If P/V is kept constant at 4.93 hp/1000 gal:

$(n_c/1.31)^3 = (6.667/8)^5 = 0.402$

$n_c = 0.97$ r/s or 58 r/min

9.11 (a) $n = 600/60 = 10$ r/s

$D_a = 4/12 = 0.333$ ft $\qquad D_t = 1$ft

$\rho = 70$ lb/ft^3 $\qquad \mu = 3$ cP

$$Re = \frac{0.333^2 x10x70}{3x6.72x10^{-4}} = 3.9x10^4$$

Flow is turbulent. From Table 9.2, for curved blades, estimate $N_P = K_T = 4.80$. Thus

$$P = \frac{4.8x10^3 x0.333^5 x70}{32.174x550} = 0.078hp$$

For scaleup keep P/V constant. Volume of small reactor:

$V_1 = (\pi/4) \times 1^2 \times 1 \times 7.48 = 5.88$ gal

Large reactor: $V_2 = 8000$ gal

Power in large reactor:

$P_2 = 0.078 \times 8000/5.88 = 106$ hp

(b) From Tables 9.2 and 9.3 for $D_t/D_a = 3$, S is almost the same for various agitators, but N_P is greatly different. Hence the critical speed would be about the same with different agitators of the same diameter but the power consumption would be much less if a paddle or propeller were used.

9.12 (a) $V = (\pi/4)(3)^2(3) = 21.2\ m^3$

If a standard turbine is used with $D_a/D_t = 1/3$,

$N_P = 5.8$

$D_t = 3.0m \qquad D_a = 1.0m$

$S = \pi \times 3^2/4 = 7.07\ m^2$

Choose a stirrer speed that will give vigorous agitation. Choose $P_g/V = 600\ W/m^3$ (3.05 HP/1000 gal)

$\bar{V}_s = 800/(3600 \times 7.07) = 0.0314$ m/s or 31 mm/s

From Fig 9.21, $P_g/P_o \cong 0.33$

$P_o = 600 \times 21.2/0.33 = 3.85x10^4$ W

$P_o = N_P n^3 D_a^5 \rho$

From Appendix 6, at 110°C (230°F), $\rho = 951\ kg/m^3$

$$n^3 = \frac{3.85x10^4}{5.8x10^5 x951} = 6.98$$

$n^3 = 1.91\ s^{-1} = 115$ rpm

(b) $P_g = 600(21.2) = 1.27x10^4 = 12.7$ kW

(c) From Eq (9.46)

$$\bar{V}_{s,c} = 0.114\left(\frac{P_g}{V}\right)\left(\frac{D_t}{1.5}\right)^{0.17} = 0.114(0.60)(3/1.5)^{0.17} = 0.077 m/s$$

Using the alternate correlation, Eq. (9-48)

$$\bar{V}_{s,c} = \frac{120}{\pi g}(n^3 D_a{}^2)(D_a / D_t)^{5.5}$$

$$\bar{V}_{s,c} = \frac{120}{\pi x 9.98}(1.91^3 1^{\ 5})(1/3\ )^{5.5} = 0.065 \text{ m/s}$$

The two correlations are in reasonable agreement, and show that the flooding velocity is over twice the calculated gas velocity based on ethylene flow to the vessel. Based on the lower value of $\bar{V}_{s,c}$, the gas flow rate at incipient flooding is

$$q = \frac{0.065}{0.0314} x800 = 1660 m^3 / h$$

However, the actual gas rate near the impeller may be considerably greater than 800 m^3/h, because the gas bubbles will very quickly become saturated with water vapor, raising the total gas flow even as the ethylene slowly dissolves and reacts. Since the vapor pressure of water at 110°C is 1.41 atm, the ethylene pressure is 1.59, and immediate saturation of the gas bubbles with water would increase the gas flow rate by a factor of 3/1.59 = 1.89.

9.13 (a) Since Re is large, K_T is constant and P varies with $n^3 D_a^5$.

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_{a1}}{D_{a2}}\right)^5$$

$D_{a2} = 2D_{a1}$

$$\left(\frac{n_2}{n_1}\right)^3 = \left(\frac{1}{2}\right)^5 = 0.03125$$

$n_2 = 0.315\ n_1$

Since n and p were not stated in Example 9.6, they are calculated assuming H = 6ft, K_T = 5.8 and ρ = 1000 kg/m^3.

$V = (\pi/4) \times 6^2 \times 6 = 169.6\ ft^3$

$D_a = 2\ ft = 0.61\ m$

$V = 169.6 \times 7.48 = 1269\ gal$

$P = 2 \times 1269 = 2.54\ HP = 1894\ W$

$P = K_T n^3 D_a^5 \rho$

$$n^3 = \frac{1894}{5.8(0.61)^5 1000} = 3.87$$

$n_1 = n = 1.57\ m/s = 94\ rpm$

$n_2 = 0.315(94) = 30\ rpm$

(b) From Eq (9.30)

$$Nt_T = 4.3\left(\frac{H}{D_t}\right)\left(\frac{D_t}{D_a}\right)^2$$

$$\frac{t_{T2}}{t_{T1}} = \left(\frac{D_{a1}}{D_{a2}}\right)^2 \left(\frac{n_1}{n_2}\right) = \left(\frac{1}{2}\right)^2 \frac{1}{0.315} = 0.794$$

$t_{T2} = 0.794(49.5) = 39.3\ s$

The predicted blending time is 20% shorter when the larger impeller is used.

9.14 (a) increase D_T, D_a by a factor of 10

P/V constant, $\dfrac{n^3 D_a^{\ 5}}{D_t^{\ 3}}$ constant

For same D_a/D_t, $n^3 D_a^{\ 2}$ is constant

$$\left(\frac{n_2}{n_1}\right)^3 = \left(\frac{D_{a1}}{D_{a2}}\right)^2 = \left(\frac{1}{10}\right)^2 = 0.01$$

$$\frac{n_2}{n_1} = (0.01)^{1/3} = 0.215$$

The maximum shear rate is proportional to n, so the maximum shear rate in the large tank is only about one fifth that in the small tank if P/V and D_a/D_t are constant.

(b) The maximum shear rate could be increased by using a smaller agitator operating at higher speed, but there is no reason to do this with a shear-sensitive suspension.

Chapter 10

10.1 Basis: 1 m^2. Let A refer to fireclay brick, B to kaolin brick, and C to steel. From Appendixes 10 and 11, thermal conductivities are

	K, W/m-°C
Fireclay brick	1.7
Kaolin brick	0.11
Steel	45

Heat loss:

$$q = \frac{T_h - T_c}{\frac{x_A}{k_A} + \frac{x_B}{k_B} + \frac{x_C}{k_C} + \frac{(x_B)_{air\_equiv.}}{0.11}}$$

$$300 = \frac{1150 - 30}{\frac{0.200}{1.7} + \frac{0.100}{0.11} + \frac{0.006}{45} + \frac{(x_B)_{air\_equiv.}}{0.11}}$$

$(x_B)_{air\_equiv.} = 0.298$ m

10.2 Pipe diameters (Appendix 3):

$D_0' = 1.315$ in. $\qquad D_i' = 1.049$ in.

$\bar{D}_L'$ (pipe) $= (1.315 - 1.049)/ \ln 1.315 / 1.049$

$= 1.177$ in.

$\bar{D}_L'$ (magnesia) $= (5.315 - 1.315)/ \ln 5.315 / 1.315$

$= 2.86$ in.

$\bar{D}_L'$ (cork) $= (6.315 - 5.315)/ \ln 6.315 / 5.315$

$= 5.80$ in.

Thermal conductivities, in Btu/h-ft-°F,

Cork: $k_A = 0.03$

Magnesia: $k_B = 0.034$

Steel: $k_C = 26$

Thermal resistances, based on 100 ft of pipeL

$$R_A = \frac{0.5/12}{0.03x\pi(5.80/12)x100} = 0.00915$$

$$R_B = \frac{2/12}{0.034x\pi(2.86/12)x100} = 0.06547$$

$$R_C = \frac{0.133/12}{26x\pi(1.177/12)x100} = 0.000014$$

Heat loss:

$$q = \frac{249-90}{0.00915+0.06547+0.000014} = 2130 \text{ Btu/h}$$

Let T_A be the temperature at the boundary between cork and magnesia and T_B that at the boundary between cork and steel. The total resistance $R_A + R_B + R_C = 0.074634$. Then

$T_A = 90 + (0.00915/0.074634)(249-90) = 109.5°F$

$T_B = 249 - (0.000014/0.074634)(249-90) = 248.97°F$

10.3 Within the shell ($r_1 < r < r_2$):

$$q = -k\frac{dT}{dr}(4\pi r^2)$$

$$\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{4\pi k}{q}\int_{T_2}^{T_1} dT$$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{4\pi k(T_1 - T_2)}{q}$$

$$q = 4\pi k(T_1 - T_2)(\frac{r_1 r_2}{r_2 + r_1})$$

$$= 4\pi r_m{}^2 \frac{k(T_1 - T_2)}{r_2 - r_1}$$

where r_m = $\sqrt{r_1 r_2}$. Hence use the geometric mean radius to calculate area in contrast to the logarithmic mean radius for a thick-walled cylinder.

10.4 Use Eq. (10.37). Interpret "change significantly" as "by 1 percent" and let the penetration distance x_P equal half the film thickness: x_P = s = 0.004/2 = 0.002 m

(a) From Eq. (10.37),

$t = x_P{}^2/(3.64^2 \times 0.00035)$

$= 8.63 \times 10^{-4}$ h or 3.11 s

(b) $F_o = 0.00035 \times 8.63 \times 10^{-4}/0.002^2$

$= 0.0755.$

The unaccomplished temperature change may be read from Fig. 10.6, or found more accurately from Eq. (10.20), as follows. Since $a_1 = (\pi/2)^2 = 2.467$,

$$\frac{T_s - \bar{T}_b}{T_s - T_a} = \frac{8}{\pi^2}(e^{-(2.467x0.0755)} + (1/9)e^{-(9x2.467x0.0755)} + (1/25)e^{-(25x2.467x0.0755)})$$

$= 0.690$

Hence $\bar{T}_b$ $= T_s - 0.690\ (T_s - T_a)$

$= 102 - 0.690(102 - 20) = 45.4°C$

10.5 From Eq. (10.24), since α and the unaccomplished temperature change are the same, t_T varies with r_m^2. If the film resistance of the quench liquid is negligible,

(a) $t_T = (2.5/1)^2 \times 4 = 25$ min

(b) $t_T = (5/1)^2 \times 4 = 100$ min

10.6 (a) Use Eq. (10.22). Quantities needed are:

$r_m = 1.5/12 = 0.125$ ft $\quad T_a = 700°F$

$T_s = 125°F \quad \alpha = 26/(486x0.11) = 0.486$

$F_0 = 0.486t/0.125^2 = 31.104t$

From Eq. (10.22),

t,h	F_0	$\dfrac{T_s - \bar{T}_b}{T_s - T_a}$	$\bar{T}_b$
0.00278	0.0865	0.2639	276.7
0.01667	0.5185	0.00364	127.09
0.10	3.1104	4.65×10^{-14}	$125 + (2.7 \times 10^{-11})$

(b) $(T_s - \bar{T}_b)/(T_s - T_a) = 0.01$.

From Eq. (10.25),

$$t_T = \frac{0.125^2}{9.87x0.486}\ln(0.608/0.01)$$

= 0.0134 h or 48.2 s

10.7 Use Eq. (10.40). Total quantity of heat lost:

$Q_T / A = 2 \times 0.7(-20-5)(12/(\pi x0.0011))^{0.5}$

$= -2062$ Wh/m^2 (inherently negative)

12 hours:

$= -171.8\ W/m^2$

e unaccomplished

9

2

3

with $s = r_m$, the

the sphere, 88% with

ith the slab. If $T_s >$

e sphere and lowest

der in between. (If T_s

he sphere.)

change is greatest

enters (or leaves)

directions along the

nfinitely long

ng the y- and z-axes,

z-axis only. Also the

for a sphere, $2/r_m$

a slab, so the

this order.

the common types of

7th ed., p. 2-335:

.3 W/m-°C

$R_{overall} = \Delta T/(q/A)$

$= (50-20)/7400 = 0.004054\ m^{2-}{}^\circ C/W$

$R_{wall} = x_w/k_m = 0.012/16.3 = 0.000736$

$\Delta T_{wall} = (0.000736/0.004054) \times 30$

$= 5.45^\circ C$

(b) For the polymer, $R_P = x_p/0.16$

$R_P = (30-5.45)/7400 = 0.003318$

$X_P = 0.00318 \times 0.16 = 0.00053$ m or 0.53 mm.

The actual thickness would be somewhat less, since part of the temperature drop is in the liquid.

(c) For steel, $k = 45\ W/m^{-\circ}C$. For the composite wall:

$R_{comp} = (0.003/16.3) + (0.009/45)$

$= 0.000384$

The new $R_{overall} = R_p + R_{comp}$

$= 0.003318 + 0.000384 = 0.003702$

The heat flux increases inversely with the ratio of the overall resistances:

Increase $= 0.004054/0.003702 = 1.095$ or 9.5%.

10.10(a) Thermal conductivities in Btu/h-ft-°F:

Water at 100°F: $k_w = 0.362$ (Appendix 6)

Air at 100⁻°F: $k_a = 0.0157$ (from Appendix 12)

$K_w/k_a = 0.362/0.0157 = 23$

(b) Assume the heated fluid is at (100+50)/2 = 75°F (535°R). For air at 75°F:

k (by interpolation) = 0.01505 Btu/h-ft-°F

ρ = (29 x 492)/(359 x 535) = 0.0742 lb/ft^3

c_p = 0.25 Btu/lb-°F

$\alpha = k/\rho c_p$ = 0.01505/(0.0742 x 0.25)

= 0.811 ft^2/h

For water at 75°F, k = 0.349 Btu/h-ft-°F

ρ = 42.24 lb/ft^3

c_p = 1.0 Btu/lb-°F

α = 0.349/(62.26 x 1.0) = 0.0561 ft^2/h

From Eq. (10.37), $x_P = 3.64(\alpha t)^{0.5}$

For air: x_p = $3.64(0.811 \times 10/3600)^{0.5}$

= 0.173 ft or 2.08 in.

For water: x_p = $3.64(0.00561 \times 10/3600)^{0.5}$

= 0.0144 ft or 0.173 in.

Even though air is a poorer conductor of heat than water, it has a much lower heat capacity and the heat "penetrates" much farther than in water for the same exposure time.

10.11 (a) $\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_0} + r_{wall} + r_{insulation}$

neglect $1/h_i$ and r_{wall}

From Appendix 11, for fiberglass insulation

k ≅ 0.027 Btu/h-ft^2-°F x 1.73073 = 0.0467 W/mK

$$r = \frac{0.050m}{0.0467} = 1.07$$

$$\frac{1}{U} = \frac{1}{20} + 1.07 = 1.12$$

U = 0.893 W/m^2°C

W = 0.893(150-20) = 116 W/m^2

(b) doubling the insulation thickness gives

$$\frac{1}{U} = \frac{1}{20} + 1.07x2 = 2.19$$

U = 0.457

Heat loss is reduced by a factor of 0.457/0.893 = 0.51.

10.12 Let A refer to firebrick, B to steel, C to magnesia. From Appendix 11, k_A = 0.98 Btu/hr-ft

From Appendix 10, k_B = 26

As in Example 10.2, k_C = 0.034

(a) The outside surface of the magnesia can be warm but not so hot as it be dangerous to the touch. Assume its temperature is 120°F. Then the overall ΔT = 1200 - 120 = 1080°F The resistances are:

R_A = 4(12 x 0.98) = 0.34

R_B = 0.5/(12 x 26) = 0.06

R_C = 3/(12 x 0.034) = 7.35

R = 7.696

As in Example 10.2,

$$\frac{\Delta T_A}{0.34} = \frac{1080}{7.696}$$

$\Delta T_A = 48°F$

$T_B = 1152°F$ (the temperature drop in the steel is negligible).

(b) ΔT is now 1300 − 120 = 1180°F. The heat transfer rate q/A would be slightly reduced, but assume it is unchanged. Let x be the new thickness of the magnesia layer.

Then R_C = x/(12 x 0.034) = 2.45x + 0.346

From (a), q/A = 1080/7.696 = 120.33 Btu/ft^2-h

Here q/A = 1180/(2.45x + 0.346) = 140.33

2.45x + 0.346 = 1180/140.33

From which x = 3.29 in.

10.13 Assume all resistance to heat transfer is in the gas film.

$d_P = 50\mu m$ $\qquad$ $k = \frac{2h}{D_P} = \frac{k}{R}$ $\qquad$ $R = 25 \times 10^{-6} m$

$T_O = 25°C$

$T_a = T_{air} = 300°C$ $\qquad$ $T_{ave} = 162°C$

$$\frac{4}{3}\pi r^3 \rho_s C_s \frac{dT_s}{dt} = 4\pi r^2 h(T_a - T_b)$$

$$\int \frac{dT_s}{T_a - T_b} = \int \frac{3hdt}{r\rho_s C_s}$$

$$\ln \left(\frac{T_a - T_b}{T_a - T_s}\right) = 3\left(\frac{kt}{R^2 \rho_s C_s}\right)$$

From Appendix 12, k_{air} = 0.0211 Btu/ft-h-°F = 0.0366 W/m°C.

$\rho_s \cong 2$ g/cm^3 = 2 x 10^6g/m^3

$c_s \cong 0.5$ cal/g-°C = 2.08 J/g-°C

$$\ln\left(\frac{275}{5}\right) = 4.01 = \frac{3t(0.0366)}{(2.5x10^{-6})^2(2x10^6)(2.08)} = 42.2t$$

t = 0.095 sec

The density and heat capacity of coal are only approximate, so the answer should be given as t ≅ 0.1 sec.

Chapter 11

11.1 Case 1: From Appendix 4:

x_w = 0.065 x 0.0254 = 0.00165 m

D_0 = 0.75 x 25.4 = 19.1 mm

D_i = 0.620 x 25.4 = 15.7 mm

$\bar{D}$ = (19.1 - 15.7)/ln(19.1/15.7) = 17.3 mm

From Eq. (11.32),

$$U_0 = \frac{1}{\frac{19.1}{12000x15.7} + \frac{0.00165x19.1}{120x17.3} + \frac{1}{14000}}$$

= 1/0.0001878 = 5320 W/m^2-°C

U_i may be calculated similarly from Eq. (11.33), but it is more easily found as follows:

$U_i = U_0D_0/D_i$ = 5320 x 19.1/15.7

= 6470 W/m^2-°C

Case 2:

X_w = 0.0035 m

From Appendix 12, k_{air} = 0.0211 Btu/ft-h-°F = 0.0366 W/m°C.

$\rho_s \cong 2$ g/cm^3 = 2 x 10^6g/m^3

$c_s \cong 0.5$ cal/g-°C = 2.08 J/g-°C

$$\ln\left(\frac{275}{5}\right) = 4.01 = \frac{3t(0.0366)}{(2.5x10^{-6})^2(2x10^6)(2.08)} = 42.2t$$

t = 0.095 sec

The density and heat capacity of coal are only approximate, so the answer should be given as t ≅ 0.1 sec.

Chapter 11

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$U_i = U_0D_0/D_i$ = 5320 x 19.1/15.7

= 6470 W/m^{2}-°C

Case 2:

X_w = 0.0035 m

D_0 = 0 $\bar{D}_L$ = 0.00213m

U_0 = –

(

= 1

U_i = 1

= 1

Case 3:

D_0 = 1

$\bar{D}_L$ =) in (Appendix 3)

3/12 = 0.0111 ft

U_0 = –

h

= 9

U_i = 98

= 1

11.2 Use Eq. (11

Case 1: ΔT_o

Δ

.3°C

Wall temper

T_{wc} =

Wall temper

ΔT_0 =

T_{wh} =

Case 2:

Normal b. pt. of benzene = 80.1°C

$\Delta T = 80.1 - 15 = 65.1°C$

$$\Delta T_i = \frac{65.1(0.025/0.018x20)}{1/14.2} = 64.2°C$$

$T_{wc} = 15 + 64.2 = 79.2°C$

$$\Delta T_0 = \frac{65.1(1/1200)}{1/14.2} = 0.77°C$$

$T_{wh} = 80.1 - 0.77 = 79.33°C$

Case 3: Steam temperature (Appendix 7) at 64.7 $lb_f/in.^2$ abs. Is 297.5°F

$\Delta T_{0verall} = 297.5 - 100 = 197.5°F$

$$\Delta T_i = \frac{197.5(1.315/(1.049x130))}{1/98.1} = 186.8°F$$

$T_{wc} = 100 + 186.8 = 286.8°F$

$$\Delta T_0 = \frac{197.5(1/14000)}{1/98.1} = 1.38°F$$

$T_{wh} = 297.5 - 1.38 = 296.12°F$

11.3 Let subscript $\underline{h}$ refer to aniline, subscript $\underline{c}$ refer to toluene. Assume average temperature of aniline is 175°F and that of toluene is 135°F.

From Appendix 15

$c_{ph} = 0.545$ Btu/lb°F $\quad$ $c_{pc} = 0.44$ Btu/lb°F

(a) From Eq. (11.6)

$Q = 10000 \times 0.545 \times (200-150)$

$= 272,500$ Btu/h

Also, $T_{cb} = \dfrac{q}{m_c c_{pc}} + T_{ca}$

$$= \frac{272500}{8600 x 0.44} + 100 = 172.0°F$$

From Eq. (11.15)

$$\bar{\Delta T} = \frac{(150-100)-(200-172)}{\ln \frac{50}{28}}$$

$$= 37.9°F$$

Overall coefficient must be, from Eq. (11.14),

$$U = \frac{272500}{70 x 37.9} = 102.7 \text{ Btu/h-ft}^2\text{-°F}$$

(b) Cannot cool 10000 lb/hr of aniline to 150°F using parallel flow since T_{cb} would have to be 172.0°F and it could not exceed 150°F even in an infinitely long parallel-flow exchanger.

11.4 Assume countercurrent flow, $\dot{m}_c = 8600$ lb/h

Find $\dot{m}_h$. From Eq. (11.6),

$$q = \dot{m}_h \times 0.545 \times (200-150) = 27.25\ \dot{m}_h$$

Also from Eq. (11.6)

$$q = 8600 \times 0.44\ (T_{cb} - 100) = 27.25\ \dot{m}_h$$

From this

$$T_{cb} = 0.0072\,\dot{m}_h + 100$$

From Eq. (11.15)

$$\bar{\Delta T_L} = \frac{50-(200-T_{cb})}{\ln\dfrac{50}{200-T_{cb}}}$$

$$= \frac{50-(100-0.0072\dot{m}_h)}{\ln\dfrac{50}{100-0.0072\dot{m}_h}}$$

By trial to satisfy equations (A) and (B), $\dot{m}_h$ is found to be 8172 lb/h. T_{cb} = 158.84°F, and $\bar{\Delta T_L}$ = 45.44°F.

Check: q = 8172 x 27.25 = 222,690 Btu/h

$$U = \frac{222{,}690}{70x45.44} = 70 \text{ Btu/h-ft}^2\text{-°F as given.}$$

11.5 For CCl_4, c_p = 0.20 cal/g-°C

= 0.20 x 4.1868 = 0.837 J/g-°C

(a) $\dot{m}$ = 19000 x 1000/3600 = 5277.8 g/s

q = mc_p (T_{ha} - T_{hb})

= 5277.8 x 0.837(85-40) = 198,788 W

For water, $\dot{m}$ = 13500 x 10000/3600 = 3750 g/s

If there is no heat loss, the exit water temperature is

T_{cb} = 20 + 198788/(4.1868 x 3750) = 32.7°C

Here 1/U = $1/h_0 + 1/h_i$

= 1/1700 + 1/11000 = 0.0006791

U = 1/0.0006791 = 1473 W/m^2-°C

ΔT_1 = 85 - 32.7 = 52.3°C

ΔT_2 = 40 - 20 = 20°C

$\Delta \bar{T}_L$ = (52.3-20)/ln(52.3/20) = 33.6°C

Area A = 198788/(1473 x 33.6) = 4.02 m^2

(b) With parallel flow, ΔT_1 = 85 - 20 = 65°C

ΔT_2 = 40 - 32.7 = 7.3°C

$\Delta \bar{T}_L$ = (65 - 7.3)/ln(65/7.3) = 26.3°C

Factor for increase in area: 33.6/26.3 = 1.28

11.6 From Appendix 3, D_i = 1.019 $\dot{m}$ D_0 = 1.315 $\dot{m}$

x_w = 0.133 $\dot{m}$ $\bar{D}_L$ = 1.115 $\dot{m}$

(a) U_i = 1840 W/m^2-°C

H_i = 4800 W/m^2-°C k_m = 45 W/m-°C

$$\frac{x_w}{k_m}\left(\frac{D_i}{D_L}\right) = \frac{0.133x1.049}{39.37x45x1.115} = 7.05x10^{-5}$$

From Eq. (11.33)

$$\frac{1}{h_0}\left(\frac{D_i}{D_L}\right) = \frac{1}{1840} - \frac{1}{4800} - 7.05x10^{-5} = 2.65x10^{-4}$$

$$h_0 = \frac{1}{2.65x10^{-4}} x \frac{1.049}{1.315} = 3010$$

% resistance in outside film: $\frac{1}{h_0}\left(\frac{D_i}{D_L}\right) x U_i x100 = 48.7\%$

in well: 7.05 x 10^{-5} x U_i x 100 = 48.7%

inside film: $\frac{U_i}{h_i} x100$ = 38.3%

(b) maximum error in h_0 is when U_i =1.1 x 1840 = 2024 and h_i = 0.9 x 4800 = 4320

$$\frac{1}{h_0}\left(\frac{D_i}{D_0}\right) = \frac{1}{2024} - \frac{1}{4320} - 7.05x10^{-5} = 1.92x10^{-4}$$

h_0 = 4154 (38% greater)

11.7 (a) From Appendix 7, λ = 945 Btu/lb

q_{steam} = 22,100 x 945 = 20,884,500 Btu/h

Flow rate of water at an average temperature of 33°C (91°F)

ρ = 62.1 lb/ft^3 (Appendix 6)

$\dot{m}$ = 870 x 60 x 0.13368 x 62.1 = 433,340 lb/h

q_{water} = 433,340(46-20)x1.8 = 20,280,300 Btu/h

q_{steam}/q_{water} = 1.03 or within 3%

(c) Outside area, from Appendix 4,

A_0 = 0.1963 x 8 x 120 = 188.5 ft^2

q_w = 20,582,000 Btu/h

ΔT_1 = 120 - 20 = 100°C or 180°F

ΔT_2 = 120 - 46 = 74°C or 133.2°F

$\overline{\Delta T_L}$ = (180-133.2)/ln(180/133.2)= 155.4°F

From Eq.(11.14),

U = 20,582,000/(188.5x155.4)

= 702 Btu/h-ft^2-°C (3,990 W/m^2-°C)

11.8 Must estimate the outlet temperature of one of the streams in parallel flow. Assumed constant are the water temperatures, flow rates, specific heats, U and A.

For countercurrent flow:

$$Q = UA = \dot{m}_L c_{ph}(120-30) = 90\,\dot{m}_L c_{ph}$$

$$(\Delta T_L) = \frac{(60-10)}{\ln\frac{60}{10}} = 27.9°C$$

$$UA = \frac{90\,\dot{m}_h c_{ph}}{27.9} = 3.226\,\dot{m}_h c_{ph}$$

$$\dot{m}_h c_{ph}(120-30) = \dot{m}_c c_{pc}(60-20)$$

For parallel flow, $120 - T_h = 2.25(T_c-20$, where T_h and T_c are the outlet temperatures:

$$\dot{m}_c c_{pc} = 2.25(T_h - 20)$$

$$T_h = 50.77°C$$

Try $T_h = 52°C$. $T_c - 20 = \frac{120-52}{2.25} = 30.2$

$$T_c = 50.2$$

$\Delta T_m = 120-20 = 100°C$ $\quad \Delta T_{out} = 1.8$ $\quad \Delta \bar{T}_L = 24.4$

$$q = (120-52)\,\dot{m}_h c_{ph} = 68\,\dot{m}_h c_{ph}$$

$$UA = \frac{68\,\dot{m}_h c_{ph}}{24.4} = 2.79\,\dot{m}_h c_{ph}$$ too small

With $T_h = 51°C$, $\Delta \bar{T}_L = 17.4°C$, $UA = 3.97\,\dot{m}_h c_{ph}$

Too large

Find $(\Delta\bar{T}_L)_2$ for Section II, from $\Delta T_b = 80 - T_2$,

$\Delta T_c = 10°C$

Then $A_1 = q_1/V_1(\Delta\bar{T}_L)_1$

$A_2 = q_2/V_2(\Delta\bar{T}_L)_2$

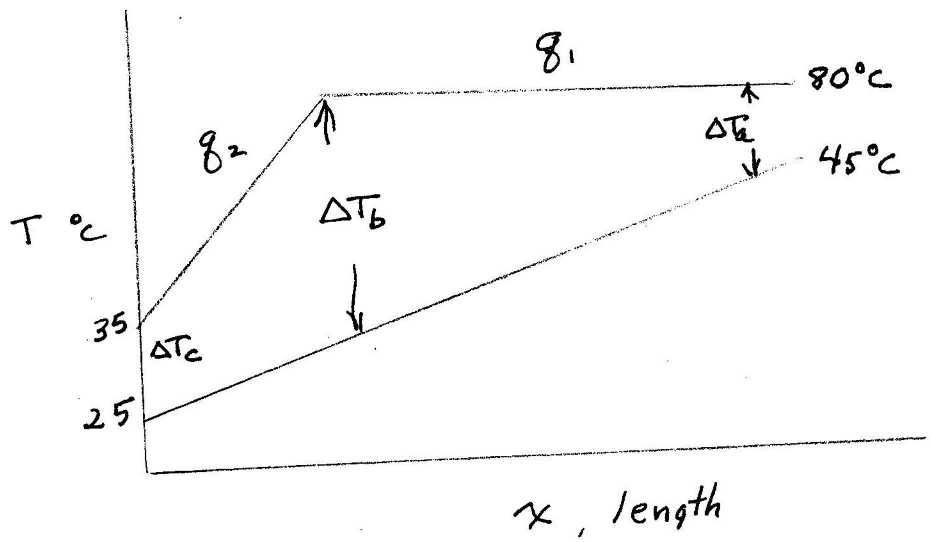
$A = A_1 + A_2$

With T_h = 51.5°C, $\overline{\Delta T_L}$ = 21.76, UA = $3.15\dot{m}_h c_{ph}$

Close enough

The hot stream is cooled to 51.5°C, cold stream heated to 50.4°C.

11.9



Countercurrent flow must be used. Let $\dot{m}$ be the condensation rate of vapor and $\dot{m}_w$ the flow rate of cooling water. Set $\dot{m}$, find $\dot{m}_w$ from q_1.

$$q_1 = \dot{m}\lambda = \dot{m}_w (45-T_2)$$

$$q_2 = \dot{m}_{cp} (80-35) = \dot{m}_w (T_2-25)$$

Solve for T_2.

Find $(\overline{\Delta T_L})_1$ for Section I, from ΔT_a = 80-45 = 35°C

$$\Delta T_b = (80-T_2)$$

12.1 Use Eqs. (12.24) and (12.27). The outlet temperature must be found by trial; assume it is 30°C. Then the average bulk temperature is (15+30)/2 = 22.5°C. Needed quantities are, from Appendixes 9,13, and 15:

μ = 900 cP $\qquad\qquad$ μ_w = 6.7 cP

k = 0.165 x 1.73073 = 0.286 W/m-°C

c_P = 0.575 x 4186.8 = 2407 J/kg-°C

$$Gz = \frac{700x2407}{3600x0.286x2.5} = 655$$

(a) From Eq. (12.27)

$N_u = 2 \times 655^{1/3}(900/6.7)^{0.14} = 34.49$

From Eq. (12.24)

$$\ln\frac{T_w - T_a}{T_w - \bar{T}_b} = 34.49\pi/655 = 0.1654$$

$$\frac{T_w - \bar{T}_b}{T_w - T_a} = \exp(-0.1654) = 0.848$$

$$\bar{T}_b = 115 - 0.848(115-15) = 30.2°C$$

This is close enough to the assumed value.

(b) For plug or rod-like flow, with G_Z = 655, Nu = 35 from Fig. 12.2. From Eq. (12.24)

$$\ln\frac{T_w - T_a}{T_w - \bar{T}_b} = 35\pi/655 = 0.1679$$

from which T_b = 115-0.845(115-15) = 30.5°C

(c) Set $\bar{T}_b$ = 114°C. Then

$$\ln \frac{115-15}{115-114} = 4.605 = \mathrm{Nu}\pi/G_z$$

Assume average Nu ≅ 4.0. Then

$$Gz = 4\pi/4.605 = 2.73 = \dot{m}c_p/kL$$

$$L = \frac{700x2407}{3600x0.286x2.73} = 600\text{m}$$

12.2 First establish the type of flow at the entrance. From Appendix 3,

$$\bar{V} = \frac{150}{3600x7.48x0.02330} = 0.239 \text{ ft/s}$$

$$\mathrm{Re} = \frac{2.067x0.239x0.79x62.3}{12x18x6.72x10^{-4}} = 167.5$$

Flow is laminar. Use Eqs. (12.27) and (12.75), with L = 60 ft. The exit temperature must be less than 120°F. Assume it is 78°F, giving an average oil temperature of 64°F. The estimated average properties are

μ = 17 cP　　　　c_p = 0.75 Btu/lb-°F

k = 0.072 Btu/h-ft-°F

ρ = 0.785 x 62.3 = 48.9 lb/ft^3

For substitution in Eq. (12.27):

$\dot{m}$ = 150 x 48.9/7.48 = 981 lb/h

G_z = (981 x 0.75)/(0.072 x 60) = 170.3

$G_z^{1/3}$ = 5.54

$\phi_v^{0.14}$ = $(17/8)^{0.14}$ = 1.111

From Eq. (12.27), since Nu = h_iD_i/k,

$$h_i = \frac{2x12x0.072x5.54x1.111}{2.067}$$

$$= 5.15 \text{ Btu/h-ft}^2\text{-°F}$$

From Eq. (12.70), since $\bar{\rho}_a = (0.79 + 0.74)/2 =$ 0.765 g/cm^3,

$$\beta = \frac{0.79 - 0.74}{0.765(120 - 60)} = 0.00109$$

$$\Delta T_i = 120 - 64 = 56°F$$

$$Gr = \frac{(2.067)^3 x32.174x48.9^2 x0.00109x56}{12^3 x(17x6.72x10^{-4})^2}$$

$$= 183{,}920$$

$$Re = \frac{4x987x12}{2.067\pi x17x2.42} = 177$$

From Eq. (12.75)

$$\phi_n = \frac{2.25(1 + 0.10x183920^{1/3})}{\log 177}$$

$$= 1.570$$

Corrected coefficient:

$$h_i = 5.15 \text{ x } 1.570 = 8.08 \text{ Btu/h-ft}^2\text{-°F}$$

Let $\bar{T}_b$ be the outlet oil temperature. Area is

$$A = \pi \text{ x } (2.067/12) \text{ x } 60 = 32.5 \text{ ft}^2$$

$$q = 987 \text{ x } 0.75(\bar{T}_b - 60)$$

$$= 740.3\bar{T}_b - 44415 \text{ Btu/h}$$

$$\Delta \bar{T}_L = \frac{60-(120-\bar{T}_b)}{\ln(60/(120-\bar{T}_b))} = \frac{\bar{T}_b - 60}{\ln(60/(120-\bar{T}_b))}$$

Also, $q = h_i A \Delta \bar{T}_L$, so

$$740.3\ T_b - 44415 = 8.08 \times 32.5 \Delta \bar{T}_L$$

$$= 262.6(\bar{T}_b - 60)/\ln(60/(120-T_b))$$

By trial $\bar{T}_b$ = 77.9°F, and the average oil temperature is 64.0°F. These are close enough to the assumed values that a second trial is not needed.

12.3 Check type of flow.

$\mu = 2.1 \times 10^{-3}$ kg/m-s $\quad D_i = 0.075$m

$Re = 0.075 \times 880/2.1 \times 10^{-3} = 31430$

Since flow is turbulent, combine Eqs. (12.49) and (12.52), neglecting viscosity correction, to give

$h_i = 0.023Re^{-0.2}Pr^{-2/3}c_pG$

$G = 1 \times 880 = 880$ kg/m^2-s

$C_p = 2.17 \times 10^3$ J/kg-°C

$Pr = 2.17 \times 10^3 \times 2.1 \times 10^{-3}/0.135 = 33.8$

$$h_i = \frac{0.023 x 2.17 x 10^3 x 880}{31430^{0.2} x 33.8^{2/3}}$$

$= 530$ W/m^2-°C

Use Eq. (11.33); neglect fouling factors.

For the steel wall, $k_m = 45$ W/m-°C

From Appendix 3, $x_w = 0.216/39.37 = 0.0055$ m

$D_0 = 0.075 + (2 \times 0.0055) = 0.086$ m (approx)

D_L = (0.086 - 0.075)/ln(0.086/0.075)

= 0.0804 m

From Eq. (11.33)

$$U_i = \frac{1}{\dfrac{1}{530} + \dfrac{0.0055x0.075}{45x0.0804} + \dfrac{0.075}{11000x0.086}}$$

= 481 W/m^2-°C

<u>Second part</u> From Eqs. (11.9) and (11.10)

$dq/dA_o = U_o\Delta T = U_i D_i \Delta T/D_o$

= 481 x 0.075(120-50)/0.086

= 29400 W/m^2

12.4 Use Eq. (11.32). h_o = 300 Btu/h-ft^2-°F

For steel, k = 26 Btu/h-ft-°F (Appendix 10)

Tube dimensions (Appendix 4)

X_w = 0.065/12 = 0.0054 ft

D_0 = 0.75/12 = 0.0625 ft

D_i = 0.0620/12 = 0.0517 ft

$\bar{D}_L$ = (0.0625 - 0.0517)/ln(0.0625/0.0517)

= 0.0569 ft

For the inside ocefficient:

ρ = 0.805 x 62.3 = 50.2 lb/ft^3

c_p = 0.583 Btu/lb-°F

k = 0.0875 Btu/h-ft-°F

μ = 1.5 x 2.42 = 3.63 lb/ft-h

$\bar{V} = 8 ft/s$

G = 8 x 50.2 x 3600 = 1.446 x 10^6 lb/h-ft^2

Use Eqs. (12.49) and (12.52).

$Re = 0.0517 \times 1.446 \times 10^6/3.63$

$= 20590$

$Pr = 0.583 \times 3.63/0.0875 = 24.19$

$$hi = \frac{0.023x0.583x1.446x10^6}{20590^{0.2}x24.19^{2/3}}$$

$= 318$ Btu/h-ft^2-°F

From Eq. (11.32)

$$U_0 = \frac{1}{\dfrac{0.0625}{318x0.0517} + \dfrac{0.0054x0.0625}{26x0.0569} + \dfrac{1}{300}}$$

$= 1/(0.0003802 + 0.000228 + 0.003333)$

$= 135.8$ Btu/h-ft^2-°F

12.5 For water at 110°F, from Appendix 6,

$\mu = 0.62$ cP or 1.50 lb/ft-h

$k = 0.368$ Btu/h-ft-°F

$\rho = 61.85$ lb/ft^3

From Appendix 15, $c_p = 1.0$ Btu/lb-°F

$Pr = 1.0 \times 1.50/0.368 = 4.08$

$G = 8 \times 61.85 \times 3600 = 1.781 \times 10^6$ lb/h-ft^2

$Re = 0.0517 \times 1.781 \times 10^6/1.50$

$= 6.14 \times 10^4$

From Eqs. (12.49) and (12.52), neglecting the viscosity correction,

$$hi = \frac{0.023x1.0x1.781x10^6}{61400^{0.2}x4.08^{2/3}}$$

$= 1769$ Btu/h-ft^2-°F

From Eq. (11.32)

$$U_0 = \frac{1}{\frac{0.0625}{1769x0.0517} + 0.000228 + 0.003333}$$

$= 1/0.004244 = 235.6$ Btu/h-ft^2-°F

Increase: (235.6-135.8)/135.8 = 0.735 or 73.5%

12.6 Use Eq. (11.37). Add the fouling resistances to the denominator of the expression for U_0 in Problem 12.5. Outside: 1/200 = 0.005000

Inside: 0.0625/(0.0517 x 330) = 0.003663. Then

$U_0 = 1/(0.004244 + 0.005000 + 0.003663)$

$= 77.5$ Btu/h-ft^2-°F

Decrease: (235.6 - 77.5)/235.6 = 0.671 or 67.1%

12.7 Let subscript s refer to the steel pipe and c to the copper tube. The Colburn analogy (Eq.(12.52)) relates coefficient h with friction factor f. The other factors in Eq. (12.52) are all the same except G which differs somewhat because of the difference in diameters.

$D_{i,s} = 1.049$ in. (Appendix 3)

$D_{i,c} = 0.870$ in. (Appendix 4)

Since the Reynolds number is the same in both cases, $D_{i,s}G_s = D_{i,c}G_c$

$G_s/G_c = 0.870/1.049 = 0.829$

Find friction factors from Fig. 5.10. At Re = 4 x 10^4, for smooth tubing, f = 0.0054. For steel pipe, k/D = 0.00015/(1.049/12) = 0.0017, and f_s = 0.0067. From Eq. (12.52), written for both cases,

$$\frac{h_s}{h_c} = \frac{f_s G_s}{f_c G_c} = \frac{0.0067}{0.0054} x 0.829 = 1.029$$

Increase is 0.029 or 2.9%. The actual effect of pipe roughness is much less than indicated by the ratio of the friction factors, and h_s would in fact be less than h_c, not slightly greater.

12.8 Find the heat flow. $\dot{m}$ = 3500/3600 = 0.972 kg/s

q = 0.972 x 4190(50-15) = 142,540 W

$$\Delta \bar{T}_L = \frac{(110-15)-(110-50)}{\ln 95/60} = 76.2°C$$

q = 142,540 = $\pi D_i L U_i \Delta \bar{T}_L$

$D_i L U_i$ = 142540/(π x 7.62) = 595.4 (A)

G = $\dot{m}/(\pi D_i^2/4)$ = 0.972 x $4/\pi D_i^2$

= $1.238/D_i^2$

Estimate T_w = 93°C (not a critical decision)

μ_w = 0.305 cP

$(\mu/\mu_w)^{0.14} = (0.78/0.305)^{0.14} = 1.14$

From Eq.(A) above, $U_i = 595.4/D_i L$. Since the wall is thin and h_0 = 11000 W/m^2-°C,

$$U_i = \frac{1}{1/h_i + 1/h_o} = \frac{1}{1/h_i + 0.000091}$$

Hence $D_i L/595.4 = (1/h_i) + 0.000091$

$$L = \frac{595.4}{D_i}\left(\frac{1}{h_i} + 0.000091\right) \quad \text{(B)}$$

Assume D_i/L is small. From Eq. (12.34)

$$h_i = \frac{0.023 c_p G(\mu/\mu_w)^{0.14}}{\Pr^{2/3}(D_i G/\mu)^{0.2}}$$

$\Pr = 4187 \times 0.78 \times 10^{-3}/0.61 = 5.354$

$D_i G/\mu = D_i \times (1.238/D_i^2)/(0.78 \times 10^{-3})$

$= 1587/D_i$

$$h_i = \frac{0.023 x 4187 x (1.238/D_i^2) x 1.14}{5.354^{2/3} x (1587/D_i)^{0.2}}$$

$= 10.17/D_i^{1.8}$

Substituting for h_i om Eq. (B) above:

$L = (595.4/D_i)((D_i^{1.8}/10.17) + 0.000091)$

$= 58.54 D_i^{0.8} + 0.05418 D_i^{-1}$

Differentiate and set the derivative equal to zero to find minimum L.

$dL/dD_i = 46.83 D_i^{-0.2} - 0.05418 D_i^{-2} = 0$

From this $D_i = (1/864.34)^{-1.8} = 0.0234$ m

$L = 5.54 \times 0.0234^{0.8} + (0.05418/0.0234)$

$= 5.22$ m

Check: $D_i/L = 0.0234/5.22 = 0.045$. This is very small, so Eq. (12.34) applies.

12.9 (a) Neglecting the viscosity ratio term, for a given pipe E.(12.34) shows:

$$h_i \;\alpha\; c_p G\left(\frac{k}{c_p \mu}\right)^{2/3}\left(\frac{\mu}{G}\right)^{0.2}$$

at constant G: $h_i \; \alpha \; c_p^{1/3} k^{2/3} \mu^{-0.47}$

The values of c_p, k and μ increase with temperature. Data for air at 1 atm are given below; they can be fitted to an exponential dependence on the absolute temperature, and exponents for two temperature ranges are shown.

T		c_p J/g-°C	Exponent	μ, cP	Exponent
°C	°K				
0	273	1.001		0.169	
500	773	1.093	0.083	0.035	0.70
1000	1273	1.185	0.163	0.049	0.67

T		k W/m-°C	Exponent	Pr
°C	°K			
0	273	0.0249		0.705
500	773	0.0560	0.81	0.683
1000	1273	0.0791	0.69	0.734

For 0° to 500°C, $h \; \alpha \; T^n$ where

$$n = 0.083(1/3) - 0.70(0.47) + 0.81(2.3) = 0.239$$

For 500° to 1000°C,

$$n = 0.165(1/3) - 0.67(0.47) + 0.69(2/3) = 0.20$$

Thus h_i increases with about the 0.22 power of the absolute temperature. Nearly the same result is obtained by neglecting the small changes in Pr and saying that $h_i \alpha c_p\mu^{0.2}$, or

$$n = 0.083 + 0.2(0.70) = 0.223 \text{ (0-500°C)}$$

$$n = 0.163 + 0.2(0.67) = 0.297 \text{ (500-1000°C)}$$

Thus h_i does increase with viscosity <u>at constant</u> Pr, but this means that k and c_p are increasing also. If another gas or gas mixture had the same k and c_p as air but a different μ, h_i would vary with $\mu^{-0.47}$.

(b) If $\bar{V}$ is kept constant, G decreases with temperature. For an ideal gas $\rho \alpha 1/T$; hence $G \alpha /T$

Since $h_i \alpha G^{0.8}T^{0.24}$

$h_i \alpha T^{-0.56}$

NOTE: For more complex molecules than N_2 and O_2, c_p increases more rapidly with temperature and the exponent on T is greater than 0.24.

12.10 From Eq. (12.37)

$$h_i \alpha \frac{k^{2/3}G^{0.8}c_p^{\ 1/3}}{D^{0.2}\mu^{0.47}}$$

Also: $q\sqrt{\Delta T_L} = h_i A$

(a) No change. G is constant, A is constant.

(b) $\dfrac{h_{i2}A}{h_{i1}A} = 2^{0.8} = 1.741$ 74% increase in $q\sqrt{\Delta T_L}$

(c) When number of tubes is doubled, G is cut in half, and A is doubled.

$\dfrac{h_{i2}A_2}{h_{i1}A_1} = \dfrac{2}{2^{0.8}} = 1.149$. 14.9% increase in $q\sqrt{\Delta T_L}$

(d) When diameter is cut in half, G is quadrupled, and A is halved.

$\dfrac{h_{i2}A_2}{h_{i1}A_1} = \dfrac{4^{0.8}}{0.5^{0.2}x2} = 1.741$ 74% increase in $q\sqrt{\Delta T_L}$

12.11 Use Eq. (12.60)

$G = \bar{V}\rho = 32 \times 3600 \times 45 = 5.184 \times 10^6$ lb/ft^2-h

$Re = \dfrac{0.5x5.184x10^6}{12x0.16x2.42} = 5.58 \times 10^5$

From Fig. 12.4 $(\varepsilon_M/\nu)_m = 1300$

$Pe = Gc_pD/k$

$= 5.184 \times 10^6 \times 0.21 \times 0.5/(12 \times 179)$

$= 253.4$

From Eq. (12.60)

$Nu = 7.0 + 0.025 \times (253.4 - 1.82 \times 5.58 \times 10^5/1300^{1.4}))^{0.8}$

$= 8.79$

$h_i = 8.79 \times 179 \times 12/0.5$

$= 37760$ Btu/h-ft^2-°F. For one tube

$\dot{m} = GS = 5.184 \times 10^6 \times (\pi/4) \times (0.5/12)^2$

$= 7069$ lb/h

$A \quad = \pi DL = 3 \times \pi \times 0.5/12 = 0.3927\ ft^2$

$q \quad = \dot{m} c_p\ (T_{cb} - T_{ca}) = h_i A \Delta \bar{T}_L$

$7069 \times 0.21(T_{cb} - 580) = 37760 \times 0.3927 \Delta \bar{T}_L$

From this, T_{cb} = 719.99°F. The temperature rise is 719.99 - 580 = 139.99°F. The energy gain is 139.99 x 0.21 = 29.4 Btu/lb.

12.12 Properties of air at 1 atm, 700°C

$\rho = 29 \times 273/(22.4 \times 973) = 0.363\ kg/m^3$

$\mu = 4.1 \times 10^{-5}$ kg/m-s (Appendix 8)

$k = 0.0654$ W/m-°C (Perry, 6th ed., p. 3-254)

$c_p = 0.27 \times 4187 = 1130$ J/kg-°C (Appendix 14)

$Pr = 1130 \times 4.1 \times 10^{-5}/0.0654 = 0.708$

$D_p = 5 \times 10^{-5}$ m. Assume $\rho_p - \rho = 960\ kg/m^3$

If Re < 1, from Stokes' law (Eq. (7.40)):

$U_t = g D_p^2 (\rho_p - \rho)/18\mu$

$$= \frac{9.80665 x (5 x 10^{-5})^2 x 960}{(18 x 4.1 x 10^{-5})} = 0.0319\ m/s$$

$Re_p \quad = 5 \times 10^{-5} \times 0.0319 \times 0.363/4.1 \times 10^{-5}$

$= 0.014$

Hence Eq. (7.40) applies. From Eq. (12.64)

$h_0 D_p/k = 2 + 0.6(0.014)^{0.5}(0.708)^{1/3} = 2.063$

$h_0 = 2.063 \times 0.0654/(5 \times 10^{-5})$

$= 2698\ W/m^2\text{-}°C$

By a heat balance on a solid particle, over time interval dt,

$\rho_s(\pi D_p^3/6)c_s(dT_s) = h_0\pi D_p^2(T_g-T_s)dt$. Then

$$\int_{T_s}^{T_{so}} \frac{dT_s}{T_g - T_s} = \ln\frac{T_g - T_{so}}{T_g - T_s} = \frac{6h_0 t}{D_p\rho_s c_s}$$

where T_{so} is the original temperature of the particle. For silica at 700°C, c_s = 1090 J/kg-°C. Use this value for the catalyst. Then

$$t = \frac{5x10^{-5}x960x1090}{6x2698}\ln\frac{700-600}{700-695}$$

$$= 0.003232 \ln 20 = 0.0097\text{s}$$

12.13 a) For laminar flow of oil in the pipe, the inside coefficient will be quite low compared to the outside coefficient, since the water will probably be in turbulent flow. Therefore assume U $\cong h_i$ and base the analysis on changes in h_i and A_i. Since the change in water temperature is small, use the solution for constant wall temperature. For laminar flow, the fractional approach to the wall temperature depends on G_z, as shown in Eq. (12.18).

From Eq. (12.13),

$$G_z = \frac{\pi\rho V c_p D^2}{4kL} = \dot{m}\, c_p/kL$$

Since G_z does not depend on D for a given mass flow rate, increasing D <u>does not change the outlet temperature of the oil.</u>

This can also be seen from Fig. 12.2, which shows Nu as a function of G_z. If Nu = hD/k is constant, increasing D lowers h so that the increase in A is cancelled by the decrease in h.

(b) For turbulent flow of oil $h_i \, \alpha \, G^{0.8}/D^{0.2}$. Since

$G \, \alpha \, \dot{m}/D^2$, $h_i \, \alpha \, (1/D^{1.6})(1/D^{0.2}) = D^{-1.8}$.

The decrease in h_i will more than offset the increase in A_i, and cooling will be less effective with the larger pipe.

To find the new outlet temperature, use the data For the 1-inch pipe. Here

$$q = \dot{m} c_p (200-110) = 90 \dot{m} c_p$$

$$\Delta \bar{T}_L = (170-80) \ln(170/80) = 119°C$$

$$U \cong h = q/A \Delta \bar{T}_L = 90 \dot{m} c_p / 119 \pi DL$$

$$UA = 0.756 \; \dot{m} c_p$$

If D is increased by 1.5, UA becomes

$$(UA)_{new} = 1.5 \times 0.756 \; \dot{m} c_p / 1.5^{1.8}$$

$$= 0.547 \; \dot{m} c_p$$

$$q_{new} = \dot{m} c_p (200 - T_b) = (UA)_{new} \Delta \bar{T}_L$$

$$= 0.547 \dot{m} c_p \Delta \bar{T}_L$$

where T_b is the new oil outlet temperature.

$$200 - T_b = \frac{0.547((200-30)-(T_b-30))}{\ln(170/(T_b-30))}$$

from which $T_b = 128.4°C$

12.14. Use the properties of air at 500°C

ρ = 29 x 273/(22.4 x 773) = 0.457 kg/m^3

μ = 3.6 x 10^{-5} kg/m-s (Appendix 8)

k = 0.0569 W/m-°C

c_p = 0.265 x 4187 = 1110 J/kg-°C

Pr = 1110 x 3.6 x 10^{-5}/0.0569 = 0.702

u_0 = 20/3.281 = 6.10 m/s

Re = 0.5 x 10^{-3} x 6.10 x 0.457/(3.6 x 10^{-5})

= 38.7

(a) Use Eq. (12.63) for flow normal to a cylinder. (Coefficients for a screen will be slightly higher than for single wires because of the increase in the actual mass velocity.)

$$Nu \times 0.702^{-0.3} = 0.35 + 0.56(38.7)^{0.52}$$

$$= 4.098 \times 0.8993 = 3.685$$

$$h = 3.685\ k/D = 3.685 \times 0.0569/(0.5 \times 10^{-3})$$

$$= 419\ W/m^2\text{-}°C$$

(b) Mass velocity:

$$G_0 = u_0\rho = 6.10 \times 0.457 = 2.79\ kg/m^2\text{-}s$$

Let T be the final air temperature and T_w the surface temperature of the wire. Assume plug flow. Then

$$G_0 c_p dT = h\ dA(T_w - T)$$

$$\frac{T_w - T_0}{T_w - T} = \exp \frac{419x3.7}{1110x2.79} = 1.650$$

$$T = 900 - (900-500)/1.650 = 658°C$$

12.15 Use Eqs. (12.63) and (12.64). For these:

$D_o = 0.025\text{m}$ $\quad T_f = (120 + 15)/2 = 67.5°\text{C}$

At 15°C, $\rho = 62.38 \times 16.018 = 999.2\ \text{kg/m}^3$

$\bar{V} = 1\ \text{m/s}$ $\quad G = 999.2\ \text{kg/m}^2\text{-s}$

At 67.5°C (153.5°F), from Appendix 6,

$\mu_f = 0.423\ \text{cP} = 4.23 \times 10^{-4}\ \text{kg/m-s}$

$\rho_f = 61.12 \times 16.018 = 979.0\ \text{kg/m}^3$

$k_f = 0.382 \times 1.73073 = 0.661\ \text{W/m-°C}$

$c_p = 4187\ \text{J/kg-°C}$

$D_0G/\mu_f = 0.025 \times 999.2/(4.23 \times 10^{-4})$

$= 59050$

$c_p\mu_f/k_f = 4187 \times 4.23 \times 10^{-4}/0.661 = 2.679$

(a) From Eq. (12.63)

$$h_o = \frac{0.661x2.679^{0.3}[0.35 + 0.56(59050)^{0.52}]}{0.025}$$

$= 6036\ \text{W/m}^2\text{-°C}$

$q/A = hT = 6036(120-15)/1000 = 633.8\ \text{kW/m}^2$

(b) From Eq. (12.64)

$$h_o = \frac{0.661[2 + (0.6x59050^{0.5}x2.679^{1/3})]}{0.025}$$

$= 5407\ \text{W/m}^2\text{-°C}$

$q/A = 5407(120-15)/1000 = 567.7\ \text{kW/m}^2$

12.16 Use Eq. (12.75). Average water temperature is

$\bar{T} = (15 + 65)/2 = 40°\text{C}$

$\overline{\Delta T} = 120 - 40 = 80°\text{C}$

$T_f = 40 + (80/2) = 80°\text{C}$

ρ at 40°C = 61.94 x 16.018 = 992.2 kg/m^3

ρ_w at 120°C = 58.86 x 16.018 = 942.8 kg/m^3

$\bar{\rho}$ = (992.2 + 942.8)/2 = 967.5 kg/m^3

ρ_f at 80°C = 60.66 x 16.018 = 971.7 kg/m^3

From Eq. (12.70)

β = (992.2 - 942.8)/(967.5 x 80)

= 6.38 x 10^{-4} °C^{-1}

μ_f = 3.58 x 10^{-4} kg/m-s $\qquad$ D_i = 0.050 m

$g\beta\Delta T$ = 9.80665 x 6.38 x 10^{-4} x 80 = 0.5005

$$Gr_f = \frac{0.050^3 x 971.7^2 x 0.5005}{(3.58 x 10^{-4})^2}$$

= 4.61 x 10^8

Re = 450

From Eq. (12.75)

ϕ_n = 2.25[1 + (0.01(4.61 x 10^8)$^{1/3}$)]/log 450

= 740 or a 640% increase

This compares with the 34% increase when heating air, as shown in Example 12.4.

12.17 Steam temperature = 212°CF

Assume outside wall temperature = 190°F

ΔT_0 = 190 - 80 = 110°F

T_f = (190+80)/2 = 135°F

From Appendix 6:

μ_f = 0.492 x 2.42 = 1.19 lb/ft-h

k_f = 0.376 Btu/h-ft-°F

ρ_f = 61.46 lb/ft^3

ρ at 80°F = 62.22 lb/ft^3

ρ at 180°F = 60.58 lb/ft^3

From Eq. (12.70)

β_f = (42.22 - 60.58)/(61.40 x 100) = 2.67 x 10^{-4}/°F

D_o = 3.50/12 = 0.292 ft

Pr_f = 1.0 x 1.19/0.376 = 3.165

$$Gr_f = \frac{0.292^3 x 61.46^2 x 2.67 x 10^{-4} x 4.17 x 10^8 x 110}{1.19^2}$$

= 8.13 x 10^8

From Eq. (12.72)

$Nu = 0.53(\ 8.13 \times 10^8 \times 3.165)^{0.25}$

= 119.4

$\bar{h}$ = 0.376 x 119.4/0.292 = 153.7 Btu/h-ft^2-°F

From Appendix 3, area per foot of length:

A_0 = 0.916 ft^2/ft

q = 153.7 x 110 x 0.916 = 15,490 Btu/h-ft

Check on wall temperature. Assume steam coefficient is 2000 Btu/h-ft^2-°F = h_i

From Eq. (11.32)

$$U_0 = \frac{1}{\dfrac{3.50}{3.068 x 2000} + \dfrac{0.216 x 3.50}{12 x 26 x 3.297} + \dfrac{1}{153.7}}$$

= 1/(0.00057 + 0.00073 + 0.00650)

= 1/0.00780 = 128.2 Btu/h-ft^2-°F,

T_w = 212 - (212-80) x 0.00130/0.00780 = 190°F

12.18 For the glass pane, x_g = 1/(8 x 12) = 0.0104 ft

$k_g/x_g = 0.4/0.0104 = 38.46$ Btu/h-ft^2-°F

This is so much larger than the probable values of the individual coefficients h_i and h_0 that the temperature drop in the glass will be very small. Coefficients h_i and h_o will be approximately equal; hence the temperature of both glass surfaces will be about 35°F. Length L = 4ft. Film temperatures:

$T_{fi} = (70 + 35)/2 = 52.5$°F or 512.5°R

$T_{fo} = 35/2 = 17.5$°F or 477.5°R

Film properties:

$k_{fi} = 0.0151$ Btu/h-ft-°F (Appendix 12)

$k_{fo} = 0.0143$ Bt/h-ft-°F

$c_{pi} = c_{po} = 0.25$ Btu/lb-°F

$\rho_{fi} = 29 \times 512.5/(359 \times 492) = 0.0841$ lb/ft^3

$\rho_{fo} = 29 \times 477.5/(359 \times 492) = 0.0784$ lb/ft^3

$\beta_i = 1/512.5$ $\qquad$ $\beta_o = 1/512.5$

$\mu_{fi} = 0.018$ cP $\qquad$ $\mu_{fo} = 0.017$ cP (Appendix 8)

$Pr_{fi} = 0.25 \times 0.018 \times 2.42/0.0151 = 0.721$

$Pr_{fo} = 0.25 \times 0.017 \times 2.42/0.0143 = 0.719$

$$Gr_i = \frac{4^3 x 0.0841^2 x 4.17 x 10^8 \Delta T_i}{512.5 x (0.018 x 2.42)^2}$$

$$= 1.94 \times 10^8 \Delta T_i$$

$$Gr_o = \frac{4^3 x 0.0784^2 x 4.17 x 10^8 \Delta T_o}{477.5 x (0.017 x 2.42)^2}$$

$$= 2.02 \times 10^8 \Delta T_o$$

Since ΔT will be about 35°F,

$$(GrPr)_i = 1.94 \times 10^8 \times 0.721 \times 35$$
$$= 4.9 \times 10^9$$

Use Eq. (12.74). From Table 12.5, b = 0.13 and n = 0.333. From Eq. (12.74)

$$h_i = \frac{0.13x0.0151}{4}\left(1.94x10^8 x0.721\right)^{0.333} x\Delta T_i^{0.333}$$

$$= 0.255\Delta T_i^{0.333}$$

$$h_o = \frac{0.13x0.0143}{4}\left(2.02x10^8 x0.719\right)^{0.333} x\Delta T_o^{0.333}$$

$$= 0.244\Delta T_o^{0.333}$$

Thus the two coefficients are nearly equal.

