PLANT DESIGN AND ECONOMICS FOR CHEMICAL ENGINEERS

Solutions Manual

McGRAW-HILL INTERNATIONAL EDITIONS
Chemical And Petroleum Engineering Series

FOURTH EDITION

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CHAPTER 02

PROCESS DESIGN DEVELOPMENT

Pb# 01

The production of formaldehyde still relies on the chemical process developed nearly 50 years ago. Nearly 70% of new installations in the US utilize a metal oxide catalyst to form formaldehyde by the oxidation of methanol

$$CH_3OH + \frac{1}{2} O_2 \rightarrow HCHO$$

Process 1

In this process shown above, vaporized methanol is mixed with air and recycle tail gas and passed through catalyst filled tubes in a heat exchanger reactor. Heat released by the exothermic reaction is removed by vaporization of a high-boiling heat transfer fluid on the outside of the tubes. Steam is normally produced by condensing the heat transfer fluid. Product leaving the bottom of the reactor is cooled and then transferred to the base of an absorber. Formaldehyde concentration in the product is adjusted by controlling the amount of water added to the top of the absorber. A product of 1 to 55% formaldehyde can be produced. Byproducts in the reaction include CO, CO_2 , dimethylether, and formic acid. The latter is removed by ion exchange. Plant yields between 88 – 92% are reported.

Reference: "Kirk-Othmer Encyclopedia of Chemical Technology", Vol. II, p. 929, J. Wiley, New York, 1994.

Process 2

A feed mixture is generated by sparging air into a pool of heated methanol and combining the vapor with steam. The mixture passes through a superheater to a catalyst bed of silver crystals or layers of silver gauze. The reactor product is rapidly cooled in a steam generator followed by a water-cooled heat exchange and then fed to the bottom of an absorption tower. Most of the methanol, water, and formaldehyde is condensed in the water-cooled bottom section of the absorber. Almost complete removal of the remaining methanol and formaldehyde from the tail gas occurs at the top of the tower by countercurrent contact with the process water. Absorber bottoms go to a distillation column where methanol is recovered for recycle to the reactor. The bottoms stream from the distillation tower is usually sent to an anion exchanger which reduces the formic acid to specifications. The product contains up to 55% formaldehyde and less than 1.5% methanol.

Reference: J.H. Martin and M.T. Butter, *Oil & Gas J.* **72**, 71(Mar. 11, 1974).

Process 3

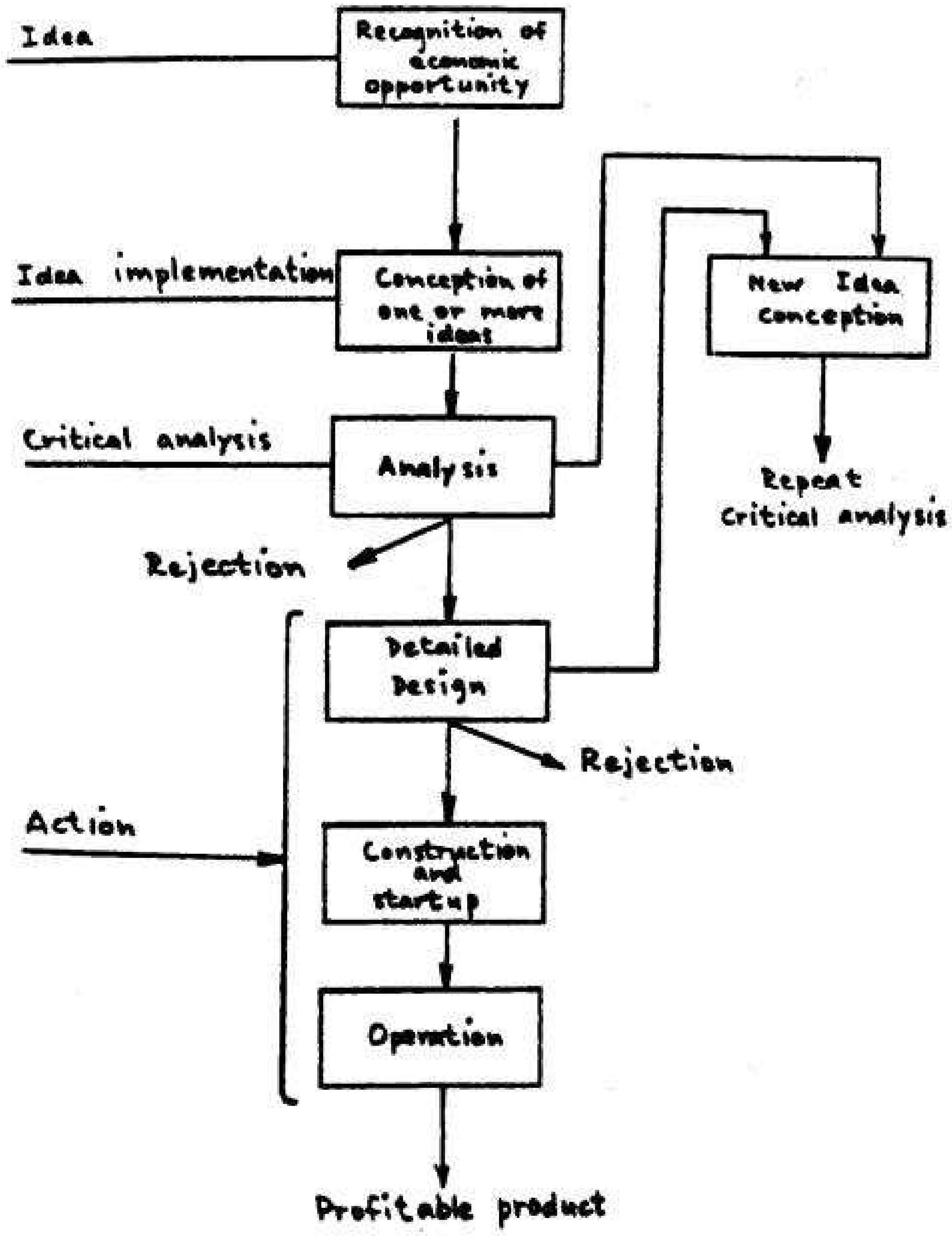
Another process to produce formaldehyde involves the partial oxidation of methane. The incentive for such a process is the reduction of raw material costs by avoiding the capital and operating expense of initially producing the methanol from methane. In this process methane gas is mixed with excess air, preheated to around 600°C and fed into a vertical, tubular reactor with a conical bottom, filled with ceramic packing saturated with a $K_2B_4O_2$ solution. The conversion of methane depends on the temperature of the feed mixture and its content of nitrogen oxides. Exit gases are rapidly cooled to 400°C in a tubular shell condenser with water. The gas is further cooled to approximately 33°C in a heat exchanger and transferred to a scrubber where formaldehyde is absorbed in a dilute solution of formaldehyde solution to provide formaldehyde of the desired concentration. Since energy requirements are quite high, the processing plant is best coupled with a cheap source of power.

Reference: M. Brown and N. Parkyns, *Catal. Today* **5**(3), 305 (1991).

J. Hargreaves, G.Hutchings, and R. Joyner, *Nature* **348**, 28 (Nov. 1990).

Pb# 02

A general outline showing the sequence of steps in the development of a formaldehyde plant is shown below:



Recognition of economic opportunity – this can come from a variety of sources. Technical services may recognize a customer's need in a resin or adhesive. Market research may uncover another use of formaldehyde as a feed product or intermediary requirement in a process.

Conception of one of more ideas – again this can come from a variety of sources, namely, the chemist, process engineer, plant operator, technical service personnel, maintenance crew, etc. The idea can be one of an entirely new process like the utilization of natural gas for a raw material (see Problem 3-1) rather than methanol. On the other hand, the idea can involve a change in operating conditions and arrangement of equipment while using the same raw materials as demonstrated by the Formox process developed by Reichhold Chemicals (see *Hydrocarbon Process.*, **44**(11), 215 (1965) or the Montecatini process (see *Hydrocarbon Process.*, **44**(11), 216 (1965).

Analysis – this step requires the greatest technical interaction on the part of the design engineer. For the production of formaldehyde this step includes the following considerations:

1. Raw materials (availability, quantity, quality, cost)
2. Thermodynamics and kinetics of the chemical reaction of, for example, the Formox process

   $$2CH_3OH+O_2 \rightarrow 2HCHO+2H_2O$$

   in terms of equilibrium, yields, rates, optimum conditions, etc.
3. Facilities and equipment available and what must be purchased
4. Estimation of production costs and total investment for various alternative designs
5. Profits (probable and optimum per kilogram of formaldehyde as well as annually, return on investment)
6. Materials of construction
7. Safety considerations

8. Markets (present and future supply and demand, present uses, new uses, etc.)
9. Competition (overall production statistics, comparison of various manufacturing processes, product specification, e.g. 37% formaldehyde)
10. Properties of formaldehyde, properties of byproducts, disposal of waste products – this can be rather difficult in the case of the formaldehyde process.
11. Sales and sales service
12. Shipping restrictions, labeling, container selection
13. Plant location
14. Patent situation and legal restrictions

The analysis step may indicate that there is insufficient data which may require additional research. The analysis step involves the development of a preliminary design. This may lead to rejection of the entire project or it may initiate the development of additional new ideas. The latter should undergo further critical analysis to determine whether these, in turn, are feasible and should be pursued further or also rejected.

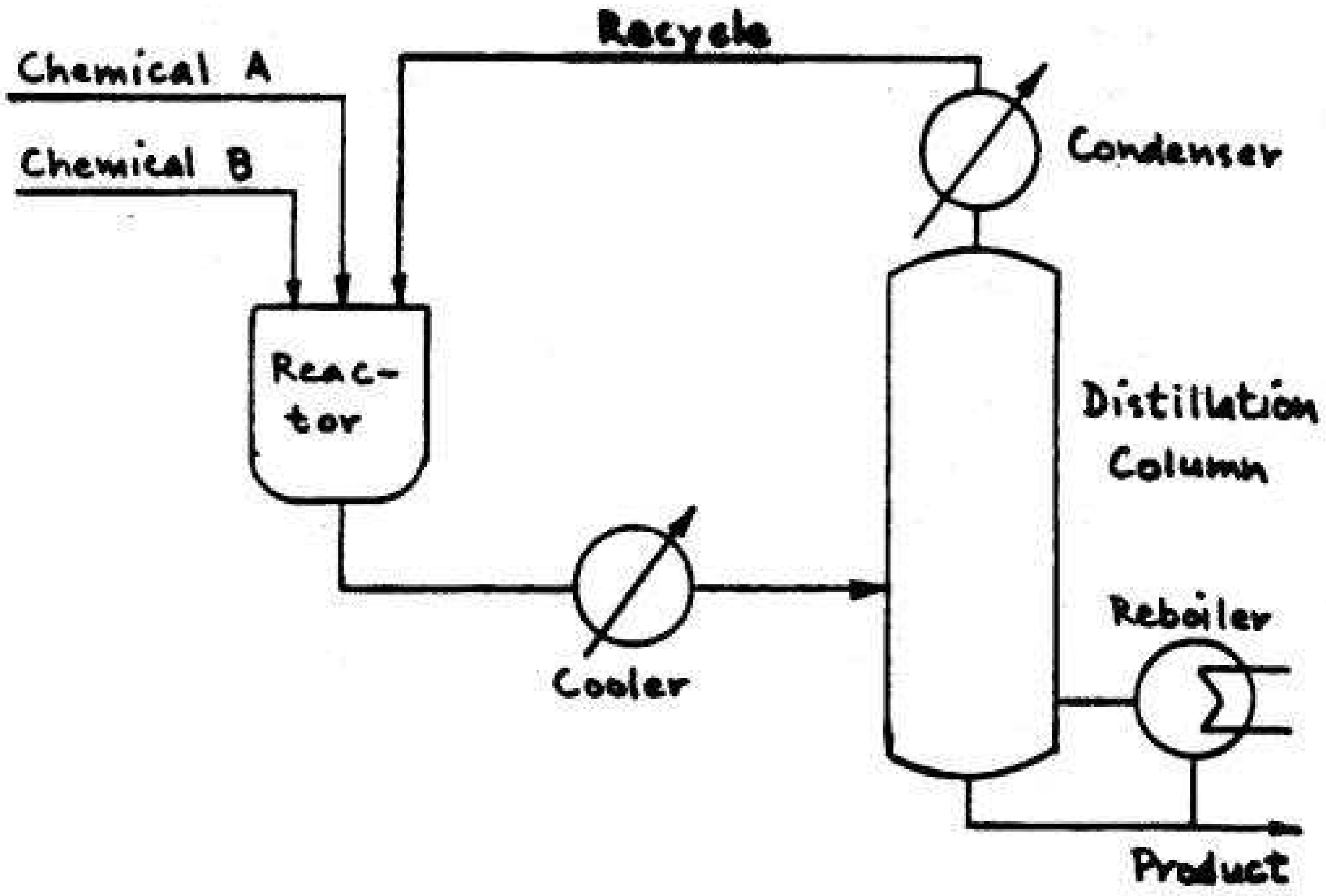
Detailed design – the following factors should be established within narrow limits before a detailed design is performed:

1. Selected process
2. Material and energy balances
3. Temperature and pressure ranges
4. Raw material and product specifications
5. Yields, reaction rates
6. Materials of construction
7. Utilities requirements
8. Plant site

If the detailed design indicates a profitable venture that meets the return criteria set forth by company managers, a recommendation will be made to go ahead with the construction if funds are available.

Construction – the final design should be developed in such a manner that items requiring the longest time for completion can be started first. Careful coordination between the design engineer and the contractor will help minimize construction delays and assist with more orderly plant startup. Detailed operating procedures need to be developed not only for startup procedures, but also for eventual continuous operation. During operation, engineers and operating personnel should be encouraged to continuously monitor the process to see where the process can be improved or simplified.

Pb# 03



Information needed for the design:

Raw materials – chemicals involved, stream capacities, thermal properties of materials, physical properties of materials, temperature, pressure, material concentrations, environmental hazards

Product – some considerations listed for the raw materials above, product purity, build-up of byproducts

Reactor – rate expression, conversion under various operating conditions, heats of reaction, thermal and physical properties of entering and exiting materials, temperature, pressure, stream capacities

Cooler – quenching temperature, vapor-liquid equilibria, heat load, thermal and physical properties

Distillation column – vapor-liquid equilibria, heat of vaporization, thermal and physical properties, stream capacities, temperature, pressure, reflux ratio, plate efficiency, stream capacities

Condenser and reboiler – heat of condensation and vaporization, heat loads, thermal and physical properties, temperature, pressure, heat transfer coefficient, stream capacities

Pb# 05

Acetylene is currently produced by the thermal cracking of hydrocarbon feeds. This thermal cracking requires considerable quantities of energy since the reactions are strongly endothermic. Further dissociation into carbon and hydrogen of all hydrocarbons begins at relatively low temperatures. Thus, the reactions must be allowed to proceed very rapidly and quenched very quickly to stable temperatures.

One method of distinguishing between processes for acetylene production is by the manner in which the energy for cracking is supplied.

1. Regenerative method

This method involves periodic cycles of heating a hot surface of ceramic materials and then cracking the hydrocarbons on these hot surfaces. An example of this method is the Wulff process. High purity acetylene is possible from a feedstock of ethane to gas oil. Byproducts are an aromatic distillate fraction and a hydrogen-rich off gas. The reaction is a vapor phase reaction in the presence of steam. Low conversion between 12 to 32 wt% is a major drawback of this method. Also, startup and shutdown are quite long compared to some of the other processes.

2. Partial oxidation

Acetylene is obtained from the incomplete combustion of methane. Oxygen and natural gas are reacted and about one-third of the entering methane is cracked and the rest is combusted to provide the necessary heat for the cracking process. The yield based on the carbon in the natural gas is usually about 31 wt%. The process generates considerable soot (carbon) that must be removed continuously for good operation. The process is licensed by BASF.

3. Two-step procedure

This process involves the generation of combustion gases at about 1500K and subsequent injection of hydrocarbons into these hot combustion gases. The feedstock normally is naptha cuts to middle distillates. High purity acetylene is obtained with a high yield of 50 to 54 wt% from this feedstock. The process generally is identified as the Hoechst HTP process.

4. Submerged-flame process

In this process the reaction between hydrocarbons and oxygen occurs in a submerged-flame burner where the burners remain submerged in oil during operation. The burner projects a flame into which oil and oxygen are fed. Combustion and cracking of the oil takes place at the boundaries of the flame. The gases are rapidly quenched by the surrounding oil which is continuously cooled. The feed is crude oil rather than natural gas. The process is designed to recycle the soot and combust it in the burner. Product yields are in the same range as those obtained for processes that use the light hydrocarbons for feed stock.

Since the flowsheets for all of these four processes are readily available in the literature along with useful information on the operating details and procedures, they will not be reproduced here. Some of the earlier references to the processes listed above provide much more useful information than is available from later references. Accordingly, these references are listed rather than the more recent references.

1. Regenerative method: *Hydrocarbon Process.* **46**(11), 139 (1967)
2. Partial oxidation: *Hydrocarbon Process.* **48**(11), 142 (1969)
3. Two-step procedure: *Hydrocarbon Process.* **46**(11), 138 (1969)
4. Submerged-flame process: *Chem. Eng.* **76**(3), 82 (1969)

The choice of one process over another is largely dependent upon the projected needs. The availability of feed stocks, the desired product purity, and byproduct marketability will be overriding considerations. As noted above, one basic difference is the form of energy input. The availability of one form of energy over another could make one process more attractive than another. The licensing and fees requirements could also provide advantages of one process over another. The principal design problems involve the chemical reactions and energy requirements.

The latter will be a major part of the operating costs and will be reflected in the sales price. Material and energy balances around the reactor or burner will be necessary. However, complete kinetic data for these complex reactions will probably not be available. In the case of these well-known processes, the information needed would be available from the licensor.

The other area of concern in all of these processes is in the separation reaction. Most require the use of selective solvents. The mass transfer calculations for these solvents would be needed. Again, data would be available from the licensor, though approximations can be made with today's sophisticated computer software.

There are many similarities in the various separation techniques. Thus, comparison of the processes can provide economic information in some cases without making an actual detailed cost estimate.

Pb# 06

Some of the most important and widely employed processes for ethylene production utilize thermal cracking of a hydrocarbon feed stock in the presence of a steam atmosphere. As noted in the preliminary design in Chapter 3, the ethylene process based on thermal cracking consists basically of four distinct process units, namely (1) thermal cracking and quenching, (2) compression and acid gas removal, (3) subcooling and product separation, and (4) refrigeration. Selection of a feed stock for an ethylene plant is primarily dependent on feed stock availability, relative feed price, yield structure, plant construction cost, as well as the sale price for the byproducts. The thermal cracking of ethane and propane by the fixed tubular heater is the most common process in the United States, and the thermal cracking of naptha by the fixed tubular heater is the most common process in Europe and Japan.

The capital cost of an ethylene plant increases as the molecular weight of the feed stock increases since the ethylene yield decreases with increasing molecular weight. In theory, an ethylene plant can be designed to process a full range of feed stocks from ethane to gas oil. However, in practice, complete feed stock flexibility to cover so broad a range is neither technically practical nor economically attractive because of various constraints in equipment design.

Distribution of product yields depends on the type of feed stock, hydrocarbon partial pressure, residence time, as well as cracking severity or coil outlet temperature. Typical yield distribution patterns for a variety of feedstocks is shown in the table below. The cracked gas recovery section in an ethylene plant is complex and allows a variety of possible flow schemes. In the separation section it is necessary to separate hydrogen and methane fractions, high-purity ethylene and propane, ethane and propane fractions, C_4s and gasoline. The design must provide for removal of acetylene and methyl acetylene/propadiene (MAPD). For the separation process, there are maximum temperature limits to prevent polymerization fouling, and minimum temperature limits to prevent hydrocarbon freezing or hydrate formation. Within the separation process, there are several possible variations. Acetylene can be removed from the cracked gas stream in the

Typical pyrolysis yields of ethylene plant feedstocks (in mass percent).

Feedstock	Ethane	Ethane	Propane	Propane	n-Butane	Isobutane	n-Pentane	Isopentane	Naphtha	Naphtha	Kerosene	Gas Oil	Gas Oil (Vac)
Conversion%	50	60	75	90	95	92	95	95					
Severity									Medium	High	High	High	High
Comp. Yield													
H_2	3.06	3.55	1.08	1.29	1.0	1.39	0.8	0.9	0.63	0.8	0.63	0.65	0.47
CH_4	2.6	4.2	18.83	24.7	21.8	23.6	17.52	20.6	11.7	15.3	11.4	10.6	7.9
C_2H_2	0.12	0.25	0.17	0.33	0.4	0.46	0.55	0.42	0.3	0.75	0.37	0.35	0.2
C_2H_4	41.65	48.2	25.0	34.5	35.8	10.3	36.5	17.95	24.0	29.8	23.2	24.0	19.5
C_2H_6	50.0	40.0	5.15	4.4	5.1	0.73	5.6	3.2	3.9	3.75	3.4	3.25	2.83
C_3H_4	0.1	0.02	0.22	0.34	0.55	2.46	0.95	1.7	0.76	1.1	0.85	1.0	0.38
C_3H_6	0.89	1.11	18.86	13.96	16.4	21.2	19.8	19.0	15.75	14.1	13.8	14.45	11.3
C_3H_8	0.14	0.17	25.0	10.0	0.15	0.33	0.55	0.55	0.41	0.25	0.45	0.45	0.36
C_4H_6	0.5	1.07	1.05	2.65	3.4	1.94	3.7	4.6	4.6	4.85	4.4	4.7	5.5
C_4H_8	0.25	0.21	1.28	1.0	1.7	16.7	2.6	14.38	6.85	4.2	4.0	4.5	4.88
C_4H_{10}	0.35	0.27	0.07	0.05	5.0	8.0	0.1	0.1	1.0	0.25	0.1	0.1	0.09
C_5s	0.2	0.27	1.43	1.81	1.65	2.12	6.1	7.5	5.0	2.3	3.0	3.3	5.0
C_6H_6	0.2	0.48	1.09	2.2	2.58	3.42	1.8	4.1	3.15	6.3	7.2	5.8	4.16
C_7H_8	0.03	0.06	0.33	0.48	1.12	1.51	0.6	1.25	4.4	4.9	4.0	3.15	2.5
C_8H_8					0.45	0.67	0.1	0.25	2.65	2.0	2.2	1.2	1.28
C_8H_{10}							0.06	0.15	0.52	0.7	0.75	0.7	0.5
C_6-C_8 PON*		0.14	0.14	1.44	1.2	3.18	0.1	0.1	8.83	2.0	1.9	1.8	2.0
C_9-200^0 C							1.48	1.75	3.75	2.8	4.85	2.4	2.0
Fuel oil			0.3	0.85	1.7	1.99	1.09	1.5	1.8	3.8	13.5	17.6	29.15

* PON (Paraffin, Olefin, Naphthene).

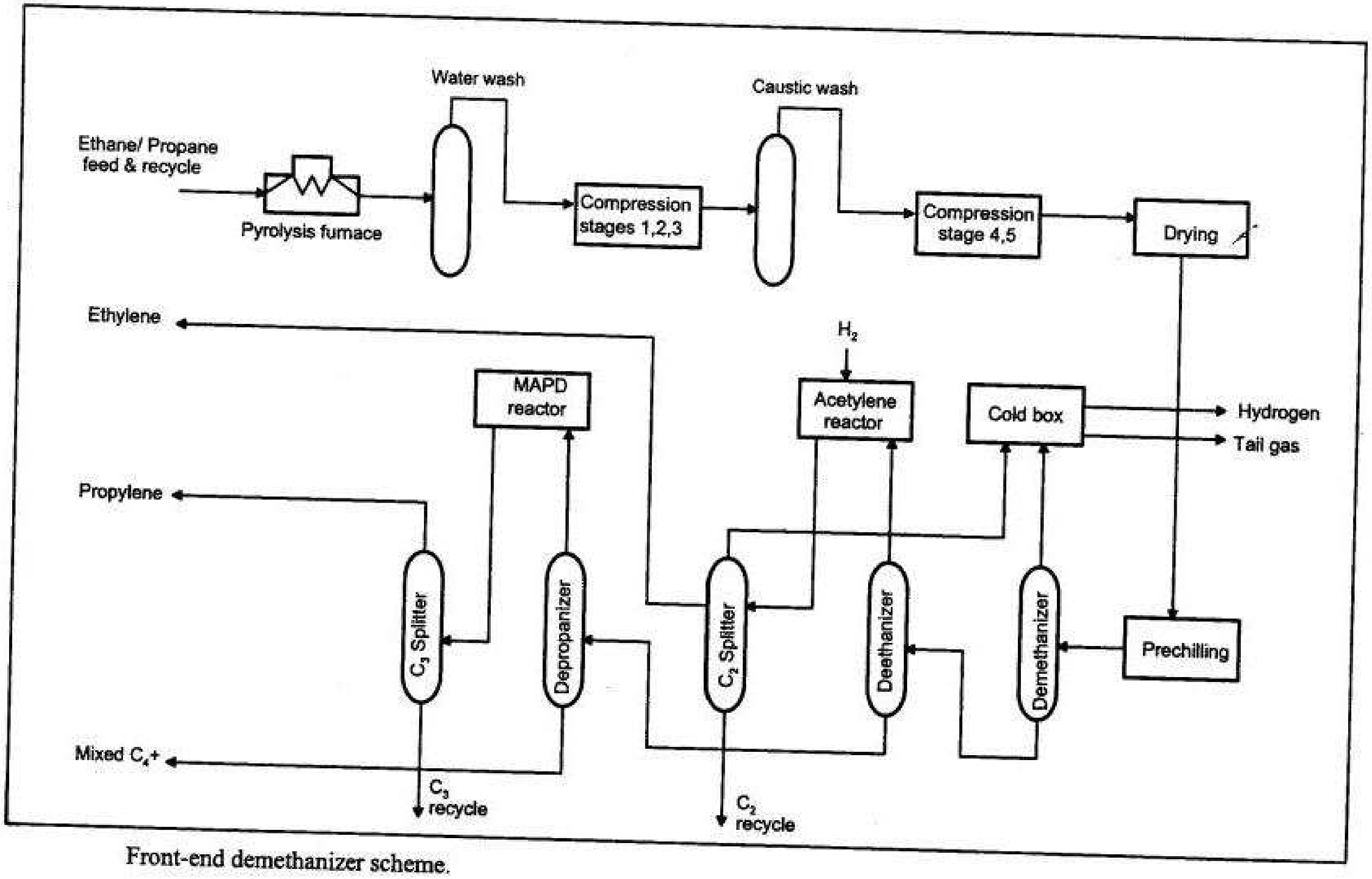
presence of the hydrogen produced in the furnaces or from a concentrated demethanized and depropanized C_2 cut. Such process arrangements are identified as the front-end demethanizer, the front-end deethanizer, and the front-end deproponizer scheme. They are briefly described below.

Front-end demethanizer

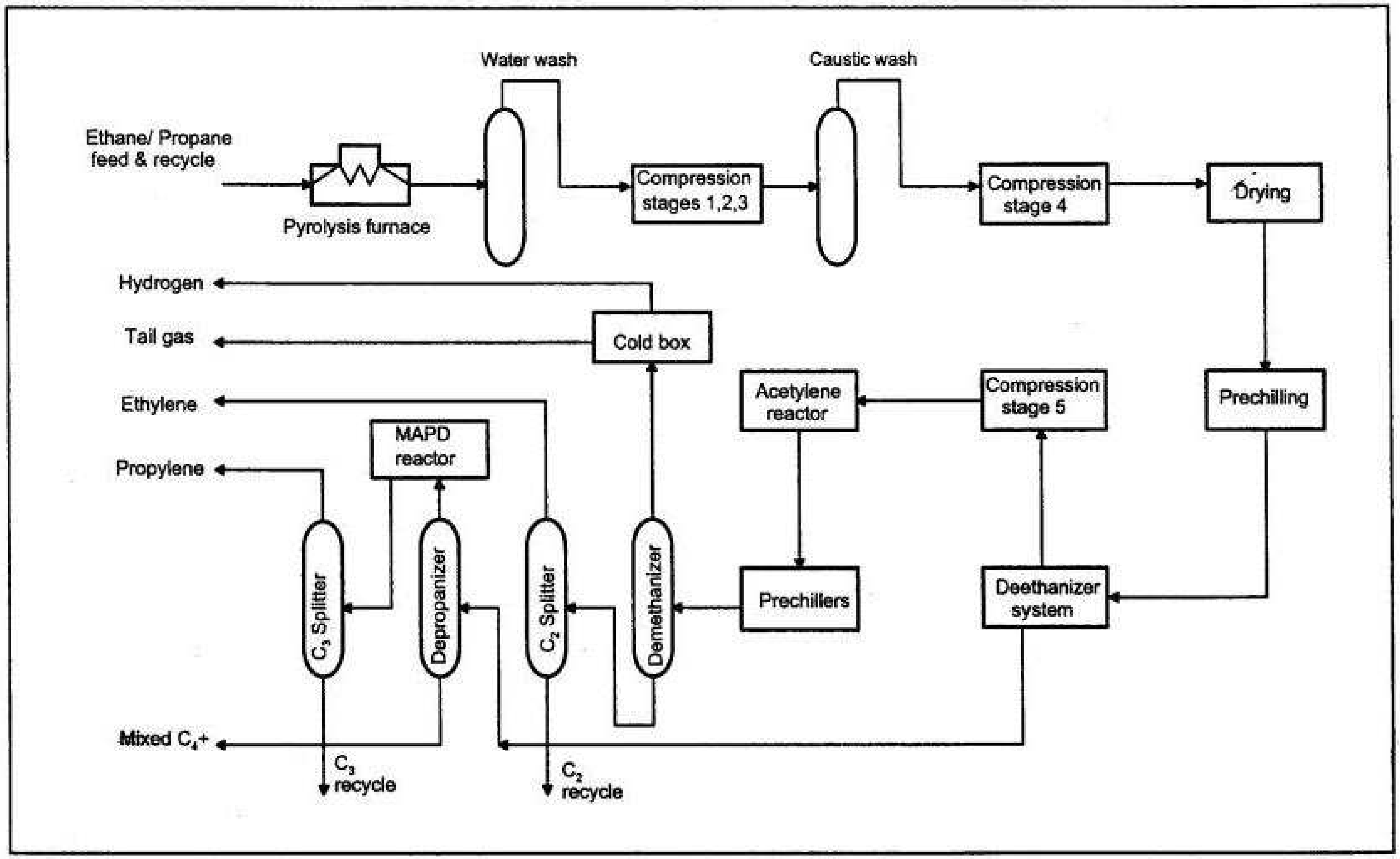
In this scheme the dried cracked gas is chilled and sent to the demethanizer/cold-box system as shown in the attached figure. The demethanizer bottoms are sent to the deethanizer. The overhead from the deethanizer is hydrogenated to remove the acetylene. From the acetylene converter, the ethylene-ethane stream passes to the C_2 splitter. The deethanizer bottoms are sent to a depropanizer which separates the C_3 components from the heavier C_4 components. The overhead from the depropanizer is fed to a methyl acetylene/propadiene hydrogenation reactor. A C_3 splitter separator separates the propylene from propane. The bottoms stream from the depropanizer passes to a debutanizer where the C_4 fraction is separated as an overhead from the C_5 and higher components.

Front-end deethanizer

As noted in the figure showing the front-end deethanizer, the cracked gas is dried and chilled before it is sent to the deethanization section. Overhead from the deethanization section is compressed and sent to a front-end acetylene reactor where acetylene is hydrogenated to ethylene and ethane. Following the acetylene reactor, the C_2 and lighter components are chilled and sent to the demethanizer which separates methane and hydrogen from the heavier components. Hydrogen and methane are separated in the cold-box, while the demethanizer bottoms are fractionated in the C_2 splitter to produce ethylene product and ethane for recycle. The bottoms from the deethanizer are depropanized, and the mixed C_3s are sent to the methyl acetylene/propadiene reactor for hydrogenation. The C_3 reactor effluent goes to a C_3 splitter to produce polymer grade propylene and propane for recycle.



Front-end demethanizer scheme.



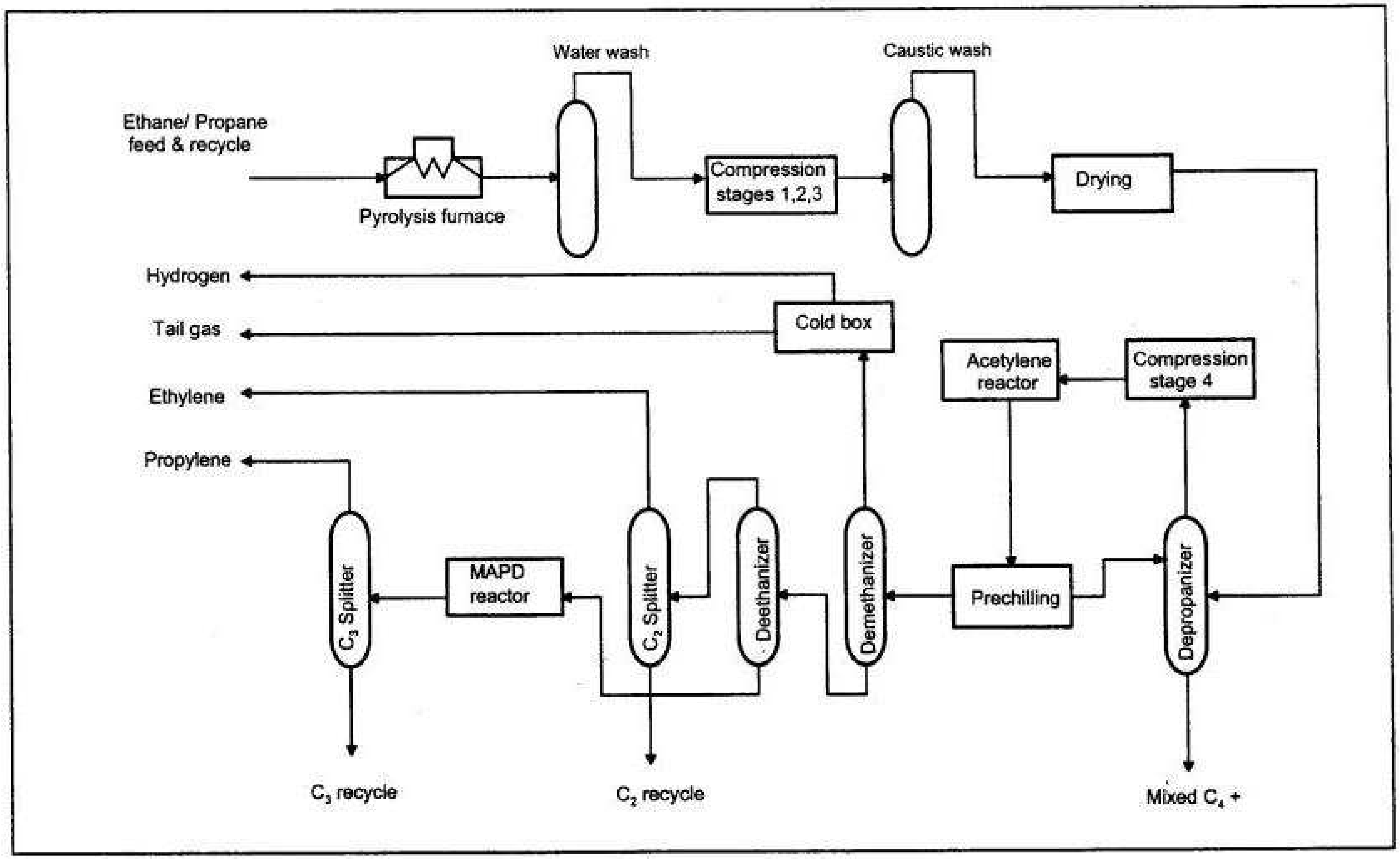
Front-end deethanizer scheme.

Front-end depropanizer

The cracked gas in this scheme shown on the next figure is compressed before being caustic-washed and dried. The dried cracked gas is sent to the depropanizer where the overhead is compressed. It then goes to a front-end acetylene reactor where the acetylene is hydrogenated to ethylene and ethane. After hydrogenation, the gases are partially condensed to provide reflux for the depropanizer. The net overhead liquid and vapor, which contain the C_3 and lighter components, go forward to the demethanizer prechillers. The net C_3 and lighter stream from the depropanizer are fractionated in the demethanizer, with the resulting bottoms stream being further fractionated in a deethanizer to a methyl acetylene/propadiene hydrogenation reactor. The reactor effluent is sent to the C_3 splitter.

In addition to the location of the demethanizer, deethanizer, and depropanizer, the location of the reactor for hydrogenating the C_2 and C_3 acetylenes is dictated by the scheme selected. In both the front-end deethanization and front-end depropanizer separation schemes, the C_2 acetylene hydrogenation converters are located ahead of the cracked gas chilling train and demethanizer. By contrast, in the front-end demethanization scheme, the acetylene converter follows the demethanizer and deethanizer. In this scheme, the hydrogen in the cracked gas has already been removed in the gas chilling train and demethanizer and an external addition of hydrogen is necessary in the acetylene converter. However, hydrogenation of the C_3 acetylenes in all schemes requires an external addition of hydrogen.

Separation of the ethylene-ethane mixture also provides several choices. This separation is commonly achieved in a high-pressure column, utilizing close-cycle propylene refrigeration. This process can also be carried out by low-pressure fractionation, using open-cycle ethylene refrigeration. Alternatively, the selection can also be achieved with absorption using highly selective solvents. The latter separation technique is seldom used. However, the low-pressure fractionation has received some support since the relative volatility between ethylene and ethane is higher and the separation is easier, resulting in a lower reflux ratio and fewer trays than in the



Front-end depropanizer scheme.

high-pressure column. However, this fractionation scheme requires the use of ethylene refrigeration instead of propylene refrigeration. Both fractionation schemes have advantages and disadvantages. For example, the lower design pressure reduces the wall thickness of the column with a resultant cost saving. However, this cost reduction is offset by the higher material cost of equipment and piping due to the lower design temperature.

A good example of the front-end demethanizer scheme is the licensed processes developed by the M.W. Kellogg Company, *Hydrocarbon Process.* **68**(11), 102(1989), and by the Stone and Webster Engineering Company, *Hydrocarbon Process.*, **64**(11), 137(1985). The process developed by Linde A.G., *Hydrocarbon Process.* **64**(11), 137(1985) for producing ethylene from hydrocarbon liquids is an example of the deethanizer scheme. The advantages and disadvantages of these processes are listed below. (Other advantages and disadvantages of these processing schemes were listed earlier.)

Kellogg scheme advantage:
High olefin yields due to the very small (less than 0.1 second) reaction times in the pyrolysis furnace unit. Process handles feeds ranging from ethane to vacuum gas oils. Provides ultra-purified hydrogen stream for catalytic conversion. Generates excess high pressure steam for use elsewhere in a multi-purpose plant.

Kellogg scheme disadvantages:
Steam pyrolysis is performed at temperatures higher than most other ethylene processes creating more coking potential. Process requires two separate water quench exchangers.

Stone and Webster scheme advantages:
Handles feeds ranging from gases to liquid hydrocarbons. Process is characterized by a highly selective pyrolysis coil. Steam pyrolysis temperature not as high as the Kellogg process reducing

the coking potential. Operating history of quench exchanger shows it to be basically non-fouling. Removes acid gas with patented amine process.

Stone and Webster scheme disadvantages:

Requires an oil quench (with liquid feed stocks) and also a water quench. Ethylene yields slightly less than that obtained with the Kellogg process.

Linde AG advantages:

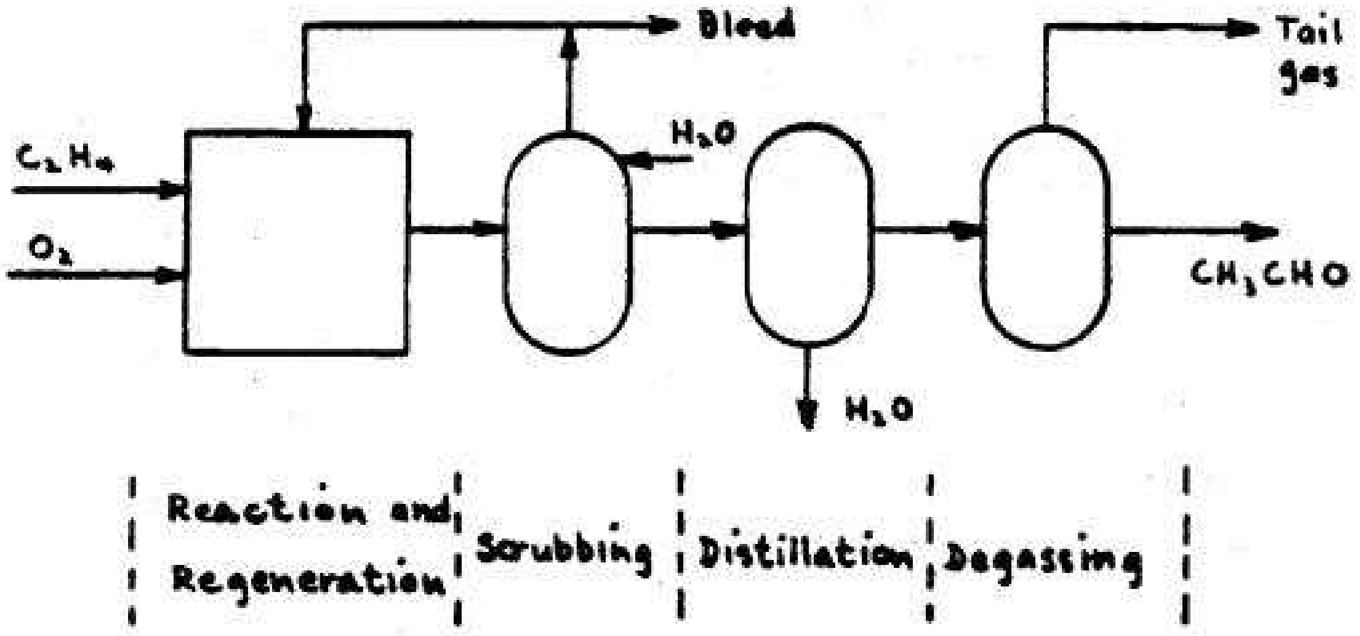
Feed is cracked in more conventional tubular furnaces which can be adapted to meet specific requirements of high capacity, high severity, or high selectivity. Recompression of cracked gases is to a lower pressure than other processes.

Linde AG disadvantages:

Operates only on hydrocarbon liquids. Requires an oil and a water quench.

Pb# 08

The simplified flow sequence for producing acetaldehyde from ethylene is given below:



Reaction and regeneration:

The main considerations are the reaction kinetics, equilibrium relations, and heat of reaction. The relationship of these factors will determine the reactor configuration and the cooling water requirements. Obviously, physical and thermal data for all the flow streams will be needed. Catalyst activity with information on its decline with time and reactivation time will be needed for regeneration design purposes.

Scrubbing:

The equilibrium relationship between the various reactor components and water will be the important design factor in this unit. Physical and thermal data for all the flow streams will be needed. Build up of inerts is controlled by a bleed stream. The composition of the latter must be known to determine the requirements for an auxiliary reactor designed to complete the oxidation of unconverted ethylene.

Distillation:

The distillation unit requires equilibrium and vapor pressure data for the mass transfer calculations. Separation of key components must be specified and separation efficiencies need to be established. Physical and thermal data for all the flow streams will again be needed. Heat transfer coefficients will be required for the design of heat exchangers, condenser, and reboiler.

Degassing:

This unit will require much of the same information that is required for the distillation unit only with different compositions and flows.

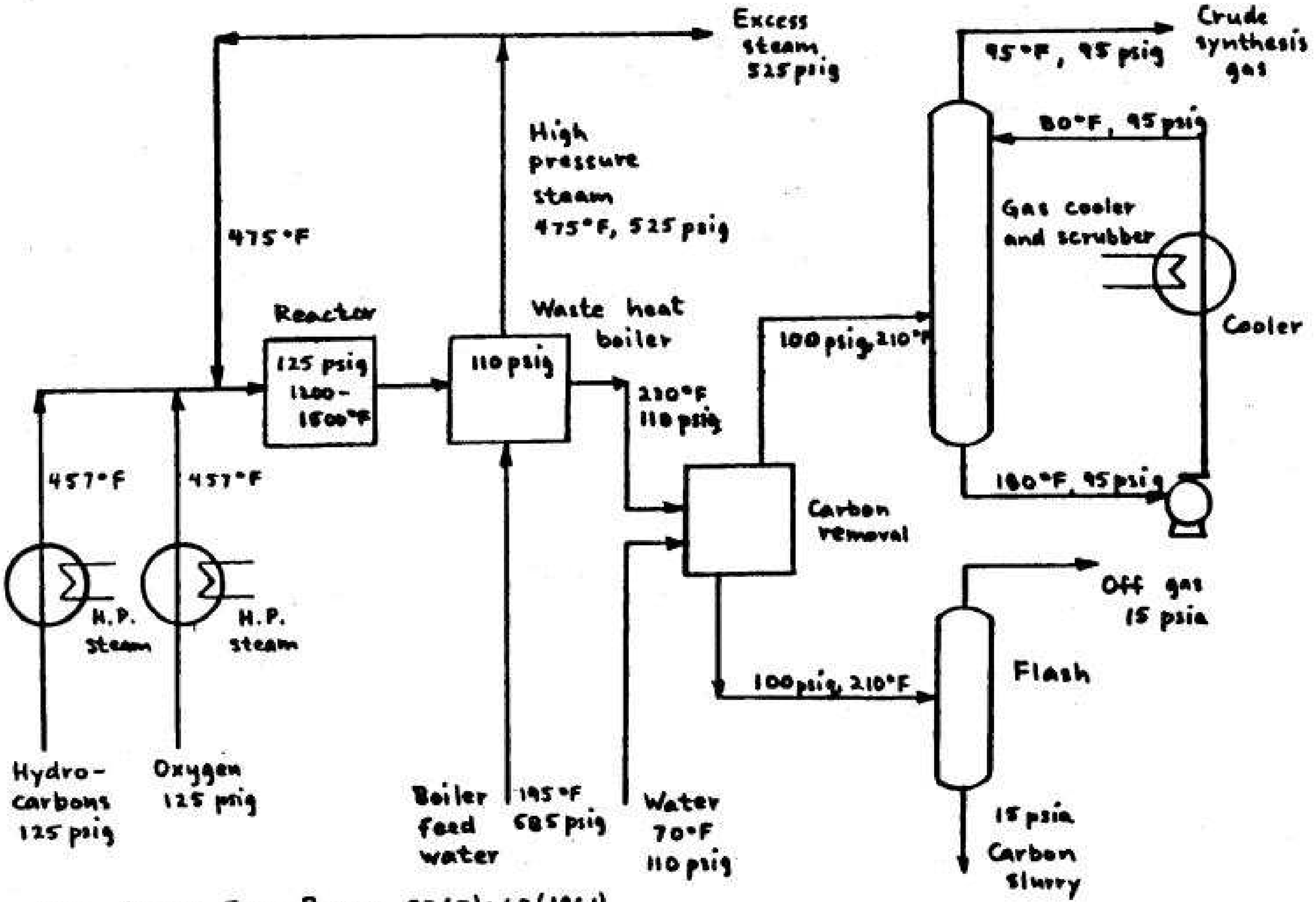
A simplified equipment flow sheet for the acetaldehyde process is given in *Hydrocarbon Process.* **44**(11), 159(1965). Since this is a proprietary process, only sketchy information is supplied with regards to temperature, pressure, and stream composition at each piece of equipment. Some of this information can be deduced from the information supplied for the raw materials and utilities requirements per short ton of acetaldehyde.

Pb# 12

A simplified equipment flow sheet for the acetaldehyde process is given in *Hydrocarbon Process.* **46**(11), 135(1967). In this variation of the process, the spent catalyst is reoxidized with air in a separate regenerator. Ethylene is converted to acetaldehyde in a single-pass contact with the catalyst solution at a slightly higher pressure than used in a single-stage unit. In a separator following the reactor, the pressure is reduced and reaction heat vaporizes the acetaldehyde product from the catalyst solution. The spent catalyst from the separator is pumped to the regenerator where it is reoxidized by contact with air. A comparison of raw materials and utilities required for the two-stage acetaldehyde process may be made with that given for an actual 75,000 short ton/year plant discussed in the same reference noted above. Since the solution of this problem is quite long, it is not presented here.

Pb# 13,14

A process flow sheet for preparing synthesis gas from any hydrocarbon is given below.

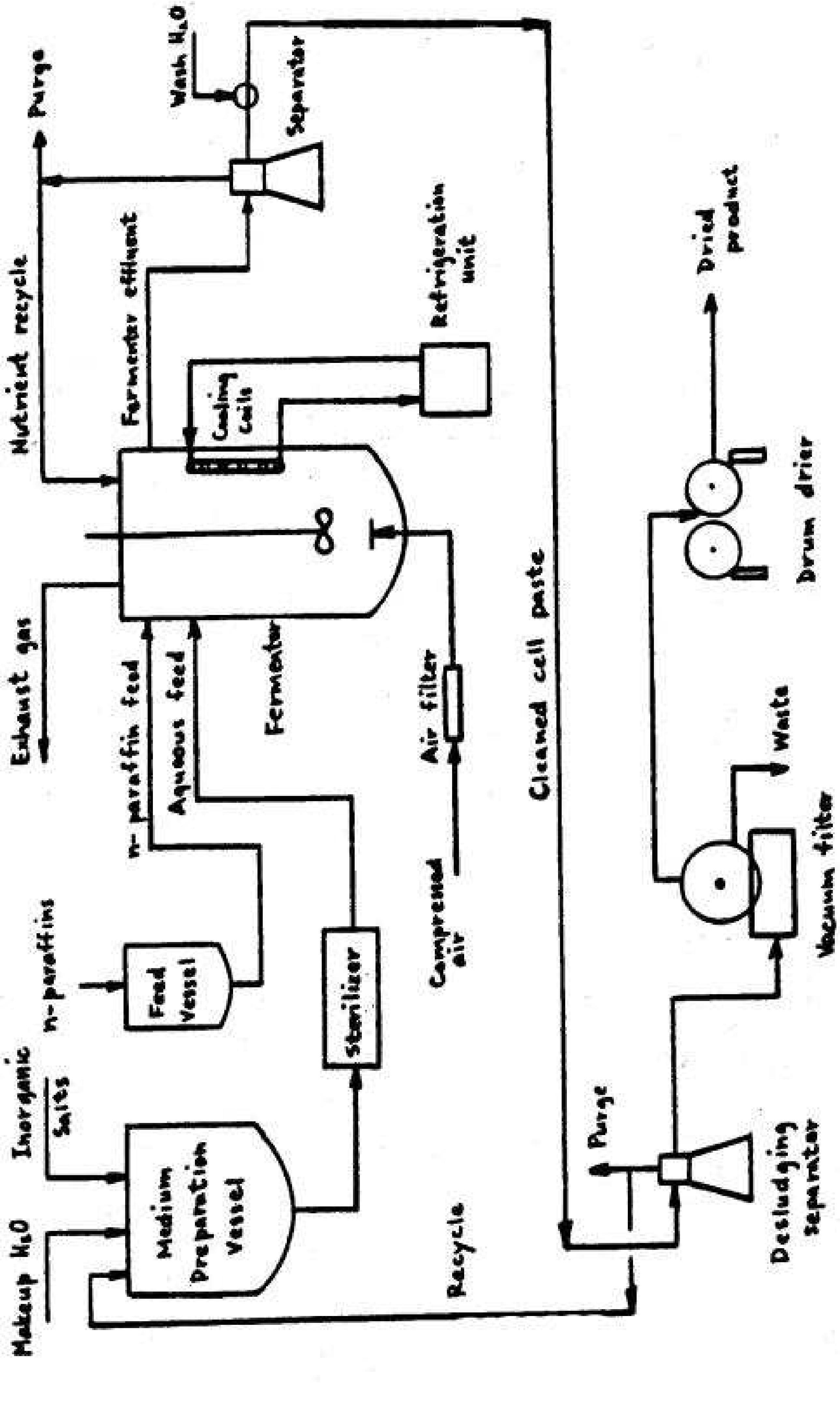


see Chem. Eng. Progr., 57(7): 68(1961)

The operating conditions for the synthesis gas process using the given feed stock, a heavy fuel oil, are presented in an article by S.C. Singer and L.W. ter Haar in *Chem. Eng. Progr.*, 57(7), 68(1961). The material balance may be made either with software programs by ASPEN PLUS or CHEMCAD and using these operating conditions and the final composition of the crude synthesis gas.

Pb# 19

A flow sheet outlining the conversion of petroleum to food supplements using organic microorganisms is presented below;



In the process an aqueous feed containing the necessary inorganic nutrients is mixed in a vessel before being sent to the fermentor with a metered amount of paraffinie feed stock. Compressed and sterilized air serves as the oxygen supply for the growing cells in the fermentor.

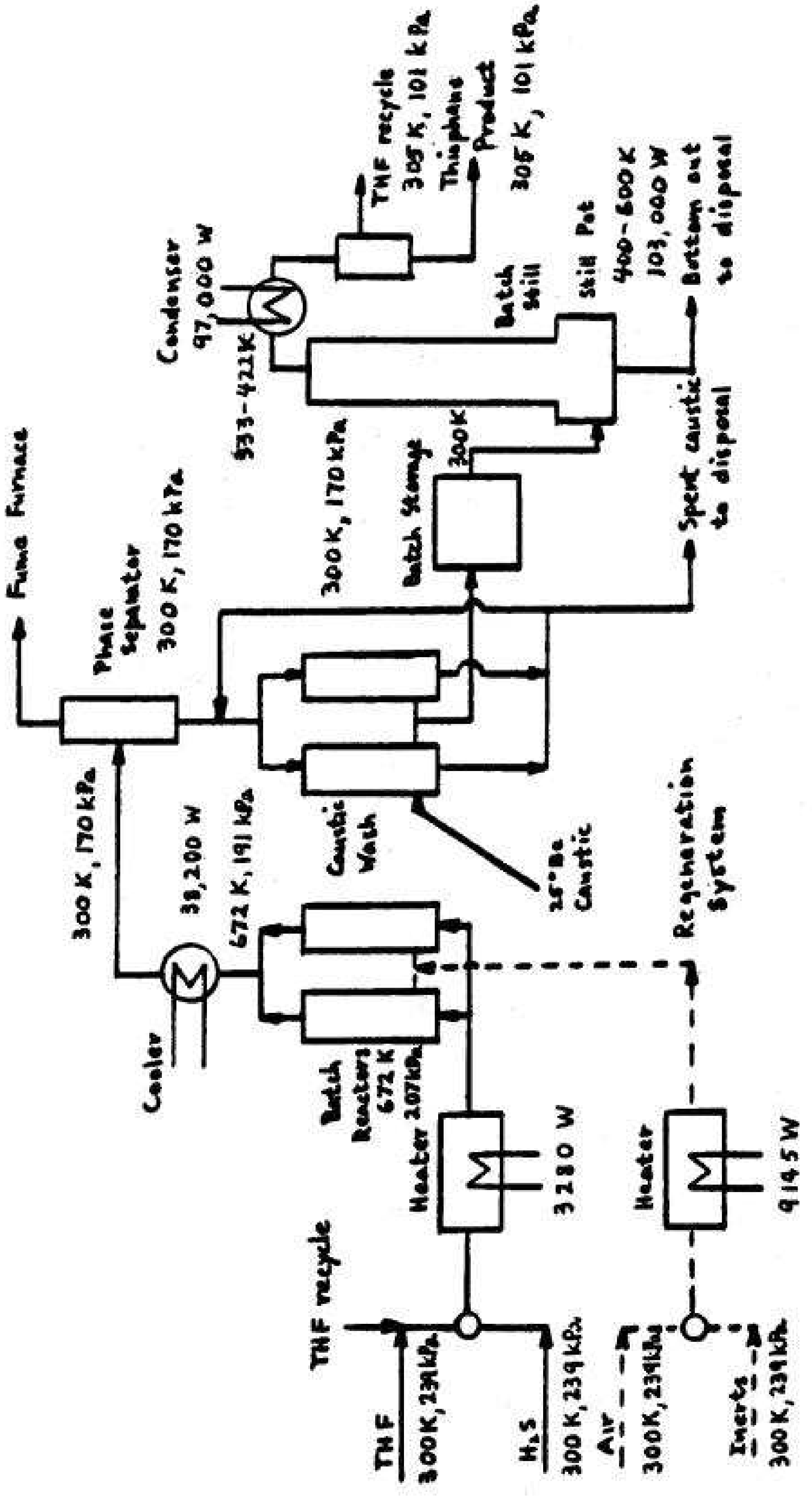
The growth of single cells is an autolytic process. The exponential growth rate can be maintained as long as the concentrations of all nutrients are kept above their respective critical values in the fermenting brew. The ratio of the paraffins to aqueous feedrates need to be maintained as high as possible. The temperature during fermentation must be kept constant by the careful control of heat removal.

At steady state, the product stream is continuously removed from the fermentor at a rate equal to feedrate. The product stream is concentrated in a desludging separator to a paste of 20 to 30% dry solids. After washing with warm water to remove adsorbed or absorbed hydrocarbons, a second desludging separator produces a cleaned paste that is further dehydrated in a rotary vacuum filter. A final drying step in an atmospheric drum dryer reduces the moisture content to less than 10%, and the powdered product is packaged for the customer.

The potential of using crude oil to produce human or animal-feed supplements has been recognized since the early 1960s. Many of the problems to accomplish this feat were listed by Wang in a *Chem. Eng.* 75(18), 99 (1968) publication. Several of these problems include separation of straight-chain hydrocarbons from isoparaffins, cyclic paraffins, and napthenes, development of a continuous non-aseptic processing technique, better understanding of the mass transfer characteristics of the process, control of the heat of fermentation, establishment of the required cell separation and purification technique, and improvement of the overall economics of the system. A number of these problems have been addressed over the past 35 years and suitable solutions have been formulated. However, the high demand for crude oil in meeting the world's transportation needs has increased the cost for crude oil to such a level that it is impractical to consider the use of this commodity to produce human or animal-feed supplements.

Pb# 20

The process flow diagram for the production of 1.40 x 10^{-2} kg/s of thiophane is presented below with appropriate temperature, pressure, and heat exchanger duties. The material balance can be developed with the aid of suitable computer software based on the conditions listed on the flow sheet.



CHAPTER 03

GENERAL DESIGN CONSIDERATIONS

Pb# 03

The vapor pressure of mercury is obtained from <u>Perry's Chemical Engineers' Handbook</u>, 6th ed., p. 3-201 as

T, °C	Vapor press, bar
10	7.101×10^{-7}
20	1.729×10^{-6}
30	3.968×10^{-6}

By interpolation, the vapor pressure of mercury at 23°C is 2.2×10^{-6} bar. For low pressures, at equilibrium, the vapor phase mol fraction of mercury can be obtained with the use of Dalton's law, or

$y = p_v^{\circ}/p$ where p = system pressure of 1.013 bar

$y = 2.2 \times 10^{-6}/1.013 = 2.17 \times 10^{-6}$ kg mols Hg / kg mols air

but $y =$ mols Hg/(mols air + mols Hg) $\cong$ mols Hg/mols air

The specific volume of air is obtained from the ideal gas relation $v = RT/p = (8314)(296.1)/101{,}300 = 24.305 \text{ m}^3/\text{kg mol}$

The concentration of mercury in air at equilibrium is then

$$c = (2.17 \times 10^{-6}/24.305)(200.6) = 17.9 \times 10^{-6} \text{ kg Hg/m}^3$$
$$= 17.9 \text{ mg Hg/m}^3$$

Since the PEL (permissible exposure limit is 0.1 mg Hg/m³, the concentration of mercury in the air greatly exceeds this limit and therefore is <u>not acceptable.</u>

Answer

Pb# 04

The density of benzene at 22°C is 879 kg/m³. The molecular weight is 78.1 kg/kg mol.

