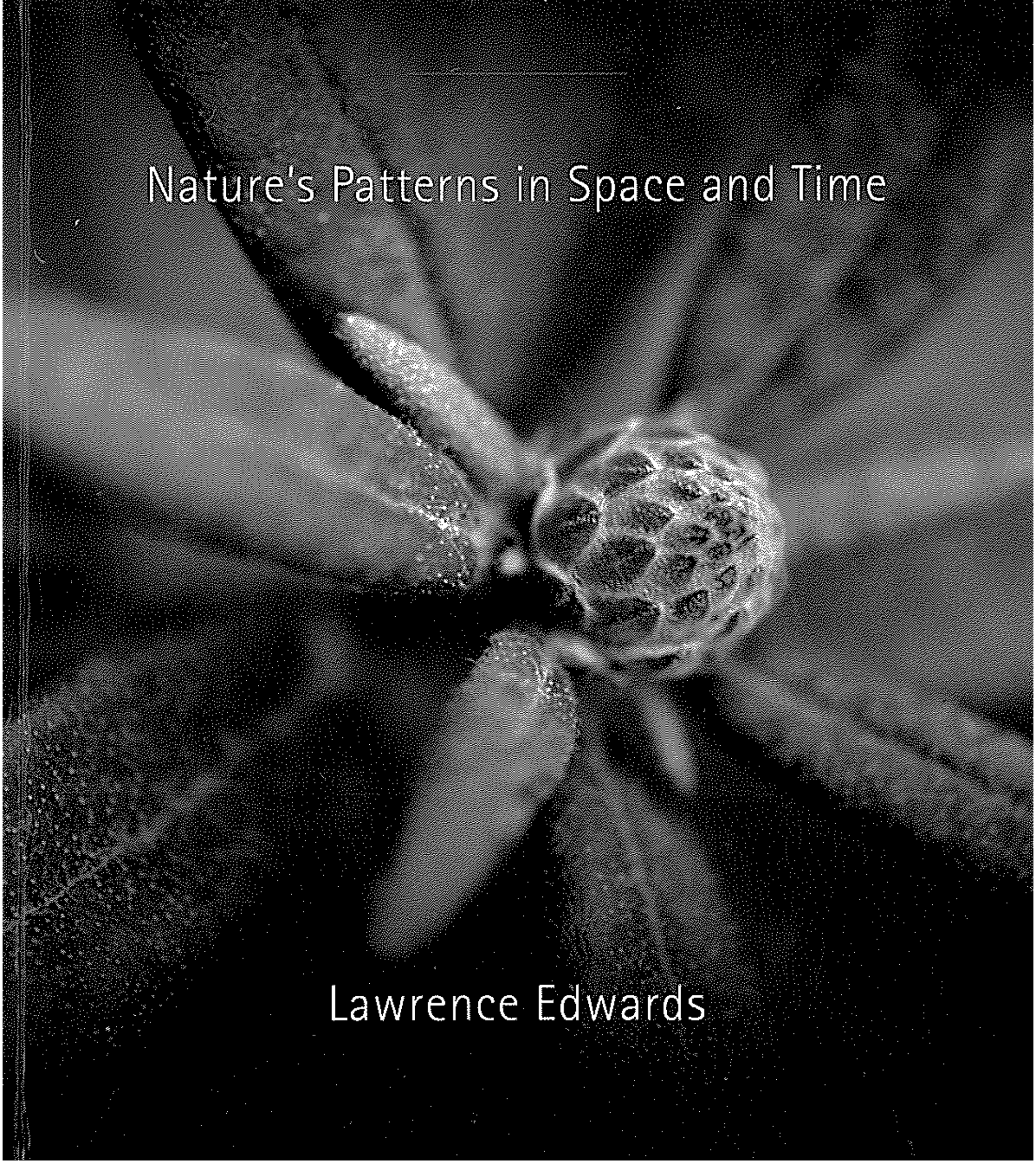
THE VORTEX OF LIFE

Nature's Patterns in Space and Time

Lawrence Edwards



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Floris Books

This edition revised by Graham Calderwood

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Foreword

In the early 1970s Lawrence Edwards came to the Camphill Community at Aberdeen in Scotland to tell us something of his work on the form of pine cones and eggs and buds and hearts. I was in the audience. We learned of the existence of the path curve and heard that this marvellous entity represents the most elementary and fundamental transformation of space into itself. He said that this most simple of all forms is taken up, not approximately, but exactly, by certain living things as they come into physical being. He said he had measured a good many of them, and found it so.

Now, what was this? Did he truly mean to say that he could, from path curve geometry — just the geometry — and a few measurements of the actual item predict all the dimensions of a real bud on a real tree, or of a real egg laid by a real hen; all of them? The implications were profound, so I listened with mounting wonder, and was at last convinced that he truly did mean what he said, and could do it.