Also,

$$q/A = h_i(70-T_i) = h_0(T_o-0)$$

If $\Delta T_i = \Delta T_o = 35°F$, as assumed,

$$q/A = 0.244\Delta T_o^{1.333}$$
$$= 27.90 \text{ Btu/h-ft}^2$$
$$U = 27.90/70 = 0.40 \text{ Btu/h-ft}^2\text{-°F}$$

This is probably a good estimate of U. The temperature drop ΔT_g through the glass is

$$\Delta T_g = qx_g/Ak_g$$
$$= 27.90/38.46 = 0.73°F$$

Actual surface temperatures (assuming $h_0=h_i$)

$T_i = 35.365°F$ $\qquad$ $T_o = 34.635°F$

(c) The Thermopane window consists of two glass panes, each 1/8 inch thick, separated by a layer of air (k_{air} = 0.0145 Btu/h-ft-°F). If there is no convection in the air space, the resistance of the composite window is $2x_g/k_g + x_{air}/k_{air}$.

$x_{air} = 1/(2 \times 12) = 0.04167$ ft

$h_o = h_i = 0.244(34.635 - 0)^{0.333}$

$= 0.794$ Btu/h-ft^2-°F

Hence

$$\frac{1}{U} = \frac{2}{0.794} + \frac{2x0.0104}{0.4} + \frac{0.04167}{0.0145}$$

$U = 1/5.445 = 0.184$ Btu/h-ft^2-°F

If natural convection does occur, a rough estimate may be made by assuming that the film coefficients on each side of the ½-inch gap can be evaluated from Eq. (12.74). From above, h = 0.794 Btu/h-ft^2-°F. For a ΔT of about 17°F

$h = 0.794(17/35)^{0.333} = 0.624$ Btu/h-ft^2-°F

With four gas films in series, each with the same h, plus two thicknesses of glass,

$1/U = (4/0.624) + (2/38.46) = 6.46$

$U = 0.155$ Btu/h-ft^2-°F

For stagnant air, U = 0.184 Btu/h-ft^2-°F. Thus there can be no convection, since the resistance of the gap cannot exceed that of stagnant air.

12.19 As long as there is no convection in the air space,

$$\frac{1}{U} = \frac{2}{0.794} + \frac{2x0.0104}{0.4} + \frac{x_{air}}{0.0145}$$

$= 2.561 + 68.97x_{air}$

$U = 1/(2.561 = 68.97x_{air})$

If x_g = 1 inch, U = 0.12 Btu/h-ft^2-°F, but allowing for natural convection as in Prob. 12.18 gives U = 0.15 Btu/h-ft^2-°F. Here natural convection does exist and increases U. Other calculations give the following:

Spacing in.	Overall coefficient U Btu/h-ft^2-°F	Convection active?
0.0	0.40	No
0.25	0.24	No
0.5	0.184	No
1.0	0.15	Yes

Chapter 13

13.1 (a) Use Fig 13.2 The quantities needed are

T_w = 25°CT_h = 78.4°C (Perry, 7th ed., p. 2-37)

ΔT_o = 78.4 - 25 = 53.4°C

From Eq. (13.11),

T_f = 78.4 - 0.75 x 53.4 = 38.4°C

μ_f = 0.85 cP = 8.5 x 10^{-4} kg/m-s (Appendix 6)

k_f = 0.105 x 1.73073 = 0.182 W/m-°C (Appendix 13)

λ = 204 x 4184 = 8.54 x 10^5 J/kg (Perry, 6th ed., p. 3-126)

ρ_f = 0.773 x 998 = 771 kg/m^3 (Perry 7th ed., p. 2-43)

c_p = 0.63 x 4.184 = 2.64 J/g-°C (Appendix 16)

$$Pr = \frac{2.64x10^3 x8.5x10^{-4}}{0.182} = 12.3$$

Assume Re = 1000. From Fig 13.2, Nu$'$ = 0.30

$$\frac{h}{k}\left(\left(\frac{\mu_f}{\rho_f}\right)^2 \frac{1}{g}\right)^{1/3} = 0.30$$

$$\frac{h}{0.182}\left(\left(\frac{8.5}{7.71}x10^{-6}\right)^2 \frac{1}{9.80665}\right)^{1/3} = 0.30$$

h = 0.30 x 0.182/(0.1234 x 10^{-12})$^{1/3}$

= 1097 W/m-°C

$$\dot{m}_T = q/\lambda = hA\Delta T_o/\lambda$$

= 1097 x π x 0.0254 x 3 x 53.4 x 3600 / 8.54 x 10^5

= 59.1 kg/h

$Re = 4\Gamma/\mu_f = 4\dot{m}_T/\pi D_o \mu_f$

$= 4 \times 59.1 / (\pi \times 0.0254 \times 8.5 \times 10^{-4} \times 3600) = 956$

From Fig. 13.2, Nw′ = 0.30. So $\dot{m}_T = 59.1$ kg/h

(b) Use Eq. 13.15

$$h = 0.729\left(\frac{0.182^3 x 771^2 x 9.80665 x 8.54 x 10^5}{53.4 x 0.0254 x 8.5 x 10^{-4}}\right)^{1/4}$$

$= 0.729(26.036 \times 10^{12})^{1/4} = 1646$ W/m^2-°C

$\dot{m}_T = 59.1 \times (1646/1097) = 88.7$ kg/h

13.2 $D_o = 0.031$ $L_p = \pi D_o = 0.0974$m

$q = 2100 \times 856 \times 1000/3600 = 499{,}300$ W

$T_h = 78.4$°C $\Delta T_o = 78.4 - 30 = 48.4$°C

Let N = number of tubes

$\Gamma = 2100 / 0.0974N = 21560/N$ kg/h-m

$A_o = 3 \times 0.0974N = 0.2922N$

From Eq. (11.32)

$$U_o = \frac{1}{\frac{1}{h_o} + \frac{31}{27 x 2800}} = \frac{1}{\frac{1}{h_o} + 0.00041}$$

Also, $U_o = q/A_o \Delta T_o = 499300/(0.2922N \times 48.4)$

$= 35305/N$

Assume outside wall temperature $T_w = 45$°C.

From Eq. (13.11)

$T_f = 78.4 - 0.75(78.4-45) = 53.4$°C

$k_f = 0.105 \times 1.731 = 0.182$ W/m-°C (Appendix 13)

$\mu_f = 0.70 \times 10^{-3}$ kg/m-s (Appendix 5)

Use Fig 13.2. Guess Re = 500

As in Problem 13.1, Pr = 12.3

At Re = 500, Nu′ = 0.32. Hence,

$$\frac{h_o}{0.182}\left(\frac{7.0x10^{-4}}{7.69x10^{2}x9.80665}\right)^{1/3} = 0.32$$

$h_o = 1327$

$$U_o = \frac{1}{(1/1327)+0.00041} = 859 = \frac{35305}{N}$$

N = 41.1 say 42 tubes Γ = 21560/42 = 513.3

$$\text{Check: Re} = \frac{4x513.3}{3600x8.5x10^{-4}} = 671$$

Second try; from Fig. 13.2, Nu′ = 0.31

h = 1327 x (0.31/0.32) = 1285

$U_o = 841$

N = 41.9 or 42 tubes.

Check on T_w:

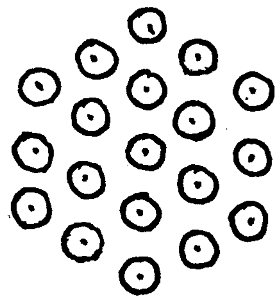
$U_o = 841.5\ W/m^2\text{-}°C$

$h_o = 1205\ W/m^2\text{-}°C$

$T_w = 78.4 - (78.4 - 30)(841/1285)$

$= 46.7\ °C$

13.3



Assume orientation of tubes as shown

Maximum capacity of condenser will be based on a very large flow of cooling water, so h_i is infinite and the exit water temperature is 70°F.

D_o = 1.5/12 = 0.125 ft

$\bar{D}_L$ = 1/12[(1.5 - 1.3)/(ln 1.5/1.3)] = 0.1165 ft

Resistance of metal wall:

$$\frac{x_w D_o}{k_m D_L} = \frac{0.1x0.125}{12x26x0.1165} = 0.00034$$

Estimate surface temperature T_{wo} to be 73°F (found by trial). T_o = 82 - 73 = 9°F

T_f = 82 - (0.75 x 9) = 75.3°F

μ_f = 0.11 x 2.42 = 0.266 lb/ft-h (Appendix 9)

k_f = 0.29 Btu/h-f-°F (Appendix 13)

ρ_f = (1/0.00165)/16.0185 = 37.84 lb/ft

(Perry, 7th ed., p. 2-214)

λ = 500 Btu/lb

From Eq. (13.17), for each stack of tubes,

$$h_N = \frac{0.725}{N^{1/4}} x \left(\frac{0.29^3 x 37.84^2 x 4.18 x 10^8 x 500}{9.3 x 0.125 x 0.266} \right)^{1/4}$$

$= 1597/N^{1/4}$

For two stacks of 3 tubes: $h_N = 1597/3^{1/4} = 1213$

For two stacks of 4 tubes: $h_N = 1597/4^{1/4} = 1129$

For two stacks of 5 tubes: $h_N = 1597/5^{1/4} = 1068$

Average film coefficient:

$\bar{h}_o = [(6 \times 1213) \times (8 \times 1129) + (5 \times 1068)]/19$

$= 1139$ Btu/h-ft^2-°F

$U_o = 1/[(1/1139) + 0.00034)]$

$= 821$ Btu/h-ft^2-°F

Check on assumed surface temperature. From Eq. (11.33), $\Delta T_o = U_o \Delta T / h_o$

$= 821(82-70)/1139 = 8.65°F$

$T_{wo} = 82 - 8.65 = 73.35°F$ (close enough)

Capacity. From Eq. (13.6)

$\dot{m}_T = 1139\pi \times 0.125 \times 14 \times 8.65 \times 19/500$

$= 2058$ lb/h

13.4 $1/U_o$ at $\bar{V} = \infty$ equals sum of the resistances of metal, steam condensate, and scale.

For metal $D_o = 1.00$ in. $\bar{D}_L = 0.950$ in.

$$\frac{x_w D_o}{k_m \bar{D}_L} = \frac{0.098 x 1.000}{12 x 63 x 0.950} = 0.000136$$

(a) Clean tube, with $\bar{V} = \infty$ and $h_i = \infty$

From Eq. (11.32)

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{x_w D_o}{k_m \bar{D}_L} = 0.00040$$

$1/h_0 + 0.000136 = 0.00040$

$h_o = 3788$ Btu/h-ft^2-°F

(b) Clean tube at $\bar{V} = 1$ ft/s; $D_o/D_i h_i = 1/268\bar{V}^{0.8}$

$$\frac{D_o}{D_i h_i} = \frac{1}{268x1} = 0.00373$$

$$h_i = \frac{1}{0.902x0.00373} = 297.2 \text{ Btu/h-ft}^2\text{-°F}$$

(c) Fouled tube at $\bar{V} = \infty$

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{do}} + \frac{x_w D_o}{k_m \bar{D}_L} = 0.00092$$

$$0.00040 + \frac{1}{h_{do}} = 0.00092$$

$h_{do} = 1923$ Btu/h-ft^2-°F

13.5 Calculate $G_r P_r$ and use Fig 12.8.

$D_o = 1/12$ ft $\quad \mu_f = 0.284$ cP (Appendix 6)

$\rho_f = 1/0.01675 = 59.8$ lb/ft^3 (Appendix 7)

$k_f = 0.393$ Btu/h-ft-°F (Appendix 6)

From Appendix 7, for 180°F and 240°F,

$$\beta = \frac{0.01692 - 0.01651}{60x((0.01692 + 0.01651)/2)} = 4.09 \text{ x } 10^{-4}/°\text{F}$$

$$G_r = \frac{(1/12)^3 (59.8)^2 (4.09x10^{-4})(32.174)(1°F)}{(0.284x6.72x10^{-4})^2}$$

$= 7.48 \times 10^5$

$P_r = (1 \times 0.284 \times 2.42)/0.393 = 1.75$

$G_rP_r = 1.75 \times 7.48 \times 10^5 = 1.309 \times 10^6$

$\log_{10}G_rP_r = 6.117$

From Fig. 12.8, $\log_{10}Nu = 1.22$

$Nu = 16.6 = h_oD_o/k$

$h_o = 16.6 \times 0.393 \times 12/1$

$= 78$ Btu/h-ft-°F

Fig. 13.5 indicates $h_o = 700$ Btu/h-ft-°F when $\Delta T =$ 1°F. If D_o were 0.01 inch, then G_rP_r would be $0.01^3 \times 1.309 \times 10^6 = 1.309$, $Nu = \log^{-1}0.04 - 1.1$. Hence h_o would be $1.1 \times 0.393 \times 12/0.01 = 517$ (about as predicted from Fig. 13.5).

The high coefficients in Fig. 13.5 are correct only for small-diameter wires. Coefficients for horizontal tubes would be much lower.

13.6 At 100°C,

$\sigma = 78.38\ (1-0.0025 \times 100)$

$= 58.8$ dyn/cm $= 58.8 \times 10^{-3}$ N/m

$\lambda = 970.3$ Btu/lb $\times 1055.06/0.4536$

$= 2.257 \times 10^6$ J/kg

From Appendix 7

$\rho_L = 16.0185 \times 1/0.01672 = 958.0$ kg/m^3

$\rho_V = 16.0185 \times 1/26.80 = 0.598$ kg/m^3

(a) From Eq. (13.20),

$(q/A)_{max} = 0.12 \times 2.257 \times 10^6[58.8 \times 10^{-3} \times 9.80665 \times (958.0 - 0.598)]^{1/4} \times 0.598^{1/2}$

$= 1.015 \times 10^6 \ W/m^2$

(b) Use Eq. (13.22) for film boiling, since

$\Delta T = 210 - 100 = 110°C$

$\lambda = 2.257 \times 10^6 \ J/kg$

c_p at 155°C = 0.46 x 4186.8 = 1926 J/kg-°C (Appendix 14)

From Eq. (13.23)

$$\lambda' = 2.257 \times 10^6 \left(1 + \frac{0.34x1926x110}{2.257x10^6}\right)^2$$

$$= 2.403 \times 10^6 \ J/kg$$

From Eq. (13.24)

$$\lambda_c = 2\pi\left(\frac{58.8x10^{-3}}{9.80665x957.4}\right)^{1/2}$$

$$= 0.0157m$$

$k_v = 0.0136 \times 1.731 = 0.0235 \ W/m\text{-}°C$ (Appendix 12)

μ_v (at 100°C) = 0.0127×10^{-3} kg/m-s (Appendix 8)

From Eq (13.22)

$$h_o\left(\frac{0.0156x0.0127x10^{-3}x110}{0.0235^3x0.598x957.4x2.403x10^6x9.8055}\right)^{1/4}$$

$$= 0.59 \times (0.069 \times 0.0157)/0.025$$

From this, $h_o = 0.633/3,340 \times 10^{-3} = 189.5 \ W/m^2\text{-}°C$

$q/A = h_o\Delta T = 189.5 \times 110 = 20845 \ W/m^2$

13.7 If no air is present, $T_h = 100°C$

Neglecting the temperature change in the cooling water, T = 30°C and $\Delta T = 70°C$

Guess that $T_w = 82°C$.

From Appendices 6 and 16 (using fps units),

$Pr = 1 \times 0.347 \times 2.42/0.388 = 2.16$

$k = 0.388 \times 1.73073 = 0.672$ W/m-°C

$\rho = 60.58 \times 16.018 = 970.4$ kg/m^3

Guess Re = 2000. From Fig 3.2, Nu = 0.2. Then

$$\frac{h_i}{0.672}\left(\frac{(3.45x10^{-4})^2}{(970.4)^2 x 9.80665}\right)^{1/3} = 0.2$$

$h_i = 5728$ W/m^2-°C

$1/U = 1/5728 + 1/2500 = 0.0005745$

$U = 1740$ W/m^2-°C

$T_h - T_w = 70(1740/5728) = 21.3$°C

$T_f = 100 - (0.75 \times 21.3) = 84$°C

This is close enough. Now, for one tube,

$q = 1740\pi \times 0.025 \times 3 \times 70 = 28700$ W

From Appendix 7, $\lambda = 970.3 \times 0.55556 \times 4.1868 =$ 2247 J/g. $\dot{m} = q/\lambda = 28700/2247 = 12.77$ g/s

$\Gamma = 12.77/(\pi \times 0.025) = 162.6$ g/s-m

At 84°C, from Appendix 6,

$\mu_f = 0.345$ cP = 0.345 g/m-s

Re = 4 x 162.6/0.345 = 1886. Close enough.

From Fig. 13.2, Nu′ = 0.20.

<u>Reynolds number of vapor at inlet:</u>

For steam at 100°C, from Appendix 8,

$\mu = 0.0125$ g/m-s

Cross-sectional area of 25-mm tube:

$S = \pi \times 0.025^2/4 = 0.000491$ m^2

$G = 11.820/0.000491 = 24073\ g/s\text{-}m^2$

$Re = 0.025 \times 24073/0.0125 = 48150$

Flow is turbulent. Find Re at the outlet, after 96% of the steam has condensed.

<u>Basis:</u> 100g mols of inlet gas

At entrance,	98×18	= 1764	g steam
	2×29	= 58	g air
	Total	= 1822	g
At exit,	0.04×1764	= 70.6	g steam
		= 58	g air
	Total	= 128.6	g

$G_{exit} = (128.6/1822) \times 24073 = 1699\ g/s\text{-}m^2$

Equilibrium temperature, when

$$y_{steam} = \frac{(70.6/18)}{(70.8/18)+2} = 0.662$$

$\rho_{water} = 0.662 \times 14.69 = 9.72\ lb_f/in.^2$

From Appendix 7, $T_{outlet} = 191.7°F = 88.7°C$

At 88.7°C, $\mu_{steam} = 0.012$ cP

$\mu_{air} = 0.02$ cP

$\mu_{mixture} \approx 0.015$ cP

$Re = 0.025 \times 1699/0.015 = 2830$

The equilibrium condensation temperature at the outlet is the dew point, 88.7°C. Since there is mass transfer from the gas phase to the condensate, the mole fraction of water in the gas at the condensate surface must be lower than that in the bulk of the gas phase, and the equilibrium temperature at the condensate surface will be lower than 88.7°C.

13.8 (a) For T = 100°C and T_w = 180°C, ΔT = 80°C or 144°F

Since $\Delta T \gg \Delta T_c$ (see Fig 13.4), film boiling occurs

From Eq. (13.22)

$$h_o\left(\frac{\lambda_c \mu_v \Delta T}{k_v^{\,3}\rho_v(\rho_L-\rho_v)\lambda' g}\right)^{1/4} = 0.59 + 0.069\frac{\lambda_c}{D_o}$$

Eq (13.23) $\qquad \lambda' = \lambda\left(1+\dfrac{0.34c_p\Delta T}{\lambda}\right)^2$

Eq (13.24) $\qquad \lambda_c = 2\pi\left(\dfrac{\sigma}{g(\rho_L-\rho_v)}\right)^{1/2}$

For water at 100°C σ = 58.8 dyn/cm= 58.8 x 10^{-3}N/m

Appendix 8 $\qquad \mu_v$ = 0.0127cp = 1.27 x 10^{-5}Pa.s

$\qquad$ K = 0.0235 W/m.°C

Appendix 12 $\qquad \rho_v$ = 0.598 kg/m^3, ρ_L = 959 kg/m^3,

$\qquad \rho_L - \rho_V$ = 958

$$\lambda_c = 2\pi\left(\frac{58.8x10^{-3}}{9.8x959}\right)^{1/2} = 0.0157$$

λ = 970.3 Btu/lb = 2.257 x 10^6 J/kg

c_p = 1.0 cal/g.°C = 4180 J/kg

$$\lambda' = 2.257 \text{ x } 10^6\left(1+\frac{0.34(4180)(80)}{2.257x10^6}\right)^2 = 2.49 \text{ x } 10^6$$

$$(\quad)^{1/4} = \left(\frac{0.0157(1.27x10^{-5})(80)}{0.0235^3x0.598x958x2.49x10^6x9.8}\right)^{1/4} = 3.06 \text{ x } 10^{-3}$$

For D_o = 0.0127m,

$$h_o(3.06 \text{ x } 10^{-3}) = 0.59 + \frac{0.069(0.0157)}{0.0127} = 0.675$$

$h_o = 220\ W/m^2.°C$ (38.7 $Btu/h.ft^2.°F$)

For D_o = 1″=0.0254m, h_o = 207 W/m^2-°C

For D_o = 2″=0.0508m, h_o = 200 W/m^2-°C

A 4 -fold increase in D_o reduces h_o only 9%

(b) For steam condensing on horizontal tubes, Eq.(13-15) shows that h varies with $D_o^{-1/4}$, or a 4-fold increase in D_o would reduce h by 29%

(c) Neglecting the small second term in the right-hand side of Eq (13.22), the film boiling coefficient varies as

$$h_o \ \alpha \ \left(\frac{k^3 \rho_v \rho_L \lambda g}{\mu \Delta T}\right)^{1/4}$$

and the condensing coeffiecient varies as

$$h_o \ \alpha \ \left(\frac{k^3 \rho_f^{\ 2} \lambda g}{D_o \mu \Delta T}\right)^{1/4}$$

13.9 (a) For 7/8-in tubes, D_o = 0.875 in = 0.0222m

Benzene boils at 80.1°C

Assume average water temperature is 40°C

Neglect tube and water film resistance

ΔT_o = 80 - 40 = 40°C

T_f = 80 - 3/4(40) = 50°C

Evaluate liquid properties at 50°C

From Appendix 13, $k_f \cong 0.150$ W/m°C

From Appendix 9, μ_f = 0.36 c_p = 3.6 x 10^{-4}Pa.s

From handbooks, $\rho = 879$ k/m^3 at 20°C

$\rho_f \cong 853$ kg/m^3 at 50°C

From example 21.2

$\lambda = 7360$ Cal/g mol $= 3.94 \times 10^5$ J/kg

$g = 9.8$ m/s^2

$$\text{Eq (13.15)}\quad h = 0.729\left(\frac{k_f^{\,3}\rho_f^{\,2}g\lambda}{\Delta T_o D_o \mu_f}\right)^{1/4}$$

$$h = 0.729\left(\frac{0.150^{\,3}(853)^{\,2}9.8(3.94x10^5)}{40(0.0222)3.6x10^{-4}}\right)^{1/4}$$

$= 1700$ W/m^2.°C or 300 Btu/h-ft^2-°F

(b) For N = 10, $h_N = \dfrac{1700}{10^{1/4}} = 956$ W/m^2°C

13.10 From Fig. 13.9

At $\Delta T = 10$°F, $h \cong 1600$ for a single tube

$h \cong 600$ for a tube bundle

At a low volume fraction bubbles, an increase in the fraction bubbles causes a greater upward velocity of the liquid and greater turbulence near the tube surface. The upper tubes in the bundle benefit from vapor generated by the lower tubes.

At a high ΔT, where the vapor volume is greater, vapor from lower tubes blankets the upper tubes leading to a lower average coefficient for the tube bundle.

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Chapter 14

14.1. $T_a = 500°C$ or 773 K $T_b = 200°C$ or 473 K

$\epsilon_a = 0.90$ $\epsilon_b = 0.25$

(a) From Eq. (14.38),

$$\mathcal{F}_{ab} = \frac{1}{(1/0.90) + (1/0.25) - 1} = 0.243$$

From Eq. (14.38)

$$\frac{q_{ab}}{A_a} = \frac{q_{ab}}{A_b} = 5.672 \times 0.243(7.73^4 - 4.73^4)$$

$$= 4231 \text{ W/m}^2$$

(b) Use Eq. (14.39). Let subscript 1 refer to surface B and subscript 2 to surface A.

$$\mathcal{F}_{ba} = \frac{1}{(1/0.25) + (1^2/0.90^2)((1/0.90) - 1)}$$

$$= 0.250$$

$$q_{ab}/A_b = 5.672 \times 0.250(7.73^4 - 4.73^4)$$
$$= 4353 \text{ W/m}^2$$

(c) From Fig. 14.6, Curve 2, F = 0.20. From Eq. (14.40), since $A_1 = A_2$,

$$\mathcal{F}_{ba} = \mathcal{F}_{ab} = \frac{1}{\dfrac{1}{0.20} + \left(\dfrac{1}{0.25} - 1\right) + \left(\dfrac{1}{0.90} - 1\right)}$$
$$= 0.123$$
$$q_{ab}/A_b = 5.672 \times 0.123(7.73^4 - 4.73^4)$$
$$= 2142 \text{ W/m}^2$$

(d) From Eq. (14.39),

$$\mathcal{F}_{ba} = \frac{1}{\dfrac{1}{0.25} + \dfrac{275}{300}\left(\dfrac{1}{0.90} - 1\right)}$$
$$= 0.244$$
$$q_{ab}/A_b = 5.672 \times 0.244(7.73^4 - 4.73^4)$$
$$= 4249 \text{ W/m}^2$$

(e) F = 0.66. Including the total tube area, $A_b = \pi/2$ m^2 per m^2 of A_a, where subscript a refers to surface A and b to surface B. From Eq. (14.40),

$$\mathcal{F}_{ab} = \frac{1}{\dfrac{1}{0.66} + \left(\dfrac{1}{0.90} - 1\right) + \dfrac{2}{\pi}\left(\dfrac{1}{0.25} - 1\right)}$$
$$= 0.283$$
$$q_{ab}/A_a = 5.672 \times 0.283(7.73^4 - 4.73^4)$$
$$= 4928 \text{ W/m}^2$$
$$q_{ab}/A_b = 4928 \times 2/\pi = 3137 \text{ W/m}^2$$

(f) F = 0.88.

$$\mathcal{F}_{ab} = \frac{1}{\dfrac{1}{0.88} + \left(\dfrac{1}{0.90} - 1\right) + \dfrac{2}{\pi}\left(\dfrac{1}{0.25} - 1\right)}$$

$$= 0.317$$

$$q_{ab}/A_a = 3137 \times 0.317/0.283 = 3514 \text{ W/m}^2$$

(g) $A_b/A_a = 4/\pi$. F = 0.98

$$\mathcal{F}_{ab} = \frac{1}{\dfrac{1}{0.98} + \left(\dfrac{1}{0.90} - 1\right) + \dfrac{4}{\pi}\left(\dfrac{1}{0.25} - 1\right)}$$

$$= 0.202$$

$$q_{ab}/A_a = 4928 \times 0.202/0.283 = 3518 \text{ W/m}^2$$

$$q_{ab}/A_b = 3518 \times 4/\pi = 4479 \text{ W/m}^2$$

14.2. Heat from sun to roof equals heat radiated to air plus heat transferred to air by conduction-convection.

Sun to roof:

$$q_T/A = 300 \times 0.8 = 240 \text{ Btu/h-ft}^2$$

Assume the radiation goes to an effective temperature of -70°C (-94°F). (See Problem 14.6)

From Eq. (14.44), since T = 460 - 94 = 366°R, and $h_c \Delta T = q_c/A$,

$$240 = h_c\ (T_w - 68) + 0.1713 \times 0.9\left[\left(\frac{T_w + 460}{100}\right)^4 - (3.66)^4\right]$$

$$= 0.38\ (T_w - 68)^{1.25} + 0.1542\left(\frac{T_w + 460}{100}\right)^4 - 27.66$$

By trial, $T_w = 135.6°F$

14.3. $q_T/A = 300 \times 0.5 = 150 \text{ Btu/h-ft}^2$

From Eq. (14.44),

$$150 = 0.38\ (T_w - 68)^{1.25} + 0.1542\left(\frac{T_w + 460}{100}\right)^4 - 27.66$$

$T_w = 98.7°F$

14.4. Use Eq. (14.44). Assume $T_w = T_{steam}$; from Appendix 7, $T_w = 329.8^\circ F$ or $165.4^\circ C$ at 7 atm abs (6 atm gauge). $T = 25^\circ C$ $\epsilon_w = 0.70$

Compute h_c from Eq. (12.72) or Fig. 12.8.

Grashof number:

$$T_f = (165.4 + 25)/2 = 95.2^\circ C \text{ or } 368.2 \text{ K}$$

From Eq. (12.71), $\beta_f = 1/368.2 = 0.00272$

$$D_o = 3.50 \times 0.0254 = 0.0889 \text{ m (Appendix 3)}$$

$$\Delta T_o = 165.4 - 25 = 140.4^\circ C$$

$$\mu_f = 0.021 \text{ cP at } 95.2^\circ C \text{ (Appendix 8)}$$

$$\rho_f = (29/22.4) \times (273/368.2) = 0.96 \text{ kg/m}^3$$

$$Gr = \frac{0.0889^3 \times 0.96^2 \times 0.00272 \times 9.80665 \times 140.4}{(0.021 \times 10^{-3})^2}$$

$$= 5.50 \times 10^6$$

Prandtl number: from Appendix 16, Pr = 0.69.

$$GrPr = 0.69 \times 5.50 \times 10^6 = 3.80 \times 10^6$$

Since log GrPr > 4, use Eq. (12.72).

$$Nu = 0.53 \times (3.80 \times 10^6)^{1/4} = 23.4$$

$$k_f = 0.0182 \text{ Btu/h-ft-}^\circ F \text{ (Appendix 12)}$$

$$= 0.0182 \times 1.73073 = 0.0315 \text{ W/m-}^\circ C$$

$$h_c = 23.4 \times 0.0315/0.0889 = 8.29 \text{ W/m}^2\text{-}^\circ C$$

Use Eq. (14.44), assuming the surroundings are at $25^\circ C$ (298 K). $T_w = 165.4^\circ C$ or 438.4 K

$$q_T/A = 8.29(165.4 - 25)$$

$$+ 5.672 \times 0.70(4.384^4 - 2.98^4)$$

$$= 1164 + 1153 = 2317 \text{ W/m}^2$$

$$A = 70 \times 0.916 \times 0.3048 = 19.54 \text{ m}^2$$

$$q_T = 2317 \times 19.54/1000 = 45.3 \text{ kW}$$

For steam at 6 atm gauge, $\lambda = 887$ Btu/lb or

887 x 0.55556 x 4.1868 = 2063 J/g (Appendix 7)

Steam condensed: $\dot{m} = 45.3/(2063/1000)$

$$= 21.96 \text{ g/s or } 79.06 \text{ kg/h}$$

Percent lost by conduction-convection:

1164 x 100/2317 = 50.2%

14.5. Heat transferred to floor comes entirely from the ceiling by radiation, since $T_{air} = T_{floor}$.

Let subscript c refer to ceiling, a to air, f to floor. From Eq. (14.37),

$$q_{cf} = A\sigma\mathcal{F}_{cf}(T_c^4 - T_f^4)$$
$$= 3500 \text{ Btu/h}$$
$$T_f = 525^\circ R \qquad \epsilon_c = 0.93$$
$$a_f = 0.63 = \epsilon_f \text{ (assumed)}$$

Use Eq. (14.40). From Fig. 14.6, Curve 6, $\bar{F} = 0.71$. $A_1 = A_2 = 18^2 = 324 \text{ ft}^2$

$$\mathcal{F}_{cf} = \frac{1}{\dfrac{1}{0.71} + \left(\dfrac{1}{0.93} - 1\right) + \left(\dfrac{1}{0.63} - 1\right)}$$

$= 0.483$. Hence

$$3500 = 324 \times 0.1713 \times 0.483\left[\left(\frac{T_c}{100}\right)^4 - 5.25^4\right]$$

$$T_c = 546^\circ R \text{ or } 86^\circ F$$

Heat transferred to air:

$$q_{ca} = h_c A(T_c - T_a)$$
$$= 0.20 \times 324 \times (86 - 65)^{1.25}$$
$$= 2913 \text{ Btu/h}$$

14.6. Let subscript a refer to air, w to water, s to space.

$$T_a = 15^\circ C = 288K \qquad T_s = 273 - 70 = 203K$$

At equilibrium convective heat transfer from air to water (or ice) equals radiant heat loss from water (or ice) to space. Hence

$$h_c\ (T_a - T_w) = \sigma\mathcal{F}_{ws}\ (T_w^4 - T_s^4)$$

Since $F = 1$, $\mathcal{F}_{ws} = \varepsilon_w$. Hence

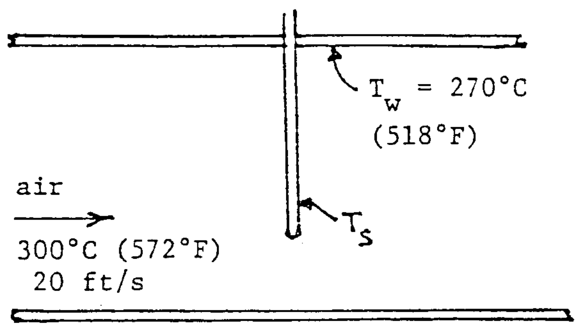
$$2.6\ (288 - T_w) = 5.672\ \varepsilon_w\left[\left(\frac{T_w}{100}\right)^4 - 2.03^4\right]$$

Emissivity of water ε_w = 0.95 (Perry, 7th ed., p. 5.29)

By trial, T_w = 246.6K = - 26.4°C

Hence ice will form, since T_w is well below 0°C.

14.7.



Let A = outside area of thermowell

Heat balance:

$$h_o A\,(572 - T_s) = h_r A\,(T_s - 518)$$

Use Fig. 12.6 to find h_o. Properties of air:

$$\rho = \frac{29}{359} \times \frac{1.5}{1} \times \frac{273}{(300 + 273)} = 0.0577 \text{ lb/ft}^3$$

$$\mu = 0.0282 \text{ cP} \quad \text{(Appendix 8)}$$

$$Re = \frac{0.5 \times 25 \times 0.0577}{12 \times 0.0.0282 \times 6.72 \times 10^{-4}}$$

$$= 3172$$

From Fig. 12.6, $h_o D/k$ = 31

From Perry, Chemical Engineers' Handbook, 6th ed., p. 3-254, k = 0.0452 W/m-K or 0.0261 Btu/h-ft-°F. Hence

h_o = 31 x 0.0261/(0.5/12) = 19.4 Btu/h-ft^2-°F

Use Eq. (14.49) to find h_r, since A_{wall} >> $A_{thermowell}$. Use the average of T_w and T_s (see p. 426). Estimate T_s = 294°C; T_{ave} = 282°C or 539.6°F or 999.6°R. Estimate that ϵ_w = 0.9. Then from Eq. (14.49)

$$h_r = 4 \times 0.1713 \times 10^{-8} \times 0.9 \times 999.6^3$$
$$= 6.16 \text{ Btu/h-ft}_2\text{-}^\circ\text{F}$$

From a heat balance,

$$19.4(572 - T_s) = 6.16\ (T_s - 518)$$

$T_s = 559^\circ$F or 293°C. (Close enough to the estimated value.) The error in temperature measurement is $300 - 293 = 7^\circ$C.

14.8. Let subscript 1 refer to the inside wall and subscript 2 to the outer wall. The average air temperature in the gap is $(18 - 9)/2 = 4.5^\circ$C or 277.5 K. Use Eq. (14.45) with

$$h_c = \frac{1}{(1/3.9) + (1/3.9)} = 1.95 \text{ W/m}^2\text{-}^\circ\text{C}$$

Assume $\epsilon_1 = \epsilon_2 = 0.9$. Find h_r from Eq. (14.49) with ϵ_w found as noted on p. 426.

$$\epsilon_w = \frac{1}{(1/0.9) + (1/0.9) - 1} = 0.818$$

$$h_r = 4 \times 5.672 \times 10^{-8} \times 0.818 \times 277.5^3$$
$$= 3.97 \text{ W/m}^2\text{-}^\circ\text{C}$$

From Eq. (14.45)

$$q_T/A = (1.95 + 3.97)(18 - (-9)) = 159.8 \text{ W/m}^2$$

Part 2. With the inside wall covered with aluminum foil, estimate $\epsilon_1 = 0.1$. Then

$$\epsilon_w = \frac{1}{(1/0.1) + (1/0.9) - 1} = 0.099$$

$$h_r = 3.97 \times 0.099/0.818 = 0.48 \text{ W/m}^2\text{-}^\circ\text{C}$$
$$h_c \text{ (unchanged)} = 1.95 \text{ W/m}^2\text{-}^\circ\text{C}$$
$$h_c + h_r = 2.43 \text{ W/m}^2\text{-}^\circ\text{C}$$

Hence q_T/A will be $2.43/(3.97 + 1.95) = 0.41$ of the previous value (a 59% reduction).

Part 3. With foil in the center, $(h_c + h_r)$ for transfer from T_1 to the foil (and from the foil to T_2) will be the same as before, 2.43 W/m^2. The foil will be at 4.5°C (277.5 K). so the ΔT is $18 - 4.5 = 13.5°C$ instead of 27°C as before. Hence q_T/A will be cut in half, to 20.5% of the first value.

14.9. Assume that the tube is gray and the surrounding liquid is black. $\epsilon_1 = \epsilon_w = 0.8$

From Eq. (14.39), $\mathcal{F}_{12} = \epsilon_w = 0.8$.

$$h_o = 32 \text{ Btu/h-ft}^2\text{-°F}$$

$$T_1 = 410 + 460 = 870°R$$

$$T_2 = 212 + 460 = 672°R$$

From Eq. (14.36),

$$q_r/A = 0.1713 \times 0.8(8.7^4 - 6.72^4)$$

$$= 505.6 \text{ Btu/h-ft}^2$$

From Eq. (14.46),

$$h_r = 505.6/(410 - 212) = 2.55 \text{ Btu/h-ft}^2\text{-°F}$$

From Eq. (14.50),

$$h_c = 32\left(\frac{32}{h_c + 2.55}\right)^{1/3}$$

By trial, $h_c = 31.4$ Btu/h-ft²-°F

Without radiation, the heat flux is

$$q_T/A = 32(410 - 212) = 6336 \text{ Btu/h-ft}^2$$

With radiation, $q_T/A = q_c/A + q_r/A$

$$= (31.4 + 2.55)(410 - 212)$$

$$= 6722 \text{ Btu/h-ft}^2$$

Increase because of radiation:

6722/6336 = 1.06 or a 6% increase

Chapter 15

15.1. Use Eq. (15.6). Since there is no flow parallel to the tubes, use $G_e = G_c = \dot{m}/S_c$.

$$S_c = 10(0.075 - 0.025) \times 3.5 = 1.75 \text{ m}^2$$

$$\dot{m} = 3 \times 29/22.4 = 3.884 \text{ kg/s}$$

$$G_c = 3.884/1.75 = 2.22 \text{ kg/m}^2\text{-s}$$

Average air temperature = 30°C

μ = 0.0183 x 10^{-3} kg/m-s (Appendix 8)

k = 0.0150 x 1.73073 = 0.0260 W/m-°C (Appendix 12)

D_o = 0.025 m

$D_o G_c/\mu$ = 0.025 x 2.22/0.0183 x 10^{-3} = 3033

c_p = 0.24 x 4.168 x 10^3 = 1000 J/kg-°C (Appendix 15)

Pr = 1000 x 0.0183 x 10^{-3}/0.0260 = 0.704

From Eq. (15.6), neglecting $(\mu/\mu_w)^{0.14}$,

$$h_o = \frac{0.0263 \times 0.2 \times 3033^{0.6} \times 0.704^{0.33}}{0.025}$$

= 23.0 W/m²-°C

Neglecting the resistance of the condensate and tube wall, $h_o = U_o$. The heat load is

q = 3.884 x 1000 x (40 - 20) = 77680 W

A_o = 100 x 3.5 x π x 0.025 = 27.5 m²

$\overline{\Delta T_L} = q/U_o A_o$ = 77680/(23.0 x 27.5) = 122.8°C. Therefore

$$\frac{(T_s - 20) - (T_s - 40)}{\ln \dfrac{T_s - 20}{T_s - 40}} = 122.8$$

From this, T_s = 153.1°C (307.6°F). From Appendix 7 the steam pressure is 75.2 $lb_f/in.^2$ absolute or 5.12 atm.

15.2. Use Eq. (15.6). Put the bottom product in the shell. Assume f_b = 0.1955 (see Example 15.3).

Shell side

N_b = 324 x 0.1955 = 63 tubes

$$S_b = \frac{0.1955\pi}{4}\left(\frac{23.25}{12}\right)^2 - \frac{63\pi}{4}\left(\frac{0.75}{12}\right)^2$$

= 0.3831 ft² (0.03559 m²)

From Eq. (15.4), since $P = 9/12 = 0.75$ ft, $p = 1$ in., and $D_o = 0.75$ in.:

$$S_c = \frac{0.75 \times 23.25}{12}\left(1 - \frac{0.75}{1}\right)$$

$$= 0.3633 \text{ ft}^2 \ (0.03375 \text{ m}^2)$$

$$G_b = 129{,}000/(3600 \times 0.03559)$$

$$= 1007 \text{ kg/m}^2\text{-s}$$

$$G_c = 129{,}000/(3600 \times 0.03375)$$

$$= 1062 \text{ kg/m}^2\text{-s}$$

$$G_e = (1007 \times 1062)^{1/2} = 1034 \text{ kg/m}^2\text{-s}$$

$$\mu = 5.2 \times 10^{-3} \text{ kg/m-s}$$

$$D_o = 0.75/39.37 = 0.01905 \text{ m}$$

$$Re = \frac{0.01905 \times 1034}{5.2 \times 10^{-3}} = 3788$$

$$Pr = \frac{2.20 \times 5.2}{0.119} = 96.1$$

From Eq. (15.6), neglecting $(\mu/\mu_w)^{0.14}$,

$$h_o = \frac{0.119 \times 0.2 \times 3788^{0.6} \times 96.1^{0.33}}{0.01905}$$

$$= 790 \text{ W/m}^2\text{-°C}$$

Tube side

Inside sectional area per pass:

$$(324/2) \times 0.00186 = 0.301 \text{ ft}^2 \text{ or } 0.0280 \text{ m}^2$$

$$Pr^{2/3} = (1.99 \times 2.9/0.137)^{2/3} = 12.11$$

$$G = 150{,}000/(3600 \times 0.0280) = 1488 \text{ kg/m}^2\text{-s}$$

$$D_i = 0.584/39.37 = 0.01483 \text{ m} \quad \text{(Appendix 4)}$$

$$\left(\frac{D_i G}{\mu}\right)^{0.2} = \left(\frac{0.01483 \times 1488}{2.9 \times 10^{-3}}\right)^{0.2} = 5.97$$

From Eqs. (12.51) and (5.53),

$$h_i = \frac{0.023 \times 1.99 \times 1488 \times 10^3}{5.97 \times 12.11}$$

$$= 942 \text{ W/m}^2\text{-°C}$$

Metal wall

$$k = 26 \times 1.73073 = 45 \text{ W/m-}^\circ\text{C}$$

$$x_w = 0.083/39.37 = 0.00211 \text{ m} \quad \text{(Appendix 4)}$$

$$\bar{D}_L = \frac{0.75 - 0.584}{\ln (0.75/0.584)} = 0.664 \text{ in. or } 0.0169 \text{ m}$$

With clean tubes, from Eq. (11.34),

$$\frac{1}{U_o} = \frac{0.01905}{0.01483 \times 942} + \frac{0.00211 \times 0.01905}{45 \times 0.0169} + \frac{1}{790}$$

$$U_o = 1/0.002676 = 374 \text{ W/m}^2\text{-}^\circ\text{C}$$

Coefficient needed to transfer desired heat:

$$\overline{\Delta T}_L = \frac{(146 - 57) - (107 - 20)}{\ln (89/87)} = 88^\circ\text{C}$$

$$Z = \frac{146 - 107}{57 - 20} = 1.05 \quad \text{[Eq. (15.1)]}$$

$$\eta_H = \frac{57 - 20}{146 - 20} = 0.294 \quad \text{[Eq. (15.2)]}$$

From Fig. 15.6a.

$$F_G = 0.96$$

$$A = 324 \times 0.1963 \times 12 = 763 \text{ ft}^2 \text{ or } 70.9 \text{ m}^2$$

$$q_{crude} = 1.99 \times 10^3 (57 - 20)(150{,}000/3600) = 3.068 \times 10^6 \text{ W}$$

$$q_{product} = 2.20 \times 10^3 (146 - 107)(129{,}000/3600) = 3.075 \times 10^6 \text{ W}$$

$$\bar{q} = 3.0715 \times 10^6 \text{ W}$$

$$U_o \text{ needed} = 3.0715 \times 10^6/(70.9 \times 88 \times 0.96) = 513 \text{ W/m}^2\text{-}^\circ\text{C}$$

The exchanger would not be suitable, since even with clean tubes U_o is only 374 W/m²-°C.

15.3. Average oil temperature: (60 + 150)/2 = 105°F

Oil properties:

$$\rho = 0.840 \times 62.3 = 52.3 \text{ lb/ft}^3 \text{ at } 60^\circ\text{F}$$

Estimate ρ = 51.0 lb/ft^3 at 105°F

k = 0.078 Btu/h-ft-°F

c_p = 0.480 Btu/lb-°F

μ = 5.0 cP at 520°R, 1.8 cP at 610°R

From $\mu = KT^n$, μ = 2.94 cP at 105°F (610°R)

Steam temperature = 338°F (Appendix 7)

Tube dimensions (Appendix 4):

D_i = 0.870 in. D_o = 1.00 in.

Inside sectional area = 0.00413 ft^2

Number of tubes, N:

Volume flow = 150 x 42/(3600 x 7.48 gal/ft^3)

= 0.234 ft^3/s

Make N = 14 tubes. Then

$\overline{V}$ = 0.234/(0.00413 x 14) = 4.05 ft/s

G = 4.05 x 51.0 x 3600 = 743,600 $lb/h\text{-}ft^2$

<u>Heat-transfer coefficients</u>:

Inside tubes:

$$Re = \frac{(0.870/12) \times 4.05 \times 51.0}{2.94 \times 6.72 \times 10^{-4}} = 7580$$

$$Pr = \frac{0.480 \times 2.94 \times 2.42}{0.078} = 43.8$$

From Eqs. (12.51) and (5.53),

$$h_i = \frac{0.023 \times 0.480 \times 743600}{7580^{0.2} \times 43.8^{2/3}}$$

= 110.7 $Btu/h\text{-}ft^2\text{-}°F$

Metal wall. From Appendix 10,

k_m for steel = 26 Btu/h-ft-°F

k_m/x_w = 12 x 26/0.065

= 4800 $Btu/h\text{-}ft^2\text{-}°F$

Outside tubes (condensing steam). Heat load is:

q = 150 x 42 x 51.0 x 0.480 x (150 - 60)/7.48

= 1,856,000 Btu/h

From Appendix 7, λ = 881 Btu/lb

$\dot{m}$ = 1,856,000/881 = 2107 lb/h condensed

With 14 tubes in each pass,

$$\Gamma_b = \frac{2107 \times 12}{1.00 \times 28\pi} = 287.4 \text{ lb/h-ft}$$

Assume average film temperature of water = 330°F

μ_f = 0.125 cP (Appendix 9)

k_f = 0.396 Btu/h-ft-°F (Appendix 6)

ρ_f = 56.35 lb/ft^3 (est. from Appendix 6)

$Re_f = 4\Gamma_b/\mu_f$

$= 4 \times 287.4/(0.125 \times 2.42) = 3800$

$Pr = (0.480 \times 0.125 \times 2.42)/0.396 = 0.37$. From Fig. 13.2,

$Nu' = 0.16$. $h_o = (0.16 \times 0.396)\left[56.35^3 \times 32.17/0.125^2 \times 6.72^2 \times 10^{-8}\right]^{1/3}$

$= 1541$ Btu/h-ft^2-°F

Overall coefficient, from Eq. (11.32), with $\overline{D}_L$ = 0.933 in.:

$$U_o = \frac{1}{\dfrac{1.00}{0.870 \times 110.7} + \dfrac{1.00}{0.933 \times 4800} + \dfrac{1}{1541}}$$

$= 88.9$ Btu/h-ft^2-°F

$\Delta T_1 = 338 - 150 = 188$°F

$\Delta T_2 = 338 - 60 = 278$°F

$$\overline{\Delta T}_L = \frac{278 - 188}{\ln (278/188)} = 230\text{°F}$$

Heat transfer area

$A = 1{,}856{,}000/(88.9 \times 230) = 90.8$ ft^2

Area per tube = 0.2618 ft^2/ft (Appendix 4)

Length of tubes:

$L = 90.8/(2 \times 14 \times 0.2618) = 12.4$ ft

Check on outside temperature drop:

$\Delta T_o = (1/1541)(230 \times 88.9) = 13.3$ °F

Average temperature of water film:

$T_f = 338 - (13.3/2) = 331.4$ °F (close enough to assumed value of 330°F)

15.4. Tube dimensions (Appendix 4):

$D_i' = 0.620$ in. $D_o' = 0.750$ in.

Wall thickness = 0.065 in.

Inside cross-sectional area = 0.00210 ft^2

Inside surface area = 0.1623 ft^2/ft of length

Gal/min at 1 ft/s velocity = 0.9425

Reynolds number at entrance:

Set $\bar{V}$ at 4 ft/s. At 100°F,

$$\nu = \mu/\rho = 36.6 \times 10^{-5} \ ft^2/s$$

$$Re = \frac{0.620 \times 4}{12 \times 36.6 \times 10^{-5}} = 565$$

Flow is laminar. The number of tubes is

$$N = 200/(0.9425 \times 4) = 53 \text{ tubes per pass}$$

Steam temperature = 307°F (Appendix 7)

Use Eq. (12.27). Assume wall temperature = 300°F. Total mass flow:

$$\dot{m} = \frac{200 \times 60 \times 55.25}{7.48 \ gal/ft^3} = 88{,}636 \ lb/h$$

Mass flow per tube: 88,636/53 = 1672.4 lb/h

Average oil temperature = 140°F

$$c_p = 0.477 \ Btu/lb\text{-}°F$$

$$k = 0.0733 \ Btu/h\text{-}ft\text{-}°F$$

$$\mu = 14.4 \times 10^{-5} \times 54.37$$

$$= 7.83 \times 10^{-3} \ lb/ft\text{-}s$$

From an exponential temperature relation,

$$\mu_w = 1.06 \times 10^{-3} \ lb/ft\text{-}s$$

$$\phi_v = (7.83/1.06)^{0.14} = 1.323$$

Set L = 15 ft, since the temperature profile is re-established at the start of each pass.

$$Gz = \frac{1672.4 \times 0.477}{0.0733 \times 15} = 725.5$$

From Eq. (12.27),

$$h_i = \frac{2 \times 0.0733 \times 12}{0.620} \times 725.5^{1/3} \times 1.323$$

$$= 33.7 \text{ Btu/h-ft}^2\text{-}^\circ\text{F}$$

Heat load: $q = 88{,}636 \times 0.477(180 - 100)$

$$= 3.382 \times 10^6 \text{ Btu/h}$$

Overall coefficient. Assume steam-film coefficient = 1200 Btu/h-ft^2-°F

From Eq. (11.33),

$$U_i = \frac{1}{\dfrac{1}{33.7} + \dfrac{0.620 \times 0.065}{0.690 \times 12 \times 26} + \dfrac{0.620}{0.750 \times 1200}}$$

$$= 1/(0.029674 + 0.000187 + 0.000689)$$

$$= 1/0.030550 = 32.7 \text{ Btu/h-ft}^2\text{-}^\circ\text{F}$$

Surface area per pass:

$$A = 53 \times 0.1623 \times 15 = 129.0 \text{ ft}^2$$

$$\Delta T_1 = 307 - 100 = 207^\circ\text{F}$$

$$\Delta T_2 = 307 - 180 = 127^\circ\text{F}$$

$$\overline{\Delta T}_L = (207 - 127)/\ln(207/127) = 163.8^\circ\text{F}$$

Since $q = U_i A \overline{\Delta T}_L$,

$$3.382 \times 10^6 = (32.7 \times A \times 163.8)$$

$$A = 631.4 \text{ ft}^2$$

Number of passes: 631.4/129.0 = 4.9

Check on wall temperature

$$T_w = 307 - ((0.000187 + 0.000689) \times (307 - 140)/0.030550)$$

$$= 307 - 5 = 302^\circ\text{F} \quad \text{Close enough.}$$

Use 6 passes to make an even number.

NOTE: Since the steam temperature is constant, no correction is needed for the fact that this is a multi-pass heater.

15.5. Eq. (15.6) for an exchanger:

$$\frac{h_o D_o}{k} = 0.2 \left(\frac{D_o G_e}{\mu}\right)^{0.6} \left(\frac{c_p \mu}{k}\right)^{0.33} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Eq. (12.63) for a single tube

$$\frac{h_o' D_o}{k} = \left(0.35 + 0.56 \left(\frac{D_o G}{\mu}\right)^{0.52}\right) \left(\frac{c_p \mu}{k}\right)^{0.3}$$

Neglecting the wall-viscosity term, the ratios of the coefficients for the exchanger and a single tube are:

Re	h_o / h_o'		
	Pr = 1	Pr = 10	Pr = 100
100	0.49	0.53	0.56
1000	0.61	0.65	0.70
10000	0.74	0.79	0.85

Over the normal range of Reynolds numbers, the outside coefficients for the exchanger tubes are about 50 to 85% of the values for single tubes. However, the mass velocity G_c for crossflow is based on the minimum area between tubes at the centerline of the exchanger, and the average mass velocity normal to the tubes is appreciably lower than the value at the centerline. Also, the Donohue equation allows for some leakage through and around the baffles.

15.6. With an aqueous solution of low viscosity, flow will be turbulent. The inside coefficient varies with the 0.8 power of the flow rate, compared with the 0.6 power for the outside coefficient. Therefore putting the stream with the larger flow rate (the cooling water) inside the tubes will give the higher overall coefficient.

Possible values. With cooling water in the tubes, say

$$h_i = h_o = 3000 \text{ W/m}^2\text{-}^\circ\text{C}$$

$$U = 1/((1/3000) + (1/3000)) = 1500$$

With cooling water in the shell:

$$h_i' = 3000 \times 0.5^{0.8} = 1723 \text{ W/m}^2\text{-}^\circ\text{C}$$

$$h_o' = 3000 \times 2^{0.6} = 4547 \text{ W/m}^2\text{-}^\circ\text{C}$$

$$U' = 1/((1/1723) + (1/4547)) = 1250$$

The overall coefficient would be about 20% larger with the cooling water in the tubes. This analysis neglects the minor effects of the dirt and metal-wall resistances, and of the ratio D_o/D_i for the tubes.