Since the pressure is low, assume the ideal gas law is applicable to the air stream that is used for ventilation.

$$\dot{n}_{Bz} = \left(2.0 \times 10^{-6} \frac{m^3}{min}\right)\left(879 \frac{kg}{m^3}\right) \Big/ 78.1 \frac{kg}{kg\,mol}$$

$$= 2.2506 \times 10^{-5} \text{ kg mol/min}$$

$$\dot{m}_{v,Bz} = nRT/p = \left(2.2506 \times 10^{-5} \frac{kg\,mol}{min}\right)\left(8314 \frac{J}{kg\,mol \cdot K}\right)(295\ K)$$

$$6.27 \times 10^{-4} \text{ m}^3/\text{min}$$

Assume complete mixing of the benzene with the dilution air

$$\dot{m}_{v,air} = \left(6.27 \times 10^{-4} \frac{m^3\ Bz}{min}\right)\left(10^6 \frac{m^3\ air/min}{m^3\ Bz/min}\right)$$

$$= \underline{\underline{627 \text{ m}^3 \text{ air/min}}}$$

Answer

Pb# 05

This problem has been simplified considerably with a number of assumptions. Actually, the corrosion rate for the iron (carbon steel) over a year is not in a fixed oxygen level atmosphere. It is most likely that the corrosion rate of iron is limited by the diffusion rate of oxygen to the tank surface. Under the circumstances, the analysis would require a corrosion model, information on the initial rate before corrosion product buildup, and need some indication of the diffusivity of the oxygen in the corrosion layer. As indicated in the problem, the reaction rate is a function of the surface are inside the tank. For a 7.6-m (inside) diameter spherical container, the inside area is given by

$$A = 4\pi r_2^2 = \pi D_2^2 = \pi (7.6)^2 = 181.5 \text{ m}^2$$

Iron corrosion $= (181.5)(0.127 \times 10^{-3})(7850)$ $\quad \rho_{Fe} = 7850 \text{ kg/m}^3$

$= 180.9 \text{ kg/y}$

$= (180.9)/(365)(24) = 0.02065 \text{ kg/h}$

$= 0.02065/55 = 3.7545 \times 10^{-4} \text{ kg mol/h}$

Oxygen rate $= (3.7545 \times 10^{-4})(1.5 \text{ kg mol } O_2 / 2.0 \text{ kg mol Fe})$

$= 2.816 \times 10^{-4} \text{ kg mol/h}$

Volume of sphere $= (4/3)\pi r_2^3 = \pi D_2^3/6 = \pi (7.6)^3/6 = 229.8 \text{ m}^3$

At room temperature (21.1°C) and 1 atm, the specific volume inside the tank, assuming an ideal gas, is

$$v = (22.4)(294.2/273.1) = 24.147 \text{ m}^3/\text{kg mol air}$$

The oxygen concentration under these conditions is $(0.21)(24.147) = 8.696 \text{ kg mol } O_2/\text{m}^3$

For a first order reaction,

$\dfrac{dc}{d\theta} = -kc$ where k = reaction rate constant, θ = time, and c = concentration

When $c = 8.696 \times 10^{-4}$ kg mol/m^3, the O_2 rate is 2.816×10^{-4} kgmol/h, and the tank volume is 229.8 m^3. At this condition

$$\frac{dc}{d\theta} = \frac{-2.816 \times 10^{-4}}{229.8} = -1.225 \times 10^{-6} \text{ kgmol/m}^3\cdot\text{h}$$

Solving for the reaction rate constant gives

$$k = -\left(\frac{dc}{d\theta}\right)/c = (1.225 \times 10^{-6})/8.696 \times 10^{-3} = 1.408 \times 10^{-4} \text{ h}$$

Now solve for the time when O_2 concentration is reduced to 19.5 mol percent. Integrating the original rate equation results in

$$\ln(c_1/c_2) = k(\theta_2 - \theta_1) \qquad \text{where } \theta_1 = 0$$

$$\theta_2 = \ln(c_1/c_2)/k$$

$$= \ln(0.21/0.195)/1.408 \times 10^{-4}$$

$$= \underline{\underline{526 \text{ h or approx. 22 days}}}$$

Answer

Pb# 06

Assume that the dilution air is available at a temperature of 20°C and a pressure of 1 atm. The evaporation rate is 7.5 g/min or 0.0075 kg/min; Thus,

$$\dot{n}_{vc} = 0.0075/78 = 9.62 \times 10^{-5} \text{ kg mol vinyl chloride/min}$$

At 1 atm use the ideal gas to obtain the volumetric flow rate

$$\dot{m}_{v,vc}\ \dot{n}_{cv} RT/p = (9.62 \times 10^{-5})(8314)(293/1.013 \times 10^{5}$$

$$= 2.31 \times 10^{-3} \text{ m}^3/\text{min}$$

Since the PEL for vinyl chloride is 1ppm,

$$\dot{m}_{v,air} = (\dot{m}_{v,vc})(10^6) = (2.31 \times 10^{-3})(10^6)$$

$$= 2.31 \times 10^{3} \text{ m}^3/\text{min}$$

If a safety factor of 5 is required,

$$\dot{m}_{v,air} = (5)(2.31 \times 10^{3})/60 = \underset{\text{Answer}}{\underline{192.5 \text{ m}^3/\text{s}}}$$

This is a fairly high flow rate and would need one or two large blowers.

Pb# 07

To determine the length of time the cannister will be usable, we will need to find a relationship between the amount adsorbed and $(T/V)\log(f_s/f)$. Let $x = (T/V)\log(f_s/f)$ and y the amount adsorbed in cm^3(liq)/100 g charcoal. From the data given,

when $y = 1 \quad x = 21$
$y = 10 \quad x = 11$

assuming a linear relation of $\log y = ax + b$

$\log 1 = 21a + b$
$\log 10 = 11a + b$
or $\log(1/10) = 10a \quad a = -0.1$
$b = \log 10 - 11(-0.1) \quad b = 2.1$

The molecular weight of dichloropropane (DCP) is 113, density is 1160 kg/m³, and molar volume = 0.1 m³/kg mol. The vapor pressure from Perry's Chemical Engineers' Handbook, 6 ed., p.354,

Temp., °C	Vapor press., mm Hg
19.4	40
28	60

Assuming the Antoine equation can be used to represent the vapor pressure of DCP

$\log p = A + B/T$
$\log 40 = A + B/292.55$
$\log 60 = A + B/301.15$

Solving the latter two equations simutaneously results in

$B = -1802 \qquad A = 7.76$

At 26.9°C or 300.05 K

$\log p = 7.76 + (-1802/300.05) = 1.755$
$p = 56.9$ mm Hg

Determine the fugacity of DCP by assuming that it is essentially equal to the partial pressure. Thus,

$f \approx p$

$p = y\,p_T$ where p_T = total pressure and $y = 750/10^6$

$p = (750/10^6)(760) = 0.57$ mm Hg

therefore, $f_s/f = 56.9/0.57 = 99.82$

$x = (T/V)\log(f_s/f) = (300/100)\log(99.82) = 5.99$

Now determine how much gas has been adsorbed

$\log y = -0.1x + 2.1 = (-0.1)(5.99) + 2.1 = 1.503$

$y = 31.84\ \text{cm}^3/100\ \text{g}$ charcoal

For 100% saturation, amount adsorbed per 100g charcoal is

$m = (31.84\ \text{cm}^3/100\text{g C})(1.16\ \text{g/cm}^3) = 36.93\text{g}$

At 82% saturation, amount adsorbed before breakthrough is

$m_{82\%} = (36.93)(0.82) = 30.3\ \text{g}$

Now determine volume of air associated with 30.3 g DCP

$$n_{air} = \left(\frac{30.3\ \text{g DCP}}{113\ \frac{\text{g DCP}}{\text{g mol DCP}}}\right)\left(\frac{10^6\ \text{g mol air}}{750\ \text{g mol DCP}}\right) = 357.5\ \text{g mol air}$$

$$\text{Vol of air} = nRT/p = \frac{(357.5\ \text{g mol})}{1\ \text{atm}}\left(82.06\ \frac{\text{atm}\cdot\text{cm}^3}{\text{g mol}\cdot\text{K}}\right)(300\ \text{K})$$

$$= 8.8 \times 10^6\ \text{cm}^3 = 8{,}800\ \ell = 8.8\ \text{m}^3$$

At 45 ℓ/min

$$\theta = (8{,}800\ \ell)/(45\ \ell/\text{min}) = \underline{\underline{195\ \text{min} = 3.25\text{h}}}$$

Answer

Cannister should last 3.25h before break-through occurs.

Pb# 08

At equilibrium, acetone, benzene, ethyl ether, and n-pentane are too rich to ignite. Carbon disulfide and methyl alcohol will always be in the flammable range. It should be noted that all of the vaporizing liquids can be ignited if not at equilibrium. Some are also quite toxic.

Answer

The types of fire extinguishers recommended when a small spill is ignited are as follows:

Liquid	Extinguishers recommended
acetone	alcohol foam, dry chemical, CO_2
benzene	foam, dry chemical, CO_2
carbon disulfide	foam, water blanket
ethyl ether	alcohol foam, dry chemical, CO_2
methyl alcohol	alcohol foam, dry chemical, CO_2
n-pentane	foam, dry chemical, CO_2

Answer

Pb# 09

At equilibrium, $f_{i,L} = f_{i,v}$

Since

$$f_{i,L} = \gamma_i f_{o,L} x_i$$

$$f_{i,v} = \varphi_i f_{o,v} y_i$$

$$\gamma_i f_{o,L} x_i = \varphi_i f_{o,v} y_i$$

At low pressures

$$f_{o,v} = p$$

$$f_{o,L} = p_v^{\circ}$$

$$\varphi_i = 1.0$$

Therefore $\gamma_i p_v^{\circ} x_i = p y_i$

For an ideal solution, $\gamma_i = 1$

and $x_i p_v^{\circ} = p y_i$

Definitions

$f_{i,L}$ = fugacity of liquid

$f_{i,v}$ = fugacity of vapor

γ_i = liquid phase activity coeff.

φ_i = vapor phase activity coeff.

$f_{o,L}$ = pure comp. liq. fugacity

$f_{o,v}$ = pure comp. vapor fugacity

p = total pressure

p_v° = vapor pressure

If the acetone has no air dissolved in it, $x_i = 1.0$ and y_i is the LFL or $y_i = 0.0255$.

At a total pressure of 1 atm,

$$p_v^{\circ} = y_i p / x_i$$

$$= (0.0255)(1.0)/1.0$$

$$= 0.0255 \text{ atm} = 19.38 \text{ mm Hg}$$

Acetone has a vapor pressure of 19.38 mm Hg at a temperature of −21.4 °C (Perry's Chemical Engineers' Handbook, 6th ed., p. 3-50.

$$T_{flash} \text{ (calculated)} = \underline{\underline{-21.4\,^{\circ}\text{C}}}$$

Answer

The handbook Dangerous Properties of Industrial Materials provides a measured flash point for acetone of −17.8°C.

Pb# 10

If the mixture is at the flash point

$$\sum \frac{y_i}{LFL_i} = 1.0$$

Assuming Raoult's law applies

$$y_i = p_{v,i}\, x_i / p$$

where $p_{v,i}$ is the vapor pressure of component i, p the pressure, and x_i and y_i the mol fraction of component in the liquid and vapor, respectively. Vapor pressure of C_8 and C_9 are available from Perry's Chemical Engineers' Handbook, 6th ed., p. 3-58

Assume flash temperature of 18°C

Comp	x_i	$p_{v,i}$, atm	y_i	LFL_i
C_8	0.5	0.0123	0.00615	0.01
C_9	0.5	0.00365	0.00183	0.008

$$\sum \frac{y_i}{LFL_i} = \frac{0.00615}{0.01} + \frac{0.00183}{0.008} = 0.844$$ temp. too low

Assume flash temperature of 21.1°C

Comp	x_i	$p_{v,i}$, atm	y_i	LFL_i
C_8	0.5	0.0145	0.00725	0.01
C_9	0.5	0.0044	0.0022	0.008

$$\sum \frac{y_i}{LFL_i} = \frac{0.00725}{0.01} + \frac{0.0022}{0.008} \simeq 1.0$$

The flash point for this mixture is <u>21.1°C</u>
Answer

Pb# 11

Since the lower flammability limit (LFL) for acetone vapor in air is 2.55%, the maximum vapor pressure the acetone could have would be

$$p_v^\circ = (0.0255)(760) = 19.38 \text{ mm Hg}$$

The temperature equivalent to this vapor pressure may be determined utilizing the experimental data provided in Perry's Chemical Engineers' Handbook, 6th ed., p. 3-50 and the use of the Antoine equation with the form

$$\ln p_v^\circ = a - b/(t + 273)$$

The vapor pressure of acetone in the range of interest is

t, °C	p_v°, mm Hg
-40.5	5
-20.8	20

Substituting these values in the expression

$$\ln 5 = a - b/(-40.5 + 273)$$

$$\ln 20 = a - b/(-20.8 + 273)$$

Solving the equations simultaneously gives $a = 19.36$ and $b = 4126.3$. Thus,

$$\ln 19.38 = 19.36 - 4126.3/(t + 273)$$

$$t = -21.3°\text{C}$$

The acetone would have to be cooled below -21.3°C to avoid an acetone-air mixture that could be ignited with a discharge of static electricity. Such cooling is not very practical and the use of an inert gas blanket should be considered during the filling operation.

Answer

Pb# 12

The stoichiometric balance for the reaction is given by

$$C_3H_8 + 5O_2 + 5(3.76)N_2 \longrightarrow 3CO_2 + 4H_2O + 5(3.76)N_2$$

For 200% theoretical air, the balance becomes

$$C_3H_8 + 10O_2 + 10(3.76)N_2 \longrightarrow 3CO_2 + 4H_2O + 5O_2 + 10(3.76)N_2$$

Since all the energy generated from the combustion process is absorbed by the product gases, the final temperature of the products is determined by means of an adiabatic flame temperature calculation. The heat capacities for each component is a function of the temperature and requires a knowledge of the C_p-temperature relation of those components. Present software is available and can be used to rapidly converge with a heat balance on the appropriate flame temperature. When this is done, a temperature of <u>1527 K</u> is obtained.

Answer

The iterative calculation can also be done by using an enthalpy table as a function of temperature for each of the coponents. The latter is available in Holman's <u>Thermodynamics</u>, 4th ed., Table A-18, pp 755-6 and is used in the hand calculation of this problem. The energy balance is

$$(\bar{h}_f^\circ)_{C_3H_8} = 3(\bar{h}_f + \bar{h}_T - \bar{h}_{298})_{CO_2} + 4(\bar{h}_f^\circ + \bar{h}_T - \bar{h}_{298})_{H_2O} + 5(\bar{h}_f^\circ + \bar{h}_T - \bar{h}_{298})_{O_2}$$
$$+ 37.6(\bar{h}_f + \bar{h}_T - \bar{h}_{298})_{N_2}$$

Where $\bar{h}_f^\circ$ is the molar heat of formation, $\bar{h}_T$ the molar enthalpy at temperature T, and $\bar{h}_{298}$ the molar enthalpy at 298 K for the components as designated by the subscripts. Molar heats of formation for these components are from Table A-6 of the same text.

Heat losses to the surroundings are assumed to be negligible.

To obtain the final temperature, assume different values of temperature until the energy balance is consistent. Units in kJ/kg mol.

Assume T = 1555 K

$$-103{,}925 = 3(-393{,}782 + 74{,}392 - 9374) + 4(-241{,}997 + 60{,}680 - 9904)$$
$$+ 5(51{,}307 - 8664) + 37.6(49{,}034 - 8676)$$

$$-103{,}925 \neq -20{,}500$$

Temperature assumed is too high; assume T = 1500 K

$$-103{,}925 = 3(-393{,}782 + 71{,}131 - 9374) + 4(-241{,}997 + 58{,}050 - 9904)$$
$$+ 5(49{,}272 - 8664) + 37.6(47{,}092 - 8676)$$

$$-103{,}925 \neq -123{,}997$$

Temperature assumed is too low. By interpolation, the final temperature under adiabatic conditions is 1511 K

The ideal gas law is valid at low pressures and high temperatures. Thus,

$$V_f = n_f RT/p$$

$$n_f = 3 + 4 + 5 + 37.6 = 49.6 \text{ kg mol/kg mol fuel}$$
$$= 4960 \text{ kg mol/100 kg mol fuel}$$

$$V_f = (4960)(8314)(1511)/101{,}300 = 615{,}000 \text{ m}^3$$
$$= \underline{\underline{6.15 \times 10^5 \text{ m}^3}}$$

Answer

Ratio of final volume to initial volume is given by

$$V_f/V_i = \frac{n_f T_f}{n_i T_i} = \frac{(4960)(1511)}{(4860)(298)} = 5.17$$

Pb# 14

The rate at which energy is produced in a reactor is given by the relation

$$\frac{dq}{d\theta} = \Delta H_{rx}\, k_o C V e^{-(E/RT)}$$

where $dq/d\theta$ is the heat liberated over time, ΔH_{rx} the heat of reaction, C the concentration of the reactant, V the volume of the reactor, E the activation energy, R the gas constant, and T the temperature.

When the reaction is exothermic, heat must be removed from the reactor to keep the temperature from increasing beyond control. The heat transfer rate can be written as

$$\frac{dq}{d\theta} = UA(T - T_o)$$

where U is the overall heat transfer coefficient, A the heat transfer area, and T_o the coolant temperature. If heat can be removed as rapidly as heat is generated by the reaction, the reaction can be kept under control. Thus, at steady state

$$\Delta H_{rx} k_o C V e^{-(E/RT)} = UA(T - T_o)$$

To simplify the relation, let $B = \Delta H_{rx} k_o C V$ and $K = UA$. Thus,

$$B e^{-(E/RT)} = K(T - T_o)$$

or $$B e^{-(E/RT)} - K(T - T_o) = 0$$

The maximum temperature where the steady-state energy balance is valid can be found by setting its derivative with respect to temperature equal to zero. Doing this, resuts in

$$B e^{-(E/RT)} (E/RT_c^2) - K = 0$$

where T_c is the critical or maximum temperature. However,

$$B e^{-(E/RT)} = K(T_c - T_o)$$

at the maximum temperature. Substituting this into the previous equation and solving for K gives

$$K(T_c - T_0)(E/RT_c^2) = K$$

or $(T_c - T_0) = RT_c^2/E$ which can be used to find T_c

Substituting values

$$(T_c - 288.1) = 1.987\,T_c^2/28{,}000$$

$$T_c = 294.3\text{ K}$$

This means that it the temperature in the reactor rises above 294.3 K (21.1°C), the reactor will begin to generate energy faster than it can be removed and the reaction will run away. Note that only a 6.2°C rise in temperature in the reactor can be tolerated.

Answer

For safe operation of the reactor some or all of the following actions can be taken:

1. Increase the size of the heat exchanger
2. Inject an inhibitor into the reactor or quench the reaction if it is ready to go out of control
3. Reduce the feed to the reactor
4. Provide adequate venting for the reactor for emergency use
5. Add diluents to the reactor to reduce the reaction rate. (These will probably have to be separated from the product after the reaction is completed)

Pb# 15

The decomposition reaction is the following

$$C_2H_2 \rightleftharpoons 2C + H_2$$

Component	mols in	mols at temp. T	mol fraction
C_2H_2	1	$1-x$	$1-x$
C	0	$2x$	0^*
H_2	0	x	x

*Assumes solid carbon product

Using the nomenclature and methodology from Sandler's Chemical and Engineering Thermodynamics

$$K_a(T) = \frac{a_C^2 a_{H_2}}{a_{C_2H_2}} \simeq \frac{y_{H_2}}{y_{C_2H_2}} = \frac{x}{1-x} \quad \text{where } x = \text{extent of reac.}$$

$$K_a(T) = e^{-(\Delta G_{rx}(T)/RT)}$$

Assume for both carbon and hydrogen

$$\Delta G_f(25°C) = 0 \quad \text{and} \quad \Delta H_f(25°C) = 0$$

$$\Delta G_{rx}(25°C) = -\Delta G_f(25°C)_{C_2H_2} = -50{,}000 \text{ cal/g mol}$$

$$\Delta H_{rx}(25°C) = -\Delta H_f(25°C)_{C_2H_2} = -54{,}194 \text{ cal/g mol}$$

The heat capacities for C, H_2 and C_2H_2 fit the relation $Cp_i = a_i + b_iT + c_iT^2 + d_iT^3 + e_i/T^2$ cal/g mol·K where T is in kelvins.

Comp.	a	$b\times10^2$	$c\times10^5$	$d\times10^7$	e
C	2.673	0.2617	–	–	116,900
H_2	6.952	−0.04576	0.09563	−0.2079	–
C_2H_2	5.21	2.2008	−1.559	4.349	–

Data are from Sadler and Perry's Chemical Engineers' Handbook

For heats of reaction at other temperatures besides 25°C or T_1

$$\Delta H^\circ_{rx}(T) = \Delta H^\circ_{rx}(T_1) + \Delta a(T-T_1) + (\Delta b/2)(T^2-T_1^2) + (\Delta c/3)(T^3-T_1^3) + (\Delta d/4)(T^4-T_1^4) - \Delta e\left(\frac{1}{T}-\frac{1}{T_1}\right)$$

Where $\Delta a = \sum \nu_i a_i$, etc. and ν_i = stoichiometric coefficient

$K_a(T)$ for other temperatures besides at T_1 or 25 °C

$$\frac{\ln K_a(T)}{\ln K_a(T_1)} = \frac{\Delta a}{R}\ln\frac{T}{T_1} + \frac{\Delta b}{2R}(T-T_1) + \frac{\Delta c}{6R}(T^2-T_1^2) + \frac{\Delta d}{12R}(T^3-T_1^3) + \frac{\Delta e}{2}\left(\frac{1}{T^2}-\frac{1}{T_1^2}\right) + \frac{1}{R}\left\{-\Delta H^\circ_{rx}(T_1) + \Delta a T_1 + \frac{\Delta b}{2}T_1^2 + \frac{\Delta c}{3}T_1^3 + \frac{\Delta d}{4}T_1^4 - \frac{\Delta e}{T_1}\right\}\left(\frac{1}{T}-\frac{1}{T_1}\right) \quad (1)$$

For this reaction $K_a(T) = x/(1-x)$ or

$$x = \frac{K_a(T)}{1+K_a(T)} \quad (2)$$

From an energy balance for an adiabatic constant volume reaction

$$x = \frac{-\sum \int n_{i,in}\, C_{v_i}\, dT}{\Delta H^\circ_{rx}(T)} \quad (3)$$

where $n_{i,in}$ are the mols that enter into the reaction. Using the heat capacity constants tabulated earlier

$$\Delta a = 7.088$$
$$\Delta b = -1.7232 \times 10^{-2}$$
$$\Delta c = 1.6546 \times 10^{-5}$$
$$\Delta d = -4.5569 \times 10^{-9}$$
$$\Delta e = 2.338 \times 10^{5}$$

$$K_a(25\,°C) = e^{-[(-50{,}000)/(1.987)(298.1)]}$$
$$= 4.5 \times 10^{36}$$

Since the final temperature will be relatively high, the ideal gas law may be assumed to be valid. Thus,

$$C_{v,i} = C_{p,i} - R = a_i - R + b_i T + c_i T^2 + d_i T^3 + e_i/T^2$$

$$\sum[n_i(a_i - R)]_{in} = 3.223 \quad \sum(n_i b_i)_{in} = 2.2008 \times 10^{-2} \quad \sum(n_i c_i)_{in} = -1.559 \times 10^{-5}$$
$$\sum(n_i d_i)_{in} = 9.34 \times 10^{-9} \quad \sum(n_i e_i)_{in} = 0$$

With this information, Eqs. (1), (2) and (3) can be solved simultaneously. The results are

$$T = 2967\ \text{K} \quad \text{and} \quad x = 0.984$$

The pressure will then be obtained from

$$p(T) = p_1(T_2/T_1) = 10(2967/298) = 99.6 \text{ atm}$$

With a safety factor of 4, the tank should be able to withstand a pressure of essentially 400 atm.

For spherical containers, Table 12-10 provides a relation to obtain the minimum wall thickness t of the tank as

$$t = \frac{Pr_i}{2SE_J - 0.2P}$$

Where P is the maximum allowable pressure, r_i the inside radius, S the maximum allowable working stress, E_J the efficiency of the welded joint. Assume that the carbon steel used in the tank is SA-442 Grade 55 with an S value of 94.7 MPa and that the welds are butt joints with complete penetration and spot examined resulting in a weld efficiency of 85%.

The diameter of the spherical tank was not given. Assume inside diameters of 1, 2, and 5 meters. For a 1-m diameter sphere

$$t = \frac{(400)(1.013 \times 10^5)(0.5)}{2(94.7 \times 10^6)(0.85) - (0.2)(400)(1.013 \times 10^5}$$

$$= 0.1324 \text{ m} \quad \text{or} \quad 5.21 \text{ in}$$

For 2 and 5 m inside diameters, the minimum thicknesses are 0.265 m and 0.662 m.

Answer

The results show the advantage of having a properly designed relief valve and burst disk assembly closely tied in with the reactor containing the decomposition reaction of acetylene. It would be imprudent to recommend a spherical container with a wall thickness greater than 0.0125 m because of the reactor costs involved.

Pb# 18

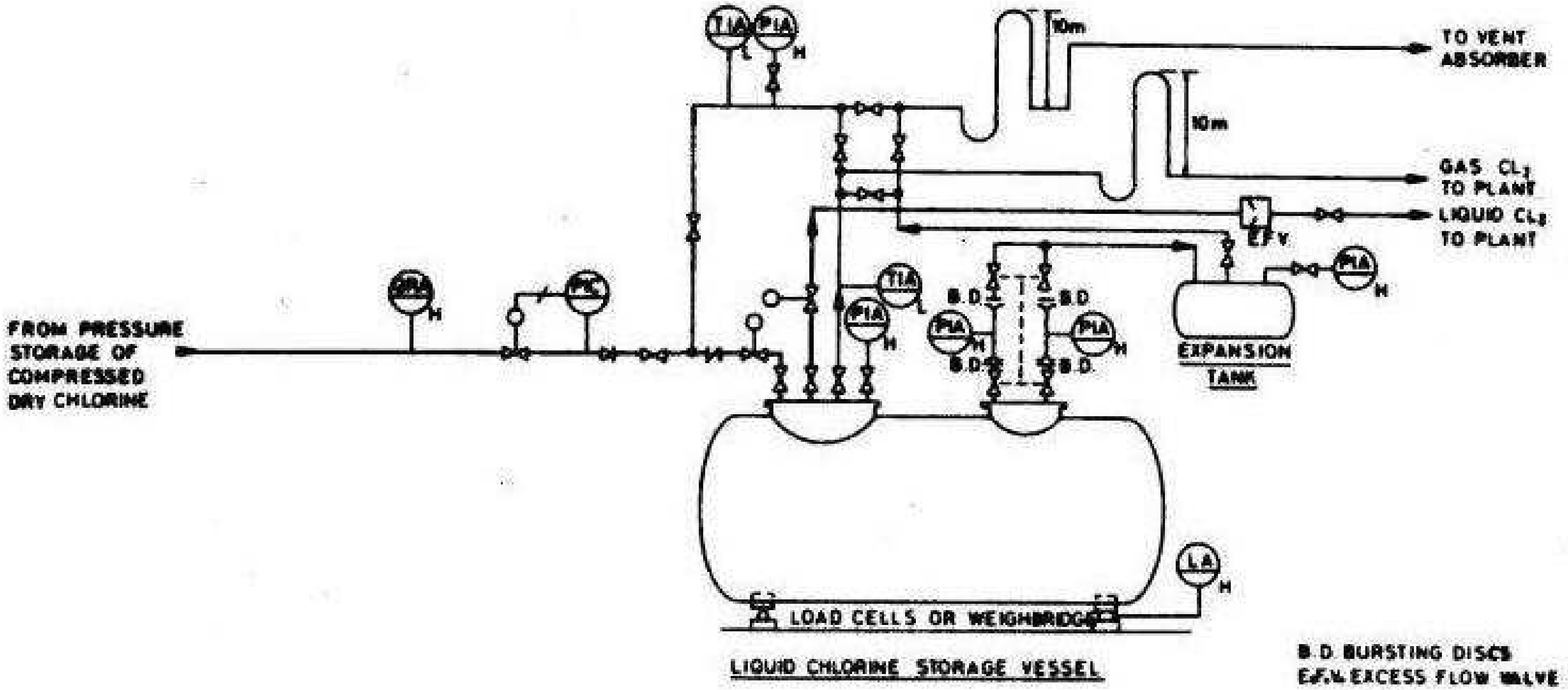
A system for transferring liquid chlorine from a chlorine storage container with pressurized chlorine gas is shown below. A feature of this design is a relief system that utilizes a duplicate burst system to simplify replacement of the discs with a mechanical interlock between the two burst systems. Bottom outlets have been avoided so that joints are not exposed to liquid chlorine and the only leakage will be of a gaseos nature.

The severity of the leak is reduced by lowering the pressure within the storage container or system. A facility must be available to accomplish this pressure reduction during chlorine transfer as well. A waste chlorine absorption unit that uses caustic or some simular chlorine consuming process is recommended. In addition, an expansion vessel is provided with an alarm to handle any pressure build-up in the container. The expansion tank is automatically vented to the absorption system (see figure).

The recompressed dry chlorine can be stored in a pressure storage receiver fitted with a relief valve set to discharge to the waste chlorine absorption unit at a predetermined pressure.

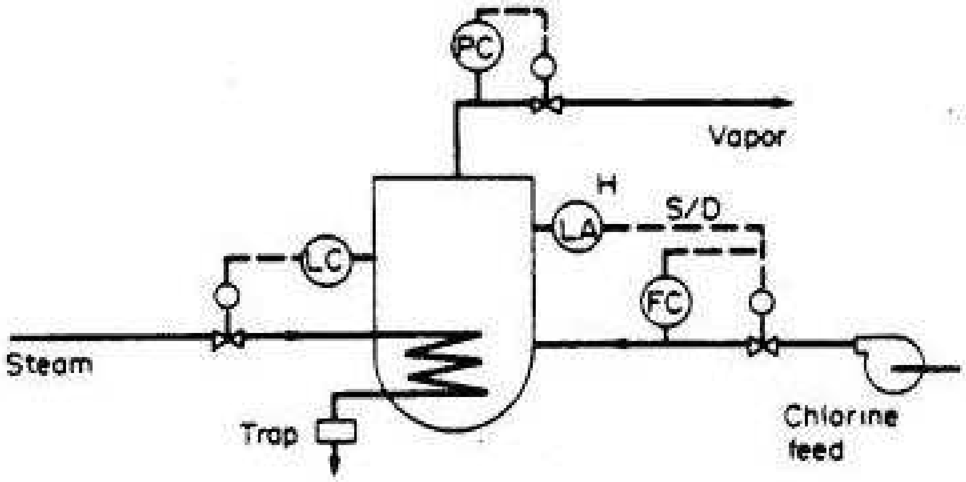
Pressurization with vaporized chlorine can function with a pressure storage tank that can utilize an external heater to vaporize the liquid chlorine. Control of the vaporization heater is tied to the pressure control system for the storage tank. Relief valves must permit over-pressurization to be relieved with excess chlorine vapor going to the waste chlorine absorption unit described above.

One possible instrumented arrangement for the chlorine vaporizer is shown in the second figure.



One possible arrangement with appropriate controls and alarm system for transferring liquid chlorine by using recompressed chlorine vapor.

Answer



Answer

Pb# 19

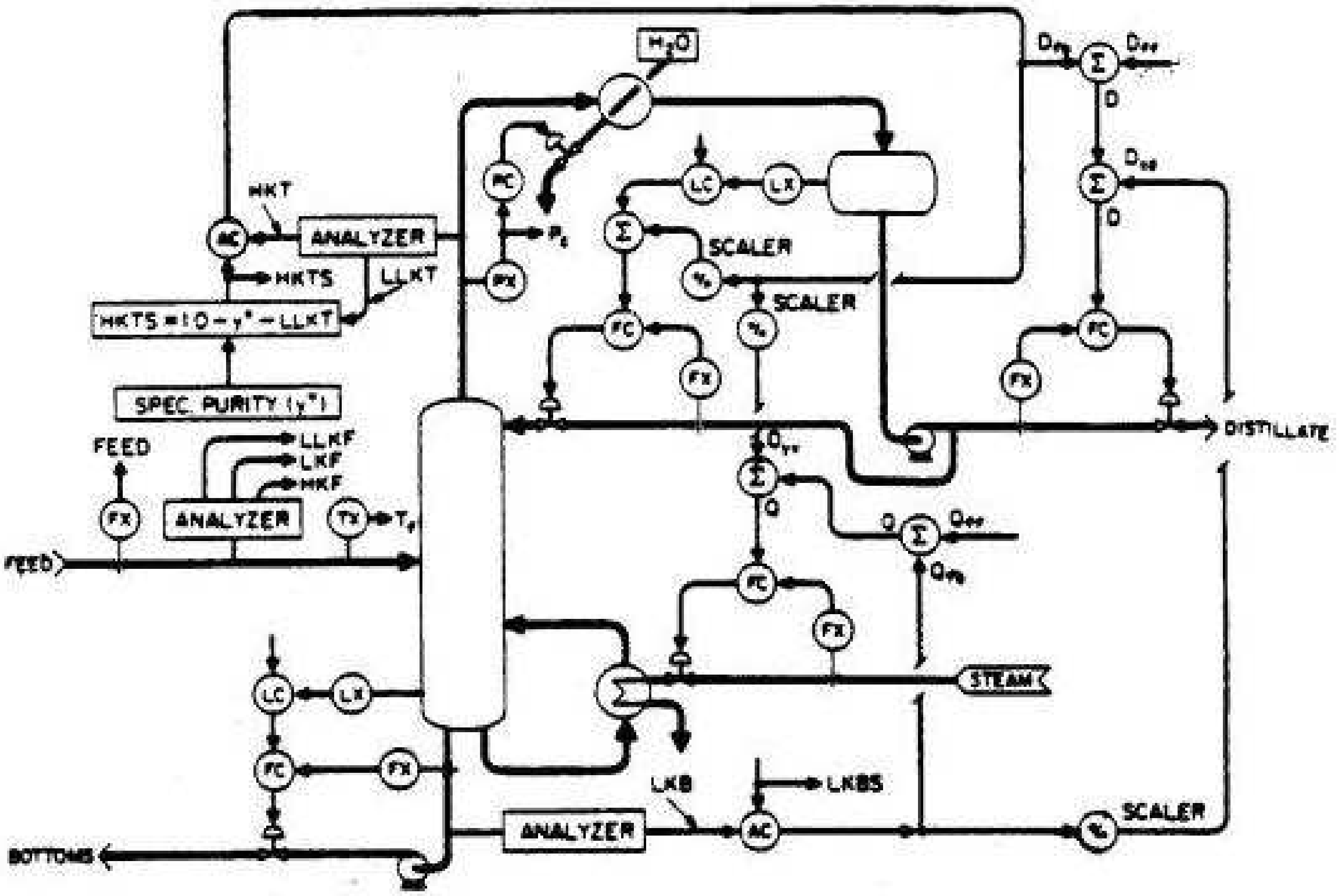
Reducing the inventory of a hazardous material in a process includes, but is not limited to the following items:

1. Investigate the possibility of producing the end product by another process that does not the use of a very hazardous material.
2. Determine whether the hazardous material may be reacted or combined before the distillation step to form a non-hazardous material. If the hazardous material is a reactant, recycle the stream back to the process to reduce the hazardous material content before the distillation step.
3. Depending upon how the hazardous material is used in the process, determine whether there are more efficient ways of absorbing, adsorbing, trapping, complexing, etc, the hazardous materials to avoid using a distillation operation that generally has considerably greater difficulty in attaining ppm concentrations in a product stream.
4. If elimination of the hazardous material is not possible, operate the distillation operation on a continuous basis. This will require less material being handled, smaller equipment used with possible elimination of the buffer storage, and improved chances of containment should there be a leak.
5. Improve the efficiencies of the column to reduce build-up of hazardous materials. Guidelines for such improvements are given by K.E. Nelson, Hydrocarbon Processing 69(3), 93-98 (1990).

Answer

A distillation column can be controlled in many ways. Generally, there is no one "best" way. Which one is to be prefered depends on the purpose of the separation, the types of controls available for each service, the variability of the feed, and the relative importance of the product recovery and product purity. Each case needs to be analyzed separately.

A typical distillation column with appropriate instrumentation in which basic controls are assisted with predictive and analyzer loops is outlined in <u>Perry's</u> <u>Chemical</u> <u>Engineers'</u> <u>Handbook</u>, 6th ed., p. 22-118 and is reproduced below with the permission of McGraw-Hill.



Nomenclature Summary for Fig.

Symbol	Explanation
D_{fb}	Contribution of feedback control to distillate-flow set point
D_{ff}	Contribution of predictive control
D_{sd}	Contribution of noninteractive decoupler to distillate-flow set point
D_{ys}	Contribution of noninteractive decoupler to reboiler-heat set point
HKF	Heavy key feed-stream component
HKT	Heavy key overhead-product-stream-component concentration, weight percent
HKTS	Specified concentration of heavy key component in overhead product, weight percent
LKBS, x^*	Specified concentration of light key component in bottoms product, weight percent
LKF, z	Light-key-feed-stream-component concentration, weight percent
LLKF	Lighter-than-light-key-feed-stream-component concentration, weight percent
LLKT	Lighter-than-light-key-overhead-product-stream-component concentration, weight percent
Q	Reboiler heat, BTU/unit time
Q_{fb}	Contribution of feedback control to steam-flow set point
Q_{ff}	Contribution of predictive control to steam-flow set point
Q_r/F	Weight ratio, reboiler-heat-to-feed
x^*, *LKBS*	Specified concentration of light key component in bottoms product, weight percent
z, *LKF*	Light-key-feed-stream-component concentration, weight percent

Pb# 20

HAZOP for Line L–102

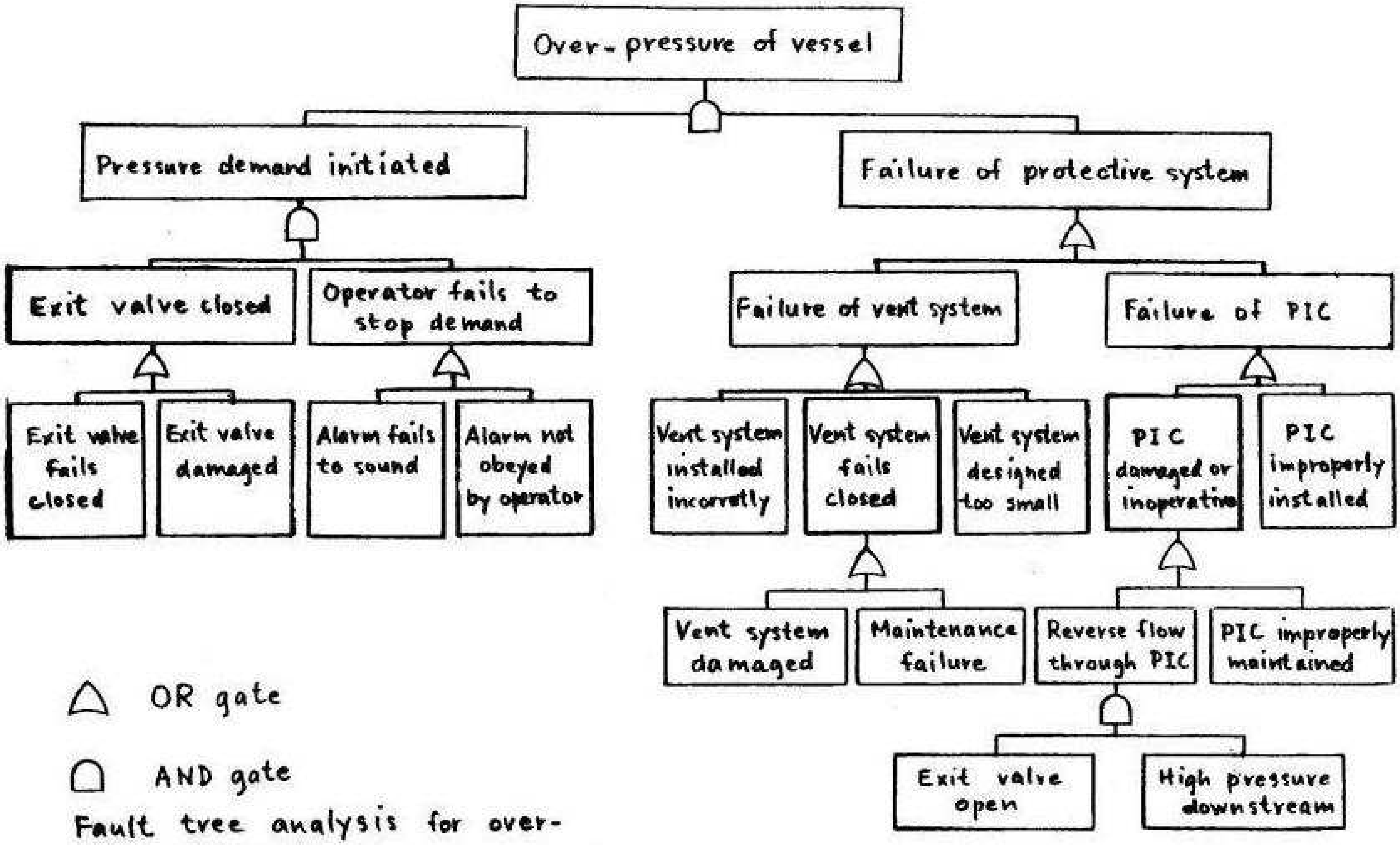
Guide word	Property	Possible cause	Possible consequence	Action required
More	Temperature	A	Pump seal failure, vapor lock	Install a feed–back line
Less	Temperature	Low ambient temperature	B	steam tracing
More	Flow	Line fracture	Spillage (possible large explosion)	C
		Pump seal failure	Spillage (possible small explosion)	D
		Control fault		Consider bypass
No	Flow	E	Shutdown	Low level alarm
		F	Shutdown	Automatic start–up of standby pump
Reverse	Flow	Pump failure	Backpressure on storage vessel	G
As well as	Impurities	H	Possible small detonation	Priming line

The missing items below complete the HAZOP study above:

A. Steam tracing operating incorrectly or downstream valve closed
B. Thermal stress on line
C. Install flow control with automatic shutoff
D. Flow diverted through standby pump capable of automatic startup
E. Tank empty
F. Pump failure
G. Relieve backpressure, divert flow through standby pump with automatic startup
H. Pump drawing in air or operator error

Answer

Pb# 21



Fault tree analysis for over-pressurizing vessel

Pb# 23

The diluted waste sample contains 15g of waste and 485g of dilution water. The total oxygen available for consumption by the waste is

$$g \text{ of } O_2 = (485)(9)/10^6 = 4.365\times10^{-3} g$$

The final oxygen content of the diluted sample is

$$g \text{ of } O_2 = (500)(4)/10^6 = 2.0\times10^{-3} g$$

The oxygen depletion due to the 15 g waste sample is

$$O_2 \text{ depletion} = 4.365\times10^{-3} - 2.0\times10^{-3} = 2.365\times10^{-3} g$$

The BOD, defined as the oxygen depletion in parts oxygen consumed per 10^6 parts of waste is then

$$BOD = (2.365\times10^{-3})(10^6/15) = \underline{\underline{158 \text{ parts } O_2/10^6 \text{ parts waste}}}$$

Answer

Pb# 27

The important factors to be considered for plant location in the fertilizer industry are (1) raw materials, (2) markets, (3) transportation facilities, (4) power and fuel availability, (5) waste disposal, and (6) water supply. The order of importance varies with the fertilizer considered and to some extent to the size of plant contemplated. Analysis of this situation for ammonia, urea, and phosphate fertilizer requires a knowledge of the raw materials required and the most commonly used process for each fertilizer. A concise review of these requirements is given by G.T. Austin in Shreve's Chemical Process Industries, 5th ed. About 90% of US ammonia production uses natural gas as the feedstock. The reactions are

$$CH_{4(g)} + H_2O \xrightarrow[2960\text{ kPa},\ 515^\circ C]{\text{Ni catalyst}} CO_{(g)} + 3\,H_{2(g)}$$

$$CH_{4(g)} + \text{air} \xrightarrow[670^\circ C]{\text{Ni catalyst}} CO_{(g)} + 2H_{2(g)} + N_{2(g)}$$

$$1/2\,N_{2(g)} + 3/2\,H_{2(g)} \xrightarrow[\text{pressure}]{\text{Fe catalyst}} NH_{3(g)}$$

The process requires large amounts of power and cooling water. Since natural gas is easily transported by pipeline, the plant need not be located near the gas well. Rather, it is more profitable to locate the plant close to the markets. Thus, the latter is the most important factor in plant location. Adequate power and water supply would also rank quite high. As the size of the plant increased, more consideration would have to be given to transportation facilities since the markets would be from the plant. Adequate supply of natural gas would also become important with plant size.

The commercial processes in current use for the manufacture of urea are based on two reactions, namely

$$CO_{2(g)} + 2NH_{3(l)} \xrightarrow[185^\circ C - 193^\circ C]{22\text{ MPa}} NH_4CO_2NH_{2(l)}$$

$$NH_4CO_2NH_2 \rightleftharpoons NH_2CONH_{2(l)} + H_2O_{(l)}$$

This process requires approximately 1/4 the power and cooling water of that required for the ammonia process per unit weight of fertilizer produced. Since ammonia is one of the principal raw materials required for the process, it is much more important to consider the source of the raw materials in a selection of the plant site for the production of urea. This factor will be the most important consideration even with an increase in plant size. However, since the fertilizer markets are not as large as for the raw material, careful consideration must be given to markets and the transportation necessar to reach those markets.

Phosphate fertilizer is obtained from phosphate rock ore. The accidulation of phosphate rock to produce superphosphate fertilizer can probably best be described by the following reaction:

$$CaF_2 \cdot 3Ca_3(PO_4)_2 + 7\,H_2SO_4 + 3H_2O \longrightarrow 3CaH_4(PO_4)_2 \cdot H_2O + 2HF + 7CaSO_4$$

Since the phosphate rock must be concentrated after it is mined, plant location is determined by the location of the phosphate rock ore regardless of plant size. The aspects of waste disposal of the non-phosphate rock is directly related to the mining operation. Markets directly affect plant size.

<u>Answer</u>

Pb# 31

The solution to this problem involves some of the same safety aspects as considered in the solution of Problem 2-13.

The most common method to vaporize chlorine is to use steam in a heating coil immersed in the chlorine liquid in the vaporizer. A schematic with typical instrumentation is shown in the solution of Problem 2-13 and will not be repeated here.

The disadvantage of using hot water to vaporize the chlorine is the need for a larger coil and a greater potential for freezing of the coil (NBP of Cl_2 is - 34°C). Economics is probably the major deterrent for not using a closed circuit heating with a heat transfer fluid that is inert to chlorine.

The use of a heating coil immersed below the liquid level in a chlorine vaporizer is the preffered arrangement to accomplish the vaporization. Not only is the coiled tube less expensive, it is also less susceptible to leaks and failures.

Answer

Pb# 34

The net heat transfer flux to a storage tank during a fire comes from radiation and convection attributed to the fire that surrouds tank and can be approximated by

$$q_{net} = q_r + q_c + q_{rr}$$

where q_r is the radiation flux from the fire, q_c the convection flux from the fire, and q_{rr} the radiation flux back from the tank. The radiation fluxes are proportional to the differences in the fourth powers of the absolute temperatures for the radiating and absorbing surfaces. The convection flux can be estimated of natural convection heat transfer. Since the surface of interest in this problem is large and vertical, the relations for natural convection on vertical plane surfaces are the most appropriate ones to use. The flame can be assumed to have transport properties equal to that of hot air in the flame and the fact that most of the gas in the flame is nitrogen. Flame temperatures vary from fuel to fuel, but a temperature of about 1500K will provide a reasonable average value for most situations.

If the benzene tank, assumed to be a vertical cylinder, is to store 1600 m^3, its minimum height is given by

$$V = \pi D_i^2 Z/4$$
$$Z = 4V/\pi D_i^2 = (4)(1600)/\pi(15)^2 = 9.05 \text{ m}$$

Storage tanks generally have an ullage volume of 10% or more to take care of liquid expansion and provide additional volume for some vaporization to occur without appreciably increasing the tank pressure. Heat transfer to the surface area surrounding the ullage space will be minimal compared to the surface area wetted by the full tank contents. Thus, only the latter area will be used in the heat transfer calculation. Also assume that the temperature of the wetted wall to be at the boiling point temperature of the fluid being stored in the tank. There will be a temperature gradient through the tank wall, but this will be small and have minimal effect on the required venting rate during a fire.

$$q_r = 95{,}000 \ W/m^2$$

is given in the problem statement and assumes that the external fire completely surrounds the storage tank. The convective flux from the fire is given by

$$q_c = h(T_f - T_w)$$

where T_f is the flame temperature and T_w is the wall temperature, assumed to be the same as the normal boiling temperature of the fluid in the tank. The heat transfer coefficient in US customary units for natural convection along a vertical surface may be obtained numerically from

$$Nu = \left\{ 0.825 + \frac{0.387 \ Ra^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2$$

as given in <u>Perry's Chemical Engineers' Handbook</u>, 6th ed., where $Nu = hL/k$, $Pr = C_p \mu / k$, $Ra = Gr\,Pr$, and $Gr = L^3 \rho^2 g \beta \Delta T / \mu^2$. Since this calculation is rather lengthy and natural convective heat transfer coefficients for air generally have values of 5 to 10 $W/m^2 \cdot K$, we will assume an average value of 7.5 $W/m^2 \cdot K$ and only calculate a better value if analysis warrants it.

Using this assumption

$$q_c = (7.5)(1500 - 353) = 8600 \ W/m^2 \cdot K$$

The heat flux radiated back from the tank surface to space can be estimated from

$$q_{rr} = \epsilon \sigma T_w^4 \qquad \text{Assume } \epsilon = 1.0$$

$$= (1.0)(0.972 \times 10^{-8})(353)^4 = 150 \ W/m^2 \cdot K$$

From these calculations

$$q_{net} = 950{,}000 + 8600 - 150 = 103{,}450 \ W/m^2 \cdot K$$

The vapor boil-off rate is then

$$\dot{m} = (q_{net})A/\Delta H_v \qquad \Delta H_v = \text{heat of vaporization}$$

$$= (q_{(net)})\pi D Z/\Delta H_v$$

$$= (103{,}450)\pi(15)(9.05)/3.943\times10^5 = 111.9 \text{ kg/s}$$

At the NBP of benzene, the vapor density is 2.755 kg/m^3. Thus

$$\dot{m}_v = \dot{m}/\rho = 111.9/2.755 = 40.65 \text{ m}^3/\text{s}$$

The vent area needed for this volumetric flow rate can be determined by use of an orifice equation of the form

$$A = \frac{\dot{m}}{0.61[(2)\Delta p\,\rho]^{1/2}} \qquad \Delta p = 250 \text{ Pa}$$

$$= \frac{111.9}{(0.61)[(2)(250)(2.755)]^{1/2}} = 4.94 \text{ m}^2$$

The area for the vent is much too large for the tank. Therefore a weak seam roof <u>should be specified</u>

Answer

Several assumptions were made in the solution of this problem and need to be verified. First, the convective flux is only 9% of the total heat flux. Even if it were changed ±100%, the overall result would only be changed slightly. Thus, an exact calculation of the heat transfer coefficient would not change the final conclusion. There is some resistance to heat transfer through the walls of the tank. For a tank wall of 0.0125m, the temperature gradient is

$$\Delta T = q_{(net)}\,t_w/k_w = (103{,}450)(0.0125)/45 = 28.7\,°\text{C}$$

This will have a negligible effect on the results, and thus this assumption is acceptable. However, there is also a thermal resistance with the film on the inside of the tank. Typical heat transfer coefficients are on the order of several thousand W/m^2·K. Assuming a value of

2,000 $W/m^2 \cdot K$, the ΔT across the film is

$$\Delta T = q_{(net)}/h = 103{,}450/2000 = 51.7\,°C$$

The combination of these two resistances only reduces the convective flux rate by approximately 6%. This change would have negligible effect on the final answer.

As a footnote, the NFPA Fire Protection Handbook and API 650 all recommend the following equation for the net heat flux to a tank subjected to an external fire:

$$q_{(net)} = 43{,}200\ A^{0.82}$$

where $q_{(net)}$ is in watts and A is the wetted area in m^2. For this problem

$$q_{(net)} = 43{,}200\,[\pi D_i Z]^{0.82} = 43{,}200\,[\pi(15)(9.05)]^{0.82}$$

$$= 6.19 \times 10^6\ W$$

from this

$$\dot{m}_v = q_{(net)}/\Delta H_v \rho$$

$$= 6.19 \times 10^6/(3.943 \times 10^5)(2.755) = 5.6\ m^3/s$$

The major difference in results is that the heat flux in the problem statement applies to a fire that completely surrounds the tank. Generally fires are seldom large enough to surround large tanks. In addition a soot layer builds up on the cool tank walls which further restricts the heat transfer. Thus, the total heat flux is reduced substantially. However, the venting rate even under these conditions will be large and a tank with a weak roof seam would be recommended.

CHAPTER 06

Cost Estimation

Items P ☆

Cost Estimation

Example 1 Estimation of fixed capital Investment using ranges of process plant component costs.

Make a study estimate of the fixed capital investment for a process plant if the purchased equipment cost is $ 100,000. Use the ranges of process plant component cost outlined in Table B for a process plant handling both solids & fluids with a high degree of automatic controls & essentially outdoor operation.

Soln

Table B.

Component	Range %
Direct costs	
① Purchased equipment	15-40
② Purchased equipment installation	6-14
③ Instrumentation & controls (installed)	2-8
④ Piping (installed)	3-20
⑤ Electrical (installed)	2-10
⑥ Buildings (including services)	3-18
⑦ Yard improvement	2-5
⑧ Service facilities (installed)	8-20
⑨ Land	1-2
Indirect costs	
① Engineering & supervision	4-21
② Construction expense	4-16
③ Contractor's fee	2-6
④ Contingency	5-15
	57 . 195

Now we will take middle ranges of all the %ag

Purchased equipment cost = 100,000.

As we know that the purchased equipment cost is app 25% of the Total investment.

$\frac{25}{100}$ × Total Investment = Purchased equipment cost

$$\frac{\text{Purchased equipment cost}}{0.25} = \text{Total Investment.}$$

Total Investment (app) = $\frac{100,000}{0.25}$ = 400,000 $

Now we will calculate approximately actual total Investment & their ratios.

Purchased equipment installation = 9% of T.I

= 0.09 × 400,000 = 36,000

Components	Assumed % of total	Cost	Ratiod % of total.
① Purchased equipment.	25	100,000	23 = $\frac{100,000}{436,000}$ ×100
② Purchased equipment installation	9	36,000	8.3
③ Instrumentation (installed)	7	28,000	6.4
④ Piping (installed).	8	32,000	7.3
⑤ Electrical (installed).	5	20,000	4.6
⑥ Buildings.	5	20,000	4.6
⑦ Yard improvement.	2	80,000	1.8
⑧ service facilities	15	60,000	13.8
⑨ Land.	1	4,000	0.9
⑩ Engg & supervision	10	40,000	9.2
⑪ Construction expense	12	48,000	11
⑫ Contractor's fee	2	8000	1.8
⑬ contingency.	8.	32,000	7.3.
	119.	436,000	100

Note : Range = 371,000 → 501,000.

If inflation is taken. Range = 436,000 → 566,000.

—x— —x— —x—

Estimating equipment cost by scaling.

$$\text{cost of equip a} = \text{cost of equip b}\left(\frac{\text{capacity of equip a}}{\text{capacity of equip b}}\right)^{x}$$

→ x is calculated from table 5. when years are same.

x = exponent.

$$\text{Years ratio} = 1 \qquad \frac{\text{Year in which to be calculated}}{\text{Year in which it is given (cost)}}$$

Example 2 Estimating cost of equipment using scaling factor & cost index.

The purchased cost of a 50gal glass lined, jacketed reactor (without drive) was $8350 in 1981. Estimate the purchased cost of a similar 300 gal, glass lined, jacketed reactor (without drive) in 1986. Use the annual avg Marshall & swift Index (all industry) to update the purchase cost of the reactor.

Soln capacity in 1981 = 50 gallons.

cost in 1981 (50gallon) = 8350$

capacity in 1986 = 300gallons

cost in 1986 = ?

$$\text{cost in 1986} = \text{cost in 1981}\left(\frac{\text{cap in 1986}}{\text{cap in 1981}}\right)^{x}\left(\frac{\text{cost index in 1986}}{\text{cost index in 1981}}\right)$$

163 Q

Table 3

cost index in 1981 (Marshell & swift index, all Industry) = 721

Cost index in 1986 = 798.

exponent for reactor glass lined, jacketed (without drive) = 0.54

Table 5 (170)

cost in 1986 = $8350\left(\frac{300}{50}\right)^{0.54} \times \left(\frac{798}{721}\right)$

= 24319.

→ The exponent for both the values (capacities) should lie in the same range.

e.g. Reactor 500 gallons. cost in 1980 = 3000.

3000 gallons, cost in 1980 = ?

500 - 1000 capacity (gallons), exponent = 0.5.

1000 - 3000, exponent = 0.55.

Soln since we have to calculate the (capacity) gallons from 500 to 3000 but both have different exponents,

so we should first bring them to same range.

so first calculate cost for 1000 capacity (gallons) using 500 (capacity) gallons.

~~capacity~~ cost of 1000 = ~~cap~~ cost of 500 $\left(\frac{\text{cap of } 1000}{\text{cap of } 500}\right)^{0.5}$

$3000\left(\frac{1000}{500}\right)^{0.5} = 4242$

Now we will calculate cost of 3000 gallons as 1000 & 3000 are both in same range.

cost of 3000 = cost of 1000 $\left(\frac{\text{cap of } 3000}{\text{cap of } 1000}\right)^{0.55}$

$= 4242\left(\frac{3000}{1000}\right)^{0.55} \Rightarrow$ cost = 7762 $

★ **Example3** Estimation of fixed capital Investment by percentage of delivered equipment cost.

Prepare a study estimate of the fixed capital investment for the process plant described in exampl, if the delivered equipment cost is 100,000.

Soln Now we will use Ratioed factors outlined in Table 17. → Solid & fluid Processing plants (Ex-1)
Tab6 6.9 5th edition

Components	Cost ($)
① Purchased equipment (delivered) E	100,000
② Purchased equipment installation 39% E	39,000
③ Instrumentation (installed), 28% E	28,000
④ Piping (installed), 31% E	31,000
⑤ electrical (installed), 10% E	10,000
⑥ Buildings (including services) 22% E	22,000
⑦ Yard improvements 10% E	10,000
⑧ service facilities (installed), 55% E	55,000
⑨ Land 6% E	6,000
Total direct costs (D)	301,000
Engineering & supervision 32% E	32,000
Construction expenses 34% E	34,000
Total Direct & Indirect costs (D+I)	367,000
Contractor's fee, 5% (D+I)	18,000
Contingency 10% (D+I)	37,000
Fixed capital Investment	422,000

Example 4. Estimating relative costs of construction labour as a function of geographical area.

If a given chemical Process plant is constructed near Dallas (southwest area) with a construction labour cost of $100,000. What would be the construction labor cost of an identical plant if it were to be erected at the same time near Los Angeles (Pacific Coast area) for the time when the factors given in Table 20 apply?

Soln

Table 20 Relative labor rate & productivity indexes in the chemical & allied products industries for the United states.

Geographical area	Relative labor rate	Relative productivity factor.
New England	1.14	0.95
Middle Atlantic	1.06	0.96
South Atlantic	0.84	0.91
Midwest	1.03	1.06
Gulf	0.95	1.22
Southwest	0.88	1.04
Mountain	0.88	0.97
Pacific coast	1.22	0.89.

Soln Construction labor cost in Dallas (south west area) = 100,000

construction labor cost in Los Angles (Pacific coast area) = ?

Relative labor rate in Southwest area = 0.88

Relative labor rate in Pacific cost area = 1.22.

$$\text{Relative labor rate ratio} = \frac{1.22}{0.88} = 1.3864$$

Relative productivity factor in southwest = 1.04

" " " " Pacfic cost = 0.89.

Relative productivity ratio factor $= \frac{0.89}{1.04} = 0.8558.$

$$\left(\begin{array}{l}\text{Construction labor cost of} \\ \text{southwest to Pacific coast}\end{array}\right) = \frac{1.3864}{0.8558} = 1.620$$

Construction Labor cost at Pacific coast (Los Angles) = 1.620 × 100,000

= 162,000

Problem I The purchased cost of a shell & tube H.E (floating head & carbon steel tubes) with 100 ft² of heating surface was \$ 3000 in 1980. What will be the purchased cost of a similar H.E with 200 ft² of heating surface in 1980 if the purchased cost capacity exponent is 0.6 for the surface area ranging from 100 → 400 ft².

b) If the purchased ~~equipment~~ cost capacity exponent for this type of H.E is 0.81 for surface areas ranging from 400 → 2000 ft², what will be the purchased cost of H.E with 1000 ft² of heating surface in 1985?