15.7 Calandrias are discussed in Chap. 13, pp. 400-1.

(a) From Perry, 6th ed., p. 3.126:

$$\lambda = 77.59 \times 4.184 = 324.6 \text{ J/g}$$

Estimate from Table 13.2, $U = 1000 \text{ W/m}^2\text{-}^\circ\text{C}$

Assume $\Delta T = 40^\circ\text{C}$ between steam and chlorobenzene.

$$q = 5000 \times 324.6 \times 1000/3600 = 450{,}800 \text{ W}$$

$$A = 450{,}800/(1000 \times 40) = 11.3 \text{ m}^2$$

(b) Because of the increased vapor volume, estimate $U = 800 \text{ W/m}^2\text{-}^\circ\text{C}$. If $\Delta T = 40^\circ\text{C}$ as before,

$$A = 450{,}800/(800 \times 40) = 14.1 \text{ m}^2$$

15.8. Use Eq. (15.26). Properties of styrene, from "Professors' Workshop on Industrial Monomer and Polymer Engineering," AIChE, copyright Dow Chemical Company, 1968, pp. 33-36. NOTE: Reasonable answers would be obtained by using data for ethyl benzene from Appendixes 9 and 15.

$$\rho = 0.8702 \times 998 = 868.5 \text{ kg/m}^3$$

$$c_p = 0.4329 \times 4.18668 \times 10^3 = 1812 \text{ J/kg-}^\circ\text{C}$$

$$\mu = 0.47 \times 10^{-3} \text{ kg/m-s}$$

Assume $T_w = 100^\circ\text{C}$. Then

$$\mu_w = 0.326 \times 10^{-3} \text{ kg/m-s}$$

The thermal conductivity at 60°C is estimated from the benzene derivatives in Appendix 13 to be

$$k = 0.08 \times 1.73073 = 0.14 \text{ W/m-}^\circ\text{C}$$

(a) $D_a = 0.667$ m $\quad D_t = 2$ m

$$n = 140/60 = 2.333 \text{ r/s}$$

$$Re = \frac{0.667^2 \times 2.333 \times 868.5}{0.47 \times 10^{-3}} = 1.92 \times 10^6$$

$$Pr = 1812 \times 0.47 \times 10^{-3}/0.14 = 6.08$$

From Eq. (15.26),

$$h_j = (0.14 \times 0.76/2) \times (1.92 \times 10^6)^{2/3}$$
$$\times 6.08^{1/3} \times (0.47/0.326)^{0.24}$$
$$= 1638 \text{ W/m}^2\text{-}^\circ\text{C}$$

(b) From Table 9.2, since flow is turbulent,

$$K_{T,1} = N_{P,1} \text{ for pitched blades} = 1.63$$

$$K_{T,2} = N_{P,2} \text{ for straight blades} = 5.75$$

$$\frac{N_{P,1}}{N_{P,2}} = \frac{(P/n^3 D_a^5 \rho)_1}{(P/n^3 D_a^5 \rho)_2} = \frac{1.63}{5.75}$$

Since P is held constant only n changes, and

$$n_1 = n_2 (N_{P,1}/N_{P,2})^{1/3}$$
$$= 2.333 \times (5.75/1.63)^{1/3} = 3.55 \text{ r/s}$$

$$Re = 1.92 \times 10^6 \times 3.55/2.333 = 2.92 \times 10^6$$

As indicated on p. 460, jacket coefficients for a pitched-blade turbine are 0.9 times those for a straight-blade turbine at the same Reynolds number in a baffled tank. Using this factor:

$$h_j = 1638(2.92/1.92)^{2/3} \times 0.9$$

$= 1950\ W/m^2\text{-}^\circ C$ (1.19 times the value for a straight-blade turbine)

15.9. Use Eq. (15.26) assuming a straight-blade turbine. The quantities needed are:

$D_a = 0.667$ m $\quad D_t = 2$ m

$n = 140/60 = 2.333$ rev/s

At 40°C (104°F), from Appendix 6,

$c_p = 1 \times 4187 = 4187$ J/kg-°C

$\rho = 61.94 \times 16.0185 = 992$ kg/m^3

$\mu = 0.657 \times 10^{-3}$ kg/m-s

$k = 0.364 \times 1.731 = 0.630$ W/m-°C

Estimated $T_w = 64$°C (147°F); $\mu_w = 0.425 \times 10^{-3}$ kg/m-s

Also:

$$\left(\frac{D_a^2 n\rho}{\mu}\right)^{2/3} = \left(\frac{0.667^2 \times 2.333 \times 992}{0.657 \times 10^{-3}}\right)^{2/3}$$

$$= 13490$$

$$\left(\frac{c_p \mu}{k}\right)^{1/3} = \left(\frac{4187 \times 0.657 \times 10^{-3}}{0.630}\right)^{1/3} = 1.635$$

$$(\mu/\mu_w)^{0.24} = (0.657/0.425)^{0.24} = 1.11$$

From Eq.(15.~~13~~ 26):

$$h_i = \frac{0.630 \times 0.76 \times 13490 \times 1.635 \times 1.11}{2}$$

Eq. (15.26),

Use Eq. (11.33) for U_t. $D_i = 2.0$; $D_o = 2.02$; $\bar{D}_L = 2.01$

$k = 26 \times 1.731 = 45$ W/m-°C. From Eq. (11.33),

$$U_i = \frac{1}{\dfrac{1}{5861} + \dfrac{2 \times 0.010}{2.01 \times 45} + \dfrac{2}{2.02 \times 10000}}$$
$$= 2037 \text{ W/m}^2\text{-°C}$$

Since $\Delta T = 110 - 40 = 70°C$ and $A = 12 \text{ m}^2$

$$q = 2037 \times 12 \times 70/1000 = 1711 \text{ kW}$$

Check on T_w:

$$T_w = 40 + (70 \times 2037/5861) = 64.3°C$$

15.10. (a) Use Eq. (15.29) rearranged to give

$$t_T = \frac{mc_p}{UA} \ln \frac{T_s - T_a}{T_s - T_b}$$
$$= \frac{5500 \times 4187}{2037 \times 12} \ln \frac{110 - 20}{110 - 60}$$
$$= 554 \text{ s or } 9.2 \text{ min}$$

(b) Average temperature is now 80°C. Physical properties are now:

$$\mu = 0.358 \times 10^{-3} \text{ kg/m-s}$$
$$k = 0.388 \times 1.731 = 0.672 \text{ W/m-°C}$$
$$\rho = 60.66 \times 16.0185 = 972 \text{ kg/m}^3$$

From Eq. (15.26) holding μ_w and all quantities other than k, ρ, and μ constant:

$$h_j \sim k^{2/3} \rho^{2/3} \mu^{-0.093}$$

Thus at 80°C

$$h_j = 5861\left(\frac{0.672}{0.630}\right)^{2/3}\left(\frac{972}{992}\right)^{2/3}\left(\frac{0.657}{0.358}\right)^{0.093}$$
$$= 6387 \text{ W/m}^2\text{-°C}$$

Substitution in Eq. (11.33) as in Problem 15.9 gives:

$$U_i = \frac{1}{\dfrac{1}{6387} + \dfrac{2 \times 0.01}{2.01 \times 45} + \dfrac{2}{2.02 \times 10000}}$$

$$= 2096 \text{ W/m}^2\text{-}^\circ\text{C}$$

From Eq. (15.29),

$$t_T = \frac{5500 \times 4187}{2096 \times 12} \ln \frac{110 - 60}{110 - 100}$$

$$= 1474 \text{ s or } 24.6 \text{ min}$$

15.11. Let subscript 1 refer to the base case (1-inch tubes) and subscript 2 to 1 1/2-inch tubes. At the maximum temperature, the local rate of heat generation equals the rate of heat transfer since the sensible heat of the gas is not changing. Then

$$q_1 = U_1 A_1 \Delta T_1$$

where

q_1 = rate of heat generation per unit volume of the tube, and A_1 is the area per unit volume

Since $D_2 = 1.5D_1$, the new area per unit volume is

$$A_2 = A_1 \left(\frac{D_2}{D_1}\right)\left(\frac{D_1}{D_2}\right)^2 = A_1/1.5$$

Since $h_{w1} \approx h_{bed,1}$, if the wall and jacket resistances are neglected:

$$U_1 = h_{w1}/2$$

Since $h_{bed} = 4k_e/r$ [Eq. (15.34)],

$$h_{bed,2} = h_{bed,1}/1.5$$

Assuming that h_w does not change,

$$\frac{1}{U_2} = \frac{1}{h_{w1}} + \frac{1}{h_{bed,2}} = \frac{1}{h_{w1}} + \frac{1.5}{h_{bed,1}} = \frac{2.5}{h_{w1}}$$

$$U_2 = h_{w1}/2.5$$

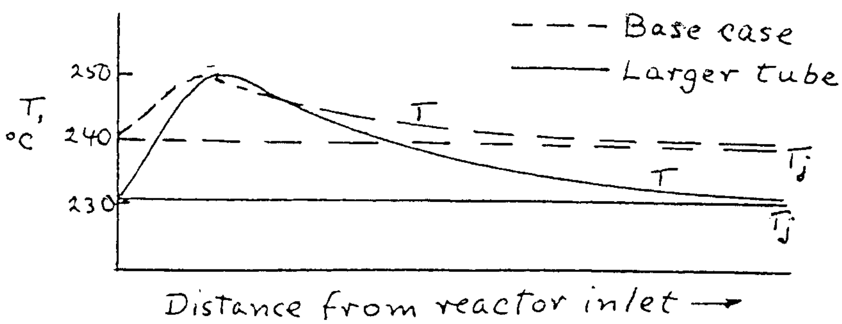
At the same maximum temperature of 250°C, the rate of heat generation is assumed to be the same in the two cases (though it would probably be slightly less in the 1 1/2-inch tube if the feed temperature were lowered). Then

$$\frac{\Delta T_2}{\Delta T_1} = \frac{q_2}{q_1}\frac{U_1 A_1}{U_2 A_2} = 1 \times \frac{2.5}{2.0} \times \frac{1.5}{1} = 1.875$$

$$\Delta T_2 = 1.875\ (250 - 240) = 18.8°C \text{ say } 19°C$$

$$T_j = 250 - 19 = 231°C$$

The temperature profiles for the two cases are as follows:



From Perry, "Chemical Engineers' Handbook, 7th ed., p. 2-305,

Sat'd steam at 240°C (464°F) is at 483 lb_f/in^2 abs

At 231°C (448°F) it is at 397 lb_f/in^2 abs

15.12 For the empty pipe, the Sieder-Tate equation [Eq. (12.33)] can be used with the viscosity term omitted, since the viscosity of a gas does not change much with temperature.

Data: $Re = 12{,}000$ $D_t = 0.05$ m

$Pr = 0.69$ (Appendix 16)

k is not needed, since it will cancel in the final comparison.

From Eq. (12.33):

$$h_i = (1/0.05)k \times 0.023 \times 12000^{0.8} \times 0.69^{1/3}$$
$$= 745.4k$$

With the packed tube, from Eq. (15.33),

$$1/h = (1/h_{bed}) + (1/h_w)$$

From Eq. (15.36),

$$h_w D_p/k = 1.94 N_{Re,p}^{0.5} N_{Pr}^{0.33}$$

Since $D_p = 12D_t/50$, $Re_p = 12000 \times 12/50$
$= 2880$

$$h_w = 1.94 \times 2880^{0.5} \times 0.69^{0.33} \times k/0.012$$
$$= 7676k$$

From Eq. (15.35),

$$k_e/k = 5 + (0.1 \times 2880 \times 0.69) = 204$$

From Eq. (15.34),

$$h_{bed} = 4 \times 204k/0.025 = 32{,}640k$$

$$h = \frac{1}{\dfrac{1}{7676k} + \dfrac{1}{32640k}} = 6217k$$

Factor of increase: $h/h_i = 6217/754.4 = 8.24$

Chapter 16

16.1. Basis: 1 hour

Water in feed: 0.92 x 20000 = 18400 kg
Solids in feed: 0.08 x 20000 = 1600
Water in concentrate: 1600 x 0.55/0.45 = 1956 kg
Evaporation: 16444 kg

Latent heat of the solution at 102 mm Hg (1.958 atm) and 51.7°C, from Appendix 7,

$$= 1022.9 \times 2326 = 2{,}379{,}000 \text{ J/kg}$$

Latent heat of steam at 120.5°C (249°F):

$$\lambda_s = 946 \times 2326 = 2{,}200{,}000 \text{ J/kg}$$

$$\Delta T = 120.5 - 51.7 = 68.8°C$$

(a) Net heat added by feed is zero.

Steam consumption:

$$\dot{m}_s = 16444 \times \frac{2{,}379{,}000}{2{,}200{,}000} = 17782 \text{ kg/h}$$

Economy: 16444/17782 = 0.925

Area: $$A = \frac{16444}{3600} \times \frac{2{,}379{,}000}{2800 \times 68.8} = 56.4 \text{ m}^2$$

(b) Net heat that must be added to the feed:

$$\frac{20000}{3600} \times 3.77 \times 1000 \times (51.7 - 21.1)$$

$$= 640{,}900 \text{ W}$$

Heat of vaporization:

$$\frac{16444}{3600} \times 2{,}379{,}000 = 10{,}866{,}700 \text{ W}$$

Total: 640,900 + 10,866,700 = 11,507,600 W

Steam consumption:

$$m_s = \frac{11{,}507{,}600}{2{,}200{,}000} = 5.23 \text{ kg/s or } 18{,}831 \text{ kg/h}$$

Economy: 16444/18831 = 0.873

Area: $$A = \frac{11{,}507{,}600}{2800 \times 68.8} = 59.7 \text{ m}^2$$

(c) Net heat brought in by feed:

$$\frac{20000}{3600} \times 3.77 \times 1000 \times (51.7 - 93.3)$$

$$= - 871{,}300 \text{ W}$$

Heat of vaporization (as in (b)):

$$= 10{,}866{,}700 \text{ W}$$

Net: 10,866,700 - 871,300 = 9,995,400 W

Steam consumption:

$$m_s = \frac{9{,}995{,}400}{2{,}200{,}000} = 4.54 \text{ kg/s or } 16{,}356 \text{ kg/h}$$

Economy: 16444/16356 = 1.005

$$\text{Area: } A = \frac{9{,}995{,}400}{2800 \times 68.8} = 51.9 \text{ m}^2$$

16.2. Steam temperature, from Appendix 7,

T_s = 200.6°F or 93.7°C

Feed temperature = 60°F or 15.6°C

Boiling temperature, T, at 14.69 x 100/760 = 1.93 $lb_f/in.^2$ is 124°F or 51.1°C

Latent heat of steam at 200.6°F:

$_s$ = 977.6 x 2326 = 2,273,900 J/kg

Latent heat of solution at 124°F:

= 1023 x 2326 = 2,379,500 J/kg

c_p = 0.93 x 4187 = 3894 J/kg-°C

T = 93.7 - 51.1 = 42.6°C

U = 1700 W/m²-°C

Basis: 100 kg/h of feed

Water in feed:		85 kg
Solids in feed:		15 kg
Water in concentrate:	15 x 50/50 =	15 kg
Water evaporated:	85 - 15 =	70 kg

Total heat load:

$$q = \frac{70 \times 2379500}{3600} + \frac{100 \times 3894 \times (51.1 - 15.6)}{3600}$$

= 50,100 W for 70 kg evaporated

Heat load for evaporation of 25,000 kg/h:

$$q = 25000 \times 50100/70 = 17{,}900{,}000 \text{ W}$$

Area required:

$$A = 17{,}900{,}000/(1700 \times 42.6) = 247 \text{ m}^2$$

Steam consumption:

$$\frac{17{,}900{,}000}{2{,}273{,}900} = 7.872 \text{ kg/s or } 28{,}340 \text{ kg/h}$$

16.3. Steam temperature: $T_s = 290.9^\circ F$ or $143.8^\circ C$
Pressure in vapor head at $40^\circ C$: 1.081 $lb_f/in.^2$
BPE of 65% NaOH (Fig. 16.3): $125^\circ F$ or $69.4^\circ C$
Temperature in evaporator: $40 + 69.4 = 109.4^\circ C$

$$T = 143.8 - 109.4 = 34.4^\circ C$$

Enthalpies (from Appendix 7)

Vaporization of steam	917.6	Btu/lb
Vapor from solution (Appendixes 7 and 15) $1121.7 + (125 \times 0.45) =$	1178	Btu/lb
Feed solution, H_f (Fig. 16.6)	105	Btu/lb
Conc. solution, H (Fig. 16.6)	330	Btu/lb
Heat of superheat in vapor $125 \times 0.45 =$	56.3	Btu/lb

Flow rates

Feed:	60000 kg/h
Conc. liquor: $60000 \times 0.44/0.65 =$	40615 kg/h
Evaporation: $60000 - 40615 =$	19385 kg/h

Enthalpy balance

Out with vapor: $19385 \times 2.204 \text{ kg/lb} \times 1178$
$$= 50.33 \times 10^6 \text{ Btu/h}$$

Out with conc. liquor: $40615 \times 2.204 \times 330$
$$= 29.54 \times 10^6 \text{ Btu/h}$$

Total out: 79.87×10^6 Btu/h

In with feed: $60000 \times 2.204 \times 105$
$$= 13.89 \times 10^6 \text{ Btu/h}$$

Net from steam: $(79.87 - 13.89) \times 10^6$
$$= 65.98 \times 10^6 \text{ Btu/h}$$

(a) Steam consumption:

$$65.98 \times 10^6/917.6 = 71900 \text{ lb/h or } 32625 \text{ kg/h}$$

(b) Area required:

$$65.98 \times 10^6 \times 0.29307/(2000 \times 34.4) = 281 \text{ m}^2$$

16.4. (a) From Eq. (4.51), $Re = 4\Gamma/\mu$

$\Gamma = 3000/0.1\pi = 9549$ kg/m-h

$$Re = \frac{4 \times 9549}{2.5 \times 10^{-3} \times 3600} = 4244; \text{ turbulent flow}$$

If flow were laminar, the film thickness would be obtained from Eq. (4.50) with $\cos\beta = 1.0$

$$\delta = \left(\frac{3\mu\Gamma}{\rho^2 g}\right)^{1/3}$$

$$= \left(\frac{3 \times 2.5 \times 10^{-3} \times 9549}{998^2 \times 9.80665 \times 3600}\right)^{1/3} = 1.27 \times 10^{-3} \text{ m}$$

Since the film thickness increases with about the 0.6 power of the flow rate for $Re > 1200$, the actual thickness is greater than for laminar flow (see p. 81).

The correction factor is

$$\frac{4244}{1200}^{(0.6 - 0.33)} = 1.41$$

At $Re = 4244$, $\delta = 1.27 \times 10^{-3} \times 1.41$

$= 1.79 \times 10^{-3}$ m

With no evaporation, the residence time is

$$t = \frac{\text{holdup}}{\text{feed rate}} = \frac{\pi DL\delta}{\pi D\Gamma/\rho} = \frac{\rho L\delta}{\Gamma}$$

$$= \frac{998 \text{ kg/m}^3 \times 6 \text{ m} \times 1.79 \times 10^{-3} \text{ m}}{(9549/3600) \text{ kg/m-s}}$$

$= 4.0$ s

(b) $q = UA\Delta T$ $\quad \Delta T = 90°C$ $\quad U = 3500$ W/m^2-°C

$\lambda = 970 \times 2326 = 2.256 \times 10^6$ J/kg (est.)

Area per tube: $A = 6 \times 0.1\pi = 1.885$ m^2

Evaporation per tube:

$\dot{m} = q/\lambda = 3500 \times 1.885 \times 90/(2.256 \times 10^6)$

$= 0.263$ kg/s or 947 kg/h

% evaporated: $947 \times 100/3000 = 31.6\%$

(c) For the same total amount evaporated:

Liquid flow at bottom of tube = 3000 kg/h

At top of tube: $3000 - 947 = 2053$ kg/h

Average flow: $(3000 + 2053)/2 = 2526.5$ kg/h

The total volume of one tube is

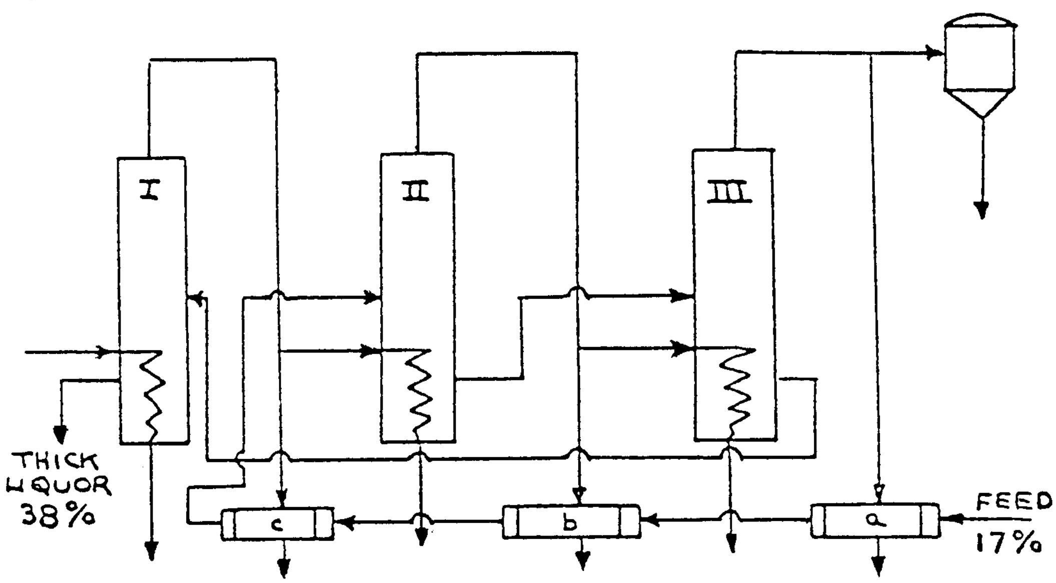
$$0.1 \times 6 \times \pi/4 = 0.0471 \text{ m}^3$$

If the liquid occupies 2/3 of this, the average residence time is:

$$t = 0.0471 \times (2/3) \times 998/(2526.5/3600) = 44.7 \text{ s}$$

Correcting the residence time in the falling-film evaporator (part (a)) for the decrease in liquid rate would increase the residence time to 4 1/2 or 5 seconds, still about an order of magnitude smaller than in an upflow evaporator.

16.5.



BPE: I = 15°F; II = 1°; III = 5°

<u>Basis: 1 hour</u>

Sp. Gr. of 17% solution: 1.041

Feed $= 35000 \times 8.33 \times 1.041 = 303{,}500$ lb

Thick liquor $= 303500 \times (0.17/0.38) = \underline{135{,}800}$

Evaporation $= 167{,}700$ lb

Temperatures:

Steam	=	230°F
Condenser (at 2 in. Hg abs)	=	101
Total ΔT	=	129°F
$\Sigma BPE = 1 + 5 + 15$	=	21
Net ΔT	=	108°F

Assume equal evaporation in each effect.

Evaporation per effect= 167700/3 = 55,900 lb/h

$$\text{Concentration in II} = \frac{0.17 \times 303500}{3035\text{-}0 - 55900} = 0.208$$

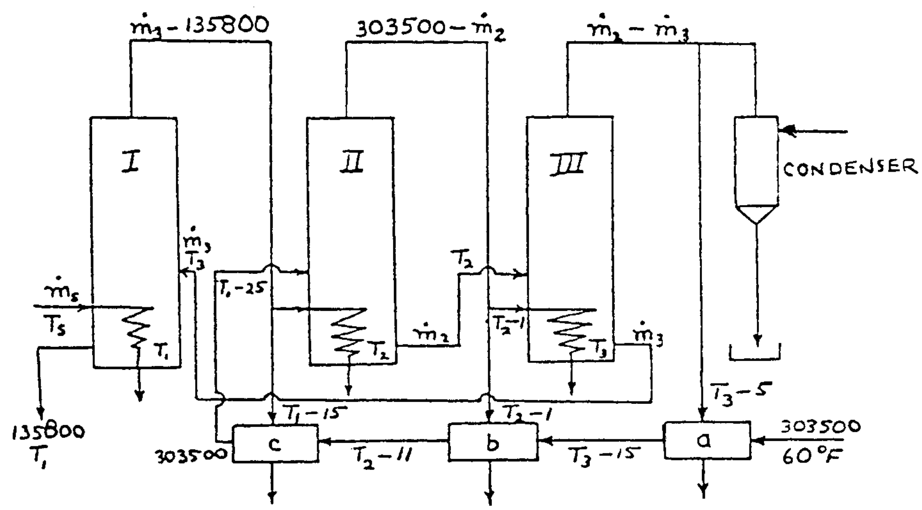
$$\text{Concentration in III} = \frac{0.17 \times 303500}{303500 - 2(55900)} = 0.269$$

Let $\dot{m}_2$ = flow rateof liquor from II to III, lb/h

$\dot{m}_3$ = flow rate of liquor from III to I, lb/h

$\dot{m}_s$ = flow rate of steam to I, lb/h

Enthalpy-balance diagram:



Assume $\Delta T_1 = 49°F$; $\Delta T_2 = 25°F$; $\Delta T_3 = 34°F$ (these are first estimates). From Appendix 7 and Table 16.5

Stream	Concen-tration	Temp., °F	λ, Btu/lb	c_p
Steam to I	--	230	$959 = \lambda_s$	--
Liquor in I	0.38	$181 = T_1$		--
Steam to II	--	166	$998 = \lambda_1$	--
Liquor in II	0.208	$141 = T_2$		0.93
Steam to III	--	140	$1014 = \lambda_2$	--
Liquor in III	0.269	$106 = T_3$		0.90
Steam to condenser	--	101	$1036 = \lambda_3$	--
Feed to II	0.17	156		0.95

Heat balances, assuming that λ for the solution = λ for the steam in the succeeding effect:

Over I:

$$959\,\dot{m}_s = (\dot{m}_3 - 135{,}800)\,998 + \dot{m}_3\,(181 - 106)\,0.90$$

$$\dot{m}_s = 1.111\,\dot{m}_3 - 141{,}300$$

Over II + c:

$$(\dot{m}_3 - 135800)\,998 = (303500 - \dot{m}_2)\,1014 + (303500 \times (141-156) \times 0.95) + (303500 \times (156-130)\,0.95)$$

$$\dot{m}_3 + 1.016\,\dot{m}_2 = 447{,}300$$

Over III + b:

$$(303500 - \dot{m}_2)\,1014 = (\dot{m}_2 - \dot{m}_3)\,1036 + \dot{m}_2\,(106-141)\,0.93 + 303500\,(130-91)\,0.95$$

$$\dot{m}_3 = 1.947\,\dot{m}_2 - 286{,}200$$

From these, $\dot{m}_2 = 247600$; $\dot{m}_3 = 195800$; $\dot{m}_s = 76230$

$$q_1 = 76230 \times 959 = 73{,}100{,}000 \text{ Btu/h}$$

$$A_1 = 73{,}100{,}000/(450 \times 49) = 3315 \text{ ft}^2$$

Heat transfer heaters:

$q_b = 303500 \times 0.95 \times (130 - 91) = 11{,}240{,}000$ Btu/h

$q_c = 303500 \times 0.95 \times (156 - 130) = 7{,}500{,}000$

$q_2 = (195800 - 135800)\ 998 - 7{,}500{,}000 = 52{,}380{,}000$

$A_2 = 52{,}380{,}000/(700 \times 25) = 2993\ ft^2$

$q_3 = (303500 - 247600)\ 1014 - 11{,}240{,}000$
$= 45{,}440{,}000$ Btu/h

$A_3 = 45{,}440{,}000/(500 \times 34) = 2673\ ft^2$

Average area = $2994\ ft^2$

Correct temperature drops in proportion to areas.

$\Delta T_1 = 49 \times 3315/2994 = 54$ (reduce to 53)

$\Delta T_2 = 25$

$\Delta T_3 = 34 \times 2673/2994 = 30$

NOTE: $\Delta T_1 + \Delta T_2 + \Delta T_3 = \Delta T = 108$

Stream	Concentration	Temp., °F	λ, Btu/lb	c_p
Steam to I	--	230	959	
Liquor in I	0.38	177	--	
Steam to II	--	162	1000	
Liquor in II	0.208	137	--	0.93
Steam to III	--	136	1016	
Liquor in III	0.269	106	--	0.90
Steam to condenser	--	101	1036	
Feed to II	0.17	152	--	0.95

Heat balances:

Over I:

$$959\ \dot{m}_s = (\dot{m}_3 - 135800)\ 1000 + \dot{m}_3\ (177-106)\ 0.90$$

$$\dot{m}_s = 1.109\ \dot{m}_3 - 141{,}600$$

Over II + c:

$$(\dot{m}_3 - 135800)\ 1000 = (303500 - \dot{m}_2)\ 1016 + (303500(137-152) \times 0.95) + (303500(152-126)\ 0.95)$$

$$\dot{m}_3 + 1.016\ \dot{m}_2 = 447{,}300$$

Over III + b:

$$(303500 - \dot{m}_2)\ 1016 = (\dot{m}_2 - \dot{m}_3)\ 1036 + \dot{m}_2\ (106-137)\ 0.93 + 303500\ (126-91)\ 0.95$$

$$m_3 = 1.953 m_2 - 288{,}100$$

From these:

$$\dot{m}_2 = 247{,}700 \qquad \dot{m}_3 = 195{,}600$$

$$\dot{m}_s = 75{,}320$$

$q_1 = 75320 \times 959 = 72{,}230{,}000$ Btu/h

$A_1 = 72{,}230{,}000/(450 \times 53) = 3029\ \text{ft}^2$

Heat transfer in heaters:

$q_b = 303500 \times 0.95(126 - 91) = 10{,}090{,}000$

$q_c = 303500 \times 0.95(152 - 126) = 7{,}500{,}000$

$q_2 = ((195600 - 135800) \times 1000) - 7{,}500{,}000$
$= 52{,}300{,}000$ Btu/h

$A_2 = 52{,}300{,}000/(700 \times 25) = 2988\ \text{ft}^2$

$q_3 = ((303500 - 247700) \times 1016) - 10{,}090{,}000$
$= 46{,}600{,}000$ Btu/h

$A_3 = 46{,}600{,}000/(500 \times 30) = 3107\ \text{ft}^2$

Average area = 3041 ft^2. The individual areas are all close enough to the average area. The answers are:

(a) Steam consumption: 75,320 lb/h

(b) Area per effect: 3041 ft^2

(c) Economy: 167,700/75,320 = 2.23 lb/lb

(d) Latent heat removed in condenser:

$$((247{,}700 - 195{,}400) \times 1036) - (303{,}500 \times 0.95(91 - 60))$$
$$= 45{,}040{,}000\ \text{Btu/h}$$

16.6 Use Eq. (15.21).

$$h_i = 2\left(\frac{k\rho c_p nB}{\pi}\right)^{1/2}$$

$$n = 400 \times 60 = 24{,}000 \text{ r/h} \qquad B = 4$$

$$h_i = 2 \times (0.15 \times 60 \times 0.3 \times 24000 \times / \ )^{1/2}$$

$$= 574 \text{ Btu/h-ft}^2\text{-}^\circ\text{F}$$

From Table 16.1, U for 100-cP liquids typically equals 300 Btu/h-ft^2-°F, roughly half the estimated value of h_i. Much of the difference may be in the metal-wall resistance, which may be fairly large in an agitated-film evaporator (for a 1/4-in. thick stainless steel wall the resistance is about 1/450 h-ft^2-°F/Btu). Also, as noted on p. 456, Eq. (15.21) predicts higher individual coefficients than are actually observed because the heated liquid is not all well mixed with the bulk of the fluid but is partly redeposited behind the scraper blades.

16.7. (a) Basis: 100 lb feed solution with 75 lb H_2O, 25 lb NaOH

Product 25 lb NaOH, 25 lb H_2O, 50 lb total

Evaporation: 75-25 = 50 lb

Remove 50/3 = 16.67 lb H_2O in each effect

Feed is backward, which means feed to No 3. The flow out of each effect is:

Effect	1	2	3
lb NaOH	25	25	25
lb H_2O	25	41.66	58.33
Total	50	66.66	83.33
% NaOH	50	37.5	30.0

Get the boiling point rise from Fig. 16.3 assuming 225, 175 and 120°F as the boiling temperatures of water at the pressures in effects 1, 2, and 3.

Effect	1	2	3
T_{H_2O}, °F	225	175	120
T, °F	300	215	143
BPE, °F	75	40	23

Net ΔT for heat transfer = (320-120) - (75+40+23) = 62°

If ΔT is about same for each effect,

$\Delta T_1 = 21$, $\Delta T_2 = 21$, $\Delta T_3 = 20$

$T_1 = 320-21 = 299°F$, $T_{sat.vapr} = 299-75 = 224°F$

$T_2 = 224-21 = 203°F$, $T_{sat.vapr} = 203-40 = 163°F$

$T_3 = 163-20 = 143°F$, $T_{sat.vapr} = 143-23 = 120°F$

Since the boiling point elevation varies only slightly with temperature, the vapor temperature of 224 and 163°F are close enough to the assumed values of 225 and 175°F and the calculation need not be repeated.

(b) If 4 effects are used and 12.5 lb H_2O removed in each effect, the flows are

Effect	1	2	3	4
lb NaOH	25	25	25	25
lb H_2O	25	37.5	50	62.5
Total	50	62.5	75	87.5
% NaOH	50	40	33.3	28.6
Approx T_{H_2O}, °F	225	175	150	120
T, °F	300	224	178	137
BPE, °F	75	49	28	17

Net ΔT for heat transfer = 320-120-(75+49+28+17) = 31°F

Operation is possible, but ΔT per effect is only 31/4 = 7.8°F.

A similar calculation for 5 effects shows boiling point elevations of about 75, 50, 37, 22, and 15 for a total of 199°F. Since this is essentially the same as the overall driving force of 200°F, the production rate would be zero.

The maximum number of effects is 4.

16.8. Basis: 100 kg feed

Water in feed: 96 kg

Water in conc.: 0.65 x 4/0.35 = 7.43 kg

Evaporation: 96 - 7.43 = 88.57 kg

Steam temperature: 120°C

Condenser temperature: 40°C

Total ΔT: 120 - 40 = 80°C

By trial to give equal area in the three effects, find ΔT_1 = 18.5°C, ΔT_2 = 20.0°C, ΔT_3 = 41.5°C

Stream	Temperature, °C	Latent Heat*, J/g	U, W/m^2-°C
Steam	120	2200	
Liquid in I	101.5 = T_1	2249 = λ_1	2950
Liquid in II	81.5 = T_2	2303 = λ_2	2670
Liquid in III	40 = T_3	2405 = λ_3	1360
Feed to I	90		

* From Appendix 7

Per 100 kg/h feed, let $\dot{m}_s$ = kg/h steam; $\dot{m}_1$ = kg/h liquid flowing from I to II; $\dot{m}_2$ = kg/h liquid flowing from II to III.

Flow of concentrate:

4/0.35 = 11.43 kg/100 kg feed

$$2200\dot{m}_s = 224900 - 2249\dot{m}_1 + 100(101.5 - 90) \times 4.2$$

$$\dot{m}_s = 104.4 - 1.022\dot{m}_1$$

$$224900 - 2249\dot{m}_1 = 2303(\dot{m}_1 - \dot{m}_2) - 84\dot{m}_1$$

$$\dot{m}_1 = 50.34 + 0.513\dot{m}_2$$

$$2303(\dot{m}_1 - \dot{m}_2) = 2405\dot{m}_2 - 27489 - 174.3\dot{m}_2$$

$$\dot{m}_1 - 1.969\dot{m}_2 = -11.94$$

From these,

$\dot{m}_s = 30.53 \qquad \dot{m}_1 = 72.28 \qquad \dot{m}_2 = 42.77$

$q_1 = 2200 \times 30.53/3600 = 18.65$ kW

$q_2 = (100 - 72.28) \times 2249/3600 = 17.32$ kW

$q_3 = (72.28 - 42.77) \times 2303/3600 = 18.88$ kW

$A_1 = 18.65/(18.5 \times 2.950) = 0.342\ m^2$

$A_2 = 17.32/(20 \times 2.670) = 0.324\ m^2$

$A_3 = 18.88/(41.5 \times 1.360) = 0.335\ m^2$

Average area/100 kg feed = $0.334\ m^2$

Total area = $140\ m^2$

(a) Feed rate: $100 \times 140/0.334 = 41{,}920$ kg/h

(b) Steam consumption:

$41920 \times 30.53/100 = 12{,}800$ kg/h

16.9. In vapor space and feed line to compressor, $T_a = 212°F$; $p_a = 14.696\ lb_f/in.^2$

$\lambda = 970.3$ Btu/lb (Appendix 7)

$q = 20000 \times 970.3 = 19{,}406{,}000$ Btu/h

Area $A = q/U\,\Delta T$, where U is estimated from Table 13.2. A paper by Frank and Prickett, Chem. Eng. **80**(20) 107 (1973), showed that the heat flux rises very rapidly with ΔT, especially for ΔTs above 20°F. Estimate U = 225 Btu/h-ft²-°F for small ΔTs, 300 for a ΔT about 30°F, and 350 for a ΔT of 40°F. (Exact values are not important).

The work required is found from Eq. (8.27) using the following quantities:

$\gamma = 1.325 \qquad \rho_a = 1/26.8 = 0.0373\ lb/ft^3$

Then, from Eq. (8.25),

$$W_{pr} = \frac{1.325 \times 14.696 \times 144}{(1.325 - 1) \times 0.0373}\left[\left(\frac{p_b}{14.696}\right)^{1-1/1.325} - 1\right]$$

$$= 231844 \left[(p_b/14.696)^{0.2447} - 1\right] \text{ ft-lb}_f/\text{lb}$$

Power, in hp:

$$P = \frac{W_{pr}\dot{m}}{\eta} = \frac{231844 \times 20000}{3600 \times 550 \times 0.70}\left[(p_b/14.696)^{0.2447} - 1\right]$$

$$= 3345.5\left[(p_b/14.696)^{0.2447} - 1\right]$$

For various assumed values of T_b, the results are:

T_b, °F	p_b, lbf/in.^{2}	ΔT, °F	U, Btu/h-ft^2-°F	$(p_b/p_a)^{0.2447}$	P, hp	A, ft^2
220	17.188	8	225	1.0391	130.8	10781
225	18.950	13	225	1.0642	214.8	6634
230	20.780	18	225	1.0885	296.1	4791
240	24.97	28	300	1.1385	463.4	2310
250	28.82	38	350	1.1792	599.5	1459

Based on 8000 hours per year, annual electricity cost is 1.340 kW/hp x 0.03 x 8000P = 321.6P. Capital and annual costs, assuming fixed charges are 30% of capital cost, are:

T_b, °F	Capital cost, $	Fixed charges, $/yr	Electricity cost, $/yr	Total annual cost, $/yr
220	754,670	226.401	42,065	268,466
225	464,380	139,314	69,080	208.394
230	335,370	100,611	95,226	195,837
240	161,700	48,510	149,029	197,539
250	102,130	30,639	192,799	223,438

Minimum costs are when T_b is about 230 °F; at this temperature, λ = 958.8 Btu/lb. The optimum absolute pressure is 20.78 $lb_f/in.^2$

Equivalent of power input:

296.1 x 2544 Btu/hp-h = 753,278 Btu/h or

753,278/958.8 = 785.6 lb steam/h

Actual evaporation is 20000 lb/h. Based on an economy of 1.0 in each effect, the equivalent number of effects, N = 20000/785.6 = 25.5. However, the efficiency of making electricity from steam is only about 35%, so the actual equivalent amount of steam is more nearly 785.6/0.35 = 2245 lb/h and the equivalent number of effects is 8.9.

17.1 (a) Use Eq. (17.19) $B_T = 3m$

$y_i = y_{Ai} = 0.2 \qquad y_2 = y_A = 0.02$

$$D_v = 0.144 \times 3600 \times 10^{-4} = 0.0518 \text{ m}^2/\text{h}$$

$$\rho_m = \frac{1}{22.4} = 0.0446 \text{ kg mol/m}^3$$

From Eq. (17.19):

$$J_A = (0.0518/22.4)(0.2 - 0.02)/3 = 1.388 \times 10^{-4} \text{ kg mol/m}^2 - \text{h}$$

(b) Mass fluxes:

$$CO_2 = 44 \times 1.388 \times 10^{-4}$$

$$N_2 = \underline{28 \times 1.388 \times 10^{-4}}$$

$$\text{Net} = 16 \times 1.388 \times 10^{-4} = 2.221 \times 10^{-3} \text{ kg/m}^2 - \text{h}$$

(c) Velocities depend on position. Here $J_A = N_A = N_B$, since diffusion is equimolal.

Station	1	Midpoint
Mol fraction CO_2, y_A	0.2	0.11
Conc. CO_2, $c_A = y_A\rho_m$	8.92×10^{-3}	4.91×10^{-3}
u_A [from Eq. (17.2)]	1.556×10^{-2}	2.826×10^{-2}
$c_B = (1-y)\,\rho_m$	3.568×10^{-2}	3.969×10^{-2}
u_B [from Eq. (17.3)]	3.890×10^{-3}	3.497×10^{-3}

Station	2
Mol fraction CO_2, y_A	0.2
Conc. CO_2, $c_A = y_A\rho_m$	8.92×10^{-4}
u_A [from Eq. (17.2)]	1.556×10^{-1}
$c_B = (1-y)\,\rho_m$	4.371×10^{-2}
u_B [from Eq. (17.3)]	3.175×10^{-3}

Let u_o be the velocity of the observer moving in the direction of CO_2 flow. Then $(u_A - u_o)$ gives a net mass flux $(\dot{m}_A)_{eff}$ equal to that in the opposite direction $[(\dot{m}_B)]_{eff}$ corresponding to $(u_B + u_o)$.

$$(\dot{m}_A)_{eff} = 44\,(N_A)_{eff} = 44\,(u_A - u_o)c_A$$

$$(\dot{m}_B)_{eff} = 28\,(N_B)_{eff} = 28\,(u_B + u_o)c_B$$

Since $(\dot{m}_A)_{eff} = (\dot{m}_B)_{eff}$ and $N_A = u_B c_A = N_B = u_B c_B$,

$$u_o = \frac{44\,u_A c_A - 28\,u_B c_B}{28\,c_B + 44\,c_A} = \frac{N_A(44-28)}{28\,c_B + 44\,c_A}$$

From this:

Station	y_A	u_o, m/h
1	0.2	1.60 x 10^{-3}
Midpoint	0.11	1.67 x 10^{-3}
2	0.02	1.76 x 10^{-3}

(d)

Station	u_B, m/h
1	3.890 x 10^{-3}
Midpoint	3.497 x 10^{-3}
2	3.175 x 10^{-3}

(e) From Eq. (17.2)

$$(N_A)_{eff} = (u_A + u_B)c_A = u_A c_A + u_B c_A = N_A + u_B c_A$$

$$N_A = 1.388 \times 10^{-4}\ \text{kg mol / m}^2 - \text{h}\ \ \text{(from part (a))}$$

Station	$u_B c_A$	$(N_A)_{eff}$
1	1.60 x 10^{-3}	1.736 x 10^{-4}
Midpoint	1.67 x 10^{-3}	1.560 x 10^{-4}
2	1.76 x 10^{-3}	1.416 x 10^{-4}

17.2 Use Eq. (17.24)

$$A = \pi \times 8^2/4 = 50.265 \text{ m}^2 \qquad B = 0.005 \text{ m}$$

$$y_A = 0 \qquad y_{Ai} = 20/760 = 0.0263 \qquad \text{Mol. Wt.} = 60.09$$

For n-propanol at 0° C (from Perry, 6th ed., p. 3-257):

$$D_v = 0.085 \times 3600 \times 10^{-4} = 0.0306 \text{ m}^2/\text{h}$$

From Eq. (17-28) and Appendix 19, at 25° C

$$\in/k = 576.7 \qquad \sigma = 4.678 \qquad kT/\in = 298/576.7 = 0.516$$

$$\text{At } 0^\circ \text{ C}, \quad kT/\in = 273/576.7 = 0.473$$

Then $\Omega_D = 2.041$ at 25°C, and 2.124 at 0°C

$$\text{at } 25^\circ \text{ C}, \; D_v = 0.0306 \times (298/273)^{3/2} \times (2.124/2.041) = 0.0363$$

$$\text{Also}, \; \rho_m = 273/(298 \times 22.4) = 0.0409 \text{ kg mol/m}^3$$

From Eq. (17.24)

$$N_A = \frac{0.0363 \times 0.0409}{0.005} \ln \frac{1}{1 - 0.0263} = 0.00791 \text{ kg mol/h-m}^2$$

$$\text{or } 0.00791 \times 60.09 = 0.476 \text{ kg/h-m}^2$$

$$\rho_{liq} = 0.80 \times 1000 = 800 \text{ kg/m}^3$$

$$\text{Cost} = \frac{0.476 \times 50.265 \times 24 \times 1.20}{800 \times 10^{-3}} = \$861 \text{ per day}$$

Recovery of the approximately $300,000 annual value of the propanol should be economically worthwhile.

17.3 Use Eq. (17.24) for unicomponent diffusion.

$$A = 10 \text{ m}^2$$

At 25° C, $D_v = 0.15 \times 3600/10^4 = 0.054 \text{ m}^2/\text{h}$

At 760 mm Hg, 97° C,

$$\rho_m = 273/(370 \;/ 22.4) = 0.0329 \text{ kg mol}/\text{m}^3$$

At 97° C, using Eq. (17.28), but with an exponent of 1.75

$$D_v = 0.54 \times (370/298)^{1.75} = 0.079 \text{ m}^2/\text{h}$$

$$y_A = 0.80 \qquad y_{Ai} = 0.10 \qquad B_T = 1 \times 10^{-4} \text{ m}$$

Rate of diffusion to the interface, from Eq. (17.24),

$$N_A = \frac{0.079 \times 0.0329}{10^{-4}} \ln \frac{1-0.10}{1-0.80} = 39.09 \text{ kg mol}/\text{h - m}^2$$

or $39.09 \times 10 \times 46 = 17{,}980$ kg alcohol / h

17.4 Use Eq. (17.19) for equimolal diffusion. From Problem 17.3:

$$D_v = 0.079 \times (368/370)^{1.75} = 0.0783 \text{ m}^2/\text{h}$$

$$A = 10 \text{ m}^2 \qquad B_T = 1 \times 10^{-4} \text{ m}$$

At 95° C,

$$\rho_m = 273/(368 \times 22.4) = 0.0331 \text{ kg mol}/\text{m}^3$$

$$y_A = 0.80 \qquad y_{Ai} = 0.10$$

$$N_A = 0.0783 \times 0.0331(0.80 - 0.10)/10^{-4}$$

$$= 18.13 \text{ kg mol}/\text{h - m}^2$$

or $18.13 \times 10 \times 46 = 8340$ kg alcohol / h

and $18.13 \times 10 \times 18 = 3263$ kg water / h

17.5 From Eq. (17.46)

$$N_A = \frac{k_y\left(y_{Ai} - y_A\right)}{\left(1 - y_A\right)_L} = k_y \ln\frac{1 - y_A}{1 - y_{Ai}}$$

From Eq. (17.66)

$$k_y = 0.023\frac{\left(D_v \rho_m\right)}{D}\left(\frac{DG}{\mu}\right)^{0.81}\left(Sc\right)^{0.44}$$

Let subscript 1 refer to water, subscript 2 to butyl alcohol. Then, since D and μ are constant,

$$\frac{N_{A2}}{N_{A1}} = \frac{\left(k_2 \ln\frac{1-y_2}{1-y_{i2}}\right)}{\left(k_1 \ln\frac{1-y_1}{1-y_{i1}}\right)} = \left(\frac{D_{v2}\,\rho_{m2}}{D_{v1}\,\rho_{m1}}\right)\left(\frac{G_2}{G_1}\right)^{0.81}\left(\frac{Sc_2}{Sc_1}\right)^{0.44}\left(\frac{\ln\frac{1-y_2}{1-y_{i2}}}{\ln\frac{1-y_1}{1-y_{i1}}}\right)$$

The diffusivities and Schmidt numbers for 0° C and 1 atm are obtained from Appendix 18. Since only the ratios are used, the values do not have to be converted to the operating conditions

$$D_{v1} = 0.853\ \text{ft}^2/\text{h} \qquad Sc_1 = 0.60$$
$$D_{v2} = 0.273\ \text{ft}^2/\text{h} \qquad Sc_2 = 1.88$$

From Eq. (17.28), D_v varies inversely with pressure at a constant temperature. Since ρ_m varies directly with pressure, the product $D_v\rho_m$ is independent of pressure at a given temperature.

$$G_2/G_1 = 100/120 = 0.833$$
$$y_{i2} = 54.5/820 = 0.0665$$
$$y_2 = 30.5/820 = 0.0372$$
$$y_{i1} = 138/518 = 0.2664$$
$$y_1 = 76/518 = 0.1467$$

$$\frac{N_{A2}}{N_{A1}} = \frac{0.273}{0.853} \times 0.833^{0.81} \times \left(\frac{1.88}{0.60}\right)^{0.44} \left(\frac{\ln\frac{1-0.0372}{1-0.0665}}{\ln\frac{1-0.1467}{1-0.2664}} \right) = 0.0933$$

Molecular weight of n-butyl alcohol = 74

Vaporization rate of n-butyl alcohol:

$$13.1 \times 0.0933 \times \frac{74}{18} = 5.02 \ \text{g/min}$$

NOTE: If the exponents in Eq. (17.62) are used, (neglecting the term $(\mu/\mu_w)^{0.14}$:

$$N_{A2}/N_{A1} = 0.0933 \times (0.833^{0.8} / 0.833^{0.81}) \times$$
$$(1.88/0.60)^{1/3} / (1.88/0.60)^{0.44} = 0.0828$$

Vaporization rate:

$$5.02 \times 0.0828 / 0.0933 = 4.46 \text{ g/min}$$

17.6 Use Eq. (17.74) to find k_c (and k_y)

From Appendix 8:

$$\mu = 0.0183 \times 10^{-3} = 1.83 \times 10^{-5} \text{ kg/m-s}$$
$$D_p = 0.5/39.37 = 0.0127 \text{ m}$$

$$\rho = (29 \times 273 \times 2)/(22.4 \times 313) = 2.258 \text{ kg/m}^3$$

$$\rho_m = 2.258/29 = 0.0779 \text{ kg mol/m}^3$$

$$\overline{V} = 2 \text{ m/s} \quad G = \overline{V}\rho = 2 \times 2.258 = 4.516 \text{ kg/s-m}^2$$

$$\text{Re} = 0.0127 \times 4.516 / 1.83 \times 10^{-5} = 3134$$

From Appendix 18, D_V at 0° C = 0.199 ft^2/h

or $0.199/(3.2808^2 \times 3600) = 5.14 \times 10^{-6} \text{ m}^2/\text{s}$

At 40°C and 2 atm, From Perry, 7th ed., p. 5-48, Eq. (5-196),

$$D_v \,\alpha\, T^{1.75} \times \frac{1}{P}$$

Hence,

$$D_v = 5.14 \times 10^{-6} \times (313/273)^{1.75} \times 1/2 = 3.26 \times 10^{-6}\ \mathrm{m^2/s}$$

$\overline{M} = 29$, assuming the naphthalene content of the air is negligible.

$$Sc = 1.83 \times 10^{-5} / 2.258 \times 3.26 \times 10^{-6} = 2.49$$

From Eq. (17.74)

$$k_c = 1.17\,(D_v/D_p)\,Re^{0.585}\,Sc^{1/3}$$

$$= \frac{1.17 \times 3.26 \times 10^{-6}}{0.0127} \times 3134^{0.585} \times 2.49^{1/3}$$

$$= 0.0452\ \mathrm{m/s},\ \text{or}\ 162.6\ \mathrm{m/h}$$

From Eq. (17.38)

$$k_y = k_c \rho_m = 162.6 \times 0.0779$$

$$= 12.67\ \text{kg mol/h-m}^2 - \text{unit mole fraction}$$

Basis: 1 m^3 of packed bed

Volume of solid = 0.6 m^3

Area/volume for spheres: $6 / D_p$

Surface area: 0.6 x 6 / 0.0127 = 283.5 m^2

$$y_A = 0 \qquad y_{Ai} = 0.35/(2 \times 760) = 0.00023$$

From Eq. (17.46) for unicomponent diffusion, as in Problem 17.5,

$$N_A = k_y \ln\frac{1-y_A}{1-y_{Ai}} = 12.67 \ln\frac{1}{1-0.00023} = 0.00291\ \text{kg mol/h-m}^2$$

Mol. wt. of naphthalene: 128.2

Initial rate of evaporation:

$$0.00291 \times 283.5 \times 128.2 = 105.8 \text{ kg / h}$$

17.7 Use Eq. (17.19). Assume the pressure is 1 atm, and that y_A in the air is zero. Then

$$y_A = 0 \qquad y_{Ai} = 0.05 \qquad B_T = 1 \text{ cm}$$

$$D_v = 0.071 \text{ cm}^2 / \text{s} \quad A = \pi \times (0.2)^2 / 4 = 0.03142 \text{ cm}^2$$

$$\rho_m = (1/22400)(273/294) = 4.145 \times 10^{-5} \text{ g mol / cm}^3$$

From Eqs. (17.39) and (17.42),

$$k_y = k_c \rho_m = \left(\frac{D_v \rho_m}{B_T} \right) = 0.071 \times 4.145 \times 10^{-5} = 2.94 \times 10^{-6}$$

Use Eq. (17.46)

$$(1 - y_A)_L = \frac{(1-0)-(1-0.05)}{\ln \dfrac{1}{0.95}} = 0.975$$

$$N_A = 2.94 \times 10^{-6} (0.05 - 0) / 0.975 = 1.51 \times 10^{-7} \text{ g mol / s - cm}^2$$

Mol. Wt. of n-heptane: $M_A = 100.2$

Mass rate of evaporation:

$$\dot{m} = N_A A M_A = 1.51 \times 10^{-7} \times 0.03142 \times 100.2$$
$$= 4.75 \times 10^{-7} \text{ g / s}$$

Volume rate of evaporation:

$$q = \dot{m}/\rho = 4.75 \times 10^{-7}/0.66 = 7.20 \times 10^{-7}\ \mathrm{cm^3/s}$$

Rate of decrease of liquid level:

$$\frac{dZ}{dT} = \frac{q}{A} = \frac{7.20 \times 10^{-7}}{0.03142} = 2.29 \times 10^{-5}\ \mathrm{cm/s}$$

$$\text{or,}\quad 0.0824\ \mathrm{cm/h}$$

With a small tube there are errors because of the highly curved meniscus, which makes the vapor pressure less than for a flat surface and makes the diffusion distance less than the nominal distance of 1 cm for the region near the wall. With a large tube these errors are decreased, but there is more chance for natural convection and external turbulence to disturb the concentration profile.

17.8 Since circulation in the bubble is rapid, the penetration theory applies (see p. 539). Use Eq. (17.75) to find k_c. Data required are:

$$D_p = 0.4\ \mathrm{cm} \qquad u_o = 27\ \mathrm{cm/s}\ \text{(From Fig. 7.8)}$$

$$D_v = 2.5 \times 10^{-5}\ \mathrm{cm^2/s}\ \text{at } 25^\circ\ \mathrm{C}\ \text{(Perry, 6th ed., p.3 - 259)}$$

$$\mu = 1.01\ \mathrm{cP\ at\ 20^\circ\ C},\ 0.90\ \mathrm{cP\ at\ 25^\circ\ C}$$

At 20° C, using Eq. (17.31)

$$D_v = 2.5 \times 10^{-5}(293/298) \times 0.90/1.01 = 2.19 \times 10^{-5}\ \mathrm{cm^2/s}$$

From Eq. (17.75)

$$k_c = 2\sqrt{\frac{2.19 \times 10^{-5} \times 27}{\pi \times 0.4}} = 4.34 \times 10^{-2}\ \mathrm{cm/s}$$

To find the fraction absorbed, the distance traveled is 1 m. The time of travel is

$$t_T = 100 / 27 = 3.70 \text{ s}$$

Neglect the change in pressure with depth and assume P = 1 atm.

From Perry, 7th ed., p. 2-127, the Henry's – law constant at 20° C is 4.01×10^4 atm / mol fraction. At 1 atm total pressure in the bubble,

$$P_{O_2} = 0.209 \text{ atm}$$

and the equilibrium mole fraction in the liquid is:

$$x_i = 0.209 / (4.01 \times 10^4) = 5.21 \times 10^{-6}$$

The equilibrium concentration at the gas liquid interface is then

$$c_{Ai} = (5.21 \times 10^{-6}) / 18.02 = 2.89 \times 10^{-7} \text{ mol / cm}^3$$

Given that $c_A = 0$; assume P_{O_2} and c_A are constant.

The mass transfer rate is:

$$N_A = k_c (c_{Ai} - c_A) =$$

$$0.0434 \times 2.89 \times 10^{-7} = 1.25 \times 10^{-8} \text{ mol / cm}^2 - \text{s}$$

The surface area of the bubble is $A = \pi \times 0.4^2 = 0.503$ cm^2

The number of moles transferred in time t_T is

$$N_A A t_T = 1.25 \times 10^{-8} \times 0.503 \times 3.70 =$$

$$2.33 \times 10^{-8} \text{ moles}$$

The number of moles originally in the bubble is found from the bubble volume

$$V_{\text{bubble}} = \pi \times (0.4)^3/6 = 0.0335\ \text{cm}^3$$

At $20^\circ\text{C}, 1\ \text{atm}$,

$$\rho_m = \frac{1}{22400} \times \frac{273}{293} = 4.16 \times 10^{-5}\ \text{mol}/\text{cm}^3$$

Moles O_2 initially in bubble:

$$0.0335 \times 4.16 \times 10^{-5} \times 0.209 = 2.91 \times 10^{-7}\ \text{moles}$$

Moles N_2 in bubble:

$$(0.791/0.209) \times 2.91 \times 10^{-7} = 11.013 \times 10^{-7}\ \text{moles}$$

Fraction of O_2 removed : $(2.33 \times 10^{-8})/(2.91 \times 10^{-7}) =$
0.080, or 8%

The removal of the oxygen lowers P_{O_2} at the end to 0.196 atm.
Basing the rate of transfer on the average value of P_{O_2}, 0.2025 atm, gives 2.26×10^{-8} moles transferred or 7.8% of the original amount.
(A negligible correction.)

17.9

$$Sh = k_c D_p / D_v = 4$$

$$k_c = 4\left(\frac{D_v}{D_p}\right)$$

Since $c_A = 0$, from Eq. (17.36) $J_A = k_c c_{Ai} = 4 D_v c_{Ai} / D_p$

The mass flux, $\dot{m}/A = M_A J_A$, where M_A is the molecular weight of benzoic acid. The rate of volume change is then

$$\frac{dV}{dt} = \frac{\dot{m}}{\rho} = \frac{4 M_A A D_v c_{Ai}}{D_p \rho} = \frac{4\pi D_p M_A D_v c_{Ai}}{D_p \rho}$$

Also,

$$\frac{dV}{dt} = -\frac{d}{dt}\left(\frac{\pi D_p^{\,3}}{6}\right) = -\frac{\pi D_p^{\,2}}{2}\frac{dD_p}{dt}$$

Hence

$$\frac{4\pi D_p^{\,2} M_A D_v c_{Ai}}{D_p \rho} = -\frac{\pi D_p^{\,2}}{2}\frac{dD_p}{dt}$$

$$dt = -\frac{\rho D_p dD_p}{8 M_A D_v c_{Ai}}$$

Integrating from $t = 0$, $D_p = D_I$ to $t = t_T$, $D_p = 0$

$$t_T = \frac{\rho}{16 M_A D_v c_{Ai}}(D_I)^2$$

The time for dissolution depends on the square of the initial size.

For $D_p = 0.01\,\text{cm}$; $T = 25^\circ\,\text{C}$; $M_A = 122.12$

$\rho = 1.265\,\text{g}/\text{cm}^3$ $\quad D_v = 1.21 \times 10^{-5}\ \text{cm}^2/\text{s}$

(From Satterfield, "Mass Transfer and Heterogeneous Catalysis, "MIT Press, 1970). Solubility at 25°C is 0.43 wt % (est. from Perry, 7th ed., p. 2-30, and the International Critical Tables). Then,

$$c_{Ai} = (0.0043/122.12)\ /1 = 3.52 \times 10^{-5}\,\text{g mol}/\text{cm}^3$$

$$t_T = \frac{1.265 \times 0.01^2}{16 \times 122.12 \times 1.21 \times 10^{-5} \times 3.52 \times 10^{-5}} = 152\,\text{sec or } 2.53\,\text{min.}$$

17.10 From Appendix 18

$$D_v = 0.853\,\text{ft}^2/\text{h at } 0^\circ\,\text{C, } 1\,\text{atm} \times 0.2581 = 0.220\,\text{cm}^2/\text{s}$$

$$T = 50^\circ\,\text{C} = 323\text{K}$$

$$P = 20\,\text{mmHg} = {}^{20}\!/_{760} = 0.0263\,\text{atm}$$

Since D_v varies with about $T^{1.75}P^{-1}$

$$D_v = 0.220\left(\frac{323}{273}\right)^{1.75}\frac{1}{0.0263} = 11.23\,\text{cm}^2/\text{s} = 11.23\times10^{-4}\ \text{m}^2/\text{s}$$

Use Appendix 8 and assume gas is mostly air

$$\mu = 0.019\,\text{cp} \;\; \text{at}\; 50^\circ\,\text{C} = 1.9 \times 10^{-5}\ \text{Pa}\cdot\text{s}$$

From gas laws

$$\rho = \frac{MP}{RT} = \frac{29(0.0263)}{82.056 \times 10^{-3}(323)} = 0.0288\,\text{kg}/\text{m}^3$$

$$Sc = \frac{\mu}{\rho D_v} = \frac{1.9 \times 10^{-5}}{0.0288 \times 11.23 \times 10^{-4}} = 0.587$$

Note that Sc for gases is almost independent of temperature.

17.11 Use Eq. (17.28) and Appendix 19

$$\text{For He, } \epsilon/k = 10.22,\ \sigma = 2.551$$

$$\text{For } CH_4, \epsilon/k = 148.6,\ \sigma = 3.758$$

$$\epsilon_{AB}/k = (10.22 \times 148.6)^{0.5} = 38.97$$

$$\sigma_{AB} = (2.551 + 3.758)/2 = 3.1545$$

$$\text{At } 20^\circ\,\text{C} = 293\text{K},\ kT/\epsilon = \frac{293}{38.97} = 7.52$$

By interpolation in the collision integral table, Ω_D=0.780

$$D_{AB} = \frac{0.001858(293)^{1.5}\left[(4+16)/(4\times 16)\right]^{0.5}}{5 \times (3.1545)^2 \times 0.780} = 0.134\,\text{cm}^2/\text{s}$$

$$\text{At } 250^\circ\text{C} = 523\text{K},\ kT/\epsilon = \frac{523}{38.97} = 13.42$$

From table, Ω_D=0.716

$$\text{by ratio, } D_{AB} = 0.134\left(\frac{523}{293}\right)^{1.5} \times \frac{0.780}{0.716} = 0.348\,\text{cm}^2/\text{s}$$

$$\frac{0.348}{0.134} = \left(\frac{523}{293}\right)^n = 2.598 = 1.785^n \qquad n = 1.65$$

The exponent of 1.65 is smaller than the value of 1.75 recommended for extrapolation of diffusivities in air. The low value of $\in / k$ for helium results in large value of $kT/\in$, and a slightly lower temperature dependence.

17.12 For water,

$$\mu = 1.794\ \text{cP at } 0°\text{C}$$
$$\mu = 2.84\ \text{cP at } 100°\text{C}$$

From the Wilke-Chang equation (Eq. 17.31)

$$\frac{D_{100}}{D_0} = \frac{373}{273} \times \frac{1.794}{0.284} = 8.63$$

From Eq. (17.32), D varies with $\mu^{1.14}$

$$\frac{D_{100}}{D_0} = \left(\frac{1.794}{0.284}\right)^{1.14} = 8.18$$

17.13 From Table 17.1,

$$\text{for } H^+,\ \lambda_+^\circ = 349.8$$
$$Na^+,\ \lambda_+^\circ = 50.1$$
$$Cl^-,\ \lambda_-^\circ = 76.3$$

From Eq. (17.33) and T=323K

$$D_{HCl} = \frac{2(8.314)(323)}{\left[\frac{1}{349.8} + \frac{1}{76.3}\right](96{,}500)^2} = 3.61 \times 10^{-5}\ \text{cm}^2/\text{s}$$

$$D_{NaCl} = \frac{2(8.314)(323)}{\left[\frac{1}{50.1} + \frac{1}{76.3}\right](96{,}500)^2} = 1.74 \times 10^{-5}\ \text{cm}^2/\text{s}$$

17.14 For nitrobenzene

$$C_6H_5NO_2,\ \rho_1 = 1.203\ \text{g/cm}^3,\ M = 123.1$$

$$\text{asume } 20^\circ\text{C} \quad \rho_2 = 0.998\ \text{g / cm}^3$$

$$\Delta\rho = 0.205\ \text{g / cm}^3$$

$$\mu_2 = 1.00\ \text{cP}$$

assumed spherical stagnant drop

guess $u_t = 10\ \text{cm / s}$

$$\text{Re} = \frac{0.1(10)(0.998)}{0.01} = 99.8$$

From Fig. 7.3,

$c_D = 1.1$

From Eq. 7.37 $u_t = \left(\frac{4g\Delta g D_p}{3c_D\rho}\right)^{0.5} =$

$$\left(\frac{4(980)(0.205)(0.1)}{3 \times 1.1 \times 0.998}\right)^{0.5} = 4.94\ \text{cm / s}$$

$$\text{revised Re} = {}^{4.94}\!/_{10} = 99.8 = 49.3$$

at Re = 49, $c_D = 1.6$, but actual Re < 49

Guess $c_D = 1.7$

$$u_t = 4.94\left(\frac{1.1}{1.7}\right)^{0.5} = 3.97\ \text{cm / s}$$

$$\text{Re} = 0.1(3.97)(0.998)/0.01 = 39.7$$

$c_D \cong 1.7$ Ok

Estimate D_v using Wilke-Chang equation (Eq. (17.31))

$$V_A = \frac{123.1}{1.203} = 102.3$$

$$D_v = \frac{7.4\times10^{-8}(2.6\times18)^{0.5}\times293}{1.0\times102.3^{0.6}} = 9.23\times10^{-6}\ \text{cm}^2/\text{s}$$

$$Sc = \frac{0.01}{0.998\times9.23\times10^{-6}} = 1086$$

From Eq. (17.71)

$$Sh = 2.0 + 0.6Re^{1/2}Sc^{1/3} = 2.0 + 0.6(21.1)^{0.5}(1086)^{1/3} = 30.3$$

$$k_c = \frac{30.3\left(9.23\times10^{-6}\right)}{0.1} = 2.80\times10^{-3}\ \text{cm/s}$$

For a circulating drip, the terminal velocity is higher than for a rigid drop, but this difference is ignored, and kc is predicted using the penetration theory and the time for the drop to fall a distance equal to its diameter

$$t = \frac{0.1\,\text{cm}}{3.97\,\text{cm/s}} = 0.0252\,\text{s}$$

From Eq. (17.50)

$$\overline{k_c} = 1.13\left(9.23\times10^{-6}/0.0252\right)^{1/2} = 2.16\times10^{-2}\ \text{cm/s}$$

The external mass transfer coefficient for the circulating drop is 7.7 times that for the rigid drop.

17.15 For water inside the fibers

$$D = 500 \times 10^{-6}\ m = 0.05\, cm$$
$$u = 0.50\, m/s$$
$$\rho = 1000\, kg/m^3$$
$$\mu = 10^{-3}\ Pa \cdot s$$
$$Re = \frac{500 \times 10^{-6}(0.50)(1000)}{10^{-3}} = 250$$

For oxygen in water,

$$D_v = 2.41 \times 10^{-5}\ cm^2/s \text{ at } 26^\circ\ C$$
$$\text{at } 20^\circ\ C, D_v = 2.41 \times 10^{-5}\left(\frac{293}{298}\right)\left(\frac{0.89}{1.0}\right) = 2.11 \times 10^{-5}\ cm^2/s$$
$$Sc = \frac{10^{-3}}{1000} \times \frac{1}{2.11 \times 10^{-9} m^2/s} = 474$$

From Eq. (17.60) and (17.61)

$$G_z' = \frac{\pi}{4} Re\, Sc \frac{D}{L} = \frac{\pi}{4}(250)(474)\frac{5 \times 10^{-4}}{1} = 46.5$$
$$Sh = 1.62\,(46.5)^{1/3} = 5.83$$
$$k_c = \frac{5.83\,(2.11 \times 10^{-5})}{0.05} = 2.46 \times 10^{-3}\ cm/s$$

17.16 From Eq. (17.57)

$$\frac{1}{K_y} = \frac{m}{k_x} + \frac{1}{k_y}$$
$$m \cong 1.0 \text{ for } NH_3 - \text{air} - \text{water system at } 20^\circ\ C$$

Eq. (17.38):

$$k_y = k_c \rho_m = k_c \rho / RT$$

Eq. (17.39):

$$k_x = k'_c \rho_x / \overline{M}$$

where k'_c refers to the liquid phase

From the penetration theory,

$$\frac{k_c}{k'_c} = \left(\frac{D_{gas}}{D_{liq}} \right)^{1/2}$$

in air $\quad D_{NH_3} = 0.216\,\text{cm}^2/\text{s at } 0^\circ\text{C}, 1\,\text{atm (Appendix 18)}$

$$\cong 0.216 \left(\frac{293}{273} \right)^{1.75} = 0.244\,\text{cm}^2/\text{s at } 20^\circ\text{ C}, 1\,\text{atm}$$

in water,

$$D_{NH_3} = 1.64 \times 10^{-5}\ \text{cm}^2/\text{s at } 285\text{K}$$

$$D_{NH_3} \cong 2.08 \times 10^{-5}\ \text{cm}^2/\text{s at } 293\text{K}$$

$$\frac{k_c}{k'_c} = \left(\frac{0.244}{2.08 \times 10^{-5}} \right)^{1/2} = 108$$

For water,

$$\rho_x / \overline{M} = 1000/18 = 55.6\ \text{moles}/1$$

For air,

$$\rho_m = \frac{1}{22.4} \times \frac{273}{293} = 0.0416\ \text{moles}/1$$

$$k_x = \frac{k_c}{108} \times 55.6 = 0.515 k_c$$

$$k_y = k_c \rho_m = 0.0416 k_c$$

Since m=1.0,

$$\frac{1}{K_y} = \frac{1}{k_x} + \frac{1}{k_y} = \frac{1}{0.515 k_c} + \frac{1}{0.0416 k_c} = \frac{25.98}{k_c}$$

The liquid phase resistance is

$$\frac{1}{0.515 k_c} \text{ or } \frac{1.94}{k_c}$$

which is $\frac{1.94}{25.98}$ or 7.5% of the overall resistance

The gas phase has $\frac{1}{0.046} \times \frac{1}{25.98} \times 100 = 92.5\%$ of the resistance

Note that the gas resistance dominates in spite of the 10^4- fold higher diffusivity. Thee is only a 100-fold difference in the $D_v^{0.5}$ values, and the molar concentration in the liquid is over 1000 times greater.

17.17

$$\frac{1}{K_y} = \frac{m}{k_x} + \frac{1}{k_y}$$

Assume

$k_y = 1$ and $m/k_x = 5$ Liquid film "controls"

$\frac{1}{K_y} = 6 \quad K_y = 0.167$

If k_x is doubled, $m/k_x = 2.5$

$\frac{1}{K'_y} = 2.5 + 1 = 3.5 \quad K'_y = 0.286$

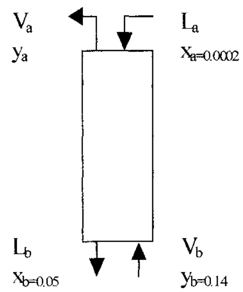
K'_y is $1.71 \times K_y$ 71% increase

If k_y is doubled, $k_y = 2$

$\frac{1}{K''_y} = 5 + 0.5 = 5.5 \quad K''_y = 0.182$

K''_y is 9% greater than K_y

Note: If the liquid-film resistance was 20 times that of the gas film, doubling k_x would increase K_y by 91 percent, and doubling K_y by only 2.5 percent.



18.1

Basis: 1 hour

$$V_b = \frac{500}{359} \times 60 = 83.57 \text{ lbmol / h}$$

$$\text{Acetone in}: 0.14 \times 83.57 = 11.70$$

$$\text{Acetone out}: 0.05 \times 11.70 = 0.585$$

$$\text{Acetone absorbed} = 11.115$$

$$\text{Moles air in}: 83.57(1 - 0.14) = 71.87$$

$$y_a = 0.585/(71.87 + 0.585) = 0.008074$$

$$L_b = L_a + 11.115$$

$$0.05 L_b - 0.0002 L_a = 11.115$$

$$L_a = 212.03 \text{ mol / h} \quad L_b = 223.145$$

(a) Water to tower:

$$212.03(1\text{-}0.0002) \times 18.02 = 3820 \text{ lb / h}$$

(b) For the operating line, use a mass balance like Eq. (18.2), based on the moles of air, V' and water, L'. (These don't change through the column). Then

$$V'\left(\frac{y}{1-y} - \frac{y_a}{1-y_a}\right) = L'\left(\frac{x}{1-x} - \frac{x_a}{1-x_a}\right)$$

From this:

$$y = F(x) / (1 + F(x))$$

where

$$F(x) = \frac{y_a}{1 - y_a} + \frac{L'}{V'}\left(\frac{x}{1-x} - \frac{x_a}{1-x_a}\right)$$

$$y_a = 0.008074 \quad x_a = 0.0002$$

$$L' = 3820 / 18.02 = 211.98 \quad V' = 71.87$$

Equilibrium relationship:

$$y^* = P'_A \gamma_A x = 0.33e^{1.95(1-x)^2} x$$

For values of x between 0.0002 and 0.08, calculate y and y* and solve Eq. (18.21) by numerical integration. The computer solution is $N_{oy} = 9.34$.

(c) Calculate the operating mass velocities from the flooding velocities based on flow rates at the bottom of the column (where both rates are greatest). The mass flow rates,

$\dot{m}_x$ and $\dot{m}_y$ are

$$\text{Liquid} : \dot{m}_x = 3820 + (11.115 \times 58.1) = 4465 \text{ lb / h}$$

$$\text{Gas} : \dot{m}_y = (71.87 \times 29) + (11.7 \times 58.1) = 2764 \text{ lb / h}$$

Use Fig 18.6

For 7% acetone in air, $\rho_y = 0.0791$. Assume $\rho_x = 60.5$

$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}} = \frac{4465}{2764}\sqrt{\frac{0.0791}{60.42}} = 0.0584$$

From Table 18.1, for 1-inch Raschig rings, F_p=155. Since this is greater than 60, the pressure drop at flooding, as recommended on p. 553, is 2.0 in. H_2O / ft. By extrapolation of the data in Fig. 18.6 to this ΔP,

$$\frac{G_y^2 F_p (\mu_x)^{0.1}}{g_c \rho_y (\rho_x - \rho_y)} = 0.115$$

For water at 80° F, μ_x=0.862 cP (Appendix 6)

$$G_y = \sqrt{\frac{0.115 \times 4.17 \times 10^8 \times 0.0791 \times (60.5 - 0.0791)}{155 \times 0.862^{0.1}}}$$
$$= 1225 \text{ lb / ft}^2 - \text{h}$$

Actual rates:

$$\text{Gas: } G_y = 1225/2 = 613 \text{ lb / ft}^2 - \text{h}$$
$$\text{Liquid: } G_x = 613 \times 4465 / 2764 = 990 \text{ lb / ft}^2 - \text{h}$$

Calculate HTU's, using Eq. (18.44) for H_x. Find D_{vx} from Eq. (17.31).

At 80° F (26.7° C), $\mu_x = 0.862$ cP or 2.086 lb / ft – h
(assuming μ is the same as for water). Also

$T = 299.7\text{ K} \quad M_B = 58.1$

ρ_B at 20° C : 0.792 g / cm^3 or 0.792 / 58.1 = 0.0136 g mol / cm^3

Estimated ρ_x at normal boiling point (56.1°C),

is 0.0128 g mol/cm^3.

Hence $V_A = 1 / 0.0128 = 78.1 \text{ cm}^3 / \text{g mol}$
(see Perry, 5th ed., p. 3 - 230).
Substitution in Eq. (17.31) gives, since
$\psi_B = 2.6$,

$$D_{vx} = \frac{7.4 \times 10^{-8} \times (2.6 \times 18)^{1/2} \times 299.7}{0.862 \times 78.1^{0.6}} = 1.29 \times 10^{-5} \text{ cm}^2 / \text{s}$$

or 5.0×10^{-5} ft^2 / h (Appendix 1)

From Fig. 18.21,

$$H_{x,O_2} = 0.68 \text{ ft}$$

For acetone in water,

$$Sc = 2.086 / (60.5 \times 5.0 \times 10^{-5}) = 689$$

For O_2, Sc=381 (see p. 582)

μ for water at 25° C: 0.90 cP (Appendix 6)

From Eq. (18.44), using n=0.3

$$H_x = 0.68\,(0.9/0.862)^{0.3}(689/381)^{0.5} = 0.93 \text{ ft}$$

To find H_y, use Fig. 18.22. for G_y =613 and G_x=990 lb / h-ft^2, by interpolation:

$$H_{y,NH_3} = 2.0 \text{ ft}$$

Use Sc for acetone-air for 0° C (Appendix 18). In view of the uncertainties in the correlations, correction to 80° F is not worthwhile. Then Sc=1.6. From Eq. (18.47)

$$H_y = 2.0 \times (1.6 / 0.66)^{0.5} = 3.11 \text{ ft}$$

Find the average slop of the equilibrium curve for use in Eq. (18.28). From the equation given in part (b),

$$\text{at } x = 0.05, \quad y^* = 0.09589$$

$$x = 0.0002, \quad y^* = 0.000464$$

$$\bar{m} = \frac{0.09589 - 0.000464}{0.05 - 0.0002} = 1.916$$

$$\bar{M}_y = [(0.008074 + 0.14)/2] \times 58.1 + 0.926 \times 29 = 31.15$$

$$\bar{M}_x = (0.0251 \times 58.1) + (0.9749 \times 18.02) = 19.02$$

$$G_M = 613 / 31.15 = 19.68 \text{ lbmol / h} - \text{ft}^2$$

$$L_M = 990 / 19.02 = 52.05$$

From Eq. (18.28)

$$H_{oy} = 3.11 + 1.916\left(19.68 / 52.05\right) \times 0.93 = 3.78 \text{ ft}$$

Column diameter $D = (4m_y / \pi G_y)^{1/2}$

$$= ((4 \times 2764) / (\pi \times 613))^{1/2} = 2.4 \text{ ft}$$

NOTES: Since the equilibrium and operating lines are nearly parallel the number of transfer units about equals the number of theoretical stages. The column height is 3.78 x 9.34 = 35.3 ft.

18.2 y_b=0.20 x_a=0 P=1 atm

Basis: 1 mole of entering gas

Moles air entering: 0.80

Moles NH_3 leaving: $0.01 \times 0.20 = 0.002$

$y_a = 0.002 / (0.80 + 0.002) = 0.00250$

Use the table in Example 18.4, p. 568, and Perry, 7th ed., p. 2-125, for equilibrium data.

(a) For minimum water rate, when y_b=0.20, from graph of y^* vs. x, $x_b = 0.118$

Basis: 1 mole of entering gas. V' =0.80 (see solution to Problem 18.1). Let $y = y_b$, $x = x_b$.

$$L'_{min}\left(\frac{0.118}{1-0.118}\right) = 0.80\left(\frac{0.2}{1-0.2} - \frac{0.0025}{1-0.0025}\right)$$

L'_{min} = 1.480 mol water / mol entering gas

(b) Actual water rate:

$L'_{min} = 1.4 \times 1.480 = 2.072$ mol water / mol entering gas.

Equation for the actual operating line, as in the solution to Problem 18.1, is:

$$2.072\left(\frac{x}{1-x}\right)=0.80\left(\frac{y}{1-y}-\frac{0.0025}{1-0.0025}\right)$$

Use Eq. (18.23). The values needed are:

x	y	y*
0.0000	0.00250	0.0000
0.0040	0.01274	0.0050
.0080	.02286	.0100
.0200	.05246	.0250
.0320	.08100	.0400
.0460	.11300	.0600
.0600	.14370	.0820
.0800	.18550	.1190
.0871	.20000	.1320

Numerical integration of 1 / (y-y*) vs. y, between y = 0.00250 and y = 0.200 gives N_{oy} = 8.4 transfer units.

18.3 Both the equilibrium and operating lines are linear functions of x and y, since the gas is dilute. Equation (18.28) becomes:

$$H_{Oy}=H_y+\frac{mG_M}{L_M}H_x=H_y+(mV/L)H_x$$

Use Eq. (18.19). From conditions of problem

$y_a^* = 0$ $\qquad y_b^* = 0.06 \times 0.08 = 0.0048$

$y_a = 0.001$ $\qquad y_b = 0.009$

By a material balance

$$V(0.009 - 0.001) = L(0.08 - 0)$$
$$V / L = 0.08/0.008 = 10$$
$$mV / L = 0.06 \times 10 = 0.6$$
$$\overline{\Delta y_L} = \frac{(0.009 - 0.0048) - (0.001 - 0)}{\ln \dfrac{0.009 - 0.0048}{0.001}}$$
$$= 0.00223$$
$$N_{Oy} = \frac{0.009 - 0.001}{0.00223} = 3.59$$

From Eq. (18.28)

$$H_{Oy} = 0.36 + (0.6 \times 0.24) = 0.504\ \text{m}$$

Packed height: $Z_t = 3.59 \times 0.504 = 1.81$ m

18.4 NOTE: This problem should be at the end of Chapter 20.

(a) Since the gas is dilute, use Eq. (20.24) to find y^*_b and x_b.

$P'_A = 1.92$ atm $\qquad\qquad$ $P = 1$ atm

Equilibrium line, from Eqs. (22.2) and (22.3)

$$y^* = P'x / P = 1.92\ x$$

Terminal concentrations:

$y_b = 0.05 \qquad x_a = 0 \qquad y_a^* = 0$

Basis: 1 mole of entering gas
Unabsorbed butane = $0.05 \times 0.05 = 0.0025$ moles

$$y_a = \frac{0.0025}{0.95 + 0.0025} = 0.00262$$

From Eq. (20.24)

$$8 = \frac{\ln \dfrac{0.05 - y_b^*}{0.00262 - 0}}{\ln \dfrac{0.05 - 0.00262}{y_b^* - 0}}$$

By trial, $y_b^* = 0.0402$

$x_b = 0.0402/1.92 = 0.0209$

By a butane material balance, as in the solution to Prob. 18.1

$$L'\left(\frac{0 - 0.0209}{1 - 0.0209}\right) = V'\left(\frac{0.00262}{1 - 0.00262} - \frac{0.05}{1 - 0.05}\right)$$

$L' / V' = 2.343$ mol butane-free oil/mol butane-free air

Molecular weight of butane = 58.12

Ratio of oil to recovered butane:

$$= 0.95 \times 0.05 \times 58.12 / \left(580 \times 10^3\right)$$
$$= 4.76 \times 10^{-6} \text{ m}^3$$

Butane-free oil:

$$\frac{2.343 \times 0.95 \times 250}{0.90 \times 998 \times 10^3} = 620 \times 10^{-6} \text{ m}^3$$

Ratio: 620 / 4.76 = 130.3 m^3 oil / m^3 recovered butane

(b) Total pressure: 3 atm

Since $y_a^* = 0$, Eq. (20.24) is not affected and $y_b^* = 0.0402$

x_b is three times that in part (a), or

$x_b = 3 \times 0.0209 = 0.0627$

$$L'\left(\frac{0.0627}{1-0.0627}\right)=V'\left(\frac{0.00262}{1-0.00262}-\frac{0.05}{1-0.05}\right)$$

$L' / V' = 0.748$

The ratio of oil to recovered butane is:

$0.748 \times 130.3 / 2.343 = 41.6$ m^3 oil / m^3 recovered butane

18.5 Basis: 1 hour

Molecular weight of benzene: 78.11

Average molecular weight of entering gas:

$$\overline{M}_b = (0.05\times 78.11)+(0.95\times 29)=31.46$$

Rate of entering gas:

$$\frac{0.30\times 273}{22.4\times 299}=0.01223 \text{ kg mol / s}$$

or 44.028 kg mol / h

Cross-sectional area of packed section:

$S = 0.01223 \times 31.46 / 1.1 = 0.35$ m^2

Diameter of packed section:

$D = (4 \times 0.35 / \pi)^{1/2} = 0.67$ m

Packed Height   Assume Raoult's Law applies.

$y_b = 0.05$

Benzene absorbed:

$N_A = 0.90 \times 0.05 \times 0.01223$
$= 5.5 \times 10^{-4}$ kg mol / s or 1.98 kg mol / h

$$y_a = \frac{0.10\times 0.05\times 0.01223}{0.01223-0.00055}=0.00524$$

Oil rate:

$$(1\text{-}0.002) \times 2000 / 230 = 8.678 \text{ kg mol / h of benzene-free oil}$$

Benzene in lean oil:

$$0.002 \times 2000 / 78.11 = 0.0512 \text{ kg mol / h}$$

$$x_a = 0.0512 / (8.678 + 0.0512) = 0.00587$$

$$x_b = \frac{1.98 + 0.0512}{8.678 + 1.98 + 0.0512} = 0.190$$

From Eqs. (22.2) and (22.3), $y_A = P_A' x_A / P$

$$y_b^* = 100 \times 0.190 / 760 = 0.025$$

$$y_a^* = 100 \times 0.00587 / 760 = 0.000772$$

<u>Column Height</u>

Find H_y and H_x from the data for ammonia-water and oxygen-water with 1 ½ - inch Raschig rings. At the tower bottom:

$$\text{Area } S = \pi \times (0.67^2) / 4 = 0.353 \text{ m}^2$$
$$G_x = (2000 + (1.98 \times 78.11)) / 0.353$$
$$= 6104 \text{ kg / h-m}^2 \text{ or } 1250 \text{ lb / h-ft}^2$$

$$G_y = (0.01223 \times 3600) \times 31.46 / 0.353$$
$$= 3924 \text{ kg / h-m}^2 \text{ or } 804 \text{ lb / h-ft}^2$$

From Fig. 18.22 at $G_y = 804$ and $G_x = 1250$, by interpolation, $H_{y,NH_3} = 1.6$ ft

From Table 18.1, for 1-inch Intalox saddles, $f_P = 1.54$

From Appendix 18, for benzene-air, Sc = 1.71

From Eq. (18.47),

$$H_y = (1.6 / 1.54)(1.71 / 0.66)^{0.5} = 1.67 \text{ ft}$$

From Fig. 18.21, for Gx = 1250,

$$H_{x,O_2} = 0.81 \text{ ft}$$

From Eq. (17.31), assuming $\Psi = 1.0$, T = 299 K,

Here $V_A = 96.5 \text{ cm}^3 / \text{mol}$ (Example 17.3, p. 522)

$$D_{v,x} = \frac{7.4\times10^{-8}\times230^{1/2}\times299}{4.0\times96.5^{0.6}} = 5.41\times10^{-6} \text{ cm}^2/\text{s}$$

Based on hexane, assume $\rho = 0.77 \text{ g/cm}^3$

$$Sc = \frac{\mu}{\rho D_v} = \frac{4\times0.01}{0.77\times5.41\times10^{-6}} = 9600$$

From Eq. (18.44), H_x varies with $(1/\mu)^{0.3}\ Sc^{0.5}$

$$H_x = \frac{0.81}{1.54}\left(\frac{1}{4}\right)^{0.3}\left(\frac{9.60\times10^3}{381}\right)^{0.5} = 1.74\text{ft}$$

Use Eq. (18.28) to find H_{oy}. When Raoult's Law holds, Eqs. (22.2) and (22.3) show that y = P'x / P; m = P' / P. Here P' = 100 mm Hg; P = 760 mm Hg. m = 0.132. At the bottom of the tower

$$G_M = 44.028 / 0.353 = 124.73 \text{ kg mol / h-m}^2$$
$$L_M = (8.678 + 1.98 + 0.0512) / 0.353 = 30.34 \text{ kg mol / h-m}^2$$

From Eq. (18.28)

$$H_{Oy} = 1.67 + \frac{0.132\times124.73\times1.74}{30.34} = 1.67 + 0.94 = 2.51 \text{ ft}$$

For N_{oy} use Eq. (18.19) with $y_b = 0.05$, $y_a = 0.00524$:

$$\overline{\Delta y_L} = \frac{(0.05-0.025)-(0.00524-0.000772)}{\ln\dfrac{0.05-0.025}{0.00524-0.000772}} = 0.01192$$

$N_{oy} = (0.05 - 0.00524) / 0.01192 = 3.76$

From Eq. (18.17)

$Z_T = 2.61 \times 3.76 = 9.81$ ft or 3.0 m

Recommended height of tower:

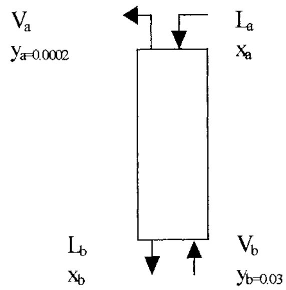
	Height, m
Base section, for gas inlet and liquid outlet	0.5
Packed section	3.0
Top section, for gas outlet, liquid inlet and distributor	0.7
Dished head top	0.2
Total	4.4

18.6 (a) Data for benzene are taken from Perry's Handbook, 6th ed., p. 3-50 for vapor pressure and p. 3-125 for H_v.

T, °C	P', mm Hg
15.4	60
26.1	100
42.2	200

$H_v = 103.57$ cal / g $\times 78.11 \times 1.8 = 14{,}560$ Btu / lb mol

By interpolation, at 25 °C, P' = 95 mm Hg
Assuming ideal solutions, p = P'x
For operation at 25 °C, $y^* = p / 760 = 95 / 760x = 0.125x$



Oil in : 14, 000 lb / h

Neglecting the small amount of benzene in oil fed,

L_a = 14000 / 250 = 56 lb mol / h

$$x_a = \frac{14000 \times 0.00015}{78.11 \times 56} = 4.80 \times 10^{-4}$$

$L_a x_a$ = 0.0269 lb mol / h

V_b = (1500 × 273) / (359 × 298) = 3.828 mol / min or 229.7 lb mol / h

$V_b y_b$ = 229.7 × 0.03 = 6.891 lb mol / h

If n mol / h are absorbed,

6.891 – n = 0.0002 (229.7 – n)
n = 6.846 lb mol / h or 534.7 lb / h

V_a = 229.7 – 6.846 = 222.85

L_a / V_a = 56 / 222.85 = 0.251

$x_b = (L_a x_a + n) / L_b$ = (0.0269 + 6.846) / (56 + 6.846) = 0.1094

y_b^* = 0.125 × 0.1094 = 0.0137

$y_b - y_b^*$ = 0.03 – 0.0137 = 0.0163

$y_a^* = 0.125 \times 4.80 \times 10^{-4} = 6 \times 10^{-5}$

$y_a - y_a^* = 2 \times 10^{-4} - 6 \times 10^{-5} = 1.4 \times 10^{-4}$

$$\overline{\Delta y_L} = \frac{0.0163 - 1.4\times10^{-4}}{\ln\left(0.0163/1.4\times10^{-4}\right)} = 3.40\times10^{-3}$$

Use Eq. (18.19) since for these low concentrations the operating line would be almost straight.

$$N_{oy} = (0.03 - (2 \times 10^{-4})) / (3.40 \times 10^{-3}) = 8.76$$

(b) For adiabatic operation, estimate the temperature rise of the liquid.

Benzene absorbed: 6.846 lb mol / h
Heat released: $6.846 \times 14560 = 99678$ Btu / h

To estimate heat capacity of oil, use data from Appendix 15 at 80 °F:

For decane,	$c_P = 0.43$
For diphenyl,	$c_P = 0.40$
For naphthalene,	$c_P = 0.38$

Assume $c_P = 0.4$ Btu / lb – °F for absorber oil

If the gas stream leaves at 25 ° C and none of the solvent evaporates, all the heat of absorption is used to heat the liquid.

$$\text{Temp. rise} = \frac{99678}{(14000 + 534.7)\times 0.4} = 17.1^\circ\text{F or } 9.5^\circ\text{C}$$

$$T_{out} = 25 + 9.5 = 34.5^\circ\text{ C} \qquad P' = 142 \text{ mm Hg}$$

$$y_b^* = (142\times0.1094)/760 = 0.0204$$

$$y_b - y_b^* = 0.03 - 0.0204 = 0.0096$$

When half the benzene has been absorbed into the liquid (subscript m = midpoint):

$$X_m = [(6.846/2) + 0.0269] / [56 + (6.846 / 2)] = 0.0581$$

Assume $T = 25 + (9.5 / 2) = 29.75\ ^{o}C$

$P' = 117$ mm Hg

$y_m^* = (117 \times 0.0581) / 760 = 0.00894$

$y_m = [6.891 - (6.846 / 2)] / [229.7 - (6.846 / 2)] = 0.0153$

$y_m - y_m^* = 0.0153 - 0.00894 = 0.00636$

Divide the column into two parts and use Eq. (18.19) for each part.

At top $\Delta y = 0.00014$
At midpoint $\Delta y = 0.00636$

$$\overline{\Delta y_L} = 0.00163$$

$$\text{Top part}: N_{Oy} = \frac{0.0153 - 0.0002}{0.00163} = 9.26$$

At midpoint $y = 0.00636$

At bottom $y = 0.00960$

$$\overline{\Delta y_L} = 0.00787$$

$$\text{Bottom}: N_{Oy} = \frac{0.03 - 0.0153}{0.00787} = 1.87$$

Total $N_{Oy} = 9.26 + 1.87 = 11.13$

NOTE: If only the terminal driving forces were used,

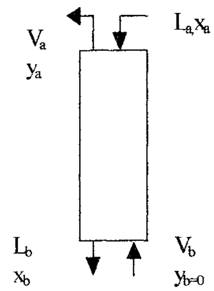
$$\overline{\Delta y_L} = 0.00224 \quad \text{and}$$

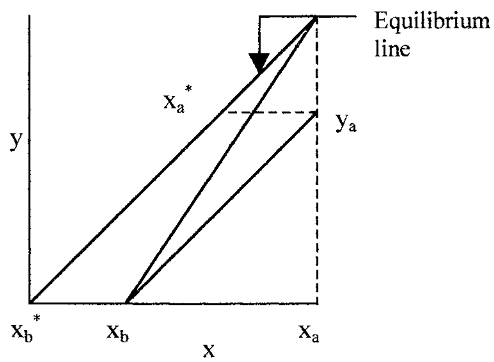
$$N_{Oy} = \frac{0.03 - 0.0002}{0.00224} = 13.3$$

This is too high because of the curvature of the equilibrium line.

(c) If a lower molecular weight oil were used, x_b would be lower for the same mass flow rate, and N_{Oy} would be smaller. If the same x_b were desired, less oil would be used. However, the oil with lower molecular weight would have a higher vapor pressure, and more oil would be lost as vapor in the absorber and in the stripping column.

18.7





Basis: 1 kg $H_2O + NH_3$

10	g NH_3 / 17 =	0.588 mol
990	g H_2O / 18 =	55.000 mol
	Total	55.588 mol

$x_a = 0.588 / 55.588 = 0.01058$

$x_b \cong 0.01 x_a \cong 0.000106$

At 20 °C, from Example 18.4, p. 568,

$y^* / x = 0.239 / 0.308 = 0.776$

g mol NH_3 removed $= 0.99 \times 0.588 = 0.582$

At the minimum air rate, $y_a = 0.776\, x_a$

$= 0.776 \times 0.01059 = 0.008218$

Since y_a = mol NH_3 removed / V_a,

$$V_a = 0.582 / 0.008218 = 70.82 \text{ g mol}$$
$$V_b = 70.82 - 0.582 = 70.238 \text{ g mol air}$$

Minimum air rate:

$$V_{b,\,min} = 70.238 \times 29 / 1000$$
$$= 2.037 \text{ kg air / kg water}$$

At $V_b = 2\ V_{b,\,min} = 2 \times 70.238 = 140.48$

$$y_a = 0.582 / (140.48 + 0.582) = 0.00413$$

$$x_a^* = 0.00413 / 0.776 = 0.00532$$

$$x_a - x_a^* = 0.01059 - 0.00532 = 0.00527$$

$$x_b - x_b^* = x_b - 0 = 0.000106$$

$$\overline{\Delta x_L} = 0.00132$$

$$N_{Ox} = (0.01059 - 0.000106) / 0.00132 = 7.94$$

18.8 (a) $G_x / G_y = 8.5$ $\quad F_p = 110$ (Table 18.1)

$$\frac{\Delta P}{L} = \frac{24}{20} = 1.2 \text{ in } H_2O/\text{ft}$$

$$\rho_y = \frac{29}{359} \times 1.5 \times \frac{273}{313} = 0.106 \text{ lb/ft}^3$$

Assume $\rho_x = 50 \text{ lb/ft}^3$

$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x - \rho_y}} = 8.5\left(\frac{0.106}{49.9}\right)^{0.5} = 0.39$$

From Fig. 18.6, ordinate = 0.04 at 1.2 in. H_2O / ft

For the flooding ordinate, assume $\Delta P_{flood} = 2.0$ in. H_2O / ft (see p. 553). By extrapolation from Fig. 18.6, the flooding ordinate is

$$\frac{G_y^2 F_p \mu_x^{0.1}}{g_c \rho_y (\rho_x - \rho_y)} = 0.044$$

With 1.5 – in. Intalox saddles, $F_p = 52$

new ordinate $= 0.04 \times 52 / 110 = 0.0189$

new $\Delta P/L \cong 0.32$ in H_2O / ft

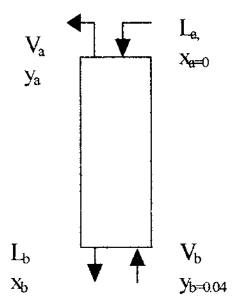
$$\Delta P = 0.32\ (20) = 6.4 \text{ in } H_2O$$

(b) If G_x / G_y remains the same, the abscissa is not changed. G_y^2 can be increased by the ratio of packing factors

G_y^2 increased $110/52 = 2.12$ times

G_y increased 1.45 times

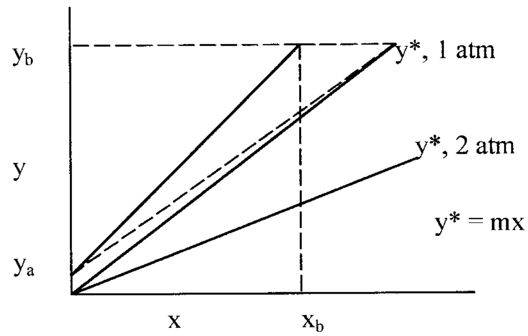
18.9



Basis $V_b = 100$ moles

$V_b y_b = 4$ moles A fed

$0.99\ (4) = 3.96$ moles A absorbed



$V_a = 100 - 3.96 = 96.04$

$y_a = (4 - 3.96) / 96.04 = 0.000416$

At L_{min}, $x_b = y_b / m = 3.96/ L_{min}$

$L_{min} = 3.96m / 0.04 = 99m$

(a) At $L = 1.5L_{min} = 148.5m$

$x_b = 3.96 / 148.5m = 0.0267 / m$

$y_b^* = mx_b = 0.0267$

$y_b = 0.04$

$y_b - y_b^* = 0.04 - 0.0267 = 0.0133$

$y_a - y_a^* = y_a - 0 = 0.000416$

$\overline{\Delta y_L} = 0.00372$

$N_{Oy} = (0.04 - 0.000416) / 0.00372 = 10.6$

(b) For operation at 2 atm, $y^* = p^* / 2$ or $y^* = 0.5mx$

$x_b = 0.0267 / m$

$y_b^* = 0.5m\,(0.0267/m) = 0.01335$

$y_b - y_b^* = 0.04 - 0.01335 = 0.02665$

$y_a - y_a^* = 0.000415$ as before

$\overline{\Delta y_L} = 0.00631$

$N_{Oy} = (0.04 - 0.000416) / 0.00631 = 6.3$

For operation at 4 atm

$y_b^* = 0.0267 / 4 = 0.006675$

$y_b - y_b^* = 0.03333$

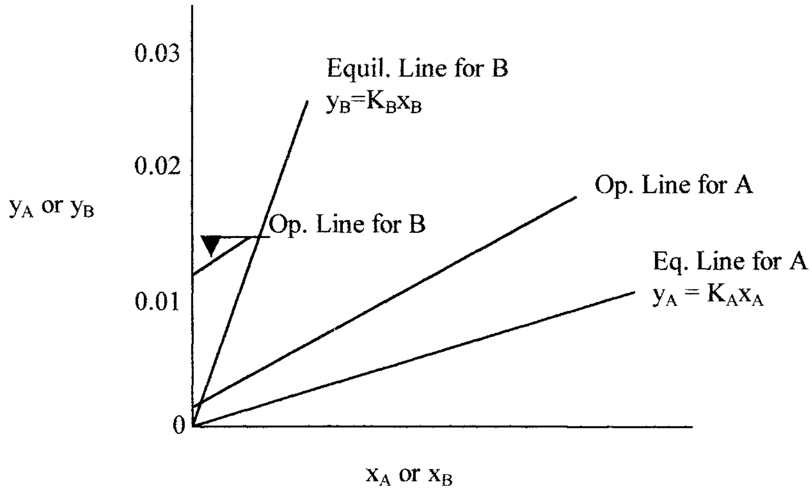
$\overline{\Delta y_L} = 0.00751$

$N_{Oy} = 5.3$

(c) Would H_{Oy} change with total pressure? The diffusivity in the gas phase varies with 1/P, but the concentration driving force is proportional to ΔyP, so the rate of diffusion across a gas film of a certain thickness is independent of pressure. Using Eq. (18.47) gives the same answer, since the Schmidt number for gases is independent of pressure.

H_y and H_{Oy} should not depend on pressure.

18.10 The gases can be nearly completely separated by operating the first column at a liquid rate only slightly greater than the minimum for A, the more soluble gas. The operating line slope L/V is the same for both A and B, so this slope is then much less than the slope of the equilibrium line for B. This results in a pinch for B at the bottom of the column, and only about 20% of the B is absorbed. With a tall enough column, 98% or more of the A can be absorbed in the first column. The second column is operated at a much higher L/V to give nearly complete removal of the B.



To get x_A and x_B, assume 98% recovery of A and L/V = 1.2 K_A.
Material balance on A:

$$V(0.02)(0.98) = L\, x_{Ab}$$

$$x_{Ab} = \frac{0.02(0.98)}{L/V} = \frac{0.0196}{1.2K_A} = 0.0163/K_A$$

The liquid from the first absorber comes practically to equilibrium with the feed gas

$$y_{Bb} = 0.01 = K_B x_{Bb} = 5K_A x_{Bb}$$

$$x_{Bb} = 0.01/5K_A = 0.002/K_A$$

$$x_{Ab}/x_{Bb} = 0.0163 / 0.002 = 8.15$$

18.11 Use the data for Intalox saddles to set G_y for $\Delta P = 0.5$ in. H_2O / ft at the normal conditions of 1 atm and 20° C. Compare this value with that predicted using the generalized correlation and obtain a correction factor. Use the generalized correlation and the correction factor to predict G_y' for 10 atm and 50 °C.

From Fig. 18.4 at $G_x / G_y = 50$:

At $G_y = 800$, $G_x = 4000$, $\Delta P \cong 0.40$

At $G_y = 900$, $G_x = 4500$, $\Delta P \cong 0.54$

By interpolation, at $G_y = 870$, $G_x = 4350$, $\Delta P = 0.50$

From Table 18.1, $F_p = 92$. Using Fig. 18.6,

$$\frac{G_x}{G_y}\left(\frac{\rho_y}{\rho_x - \rho_y}\right)^{0.5} = 5\left(\frac{0.0075}{62.1}\right)^{0.5} = 0.17$$

At $\Delta P = 0.5$ the ordinate is 0.041. Hence

$$G_y^2 = \frac{0.0431 \times 32.174 \times 62.1 \times 0.075}{92 \times 1^{0.1}} = 0.0669$$

$$G_y = 0.258 \text{ lb / ft}^2 - \text{s} \text{ or } 929 \text{ lb / ft}^2 - \text{h}$$

Correction factor: $870 / 929 = 0.936$

At 10 atm and 50° C, $\rho_y = 0.683 \text{ lb / ft}^3$ and $\mu_x = 0.59$ cP.

$$\frac{G_x}{G_y}\left(\frac{\rho_y}{\rho_x - \rho_y}\right)^{0.5} = 5\left(\frac{0.683}{61.6}\right)^{0.5} = 0.53$$

At $\Delta P = 0.5$ the ordinate is 0.022 (Fig. 18.6)

$$G_y'^2 = \frac{0.022 \times 32.174 \times 61.6 \times 0.683}{92 \times 0.59^{0.1}} = 0.341$$

$G_y' = 0.584 \text{ lb / ft}^2 - \text{s}$ or $2103 \text{ lb / ft}^2 - \text{h}$

Corrected $G_y' = 2103 \times 0.936 = 1968 \text{ lb / ft}^2 - \text{h}$

18.12 (a) For air at 1 atm and 20° C (assumed temperature)

$$\rho_y = \frac{29}{359} \times \frac{273}{293} = 0.0753 \, \text{lb} / \text{ft}^3 \text{ or } 1.206 \, \text{kg} / \text{m}^3$$

$$\rho_x = 62.3 \, \text{lb} / \text{ft}^3$$

$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}} = 8\left(0.075 \big/ 62.3\right)^{1/2} = 0.278$$

From Fig 18.8 for Mellapak 250Y

$$u_{o,f}\left(\rho_y \big/ \rho_x - \rho_y\right)^{1/2} = 0.065$$

$$u_{o,f} = 0.065(62.2 / 0.0753)^{1/2} = 1.87 \text{m} / \text{s}$$

$$u_o = 0.6 \times 1.87 = 1.12 \text{ m} / \text{s}$$

$$G_y = 1.12 \times 1.206 = 1.35 \text{ kg} / \text{m}^2 - \text{s} \quad (995 \text{ lb} / \text{h} - \text{ft}^2)$$

(b) at P = 2 atm $\rho_y = 0.1506 \text{ lb} / \text{ft}^3$ or $2.41 \text{ kg} / \text{m}^3$

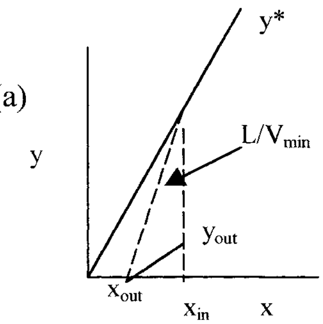
$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}} = 8\left(0.1506 \big/ 62.3\right)^{1/2} = 0.393$$

From Fig. 18.8, $u_{o,f} = 0.058 \, (62.1 / 0.1506)^{1/2} = 1.18 \text{ m} / \text{s}$

$$G_y = 0.6 \, (1.18)(2.41) = 1.71 \text{ kg} / \text{m}^2 - \text{s}$$

$$(1260 \text{ lb} / \text{h} - \text{ft}^2)$$

18.13 (a)



$M_{toluene} = 92.13$

$y^* = P_{tol}/P = 256x \ / \ 1.1$
$= 233 \ x$

1 liter has 1000 / 18 = 55.56 g mol H_2O

at 60 ppm, toluene concentration

$$= \frac{60 \times 10^{-6} \times 1000}{92.13} = 6.51 \times 10^{-4} \text{ g mol / liter}$$

$$x_{in} = \frac{6.51 \times 10^{-4}}{55.56} = 1.17 \times 10^{-5}$$

$$\Delta x = 58 / 60 \times x_{in} = 1.13 \times 10^{-5}$$

By material balance, $L \triangle x = V \triangle y$

at V_{min}, $\triangle y = y^*$ for x_{in}

$$V_{min} = \frac{L\Delta x}{233 x_{in}} = \frac{L}{233} \times \frac{58 \text{ ppm}}{60 \text{ ppm}} = 4.15 \times 10^{-3} \text{ L}$$

$$L = 100 \text{ l / min} \times 55.56 \text{ mol / l} = 5556 \text{ mol / min}$$

$$V_{min} = 5556 \times 4.15 \times 10^{-3} = 23.06 \text{ mol / min}$$

(b) at $V = 2 \ V_{min} = 46.11$ mol / min

$$y_{out} = \frac{L\Delta x}{V} = \frac{5556 \times 1.13 \times 10^{-5}}{46.11} = 1.36 \times 10^{-3}$$

$x^* = y_{out} / 233 = 5.84 \times 10^{-6}$

At top, $x\text{-}x^* = 1.17 \times 10^{-5} - 5.84 \times 10^{-6} = 5.86 \times 10^{-6}$

At bottom, $x\text{-}x^* = 2/60\ (1.17 \times 10^{-5}) = 3.9 \times 10^{-7}$

$$\overline{\Delta x}_{LM} = 20.2 \times 10^{-7}$$

$$N_{Ox} = \frac{\Delta x}{\lim\left(x - x^*\right)} = \frac{1.13 \times 10^{-5}}{20.2 \times 10^{-7}} = 5.59$$

(c) Use Eq. (18.29)

$$H_{Ox} = H_x + \frac{L}{mG} H_y$$

$$H_{Ox} = 0.6 + \frac{5556}{233 \times 46.11} \times 0.7 = 0.962\ \text{m}$$

$$Z = 5.59 \times 0.962 = 5.38\ \text{m}$$

18.14 Replace 1-in. Berl saddles with 1-in. metal Pall rings. From Table 18.1

	1-in. Berl saddles	1-in metal Pall rings
F_p	110	56
f_p	1.36	1.54

If the flow rates are not changed, the gas pressure drop will be reduced because of the lower value of F_p. Estimating the change using Figure 18.6, assume $\Delta P = 0.50$ in. H_2O at air abscissa value of 0.2.

$$X = \frac{G_x}{G_y}\left(\frac{\rho_y}{\rho_x - \rho_y}\right)^{0.5} = 0.2$$

$$Y = \frac{G_y^2 F_p \mu_g^{0.1}}{g \Delta\rho \rho_y} = 0.04$$

With the lower F_p, $Y = 0.04\ (56 / 110) = 0.0204$

$\triangle P \cong 0.25$ in H_2O
50 % decrease in $\triangle P$

If the original value of $\triangle P$ was 1.0 in. H_2O, the change in Y from 0.055 to 0.028 would reduce $\triangle P$ to about 0.35 in. H_2O, a 65% decrease.

The change to Pall rings will also change the overall mass transfer coefficient by a factor of about (1.54 / 1.36) or 1.13, giving slightly greater solute recovery in the absorber.

The switch to Pall rings would permit higher gas and liquid rate at the same pressure drop. For the case where Y in Figure 18.6 is 0.04, lowering F_p from 110 to 56 would permit G_y to increase by $(110 / 56)^{1/2}$ or 1.4 times, assuming G_x / G_y was kept constant. However, the increase in G_x and G_y might mean an increase in H_{Oy} in spite of the slight increase in f_p.

Chapter 19

19.1 (a) Percentage humidity lines

Mol. Weight of acetone: $58.08 = M_A$

For 100% humidity, $\mathcal{H} = \mathcal{H}_s$

From Eq. (19.3), with P'_A in mm Hg,

$$\mathcal{H} = \mathcal{H}_s = (58.08P'_A) / [29\ (760 - P'_A)]$$

Also at 50%, $\mathcal{H} = \mathcal{H}_s / 2$. Find P'_A from Table 19.1, p. 620.

Temperature °C	P'_A mmHg	$760 - P'_A$ mmHg	$\mathcal{H}_A = 100$ $\mathcal{H}$	$\mathcal{H}_A = 50$ $\mathcal{H}$
10	115.6	644.4	0.359	0.180
20	179.6	580.4	0.620	0.310
30	281.0	479.0	1.175	0.588
40	420.1	339.9	2.475	1.238
50	620.9	139.1	8.940	4.470
56.1	760.0	0	----	----

(b) Saturated volume

From Eq. (19.7b), for $\mathcal{H}_A = 100$ (Note that T is in Kelvins):

$$v_H = [(0.0224 \times T) / 273)][(1/29) + (\mathcal{H}/58.08)]$$

$$= (8.21 \times 10^{-5}\ T)(0.03448 + 0.01722\mathcal{H})\ m^3 / g$$

Temperature °C	Temperature K	$v_H \times 10^4$ m^3 / g
10	283	9.45
20	293	10.86
30	303	13.61
40	313	19.81
50	323	49.97

(b) Latent heat – given in Table 19.1, p. 620

(c) Humid heat. In this temperature range, c_p for air, from Appendix 14, is $0.25 \times 4.187 = 1.047\ J / g - {}^{o}C$

From Eq. (19.6)

$$c_s = 1.047 + 1.47\mathcal{H}$$

When $\mathcal{H} = 0$, $c_s = 1.047$

$\mathcal{H} = 6$, $c_s = 1.047 + 1.47 \times 6 = 9.867$

Draw a straight line through points (0, 1.047) and (6, 9.867)

(e) Adiabatic cooling lines

From Eq. (19.11)

$$T = T_s + \lambda_s (\mathcal{H}_s - \mathcal{H}) / c_s$$

When $T_s = 20^o$ C, $\mathcal{H}_s = 0.620$, $\lambda_s = 552$ J/ g

$T_s = 40^o$ C, $\mathcal{H}_s = 2.475$, $\lambda_s = 536$ J/ g

Choose values of $\mathcal{H}$

T_s, °C	$\mathcal{H}$	T, °C
20	0.600	25.72
20	0.575	33.13
20	0.550	40.82
20	0.525	48.83
20	0.500	57.17
40	2.400	48.79
40	2.350	54.88
40	2.300	61.18
40	2.250	67.70

(f) Wet-bulb temperature lines

From Eq. (19.16), with $h_y / M_B k_y = 1.7$ J / g – °C:

$$\mathcal{H} = \mathcal{H}_w - (h_y / M_B k_y \lambda_w)(T - T_w)$$

$$= \mathcal{H}_w - (1.7 / \lambda_w)(T - T_w)$$

When $T_w = 20$ °C, $\mathcal{H}_w = 0.620$, $\lambda_w = 552$ J / g

$T_w = 40$ °C, $\mathcal{H}_w = 2.475$, $\lambda_w = 536$ J / g

Choose values of T.

T_W, °C	T, °C	$\mathcal{H}$
20	25	0.605
20	30	0.589
20	35	0.574
20	40	0.558
20	45	0.543
20	50	0.528
40	45	2.459
40	50	2.443
40	55	2.427
40	60	2.412

When the wet-bulb temperature is 20 °C, the wet-bulb and adiabatic cooling lines very nearly coincide.

19.2 (a) Use Fig. 12.38 from Perry, "Chemical Engineers' Handbook, " 7th ed., p. 12.29. Draw a psychrometric line through $\mathcal{H} = 0.7$, T = 70 °C (158 °F), parallel to the adjacent psychrometric lines. This line intersects the saturated humidity line at T = 108 °F (42.2 °C).

(b) $\mathcal{H}_b = \mathcal{H}_s$ at 15 °C (59 °F) = 0.23 kg / kg

(c) For $\mathcal{H} = 0.7$, $c_s = 0.46$ Btu / lb dry air -^{o} F or $0.46 \times 4.1868 =$ 1.93 kJ / kg-^{o}C. At $\mathcal{H} = 0.7$, $T_s = 98^o$ F (36.7^o C). Then, for 1 kg of dry air, cooled to 36.7 ^{o}C,

$$Q_{\text{cooling I}} = 1.93\ (70 - 36.7) = 64.3 \text{ kJ}$$

The amount condensed is $\mathcal{H} - \mathcal{H}_b = 0.7 - 0.23 = 0.47$ kg. From Perry, p. 12.29, $\lambda_{36.7^o C} = 182$ Btu / lb or 423 kJ / kg. $\lambda_{15^o C} = 435$ kJ / kg. Average value:

$$\overline{\lambda} = 429 \text{ kJ / kg}$$

$$Q_{\text{condensation}} = 0.47 \times 429 = 201.6 \text{ kJ}$$

To cool air and uncondensed benzene from 36.7 ^{o}C to 15 ^{o}C:

$$\overline{\mathcal{H}} = (0.23 + 0.70) / 2 \ \ 0.465$$

$$c_s = 0.37 \text{ Btu / lb} - {}^o\text{F or } 1.55 \text{ kJ / kg-}{}^o\text{C}$$

$$Q_{\text{cooling II}} = 1.55\ (36.7 - 15) = 33.6 \text{ kJ}$$

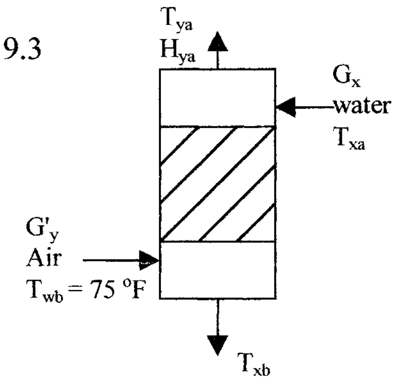
To cool condensed benzene from average condensation temperature of (36.7 + 15) / 2 = 25.85 ^{o}C to 15 ^{o}C, with a specific heat of 2.09 kJ / kg:

$$Q \text{ cooling III} = 0.47 \times 2.09 \times 10.85 = 10.7 \text{ kJ}$$

Total heat transferred:

$$Q_T = 64.3 + 201.6 + 33.6 + 10.7 = 310.2 \text{ kJ / kg air}$$

19.3



Approach = 10 °F = T_{xb}-T_{wb}
$T_{xb} = 85\ ^\circ F$

Range = $T_{xa} - T_{xb} = 17\ ^\circ F$
$T_{xa} = 85 + 17 = 102\ ^\circ F$

Cross-section of tower = $36 \times 36 = 1296\ ft^2$

(a) Although T_{yb} was not specified, the enthalpy can be determined from T_{wb}, since the adiabatic saturation temperature for the air-water system is the same as the wet-bulb temperature.

From Fig. 19.8, $T_{wb} = 75\ ^\circ F$, $H_{yb} = 31$ Btu / lb

For $T_x = 85\ ^\circ F$, $H^* = 42$ Btu / lb
For Tx = 102 °F, $H^* = 68$ Btu / lb

From Eq. (19.27)

$$G'_y \Delta H = G_x c_L (T_{xa} - T_{xb})$$

If $T_{yb} \cong 90\ ^\circ F = 550\ ^\circ R$,

$$\rho_y \cong (29 / 359) \times (492 / 550) = 0.0723\ lb / ft^3$$

$$G'_y = (523{,}000 \times 0.0723 \times 60) / 1296 = 1751\ lb / h\text{-}ft^2$$

$$G_x = (6000 \times 8.33 \times 60) / 1296 = 2314\ lb / h\text{-}ft^2$$

$$\Delta H = (2314 \times 1.0 \times 17) / 1751 = 22.5\ Btu / lb$$

(b) $H_{ya} = 31 + 22.5 = 53.5$ Btu / lb

At bottom, $H^* - H = 42 - 31 = 11$ Btu / lb

At top, $H^* - H = 68 - 53.3 = 14.5$

$$\overline{(H^* - H)_L} = 12.7$$

$$N_{Oy} = \frac{H_{ya} - H_{yb}}{\overline{\Delta H_L}} = \frac{22.5}{12.7} = 1.77$$

$H_{oy} = 3$ ft / 1.77 = 1.69 ft

(c) $T_{wb} = 78\,^{\circ}F$ $H_{yb} = 34$ Btu / lb

For a 7 °F approach, $T_{xb} = 78 + 7 = 85\,^{\circ}F$

If $T_{xa} = 102\,^{\circ}F$, as in part (a) and range = 17 °F

$\Delta H = 22.5$ Btu / lb, as in part (a)

at top, $H_{ya} = 34 + 22.5 = 56.5$,

$H^* - H_{ya} = 68 - 56.5 = 11.5$ Btu /lb

at bottom, $H^* - H_{yb} = 42 - 34 = 8$ Btu / lb

$$\overline{(H^* - H)_L} = 9.64$$

$N_{Oy} = 22.5 / 9.64 = 2.33$

Z needed = $2.33 / 1.77 \times 3$ ft = 3.95

Need an additional foot of fill.

19.4 New $G'_y = 1.2 \times 1751 = 2101$

The new operating line has a lower slope, which increases $H^* - H_{ya}$. If N_{Oy} is about the same, T_{xb} will decrease to give a lower value of $H^* - H_{yb}$.

Guess range = 19 °F, T_{xb} = 102 – 19 = 83 °F

$$H^* = 40$$

$$\Delta H = G_x \Delta T / G'_y = (2314 \times 19) / 2101 = 20.9$$

$$H_{ya} = 31 + 20.9 = 51.9$$

At bottom, $(H^* - H) = 40 - 31 = 9$

At top $(H^* - H) = 68 - 51.9 = 16.1$

$$\overline{(H^* - H)_L} = 12.2$$

$N_{Oy} = 20.9 / 12.2 = 1.71$

This is about the same at 1.77, so the exit water temperature should drop about 2 °F to 83 °F.