Soln (a) Purchased cost of 100 ft² H.E = 3000 (1980).

Purchased cost of 200 ft² H.E (1980) = ?

Purchased cost capacity exponent = x = 0.6

$$\overset{\text{cost}}{\cancel{\text{capacity}}} \text{ of } b = \overset{\text{cost}}{\cancel{\text{capacity}}} \text{ of } a \left(\frac{\text{capacity of } b}{\text{capacity of } a}\right)^{x}$$

Since years are same, so their index = 1

$$\text{cost of } 200\,\text{ft}^2 \text{ H.E in, 1980} = 3000\left(\frac{200}{100}\right)^{0.6}$$

$$= 4547 \,\$.$$

b) Purchased cost of $100\ ft^2$ H·E = 3000 \$ (1980)

Purchased cost of $1000\ ft^2$ H E = ? (1985).

Purchased cost exponent for heating surface $400 - 2000\ ft^2 = 0.81$

Since we have to calculate the value from 1980 → 1985, so first of all their cost indexes should be calculated from table 3. (163)

Cost Index (Process Industry) in 1985 = 813

Cost Index in 1980 = 675

↑ Marshall & Swift index.

As the range of new exponent is from $400 \rightarrow 2000\ ft^2$ but we have cost of range of $100\ ft^2$ so we can not calculate directly from $100\ ft^2 \rightarrow 1000\ ft^2$

1) Either calculate for $400\ ft^2$ in 1980.

2) or for $400\ ft^2$ in 1985 & then convert then $400\ ft^2$ in 1985 to $1000\ ft^2$ in 1985.

From $100 \rightarrow 400\ ft^2$ in 1980 $\boxed{x = 0.6}$

$$\text{cost of } b = \text{cost of } a \left(\frac{\text{cap of } b}{\text{cap of } a}\right)^x$$

$$= 3000\left(\frac{400}{100}\right)^{0.6} = 6892$$

From $\underset{1980 \rightarrow 1985}{400 \rightarrow 1000\ ft^2}$ $x = 0.81\ (400 \rightarrow 2000\ ft^2)$

$$\text{cost of } b = \text{cost of } \overset{(400ft^2)}{a}\left(\frac{\text{Index in } 1985}{\text{Index in } 1980}\right)\left(\frac{\text{cap of } b}{\text{cap of } a}\right)^x.$$

$$= 6892\left(\frac{813}{675}\right)\left(\frac{1000}{400}\right)^{\boxed{0.81}}$$

$$= 17436$$

———x———x———x———

③

P 6.3 The purchased & installation costs of some pieces of equipment are given as a function of wt rather than capacity. An example of this is the installed costs of large tanks. The 1980 costs for an installed aluminum tank wt 100,000 lb was 390,000 $. For a size range from 100,000 to 1,000,000 lb the installed cost weight exponent for Al tanks is 0.93. If an aluminium tank weighing 700,000 lb is required, what is the present capital required? (1998)

Soln cost of 100,000 lb of Aluminium tank (1980) = 390,000

cost of 700,000 lb, of Al tank (1998) = ?

cost weight exponent for Al (100,000 → 1,000,000) = 0.93

cost Index, 1980 = 675 (Process Industry)

cost Index 1998 = 1200 → Table 3. (163)

(Unknown)

Note Unless stated any other index, Use Marshall & Swift index.

$$\text{cost of } b = \text{cost of } a\left(\frac{\text{wt of } b}{\text{wt of } a}\right)^{x}\left(\frac{\text{Index in 1998}}{\text{Index in 1980}}\right)$$

$$= 390{,}000\left(\frac{700{,}000}{100{,}000}\right)^{0.93}\left(\frac{1200}{675}\right)$$

$$= 4.24 \times 10^{6}$$

Problem 6.4 What weight of the installed stainless steel tank could have been obtained for the same capital investment as in the previous problem? The 1980 cost for an installed 304 stainless steel tank weighing 300,000 lb was $670,000. The installed cost weight exponent for

stainless steel tanks is 0.88 for a size ranges fro
300,000 to 700,000 ?

Soln capital investment for previous problem = 4.24×10^6

wt of 1998 (4.24×10^6 \$) = ? [illegible] in 1998.

cost in 1980 = 670,000 (300,000 Lb)

cost to weight exponent for stainless steel tank = 0.88.
(300,000 → 700,000)

$$\underset{4.24\times10^6}{\text{cost of } b} = \text{cost of } a\left(\frac{\text{Index of } b}{\text{Index of } a}\right)\left[\frac{\text{wt of } b}{\text{wt of } a}\right]^{0.88}$$

$$4.24\times10^6 = 670{,}000\left(\frac{1200}{675}\right)\left(\frac{\text{wt of } b}{300{,}000}\right)^{0.88}$$

$$\left(\frac{\text{wt of } b}{300{,}000}\right)^{0.88} = 3.559$$

$$\left(\frac{\text{wt of } b}{300{,}000}\right)^{0.88\times\frac{1}{0.88}} = (3.559)^{1/0.88}$$

$$= 4.23$$

$$\text{wt of } b = 4.23 \times 300{,}000$$

$$= 1269{,}000$$

$$= 1.269 \times 10^6$$

~~Calculation of No of stages (N).~~ ⑤

$$\cancel{\frac{X_2 - X_1}{X_2 - Y_1/m} = \frac{\left(\frac{1}{A}\right)^{N+1} - \frac{1}{A}}{\left(\frac{1}{A}\right)^{N+1} - 1}}$$

A P

Problem 6.5 The purchased cost of a 1400 gal stainless steel tank in 1980 was \$7500. The tank is cylindrical with flat top & bottom, & the diameter is 6 ft. If the entire outer surface of the tank is to be covered with 2 in thickness of magnesia block, estimate the present total cost for the installed & insulated tank. The January 1, 1980 cost for the 2 in magnesia block was \$ 2.20/ft². while the labor for installing the insulation was \$5/ft².
(cost of installation & insulation should also be used)

soln As the entire outer surface is to be installed & installed. so first we will be calculate the outer surface area & then will calculate its ~~insta~~ insulation cost & labor cost & then we will calculate total installed cost

$$\text{Surface area} = 2(\pi r^2) + \overset{2\pi r h}{\overset{D}{\underline{\pi\, D h}}}$$

for top & bottom

$$r = \frac{\text{Dia}}{2} = \frac{6\,\text{ft}}{2} = 3\,\text{ft}.$$

$h = ?$

As volume of the tank $= \pi r^2 h$

$$h = \frac{\text{volume}}{\pi r^2} = \frac{1400\,\text{gal}}{3.1416 \times (3)^2\,\text{ft}^2} \Bigg| \frac{1\,\text{ft}^3}{7.48\,\text{gallons}}$$

$= 6.62\,\text{ft}$

$$\text{Now surface area} = 2(3.1416 \times 3^2) + 3.1416 \times \overset{D}{6\,\text{ft}} \times \overset{h}{6.62\,\text{ft}^2}$$

$= ~~[illegible]~~ 181.33\,\text{ft}^2$

→ As cost of insulation of 2 in magnesia block = 2.20\$/ft²

→ cost of insulation $= \frac{2.20\$}{\text{ft}^2} \times 181.33\,\text{ft}^2 = 399\$$.

→ Cost of Labor = \$5/ft²

Total cost of Labor = 5 × 181.33 = 906.6 \$

(As total cost of equipment installed & insulated) = cost of equipment + cost of installation + Labor charges + Insulation costs

Cost of installation as a function of ~~total~~ purchased equipment cost is expressed in table 6 [Page 171]

→ cost of installation = 45% of cost of purchasing equipment

= 0.45 × 7500 = 3375 \$

(Total cost of installed & insulated equipment) = 7500 + 3375 + 906.6 + 399 ?

= 12180 \$ (in 1980)

Present cost of equipment in 1998 = ?

$$\text{Present cost} = \text{cost in } 1980 \left[\frac{\text{cost index } 1998}{\text{cost index } 1980} \right]$$

As capacity/wt is same so their ratio = 1

$$\text{Present cost} = 12180 \times \frac{1200}{675} = 21653$$

Pb 6.6 A one story ware house 120 by 60 ft is to be added to an existing plant. An asphalt pavement service area 60 by 30 ft will be added adjacent to the ware house. It will also be necessary to put in 500 linear ft of railroad siding to service the ware house. Utility service lines are already available at the warehouse site. The proposed ware house has a concrete floor & steel frame, walls & roof. No heat is necessary but the lighting & sprinklers must be installed. Estimate the total cost of the proposed addition. (Consult Appendix B for necessary cost data).

Soln Appendix B (Page 803, 804, 805, 806)

First we will calculate area of ware house (120×60) & then multiply with its cost/ft^2 & then area of service road is calculated by multiplying its cost/ft^2.

→ Cost of railroad = $63/lin ft (805)

Total cost of railroad = $\frac{63}{\text{lin ft}} \times 500\,\text{lin ft} = \underline{31500}$ $

Area of ware house = 120 × 60 = 7200 ft^2

→ cost of building & construction = $28/$ft^2$. (804)

Total cost of building & construction of ware house = 7200×28

= 201600 $

Area of asphalt pavement = 60 × 30 = 1800 ft^2.

→ cost of asphalt pavement = 12.9 $/$yd^2$ (806)

Area of asphalt pavement in yards = $\frac{1800\,ft^2}{} \Big| \frac{1\,yd^2}{(3)^2\,ft^2}$

= 200 yd^2

Total cost of asphalt pavement = 200 × 12.9

= 2580 $.

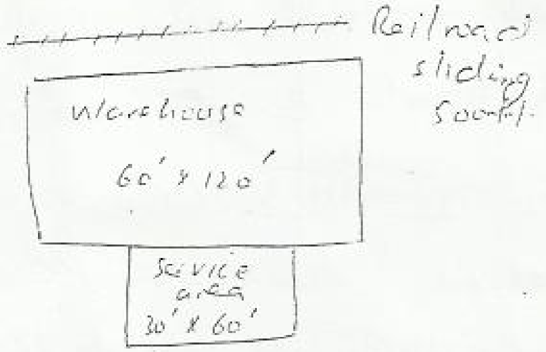
→ Sprinkling cost $\overset{\text{Dry (No heat)}}{}$ = 2.1 $/$ft^2$ (803)

Total cost of sprinkling = $\frac{\text{cost of sprinkling}}{ft^2}$ × Area of ware house

= 2.1 × 7200 = 15120 $

~~Dry system / cost = 2.1 $/ft²~~
~~(at least)~~

~~Total cost = 2.1 × 7200~~
~~= 15120 $~~



(Total cost of the ware house) = Railroad + building construction + asphalt pavement + sprinkling system ~~+ Dry system~~

= 31500 + 201600 + 2580 + 15120

= 250800 $

Imp 17 table

⑦ The purchased cost of equipment for a solid processing plant is 500,000. The plant is to be constructed as an addition to an existing plant. Estimate the total capital investment & the fixed capital investment for the plant. What %age & amount of the fixed capital investment is due to cost for land & contractor's fee ?

Soln Table 17. (183)

Total equipment cost = 500,000 $

→ Installation cost (45% of equipment cost) = 0.45 × 500,000
= 225000

→ Instrumentation & Process control, (9%) = 45,000

→ Piping cost = (16%) = 0.16 × $\overset{500}{\cancel{60}}$,000 = 80,000

→ Electrical cost, = 10% = 0.10 × 500,000 = 50,000

→ Building. 25% = 0.25 × 500,000 = 125,000

→ Yard Improvement = 0.13 × 500,000 = 65,000

→ Service facilities = 0.4 × 500,000 = 200,000

→ Land = 0.06 × 500,000 = 30,000

$\boxed{\text{Total Direct cost} = 1320{,}000}$

→ Engineering & supervision = 0.33 × 500,000 = 165,000

→ construction expenses = 0.39 × 500,000 = 195,000

$\boxed{\text{Total cost}}$ = Direct cost + Indirect cost
= 1320,000 + 165,000 + 195,000 = 1680,000

Contractor's fee is 5% of total direct & Indirect cost.

→ Contractor's fee = $\frac{5}{100}$ × 1680,000 = 84,000.

→ Contingency = $\frac{10}{100}$ × 1680000 = 168,000

$\boxed{\text{Fixed capital Investment}}$ = Direct cost + Indirect cost + Contractor's fee + Contingency
= 1680,000 + 84,000 + 168,000
= 1932,000

from the table 7, total capital Investment is 455.

→ [Fixed capital Investment = 387,
→ working capital = 68
Total capital Investment = 455/) → ~~[illegible]~~ % of purchased cost

$\boxed{\text{Total capital Investment}}$ = $\frac{455}{100}$ × 500,000
= 22 750,00

[or Working capital = 68% of purchase cost
= 0.68 × 509 000 = 340,000
Total capital Investment = F.C.I + ~~Total~~ w.c].

%age of fixed capital investment due to land $= \frac{}{F.C.I} \times 100$

$= \frac{30,000}{1932,000} \times 100 = 1.553\%$ land of F.C.I

%age of F.C.I due to contractor's fee $= \frac{86000}{1932,000} \times 100$

$= 4.35\%$

Total %age due to contractor fee & land $= 4.35 + 1.553$

$= 5.903\%$

<u>OR</u> T.C.I = 4.55 × 500,000 = 2275,000

F.C.I = 3.87 × 500,000 = 1932,000.

Land & contractor's fees are 23% of purchased equipment cost = 0.23 × 500,000 = 115 000

% = 5.95%

---

⑧ The purchased equipment cost for a plant which produces pentaerythritol (solid-fluid processing plant) is \$ 300,000. The plant is to be an addition to an existing formaldehyde plant. The major ~~plant~~ part of the building cost will be for indoor construction & contractor's fee will be 7% of the direct plant cost. All other costs are close to average values found for typical chemical plant. On the basis of this, estimate the following.

a) the total direct plant cost.

b) the fixed capital investment.

c) the total capital investment.

<u>Soln</u> <u>Table. 17</u>

Total Direct costs for <u>solid - fluid processing</u> plant = 293 of purcha equipment cost

Land cost = 6 %

As land is not required, cg the plant i added to an existing plant, so

Total direct cost = 293 - 6 = 287 %

As Total direct cost is 287 % of purchase equipment cost

so actual direct cost = $\frac{287}{100} \times 300{,}000 = 861{,}000$

$\boxed{\text{Total direct cost} = 861{,}000}$

b) Fixed capital Investment

Fixed capital Investment = 413 % of equipment cost.

As land is not required so substract 6%, and contractor's fee is 18 % of equipment cost so now add contractor fee given 7%.

Fixed capital Investment = $413 - 6 - 18 + 7 = 396\ \%$

Total F.C.I = $\frac{396}{100} \times 300{,}000 = 1188{,}000$

$\boxed{\text{Total F.C.I} = 1{,}188{,}000}$

c) the total capital investment.

Total capital Investment = 487 %

Land (not required) = 6 %

Contractor fee (Actual) = 18 %

Contractor fee (used) = 7 %

Total capital Investment = $487 - 6 - 18 + 7$

$= 470$

Total capital Investment = $\frac{470}{100} \times 300{,}000$

$\boxed{= 1410{,}000\ \$}$

soln 6.9 All Table 19 SPX

Estimate by the turnover ratio method the fixed capital investment required for a proposed sulfuric acid plant (battery limit) which has a capacity of 140,000 tons of 100% sulphuric acid per year (contact catalytic process) using the data from table 19 for 1990 with sulphuric acid costs at $72/tonn. The plant may be considered as operating full time. Repeat using the cost capacity exponent method with data from table 19?

Soln ~~Capacity~~ = 140,000

cost of H_2SO_4 = 72/tonn.

$$\text{Turnover ratio} = \frac{\text{gross annual sales}}{\text{Fixed capital investment}}$$

from table 19 (1986) → For capacity of 100,000 tonns/year,

→ the F.C.I required = 3000,000 = 3 Million

cost of sulphuric acid/tonn = 72

$$\text{Turnover ratio} = \frac{\text{Price/tonn} \times \text{total Production}}{\text{F.C.I}} = \frac{\frac{72}{\text{tonn}} \times \overset{\text{Table ?}}{\left(\frac{100{,}000\,\text{ton}}{\text{year}}\right)}}{3000{,}000}$$

$$= 2.4$$

Now for capacity of 140,000, F.C.I = ?

$$\text{F.C.I} = \frac{\text{Price/tonn} \times \text{total production}}{\text{Turnover ratio}}$$

$$= \frac{\overset{\text{Actual}}{140{,}000} \times 72}{2.4} = 420{,}0000$$

b) <u>For cost capacity method.</u> Since Year is same, so Year ratio = 1.

$$\text{cost for } b = \text{cost for } a \left(\frac{\text{capacity of } b}{\text{capacity of } a} \right)^{x}$$

x = Power-factor for plant capacity ratio = 0.65 (From table 19 / 186)

$$\text{cost for } b = 3000{,}000 \left(\frac{140{,}000}{100{,}000} \right)^{0.65}$$

$$= 373{,}3399 \ \$$$

Pb 6.10 The total capital investment for a chemical plant is \$1million & the working capital is \$100,000. If the plant can produce an avg of 8000kg of final product per day ~~using~~ during a 365 day of year, what selling price in dollars per kg of product would be necessary to give a turnover ratio of 1?

Soln

$$\boxed{\text{Turn over ratio} = \frac{\text{Annual gross sales}}{\text{fixed capital Investment}}}$$

Annual gross sales = Price x Total production.

As turn over ratio = 1,

Total capital Investment = 1000,000

working capital = 100,000

Total capital Investment = F.C.I + W.C.

F.C.I = Total capital Investment - W.C

= 1000,000 - 100,000 = 900,000

Annual gross sales = Turn over ratio x F.C.I

= 1x 900,000 = 900,000

As annual gross sales = Price x total production

$$\frac{900{,}000\,\$}{\text{year}} = \text{Price} \times \frac{8000\,\text{kg}}{\text{day}}.$$

$$\text{Price} = \frac{900{,}000/\text{year}}{8000\,\text{kg}/\text{day}} = \frac{900{,}000}{\text{year}}\bigg|\frac{\text{day}}{8000\,\text{kg}}\bigg|\frac{1\,\text{year}}{365\,\text{day}}\bigg|$$

$$= 0.308 \text{ dollar}/\text{kg}$$

---

soln

Problem 6.11 A process plant was constructed in the philadelphia area (Middle Atlantic) at a labor cost of $200,000 in 1980? what would the average costs for the same plant to be in the Miami, Florida area (South Atlantic) if it were constructed in late 1988? Assume for simplicity, that the relative labor rate to relative productivity factor remains essentially constant?

Soln Labor cost in Middle atlantic in 1980 = 200,000

Labor cost for Miami, Florida (south atlantic) 1988 = ?

→ Relative Labor rate in Middle atlantic = 1.06

(188) Table-18 → Relative Labor rate in south atlantic = 0.84

$$\text{Relative labour rate ratio} = \frac{\text{Relative labor rate ratio in south atlantic}}{\text{Relative labor rate ratio in Middle atlantic}}$$

$$= \frac{0.84}{1.06} = 0.792.$$

→ Relative productivity rate in south atlantic = 0.91

→ Relative productivity rate in middle atlantic = 0.96

$$\text{elative productivity rate}\} = \frac{\text{in south atlantic}}{\text{in Middle atlantic}} = \frac{0.91}{0.96} = 0.948$$

①

~~Labor cost of M.A to S.A~~ = $\frac{0.792}{0.948} = 0.83552$

$\frac{\text{Relative labor rate ratio}}{\text{Relative productivity factor ratio}}$

Labor cost in South atlantic = 0.83552 X 200,000
(in 1980)
= 167103.3

As the relative labor rate & relative productivity factor remains essentially constt. so cost in 1980 is 167103 $

(for philadephia) Table # 24 201

Labor cost at S.A in 1988 = $167103 \times \frac{\text{Index of 1988}}{\text{Index of 1980}}$

$= 167103 \times \frac{454}{267} = 284,138$ $

Table 27

B ★

6.13 The total capital investment for a conventional chemical plant is 1,500,000 & the plant produces 3 million kg of product annually. The selling price of the product is $0.82/kg. Working capital amount to 15% of the total capital investment. The investment is from company funds & no interest is charged. Raw materials costs for the product are $0.09/kg, Labor 0.08 $/kg, utilities $0.05/kg & packaging 0.008 $/kg. Distribution costs are 5% of the total product cost. Estimate the following.

a) Manufacturing cost per kg of product.

b) Total production cost per year.

c) Profit per kilogram of product before taxes.

d) Profit per kg of product after taxes (use current rate).

Total capital Investment (TCI) = 1500,000

→ Production/year = 3×10^6 kg.

→ selling price = \$ 0.82/kg.

→ working capital = 15% of TCI = 0.15 × 1500,000 = 225,000

→ Interest rate = 0

→ Raw material cost = 0.09/kg

→ Labor cost = 0.08/kg

→ utilities = 0.05/kg

→ Packaging = 0.008/kg

Distribution costs = 5% of total product cost.

Ⓐ Manufacturing cost = Direct ~~cost~~ production cost + fixed charges + plant overhead

Table 27.

① Direct Production cost

We will calculate all the costs per kg of product.

→ F.C.I = T.C.I − W.C = 1500,000 − 225,000 → W.C = 15% of T.C.I

= 1,275,000

$\frac{F.C.I}{kg}$ = 1,275,000 / 3×10^6 = 0.425 \$/kg.

	cost/kg \$
→ 2) Raw material	0.09
→ 2) Operating labor	0.08
→ 3) supervisory & clearical labor (15% of operating labor)	$\frac{15}{100} \times 0.08 = 0.012$
→ 4) Utilities	0.05
→ 5) Maintenance & repairs 6% $\frac{FCI}{kg}$	$0.425 \times \frac{6}{100} = 0.0255$
→ 6) Operating supplies (15% maintenance & repairs)	$\frac{15}{100} \times 0.0255 = 0.0038$
→ 7) Laboratory charges (15% operating labor)	0.15 × ~~0.0255~~ 0.08 = 0.012

→ ⑧ Patents & royalities (3% of $\frac{T.P.C}{kg}$) = 0.000
Total product is not known to us.
total product cost /kg.

→ ⑨ Packagings

0.008

Total = 0.2813 %

Ⓑ Fixed charges.

cost /kg

→ 1) Depriciation 10% $\frac{FCI}{kg}$ = 0.1×0.425 = 0.0425.

→ 2) Local tax (2.5% of $\frac{FCI}{kg}$) = $\frac{9.5}{100}$×0.425 = 0.010625

→ 3) Insurance $\frac{0.7\%\ FCI}{kg}$ = $\frac{0.7}{100}$×0.425 0.00298

→ 4) Rent (8-12% value of Rented land & building) 0

It is not told about rented buildings, so ignore them.

Total fixed charges = 0.056105

Ⓒ Plant overhead costs

$\underline{POH}$ = (Overhead, supervision, maintenance (60% of operating labor, supervison & maintenance)) = 0.6 × (0.08 + 0.012 + 0.0255)
operating labor, supervision, Maintenance.
= 0.0705

Manufacturing cost = Direct plant cost + fixed charges + plant overheads cost

= 0.2813 + 0.056105 + 0.0705

$\boxed{= 0.408\ \$/kg}$ part a.

(b) Total product~~co~~ cost per year.

Total product cost = Manufacturing cost + General expenses.

Manufacturing cost = ~~0.385~~ $\frac{\$}{kg}$
0.408

General expenses.

→ 1) Administrative costs (15% operating labor, supervision & maintenance) $= 0.15\times(0.08+0.012+0.0255) = 0.0176$

→ 2) Distribution & selling cost (5% of total product cost) $= \overset{0.05}{\underline{0.5}}\times TPC$

→ 3) Research & Development (5% of TPC) $= \overset{0.05}{\underline{0.5}}\times T.P.C$

→ 4) $\overset{\text{financing}}{\uparrow}$ (Interest) = (No interest) 0

Total product cost/kg = Manufacturing cost/kg + general expense/kg

$$TPC = 0.408 + 0.0176 + (0.5+0.5)TPC$$

$$TPC - 0.10\,TPC = 0.408 + 0.0176$$

$$TPC = \frac{0.4256}{0.9} \Rightarrow \boxed{TPC = 0.473\ \frac{\$}{kg}}$$

$$\Rightarrow \boxed{\text{Total product cost/year}} = 0.473\ \frac{\$}{kg} \times \frac{3\times10^6\ kg}{year} = \boxed{1.42\times10^6\ \frac{\$}{year}}$$

Part b.

(c) Profit per kg of product before taxes.

Selling price per kg = $ 0.82/kg.

Total product cost/kg = 0.473 $/kg

⇒ $\boxed{\text{Profit}}$ = selling ~~price~~ price − Total product cost
$= 0.82 - 0.473 \Rightarrow \boxed{\text{Profit} = 0.347}$ Part c

(d) **Profit /kg of product after taxes.** (net profit $= .347 - .34 \times .347$ $= .2290\,\$/kg$) ⑬

~~for Income tax rate see table 2, chapter 8 (for 1988).~~

~~Income tax~~

Profit before taxes $= 0.347 \dfrac{\$}{kg}$

For 3 million units, Profit before taxes $= 0.347 \times 3\times10^6$

$= \boxed{1041000}$

Profit after taxes. standard rate will be used

Tax $=$ 34 % of gross earnings.

$= 0.34 \times 0.347 = 0.11798\ \$/kg$

For 3 million units $= 0.11798 \times 3\times10^6$

$= \boxed{353940\ \$}$

Net profit after taxes = Profit before taxes − taxes.

$= 1041000 - 353940$

$= 687060\ \$$

$\boxed{\text{Net Profit after taxes /kg}} = \dfrac{687060\ \$}{3\times10^6\ kg} = 0.22902 \dfrac{\$}{kg}$

Pb-14 on register **ASSIGNMENT**

*★ Pb 12 A company has been selling a soap containing 30% by wt water at a price of \$10 per 100 lb f.o.b (i.e freight on board, which means the laundary pays the freight charges). The company offers an equally effective soap containing only 5% water. The water content is of no importance to the laundary, so it is willing to accept the soap

containing 5% water if the delivered costs are equivalent. If the freight rate is 70 cents/100lb, how much should the company charge the laundary per 100 f.ob for the soap containing 5% water?

Sol^n Basis 100 lbs of soap.

Total cost of soap/100 lbm = 10 $ + 0.70

= 10.70 $

95 lb (95% soap is equivalent to 70% soap)

$\frac{70}{0.95}$ = 73.68 lb of 95% soap equivalent to 70% soap.

Freight on 95% soap

100 lb freight = 0.7 $

1 ———— = $\frac{0.7}{100}$

73.68 ———— = $\frac{0.7}{100}$ × 73.68 = 0.51576 $

Maximum f.ob cost of 95 soap = 10.70 − 0.51

= 10.18 $ lbm

Maximum price of 95% soap

For 73.68 lb of 95% soap price = 10.1824 lbs

1 ———— = $\frac{10.1824}{73.68}$

100 ———— = $\frac{10.1824 \times 100}{73.68}$

= 12.8 $/100 ll....

3 Assignment Two

blem 15: A company has direct production costs equal 50 % of total annual sales & fixed charges, overhead & general expenses equal to $ 200,000. If management proposes to increase the present annual sales of $ 800,000 by 30% with a 20% increase in fixed charges, overhead & general expenses, what annual sales in dollar is required to provide the same gross earings as the present plant operation?

What would be the net profit if the expanded plant was operated at full capacity with an income tax on gross earnings fixed at 34 % ?

What would be the net profit for the enlarged plant if total annual sales remained the same as at present? What would be the net profit for the enlarged plant if the total annual sales actually decreased to $700,000

C **Soln** Direct Production cost = 50 % of total annual sales

fixed charges, overhead & general expenses = 200,000

Present annual sales = 800,000

Case-A Proposed increase in present annual sales = 30%

Increase in fixed charges, overhead & general expenses = 20%

T.F.O Annual sales in dollar = ? if gross earnings is same as the present plant operation

sales = 800,000 ↳ F.C, O.H, G.E = 200,000

Soln Now he demands the annual sales (new) if profit is same as for conditions of 800,000 & 200,000

Gross earnings = Total annual sales − Total product cost

Gross earnings = x − Total product cost — ①

for total product cost = (Manufacturing) + (General expenses)

= (Direct Production + Fixed charges + plant overhead + (General Expenses)

Now the direct production cost is 50% of total annual sales. & Total annual sales are not known so denoted by x

Total annual sales 50% of Direct production

direct production = 0.50 x.

& there is an increase of 20% in fixed charges, overhead & general Expenses

Fixed charges, overhead & General Expenses = 20% increased

= 1.20 × 200,000 = 240,000

so total product cost = 0.50x + 240,000

Put in eq ①

Gross Earnings = x − (0.50 x + 240,000) — ②

Now Gross Earnings are to be calculated from previous data (present data)

= Total sales − Total product cost

Gross earnings = (Total sales) − (Direct production + F.C + Overhead + G.E)

= 800,000 − [(50% of Total annual sales) + (200,000)]

= 800,000 − ((0.5 × 800,000 + 200,000))

Gross earnings = ~~800,000~~ 200,000

Now put in eq ②

$$200,000 = x - 0.50x - 240,000$$

$$200,000 + 240,000 = x(1-0.5)$$

$$\boxed{x = 880,000}$$

Gross annual sales if profit is to remain same

Case B Net Profit if the expanded plant is operated at full capacity (Income tax = 34%)

a) Net profit = Gross profit × 0.66 — ③

c) Gross profit = Total annual sales − Total product cost — ④

Now since the plant is operated at full capacity, so we will take that there is an increase of 30% in sales cost or 20% of F.C, G.E etc

Total annual sales = 30% increase of 800,000

= 1.3 × 800,000 = 104,000,0

c) Total product cost = (Direct production cost) + (G.E + F.C + overhead)

50% of total annual sales

= (0.5 × 104,000,0) + (240,000)

= 520,000 + 240,000 = 760,000

Now put in eq ④

Gross profit = 104,0000 − 760,000

= 280,000

Taxes (34%)
~~Net Profit~~ = 0.34 × 280,000

= 952,800

Net profit = 280,000 − 952,00

= 1,848,00

ⓒ What would be the net profit for the e plant if the total annual sales remained the same present

Soln As it is told that annual sales remained the same as at, present (present annual sales = 800,000) & the rest of the data is told that it is for enlarged plant.

Gross Earnings = Annual sales − Total product cost

$= 800{,}000 - (\underset{\text{50\% of Annual sales}}{\text{Direct cost}} + F.C + \text{Overhead} + \underset{\text{24\% increase}}{G.E})$

$= 800{,}000 - (0.5 \times 800{,}000 + 240{,}000)$

$= 160{,}000$

Net Profit = $\overset{(100-34)}{0.66} \times 160{,}000 = 105600$

ⓓ What would be the net profit for the enlarged plant if the total annual sales actually decreased to 700,000

Soln Annual sales = 700,000

Direct cost = 50% of Annual sales

$= 0.5 \times 700{,}000 = 350{,}000$

since it is asked that it is enlarged plant so its Overhead, G.E F.Charges = 240,000

Gross profit = $700{,}000 - (350{,}000 + 240{,}000)$

$= 110{,}000$

Net Profit = $0.66 \times 110{,}000$

$= 72600$

Q#164 A process plant making 2000 tons/year of a ⓘ
uct selling for $0.80/lb has annual direct production costs
f $2 Million at 100% capacity & other fixed costs of
700,000. What is the fixed costs/lb at the break even
point?

b) If the selling price of the product is increased by 10%, what is the dollar increase in net profit at full capacity if the income tax rate is 34% of gross earnings?

Soln

At break even point   Gross profit = 0 &

∴ Gross profit = Total annual sales − Total product cost

0 = Total annual sales − Total product cost

so $\boxed{\text{Total annual sales} = \text{Total product cost}}$

~~Total annual sa~~

Fixed charges + plant overhead + General expenses

Total product cost = Direct cost + Fixed expenses

∴

Total production = 2000 tons/year

~~Total~~ product cost/lb = 0.80 $/lb

Annual direct production costs = 700,000

(case a) Fixed cost/lb at break even point = ?

Soln

$$\frac{\text{Fixed cost}}{\text{lb}} = \frac{\text{Total fixed cost}}{\text{Total production at break even point}}$$

$$= \frac{700,000}{\text{Total production at break even point}} \quad ⓘ$$

At breakeven point

Total annual $\overset{\text{sales}}{\text{cost}}$ = total $\overset{\text{product cost}}{\text{~~[illegible] sale~~}}$ —— ②

Total annual sales cost = Total production in lb × cost / lbs —— ⓐ

Let total production in lbs is 'x'.

~~also~~ and $\frac{\text{cost}}{\text{lb}}$ = 0.8 \$/lb

Total product cost = Direct $\overset{\text{production}}{\text{cost}}$ + Fixed cost —— ⓑ

where ~~x × total production~~

Direct production cost = $\frac{\text{Direct product cost}}{\text{lb}}$ × total production in lbs —— ⓗ

also ~~where~~

$\frac{\text{Direct product cost}}{\text{lb}}$ = $\frac{\text{total production cost}}{\text{Total products in lbs}}$ —— ⓘ

where

Total products in lbs = $\frac{2000\text{ tons}}{\text{year}}\Big|\frac{2000\text{ lbs}}{1\text{ tonn}}\Big|$ = 4000,000 lbs/year

so ⓘ

Direct product cost/lb = $\frac{2\text{ Million }\$}{4000{,}000\text{ lbs/year}}$ = 0.5 \$/lb.

so eq ⓗ

$D.P.C = (0.5)x$ — ⓙ

so eq ⓑ $T.P.C = (.5)x + F.C$ — ⓒ

so put ⓒ & ⓐ in ②

Total annual sales = total product cost

$x \times \underset{\text{selling}}{\text{~~[illegible]~~}} = x \times \frac{\text{Direct product cost}}{\text{lb}}$ + fixed charges

$x \times 0.8 = x \times 0.5 + 700{,}000$

$x(0.8 - 0.5) = 700{,}000$

$x = 2.33 \times 10^6$ lbs at Break even point

total production in lbs at break even point $= 2.33 \times 10^6$ lbs ⓒ
so eq ①

$$\frac{\text{Fixed cost}}{\text{lb}} = \frac{700,000}{2.33 \times 10^6} = 0.3 \text{ \$/lb}$$

ⓑ If selling price of the product is increased by 10%, what is the dollar increase in net profit

Soln Since increase in net profit is demanded, so firstly we will calculate selling price without increasing the selling price by 10% & then by increasing 10%.

Gross Earnings = total sales - Total product cost

= total sales - (Direct costs + F.C)

= (4000,000 × 0.8) - (2000,000 + 700,000)

= 500,000

$$\boxed{\text{Profit} = 500,000 \times 0.66 = 330,000}$$

If selling price rises 10%,

Gross earings = total sales - (D.P.C + f.C)

$$= (4000,000 \times 0.8 \times \underset{10\% \text{ increased}}{1.1}) - (2000,000 + 700,000)$$

= 820,000

$$\boxed{\text{Net Profit} = 820,000 \times 0.66 = 541,200}$$

Increase in Income = 541,200 - 330,000

$$\boxed{= 211,200}$$

Ⓐ Cost Index :- updating equipment cost same size

$$\text{Present cost} = \text{Original cost}\left(\frac{\text{index value at present}}{\text{index value at original cost was obtained}}\right)$$

Case B:-

Capacity = $0.2\ m^3$
Cost in 1991 = \$ 10,000

Case A:

Capacity = $1.2\ m^3$
Cost in 1996 = ?

Ⓑ Scaling Factor :-

$$\underset{\text{at present}}{\text{Cost of a}} = \left(\begin{matrix}\text{Cost}\\ \text{of b}\\ \text{present}\end{matrix}\right) \times^{0.6}$$

Sol:

$$\left(\begin{matrix}\text{Cost of a}\\ \text{in 1996}\end{matrix}\right) = \left\{\left(\begin{matrix}\text{Cost of b}\\ \text{in 1996}\end{matrix}\right)\right\}\left(\frac{\text{Size of a}}{\text{Size of b}}\right)^{\text{Exponent}}$$

$$= \left\{\$10{,}000 \times \frac{381.7}{361.3}\right\}\left(\frac{1.2}{0.2}\right)^{0.54}$$

$$= \$\ 27{,}850$$

$(bx^{0.6})$

$\log a = \log(bx^{0.6})$

$\log a = \log b + 0.6 \log x$

$y = mx + c$

CHAPTER 07

Interest and Investment Costs

Interest & Investment costs.

DANIAL KAMRAN

Example 1 Applications of different types of interest.

It is desired to borrow \$1000 to meet a financial obligation. This money can be borrowed from a loan agency at a monthly interest rate of 2%. Determine the following

a) the total amount of principal plus simple interest due after 2 years if no intermediate payments are made.

Length of one interest period = 1 month

No of interest periods in 2 years = 24

Money borrowed = \$ 1000 (P)

$i = 2\% = 0.02$

since no intermediate payments are made, so net interest will be paid at the end of 24 periods.

$S = P + I \Rightarrow S = P + Pin$

$$\boxed{S = P(1+in)}$$

simple interest rate method.

$S = 1000(1 + 0.02 \times 24)$

$$\boxed{S = 1480\$}$$

b) The total amount of principal plus compounded interest due after 2 years if no intermediate payments are made.

For compounded interest rate, $S = P(1+i)^n$.

$i = 0.02$ $n = 24$

n of payments desired in 2 years

$S = 1000(1+0.02)^{24} \Rightarrow \boxed{S = 1608\$}$

c) The nominal interest rate when the interest is compounded monthly.

As it is 2% monthly, so nominal interest rate per year compounded monthly = $2 \times 12 = 24\%$

d) Effective interest rate when the interest is compo monthly.

No of interest periods per year = m = 12.

Nominal interest rate $\overset{=r}{}$ = 0.24

Effective interest rate = $\left(1+\frac{r}{m}\right)^m - 1$

$$i_{eff.} = \left(1+\frac{0.24}{12}\right)^{12} - 1$$

$$= 0.2682 = 26.82\%$$

Simpler way

Example 2 Calculation with continuous interest compounding.

For the case of a nominal annual interest rate of 20%, determine a) the total amount to which one dollar of initial principal would accumulate after one 365 day year with daily compounding

Soln nominal interest rate, r = 20% = 0.20

S (with continuous interest compounding after one 365 day year with daily compounding) = ?

Principal amount $\overset{=P}{=}$ \$ 1

As compounding is done daily, so no of compounding periods per year, m = 365

$$S = P\left(1+\frac{r}{m}\right)^{m \nearrow n=1} = 1\left(1+\frac{0.2}{365}\right)^{365}$$

$= P(1+i_{eff})^n$

$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$

$$= \$\ 1.2213$$

b) The total amount to which one dollar of initial principal would accumulate after one year with continuous compounding.

for continuous compounding, $S = P e^{rn}$.

$n = 1$ year. $S = 1 e^{0.20 \times 1}$

$$\boxed{S = 1.2214 \$}$$

c) the effective annual interest rate if compounding is continuous.

for continuous compounding, $i_{eff} = e^{r} - 1$

$$= e^{0.20} - 1 = 1.2214 - 1$$

$$= 0.2214 = 22.14\%$$

- It means that the nominal annual interest rate of 22 % is equal to 22.14 % effective interest rate when compounding is done continuously.

* * *

Example 4: Determination of present worth & discount.

A bond has a maturity value of $1000 & is paying discrete compound interest at an effective annual rate of 3 %, Determine the following at a time 4 years before the bond reaches maturity value.

a) Present value

Maturity value, $S = 1000$

Effective annual interest rate, $i_{eff} = 3\% = 0.03$.

Present value (4 years before) = ?

Since effective interest rate is given, so use

$$S = P(1 + i_{eff})^{n}.$$

$$P = \frac{S}{(1 + i_{eff})^{n}} = \frac{1000}{(1 + 0.03)^{4}}$$

$$= 888.48 \$$$

b) Discount.

Discount Difference b/w ~~prese~~ future value & present.

Future value = 1000 Present value = 888.48

$$\text{Discount} = 1000 - 888.48 = 111.52$$

c) Discrete compound rate of effective interest which will be received by a purchaser if the bond were obtained for \$ 700.

Since bonds are obtained for 700, so $P = 700$

$S = 1000$ $i_{eff} = ?$

$$S = P(1 + i_{eff})^n \qquad n = 4 \text{ years}$$

$$1000 = 700(1 + i_{eff})^4 \Rightarrow \frac{1000}{700} = (1 + i_{eff})^4$$

$$(1.429)^{1/4} = 1 + i_{eff} \Rightarrow i_{eff} = 1.0933 - 1$$

$$i_{eff} = 0.09333 \Rightarrow \boxed{i_{eff} = 9.33\%}$$

d) Repeat part a) for the case where the nominal bond interest is 3% compounded continuously.

$S = 1000,$ $P = ?$ $r = 3\% = 0.03$ $n = 4$

$$S = Pe^{rn} \Rightarrow P = \frac{S}{e^{rn}} = \frac{1000}{e^{0.03 \times 4}}$$

$$= 886 \,\$$$

mple 5 P Application of annuities in determining amount of depriciation with discrete interest compounding.

A piece of equipment has an initial installed value of \$ 12,000. It is estimated that its use-ful life period will be 10 years & its scrap value at the end of the use-ful life will be \$ 2000. The depriciation will be charged as a cost by making equal charges each year, the first payment being made at the end of the first year. The depriciation fund will be accumulated at an annual interest rate of 6%. At the end of life period, enough money must have been accumulated to account for the decrease in equipment value. Determine the yearly cost due to depriciation under these conditions.

Note This method for determining depriciation is based on an ordinary annuity & is known as sinking fund method.

Soln Installed value = $V = 12,000$ $n = 10$ years

Scrap value, $V_s = 2000$.

$r = 6\% = 0.06$.

Yearly cost due to depriciation, $R = ?$

It means that in future after 10 years, we will require $12,000 - 2000 = 10,000$ \$ to get the new equipment (purchase)

$S = 10,000$.

For ordinary annuity, periodic amount R,

$$S = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$10,000 = \frac{R\left[(1+0.06)^{10} - 1\right]}{0.06} \Rightarrow R = \frac{10,000 \times 0.06}{(1.06)^{10} - 1}$$

Yearly cost due to depriciation = 759 $/year

Example-6 Application of annuities in determining amount of depriciation with continuous cash flow & interest compounding.

Repeat example 5 with continuous cash flow & nominal annual interest of 6% compounded continuously.

Soln $V = 12{,}000$ $V_s = 2{,}000$ $n = 10$ years.

$S = V - V_s = 12{,}000 - 2000 = 10{,}000$

So it means that we want to save 10,000 $ in 10years using continuous compounding by the use of annuity method

$r = 6\% = 0.06$

$$S = \bar{R}\left(\frac{e^{rn}-1}{r}\right)$$

$$10{,}000 = \bar{R}\left(\frac{e^{0.06\times10}-1}{0.06}\right) \Rightarrow \bar{R} = \frac{10{,}000\times0.06}{e^{0.06\times10}-1}$$

$$\boxed{\bar{R} = 730\ \$/\text{year}}$$

For Discrete interest $S = \frac{R\left[(1+i)^n-1\right]}{i}$

For compounded interest, $S = \frac{\frac{R}{m}\left[(1+r/m)\right]^{\frac{m}{r}rn}-1}{r/m}$

For continious companding. $S = \bar{R}\left(\frac{e^{rn}-1}{r}\right)$

Problems 1) It is desired to have ~~obtain~~ \$9000 available 12 years from now. If \$5000 is available for investment at the present time, what discrete annual rate of compound interest on the investment would be necessary to give the desired amount?

$S = 9000$ $n = 12$ years $P = 5000.$

$i = ?$

As $S = P(1+i)^n$ for discrete annual rate of compound ~~return~~ interest

Compound Interest

when interest can be obtained from the principal is interest accumulated after each year.

$$9000 = 5000(1+i)^{12} \Rightarrow (1.8)^{1/12} = 1+i$$

$$i = 1.050 - 1 \Rightarrow 0.050$$

$$\boxed{i = 5\%}$$

---

2) P What would be the total amount available 10 years from now if \$2000 is deposited at the present time with nominal interest at the rate of 6% compounded semiannually.

$P = 2000$ $S = ?$ $n = 10$ years

$r = 6\% = 0.06$

For semiannual compounded, No of compoundings/year = $2 = m$

$\cancel{S = P\left(1 + \frac{r}{m}\right)^{mn}}$

$\cancel{S = 2000\left(1 + \frac{0.06}{2}\right)}$

$\cancel{S = P\left(1 + \frac{r}{m}\right)^{m}}$

$r = 6\%$
$= 0.06$ $\cancel{m = 2 \times 10 = 20}$

$\cancel{S = 2000\left(1 + \frac{0.06}{20}\right)^{20}}$

$$S = P(1 + i_{eff})^n$$

$n = 10$

$i_{eff} = \left(1+\frac{r}{m}\right)^{m} - 1 \Rightarrow \left(1+\frac{0.06}{2}\right)^{2} - 1$

m = no of compoundings/year = 2

$i_{eff} = 0.0609.$

$S = 2000\,(1+0.0609)^{10} = 3612.$

OR $S = P\left(1+\frac{r}{m}\right)^{mn}.$

$S = 2000\left(1+\frac{0.06}{2}\right)^{2\times 10} = 3612.$

Problem 3 An original loan of $2000 was made at 6% simple interest per year for 4 years. At the end of this time, no interest had been paid & the loan was extended for 6 more years at a new, effective, compound interest rate of 8% per year. What is the total amount owed at the end of the 10 years if no intermediate payments are made.

Soln Case 1 → Simple Interest rate

P = 2000 i = simple interest rate = 0.06

n = 4 years.

Future amount after 4 years = ?

Simple Interest rate when the interest is paid only on the original capital after each period.

$S = P + I \qquad I = Pin.$

$S = P + Pin \Rightarrow S = P(1+in)$

$S = 2000\,(1+0.06\times 4)$

$\boxed{S = 2480\$}$

Case - 2 Compound interest rate when the interest is paid on the amount dued (original capital + Interest) after each period.

Now principal amount after 4 years = 2480 \$

$n = 6$ years $\qquad i_{eff} = 8\% = 0.08$

$S = P(1 + i_{eff})^n = 2480(1 + 0.08)^6$

$= 3935$ \$

P

4) A concern borrows \$ 50,000 at an annual, effective compound-interest rate of 10%. The concern wishes to pay off the debt in 5 years by making equal payments at the end of each year. How much each payment will have to be?

Since he has to pay this amount (Present amount) so it will be P not S.

Present amount = 50,000. (P)

$i_{eff} = 0.10 \qquad n = 5$ years.

$R = ?$ [per year].

$S = P(1+i)^n \qquad S = \frac{R[(1+i)^n - 1]}{i}$

$P(1+i)^n = \frac{R[(1+i)^n - 1]}{i}$

for compound interest, $P = \frac{R(1+i)^n - 1}{i(1+i)^n}$

$$50{,}000 = \frac{R[(1+0.10)^5 - 1]}{0.10(1+0.10)^5} \Rightarrow \frac{50{,}000(0.10)(1.10)^5}{(1+0.10)^5 - 1} = R$$

$$R = \frac{8.052.55}{0.61051} \Rightarrow \boxed{R = 13189 \text{ \$}}$$

Note calculation of annuity has 2 formulas, one from present value to other from future value (S).

Loan will have to paid for at present whereas depriciation etc are used to be paid (colled) in future so future formula will be used.

Problems The orginal cost for a distillation tower is \$ 24,000 the useful life of the tower is estimated to be 8 years. The sinking fund method for determining the rate of depriciation is used & the effective annual interest rate for the depreciation fund is 6%. If the scrap value of the distillation tower is 4000, determine the asset value (i.e total book value of equipment) at the end of 5 years?

Soln $V = 24,000$ $n = 8$ $i_{eff} = 6\% = 0.06$

$V_s = 4000$ Asset value = ? (after 5 years).

First we will use annuity formula to calculate the amount which we will have to deposit at 6% inorder to get 20,000.

Depreciable amount, $S = 24,000 - 4,000 = 20,000$

future value of annuity which I want to get in 8 years

When R is calculated, now calculate its value accumulated in 5 years, subtracting it from initial value, we will get book value.

Calculation of annuity per year at 6% effective interest rate

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$20,000 = R\left[\frac{(1+0.06)^8 - 1}{0.06}\right] \Rightarrow R = \frac{20,000 \times 0.06}{(1.06)^8 - 1}$$

$$\boxed{R = 2021 \ \$/\text{year}}$$

For Book value after 5 years, we should have the future value of the equipment after 5 years.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 2021\left(\frac{(1+0.06)^{\overset{5}{8}} - 1}{0.06}\right)$$

$\boxed{S = 11393}$ This is not the future value of the equipment but it is the amount regained by us (it is depriciable amount which have been collected)

Now this is the amount accumulated after 5 years [i.e. depreciable amount accumulated].

Future amount after 5 years = 11393 \$

Book value = original value − Depreciable amount accumulated in 5 years

= 24000 − 11393

$\boxed{\text{Book value} = 12607}$

* * *

Problem 6 An annuity due is being used to accumulate money. Interest is compounded at an effective annual rate of 8% & \$1000 is deposited at the beginning of each year. What will the total amount of the annuity due be after 5 years?

Note
Now \$1000 is deposited at the start of each year, so the general formula of annuity cannot be used coz in that annuity is deposited at the end of each year. Now use general method,

Annuity deposited at start of ~~on~~ first year will have a profit for 5 years, for 2nd will have 4 years, & the annuity paid at the start of last year will have a profit of 1 year after 5th year.

$$\Rightarrow S = R(1+i)^{n-1} + R(1+i)^{n-2} + R(1+i)^{n-3} + \ldots + R(1+i) + R$$

$$S = R\left[(1+i)^5 + (1+i)^4 + (1+i)^3 + (1+i)^2 + (1+i)\right]$$

$$= 1000\left[(1+0.08)^5 + (1+0.08)^4 + (1+0.08)^3 + (1+0.08)^2 + (1+0.08)\right]$$

= ~~[illegible]~~ 6335 \$.

* * *

~~$S = R[(1+i)^6 - 1]$~~

Q. Problem: For total yearly payment of \$ 5000 for 10 yrs compare the compound amount accumulated at the end of 10 years if the payments are a) end of year b) weekly c) continuous

The effective (annual) interest ~~rate~~ is 20% & payments are uniform.

Soln Yearly payment = $R = 5000$ $n = 10$ years.

$S = ?$

a) end of year. [Payments are made at end of each year].

Note when compounding is done once in a year, $i_{eff} = i_{nominal}$. $i_{eff} = r$.

so no of interest per period = 1.

thus $$S = \frac{R[(1+i)^n - 1]}{i}$$ $i = 20\%$

$$S = \frac{5000((1+0.2)^{10}-1)}{0.2} \Rightarrow \boxed{S = 129,793}$$

b) Payments are paid at end of each week.

No of compoundings per year $= \frac{365}{7} = 52$.

$m = 52$

$$S = \frac{R}{m}\left[\frac{(1+r/m)^{mn} - 1}{r/m}\right]$$

r = Nominal interest rate $i_{eff} = 0.2$

$$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$$

$$0.2 = \left(1+\frac{r}{52}\right)^{52} - 1 \Rightarrow (1.2)^{1/52} = \left(1+\frac{r}{52}\right)$$

$$1.0035 - 1 = \frac{r}{52} \Rightarrow 0.0035 \times 52 = r$$

$$\boxed{r = 0.182} \Rightarrow 18.2\%$$

$$S = \frac{\frac{5000}{52}\left[\left(1+\frac{0.182}{52}\right)^{52\times10} - 1\right]}{0.182/52}$$

141538

$$= \frac{96.15\,(5.1522)}{0.0035} \Rightarrow \boxed{S = 141538}$$

c) Payments are made continuous.

$$S = \bar{R}\left[\frac{e^{rn}-1}{r}\right].$$

~~calc~~

r = Nominal interest, It will be different than the nominal interest calculated ~~from~~ when compounding is done once in a week.

$$i_{eff} = e^{r}-1 \Rightarrow 0.2 = e^{r}-1$$

$$1.2 = e^{r} \Rightarrow \ln 1.2 = r$$

$$\boxed{r = 0.182}$$

$$S = 5000\left[\frac{e^{0.182\times10}-1}{0.182}\right] \Rightarrow \boxed{S = 142084.}$$

Q10) For the conditions of problem 9, determine the present worth at time zero for each of the 3 types of payments.

a) Present value for end of year payments.

$$\text{As} \quad S = R\left(\frac{(1+i)^n-1}{i}\right) \qquad S = (1+i)^n \cdot P.$$

$$(1+i)^n P = R\left(\frac{(1+i)^n-1}{i}\right) \Rightarrow P = \frac{R(1+i)^n-1}{i(1+i)^n}$$

$$P = \frac{5000\times\left[(1+0.20)^{10}-1\right]}{0.2(1+0.2)^{10}} \Rightarrow \boxed{P = 20962}$$

b) When payments are made at the end of wee

As ~~P~~ $s = \frac{R}{m}\left[\frac{(1+\frac{r}{m})^{mn}-1}{r/m}\right].$

r = Nominal interest rate

$i_{eff} = (1+\frac{r}{m})^{m}-1 \implies 0.2 = (1+\frac{r}{52})^{52}-1$

$$\boxed{r = 0.182}$$

Also ~~P~~ $s = P(1+\frac{r}{m})^{mn}.$

so $P(1+\frac{r}{m})^{mn} = \frac{R}{m}\left[\frac{(1+\frac{r}{m})^{mn}-1}{r/m}\right].$

$$P = \frac{R/m\left[(1+r/m)^{mn}-1\right]}{(r/m)(1+\frac{r}{m})^{mn}}$$

m = 52 weeks so 52 compoundings / year.

$$= \frac{\frac{5000}{52}\left[(1+\frac{0.182}{52})^{52\times 10}-1\right].}{\frac{0.182}{52}(1+\frac{0.182}{52})^{52\times 10}}$$

$$= \frac{96.15(5.1522)}{(0.0035)(6.1522)}$$

$$\boxed{P = 23006\ \$}$$

c) For continuous compounding

$s = R\left(\frac{e^{rn}-1}{r}\right].$ $\qquad s = Pe^{rn}.$

$Pe^{rn} = R\left[\frac{e^{rn}-1}{r}\right]. \implies P = \frac{R[e^{rn}-1].}{re^{rn}}$

$\boxed{r = 0.182}$

$i_{eff} = e^{r}-1$

$$P = \frac{5000\left[e^{\overset{0.182}{\cancel{52}}\times 10}-1\right].}{0.182\,e^{\cancel{52}\times 10\ 0.182\times 10}} \implies \boxed{P = 23021}$$

Problem 11 P/ A heat exchanger has been designed for use in a chemical process. A standard type of heat exchanger with a negligible scrap value costs \$ 4000 & will have a useful life of 6 years. Another proposed H.E of equivalent design capacity costs \$ 6800 but will have a useful life of 10 years & a scrap value of \$ 800. Assuming an effective compound interest rate of 8% per year, determine which H.E is cheaper by comparing capitalyed costs.

Soln

Note That H.E is cheaper whose capitilyed cost is less.

Exchanger 1 $V_s = 0$ $V = 4000$ $n = 6$ years.

Replacement cost $= V - V_s = 4000 - 0 = 4000$

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

$C_V =$ original cost $C_R =$ replacement cost

$C_V = 4000$. $C_R = 4000$

$$K = 4000 + \frac{4000}{(1+0.08)^6 - 1} \qquad i = 8\%$$

$$K = 4000 + 6815$$

$$\boxed{K = 10815}$$

Exchanger 2. $V = 6800$ $V_s = 800$

Replacement cost, $C_R = 6800 - 800 = 6000$

$n = 10$ years

Original cost, $C_V = 6800$

$$K = C_V + \frac{C_R}{(1+i)^n - 1} \Rightarrow 6800 + \frac{6000}{(1+0.08)^{10} - 1}$$

$$\boxed{K = 11977}$$

As the capatilyzed cost of exchanger 1 is small, it is cheaper.

---

12) A new storage tank can be purchased & installed for 10,000. This tank would last for 10 years. A worn out storage tank of capacity equivalent to the new tank is available, & it has been proposed to repair the old tank instead of buying the new tank. If the tanks were repaired, it would have a useful life of 3 years before the same type of repairs will be needed again. Neither tank has any scrap value. Money is worth 9% compounded annually. On the basis of equal capitalized costs for the 2 tanks, how much can be spent for repairing the existing tank?

soln

Old tank

- $n = 3$ years.

$V_s = 0$

New tank

Initial cost = 10,000

$n = 10$ years.

$V_s = 0$

$i = 9\% = 0.09$ [As the compounding is done annually so i_{eff} = nominal interest rate]

→ capitilyzed costs for 2 tanks are equal

For New tank.

capitilyzed cost, $K = C_V + \dfrac{C_R}{(1+i)^n - 1}$

C_V = original cost = C_R = replacement cost = 10,000

$$K = 10{,}000 + \frac{10{,}000}{(1+0.09)^{10} - 1} \Rightarrow \boxed{K = 17313}$$

$$K = C_v + \frac{C_R}{(1+i)^n - 1}$$

original cost = C_v

Replacement cost = original cost - salvage value

$$C_R = C_v - 0 \Rightarrow C_v = C_R$$

so $$K = C_v + \frac{C_v}{(1+i)^n - 1}$$

$n = 3$.

$$K = C_v + \frac{C_v}{(1+0.09)^3 - 1}$$

As the capitilyzed costs are equal so,

$$17313 = C_v + \frac{C_v}{(1+0.09)^3 - 1}$$

$$17313 = \frac{0.2950\ C_v + C_v}{0.2950}$$

$$17313 \times 0.2950 = C_v(1.2950)$$

$$C_v = \frac{5107.8}{1.2950} \Rightarrow \boxed{C_v = 3944}$$

<u>Problem 13</u> Equation 28 is the expression for capitalyzed cost based on discrete interest compounding. For continuous interest compounding, the expression becomes.

$$K = C_v + \frac{C_R}{e^{rn} - 1}$$

Present a detailed derivation of this continuous interest relationship going through each of the equivalent steps used in deriving eq 28. → No need to derive eq 28, but counter part of eq 28 for continuous compoundi

eq 28 $$K = C_v + \frac{C_R}{(1+i)^n - 1}$$

For continuous compounding, Future amount (S) of the present principal P is,

$$S = Pe^{rn}.$$

If the perpetuation is to occur, the amount S accumulated after n periods minus the cost for the replacement must equal the present worth P.

value deposited at the start of period when compounded, will give replacement value & the value which we deposit will remain same, & the process goes on.

ie Present worth = S − Replacement cost.

$$P = S - C_R$$

$$P = Pe^{rn} - C_R.$$

$$P - Pe^{rn} = -C_R \Rightarrow P(1-e^{rn}) = -C_R$$

$$P = \frac{-C_R}{1-e^{rn}} \Rightarrow P = \frac{C_R}{e^{rn}-1}$$

~~285~~
3073 305 40437 06

Capitilized cost = original cost of the equipment + Present value of the renewable perpetuity

$$K = C_V + \frac{C_R}{e^{rn}-1}$$

Problem 14 The total investment required for a new chemical plant is estimated to be $ 2 million. Fifty % of the investment will be supplied from company's own capital. Of the remaining investment, half will come from a loan at an effective intrest rate of 8%, & the other half will come from an issue of preferred stock paying dividends at a stated effective rate % . The income tax rate for the company is of pre tax earnings. Under these conditions, how dollars /year does the company actually lose (i.e

CHAPTER 09

Depreciation

obsolence depriciation due to tech. (a) advancements A

DEPRICIATION DANIAL KAMRAN.

The value of machine or asset constantly reduces with the lapse of time during use, which is known as depriciation.

OR

The reduction of value of an asset due to physical deterroration, technological advances, economich changes or other factors which ultimately will cause retirement of the property

METHODS OF DEPRICIATION CALCULATIONS.

- Arbitrary methods. (with no interest).
  - Straight line method.
  - Sum of the years digits method.
  - Declining balance methods
    - Text book declining method
    - Double declining method.
- With interest
  - Sinking fund method
  - Present worth method.