19.5 For small drops in air, check to see when Stokes' law holds (Eq. 7.40)). For air at 1 atm, 140°F (600 °R):

$\mu = 0.0192$ cP $= 1.92 \times 10^{-5}$ Pa-s (Appendix 8)
$\rho_g = (29 \times 492) / (22.4 \times 600) = 1.06$ kg / m^3
$\rho_l = 993$ kg / m^3

$$u_t = \frac{9.80 D_p^2 (993 - 1.06)}{18 \times 1.92 \times 10^{-5}} = 2.81 \times 10^7 D_p^2$$

$$Re_p = \frac{D_p (2.81 \times 10^7 D_p^2) 1.06}{1.92 \times 10^{-5}}$$

$$= 1.55 \times 10^{12} D_p^3$$

For $Re_p = 1$, $D_p = (1 / 1.55 \times 10^{12})^{1/3}$

$$= 8.64 \times 10^{-5} \text{ m or } 86\,\mu\text{m}$$

Thus Stokes' law holds for drop smaller than about 80 micrometers. From Eq. (12.64)

$$Nu = 2 + 0.6(Re_p)^{1/2}\, Pr^{1/3}$$

From Appendix 16, $Pr = 0.69$; $Pr^{1/3} = 0.884$

For $D_p = 50\,\mu\text{m}$

$$Re_p = 1.55 \times 10^{12}\,(50 \times 10^{-6})^{3} = 0.194$$

$$Nu = 2 + 0.6(0.194)^{1/2}\,(0.884) = 2.23$$

As a drop evaporates, the Nusselt number gets smaller and approaches 2.0. To simplify the analysis, assume Nu = 2.0. The drops injected into warm air soon reach the wet-bulb temperature. Heat transferred to a drop:

$$Q = hA(T_g - T_w)$$

$$H = 2k / D_p \qquad A = \pi D_p^2$$

$$Q = 2k\pi\, D_p^2\,(T_g - T_w) / D_p$$

The heat transferred to the drop is used to evaporate water. Hence

$$Q = -\lambda_w \frac{d}{dt}\left(\frac{\pi\, D_p^3\, \rho_l}{6}\right) = -\lambda_w \frac{\pi\, D_p^2}{2} \frac{dD_p \rho_l}{dt}$$

From the two equations for Q,

$$2\,k\,\pi\,D_p^2(T_g - T_w)/D_p = \frac{-\lambda_w \pi D_p^2 \rho_l}{2}\frac{dD_p}{dt}$$

$$\int_{D_{p,1}}^{D_{p,2}} -D_p dD_p = \int_0^t \frac{4k(T_g - T_w)dt}{\lambda_w \rho_l}$$

$$D_{p,1}{}^2 - D_{p,2}{}^2 = \frac{8k(T_g - T_w)t}{\lambda_w \rho_l}$$

For complete evaporation of the drop $D_{p,2} = 0$.

$$t = \frac{D_{p,1}{}^2 \lambda_w \rho_l}{8k(T_g - T_w)}$$

[redacted]) The evaporation time depends on the wet-bulb temperature. For $T_w = 110\ ^\circ F$, $T_g = 140\ ^\circ F$, $T_g - T_w = 30\ ^\circ F$:

$k = 0.0166$ Btu / ft-h-°F (Appendix 12)

At 120 °F, $\lambda_w = 1025.5$ Btu / lb (Appendix 7)

$D_p = 50 \times 10^{-6} / 0.3048 = 1.64 \times 10^{-4}$ ft

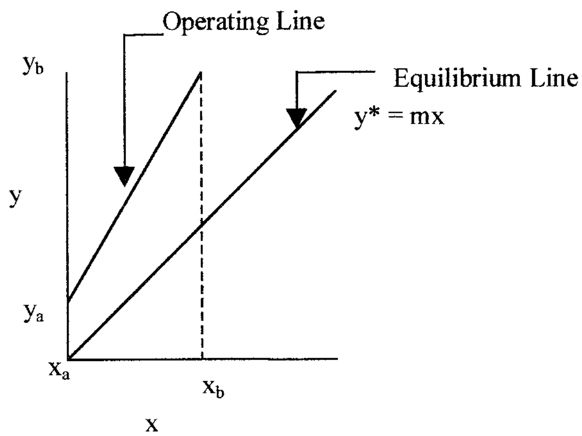
$$t = \frac{(1.64 \times 10^{-4})^2(1025.5)(62.0)(3600)}{8 \times 0.0166 \times 30} = 1.55\ s$$

) $a = 6\phi D_p$, where ϕ = volume fraction drops
$h = 2k / D_p = 2 \times 0.0166 / 1.64 \times 10^{-4} = 202$ Btu / ft^2-h-°F

$ha = 12k\phi / D_p^2 = (12 \times 0.0166 \times 0.01) / (1.64 \times 10^{-4})^2$

$= 7.41 \times 10^4$ Btu / ft^3-h-°F

20.3 $N = 0.75 \times 8 = 6$ ideal stages



Assume $x_a = 0$, then $y_a^* = 0$

$y_a = 0.05\ y_b$ for 95% removal

$$A = \frac{L}{mV}$$

assume dilute gas so L and V are constant

Guess $A = 2.0$ or $L/V = 2\ m$

By material balance, $Lx_b \cong V(y_b - y_a) = 0.95\ y_b V$

$$x_b = 0.95 y_b \frac{V}{L} = \frac{0.95 y_b}{2\,m}$$

$$y_b^* = mx_b = \frac{0.95 y_b}{2}$$

$$y_b - y_b^* = y_b(1 - 0.475) = 0.525 y_b$$

$$y_a - y_a^* = y_a = 0.05 y_b$$

From Eq. (20-22)

$$N = \frac{\ln\left[\left(y_b - y_b^*\right)/\left(y_a - y_a^*\right)\right]}{\ln A} = \frac{\ln(0.525/0.05)}{\ln 2} = 3.39$$

After a few trials, for A = 1.34

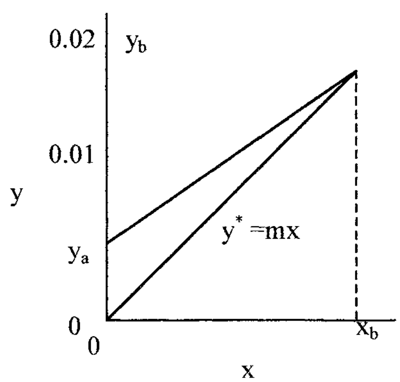
$$x_b = \frac{0.95 y_b / m}{1.34} = 0.709 y_b / m$$

$$y_b - y_b^* = y_b(1 - 0.709) = 0.291 y_b$$

$$N = \frac{\ln(0.291/0.05)}{\ln 1.34} = 6.02 \quad \text{O.K.}$$

The L/V ratio must exceed the slope of the equilibrium line by a factor of 1.34.

20.4



$$A = \frac{L}{mV} = 0.9, \quad \frac{L}{V} = 0.9m$$

Note pinch at bottom, since A < 1.0

Assume $x_a = 0$

Lowest possible value of $y_a = 0.1\ (y_b) = 0.002$ or 90% absorbed

Guess $y_a = 0.003$ or 85% absorbed

$Lx_b = V\ (0.02\text{-}0.003) = 0.017\ V$

$x_b = 0.017\ V/\ L = 0.017/\ 0.9m$

$$y_b^* = mx_b = 0.01889$$

$$y_b - y_b^* = 0.02 - 0.01889 = 0.00111$$

$$y_a - y_a^* = 0.003$$

From Eq. (20.22)

$$N = \frac{\ln(0.00111/0.003)}{\ln(0.9)} = 9.44$$

NOTE: It would be better to use Eq. (20.26).

For 84% absorbed $y_a = 0.0032$

$$x_b = \frac{V \Delta y_b}{L} = \frac{\Delta y}{0.9m}$$

$$x_b = 0.018667 / m$$

$$y_b - y_b^* = 0.02 - 0.018667 = 0.001333$$

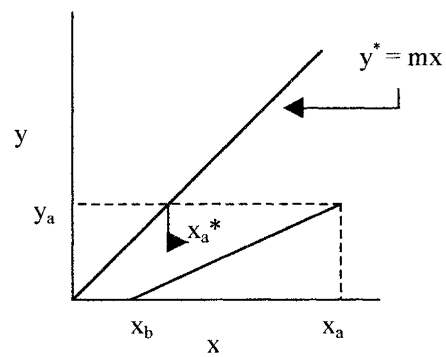
$$N = \frac{\ln(0.0013333/0.0032)}{\ln(0.9)} = 8.31$$

Similar calculations give the following results:

% Absorbed	N
78	4.75
80	5.58
82	6.31
83	7.42
84	8.31
85	9.44
86	10.89

At N = 5, about 79% of the NH_3 could be absorbed, and at N = 10, about 85% would be absorbed.

20.5 (a)



$N = 8$

$$S = \frac{mV}{L} \qquad \frac{L}{V} = \frac{m}{S}$$

$x_b = 0.02x_a$ for 98% removal

Assume $y_a = 0$ (no hydrocarbon in inlet air)

Try $S = 1.5$ $\quad L\Delta x = V\Delta y = Vy_a$

$$y_a = \frac{L}{V}(0.98)x_a = \frac{m}{1.5}(0.98x_a) = 0.6533mx_a$$

$$x_a^* = ya/m = 0.6533x_a$$

From Eq. (20.28),

$$N = \ln\left[\left(x_a - x_a^*\right)/\left(x_b - x_b^*\right)\right]/\ln S$$

$$N = \ln\left[\frac{x_a(1-0.6533)}{0.02x_a}\right]/\ln 1.5 = 7.04$$

For S = 1.4, $y_a = 0.70mx_a$

$$N = \ln\left[\frac{x_a(1-0.7)}{0.02x_a}\right]/\ln 1.4 = 8.05 \quad \text{O.K.}$$

A stripping factor of 1.4 is needed for 98% removal.

(b) At S = 2.0 guess 99% removal, $x_b = 0.01x_a$

$$y_a = \frac{L}{V}(0.99x_a) = \frac{m}{2.0}(0.99x_a) = 0.495mx_a$$

$$x_a^* = 0.495x_a$$

$$N = \frac{\ln\left[\frac{x_a(1-0.495)}{0.01x_a}\right]}{\ln 2.0} = 5.63$$

Try 99.7% removal or $x_b = 0.003x_a$

$$y_a = \frac{m}{2.0}(0.997x_a) = 0.4985mx_a$$

$$x_a^* = 0.4985x_a$$

$$N = \frac{\ln\left(\frac{1-0.4985}{0.003}\right)}{\ln 2} = 7.39 \text{ stages}$$

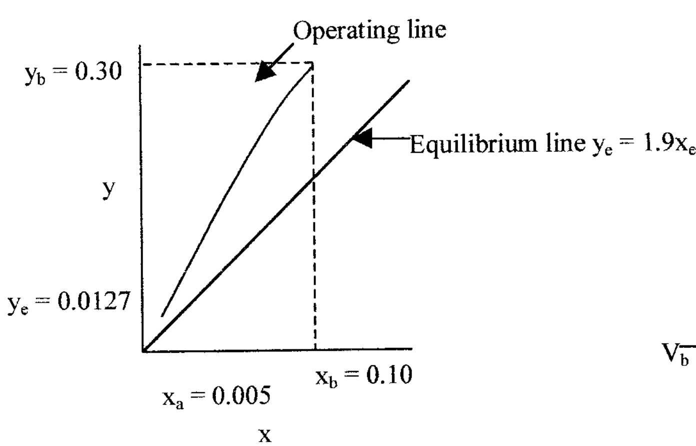
At 99.8% removal $\qquad x_b = 0.002x_a$

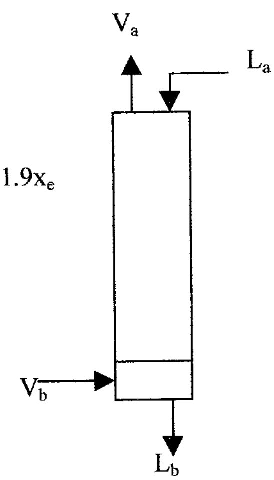
$$x_a^* = 0.499x_a$$

$$N = \frac{\ln\left(\frac{1-0.499}{0.002}\right)}{\ln 2} = 7.97$$

99.8% removal could be achieved at S = 2.0 with N = 8.

20.6





$$y_a^* = 1.9(0.005) = 0.0095$$

$$y_a = 0.0127 \text{ (from Example 20.1)}$$

$$y_a - y_a^* = 0.0127 - 0.0095 = 0.0032$$

The driving force at the top is only ¼ the previous value.

Basis: 100 moles entering gas with 30 mol % acetone

3% not absorbed = 0.9 mol, 29.1 mol absorbed

$29.1 + 0.005 (L_a) = 0.1L_b$

$L_a = L_b - 29.1$

$L_b = 28.95 / 0.095 = 304.7$ $L_a = 304.7 - 29.1 = 275.6$

$L_a(0.005) = 1.38$ moles acetone in incoming oil

When 10 moles of acetone are left in the gas, 9.1 moles have been absorbed

$$y = \frac{10}{10+70} = 0.125$$

$$x = \frac{9.1+1.38}{275.6+9.1} = 0.0368$$

This gives an intermediate point on the operating line, which is curved and slightly below the line in Fig. 20.6

5.6 ideal stages are needed.

21.1 (a) Let A refer to ethylbenzene, B to toluene, W to water. There are two liquid phases: water and organic (ethylbenzene plus toluene).

Let x' = mol fraction on a water-free basis.

In feed, $x'_A = 40 / 65 = 0.615$

$x'_B = 1 - 0.615 = 0.385$

Then

$$x'_A P'_A + x'_B P'_B + P'_w = P$$

$$0.615\ P'_A + 0.385\ P'_B + P'_w = 0.5 \times 760 = 380 \text{ mm Hg}$$

Temp, °C	P'_w	$0.615\ P'_A$	$0.385\ P'_B$	P
60	149.4	34.1	53.7	237.2
70	233.7	52.2	77.9	363.8
71.3	244.5	54.0	81.5	380.0
80	355.1	77.4	111.5	544.0

From a plot of P vs. temperature the pressure equals 380 mm Hg at 71.3 °C

Liquid composition:

$x_A = 0.40$ $x_B = 0.25$ $x_W = 0.35$

Vapor composition: $y_A = 54 / 380 = 0.142$

$y_B = 81.5 / 380 = 0.214$

$y_W = 244.5 / 380 = 0.644$

(b) Assume no water condenses at the dew point. With the vapor composition the same as the liquid composition:

$p_A = 0.40 \times 380 = 152 = x_A P'_A$

$x_A = 152 / P'_A$

$P_B = 0.25 \times 380 = 95 = x_B P'_B$

$x_B = 95 / P'_B$

$p_W = 0.35 \times 380 = 133$

Find the temperature T at which $x_A + x_B = 1.0$

By trial, T = 91 °C

$x_A = 152 / 198 = 0.768$

$x_B = 95/409.5 = 0.232$

At 91 °C, P'_w is about 530 mm Hg, well above p_w. Hence no water will condense at 87 °C.

(c) Basis: 1 mole of feed (0.35 moles water, 0.65 mol organic). Must assume that some liquid water will be present, exerting its own vapor pressure at whatever the temperature is. Find this temperature by trial.

By trial 71.8 °C. $p_W = 250$ mm Hg

For the organic, P' = 380 –250 = 130 mm Hg

Moles of water in vapor: $(250 / 380) \times 0.5 = 0.329$

Moles of organic in vapor: $0.5 - 0.329 = 0.171$

Fraction of the organic vaporized:

$f = 0.171 / 0.65 = 0.271$

Composition of organic liquid fed:

$x'_{FA} = 0.40 / 0.65 = 0.6154$

$x'_{FB} = 0.25 / 0.65 = 0.3846$

From Eq. (18.2),

$$y'_B = -\frac{1-f}{f}x'_B + \frac{x'_{FB}}{f} = 1.463 - 2.802x'_B$$

From p. 665

$y'_B = P'_B\, x'_B / 130$

$1.463 - 2.802\, x'_B = P'_B\, x'_B / 130$

$$x'_B = \frac{1.463}{2.802 + (P'_B / 130)}$$

Similarly,

$y'_A = 2.340 - 2.802\, x'_A = P'_A\, x'_A / 130$

$$x'_A = \frac{2.360}{2.802 + (P'_A / 130)} = 1 - x'_B \qquad \text{(A)}$$

$$x'_A = 1 - \frac{1.463}{2.802 + (P'_B / 130)} \qquad \text{(B)}$$

At 71.8 °C, $P'_A = 88.5, P'_A = 211.7$

From these, Equation (A) gives $\quad x'_A = 0.670$

Equation (B) gives $\quad x'_A = 0.671$

These are close enough. Hence T = 71.8 °C

Composition of organic liquid:

$x'_A = 0.671, \quad x'_B = 0.329$

Overall liquid composition

Moles water in – moles water out = 0.35 – 0.329 = 0.021

$x_W = 0.021 / 0.5 = 0.042$

$x_A = (1 - 0.042) \times 0.671 = 0.643$

$x_B = (1 - 0.042) \times 0.329 = 0.315$

Vapor composition:

$y_W = 0.329 / 0.5 = 0.658$

$y'_A = 2.340 - 2.802 \times 0.671 = 0.460$

$y_A = (1 - 0.658) \times 0.460 = 0.157$

$y'_B = 1.463 - 2.802 \times 0.329 = 0.541$

$y_B = (1 - 0.658) \times 0.541 = 0.185$

21.2 $\quad x_F = 0.75 \quad q = 1.15 \quad x_D = 0.9999 \quad x_B = 0.00002$

From Eq. (21.31), slope of feed line =

$-1.15 / (1 - 1.15) = 7.667$

Slope of rectifying line: 1.4 / 2.4 = 0.58333

Draw stripping line through (0.00002, 0.00002) and intersection of feed and rectifying lines.

The equilibrium line is straight when $x > 0.90$. Use Eq. (20.24) for the upper end of the column and a McCabe-Thiele diagram for the remainder (see note at end).

Equation for upper end of equilibrium line, found from coordinates (1,1) and (0.90, 0.959) is

$$y_e = 0.410x_e + 0.590$$

From this

$$y_D = 0.409959 + 0.590 = 0.999959 = y_a^*$$

(a) Minimum number of plates. With total reflux, $y = x$.

(i) When $x > 0.90$, use Eq. (20.24).

$y_b = x_b = 0.90$ $\qquad$ $y_b - y_b^* = 0.90 - 0.959 = -0.059$

$y_a = x_a = 0.9999$ $\qquad$ $y_a - y_a^* = 0.9999 - 0.999959 = -0.000059$

$$\frac{\log(0.059 / 0.000059)}{\log\left(\dfrac{0.90 - 0.9999}{0.959 - 0.999959}\right)} = N_1 = 7.75 \text{ plates}$$

(ii) From $x = 0.005$ to $x = 0.90$, from diagram:

$N_2 = 4.1$ plates

(iii) From $x = 0.00002$ to $x = 0.005$ use Eq. (20.24).

$x_b = y_b = 0.005$

$y_b^* = 0.035$ (from diagram)

$x_a = y_a = 0.00002$

Since the equilibrium line is straight in this region,

$$y_a^* = 0.035\, x_a / 0.005 = 0.00014$$

From Eq. (20.24), interchanging y and y* since the equilibrium line is above the operating line,

$$N_3 = \frac{\ln \dfrac{0.035 - 0.005}{0.00014 - 0.00002}}{\ln \dfrac{0.035 - 0.00014}{0.005 - 0.00002}} = 2.84$$

or 1.84 plates plus reboiler

Total minimum number of ideal plates:

7.75 + 4.1 + 1.84 = 13.69 plates plus reboiler

(b) Minimum reflux ratio:

At the intersection of the feed line with the equilibrium curve,

$x' = 0.77 \qquad y' = 0.9017$

From Eq. (21.44)

$$R_{Dm} = \frac{0.9999 - 0.9017}{0.9017 - 0.77} = 0.746$$

(c) Draw an effective equilibrium line with $\eta_M = 0.72$. Slope of operating line: 1.4 / 2.4 = 0.58333

(i) When x > 0.90, use Eq. (20.24) and (1-y) in place of y. Rectifying line equation is

$$y = 0.5833x + 0.9999(1 - 0.58333)$$

$x_b = 0.90 \qquad y_b = 0.9416 \qquad 1 - y_b = 0.0584$

$x_a = y_a = 0.9999 \qquad 1 - y_a = 0.0001$

$(y_b^*)' = 0.9416 + 0.72(0.959 - 0.9416) = 0.9541$

$1 - (y_b^*)' = 0.0459$

$(y_a^*)' = 0.9999 + 0.72(0.999959 - 0.9999)$

$= 0.9999424$

$$1 - (y_a^*)' = 0.0000576$$

$$N_1 = \frac{\ln \dfrac{0.0584 - 0.0459}{0.0001 - 0.0000576}}{\ln \dfrac{0.0584 - 0.0001}{0.0459 - 0.0000576}} = 23.7 \text{ plates}$$

(ii) For $x = 0.005$ to $x = 0.90$, from diagram using an effective equilibrium line with

$\eta_M = 0.72$, $N_2 = 9.2$ actual plates

(iii) For $x = 0.00002$ to $x = 0.005$, use Eq. (20.24).

$x_b = y_b = 0.005$

$(y_b^*)' = 0.72(0.035 - 0.005) + 0.005 = 0.0266$

$x_a = y_a = 0.00002$

$(y_a^*)' = 0.72(0.00035 - 0.00002) + 0.00002 = 0.0002576$

As in (iii), part (a),

$$N_3 = \frac{\ln \dfrac{0.0266 - 0.005}{0.0002576 - 0.00002}}{\ln \dfrac{0.0266 - 0.0002576}{0.005 - 0.00002}} = 2.71 \text{ plates}$$

or 1.71 plates plus reboiler

Total number of plates:

$23.7 + 9.2 + 1.71 = 34.61$ actual plates

Say 35 plates plus reboiler

(c) A partial condenser is the equivalent of one theoretical plate or, in this case, $1/0.72 = 1.39$ actual plates (see p. 654). Hence the number of plastes is $34.61 - 1.39 = 33.22$.

NOTE: The problem can be solved directly over the entire range by a computer program with the following empirical correlation for relative volatility:

$$\alpha = 7.563354 - 12.0874x + 9.083008x^2 - 1.975098x^3$$

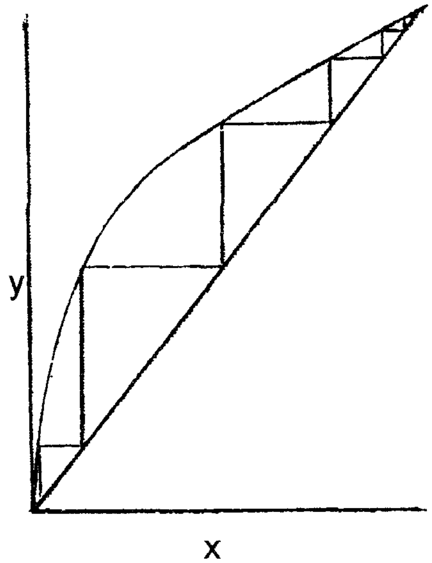
This gives the following results:

(a) 13.86 total plates plus reboiler

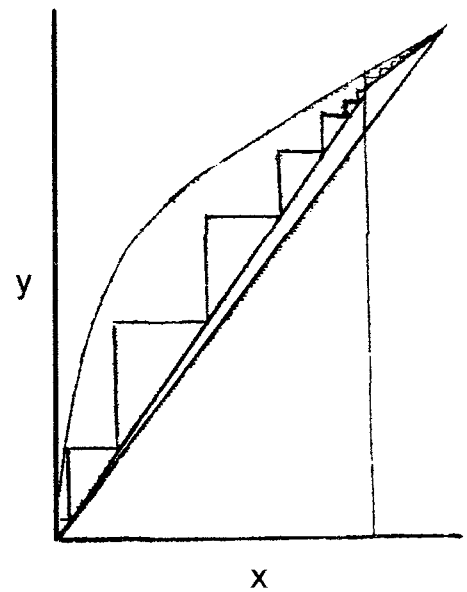
(b) $R_{Dm} = 0.746$

(c) 34.49 total plates plus reboiler

(d) 33.10 total plates plus reboiler



Problem 21.2 (a)



Problem 21.2 (c)

21.3 Feed is 30% liquid, 70% vapor.

$$f = 0.70$$

Slope of feed line:

$$-\frac{1-f}{f} = \frac{0.30}{0.70} = -0.429$$

From x-y diagram, the coordinates of the intersection of the feed line with the equilibrium line are:

$y' = 0.284, \qquad x' = 0.171$

The diagram shows that Eq. (21.44) for minimum reflux ratio may be used.

$$R_{Dm} = \frac{x_D - y'}{y' - x'} = \frac{0.780 - 0.284}{0.284 - 0.171} = 4.39$$

At twice the minimum

$$R_D = 2 \times 4.39 = 8.78$$

Slope of rectifying line = 8.78 / 9.78 = 0.898

Draw stripping line from point (0.01, 0.01) and the intersection of the rectifying and feed lines. Draw the effective equilibrium line using Eq. (21.70), using $\eta_M = 0.70$.

From the diagram

(a) Number of actual plates above feed plate = 25.
Number of actual plates in stripping section = 8.

(b) Basis: 1 mole of overhead product (D = 1)
From Eq. (21.5), inverted:

$$F/D = F = \frac{0.78 - 0.01}{0.25 - 0.01} = 3.21$$

From Eq. (21.3),

$$B = 3.21 - 1 = 2.1$$

$$R_D = L/D = 8.78$$

$$V = 8.78 + 1 = 9.78$$

Since the feed is 70% vapor, and 1 cal / g mol = 1.8 Btu / lb mole, the vapor in the stripping section is:

$$\overline{V} = 9.7 - (0.7 \times 3.21) = 7.53 \text{ mol}$$

$$\lambda = 1.8 \times 7700 = 13860 \text{ Btu / lb mol}$$

Heat required at reboiler:

$$q_r = 7.53 \times 13860 = 104{,}400 \text{ Btu / lb mol of overhead product.}$$

(c) Basis: 1 mole of overhead product (D=1) Let R' = lb moles of reflux added by condensing vapor on the top plate by the cold returned reflux. Total liquid in rectifying section:

L = 8.78 lb mol
Returned relux: 8.78 - R' lb moles
Latent heat supplied in forming R' : 13860 R' Btu

Sensible heat to heat returned reflux:

$$(8.78 - R')\overline{C}(T_b - T) = 13860R'$$

where $\overline{C}$ = molal specific heat of returned reflux, Btu/lb mole $-^\circ$ F

T_b = bubble point of returned reflux, 55.1 °C

T = temperature of returned reflux, 25 °C

Specific heats from Appendix 15, for

(55.1 + 25) / 2 = 40.05 °C (104 °F)

	c_p,	Mol.	C,
Liquid	Btu / lb -°F	Wt.	Btu / lb mole-°F
Acetone	0.541	58.08	31.42
Methanol	0.620	32.04	19.86

$\overline{C} = (0.78 \times 31.42) + (0.22 \times 19.86) = 28.88$ Btu / lb mole -°F

Then

$$(8.78 - R') \times 28.88 \times 1.8\,(55.1 - 25) = 13860\,R'$$

$$R' = 0.89$$

Returned reflux: 8.78 – 0.89 = 7.89 mol

Vapor to condenser: 7.89 + 1 = 8.89 mol

Heat removed in condenser:

Latent heat: $8.89 \times 13860 =$ 123,200 Btu

Sensible heat: $8.89 \times 28.88 \times 1.8 \times (55.1\text{-}25) = \underline{13{,}900}$

137,100

Total, $-q_c = 137{,}100$ Btu / lb mole of overhead product

21.4 Construct equilibrium diagram from the data in Fig. 21.2 and Table 21.1

Assume the reflux is saturated liquid.

$x_D = 0.98 \qquad x_F = 0.50$

Minimum reflux ratio, from Eq. (21.44):

$x' = x_F = 0.50 \qquad y' = 0.71$

$$R_{Dm} = \frac{0.98 - 0.71}{0.71 - 0.50} = 1.29$$

Basis: 100 moles feed

$R_D = L / D \qquad L + D = V = \bar{V} = 120$

$F = 100$

$D = 120 / (1 + R_D)$

$B = 100\text{-}120 / (1 + R_D)$

From Eq. (21.6)

$$\frac{B}{100} = \frac{0.98 - 0.50}{0.98 - x_B} = 1 - 1.20 / (1 + R_D)$$

$$x_B = 0.98 - \frac{0.48}{1 - \dfrac{1.20}{1 + R_D}}$$

Assume values of R_D, between 1.5 and 4. Compute x_B.

R_D	x_B
1.5	0.057
2	0.180
2.5	0.250
3	0.294
4	0.348

From McCabe-Thiele diagrams, drawn with these reflux ratios, compute the number of ideal plates. Estimate the total number of actual plates, using $\eta_o = 0.5$.

Number of Actual Plates Required*

R_D	In Strippping Section	In Rectifying Section
	(Not Counting Reboiler)	
1.5	13.66	18.20
2	4.96	13.78
2.5	2.54	12.20
3	1.38	11.38
4	0.26	10.48

*These numbers were generated by a computer program using the following empirical equation for relative volatility:

$a = 2.3407 + 0.2709x$

With feed on Plate 11 from the top, there are only 10 plates in the rectifying section. Cannot feed on Plate 11 even with $R_D = 4$.

With feed on Plate 17, there are 16 plates in rectifying section and 8 in the stripping section. Thus feed on Plate 17, and use a reflux ratio greater than 1.5. Plot number of plates required in each section versus R_D, and find the reflux ratio which is needed for no more than 16 plates in the rectifying section and no more than 8 in the stripping section. this R_D is 1.74 for both sections (for this condition, $x_B = 0.126$; 15.30 plates are needed in the rectifying section and 7.92 in the stripping section). Then

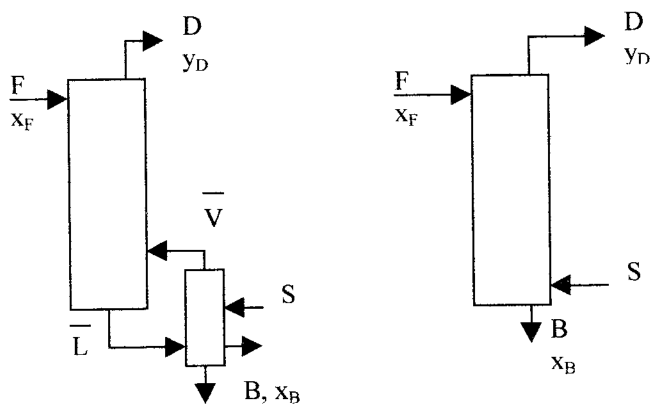
$L / D = 1.74$

$L = 1.74D$

$(1.74 + 1) = V = 120$

$D = 120 / 2.74 = 43.8$ moles / h

21.5



$x_F = 0.0794$ $x_D = y_D = 0.1125$ $D = 0.562F$

$\overline{V} = 0.562F = 0.562\overline{L}$

$$\text{Slope of operating line} = \frac{\overline{L}}{\overline{V}} = \frac{1}{0.562} = 1.779$$

Method 1. Basis: 1 mole of feed (F = 1)

From Eq. (18.5)

$$D/F = 0.562 = \frac{0.0794 - x_B}{0.1125 - x_B}$$

$x_B = 0.0370$

From Eq. (21.6)

$$B = (0.1125 - 0.0794) / (0.1125 - 0.0370) = 0.437$$

Method 2. $B = F = 1$ $D = 0.562$

Let S = moles steam / mole feed = 0.562

By a material balance:

$$(0.562 \times 0.1125) + x_B = 0.0794$$

$$x_B = 0.0162$$

McCabe-Thiele diagrams use the same operating line for the two methods, but in Method 2 it extends to the point (0, 0.0162).

Disadvantages of Method 2:

Requires 4 theoretical plates (instead of 2 plus a reboiler for Method 1).

Advantages of Method 2:

Dirtier liquid can be handled without fouling a reboiler. Also the liquid leaving the bottom contains a smaller percentage of more volatile component (though the total amount is the same by either method). This may or may not be an advantage.

21.6 Assume air is 79 mol % N_2 , 21 mol % O_2

Let x = mass fraction of N_2

Feed:

$$x_F = \frac{0.79 \times 28.02}{0.79 \times 28.02) + (0.21 \times 32)} = 0.767$$

Also, $x_B = 1\text{-}0.51 = 0.49$

x_D must be found by trial.

$R_D = 2.6 = La / D$

Follow the method of Example 21.5, assuming values for x_D of 0.90, 0.95, 0.99. Read values of H from plots of the data in Table 21.9. Find D from Eq. (21.3).

For intermediate points, based on F = 100 kg:

x_D (assumed)	0.90	0.95	0.99
Mol. wt.	28.42	28.22	28.06
H_D, cal/g(= $H_{x,a}$)	10.45	9.96	9.55
$y_a = x_D$	0.90	0.95	0.99
D	67.6	60.2	55.4
$V_a = 3.6D$	243.4	216.7	199.4
$H_{y,a}$	51.0	50.5	50.22
$R = 2.6D$	175.8	156.5	144.0
x_n (assumed)	0.83	0.86	0.90
Mol. wt.	28.70	28.57	28.42
y_{n+1} (est. with $L/V = 2.6/3.6 = 0.722$)	0.850	0.885	0.926
$H_{x,n}$	11.01	10.74	10.45
$H_{y,\, n+1}$	51.4	51.2	50.8
V_{n+1} (Eq. (21.48))	243.5	219.4	199.9
$L_n = V_{n+1}$-D	176.9	159.2	144.5
y_{n+a} (Eq. 21.49)	0.850	0.885	0.925

The calculated values of y n+1 are almost the same as estimated, and no further calculations are needed for the intermediates.

At the feed plate. Since the feed line is not vertical, it is necessary to estimate x_n and y_{n+1} for the feed plate from a McCabe-Thiele diagram. The feed line is drawn through (0.767, 0.767) with a slope of -(1-0.30) / 0.30 = 2.33. Operating lines are drawn through x_D, x_D for each assumed value with a slope of 2.6 / 3.6 = 0.722. The intersections with the feed line give the estimated values of x_n and y_{n+1}.

At the feed point

x_D (assumed)	0.90	0.950	0.99
x_n (estimated)	0.755	0.750	0.748
Mol. wt.	29.00	29.02	29.02
y_{n+1} (estimated)	0.793	0.806	0.814
Mol. wt,.	28.84	28.79	28.76
$H_{x,n}$ cal/g	11.41	11.44	11.47
$H_{y,\, n+1}$	51.94	51.82	51.74
V_{n+1} (Eq. (21.48))	240.7	215.4	198.7
$L_n = V_{n+1}$-D	173.1	155.2	143.3
y_{n+a} (Eq. 21.49)	0.796	0.806	0.815

Again, these are close enough to the assumed values that no additional calculation is needed. The operating lines are straight, and the same as calculated based on a constant mass flux (V changes from 217.0 to 220.3).

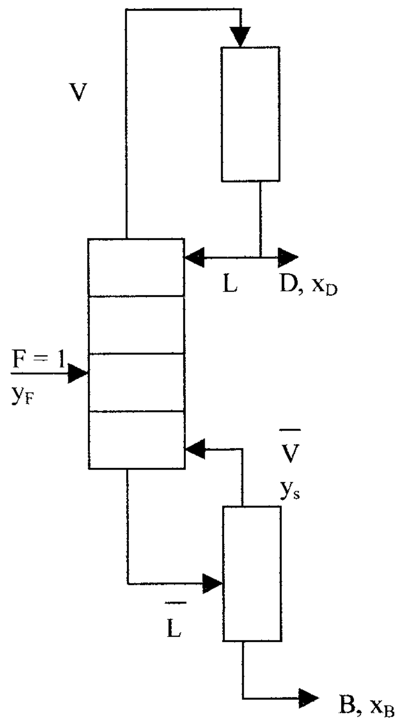
From the diagram, the estimated numbers of plates are:

x_D	0.90	0.95	0.99
Number (excluding reboiler)	1.49	2.50	4.75

For a reboiler and 6 ideal plates, by extrapolation, $x_D = 0.996$.
Overhead composition: 99.6 wt % N_2, 0.4 wt % O_2

(Note: A computer solution using constant mass overflow and α (based on mass) of 2.811, the average for the part of the diagram of interest, gives N = 6.00 when $x_D = 0.99568$.)

21.7 1 mol feed (F = 1)



$L = \bar{L} = 1.35$

$\bar{V} = 0.7$

$V = 1 + 0.7 = 1.7$

$D = 1.7 - 1.35 = 0.35$

$B = 1.35 - 0.7 = 0.65$

Since the plates are ideal, $y_n = 12.6x_n$

From Eq. (21.9), since L and V are constant,

$$Vy_{n+1} = Lx_n + Dx_D$$

$$y_{n+1} = (1.35x_D / 1.7) + (0.35x_D / 1.7)$$

$$= 0.749x_n + 0.2059x_D$$

Plate 1: $y_1 = x_D$ $x_1 = y_1 / 12.6 = x_D / 12.6$

Plate 2: $y_2 = 0.794x_1 + 0.205x_D$

$= ((0.0794 / 12.6) + 0.2059)\, x_D = 0.269\, x_D$

$x_2 = y_2 / 12.6 = 0.0213\, x_D$

Plate 3: $y_3 = 0.794x_2 + 0.2059\, x_D$

$= ((0.794 \times 0.0213) + 0.2059)\, x_D = 0.2228\, x_D$

$x_3 = 0.2228\, x_D / 12.6 = 0.01768\, x_D$

Let y_s = mol fraction in vapor from reboiler

By a material balance around reboiler:

$$1.35x_3 = 0.7y_s + 0.65\, x_B$$

Also $y_s = 12.6\, x_B$

Thus

$$1.35 \times 0.01768\, x_D = ((12.6 \times 0.7) + 0.65)\, x_B$$

$$x_D = 396.8\, x_B$$

Overall ammonia balance:

$$0.004 = 0.35\, x_D + 0.65\, x_B$$

$$= ((0.35 \times 396.8) + 0.65)\, x_B = 139.5\, x_B$$

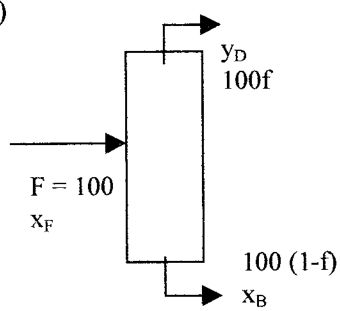
Answers:

(a) $x_B = 0.004 / 139.5 = 0.0000286$

(b) $x_D = 396.8 \times 0.0000286 = 0.01135$

(c) $x_3 = 0.01768 \times 0.01135 = 0.0002006$

21.8 (a)



$x_F = 0.68$

$x_D = 0.80$

<u>Basis: 100 moles feed</u>

From Eq. (21.1)

$$0.68 = 0.80f + (1-f)x_B$$

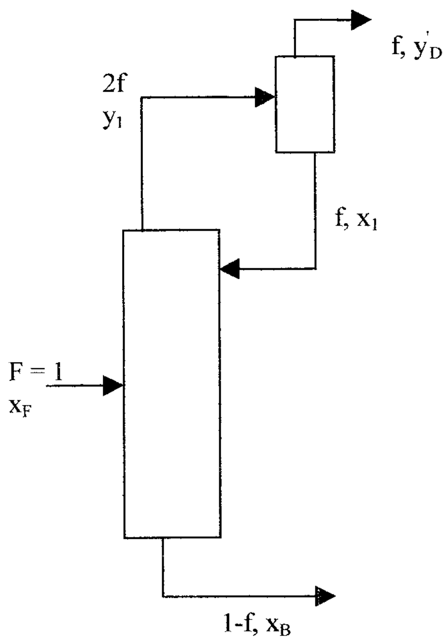
From Fig. 21.2, when $y_D = 0.80$, $x_B = 0.62$, and

$$0.68 = 0.80f + 0.62(1-f)$$
$$f = 0.333$$

Product: 33.3 mol / 100 mol feed

Moles vaporized: 33.3 mol / 100 mol feed

(b)



$y'_D = 0.80$

$x_F = 0.68$

Basis: 1 mole feed

Since y'_D and x_l are in equilibrium, when $y'_D = 0.80$, from Fig. 21.2, $x_l = 0.62$. By a material balance around the partial condenser: $2fy_l + 0.80f + 0.62f$

$$y_l = 0.71$$

From Fig. 21.2, when $y_l = 0.71$, $x_B = 0.50$

By a material balance around the entire still:

$$0.68 = 0.80f + 0.50\,(1-f)$$

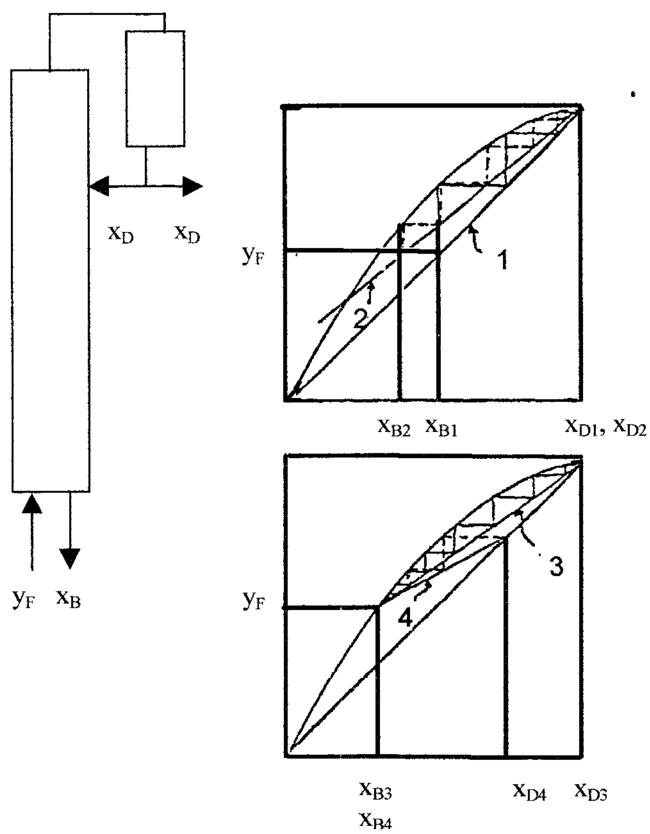
$$f = 0.600$$

Product: 60.0 mol / 100 mol feed

Moles vaporized:

$$2 \times 60.0 = 120.0 \text{ mol / 100 mol feed}$$

21.9 The solution to this problem is illustrated in the following diagram:



(a) Under Condition 1, the operating line is the 45-degree line, and the overhead product (assuming a normal x-y diagram with no azeotrope present) is pure low boiler. The invariant zone or pinch point is at the top of the column. The bottoms liquid composition, x_{B1}, equals y_F.

(b) When overhead product is withdrawn, the reflux ratio falls and the slope of the operating line decreases. With a low rate of overhead removal (Condition 2), the overhead is still pure low boiler and the invariant zone is at the top of the column, as in Condition 1.

The bottoms, however, becomes leaner: $x_{B2} < x_{B1}$. As overhead is withdrawn at higher and higher rates, a point is reached where the lower end of the operating line intersects the equilibrium line at y_F (Condition 3). Here x_{B3} and y_F are equilibrium compositions. The overhead is still pure low-boiler, but there are two invariant zones, one at the bottom and one at the top of the column.

As product is withdrawn at still higher rates, the operating line still passes through the equilibrium line at y_F, but its upper end moves downward and to the left. The bottoms product has a composition in equilibrium with y_F ($x_{B4} = x_{B3}$), but the overhead is no longer pure (Condition 4). Whereas x_{D1} . x_{D2}, and $x_{D3} \cong 1.0$, $x_{D4} < 1.0$. The single invariant zone is at the bottom of the column.

21.10 Rate of vaporization:

$$\frac{dn}{dt} = -\frac{q}{\Delta H} = -\frac{4}{40} = -0.10 \text{ g mol/s}$$

Integrating between limits gives

$$n = n_o - (qt / \Delta H)$$

Initial charge:

$$n_o = \frac{10,000}{(0.70 \times 40.5) + (0.30 \times 18)} = 296.3 \text{g mols}$$

From Eq. (21.82) , rearranging and dividing both sides by dt:

$$\frac{dx}{(y-x)\,dt} = \frac{1}{n}\frac{dn}{dt}$$

Thus $$\int_{0.7}^{x(t)} \frac{dx}{y-x} = \int_{0}^{t} \frac{-q/\Delta H dt}{[296.3-(q/\Delta/\ )t]}$$

$= \ln(1 - qt / 296.3\Delta H)$

$= \ln(1- 4t / (296.3 \times 40))$

$= \ln(1 - 3.374 \times 10^{-4}\, t)$

Set the integral $\int dx / (y-x) = I$; then

$1 - 3.374 \times 10^{-4}\, t = e^{I}$

and $t= 1 - e^{I} / (3.374 \times 10^{-4})$. Numerical integration, using the equilibrium diagram* to find y for various assumed vaolues of x, gives the data in Table A below. (*Here the results wer found by computer using the correlation for α given in the solution to Problem (21.2).

TABLE A

x	y	1/ (y-x)	I	t, seconds
0.70	0.870	5.882	0	0
0.65	0.848	5.051	-0.273	709
0.60	0.826	4.425	-0.510	1184
0.55	0.803	3.953	-0.720	1521
0.50	0.780	3.571	-0.908	1768
0.45	0.756	3.268	-1.079	1956
0.40	0.730	3.030	-1.236	2103
0.35	0.701	2.849	-1.383	2221
0.30	0.668	2.717	-1.522	2317

Note: All the liquid is boiled off at t = 296.3/0.10 = 2936 seconds.

Volume V of liquid remaining at any time is:

$$V = n[40.5x + 18(1 - x)]$$

Since $n = 296.3 - 4t / 40 = 296.3 - 0.1t$,

$$V = (296.3 - 0.1t)\,[40.5x + 18(1 - x)]$$

Setting $V = 5000$ and solving for x gives

$$x = \frac{222.2}{296.3 - 0.1t} - 0.8 \qquad \text{(B)}$$

(a) Plotting the data from Table A and Eq. (B) on a graph of x vs. t gives two lines intersecting at $t_T = 1350$ s or 22.5 min.

(b) From the plot of the data in Table A:

When $t = t_T / 2 = 675$ s, $x = 0.654$

When $t = 3t_T / 4 = 1012.5$ s, $x = 0.620$

When $t = t_T = 1350$ s, $x = 0.576$

(c) Use an overall material balance. At $t_T = 1350$ s, $n = 296.3 - (0.1 \times 1350) = 161.3$ mol. Total moles vaporized: $296.3 - 161.3 = 135.0$

Mol methanol in charge: $0.70 \times 296.3 =$	207.41
Mol methanol remaining: $0.576 \times 161.3 =$	<u>92.91</u>
Mol methanol vaporized:	114.50

Thus the fraction of methanol in the distillate at time $t_T = 114.5 / 135.0 = 0.848$

21.11 (a) The sidestream product is richer than the feed, so the intermediate section of the column is above the feed plate and between the feed plate and the plate where the sidestream S is withdrawn. The operating line for the intermediate section is obtained by making a material balance from the top of the column to a plate s in the intermediate section:

$$V_{s+1} = y_{s+1} = L_s\, x_s + Dx_D + Sx_s$$

Assuming constant molal overflow in this section, the equation for the operating line is

$$y_{s+1} = \left(\frac{L}{V}\right)_s x_s + \frac{Dx_D + Sx_s}{V_s}$$

The upper operating line is given by Eq. (21.13):

$$y_{n+1} = \left(\frac{L}{V}\right)_n x_n + \frac{Dx_D}{V_n}$$

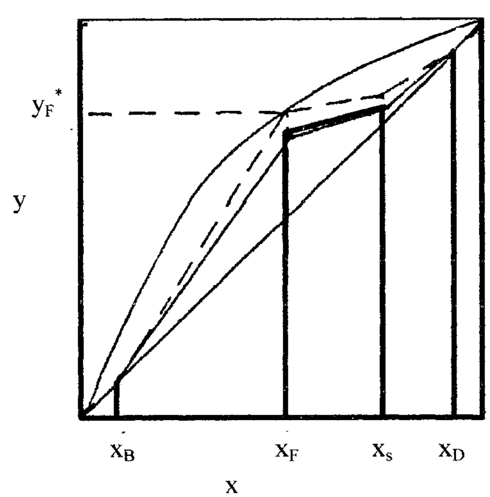
The lower operating line is given by Eq. (21.16):

$$y_{m+1} = \left(\frac{L}{V}\right)_m x_m + \frac{Bx_B}{V_m}$$

If the sidestream is withdrawn as a liquid, the flow of liquid down the column is reduced, and there is no change in the vapor rate. Therefore,

$$\left(\frac{L}{V}\right)_s < \left(\frac{L}{V}\right)_n$$

The operating lines for the 3 sections are as shown in the diagram. Dashed lines are for $(L / D)_{min}$



(b) Basis: F = 100 moles

$x_F = 0.5$, $x_D = 0.985$, $x_B = 0.02$, $x_B = 0.80$

$Fx_F = Dx_D + Bx_B + Sx_s$

Since $Sx_s = 0.4\ Fx_F = 0.4 \times 100 \times\ 0.5 = 20$

$S = 20 / 0.80 = 25$

Since $F = D + B + 25 = 100$, $B = 75 - D$ and

$$50 = 0.985D + (75 - D)(0.02) + 20$$

$$D = 29.53 \qquad B = 45.47$$

At the minimum reflux ratio, a pinch will occur at the feed plate. For the middle section,

$$Y_{s+1} = \left(\frac{L}{V}\right)_s x_s + -\frac{Dx_D + Sx_s}{V_s}$$

$$V_s = L_s + D + S = L_s + 29.53 + 25 = L_s + 54.53$$

$$Dx_D + Sx_s = 29.53\ (0.985) \times 25\ (0.80) = 49.09$$

$$y_{s+1} = \frac{L_s x_s}{L_s + 54.53} + \frac{49.09}{L_s + 54.53} \qquad \text{(A)}$$

Since $y^* = \dfrac{\alpha x}{1 + (\alpha - 1)\,x}$ and $y_{s+1} = y^*_F$ at the pinch

$x_s = 0.50$

$$y^*_F = \frac{2.3 \times 0.5}{1 + 1.3(0.5)} = 0.697 = y_{s+1}$$

From Eq. (A):

$$0.697 = \frac{L_s \times 0.5}{L_s + 54.53} + \frac{49.09}{L_s + 54.53}$$

From this, $L_s = 15.9 / 0.283 = 56.18$

Minimum $L_n = L_s + S = 56.18 + 25 = 81.18$ moles reflux

(c) For only two products with $x_B = 0.02$, $x_D = 0.985$, from Eq. (21.44):

$$R_{Dm} = \frac{x_D - y'}{y' - x'} = \frac{0.985 - 0.697}{0.697 - 0.50} = 1.46$$

From Eq. (21.5),

$$\frac{D}{F} = \frac{0.5 - 0.2}{0.985 - 0.02} = 0.497$$

Minimum Ln = $1.46 \times 100 \times 0.497 = 72.56$ moles reflux

With a side-stream drawoff the minimum reflux is 81.18 / 72.56 = 1.119 times that with no side-stream drawoff.

21.12 (a) Plot ln P' vs 1/T for n-C_4H_{10} and n-C_5H_{12}; where T is in °K, this gives linear plots. Assume pure butane at the top of the column and pure pentane at the bottom to get the temperatures at each end. Neglect the pressure drop through the column.

For P = 1 atm,

at top, T = - 0.5 °C,

$\alpha = P_A / P_B = 1 / 0.249 = 4.02$

at bottom, T = 36.1 °C, $\alpha = 3.22 / 1 = 3.22$

Average $\alpha = (4.02 \times 3.22)^{1/2} = 3.60$

For P = 2 atm,

at top, T= 18.8 °C, $\alpha = 2 / 0.549 = 3.64$

at bottom, T = 58.0 °C, $\alpha = 5.99 / 2 = 2.99$

Average $\alpha = (3.64 \times 2.99)^{1/2} = 3.60$

For P = 8 atm,

at top, T= 69.5 °C, $\alpha = 8 / 2.729 = 2.94$

at bottom, T = 114.6 °C, $\alpha = 19.7 / 8 = 2.46$

Average $\alpha = (2.94 \times 2.46)^{1/2} = 2.69$

(b) Use Fenske equation, Eq. (21.42):

For 1 atm,

$$N_{min} = \frac{\ln[0.99(0.99)/0.01(0.01)]}{\ln 3.60} - 1 = 6.17$$

For 2 atm, $N_{min} = 6.70$

For 8 atm, $N_{min} = 8.29$

Carrying out the separation at high pressure requires more plates but permits the use of air-cooled or water-cooled condensers.

21.13 Basis: 1 hour (Ethylbenzene = EB; styrene = S)

Mol. wt. of EB = 106.16; of S = 104.14

Feed: $30000 \times 0.54 / 106.16 = 152.6$ mol EB / h

$30000 \times 0.46 / 104.14 = \underline{132.5}$ mol S / h

Total 285.1

$x_F = 152.6 / 285.1 = 0.535$

$x_B = 0.002$ (not worth correcting to mol fraction)

$x_D = (97 / 106.16) / (97 / 106.16 + 3 / 104.14) = 0.969$

From Eq. (21.5)

$D / F = (0.535 - 0.002) / (0.969 - 0.002) = 0.551$

$D = 0.551 \times 285.1 = 157.1$ kg mol / h

$B = 285.1 - 157.1 = 128.0$ kg mol / h

Vapor flow in rectifying section:

$R / D = 6.15$

$V = (R + 1)D = 7.15 \times 157.1 = 1123.3$ kg mol / h

Vapor density at top of column, based on EB (58 °C)

$$\rho_v = \frac{106.16}{22.4} \times \frac{50}{760} \times \frac{273}{331} = 0.257 \text{ kg / m}^3$$

(a) Column diameter

Let $F = 2.8 = u_o\sqrt{\rho_v}$ (m / s) (kg / m^3)$^{0.5}$

$u_o = 2.8 / 0.257^{0.5} = 5.52$ m / s (18.1 ft / s)

Volumetric flow rate, q_v:

$$q_v = 1123.3 \times 106.16 / (3600 \times 0.257) = 128.9 \text{ m}^3 / \text{s}$$

Area of column: $A = q_v/u_o = 128.9 / 5.52 = 23.35 \text{ m}^2$

Diameter: $D = (4 \times 23.35 / \pi)^{0.5} = 5.45 \text{ m (17.9 ft)}$

(b) <u>At bottom of column</u>

$P = 50 + (70 \times 2.5) = 225$ mm Hg

Boiling point of styrene at this pressure:

107 °C (380 K)

$$\rho_v = \frac{104.14}{22.4} \times \frac{225}{760} \times \frac{273}{380} = 0.989 \text{ kg / m}^3$$

For the same total vapor flow:

$$q_v = 1123.3 \times 104.14 / (3600 \times 0.989) = 32.85 \text{ m}^3 / \text{s}$$

For the same column area (23.35 m^2)

$$u_o = q_v / A = 32.85 / 23.35 = 1.41 \text{ m / s}$$
$$F = 1.41\,(0.989)^{0.5} = 1.40$$

(c) The diameter of the lower section should be such that F at the top of the lower section (say halfway up the column) is 2.8. At this point, assume:

$$P = 50 + 35\,(2.5) = 137.5 \text{ mm Hg}$$
$$T = (107 + 58) / 2 = 82.5 \text{ °C (335.5 K)}$$
$$\rho_v = \frac{105}{22.4} \times \frac{137.5}{760} \times \frac{273}{355.5} = 0.651 \text{ kg / m}^3$$
$$q_v = 1123.3 \times 105 / (3600 \times 0.651) = 50.3 \text{ m}^3 / \text{s}$$

For $F = 2.8$, $u_o = 2.8 / 0.651^{0.5} = 3.47$ m / s

Area: $A = 50.3 / 3.47 = 14.5 \text{ m}^2$

$$D = (4 \times 14.5 / \pi)^{0.5} = 4.30 \text{ m (14.1 ft)}$$

21.14 Consider the combined feed first to choose a reflux rate.

$x_F = (0.37 + 0.68) / 2 = 0.525$

for $F = 100$, $x_D = 0.98$, $x_B = 0.01$, from

Eq. (21.15):

$$\frac{D}{F} = \frac{0.525 - 0.01}{0.98 - 0.01} = 0.531$$

$D = 53.1$, $B = 100 - 53.1 = 46.9$

With liquid feed at its boiling point, use Eq. (21.44) for R_{Dm}. See Fig. 21.2 for equilibrium data, which gives

$x = x_F = 0.525$; $y = 0.73$

$$R_{Dm} = \frac{0.98 - 0.73}{0.73 - 0.525} = 1.22$$

Use a reflux ratio 1.2 times R_{Dm}.

$R_D = 1.2 \times 1.22 = 1.46$

$R = 1.46 \times 53.1 = 77.5$

Assume constant molal overflow for simplicity and because the error from this assumption would be about the same for both cases.

The upper operating line is drawn from 0.98, 0.98 to $x = 0$, $y = 0.98 / 2.46 = 0.398$. The lower operating line is drawn from 0.01, 0.01 to the intersection of the upper operating line and the vertical q line from $x_F = 0.525$. From the McCabe-Thiele diagram (by computer program as in Problem 21.4):

$N = 18.7$ ideal plates plus a reboiler

<u>With two feeds:</u> The upper operating line is unchanged, but this line ends at $x = 0.68$, the concentration of the richer feed. For the middle section:

$L = 77.5 + 50 = 127.5$

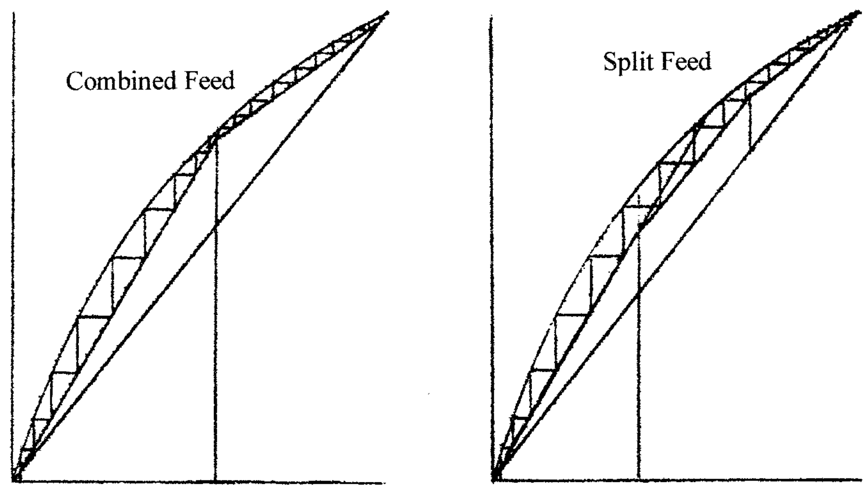
$V = R + D = 77.5 + 53.1 = 130.6$

$L / V = 127.5 / 130.6 = 0.976$

The slope of the upper operating line is 1.46 / 2.46 = 0.593. A line with slope 0.976 is drawn from x = 0.68, y = 0.98 - (0.98 - 0.68)0.593 = 0.802 until it intersects the lower operating line. The middle and lower lines intersect as expected at x = 0.37, the concentration of the lower feed.

From the McCabe-Thiele diagram (computer solution), N = 16.0 ideal plates plus a reboiler

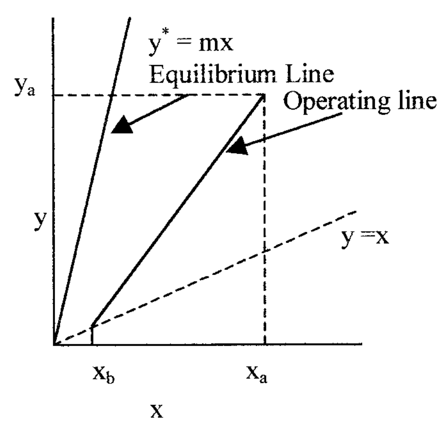
The use of separate feed points could permit reducing the number of ideal plates by 2.7 or the number of actual plates by 4 or 5.



21.15

$$y^* = \frac{\alpha x}{1+(\alpha-1)x}$$

Since x_F is about 3.5×10^{-3} mole fraction, assume a straight equilibrium line even though $\alpha = 120$.



$y^* \cong 120x$

Since the liquid and the vapor are mostly toluene, use ppm (by wt) for convenience, and the slope of the equilibrium line is still 120 (ppm / ppm)

Use Eq. (20.28).

$$N \frac{\ln\left[(x_a - x_a^*)/(x_b - x_b^*)\right]}{\ln S}$$

where S = mV/L = 120 × 0.25 = 30

If x_b is taken as the product composition x_B ,N in Eq. (20.28) is the total number of stages including the reboiler.

$x_a = 680$

$x_a^* = y_a / m$

$y_a = y_b + \frac{L}{V}\Delta x$

$= 0.3 + (1 / 0.25)(680 - 0.3) = 2719$

$x_a^* = 2719 / 120 = 22.7 \qquad x_b = 0.3$

$x_b^* = 0.3 / 120 = 2.5 \times 10^{-3}$

$$N = \frac{\ln\left[(680 - 22.7) / (0.3 - 2.5\times10^{-3}\right]}{\ln 30} = 2.26$$

NOTE: Because of the low value of m, the plate efficiency would be expected to be low, as explained in Chapetr 17, and a large number of plates would be used in practice.