STRAIGHT LINE METHOD.

→ Its assumed that value of property reduces linearly with time.

Depriciation calculated in each year is constant.

$$\text{Depriciation per year} = \frac{\text{Initial value} - \text{Salvage value}}{\text{No of use ful life years}}$$

→ Initial value is only that amount which has to be depriciated. e.g Total Initial investment is 10,000 but out of it, land is bought for 5000, so it means that land is not depriciated item so Amount to be depriciated is equal to total investment minus amount which is non-depriciable

Amount to be depriciated = 10,000 − 5,000 = 5,000

Asset value or book value $= V_a = V - ad$

V = orginal value.
a = number of years in actual use
d = dep charged / year

Sum of the years digits method ✓

Depriciation for any year a.

$$Dep = \frac{n-a+1}{\Sigma n}(V - V_s)$$

when n is not given depriciation for 2nd year is given

$$Dep = \frac{n-2+1}{\frac{n(n+1)}{2}}(V - V_s)$$

a = Year in which depriciation is required.

For example if service life is 5 years. $n = 5$

$$\Sigma n = 1+2+3+4+5 = 15$$

For first year $Dep = \frac{5-1+1}{15}(V - V_s) = \frac{5}{15}(V - V_s)$

For 2nd year $Dep = \frac{5-2+1}{15}(V - V_s) = \frac{4}{15}(V - V_s)$

For 5th year $Dep = \frac{5-5+1}{15}(V - V_s) = \frac{1}{15}(V - V_s)$.

Accelerated Depriciation method.

Declining balance method It has 2 types.

a). text book declining balance method.

b) Double declining balance method.

✓Text book Declining balance method.

Fixed percentage factor, $f = 1 - \left(\frac{V_s}{V}\right)^{1/n}$

Asset value for any year. $V_a = V(1-f)^a$.

7x2 = 14

7x7x4 = 196

49x7x4 = 1372

Depriciation for any particular year it is calculated by subtracting the ~~depriciation~~ asset value of the previous year & of that year. e.g if I require the ~~asset value~~ depriciation for 5th year it means that it's asset value will be low for 5th year than for 4th year ~~eg~~ by an amount which has been depriciated.

$$Dep = V_4 - V_5.$$

For 5th year ↪ asset value for fifth year.

→ Method totally based on salvage value, when salvage value is equal to zero, $f = 1$ & $V_a = 0$. [Salvage value can not be estimated exactly

Double Declining balance method

$$V_a = V(1-\rho)^a.$$

$$\rho = \frac{2d}{V} \qquad d = \frac{V - V_s}{n}$$

SINKING FUND METHOD.

$$V - V_s = \frac{R[(1+i)^n - 1]}{i}.$$

So annual depriciation rate, $R = \dfrac{(V-V_s)i}{[(1+i)^n - 1]}.$

Asset value or book value at any time, $V_a = V - (V-V_s)\left(\dfrac{(1+i)^a - 1}{(1+i)^n - 1}\right)$

since it is for a-th year so it will be $(1+i)^a$ not $(1+i)^n$ in numerator

for Declining balance methods for calculating the depriciation for first year, we will calculate its value before the start of operation (year before it / zeroth year) & 1st asset value after first year & the difference is the depriciation for first year)

$$V - V_a = \frac{R[(1+i)^a - 1]}{i}$$

~~Of~~ Now put value of R

$$V - V_a = \frac{(V-V_s)\cancel{i}}{(1+i)^n - 1}\,\frac{[(1+i)^a - 1]}{\cancel{i}}.$$

$$V_a = V - \frac{(V-V_s)[(1+i)^a - 1]}{(1+i)^n - 1}$$

ordinary annuity Payment due at the end of each year

Annuity due · Payment due at the start of each year

Deferred annuity Payment is due after a certain no of years.

Book value Present value of equipment (Initial cost – Amount depreciated up till now)

Net Realizeable value Actual maket value of the equipment

Unamortize value Difference b/w book value & net realizeable value

Profitability General term for the measure of (amount of profit that can be obtained from a given situation)

capitalized cost original cost + ~~req~~ present cost of the renewable property.

$$K = Cv + \frac{CR}{(1+i)^n - 1}$$

$$K = Cv + P \qquad P = S - CR$$

$$P = P(1+i)^n - CR$$

$$P = \frac{CR.}{(1+i)^n - 1}$$

Perpetuity is an annuity in which periodic payment continues indefinitely.

Annuity Series of equal payments occurring at regular interval of time

Amortization Amount of the property which has been recovered as a result of depreciation

ECONOMICS.

Problems A reactor of special design is the major item of equipment in a small chemical plant. The initial cost of a completely installed reactor is \$60,000 & the salvage value at the end of the useful life is estimated to be \$10,000. Excluding depriciation costs for the reactors, the total annual expenses for the plant are \$100,000. How many years of useful life should be estimated for the reactor if 12% of the total annual expenses for the plant are due to the cost for reactor depriciation? The straight line method for determining depriciation should be used?

Soln Initial cost of the reactor = 60,000. = V

Salvage ~~rate~~ value = 10,000. = V_s.

Total annual expenses (excluding depriciation) = 100,000.

Useful life = n = ?

12% of the total annual expenses of the plant are due to reactor depriciation.

Depriciation using straight line method

$$Dep = \frac{V - V_s}{n} = \frac{60,000 - 10,000}{n}$$

$$Dep = 50,000/n \quad — ①$$

As, total annual expenses = (Annual expenses without depriciation) + (Depriciation)

$$= 100,000 + 50,000/n.$$

As Depriciation amounts to 12% of total annual expenses;

Dep = 0.12 (Total annual expenses)

$$Dep = 0.12(100,000 + 50,000/n) \quad — ②$$

Comparing eq ① & ②. $\frac{50,000}{n} = 0.12\left(100,000 + \frac{50,00}{n}\right.$

$$\frac{50,000}{n} = 12000 + \frac{6000}{n}.$$

$$\frac{50,000}{\cancel{n}} = \frac{12000n + 6000}{\cancel{n}} \Rightarrow 50,000 = 12,000^{n} + 6,000$$

$$12000\,n = 50,000 - 6,000.$$

$$n = \frac{44,000}{12,000} \Rightarrow \boxed{n = 3.67 \text{ years}}$$

Pblem 2 The initial installed cost for a new piece of equipment is 10,000 & its scrap value at the end of useful life is estimated to be 2,000. The useful life is estimated to be 10 years. After the equipment has been in use for 4 years, it is sold for $ 7,000. The company which originally owned the equipment employs the straight line method for determining depriciation costs. If the company had used an alternative method for determining depriciation costs, the asset value (or book value) of the piece of equipment at the end of 4 years would have been $ 5240. The total income tax rate for the company is 34% of all gross earnings. capital gain tax amount to 34% of the gain. How much net savings after taxes would the company have achieved by using the alternative (in this case, reducing-balance) depriciation method instead of the straight line depriciation method ?

Soln

Initial cost = 10,000 (V)

scrap value = V_s = 2,000.

Use-ful life = n = 10 years.

sold for 7,000 at the end of 4 years

Asset value at the end of 4 years (using reducing balance method) = 5240.

sale tax rate = 34%.

Net savings after tax = ?

②

Net savings after tax = Fraction of tax × Reduction in gross earnings ——①

Reduction in gross earnings = Difference in depriciation of 2 methods.

Depriciation using straight line.

$$Dep = \frac{V - V_s}{n} = \frac{10,000 - 2000}{10} = \frac{8,000}{10} = 800/\text{year}.$$

Depriciation after 4 years = 800 × 4 = 3200

By Declining balance method.

As Depriciation = Initial value - Asset value at that time.

$= V - V_a.$

$Dep = 10,000 - 5240. \Rightarrow \boxed{Dep = 4760.}$

or Depriciation can be calculated as.

Asset value at any particular year a $\quad V_a = V(1-f)^a.$

where $f = 1 - \left(\frac{V_s}{V}\right)^{1/n} = 1 - \left(\frac{2000}{10,000}\right)^{1/10}$

$= 0.1487$

Asset value after 4 years $= V_4 = 10,000(1-0.1487)^4$

$= 5252.$

So $Dep = V - \cancel{Vd}. V_a.$

$= 10,000 - 5252 = 4748.$

$\boxed{Dep = 4748}$

So Difference in depriciation of 2 methods = 4760 - 3200 (↗ as given.)

Reduction in gross earnings = 1560

Net savings after tax = 0.34 × 1548

$\boxed{= \$\ 530}$ → Fin.

(from sir notes)

capital gain effects

Net savings = 1560 - 530

= 1030$

Problem 9.3 A piece of equipment originally costing $40,000 was, into use 12 years ago. At the time, the equipment was put into use, the service life was estimated to be 20 years and the salvage & scrap value at the end of the service life were assumed to be zero. On this basis a straight line depriciation fund was set up. The equipment can now be sold for $10,000 & a more advanced model can be installed for $55,000. Assuming the depriciation fund is available for use, how much new capital must be supplied to make the purchase?

Soln

Initial cost = 40,000 = V.

Salvage value = $V_s = 0$

Useful life = $n = 20$ years.

Asset value when sold after 12 years = 10,000.

Cost of advanced model = 55,000.

New capital required = ?

New capital required = (Cost of new installment) − (Asset value when sold + Depriciation amount in 12 years).

Depriciation straight line depriciation method.

$$\text{Dep} = \frac{V - V_s}{n} = \frac{40,000 - 0}{20} = 2,000/\text{year}.$$

Depriciation fund after 12 years = 2,000 × 12 = 24,000

New capital required = 55,000 − (24,000 + 10,000)

= 21,000 $.

③

The original investment for an asset was 10,000 \$ & the asset was assumed to have a service life of 12 years ~~&~~ with \$ 2,000 salvage value at the end of service life. After the asset has been in use for 5 years, the remaining service life & final salvage value are restimated at 10 years & 1000 respectivly. Under these conditions what is the depriciation cost during the sixth year of the total life if straight line depriciation is used?

Soln original cost = 10,000 (V)
salwage value = V_s = 2,000.
service life = n = 12 years.

After 5 years New service life = 10 years.
New salwage value = V_s = 1000.

Depriciation cost during 6th year.

Note: first we will calculate depriciation cost per year for actual service life & will calculate the asset value after 5 years & from that we will calculate the depriciation of new year [6th year of total life].

$$\text{Depriciation} = \frac{V - V_s}{n} = \frac{10,000 - 2000}{12} = 666.67/\text{year}$$

Depriciation cost after 5 years = 5 × 666.67
= 3333.33

Asset value after 5 years = 10,000 − 3333.33
= 6666.67

Reestimated ~~New~~ New initial cost = 6666.67.

$$\text{depriciation for 6th year} = \frac{V - V_s}{n} = \frac{6666.67 - 1000}{10}$$

= 567 ~~rps~~ \$

9.5 A property has an initial value of \$ 50,000, service life of 20 years, and final salvage value of \$ 4000. It has been proposed to depriciate the property by the text-book declining balance method. Would this method be acceptable for income tax purposes if the income tax laws donot permit annual depriciation rates greater than twice the minimum annual rate with the straight-line method?

Soln It means that if we multiply the depriciation rate of straight line method by 2 & it comes less than that by the declining balance method, then we can not use it.

Initial value = $V = 50,000$

Service life = $n = 20$ years

Salvage life = $V_s = 4000$

Rate = dep / Initial value

dep = $\frac{50,000 - 4000}{20}$

= 2300

Rate = $\frac{2300}{50,000} = 0.046$

2 times the rate $2 \times 0.046 = 0.092$ = fixed percentage factor

~~Depriciation rate for straight line method~~ $= \frac{V_s}{V}$

$= \frac{4000}{50000} = 0.08$

By Declining balance method $f = 1 - (V_s/V)^{1/n}$

$= 1 - \left(\frac{4000}{50,000}\right)^{1/20} = 0.1186$

Since minimum rate is 0.092 but it is greater than 0.1186 so this method of calculation of dep using text book declining is not acceptable

~~As Minimum annual rate = 0.08~~

~~2 times the minimum annual rate = 2×0.08~~

~~$= 0.16 >$ that by Declining balance method~~

Feasibility report As depriciation rate found by text book declining method is less than twice the minimum annual depriciation rate by straight line method ($0.1186 < 2 \times 0.08$) so this method is acceptable for income tax purposes.

— x — x — x — x —

A piece of equipment having a negligible salvage or scrap value is estimated to have a service life of 10 years. The original cost of the equipment was \$40,000.

Determine the following.

a) the depriciation charge for the fifth year if double declining balance depriciation is used.

b) the depriciation charge for the fifth year if sum of the years digits depriciation is used.

c) the % of the original investment paid off in the first half of the service using the double declining balance method.

d) the % of the original investment paid off in the first half of the service life using the sum of the years digits method.

Soln $n = 10$ years. Original cost $= V = 40,000$.

salvage value $= V_s = 0$.

a) Depriciation charge for fifth year using Double declining balance method.

We will calculate the asset value for fourth & fifth year, subtraction will give the depriciation charge for fifth year.

(For calculating depriciation for 1st year, we will calculate its value before the start (zero year) & 1st year's difference in dep for first year)

$$V_a = V(1-f)^a.$$

$f = 1 - \left(\frac{V_s}{V}\right)$ for text book declining balance method

$f = \frac{2d}{V}$ for double declining balance method.

$$d = \frac{V - V_s}{n} = \frac{40000 - 0}{10} = 4,000$$

$$f = \frac{2d}{V} = \frac{2 \times 4000}{40,000} = 0.2$$

$$V_4 = V(1-0.2)^4 = 40,000(1-0.2)^4 = 16384.$$

$$V_5 = V(1-0.2)^5 = 40,000(1-0.2)^5 = 13107$$

As the asset value for fifth year is less than that in fourth year. It means that this amount has been depreciated by us.

So Depriciation for fifth year = $V_4 - V_5$

$$= 16384 - 13107 = 3277$$

b) Using sum of the years digits method.

Depriciation for any year = $\dfrac{(n - a + 1)}{\Sigma n} (V_0 - V_5)$

As $n = 10$, $a = 5$ as we are finding for 5th year.

$$Dep = \frac{10 - 5 + 1}{55} (40{,}000) = 4363$$

$\Sigma n = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55.$

For first year $\dfrac{10}{55} \times V_0 - V_5$ For 2nd year $\dfrac{9}{55} \times V - V_5$

& so on.

c) % of original investment paid off in 5 years.

Asset value after 5 years = 13107 Since we have calculated asset value after 5 years, so if we subtract it from 40,000 the result will be amount depriciated in 5 years

Initial cost = 40,000

Amount depriciated = 40,000 − 13107 = 26893

% of investment paid off = $\dfrac{26893}{40{,}000} \times 100 = 67.3\ \%$

d) % of original Investment paid off in 5 years for sum of years digits method [Asset value can not be directly calculated but depriciation can be calculated directly so the total amount depriciated in 5 years will be added].

Total depriciated amount in 5 years is obtained by adding the depriciation for each year.

Total Depriciation for 5 years = $\dfrac{10 + 9 + 8 + 7 + 6}{55} (V - V_5)$

$$= \frac{10 + 9 + 8 + 7 + 6}{55} (40{,}000) = 29090.$$

% investment paid off = $\frac{29090}{40,000} \times 100 = 72.72\%$

Alternative method

Depriciation cost for 1st year = $\frac{10}{55} \times 40,000 = 7273$

" " " 2nd " = $\frac{9}{55} \times 40,000 = 6545$

" " " 3rd " = $\frac{8}{55} \times 40,000 = 5818$

" " " 4th " = $\frac{7}{55} \times 40,000 = 5091$

" " " 5th " = $\frac{6}{55} \times 40,000 = 4364$

Total depriciation in 5 years = $7273 + 6545 + 5818 + 5091 + 4364$
$= 29091$

% paid off = $\frac{29091}{40,000} \times 100 = 72.72\%$

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7) The original cost of a property is \$30,000 & it is depriciated by a 6% sinking-fund method. What is the annual dep charge if the book value of the property after 10 years is the same as if it had been depriciated at \$2500/year by the straight line method?

Soln original cost = \$ 30,000
Depriciation /year using straight line = 2500

$$S = \frac{R\left[(1+i)^n - 1\right]}{i}$$

S = Principal ~~amount~~ value
S = V − Vs.

$$V - V_s = \frac{R\left[(1+i)^n - 1\right)}{i} \Rightarrow R = \frac{(V - V_s)\, i}{(1+i)^n - 1} \quad \text{—} \quad ①$$

For given years

$$R = \frac{(V - V_a)\, i}{(1+i)^a - 1}$$

i = Depriciating per centage

Rearrage $$V - V_a = \frac{R\left[(1+i)^a - 1\right]}{i}$$

Put R from eq ①

$$V - V_a = \frac{(V - V_s)\cancel{i}}{(1+i)^n - 1} \times \frac{(1+i)^a - 1}{\cancel{i}}$$

$$V_a = V - \frac{(V - V_s)\left[(1+i)^a - 1\right]}{(1+i)^n - 1}$$

Soln

$$V - V_a = \frac{R\left[(1+i)^a - 1\right]}{i}$$

$V = 30{,}000$ $i = 6\% = 0.06$

Asset value ~~of~~ after 10 years.

Depriciation using straight line method = 2500/year

for 10 year = $\frac{2500}{\text{year}} \times 10\,\text{year} = 25000.$

Scrap ~~Asset~~ value = V − Depriciation for 10 years $V_a = V - ad$

$= 30{,}000 - 25{,}000 = 5{,}000$

R = ?

↳ Annual depriciation charg.

$$30{,}000 - 5{,}000 = \frac{R\left[(1+0.06)^{10} - 1\right]}{0.06}$$

$$R = \frac{25{,}000 \times 0.06}{(1+0.06)^{10} - 1} = 1896$$

$\boxed{R = 1896}$ so the depriciation charge per year for sinking fund method will be 1896 $/year whereas for straight line method it will be 2500 $/year

So it means that by this method (sinking fund method) the depriciation has been reduced to 1896 instead

2500/year so 604 $ has been kept as a profit per year, so greater dividened can be given to share holder & greater profit can be retained. by invester

9.8 A concern has a total income of $ 1 million/year. & all expenses except depriciation amount to $ 600,000/year. At the start of the first year of the concern's operation, a composite account of all depriciable items shows a value of $ 850,000. & overall service life is estimated to be 20 years. The total salvage value at the end of the service life is estimated to be $50,000. Thirty % of all profits before taxes must be paid out as income taxes. What would be the reduction in income tax charges for the first year of operation if the sum of the years digits method were used for depriciation accounting instead of the straight line method?

Soln Total Income = 1000,000 (1 million)

Total expenses without depriciation = 600,000

$V = 850,000$ $V_s = 50,000$ $n = 20$ years.

Reduction in income tax = Difference in Income tax of the 2 methods

Using straight line depriciation method.

Depriciation = $\frac{V - V_s}{n} = \frac{850,000 - 50,000}{20}$

$= \frac{800,000}{20} = 40000$/year

Depriciation for first year = 40,000

Total Revenue = Total Income − (Total expenses)

= Total Income − (Expenses without depriciation + Depriciation)

= 1000,000 − (600,000 + 40,000) = 360,000

Net revenue / Profit = 360,000.

~~Sales~~ Income tax paid = $0.3 \times 360,000 = 108,000$

Using sum of the years digits method

$En = 1+2+3+\dots+20 = 210$

$$\text{Depriciation} = \frac{n-a+1}{En} \times (V - V_s)$$

for First year $a = 1$

$$\text{Dep} = \frac{20-1+1}{210} \times (850,000 - 50,000) = 76190$$

Total Revenue = Total Income − Total Expenses.

= 1000,000 − (Expenses without dep + Depriciation)

= 1000,000 − (600,000 + 76,190)

Net Profit = 323810

~~Sales~~ Income tax = $0.3 \times 323810 = 97143$

Reduction in ~~sales~~ Income tax = $108,000 - 97143$

= 10857

$$\% \text{ Reduction} = \frac{\text{Total Reduction}}{\text{Actual tax}} \times 100$$

$$= \frac{10857}{108,000} \times 100 = 10.05\%$$

b) Calculate income tax ratio in 18th year.

Sum of the years digits method

$$\text{Depriciation in 18th year} = \frac{n-a+1}{210} \times (V - V_s)$$

$$= \frac{20-18+1}{210} \times (850,000 - 50,000)$$

$= 11428$

Total Profit = 1000,000 − (600,000 + 11428)

= 388571

~~Sales~~ tax = $0.3 \times 388571 = 116571$

<u>For Straight line method</u> Dep = $\frac{850,000 - 50,000}{20} = 40,000$

Net Profit = $1000,000 - (600,000 + 40,000)$

$= 360,000$

Sales tax = $0.3 \times 360,000 = 108,000$

Now after 18 years, the income tax for sum of the years digits methods is greater than that for straight line method coz the reason is that for the straight line method, as the profit is constant so income tax will be constant as the depriciation is also constant but for the sum of the years digits ~~balance~~ method, depriciation is decreasing & the total profit is constant so net profit is increasing & ~~sales~~ income tax is also increasing.

9.10 A profit producing property has an initial value of \$50,000 a service life of 10 years & 0 salvage & scrap value. By how much would annual profits before taxes be increased if a 5% sinking fund method were used to determine depriciation costs instead of the straight line method?

Initial value = $V = 50,000$

Salvage value = $V_s = 0$

Increase in annual profits before taxes = ?

<u>Profits using straight line method</u>

Profit = Income − expenses — ①

Since Income is not given so it will be retained as such & only depriciation expenses can be calculated.

$$\text{Depriciation} = \frac{V - V_s}{n} = \frac{50{,}000 - 0}{10} = 5{,}000$$

$$\text{Profit} = \text{Income} - 5{,}000 \quad \text{—} \quad ②$$

Using sinking-fund method.

Depriciation rate, $R = ?$

$$V - V_s = \frac{R((1+i)^n - 1)}{i}$$

$$50{,}000 - 0 = \frac{R\left[(1+0.05)^{10} - 1\right]}{0.05} \qquad i = 5\% = 0.05$$

$$R = \frac{50{,}000 \times 0.05}{(1.05)^{10} - 1} \Rightarrow \boxed{R = 3975}$$

$$\text{Profit} = \text{Income} - \text{Dep.}$$
$$= \text{Income} - 3975$$

$$\left(\begin{matrix}\text{Increase in annual profits using} \\ \text{sinking fund method}\end{matrix}\right) = \begin{matrix}\text{Profits of} \\ \text{sinking} \\ \text{fund}\end{matrix} - \begin{matrix}\text{Profits of} \\ \text{straight line}\end{matrix}$$

$$= (\text{Income} - 3975) - (\text{Income} - 5000)$$
$$= \text{Income} - 3975 - \text{Income} + 5000$$
$$= 1025 = 1025$$

(or we can say that profit will increase for sinking fund method when its expenses (depriciation exp) decreases)

So an extra profit of 1025 ~~rps~~ $ will be available when we use sinking-fund method instead of straight line method.

→ The reson is that the amount which we are depriciating is also giving us some profit coz it will be placed in bank so some interest will be available to us.

——————— × ——————— × ——————— × ———————

3 The total value of a new plant is $ 2 million. A certificate of necessity has been obtained permitting a write off of 60% of the initial value in 5 years. The balance of the plant requires a write off period of 15 years. Using the straight line method & assuming negligible salvage & scrap value, determine the total depriciation cost during the first year?

Total Investment = 2 Million = 2000,000

For First 5 years = 60% of Total value = $0.6 \times 2 \times 10^6$

$= 1.2 \times 10^6$

For remaining 15 years, = 40% of Total value = $0.4 \times 2 \times 10^6$

$= 8 \times 10^5$

Salvage value = 0

Total Depriciation cost for first year = (Depriciation due to necessity) + (Depriciation due to writt off)

(Depriciation due to necessity) As this depriciation cost has a total life of 5 years ~~& its end cost is that of the 15 years i.e 8 x 10^5~~ Both have zeo salvage value. one has service life 5 years, & other has 15 years from start period

$$Dep = \frac{1.2 \times 10^6 - 0}{5} = 0.24 \times 10^6 = 2.4 \times 10^5$$

Depriciation due to write off. It is for 15 years & it, salvage value is 0. $V = 8 \times 10^5$

$$Dep = \frac{8 \times 10^5 - 0}{15} = 53\,333.$$

Total Depriciation = ~~80,000~~ 2.4×10^5 + 53,333

= ~~133,333~~ = 293 300

9.11 In order to make it worth while to purchase a new piece equipment, the annual depriciation costs for the equipment cannot exceed $ 3000 at any time. The original cost of the equipment is $ 30,000 & it has get salvage or scrap value. Determine the length of the service life necessary if the equipment is depriciated a) by the sum of the years digits method. and b) by the straight line method.

Maximum annual depriciation = 3,000.

Initial Investment = 30,000

Salvage value = 0.

Service life = ?

a) Sum of the years digits methods

~~Dep~~ As the maximum depriciation will be for first year in sum of years digits methods.

$$Dep = \frac{n - a + 1}{\Sigma n}(V - V_s)$$

$$3000 = \frac{n - \cancel{1} + \cancel{1}}{n\left(\frac{n+1}{2}\right)}(30,000 - 0)$$

$$3000\left[\frac{n^2 + n}{2}\right] = n(30,000)$$

$$\frac{n^2 + n}{2} = n \times 10 \Rightarrow n^2 + n = 20n$$

$$n^2 - 20n + n = 0.$$

$\left(\begin{matrix} n = 0 \\ n = 19 \end{matrix}\right)$

$$n^2 - 19n = 0 \Rightarrow n(n-19) = 0$$

$\boxed{n = 0}$ Not possible

$n - 19 = 0$

$\boxed{n = 19}$

Verificiation Dep for year 1 $= \frac{19 - 1 + 1}{190}(30,000)$

$\Sigma 19 = 1 + 2 + 3 + \cdots + 19 = 190$

$= 3,000$

which verfies that the service life is true of maximum priciation can only be possible in the first year which is same as calculated by us.

b) <u>By straight line method.</u>

$$Dep = \frac{V - V_s}{n} \Rightarrow n = \frac{V - V_s}{Dep}$$

$$n = \frac{30,000 - 0}{3000} \Rightarrow \boxed{n = 10}$$

As depriciation for straight line method will be same in each year.

9.15 An asset with an original cost of 10,000 & no salvage value has a depriciation charge of \$ 2381 during its 2nd year of service when depriciated by the sum of digits method. What is its expected useful life?

Initial cost $(V) = 10,000$

salvage value $(V_s) = 0$.

Depriciation charge for 2nd year = 2381.

$n = ?$

As $$Dep = \frac{n - a + 1}{\Sigma n}(V_0 - V_s)$$

for 2nd year, $a = 2$.

$$2381 = \frac{n - 2 + 1}{n\left(\frac{n+1}{2}\right)}(10,000) \Rightarrow \frac{n^2+n}{2}\frac{(2381)}{10,000} = n - 1$$

$$\frac{n^2+n}{2}(0.238) = n - 1$$

$$0.238n^2 + 0.238n = 2n - 2$$

$$0.238n^2 + 0.238n - 2n + 2 = 0$$

$$0.238n^2 \cancel{+} - 1.762n + 2 = 0$$

$$n^2 - 7.40n + 8.40 = 0$$

$n = 6$, $n = 1.4$

As service life of 1.4 years is not possible coz depriciation for 2nd year is given to us.

so service life = n = 6 years.

verification

Dep for 2nd year = $\frac{6-2+1}{21} \times 10,000$

$\Sigma n = 1+2+3+4+5+6 = 21$ $= 2381$

so the service life of 6 years is correct

9.12 The owner of a property is using the unit of production method for determining the depriciation costs. The original value of the property is 55,000. It is estimated that this property can ~~be~~ produce 5500 units before its value is reduced to zero. ie the depriciation cost per unit is 10. The property produces 100 units during the first year, & production rate is doubled each year for the first 4 years. The production rate obtained in the 4th year is then held constant until the value of the property is paid off. what would have been the annual depriciation cost if the straight line method based on time had been used?

Soln Depriciation cost per unit is given but the depriciation cost per year is to be calculated.

$V = 55,000$ $n = ?$ $V_s = 0$

Dep per year = $\frac{V - V_s}{n}$

for n

Year	No of units		Total units produced
1	100		100
2	200	double	300
3	400	double	700
4	800	double	1500
5	800	constant. production	2300
6	800		3100
7	800		3900
8	800		4700
9	800		5500

N = 9 years.

∴ 5500 units are produced in 9 years.

$$\text{Depriciation per year} = \frac{55000 - 0}{9} = 6111.11\ ₹/\text{year}$$

<u>Example 1</u> Page-282 The original value of a piece of equipment is 22,000 completely installed & ready for use. Its salvage value is estimated to be 2000 at the end of service life estimated to be 10 years. Determine the asset or book value of the equipment at the end of 5 years using,

a) Straight line method.

b) Text book declining balance method.

c) Double declining balance (200%) method

[ie the declining balance method using a fixed percentage factor giving a depriciation rate equivalent to twice the minimum rate with the straight line method]

<u>Soln</u> original value = 22,000
salvage value = 2000
service life = 10 years
Asset value after 5 years = ?

a) <u>Straight line method</u>

$$\text{Depriciation per year} = \frac{V - V_s}{n} = \frac{22,000 - 2000}{10}$$

$$= 2,000\,/\text{year}$$

Depriciation for 5 years = 2000 × 5 = 10,000

Asset value after 5 years = Initial value − Depriciation of 5 years

$$= 22000 - 10{,}000 = 12{,}000$$

$V_a = V - ad$
$= 22000 - 5 \times 2000$
$= 12{,}000$

b) <u>Text book declining balance method</u>

$$V_a = V(1-f)^a$$

a = year at which value is to be obtained

a = 5 in this case

$$V_5 = V(1-f)^5 \quad — ①$$

$$\left(\frac{\quad}{V}\right) = 1-\left(\frac{\quad}{22,000}\right) \Rightarrow f = 0.213$$

Put in eq ①

$$V_5 = 22,000\,(1-0.213)^5$$
$$= 6641$$

c) <u>Using double declining balance method</u>

$$V_a = V(1-f)^a$$

$$f = \frac{2d}{V} \qquad d = \frac{V - V_s}{n}$$

$$d = \frac{22,000 - 2000}{10} = 2,000$$

$$f = \frac{2000 \times 2}{22,000} \Rightarrow \boxed{f = 0.1818}$$

$$V_5 = 22,000\,(1-0.1818)^5$$

$$\boxed{V_5 = 8066}$$

---

due to tech

ample 7 Determination of capitalized cost

A new piece of completely installed equipment costs 12,000 & will have a scrap value of 2000 at the end of its useful life. If the useful life period is 10 years & the interest is compounded at 6% per year, what is the capitalization cost of the equipment ?

Soln original value, C_V = 12,000

scrap value = V_s = 2000 $\quad n = 10$ years.

$i = 6\% = 0.06$.

so Replacement value = original value − scrap value

$$C_R = 12{,}000 - 2{,}000 \Rightarrow \boxed{C_R = 10{,}000}$$

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

capitalized cost compound interest

$$= 12{,}000 + \frac{10{,}000}{(1+0.06)^{10} - 1} \Rightarrow K = 12000 + 12644$$

$$\boxed{K = 24644}$$

It means that if we put 24644 Rps in bank at the start of project ~~life after 10 years, we can replace the equipment~~ & buy equipment of 12,000

so net amount remaining in the account is so it will become 22644 after 10 years, we made replacement to the equipment of 10,000 remains again. Again after 10 years, it will be 22644 & 10,000 is get as a replacement cost

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Example 8 Comparisons of alternative investments usi capitalized costs.

A reactor which will contain corrosive liquids has been designed. If the reactor is made of mild steel, the initial cost will be $ 5000 & the useful period will be 3 years. nce stainless steel is highly resistant to corrosion action of the liquids, stainless steel as the material of construction has been proposed as an alternative to mild steel. The stainless steel reactor would have an initial installed cost of \$15,000. The scrap value at the end of the useful life would be zero for either type of reactor & replacement cost of both is equal to the original price. On the basis of equal capitalized costs for both type of reactors, what should be the useful life period for the stainless steel reactor if money is worth 6% compounded annually.

Soln For reactor of mild steel $C_V = 5000$ $V_S = 0$

$C_R = 5,000$ $n = 3$ years

stainless steel reactor $C_V = 15,000$ $V_S = 0$

$C_R = 15,000$ $i = 6\% = 0.06$

annual compounding, i_{eff} = nominal interest

As capitilized costs of both are equal,

K of mild steel = K of stainless steel.

$$C_V + \frac{C_R}{(1+i)^n - 1} = C_V + \frac{C_R}{(1+i)^n - 1}$$

$$5000 + \frac{5000}{(1+0.06)^3 - 1} = 15000 + \frac{15000}{(1+0.06)^n - 1}$$

$$31175 = 15000 + \frac{15000}{(1+0.06)^n - 1}$$

$$16175 = \frac{15000}{(1+0.06)^n - 1}$$

$(1+0.06)^n - 1 = 15000/16175 \Rightarrow (1+0.06)^n = 1 + 0.927$

$(1+0.06)^n = 1.9273$ Taking ln on both sides

$n \ln(1+0.06) = \ln 1.9273$

$$n = \frac{\ln 1.9273}{\ln 1.06} \Rightarrow n = \frac{0.656}{0.0583}$$

$$\boxed{n = 11.3 \text{ years}}$$

CHAPTER 10

Profitability, Alternative Invesments, and Replacements

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Malik Zeeshan AFZAL

PROFITABILITY

It is term used for measure of the amount of profit that can be made from a given situation.

PROFIT

unamortized value = B.V − net realizable value

It is the difference between income and expenses. Profit depends on the quantity of good or services produced and the selling price. Amount of profit can also be affected by the economic efficiency of the plant or operation.

METHODS FOR PROFITABILITY EVALUATION

Following methods are used for profitability evaluation.

1) rate of return on investment
2) Discounted cash flow based on full-life performance
3) Net present worth.
4) Captilized cost.
5) Payout period.

① RATE OF RETURN ON INVESTMENT

It is normally expressed on an annual percentage basis. It is defined as

$$\text{Fractional return} = \frac{\text{yearly profit}}{\text{total initial investment}}$$

$$\text{percent return on investment} = \frac{\text{yearly profit}}{\text{total initial investment}} \times 100$$

ALTERNATIVE INVESTMENTS

333-5002419

GENERAL RULES

The minimum investment which will give the necessary functional results and required rate of return should always be accepted unless there is no specific reason for accepting an alternative investment requiring more initial capital when alternatives are available, the base plan would be that requiring the minimum acceptable investment.

EXAMPLE. 1

A chemical company is considering adding a new production unit which will require a total investment of \$ 1,200,000 and will yield a profit of \$ 240,000
An alternative addition has been proposed requiring an investment of \$ 2 million and yielding an annual profit of \$ 300,000 Minimum rate required for the new investment is 14%. which investment is recomended?

SOLUTION

For first investment

initial investment = \$ 1,200,000

annual profit = \$ 240,000

$$\% \text{ rate of return} = \frac{\text{yearly profit}}{\text{total initial investment}} \times 100$$

$$\% \text{ rate of return} = \frac{240,000}{1,200,000} \times 100 = 20\%$$

For second investment

initial investment = \$ 2 million.

annual profit = \$ 300,000

$$\% \text{ rate of return} = \frac{300,000}{2,000,000} \times 100$$

$$\% \text{ rate of return} = 15\%$$

Both these returns exceed the minimum required value and it might appear that second investment is recommended because it yields greater amount of profit per year. If we compare initial investment, we see that an extra investment of \$ 800,000 gives a profit of only \$ 60,000

[illegible]

So if company has \$ 2 million, it would be better to investment \$ 1,200,000 for first investment and \$ 800,000 in another investment at 14% return.

20 ③

EXAMPLE 4

Pak-2 Pak-2

A plant is being designed in which 450,000 lb/day of a water-caustic-soda liquor containing 5% by weight caustic soda must be concentrated to 40% by weight. A single or multiple-effect evaporator will be used and a single effect evaporator of required capacity requires an initial investment of $18000. The same investment is required for each additional ~~[illegible]~~ effect. The service life is to be estimated 10 years and the salvage value of each effect at the end of service life is estimated to be $6000. Fixed charges minus depreciation amount to 20% yearly based on initial investment. Steam costs $0.6 per 1000 lb and administration, labor and other costs are $40 per day no matter how many effects are used. When X is the number of evaporator effects, 0.9X equals the number of pounds of water evaporated per pound of steam. [illegible] on 300 days (operating) in a year. If the minimum acceptable return on any investment is 15%, how many effects must be used?

SOLUTION

Dep/day = $\frac{\text{Dep}}{\text{year}} \times \frac{1\ \text{year}}{300\ \text{day}}$

Dep/day of X eff. = $\frac{\text{Dep}}{300\ \text{day}} \times X$

$\frac{V-V_s}{n\,300}\ X$

Basis = 1 operating day.

here

X = total number of evaporator effects.

From straight-line method

$$\text{depreciation per day} = \frac{V-V_s}{n \times 300} \qquad \text{— (A)}$$

[illegible]

V = original cost = $18000

V_s = salvage value = $6000

n = 10 years

In above relation 300 days are operating days. Put values in eq. (A)

$$\boxed{\begin{aligned} d &= X\,\frac{(18000-6000)}{10 \times 300} \\ d &= \$4.00\ X/\text{day} \end{aligned}}$$

$v.\ \Sigma^{n}$ — (B)

$$\text{Fixed charges} - \text{depreciation} = \frac{X(18000)(0.2)}{300} = \$12\ X/\text{day} \qquad \text{— (C)}$$

④

In order to calculate wt. of evaporated water, apply materi balance.

Overall material balance

$$F = D + W$$

$$450000 = D + W$$

Solid balance

$$.05F = 0(D) + 0.4(W)$$

$$0.05(450000) = 0.4W.$$

$$W = 56250 \text{ lb/day}$$

$$450000 = D + 56250$$

$$D = 450000 - 56250$$

$$D = 393750 \text{ lb/day.}$$

Steam used = $\frac{393750}{0.9X}$ lb/day ←

Economy = $\frac{D}{m_s}$ — Ⓔ

$= \frac{393750}{0.9X}$

put in Ⓔ

Cost of steam = \$0.6 per 1000 lb ← from statement

Cost of 1000 lb of steam = \$0.6

" " $\frac{393750}{0.9X}$ " " = $\frac{0.6 \times 393750}{0.9X1000}$ = $\frac{\$262.5}{X}$ per day

1000 LS — 0.6

1 — $\frac{0.6}{1000}$

$\frac{393750}{0.9X}$ — $\frac{0.6 \times 393750}{1000 \times 0.9X}$

$= \frac{236250}{900X} = \frac{262.5}{X}$ — Ⓒ

	X no. of effect	Steam Cost per day. (eq. D)	Fixed charges - dep. charges per day. (eq. C)	Dep. per day. eq. Ⓑ	Lab. etc per day.	total cost per day (= sum of costs)
	1)	\$ 262.5	\$ 12 = ,	\$ 4	\$ 40	\$ 318.5 = 262+12+4+40, 0K
:42/2	2)	\$ 131.3	\$ 24 - 2×c	\$ 8	\$ 40	\$ 203.3
2-5/3	3)	\$ 87.5	\$ 36	\$ 12	\$ 40	\$ 175.5
2-5/4	4)	\$ 65.6	\$ 48	\$ 16	\$ 40	\$ 169.6
2-5/5	5)	\$ 52.5	\$ 60	\$ 20	\$ 40	\$ 172.5
5/6	6)	\$ 43.8	\$ 72	\$ 24	\$ 40	\$ 179.8

1/& 2) Now compare two effects with one effect.

Percent return = $\frac{(318.5 - 203.3)(300)(100)}{36000 - 18000}$ = 192%

3/& 4/ So two effects are better than one effect.

Comparing four effects with three effect.

Percent return = $\frac{(175.5 - 169.6)(300)(100)}{72000 - 54000}$ = 9.8%

Compare two effects with three effects

Percent return = $\frac{(203.3 - 175.5)(300)(100)}{54000 - 36000}$ = 46.3%

Three effects are better than two

Comparing five effects with three effects.

$$\text{return} = \frac{(175.5-172.5)(300)(100)}{(90,000-54000)} = 2.5\%$$

From above comparisons, it is found that three effects are useful for given conditions

② DISCOUNTED CASH FLOW P

This method takes into account the time value of money and is based on the value of investment that is unreturned at the end of each year during the estimated life of the project.) A trial and error procedure is used to establish a rate of return which can be applied to yearly cash flow so that original investment is reduced to zero during the project life; So rate of return by this method is equivalent to the maximum interest rate at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal.

Example Ex. ②

Find annual interest rate by discounted cash flow has the following data.

Initial fixed capital investment = \$ 100,000

Working capital investment = \$ 10,000

Service life = n = 5 years.

Salvage value at the end of service life = V_s = \$ 10,000

YEAR	Estimated cash flow
0	\$ 110,000
1	30,000
2	31,000
3	36,000
4	

⑤ B

4	40,000
5	43,000

SOLUTION

At the end of five years, the cash flow to the project compounded on the basis of end-of-year income is

$$S = P(1+i)^{n-1}$$

$$S = 30{,}000(1+i)^4 + 31000(1+i)^3 + 36000(1+i)^2 + 40000(1+i) + 43000 \quad Ⓐ$$

Here S represents the future worth and must equal to Ⓐ future worth of the initial investment compounded at a interest rate i corrected for salvage value and working capital. So

$$S = P(1+i)^n - W.C. - \text{Salvage value}$$

$$S = 110{,}000(1+i)^5 - 10{,}000 - 10{,}000 \quad Ⓑ$$

Equating eq: Ⓐ + Ⓑ

$$110{,}000(1+i)^5 - 20000 = 30000(1+i)^4 + 31000(1+i)^3 + 36000(1+i)^2 + 40000(1+i) + 43000$$

Now suppose such a value of i so that both sides of above equations are satisfied

Let us assume

$i = 0.207$ put in above equation

$$261792 = 63672.3 + 54510.9 + 52446 + 48280 + 43000$$

$$261792 = 261908$$ (nearly equal)

So

After taxes discounted cash flow rate of return $= i = 20.7\%$

③ NET PRESENT WORTH

In this method, cost of capital can be taken as the average value of return, the company earns on its capital or it can be designated as the minimum acceptable return for the project. The net present worth of the project is then difference between the present value of the annual cash flows and the initial required investment.

⑤. PAYOUT PERIOD R

minimum length of time theoretically necessary to recover the original capital investment in the form of cash flow to the project based on total income minus all costs except depreciation. (In this method original capital investment means only the original depreciable fixed capital investment and interest effects are neglected.
In equation form

$$\text{Payout period (year)} = \frac{\text{Depreciable fixed capital investment}}{\text{Avg profit/year} + \text{Avg dep./year}}$$

⑥ CAPTILIZED COSTS D

Captilized cost represents the amount of money that must be available initially to purchase the equipment and to provide sufficient fund for interest accumulation to permit perpetual replacement of the equipment.)
The basic equation for captilized cost for equipment is

$$K = \frac{C_R(1+i)^n}{(1+i)^n-1} + V_s = C_V + \frac{C_R}{(1+i)^n-1}$$

where

K = captilized cost.
C_R = replacement cost of the equipment.
V_s = salvage value at the end of service life.
n = estimated service life of equipment.
i = interest rate.
$\frac{(1+i)^n}{(1+i)^n-1}$ = captilized cost factor.

Example 1 (P.3[illegible])

⑦

A proposed manufacturing plant requires an initial ~~initial~~ fixed-capital investment of $900,000 and $100,000 of working capital. It is estimated that the annual income will be $800,000 and the annual expenses including depreciation will be $520,000 before income taxes. A minimum annual return of 15 percent before income taxes is required before the investment will be ~~worthwhile~~. Income taxes amount to 34% of all pre-tax profits.

North wile

Determine

a) annual % return on the total initial investment before income taxes.

F.C.I. = Cost of plant = $ 900,000 = fixed capital investment.

Annual profit = $ 800,000.

Annual expenses = $ 520,000

Net profit = Annual profit − Annual expences.

" " = $ 800,000 − $ 520000

" " = $ 280,000

W.C. = working capital = $ 100,000

total capital investment = F.C.I. + W.C.

" " " = 900,000 + 100,000

" = $ 1000,000

$$\% \text{ rate of return} = \frac{\text{Annual profit}}{\text{total cap. invest.}} \times 100$$

$$\% \text{ rate of return} = \frac{280,000}{1000,000} \times 100 = 28\%$$

b) The rate of return after taxes

Net Annual profit = $ 280,000

Income tax rate = 34%

Income tax paid = 280,000 x 0.34 = $ 95200

So net annual profit after taxes = 280,000 − 95200

= $ 184800

$$\% \text{ rate of return} = \frac{184800}{1000,000} \times 100$$

% rate of return = 18.48%

f) % annual percent return on the total capital investment before income taxes based on capital recovery with minimum profit.

Required rate of return = 15% (900000+100000)

minimum profit required = (1000,000)(0.15) = \$150,000

Fictitious expenses = 520,000 + 150,000 = \$670,000

Now net profit = 800,000 − 670,000 = \$130,000.

So

$$\% \text{ rate of return} = \frac{130,000}{1000,000} \times 100 = 13\%$$

g) Annual percent return on average investment before income taxes assuming straight line depreciation and zero salvage value.

$$\text{average fixed capital investment} = \frac{\$900,000}{2} = \$450,000$$

total capital investment = 450,000 + 100,000
= \$550,000

$$\% \text{ rate of return} = \frac{280,000}{550,000} \times 100 = 51\%$$

EXAMPLE

Choice the most suitable design with given data.

DESIGN	1	2	3	4
Initial installed cost	10,000	16,000	20,000	26,000
Operating cost	100	100	100	100
Fixed charges %	20	20	20	20
Value of heat saved	4100	6000	6800	8850

SOLUTION

The net annual savings equals the value of heat saved minus the sum of the operating cost and fixed charges.

Value of heat saving = \$4100

operating cost = \$100

Fixed charges = 0.2(10,000) = \$2000

Net saving = 4100 − 2000 − 100 = \$2000

$$\text{Annual } \% \text{ return} = \frac{2000}{10000} \times 100 = 20\%$$

For 2nd design

Value of heat saved = \$ 6000

operating cost = \$100

Fixed charges = 0.2(16,000) = \$ 3200

Net saving = 6000 - 3200 - 100 = \$2700

Annual % return = $\frac{2700}{16000} \times 100 = 16.9\%$.

For 3rd design

Value of heat saved = \$6900

operating cost = \$100

Fixed charges = 0.2(20000) = \$4000

Net saving = 6900 - 4000 - 100 = \$ 2800

Annual % return = $\frac{2800}{20000} \times 100 = 14\%$.

For 4th design

Value of heat saved = \$ 8850

operating cost = \$100

Fixed charges = 0.2(26000) = \$5200

Net saving = 8850 - 5200 - 100 = \$ 3550

annual % return = $\frac{3550}{26000} \times 100 = 13.6\%$.

As the annual percent rate of return is above minimum 10% so any one of above designs will be acceptable.

ANALYSIS BY INCREMENTAL INVESTMENT

Comparing design 1 with 2

$$\% \text{ return} = \left(\frac{2700-2000}{16000-10000}\right) \times 100 = 11.67\%.$$

Design no. 2 is acceptable

Comparing design 2 and 3

$$\% \text{ return} = \left(\frac{2800-2700}{20000-16000}\right) \times 100 = 2.5\%.$$

Design 3 is rejected.

ECONOMICS

CH.10 ⑯

PROBLEM 10.1

PROFITIBILITY

A proposed chemical plant will require a fixed capita investment of $10 million. It is estimated that the work capital will amount to 25% of total investment and depreciation costs amount to 10% of the fixed capital investment. If the annual profit will be $3 million, determ the standard percent return on the total investment and the minimum payout period.

SOLUTION :-

Fixed capital investment = $10 million = $\$10 \times 10^6$

Working capital = 25% of total investment.

Depreciation = 10% of fixed capital investment

Annual profit = $3 million

% return = ?

Minimum payout period = ?

Let total capital investment = x.

so working capital = 0.25x

As

total capital investment = Fixed capital investment + $\overset{w}{c}$ working capital

$$x = 10 \times 10^6 + 0.25x$$

$$x - 0.25x = 10 \times 10^6$$

$$0.75x = 10 \times 10^6$$

$$x = \frac{10 \times 10^6}{0.75}$$

total capital investment = x = $\$13.33 \times 10^6$

As

$$\% \text{ return} = \frac{\text{Profit}}{\text{total investment}} \times 100$$

$$\% \text{ return} = \frac{3 \times 10^6}{13.33 \times 10^6} \times 100$$

$$\% \text{ return} = 22.5\%$$

As

$$\text{Payout period} = \frac{\text{F.C.I.}}{\text{Profit + Depreciation}}$$

$$\text{Payout period} = \frac{10 \times 10^6}{3 \times 10^6 + 0.1 \times 10 \times 10^6}$$

Payout period = 2.5 years

PROBLEM 10.2

Pak'07

An investigation of proposed investment has been made the following results has been presented to management. The minimum payout period based on capital recovery using a minimum return of 10% as a fictitious expense is 10 years, annual depreciation cost amount to 8% of the total investment. Using this information, determine the standard rate of return on the investment.

SOLUTION

% return = 10%

minimum payout period = 10 years

Annual depreciation cost = 8% of total investment

Standard rate of return on investment = ?

Since payout period is :

$$\text{Payout period} = \frac{\text{F.C.I.}}{\text{Annual profit + Depreciation cost}} \quad \text{--- (A)}$$

Since W.C = 0

→ T.C.I. - F.C.I = 0

$\boxed{\text{TCI} = \text{FCI}}$

As

$$\%\ \text{Return} = \frac{\text{Annual profit}}{\text{total investment}} \times 100$$

$$\text{annual profit} = \frac{\%\text{R}}{100}(\text{F.C.I}) \quad \text{(B)}$$

As

total investment = F.C.I. + W.C.

total investment = F.C.I. + 0

total investment = F.C.I.

As

$$\text{Annual profit} = \frac{\%\text{R}}{100}(\text{F.C.I.}) - \underbrace{0.1(\text{F.C.I.})}_{\text{fictitious expense}}$$

Let $\frac{\%\text{R}}{100} = x$

Put values in Ⓐ

$$\text{Payout period} = \frac{\text{F.C.I.}}{x(\text{F.C.I.}) - 0.1(\text{F.C.I}) + 0.08(\text{F.C.I})} \times 100$$

$$10 = \frac{\text{F.C.I.}}{\text{F.C.I.}(x - 0.1 + 0.08)}$$

$$10 = \frac{1}{x - 0.1 + 0.08}$$

$$10x - 1 + 0.8 = 1$$

$$10x = -0.8 + 2$$

$$x = \frac{1.20}{10} = 0.12$$

$$x = \frac{\%\,R}{100}$$

$$\%\,R = x \times 100$$

$$\%\,R = 0.12 \times 100 = 12\%$$

PROBLEM 10.3

The information given in Prob. 2 applies to conditions before income taxes. If 34% of all profits must be paid out for income taxes, determine the standard rate of return after taxes using the figures given in prob. 2

SOLUTION

Income taxes paid = 34%

% return = 12%

Standard rate of return after taxes = ?

Return after tax = Return before tax – 0.34 Return ~~after~~ Before tax

Return after tax = Return before tax (1 – 0.34)

Return after tax = 0.12(1 – 0.34)

Return after tax = 0.0792

Return after tax = 7.92%

PROBLEM 10.4 ✓

(19)

(1997)

A heat exchanger has been designed and insulation is being considered for the unit. The insulation can be obtained in thickness of 1, 2, 3, 4 inches. Following data has been determined for the different insulation thickness :

SOLUTION

Insulation	1"	2"	3"	4"
Btu/hr saved	300,000	350,000	370,000	380,000
Cost for installed Ins.	$1200	$1600	$1800	$1870
Annual fixed cost	10%	10%	10%	10%

What thickness of insulation should be used. The value of heat is 30 cents/1000000 Btu. An annual return of 15% on the fixed capital investment required for any capital put into this type of investment. The exchanger operates 300 days per year.

SOLUTION

1 $ = 100 cents

Annual return = 15%

Value of heat = 30 cents/1000,000 Btu

working days = 300 days

Amount saved due to insulation = Heat saved × cost of heat

$$= 300000\,\frac{\text{Btu}}{\text{hr}} \times \frac{30\ \text{cents}}{1000000\ \text{Btu}}$$

$$= 9\ \text{cents/hr}$$

Amount saved ~~per day~~ $= 9\,\frac{\text{cents}}{\text{hr}}\,\Big|\,\frac{24\ \text{hr}}{1\ \text{day}} = 216\ \text{cents/day}$

$$= 216\,\frac{\text{cents}}{\text{day}}\,\Big|\,\frac{1\,\$}{100\ \text{cents}}\,\Big|\,\frac{300\ \text{days}}{\text{one year}}$$ ← working days

$$= \$648/\text{year}$$

Net Savings = Amount saved — annual fixed cost (on cost for insulation)

$= 643 - 0.1 \times 1200$

Net Savings $= \$528/\text{year}$

2)

Amount saved due to insulation $= 350000\,\frac{\text{Btu}}{\text{hr}} \times \frac{30\ \text{cents}}{1000,000\ \text{Btu}}$

$$= 10.5\ \text{cents/hr}$$

[illegible]

⑳

Amount saved per year $= \frac{19.5\ \text{cents}}{\text{hr}} \left| \frac{1\$}{100\ \text{cents}} \right| \frac{24\ \text{hr}}{1\ \text{day}} \left| \frac{300\ \text{day}}{1\ \text{year}} \right|$

$= \$756.0/\text{year}$

Cost for insulation $= 0.1 \times 1600 = \$160/\text{year}$

Net amount saved $= 756.0 - 160 = 596.0/\text{year}$

$\frac{596}{400} = 17\%$
2nd is preferred

*. In both heat saved is 350,000 Btu/hr. but I have taken 370,000 Btu/hr. so use the correct value 350,000 Btu/hr. for this part.

c)

Amount saved $= \frac{370000\ \text{Btu}}{\text{hr}} \left| \frac{30\ \text{cents}}{1{,}000{,}000\ \text{Btu}} \right| \frac{24\ \text{hr}}{1\ \text{day}} \left| \frac{300\ \text{day}}{1\ \text{year}} \right| \frac{1\$}{100\ \text{cents}}$

Amount saved $= \$799.2/\text{year}$

Cost for insulation $= 0.1 \times 1800 = \$180$ 590

So

Net amount saved $= 799.2 - 180 = \$619.2/\text{year}$

d)

Amount saved $= \frac{380{,}000\ \text{Btu}}{\text{hr}} \left| \frac{30\ \text{cents}}{1000{,}000\ \text{Btu}} \right| \frac{1\$}{100\ \text{cents}} \left| \frac{24\ \text{hr}}{1\ \text{day}} \right| \frac{300\ \text{days}}{1\ \text{year}}$

Amount saved $= \$820.8/\text{year}$

Cost for insulation $= 0.1 \times 1870 = \$187/\text{year}$

Net amount saved $= 820.8 - 187 = \$633.8/\text{year}$

$\frac{633.8 - 596}{1870 - 1600} = 14\%$ out

COMPARISON

Comparing a) & b)

$\%\ \text{saving} = \frac{596 - 528}{1600 - 1200} \times 100 = 17\%$

Comparing b) & c)

$\%\ \text{savings} = \frac{619.2 - 596}{1800 - 1600} \times 100 = 11.6\%$

Comparing b) & d)

$\%\ \text{savings} = \frac{633.8 - 596}{1870 - 1600} \times 100 = 14\%$

So comparing, we found that 2" insulation is better than other insulations.

PROBLEM 10.5 2/7

A company must purchase one reactor to be used in overall operation. Four reactors have been designed, all of which are equally capable of giving the required service. The following data apply to the four designs

SOLUTION

	1	2	3	4
Fixed capital Investment	\$10,000	\$12,000	\$14000	\$16000
Sum of operating & fixed costs per year.	\$3,000	\$2800	\$2350	\$2100

rate of return = 15%

Comparing 1) + 2)

$$\% \text{ return} = \left(\frac{3000-2800}{12000-10000}\right)\times 100 = 10\%$$

Comparing 1) & 3)

$$\% \text{ return} = \left(\frac{3000-2350}{14000-10000}\right)\times 100 = 16.25\%$$

Comparing 3) + 4)

$$\% \text{ return} = \left(\frac{2350-2100}{16000-14000}\right)\times 100 = 12.5\%$$

From comparison, we see that design 3 will be better.

PROBLEM 10.6 2/7

The capitalized cost for a piece of equipment has been found to be \$55,000. This cost is based on the original cost plus the present value of an indefinite number of renewals. An annual interest rate of 12% was used in determining the capitalized cost. The salvage value at the end of the service life was taken to be zero and service life was estimated to be 10 years. Under these conditions what would be the original cost of the equipment?

H

$$K = \frac{C_R(1+i)^n}{(1+i)^n-1} + \cancel{V_s}$$

㉒

$$C_R = V - \cancel{V_s}^{0} \rightarrow C_R = V$$

SOLUTION

~~Vs~~

Capitilized cost = K = \$ 55,000

$i = 12\%$

$V_s = 0$

$n = 10$ years

$C_V = ?$

As we know

$$\boxed{K = \frac{C_V(1+i)^n}{(1+i)^n - 1} + V_s}$$

$$55000 = \frac{C_V(1+.12)^{10}}{(1+.12)^{10}-1} + 0$$

$C_V = \$ 37291.47$

PROBLEM 10.8 Imp (2002) 2 mrs (2nd)

An existing warehouse is worth \$ 500,000 and the avg ~~storage~~ value of goods stored in it is \$ 400,000. The salvage insurance rate on it is 1.1 percent and the annual rate on the stored goods is 0.95 percent. If a proposed insurance sprinkling system is installed in the warehouse, both insurance rates would be reduced to three quarters of the original rate. The installed sprinkling system would cost \$ 20,000 and the additional annual cost of maintainace, inspection and taxes would be \$ 300. The required write off period for the entire investment in the sprinkler system is 20 years. The capital necessary to make the investment is available. The operation the warehouse is now giving an 8% return on the original investment. Give reasons why you would recomend installing the sprinkler system.

SOLUTION

ware house worth = \$ 500,000

Avg. value of goods = \$ 400,000

Annual insurance rate of ware house = 1.1%

" " " " goods = 0.95%

Aft- the installation of warehouse with sprinkler syst

Insurance rate of goods = $0.95 \times \left(\frac{3}{4}\right) = 0.7125\%$

" " " ware house = $1.10 \times \frac{3}{4} = 0.825\%$

Cost of installed sprinkler system = \$ 20,000

other costs = \$ 300

Required write off period = 20 years.

% return = 8% on original investment.

Now

Insurance cost without sprinkler system

Cost of ware house $= (500,000)(0.011)(20)$

$= \$0.11 \times 10^6$

Cost of goods $= (400,000)(0.0095)(20)$

Cost of goods $= \$76,000$

total cost = Cost of W.H. + cost of goods

total cost $= 0.11 \times 10^6 + 76,000$

total cost $= \$180,000$

Now after the installation of sprinkler system

Cost of W.H. $= (500,000)(.00825)(20) = \$82500$

Cost of goods $= (400,000)(0.007125)(20) = \$57,000$

total cost $= 82500 + 57000 = \$139500$

For 20 years:-

Installation + maintenance cost $= 20,000 + 20(300)$

$= 20000 + 6000$

$= \$26000$

total cost of sprinker system $= 26000 + 139500$

$= \$165500$

So, savings in 20 years due to sprinkler system $= 186000 - 165500$

$= \$20500$

So, saving in 1 year $= \frac{20500}{20} = \$1025$

$\% \text{ return} = \frac{1025}{20000} \times 100 = 5.125\%$

We get more return on non-sprinkler system, so it is better to use non-sprinkler system.

Additional cost savings $= 60{,}000 - 74{,}000 - d_2$

$= -14000 - d_2$

labor cost saving $= 150{,}000 - 120{,}000 = \$\,30000$

total saving $= -14000 - d_2 + 30000$

total saving $= 16000 - d_2$

As

$$\%\ \text{return} = \frac{\text{Profit}}{\text{total investment}} \times 100$$

$$9 = \frac{16000 - d_2}{200{,}000 - 130{,}000} \times 100$$

$$\therefore .09 = \frac{16000 - d_2}{70000}$$

$$d_2 = 16000 - 6300$$

$$d_2 = \$\,9700$$

Using straight line method

$$d_2 = \frac{V - V_1}{n}$$

$$n = \frac{V - V_1}{d}$$

$$n = \frac{200{,}000 - 0}{9700}$$

$$n = 20.61 \text{ years}$$

? PROBLEM 10.10 Malik Zeeshan AFZAL

Malik Zeeshan

(27)

Rashid

SOLUTION :-

not Cafang CF

For Batch Wise Reactor

Cost = \$40,000

$n = 15$

V_s = \$1000

After 5 years

Partial Sale value = 5500 Remaining service life = 10 years

minimum return = ~~\$1000~~ 15%.

For continuous reactor

Cost = \$70,000

V_s = \$1000

n = 12 years.

Labor saving = \$7000/year.

Add. cost = \$1000

Increase in income = 5000

For batch reactor

After 5 years — annual depreciation $= \dfrac{V - V_s}{n} = \dfrac{5000 - 1000}{10}$

d_1 = \$400/year.

For continuous reactor

$$d_2 = \frac{V - V_s}{n} = \frac{70,000 - 1000}{12} = \$5750$$

Increase in depreciation = 5750 − 400 = \$5350/year.

Savings for new plant = Increase in income + labor saving − add. cost − decrease in depreciation

$$\boxed{\text{Saving} = 5000 + 7000 - 1000 - 5350 = \$5650}$$

As

$$\%\ \text{return} = \frac{\text{Savings}}{\text{total Investment}} \times 100$$

$$\%\ \text{return} = \frac{5650}{70000 - 5000} \times 100$$

$$\%\ \text{return} = 8.692\ \%$$

∴ % return is 8.692% which is less than required 15%, so there is no need of replacement.

Zeeshan

Malik Zeeshan CHAN. Or.

malik Zeeshan Afzal Chan

Malik Zeeshan Afzal

malik Zeeshan Chan Eng.

malik

OBLEM 10.11

V = original value = \$ 600

n = 5 years

Salvage value = V_s = \$ 100

As

$$d = \frac{V - V_s}{n} = \frac{600 - 100}{5} = \$100/\text{year}$$

SOLUTION

For new plant

V = \$ 6000

V_s = 0

n = 10 years

$$d = \frac{V - V_s}{n} = \frac{6000 - 0}{10} = \$600/\text{year}$$

Increase in depreciation cost = 600 − 100 = \$ 500/year

Decrease in maint and labor cost = \$ 1000/year

Increase in insurance and taxes = \$100/year

total savings = 1000 − 100 − 500 = \$400/year

Net F.C.I = 6000 − 600 = \$ 5400

$$\% \text{ return} = \frac{400}{5400} \times 100 = 7.41\%$$

Name. Zeeshan Afzal.

Chemical Engineering.

NFC IET Multan

no specific [illegible] main should always

reason for [illegible]

CHAPTER 11

OPTIMUM DESIGN
AND
DESIGN STRATEGY

PROBLEM # 01: A multiple-effect evaporator is to be used for evaporating 400,000 lb of water per day from a salt solution. The total initial cost for the first effect is $18,000, and each additional effect costs $15,000. The life period is estimated to be 10 years, and the salvage or scrap value at the end of the life period may be assumed to be zero. The straight-line depreciation method is used. Fixed charges minus depreciation are 15 percent yearly based on the first cost of the equipment. Steam costs $1.50 per 1000 lb. Annual maintenance charges are 5 percent of the initial equipment cost. All other costs are independent of the number of effects. The unit will operate 300 days per year. If the pounds of water evaporated per pound of steam equals 0.85 x number of effects, determine the optimum number of effects for minimum annual cost.

DATA:- water to be evaporated = 400,000 lb/day

cost of 1st effect = $ 18,000

cost for each additional effect = $ 15000

service life = n = 10 years.

salvage or scrap value = $V_s = 0$

Annual fixed charges − dep. = 0.15 (cost of Equip)

cost of steam = $1.50 / 1000 lb steam

Annual maintenance charges = 0.05 (cost of Equip)

1 year = 300 days

$$\frac{\text{lbs. of water Evaporated}}{\text{lb of steam}} = (0.85)(\text{No. of effects})$$

T.F.O

optimum no. of effects for minimum annual cost = ?

Soln:-

Now; let No. of effects = x.

then

cost of Equip (initially) = $V = 18000 + (x-1)15000$

Annual Dep. charges = $\frac{V - V_s}{n} = \frac{18000 + (x-1)15000 - 0}{10}$

$$\boxed{d = 1800 + (x-1)1500} \longrightarrow ⓐ$$

Now; annual fixed charges = $0.15(V) = 0.15(18000 + (x-1)15000)$

$$\boxed{F.C = 2700 + (x-1)\,2250} \longrightarrow ⓑ$$

Annual maintenance charges $= 0.05\,(V)$

$$= 0.05\,(18000 + (x-1)\,15000)$$

$$\boxed{\text{Annual maint. charges} = 900 + (x-1)\,750} \longrightarrow ⓒ$$

STEAM COST :- (Annual)

$$\because \frac{\text{lbs of water evaporated}}{\text{lb of steam}} = 0.85x.$$

$$\text{lb of steam} = \frac{\text{lbs of water evaporated}}{0.85x} \longrightarrow Ⓐ$$

Now;

$$\text{Steam cost} = (\text{lb of steam}) \times \left(\frac{\$1.5}{1000\,lb}\right)$$

Ⓐ ⇒

$$\text{Steam cost} = \frac{\text{lbs of water Evap.}}{0.85x} \times \frac{\$1.5}{1000\,lb}$$

$$^{*}\ \text{Steam cost} = \frac{400{,}000\ \cancel{lb}}{\cancel{day}} \left| \frac{300\ \cancel{day}}{1\,yr} \right| \frac{1}{0.85x} \left| \frac{\$1.5}{1000\ \cancel{lb}} \right.$$

$$\boxed{\text{Steam cost} = \frac{211765}{x}} \longrightarrow ⓓ \quad \$/yr$$

ⓐ, ⓑ, ⓒ & ⓓ ⇒

$$\text{Total annual cost} = C_T = \text{Dep} + F.C + \text{Maint} + \text{Steam cost}$$

$$C_T = 1800 + (x-1)\,1500 + 2700 + (x-1)\,2250 + 900 + (x-1)\,750 + \frac{211765}{x}.$$

$$C_T = 5400 + (x-1)(4500) + \frac{211765}{x}$$

$$C_T = 5400 + 4500x - 4500 + 211765/x$$

$$C_T = 900 + 4500x - 211765/x$$

$$\frac{dC_T}{dx} = 4500 - \frac{211765}{x^2}$$

For optimum values, $\frac{dC_T}{dx} = 0$

so.

$$4500 - \frac{211765}{x^2_{opt}} = 0$$

$$4500 = \frac{211765}{x^2_{opt}}$$

$$x_{opt} = \left(\frac{211765}{4500}\right)^{1/2}$$

$$x_{opt} = 6.85 \approx 7$$

Hence;

$$\boxed{\text{No. of Effects for min. total annual cost} = 7}$$ → Ans

* * *

3.7 optimum Economy 7.72

Steam cost:-

$$\frac{400{,}000\ \cancel{\text{lb } H_2O}}{\cancel{\text{day}}} \left| \frac{\cancel{\text{lb of steam}}}{0.85x\ \cancel{\text{lb } H_2O}} \right| \frac{\$1.50}{1000\ \cancel{\text{lb steam}}} \left| \frac{300\ \cancel{d}}{1\ \text{yr}} \right.$$

PROBLEM # 02: Determine the optimum economic thickness of insulation that should be used under the following conditions: Saturated steam is being passed continuously through a steel pipe with an outside diameter of 10.75 in. The temperature of the steam is 400°F, and the steam is valued at \$1.80 per 1000 lb. The pipe is to be insulated with a material that has a thermal conductivity of 0.03 Btu/(h)(ft²)(°F/ft). The cost of the installed insulation per foot of pipe length is \$4.5 x I_t, where I_t is the thickness of the insulation in inches. Annual fixed charges including maintenance amount to 20 percent of the initial installed cost. The total length of the pipe is 1000 ft, and the average temperature of the surroundings may be taken as 70°F. Heat-transfer resistances due to the steam film, scale, and pipe wall are negligible. The air-film coefficient at the outside of the insulation may be assumed constant at 2.0 Btu/(h)(ft²)(°F) for all insulation thicknesses.

DATA :-

outside pipe diameter = 10.75"

steam temp. = T_s = 400°F

steam cost = \$1.80 / 1000lb

Thermal conductivity = K = 0.03 Btu/hr. ft².(°F/ft)

Installed insulation cost / foot of pipe = \$4.5 I_t

I_t = insulation thickness in inches.

Annual fixed charges = 0.2 (installation cost)

pipe length = L = 1000 ft

Temp. of surroundings = $T_{surr.}$ = 70°F

air - Film coefficient = h_o = 2.0 Btu/hr.ft².°F

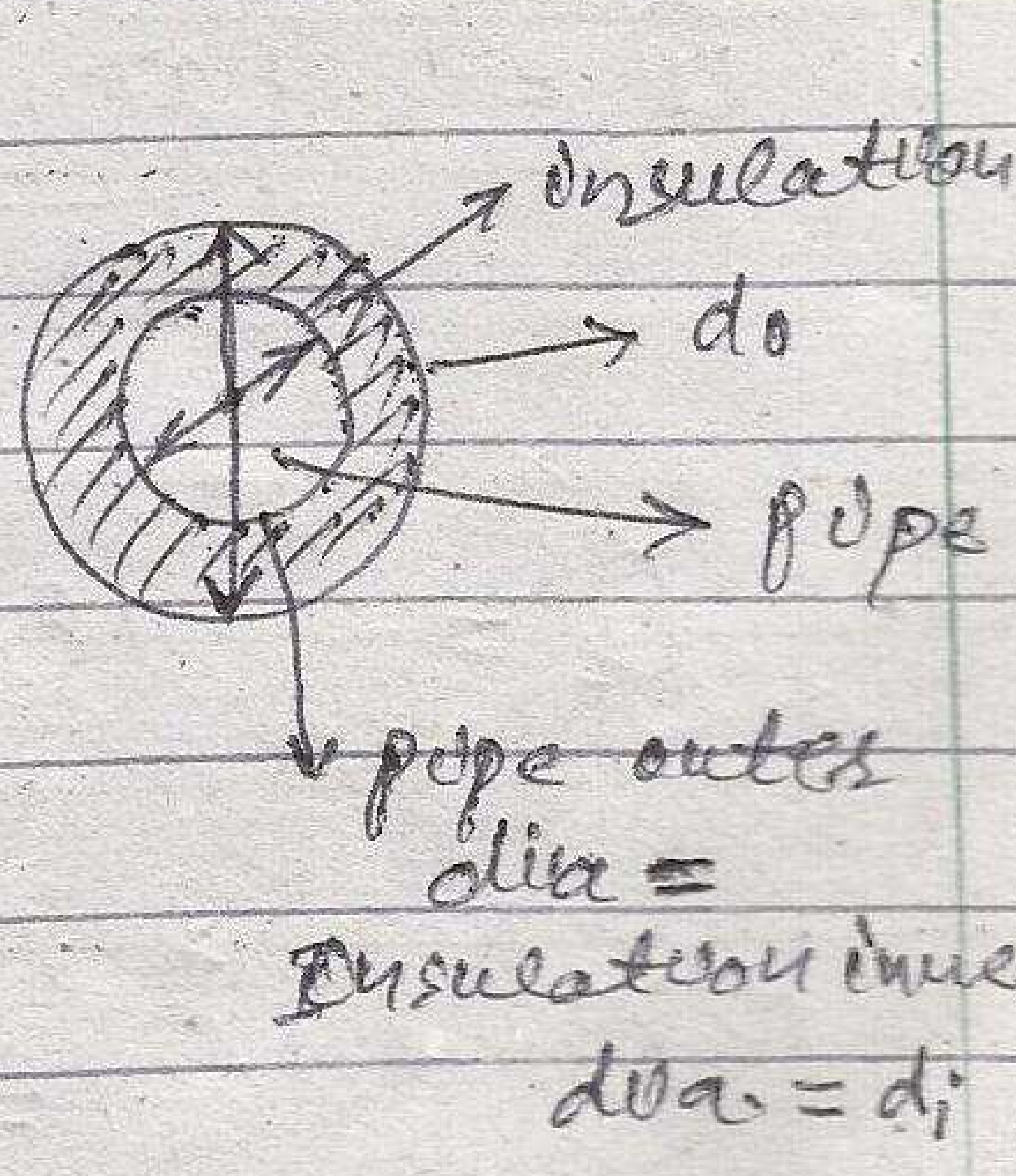
at 400°F ; * λ_s = 826 Btu/lb (Table V) Page 885

T.F.D :-

optimum Economic thickness = I_t = ?

Soln :

Now



Insulation inner dia = pipe outer dia

di = 10.75"

Insulation outer dia = do = ?

So,

average insulation thickness would be = $\overset{*}{I_t} = \frac{do - di}{2}$

Installation cost $= \$4.5 \times I_t \times L$
(For whole pipe)

$$= 4.5\left(\frac{d_o - d_i}{2}\right) \times 1000$$

$$= 2250\,(d_o - 10.75)$$

Annual fixed charges $= 0.2\,(2250(d_o - 10.75))$

$$= 450\,(d_o - 10.75)$$

$$\boxed{\text{Annual F.C} = 450 d_o - 4838} \longrightarrow ①$$

*

Annual steam cost = (mass of steam) $\left(\frac{\$1.80}{1000\,\text{lb steam}}\right) \longrightarrow ⓐ$

Now; for thick walled cylinders,

$$^{*}Q = \frac{2\pi K L (T_i - T_o)}{\ln(d_o/d_i)}$$

*where;

$T_i = T_s =$ Temp. at inner surface of insulation

$T_o =$ " " outer " " "

$$Q = \frac{(2\pi)(0.03)(1000)(400 - T_o)}{\ln(d_o/10.75)} \longleftarrow \text{units cancel out}$$

$$^{*}Q = \frac{188.5\,(400 - T_o)}{\ln(d_o/10.75)} \longrightarrow ⓑ \;(\text{Btu/hr})$$

also; $^{*}Q = UA\Delta T$ (convection)

or in this case; $^{*}Q = (h_o)(\pi d_o L)(T_o - T_{surr})$.

→ do

been converted in feet.

$$Q = (2)(\pi)(d_o)(1000)(T_o - 70)\,\frac{\text{Btu}}{\text{hr}\,\cancel{\text{ft}^2}\,\cancel{{}^\circ\text{F}}}\;\Big|\;\text{inch}\;\Big|\;\cancel{\text{ft}}\;\Big|\;\cancel{{}^\circ\text{F}}\;\Big|\;\frac{\cancel{1\text{ft}}}{12''}$$

$$Q = \frac{(2\pi)(d_o)(1000)(T_o - 70)}{12} \quad \text{Btu/hr}$$

∵ do is in inches

$$Q = 524\,d_o\,(T_o - 70)$$

$$T_o - 70 = Q/524\,d_o$$

$$\boxed{T_o = \frac{Q}{524\,d_o} + 70} \longrightarrow ©$$

putting the value of "To" from © into ⓑ

ⓑ ⇒

$$Q = \frac{188.5}{\ln\left(\frac{d_o}{10.75}\right)}\left[400 - \left(\frac{Q}{524\,d_o} + 70\right)\right]$$

$$Q \cdot \ln\left(\frac{d_o}{10.75}\right) = 188.5\left[400 - \left(\frac{Q}{524\,d_o} + 70\right)\right]$$

$$Q \cdot \ln\left(\frac{d_o}{10.75}\right) = 75400 - \frac{0.4\,Q}{d_o} - 13195$$

$$Q \cdot \ln\left(\frac{d_o}{10.75}\right) + 0.4\,\frac{Q}{d_o} = 62205$$

$$Q\left[\ln\left(\frac{d_o}{10.75}\right) + \frac{0.4}{d_o}\right] = 62205$$

$$Q = \frac{62205}{\ln\left(\frac{d_o}{10.75}\right) + \left(\frac{0.4}{d_o}\right)} \quad (\text{Btu/hr})$$

∵ mass of steam = m_s = Q/λ_s

$= 1/d_o - 0.4/d_o^2$

ⓐ ⇒ Annual steam cost $= \left(\frac{Q}{\lambda_s}\right)\left(\$1.80/1000\ \text{lb steam}\right)$

$$= \frac{62205}{\ln\left(\frac{d_o}{10.75}\right)+\left(\frac{0.4}{d_o}\right)}\ \frac{\cancel{\text{Btu}}}{\cancel{\text{hr}}}\ \Bigg|\ \frac{\cancel{\text{lb steam}}}{826\ \cancel{\text{Btu}}}\ \Bigg|\ \frac{\$1.80}{\cancel{\text{lb steam}}}$$

$$* \Bigg|\ \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}}\ \Bigg|\ \frac{365\ \cancel{\text{days}}}{1\ \text{yr}}$$

$$\text{Annual steam cost} = \frac{1188}{\ln\left(\frac{d_o}{10.75}\right)+\left(\frac{0.4}{d_o}\right)} \longrightarrow ②$$

Now, From ① & ②

$$\text{Total Annual cost} = C_T = (450 d_o - 4838) + \left(\frac{1188}{\ln\left(\frac{d_o}{10.75}\right)+\left(\frac{0.4}{d_o}\right)}\right)$$

$$\frac{dC_T}{d(d_o)} = 450 + \frac{d}{d(d_o)}\left[\frac{1188}{\left(\ln\left(\frac{d_o}{10.75}\right)+\frac{0.4}{d_o}\right)}\right]$$

$$\frac{dc_T}{d(d_o)} = 450 - \frac{1188}{\left[\ln\left(\frac{d_o}{10.75}\right)+\left(\frac{0.4}{d_o}\right)\right]^2}\left[{}^{*}\ \frac{1}{d_o} - \frac{0.4}{d_o^2}\right].$$

For optimum values of 'd_o', $\frac{dc_T}{d(d_o)} = 0$

$$450 = \frac{1188}{\left[\ln\left(\frac{d_o}{10.75}\right)+\left(\frac{0.4}{d_o}\right)\right]^2} \times \left[\frac{1}{d_o} - \frac{0.4}{d_o^2}\right]$$

For trial & Error soln.

put $d_o = 16.5''$ ✓

$\Rightarrow \quad 450 \neq 343$

Now put $d_0 = 16''$ ✓

$$450 \neq 383$$

Now put $d_0 = 15.7''$ ✓

$$450 \approx 451$$

So the optimum value of do i.e

$$\boxed{d_{o,opt} = 15.7''}$$

Now

$$I_t = \frac{d_o - d_i}{2} = \frac{15.7 - 10.75}{2}$$

$$I_t = 2.475 \approx \underline{\underline{2.5''}}$$

Hence;

$$\boxed{\text{The optimum Economic insulation thickness} = I_t = 2.5''}$$

↓

ANS.

PROBLEM # 03: An absorption tower containing wooden grids is to be used for absorbing SO_2 in a sodium sulfite solution. A mixture of air and SO_2 will enter the tower at a rate of 70,000 ft^3/min, temperature of 250 ^{0}F, and pressure of 1.1 atm. The concentration of SO_2 in the entering gas is specified, and a given fraction of the entering SO_2 must be removed in the absorption tower. The molecular weight of the entering gas mixture may be assumed to be 29.1. Under the specified design conditions, the number of transfer units necessary varies with the superficial gas velocity as follows:

$$\text{Number of transfer units} = 0.32\mathbf{G_s}^{0.18}$$

Where $\mathbf{G_s}$ is the entering gas velocity as lb/(h)(ft^2) based on the cross-sectional area of the empty tower. The height of a transfer unit is constant at 15 ft. The cost for the installed tower is \$1 per cubic foot of inside volume, and annual tied charges amount to 20 percent of the initial cost. Variable operating charges for the absorbent, blower power, and pumping power are represented by the following equation:

$$\text{Total variable operating costs as } \$/\text{h} = 1.8G_s^2 \times 10^{-8} + \frac{81}{G_3} + \frac{4.8}{G_s^{0.8}}$$

The unit is to operate 8000 h/year. Determine the height and diameter of the absorption tower at conditions of minimum annual cost.

DATA:-

Mixture flow rate = Q = 70,000 ft^3/min

$T = 250^{\circ}F + 460 = 710^{\circ}R$, $T_{s.c} = 32 + 460 = 492^{\circ}R$

P = 1.1 atm , $P_{s.c}$ = 1.0 atm

Molecular wt = M = 29.1 lbm/lb.mole

No. of Transfer units = NTU = $0.32\, G_s^{0.18}$

G_s = Gas entering velocity $\left(\frac{lb}{hr.ft^2}\right)$

HTU = 15 ft

Installation cost = $\frac{\$1}{ft^3 \text{ of inside volume}}$

Annual fixed charges = (0.2)(Installation cost)

Total variable ops cost $= 1.8\, G_s^2 \times 10^{-8} + \frac{81}{G_s} + \frac{4.8}{G_s^{0.8}} \left(\frac{\$}{hrs}\right)$

1 year = 8000 hrs

T.F.O: Height $= z = ?$

$D_{UA,opt} = D_{opt} = ?$

Solu:

$$\because \rho = \frac{M \cdot P \cdot T_{s.c}}{359 \cdot P_{s.c} \cdot T} = \frac{(29.1)(1.1)(492)}{(359)(1)(710°R)}$$

$$\rho = 0.0618 \ \text{lbm/ft}^3$$

Now;

$$\dot{m} = Q \times \rho = \frac{70{,}000 \ \cancel{\text{ft}^3}}{\cancel{\text{min}}} \Bigg| \frac{0.0618 \ \text{lbm}}{\cancel{\text{ft}^3}} \Bigg| \frac{60 \ \cancel{\text{min}}}{1 \ \text{hr}}$$

$$\boxed{\dot{m} = 259560 \ \text{lbm/hr}}$$

mass velocity $= G_s = \dot{m}/\text{Area}$

$$G_s = 259560 / \tfrac{\pi}{4} D^2$$

$$G_s = \frac{\cancel{330482}}{\cancel{D^2}} \ \frac{330650}{D^2} \ \text{lbm/hr.ft}^2$$

$$\Rightarrow \boxed{D^2 = \frac{330650}{G_s}} \longrightarrow ⓐ$$

$\because$ volume $=$ Area $\times$ Height

✓ Area $= \frac{\pi}{4} D^2$

✓ Height $=$ HTU $\times$ NTU $= (15)(0.32 \ G_s^{0.18})$

$= 4.8 \ G_s^{0.18}$

So; volume $= (\frac{\pi}{4} D^2)(4.8 \ G_s^{0.18})$

volume $= 3.8 \ D^2 \ G_s^{0.18}$

$$ⓐ \Rightarrow \text{volume} = (3.8)\left(\frac{\overset{330650}{\cancel{330482}}}{G_s}\right)\left(G_s^{0.18}\right)$$

$$\text{volume} = \frac{\overset{1256470}{\cancel{[illegible]}}}{q_s^{0.82}} \quad ft^3$$

$$\begin{array}{l}\text{Installation} \\ \text{cost}\end{array} = (\text{volume}) \times \left(\frac{\$\,1}{1 ft^3 \text{ of vol.}} \right)$$

$$= \frac{1256470}{q_s^{(0.82)}} \times \frac{\$\,1}{\cancel{ft^3}} \Big| \frac{8}{7} \Big|$$

*

$$\begin{array}{l}\text{Installation} \\ \text{cost}\end{array} = \frac{1256470}{q_s^{0.82}} \left(\frac{\$}{q_s} \right) \cancel{[illegible]}$$

*

$$\begin{array}{l}\text{Annual fixed} \\ \text{charges}\end{array} = (0.2) \left(\frac{1256470}{q_s^{0.82}} \right) \frac{\$}{q_s} \Big| \frac{1 \text{ year}}{8000 \text{ hrs.}}$$

*

$$\boxed{\text{Annual F.C} = \frac{31.41}{q_s^{0.82}} \quad (\$/\text{hr})}$$

Now;

$$\begin{array}{l}\text{Total} \\ \text{cost}\end{array} = C_T = \begin{array}{c}\text{operating} \\ \text{cost}\end{array} + \begin{array}{c}\text{fixed} \\ \text{charges}\end{array}$$

$$C_T = 1.8 q_s^2 \times 10^{-8} + \frac{81}{q_s} + \frac{4.8}{q_s^{0.8}} + \frac{31.41}{q_s^{0.82}}$$

Now;

$$\frac{dC_T}{dq_s} = 3.6 \times 10^{-8} q_s - \frac{81}{q_s^2} - \frac{(0.8)(4.8)}{q_s^{1.8}} - \frac{(0.82)(31.41)}{q_s^{1.82}}$$

$$\frac{dC_T}{dq_s} = 3.6 \times 10^{-8} q_s - \frac{81}{q_s^2} - \frac{3.84}{q_s^{1.8}} - \frac{25.8}{q_s^{1.82}}$$

for optimum values; $\frac{dC_T}{dG_s} = 0$

$$0 = 3.6\times10^{-8} G_s - \frac{81}{G_s^2} - \frac{3.84}{G_s^{1.8}} - \frac{25.8}{G_s^{1.82}}$$

By trial and Error soln:-

$$\boxed{G_s = 1750 \text{ lb/hr.ft}^2}$$ * 1762

Eq ⓐ ⇒

$$D^2 = \frac{330650}{1750} = 188.94$$

$$\boxed{D_{opt} = 13.74 \text{ ft}}$$ → Ans ①

also;

$Z = HTU \times NTU$

$$Z = (15)(0.32)(1750)^{0.18}$$

$$\boxed{Z = 18.41 \text{ ft}}$$ → Ans.

Hence.

Optimum diameter = 13.74 ft ✓

optimum height = 18.41 ft ✓

* $G_s = 1770$ would be more near approx.

1750 → −0.0028

1770 → 0.0015

PROBLEM # 04: Derive an expression for the optimum economic thickness of insulation to put on a flat surface if the annual fixed charges per square foot of insulation are directly proportional to the thickness,

(a) neglecting the air film

(b) including the air film. The air-film coefficient of heat transfer may be assumed as constant for all insulation thicknesses.

~~(a)~~- let.

I_t = Insulation thickness in inches.

q = Heat rate per square foot of area, A

k = thermal conductivity; $\frac{Btu}{hr.ft^2(°F/ft)}$

h_a = air film coefficient.

ΔT = Temp. driving force, °F

$\longrightarrow$ H_y = hours of opr. per year.

$\longrightarrow$ C_q = Value of the Heat for the cost of Heat, \$/Btu

(a)- $\because \quad Q = UA\Delta T$

or $\quad q = U.\Delta T \longrightarrow ①$

$$\frac{1}{U} = \frac{I_t}{k} + \frac{1}{h_a}$$

$\because \quad h_a = 0$

so $\quad \frac{1}{U} = \frac{I_t}{k}$

or $\quad U = k/I_t$ $\quad \because$ thickness is in inches

so divide I_t by 12,

$U = k/_{I_t/12}$ or $\boxed{U = \frac{12k}{I_t}}$

Eq. ① ⇒ $q = \frac{12\,K\,\Delta T}{I_t}$. $(\text{Btu}/(\text{hr})\text{ft}^2)$

~~cost of Heat per year~~ (per sq. foot) $= \frac{12\,K\,\Delta T}{I_t} \times H_y \times C_q$ $\quad \frac{\text{Btu}}{\text{hr}} \Big| \frac{\text{hr}}{\text{yr}} \Big| \frac{\$}{\text{Btu}}$

$= \frac{12 K \Delta T C_q H_y}{I_t} \quad (\$/\text{yr.ft}^2)$ $\longrightarrow$ ⓐ

Now as Given in statement;

yearly fixed charges $= C_{Fi} \times I_t$ $\quad (\$/\text{yr})$ $\longrightarrow$ ⓑ

where;

C_{Fi} = Annual cost of insulation per ~~foot of~~ sq. foot of area per inch of thickness

ⓐ & ⓑ ⇒

Total cost $= C_T = \frac{12\,K\Delta T C_q H_y}{I_t} + C_{Fi} I_t \quad \left(\frac{\$}{\text{yr.ft}^2}\right)$

Differentiating the above Eq.

$$\frac{dC_T}{dI_t} = -\frac{12 K \Delta T C_q H_y}{I_t^2} + C_{Fi}$$

For optimum insulation thickness per sq. foot of area; $\frac{dC_T}{dI_t} = 0$.

$$\frac{12 K\Delta T C_q H_y}{I_t^2} = C_{Fi}$$

or $\boxed{I_t = \left(\frac{12\,K\,\Delta T\,C_q\,H_y}{C_{Fi}}\right)^{1/2}}$ → ANS

(b) Including Air film :-

Now;
$$\frac{1}{U} = \frac{I_t}{K} + \frac{1}{h_a}$$

or
$$\frac{1}{U} = \frac{I_t/12}{K} + \frac{1}{h_a}$$

$$\frac{1}{U} = \frac{I_t}{12K} + \frac{1}{h_a}$$

$$\frac{1}{U} = \frac{h_a I_t + 12K}{h_a . 12K}$$

$$U = \frac{12 . h_a . K}{I_t h_a + 12K}$$

Eq ① $\Rightarrow$

$$q = \frac{12K . h_a}{I_t h_a + 12K} \Delta T$$

cost of Heat per year per sq. foot $= \dfrac{12 K h_a \Delta T}{I_t h_a + 12K} C_q \times H_y$

$$= \frac{12 K h_a \Delta T C_q H_y}{I_t h_a + 12K} \longrightarrow ⓒ$$

year fixed charges per sq. foot of area $= C_{FO} \times I_t \longrightarrow ⓓ$

ⓒ & ⓓ $\Rightarrow$

$$C_T = \frac{12 K h_a \Delta T C_q H_y}{(I_t h_a + 12K)^1} + C_{FO} I_t$$

NOTE

$$\frac{dC_T}{dI_t} = - \frac{12 K h_a \Delta T C_q H_y}{(I_t h_a + 12K)^2} \times \underline{h_a} + C_{Fi}$$

for optimum conditions;

$$\frac{dC_T}{dI_t} = 0$$

$$C_{Fi} = \frac{12 K h_a^2 \Delta T C_q H_y}{(I_t h_a + 12K)^2}$$

$$(I_t h_a + 12K)^2 = \frac{12 K h_a^2 \Delta T C_q H_y}{C_{Fi}}$$

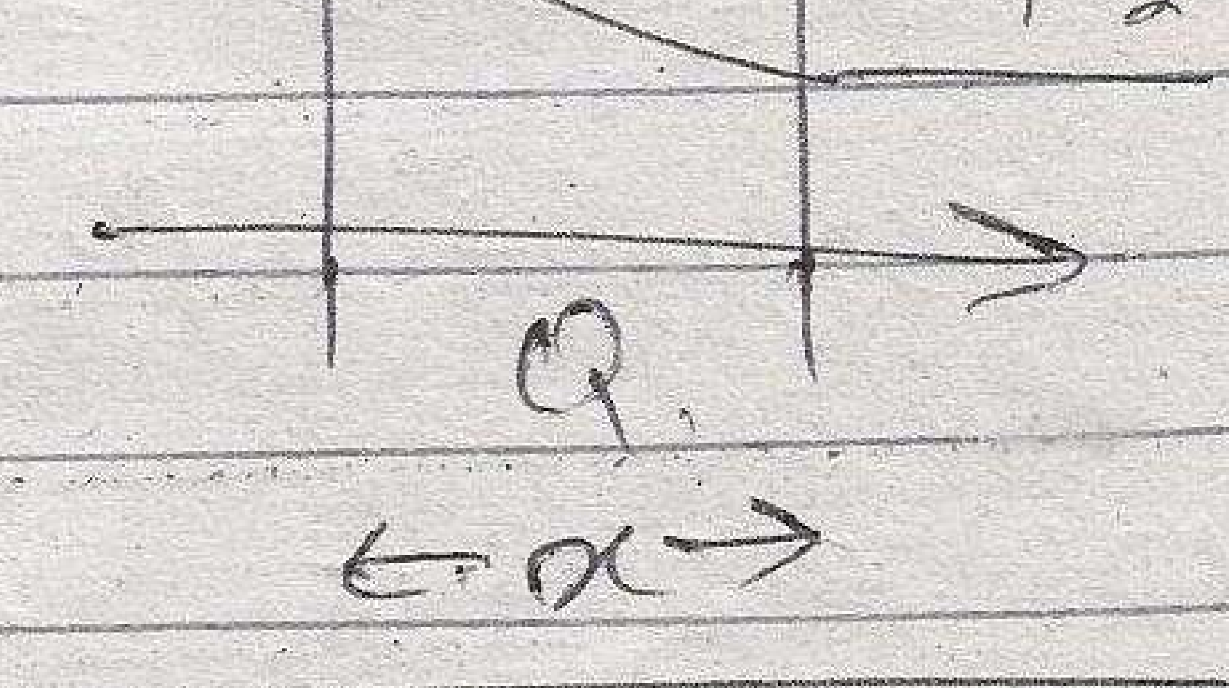
$$I_t . h_a + 12K = \sqrt{\frac{12 K h_a^2 \Delta T C_q H_y}{C_{Fi}}}$$

$$I_t h_a = \sqrt{\frac{12 K h_a^2 \Delta T C_q H_y}{C_{Fi}}} - 12K$$

$$\Rightarrow I_t = \frac{1}{h_a}\sqrt{\frac{12 K h_a^2 \Delta T C_q H_y}{C_{Fi}}} - \frac{12K}{h_a}$$

$$\boxed{I_t = \sqrt{\frac{12 K \Delta T C_q H_y}{C_{Fi}}} - \frac{12K}{h_a}} \rightarrow \text{ANS}$$

HINTS FOR PROBLEM 2:- From Fourier law for flat surfaces.



$$Q = -\frac{KA(T_1 - T_2)}{x}$$

$$q = -\frac{K(T_1 - T_2)}{x}$$

$$q = \frac{(T_1 - T_2)}{(x/K)}$$

$$\text{so } q = \frac{(T_1 - T_2)}{R} \Rightarrow \boxed{U(T_1 - T_2)}$$

$\because x/K = R$

& $1/R = U$.

Now;

Total Annual cost $= C_T =$ Fixed cost + opr cost

$C_T = 8800 F_{sf}^2 - 51{,}000 F_{sf} + 110{,}000 + 18{,}000 F_{sf}$

Differentiating;

$\frac{dC_T}{dF_{sf}} = 17{,}600 F_{sf} - 51{,}000 + 18{,}000$

For optimum value; $\frac{dC_T}{dF_{sf}} = 0$

$176{,}00 F_{sf} = 51{,}000 + 18000$

$F_{sf} = 33{,}000 / 176{,}000$

$\boxed{F_{sf} = 1.875}$

① ⇒

solvent $= (450{,}000)(1.875)$ ft³/yr

$\boxed{\text{solvent} = 843750 \text{ ft}^3/\text{yr}}$

Total Flow through Tower $= Q =$ Feed + solvent *

$= Q = \frac{1293750 \text{ ft}^3}{\text{yr}} \left| \frac{1 \text{ yr}}{300 \text{ day}} \right| \frac{1 \text{ day}}{24 \text{ hrs}}$

$\boxed{Q = 180 \text{ ft}^3/\text{hr}}$

Now;

Flowrate = velocity x Area

$Q = V \times A$

$\frac{180 \text{ ft}^3}{\text{hr}} = \frac{40 \text{ ft}^3}{\text{ft}^2 \cdot \text{hr}} \times \left(\frac{\pi}{4}\right) D^2$

$D^2 = \left(\frac{180}{40}\right)\left(\frac{4}{\pi}\right) \text{ft}^2 \Rightarrow \boxed{D_{opt} = 2.39 \text{ ft}}$

PROBLEM # 07: Prepare a plot of optimum economic pipe diameter versus the flow rate of fluid in the pipe under the following conditions:
Costs and operating conditions ordinarily applicable in industry may be used.
The flow of the fluid may be considered as in the turbulent range.
The viscosity of the fluid may range from 0.1 to 20 centipoises.
The plot is to apply for steel pipe.
Express the diameters in inches and use inside diameters.
The plot should cover a diameter range of 0.1 to 100 in.
Express the flow rate in 1000 lb/h.
The plot should cover a flow-rate range of 10 to 100,000 lb/h.
The plot should be presented on log-log coordinates.
One line on the plot should be presented for each of the following fluid densities:
100, 50, 10, 1, 0.1, 0.01, and 0.001 lb/ft³.

Personal Notes:- "Such an awkward & lengthy question to be solved. If this ~~as~~ FUCKIN ASS COME Appear on papers cut your dick and put it into your ass instead of shitting rest of your paper."

Solu:

Two Eqs of Turbulent flow will be used:

① For $D_i \geq 1''$

$$D_{,opt} = \frac{2.2 W_m^{0.45}}{\rho^{0.32}} \longrightarrow (A)$$

② For $D_i < 1''$

$$D_{,opt} = 4.7 q_f^{0.49} \rho^{0.14}$$

$\because \dot{m} = q \times \rho$

$q = \dot{m}/\rho$

$\dot{m} \Rightarrow W_m$

$$= (4.7)\left(\frac{\dot{m}^{0.49}}{\rho^{0.49}}\right)(\rho^{0.14})$$

$$D_{opt} = \frac{4.7 W_m^{0.49}}{\rho^{0.35}} \longrightarrow (B)$$

PROBLEM # 06: A solvent-extraction operation is carried out continuously in a plate column with gravity flow. The unit is operated 24 h/day. A feed rate of 1500 ft³/day must be handled 300 days per year. The allowable velocity per square foot of cross-sectional tower area is 40 ft³ of combined solvent and charge per hour. The annual fixed costs for the installation can be predicted from the following equation:

$$C_F = 8800F_{sf}^2 - 51{,}000F_{sf} + 110{,}000 \ \$/\text{year}$$

where F_{sf} = cubic feet of solvent per cubic foot of feed.
Operating and other variable costs depend on the amount of solvent that must be recovered, and these costs are \$0.04 for each cubic foot of solvent passing through the tower. What tower diameter should be used for optimum conditions of minimum total cost per year?

DATA:

Feed rate = 1500 ft³/day

opr. day = 300 days/yr

opr. hours = 24 hrs/day,

allowable velocity = V = 40 ft³/ft².hr

Annual F.C = $C_F = 8800 F_{SF}^2 - 51{,}000 F_{SF} + 110{,}000 \ (\$/yr)$

$F_{SF} = \dfrac{\text{ft}^3 \text{ of solvent}}{\text{ft}^3 \text{ of feed}}$

ops & other costs = \$0.04/ft³ of solvent

optimum diameter = ?

solu:

$\text{Feed} = \dfrac{1500 \text{ ft}^3}{\text{day}} \Big| \dfrac{300 \text{ day}}{1 \text{yr}}$ *

$\boxed{\text{Feed} = 450{,}000 \text{ ft}^3/\text{yr}}$

solvent = $(450{,}000)(F_{sf}) \ (\text{ft}^3/\text{yr}) \longrightarrow$ ①

opr. costs = $\dfrac{\$ \ 0.04}{\text{ft}^3 \text{ of solvent}} \times \dfrac{450{,}000 F_{sf} \text{ ft}^3}{\text{yr}}$

opr. costs = $18{,}000 F_{sf} \ \$/\text{yr}$

Steps:— ① - Took arbitrarily some values of $\dot{w}_m$ from a range of 10 — 100,000 lb/hrs.

② - Find out D_{opt} from ~~both~~ Eqs. Ⓐ & on other part with Ⓑ.

③ - Fixed density each time i.e first fix $\rho = 100\ lb/ft^3$ & found some values of D_{opt}

④ - Next time change the density & find ~~oth~~ values again with the same method.

⑤ Finally draw graph b/w. Flow rate ($\dot{w}_m$) vs optimum dia (D_{opt}).

e.g.—

TABLE:— For $\rho = 100\ lb/ft^3$

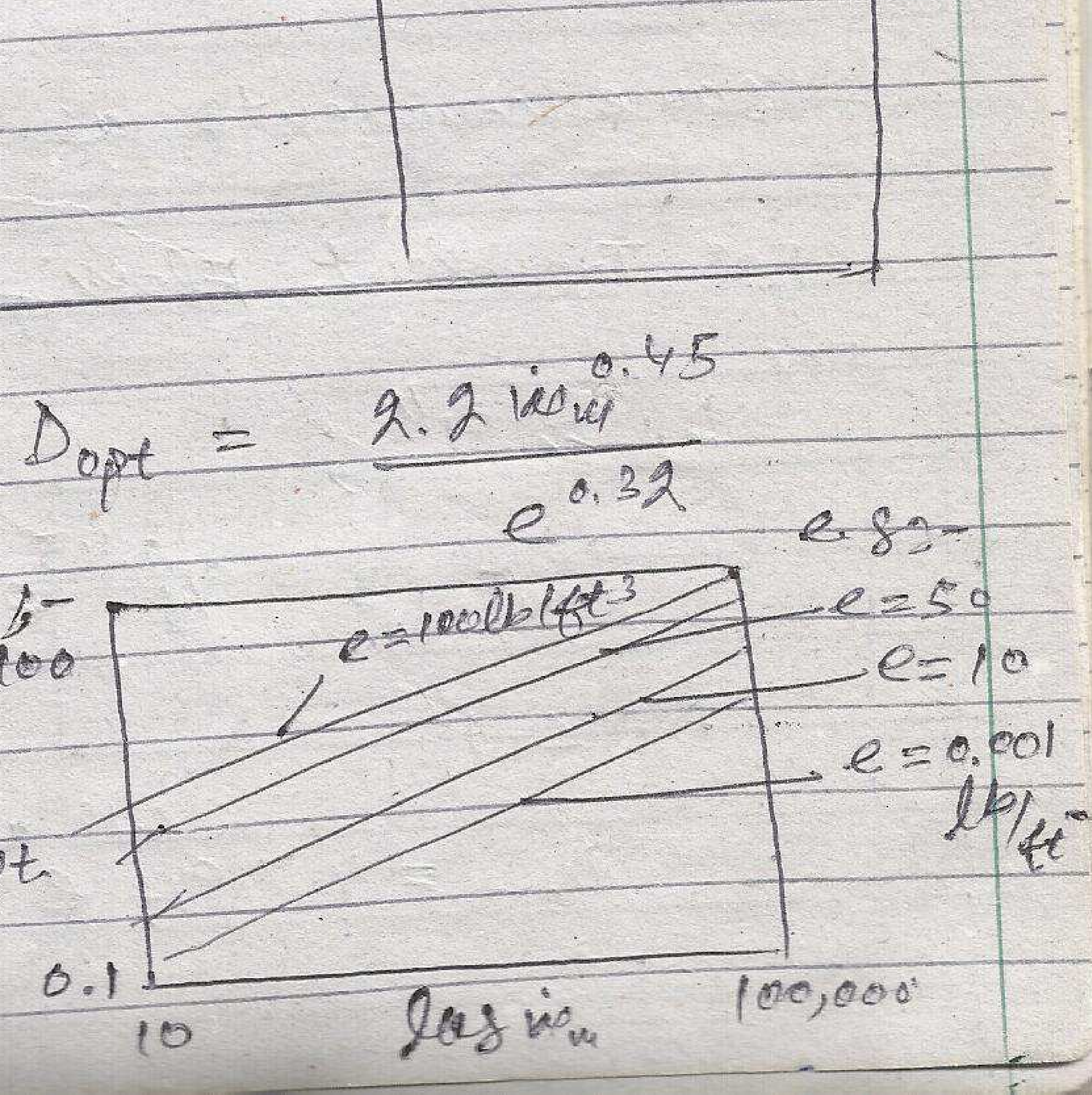
$\dot{w}_m$	D_{opt}	$\log \dot{w}_m$	$\log D_{op}$
10			
–			
–			
100,000			

Eq:— $$D_{opt} = \frac{2.2\,\dot{w}_m^{0.45}}{\rho^{0.32}}$$

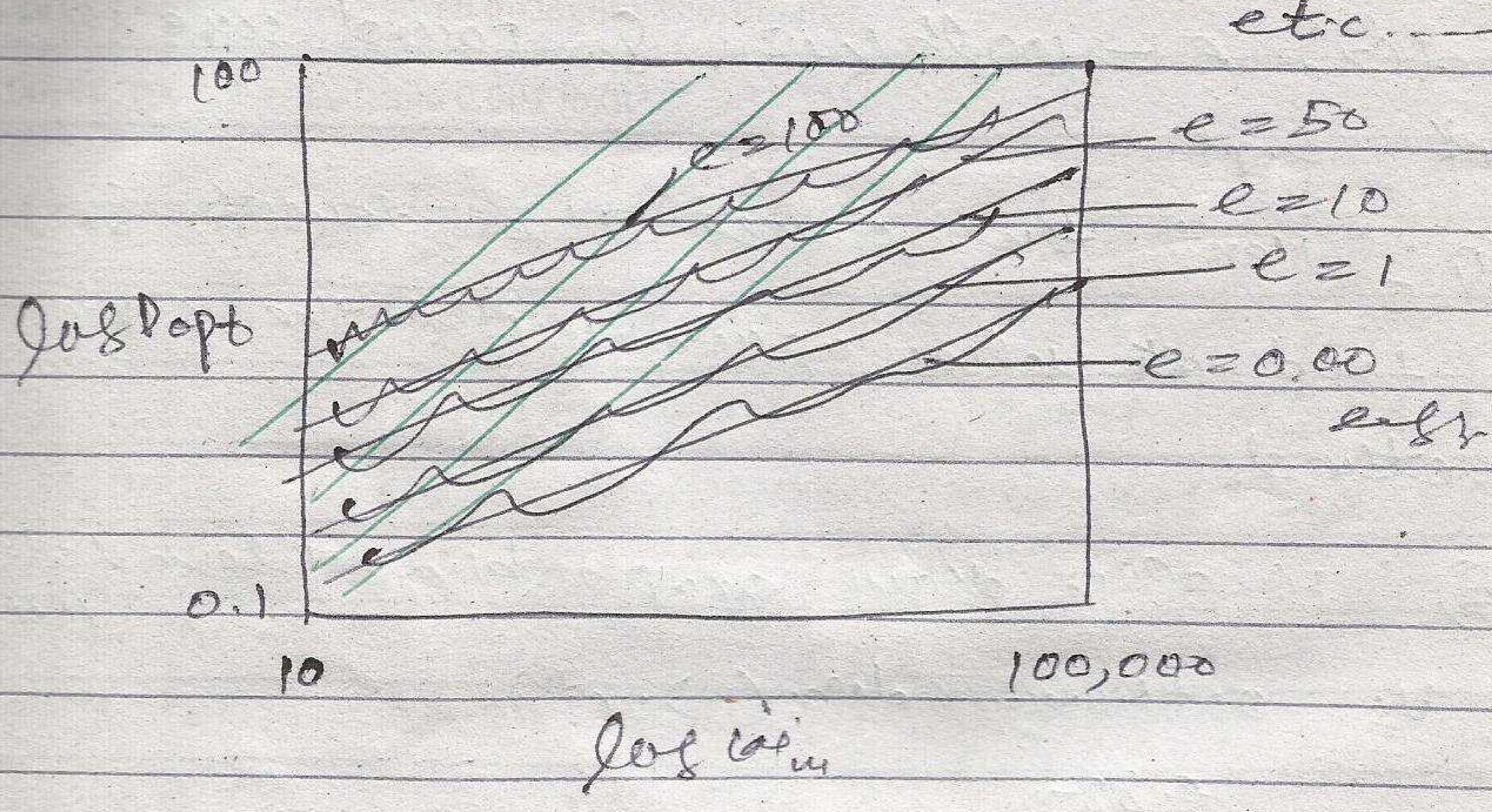
Now draw graph

Next $\rho = 50\ lb/ft^3$

$\dot{w}_m$	D_{op}	$\log \dot{w}_m$



Similar procedure is done with using Eq Ⓑ & again plot a graph;



PROBLEM # 08: For the conditions indicated in Prob. 7, prepare a log-log plot of fluid velocity in feet per second versus optimum economic pipe diameter in inches. The plot should cover a fluid-velocity range of 1 to 100 ft/s and a pipe-diameter range of 1 to 10 in.

* same comments as previous :P

For Turbulent Flow;-

For $D_i \geq 1''$;

$$D_{opt} = 3.9\, q_f^{0.45}\, e^{0.13} \longrightarrow ①$$

$D_i < 1''$

$$D_{opt} = 4.7\, q_f^{0.49}\, e^{0.14} \longrightarrow ②$$

If we substitute $q_f = V.\frac{\pi}{4}.D_{opt}^2$

① =>

$D_{opt} = (3.9)(V^{0.45})(0.9)(D_{opt}^{0.9})(e^{0.13})$

$D_{opt}^{0.1} = 3.51\, V^{0.45} e^{0.13}$ taken (1/0.1 power)

or $^{*}D_{opt} = \underbrace{283838}\, V^{4.5} e^{1.3} \longrightarrow$ ①'

similarly;

② =>

$D_{opt} = (4.7)(V^{0.49})(0.9)(D_{opt}^{0.98})(e^{0.14})$

$D_{opt}^{0.02} = 4.23\, V^{0.49} e^{0.14}$ taken (1/0.02 power)

$^{*}D_{opt} = (2.075 \times 10^{31})(V^{24.5})(e^{7}) \longrightarrow$ ②'

①' & ②' may seems to have dia very much large when we put e=100 & velocity = 100 ft/sec but rembes when we will take log of Dopt it will not exceed from 100 so, & graph will be possible.

so;

V	Dopt	logv	log Dop
1	—		
20	—		
40	—		
100	—		

P-404

Problem 9.7 Optimum Pipe Diameter vs Flow Rate

For the range of viscosity considered, the viscosity term will range from 1 to 1.08 in the equation for diameters less than 0.0254m and from 1 to 1.077 in the equation for diameters greater than 0.0254 for the stated viscosity range. Therefore, we will plot $y = D_{I,opt}/\mu_{CP}{}^{0.025}$ and $z = D_{I,opt}/\mu_{CP}{}^{0.025}$ in the two cases where μ_{CP} is in cP.

From the equations on page 404 modified to account for viscosity in cP,

Turbulent Flow with Diameter greater than or equal to 0.0254

$$y(F_m, \rho) := 0.363 \cdot (1000)^{0.025} \cdot F_m{}^{0.45} \cdot \rho^{-0.32} \quad = D_{I,opt}/\mu_{CP}{}^{0.025}$$

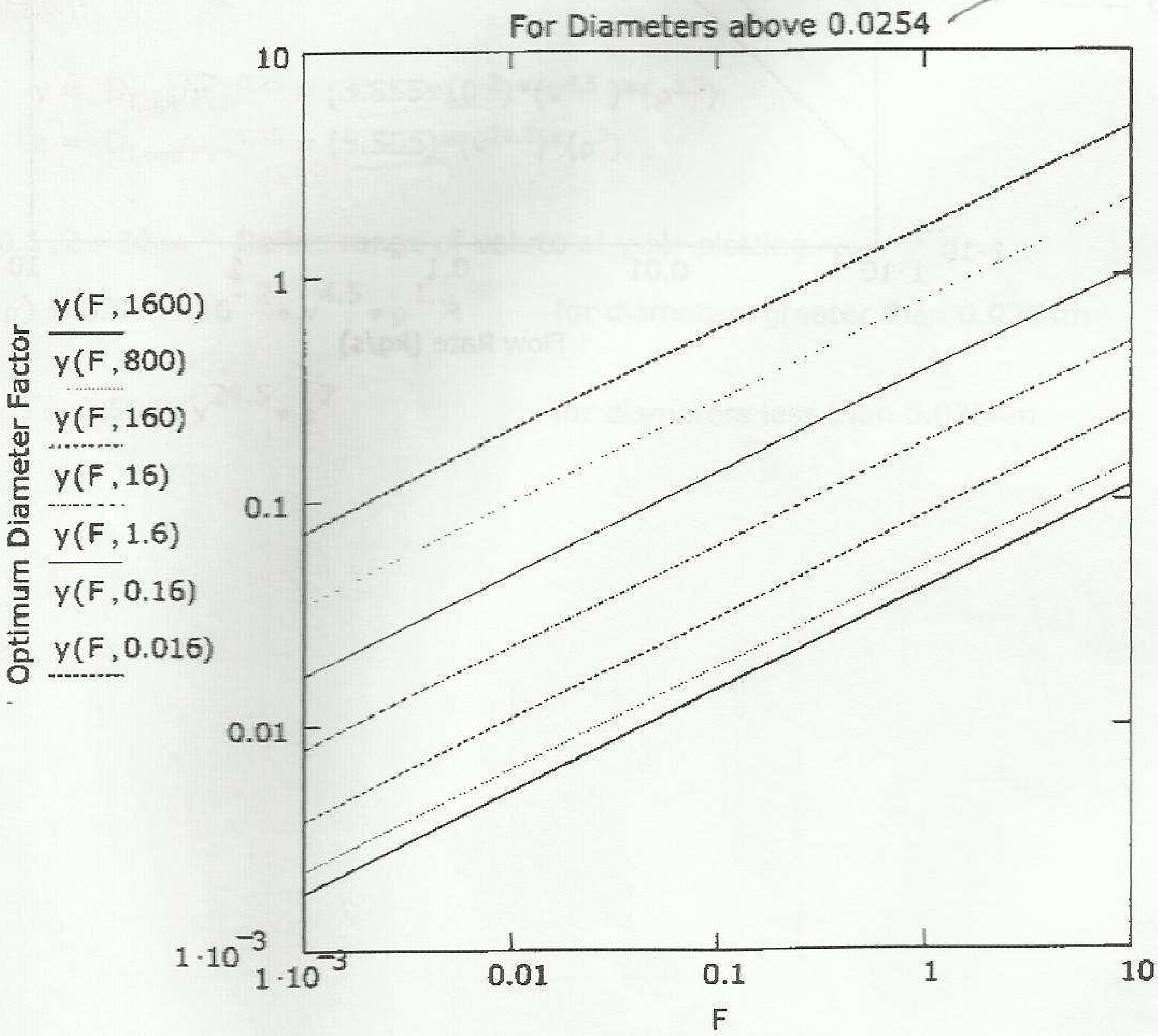
$\dot{m} = q \times \rho$

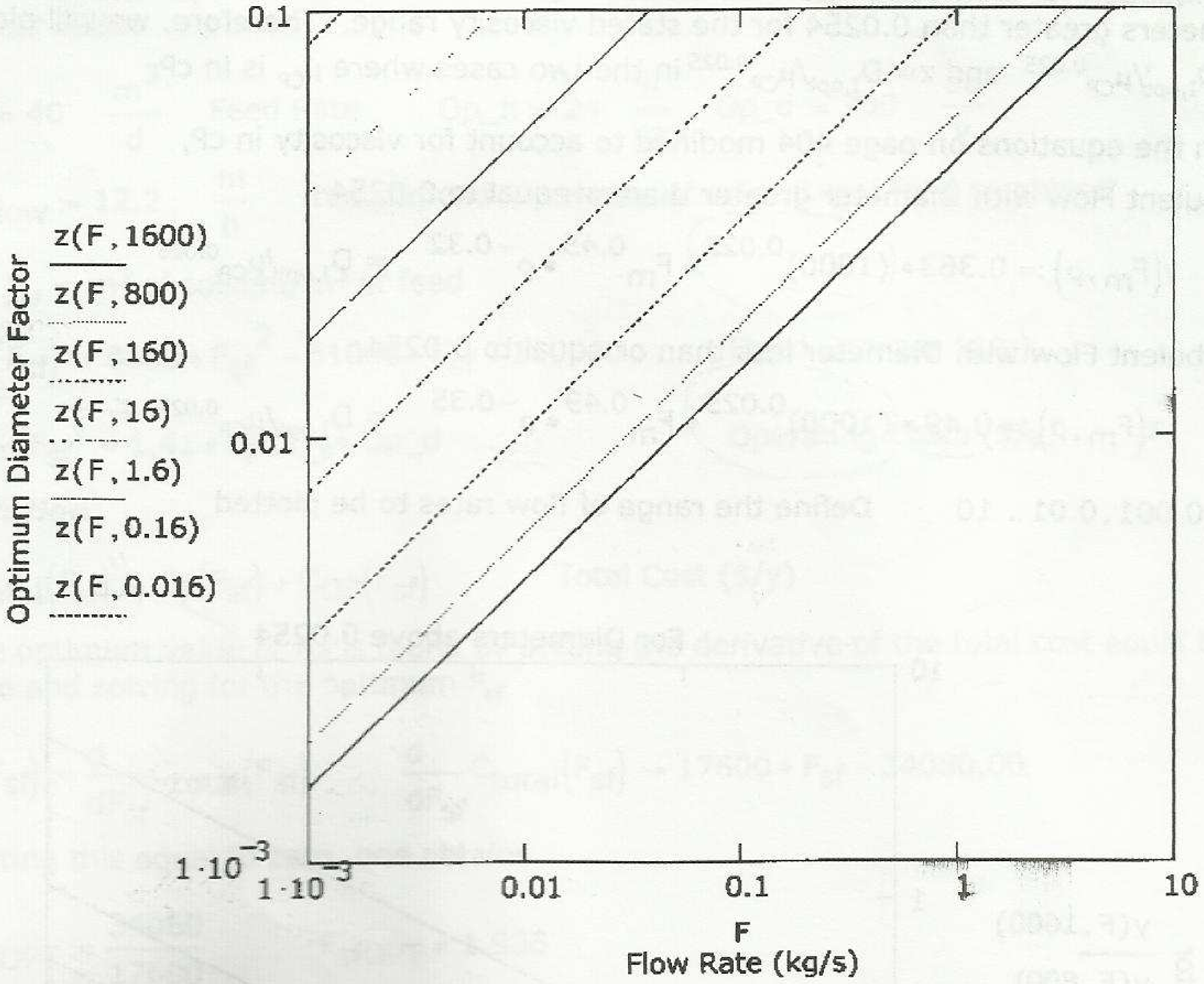
Turbulent Flow with Diameter less than or equal to 0.0254

$$z(F_m, \rho) := 0.49 \cdot (1000)^{0.025} \cdot F_m{}^{0.49} \cdot \rho^{-0.35} \quad = D_{I,opt}/\mu_{CP}{}^{0.027} \longleftarrow \quad q = \dot{m}/\rho$$

$F := 0.001, 0.01 .. 10$ Define the range of flow rates to be plotted

" 1.0





For Diameters less than 0.0254

Problem 9.8 Optimum Pipe Diameter vs Velocity

Math Cad File

The basis will be eqs. 9-76 and 9-77 on p.404, namely,

TURBULENT FLOW and diameter greater than or equal to 0.0254m

(1) $D_{I,opt}=0.363*(q_f^{0.45})*(\rho^{0.13})*(\mu^{0.025})$ with q_f in m^3/s

ρ in kg/m^3

μ in Pa-s

TURBULENT FLOW and diameter less than or equal to 0.0254m

(2) $D_{I,opt}=0.49*(q_f^{0.49})*(\rho^{0.14})*(\mu^{0.02})$ with q_f, ρ, μ in same units

If we substitute $q_f = v*\pi*(D_{I,opt})^2$ and $\mu = \mu_{CP}$(to place viscosity in units of cP) into the above equations, we obtain, respectively,

(1') $(D_{I,opt})^{0.1}=\underline{(7.221x10^{-2})}*(v^{0.45})*(\rho^{0.13})*(\mu_{CP}^{0.025})$

(2') $(D_{I,opt})^{0.02}=\underline{(1.0347)}*(v^{0.49})*(\rho^{0.14})*(\mu_{CP}^{0.027})$

Solving these equations for $D_{I,opt}$ and, similar to Problem 9.7, defining $y = D_{I,opt}/\mu_{CP}^{0.25}$ and $z = D_{I,opt}/\mu_{CP}^{1.35}$ to keep only ρ and v on the right hand side, we obtain:

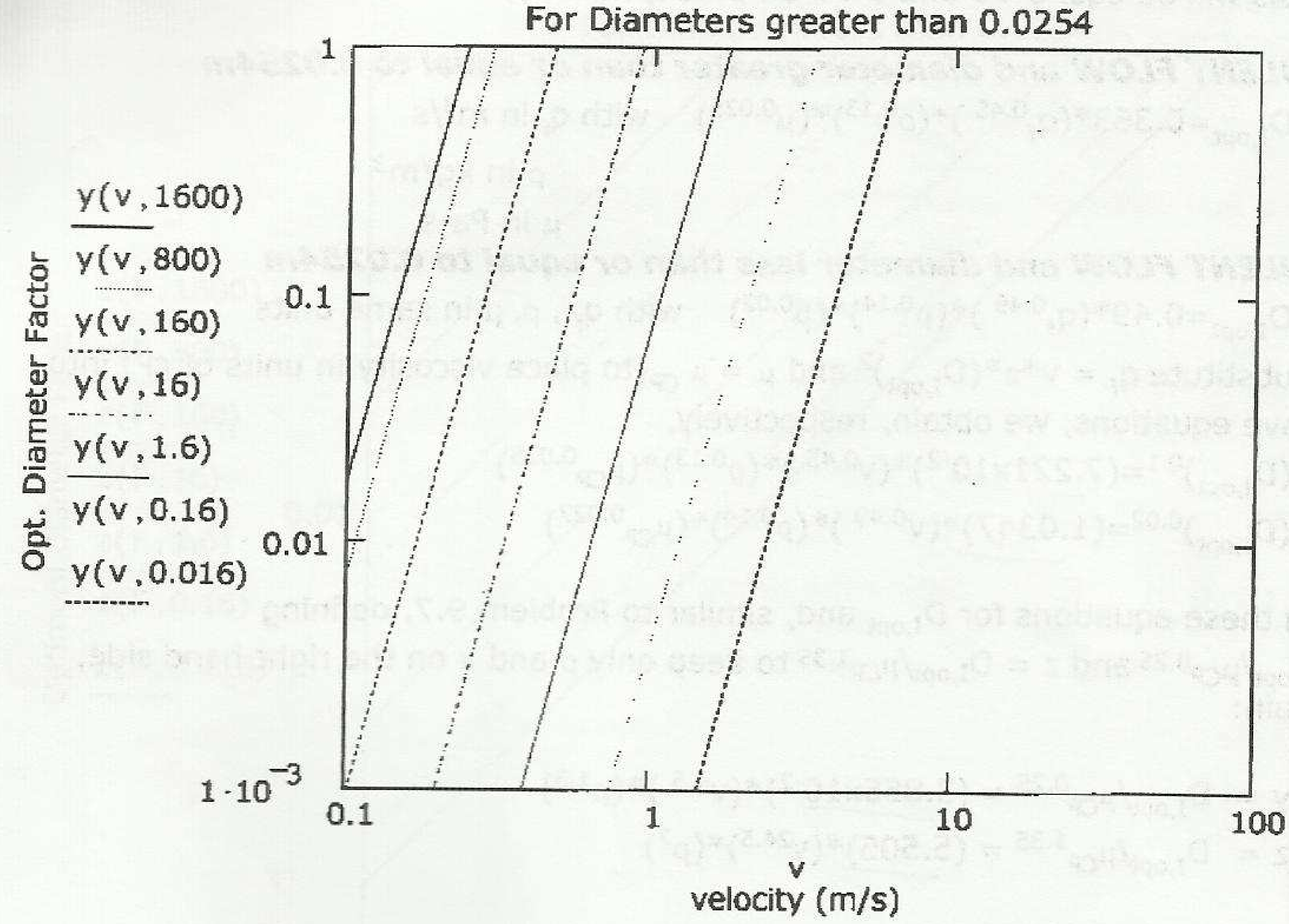
(1") $y = D_{I,opt}/\mu_{CP}^{0.25} = \underline{(3.855x10^{-2})}*(v^{4.5})*(\rho^{1.3})$

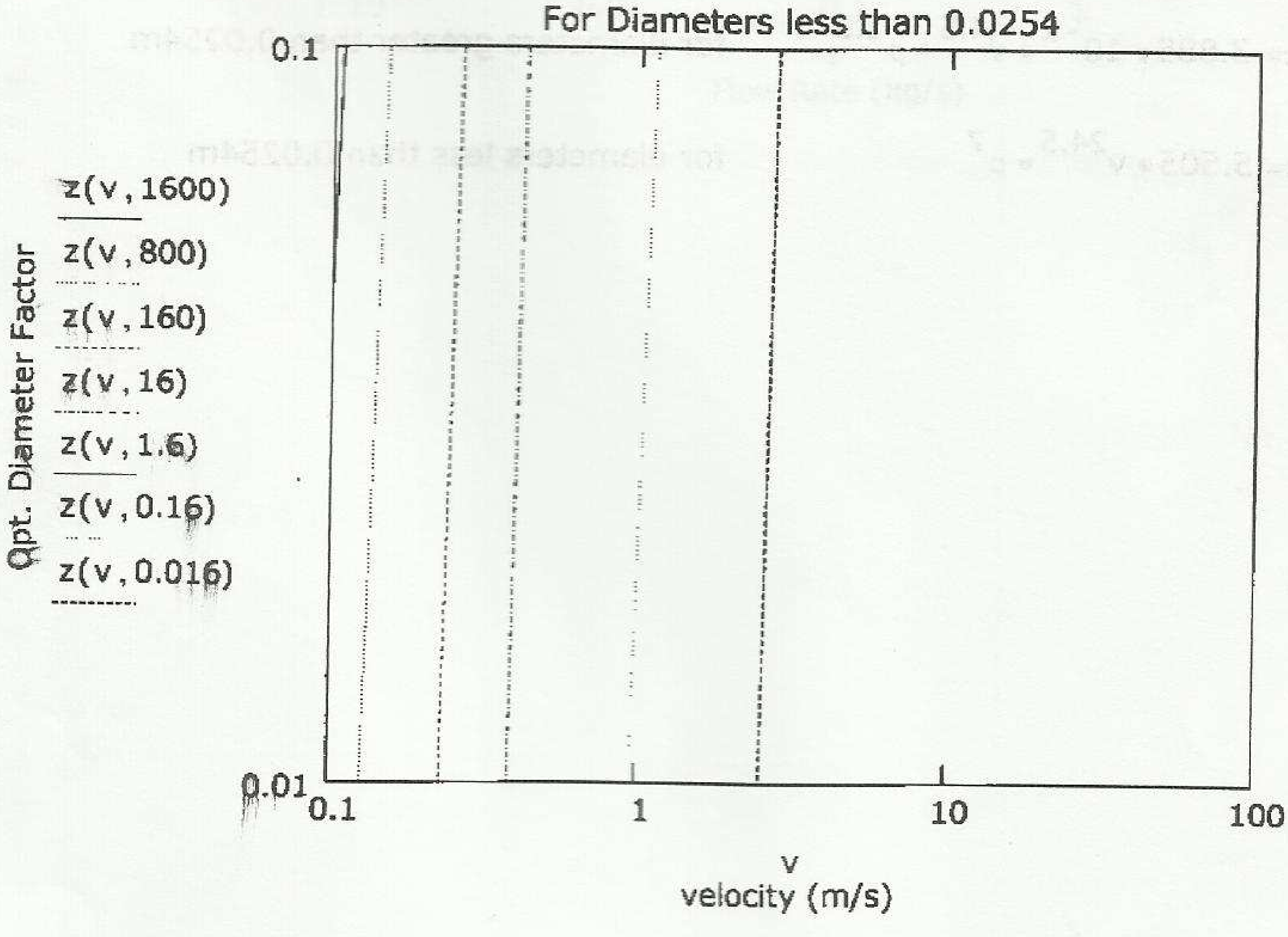
(2") $z = D_{I,opt}/\mu_{CP}^{1.35} = \underline{(5.505)}*(v^{24.5})*(\rho^7)$

$v := 0.1, 2 .. 30$ Define range of values of v for plotting

$y(v,\rho) := 3.885 \cdot 10^{-2} \cdot v^{4.5} \cdot \rho^{1.3}$ for diameters greater than 0.0254m

$z(v,\rho) := 5.505 \cdot v^{24.5} \cdot \rho^{7}$ for diameters less than 0.0254m


For Diameters greater than 0.0254
Opt. Diameter Factor
y(v,1600)
y(v,800)
y(v,160)
y(v,16)
y(v,1.6)
y(v,0.16)
y(v,0.016)
1
0.1
0.01
$1\cdot10^{-3}$
0.1
1
10
100
v
velocity (m/s)


For Diameters less than 0.0254
Opt. Diameter Factor
z(v,1600)
z(v,800)
z(v,160)
z(v,16)
z(v,1.6)
z(v,0.16)
z(v,0.016)
0.1
0.01
0.1
1
10
100
v
velocity (m/s)

PROBLEM # 09: A continuous evaporator is being used to concentrate a scale-forming solution of sodium sulfate in water. The overall coefficient of heat transfer decreases according to the following expression:

$$\frac{1}{U^2} = 8 \times 10^{-6}\theta_b + 6 \times 10^{-6}$$

where U = overall coefficient of heat transfer, Btu/(h)(ft²)(°F),
and θ_b= time in operation, h.