21.16 Basis: 100 moles of feed (F = 100)

$x_F = 0.40 \qquad x_D = 0.99$

From the data in Table 21.6

$x' = x_F = 0.40 \quad y' = 0.729$

From Eq. (21.44)

$$R_{Dm} = \frac{0.99 - 0.729}{0.729 - 0.40} = 0.793$$

If x_B is very small, Eq. (21.5) gives

$$D = 100\ (0.40 / 0.99) \cong 40$$

Use $\quad L_n = 1.2 \times 0.793 \times 40 = 38.1$

$$V_n = 38.1 + 40 = 78.1$$

The mol wt. of methanol is 32. At the top of the column where T = 65 °C (338 K):

$$\rho_v = \frac{32}{359} \times \frac{273}{338} = 0.072 \text{lb} / \text{ft}^3\ (1.153\ \text{kg/m}^3)$$

$$\rho_L \cong 46.2\ \text{lb} / \text{ft}^3\ (\ 740\ \text{kg} / \text{m}^3\ )$$

Use Eq. (21.66) and Fig. 21.29

$$\frac{L}{V}\left(\frac{\rho_V}{\rho_L}\right)^{0.5} = \frac{38.1}{78.1}\left(\frac{0.072}{46.2}\right)^{0.5} = 1.93 \times 10^{-2}$$

From Fig. 21.29, for a 24-inch (610-mm) plate spacing, $K_V = 0.39$.

From Eq. (21.66), since $\sigma = 19.5$ dyne / cm

$$u_c = 0.39\left(\frac{46.2 - 0.072}{0.072}\right)^{0.5}\left(\frac{19.5}{20}\right)^{0.2} = 9.82\ \text{ft/s}(2.99\ \text{m/s})$$

At top of column,

$$F = u_c \rho_V^{0.5} = 2.99(1.153)^{0.5} = 3.21(\text{m/s})(\text{kg/m}^3)^{0.5}$$

(See Fig. 21.35 for evidence that this is near the maximum allowable value.)

With 8% open area in the plates,

$$u_o = 2.99 / 0.08 = 37.4\ \text{m / s}$$

From Fig. 21.28, for a tray thickness 0.2 times the hole diameter, Co = 0.70. From Eq. (21.59),

$$h_d = 51.0\,(\frac{37.4}{0.70})^2(\frac{1.153}{740}) = 228 \text{ mm of liquid}$$

To get h_l, a column diameter and q_L are needed. Assume a column diameter of 1.2 m with weir lengths L = 0.8 times the diameter or 0.96 m. The downcomer area is then 0.14 x column area. (Perry, 6th ed., p. 1-26) Total vapor flow:

$$q_V = 2.99(\frac{\pi \times 1.2^2}{4})(1-(2\times 0.14)) = 2.43 \text{ m}^3/\text{s}$$

Liquid flow:

$$q_L = q_V(\frac{L}{V})(\frac{\rho_V}{\rho_L})$$

$$= 2.43(\frac{38.1}{78.1})(\frac{1.153}{740}) = 1.86\times 10^{-3}\text{m}^3/\text{s}$$

Assume weir height h_w = 50 mm

From (21.61), (21.60), and (21.58) with $\beta = 0.6$

$h_{ow} = 43.4\ (1.86 \times 10^{-3} \times 60 / 0.96\ )^{2/3} = 10.3$ mm

$h_l = 0.6\ (50 + 10.3) = 36.2$ mm

$h_t = 228 + 36.2 = 264.2$ mm liq

or 264.2 (46.2 / 62.3) = 196 mm H_2O

The height of clear liquid in the downcomer should be calculated as a check on the flooding limit from Fig. 21.29

From Eq. (21.63), neglecting $h_{f,L'}$

$Z_c = 2 \times 0.6 \times (50 + 10.3) + 228 = 300$ mm

The downcomers are only half filled with clear liquid. However, if the average froth density is half the liquid density, the froth height would be 600 mm compared to the plate spacing of 610 mm, and the column would be almost flooded.

(b) Near the bottom of the column the temperature is about 100 °C and the vapor nearly all water. A rough estimate of the vapor flow is obtained from the flow at the top and the ratio of the heats of vaporization:

$$\frac{V_m}{V_n} = \frac{8410 \text{ cal / mol}}{9710} = 0.866$$

At the bottom of the column:

$$u = 2.99 \text{ m / s} \times 0.866 \times (373 / 338) = 2.86 \text{ m / s}$$

$$\rho_V = \frac{18 \times 273}{22.4 \times 373} = 0.588 \text{kg/m}^3$$

$$F = 2.86 \times 0.588^{0.5} = 2.19 \text{ (m / s) (kg / m}^3\text{)}^{0.5}$$

$$\rho_L = 59.8 \times 16.0185 = 957.9 \text{ kg / m}^3 \text{ (Appendix 14)}$$

As in Part 9(a)

$$h_d = 51.0 (35.8 / 0.7)^2 (0.588 / 957.9) = 81.9 \text{ mm } H_2O$$

$$q_v = 2.86(\pi \times 1.2^2 / 4) (1\text{-}0.72) = 2.33 \text{ m / s}$$

Per 100 moles of feed, $V_m = 0.866 \times 78.1 = 67.6$

From a material balance over the column,

$$L_m = L_n + F + V_m - V_n = 38.1 + 100 + 67.6 - 78.1 = 127.6$$

$$q_L = 2.33 (127.6 / 67.6)(0.588 / 957.9)\text{=}2.70 \times 10^{-3} \text{ m}^3 \text{ / s}$$

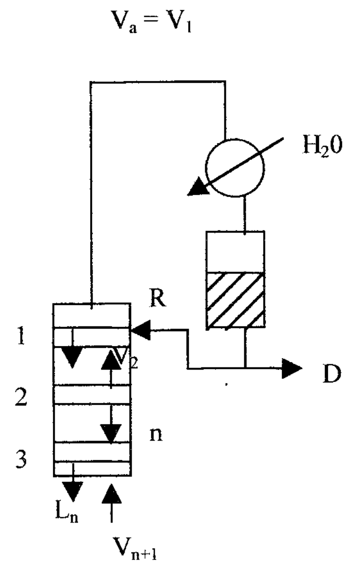
$$h_{OW} = 43.43(2.70 \times 10^{-3} \times 60 / 0.96)^{2/3} = 13.3 \text{ mm}$$

$$h_l = 0.6(50 + 13.3) = 38.0 \text{ mm}$$

$$h_t = 81.9 + 38.0 = 119.0 \text{ mm}$$

This is much less than the 196 mm in the top of the column. The column will flood near the top because of the higher vapor density and the slightly higher vapor velocity in the upper section (2.99 vs. 2.86 mm / s).

21.17



$V_1 < V_2$ because cold reflux causes some vapor to condense on plate1. If R/D is kept constant, both R and D will decrease $V_{n+1} = L_n + D$ as before

If the vapor rate to the column is not changed, the decrease in D means an increase in Ln, even though R has decreased, since $L_n = V_{n+1} - D$.

Therefore $(L/V)_n$ increases and the slope of the operating line increases. The overhead product becomes purer, since the upper operating line is further below the equilibrium line, but the value of x_B increaes because less methanol is included in the overhead product.

$F x_F = Dx_D + Bx_B$

For example, $F = 100$, $x_F = 0.50$, $x_D = 0.99$, $x_B = 0.01$, $D = B = 50$
If D is lowered to 48 because of cold reflux, the maximum $Dx_D = 48$, leaving $Bx_B = 52$, Then $x_B \cong 2/52 = 0.038$.

22.1 Basis: 1 mole of feed. The mass balances are:

Component 2 in distillate:	$0.99 \times 0.42 = 0.4158$
Component 2 in bottoms:	$0.42 - 0.4158 = 0.0042$
Component 3 in bottoms:	$0.98 \times 0.46 = 0.4508$
Component 3 in distillate:	$0.46 - 0.4508 = 0.0092$

Assume all of component 1 is in the distillate, and all of component 4 is in the bottoms. Then:

	Distillate	Distillate	Bottoms	Bottoms
Component	Mol	x_D	Mol	x_B
1	0.0500	0.1053	-	-
2	0.4158	0.8754	0.0042	0.0080
3	0.0092	0.0193	0.4508	0.8587
4	-	-	0.0700	0.1333
Total	0.4750	1.0000	0.5250	1.0000

Use the Fenske equation [Eq. (19.13)] on components 2 and 3, with i = 2 and j = 3.

$$N_{min} = \frac{\ln \dfrac{0.8754/0.0080}{0.0193/0.8587}}{\ln (1.7/1.0)} - 1 = 15.00 \text{ plates plus reboiler}$$

Check the terminal concentrations of components 1 and 4. The concentrations of component 4 in the condenser, which was assumed zero above, can be calculated by applying Eq. (22.13) to components 3 and 4, using i = 3 and j =4.

$$16.00 = \frac{\ln \dfrac{0.0193/0.8587}{x_{D4}/0.1333}}{\ln (1.0/0.65)}$$

This gives $x_{D4} = 3.0 \times 10^{-6}$, which is satisfactory. The concentration of component 1 in the reboiler is found using i = 1 and j = 2.

$$16.00 = \frac{\ln \dfrac{0.1053/x_{B1}}{0.8754/0.0080}}{\ln(2.1/1.7)}$$

Thus $x_{B1} = 3.3 \times 10^{-5}$

22.2 Use component 2 as the light key (A) and component 3 as the heavy key (B). From Eq. (22.15), since the fractional recoveries of A and B in the distillate are 0.99 and 0.02, respectively:

$$\frac{L_{min}}{F} = \frac{0.99 - 1.7(0.02)}{1.7 - 1} = 1.366$$

On the basis o 1 mole of feed and a recovery of A of 0.99, the results of Problem 22.1 give:

$$0.8754D / (1 \times 0.42) = 0.99$$

$$D = 0.99 \times 0.42 / 0.8754 = 0.475$$

Hence $R_{Dm} = L_{min} / D = 1.366 / 0.475 = 2.88$

Pseudo-Binary

Considering only components 2 and 3 and using x' and y' for the pseudo-binary system:

$$x_F = 0.42 / (0.42 + 0.46) \qquad = 0.4773$$
$$x_D = 0.4158 / (0.4158 + 0.0092) \qquad = 0.9784$$
$$x_B = 0.0042 / (0.0042 + 0.4508) \qquad = 0.0092$$

In equilibrium with x_F',

$$y' = \frac{\alpha x'}{1 + (\alpha - 1)x'} = \frac{1.7 \times 0.4773}{1 + (0.7 \times 0.4773)} = 0.6082$$

From Eq. (18.43),

$$R_{Dm} = \frac{0.9784 - 0.6082}{0.6082 - 0.4773} = 2.83$$

Underwood method

Since $\alpha_{2\text{-}3} = 1.7$, ϕ in Eqs. (22.29) and (22.30) must be between 1 and 1.7. By trial, as in Example 22.4, $\phi = 1.269$. Then setting g_i as the right-hand side of Eq. (22.3), the results are:

Component	αi	x_{Fi}	f_i, ϕ=1.269	x_{Di}	g_i
1	2.1	0.05	0.1264	0.1053	0.2534
2	1.7	0.42	1.6566	0.8754	3.4529
3	1.0	0.46	-1.7100	0.0193	-0.0717
4	0.65	0.07	-0.0735	-	-

$$\sum f_i = -0.0006 \qquad \sum g_i = 3.6346$$

From Eq. (22.30), R_{Dm} = 3.6346 - 1 or 2.635. This is 7% less than predicted from the pseudo-binary and 8.7% less than found from Eq. (22.15). Using the Gilliland correlation (Fig. 22.5), with R_{Dm} = 2.635 and N_{min} from Problem 22.1 equal to 15.00 + 1 or 16.00:

$$R_D = 1.3\ 2.635 = 3.426$$

The abscissa is (3.426 - 2.635) / 4.426 = 0.1787
From Fig. 19.5, $(N - N_{min}) / (N + 1) = 0.46$

$$N = 16.46 / (1 - 0.46) = 30.5$$

or 29.5 plates plus a reboiler.

From the pseudo-binary system with α =1.7 and R = 1.3 × 2.83 = 3.679, a computer solution gives N = 29.48 plus a reboiler.

22.3 From Perry, 7th ed., p. 2-50, make a plot of vapor pressure vs. temperature. The vapor pressure of propane is 270 psia (K = 1) at 121 °F; for iso-butane it is 270 psia at 198°F. As in Example 22.1, temperature T is found so that Eq. (22.7) is satisfied. By trial T = 134 °F. The results are:

Component	x_i	K_i	$x_iK_i = y_i$
C_2	0.10	2.90	0.290
C_3	0.45	1.11	0.500
i-C_4	0.30	0.53	0.159
n-C_4	0.15	0.39	0.059

$\sum = 1.008$

<u>Fraction vaporized</u>

The feed is heated to 134 + (1.8 20) = 170 °F. Average temperature is (134 + 170) / 2 = 152 °F. From J.B. Maxwell, Data Book on Hydrocarbons, D. Van Nostrand, New York, 1950, the specific heats and heats of vaporization are found. The heat to heat one mole of feed from 134 °F to 170°F is

Component	C_p, Btu/lb mol-°F	Moles in Feed	Heat Required, Btu
C_2	14.58	0.10	52.5
C_3	20.76	0.45	336.3
i-C_4	27.36	0.30	295.5
n-C_4	27.65	0.15	149.3
			Total = 833.6

The vapor is assumed to have the composition shown by x_iK_i. The heats of vaporization, at the appropriate partial pressure of each component, and the heat required to vaporize 1.008 moles are

Component	y_i	MW	ΔH_V, Btu / lb	Q, Btu
C_2	0.290	30.07	90	785
C_3	0.500	44.09	115	2535
i-C_4	0.159	58.12	135	1248
n-C_4	0.059	58.12	135	463
				5031

Average heat of vaporization:

5031 / 1.008 = 4991 Btu / lb mole

Fraction vaporized: 833.6 / 4991 = 0.167 or 16.7%

NOTE:This amount of vaporization would change the liquid and vapor compositions, and the fraction actually vaporized would be slightly different.

22.4 Basis: 1 mole of feed. Assume none of Component 1 in the distillate. The split of component 3 between overhead and bottoms must be found by trial. Assume 60% of component 3 is in the distillate.

	α_i	Moles	Dist.	Dist.	Bottoms	Bottoms
Comp.		Feed	Moles	x_{Di}	Moles	x_{Bi}
1	2.6	0.06	0.0600	0.1224	-	-
2	1.9LK	0.40	0.3960	0.8078	0.0040	0.0078
3	1.5	0.05	0.030	0.0612	0.0200	0.0392
4	1HK	0.42	0.0042	0.0086	0.4158	0.8156
5	0.6	0.07	-	-	0.0700	0.1374
			0.4902	1.0000	0.5098	1.0000

From Eq. (22.13)

$$N_{min}+1=\frac{\ln\dfrac{0.8087/0.0078}{0.0086/0.8156}}{\ln 1.9}=14.32$$

The ratio of x_D / x_B for component 3 is then estimated Eq. (22.13). With i = 3 and j = 4.

$$14.32=\frac{\ln\dfrac{x_{D3}/x_{B3}}{0.0086/0.8156}}{\ln 1.5}$$

x_{D3} / x_{B3} = 3.508

Check: With i = 2, j = 3, and $\alpha = 1.9 / 1.5$,

$$x_{D3} / x_{B3} = 3.508$$

By trial, the split of moles of component 3 to give this is 3.49: 1. This gives

	Dist.	Dist.	Bottoms	Bottoms
Comp.	Moles	x_{Di}	Moles	x_{Bi}
1	0.0600	0.1202	-	-
2	0.3960	0.7934	0.0040	0.0080
3	0.0389	0.0779	0.0111	0.0222
4	0.0042	0.0085	0.4158	0.8301
5	-	-	0.0700	0.1397
	0.4991	1.0000	0.5009	1.0000

Here $x_{D3} / x_{B3} = 0.0779 / 0.0222 = 3.509$

Check on N_{min}

$$N_{min} + 1 = \frac{\ln \dfrac{0.7934/0.0080}{0.0085/0.8301}}{\ln 1.9} = 14.34$$

Check on component 1 in bottoms. i = 1, j = 2

$$14.34 = \frac{\ln \dfrac{0.1202/x_{B1}}{0.7934/0.0080}}{\ln (2.6/1.9)}$$

$$x_{B1} = 1.34 \times 10^{-5}$$

Check on component 5 in distillate. i = 4, j = 5

$$14.34 = \frac{\ln \dfrac{0.0042/0.4158}{x_{D5}/0.0070}}{\ln (1.0/0.6)}$$

$$x_{D5} = 4.7 \times 10^{-7}$$

At minimum reflux with q = 1, from Eq. (22.29)

$$\sum f_i = 0 = \sum \frac{\alpha_i x_{Fi}}{\alpha_i - \varphi}$$

There are two values of ϕ between 1.0 and 1.9 that satisfy Eq. (22.29), since there is one component between the light and heavy keys. By trial, these are found to be 1.267 and 1.550.

Comp.	α_i	f_i $\phi = 1.267$	f_i $\phi = 1.266$	f_i $\phi = 1.550$	f_i $\phi = 1.549$
1	2.6	0.1170	0.1169	0.1486	0.1484
2	1.9	1.2006	1.1987	2.1714	2.1652
3	1.5	0.3219	0.3205	-1.5000	-1.5306
4	1.0	-1.5730	-1.5789	-0.7636	-0.7650
5	0.6	-0.0630	-0.0631	-0.442	-0.0443
		0.0035	-0.0059	0.0122	-0.0263

When these two values of ϕ are substituted into Eq. (22.30) to get R_{Dm}, different values result if the values of x_{Di} calculated for N_{min} are used.

$$R_{Dm} = \sum \frac{\alpha_i x_{Di}}{\alpha_i - 1.267} - 1 = 2.086$$

$$R_{Dm} = \sum \frac{\alpha_i x_{Di}}{\alpha_i - 1.550} - 1 = 1.152$$

This difference comes mainly from an incorrect value for x_{D3}, the component between the keys, since the other concentrations cannot be altered significantly. If the 0.05 moles of component 3 are split with in the distillate and 0.05 $-\beta$ in the bottoms, and if the slight changes in x_{D1}, x_{D2}, and x_{D4} are neglected, the distillate product is about 0.50 moles, and the two summations can be written as follows.

$$\sum_1 = \frac{2.6(0.1202)}{2.6-1.267} + \frac{1.9(0.7934)}{1.9-1.267} + \frac{1.5(\beta/0.50)}{1.5-1.267} + \frac{1.0(0.0085)}{1.0-1.267}$$

$$\sum_2 = \frac{2.6(0.1202)}{2.6-1.55} + \frac{1.9(0.7934)}{1.9-1.55} + \frac{1.5(\beta/0.50)}{1.5-1.55} + \frac{1.0(0.0085)}{1.0-1.55}$$

$$\sum_1 = 2.584 + 12.88\beta$$
$$\sum_2 = 4.589 - 60\beta$$

For $\sum_1 = \sum_2$, $\beta = 0.0275$

The x_{Di} values are revised and used in Eq. (22.30)

Comp.	Moles in D	x_{Di}	$\frac{\alpha_i x_{Di}}{(\alpha_i - 1.267)}$	$\frac{\alpha_i x_{Di}}{(\alpha_i - 1.550)}$
1	0.06	0.1230	0.2399	0.1484
2	0.396	0.8120	2.4373	2.1652
3	0.0275	0.0564	0.3631	-1.5306
4	0.0042	0.0086	-0.0322	-0.7650
5	-	-	-	-0.0443
	0.4877	1.000	Σ =3.008	Σ =3.007 checks

From Eq. (22.30), $R_{Dm} = 3.008 - 1 = 2.008$

$L = R = 2.008\ (0.4877) = 0.9793$
$V = R + D = 1.467$
$(L/V) = 0.9793 / 1.467 = 0.6676$

$\overline{L} = L + F = 1.9793$

$\overline{L}/\overline{V} = 1.9793/1.467 = 1.349$

As R is decreased towards R_m and N increased towards infinity, the following concentration changes take place.

a. The value of x_{D3} decreases from 0.0779 towards the final value of 0.0564, which corresponds to a more nearly equal split of this component between the distillate and bottoms.

b. The value of x_{D5} decreases from the already low value of 4.7×10^{-7} to zero, since each plate above the pinch will lower the value of y_5 according to Eq. (22.17). (Since $a = 0.6$ based on component 4, and $K_4 \cong 0.7$ at the feed bubble point, $K_5 \cong 0.42$, definitely less than L/V, which is 0.667).

c. The value of x_{B1} decreases from 1.34×10^{-5} to zero as R approaches R_m, since $K_1 \cong 1.8$, which is much greater than $\bar{L}/\bar{V}$ (see Eq. (22.19))

d. The concentrations of the keys shift slightly as the split of component 3 changes, but the percent recovery for each key does not change.

22.5 Basis: 1 mole of feed. Assume $x_{B1} = 0$, $x_{D5} = 0$, and 99.7% of component 2 appears in the distillate. (This split must be found by trial.)

	Feed	Dist.	Dist.	Bottoms	Bottoms
Comp.	Moles	Moles	x_{Di}	Moles	x_{Bi}
1	0.054	0.05400	0.0706	-	-
2	0.221	0.220337	0.2878	0.000663	0.0028
3	0.488	0.48312	0.6311	0.00488	0.0208
4	0.212	0.00806	0.0105	0.20394	0.8698
5	0.025	-	-	0.02500	0.1066
		0.765517	1.0000	0.234483	1.0000

From Eq. (22.13), with i = 3, j = 4, $\alpha = 1.13/1.0$

$$N_{min} + 1 = \frac{\ln \dfrac{0.6311/0.0208}{0.0105/0.8698}}{\ln 1.13} = 64.06$$

Check on x_{B2}, with i = 2, j = 3, $\alpha = 1.15 / 1.13$

$$64.06 = \frac{\ln \dfrac{x_{D2}/x_{B2}}{0.6311/0.0208}}{\ln (1.15/1.13)}$$

x_{B2} (predicted): 0.2878 / 93.35 = 0.0030

x_{B2} (from Table): 0.0028

Close enough

Check on x_{D5}, with i = 4, j = 5, $\alpha = 1.0 / 0.7$

$$64.06 = \frac{\ln \dfrac{0.0105/0.8698}{x_{D5}/0.1066}}{\ln (1.0/0.7)}$$

$x_{D5} = 1.5 \times 10^{-13}$

Check on x_{B1}, with i = 1, j = 3, $\alpha = 1.23 / 1.13$

$$64.06 = \frac{\ln \dfrac{0.0706/x_{B1}}{0.6311/0.0208}}{\ln (1.23/1.13)}$$

$x_{B1} = 1.0 \times 10^{-5}$

For minimum reflux, use Eq. (22.29) and (22.30) solve for ϕ which must be between 1.0 and 1.13. From Eq. (22.29):

Comp.	α_i	xFi	f_i $\phi = 1.03$	f_i $\phi = 1.027$
1	1.23	0.054	0.332	0.327
2	1.15	0.221	2.118	2.066
3	1.13	0.488	5.514	5.354
4	1.0	0.212	-7.067	-7.852
5	0.7	0.025	-0.053	-0.054
			0.844	-0.159

Further trials give $\phi = 1.02745$. From Eq. (22.30)

Comp.	x_{Di}	α_i	Right-hand Side
1	0.0706	1.23	0.4287
2	0.2878	1.15	2.7007
3	0.6311	1.13	6.9541
4	0.0105	1.0	-0.3825
5	-	0.7	-
			9.7010

$R_{Dm} = 9.70 - 1 = 8.70$

Compare with Eq. (22.15) with $\alpha_{3\text{-}4} = 1.13$ and recoveries of 3 and 4 of 0.99 and 0.038 respectively.

$$L_{min} / F = (0.99 - (1.13 \times 0.038)) / (1.13 - 1) = 7.285$$

For F = 1, D = 0.7655 moles.
Assume $L_{min} = R_m = 7.285$ moles.

From above, for $R_{Dm} = 8.70$, since $R_{Dm} = L_{min} / D$

$$R_m = L_{min} = 8.70 \times 0.7655 = 6.66 \text{ moles.}$$

The agreement with 7.285 is fair.

Reflux ratio for 100 ideal plates

$N = 100 \qquad\qquad N_{min} = 64.06$

$$\frac{N - N_{min}}{N+1} = \frac{100-64.06}{101} = 0.356$$

From Fig. 22.5, $(R_D - R_{Dm}) / (R_D + 1) = 0.35$

$R_{Dm} = 8.7$; $R_D = 13.92$

22.6

Comp.	x_F	b.p., °C	P' at 110 °C, mm Hg	α
LK Benzene	0.30	80.1	1800	2.43
HK Toluene	0.25	110.6	740	1.0
Ethylbenzene	0.45	136.2	360	0.486

Basis: 100 moles feed

Comp.	Moles Feed	Moles Distillate	Moles Bottoms	Mole Fraction Distillate	Mole Fraction Bottoms
Benzene	30	29.4	0.6	0.9916	0.0085
Toluene	25	0.25	24.75	0.0084	0.3518
Ethylbenzene	45	~0	~ 45	0	0.6397
		29.65	70.35		

Benzene : $30 \times 0.98 = 29.4$; 0.6 benzene in bottoms.

(a) For a first approximation, all ethylbenzene goes to the bottoms product.

Use Fenske equation (Eq. (22.13)) for N_{min}. Evaluate α at 110 °C.

$\alpha_{LK/HK} = 2.43$

$$N_{min} = \frac{\ln \dfrac{0.9916/0.0085}{0.0084/0.3518}}{\ln 2.43} - 1 = 8.57$$

(b) Apply Fenske equation to benzene-ethyl benzene separation with $\alpha = 1800 / 360 = 5.0$

$$\ln\left(\frac{29.4/0.6}{n/45}\right) = (8.57 + 1)\ln 5.0 = 15.40$$

$$\left(\frac{2205}{n}\right) = 4.89 \times 10^{6}$$

$n = 4.51 \times 10^{-4}$ moles EB in distillate

$$x_{EB} = \frac{4.51 \times 10^{-4}}{29.65} = 1.5 \times 10^{-5}$$

(c) 0.2 atm = 152 mm Hg

Comp.	T, °C	P' mm Hg	p= P' x_F	α
Benzene	60	390	117	2.79
Toluene	60	140	35	1
Ethylbenzene	60	56	25	0.40
			$\sum = 177$	

Comp.	T, °C	P' mm Hg	p= P' x_F	α
Benzene	55	322	97	2.88
Toluene	55	112	28	1
Ethylbenzene	55	44	9	0.39
			$\sum = 134$	

At the boiling point at 0.2 atm, $\alpha \cong 2.85$ compared to 2.43 at 1 atm.

From Eq. (22.13),

$$N_{min} + 1 = \frac{\ln \dfrac{29.4/0.6}{0.25/24.75}}{\ln 2.85} = 7.1$$

The heat of vaporization per mole is slightly higher at the lower pressure, but for a given number of plates, less reflux is needed because of the higher relative volatility. Whether to operate at 1 atm or 0.7 atm would depend on the temperatures of the heating medium and cooling water rather than on slight changes in total heat duty.

Chapter 23

23.1 Basis: 10 tons inert solids

$CuSO_4$ removed: $0.98 \times 1.2 = 1.176$ tons

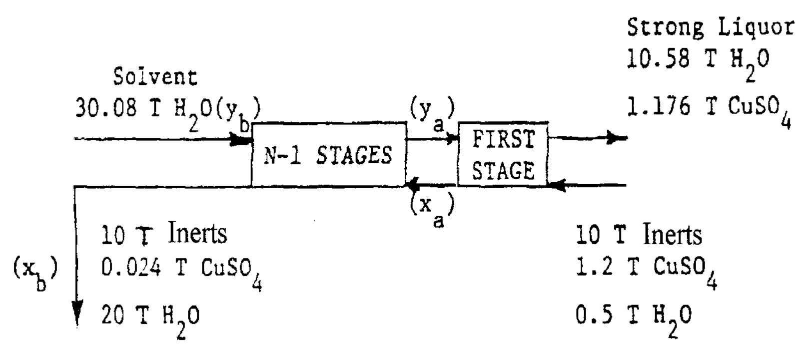
$CuSO_4$ left: $0.02 \times 1.2 = 0.24$ tons

Water required:

For strong liquor:	$1.176 \times 90/10 =$	10.58
To saturate inert solids:	$10 \times 2 = 20.00$	
Less water in feed	0.50	
Net requirement	19.50	19.50
Total water required:		30.08 tons

Water in exhausted inert solids = 20 tons; water in strong liquor = 10.08 tons.

Base calculations on tons of $CuSO_4$ per ton of water. Calculate first stage separately.



Overflow from stage 2 to stage 1

Water: 10.58 (out) + 20.00 (out) - 0.5 (in) = 30.08 tons (in)

Since the liquor leaving stage 1 with the inert solids is the same as the strong liquor:

$CuSO_4$: $1.176 + (20 \times 1.176 / 10.58) - 1.2 = 2.199$ tons

Concentration: $2.199 / 30.08 = 0.0731$

Use Eq. (20.27) for stages other than stage 1. The quantities are

$$x_a = y_a^* = 1.176 / 10.58 = 0.1112$$

$$x_b = y_b^* = 0.024 / 20 = 0.0012$$

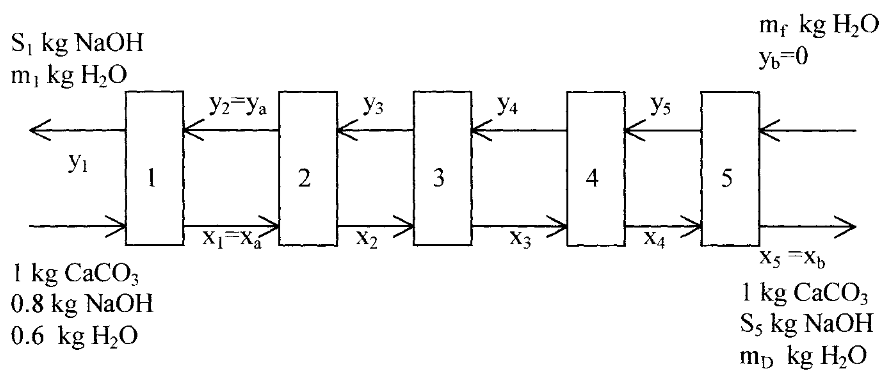
$$y_b = 0 \qquad y_a = 0.0731$$

$$N - 1 = \frac{\ln \dfrac{0.0731 - 0.1112}{0 - 0.0012}}{\ln \dfrac{0.0012 - 0.1112}{0 - 0.0731}} = 8.5$$

N = 9.5 say 10 stages

23.2 Basis: 1 kg solid ($CaCO_3$)

Let S = kg NaOH; m = kg H_2O



Mol. wt. $CaCO_3$ = 100 Mol. wt. NaOH = 40

NaOH fed = $1 \times 2 \times 40 / 100 = 0.80$ kg

$$S_1 = 0.99 \times 0.8 = 0.792$$

$$S_5 = 0.80 - 0.792 = 0.008$$

$$m_D = 1.5 - 0.008 = 1.492$$

From an over-all water balance,

$m_f + 0.6 = m_l + 1.492$

$m_l = m_f - 0.892$

$y_1 = 0.792 / (0.792 + m_l) = x_a = y_a^*$

$y_a^* = 0.792 / (0.792 + m_f - 0.892)$

$= 0.792 / (m_f - 0.1)$ (A)

$y_b = 0$

$x_b = y_b^* = 0.008 / 1.5 = 0.00533$

Overflow from stage 2 to stage 3: Since the total flow is m_f,

NaOH = $1.5x_a$ (out) + 0.792 (out) - 0.8 (in)

$= 1.5\,x_a - 0.008 = y_2 m_f$ (in)

Hence $y_2 = y_a = (1.5x_a - 0.008) / m_f$

$= (1.5\,y_a^* - 0.008) / m_f$ (B)

From Eq. (20.27)

$$N = 4 = \frac{\ln \dfrac{y_a - y_a^*}{0 - 0.00533}}{\ln \dfrac{0.00533 - y_a^*}{0 - y_a}}$$

Multiplying through by - 1 and transposing:

$$4 \ln\left(\frac{y_a^* - 0.00533}{y_a}\right) = \ln 187.62(y_a^* - y_a)$$

$$\left(\frac{y_a^* - 0.00533}{y_a}\right)^4 = 187.62(y_a^* - y_a) \qquad \text{(C)}$$

(a) Assume values of m_f, compute $x_a = y_a^*$ and also y_a from Eqs. (A) (B), and (C). By trial, Eq. (C) is satisfied when $m_f = 3.376$ kg, $y_a^* = x_a = 0.2418$, and $y_a = 0.1051$.

(b) Then: $m_1 = 3.376 - 0.892 = 2.484$ kg

$y_1 = 0.792 / (0.792 + 2.484) = 0.2418$
(the same as x_a and y_a^*)

$y_2 = y_a = 0.1051$

$x_2 = y_2 = 0.1051$

To find the concentrations of intermediate streams:

$$3.376y_3 + (0.2418 \times 1.5) = (3.376 \times 0.1051) + (1.5 \times 0.1051)$$

$y_3 = 0.0444 = x_3$

$$y_4 = x_4 = [0.0444(3.376 + 1.50) - (0.1051 \times 1.5)] / 3.376 = 0.0174$$

$y_5 = x_b = 0.00533$

23.3 Basis: 1 kg solid.

Let x = wt fraction NaOH in liquid

NaOH in feed: 0.8 kg

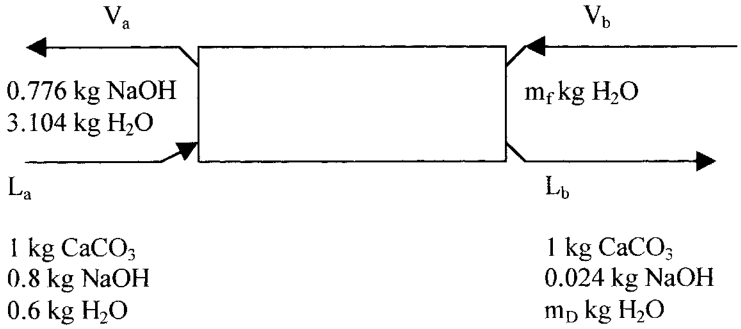
NaOH in washed solids: $0.03 \times 0.8 = 0.024$

NaOH in strong liquor: $0.8 - 0.024 = 0.776$

Water in strong liquor: $0.776 \times 0.8 / 0.2 = 3.104$

Operating line:

$y_a = 0.776 / (3.104 + 0.776) = 0.20$
$y_b = 0$
$x_a = 0.8 / (0.8 + 0.6) = 0.5714$



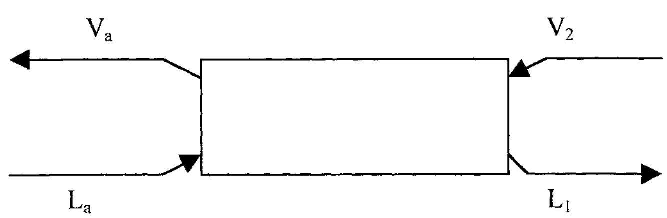
Find x_b. From an overall water balance,

$$m_D + 3.104 = m_f + 0.6$$
$$m_D = m_f - 2.504$$
$$x_b = 0.024 / (0.024 + m_D)$$

Now $(0.024 + m_D)$ is the factor (call it F) from the bottom row of Table 23.4. Hence $x_b = 0.24/F$. Find x_b and F from the table to satisfy this. By trial, this gives $F = 1.57$ and $x_b = 0.0153$.

Then $m_D = 1.57 - 0.024 = 1.546$ and $m_f = 1.546 + 2.504 = 4.050$.

For intermediate points, make a material balance for water and NaOH. First stage from feed end:



$x_1 = y_a = 0.20$

From Table 23.4, water (w) + NaOH (s) = 3.60

$s_1 = 0.20 \times 3.60 = 0.72$

$w_1 = 3.60 - 0.72 = 2.88$

Then, in stream V_2:

$s_2 = 0.776 + 0.720 - 0.8 = 0.696$

$w_2 = 3.104 + 2.880 - 0.6 = 5.384$

$y_2 = 0.696 / (5.384 + 0.696) = 0.114$

With $x_n = 0.1$ (assumed),

$w_n + s_n = 2.20$ $\quad\quad$ $s_n / (w_n + s_n) = 0.1$

$s_n = 0.1 \times 2.20 = 0.22$

$w_n = 2.20 - 0.22 = 1.98$

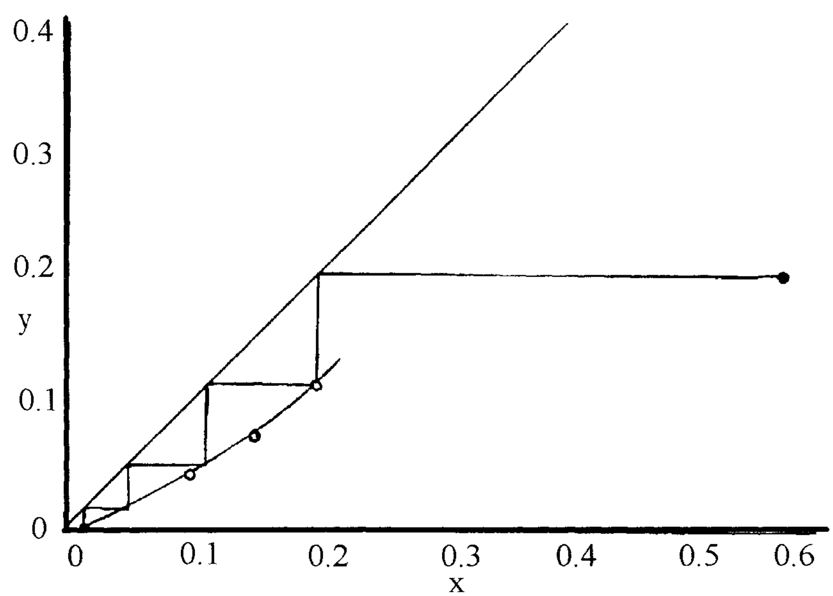
$s_{n+1} = 0.22 + 0.776 - 0.8 = 0.196$

$w_{n+1} = 3.104 + 1.980 - 0.6 = 4.484$

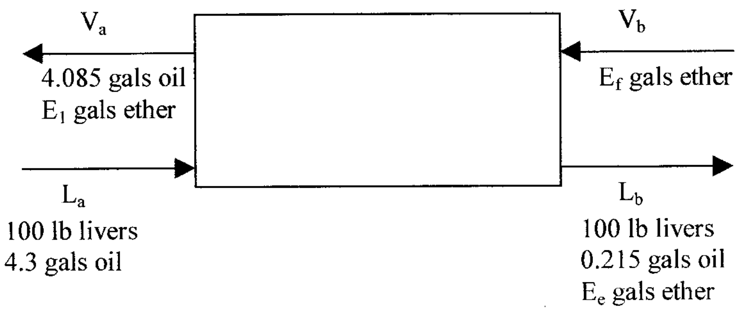
$y_{n+1} = 0.196 / (4.484 + 0.196) = 0.0419$

Similarly, with $x_n = 0.15$, $y_{n+1} = 0.0735$

At equilibrium, $x_e = y_e$. From the McCabe-Thiele diagram, 4.0 stages are needed.



23.4



Basis: 100 lb exhausted livers

Let x and y = gals oil/ gal total liquid in each phase

Oil in exhausted livers: $4.3 \times 0.05 = 0.215$ gal
Oil in product: $4.3 \times 0.95 = 4.085$ gal

From the problem statement:

$$4.085 = 0.65\,(E_l + 4.085)$$

$$E_l = 2.20 \text{ gals ether}$$

Then $x_a = 1.00$ $\quad y_a = 0.65$ $\quad y_b = 0$

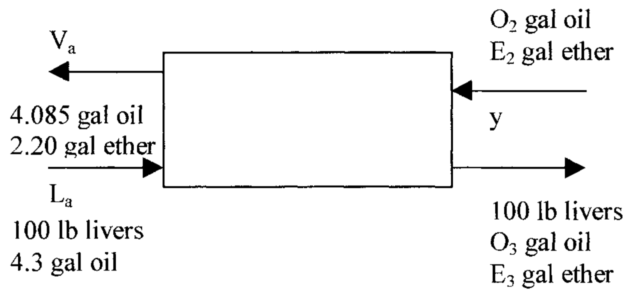
$$x_b = 0.215 / (0.215 + E_e)$$

Also, $E_e = 100C - 0.215$, where C = gals solution retained by exhausted livers, from Table 23.5. From these, values of E_e and x_b are found which satisfy these relations., The results are

$x_b = 0.055$ $\quad E_e = 3.70$ $\quad C = 0.0392$

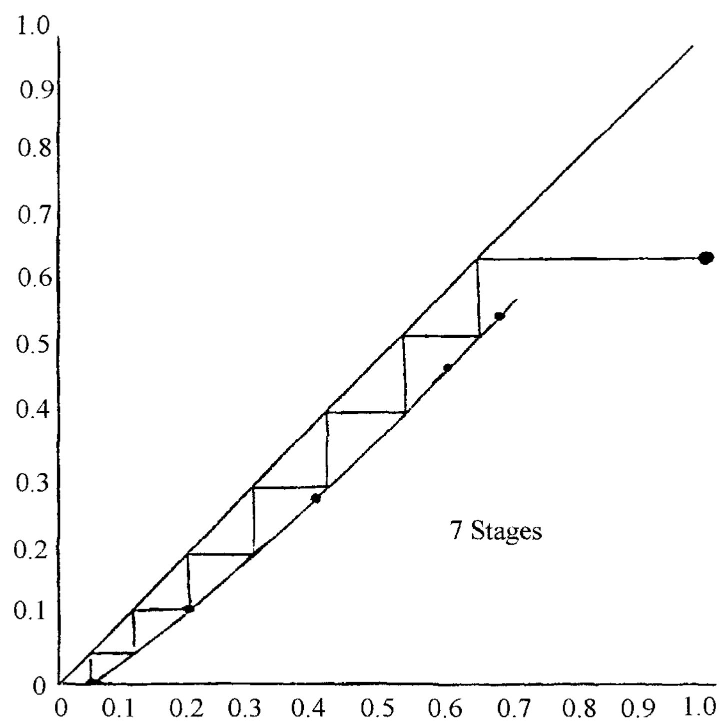
<u>Operating Line</u>

For intermediate points, assume values of x. Find the total volume of solution (call it S) from a plot of the data in Table 23.5. Calculate y as follows:



x (assumed)	0.055	0.2	0.4	0.6	0.68	1.00
S (Table 23.5)		5.0	6.8	9.9	12.0	
$O_3 = xS$		1.00	2.72	5.94	8.16	
$E_3 = S - O_3$		4.00	4.08	3.96	3.84	
$O_2 = 4.085 + O_3 - 4.30$		0.785	2.505	5.73	7.95	
$E_2 = 2.20 + E_3$		6.20	6.28	6.16	6.04	
$O_2 + E_2 = S_2$		6.98	8.78	11.89	13.99	
$y = O_2/S_2$	0	0.112	0.285	0.482	0.568	0.65

For the equilibrium line $x_e = y_e$. From the McCabe-Thiele diagram, 7 stages are required.

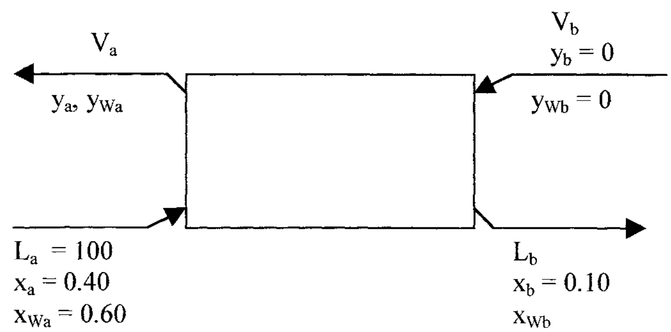


23.5 (a) Basis: 100 kg feed solution.

Let x, y refer to wt. fraction acetone: x_W, y_W to water; x_T, y_T to trichloroethane.

Plot the equilibrium line x_e, y_e.

Establish the ends of the operating line from overall material balances.



Balances:

Overall: $L_b + V_a = V_b + 100$

Acetone: $0.1\, L_b + y_a V_a = x_a L_a = 40$

At minimum solvent rate, y_a' is found from the equilibrium curve at $x_a = 0.40$ to be 0.53.

Hence $0.1 L_b + 0.53 V_a = 40$

Water: $y_{Wa} V_a + x_{Wb} L_b = 60$

When $x_b = 0.10$, from the first part of Table 23.6, $x_{Tb} = 0.0061$
From these, $V_b = 26.46$, $V_a = 63.61$

$L_b = 62.85$

The minimum solvent required is 26.46.

(b) $V_b = 1.8$ times the minimum

$= 1.8 \times 26.46 = 47.63$

As before, from a water balance:

$y_{Wa} V_a + x_{Wb} L_b = 60$

Overall: $L_b = 100 - 47.63 - V_a$

x_{Wb} is unchanged at 0.8939

Hence $L_b = (60 - 147.63 y_{Wa}) / (0.8939 - y_{Wa})$

$V_a = 147.63 - L_b$

By trial, estimate y_{Wa} to be 0.028

$L_b = 64.52$; $V_a = 83.11$

$y_a = (40 - 0.1 L_b) / V_a = 0.404$

From the equilibrium data, for $y_a = 0.404$. $y_{Wa} = 0.028$ (as estimated). Hence the upper end of the operating line is at $x_a = 0.40$, $y_a = 0.404$

The lower end is at $x_b = 0.10$, $y_b = 0$

Intermediate point. Set $x = 0.25$.

Estimate x_T to be 0.01; $x_W = 1 - 0.25 - 0.01 = 0.74$

Overall balance, from feed end:

$V = L + V_a - L_a = L + 83.11 - 100$

$= L - 16.89$ (A)

Estimate: $y = 0.22$, $y_W = 0.0089$

From an acetone balance:

$$yV = (0.404 \times 83.11) + 0.25L - 40$$

$$y = (0.25L - 6.42) / V \qquad \text{(B)}$$

Water balance:

$$y_W V = 0.74L + (0.028 \times 83.11) - 60$$

$$V = (0.25\ L - 57.67) / 0.0089 \qquad \text{(C)}$$

From Eqs. (A) and (C):

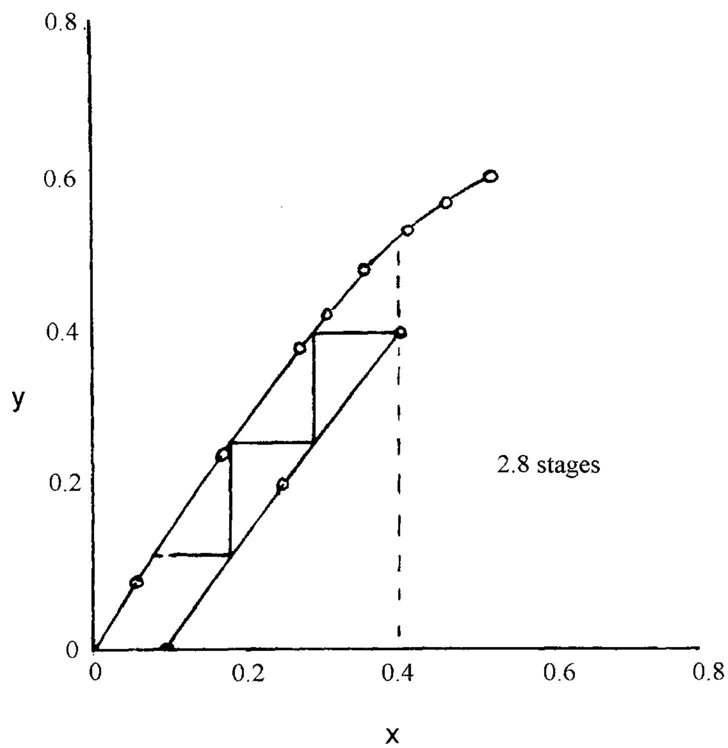
$$L = 78.69,\ V = 61.80$$

From Eq. (B), $y = 0.214$. From the equilibrium curve, $y_W = 0.0089$, as estimated. The coordinates of the intermediate point are $x = 0.25$, $y = 0.214$

The operating line is almost straight. From the diagram, 2.8 stages are needed. Use 3 stages.

(c) The flow rates, in kg / h, are

Feed: 100	Extract 83.11
Solvent: 47.63	Raffinate: 64.52



23.6 $D_p = 0.2$ cm, $\rho_p = 53 / 62.4 = 0.849$ g / cm^3

For water at 30 °C, $\rho = 0.996$ g / cm^3 , $\mu = 0.801$ cP

To get u_t, try Stokes' law [Eq. (7.40)]:

$$u_t = \frac{980(0.2)^2(0.996-0.849)}{18(0.00801)} = 39.97 \text{ cm/s}$$

$$Re_p = \frac{0.2(39.97)(0.996)}{0.00801} = 994$$

This is well beyond the range of Stokes' law, so u_t must be calculated using Eq. (7.37) and Fig. 7.6. Guess $Re_p = 100$, which gives $C_D = 1.0$

$$u_t = \left(\frac{4(980)\,(0.147)(0.2)}{3\,(1.0)\,(0.996)}\right)^{1/2} = 6.21 \text{ cm/s}$$

$$Re_p = \frac{0.2(6.21)(0.996)}{0.00801} = 154, \qquad C_D = 0.9$$

Guess $C_D = 0.85$

$u_t = 6.74$ cm / s

$Re = 167$, $C_D = 0.85$ (OK)

$$u_t = 6.74 \text{ cm / s} \times \frac{3600}{30.5} = 796 \text{ ft / h}$$

Let h = volume fraction holdup of drops

V_d, V_c = actual velocities of drops or continuous phase (both positive)

Then $u_t = V_d + V_c = 796$ ft / h

Given

$$\overline{V}_{s,d} = 2\overline{V}_{s,c}, \qquad h = 0.30$$

$$V_d = \overline{V}_{s,d}/\,h, \qquad V_c = \overline{V}_{s,c}/(1-h)$$

$$\frac{\overline{V}_{s,d}}{0.3} + \frac{\overline{V}_{s,d}}{2(0.7)} = 796$$

$$\overline{V}_{s,d}(3.33 + 0.714) = 796$$

$$\overline{V}_{s,d} = 197\text{ft} / \text{h}$$

$$\overline{V}_{s,d} + \overline{V}_{s,c} = 197 + 0.5(197) = 296$$

$$\sqrt{\overline{V}_{s,d}} + \sqrt{\overline{V}_{s,c}} = 14.0 + 9.9 = 23.9$$

Second part

For $\overline{V}_{s,d} = \overline{V}_{s,c}$ and $h = 0.30$

$$\overline{V}_{s,d} = \frac{796}{3.33 + 1.43} = 167 = \overline{V}_{s,c}$$

$$\overline{V}_{s,d} + \overline{V}_{s,c} = 334$$

and $\sqrt{\overline{V}_{s,d}} + \sqrt{\overline{V}_{s,c}} = 25.8$

For $\overline{V}_{s,d} = 0.5\overline{V}_{s,c}$ and $h = 0.30$

$$\overline{V}_{s,d} = \frac{796}{3.33 + 2.86} = 129$$

$$\overline{V}_{s,c} = 258$$

$$\overline{V}_{s,d} + \overline{V}_{s,c} = 387$$

31% higher than for

$$\overline{V}_{s,d} = 2\overline{V}_{s,c}$$

but $\sqrt{\overline{V}_{s,d}} + \sqrt{\overline{V}_{s,c}} = 11.4 + 16.1 = 27.5,$

15% higher than for $\overline{V}_{s,d} = 2\overline{V}_{sc}$

The sum of the square roots is more nearly constant than the sum of the velocities.

23.7 Assume the packing 40% voids.

$D_p = 0.015$ m

$a_v = 6(1-\varepsilon) / D_p = (6 \times 0.6) / 0.015 = 240\ m^2/m^3$

$\rho_c = 62.15 \times 16.018 = 995$ kg / m3

$\mu_c = 0.801$ cP $= 8.01 \times 10^{-4}$ Pa·s

$\sigma = 36.1 \times 10^{-3} = 0.0361$ N/ m

$\rho_d = 53.64 \times 16.018 = 859.2\ kg/m^3$

$\Delta\rho = 136.3\ kg / m^3$

Use Eq. (23.4), for which R = 2. Assume $a = 1.0$ (continuous wetting). For spheres, use $C_1 = 0.208$. Then

$$\overline{V}_{s,c}(1+R^{0.5})^2\left(\frac{a_v}{g}\right)^{0.5} = 27.35\overline{V}_{s,c}$$

$$= 1.0 \times 0.208 \times 0.4^{1.54}\left(\frac{136.3}{859.2}\right)^{0.41}\left[\frac{1}{240}\left(\frac{136.3^2 \times 9.805}{8.01\times10^{-4}}\right)^{1/3}\right]^{0.3} \times$$

$$\left[\frac{8.01\times10^{-4}}{(136.3\times0.0361/240)^{0.5}}\right]^{0.15} = 0.0615$$

$$\overline{V}_{s,c} = 0.0615 / 27.35 = 0.00225\ m/s$$

23.8 (a) Adding an equal amount of MIK to the feed gives a mixture with 0.5 MIK, 0.2 acetone, and 0.3 H_2O. A tie line through this point on Fig. 23.8 shows the extract to be 0.725 MIK, 0.232 acetone, and 0.043 H_2O. The raffinate composition is 0.023 MIK, 0.132 acetone, and 0.845 H_2O.

Per unit mass of feed, an acetone balance gives

$$0.4 = 0.232E + 0.132\ R$$

$$E + R = 1 + 1 = 2.0$$

$$0.4 = 0.232E + 0.132\ (2\text{-}E)$$

$$E = \frac{0.4 - 0.264}{0.232 - 0.132} = 1.36$$

$$\text{Fraction acetone extracted} = \frac{1.36(0.232)}{0.4} = 0.789$$

(b) If only half the MIK is added in the first step, the mixture is 0.333 MIK, 0.267 acetone, and 0.4 H_2O. The phase compositions are

	Extract	Raffinate
MIK	0.615	0.035
Acetone	0.325	0.210
Water	0.060	0.755

By a material balance, E + R = 1.5

$$0.4 = 0.325\ E + 0.21\ (1.5 - E)$$

$$E = \frac{0.4 - 0.315}{0.325 - 0.21} = 0.739, \ R = 0.761$$

Acetone extracted = 0.739 (0.325) = 0.24

Adding 0.5 parts MIK to 0.761 parts raffinate gives a mixture with the following composition

$$\text{MIK} = \frac{0.5 + 0.761(0.35)}{1.261} = 0.418$$

$$\text{Acetone} = \frac{0.21(0.761)}{1.261} = 0.127$$

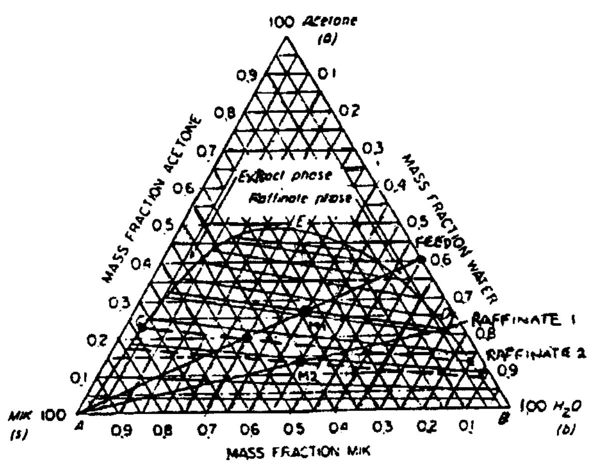
$$H_2O = \frac{0.755(0.761)}{1.261} = 0.455$$

This separates to give an extract with 0.20 acetone and a raffinate with 0.075 acetone. A acetone balance give

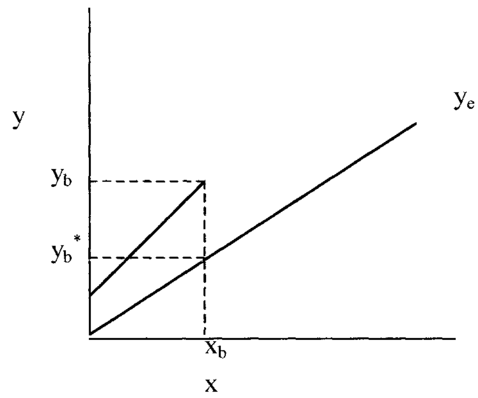
$$0.16 = 0.20E + 0.075\ (1.261 - E)$$

$$E = 0.523$$

Acetone extracted: 0.523 0.20 = 0.105
Total extracted: 0.24 + 0.105 = 3.45
Fraction acetone extracted: 0.345 / 0.4 = 0.863



23.9



$K_D = 0.15 = y_e / x_e$
$x_a = 0$

$L\Delta x = V\Delta y$ Neglect changes in L and V
$L = 0.45V$ Operating line slope = 0.45 = L/V
$y_a = 0.02y_b$ $y_a^* = 0$

For extraction,

$$E = \frac{K_D V}{L}$$

[(Eq. 23.5)], but this is for transfer into the organic phase. Here solute is extracted from the organic phase into the water, so $K'_D = 1 / K_D = 1 / 0.15 = x_e / y_e$ and appropriate factor is

$$\frac{LK_D'}{V} = \frac{0.45}{0.15} = 3.0$$

This factor corresponds to absorption factor A in absorption.

$$Lx_b = V(0.98)y_b$$

$$x_b = \frac{1}{0.45}(0.98)y_b = 2.18y_b$$

$$y_b^* = 0.15x_b = 0.15(2.18)y_b = 0.327y_b$$

Use Eq.(20.22)

$$N = \frac{\ln[(y_b - 0.327y_b)/0.02y_b]}{\ln 3.0} = 3.20 \text{ stages}$$

24.1 (a) Use Eqs. (24.1) and (24.8). Data are:

$T_{sa} = 70\ ^{o}F$, $T_{sb} = 200\ ^{o}F$ $c_{ps} = 0.48$ Btu / lb -°F

$T_{hb} = 1000\ ^{o}F$ $T_w = 150\ ^{o}F = T_v$ (assumed)

$X_a = 0.06$ $X_b = 0.004$ $c_{pL} = 1.0$

From Appendix 7, λ at 150 °F = 1008.1 Btu / lb

Estimate $c_{pv} = 0.25$ Btu / lb -°F. From Eq. (24.8),

$$N_t = 2.2 = \ln\frac{1000-150}{T_{ha}-150}$$

$$T_{ha} = 244\ ^{o}F = T_{vb}$$

From Eq. (24.1)

$$\begin{aligned} q_T/\dot{m} &= 0.48(200-70)+0.06\times1\times(150-70) \\ &\quad +(0.06-0.004)\times1008.1 \\ &\quad +0.004\times1\times(200-150) \\ &\quad +(0.06-0.004)\times0.25\times(244-15) \\ &= 125.17 \text{ Btu / lb-h} \end{aligned}$$

$q_T = 125.17 \times 18000 = 2.253 \times 10^6$ Btu / h

Mass flow rate of gas

$m_g = (2.253 \times 10^6) / (0.25 \times (1000 - 244))$

$= 11920$ lb / h

Mass velocity $G = m_g / S = 2000$ lb / h-ft^2

$S = 11920 / 2000 = 5.96$ ft^2

$D = 2.75$ say 2.8 ft

With $D = 2.8$, $S = 6.16\ ft^2$; $G = 1938\ lb / h - ft^2$

Find L from Eq. (24.23).

$\triangle T_1 = 1000 - 150 = 850\ ^oF$

$\triangle T_2 = 244 - 150 = 94\ ^oF$ $\overline{\triangle T_L} = 343\ ^oF$

$$L = \frac{2.253 \times 10^6}{0.125\pi \times 2.8 \times 1938^{0.67} \times 343} = 37.5\ ft$$

This is longer than normal design.

(b) Using $N_t = 1.8$ the same sequence gives:

$T_{ha} = 290.5\ ^oF$

$q_T/\dot{m} = 125.82$; $q_T = 2.265 \times 10^6 Btu / h$

$m_g = 12770\ lb / h$

$S = 6.39\ ft^2$

$D = 2.85$ say 3 ft; then $S = 7.079\ ft^2$

$G = 12770 / 7.069 = 1806\ lb / h - ft^2$

$T_l = 850\ ^oF$ $T_l = 140.5\ ^oF$

$T_L = 394.2\ ^oF$

$L = 32\ ft$

This is a better design length.

NOTE: Since less than half the heat transferred is used to vaporize the water (56.45 / 125.37 or 45%), Eq. (24.23) is not strictly applicable using the log mean $\triangle T$. The true mean $\triangle T$ should be found for each of the three sections - preheating, vaporization, and final solids heating - as shown in Fig. 24.2 (b).

24.2 At the critical point $X_c = 0.20 - 0.04 = 0.16$

$X_1 = 0.35 - 0.04 = 0.31$ $X_2 = 0.10 - 0.04 = 0.06$

Use Eq. (24.22)

$$7 = K[(0.31 - 0.16) + 0.16 \ln (0.16 / 0.06)]$$

where $K = m_s / AR_c$

$$K = 22.81$$

With $X_1 = 0.31$ and $X_2 = 0.05 - 0.04 = 0.01$:

$$t_T = 22.81[(0.31 - 0.16) + 0.16 \ln (0.16 / 0.01)] = 13.5 \text{ h}$$

24.3 Wt. of bone-dry solids: $m_s = 0.50 \times 5 = 2.5$ kg

$X^* = 0.05 / 0.95 = 0.053$

At start, $X_1 = 1.0 - 0.053 = 0.947$

At end, $X_2 = (0.15 / 0.85) - 0.053 = 0.123$

In the falling-rate period:

Wet-slab Weight, kg	R kg/m^2-h	X_T	X	1/R
5.3	4.4	1.120	1.067	0.227
4.2	3.9	0.680	0.627	0.256
3.3	3.4	0.320	0.267	0.294
2.9	2.0	0.160	0.107	0.500
2.7	1.0	0.080	0.027	1.000

$A = 0.6 \times 0.9 = 0.54 \text{ m}^2$, From Eq. (24.15),

$$t_T = (2.5/0.54)\int_{0.123}^{0.947} dX/R$$

By integration of a plot of 1/R vs. X

$$t_T = 4.630 \times 0.229 = 1.06 \text{ h}$$

24.4 Basis: 800 lb of wet solid

Bone-dry solid: $800 / 2.4 = 333.3$ lb $= m_s$

$X^* = 0.05$ $A = 1.1 \times 333.3 = 366.6$ ft^2

At inlet, $X_1 = 1.40 - 0.05 = 1.35$

At outlet, $X_2 = 0.20 - 0.05 = 0.15$

From Fig. 19.2, assuming T_w is constant:

Air entering: $\mathcal{H}_b = 0.0042$, $T_w = 70$ °F,

$T = 120$ °F

Air leaving: $\mathcal{H}_a = 0.012$, $T_w = 70$ °F,

$T = 87$ °F

Water entering with stock:	800 - 333.3 =	466.7 lb
Water in dry stock:	333.3× 0.20 =	66.7
Total water evaporated:		400.0 lb

Flow of dry air:

$V' = 400 / (0.012 - 0.0042) = 51280$ lb / h

The dryer consists of two sections: a constant-drying-rate section and a falling-rate section.

The critical moisture content, X_c, is $0.40 - 0.05 = 0.35$

Constant-rate section:

Water in with stock:	800 - 333.3 = 466.7 lb
Water left in stock:	$333.3 \times 0.40 = \underline{133.3}$
Water evaporated:	333.4 lb

Humidity of leaving air: $\mathcal{H}_a = 0.012$

Humidity of air entering constant-rate section:

$$\mathcal{H}_c = 0.012 - 333.4 / 51280 = 0.0055$$

T (from Fig. 19.2, at $T_w = 70\ ^\circ\text{F}$): 115 °F

T at outlet (from Fig. 19.2): 87 °F

At 70 + 3 = 73 °F, $\lambda_i = 1030$ Btu / lb (Appendix 7)
Use Eq. (24.13) with a log mean driving force.

$$\Delta T_1 = 115 - 73 = 42\ ^\circ\text{F}$$

$$\Delta T_2 = 87 - 73 = 14\ ^\circ\text{F} \qquad \overline{\Delta T_L} = 25.5\ ^\circ\text{F}$$

From Eq. (24.13)

$$R_c = (12 \times 25.5) / 1030 = 0.297 \text{ lb / h - ft}^2$$

Falling-rate section:

Assume that R is linear in X during the whole falling-rate period.
Then R' = 0 when X'= 0.

From Eq. (24.19):

$$a = R_c / X_c = 0.297 / 0.35 = 0.849.$$

Also $R_c - R_2 = a(X_c - X_2)$

$$R_2 = 0.297 - 0.849\ (0.35 - 0.15)$$

$$= 0.127 \text{ lb / h - ft}^2$$

Total drying time, from Eq. (24.22):

$$t_T = \frac{333.3}{366.6 \times 0.297}[(1.35 - 0.35) + 0.35 \ln \frac{0.35}{0.15}] = 4.0 \text{ h}$$

24.5 Assume the dryer has an equivalent diameter of D_e m and that the air temperature is 200 °F (93.3 °C). Then

$$\mu = 0.0208 \text{ cP} = 2.08 \times 10^{-5} \text{ Pa-s}$$

$$k = 0.0181 \times 1.73073 = 0.0313 \text{ W / m -°C}$$

$$Re = DG / (2.08 \times 10^{-5}) = 48077DG$$

$$Pr = 0.69$$

From Eq. (12.32)

$$h = 0.023\,(48077DG)^{0.8}\,(0.69)^{1/3}\,0.0313 / D_e$$

$$= 3.54G^{0.8} / D_e^{\,0.2} \text{ W / m}^2 \text{ -°C}$$

The above equation predicts coefficients only 3.54 / 8.8 or 40 % of those given by Eq. (24.11). The difference may be due to radiant heat transfer in the dryer plus the effect of a rough surface.

24.6 (a) Estimate u_t, Re, h for a 50 - μm drop (5×10^{-3} cm) evaporating in air at 100 °C, 1 atm

$$\mu = 0.021 \text{ cp} \quad \text{(Appendix 8)}$$

$$\rho = \frac{29}{22{,}400} \times \frac{273}{373} = 9.48 \times 10^{-4} \text{g / cm}^3$$

$$\Delta\rho \cong \rho_L = 1.0 \text{ g / cm}^3$$

Assume Stokes' law [Eq. (7.40)]

$$u_t = \frac{gD_p^2\Delta\rho}{18\mu} = \frac{980(5\times10^{-3})^2(1.0)}{18\times2.1\times10^{-4}} = 6.48\,\text{cm}/\text{s}$$

$$Re_p = \frac{(5\times10^{-3})\times6.48\times9.48\times10^{-4}}{2.1\times10^{-4}} = 0.146$$

Since $Re_p < 1$, Stokes' law applies

Use Eq. (12.64) for heat transfer to spheres

$$Nu = \frac{h_o D_p}{k_f} = 2.0 + 0.6 Re_p^{0.5} Pr^{1/3}$$

$Pr = 0.69$ for air at 100 °C (Appendix 16)

$k_f = 0.0184$ Btu / h-ft-°F (Appendix 12)

$k_f = 0.0184 / 242 = 7.60\times10^{-5}$ cal / s-cm -°C

$Nu = 2 + 0.6(0.146)^{0.5}(0.69)^{1/3} = 2.20$

This is the initial or maximum value of Nu. As the drop evaporates, u_t and Re decrease, and Nu approaches 2.0. Use Nu = 2.0 to simplify the analysis, since h then varies with D_p^{-1}. Heat balance on drop:

$$-\frac{d}{dt}\left(\frac{\pi D_p^3\rho}{6}\right) = -\frac{\pi D_p^2\rho}{2}\frac{dD_p}{dt} = \pi D_p^2 h\Delta T/\lambda$$

Since $h = 2k_f / D_p$

$$-\rho/2\,\frac{dD_p}{dt} = \frac{2k_f}{D_p}\frac{\Delta T}{\lambda}$$

$$-D_p \frac{dD_p}{dt} = \frac{4k_f}{\rho} \times \frac{\Delta T}{\lambda}$$

$$D_{po}^2 - D_p^2 = \frac{8k_f \Delta T}{\rho\lambda} \times t$$

For a dilute slurry, the final size, is small relative to D_{po} and the drying time is approximately proportional to D_{po}^2.

$$t \cong \frac{D_{po}^2 \lambda \rho}{8k_f \Delta T}$$

(b) Use λ for water at 100 °C

$$\lambda = 970.3 \text{ Btu / lb } = 539 \text{ cal / g}$$

$$t = \frac{(5\times10^{-3})^2 \times 1 \times 539}{8(7.6\times10^{-5})(50)} = 0.44 \text{ s}$$

If the 50 - μm drop becomes a 20 - μm solid particle

$$t = \frac{(5\times10^{-3})^2 - (2\times10^{-3})^2 \times 539}{8(7.6\times10^{-5})(50)} = 0.37 \text{ s}$$

24.7 $u = 8$ ft / s

From Example 24.1, $T_i = 80$ °F, $h_c = 4.97$ Btu / h-ft^2-°F

For radiant heat transfer between parallel surfaces,

From Eq. (14.37)

$$q_{12} = \sigma A_1 F_{12}(T_1^4 - T_2^4)$$

$$\sigma = 0.1713\times10^{-8} \text{Btu/ft}^2 \text{ - h - R}^4$$

From Eq. (14.38)

$$F_{12} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Assume $\varepsilon_1 = \varepsilon_2 = 0.9$

$$F_{12} = \frac{1}{1.11 + 1.11 - 1} = 0.82$$

$T_1 = 160\ ^{o}F = 620\ ^{o}R$

$T_2 = 80\ ^{o}F = 540\ ^{o}R \quad \Delta T = 80\ ^{o}F$

$$\frac{q_{12}}{A_1} = 0.1713 \times 10^{-8}(0.82)(620^4 - 540^4) = 88.1\ \mathrm{Btu / h - ft^2}$$

$h_r = 88.1 / 80 = 1.1$ Btu / h-ft^2-^{o}F

$h_c + h_r = 4.97 + 1.1 = 6.07$

The initial drying rate is increased by the factor 6.07 / 4.97 = 1.22

(b) The drying time will not be reduced by the same factor, since other resistances to heat and mass transfer in the partially dry slab lower the drying rate and make the radiation contribution less important.

24.8 $D_p = 300\ \mu m$ or 9.84×10^{-4} ft

Air in at 250 ^{o}F, $\mathcal{H}_{ya} = 0.016$ lb H_2O / lb dry air

$u_o = 1.2$ ft / s

Assume plug flow of gas through the bed, but a uniform solid temp.

$$u_o \rho \, cp \, dT = h \, a \, dL \, (T\text{-}Ts)$$

$$\int \frac{dT}{T - T_s} = \int \frac{h \, a \, dL}{u_o \rho c_p}$$

$$\ln\left(\frac{T_{in} - T_s}{T_{out} - T_s}\right) = \frac{h \, a \, L}{u_o \, \rho \, c_p}$$

Assume external void fraction of 40 % in the settled bed.

$$a = \frac{6(1-\in)}{D_p} = \frac{6(0.60)}{9.84\times10^{-4}} = 3.66\times10^{3} \ ft^2/ft^3 bed$$

As the bed expands, a decreases, but L increases, and (1- $\in$)L is constant, so the settled bed length can be used with L = 4 ft.

Estimate h for fixed bed at 1.2 ft / s and assume this value is approximately correct for the fluid bed.

at 250 ^oF, μ= 0.022 c_p (Appendix 8)

$$\rho = \frac{29}{22{,}400} \times \frac{492R}{710R} = 8.97\times10^{-4} g/cm^3 \text{ or } 0.056 lb/ft^3$$

$$Re_p = \frac{0.03\,cm \times 36.6\,cm/s \times 8.97\times10^{-4}}{2.2\times10^{-4}} = 4.48$$

From Eq. (17.74) , with Nu and Pr replacing Sh and Sc

$$Nu = 1.17 Re_p^{0.585} \, Pr^{1/3}$$

$$Nu = 1.17(4.48)^{0.585} \, (0.69)^{1/3} = 2.49$$

However, the minimum Nusselt number for a single sphere is 2.0, and it should be much higher for a sphere in a packed or fluidized bed. (See Figure 17.7 and discussion on page 538)

Assume Nu = 5.

at 212 °F, k = 0.0184 Btu / h-ft-°F

c_p = 0.25 Btu / lb-°F

(No need to correct to 250 °F, since air temperature changes in the bed, and only approximate values are needed.)

$$h \cong \frac{5(0.0184)}{9.84\times10^{-4}} = 93.5\,\text{Btu / h} - \text{ft}^2 - {}^\circ\text{F}$$

$$\ln\left(\frac{T_{in} - T_1}{T_{out} - Ts}\right) = \frac{93.5(3.66\times10^3)4}{(1.2\times3600)(0.056)(0.25)} = 2.26\times10^4$$

The exit air temperature is equal to the solid temperature, even if the value of h is an order of magnitude lower than the estimate.

(b) The solid temperature should be about the same as the wet-bulb temperature.

From Fig. 19.2, T_{wb} = 105 °F ≅ Ts

(c) Once the solid has reached the wet-bulb temperature, and the air leaves at 105 °F, saturated with water vapor,

$\mathcal{H} = 0.050$

Air flow per ft^2 = 1.2 (3600) (0.056) = 242 lb / h-ft^2

Water removed = 242 (0.050 - 0.016) = 8.23 lb / h-ft^2 cross-section

Assume $\rho_{bed} \cong 60$ lb solid / ft^3

for 1 ft^2 bed, solids ≅ 4 ft^3 × 60 = 240 lb

Initial rate of drying = 8.23 / 240 = 0.034 lb H_20 /h-lb catalyst

24.9 6 - inch atomizer at 10,000 rpm or 166.7 s^{-1}

$\rho_L = 70\ lb / ft^3 = 1124\ kg / m^3$

F = 30 lb / min at 120 °F (49 °C)

$\sigma = 67.9$ dyne / cm = 0.0679 N/m

μ_W=0.56 cp, assume μ_{slurry} =1c_p=10^{-3} Pa-s

Use Eq. (24.25)

$$r = 3(0.0254) = 0.0762\ m$$

$$L_p = 2 \times \pi \times r = 0.479\ m$$

$$\Gamma = \text{spray rate / disk periphery}$$

$$\Gamma = \frac{30}{2.2} \times \frac{1}{60}\frac{kg}{s} \times \frac{1}{0.479} = 0.475\ kg / s - m$$

$$\frac{\overline{D_s}}{r} = 0.4\left(\frac{\Gamma}{\rho_L n r^2}\right)^{0.6}\left(\frac{\mu}{\Gamma}\right)^{0.2}\left(\frac{\sigma\rho_L L_p}{\Gamma^2}\right)^{0.1}$$

$$= 0.4\left(\frac{0.475}{1124 \times 166.7 \times 0.0762^2}\right)^{0.6}\left(\frac{10^{-3}}{0.475}\right)^{0.2}\left(\frac{0.0679 \times 1124 \times 0.479}{0.475^2}\right)^{0.1}$$

$$= 0.4 \times 0.00964 \times 0.2915 \times 1.663 = 0.00187$$

$$\overline{D_s} = 0.00187(0.0762) = 1.42 \times 10^{-4}\ m = 142\ \mu m$$

(b) D_s varies with $r / r^{1.2} \times \Gamma^{\,0.6\text{-}0.2\text{-}0.2} \times L_p^{\,0.1}$ and $n^{-0.6}$

If dryer wheel is two times larger, assume capacity is eight times larger

Γ is 8 / 2 = 4 × larger

L_p is 2 × larger

$\Gamma^{0.2}$ is 1.319 × larger

$L_p^{\,0.1}$ is 1.07 × larger

$$\frac{1.319 \times 1.07}{2^{0.2}} = 1.229 \text{ larger}$$

$$\left(\frac{n_{12}}{n_{16}}\right)^{0.6} = 1.229 \qquad \frac{n_{12}}{n_{16}} = 1.41$$

use n_{12} = 1.4 × 10,000 = 14,100 rpm = 235 s^{-1}

Check: r_2 = 0.1524 m

L_{p2} = 0.958 m

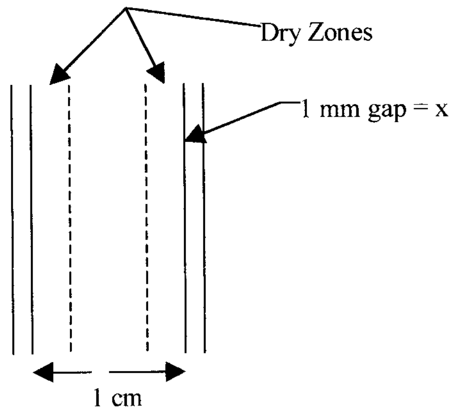
Γ_2 = (0.475 × 8) / 2 = 1.9 kg / s-m

$$\frac{\overline{D_s}}{r_2} = 0.4\left(\frac{1.9}{1124 \times 235 \times 0.1524^2}\right)^{0.6}\left(\frac{10^{-3}}{1.9}\right)^{0.2}\left(\frac{0.0679 \times 1124 \times 0.958}{1.9^2}\right)^{0.1}$$

$$= 0.4 \times 7.84 \times 10^{-3} \times 0.221 \times 1.351 = 9.36 \times 10^{-4}$$

$$\overline{D_s} = 9.36 \times 10^{-4}(0.1574) = 1.43 \times 10^{-4} \text{ m} = 143\ \mu\text{m}$$

24.10



T_h = -5 °C (heating surface) = 268 K

$k_s = 1.2 \times 10^{-4}$ cal / s-cm-K

$k_g = 4.8 \times 10^{-5}$ cal / s-cm-K

(a)

$$h_c = k_g/x = \frac{4.8\times10^{-5}}{0.1} 4.8\times10^{-4}\,\text{cal/s} - \text{cm}^2 - \text{K}$$

From Eq. (14.49), $h_r = 4\sigma\varepsilon_W T^3$, $\sigma = 5.672 \times 10^{-8}$ W / m^2-K^4

For $\varepsilon_W = 0.9$

$h_r = 4(0.9)\, 5.672 \times 10^{-8}(268)^3 = 3.93$ W / m^2-K

$$h_r = 3.93\times\frac{1\text{m}^2}{10^4\text{cm}^2}\times\frac{\text{cal}}{4.184\,\text{J}} = 0.94\times10^{-4}\,\text{cal / s - cm}^2 - \text{K}$$

$h = h_r + h_c = 5.74 \times 10^{-4}$ cal / s-cm^2-K

(b) Allow for resistance of dry zone of thickness z.

z = 1/4 cm with drying from both sides

$$h_s = \frac{1.2\times10^{-4}}{0.25} = 4.8\times10^{-4}$$

$$\frac{1}{U} = \frac{1}{h_s} + \frac{1}{h_c + h_r}$$

$$\frac{1}{U} = \frac{1}{4.8\times10^{-4}} + \frac{1}{5.74\times10^{-4}} = 3.83\times10^{3}$$

$U = 2.61\times 10^{-4}$ cal / s- cm^2-K

(c) Per cm^2, water removed = 1 $cm^3 \times 0.65 \times 1 = 0.65$ g H_2O

λ=597 cal / g at 0 °C (Appendix 7)

λ= 592 cal / g at 10 °C

estimate $\lambda \cong 602$ cal / g at -10 °C

q = 0.65 (602) = 391 cal for water evaporation

$Q = UA(T_h - T_i)$

The value of T_i is not easily determined, since it depends on the mass transfer resistances in the dry layer and gas film as well as on the heat transfer resistances.

If T_i = - 20 °C, ΔT = 15 °C

A = 2 cm^2

$Q = 2.61\times 10^{-4}$ (2) (15) = 7.83×10^{-3} cal / s

Based just on q needed for water evaporation

$$t = \frac{391}{7.83 \times 10^{-3}} \times \frac{1}{3600} = 13.9\,\text{h}$$

(d) If the layers were 0.5 cm thick, only half as much heat needs to be transferred, and the average value of U is higher

$$\frac{1}{U^1} = \frac{1}{2 \times 4.8 \times 10^{-4}} + \frac{1}{5.74 \times 10^{-4}} = 2.78 \times 10^3$$

$$U^1 = 3.59 \times 10^{-4}$$

The drying time is reduced by a factor of

$$2 \times \frac{3.59 \times 10^{-4}}{2.61 \times 10^{-4}} = 2.75.$$

25.1 Properties of MEK, $C_4H_8O_8$

T, °C	14	25	41.6	79.6
P', mmHg	60	100	200	760

$\rho = 0.805\ g / cm^3$ at 20 °C M = 72.1

Incoming air has 0.40 lb MEK / 1000 SCF

$$y = \frac{0.40 / 72.1}{1000 / 359} = 1.99 \times 10^{-3} \text{ mol fraction MEK}$$

$$p = 1.99 \times 10^{-3} (760) = 1.51 \text{ mm Hg}$$

At 25°C, p/P' = 1.51 / 100 = 0.0151

If the bed heats up to 40 °C during adsorption (a rough estimate),

$$p/P' = 1.51 / 192 = 7.86 \times 10^{-3}$$

Use Fig. 25.4 to estimate the equilibrium adsorption at 25 °C and 40 °C. From the Table of additive volume increments ("Mass Transfer in Heterogeneous Catalysis, " Satterfield, C.N., p. 16, MIT Press, 1970)

$$V = 4\ (14.8) + 8(3.7) + 9.9 = 98.7$$

at 25 °C,

$$(T / V) \log (f / f_s) = (298 / 98.7) \log (100 / 1.51) = 5.498$$

From Fig. 25.4, using the curve for n-paraffins:

Volume adsorbed = 25 cm^3 / 100 g

$$W = 0.25 \times 0.805 = 0.20 \text{ g / g carbon}$$

At 40°C,

$$(T/V) \log (f / f_s) = (313 / 98.7) \log (192.1 / 1.51) = 6.670$$

$$W = 0.20 \times 0.805 = 0.16 \text{ g / g carbon}$$

Air flow at bed inlet:

$$(12000 / 60) \times (298 / 273) = 218 \text{ ft}^3 / \text{s}$$

For $u_o = 0.5$ ft / s, area = 436 ft^2

For a cylindrical bed:

$$D_{bed} = (436 \times 4 / \pi)^{0.5} = 23.6 \text{ ft}$$

A rectangular bed 14 ft × 31.1 ft or 16 ft × 27.3 ft could also be used. Check bed depth to see if a horizontal cylinder could be used.

Per ft^2 of cross section, MEK adsorbed in 8 h is:

$$8 \times 0.5 \times 3600 \text{ x } (273 / 298) \times (0.40 / 1000) = 5.28 \text{ lb}$$

Carbon needed: 5.28 / 0.16 = 33 lb

Bed density is 30 lb / ft^3 or 30 lb / ft for 1 ft^2

$$L_{min} = 33 / 30 = 1.1 \text{ ft}$$

If half the bed capacity is used at the breakpoint, a bed 2.2 ft long would suffice. Then a rectangular bed 2 - 3 ft deep could be placed in the middle of a horizontal cylindrical vessel. This would be less expensive than a 23.6-ft diameter cylindrical bed.

Estimate N, the number of transfer units

$$\overline{D_p} = \frac{3.327 + 1.651}{2} = 2.49 \text{ mm (Appendix 5)}$$

At 25 °C μ/ρ for air = 0.152 cm^2 / s

$$Re = \frac{0.249(0.5 \times 30.48 \text{ cm/s})}{0.152} = 25.0$$

From Appendix 18, in air at 0 °C and 1 atm,

$D_v = 0.278$ ft^2/ h for ethyl acetate, M = 88.10
$D_v = 0.302$ ft^2/ h for ethyl ether, M =74.12
guess D_v =0.305 ft^2 / h for MEK, M = 72.10

From page 519, use temperature correction factor

at 25 °C, 1 atm,

$$D_v \cong 0.305 \times (298 / 273)^{1.75} \times 0.2581 = 0.092 \text{ cm}^2/\text{s}$$

$$\mu = 0.018 \text{ cp}$$

$$\rho = 1.186 \times 10^{-3} \text{g} / \text{cm}^3$$

$$S_c = \frac{1.8 \times 10^{-4}}{1.186 \times 10^{-3} \times 0.092} = 1.96$$

From Eq. (17.74) $Sh = 1.17(25)^{0.585} (1.96)^{1/3} = 9.62$

$$k_{c,ext} = \frac{9.62(0.092)}{0.249} = 3.55 \text{ cm} / \text{s}$$

For internal diffusion, assume

$$D_e = {}^{D_v}\!\!/_{10} = 9.2 \times 10^{-3} \text{cm}^2/\text{s}$$

From Eq. (25.8)

$$k_{c,int} = \frac{D_e}{D_p/10} = \frac{9.2 \times 10^{-3}}{0.0249} = 0.369 \text{ cm} / \text{s}$$

$$\frac{1}{K_c} = \frac{1}{3.55} + \frac{1}{0.363} = 2.99$$

$$K_c = 0.334 \text{ cm} / \text{s}$$

For spherical particles,

$$a = \frac{6}{D_p}(1- \in) = \frac{6(0.6)}{0.289} = 14.46 \text{ cm}^2/\text{cm}$$

$$K_c a = 0.334 \times 14.46 = 4.83 \text{ s}^{-1}$$

For a bed 2.5 ft deep

$$N = \frac{K_c a L_T}{u_o} = 4.83 \times \frac{2.5}{0.5} = 24.1$$

Based on a linear isotherm, which is conservative, Fig. 25.12 shows that c / c_o would be ~ 0.05 at $\tau \cong 0.5$. Since L_{min} is 1.1 ft for 8 h, an ideal bed 2.5 ft deep could operate for $2.5 / 1.1 \times 8 =$ 18.2 h. With $\tau \cong 0.5$, breakthrough would occur at 9.1 h, which is satisfactory. Because the adsorption is favorable, the breakthrough curve will be steeper than shown in Fig. 25.12, which gives an additional margin of safety.

25.2 Calculate $K_c a$ to get N from Eq. (25.10)

10 mesh screen opening = 1.651 mm (Appendix 5)
20 mesh screen opening = 0.833 mm

$$\overline{D}_p = (1.651 + 0.833) / 2 = 1.242 \text{ mm or } 0.1242 \text{ cm}$$

Assume $\phi_s = 0.85$ (See Table 7.1)

Assume $\varepsilon = 0.4$

$$a = \frac{6(1-\varepsilon)}{D_p \varphi_s} = \frac{6(0.6)}{0.1242 \times 0.85} = 34.1 \text{ cm}^2/\text{cm}^3 \text{ bed}$$

For water at 20 °C, $\mu = 1.0$ cp

For phenol $\rho = 1.071$ g / cm^3 at 25 °C and about 0.89 g / cm^3 at the normal boiling point, 181.8 °C

$M = 94.11 \qquad V = 94.11 / 0.89 = 105.7$

From Eq. (17.32)

$$D_v = \frac{13.26 \times 10^{-5}}{1.0^{1.14} \times 105.7^{0.589}} = 8.52 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.010}{1.0 \times 8.52 \times 10^{-6}} = 1174$$

$$u_o = 3 \text{ cm / s}$$

$$Re = \frac{0.1242 \times 3 \times 1.0}{0.01} = 37.3$$

From Eq. (17.74),

$$Sh = 1.17 \times 37.3^{\,0.585} \times 1174^{\,1/3} = 102.5$$

$$k_{c,ext} = \frac{102.5 \times 8.52 \times 10^{-6}}{0.1242} = 7.0 \times 10^{-3}\,\text{cm}/\text{s}$$

$$D_e = (8.52 \times 10^{-6})/5 = 1.70 \times 10^{-6}\,\text{cm}^2/\text{s}$$

$$k_{c,int} = \frac{10 \times 1.70 \times 10^{-6}}{0.1242} = 1.37 \times 10^{-4}\,\text{cm}/\text{s}$$

$$1/K_c = (1 / 7.0 \times 10^{-3}) + (1 / 1.37 \times 10^{-4}) = 7442$$

$$K_c = 1.34 \times 10^{-4},\ K_c a = 4.57 \times 10^{-3}\ s^{-1}$$

$$N = K_c a L_T / u_o = 4.57 \times 10^{-3} \times 4 / 0.03 = 0.61$$

Such a small value of N would give an impractical design, since even for irreversible adsorption three transfer units are needed for 95% removal at the start. If the velocity were lowered to 0.005 m/s, N would increase over 6 fold.

At 0.005 m / s,

$$k_{c\ ext} = 7.0 \times 10^{-3}\,(0.005/0.03)^{0.585} = 2.45 \times 10^{-3}\ \text{cm}/\text{s}$$

$$K_c = 1 / (1/(2.45 \times 10^{-3}) + 1 / (1.37 \times 10^{-4})) = 1.30 \times 10^{-4}$$

$$N = 1.30 \times 10^{-4} \times 34.1 \times 4 / 0.005 = 3.55$$

25.3 Vapor pressure of water at 20 °C (68 °F) = 0.340 lb_f / in^2

For air with $\mathcal{H}_R$ = 50% at 20 °C and 1 atm:

$$P_A = 0.5\,(0.340) = 0.170\ lb_f / in^2$$

$$\text{Moles } H_2O / \text{mole dry air} = \frac{0.170}{14.7 - 0.170} = 1.17 \times 10^{-2}$$

At 30 °C, $P'_A = 0.616\ lb_f / in^2 = P_A$ (since $\mathcal{H}_R = 100\%$)

$$\text{At 8 atm, moles } H_2O \text{ / mole dry air} = \frac{0.616}{(14.7 \times 8) - 0.616}$$

$$= 5.27 \times 10^{-3}$$

Water removed = $1.17 \times 10^{-2} - 0.527 \times 10^{-2} = 0.643 \times 10^{-2}$

Fraction removed = $(0.643 \times 10^{-2}) / 1.17 \times 10^{-2} = 0.550$

Air entering the bed has a relative humidity of 100%, but if the temperature rises to 50 °C, the same air would have a much lower relative humidity.

at 50 °C, $P'_A = 1.80\ lb_f / in^2$

$\mathcal{H}_R = 0.616 / 1.80 \times 100 = 34.2$

From Fig. 25.3, the capacity of silica gel is 20 lb H_20/100 lb silica.

25.4 For $D = 6$ ft, $L_T = 4$ ft, $u_o = 60$ ft / min

$\Delta p = 16$ in $H_2O = 4.0$ in H_2O / ft

If D is increased, L_T and u_o are decreased and both changes contribute to a decrease in Δp.

Use the Ergun equation (7.22) to estimate the change in Δp.

$$\overline{D}_p = \frac{3.327 + 0.991}{2} = 2.16 \text{ mm (Appendix 20)}$$

Assume $\phi_s = 0.85$, $\varepsilon = 0.4$

Assume gas is air at 25 °C, 1 atm

$$\rho = \frac{29}{22{,}4000} \times \frac{273}{298} = 1.19 \times 10^{-3}\,g/cm^3$$

$$\mu = 0.018\,cp$$

$$u_o = 60 \times 30.5/60 = 30.5\,cm/s$$

$$Re = \frac{0.216(30.5)(1.19 \times 10^{-3})}{1.8 \times 10^{-4}} = 43.6 = D_p u_o \rho/\mu$$

Equation (7.22) can be written in the following form, with $V_o = u_o$:

$$\frac{\Delta p}{L} = \frac{\rho u_o^2}{g_c \varphi_s D_p} \frac{1-\varepsilon}{\varepsilon^3} \left[\frac{150(1-\varepsilon)}{\varphi_s D_p u_o \rho/\mu} + 1.75 \right]$$

The left-hand term in the square brackets is the laminar flow term. Evaluating this gives

$$\frac{150(1-\varepsilon)}{\varphi_s D_p u_o \rho/\mu} = \frac{150(0.6)}{0.85 \times 43.6} = 2.43$$

Since this is comparable to the 1.75 turbulent flow term, the pressure drop per unit length will vary with an intermediate power of the velocity, probably about the 1.4 power, since the laminar flow term is larger. The pressure drop for other velocities is estimated more accurately from the change in u_o^2 and the change in the sum of the other two terms.

For u_o in ft / s:

$$\frac{\Delta p}{L} = \alpha u_o^2 \left[\frac{2.43}{u_o} + 1.75 \right]$$

When $u_o = 1$, $\Delta p / L = 4.0$; $a = 4 / (2.43 + 1.75) = 0.957$

For other velocities,

$$\frac{\Delta p}{L} = 0.957 u_o^2 (2.43/u_o + 1.75)$$

in. H_2O / ft

u_o	$\Delta p / L$	D_{bed}	L_T	Δp
ft / s	in.H_2O/ft	ft	ft	in. H_2O
1	4	6	4	16.0
0.8	2.93	6.71	3.2	9.38
0.7	2.45	7.17	2.8	6.86
0.6	2.00	7.75	2.4	4.80

A bed 7.2 ft diameter and 2.8 ft deep would give a pressure drop of about 6.8 in H_2O, which is somewhat less than the allowed value of 8 in H_2O. (Using the approximation that $\Delta p/L$ varies with $u_o^{1.4}$ give $\Delta p/L = 2.43$ in H_2O / ft when $u_o = 0.7$ ft / s.)

25.5 Follow Example 25.2

$$W_{sat} = \frac{\int_0^\infty u_o (1 - c/c_o) c_o \rho_M \times 18.02 dt}{\rho_b L_T}$$

From the plot, $\int_0^t (1 - c/c_o) dt = 15\text{h}$ for $t = 0$ to 15

(since $1 - c/c_o \cong 1$,) $+ 1.736$ h for $t = 15$ to $18.5 = 16.74$ h

$u_o \rho_M = 29.2$ mol / h-ft^2 = N_2 feed

$$W_{sat} = \frac{29.2 \times 16.74 \times 1490 \times 10^{-6} \times 18.02}{44.5 \times 1.44} = 0.205 \frac{\text{lb } H_2O}{\text{lb solid}}$$

Check with Fig. 25.3

$$p = 1490 \times 10^{-6} \times 760 = 1.13 \text{ mm Hg}$$

For water at 79 °F,

$$P' = 25.4 \text{ mm Hg}$$

relative humidity = 1.13 / 25.4 = 0.044 or 4.4%

From Fig. 25.3, expected $W_{sat} \cong 0.20$ lb H_2O / lb solid

At $c / c_o = 0.05$, t = 15.8 h

$$\int_0^{15.8} (1 - c/c_o)dt = 15.79$$

$$\text{Fraction of bed unused} = \frac{16.74 - 15.79}{16.74} = 0.057$$

Length of unused bed = 0.057(1.44) = 0.082 ft

25.6 For n-butanol: M = 74.12; b. pt. = 118 °C

(a) c = 365 ppm P = 737 mm Hg

$$p \cong f = 365 \times 10^{-6} \times 737 = 0.269 \text{ mmHg}$$

At 25 °C, from Perry's Handbook, 7th ed.,

p. 2-62, $P' = 7.0$ mm Hg $\cong f_s$

At 20 °C, $\rho = 0.810$ g / cm^3; at 118 °C,

$\rho = 0.70$ g / cm3

$$V = 74.12 / 0.70 = 106 \text{ cm}^3 / \text{mol}$$

$$\frac{T}{V} \log \frac{f_s}{f} = \frac{298}{106} \log \frac{7.0}{0.269} = 3.98$$

From Fig. 25.4 the curve for n-paraffins give 31 cc /100 g solid.

$W_{sat} = 0.31 \times 0.70 = 0.217$ g solute / g carbon

This is lower than the value of 0.291 g solute/g carbon determined in Example 25.2. Some of the difference may be due to differences in carbon properties, but the correlation for polar molecules such as alcohols and ketones probably lies above the correlation for n-paraffins.

(b) $p \cong f = 20 \times 10^{-6} \times 760 = 0.0152$ mmHg

At 30 °C, $P' = 9.5$ mmHg $\cong f_s$

$$\frac{T}{V}\log\frac{f_s}{f} = \frac{303}{106}\log\frac{9.5}{0.0152} = 7.99$$

From Fig. 25.4 the curve for n-paraffins gives 14 cc / 100 g solid, but since this curve gave too low a value in part (a) the value of W is corrected using a ratio from part (a).

$W_{sat} = 0.14 \times 0.70 \times (0.291 / 0.217) = 0.13$ g solute / g carbon

25.7 (a) $P_{N_2} = 0.79 \times 10 = 7.9$ atm. From p. 816, with K in atm^{-1}, $c = P_{N_2}$ and the Langmuir isotherm is given by

$$W_{sat} = W_{max}\frac{KP_{N_2}}{1+KP_{N_2}} = \frac{0.046 \times 0.5 \times 7.9}{1+(0.5 \times 7.9)}$$
$$= 0.0367 \text{ g / g or lb / lb}$$

(b) For ideal breakthrough, from Eq. (25.2) with

$W_o = 0$,

$u_o c_o t = W_{sat}\, \rho_b L$

Assume $\epsilon = 0.4$; $1-\epsilon = 0.6$

$$\rho_b = 1.3 \times 0.6 \times 62.4 = 48.7 \text{ lb / ft}^3$$

At 30 °C, 10 atm,

$$c_o = \frac{7.9 \times 273 \times 28}{359 \times 303} = 0.555\ \text{lb} / \text{ft}^3$$

$$t = \frac{0.0367 \times 48.7 \times 4.0}{20 \times 0.555} = 0.694\ \text{min} \quad \text{or}\ 38.6\ \text{s}$$

The actual time for the adsorption part of the cycle would be less than 38 s, since not all the solid is saturated when breakthrough occurs.

25.8 For 4 x 6 mesh carbon,

$D_p = (4.699 + 3.327) / 2 = 4.01$ mm

For air at 20 °C, 1 atm;

$\mu = 0.0179$ cp

$\rho = 1.206 \times 10^{-3}$ g / cm^3

$$Re = \frac{0.401 \times 75 \times 1.206 \times 10^{-3}}{1.79 \times 10^{-4}} = 203$$

For Hg, M = 200.59, b.pt. 357°C

Use Eq. (17.28). From Appendix 19, $\sigma = (2.969 + 3.711) / 2 = 3.34$

$\in / \kappa = (750 \times 78.6)^{0.5} = 242.8$

$kT/\in_{AB} = 393 / 242.8 = 1.207$

$\Omega_D = 1.317$

$$D_{AB} = \frac{0.001848(293)^{3/2}\left[(200.59+29)/(200.59\times 29)\right]^{1/2}}{1\times 3.34^2 \times 1.317} = 0.125\ \text{cm}^2/\text{s}$$

$$Sc = \frac{1.79\times10^{-4}}{1.206\times10^{-3}\times0.125} = 1.19$$

From Eq. (17.74)

$$Sh = 1.17 \times 203^{0.585} \times 1.19^{1/3} = 27.7$$

$$k_c = 27.7 \times 0.125 / 0.401 = 8.63 \text{ cm / s}$$

$$a = 6(1-\epsilon) / D_p = 6 \times 0.6 / 0.401 = 8.98$$

$$K_c a \cong k_c a = 8.63 \times 8.98 = 77.5 \text{ s}^{-1}$$

(b) From Eq. (25.13) the initial concentration profile is given by

$$\ln (c / c_o) = -K_c aL / u_o$$

For $c / c_o = 10^{-3}/ 100 = 10^{-5}$ and $u_o = 75$ cm / s,

$$L_{min} = (-\ln 10^{-5})(75) / 77.5 = 11.1 \text{ cm}$$

(c) For ideal adsorption, from Eq. (25.2),

$$t = W_{sat}\, \rho_b L/u_o c_o$$

$$W_{sat} = 0.2 \text{g / g} \qquad \text{Assume } \rho_b = 0.5 \text{ g / cm}^3$$

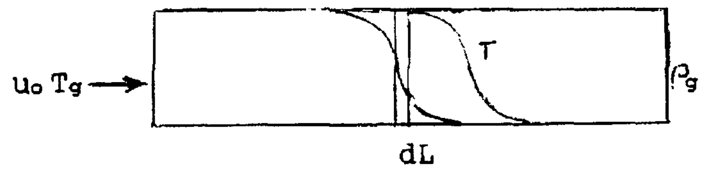
$$c_o = 100\times10^{-6} \text{g / Nm}^3 \times (273/293)$$

$$\times10^{-6} \text{cm}^3/ \text{m}^3 = 9.32\times10^{-11} \text{g / cm}^3$$

$$t = \frac{0.2\times0.5\times50}{75\times9.32\times10^{-11}\times3600} = 1.99\times10^5 \text{ h} = 22.7 \text{ years}$$

The actual length of the adsorption zone may be much greater than the estimated value of L_{min} of 11.1 cm because of internal diffusion resistance, which is hard to predict. However, even if the length of unused bed is 25 cm, the time to breakthrough would be about 10 years.

25.9



From an energy balance,

$$u_o\rho_g c_{p,g}(T_g-T_o)dt = \rho_b c_s(T_g-T_o)dL + \rho_b\, W_{sat}\, \Delta H dL$$

$$\frac{dL}{dt} = \frac{u_o\rho_g c_{p,g}}{\rho_b c_s + (\rho_b W_{sat}\Delta H/(T_g - T_o))}$$

At 200 °F, $\rho_g = 29 \times 492 / (359 \times 660) = 0.0602$ lb / ft ^{3}

From Appendix 14,

$c_{p,g} = 0.251$ Btu / lb-°F at 140 °F

Assume $\rho_b = 30$ lb / ft^3 (carbon only)

$c_s = 0.30$ Btu / lb-°F

$W_{sat} = 0.09$ lb / lb carbon

If $\Delta H_v = 300$ Btu / lb, $\Delta H = 300 \times 1.5 = 450$ Btu / lb

$$\frac{dL}{dt} = \frac{120(0.0602)(0.251)}{30(0.30) + (30 \times 0.09 \times 450/(200 - 80))} = 0.0948 \text{ ft / min}$$

(b) The rate of movement of the temperature front does not depend on the inlet temperature. Since $u_o\rho_g$ is the same,

$dL/dt = 0.0948$ ft / min as in part (a).

(c) Use Fig. 25.4, $W_{sat} = 0.09$ lb / lb. For VCM,

$\rho_L = 0.91$ g / cm^3; $M = 62.5$

Volume adsorbed: 0.09 / 0.91

$= 0.099\ cm^3$ / g carbon or $9.9\ cm^3$ / 100 g

From Fig. 25.4,

$$\frac{T}{V}\log\frac{f_s}{f} = 11$$

$V = 62.5 / 0.91 = 68.7\ cm^3 / mol$

$T = 200\ ^oF = 366\ K$

$f_s = 1.479 \times 10^4$ mmHg (calculated from Perry's Handbook, 7th ed., p. 2-53)

$$\log\frac{f_s}{f} = \frac{11 \times 68.7}{366} = 2.06$$

$f_s / f = 116$

$f = 1.479 \times 10^4 / 116 = 127.5$ mm Hg

$y = 127.5 / 760 = 0.168$

The equilibrium content is

$$c_o = \frac{0.168 \times 62.5 \times 273}{359 \times 366} = 0.0218\ lb / ft^3$$

$L/t = u_o\ c_o / W_{sat}\ \rho_b = 120 \times 0.0218 / (0.09 \times 30) = 0.97$ ft / min

Since the desorption front for a bed kept at 200 °F would move at about 1.0 ft / min, much faster than the temperature front, the rate of regeneration with hot gas is controlled by the movement of the temperature front.

(d) The main advantage of increasing the regeneration temperature is to obtain a higher concentration of VCM in the desorbing gas.

25.10 $$Ca^{2+} + 2\,NaR \rightleftharpoons CaR_2 + 2\,Na^+$$

$$K' = K'_{Ca^{2+}} / K_{Na^+} = 5.16 / 1.98 = 2.61$$

$$Mg^{2+} + 2\,NaR \rightleftharpoons MgR_2 + 2\,Na^+$$

$$K' = K'_{Mg^{2+}} / K_{Na^+} = 3.29 / 1.98 = 1.66$$

Both reaction are quite favorable, and judging from Example 25.6, for Ca^{+2} / NaR exchange, resin in equilibrium with the feed solution used be nearly completely converted to CaR_2 and MgR_2.

Bed volume = $(\pi / 4)(1)^2\ 1.5 = 1.178\ ft^3 = 33.3$ liters

Assume 2 meg / mL or 2 eq / liter of resin

If $\tau = 0.40$, bed has 2 (0.6) = 1.2 equivalents / liter

Total Capacity = 33.3 (1.2) = 40 equivalents

1.0 liter of solution has

$$1000\ g \times 20 \times 10^{-6} / 40 = 5.0 \times 10^{-4}\ g\ mol\ Ca^{+2}$$

and $$1000 \times 10 \times 10^{-6} / 24.3 = \underline{4.1 \times 10^{-4}\ g\ mol\ Mg^{+2}}$$

$$\text{Total} \quad 9.1 \times 10^{-4}$$

Need $2 \times 9.1 \times 10^4 = 1.82 \times 10^{-3}$ mole NaR / liter treated

For complete utilization of NaR,

$$V = (40 / 1.82 \times 10^{-3}) \times (1\ gal / 3.785\ l) = 5.\ 81 \times 10^3\ gal$$

If breakthrough occurs when 80% of bed capacity has been used,

$$V = 0.8\ (5.21 \times 10^3) = 4.65 \times 10^3\ gal$$

Chapter 26

26.1. (a) Use second form of Eq. (26.18) for R = 0, $\alpha = 8$:

$$y_{mix} = \frac{8 \times 0.209}{1 + (7 \times 0.209)} = 0.679 \text{ or } 67.9\%\ O_2$$

(b) Assume residue concentration x = 0.10.

At discharge end

$$y_i = \frac{8 \times 0.1}{1 + (7 \times 0.1)} = 0.471$$

At inlet end, $y_i = 0.679$

$y = (0.679 + 0.471)/2 = 0.575$

By a material balance

$0.575V + 0.1L = 0.209$

$V + L = 1.0$

$0.575V + 0.1(1 - V) = 0.209$

$V = 0.229$

Fraction O_2 recovery:

$0.229 \times 0.575/0.209 = 0.630$

Assume x = 0.11.

$$y_i = \frac{8 \times 0.11}{1 + (7 \times 0.11)} = 0.497$$

$y = (0.679 + 0.497)/2 = 0.588$

$0.588V + 0.11(1 - V) = 0.209$

$V = 0.207$

Fraction O_2 recovery:

$0.207 \times 0.588/0.209 = 0.582$

By interpolation, x = 0.105 and

$y = 0.58$ or 58% O_2

26.2 (a) Basis: 100 moles feed (50 H_2 and 50 CH_4)

H_2 balance: $50 = 0.80V + 0.42(100 - V)$

$V = (50-42)/(0.8 - 0.42) = 21.1$

On feed side,

$$\bar{P}_{H_2} = 100(0.50 + 0.42)/2 = 46 \text{ lb}_f/\text{in.}^2$$

In residue,

$$P_{H_2} = 15 \times 0.8 = 12 \text{ lb}_f/\text{in.}^2$$

$$\Delta P_{H_2} = 46 - 12 = 34$$

$$P_{CH_4} = 54 - 3 = 51$$

$$\alpha = \frac{0.8V \times 51}{0.2V \times 34} = 6.0$$

(b) Since R = 0, use Eq. (26.18)

At inlet,

$$y_i = \frac{6 \times 0.50}{1 + (5 \times 0.50)} = 0.857$$

Assume x = 0.42 as before. At exit,

$$y_i = \frac{6 \times 0.42}{1 + (5 \times 0.42)} = 0.813$$

$$\tilde{y} = (0.857 + 0.813)/2 = 0.835 \text{ or } 83.5\% \ H_2$$

26.3. For A assume molar volume = molecular weight = 150 cm³/g mol $\mu_B = 1$ cP

From Eq. (21.28),

$$D_v = \frac{13.26 \times 10^{-5}}{1^{1.14} \times 150^{0.589}} = 6.93 \times 10^{-6} \text{ cm}^2/\text{s}$$

Use Eq. (26.34) to find the effective diffusivity D_e

Molecular volume:

$$\pi D^3/6 = 150/(6.023 \times 10^{23})$$

$$D = 7.81 \times 10^{-8} \text{ cm}$$

$$\lambda = D/D_{pore} = \frac{7.81 \times 10^{-8}}{5 \times 10^{-6}} = 0.0156$$

With $\lambda = 0.0156$, the last two terms in Eq. (26.34) are:

$(1 - \lambda^2)(1 - 2.104\lambda + 2.09\lambda^2 - 0.95\lambda^3)$ = 0.938. Assume tortuosity $\tau = 2$. Then

$$D_e = \frac{6.93 \times 10^{-6} \times 0.45 \times 0.938}{2}$$

$$= 1.46 \times 10^{-6} \text{ cm}^2/\text{s}$$

$C_{A1} = 0.01 \text{ g/cm}^3 \qquad C_{A2} = 0$

From Eq. (26.5) with $z = 30 \times 10^{-4}$ cm

$$J_A = \frac{1.46 \times 10^{-6} \times 0.01}{30 \times 10^{-4}}$$

$$= 4.87 \times 10^{-6} \text{ g/cm}^2\text{-s}$$

For B, molecular diameter, if spherical:

$$D = \left(\frac{6 \times 15000}{\pi \times 6.023 \times 10^{23}}\right)^{1/3} = 3.62 \times 10^{-7} \text{ cm}$$

For this size molecule the diffusivity should be found from the Stokes-Einstein equation.

From Eq. (29.54) with T = 20°C or 293 K:

$$D_v = \frac{7.32 \times 10^{-16} \times 293}{1.81 \times 10^{-7} \times 1.0}$$

$$= 1.18 \times 10^{-6} \text{ cm}^2/\text{s}$$

But the polymer chain will probably have an effective diameter which is 1.5 to 2 times that for a compact molecule of the same molecular weight. This will reduce the bulk diffusivity and also decrease the hindrance factor.

Assume $D = 6 \times 10^{-7}$ cm

$$D_v = 1.18 \times 10^{-6} \times (3.62/6.0)$$

$$= 7.12 \times 10^{-7} \text{ cm}^2/\text{s}$$

If $D = 6 \times 10^{-7}$,

$$\lambda = 6 \times 10^{-7}/5 \times 10^{-6} = 0.12$$

From Eq. (26.34) the volume fraction times the hindrance factor is

$$(1 - 0.12)^2(1 - 2.104(0.12) + 2.09(0.12)^2 - 0.95(0.12)^3) = 0.601$$

$$D_e = \frac{7.12 \times 10^{-7} \times 0.45 \times 0.601}{2}$$

$$= 9.63 \times 10^{-8} \text{ cm}^2/\text{s}$$

$$J_B = \frac{9.63 \times 10^{-8} \times 0.01}{30 \times 10^{-4}}$$

$$= 3.21 \times 10^{-7} \text{ g/cm}^2\text{-s}$$

26.4. (a) Assume pinholes are straight cylindrical channels with a diameter of 0.01×10^{-6} m and length $\Delta L = 0.1 \times 10^{-6}$ m. For laminar flow at $\Delta P = 985$ $\text{lb}_f/\text{in.}^2$ or 6.79×10^6 Pa and $\mu = 1$ cP $= 10^{-3}$ Pa-s, using the equation at the bottom of p. 99:

$$\bar{V} = \frac{\Delta P D^2}{32 \Delta L \mu} = \frac{6.79 \times 10^6 \times 10^{-16}}{32 \times 10^{-7} \times 10^{-3}}$$

$$= 0.212 \text{ m/s}$$

$$Re = 10^{-8} \times 0.212 \times 1000/10^{-3}$$

$$= 2.12 \times 10^{-3}$$

Flow per pinhole:

$$\pi D^2 \bar{V}/4 = \pi \times 10^{-16} \times 0.212/4$$

$$= 1.665 \times 10^{-17} \text{ m}^3/\text{s}$$

$$J = 6.5 \text{ gal/ft}^2\text{-day} \times \frac{3.785 \times 10^{-3}}{0.0929 \times 24 \times 3600}$$

$$= 3.065 \times 10^{-6} \text{ m}^3/\text{m}^2\text{-s}$$

Let J_1 = flux through perfect regions

$= J - J_2$

J_2 = flux through pinholes

Salt flux:

$$(J - J_2)(0.005)C_o + J_2 C_o = J(0.03)C_o$$

$$0.995 J_2 = 0.025 J$$

$$J_2 = 0.0251 J = 7.70 \times 10^{-8} \text{ m}^3/\text{m}^2\text{-s}$$

Number of pinholes:

$$n = 7.70 \times 10^{-8}/1.665 \times 10^{-17}$$

$$= 4.62 \times 10^9/\text{m}^2 \text{ or } 4.62 \times 10^5/\text{cm}^2$$

(b) Fraction hole area:

$$n\pi D^2/4 = 4.62 \times 10^9 \times (\pi/4)10^{-16}$$
$$= 3.63 \times 10^{-7}$$

(c) Sealing the pinholes would reduce the flux to $J - J_2$. Since $J_2/J = 0.0251$, the reduced flux is:

$$(1 - 0.0251) \times 6.5 = 6.34 \text{ gal/ft}^2\text{-day}$$

26.5. From Eq. (26.47)

$$J_w = \frac{C_w D_w v_w}{RT}\left(\frac{\Delta P - \Delta\pi}{z}\right)$$

$C_w = 0.2 \text{ g/cm}^3$ $\quad D_w = 10^{-6} \text{ cm}^2/\text{s}$

$v_w = 18 \text{ cm}^3/\text{mol}$ $\quad T = 298 \text{ K}$

$R = 82.056 \text{ cm}^3\text{-atm/g mol-K}$

Assume $z = 0.1\ \mu m = 10^{-5}$ cm

$\Delta\pi = 20$ if rejection is 100%

$$J_w = \frac{0.2 \times 10^{-6} \times 18(\Delta P - 20)}{82.056 \times 298 \times 10^{-5}}$$
$$= 1.47 \times 10^{-5}(\Delta P - 20) \text{ cm}^3/\text{cm}^2\text{-s}$$

From Eq. (26.48)

$$J_s = D_s S_s(\Delta C_s/z)$$

$D_s = 10^{-9} \text{ cm}^2/\text{s}$ $\quad S_s = 0.035$

$$J_s = 10^{-9} \times 0.035\Delta C_s/10^{-5}$$
$$= 3.5 \times 10^{-6}\ C_s$$

For 80% rejection, $\Delta C_s = 0.8C_o$

$$J_s = 2.8 \times 10^{-6} C_o$$

The product has $0.2C_o$ salt concentration and

$$J_s/J_w \cong 0.2C_o$$
$$J_w = 2.8 \times 10^{-6} C_o/0.2C_o$$
$$= 1.4 \times 10^{-5} \text{ cm}^3/\text{cm}^2\text{-s}$$

If π varies linearly with concentration,

$$\Delta\pi = 0.8 \times 20 = 16$$
$$(\Delta P - 16) = 1.4 \times 10^{-5}/1.47 \times 10^{-5}$$
$$= 0.95 \text{ atm}$$
$$\Delta P = 16.95 \text{ atm}$$

For 90% rejection

$J_s = 3.15 \times 10^{-6} C_o$, $\Delta\pi = 18$

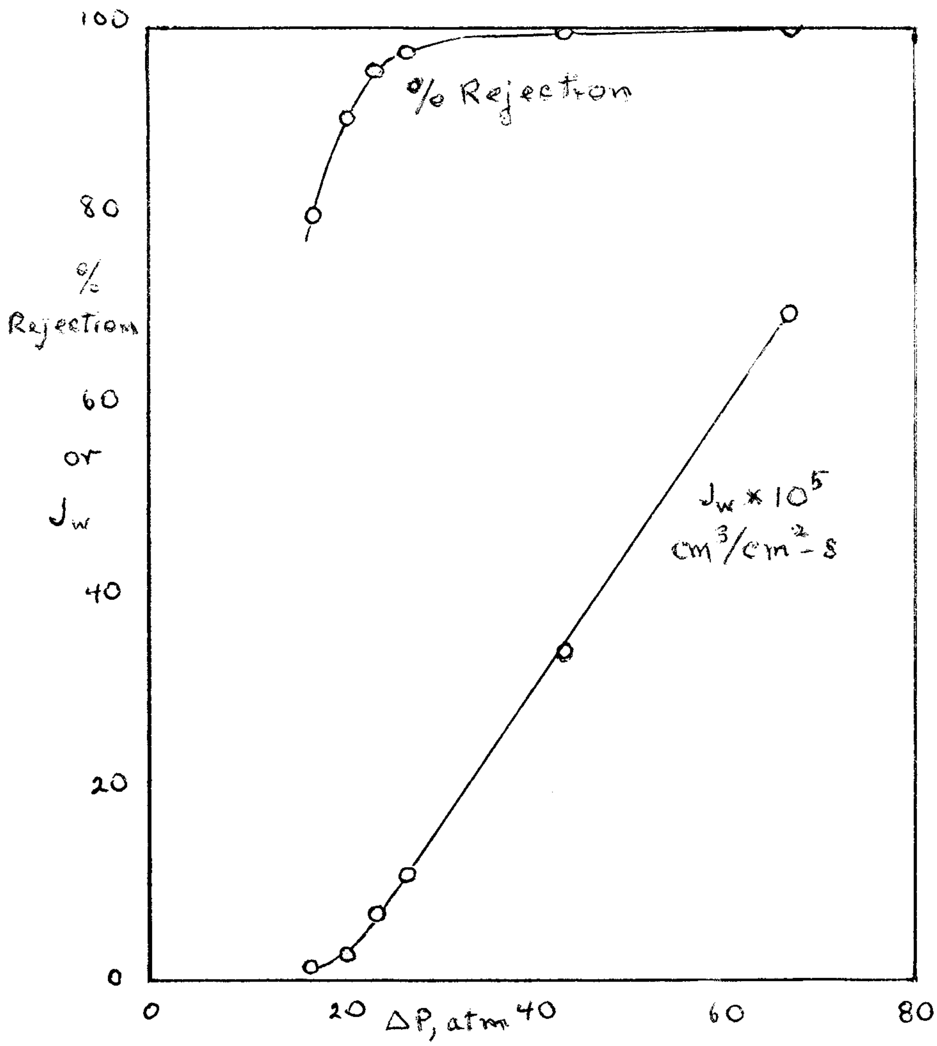
$J_w = 3.15 \times 10^{-6}/0.1 = 3.15 \times 10^{-5}$

$(\Delta P - 18) = 3.15 \times 10^{-5}/1.47 \times 10^{-5} = 2.14$

$\Delta P = 20.1$

Similar calculations give the following results:

% rejection	$\Delta\pi$	$J_s/C_o \times 10^6$	$J_w \times 10^5$	P, atm
80	16	2.8	1.4	16.95
90	18	3.15	3.15	20.1
95	19	3.33	6.66	23.5
97	19.4	3.40	11.3	27.1
99	19.8	3.47	34.7	43.4
99.5	19.9	3.48	69.6	67.2



26.6. (a) Permeate flux is based on the outer area of the fiber. For O.D. = 600 μm, L = 1 m:

Volume flow:

$$\frac{2 \times 10^{-3}}{60} \frac{m^3}{m^2-s} \times \pi(6 \times 10^{-4})$$

$$= 6.28 \times 10^{-8} \ m^3/s$$

$$\bar{V} = 6.28 \times 10^{-8}/(\pi/4)(4 \times 10^{-4})^2 = 0.50 \ m/s$$

$$\mu = 0.018 \ cP = 1.8 \times 10^{-5} \ Pa\text{-}s$$

If the flux is constant along the length of the fiber, Eq. (26.57) applies with D = 400 μm = 4×10^{-4} m. From Eq. (26.54) the term $4J_w L/D = \bar{V}$. Hence

$$\Delta P_s = \frac{32 \times 1.8 \times 10^{-5} \times 1 \times 0.50}{2 \times (4 \times 10^{-4})^2}$$

$$= 900 \ Pa \text{ or } 0.0089 \ atm$$

(b) For fibers 5 m long the exit velocity is 5 times higher and the pressure drop is 25 times greater.

$$\Delta P = 0.22 \ atm$$

26.7. Assume the permeate has 30% ethanol and the product has 100% ethanol.

Basis: F = 100 g with 90 g ethanol

$$90 = 0.3V + 1.0(100 - V)$$

$$V = 14.3 \text{ g permeate}$$

$$L = 85.7 \text{ g product at } 70^\circ C$$

Heat balance. Use liquid at 70°C as reference point.

$$100c_p(80 - 70) + Q = 14.3\ H_v + 85.7(0)$$

$$\Delta H_v = 557 \text{ cal/g for water}$$

$$\Delta H_v = 237 \text{ cal/g for ethanol}$$

$$\overline{\Delta H_v} = 0.7(557) + 0.3(237) = 461 \text{ cal/g}$$

For ethanol, $c_p = 0.77$ cal/g-°C

Total heat supplied:

$$Q = 14.3(461) - 100(0.77)(10) = 5822 \text{ cal}$$

Liquid flux decreses from 100 to 85.7 g over several stages. Use 92 g as average. Heat needed to heat liquid 10°C:

$$92(0.77)(10) = 708 \text{ cal}$$

Number of heaters needed: 5822/708 = 8.2

Use 9 stages and 8 heaters.