The only factor which affects the overall coefficient is the scale formation. The liquid enters the evaporator at the boiling point, and the temperature and heat of vaporization are constant. At the operating conditions, 990 Btu are required to vaporize 1 lb of water, the heat-transfer area is 400 ft², and the temperature-difference driving force is 70°F. The time required to shut down, clean, and get back on stream is 4 h for each shutdown, and the total cost for this cleaning operation is $100 per cycle. The labor costs during operation of the evaporator are $20 per hour. Determine the total time per cycle for minimum total cost under the following conditions:

(a) An overall average of 65,000 lb of water per 24-h day must be evaporated during each 30-day period.
(b) An overall average of 81,000 lb of water per 24-h day must be evaporated during each 30-day period.

DATA:-

$$\frac{1}{U^2} = 8\times10^{-6}\,\theta_b + 6\times10^{-6}$$

where;

U = over all coefficient of Heat tranfer Btu/hr. ft². °F

θ_b = operation time, hr

To vaporize 1 lb of water = 990 Btu req.

Heat tranfer area = A = 400 ft²

Temp. diff. driving force = Δt = 70°F

Shutdown clean time = 4 hr

cost for cleaning = C_c = $100/cycle.

labor costs = $20/hr

(a)- $\dot{w}_1$ = 65,000 lb H₂O Evap./day

(b)- $\dot{w}_2$ = 81,000 lb H₂O Evap./day

opr hours = 24

opr. days = 30.

solu- (a) → For $\dot{m}$ = 65,000 lb H_2O Evap./day.

∵ $\theta_{b,opt}$ i.e optimum cycle time for minimum total cost can be found by using the relation below;

i.e

$$\theta_{b,opt} = \frac{C_c}{S_b} + \frac{2}{aS_b}\sqrt{adC_cS_b} \longrightarrow ①$$

$$\theta_{b,opt} = \left(\frac{100}{20}\right) + \frac{2}{(8\times10^{-6})(20)}\sqrt{(8\times10^{-6})(6\times10^{-6})(100)(20)}$$

$$\theta_{b,opt} = 5 + 3.87 \Rightarrow \boxed{\theta_{b,opt} = 8.87 \text{ hrs}}$$

∵ Eq ① is independent of Q_H so a check must be made by using a relation below.

$$\rightarrow \boxed{\theta_{b,opt} + \theta_c = 12.87 \text{ hr}}$$

$$\theta_t = \frac{2AH^*\Delta t}{aQ_H}\left[(a\theta_{b,opt} + d)^{1/2} - d^{1/2}\right] \longrightarrow ②$$

*Now;

$$Q_H = \frac{65000 \text{ lb } H_2O \text{ Evap}}{\text{day}} \left| \, 30 \text{ days} \, \right| \frac{990 \text{ Btu}}{1 \text{ lb } H_2O \text{ Evap}}$$

$$Q_H = 1.93\times10^{9} \text{ Btu}.$$

* H^* = Total available time in (hours) $= 30 \text{ days} \left| \frac{24 \text{ hrs}}{1 \text{ day}} \right.$

$= 720$ hrs

Total
Available Time for opn.
↓

$$\theta_t = \frac{(2)(400)(720)(70)}{(8\times10^{-6})(1.93\times10^{9})}\left[\left[(8\times10^{-6})(8.87)+6\times10^{-6}\right]^{1/2} - \sqrt{6\times10^{-6}}\right]$$

$$\boxed{\theta_t = 16.51 \text{ hrs.}}$$

∵ Total available time i.e θ_t is greater than $\underline{\theta_{b,opt} + \theta_c}$ so; $\underline{\underline{\theta_{b,opt}}}$ may be use,

(cycle Time)

(b) - $\dot{m}_2 = 81{,}000$ lb H_2O Evap. /day.

Now using Equality relation.

$$\theta_{t_2} \cdot \dot{m}_2 = \theta_t \cdot \dot{m}_1$$

$$\theta_{t_2} = \left(\frac{\dot{m}_1}{\dot{m}_2}\right)\theta_t$$

$$\theta_{t_2} = \left(\frac{65000}{81000}\right)(16.51)$$

$$\boxed{\theta_{t_2} = 13.24 \text{ hrs}}$$

therefore, available cycle time i.e θ_{t_2} is still greater that cycle time for min cost i.e $\theta_{b,opt} + \theta_c$.

PROBLEM # 10: An organic chemical is produced by a batch process. In this process, chemicals X and Y react to form chemical Z. Since the reaction rate is very high, the total time required per batch has been found to be independent of the amounts of the materials, and each batch requires 2 h, including time for charging, heating, and dumping. The following equation shows the relation between the pounds of Z produced (lb_Z) and the pounds of X (lb_X) and Y (lb_Y) supplied:

$$lb_Z = 1.5(1.1\ lb_X\ lb_Z + 1.3\ lb_Y\ lb_Z - lb_X\ lb_Y)^{0.5}$$

Chemical X costs \$0.09 per pound. Chemical Y costs \$0.04 per pound. Chemical Z sells for \$0.80 per pound. If one-half of the selling price for chemical Z is due to costs other than for raw materials, what is the maximum profit obtainable per pound of chemical Z?

DATA:-

Raw materials (Reactants) = X, Y

product = Z

Time for each batch = 2 hr

$$lb_Z = 1.5(1.1\ lb_X lb_Z + 1.3\ lb_Y lb_Z \cancel{+} lb_X lb_Y)^{0.5}$$

selling price = S = 0.8/lb_Z.

cost of X = \$ 0.09/lb_X

cost of Y = \$ 0.04/lb_Y

profit/lb_Z = ? other cost = $\frac{1}{2}$ (selling price)

solution:

Basis:-

$lb_Z = Z = 1\ lb$

$$\therefore lb_Z = 1.5(1.1\ lb_X lb_Z + 1.3\ lb_Y lb_Z - lb_X lb_Y)^{0.5}$$

put $lb_Z = 1$ in the above Eq.

$$1 = 1.5(1.1X + 1.3Y - XY)^{0.5}$$

where; $X = lb_X/lb_Z$ & $Y = lb_Y/lb_Z$

so;

$$1 = 2.25(1.1X + 1.3Y - XY)$$

$$1 = 2.475X + 2.925Y - 2.25XY.$$

$$\Rightarrow \quad 2.475X - 2.25XY = 1 - 2.925Y.$$

or $$X(2.475 - 2.25Y) = 1 - 2.925Y.$$

$$\Rightarrow \quad X = \frac{(1 - 2.925Y)}{(2.475 - 2.25Y)} \longrightarrow ①$$

raw material cost $= C_{RM} = 0.09X + 0.04Y \longrightarrow ②$

$$\Rightarrow \quad C_{RM} = 0.09\left(\frac{1 - 2.925Y}{2.475 - 2.25Y}\right) + 0.04Y.$$

NOTE:- In this case maximizing the profit is the same as minimizing the cost. Also optimum is not a function of the selling price or cost of items other than the raw materials cost. therefore optimum could be determined by minimizing the raw material cost.

so;

$$\cancel{\frac{dC_{RM}}{dX} = \frac{d}{dX}}$$

$$\frac{dC_{RM}}{dY} = \frac{d}{dY}\left[0.09\left(\frac{1 - 2.925Y}{2.475 - 2.25Y}\right) + 0.04Y\right]$$

$$\frac{dC_{RM}}{dY} = 0.09\frac{d}{dY}\left(\frac{1 - 2.925Y}{2.475 - 2.25Y}\right) + \frac{d}{dY}\left[0.04Y\right]$$

$$\frac{dC_{RM}}{dY} = 0.09\left[\frac{(2.475-2.25Y)(-2.925)-(1-2.925Y)(-2.25)}{(2.475-2.25Y)^2}\right] + 0.04$$

$$= 0.09\left[\frac{(6.64-7.24)-(6.64-2.25)}{(2.475-2.25Y)^2}\right] + 0.04$$

$$= 0.09\left[\frac{\cancel{6.64}-7.24-\cancel{6.64}+2.25}{(2.475-2.25Y)^2}\right] + 0.04$$

$$= 0.09\left[\frac{-4.99}{(2.475-2.25Y)^2}\right] + 0.04$$

$$\frac{dC_{RM}}{dY} = \frac{-0.4491}{(2.475-2.25Y)^2} + 0.04$$

Now for optimum values of Y ; $\frac{dC_{RM}}{dY} = 0$

$$0 = \frac{-0.4491}{(2.475-2.25Y_{opt})^2} + \cancel{4}0.04$$

or $$\frac{0.4491}{(2.475-2.25Y_{opt})^2} = 0.04$$

$$(2.475-2.25Y_{opt})^2 = 11.23$$

$$5.06Y_{opt}^2 + 6.12 - 11.14Y_{opt} = 11.23$$

or $$5.06Y_{opt}^2 - 11.14Y_{opt} - 5.11 = 0.$$

solving Eq.

$$Y_{opt} = 2.59 \text{ (lbY/lbZ)}$$

Eq ① ⇒

$$X_{opt} = \frac{1 - (2.925)(2.59)}{2.475 - (2.25)(2.59)}$$

$$X_{opt} = 1.96 \quad (lb_x / lb_2)$$

Hence Eq ② ⇒

$$C_{RM,opt} = 0.09(1.96) + 0.04(2.59)$$

$$C_{RM_{opt}} = \$0.28 / lb_2$$

Now *

other cost = $0.5(0.8)$ $/lb₂

= \$0.4 /lb₂

selling price = s = \$0.8 /lb₂

so;

$$\text{profit} = \begin{pmatrix}\text{selling}\\ \text{price}\end{pmatrix} - \begin{pmatrix}C_{RM} + \text{other}\\ \text{costs}\end{pmatrix}$$

$$= (0.8) - (0.28 + 0.4)$$

$$\boxed{\text{profit} = 0.12 \ \$ / lb_2}$$ — ANS

PROBLEM # 11: Derive an expression similar to Eq. (56) for finding the optimum exit temperature of cooling water from a heat exchanger when the temperature of the material being cooled is not constant. Designate the true temperature-difference driving force by $F_G\Delta t_{lm}$,where F_G is a correction factor with value dependent on the geometrical arrangement of the passes in the exchanger. Use primes to designate the temperature of the material that is being cooled.

Let,

t_1' = warmer stream inlet temp.

t_2' = " " outlet temp.

t_1 = cold stream inlet temp.

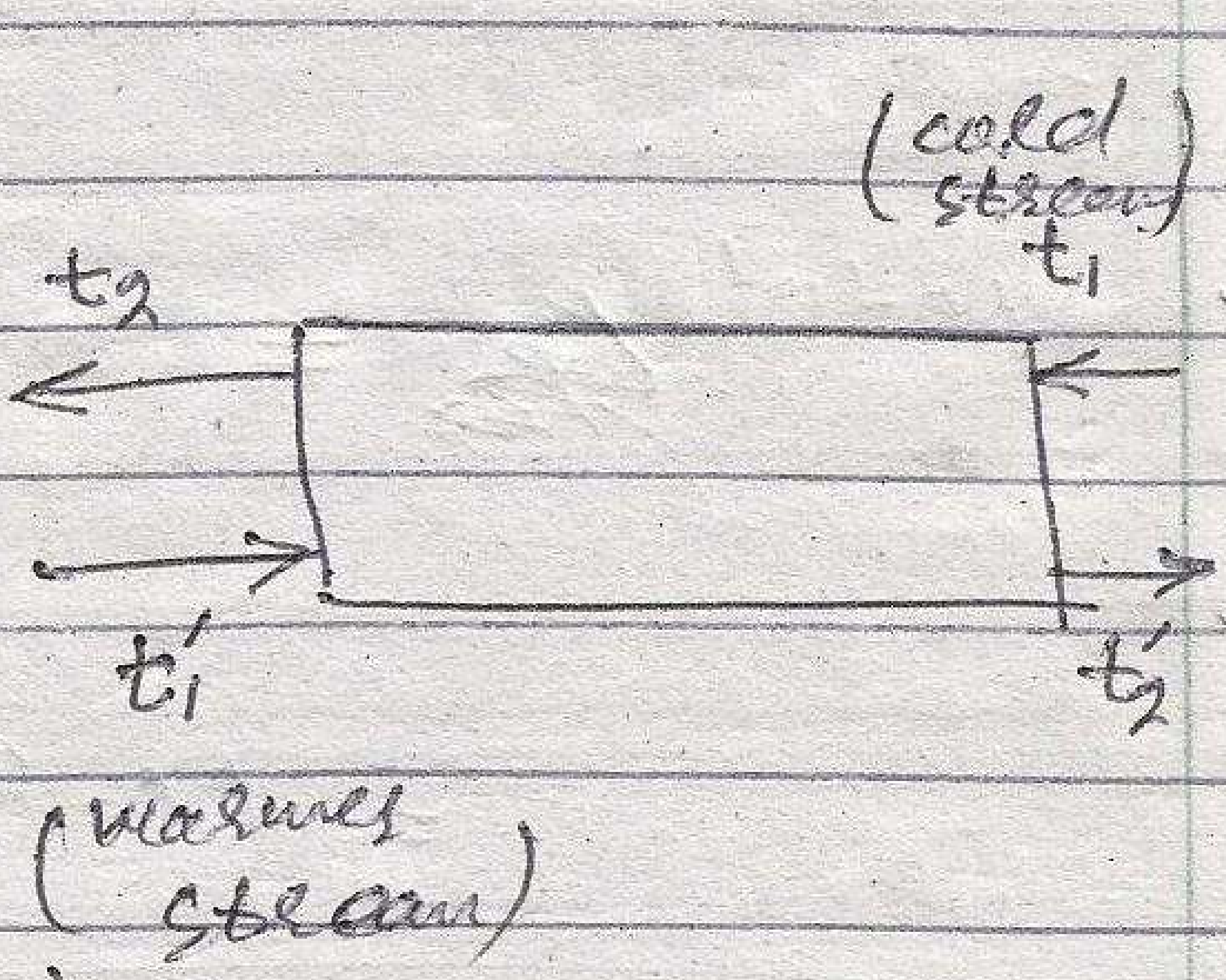
t_2 = " " outlet temp.

Now; rate of heat transfer is given by;

$$q = mc_p(t_2 - t_1) = UAF_G\Delta T_{lm} \longrightarrow ①$$

where F_G is the correction factor

& $F_G = \dfrac{\Delta T_{actual}}{\Delta T_{lm}}$



~~$\Delta T_{lm} = \dfrac{\Delta t_2 - \Delta t_1}{}$~~

$$\Delta T_{lm} = \frac{(t_1' - t_2) - (t_2' - t_1)}{\ln\left(\dfrac{t_1' - t_2}{t_2' - t_1}\right)}$$

① ⇒

$$q = mc_p(t_2 - t_1) = UAF_G \times \frac{(t_1' - t_2) - (t_2' - t_1)}{\ln[(t_1' - t_2)/(t_2' - t_1)]}$$

$$\longrightarrow ①'$$

① $\Rightarrow$

$$q = m c_p (t_2 - t_1)$$

$$m = q / c_p(t_2 - t_1)$$

or $m H_y C_w = \dfrac{q H_y C_w}{c_p(t_2 - t_1)}$ ($/yr) $\longrightarrow$ ②

$\downarrow$

(Annual cost for cold stream)

also

Annual Fixed charges = $A K_F C_A \longrightarrow$ ③

Total Annual cost $= C_T = \dfrac{q H_y C_w}{c_p(t_2 - t_1)} + A K_F C_A \longrightarrow$ ④

Eq ①' $\Rightarrow$

$$A = \frac{q \cdot \ln[(t_1' - t_2) / (t_2' - t_1)]}{F_G U[(t_1' - t_2) - (t_2' - t_1)]}$$

④ $\Rightarrow$

$$C_T = \frac{q H_y C_w}{c_p(t_2 - t_1)} + \frac{q K_F C_A \cdot \ln[(t_1' - t_2)/(t_2' - t_1)]}{F_G U[(t_1' - t_2) - (t_2' - t_1)]}$$

Diff. w.r.t 't_2'

$$\frac{dC_T}{dt_2} = \frac{d}{dt_2}\left[\frac{q H_y C_w}{c_p(t_2 - t_1)}\right] + \frac{d}{dt_2}\left[\frac{q K_F C_A [\ln (t_1' - t_2)/(t_2' - t_1)]}{F_G U[(t_1' - t_2) - (t_2' - t_1)]}\right] \longrightarrow \text{Ⓐ}$$

$$\frac{d}{dt_2}\left[\frac{q H_y C_w}{c_p(t_2 - t_1)}\right] = -\frac{q H_y C_w}{c_p (t_2 - t_1)^2} \longrightarrow \text{Ⓑ}$$

$$\frac{d}{dt_2}\left[\frac{V K_F C_A \cdot \overset{*}{\ln}\left[(t_1'-t_2)/(t_2'-t_1)\right]}{F_G U \;(t_1'-t_2)-(t_2'-t_1)}\right]$$

$$= \frac{V K_F C_A}{F_G U}\left[\frac{\left\{(t_1'-t_2)-(t_2'-t_1)\right\}\cdot\frac{(+1)}{(t_1'-t_2)}\cdot(-1) - \ln\left[\frac{(t_1'-t_2)}{(t_2'-t_1)}\right](-1)}{\left[(t_1'-t_2)-(t_2'-t_1)\right]^2}\right]$$

$$= \frac{V K_F C_A}{F_G U}\left[\frac{\left[(t_1'-t_2)-(t_2'-t_1)\right]\frac{(-1)}{(t_1'-t_2)} + \ln\left[\frac{t_1'-t_2}{t_2'-t_1}\right]}{\left[(t_1'-t_2)-(t_2'-t_1)\right]^2}\right]$$

$$= \frac{V K_F C_A}{F_G U}\left[\frac{-\left(\frac{t_1'-t_2}{t_1'-t_2}\right)+\left(\frac{t_2'-t_1}{t_1'-t_2}\right)+\ln\left[\frac{t_1'-t_2}{t_2'-t_1}\right]}{\left[(t_1'-t_2)-(t_2'-t_1)\right]^2}\right]$$

$$= \frac{V K_F C_A}{F_G U}\left[\frac{-1+\frac{t_2'-t_1}{t_1'-t_2}+\ln\left[\frac{t_1'-t_2}{t_2'-t_1}\right]}{\left[(t_1'-t_2)-(t_2'-t_1)\right]^2}\right] \longrightarrow Ⓒ$$

from

Ⓑ & Ⓒ; $\Rightarrow$

Ⓐ $\Rightarrow$

$$\frac{dC_T}{dt_2} = -\frac{V H_Y C_W}{C_p(t_2-t_1)^2} + \frac{V K_F C_A}{F_G U}\left[\frac{\frac{t_2'-t_1}{t_1'-t_2} - 1 + \ln\left(\frac{t_1'-t_2}{t_2'-t_1}\right)}{\left[(t_1'-t_2)-(t_2'-t_1)\right]^2}\right]$$

For optimum value of 't_2' ; $\frac{dC_T}{dt_2} = 0$.

So.

$$\frac{\cancel{V} K_F C_A}{F_G U}\left[\frac{\frac{t_2'-t_1}{t_1'-t_{2,opt}} - 1 + \ln\left(\frac{t_1'-t_{2,opt}}{t_2'-t_1}\right)}{\left[(t_1'-t_2)_{opt}-(t_2'-t_1)\right]^2}\right] = \frac{\cancel{V} H_Y C_W}{C_p(t_2-t_1)^2_{opt}}$$

$$\frac{(t_{2,opt} - t_1)^2}{[(t_1' - t_{2,opt}) - (t_2' - t_1)]^2}\left[\frac{t_2' - t_1}{t_1' - t_{2,opt}} - 1 + \ln\left(\frac{t_1' - t_{2,opt}}{t_2' - t_1}\right)\right] = \frac{U H_y C_w F_G}{K_F C_p C_A}$$

PROBLEM # 12: Under the following conditions, determine the optimum economic thickness of insulation for a $1\frac{1}{2}$-in. standard pipe carrying saturated steam at 100 psig. The line is in use continuously. The covering specified is light carbonate magnesia, which is marketed in whole-number thicknesses only (i.e., 1 in., 2 in., 3 in., etc.). The cost of the installed insulation may be approximated as $20 per cubic foot of insulation. Annual fixed charges are 20 percent of the initial investment, and the heat of the steam is valued at $1.50 per 1 million Btu. The temperature of the surroundings may be assumed to be 80°F.

L. B. McMillan, *Trans. ASME*, ml269 (1926), has presented approximate values of optimum economic insulation thickness versus the group $(kb_cH_y\Delta t/a_c)^{0.5}$ with pipe size as a parameter.

k = thermal conductivity of insulation, Btu/(h)(ft^2)(^{0}F/ft)

b_c = cost of heat, $/Btu

H_y = hours of operation per year, h/year

Δt = overall temperature-difference driving force, ^{0}F

a_c = cost of insulation, $/(ft^3)(year)

The following data are based on the results of McMillan, and these data are applicable to the conditions of this problem:

$\left(\frac{kb_cH_y\Delta t}{a_c}\right)^{0.5}$	Optimum economic thickness of insulation, in., for nominal pipe diameter of			
	½ in.	1 in.	2 in.	4 in.
0.1	....	0.40	0.5	0.6
0.2	0.80	0.95	1.1	1.3
0.3	1.20	1.4	1.6	1.9
0.5	1.85	2.1	2.45	2.9
0.8	2.75	3.1	3.6	4.3
1.2	3.80	4.3	4.9	

Steam pr. = 100 psig.

Insulation material = light carbonate magnesia.

cost of installed insulation = $C_{ins.}$ = \$ 20/ft^3

Annual fixed charges = $C_{F.C}$ = 0.2[Initial investment]

steam cost = \$1.50/10,00,000 Btu = b_c

Temp. of surrounding = T_{surr} = 80°F solun Now;

For Absolute pr = 100 psig + 14.7 psi

Ab. pr = 114.7 psi

At this ~~Temp~~ Pr; From steam Tables:

$T_s = 338\,°F$

also From Tables;

Thermal conductivity = $\boxed{k = 0.038 \text{ Btu/hr.ft}^2.(°F/ft)}$

$\boxed{b_c = \$1.50\times10^{-6}/\text{Btu}}$

operating Hours = $H_y = 365 \times 24$ hrs

$\boxed{H_y = 8760 \text{ hrs}}$

$\Delta t = (338 - 80)\,°F = 258\,°F$

$\boxed{\Delta t = 258\,°F}$

$a_c = (0.2)(20) = 4\ \$/\text{ft}^3.\text{yr}$

$\boxed{a_c = \$4/\text{ft}^3.\text{yr}}$

Now;

$$\left(\frac{k\, b_c H_y \Delta t}{a_c}\right)^{1/2} = \left[\frac{(0.038)(1.50\times10^{-6})(8760)(258)}{4}\right]^{1/2}$$

$\boxed{\text{Factor} = 0.18}$

Now From Table;

FACTOR:-	1"	$1\frac{1}{2}$"	2"
0.1	0.40	0.45	0.5
0.18	0.84	0.91	0.98
0.2	0.95	1.025	1.1

Now; First we have to figure out the values for 1" & 2" for factor 0.18. so by interpolation:-

→ $\overbrace{0.1 \longrightarrow 0.18 \qquad 0.2}^{0.1}$

$\underbrace{0.40 \qquad\qquad 0.95}_{0.55}$

$\Rightarrow \frac{0.55}{0.1} = (5.5)$

$\Rightarrow 5.5(0.18-0.1) = 0.44$

$\Rightarrow 0.44 + 0.4 = \boxed{0.84}$

→ $\overbrace{0.1 \longrightarrow 0.18 \qquad 0.2}^{0.1}$

$\underbrace{0.5 \qquad\qquad 1.1}$.

$\Rightarrow \frac{0.6}{0.1} = 6$

$\Rightarrow 6(0.18-0.1) = 0.48$

$\Rightarrow 0.48 + 0.5 = \boxed{0.98}$

→ $\overbrace{0.1 \longrightarrow 0.18 \qquad 0.2}^{0.1}$

$\underbrace{0.45 \qquad\qquad 1.025}_{0.575}$

$\Rightarrow \frac{0.575}{0.1} = 5.75$

$\Rightarrow 5.75(0.18-0.1) = 0.46$

$0.46 + 0.45 = 0.91"$

so optimum Economic thickness of insulation = 0.91"

∴ select $\underline{\underline{0.98"}}$ insulation available

PROBLEM # 13: A catalytic process uses a catalyst which must be regenerated periodically because of reduction in conversion efficiency. The cost for one regeneration is constant at \$800. This figure includes all shutdown and startup costs, as well as the cost for the actual regeneration. The feed rate to the reactor is maintained constant at 150 lb/day, and the cost for the feed material is \$2.50 per pound. The daily costs for operation are \$300, and fixed charges plus general overhead costs are \$100,000 per year. Tests on the catalyst show that the yield of product as pounds of product per pound of feed during the first day of operation with the regenerated catalyst is 0.87, and the yield decreases as **$0.87/(\theta_D)^{0.25}$**, where **θ_D** is the time in operation expressed in days. The time necessary to shut down the unit, replace the catalyst, and start up the unit is negligible. The value of the product is \$14.00 per pound, and the plant operates 300 days per year. Assuming no costs are involved other than those mentioned, what is the maximum annual profit that can be obtained under these conditions?

DATA:-

Cost for Regeneration = C_{regen} = \$ 800 /regen

Feed rate = F = 150 lb/day.

cost for Feed material = C_{Feed} = \$2.50 /lb

cost for operation = C_{oper} = \$300 /ops. day.

Fixed charges + over Head cost = F.C = \$100,000 /yr

product yield = $Y = \dfrac{0.87}{\theta_D^{0.25}}$ $\dfrac{\text{lb of prod.}}{\text{lb of Feed.}}$

operating time in days = θ_D

Product value = S = \$ 14.00 / lb

ops. days = 300 days / yr

Maximum Annual profit = $\text{Profit}_{(max)}$ = ?

soln:-

let;

θ_D = days of operating time per cycle.

Now;

$$\text{If time is } \overset{*\ \nearrow(\text{days})}{\theta_D}\text{, then no. of cycles} = 1$$

$$\text{" " " } 1 \text{ " " " } = \frac{1}{\theta_D}$$

$$\text{" " " } 300 \text{ " " " " } = \frac{300}{\theta_D}$$

Hence;

$$\boxed{\frac{\text{cycles}}{\text{yr}} = \frac{300}{\theta_D}}$$

Now also;

$$\text{Annual profit} = \text{Annual income} - \text{Annual product cost} \longrightarrow ①$$

also;

$$\text{Annual product cost} = \text{Annual cost for Feed} + \text{Annual opr cost} + \text{Annual reg. cost} + \text{F.C} \longrightarrow ②$$

$\Rightarrow$

$$\text{production rate} = (\text{Feed})(\text{Yield}) = \text{F.Y}$$

$$= \frac{150 \text{ lb of } \cancel{\text{Feed}}}{\text{day}} \left| \frac{0.87 \text{ lb of Prod.}}{\theta_D^{0.25} \text{ lb of } \cancel{\text{Feed}}} \right.$$

$$\text{production rate} = \frac{(150)(0.87)}{\theta_D^{0.25}} \text{ lb of Prod/day}$$

Assuming the yield for the first day is 0.87;

$$\frac{\text{Production}}{\text{cycle}} = \int_1^{\theta_D} \frac{(150)(0.87)}{\theta_D^{0.25}}\, d\theta + \underbrace{(150)(0.87)}_{\text{1st day}}$$

and so on...

$$= (150)(0.87) \int_1^{\theta_D} \theta_D^{-0.25}\, d\theta + (150)(0.87)$$

$$\frac{\text{production}}{\text{cycle}} = (130.5) \left| \frac{\theta_D^{-0.25+1}}{-0.25+1} \right|_1^{\theta_D} + 130.5$$

$$\frac{\text{production}}{\text{cycle}} = \left(\frac{130.5}{0.75}\right) \Big| \theta_D^{0.75} \Big|_{1}^{\theta_D} + 130.5$$

$$= 174\left(\theta_D^{0.75} - 1\right) + 130.5$$

$$= 174\theta_D^{0.75} - 174 + 130.5$$

$$\boxed{\frac{\text{production}}{\text{cycle}} = 174\,\theta_D^{0.75} - 43.5} \quad \text{lb/cycle}$$

Now;

$$\text{Annual income} = \frac{\text{production}}{\text{cycle}} \times \frac{\text{cycle}}{\text{yr}} \times \text{product value}$$

$$= \left(174\theta_D^{0.75} - 43.5\right)\cdot\left(\frac{300}{\theta_D}\right)\cdot(14) \quad (\$/\text{yr})$$

$$= \frac{\text{lb. of prod.}}{\text{cycle}} \times \frac{\text{cycle}}{\text{yr}} \times \frac{\$}{\text{lb. of prod}}$$

$$= \$/\text{yr}$$

=>

$$\text{Annual income} = \frac{730800\,\theta_D^{0.75}}{\theta_D} - \frac{182700}{\theta_D}$$

$$\boxed{\text{Annual income} = \frac{730800}{\theta_D^{0.25}} - \frac{182700}{\theta_D}} \quad \$/\text{yr} \longrightarrow ③$$

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$$\text{Annual cost of as Feed} = \frac{150\text{ lb}}{\text{day}} \Big| \frac{\$2.5}{\text{lb}} \Big| \frac{300\text{ day}}{1\text{ yr}}$$

$$\boxed{\text{Annual cost for Feed} = 112500\ \$/\text{yr}} \longrightarrow ⑨$$

$$\text{Annual ops. cost} = \frac{\$\,300}{\text{day}} \Big| \frac{300\,\text{day}}{\text{yr}} = 90{,}000\ \$/\text{yr}$$

$$\boxed{\text{Annual ops. cost} = 90{,}000\ \$/\text{yr}} \longrightarrow \text{ⓑ}$$

$$\text{Annual Reg. cost} = \frac{\$800}{\text{cycle}} \Big| \frac{300\ \text{cycles}}{\theta_D\ \ \text{yr}} = \frac{240{,}000}{\theta_D}\ \$/\text{yr}$$

$$\boxed{\text{Annual Reg cost} = \frac{240{,}000}{\theta_D}\ \$/\text{yr}} \longrightarrow \text{ⓒ}$$

ⓐ, ⓑ & ⓒ $\Rightarrow$

$$\text{Annual product cost} = 112500 + 90{,}000 + \frac{240{,}000}{\theta_D} + \text{F.C}^{*}$$

$$\boxed{\text{Annual product cost} = \frac{240{,}000}{\theta_D} + 302500} \longrightarrow \text{④}$$

Eq ① $\Rightarrow$

$$\text{profit/yr} = \frac{730800}{\theta_D^{0.25}} - \frac{182700}{\theta_D} - \frac{240{,}000}{\theta_D} - 302500$$

Diff. w.r.t θ_D.

$$\frac{d}{d\theta_D}(\text{profit/yr}) = \frac{(-0.25)(730800)}{\theta_D^{1.25}} + \frac{182700}{\theta_D^{2}} + \frac{240{,}000}{\theta_D^{2}}$$

$$= -\frac{182700}{\theta_D^{1.25}} + \frac{422700}{\theta_D^{2}}$$

Now for $\theta_{D,opt}$; $\frac{d}{d\theta_D}(\text{profit/yr}) = 0.$

So;

$$\frac{182700}{\theta_{D,opt}^{1.25}} = \frac{422700}{\theta_{D,opt}^{2}}$$

$$\frac{\theta_{D,opt}^{2}}{\theta_{D,opt}^{1.25}} = \frac{422700}{182700}$$

$$\theta_{D,opt}^{0.75} = 2.31 \Rightarrow \boxed{\theta_{D,opt} = 3.06 \text{ days}}$$

Eq ③ =>

$$\text{Annual Income} = \frac{730800}{(3.06)^{0.25}} - \frac{182700}{3.06}$$

$$\boxed{\text{Annual Income} = 492840 \ \$/\text{yr}}$$

Eq ④ =>

$$\text{Annual prod. cost} = \frac{240,000}{3.06} + 302500$$

$$\boxed{\text{Annual prod. cost} = 380931 \ \$/\text{yr}}$$

So finally; ahhhhhh....... :D

$$\frac{\text{Profit}_{(max)}}{\text{yr}} = 492840 - 380931$$

$$\boxed{\frac{\text{profit}_{(max)}}{\text{yr}} = 111,909 \ \$/\text{yr}} \longrightarrow \text{Ans}$$

PROBLEM # 14: Derive the following equation for the optimum outside diameter of insulation on a wire for maximum heat loss:

$$D_{opt} = \frac{2k_m}{(h_c + h_r)_c}$$

where k_m is the mean thermal conductivity of the insulation and $(h_c+ h_r)_c$ is the combined and constant surface heat-transfer coefficient. The values of k_m and $(h_c+ h_r)_c$ can be considered as constants independent of temperature level and insulation thickness.

let;

$\dot{Q}$ = rate of Heat flow

ΔT_{ov} = overall temp. driving force

$R_{th,ov}$ = overall thermal Resistance

then

$$\dot{Q} = \frac{\Delta T_{ov}}{R_{th,ov}} \longrightarrow ①$$

Now

$$R_{th,ins} = \frac{\ln(r_2/r_1)}{2\pi L K_m}$$ = thermal resist. inside of insulation.

$$R_{th,out} = \frac{1}{(h_c+h_r)_c 2\pi L r_2}$$ = thermal resist. due to conv. & radiation from outer portion of insulation.

$$\therefore \quad R_{th,ov} = \left[\frac{\ln(r_2/r_1)}{K_m} + \frac{1}{(h_c+h_r)_c r_2}\right]\frac{1}{2\pi L}$$

Eq. ① =>

$$\dot{Q} = \frac{\Delta T_{ov}}{\left[\frac{\ln(r_2/r_1)}{K_m} + \frac{1}{(h_c+h_r)_c r_2}\right] \times \frac{1}{2\pi L}}$$

or

$$\dot{Q} = \frac{2\pi L \Delta T_{ov}}{\frac{\ln(r_2/r_1)}{K_m} + \frac{1}{(h_c+h_r)_c r_2}}$$

Differentiating the Eq:- w.r.t 'r_2'

$$\frac{d\dot{Q}}{dr_2} = -2\pi L \Delta T_{ov} \times \frac{1}{\left[\frac{\ln(r_2/r_1)}{K_m} + \frac{1}{(h_c+h_r)_c\, r_2}\right]^2} \times \left\{ \quad \right\}$$

$$\boxed{\frac{1}{K_m}\cdot\frac{1}{r_2} - \frac{1}{(h_c+h_r)_c}\cdot\frac{1}{r_2^2}}$$

$$\frac{d\dot{Q}}{dr_2} = -\frac{2\pi L \Delta T_{ov}}{K_m r_2} \times \frac{1}{\left[\frac{\ln(r_2/r_1)}{K_m} + \frac{1}{(h_c+h_r)_c\, r_2}\right]^2}$$

$$+ \frac{2\pi L \Delta T_{ov}}{(h_c+h_r)_c \cdot r_2^2} \times \frac{1}{\left[\frac{\ln(r_2/r_1)}{K_m} + \frac{1}{(h_c+h_r)_c\, r_2}\right]^2}$$

For optimum values of 'r_2'

$$\frac{d\dot{Q}}{dr_2} = 0$$

So;

$$\frac{\cancel{2\pi L \Delta T_{ov}}}{K_m r_2} \times \frac{1}{\left[\frac{\ln(r_2/r_1)}{K_m} + \cancel{\frac{1}{(h_c+h_r)_c\, r_2}}\right]^2} = \frac{\cancel{2\pi L \Delta T_{ov}}}{(h_c+h_r)_c\, r_2^2} \times \frac{1}{\left[\frac{\ln(r_2/r_1)}{K_m} + \cancel{\frac{1}{(h_c+h_r)_c\, r_2}}\right]^2}$$

$$\frac{1}{K_m r_{2,opt}} = \frac{1}{(h_c+h_r)_c \cdot r_{2,opt}^2}$$

or $$r_{2,opt} = \frac{K_m}{(h_c+h_r)_c}$$

Now; ∵ $$\boxed{D_{opt} = 2 r_{opt} = \frac{2 K_m}{(h_c+h_r)_c}} \longrightarrow \text{Ans}$$

PROBLEM # 16: Using a direct partial derivative approach for the objective function, instead of the Lagrangian multiplier as was used in Eqs. (92) to (95), determine the optimum values of x and y involved in Eqs. (92) to (95).

objective function : xy

constraint : $x^2 + y^2 = 10$

Now;

$$\phi(x,y) = xy \longrightarrow ①$$

also $x^2 + y^2 = 10$

$$y^2 = 10 - x^2 \Rightarrow y = (10 - x^2)^{1/2} \longrightarrow ②$$

Eq ① $\Rightarrow$

$$\phi(x,y) = x(10 - x^2)^{1/2}$$

$$\frac{d\phi}{dx} = (x)\left(\frac{1}{2}\right)(10 - x^2)^{-1/2}(-2x) + (10 - x^2)^{1/2} \cdot 1$$

$$\frac{d\phi}{dx} = -\frac{x^2}{(10 - x^2)^{1/2}} + (10 - x^2)^{1/2}$$

for optimum values of x; $\frac{d\phi}{dx} = 0$

$$\frac{x^2_{,opt}}{(10 - x^2_{opt})^{1/2}} = (10 - x^2_{,opt})^{1/2}$$

$$x^2_{,opt} = 10 - x^2_{,opt} \Rightarrow 2x^2_{,opt} = 10$$

$$\boxed{x_{,opt} = \sqrt{5}}$$

ANS

$$② \Rightarrow \boxed{y_{opt} = \sqrt{5}}$$

where only $x \geq 0$ & $y \geq 0$ are being sought

PROBLEM # 17: Find the values of x, y, and z that minimize the function $x + 2y^2 + z^2$ subject to the constraint that $x + y + z = 1$, making use of the Lagrangian multiplier.

objective function: $\phi(x,y,z) = x+2y^2+z^2$

constraint: $\phi^N(x,y,z) = x+y+z = 1$

Forming the Lagrange Expression;

$$L(x,y,z) = (x+2y^2+z^2) + \lambda(x+y+z-1)$$

By partial differentiation;

$$\frac{\partial L}{\partial x} = (1+0+0) + \lambda(1)\cdot + 0 + 0\cdot$$

$$\frac{\partial L}{\partial x} = 1+\lambda \longrightarrow ①$$

$$\frac{\partial L}{\partial y} = 4y+\lambda \longrightarrow ②$$

$$\frac{\partial L}{\partial z} = 2z+\lambda \longrightarrow ③$$

$$\frac{\partial L}{\partial \lambda} = x+y+z-1 \longrightarrow ④$$

By putting all the respective partial derivatives w.r. equal to zero;

so

$$1+\lambda = 0 \implies \boxed{\lambda = -1}$$

$$4y+\lambda = 0 \implies \boxed{y_{opt} = 1/4}$$

$$2z + \lambda = 0 \implies \boxed{z_{opt} = 1/2}$$

$$x + y + z - 1 = 0 \implies x = 1 - 1/4 - 1/2$$

$$\boxed{x_{opt} = 1/4}$$

$$\therefore \boxed{X_{opt} = 0.25 \; ; \quad Y_{opt} = 0.25 \; ; \quad z_{opt} = 0.5}$$

$\longrightarrow$ Ans

"TrUeLiAr"

CHAPTER 14

MATERIALS TRANFER, HANDLING, AND TREATMENT EQUIPMENT DESIGN AND COSTS

Example 5: Estimation of filtering area required for a plate-and-frame filtration operation. A plate-and-frame filter press is to be used for removing the solid material from a slurry containing 5 lb of dry solids per cubic foot of solid-free liquid. The viscosity of the liquid is 1 centipoise, and the filter must deliver at least 400 ft^3 of solid-free filtrate over a continuous operating time of 2 h when the pressure-difference driving force over the filter unit is constant at 25 psi. On the basis of the following data obtained in a small plate-and-frame filter press, estimate the total area of filtering surface required.

Experimental data. The following data were obtained in a plate-and-frame filter press with a total filtering area of 8 ft^2:

Total volume of filtrate (V), ft^3	Time from start of filtration (θ), h at constant pressure difference of		
	AP = 20 psi	AP = 30 psi	AP = 40 psi
5	0.34	0.25	0.21
8	0.85	0.64	0.52
10	1.32	1.00	0.81
12	1.90	1.43	1.17

The slurry (with filter aid) was identical to that which is to be used in the large filter. The filtrate obtained was free of solid, and a negligible amount of liquid was retained in the cake.

DATA:-

$W = 5 lb/ft^3$

liquid vis. $= \mu = 1 cp = 2.42 \ lb/hr.ft$

Vol. of filterate delivered $= V = 400 ft^3$

operating time $= \theta = 2 hr$

pr. diff driving force $= \Delta P = 25 psi$

T.F.:

Area of filtering surface $= A = ?$

soln:-

$\because \quad V^2 + 2AV_FV = \dfrac{2A^2(\Delta P)^{1-s}}{\alpha' w \mu}\theta \longrightarrow ①$

From Fig 14-59 page # 547 a plot is V/A & $\theta\Delta P/V/A$ is drawn so rearranging Eq. ① to the req. form as;

♥ True Lif

$$V^2 + 2AV_FV = \frac{2A^2(\Delta P)\cdot\theta}{\alpha'(\Delta P)^s\cdot W\cdot\mu}$$

$$V^2\alpha'(\Delta P)^s W\mu + 2AV_FV\alpha'(\Delta P)^s W\mu = 2A^2\Delta P\cdot\theta$$

$$\frac{V}{2A^2}\left[V\alpha' W\mu(\Delta P)^s + 2AV_F\alpha' W\mu(\Delta P)^s\right] = \theta\cdot\Delta P$$

$$\frac{V}{A}\left[\frac{\alpha' W\mu(\Delta P)^s}{2}\cdot\frac{V}{A} + \alpha' W\mu V_F(\Delta P)^s\right] = \theta\Delta P$$

$$\frac{\theta\Delta P}{V/A} = \frac{\alpha' W\mu(\Delta P)^s}{2}\cdot\frac{V}{A} + \alpha' W\mu V_F(\Delta P)^s \longrightarrow ②$$

$$y = mx + C$$

Eq ② ⇒

$$m = \text{slope} = \frac{\alpha' W\mu(\Delta P)^s}{2} \longrightarrow ⓐ$$

$$C = \text{intescept} = \alpha' W\mu V_F(\Delta P)^s \longrightarrow ⓑ$$

From Fig 14.59 :-

ΔP, psf	slope.	intescept
20 × 144 = 2880.	2380.	70
30 × 144 = 4320	2680	80
40 × 144 = 5760.	2990.	90.

Now; values of α' & s can be obtained by ~~simultaneous soln with any~~ two ~~of othe three slopes values presented inthe above table~~

eg5 ⓐ =>

$$\alpha' = \frac{(\text{slope})(2)}{w\mu(\Delta p)^s} \longrightarrow ③$$

select the filter runs at 2880 to 4320 psf

$$\alpha' = \frac{(2380)(\cancel{2})}{(\cancel{5})(\cancel{2.42})(2880)^s} = \frac{(2680)(\cancel{2})}{(\cancel{5})(\cancel{2.42})(4320)^s}$$

$$\frac{(4320)^s}{(2880)^s} = \frac{2680}{2380}$$

$$\ln\left[\frac{(4320)^s}{(2880)^s}\right] = \ln(1.126)$$

$$\ln(4320)^s - \ln(2880)^s = \ln(1.126)$$

$$s.\ln(4320) - s\ln(2880) = \ln(1.126)$$

$$s[\ln(4320) - \ln(2880)] = \ln(1.126)$$

$$s = \frac{0.1187}{0.4054} = 0.2927 \approx 0.3$$

$$\boxed{s = 0.3}$$

Eq ③ => for Δp = 2880 psf

$$\alpha' = \frac{(2380)(2)}{(5)(2.42)(2880)^{0.3}} = 36.058$$

$$*\boxed{\alpha' = 36\ \text{ft}^2/\text{lb}}$$ ← controversial

ⓑ =>

$$v_F = \frac{\text{intercept}}{\alpha' w\mu(\Delta p)^s}$$

on the basis of Fig. 14-59 intercept for 4320 psf data.

$$V_F = \frac{80}{(36)(5)(2.42)(4320)^{0.3}}$$

$$\boxed{V_F = 0.015\ ft^3/sq.ft}$$

Eq ① =>

$$(400)^2 + (2)(A)(0.015)(400) = \frac{(2)(A^2)(25 \times 144)^{1-0.3} \times (2)}{(36)(5)(2.42)}$$

$$160{,}000 + 12A = 2.83A^2$$

or $2.83A^2 - 12A - 160{,}000 = 0$

solving for 'A'

$$A = 239.90 \approx 240\ ft^2$$

so; $\boxed{\text{area of filtering surface} = 240\ ft^2}$

Example 6: Effect of pressure difference on capacity of a rotary vacuum filter. A rotary vacuum filter with negligible filter-medium resistance delivers 100 ft^3 of filtrate per hour when a given CaC0$_3$.H,O mixture is filtered under known conditions. How many cubic feet of filtrate will be delivered per hour if the pressure drop over the cake is doubled, all other conditions remaining constant? Assume the CaCO$_3$ filter cake is noncompressible.

DATA:-

filtermedium resistance = 0.

vol. of filtrate = $V_{F_1} = 100\ ft^3/hr.$

$V_2 = ?$ if $\Delta P_2 = 2\Delta P_1$

cake is non-compressible ⇒ $s = 0$.

for rotary vacuum filters

∵

$$V_R^2 + 2 A_D \psi_f V_F V_R = \frac{2 A_D^2 \psi_f (\Delta p)^{1-s}}{\alpha' w \mu N_R}$$

By neglecting filter medium resistance & cake is noncompressible

⇒

$$V_R = A_D \sqrt{\frac{2 \psi_f \Delta P}{\alpha w \mu N_R}}$$

vol. of filtrate per unit time $= V_R \times N_R = A_D \sqrt{\frac{2 \psi_f \Delta P N_R}{\alpha w \mu}}$

$\Rightarrow V_1$

$$V_1 = A_D \sqrt{\frac{2 \psi_f N_R}{\alpha w \mu}} \sqrt{\Delta p_1} \longrightarrow ①$$

similarly;

$$V_2 = A_D \sqrt{\frac{2 \psi_f N_R}{\alpha w \mu}} \sqrt{2 \Delta p_1} \quad —— ②$$

② ÷ ① ⇒

$V_2 = \sqrt{2} \cdot V_1$ or $V_2 = \sqrt{2} \times 100 \ \frac{ft^3}{hr}$

$$\boxed{V_2 = 141.4 \ ft^3/hr}$$

Example 7: Estimation of horsepower motor required for vacuum pump on a rotary vacuum filter. A rotary vacuum-drum filter is to handle a slurry containing 20 lb of water per 1 lb of solid material. Tests on the unit at the conditions to be used for the filtration have shown that the dimensionless ratio of α/β is 0.6 and 19 lb of filtrate (not including wash water) is obtained for each 21 lb of slurry. The temperature of the surroundings and of the slurry is 70°F, and the pressure of the surroundings is 1 atm. The pressure drop to be maintained by the vacuum pump is 5 psi. The fraction of the drum area submerged in the slurry is 0.3, and the fraction of the drum area available for air suction is 0.1. On the basis of the following assumptions, estimate the horsepower of the motor necessary for the vacuum pump if the unit handles 50,000 lb of slurry per hour.

Assumptions:

- Resistance of filter medium is negligible.
- Any effects caused by air leakage are taken into account in the value given for α/β
- For air at the temperature involved, heat capacity at constant pressure divided by heat capacity at constant volume is 1.4.
- The vacuum pump and motor have an overall efficiency of 50 percent based on an isentropic compression.
- The value of β is based on the temperature and pressure of the air surrounding the filter.
- The filter removes all of the solid from the slurry.

• DATA:-

slurry charged initially = 21 lb $\begin{cases} 20 \text{ lb } H_2O \\ 1 \text{ lb solids} \end{cases}$

$\alpha/\beta = 0.6$

Filterate obtained for each 21 lb slurry (Not including wash water) = 19 lb $\begin{cases} 21 \text{ lb slurry} \\ 19 \text{ lb filterate} \\ 1 \text{ lb solid} \\ 1 \text{ lb } H_2O \text{ wasted.} \end{cases}$

Temp. of surroundings & slurry = $T = 70°F$

pr. of the surrounding = 1 atm

pr. drop = Δp = 5 psi

Fraction of area submerged in slurry = $\varphi_f = 0.3$

" " " available for air suction = $\varphi_a = 0.1$

T.F.O:-

Horsepower of motor for vacuum pump of handles 50,000 lb slurry per hours = ?

Sol^n For isentropic compression:-

$$\text{Theoratical power}_{(HP)} = \frac{3.03\times10^{-5}K}{K-1}\,P_1 q_{fm_1}\left[\left(\frac{P_2}{P_1}\right)^{K-1/K} - 1\right] \longrightarrow ①$$

Heat capacity at constt. pr divided by heat capacity at constt volume $= K = 1.4$

Efficiency = 50 percent. $(\eta = 0.5)$

P_1 = vacuum pump outake pr.

P_2 = " " delivery pr.

q_{fm_1} = ft³ of gas per min at vacuum outake conditions $= V_1$

∵

$$P_1V_1 = P_2V_2$$

or

$$V_1 = \frac{P_2V_2}{P_1} \longrightarrow ②$$

$$\boxed{P_2 = 1\,atm = 14.7\ psi = 14.7\times144\ psf = \boxed{2116.8\,psf}}$$

$\Delta P = 5\ psi$

$\Delta P = P_2 - P_1$

$P_1 = P_2 - \Delta P = 14.7 - 5 = 9.7\ psi \times 144$

$$\boxed{P_1 = 1396.8\ psf}$$

∵

$$\frac{\text{vol. of air per unit time } (V_2)}{\text{wt of dry cake per unit time } (?)} = \frac{\psi_a}{\psi_f}\cdot\frac{\mu}{\mu_a}\cdot\frac{\alpha}{2\beta w} \longrightarrow ③$$

At 70°F :-

viscosity of water $= \mu = 1 cp = 1 \times 2.42 = 2.42 \frac{lb}{ft.hr}$

" " air $= \mu_a = 0.018 cp = 0.018 \times 2.42$

$= 0.04356 \frac{lb}{ft.hr}$

W = mass of dry cake solids per unit vol. of filtrate.

i.e $W = \dfrac{\text{mass of dry cake solids}}{\text{vol}(ft^3)\text{ of filtrate}} \rightarrow$ ⓐ

mass of dry cake solids = 1 lb

mass of filtrate = m = 19 lb

Density of filtrate$_{(H_2O)}$ $= \rho = 62.4\ lbm/ft^3$

vol. of filtrate = V = ?

$\because \rho = m/V$

$\Rightarrow V = m/\rho = \dfrac{19\ lbm}{62.4\ lbm} \Big| ft^3$

$V = 19/62.4\ ft^3$ filtrate

ⓐ $\Rightarrow w = \dfrac{1}{19/62.4} = 3.28\ lb_{(solid)}/ft^3$ (filtr

wt of dry cake per unit time = ?

For 21 lb slurry, wt of cake = 1

" 1 lb " " " " $= \frac{1}{21}$

" 50,000 lb/hr " " " " $= \frac{50,000}{21}$ lb/hr wt. of dry cake per unit time = 2381 lb/hr.

Eq ③ ⇒

$$V_2 = \frac{0.1}{0.3} \times \frac{2.42}{0.04356} \times \frac{(0.6)}{(2)(3.28)^{✓}} \cdot (2381)^{✓} \; \frac{ft^3}{\cancel{lb}} \Big| \frac{\cancel{lb}}{hr}$$

(vol of air per hour)

$$\boxed{V_2 = 4033 \; ft^3/hr} \Rightarrow$$

Eq ② ⇒

$$V_1 = \frac{2116.8}{1396.8} \times 4033 \; \frac{ft^3}{\cancel{hr}} \Big| \frac{1 \cancel{hr}}{60 \, min}$$

$$\boxed{V_1 = 102 \; ft^3/min}$$

Eq ① ⇒

$$\text{theoratical power (Hp)} = \frac{(3.03 \times 10^{-5})(1.4)}{0.4} \times 1396.8 \times 102 \times \left[\left(\frac{2116.8}{1396.8}\right)^{0.4/1.4} - 1\right]$$

Theoratical power (Hp) = 1.9 hp

$$\because \; \eta = \frac{\text{Theoratical power}}{\text{Actual power.}}$$

(for pumps)

Actual always be greater due to friction effect

$$\boxed{\text{Actual power} = 1.9/0.5 = 3.8 \, hp}$$

So; A 4 hp motor would be satisfactory.

PROBLEM 1: A lean oil is to be used as the absorbing medium for removing a component of a gas. As part of the design for the absorption unit, it is necessary to estimate the size of the motor necessary to pump the oil to the top of the absorption tower. The oil must be pumped from an open tank with a liquid level 10 ft above the floor and forced through 150 ft of schedule number 40 pipe of 3-in. nominal diameter. There are five 90° elbows in the line, and the top of the tower is 30 ft above the floor level. The operating pressure in the tower is to be 50 psig, and the oil requirement is estimated to be 50 gpm. The viscosity of the oil is 15 centipoises, and its density is 53.5 lb/ft^3. If the efficiency of the pumping assembly including the drive is 40 percent, what horsepower motor will be required?

DATA:-

$Z_1 = 10ft$

Length of pipe = $L = 150ft$

pipe diameter = 3" nominal.

# of 90° Elbows = 05

$Z_2 = 30\ ft$

$P_1 = 14.7\ psia$ $P_2 = 50psig$

$Q = 50\ gal/min.$

$\mu_{oil} = 15cp$; $\rho_{oil} = 53.5\ lbm/ft^3$

$\eta = 0.4$

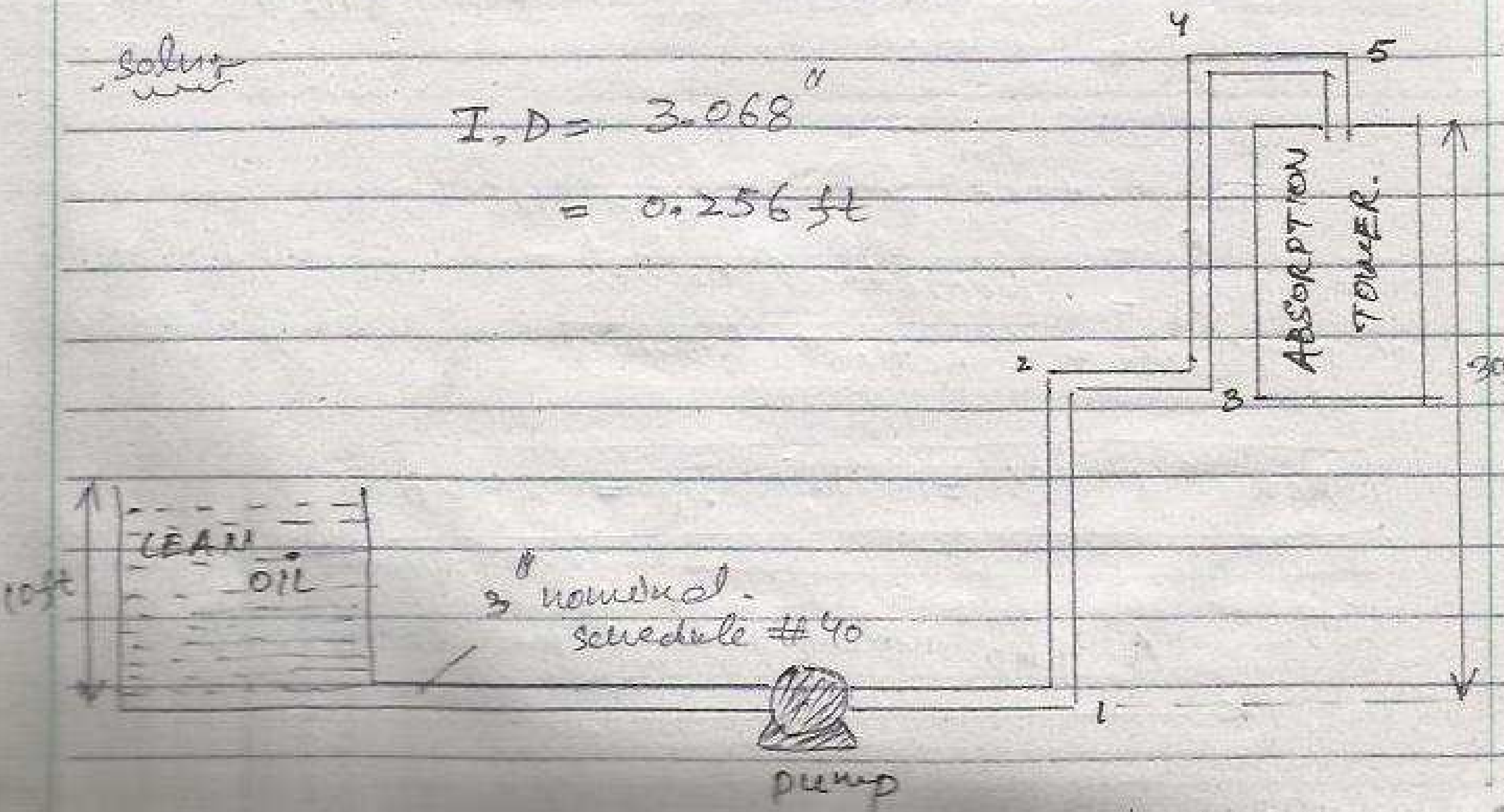
T.F.O:-

$Power_{(HP)} = ?$

Solu:-

I.D = 3.068"

= 0.256 ft



Basis:- 1lb of flowing lean oil

As we know;

$$\text{Actual power Req.} = \frac{\text{work done} \times \text{mass flowrate}}{\eta}$$

i.e

$$P = \frac{W_o \times \dot{m}}{\eta} \longrightarrow ①$$

The total mechanical Energy Balance Eq. =>

$$W_o = \frac{g}{g_c}(Z_2 - Z_1) + \nu(P_2 - P_1) + \left(\frac{V_2^2}{2\alpha g_c} - \frac{V_1^2}{2\alpha g_c}\right) + \Sigma F$$

or

$$W_o = (Z_2 - Z_1) + \nu(P_2 - P_1) + \left(\frac{V_2^2}{2g_c} - \frac{V_1^2}{2g_c}\right) + \Sigma F \longrightarrow ②$$

∵ dia of the pipe is not changing & flow rate (50gpm) is constt as well through out

so; $V_1 = V_2$.

$$\Rightarrow \left(\frac{V_2^2}{2\alpha g_c} - \frac{V_1^2}{2\alpha g_c}\right) = 0 \longrightarrow ⓐ$$

Now; $P_2 = 50 + 14.7 = 64.7 \text{ psia} \times 144 = 9316.8 \text{ psf}$

$P_1 = 14.7 \text{ psia} \times 144 = 2116.8 \text{ psf}$

$$\nu = 1/\rho_{oil} = 1/53.5$$

$$\nu(P_2 - P_1) = \cancel{64}\,\frac{(9316.8 - 2116.8)\ \text{lbf}}{\text{ft}^2}\ \bigg|\ \frac{\text{ft}^3}{53.5\ \text{lbm}}$$

$$\nu(P_2 - P_1) = 135\ \text{ft lbf/lbm} \longrightarrow ⓑ$$

$$Z_2 - Z_1 = 20\ ft\ lb_f / lb_m \longrightarrow ⓒ$$

$$\Sigma F = F_1 + F_2$$

F_1 = friction losses due to fittings

F_2 = " " " " expansion & contraction

$$F_1 = \frac{2f(L+Le)V^2}{g_c D}$$

velocity "V":-

$$Q = V \cdot A \quad ; \quad Q = V \cdot \frac{\pi}{4} D^2$$

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 50\ \cancel{gal}}{\cancel{min}} \left| \frac{}{3.14} \right| \frac{\cancel{min}}{(0.256)^2\ \cancel{ft^2}} \left| \frac{1\ ft^3}{7.48\ \cancel{gal} \times 60\ sec} \right.$$

$$\boxed{V = 2.17\ ft/sec}$$

friction factor 'f':-

$$\mu = 15cp = \frac{15 \times 2.42}{3600} = 10.1 \times 10^{-3}\ \frac{lb_m}{ft.sec}$$

$$N_{Re} = \frac{D\rho V}{\mu}$$

$$N_{Re} = \frac{(0.256)(53.5)(2.17)}{10.1 \times 10^{-3}}$$

$$\rightarrow N_{Re} = 2947$$

$$\rightarrow \varepsilon / D = \frac{0.00015}{0.256} = 0.00058$$ (based on commercial steel)

Now from Graph; 14-1 page #482. (4th Ed.)

$$\boxed{f = 0.011}$$

L+Le:- page 484 Table 1

$$\frac{L_e}{D} = (5)(32) \quad ; \quad L_e = (5)(32)(0.256)$$

$$L_e = 40.96\ ft$$

$$\boxed{L+L_e = 150 + 40.96 = 191\ ft}$$

$$F_1 = \frac{(2)(0.011)(191)(2.17)^2}{(32.174)(0.256)}$$

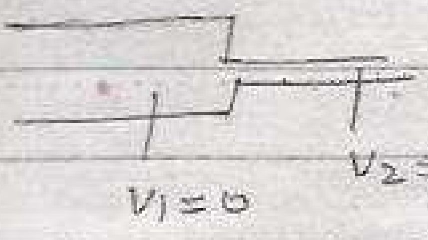
$$\boxed{F_1 = 2.40\ ft\ lb_f/lb_m}$$

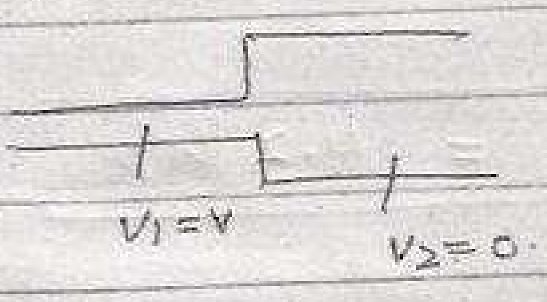
$$F_2 = F_c + F_e$$

or

$$F_2 = \frac{K_c V_2^2}{2\alpha g_c} + \frac{(V_1 - V_2)^2}{2\alpha g_c}$$

$\because$ $\alpha = 1$ $\quad N_{Re} = 2947$





$$K_c = 0.4\left(1.25 - \frac{A_2}{A_1}\right)$$

$\because$ $A_2 <<<< A_1$ i.e $A_2 \approx 0$.

so $\boxed{K_c = 0.4 \times 1.25 = 0.5}$

$$F_2 = \frac{(0.5)(2.17)^2}{(2)(32.174)} + \frac{(2.17-0)^2}{(2)(32.174)}$$

$\boxed{F_2 = 0.11\ ft\ lb_f/lb_m}$ → Just Taklus

prog.

$$\boxed{\Sigma F = 2.40 + 0.11 = 2.51 \text{ ft lbf/lbm}}$$

$\longrightarrow$ ⓓ

Eq ② $\Rightarrow$

$$W_o = 20 + 135 + 0 + 2.51$$

$$\boxed{W_o = 157.51 \text{ ft lbf/lbm}}$$

$$\dot{m} = Q \times \rho_{oil} = \frac{50 \text{ gal}}{\cancel{\text{min}}} \left| \frac{53.5 \text{ lbm}}{\text{ft}^3} \right| \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \left| \frac{1 \text{ min}}{60 \text{ sec}} \right.$$

$$\boxed{\dot{m} = 5.96 \text{ lbm/sec}}$$

Eq ① $\Rightarrow$

$$P = \frac{157.51 \text{ ft}\cancel{\text{lbf}}}{\cancel{\text{lbm}} \times 0.4} \left| \frac{5.96 \cancel{\text{lbm}}}{\cancel{\text{sec}}} \right| \frac{1 \text{ hp} \cdot \cancel{\text{sec}}}{550 \cancel{\text{ft lbf}}}$$

$$\boxed{P = 4.3 \text{ hp}}$$

PROBLEM 2: Hydrogen at a temperature of 20°C and an absolute pressure of 1380 kPa enters a compressor where the absolute pressure is increased to 4140 kPa. If the mechanical efficiency of the compressor is 55 percent on the basis of an isothermal and reversible operation, calculate the pounds of hydrogen that can be handled per minute when the power supplied to the pump is 224 kW. Kinetic-energy effects can be neglected.

- DATA:-

Temp. of H_2 = $T = 20°C$

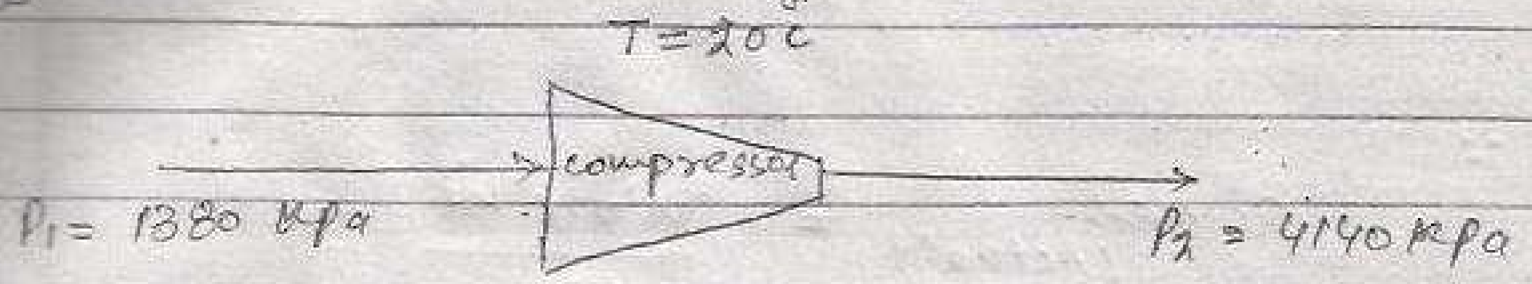
$P_1 = 1380$ kpa. ; $P_2 = 4140$ kpa

$\eta = 0.55$ Power = 224 kW

T.F.0

Hydrogen handled per unit = $\dot{m}$ = ?

solu:-



Total mechanical energy balance Eq is;

$$W_0 = \frac{g}{g_c}(z_2 - z_1) + \int_1^2 v\,dp + \left(\frac{V_2^2}{2\alpha g_c} - \frac{V_1^2}{2\alpha g_c}\right) + \Sigma F \longrightarrow ①$$

$\because$ $z_1 = z_2 \Rightarrow z_2 - z_1 = 0. \rightarrow ⓐ$

$\because$ operation is isothermal and reversible

$$\Rightarrow \left(\frac{V_2^2}{2\alpha g_c} - \frac{V_1^2}{2\alpha g_c}\right) = 0. \longrightarrow ⓑ$$

also;

$$\Sigma F = 0 \longrightarrow ⓒ$$

$\underbrace{\int_1^2 v\,dp}$:-

$\because$ $PV = nRT$

$PV = \frac{m}{M} RT$

If $m = 1$ then $V = v$

$Pv = \frac{RT}{M}$

or $v = \frac{RT}{PM}$

$$\Rightarrow \int_1^2 \frac{RT}{PM}\,dp$$

$$= \frac{RT}{M}\int_{P_1}^{P_2} \frac{1}{P}\,dP$$

$$= \frac{RT}{M}\ln\left(\frac{P_2}{P_1}\right)$$

$\Rightarrow T = 20^\circ + 273 = 293K$

$R = 8.314$ J/mol. K

$M = 2$ gm/mol.

$$= \frac{8.314\,J}{mol.\,\cancel{K}} \left| \frac{293\,\cancel{K}}{2\,gm} \right| mol \left| \ln\frac{(4140)}{(1380)} \right| \frac{1000\,gm}{1\,Kg} \left| \frac{1\,KJ}{1000\,J}\right.$$

$$= 1338\ KJ/Kg \longrightarrow ⓓ$$

Eq. ① $\Rightarrow$

$$W_0 = 1338\ \text{kJ/kg}$$

$\because$

$$\text{power}_{(\text{Actual})} = \frac{W_0 \times \dot{m}}{\eta}$$

or

$$\dot{m} = \frac{\text{power} \times \eta}{W_0}$$

$$\dot{m} = \frac{224\ \cancel{\text{kW}}}{1338\ \cancel{\text{kJ}}} \left| \frac{\cancel{\text{kg}}}{\ } \right| \frac{0.55}{\ } \left| \frac{\ }{1\ \cancel{\text{kW}}} \right| \frac{\cancel{\text{kJ}}}{\cancel{\text{sec}}} \left| \frac{2.2046\ \text{lbm}}{1\ \cancel{\text{kg}}} \right| \frac{60}{1}$$

$$\boxed{\dot{m} = 12.2\ \text{lbm/min}}$$

$\longleftrightarrow$

PROBLEM 3: For the conditions indicated in Prob. 2, determine the mechanical efficiency of the pump on the basis of adiabatic and reversible operation. A single-stage compressor is used, and the ratio of heat capacity at constant pressure to the heat capacity at constant volume for the hydrogen may be assumed to be 1.4.

$\eta_{ad} = ?$ $\quad K = 1.4$ —— $C_p/C_v = K$

solⁿ:

$$\because \eta = \frac{\text{theoretical power req.}}{\text{Actual power req.}}$$

~~From the conditions of Pb #02 :-~~

$$\eta_{iso} = \frac{\text{theoretical power}}{\text{Actual power}} = 0.55$$

Also for an iso-thermal reversible process;

$$\text{Theoratical power}_{(iso)} = P_1V_1 \ln(P_2/P_1)$$

so;

$$\text{Actual power} = \frac{P_1V_1 \ln(P_2/P_1)}{0.55} \longrightarrow ①$$

Now for an adiabatic reversible process;

$$\text{Theoratical power} = \frac{K}{K-1} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} - 1\right] \longrightarrow ②$$

so Eq ① & ② ⇒

$$\eta_{ad} = \frac{K}{K-1} \cancel{P_1V_1}\left[\left(\frac{P_2}{P_1}\right)^{K-1/K} - 1\right] \times \frac{0.55}{\cancel{P_1V_1} \ln(P_2/P_1)}$$

$$\eta_{ad} = \frac{1.4}{0.4}\left[\left(\frac{4140}{1380}\right)^{0.4/1.4} - 1\right] \times \frac{0.55}{\ln(4140/1380)}$$

$$\boxed{\eta_{ad} = 0.646}$$

$$\boxed{\text{mechanical eff. of the pump} = 64.6\%}$$

PROBLEM 4: A steel pipe of 4-in. nominal diameter is to be used as a high-pressure steam line. The pipe is butt-welded, and its schedule number is 40. Estimate the maximum steam pressure that can be used safely in this pipe.

DATA:-

Nominal diameter = 4"

schedule # 40 butt-welded

safe working pr = P_s = ?

soln:-

$$\because \text{ schedule \#} = 1000 \cdot \frac{P_s}{S_s} \longrightarrow ①$$

For butt welded;

safe working stress = S_s = 6500 psi

$$① \Rightarrow \quad P_s = \frac{(\text{schedule \#})(S_s)}{1000}$$

$$P_s = \frac{(40)(6500)}{1000} = 260$$

$$\boxed{P_s = 260 \text{ psi}}$$

also

$$P_s = \frac{2 S_s t_m}{D_m} \longrightarrow ②$$

t_m = min. wall thickness = $\frac{D_o - D_i}{2}$

for 4" nominal pipe dia.

$D_i = 4.026"$ & $D_o = 4.50"$

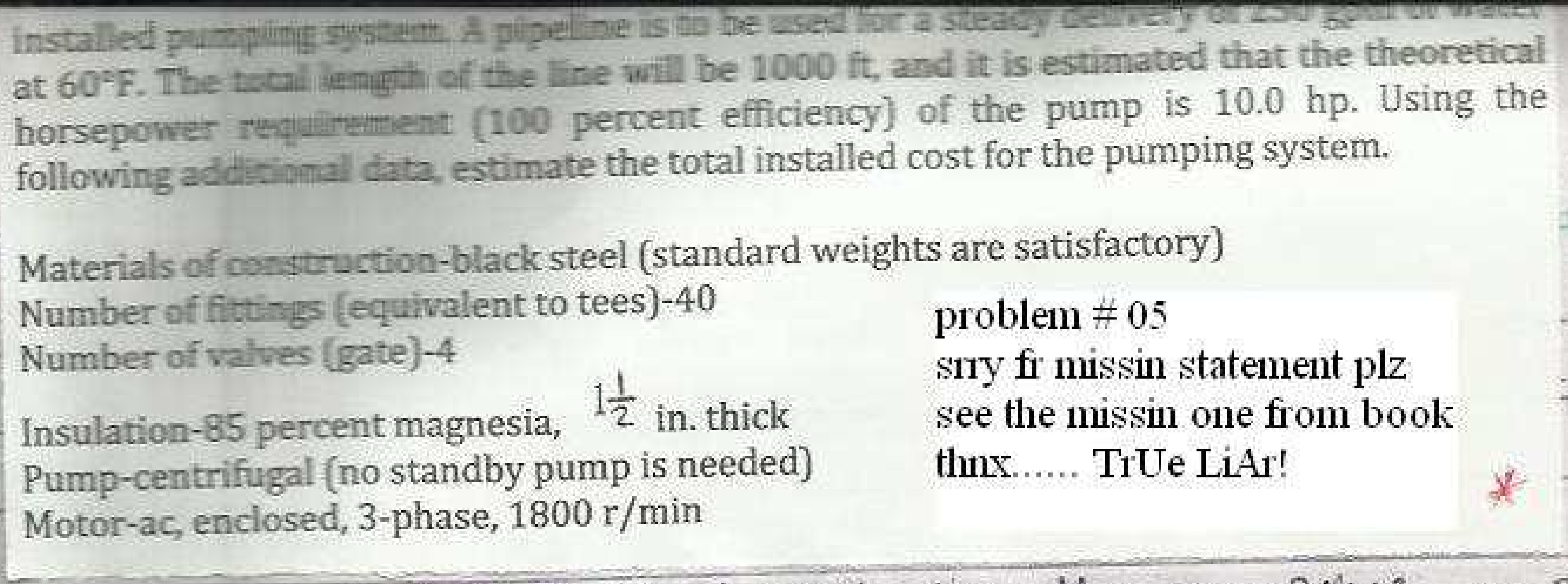
$$t_m = \frac{4.50 - 4.026}{2} = 0.237$$

$$\boxed{t_m = 0.237}$$

$$D_m = \frac{D_o + D_o}{2} = \frac{4.50 + 4.026}{2} = 4.263"$$

$$② \Rightarrow \quad P_s = \frac{(2)(6500)(0.237)}{(4.263)} = 722.7$$

$$\boxed{P_s = 723 \text{ psi}}$$ $\longrightarrow$ so the max. steam pr. that can be used safely = 723 psi



installed pumping system. A pipeline is to be used for a steady delivery of 250 gal/min of water at 60°F. The total length of the line will be 1000 ft, and it is estimated that the theoretical horsepower requirement (100 percent efficiency) of the pump is 10.0 hp. Using the following additional data, estimate the total installed cost for the pumping system.

Materials of construction-black steel (standard weights are satisfactory)
Number of fittings (equivalent to tees)-40
Number of valves (gate)-4
Insulation-85 percent magnesia, $1\frac{1}{2}$ in. thick
Pump-centrifugal (no standby pump is needed)
Motor-ac, enclosed, 3-phase, 1800 r/min

problem # 05
srry fr missin statement plz
see the missin one from book
thnx...... TrUe LiAr!

T.F-9:- Total installed cost for the pumping system = ?

DATA:-

water flow rate = $q_f = 250$ gal/min.

Temp. of water = T = 60°F

Length of line = L = 1000 ft

theoratical power = 10 hp.

Materials of construction → Black steel

\# of fittings (tees) → 40.

\# of valves (gate) → 4.

Insulation - 85 percent magnesia → 1.5" thick

pump → centrifugal (No stand by)

motor → AC, Enclosed, 3-phase, 1800 r/min.

soln:-

$$q_f = \frac{250 \text{ gal}}{\text{min}} \left| \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right| \frac{1 \text{ min}}{60 \text{ sec}} = 0.56$$

$$\boxed{q_f = 0.56 \text{ ft}^3/\text{sec}}$$

$$\boxed{\dot{m}_f = 0.56 \times 62.3 = 35 \text{ lbm/sec}}$$

(at 60°F ($\rho_{H_2O} = 62.3$)

$\mu_{H_2O} = 1.12$ cp at 60°F

Assume Turbulent flow; so then by Eq;

$$D_{i,opt} = 3.9 q_f^{0.45} \rho^{0.13} = (3.9)(0.56)^{0.45}(62.3)^{0.13}$$

$$\boxed{D_{i,opt} = 5.14''}$$

so use 5in nominal steel pipe, schedule 40

$$I.D = D_i = 5.047''$$

From Fig 14-2 page # 498 (4th Ed.)

$$N_{Re} = \frac{380 \cdot \rho \cdot q_{fm}}{D_i \cdot \mu_c}$$

where; $q_{fm} = q_f \times 60 = 33.6 \ ft^3/min.$

$$N_{Re} = \frac{(380)(62.3)(33.6)}{(5.047)(1.12)}$$

$$\boxed{N_{Re} = 140,720}$$

So flow is turbulent; ✓ (check)

To use ~~Eq.~~ Fig 12-20 to obtain cost of pump and motor need to develop equivalent pr. drop across pump:

$$\Delta p = 10hp \left| \frac{550 \ ft \ lb_f}{hp \ sec} \right| \frac{62.3 \ lb_m}{ft^3} \left| \frac{sec}{35 \ lb_m} \right| \frac{1 \ ft^2}{144 \ in^2}$$

$$\boxed{\Delta p = 68 \ psi}$$

$$\boxed{\text{capacity factor; } (gal/min) \times psi = 250 \times 68 = 17,000}$$

① → From Fig 12-20 cost of pump & motor = \$3600 (5th Ed.) (upto 150 psi above) (AVS) ⟹ ⓐ

From Fig 12-4 (5th Ed.) cost of 5.047" dia carbon steel pipe = \$7/ft

② so; pipe cost = (7)(1000) = $7000 → ⓑ

→ From Fig 14-4 (4th Ed.) the cost of fittings indexed to Jan 1, 2002 = $70/tee

so; ③ Fitting cost (tees) = (70)(40) = $2800 → ⓒ

→ From Fig 14-28 (4th Ed.) the cost of a gate valve, carbon steel, flanged, with a 125 lb rating. (Jan 1, 2002) = $310/valve

④ valve cost = (310)(4) = $1240 → ⓓ

→ From Fig 14-32 (4th Ed), cost of 5.047" 85% magnesia is = $4.8/ft

⑤ insulation cost = (4.8)(1000) = $4800. → ⓔ

→ From Table 6 page # 171 (4th Ed.) installed cost for the pump is 25 to 60% of the purchased equipment cost. Use an average of 40% or a factor of 1.4

Installed cost = (3600 + 7000 + 2800 + 1240 + 4800)(1.4)

$\boxed{\text{Installation cost} = \$27,216}$

PROBLEM 6: A centrifugal pump delivers 100 gpm of water at 60°F when the impeller speed is 1750 r/min and the pressure drop across the pump is 20 psi. If the speed is reduced to 1150 r/min, estimate the rate of water delivery and the developed head in feet if the pump operation is ideal.

DATA:-

$q_1 = 100$ gpm

$N_1 = 1750$ rev/min

pr. drop across pump $= \Delta p = 20$ psi

$N_2 = 1150$ rev/min

T.F.O.:-

$q_2 = ?$; develop head $= h_2 = ?$

Soln

$$\because \quad \frac{N_1}{N_2} = \frac{q_1}{q_2}$$

$$\Rightarrow \quad q_2 = \frac{N_2 \times q_1}{N_1} = \frac{(1150)(100\,\text{gpm})}{(1750)}$$

$$\boxed{q_2 = 65.7\ \text{gpm}} \longrightarrow (a)$$

$$\because \quad \Delta p = \frac{\rho g h}{g_c}$$

$$\text{or} \quad h_1 = \frac{\Delta p \times 144}{\rho} \leftarrow \quad (\text{lb}_f/\text{ft}^2)$$

$$h_1 = \frac{(20)(144)}{62.4}$$

$$\boxed{h_1 = 46.2\ \text{ft}}$$

Now;

$$\frac{h_1}{h_2} = \frac{N_1^2}{N_2^2}$$

$$h_2 = \left(\frac{N_2}{N_1}\right)^2 \times h_1 = \frac{(1150)^2}{(1750)^2} \times 46.2$$

$$\boxed{h_2 = 19.95\ \text{ft}} \longrightarrow (b)$$

TrUe LiAr!!

PROBLEM 7: A two-stage steam jet is to be used on a large vacuum system. It is estimated that 9 kg of air must be removed from the system per hour. The leaving vapors will contain water vapor at a pressure equivalent to the equilibrium vapor pressure of water at 15°C. If a suction pressure of 2.0 in. Hg absolute is to be maintained by the jet, estimate the pounds of steam required per hour to operate the jet.

- DATA:-

two stage steam Jet;

Air to be removed = 9 kg/hrs = 20 lb/hrs

ps. of H_2O vapors leaving = Eq. vap. ps of H_2O at 15°C

suction ps = 2 in Hg absolute

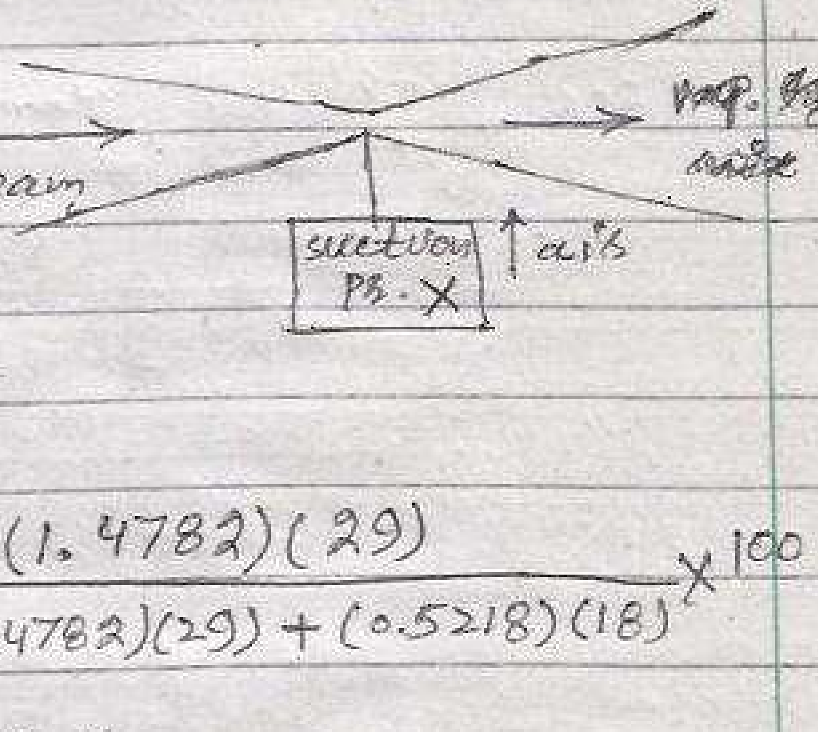
T.F.O:-

pounds of steam Req. / per hour = $\dot{m}_s$ = ? for ops.

soln:-

vap. ps. of H_2O at 15°C = $P_{(H_2O, vap)}$ = 0.5218 in Hg (From perry)

suction ps. of dry air = suction ps − $P_{(H_2O,vap)}$



= 2 − 0.5218
= 1.4782 in Hg.

Now;

$$\text{\% age dry air} = \frac{(1.4782)(29)}{(1.4782)(29) + (0.5218)(18)} \times 100$$

= 82 %

Now;

If air is 82 % then mix is = 100

" " " 1 " " " = $\frac{100}{82}$

" " " 20 lb/hrs " " " = $\frac{100 \times 20}{82}$ lb/hrs.

$\boxed{\text{Total wt of mix} = 24.4 \text{ lb/hrs}}$

From Table 03 (Page # 523)

wt % of dry air	Steam consumptio (2 stage)
100	50
82	45.2
70	42

calculations:

$$\begin{tikzcd} 100 \arrow[rr, bend left, leftrightarrow, "30"] & & 70 \arrow[ll, "(12)", "82"'] \end{tikzcd}$$

$$\begin{tikzcd} 50 \arrow[rr, bend right, leftrightarrow, "8"'] & & 42 \end{tikzcd}$$

→ $\boxed{45.2}$ by interpolat
at 82

So for a capacity of 10lb/hr at 82% steam consumption = 45.2 lb/hr

For 10lb/hr, steam req. = 45.2 lb/hr

" 1 " " = $\frac{45.2}{10}$

" 24.4 " " = $\frac{(45.2)(24.4)}{10}$

= 110 lb/hr

so,

$\boxed{\text{lbs of steam req. to opr.} = m_s = 110\ \text{lb/hr}}$

PROBLEM 8: The rate of flow of a liquid mixture is to be measured continuously. The flow rate will be approximately 40 gpm, and rates as low as 30 gpm or as high as 50 gpm can be expected. An orifice meter, a rotameter, and a venturi meter are available. On the basis of the following additional information, would you recommend installation of the orifice meter, the venturi meter, or the rotameter? Give reasons for your choice.

- Density of liquid = 58 lb/ft³
- Viscosity of liquid = 1.2 centipoises
- Diameter of venturi throat = 1 in.
- Upstream diameter of venturi opening = 2 in.
- Manometers connected across the venturi and the orifice contain a nonmiscible liquid (sp gr = 1.56) in contact with the liquid mixture
- The maximum possible reading on these manometers is 15 in.
- Orifice is square-edged with throat taps
- Diameter of orifice opening = 1 in.
- Diameter of upstream chamber for orifice meter = 3 in.
- Calibration curve for rotameter is for water at 60°F and gives the following

values:

Rotameter reading	*Flow rate, ft³/min*
2.0	2.0
4.0	4.0
6.0	6.0
8.0	8.0

- The density of the plummet in the rotameter is 497 lb/ft³.

- DATA:-

Density of liquid = ρ_L = 58 lb/ft³

viscosity of liquid = 1.2 cp = μ

dia of venturi throat = 1″

upstream dia of venturi opening = 2″

sp. gr of liquid in manometer = 1.56

max. possible reading on manometer = 15″

dia of orifice opening = 1″

upstream dia of orifice meter = 3″

Density of the plummet = 497 lb/ft³

solution:-

$\rho_L = 58$ lb/ft³

$\rho_{mL} = 1.56 \times 62.4 = 97.3$ lb/ft³

flow rate = q_f = 40 gpm (Approx.)

min flow rate = $q_{f,min}$ = 30 gpm.

max " " = $q_{f,max}$ = 50 gpm.

VENTURIMETER:-

for venturimeter;

$$q_f = C_d S_c \sqrt{\frac{2 g_c v (P_1 - P_2)}{1 - (S_c/S_1)^2}} \longrightarrow ①$$

$\Rightarrow$

$$d_c = 1'' = \frac{1}{12} ft \quad ; \quad S_c = \pi/_4 \, d_c^2$$

$$S_c = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 \Rightarrow \boxed{S_c = 5.45 \times 10^{-3} ft^2}$$

$$d_1 = 2'' = \left(\frac{2}{12}\right) ft \quad ; \quad S_1 = \pi/_4 \, d_1^2$$

$$S_1 = \frac{\pi}{4}\left(\frac{2}{12}\right)^2 \Rightarrow \boxed{S_1 = 0.0218 ft^2}$$

velocity at throat:-

$$V_c = \frac{q_f}{S_c} = \frac{40 \cancel{gal}}{\cancel{min}} \left| \frac{1 ft^3}{5.45 \times 10^{-3} \cancel{ft^2}} \right| \frac{}{7.48 \cancel{gal}} \left| \frac{1 \cancel{min}}{60 sec} \right.$$

$$\boxed{V_c = \frac{q_f}{S_c} = 16.4 \; ft/sec}$$

* calculate "C_d":-

$$\mu = 1.2 cp \times 2.42 = \frac{2.904 \, lbm}{hr \cdot ft} \left| \frac{\cancel{hr}}{3600 sec} \right. = \frac{8.1 \times 10^{-4} \, lb}{ft \cdot sec}$$

upstream $\longrightarrow V_{up} = 4.1 \, ft/sec$

$$\because N_{Re} = \frac{D \rho u}{\mu} \Rightarrow \frac{d_1 \rho_L V_{up}}{\mu} = \left(\frac{2}{12}\right)\left(\frac{58 \times \overset{4.1}{\cancel{}}}{8.1 \times 10^{-4}}\right)$$

N_{Re} ~~1,195,720~~ $\quad N_{Re} = 48930$

$\because$ N_{Re} is greater than 5000 at upstream section so; $\boxed{C_d = 0.98}$

EV ①

$$\frac{q_f}{S_c} = \frac{c_d \cdot \sqrt{2 g_c} \cdot \sqrt{v(P_1 - P_2)}}{\sqrt{1 - (S_c/S_1)^2}}$$

$$16.4 = \frac{0.98 \sqrt{2 \times 32.174} \cdot \sqrt{v(P_1 - P_2)}}{\sqrt{1 - (5.45 \times 10^{-3}/0.0218)^2}}$$

$$16.4 = 7.6 \sqrt{v(P_1 - P_2)} \text{ ——— } \textcircled{*}$$

$$\sqrt{v(P_1 - P_2)} = 2.15$$

$$v(P_1 - P_2) = 4.64 \longrightarrow \text{ⓐ}$$

$$\because \quad \Delta P = (\rho_{mL} - \rho_L)\frac{\cancel{g}}{\cancel{g_c}} \cdot \Delta h \quad \cancel{\longrightarrow} \text{ⓐ}$$

or

$$\frac{\Delta P}{\rho_L} = \frac{(\rho_{mL} - \rho_L)}{\rho_L} \cdot \Delta h$$

$$\Rightarrow \quad v \Delta p = \left(\frac{\rho_{mL} - \rho_L}{\rho_L}\right) \cdot \Delta h \longrightarrow \text{ⓑ} \qquad \because v = 1/\rho_L$$

ⓐ & ⓑ ⇒ $\qquad \Delta p = \text{lbf}/\text{ft}^2$

$$4.64 = \left(\frac{\rho_{mL} - \rho_L}{\rho_L}\right) \cdot \Delta h$$

$$\Delta h = \frac{(4.64)(58)}{(97.3 - 58)} \Rightarrow 6.85\,\text{ft} = 82''$$

$$\boxed{\Delta h = 82 \text{ in}}$$

∵ the maximum possible reading on manometer is 15" so venturimeter will not suitable for this flow rate.

also; at $q_{fm} = 30$ gpm ; $q_{fm}/S_c = V = 12.3$ ft/sec

$\textcircled{*} \Rightarrow \quad 12.3 = 7.6 \sqrt{v(P_1 - P_2)}$; $\sqrt{v(P_1 - P_2)} = 2.6$

ⓐ & ⓑ ⇒ $\boxed{\Delta h = 46.4''}$ readings in case of minimum flow;

ORIFICEMETER:- also for orificemeter;

$$q_f = C_d S_c \sqrt{\frac{2g_c v(P_1 - P_c)}{1-(S_c/S_1)^2}} \longrightarrow ②$$

=>

$d_0 = d_c = 1'' = (\frac{1}{12})\,ft$; $S_c = \pi/4\, d_c^2$

$S_c = (\frac{\pi}{4})(\frac{1}{12})^2 \Rightarrow$ $\boxed{S_c = 5.45 \times 10^{-3}\, ft^2}$

$d_1 = 3'' = (\frac{3}{12})\,ft$; $S_1 = \pi/4\, d_1^2$

$S_1 = (\frac{\pi}{4})(\frac{3}{12})^2$; $\boxed{S_1 = 0.049\, ft^2}$

velocity at throat:-

$$V_0 = \frac{q_f}{S_c} = \frac{40\, gal}{min} \Big| \frac{1\, min}{5.45 \times 10^{-3}\, ft^2} \Big| \frac{1\, ft^3}{7.48\, gal} \Big| \frac{}{60\, sec}$$

$$\boxed{V_0 = \frac{q_f}{s_c} = 16.4\, ft/sec} \longrightarrow \text{"}V_0\text{"}$$

calculate "Cd":-

$$N_{Re} = \frac{d_0 V_0 \rho}{\mu} = \left(\frac{1}{12}\right)\left(\frac{16.4 \times 58}{8.14 \times 10^{-4}}\right)$$

$N_{Re} = 97,400$

also; $\frac{d_0}{d_1} = 1/3 = 0.33$

From Fig; 14-55; $\boxed{C_d = 0.61}$

Eq ② =>

$$\frac{q_f}{S_c} = \frac{C_d \sqrt{2g_c} \cdot \sqrt{v(P_1 - P_2)}}{\sqrt{1-(S_c/S_1)^2}}$$

$$16.4 = \frac{0.61\sqrt{2\times 32.174}\cdot\sqrt{\nu(P_1-P_2)}}{\sqrt{1-\left(5.45\times10^{-3}/0.049\right)^2}}$$

$$16.4 = 5.\sqrt{\nu(P_1-P_2)}$$

$$\nu(P_1-P_2) = 10.75$$

or $$10.75 = \left(\frac{\rho_{mL}-\rho_L}{\rho_L}\right)\cdot\Delta h$$

$$\Delta h = 15.86\,ft \implies \boxed{\Delta h = 190.4''}$$

Hence orifice meter is not suitable as well;

ROTAMETER:-

For rotameter;

$$q_f = Cd.Sc\sqrt{\frac{V_p\,2g\,(\rho_p-\rho)\,\nu}{S_p\left[1-(S_c/S_1)^2\right]}} \longrightarrow \textcircled{3}$$

For H_2O: at 60°F

$$\boxed{\rho_w = 62.3\ lbm/ft^3}$$

$$\boxed{\nu_w = 1/\rho_w = 0.0160}$$

For Given liquid;

$$\boxed{\begin{aligned}\rho_L &= 58\ lbm/ft^3\\ \nu_L &= 0.0172\ ft^3/lbm\end{aligned}}$$

Eq ③ ⟹ (H_2O) case.

$$q_{f_w} = Cd.Sc\sqrt{\frac{V_p\,2g}{S_p\left[1-(S_c/S_1)^2\right]}}\cdot\sqrt{\frac{(407-62.3)}{62.3}} \longrightarrow \textcircled{a}$$

Eq ② ⟹ Liquid case;

$$q_{f_L} = Cd.Sc\sqrt{\frac{V_p\,2g}{S_p\left[1-(S_c/S_1)^2\right]}}\cdot\sqrt{\frac{407-58}{58}} \longrightarrow \textcircled{b}$$

ⓑ ÷ ⓐ

$$\frac{v_{f_L}}{v_{f_w}} = \sqrt{\frac{497-58}{58}} \times \sqrt{\frac{62.3}{497-62.3}}$$

$$\frac{v_{f_L}}{v_{f_w}} = \sqrt{\frac{439 \times 62.3}{58 \times 434.7}}$$

$$\boxed{v_{f_L} = 1.042\, v_{f_w}} \longrightarrow \circledast$$

when $v_{f_w} = 2\ ft^3/min$

$$\circledast \Rightarrow v_{f_L} = \frac{2.084\ ft^3}{min} \left| \frac{7.48\ gal}{1\ ft^3} \right. \Rightarrow 15.58$$

$$\boxed{v_{f_L} = 15.58\ gal/min}$$

similarly;

$v_{f_w} = 4\ ft^3/min$; $v_{f_L} = 31.17\ gal/min$

$v_{f_w} = 6$ " ; $v_{f_L} = 46.8\ gal/min$

$v_{f_w} = 8$ " ; $v_{f_L} = 62.35\ gal/min$

so;

rota meter	water (ft^3/min)	Liquid (gpm)
2.0	2	15.58
→ 4.0	4	31.17
→ 6.0	6	46.8
→ 8.0	8	62.35

~~As flow rate is btw 30 & 50 gpm, so~~

~~rotameter is suitable for this duty.~~

∴ the maximum flow rate i.e 52.35 gpm is in the range of 6" to 8" so. rotameter would be satisfactory.

PROBLEM 9: A spherical carbon-steel storage tank has an inside diameter of 30 ft. All joints are butt-welded with backing strip. If the tank is to be used at a working pressure of 30 psig and a temperature of 80°F, estimate the necessary wall thickness. No corrosion allowance is necessary.

- DATA:-

Inside dia of vessel = D_i = 30 ft

Joints are butt-welded with backing strip

working ps. = P_i = 30 psig

working temp. = T = 80°F

corrosion allowance = $C_c = 0$

T.F.O:- minimum wall thickness = t = ?

Soln:- From Table 4 page # 537

for spherical shells:-

$$t = \frac{P \times r_i}{2SE_j - 0.2P} + C_c \longrightarrow ①$$

Now;

P = maximum allowable internal ps., psig

S = max. allowable working stress, psi

E_j = efficiency of joints expressed as fraction

C_c = corrosion allowance

r_i = inside radius of shell, in

so; 'P' will be 10% above the normal working ps.

i.e.. $P = (1.1)(30) = 33$ psig

$S = 13{,}700$ psi (Table 4 page # 538 for carbon steel)

$E_j = 0.85$ (Assume butt weld, spot examined)

$r_i = d_i/2 = \frac{30 \times 12}{2} = 180$ in.

$C_c = 0$

so Eq ① ⟹ $t = \frac{33 \times 180}{(2)(13{,}700)(0.85) - 0.2(33)}$

$\boxed{t = 0.255''}$ minimum wall thickness

PROBLEM 10: Estimate the cost of the steel for the spherical storage tank in the preceding problem if the steel sheet costs $0.25 per pound. On the basis of the data presented in Fig. 14-56, estimate the fraction of the purchased cost of the tank that is due to the cost for the steel.

DATA:

steel sheet costs = 0.25 $/lb.

$Dia_i = 30ft$; $r_i = 15 ft$; $t_m = 0.255''$

T.B.S

cost of steel for the spherical storage tank in the preceding pb. = ?

Fraction of the purchased cost of tank due to cost of steel = ?

soln:

with the small / min. wall thickness relative to 'r_i', it is ~~sa~~ satisfactory to obtain the volume occupied by the wall by multiplying the inside area with the min. wall thickness.

$$A_{sphere} = 4\pi r_i^2 = (4)(3.14)(15)^2$$

$$\boxed{A_{sphere} = 2826\ ft^2}$$

From Table #10 (page 883)

sp. gravity of steel = 7.83

$$\boxed{\rho_{steel} = 7.83 \times 62.4 = 489\ lbm/ft^3}$$

(weight of steel)

$$\text{cost of steel} = \underbrace{(A_{sphere})(t_m)(\rho_{steel})}\ \$0.25/lb$$

$W = V \times \rho$

$$= (2826)\left(\frac{0.255}{12}\right)(489)(\$0.25/lb)$$

$$\boxed{\text{cost of steel} = 7341\ \$.} \longrightarrow ⓐ$$

$V_{sphere} = 4/3 \pi r_i^3 = (\frac{4}{3})(3.14)(15)^3 = 14130 ft^3$

$V_{sphere} = \frac{14130 ft^3 \mid 7.48 gal}{\mid 1 ft^3}$

$\boxed{V_{sphere} = 105692 \text{ gal}}$

From Fig 14-56. page # 539

For 30-psig carbon steel tank (spherical)

purchased cost. = 2×10^5 \$. {at a capacity 1.06×10^5}

so;

the fraction of cost of tank due to cost of steel = $\frac{\text{cost of steel}}{\text{purchased cost}}$

$= 7341 / 2 \times 10^5$

$= 0.037 = 3.7 \%$

PROBLEM 11: Filtration tests with a given slurry have indicated that the specific cake resistance α is 157 h²/lb. The fluid viscosity is 2.5 lb/(h)(ft), and 3 lb of dry cake is formed per cubic foot of filtrate. The cake may be assumed to be noncompressible, and the resistance of the filter medium may be neglected. If the unit is operated at a constant pressure drop of 5 psi, what is the total filtering area required to deliver 30 ft3 of filtrate in $\frac{1}{2}$ h?

DATA:

specific cake resistance = $\alpha = 157 \ h^2/lb$

fluid viscosity = $\mu = 2.5$ lb/hr.ft

lb of dry cake / ~~lb~~ ft^3 of filtrate = $w = 3 \ lb/ft^3$

cake is non-compressible = $s = 0$

Filter medium resistance negligible = $R_F = 0$

pr. drop = $\Delta P = 5$ psi ; filtrate delivered = $30 ft^3$

filtering time = $\theta = 1/2$ h

T.F.D:- filtering area req. = A = ?

Now; for Batch filters;

$$V^2 + 2A\cancel{V_F}^{\,0}V = \frac{2 \cdot A^2 \cdot \Delta P^{\cancel{1-S}\,0}}{\alpha' w \mu}\theta$$

so;

$$V^2 = \frac{2A^2\Delta P}{\alpha w \mu}\theta$$

$$A = \left[\frac{V^2 \cdot \alpha \cdot w \cdot \mu}{2 \cdot \Delta P \cdot \theta}\right]^{1/2}$$

$$A = \left[\frac{(30)^2(157)(3)(2.5)}{(2)\underbrace{(5 \times 144)}(0.5)}\right]^{1/2}$$

$(\Delta P = 5 \times 144 \text{ psf})$

$$\boxed{A = 38.4\ ft^2}$$

PROBLEM 12: A rotary filter with a total filtering area of 8 ft2 has been found to deliver 10 ft3 of filtrate per minute when operating at the following conditions:

Fraction of filtering area submerged in the slurry = 0.2

r/min = 2

Pressure drop = 20 psi

Another rotary filter is to be designed to handle the same slurry mixture. This unit will deliver 100 ft3 of filtrate per minute and will operate at a pressure drop of 15 psi and a revolving speed of 1.5 r/min. If the fraction of filtering area submerged in the slurry is 0.2, estimate the total filtering area required for the new unit. In both cases, it may be assumed that no solids pass through the filter cloth, the cake is noncompressible, and the resistance of the filtering medium is negligible.

DATA;

Small filter

$A_D = 8 ft^2$

$V_R N_R = 10 ft^3/min$

$\psi_f = 0.2$

$N_R = 2$ rev/min

$\Delta P = 20 psi$

$= 20 \times 144 psf$

Large filter

$A_D = ?$ (T.F.D)

$V_R N_R = 100 ft^3/min$

$\Delta P = 15 psi$

$= 15 \times 144 psf$

$\psi_f = 0.2$

$N_R = 1.5$ rev/min

cake is non-compressible => $s = 0$

Filter_medium resistance => $R_F = 0$

soln

vol of filtrate per unit time $= V_R N_R = A_D \sqrt{\dfrac{2 \psi_f \Delta P . N_R}{\alpha w \mu N_R}} \longrightarrow ①$

so;

small filter :- ① =>

$$10 = 8 \sqrt{\frac{(2)(0.2)(20 \times 144)(2)}{\alpha w \mu}} \longrightarrow ②$$

Large filter :- ① =>

$$100 = A_D \sqrt{\frac{(2)(0.2)(15 \times 144)(1.5)}{\alpha w \mu}} \longrightarrow ③$$

∵ α, w & μ remain constt on both filters Eq ③ by Eq ② to obtain new area; of filter.

$$\frac{100}{10} = \frac{A_D}{8} \sqrt{\frac{\cancel{(2)}\cancel{(0.2)}(15 \times 144)(1.5)}{\cancel{(2)}\cancel{(0.2)}(20 \times 144)(2)}}$$

$$\boxed{A_D = 106.67 \text{ ft}^2}$$

PROBLEM 13: A plate-and-frame filter press is.used to filter a known slurry mixture. At a constant pressure drop of 10 psi, 50 ft³ of filtrate is delivered in 10 min, starting with a clean filter. In a second run with the same slurry and filter press, 40 ft³ of filtrate is obtained in 9 min when the pressure drop is 6 psi, starting with a clean filter. What is the compressibility exponent for the cake if the resistance of the filter medium is negligible?

DATA:

RUN ①	RUN ②
$\Delta P = 10$ psi => 10×144 psf	$\Delta P = 6$ psi => 6×144 psf
$V = 50$ ft³	$V = 40$ ft³
$\theta = 10$ min $= 0.17$ hr	$\theta = 9$ min $= 0.15$ hr

Filter medium resistance = $R_F = 0$

T.F.D:-

compressibility exponent = $S = ?$

Soln:

∵ For Batch filter (plate & frame may be :p)

$$V^2 + \cancel{2AV_F V}^{\,0} = \frac{2A^2(\Delta P)^{1-S}}{\alpha' w \mu}\,\theta.$$

$$V^2 = \frac{2A^2(\Delta P)^{1-S} \cdot \theta}{\alpha' w \mu} \longrightarrow ①$$

RUN ①:- ① ⇒

$$(50)^2 = \frac{2A^2(1440)^{1-S}(0.17)}{\alpha' w \mu} \longrightarrow ②$$

RUN ②:-

$$(40)^2 = \frac{2A^2(864)^{1-S}(0.15)}{\alpha' w \mu} \longrightarrow ③$$

③ ÷ ②

$$\frac{(40)^2}{(50)^2} = \frac{(864)^{1-S}}{(1440)^{1-S}} \times \frac{0.15}{0.17}$$

$$0.64 = \frac{(864)^{1-S}}{(1440)^{1-S}} \times \frac{0.15}{0.17}$$

Taking natural log

$$\ln(864)^{1-S} - \ln(1440)^{1-S} = \ln(0.7253)$$

$$(1-S)[\ln(864) - \ln(1440)] = \ln(0.7253)$$

$$1 - S = 0.67$$

$$\boxed{S = 0.33}$$ compressibility for cake

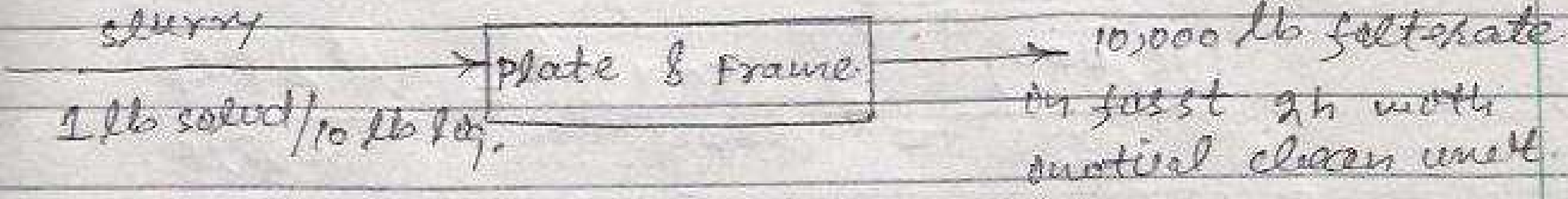
filtered with a plate-and-frame filter press having a total filtering area of 250 ft². This unit delivers 10,000 lb of filtrate during the first 2 h of filtration, starting with a clean unit and maintaining a constant pressure drop of 10 psi. The resistance of the filter medium is negligible. The time required for washing and dumping is 3 h/cycle. The pressure drop cannot exceed 10 psi, and the unit is always operated at constant pressure drop.

The filter press is to be replaced by a rotary vacuum-drum filter with negligible filter-medium resistance. This rotary filter can deliver the filtrate at a rate of 1000 lb/h when the drum speed is 0.3 r/min. Assuming the fraction submerged and the pressure drop are unchanged, what drum speed in r/min is necessary to make the amount of filtrate delivered in 24 h from the rotary filter exactly equal to the maximum amount of filtrate obtainable per 24 h from the plate-and-frame filter?

Pb#14: srry for missin statment but no worries we have the book ooo yeahhh!!! TrUe LiAr

DATA:

case ①:



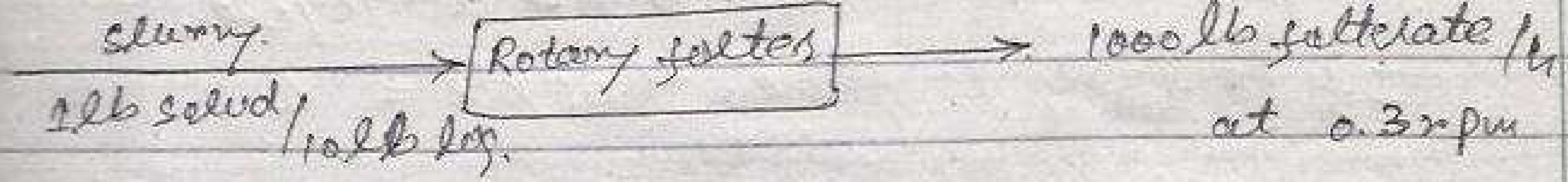
pr drop (constt) = Δp = 10 psi = 10×144 psf

filtering area = A = 250 ft²

operation time = $\boxed{\theta_1}$ = 2 h.

washing & dumping time = 3 h/cycle

case ②:



→ ψ & Δp are constt, → Filter med. resis negligible

T.F.O:

Drum speed = rpm = ?

solⁿ: For case ①;

let w_F = lb of filtrate rate/cycle } let 'θ' be the op. time...

& ρ_F = lb/ft³

i.e $V = W_F/\rho_F$ (ft³/cycle) → ⓐ

as for Batch filters

$$V^2 + \overset{\to 0}{\cancel{2V_FV}} = \frac{2A^2\Delta p^{1-s}}{\alpha' w \mu}\,\theta$$

$s \to 0$

$\alpha' \to \alpha$

$$V^2 = \frac{2A^2 \Delta P}{\alpha w \mu} \cdot \theta$$

ⓐ =>

$$\left(\frac{W_F}{\rho_F}\right)^2 = \frac{2A^2 \Delta P}{\alpha w \mu} \theta$$

$$* \quad \frac{W_F^2}{\theta} = \frac{2A^2 \Delta P \rho_F^2}{\alpha w \mu} \longrightarrow ①$$

$$\frac{(10{,}000)^2}{2} = 50 \times 10^6 = \frac{2A^2 \Delta P \rho_F^2}{\alpha w \mu} \longrightarrow ②$$

or. ① & ② →

$$\underbrace{W_F = \left[50 \times 10^6 \cdot \theta\right]^{1/2}} \longrightarrow ③$$

Best θ by

cycle time = ops time + washing/dumping time

$$= \theta + 3 \ (h/\text{cycle})$$

If $(\theta+3)$h is time, the cycle = 1

" 1 " " " " = $\frac{1}{\theta+3}$

" 24h " " " " " = $\frac{24}{\theta+3}$

so;

$$\boxed{\text{cycle}/24h = \frac{24}{\theta+3}} \ \text{h}/\text{day} \longrightarrow ④$$

③ & ④ =>

$$\text{product as lb of filtrate}/24h = \frac{\text{lb of filtrate}}{\text{cycle}} \times \frac{\text{cycle}}{24h}$$

$$= (50 \times 10^6 \cdot \theta)^{1/2} \cdot \left(\frac{24}{\theta+3}\right)$$

$$= \frac{1.697 \times 10^5 \, \theta^{1/2}}{\theta+3} \longrightarrow ⑤$$

for θ_{opt} ; $\dfrac{d(\text{product}/24h)}{d\theta} = 0.$

so; $\dfrac{d}{d\theta}\left[\dfrac{1.697\times10^{5}\,\theta^{1/2}}{\theta+3}\right] = 0$

$$1.697\times10^{5}\cdot\frac{d}{d\theta}\left[\theta^{1/2}/(\theta+3)\right] = 0$$

$$\left[\frac{(\theta_{opt}+3)(\frac{1}{2})(\theta_{opt}^{-1/2}) - \theta_{opt}^{1/2}(1)}{(\theta_{opt}+3)^{2}}\right] = 0$$

$$\frac{(\theta_{opt}+3)}{2\theta_{opt}^{1/2}} = \theta_{opt}^{1/2}$$

$$\theta_{opt}+3 = 2\theta_{opt} \Rightarrow \boxed{\theta_{opt} = 3 \text{ hrs}}$$

Eq ⑤ ⇒

$$\frac{\text{max product as lb of filtrate}}{24h} = \frac{1.697\times10^{5}\cdot3^{1/2}}{3+3}$$

$$= 48988 \text{ lb}/24\text{ h}.$$

As in the given statement;

Max amount of filtrate obtained per 24 h in plate & frame = filtrate obtained obtained in 24 h in rotary drum

case ② :- For equal product rate in the rotary filter per hour requires

$$\dot{m} = 48988/24 \text{ lb/hr}$$

$$\boxed{\dot{m} = 2041 \text{ lb/hr}}.$$

* vol. of Filtrate per unit time $= A_D \sqrt{\dfrac{2 \psi_s \Delta P N_R}{\alpha w \mu}}$

$$\text{lb of filtrate}/h = A_D \rho_F \left(\frac{2 \psi_s \Delta P}{\alpha w \mu} \right)^{1/2} \cdot (N_R)^{1/2}$$

where N_R = No. of rev. per hours

→ Form the data for the operating rotary filter.

$$1000 \text{ lb}/h = A_D \rho_F \left(\frac{2 \psi_s \Delta P}{\alpha w \mu} \right)^{1/2} [(0.3)(60)]^{1/2} \xrightarrow{} ⑥$$

→ For the required rotary filter

$$2041 \text{ lb}/h = A_D \rho_F \left(\frac{2 \psi_s \Delta P}{\alpha w \mu} \right)^{1/2} (N_R)^{1/2} \xrightarrow{} ⑦$$

⑦ ÷ ⑥ ⟹

$$\frac{2041}{1000} = \frac{N_R^{1/2}}{[(0.3)(60)]^{1/2}}$$

$$\boxed{N_R = 74.98 \text{ rev}/h.}$$

$$\boxed{N_R = 1.25 \text{ rpm}}$$

PROBLEM 15: A plate-and-frame filter press with negligible filter-medium resistance is being used to filter a water slurry of constant composition. Experimental tests show that, during continuous operation, 300 ft³ of filtrate is delivered when the pressure drop is 20 psi and 150 ft³ of filtrate is delivered when the pressure drop is 5 psi. The unit is to be operated at a constant pressure drop of 15 psi during filtration and washing. The cake will be washed with 10 ft³ of water at the end of 2 h of continuous filtration. If reverse thorough washing (i.e., wash rate = $\frac{1}{4}$ of final filtrate delivery rate) is used, estimate the time required for washing.