26.8. For CO_2/CH_4 at 80°F,

$$\alpha = 0.0413/0.00205 = 20.1$$

At 100°F, $\alpha = 0.0425/0.0029 = 14.7$

Using the equation in the text on p. 861, written in logarithmic form for Q_1 and Q_2:

$$\ln \frac{Q_2}{Q_1} = \frac{E}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) = \frac{E}{R}\left(\frac{T_2 - T_1}{T_1 T_2}\right)$$

For CO_2,

$$E = \frac{1.987 \times 540 \times 560}{20 \times 1.8} \ln \frac{0.0425}{0.0413}$$

$$= 478 \text{ cal/g mol}$$

For CH_4,

$$E = \frac{1.987 \times 540 \times 560}{20 \times 1.8} \ln \frac{0.0029}{0.00205}$$

$$= 5789 \text{ cal/g mol}$$

At 130° F,

$$\ln \frac{Q_{CO_2}}{0.0425} = \frac{478}{1.987}\left(\frac{30 \times 1.8}{560 \times 590}\right)$$

$$Q_{CO_2} = 0.0442$$

$$\ln \frac{Q_{CH_4}}{0.0029} = \frac{5789}{1.987}\left(\frac{30 \times 1.8}{560 \times 590}\right)$$

$$Q_{CH} = 0.00467$$

$$\alpha = 0.0442/0.00467 = 9.5$$

26.9. At inlet

$P_{A1} = 0.5 \times 600 = 300$ lbf/in.^{2} $\quad Q_A = 0.4$

$P_{B1} = 0.3 \times 600 = 180$ lbf/in.^{2} $\quad Q_B = 0.1$

$P_{C1} = 0.2 \times 600 = 120$ lbf/in.^{2} $\quad Q_C = 0.15$

Assume the permeate composition is the same as the feed composition to estimate the individual fluxes. Then correct the permeate composition and repeat the calculation.

From Eq. (26.8),

$J_A = 0.4[300 - 0.5(120)] = 96$

$J_B = 0.1[180 - 0.3(120)] = 14.4$

$J_C = 0.15[300 - 0.2(120)] = \underline{14.4}$

124.8

$y_A = 96/124.8 = 0.769$

$y_B = 14.4/124.8 = 0.115$

$y_C = 14.4/124.8 = 0.115$

Second try:

$J_A = 0.4[300 - 0.769(120)] = 83.1$

$J_B = 0.1[180 - 0.115(120)] = 16.6$

$J_C = 0.15[300 - 0.115(120)] = \underline{15.9}$

115.6

$y_A = 83.1/115.6 = 0.719$

$y_B = 16.6/115.6 = 0.114$

$y_C = 15.9/115.6 = 0.1375$

Continuing in this way, the fourth and fifth tries agree and give $y_A = 0.7274$ or 72.74%, $y_B = 13.93\%$ and $y_C = 13.33\%$.

26.10.(a) Use an arithmetic average driving force. From Table 16.2, p. 874, for N_2,

$$\Delta P_{ave} = \frac{72(0.21) + 71(0.43)}{2} - 31(0.08)$$

$$= 20.35 \text{ bar}$$

For CH_4,

$$\Delta P_{ave} = \frac{72(0.14) + 71(0.34)}{2} - 31(0.03)$$

$$= 16.18 \text{ bar}$$

$$\alpha_{N_2/CH_4} = \frac{J_{N_2}/\Delta P_{N_2}}{J_{CH_4}/\Delta P_{CH_4}}$$

$$= \frac{0.08V \times 16.18}{0.03V \times 20.35} = 2.1$$

(b) For H_2 the same method gives a negative driving force.

$$P_{ave} = \frac{72(0.59) + 71(0.12)}{2} - 31(0.86)$$

$$= -1.16 \text{ bar}$$

The 5-fold change in the partial pressure of hydrogen on the high-pressure side of the membrane and the high partial pressure in the permeate makes use of an average driving force invalid.

(c) Values of the permeability could be assumed for each species and a stepwise calculation carried out until the residue has 12% H_2. If the other concentrations in the residue or the permeate composition do not match the data, the permeabilities are changed and the calculations repeated.

26.11. (*a*) Find the number of fibers, n.

$A = n\pi D L_T = 5.2 \text{ m}^2$, where L_T is the total length.

$D_o = 600 \ \mu\text{m} \qquad L_T = 1 \text{ m}$

$$n = \frac{5.2}{\pi \times 600 \times 10^{-6} \times 1} = 2759$$

At STP, the exit velocity is

$$\bar{V} = \frac{3.1 \times 10^{-3} (m^3/\text{min})}{60 \times 2759 \times (\pi/4) \times (300 \times 10^{-6})^2} = 0.265 \text{ m/s}$$

At 20°C, $\bar{V} = 0.265\ (293/273) = 0.284$ m/s

μ = 0.018 cP = 1.8×10^{-5} Pa.s; ρ = 1.206 kg/m³

$$\text{Re} = \frac{300 \times 10^{-6} \times 0.284 \times 1.206}{1.8 \times 10^{-5}} = 5.71$$

Use Eq. (5.20) for laminar flow, in differential form,

$$\frac{dP}{dL} = \frac{32\bar{V}\mu}{D^2}$$

Assume a constant permeation rate over the fiber length.

$$\bar{V} = \bar{V}_{exit}\frac{L}{L_T}$$

$$dP = \frac{32\bar{V}_{exit}\,\mu L\ dL}{D^2 L_T}$$

By integration, $\Delta P = \dfrac{32\bar{V}_{exit}\ \mu\ L_T^2}{2D^2 L_T}$

For $L_T = 1$, $\Delta P = \dfrac{16 \times 0.284 \times 1.8 \times 10^{-5}}{(300 \times 10^{-6})^2} = 909$ Pa $= 9.09 \times 10^{-3}$ bar

(*b*) For 150- x 300-μm fibers, the permeate flux per fiber would be half as great, but the cross-sectional area is reduced by four, so $\bar{V}$ is two times greater. ΔP depends on $\bar{V}/D^2$. So ΔP changes to:

909 x 2 x 4 = 7270 Pa = 0.073 bar.

26.12(*a*) $R = 0.05 = P_1/P_2$ Let $P_1 = 20$ atm, $P_2 = 1$ atm

<u>Basis</u>: $F = 100$, $x_F = 0.01$, $y = 0.049$. Stage cut = 19 percent

$V = 0.19F$, $F = 19$, $L = F - V = 81$

Check CFC balance. In: 100 (0.01) = 1

Out: 19 (0.049) + 81 (0.001) = 1.012. OK.

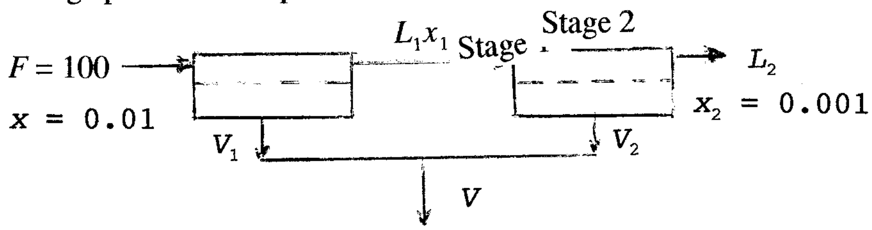
For CFC at inlet end, $p_{1,\ CFC} = 20\ (0.01) = 0.2$ atm

$p_{2,CFC} = 1\ 0.049) = 0.049$ atm

$\Delta p_{CFC} = 0.2 - 0.049 = 0.151$ atm

At the residue end, $p_{1,\ CFC} = 20\ (0.001) = 0.02$ atm

Since $p_{1,\,CFC}$ is so low, Δp_{CFC} at the residue end can't be estimated without a trial-and-error calculation. For an approximate solution, a value of α is assumed, and a 2-stage calculation made to find the true average permeate composition.



Choose $x_1 = 0.003$. Find y^* for $x = 0.01$ and $x = 0.003$, and average the values. Find L_1 and V_1 by a material balance.

Guess $\alpha = 30$, and use Eq. (26.17) for $R = 0.05$.

Stage 1. For $x = 0.01$:

$$29y^2 + (1 - 30 - 20 - (20 \times 0.01 \times 29))y + (30 \times 0.01 \times 20) = 0$$

$$29y^2 - 54.8y + 6 = 0$$

$$y = 0.117$$

At $x = 0.003$:

$$29y^2 + (-49 - (580 \times 0.003))y + (600 \times 0.003) = 0$$

$$29y^2 - 50.74y + 1.8 = 0$$

$$y = 0.0362$$

$$y_{ave} = (0.117 + 0.0362)/2 = 0.0766$$

Material balance:

$$Fx_{in} = 100 \times 0.01 = 0.003L_1 + 0.0766(100 - L_1)$$

$$L_1 = 6.66/0.0736 = 90.5$$

$$V_1 = 9.5$$

Stage 2. $x_{in} = 0.003$, $y = 0.0362$ For $x = 0.001$:

$$29y^2 + (-49 - (580 \times 0.001))y + (600 \times 0.001) = 0$$

$$29y^2 - 49.58y + 0.6 = 0$$

$$y = 0.0122$$

$$y_{ave} = (0.0122 + 0.0362)/2 = 0.0242$$

Material Balance.

$$L_1\, x_1 = 90.5 \text{ x } 0.003 = 0.242V_2 + 0.001(90.5 - V_2)$$

$$0.2715 = 0.0232V_2 + 0.0905$$

$$V_2 = 7.8$$

Total permeate: $V_1 + V_2 = 9.5 + 7.8 = 17.3$; $V/F = 0.173$

$$y_{\text{ave}} = [(9.5 \text{ x } 0.0766) + (7.8 \text{ x } 0.0242)]/17.3 = 0.053$$

These values are close to the reported values of $V/F = 0.19$ and $y = 0.49$, so selectivity α is close to 30. (Using a 3-stage calculation with $\alpha = 30$ gives $V/F = 0.18$ and $y = 0.051$, which are even closer to the reported values.)

(*b*) New feed is the permeate from Unit 1. Use a 2-stage calculation for Unit 2.

<u>Stage 1.</u> $\alpha = 30$ $x = 0.051$ $1/R = 20$

$$29y^2 + (-49 - (580 \text{ x } 0.051))y + (600 \text{ x } 0.051) = 0$$

$$29y^2 - 78.58y + 30.6 = 0$$

$$y = 0.471$$

Choose $x_1 = 0.02$. Then

$$29y^2 - 60.6y + 12 = 0$$

$$y = 0.221$$

$$y_{\text{ave}} = (0.471 + 0.221)/2 = 0.346$$

Material balance: $5.1 = 0.346V_1 + 0.02(100 - V_1)$

$$3.7 = 0.326V_1$$

$$V_1 = 9.51,\ L_1 = 90.49$$

<u>Stage 2.</u> At inlet, $y = 0.221$ (same as at outlet of Stage 1)

At outlet, $y = 0.117$ (from part (*a*))

$$y_{\text{ave}} = (0.221 + 0.117)/2 = 0.169$$

$$90.49 \text{ x } 0.02 = 0.169V_2 + 0.01(90.49 - V_2)$$

$$V_2 = 0.9049/0.159 = 5.69$$

Stage cut: $(V_2 + V_2)/\ 100 = (9.51 + 5.69)/100 = 0.152$

$$\bar{y} = \frac{(9.51 \times 0.346) + (5.69 \times 0.169)}{15.2} = 0.2797$$

Permeate is 28% CFC-11.

26.13. The permeate flow rate is not proportional to the overall pressure difference because it is made up of two components, and the oxygen and nitrogen fluxes are each proportional to partial pressure differences. When the feed pressure doubles the driving force for oxygen more than doubles, as shown by a sample calculation for a counterflow unit. Case I. $P_1 = 90 \text{ lb}_f/\text{in.}^2 = 105 \text{ lb}_f/\text{in.}^2$ absolute

<u>At inlet,</u> $P_{O_2} = 105 \times 0.21 = 22.05 \text{ lb}_f/\text{in.}^2$ absolute

On the permeate side, if $y \cong 0.40$, $P_2 = 15 \text{ lb}_f/\text{in.}^2$ absolute

$$P_{O_2} = 15 \times 0.40 = 6.0 \text{ lb}_f/\text{in.}^2 \text{ absolute}$$

$$\Delta P_{O_2} = 22.05 - 6.0 = 16.05 \text{ lb}_f/\text{in.}^2$$

Case II. $P_1 = 180 \text{ lb}_f/\text{in.}^2 = 195 \text{ lb}_f/\text{in.}^2$ absolute

$$P_{O_2} = 195 \times 0.21 = 40.95 \text{ lb}_f/\text{in.}^2 \text{ absolute}$$

If y is still about 0.40,

$$\Delta P_{O_2} = 40.95 - 6.0 = 34.95 \text{ lb}_f/\text{in.}^2$$

The driving force for oxygen is changed by the factor 34.95/16.05 = 2.18 for a 2-fold change in overall driving force.

The corresponding values for nitrogen are:

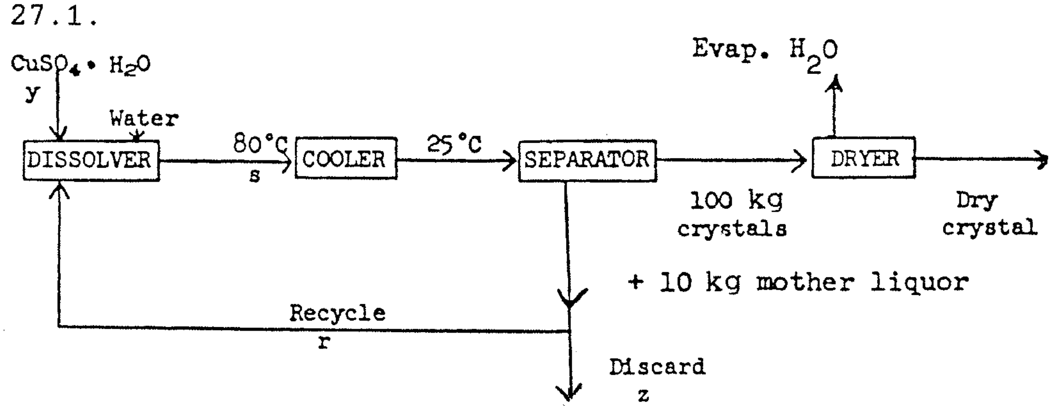
Case I. $\Delta P_{N_2} = (105 \times 0.79) - (15 \times 0.6) = 73.95 \text{ lb}_f/\text{in.}^2$

Case II. $\Delta P_{N_2} = (195 \times 0.79) - (15 \times 0.6) = 145.05 \text{ lb}_f/\text{in.}^2$

The driving force for nitrogen is changed by 1.96 times, which is slightly less than 2.0, but at the feed end the permeate flux would still increase by a factor greater than 2.0.

Chapter 27

27.1.



Basis: 100 kg crystals of $CuSO_4 \cdot 5H_2O$ from separator

Molecular weights: $CuSO_4 \cdot H_2O = 177.65$ $H_2O = 18.02$

$CuSO_4 \cdot 5H_2O = 249.71$

Crystals from separator carry 10 kg mother liquor. Let i = kg impurity and x = kg H_2O in liquor. Also s = kg solution at 80°C, y = kg feed, and z = kg of mother liquor discarded.

Since the liquor is saturated at 25°C, there are 0.4 kg $CuSO_4 \cdot 5H_2O$ per kg of free water. Then

Total dry $CuSO_4 \cdot 5H_2O$: $100 + 0.4x$

Allowable impurity: $i = 0.006(100 + 0.4x + i)$

$i = 0.60362 + 0.00241x$

Also, $x + 0.4x + i = 10$

From these equations, x = 6.700 kg H_2O/10 kg mother liquor. Then

$0.4x = 2.680$ kg $CuSO_4 \cdot 5H_2O$/10 kg mother liquor.

$i = 0.62$ kg impurity/10 kg mother liquor.

$CuSO_4 \cdot 5H_2O$ in dry crystals:

$100 + 2.68 = 102.68$ kg

Total weight of dry crystals:

$100 + 2.68 + 0.62 = 103.30$ kg

Over-all balances

Impurity: $0.035y = 0.620 + 0.620z/10$

$CuSO_4 \cdot 5H_2O$: $\frac{249.71}{177.65} \times 0.965y$

$= 102.68 + 2.680z/10$

$1.356y = 102.68 + 0.268z$

From these equations,

y = 83.00 kg feed

z = 36.82 kg discarded mother liquor

Water out:

In crystals: $102.68 \times \dfrac{5 \times 18.02}{249.71} = 37.05$ kg

Evaporation in dryer: $= 6.70$

In discard: $(36.82/10) \times 6.70$

$$+ \frac{36.82}{10} \times 2.68 \times \frac{5 \times 18.02}{249.71} = \underline{28.23}$$

Total 71.98 kg

Water in with $CuSO_4 \cdot H_2O$

$0.965 \times 83.00 \times 18.02/177.65 = 8.12$ kg

Water makeup: $71.98 - 8.12 = 63.86$ kg

<u>Material balances around dissolver</u>

Let r = kg recycle

s = kg solution at 80°C

Impurity in with $CuSO_4 \cdot H_2O$:

$0.035 \times 83.00 = 2.905$ kg

Impurity in recycle: 0.062r

Impurity in 80°C solution: 2.905 + 0.062r

Water and $CuSO_4 \cdot 5H_2O$ in 80°C solution:

s - 2.905 - 0.062r

Water balance:

$$8.12 + 63.86 = \left[\frac{100}{220} + \frac{120 \times 5 \times 18.02}{220 \times 249.71}\right] \times (s - 2.905 - 0.0620r) - (6.700/10)r - \frac{5 \times 18.02 \times (2.68/10)r}{249.71}$$

$0.6514s - 0.8071r = 73.89$

$CuSO_4 \cdot 5H_2O$ balance:

$$\frac{249.71}{177.65} \times 0.965 \times 83.00 = \frac{120}{220}(s - 2.905 - 0.0620r) - 0.268r$$

$0.5455s - 0.3018r = 114.17$

From these equations, s = 286.68 kg

r = 139.83 kg

(a) Per 100 kg impure $CuSO_4 \cdot H_2O$

Water makeup: $63.86 \times 100/83.00 = 76.94$ kg

Recycle: $139.83 \times 100/83.00 = 168.47$ kg

(b) Recovery:

$$\frac{100 \times 102.68 \times 177.65}{83.00 \times 0.965 \times 249.71} = 91.2\%$$

27.2. From Fig. 27.3:

Solubility of $MgSO_4$ at 45° F = 0.224 mass fraction

<u>Basis: 100 kg feed</u>

Feed contains 43 kg $MgSO_4$/100 kg H_2O

Mass fraction $MgSO_4 = 43/143 = 0.301$

Molecular weights: $MgSO_4 = 120.38$

$MgSO_4 \cdot 7H_2O = 246.49$

Mass fraction $MgSO_4$ in $MgSO_4 \cdot 7H_2O$

$= 120.38/246.49 = 0.488$

Enthalpies, Btu/lb

From Appendix 7,

Vapor: $1081.1 + (0.5 \times 2) = 1082.6$ (601.45 kcal/kg)

Feed (from Fig. 27.4): 45 (25 kcal/kg)

Mother liquor: − 55 (−30.56 kcal kg)

Crystals: − 162 (−90 kcal/kg)

Let V = kg vapor, C = kg crystals. Then

$100 - (V + C) =$ kg mother liquor

$MgSO_4$ balance:

$$100 \times 0.301 = 0.488C + 0.224(100 - V - C)$$

$$7.7 = 0.264C - 0.224V$$

Enthalpy balance:

$$100 \times 25 = 601.45V - 90C - 30.56(100 - V - C)$$

$$5556 = 632.01V - 59.44C$$

From these equations,

C = 39.80 kg crystals

V = 12.53 kg vapor

Feed solution for 900 kg crystals:

$(900/39.80)100 = 2261$ kg

27.3. NOTE: Shape factor $a = v_c/L^3$

Since, in the ideal product classification crystallizer, all product crystals are of the same size, all crystals are retained in the crystallizer until they have grown to size L_T. Hence at the end of the process $L = L_T = L_{pr}$. Also, the population density, n, is independent of L and is that at zero size, n°.

From Eq.(27.22) the number of crystals per cubic meter between sizes L and L + dL is ndL, or in this case n°dL, and the mass is $a\rho_c n° L^3 dL$. The total number of crystals per cubic meter is $\int n° dL = n° L_T$, but the total mass must be obtained by another integration:

$$m_c = n° a\rho_c \int_0^{L_T} L^3 dL = \frac{n° a\rho_c L_T^4}{4}$$

Also, since $n° = B°/G$ (from Eq. (27.40)),

$$m_c = \frac{a\rho_c B° L_T^4}{4G} \qquad (1)$$

The total mass of crystals per volume of liquid is same in both crystallizer and product magmas. For a volume V of product, by a particle balance, the number of crystals in the product equals the number of crystals per unit time. The mass of a single product crystal is $a\rho_c L_T^3$, and a mass balance yields

$$m_c = \frac{B° V a\rho_c L_T^3}{Q} \qquad (2)$$

where Q is the volumetric flow rate of liquid in the product. Equating the values of m_c given by (1) and (2) yields

$$\frac{V}{Q} = \frac{L_T}{4G} = \tau \qquad (3)$$

The time required to grow a crystal is L_T/G, and Eq. (3) shows that the growth time in an ideal product classification crystallizer is four times the magma retention time.

27.4. $L_T = 1.4$ mm

Growth Rate:

$G = 0.2 \times 3600/10^6 = 7.2 \times 10^{-4}$ m/h

Mass of crystals per unit volume of liquid:

$m_c = 2300 \times 0.35 = 805$ kg/m^3

The retention time, from Eq. (3) of Problem 27.3, is:

$$\tau = \frac{L_T}{4G} = \frac{1.4 \times 10^{-3}}{4 \times 7.2 \times 10^{-4}} = 0.486 \text{ h}$$

The production rate of crystals per unit volume (V = 1) of liquid in the crystallizer is, since $V/Q = \tau = 0.486$:

$Qm_c = (1/0.486) \times 805 = 1656$ kg/h-m^3

Nucleation rate from Eq. (1) of Problem 27.3

$$B° = \frac{805 \times 4 \times 7.2 \times 10^{-4}}{0.20 \times 2300 \times (1.4 \times 10^{-3})^4} = 1.312 \times 10^9 \frac{\text{nuclei}}{\text{h-m}^3}$$

27.5. Predominant size: 35 mesh. From Appendix 5, this is 0.0164 in., and $L_{pr} = 0.0164/12 = 1.367 \times 10^{-3}$ ft.

$\tau = 2$ hr, $L_{pr} = 3G\tau$ (see top 2 lines, p.914)

(b) Required growth rate, $G = L_{pr}/3\tau$

$$G = \frac{1.367 \times 10^{-3}}{3 \times 2} = 2.278 \times 10^{-4} \text{ ft/h}$$

Production rate: C = 2,000 lb/hr. Assume a = 1.

Density of crystal: 105 lb/ft^3

Liquid flow rate in product:

$$Q = \frac{0.85 \times 2{,}000}{0.15 \times 105} = 107.94 \ ft^3/h$$

Volume of liquid in crystallizer:

$$V_c = \tau Q = 2 \times 107.94 = 215.88 \ ft^3/\text{ton product}$$

Also, the nucleation rate from Eq. (27.44), is:

$$B° = \frac{9 \times 2{,}000}{2 \times 105 \times 215.88 \times (1.367 \times 10^{-3})^3}$$

$$= 1.554 \times 10^8 \ \text{nuclei/h-ft}^3 \ \text{liquid}$$

(a) For the calculation of cumulative screen analysis (see p. 916):

$$z = \frac{L}{G\tau} = \frac{L}{2.278 \times 10^{-4} \times 2} = 2.195 \times 10^3 L$$

Cumulative screen analysis, from Eq. (27.35)

$$\mu_3 = 1 - (1 + z + \frac{z^2}{2} + \frac{z^3}{6})\ e^{-z}$$

Table 1 gives calculation of mass fraction, μ_3, of product for mesh sizes on the Tyler Scale.

Table 1. Cumulative Screen Analysis

Mesh	Size, in.	$L \times 10^3$ ft	z	μ_3
150	0.0041	0.3417	0.750	0.007
100	0.0058	0.4833	1.061	0.023
80	0.0069	0.5750	1.262	0.039
65	0.0082	0.6833	1.500	0.066
60	0.0097	0.8083	1.774	0.105
48	0.0116	0.9667	2.122	0.166
42	0.0138	1.1500	2.524	0.248
35	0.0164	1.3667	3.000	0.353
32	0.0195	1.6250	3.567	0.478
28	0.0232	1.9333	4.244	0.613
24	0.0276	2.3000	5.049	0.742
20	0.0328	2.7333	6.000	0.849
16	0.0390	3.2500	7.134	0.925
14	0.046	3.8333	8.414	0.968
12	0.055	4.5833	10.060	0.990
10	0.065	5.4167	11.890	0.998

Note: A tabulation of the function μ_3 vs. z is given in the Appendix of Ref. 18.

Fig. 27.15(a) is a plot of these values of x_m (= μ_3) vs. z. The mesh sizes corresponding to values of z may be read from the table.

27.6. From Eq. (27.40) the growth rate is

$$G = B^o/n^o = 1.84/1.46 \times 10^6$$

$$= 1.26 \times 10^{-6} \text{ cm/s or } 0.0454 \text{ mm/h}$$

From Eq. (27.45), since $B^o/n^oG = 1$,

$$K_N = \frac{t_{To}}{u_T^2 \rho_c (G\tau)^5}$$

$$= \frac{35}{1350^2 \times 2.163 \times (0.0454 \times 1.80)^5}$$

$$= 2.436$$

27.7. (a) Assume Sh = 2.0 or $k_c = 2D_v/D_p$

The rate of change of mass is:

$$\frac{d(\rho_s \pi D_p^3/6)}{dt} = -\pi D_p^2 k_c \Delta c$$

$$3\rho_s D_p^2 dD_p/6 = - D_p^2(2D_v/D_p)\Delta c$$

$$\int D_p\, dD_p = -\int \frac{4D_v \Delta c}{\rho_s}\, dt$$

$$D_p^2/2 = - 4D_v \Delta ct/\rho_s$$

For complete dissolution

$$-\Delta D_p^2 = D_p^2 = 8D_v \Delta ct/\rho_s$$

$$\rho_s = 1.988 \text{ g/cm}^3$$

Use Eq. (17.33) and Table 17.1 with λ for K^+ and Cl^-. For dilute solutions at 25°C,

$$D_v = \frac{2 \times 8.314 \times 298}{(1/73.5 + 1/76.3)96{,}500^2}$$

$$= 1.99 \times 10^{-5} \text{ cm}^2/\text{s}$$

Correct to 130°F or 54.4°C, using T/μ ratios for water (see Appendix 6):

$$D_v = 1.99 \times 10^{-5}\left(\frac{327.4 \times 0.89}{298 \times 0.51}\right)$$

$$= 3.82 \times 10^{-5} \text{ cm}^2/\text{s}$$

The diffusivity of KCl in an almost saturated solution is not known, but Reid et al (Ref. 21 in Chap. 21) show that D_v decreases at first and then increases slightly with concentration in the range 0 to 2M.

Use $D_v = 3.8 \times 10^{-5}$ cm²/s

At 100°F, $c_s = 39.3$ g/100 g H_2O

$c = 39.3 + 1 = 40.3$ g/100 g H_2O

At 100°F, the mass fraction is

40.3/140.3 = 0.287. From data in Perry,

7th ed., p. 2-104, $\rho = 1.189$ g/cm³.

$c = 40.3/(140.3/1.189) = 0.342$

At 130°F, the mass fraction is 44/144 =

0.306. From Perry, $\rho = 1.195$ g/cm³.

$c = 44/(144/1.195) = 0.365$

$\Delta c = 0.365 - 0.342 = 0.023$

For $D_p = 10\ \mu m = 10^{-3}$ cm,

$$t = \frac{(10^{-3})^2 \times 1.988}{8 \times 3.8 \times 10^{-5} \times 0.023} = 0.28 \text{ s}$$

For $D_p = 50\ \mu m$,

$t = 0.28 \times (50/10)^2 = 7.0$ s

(b) Volume fraction:

$$\frac{30 \text{ g/L}}{1.198} = 15.1 \text{ cm}^3/\text{L}$$

$$= 0.015 \text{ cm}^3 \text{ solid/cm}^3 \text{ slurry}$$

Hence $\epsilon = 1 - 0.015 = 0.985$

For $D_p = 200\ \mu m = 0.02$ cm,

$a = 6(1 - \epsilon)/D_p = 6 \times 0.015/0.02$

$= 4.5$ cm²/cm³ slurry

From Fig. 17.8 for $\Delta\rho = 1$ g/cm³,

$D_p = 200\ \mu m$, $D_v = 1.0 \times 10^{-5}$ cm²/s,

$k_{cT} = 0.006$ cm/s

From part (a), $D_v = 3.8 \times 10^{-5}$

Assuming $k_c = 2k_{cT}$ because of turbulence,

$$k_c = 2 \times 0.006 \times (3.8/1.0)^{2/3}$$
$$= 0.029 \text{ cm/s}$$

The equation for rate of change of concentration is

$$dc/dt = - k_c a(c_s - c)$$

Integrating this equation between the limits $c = c_1$ at $t = 0$ to $c = c_2$ at $t = t$ gives:

$$\ln\left(\frac{c_s - c_1}{c_s - c_2}\right) = k_c at = 0.029 \times 4.5 \times 10$$
$$= 1.305$$
$$e^{1.305} = 3.69$$

Since $c_s - c_1 = 44 - 40.3 = 3.7$ g/100 g,

$$c_s - c_2 = 3.7/3.69 = 1.0 \text{ g/100 g}$$

After 10 seconds the solution has come three-fourths of the way from the original concentration to the new saturation value.

The assumption of a constant driving force in part (a) leads to an underestimate of the dissolution time for 50-μm crystals but is probably satisfactory for 10-μm crystals.

27.8. The large tank operates as an MSMPR crystallizer with an average residence time for crystals equal to the suspension volume divided by the flow rate.

$$\bar{t}_a = V_a/Q_a$$

The pilot plant uses a small tank with the DDO method to give increased residence time for crystals larger than a certain cut size, L_F. The sum of the underflow, Q_u, and the overflow, Q_o, gives the same average residence time as for the large tank.

$$Q_o + Q_u = Q_b$$

where Q_b = total flow from small tank

Residence time in small tank:

$$\bar{t}_b = V_b/Q_b = \bar{t}_a$$

Since $Q_o = 3.5Q_u$, $Q_b = 4.5Q_u$

The large crystals leave only in the underflow, so their average residence time is

$$\bar{t}_b' = V_b/Q_u = 4.5V_b/Q_b = 4.5\bar{t}_a$$

(b) If the linear growth rate is proportional to the supersaturation but independent of crystal size, as given by Eq. (27.20),

$$G = 2K(y - y_s)/\rho_M$$

For large crystals, $D_p' = G\bar{t}_b'$.

Since the large crystals were only 50/30 or 1.67 times larger than the average for MSMPR operation, even though the residence time was 4.5 times greater, the degree of supersaturation must have been much lower in DDO operation. The fact that the mean size of particles in the overflow was smaller than before (20 m versus 30 m) also indicates a lower supersaturation.

(c) Since the total amount of solids formed per unit volume of feed should be the same for both methods of operation, the combined streams from DDO operation should have 15 wt% $CaSO_4 \cdot 2H_2O$. The overflow has less than 15% and the underflow more than 15%, but the concentrations cannot be calculated from the limited data supplied.

If the overflow is assumed to have 10% solids

$$0.15Q_b = 0.1Q_o + C_u Q_u$$

$$0.15(4.5Q_u) = 0.35Q_u + C_u Q_u$$

$$C_u = 0.325 \text{ or } 32.5\% \text{ in underflow}$$

The fraction of fines in the total product would be:

$$(0.1 \times 3.5)/(0.15 \times 4.5) = 0.52$$

27.9. From Eqs. (27.28) and (27.29)

$$z = L/G\tau$$

$$n = n^o e^{-z}$$

Assume spherical particles with $L = D$ (so $D = zG\tau$). Surface area:

$$s_p = \sum n\pi D_i^2 = \pi(G\tau)^2 \int_c^\infty n^o e^{-z} z^2 dz$$

Volume:

$$v = \frac{\sum n\pi D_i^2}{6} = \frac{\pi(G\tau)^3}{6} \int_c^\infty n^o e^{-z} z^3 dz$$

Surface-volume mean size:

$$\overline{D_s} = 6v/s_p$$

$$= G\tau \frac{\int_c^\infty e^{-z} z^3 dz}{\int_c^\infty e^{-z} z^2 dz}$$

From integral tables,

$$\overline{D_s} = G\tau \times 3!/2! = 3G\tau \text{ or } z = 3$$

(b) Mass average size:

$$\overline{D_m} = \sum nD^4 / \sum nD^3$$

$$= \frac{(G\tau)^4 \int_c^\infty e^{-z} z^4 dz}{(G\tau)^3 \int_c^\infty e^{-z} z^3 dz}$$

$$= G\tau \times 4!/3! = 4G\tau \text{ or } z = 4$$

(c) The second moment gives the surface area distribution and the Eq.(27.38) gives the differential distribution.

$$\frac{d\mu_2}{dz} = \frac{z^2 e^{-z}}{2}$$

This distribution is similar to the mass distribution shown in Fig. 27.14b but has a maximum at a different position. To get the maximum in the curve, differentiate again and set the derivative equal to zero.

$$\frac{d^2\mu_2}{dz^2} = \frac{2ze^{-z}}{2} + \frac{z^2e^{-z}(-1)}{2}$$

$$0 = e^{-z}(z - (z^2/2))$$

$$z = z^2/2 \quad \text{or } z = 2$$

Thus more of the surface area is in the element dz at z = 2 than any other element, though the surface-area mean diameter corresponds to z = 3.

For the mass distribution, the derivative of the third moment is given by Eq. (27.39):

$$\frac{d\mu_3}{dz} = \frac{z^3e^{-z}}{6}$$

Differentiating again to get the maximum:

$$\frac{d^2\mu_3}{dz^2} = \frac{3ze^{-z}}{6} + \frac{z^3e^{-z}(-1)}{6}$$

$$0 = e^{-z}((z^2/2) - (z^3/6))$$

$1/2 = z/6$ or $z = 3$, as shown in Fig. 27.14b.

Chapter 28

28.1. Basis: 1 gram of +200-mesh material

From Eq. (28.11) and using the data in Table 7.1.

$$N_i = \frac{x_i}{2 \times 0.00265\ D_{pi}^3} = 188.7\ \left(\frac{x_i}{D_{pi}^3}\right)$$

From Eq. (28.7), substituting values from Table 28.1 for the 12 increments:

$\Sigma N_i \bar{D}_{pi} = 303.60$ $\qquad$ $N_T = 1659$

$\bar{D}_N = 0.183$ mm

The volume mean diameter, from substitutions in Eq. (28.9), is 0.484 mm, larger than D_N.

The arithmetic mean diameter (or number mean) is calculated by multiplying the number of particles in each size group by the average size $\bar{D}_{pi}$ for that group, and dividing the sum of these terms by the total number of particles. The volume mean is found from a similar equation which uses $\bar{D}^3_{pi}$ rather than $\overline{D_{pi}}$ as the weighting factor in the summation. Dividng by the total number of particles gives an average value of D^3_p, and the cube root gives $\bar{D}_V$. Thus the very small particles, which are in enormous numbers, affect the volume mean less than they do the number mean, since the number mean gives equal weight to each particle in the entire mass.

28.2. The logarithmic probability plot has two linear branches: one for sizes coarser than 14 mesh, the other for sizes smaller than 20 mesh. There is much more fine material of any given size in the sample than would be predicted from the distribution of the +14-mesh material.

28.3. Assume all the energy delivered to the mass appears as heat.

Volume of batch: $\dfrac{1800 \times 7.48}{70} = 192$ gal

Power load: $6000 \times 192/1000 = 1152$ hp

1 hp = 2545 Btu/h

Heat to be removed:

$1152 \times 2545 = 2.93 \times 10^6$ Btu/h

The specific heat of water is 1.0 Btu/lb-°F, and the cooling water required is

$$\frac{2.93 \times 10^6}{15 \times 1.0} = 195{,}300 \text{ lb/h}$$

or $195{,}300/(60 \times 8.33) = 391$ gal/min

28.4. Use Eqs. (28.12) and (28.13). From the data in Table 28.3:

	Run 1	Run 2
Σx_i	24.15	24.84
N	50	50
$\bar{x}$	0.4830	0.4968
$\Sigma(x_i - \bar{x})^2$	1.80945	0.50049

$$\text{Assume } \mu_p = \frac{0.4830 + 0.4968}{2} = 0.4899$$

(a) From Eq. (28.12),

Run 1: $s = (1.89045/49)^{1/2} = 0.19217$

Run 2: $s = (0.50049/49)^{1/2} = 0.10106$

From Eq. (28.13), for both runs,

$$\sigma_e = \left(\frac{0.4899\,(1 - 0.48.99)}{100}\right)^{1/2} = 0.04999$$

(b) Run 2: $\dfrac{\sigma_e}{s} = \dfrac{0.04999}{0.10106} = 0.497$ or 49.7%

28.5. NOTE: "power necessary per ton" should read "work necessary per ton."

(a) Use Eq. (28.15). From Tables 28.1 and 28.4:

D_i, mm	x_i (1)	x_i (2)
4.013	0.031	
2.845	.103	.033
2.007	.200	.082
1.409	.186	.112
1.001	.152	.123
.711	.120	.130
.503	.095	.195
.356	.065	.135
.252	.043	.085
.178		.062
.126		.040

[redacted]s 2 in. or 50.8 mm.
[redacted]i vs D_{pi} (the
[redacted]sh sizes):
[redacted]sses 2.06 mm
[redacted]sses 1.24 mm
[redacted]e 28.2)

[redacted] $9.32\left(\frac{1}{\sqrt{2.06}} - \frac{1}{\sqrt{50.8}}\right)$
[redacted]ton

[redacted] $9.32\left(\frac{1}{\sqrt{1.24}} - \frac{1}{\sqrt{50.8}}\right)$
[redacted]ton

29.6. Find the capacity of the filter press, assuming no washing. From Problem 29.4:

Filtration time:	35.9/60 =	0.598 h
Allow 10 min for cleaning:		.167 h
Total cycle:		0.765 h

Capacity: 742/0.765 = 970 lb/h = $\dot{m}_c$

Area of continuous filter. Use Eq. (29.31).

Δp = 0.7 x 14.696 x 144 = 1482 lb_f/ft^2

f = 0.25 c = 11.0 lb/ft^3 (see final table, Problem 29.3)

α = 10 x 10^{10} (by interpolation)

R_m = 1.5 x 10^{10}

μ = 6.6 x 10^{-4} x 3600 = 2.38 lb/ft-h

n = 2 x 60 = 120 rev/h

From Eq. (29.31):

$$A_T = \frac{\dot{m}_c \alpha}{\left[\dfrac{2\, c\alpha(\Delta p)\, g_c f n}{\mu} + \left(nR_m\right)^2\right]^{1/2} - nR_m}$$

$\dot{m}_c\alpha$ = 970 x 10 x 10^{10} = 9.7 x 10^{13}

$$\frac{2\, c\alpha\,(\Delta p)\, g_c f n}{\mu} = \frac{2 \times 11.0 \times 10 \times 10^{10} \times 1482 \times 4.17 \times 10^{8} \times 0.25 \times 120}{2.38}$$

$= 1.71 \times 10^{25}$

$nR_m = 120 \times 1.5 \times 10^{10} = 1.8 \times 10^{12}$

$$A_T = \frac{9.7 \times 10^{13}}{[1.71 \times 10^{25} + (1.8 \times 10^{12})^2]^{1/2} - 1.8 \times 10^{12}}$$

$= 35.8\ ft^2$

A filter 3.5 ft in diameter with a 3.5-ft face would be adequate.

29.7. Δp = 144 x 70 = 10080 lb_f/ft^2

α = 8.8 x 10^{10}[1 + 3.36 x 10^{-4} x $10080^{0.86}$]

= 1.70 x 10^{11} ft/lb

c = 3 lb/ft^3 R_m = 1.2 x 10^{10} ft^{-1}

$\mu = 6.6 \times 10^{-4}$ lb/ft-s $\qquad t = 3600$ s

$V = 1400/7.48 = 187.2$ ft^3

From Eq. (29.21),

$$\frac{1}{q_o} = \frac{6.6 \times 10^{-4} \times 1.2 \times 10^{10}}{10080 \times 32.174 \times A}$$

$$= 24.42/A$$

From Eq. (29.23),

$$K_c = \frac{6.6 \times 10^{-4} \times 3 \times 1.70 \times 10^{11}}{10080 \times 32.174 \times A^2}$$

$$= 1037.9/A^2$$

Substituting in Eq. (29.24):

$$t = 3600 = \left[\frac{1037.9}{2A^2} \times 187.2^2 + \frac{187.2 \times 24.42}{A}\right]$$

From this

$$A^2 - 1.2698A - 5051.7 = 0$$

$$A = 71.7 \text{ ft}^2$$

29.8. Find the reciprocal of the final filtration rate from Eq. (29.20).

$V/A = 187.2/71.7 = 2.61$ ft^3/ft^2

$$\frac{dt}{dV} = \frac{6.6 \times 10^{-4}}{71.7 \times 32.174 \times 10080} \times (1.70 \times 10^{11} \times 3 \times 2.61 + 1.2 \times 10^{10})$$

$$= 38.1 \text{ s/ft}^3$$

In a tank filter the wash goes through the entire cake and all the area (as does the filtrate):

Washing rate: $0.85/38.1 = 0.0223$ ft^3/s

Washing time: $187.2/(3 \times 0.0223) = 2798$ s or 0.777 h

29.9. Basis: 1 ft^2 of area From Eq. (29.16):

$$\Delta p_m = \mu u R_m / g_c$$

For the constant-rate period:

$$u = 0.6/(7.48 \times 60) = 1.337 \times 10^{-3} \text{ ft/s}$$

$$\Delta p_m = \frac{6.6 \times 10^{-4} \times 1.337 \times 10^{-3} \times 1.2 \times 10^{10}}{32.174}$$

$$= 329 \text{ lb}_f/\text{ft}^2$$

From Eqs. (29.35) and (29.36),

$$\frac{\Delta p_c}{\alpha} = \frac{\mu c t}{g_c}\left(\frac{V}{At}\right)^2 = \frac{\mu c t u^2}{g_c}$$

From this

$$\frac{d(\Delta p_c)}{\alpha} = \frac{\mu c u^2}{g_c}\,dt$$

Since $\Delta p = (\Delta p_c) + (\Delta p_m) = \Delta p_c + 329$

$$\alpha = 8.8 \times 10^{10}\,[1 + 3.36 \times 10^{-4}\,(\Delta p_c + 329)^{0.86}]$$

Hence, integrating between limits

$$\int_0^{(10080 - 329)} \frac{d(\Delta p_c)}{1 + 3.36 \times 10^{-4}\,(\Delta p_c + 329)^{0.86}} = \frac{8.8 \times 10^{10} \times 6.6 \times 10^{-4} \times 3 \times (1.337 \times 10^{-3})^2}{32.174} \int_0^{t_c} dt$$

$= 9.68\ t_c$, where t_c is the time in the constant-rate period. By numerical integration, the left-hand side equals 6616 and

$$t_c = 6616/9.68 = 683.5 \text{ s}$$

Total flow during constant-rate period:

$$V_c = 1.337 \times 10^{-3} \times 683.5 = 0.914 \text{ ft}^3$$

At the end of the constant-rate period, from Eq. (29.20),

$$\frac{dt}{dV} = \frac{\mu}{Ag_c(\Delta p)}\left(\frac{\alpha c V_c}{A} + R_m\right) = \frac{1}{q_{o,2}}$$

For the constant-pressure period, the "septum resistance" (considering the cake deposited during the constant-rate period to be part of the septum) is $(\alpha c V_c/A + R_m)$, and $1/q_o = 1/q_{o,2}$.

Integrating Eq. (29.22) between the limits t_c, V_c and t, V gives

$$t = t_c + \frac{K_c}{2}(V^2 - V_c^{\,2}) + \frac{1}{q_{o,2}}(V - V_c)$$

$$V = 1400/(7.48 \times 71.7) = 2.61$$

$$V - V_c = 2.61 - 0.914 = 1.696 \text{ ft}^3/\text{ft}^2$$

$$\Delta p = 70 \times 144 = 10080 \text{ lb}_f/\text{ft}^2$$

$$\alpha = 8.8 \times 10^{10}[1 + 3.36 \times 10^{-4} \times 10080^{0.86}]$$

$$= 1.70 \times 10^{11} \text{ ft/lb}$$

From Eqs. (29.21) and (29.23),

$$\frac{1}{q_{o,2}} = \frac{6.6 \times 10^{-4}}{1^2 \times 10080 \times 32.174} \times$$

$$(1.70 \times 10^{11} \times 3 \times 0.914 + 1.2 \times 10^{10})$$

$$= 973$$

$$K_c = \frac{6.6 \times 10^{-4} \times 3 \times 1.70 \times 10^{11}}{1^2 \times 10080 \times 32.174}$$

$$= 1037.9$$

$$t = [683.5 + (1037.9(2.61^2 - 0.914^2))/2]$$

$$+ (973 \times 1.696)$$

$$= 5435 \text{ s or } 1.51 \text{ h}$$

29.10 Use Eq. (29.31):

$$p = 50 \times 144 = 7200 \text{ lb}_f/\text{ft}^2$$

$$= 8.8 \times 10^{10}[1 + 3.36 \times 10^{-4} \times 7200^{0.86}]$$

$$= 1.49 \times 10^{11} \text{ ft/lb}$$

Volumetric rate

$V/t = 1400/(3600 \times 7.48) = 0.0520 \text{ ft}^3/\text{s}$

From Eq. (29.30),

$\dot{m}_c = 3 \times 0.0520 = 0.156 \text{ lb/s}$

$n = 1/(60 \times 3) = 0.00556 \text{ rev/s}$

<u>For use in Eq. (29.31):</u>

$$2c\alpha\Delta p g_c f n/\mu = \frac{2 \times 3 \times 1.49 \times 10^{11} \times 7200 \times 32.174 \times 0.5 \times 0.00556}{6.6 \times 10^{-4}}$$

$$= 8.72 \times 10^{17}$$

$nR_m = 0.00556 \times 1.2 \times 10^{10} = 6.67 \times 10^7$

Substituting this in Eq. (29.31),

$$A_T = \frac{0.156 \times 1.49 \times 10^{11}}{[8.72 \times 10^{17} + (6.67 \times 10^7)^2]^{1/2} - 6.67 \times 10^7}$$

$$= 26.7 \text{ ft}^2$$

29.11. Assume $u_{tan} = u = 18$ m/s.

$r = 600/2 = 300$ mm or 0.3 m

$g = 9.80665 \text{ m/s}^2$

(a) The separation factor, from Eq. (29.72), is

$F_c/F_g = 18^2/(0.3 \times 9.80665) = 110$

(b) $D_p = 20\ \mu\text{m}$

$\rho_p = 1800/16.0185 = 112.4 \text{ lb/ft}^3$

Diameter of cyclone: 600/25.4 = 23.6 in.

From Fig. 29.34, at $D_p = 20$ µm, the collection efficiency is close to 100% for particles with a density of 155 lb/ft^3. For the less dense particles used here, the collection efficiency would be expected to be 98-99%. Essentially all particles larger than 20 µm should be removed; the fraction of the smaller particles removed would depend on the particle-size distribution.

29.12. $D_p = 20 \times 10^{-6}/0.3048 = 6.56 \times 10^{-5}$ ft

$\mu = 0.018$ cP (at 70°F)

$= 0.018 \times 6.72 \times 10^{-4} = 1.21 \times 10^{-5}$ lb/ft-s

$$\rho = \frac{29}{359} \times \frac{492}{530} = 0.075 \text{ lb/ft}^3$$

$\rho_p = 112.4$ lb/ft^3

Settling regime, from Eq. (7.45)

$$K = 6.56 \times 10^{-5} \left[\frac{32.174 \times 0.075(112.4 \quad 0.075)}{(1.21 \times 10^{-5})^2}\right]^{1/3}$$

= 0.805 (Stokes' Law)

From Eq.(7.40)

$$u_t = \frac{32.174 \times (6.56 \times 10^{-5})^2(112.4 - 0.075)}{18 \times 1.21 \times 10^{-5}}$$

$= 0.0714$ ft/s

$u_o = 8 \times 3.2808 = 26.2$ ft/s

$D_b = 25/(25.4 \times 12) = 0.082$ ft

Separation number, for use in **Fig. 29.19**,

$u_t u_o / gD_b = 0.0714 \times 26.2/(32.174 \times 0.082)$

$= 0.709$

Target efficiency, from Fig. **29.19**,

$\eta_t = 0.74$

The overall collection efficiency of the first row, since the ribbons cover only 50% of the area, is 74/2 or 37%.

29.13. $\omega = 2\pi \times 1200/60 = 125.7$ radians/s

$\rho_p = 1.6 \times 998 = 1597$ kg/m^3

$\rho = 1.2 \times 998 = 1198$ kg/m^3

$\mu = 2 \times 10^{-3}$ kg/m-s $\qquad r_2 = 0.3$ m

$D_{pc} = 3 \times 10^{-5}$ m

Settling regime, from Eq. (7.45) with $r_2\omega^2$ in place of g and D_{pc} in place of D:

$\rho_p - \rho = 1597 - 1198 = 399$ kg/m^3

$$r_2\omega^2 = 0.3 \times 125.7^2 = 4740 \text{ m/s}^2 \text{ at wall}$$

$$K = 3 \times 10^{-5}\left[\frac{4740 \times 1198 \times 399}{2^2 \times 10^{-6}}\right]^{1/3}$$

$= 2.48$ (this is in the range for Stokes' Law; see p. 167)

Can therefore use Eq. (29.76).

$$b = 0.4 \text{ m} \qquad r_1 = 0.3 - 0.075 = 0.225 \text{ m}$$

From Eq. (29.76),

$$q_c = \frac{\pi \times 0.4 \times 125.7^2 \times 399}{18 \times 2 \times 10^{-3}} \times \frac{(3 \times 10^{-5})^2 \times (0.3^2 - 0.225^2)}{\ln [2 \times 0.3/(0.3 + 0.225)]}$$

$= 0.0584 \text{ m}^3/\text{s}$ or $210 \text{ m}^3/\text{h}$

29.14. Use Eq. (29.44).

$$\rho = 998 \text{ kg/m}^3$$

$$\mu = 0.9 \text{ cP} = 9 \times 10^{-4} \text{ kg/m-s}$$

$$\omega = 2\pi \times 2000/60 = 209.4 \text{ radians/s}$$

$$\alpha = 9.5 \times 10^{10} \times 2.2045/3.2808 = 6.38 \times 10^{10} \text{ m/kg}$$

$$b = 45 \text{ mm} = 0.45 \text{ m}$$

$$r_2 = 0.375 \text{ m} \qquad r_1 = 0.200 \text{ m}$$

$$r_i = (0.75/2) - 0.15 = 0.225 \text{ m}$$

$$R_m = 2.6 \times 10^{10} \text{ ft}^{-1} = 8.53 \times 10^{10} \text{ m}^{-1}$$

$$m_c = 0.45\pi(0.375^2 - 0.225^2) \times 2000 \times (1 - 0.435) = 143.8 \text{ kg}$$

From Eqs. (29.45) and (29.46)

$$\overline{A}_a = (0.225 + 0.375) \times 0.45\pi = 0.848 \text{ m}^2$$

$$\overline{A}_L = \frac{2\pi \times 0.45 \times (0.375 - 0.225)}{\ln 0.375/0.225}$$

$$= 0.830 \text{ m}^2$$

Also, $A_2 = 0.45 \times 2 \times 0.375\pi = 1.060 \ m^2$

From Eq. (29.44),

$$q = \frac{998 \times 209.4^2}{2 \times 9 \times 10^{-4}} \times \frac{(0.375^2 - 0.225^2)}{\dfrac{6.38 \times 10^{10} \times 143.8}{0.848 \times 0.83} + \dfrac{8.53 \times 10^{10}}{1.06}}$$

$$= 1.67 \times 10^{-5} \ m^3/s \quad \text{or} \quad 0.601 \ m^3/h$$

29.15.(a) Assume complete rejection of latex particles, and use Eq. (29.52) to find k_c from the slope of the semi-log plot. From Fig. 25.25,

$$\text{Slope} = \frac{270}{\ln 55/1} = 67.4 \ \text{gal/ft}^2\text{,day}$$

Convert to cm/s using Table 29.3.

$$k_c = 67.4 \times 4.72 \times 10^{-7} \times 100 = 3.18 \times 10^{-3} \ \text{cm/s}$$

NOTE: This is higher than the value of k_c for whey protein in Example 29.3, even though the latex particles are much larger.

(b) To determine an effective diffusivity for latex, compare the value of k_c with that for albumin. From Fig. 29.25 for albumin,

$$k_c' = \frac{128}{\ln 24/1} = 40.2 \ \text{gal/ft}^2\text{,day}$$

$$= 1.9 \times 10^{-3} \ \text{cm/s}$$

Assume T = 293K and use Eq. (29.54).

$$r_0 = 3.6 \times 10^{-3} \ \mu\text{m} = 3.6 \times 10^{-7} \ \text{cm}$$

$$D_v' = \frac{7.32 \times 10^{-16} \times 293}{3.6 \times 10^{-7} \times 1.0} = 5.96 \times 10^{-7} \ \text{cm}^2/\text{s}$$

Assume k_c varies with $D_v^{2/3}$ or $D_s^{2/3}$.

$$\left(\frac{D_s}{5.96\times10^{-7}}\right)^{2/3} = \frac{k_c}{k_c'} = \frac{3.18\times10^{-3}}{1.9\times10^{-3}} = 1.67$$

For latex, $D_s = 1.67^{3/2} \times 5.96 \times 10^{-7} = 1.29\times10^{-6}$ cm²/s

$$r_0 = 0.1\times10^{-3}\ \mu m = 10^{-5}\ cm$$

Normal diffusivity,

$$D_v = \frac{7.32\times10^{-16}\times293}{10^{-5}\times1.0} = 2.14\times10^{-8}\ cm^2/s$$

The shear induced diffusivity D_s is about 60 times the molecular diffusivity D_v.

(c) Use Eq. (29.65). $D_s = 0.03\gamma r^2$.

$$\gamma = \frac{1.29\times10^{-6}}{0.03\times(10^{-5})^2} = 4.3\times10^5\ s^{-1}$$

29.16. At $\bar{V}$ = 300 cm/s, ρ = 1 g/cm³, μ = 0.01 g/cm-s

Re = 2 x 300 x 1/0.01 = 60,000

Sc = 25,000, as in Example 29.3.

Sh = 0.0096 x $60000^{0.913}$ x $25000^{0.346}$ = 7352

$$k_c = \frac{7352\times4\times10^{-7}}{2} = 1.47\times10^{-3}\ cm/s$$

Choose c_1 = 20 g/L and various flow rates.

$$\ln\frac{c_s}{c_1} = \frac{v}{k_c} = \frac{1.5\times10^{-3}}{1.47\times10^{-3}} = 1.02$$

$c_s = 2.774c_1$ = 55.5 g/L

$$\Delta\pi = \pi = 4.4\times10^{-3}(55.5) - 1.7\times10^{-6}(55.5)^2 + 7.9\times10^{-8}(55.5)^3$$

= 0.252 atm

$Q_m = 6.94\times10^{-3}$ cm³/s-cm²-atm

From Eq. (29.49),

$$\Delta p - \Delta\pi = \frac{v}{Q_m} = \frac{1.5\times10^{-3}}{6.94\times10^{-3}} = 0.216\ atm$$

$$\Delta p = 0.216 + 0.252 = 0.468\ atm$$

Similar calculations give the following values:

$v \times 10^3$, cm/s	Flux, L/m^2-h	Δp, atm
1.5	54	0.47
2.0	72	0.66
2.5	90	0.93
3.0	108	1.36
3.5	126	2.17
4.0	144	3.98

The maximum flux, from Eq. (29.52), is

$$v_{max} = 1.47 \times 10^{-3} \ln \frac{400}{20} = 4.4 \times 10^{-3} \text{cm/s} = 159 \text{ L/m}^2\text{-h}$$

At Re = 60,000, from Fig. 5.10 for smooth pipe, f = 0.005. With $\bar{V}$ = 3 m/s, r = 1000 kg/m^3, using Eq. (5.9),

$$\Delta p_s = \frac{2(0.005)(1000)(3)^2}{0.02} \times 1.5 = 6750 \text{ Pa} = 0.98 \text{ lb}_f/\text{in.}^2$$

29.17. Flux = 150 L/m^2-h or $v = 4.17 \times 10^{-5}$ m/s

$$\Delta p = 2.1 \text{ atm} = \Delta p_{gel} + \Delta p_{membrane}$$

$$\Delta p_{membrane} = 2.1 \times \frac{150}{280} = 1.125 \text{ atm}$$

$$\Delta p_{gel} = 2.1 - 1.125 = 0.975 \text{ atm} = 9.88 \times 10^4 \text{ Pa}$$

Use Eq. (7.17) with $\varepsilon = 0.40$.

At 30°C, μ = 0.801 cP.

$$\frac{\Delta p}{L} = \frac{150 \times (4.17 \times 10^{-5}) \times (0.801 \times 10^{-3}) \times 0.6^2}{(0.8 \times 10^{-6})^2 \times 0.4^3}$$

$$= 4.4 \times 10^7 \text{ Pa/m}$$

$$L = \frac{9.88 \times 10^4}{4.4 \times 10^7} = 2.25 \times 10^{-3} \text{ m} = 2.25 \text{ mm}$$

The pressure drop is quite sensitive to the value of ε. If $\varepsilon = 0.35$, $\Delta p/L = 7.7 \times 10^7$ Pa/m, and L = 1.28 mm.