DATA:

RUN ①	RUN ②	Required.
$V = 300\,ft^3$	$V = 150\,ft^3$	$\Delta p = 15\ psi \times 144 = 2160\ psf$
$\Delta p = 20\,psi = 20 \times 144 = 2880\ psf$	$\Delta p = 05\ psi = 5 \times 144 = 720\ psf$	vol of wash water $= 10\,ft^3$
		$V = ?$
$\theta = 2\,hrs$	$\theta = 2\,hrs$	washing time = ?
		T.F.O $\theta = 2\,hr$

wash rate $= \frac{1}{4}$ (filtrate delivery rate)

$R_F = 0$; $V_F = 0$

solve

washing rate $= \dfrac{\text{vol used for washing}}{\text{time for washing}}$

or

washing time $= \dfrac{\text{vol used for washing}}{\text{washing rate}}$ $\longrightarrow$ ①

Also for batch filters;

$$V^2 + 2A\overset{\nearrow 0}{\cancel{V_F}}V = \frac{2A^2\Delta p^{1-s}}{\alpha' w \mu}\theta$$

$$V^2 = \frac{2A^2\Delta p^{1-s}}{\alpha' w \mu}\theta \longrightarrow ②$$

For RUN ①:- Eq ② ⇒

$$(300)^2 = \frac{2A^2(2880)^{1-s}}{\alpha' w \mu} \longrightarrow ⓐ$$

FOR RUN ②:-

$$(150)^2 = \frac{2A^2(720)^{1-s}}{\alpha' w \mu} \longrightarrow ⓑ$$

ⓑ ÷ ⓐ $\Rightarrow \dfrac{(300)^2}{(150)^2} = \dfrac{(2880)^{1-s}}{(720)^{1-s}}$

$$4 = (4)^{1-s}.$$

or $\ln 4 = 1-s. \ln 4.$

$1-s = 1. \quad \Rightarrow \boxed{s=0}$

(so cake is non-compressible)

RUN① => Eq ①

$$(300)^2 = \frac{2A^2.\Delta P}{\alpha w \mu}\,\theta$$

$$\boxed{\frac{A^2}{\alpha w \mu} = \frac{(300)^2}{(4)(2880)} = 7.81}$$

For Required filteration:

Eq ②

$$V^2 = (2)(7.81)(2160)(2)$$

$\boxed{V = 260\ ft^3}$ vol of filtrate deli

Diff Eq. ②

$$\frac{d}{d\theta}(V^2) = \frac{2A^2.\Delta P}{\alpha w \mu}\,\frac{d}{d\theta}(\theta).$$

$$2V.\frac{dV}{d\theta} = (2)(7.81)(2160)(1).$$

$$\frac{dV}{d\theta} = \frac{(2)(7.81)(2160)}{(2)(260)}$$

$\boxed{\dfrac{dV}{d\theta} = 65\ ft^3/hr}$ filtrate delivery rat wash rate = $1/4(65)$ = $16.25\ ft^3/hr$

Eq ① ⇒

Time for washing = $10/16.25$ = 0.62.

$$\boxed{\begin{aligned}\text{washing time} &= 0.62\ \text{hr} \\ &= 37\ \text{mins}\end{aligned}}$$

PROBLEM 16: A slurry is filtered, and the filter cake is washed by use of a plate-and frame filter press operated at a constant pressure drop of 40 psi throughout the entire run. Experimental tests have been carried out on this equipment, and the results for the slurry mixture used can be expressed as follows for any one pressure drop:

$$\theta \Delta P = K'\left(\frac{V}{A}\right)^2 + K''\frac{V}{A}$$

where k' and k'' are constants. At a pressure drop of 40 psi, 0.02 lb of filtrate is collected in 1.8 min for each square inch of cloth area, and 0.08 lb of filtrate per square inch of cloth area is collected in 22.2 min. Calculate the time required to filter and wash the cake formed when 0.11 lb of filtrate has been collected per square inch of cloth area if an amount of wash water equal to half the filtrate is used. The specific gravities of the filtrate and wash water are 1.0, and both are at the same temperature. Simple forward washing is used so that the washing rate is equal to the filtrate delivery rate at the end of the filtration.

case ①	case ②	case ③
$\Delta P = 40\,psi$	$\Delta P = 40\,psi$	$\theta = \theta_f + \theta_w = ?$
$W_A = 0.02\ lb/in^2$	$W_A = 0.08\ lb/in^2$	$W_A = 0.11\ lb/in^2$
$\theta = 1.8\ min$	$\theta = 22.2\ min.$	

Given Eq :-

$$\theta \Delta P = K'\left(\frac{V}{A}\right)^2 + K''\left(\frac{V}{A}\right) \longrightarrow ①$$

case ① :- From Eq. ① $\because \frac{V}{A} = W_A/e \ \left(ft^3/in^2\right)$

$$(1.8)(40) = \frac{K'}{e^2}(0.02)^2 + \frac{K''}{e}(0.02)$$

÷ the whole Eq. by 0.02

$$3600 = \frac{K'}{e^2}(0.02) + \frac{K''}{e} \longrightarrow ②$$

case ② :-

$$(22.2)(40) = \frac{K'}{e^2}(0.08)^2 + \frac{K''}{e}(0.08)$$

÷ by 0.08

$$11100 = \frac{K'}{e^2}(0.08) + \frac{K''}{e} \longrightarrow ③$$

③ − ②

$$11100 - 3600 = \frac{K'}{e^2}(0.08 - 0.02)$$

$$\boxed{K'/e^2 = 125000}$$

Eq ② $\Rightarrow$ $3600 = (125000)(0.02) + K''/e$

$$\boxed{K''/e = 1100}$$

case ③ :-

filteration time : 'θ_f' :-

Eq ① $\Rightarrow$

$$\theta_f \Delta p = \frac{K'}{e^2}(0.11)^2 + \frac{K'}{e}(0.11)$$

$$\theta_f \cdot 40 = (125000)(0.11)^2 + (1100)(0.11)$$

$$\boxed{\theta_f = 40.8 \text{ min}} \longrightarrow ⓐ$$

washing Time : 'θ_w' :-

Eq ① $\Rightarrow$

$$\theta_W \Delta p = \frac{K'}{e^2} W_A^2 + \frac{K''}{e} W_A$$

Derivation of above.

$$\frac{d}{d\theta}(\theta_W \Delta p) = \frac{d}{d\theta}\left[\frac{K'}{e^2} \cdot W_A^2\right] + \frac{d}{d\theta}\left[\frac{K''}{e} \cdot W_A\right]$$

$$\Delta p = \frac{K'}{e^2} 2W \cdot \frac{dw}{d\theta} + \frac{K''}{e} \cdot \frac{dw}{d\theta}$$

$$\Delta p = \frac{dw}{d\theta}\left[\frac{2K'}{e^2} W + \frac{K''}{e}\right]$$

$$\frac{dW_A}{d\theta} = \frac{40}{(2)(125000)(0.11) + 1100}$$

filtrate delivery rate $= \frac{dw}{d\theta} = 0.0014$ lb/in². min

$\because$ filtrate delivery rate = washing rate

$\Rightarrow$ $\boxed{\text{washing rate} = 0.0014\ lb/in^2.min}$

also From statement;

Amount of H_2O for washing $= \frac{1}{2}$[filtrate delivered)

$= \frac{1}{2}[0.11]$

$= 0.055\ lb/in^2$

$\because$

washing rate $= \dfrac{\text{Amount of } H_2O \text{ for washing}}{\text{Time for washing}}$

or

Time for washing $= \theta_w = \dfrac{0.055\ lb/in^2}{0.0014\ lb/in^2.min}$

$\boxed{\theta_w = 39.3\ min} \longrightarrow$ ⓑ

ⓐ & ⓑ $\Rightarrow$

Time req. for filtration & washing $= \theta = \theta_f + \theta_w$

$= 40.8 + 39.3$

$\boxed{\theta = 80.1\ min}$

O Yeah babe ......

CHAPTER 16

MASS-TRANSFER
AND
REACTOR
EQUIPMENT DESIGN
AND COSTS

Example 5 **Determination of holdup time in downcomer.** A valve-tray tower with 24-in. plate spacing and liquid crossflow contains straight segmental downcomers. The overflow weir at the downcomer entrance is formed by an extension of the downcomer plate. The height of this weir is 3 in. The inside diameter D of the tower is 5 ft, and the weir length is $0.6D$. If liquid with a density of 55 lb/ft^3 flows across the plate at a rate of 30,000 lb/h, estimate the residence or holdup time in the downcomer from this plate.

DATA:-

plate spacing = 24''

weir Height = h_w = 3''

Tower inside dia = D = 5 ft

weir length = l_w = 0.6D = 3 ft

Density of liquid = 55 lb/ft^3

mass Flow rate = $\dot{m}$ = 30,000 lb/hr

T.F.O

Residence time in the downcomer = ?

soln:-

$$\because \text{residence time} = \frac{\text{vol. of downcomer}}{\text{vol flow rate of downcoming liquid}} \longrightarrow ①$$

Now:-

$$\text{vol flow rate of downcoming liquid} = \frac{30{,}000\ \text{lb}}{\text{hr}}\left|\frac{\text{ft}^3}{55\ \text{lb}}\right|\frac{1\ \text{hr}}{3600\ \text{sec}}$$

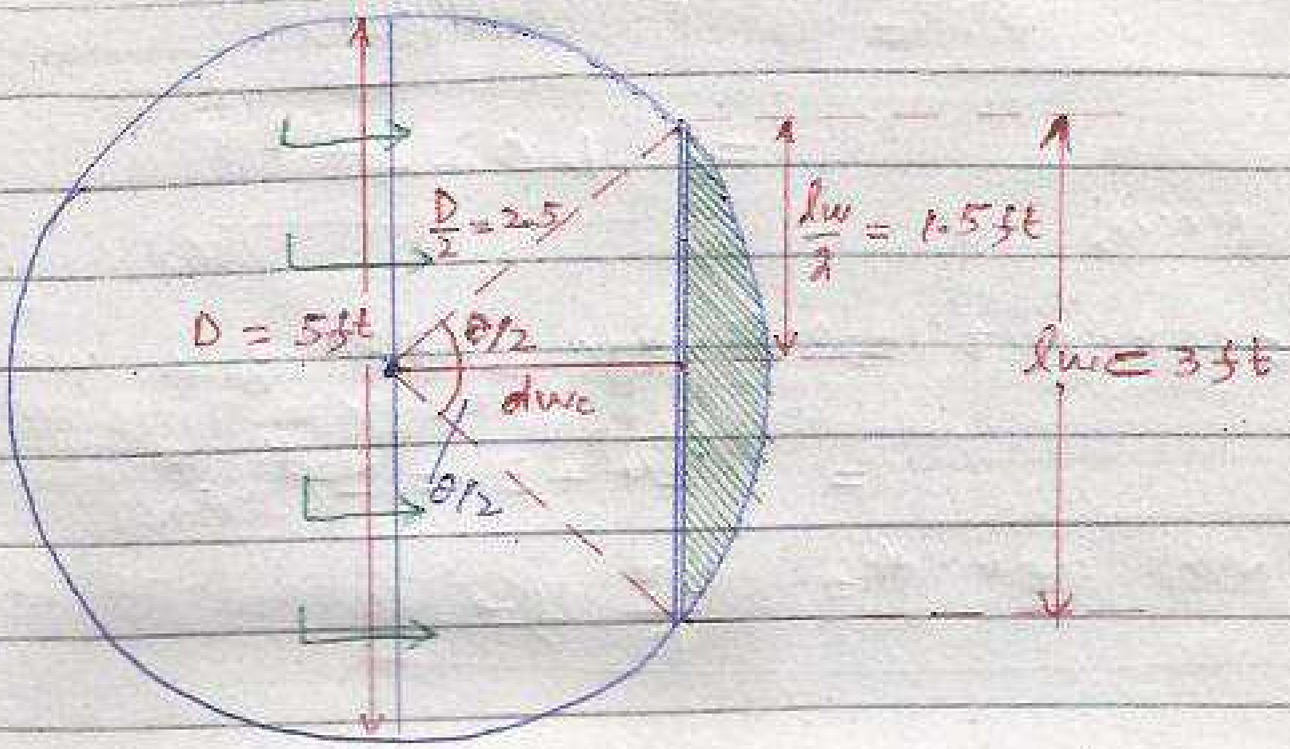
$$= 0.152\ \text{ft}^3/\text{sec} \longrightarrow ②$$

Now:

$$\text{vol of downcomer} = \text{D.C crosssectional area} \times \text{column D.C Height.} \longrightarrow ②$$

$$\therefore\ \text{D.C cross sectional area} = \frac{\pi D^2 \theta}{1440} - \frac{l_w d_{wc}}{2}$$

where; lw_c = the distance from wall to center of the tower. i.e.



Now, According to patagoras theorem;

$$(D/2)^2 = (dw_c)^2 + (lw/2)^2$$

or $$dw_c = \sqrt{(D/2)^2 - (lw/2)^2}$$

$$dw_c = \sqrt{(2.5)^2 - (1.5)^2}$$

$$\boxed{dw_c = 2ft}$$

also; If θ is the angle in degrees subtended from the center of the tower by the wall.

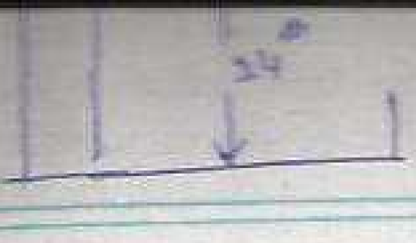
$$\sin\theta/2 = \frac{lw/2}{D/2}$$

$$\sin\theta/2 = lw/D$$

or $$\theta/2 = \sin^{-1}\left(\frac{1.5}{2.5}\right)$$

$$\theta = 2\sin^{-1}(0.6)$$

$$\Rightarrow \boxed{\theta = 73.7^\circ}$$



So;

D.C cross sectional area $= \frac{(\pi)(5)^2(73.7)}{1440} - \frac{(3)(\cancel{2})}{\cancel{2}}$

$= \boxed{1.02\ ft^2}$

Now;

D.C Height = Tray spacing + weir height

$= 24 + 3 = 27''$

$\boxed{\text{D.C Height} = \frac{27}{12}\ ft}$

Eq ② ⇒

vol of downcomer $= (1.02)\left(\frac{27}{12}\right)$

$\boxed{\text{vol of D.C} = 2.295\ ft^3} \longrightarrow$ ⓑ

ⓐ & ⓑ ⇒ in Eq ①

Residence Time $= \frac{2.295}{0.152}$ $\frac{\cancel{ft^3}}{} \Big| \frac{sec}{\cancel{ft^3}}$

$\boxed{\text{Residence Time} = 15.1\ sec}$

Pb# 01

gradient is negligible. The diameter of the perforations is $\frac{3}{16}$ in., and the superficial vapor velocity (based on the cross-sectional area of the empty tower) is 3.4 ft/s. The pressure drop due to passage of gas through the holes may be assumed to be equivalent to 1.4 kinetic heads (based on gas velocity through holes). (Tray thickness = hole diameter and active area = 90 percent of total area-see Fig. 16-12). If the liquid density is 50 lb/ft^3 and the gas density is 0.10 lb/ft^3, estimate the pressure drop per tray as pounds force per square inch.

DATA:

dia of tower = D = 5 ft

Area of columns = A_C

Area of sieve holes = A_S

$A_S = 0.1\,A_C$

Height of weir = h_w = 1.5"

Liquid head over weir = 1" = h_o

superficial vapor velocity = V = 3.4 ft/sec

kinetic heads = K.H = 1.4

Density of liquid = ρ_L = 50 lb/ft³

" " gas = ρ_G = 0.10 lb/ft³

T.F.D:

$\Delta P_T = ?$ (psi)

Soln:

∵ For sieve Tray columns

$$\Delta P_T = \frac{h_T \rho g}{144\, g_c} \longrightarrow ①$$

also;

$$h_T = h_c + h_w + h_o + 0.5\overset{0}{\nearrow}h_g$$

$$h_T = h_c + h_w + h_o \longrightarrow ②$$

For sieve trays

$$h_c = (K.H)\frac{V_c^2}{2g_c} \cdot \frac{\rho_g}{\rho_L}$$

∵ $V_1 A_1 = V_2 A_2$

or. $V A_c = V_c A_s$

$$V_c = \frac{V A_c}{A_s} = \frac{(3.4)(\cancel{A_c})}{0.1\,\cancel{A_c}} = \underline{34\ ft/sec.}$$

$$h_c = \frac{(1.4)(34)^2 (0.10)}{(2)(32.17)(50)} = 0.0503$$

$$\boxed{h_c = 0.0503 \text{ ft}} \longrightarrow ⓐ$$

Eq ② =>

$$h_T = 0.0503 + \left(\frac{1.5}{12}\right) + \left(\frac{1}{12}\right)$$

$$\boxed{h_T = 0.26 \text{ ft}}$$

Eq ① =>

$$\Delta P_T = \frac{(0.26)(50)}{144} = 0.0898 \text{ psi}$$

$$\boxed{\Delta P_T = 0.09 \text{ psi}}$$

*

2. An existing bubble-cap distillation tower has an ID of 8 ft and a tray spacing of 30 in. The tower is operating under a pressure of 70 psig, and a slot liquid seal of 2 in. is maintained. At the point of maximum volumetric vapor flow, the molecular weight of the vapor is 100, the rate of vapor flow is 1500 lb mol/h, the liquid density is 55 lb/ft^3, and the temperature is 175°F. The pressure drop through the tower is negligible, and the ideal gas law is applicable to the rising vapors. Approximately what percent of the maximum allowable flow rate is being used in the tower?

• DATA:

dia of column = 8 ft

Tray spacing = 30"

operating pr = P = 70 psig = 84.7 psia

slot liquid seal = 2"

Mol. wt of vapor = M = 100 lbm/lbmole

vap. flow rate = q = 1500 lbmol/hr

liquid density = ρ_L = 55 lb/ft^3

$T = 175\,^\circ F$

$\Delta P_T = 0$

T.F.O: what % of max. allowable flow rate is used in tower = ?

sol:-

$\because$

% of max. allowable vap. flowrate $= \dfrac{G}{G_m} \times 100 \longrightarrow ①$

$\because$

$$G_m = K_v \sqrt{\rho_G(\rho_L - \rho_G)} \longrightarrow ②$$

From Fig 16-6, page # 657

$$\boxed{K_v^{\circledast} = 0.34 \text{ ft/sec}}$$

also

$$\rho_G = \frac{M \cdot P \cdot T_{s.c}}{359 \cdot P_{s.c} \cdot T}$$

$$= \frac{(100)(84.7)(460+32)}{(359)(14.7)(460+175)}$$

$$\boxed{\rho_G = 1.24 \text{ lbm/ft}^3}$$

Eq ② $\Rightarrow$

$$G_m = (0.34)\sqrt{1.24(55-1.24)}$$

$$G_m = 2.8 \text{ lbm} / \cancel{\text{sec}}\ \text{ft}^2 \ \circledast$$

$$G_m = \frac{2.8 \cancel{\text{lbm}}}{\cancel{\text{sec}}\text{-}\cancel{\text{ft}^2}} \left| \frac{\text{lbmole}}{100 \cancel{\text{lbm}}} \right| \frac{\pi/4 (8)^2 \cancel{\text{ft}^2}}{} \left| \frac{3600 \cancel{\text{sec}}}{1 \text{hr}} \right.$$

$$\boxed{G_m = 5067 \text{ lbmole/hr}}$$

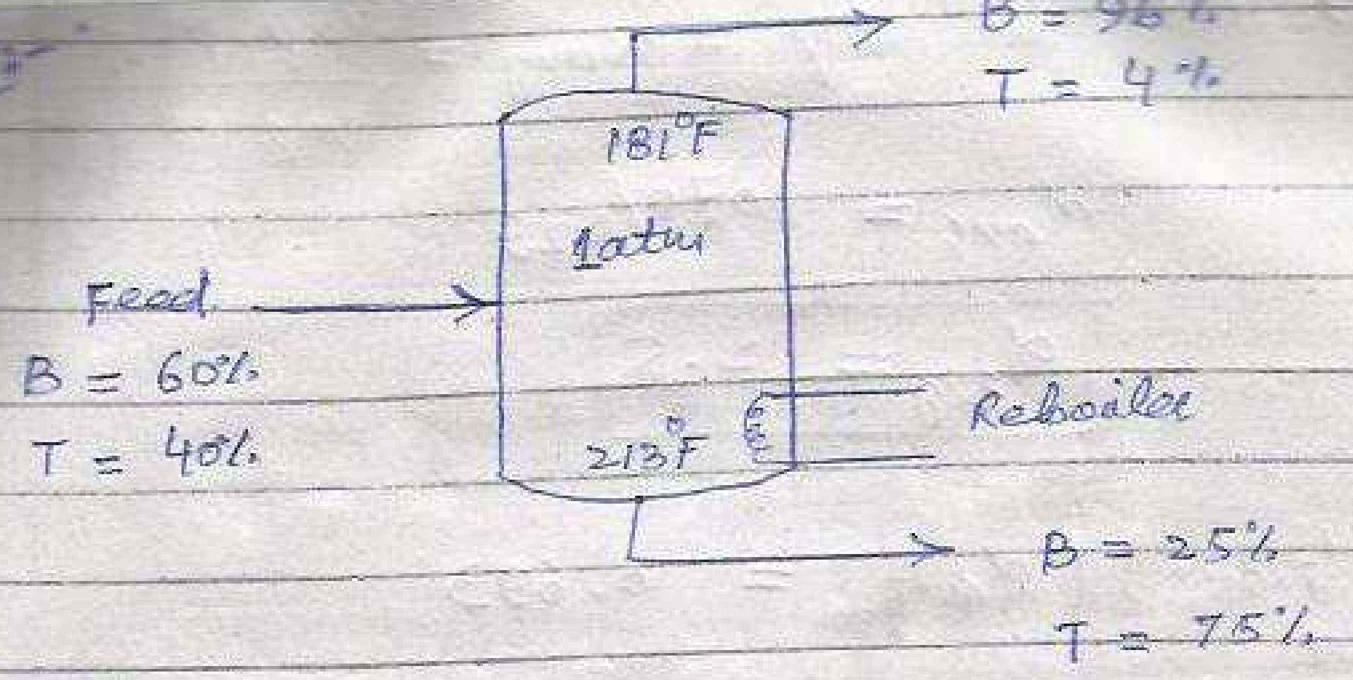
① $\Rightarrow$ % of max. allowable vap. rate $= \dfrac{1500}{5067} \times 100 = 29.6 \cong 30$

$$\boxed{= 30\%}$$

Pb# 03

which is intended to accomplish this separation
operating conditions chosen, calculations have shown that 6.1 theoretical stages are necessary. The temperature is 181°F at the top of the column and 213°F in the reboiler. Assuming the reboiler acts as one theoretical stage, estimate the number of actual trays required. Benzene and toluene mixtures may be considered as ideal. At 197°F, the vapor pressure of pure benzene is 1070 mm Hg and the vapor pressure of pure toluene is 429 mm Hg.

DATA:-



No. of theoretical stages = 6.1

vap. pr. of Benzene = 1070 mm Hg

" " " Toluene = 429 mm Hg

T.F.O:-

No. of Actual stages = ?

soln:-

$$\because \text{No. of Actual Trays} = \frac{\text{No. of theoretical Trays}}{(\eta) \text{ or } E_o} \longrightarrow ①$$

$$\text{No. of theoretical Trays} = 6.1 - 1 = 5.1 \longrightarrow ⓐ$$

$$\text{Avg Temp} = \frac{181 + 213}{2} = 197°F$$

At 197 °F:-

viscosity of Benzene $= \mu_B =$ ~~0.27cp~~ 0.27cp

" " Toluene $= \mu_T = 0.29\,cp$

$$\mu_{F,avg} = (0.6)(0.27) + (0.4)(0.29)$$

$$\boxed{\mu_{F,avg} = 0.278}$$

[illegible]

[illegible]

Relative volatility $= \alpha_{avg} = \dfrac{\text{Vap. ps of Benzene}}{\text{Vap ps. of Toluene}}$

$\alpha_{avg} = 1070/429$

$$\boxed{\alpha_{avg} = 2.5}$$

$(\mu_{F,avg})(\alpha_{avg}) = 0.693$

From Fig 16-92 page # 664

$E_o = 56\% \longrightarrow$ ⑥

Eq ① $\Rightarrow$

No. of Actual Trays $= 5.1/0.56 = 9.1$

$$\boxed{\text{No. of Actual Trays} = 9.1}$$

$\longleftrightarrow$

4. For the conditions given in Example 3, what weir height would be necessary to reduce the dimensionless ratio of liquid gradient/pressure-drop head caused by bubble-cap assemblies to the recommended maximum value of 0.4?

DATA:

$\dfrac{\text{liquid gradient}}{\text{ps. drop head}} = \dfrac{h_g}{\Delta p} = 0.4$

Exp#03

$h_c = 0.069$ ft

$h_{slot} = 0.107$ ft

$h_o = 0.133$ ft

$l_L = 4.4$ ft ; $l_c = 2.7$ ft ; $s = 0.042$ ft

$r = 4$; $C_D ; 1.25$; $F_c = 1.12$

$Q_L = 1$ ft^3/sec

T.F.O :- $h_w = 0$

soln:

∵ $h_g/\Delta P = 0.4$ ⇒ $h_g = 0.4\Delta P$ → ①

Pr. drop head due to Bubble cap assembly:

$\Delta P = h_c + h_{slot} = 0.069 + 0.107 = 0.176$ ft

① ⇒ $\boxed{h_g = 0.0704 \text{ ft}}$

∵

$$(h_g F_c)^{1.5}(1.5r - 1.4) + (h_g F_c)^{0.5}(3r)\left[h_w + h_o + \frac{s(l_s - l_c)}{l_c}\right] = \frac{0.42\, Q_L\, r^{1.5}}{C_D\, l_c}$$

⇒

$$\underset{0.0704}{(0.074\times1.12)}^{1.5}(1.5(11) - 1.4) + \underset{0.0704}{(0.074\times1.12)}^{0.5}\, 3(11)\left[h_w + 0.132 + \frac{0.042(1.7)}{2.7}\right] = \frac{0.42(13)(11)^{1.5}}{1.25\times2.7}$$

~~$0.36 + \overset{9.57}{9.299}[h_w + 0.16] = 4.54$~~

~~$h_w = 0.28$ ft~~

~~$\boxed{h_w = 3.39 \text{ in}}$~~ ×

$0.33 + 9.26[h_w + 0.16] = 4.54$

$h_w = 0.294$ ft

$\boxed{h_w = 3.53 \text{ in}}$ ✓

Pb# 05

air are at a temperature of 70°F, and the pressure
operation of the plate and a liquid gradient of 0.4 in., estimate the percent of the total pressure drop across the tray due to the head of liquid above the slots. The following specifications apply to the unit

Inside diameter = 6 ft
Minimum total cross-sectional area of riser, direction-reversal space in cap, or annular cap space = 15 percent of tower cross-sectional area
- Total slot area = 10 percent of tower cross-sectional area

Length of chord weir = 0.7 X tower diameter
Slots in caps are triangular, and slot width at base = 0.4 in.
Slot height = 1.5 in.
Static submergence = 1.0 in.

DATA:

superficial velocity = V = 2ft/sec

P = 1 atm T = 70°F

liquid gradient = h_g = 0.4"

inside dia = D = 6ft

cross sectional area of riser etc = A_{riser} = 0.15 (Area of tower)

length of weir = 0.7 × tower dia = 0.7D

slots are triangular.

slot width at base = b = 0.4"

slot height = c = 1.5"

static submergence = 1"

Q_L = 0.1 ft³/sec. dia

R.F.O:

% ΔP_T due to liquid head above slot = ?

solution:

∵ for bubble caps

$$\Delta P_T = \frac{h_T \rho g}{144\, g_c} \longrightarrow ①$$

also;

$$h_T = \underbrace{h_c + h_{slot} + s_s + h_o}_{\text{pr. drop across contactors assembly}} + 0.5 h_g \longrightarrow ②$$

∴ $h_c = \frac{3 v_c^2}{g} \frac{\rho_g}{\rho_L}$

$\because \quad V_1 A_1 = V_2 A_2$

$$VA_{column} = V_c A_{riser}$$

$$V_c = \frac{(V)(A_{column})}{(0.15 A_{column})} = 2/0.15 = \cancel{2}13.33$$

$$\boxed{V_c = 13.33 \text{ ft/sec}}$$

→ [illegible]

also; at 70°F ;

water density = ρ_L = 62.3 lbm/ft³

at;

$$\rho_g = \frac{M.P.T_{sc}}{359\ P_{sc}\ T} = \frac{(29)(1)(460+32)}{(359)(1)(460+70)}$$

$$\boxed{\rho_g = 0.075 \text{ lbm/ft}^3}$$

so;

$$h_c = \frac{(3)(13.33)^2}{32.17} \times \frac{0.075}{62.3}$$

$$\boxed{h_c = 0.02 \text{ ft}} \longrightarrow ⓐ$$

pr. drop through cap slots

$\because$ for Triangular slot;

$$h_{slot} = 1.85 \left(\frac{c\,Q_s}{b}\right)^{2/5} \left[\frac{\rho_g}{(\rho_L - \rho_g)\,g}\right]^{1/5}$$

Q_s = Gas flow rate per slot.

so; $Q_s = V_c A_{slot}$

$$A_{slot} = \frac{1}{2}(b \times c) = \frac{\frac{1}{2}(0.4 \times 1.5)}{144} = 2.1 \times 10^{-3} \text{ ft}^2$$

$$Q_s = (13.33)(2.1 \times 10^{-3}) = 0.028 \text{ ft}^3/\text{sec}$$

$$h_{slot} = (1.85)\left(\frac{(1.5)(0.028)}{0.4}\right)^{2/5} \left[\frac{0.075}{(62.3 - 0.075)\,32.17}\right]^{1/5}$$

$$(1.85) \quad (0.406) \quad (0.130)$$

$$\boxed{h_{slot} = 0.097 \text{ ft}} \longrightarrow ⓑ$$

static submergence :

$$S_w = h_{12} = 0.083 \text{ ft} \longrightarrow ⓒ$$

liquid crest over weir :

$$h_o = \left(\frac{1.7\, Q_L}{l_w \sqrt{g}} \right)^{2/3}$$

$$l_w = 0.7 \times 6 = 4.2 \text{ ft}$$

$$\overset{*}{Q_L} = \frac{0.1 \text{ ft}^3}{\text{sec}\cdot \cancel{\text{ft}}} \Big| \frac{6 \cancel{\text{ft}}^{*}}{} = 0.6 \text{ ft}^3/\text{sec}$$

$$h_o = \left(\frac{1.7 \times 0.6}{4.2 \sqrt{32.17}} \right)^{2/3}$$

$$\boxed{h_o = 0.12 \text{ ft}} \longrightarrow ⓓ$$

Eq ② $\Rightarrow$

$$h_T = 0.02 + \overset{0.097}{\cancel{0.075}} + 0.083 + 0.12 + 0.017$$

$$\boxed{h_T = 0.34 \text{ ft}}$$

$$\boxed{\Delta P_T = \frac{(0.34)(62.3)}{144} = 0.147 \text{ psi}}$$

% of ΔP_T above the liquid slot $= \dfrac{S_w + h_o + 0.5 h_s}{h_T} \times 100$

$$= \frac{0.083 + 0.12 + 0.017}{0.34} \times 100 = 64.7$$

$$= 65\,\%$$

so; ΔP_T above the liquid slot = 65%

6. A random-packed absorption tower is to be used for removing a hydrocarbon from a gas mixture by countercurrent scrubbing with an oil. Stoneware Raschig rings of 1-in. nominal size will be used as the packing. The tower must handle a gas rate of 2000 lb/h and a liquid rate of 6000 lb/h. A gas velocity equal to 70 percent of the maximum allowable velocity at the given liquid and gas rates will be used. The gas density is 0.075 **lb/ft^3**. The liquid density is 55 **lb/ft^3**. The viscosity of the oil is 20 centipoises. Under these conditions, estimate the necessary tower diameter and the pressure drop through the tower as inches of water per foot of packed height. *

DATA:

Stoneware raschig rings = 1"

Gas rate = G = 2000 lb/hr

liquid rate = L = 6000 lb/hr

Gas velocity = (0.7)(max. allowable velocity)

$\rho_G = 0.075$ lb/ft^3 $\rho_L = 55$ lbm/ft^3

$\mu_L = 20$ cp

T.F.O.:

Dia of tower = D = ?

pr. drop = $\frac{\Delta P}{h}$ = ?

Solve:

∴ Area of column = $A_c = \pi/4\, D^2$

(a) or $D = \sqrt{\frac{4 \times A_c}{\pi}} \rightarrow$ ①

Now

$$\frac{L}{G}\sqrt{\frac{\rho_G}{\rho_L}} = \frac{6000}{2000}\sqrt{\frac{0.075}{55}} = 0.11$$

from ~~table~~ fig's 16-20 page # 698

$$\frac{G_m^2 (a_p/\varepsilon^3)(\mu_L')^{0.2}}{g\,\rho_G\,\rho_L} \times \frac{1}{(3600)^2} = 0.19$$

$$G_m^2 = \frac{(0.12)(3600)^2(32.17)(0.075)(55)}{(\mu_L')^{0.2}(a_p/\epsilon^3)}$$

From Table # 03 page 690

$$a_p/\epsilon^3 = 150$$

$$G_m^2 = \frac{(0.12)(3600)^2(32.17)(0.075)(55)}{(20)^{0.2}(150)}$$

$$\boxed{G_m = 869 \text{ lbm/hr.ft}^2}$$ max. allowable gas rate.

or *

$$\boxed{G_{actual} = 0.7 \times 869 = 608.3 \text{ lbm/hr.ft}^2}$$

or

$$\text{Area} = A_c = \frac{2000 \text{ lb}}{\text{hr}} \left| \frac{\text{hr.ft}^2}{608.3 \text{ lbm}} \right. \quad *$$

$$\boxed{A_c = 3.3 \text{ ft}^2}$$

or ① =>

$$D = \sqrt{\frac{4 \times 3.3}{\pi}} = 2.05 \text{ ft}$$

$$\boxed{\text{column dia} = 2.05 \text{ ft}}$$

(b)

$$\frac{\Delta p}{L} = ?$$

$$\text{Kinematic viscosity} = \frac{\overset{\mu_L}{\text{viscosity}}}{\underset{\rho_L/\rho_{H_2O}}{\text{sp. gravity}}} = 20\,\text{cp} \Big/ {}^{55}\!/_{62.3}$$

$$= 22.65 \text{ ct. stokes}$$

Now; From ~~Fig~~ Table 7.05 page 696

$$\frac{\Delta P_F}{h} = 1.9 \text{ in water / ft of packed height}$$

also;
$$\Delta P = \Delta P_F \left(G / G_m \right)^{3.2}$$

$$\Delta P = (1.9) \times h \times \left(\frac{608.3}{869} \right)^{3.2}$$

$$\boxed{\frac{\Delta P}{h} = 0.61 \text{ in } H_2O / \text{ft of packed height}}$$

7. A blower is being used to force 200 lb of air per hour through a 12-in. inside-diameter tower countercurrent to water flowing at a rate of 6000 lb/h. The tower is packed to a height of 10 ft with 1-in. Berl saddles. The average pressure in the tower is 1 atm, and the temperature is 75°F. The blower-motor combination used at the gas inlet has an overall efficiency of 50 percent. Estimate the power cost per 8000 operating hours for forcing the air through the packing if the cost of power is $0.08 per kilowatthour.

- DATA:

Gas rate = $G' = 200$ lb/hr.

dia of column = D = 1 ft

liquid rate = $L' = 6000$ lb/hr

packing height = 10 ft

Berl saddles = 1"

$P = 1$ atm $T = 75°F$

overall efficiency of blower motor = 50%

ops hours = 8000 h.

power cost = $0.08 /kWh

∴ for corrugated packing

$$\frac{\Delta P}{h} = \gamma(10)^{\phi L/\rho_L} \times \frac{G^2}{\rho_G} \longrightarrow ①$$

So; From Table 4 page #694.

✓ $\gamma = 16\times10^{-8}$; ✓ $\phi = 3\times10^{-3}$

Area of column $= A_c = \pi/4\, D^2$

$A_c = \pi/4(1)^2 = 0.785\ ft^2$

$$\boxed{A_c = 0.785\ ft^2}$$

✓ $G = \frac{200\ lb}{hr} \Big| 0.785\ ft^2 = 255\ \frac{lbm}{hr\ ft^2}$

✓ $L = \frac{6000\ lb}{hr} \Big| 0.0785\ ft^2 = 7643\ \frac{lbm}{hr\ ft^2}$

also at 1 atm & 75 °F

$$\rho_G = \frac{M\cdot P\cdot T_{sc}}{359\cdot P_{sc}\cdot T}$$

$$\rho_G = \frac{(29)(1)(460+32)}{(359)(1)(460+75)}$$

$$\boxed{\rho_G = 0.074\ lbm/ft^3}$$

so ① ⇒

$$\frac{\Delta P}{h} = (16\times10^{-8})(10)^{\frac{(3\times10^{-3}\times7643)}{\cancel{7643}\ 62.3}} \times \frac{(255)^2}{0.074}$$

$$\frac{\Delta P}{h} = 0.328 \text{ psf / ft of packing}$$

or $\Rightarrow$ $\Delta P = 0.328 \times 10$ psf

$\Delta P = 3.28$ lb/ft²

∵ $\eta = 50\%$

* $$\Delta P\,(\text{Actual}) = \frac{3.28}{0.5} = 6.56 \text{ psf}$$

$$\boxed{\Delta P = 6.56 \text{ lbs/ft}^2}$$

$$\because \text{power} = \frac{(\text{pr. drop}) \times (\text{mass flow rate})}{(\text{density})}$$

~~(massdensity)~~ ~~mass~~

$$\text{power} = \frac{6.56 \text{ lbs}}{\cancel{\text{ft}^2}} \left| \frac{900 \text{ lb}}{\text{hr}} \right| \frac{\cancel{\text{ft}^3}}{0.074 \text{ lbm}} \left| \frac{1 \text{ hr}}{3600 \text{ sec}} \right| \frac{\text{hp} \cdot \text{sec}}{550 \text{ ft lbs}} \left| \frac{0.746 \text{ kwatts}}{\text{hp}} \right.$$

power $= 6.68 \times 10^{-3}$ kw.

or

power $= (6.68 \times 10^{-3})(8000 \text{ h})$

$$\boxed{\text{power} = 53.44 \text{ kwh}}$$

$$\text{Total power cost} = \frac{\$0.08}{\text{kwh}} \left| 53.44 \text{ kwh} \right.$$

$$\boxed{\text{power cost} = \$4.3}$$

Pb#08

Packed height = 3 ft
- Gas rate = 100 lb/h
- Liquid rate = 90 lb/h

Average molecular weight of gas = 100
Average temperature = 110°F
- Total pressure drop = 12 mm Hg

Liquid viscosity = 1.2 centipoises
Liquid density = 55 lb/ft^3

A larger distillation tower is needed for the same mixture, and it has been decided to use a 12-in. inside-diameter tower with the same type and size of packing. The packed height will be 3 ft. The condenser pressure will be 50 mm Hg, and the average temperature can be assumed to be 95°F. At this temperature, the liquid viscosity is 1.4 centipoises and the liquid density is 57 lb/ft^3. The 12-in. tower will be operated at the same percent of the maximum allowable gas mass velocity as the 6-in. tower, and the same ratio of L/G will be used. The average molecular weight of the gas is unchanged. With the given packing and fluids, the pressure drop at the flooding point can be assumed to be 16 mm Hg/ft of packed height. Neglecting the effect of temperature on fluid densities at the flooding point, estimate the gas rate as pounds per hour at the indicated operating conditions in the 12-in. tower.

DATA:

Small Tower:-

$D = 6''$

cond. pr = P_{cond} = 100 mmHg

Gas rate = $G' = 100$ lb/hr

Liquid rate = $L' = 90$ lb/hr

Mol wt = $M = 100$.

Avg Temp. = $T = 110^\circ F$

Total pr. drop = 12 mmHg

Liquid viscosity = $\mu_L = 1.2$ cp

" density = $\rho_{L_1} = 55$ lb/ft^3

Large Tower:-

$D = 12''$

packing height = 3 ft

P_{cond} = 50 mmHg

$T = 95^\circ F$

$\mu_L = 1.4$ cp.

$\rho_{L_2} = 57$ lb/ft^3

pr. drop at flooding point $= \frac{\Delta P_F}{h} = 16 \frac{\text{mmHg}}{\text{ft}}$

T.F.O:- Gas rate for 12'' tower = ?

Soln:

~~Gas~~

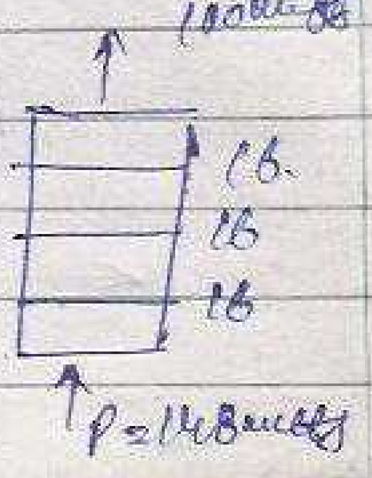
$$\because \quad \frac{G_{m_1}}{G_{m_2}} = \sqrt{\frac{\rho_{G_1}\rho_{L_1}}{\rho_{G_2}\rho_{L_2}}} \longrightarrow ①$$

Small Tower 6'' dia :-

So;

$$\text{Avg pr} = \frac{148+100}{2}$$

$$\boxed{\text{Avg Pr.} = 124 \text{ mmHg}}$$

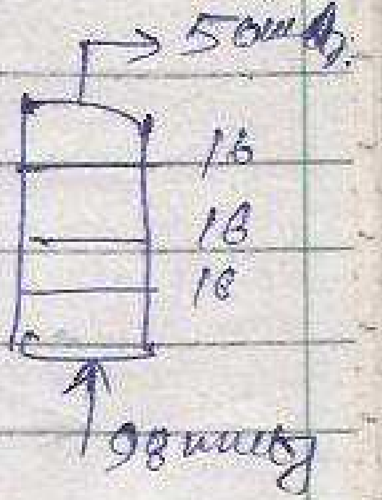


at $T = 110°F$

$$\rho_{G_1} = \frac{M \cdot P \cdot T_{sc}}{359 \cdot P_{sc} \cdot T} = \frac{(100)(124)(460+32)}{(359)(760)(460+110)}$$

$$\boxed{\rho_{G_1} = 0.039 \text{ lbm/ft}^3}$$

For $\underbrace{\text{larger Tower } 12'' \text{ dia}}$:-



$$\text{Avg Pr} = \frac{98+50}{2} = 74 \text{ mm Hg}$$

at $T = 95°F$

$$\rho_{G_2} = \frac{(100)(74)(460+32)}{(359)(760)(460+95)}$$

$$\boxed{\rho_{G_2} = 0.024 \text{ lbm/ft}^3}$$

Eq ① $\Rightarrow$

$$\frac{G_{m_1}}{G_{m_2}} = \sqrt{\frac{0.039 \times 55}{0.024 \times 57}}$$

$$\boxed{\frac{G_{m_1}}{G_{m_2}} = 1.25} \longrightarrow ②$$

For $\underline{\text{small Tower}}$;

$$\text{Area} = A_1 = \pi/4\, D^2 = \pi/4\,(6/12)^2$$

$$A_1 = 0.196 \text{ ft}^2$$

so ; ~~$G' = G_m \times A$~~

$$G_{m_1} = \frac{100 \text{ lb}}{\text{hr}} \Big/ 0.196 \text{ ft}^2 = 509.55$$

$$\boxed{G_{m_1} = 509.6 \text{ lbm/hr.ft}^2}$$

Eq ② $\Rightarrow$ $G_{m_2} = 407.64$ lbm/hr ft²

Area of larger tower $= A_2 = \pi/4\,(12/12)^2$

$A_2 = 0.785$ ft²

so * $G_2' = \dfrac{407.64 \text{ lbm}}{\text{hr-}\cancel{\text{ft}^2}} \Big| \, 0.785\ \cancel{\text{ft}^2}$

$$\boxed{G_2' = 320 \text{ lbm/hr}}$$

$\underleftrightarrow{\text{Gas rate through 12" dia tower}}$

9. A special test on the 6-in. distillation tower described in Prob. 8 indicates that the total pressure drop is 2.9 mm Hg when the gas rate is 50 lb/h. For this test, the condenser pressure was maintained at 100 mm Hg, the *L*/*G* ratio was the same as shown in Prob. 8, and temperature change was negligible. Operation can be considered to be under preloading conditions. Estimate the total pressure drop at the operating conditions indicated in Prob. 8 for the 12-in. column.

DATA:

smaller tower 6" dia:

$G' = 100$ lb/hr   {from previous pb}

$L' = 90$ lb/hr.

Total pr drop $= \Delta P = 12$ mm Hg ✓

$P_{cond} = 100$ mm Hg

$T = 110°F$, $\rho_L = 55$ lb/ft³

condition ①: when gas rate $= G' = 50$ lb/hr.

total pr. drop $= \Delta P = 2.9$ mm Hg ✓

cond. pr $= P_{cond} = 100$ mm Hg.

large tower: 12" dia:

$P_{cnd} = 50$ mm Hg

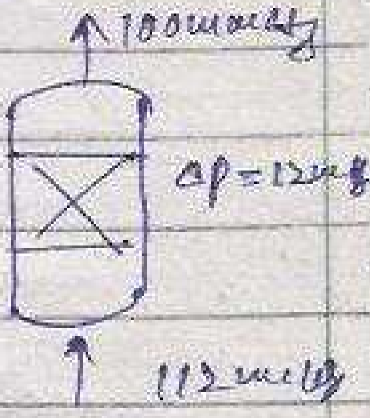
$\rho_L = 57$ lb/ft³ L/G is same as for 6" tower

$T = 95°F$

For smaller Tower :-

∴ For packed Beds & corrugated packing

we know;
$$\frac{\Delta p}{h} = \gamma(10)^{\phi L/\rho_L} \frac{G^2}{\rho_G} \longrightarrow ①$$



$$\text{avg pr.} = \frac{112+100}{2} = 106 \text{ mmHg}$$

At T = 110°F.

$$\rho_G = \frac{M.P.T_{s.c}}{359.P_{s.c}.T}$$

$$\rho_G = \frac{(100)(106)(460+32)}{(359)(760)(460+110)}$$

$$\boxed{\rho_G = 0.0335 \text{ lbm/ft}^3}$$

$$L = L'/A = \frac{90 \text{ lb}}{\text{hr}} \Bigg| \frac{}{\pi/4\,(6/12)^2 \text{ ft}^2} = 458.$$

$$\boxed{L = 458 \text{ lbm/hr ft}^2}$$

$$G = G'/A = \frac{100 \text{ lb}}{\text{hr}} \Bigg| \frac{}{\pi/4\,(6/12)^2 \text{ ft}^2} = 509$$

$$\boxed{G = 509 \text{ lbm/hr ft}^2}$$

$$\Delta p = 12 \text{ mmHg} \Bigg| \frac{14.7 \text{ lb}}{760 \text{ mmHg}} \Bigg| \frac{}{\text{in}^2} \Bigg| \frac{144 \text{ in}^2}{1 \text{ ft}^2}$$

$$\boxed{\Delta p = 33.42 \text{ Psf}}$$

h = 3 ft → packing height

Eq ① ⇒
$$\frac{33.42}{3} = \gamma(10)^{\phi \times 458/55} \cdot \frac{(509)^2}{0.0335}$$

$$11.14 = \gamma(10)^{8.33\phi} \times 7.97\times10^{6}$$

$$\gamma(10)^{8.33\phi} = 1.397\times10^{-6}$$

Taking ln of above;

$$\ln\gamma + 8.33\phi\ln(10) = \ln(1.397\times10^{-6})$$

$$\ln\gamma + 19.18\phi = -13.48 \longrightarrow Ⓐ$$

using condition ① when $G' = 50 \text{ lb/hr}$:-

$$\because L/G = 90/100 = 0.9$$

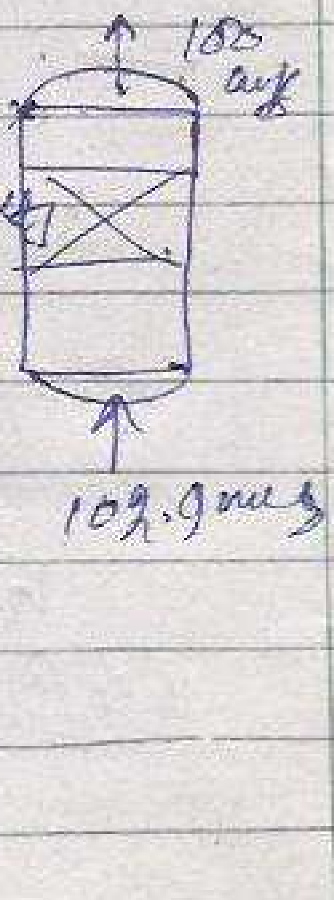
* $L/G = 0.9 \implies L' = 0.9\times50 = 45$

so $L' = 45 \text{ lb/hr}$; $G' = 50 \text{ lb/hr}$

$$L = \frac{45 \text{ lb}}{\text{hr.}} \Big| \frac{}{\pi/4(6/12)^2 \text{ ft}^2} = 229 \frac{\text{lbm}}{\text{hr. ft}^2}$$

$$G = \frac{50 \text{ lb}}{\text{hr}} \Big| \frac{}{\pi/4(6/12)^2 \text{ ft}^2} = 255 \frac{\text{lbm}}{\text{hr ft}^2}$$

$$\text{Avg. ps} = \frac{102.9+100}{2} = 101.45 \text{ mmHg})$$



at $T = 110°F$

$$\rho_G = \frac{(100)(101.45)(460+32)}{(359)(760)(460+110)}$$

$$\boxed{\rho_G = 0.032 \text{ lbm/ft}^3}$$

$$\boxed{\Delta p = 2.9 \text{ mmHg} = 8.08 \text{ psf}}$$

Eq ① ⇒

$$\frac{8.08}{3} = \gamma(10)^{\phi \times 229/55} \times \frac{(255)^2}{2032}$$

$$2.69 = \gamma(10)^{4.16\phi} \cdot 2.032 \times 10^{6}$$

$$\gamma(10)^{4.16\phi} = 1.32 \times 10^{-6}$$

$$\ln\gamma + 4.16\phi \ln(10) = \ln(1.32 \times 10^{-6})$$

$$\ln\gamma + 9.6\phi = -13.54 \longrightarrow Ⓑ$$

Ⓑ − Ⓐ ⇒

$$\cancel{\ln\gamma} + 9.6\phi - \cancel{\ln\gamma} - 19.18\phi = -13.54 + 13.48$$

$$\phi(9.6 - 19.18) = 13.48 - 13.54$$

$$\boxed{\phi = 6.3 \times 10^{-3}}$$

Eq Ⓐ ⇒ $\phi = 6.3 \times 10^{-3}$

$$\ln\gamma + (19.18)(6.3 \times 10^{-3}) = -13.48$$

$$\boxed{\gamma = 1.24 \times 10^{-6}}$$

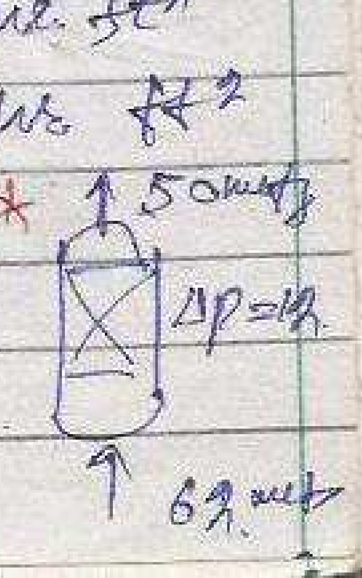
For larger Tower $12''$ in dia

$\Delta p = ?$

From Pb # 08 $\longrightarrow$ $G_m = G = 407.64$ lbm/hr. ft^2 *

& $L = 0.9G \Rightarrow L = 367$ lbm/hr. ft^2

$$\boxed{\text{Avg. pr.} = \frac{62+50}{2} = 56 \text{ mm Hg}}$$



$$\rho_G = \frac{(100)(.56)(460+32)}{(359)(760)(460+95)}$$

$$\boxed{\rho_G = 0.018 \text{ lbm/ft}^3}$$

Eq ① =>

$$\frac{\Delta p}{3} = (1.24\times10^{-6})(10)^{\frac{6.3\times10^{-3}\times 367}{574}} \times \frac{(408)^2}{0.018}$$

$$\Delta p = 37.8 \text{ psf}$$

$$\Delta p = \frac{37.8\times760}{14.7\times144} = 13.56 \text{ psf mmHg}$$

$$\boxed{\Delta p = 13.56 \text{ psf}} \text{ mmHg}$$

Total pr. drop for larger towers:

10. A random-packed tower is to be used for contacting 3000 lb/h of air with 4000 lb/h of water. The tower will be packed with l-in. chemical porcelain Raschig rings. The operating pressure is 1 atm, and the temperature is 70°F. Assuming column operation is in the preloading range, estimate the optimum diameter per foot of packed height to give minimum annual cost for fixed charges and blower operating charges. The following additional data apply:

Annual fixed charges = 0.2 times cost of installed unit

Cost of installed unit, complete with distributor plates, supports, and all auxiliaries = 2.0 times purchased cost of shell and packing

Cost of delivered power = **$0.08/kWh**

Operating time = 8000 h/year

Purchased cost of shell = **$40/ft^3** of packed volume

Purchased cost of packing = **$22.00/ft^3**

DATAS:

$G' = 3000$ lb/hr

$L' = 4000$ lb/hr

porcelain Raschig rings = 1"

ops. pr = 1 atm

$T = 70^\circ F$

column operating in preloading range.

Annual fixed charges = 0.2 (unit installation cost)

cost of installed unit = 2.0 (purchased cost of shell & packing)

opr. time = 8000 h/yr

cost of delivered power = \$0.08/kWh

purchased cost of shell = \$40/ft³ of packed vol.

purchased cost of packing = \$22/ft³

T.F.D :-

dia of column at minimum cost = D_{opt} = ?

soln :-

$$\therefore \quad \frac{\Delta P}{h} = 8(10)^{\phi L/\rho_L} * G^2/\rho_G \longrightarrow ①$$

$$G = \frac{3000\,lb}{hr} \Big| \frac{}{\pi/4\,D^2} = 3822/D^2 \; lb/hr.ft^2$$

$$L = \frac{4000\,lb}{hr} \Big| \frac{}{\pi/4\,D^2} = 5096/D^2 \; lb/hr.ft^2$$

density of water :- at 70°F

$$\rho_L = 62.4 \; lbm/ft^3$$

density of Air :-

$$\rho_G = \frac{M.\,P.\,T_{sc}}{359 \cdot P_{sc}\,T} = \frac{(29)(1)(460+32)}{(359)(1)(460+70)}$$

$$\rho_G = 0.075 \; lbm/ft^3$$

~~power cost~~ From Table #09 page # 694.

$\gamma = 32\times10^{-8}$ $\qquad \phi = 4.3\times10^{-3}$

Eq ① ⇒

$$\frac{\Delta P}{h} = 32\times10^{-8}\,(10)^{\frac{4.3\times10^{-3}\times 5096/D^2}{62.4}} \times \frac{(3892/D^2)^2}{0.075}$$

$$\frac{\Delta P}{h} = \frac{62.32}{D^4}\,(10)^{0.35/D^2}$$

$$\because \quad \text{power} = \frac{\Delta P \cdot \dot{m}_G}{\rho_G} \quad (G')$$

$$\text{power} = \frac{62.32\,(10)^{0.35/D^2}}{D^4}\cdot\frac{\cancel{lbf}}{\cancel{ft^2}\cdot ft}\left|\frac{\cancel{ft^3}}{0.075\,\cancel{lbm}}\right|\frac{3000\,\cancel{lb}}{\cancel{hr}}$$

$$\left|\frac{1\,\cancel{hr}}{3600\,\cancel{sec}}\right|\frac{1\,\cancel{hp}\cdot\cancel{sec}}{550\,\cancel{ft\,lbs}}\left|\frac{0.746\,kw}{1\,\cancel{hp}}\right.$$

$$\text{power} = \frac{0.939\,(10)^{0.35/D^2}}{D^4} \qquad \frac{kw}{ft}\left|\frac{\$0.08}{\cancel{kwh}}\right|\frac{8000\,h}{yr}$$

$$\boxed{\text{power cost} = \frac{601\,(10)^{0.35/D^2}}{D^4} \quad \$/yr.\,ft}$$ ops. charges

→ ①

⇒ packed volume $= \pi/4\, D^2 \cdot h = 0.785\, D^2 \cdot h\ (ft^3)$

packed vol / unit packed height $= \dfrac{0.785\,D^2\cdot\cancel{h}}{\cancel{h}} \quad \left(ft^3/ft\right)$

purchased cost of shell = $\frac{\$40}{\cancel{ft^3}} \Big| \frac{0.785 D^2 \cancel{ft^3}}{ft}$

$= 31.4 D^2$ ($/ft) → ⓐ

purchased cost of packing = $\frac{\$22}{\cancel{ft^3}} \Big| \frac{0.785 D^2 \cancel{ft^3}}{ft}$

$= 17.27 D^2$ ($/ft) → ⓑ

Now ⓐ & ⓑ ⇒

purchased cost of shell and packing $= (31.4 + 17.27) D^2$ ($/ft)

$= 48.67 D^2$ ($/ft) → ⓒ

ⓒ ⇒

cost of installed unit $= (2)(48.67 D^2)$

$= 97.34 D^2$ ($/ft) → ⓓ

Now ⓓ ⇒

Annual F.C $= 0.2(97.34 D^2)$

$= 19.5 D^2$ ~~$/ft~~ $/yr.ft → ②

① & ② ⇒

Total annual cost/ft $= \frac{601(10)^{0.35/D^2}}{D^4} + 19.5 D^2$

for optimum values of 'D'

$$\frac{d}{dD} \frac{601(10)^{0.35/D^2}}{D^4} + \frac{d}{dD}(19.5 D^2) = 0.$$

solving the equation; By trial & Error solu

$$\boxed{D_{opt} = 2.05 \text{ ft}}$$

Mathcad:

Dia (ft)	Annual cost ($/yr.ft)
1	4506
2	406.80
2.05	404.50
2.06	407.50
2.1	408.50

⇒ Hell Yeah!!! ⇐

I'm fuckin true Liar!