So he could do what none could before; he could say what living form is from first principles. So far as I knew until that day, all that had ever been managed by way of exact assessment of living form was statistical, and represented by Gaussian-style distributions and a collection of norms. Norms have their uses, but in the end they are just compilations of measurements, and reveal nothing of principle. No one knew how the numbers fall out as they do.

I suppose most still believe there are no principles of living form. Biologists are still apt to say with Darwin that forms come by lottery; that new forms arise at random, following no particular rule. Winners propagate forms that work best on test, and so form evolves by no more than selection of those chance productions that best aid the grapple with real life. By that bare criterion, the actual principles that might underlie these purely adventitious forms hardly matter. All that signifies is that whatever comes should work better than what came before. Lawrence Edwards showed us on that day that matters are otherwise.

I decided there and then that I had to understand this astounding science in detail enough to do it. I began correspondence with him that led to cooperation between us at several levels over decades — and, for me, a deeply valued friendship. Of course I learned far more from him than he did from me, and I was not by any means the only one to notice the importance of what he did.

There is now a kind of research fellowship for those doing science based on or assisted by projective geometry; its members hold their conferences in Strontian, Scotland, which was for so long Lawrence Edward's home, in his honour and in recognition of his fundamental and pioneering contributions to this work. The fellowship provides a focus and a point of exchange for the work, and a place from which it can be fostered. There is much still to do.

Not long ago it was suggested to Lawrence that a fresh edition of The Vortex of Life, one possibly including new research, should be prepared. Lawrence said that I should see to it — and I was very happy to do this. Now there was nothing of his original that could be improved, or needed correction, so I had in this respect an easy time of it. As for the new research, I have added a little of my own; there is a study confirming his work on the heart, and something of my attempt to understand just one of his several discoveries, the bud phase-shift.

Graham Calderwood
March 2006

Preface

It is nearly forty-five years ago that I first came in touch with the work of George Adams (1894–1963), and for many years thereafter I had the privilege of learning from him, and working with him. During this period I was able to benefit from his remarkable ability to take the most abstruse mathematical concepts and make imaginative realities of them, without sacrificing the essential precision of thought in which they had been conceived; also, with growing respect, I learned to value his almost intuitive vision as to how these realities can be conceived, and almost seen, to be at work in the spiritual-physical world around us.

After his death it was, and has always continued to be, my aim to follow this work further in every way in which I could, but especially in relation to the forms of living nature. And this book, parts of which were first published ten years ago under the title, *The Field of Form,* is a description of some of the research since then, especially Chapters 12 to 16.

In preparing this new edition I have had two main purposes in view. Firstly to give as clear and vivid an account of the work as possible to the lay reader. To this end the main body of the book has been rewritten with the non-mathematical reader wholly in mind. No hint of equation or formula has been allowed to insert itself into the text, but without, I hope, sacrificing essential accuracy and clarity of thought. It must be left to my readers to judge how successful I have been in this.

The other aim has been to encourage mathematically inclined readers to take such lines of research for themselves. This book is really little more than a set of unanswered, or at best partially answered, questions. Only consistent and detailed work will enable further progress to be made. For those who wish to essay such progress, the basic relevant mathematical tools will be found in the appendices. May good fortune attend those who wish to try!

In recent years, there has been a great change in my working methods. Up to 1987, hand measurements and calculations were all taken from analysis of photographic records of the selected buds. The methods for this, and the subsequent calculations, are described in Chapter 5 and Appendix 3. They involve measuring the diameter of the bud at a series of equally-spaced intervals along the long axis of the bud. These calculations are time-consuming and tedious, and these days it is better to use a computer.

Since the summer of 1987 I have taken no more photographs. At that time I was gifted by the Margaret Wilkinson Research Fund with an electronic scanner which eliminates the necessity for photography. This remarkable machine not only does all one's calculations but also makes the measurements on which they are based. It consists essentially of a computer with a wide range of memory, an extra monitor screen and a TV camera. When the buds, mounted on their glass measuring plate, are put into the machine, an enlarged image of them appears on the computer screen, and from there a message is sent to the computer giving, firstly the *x*-*y* co-ordinates of the tip of each bud, then of its base, and thereafter of the left- and right-hand ends of each diameter, working from the base end upwards. This data goes into a new file for each bud, numbered from 1 to 20. The computer program then goes from file to file, works out the dimensions of the mean bud of the whole set, and delivers all the required calculations.

When the machine arrived, it was found to give results satisfactorily consistent with those which were already being achieved by the most careful hand measurement and of equal reliability and accuracy. But where the machine really scores is in its speed of working. By using it, I was able to do between four and five times as much work in a set time as previously. After this, nearly all the work has been done with twenty buds per day instead of the previous ten, thus giving greatly increased reliability, whilst covering twice as many species. I should like to record here my very great thanks to the Margaret Wilkinson Research Fund for the great help to the work that this has meant.

In the autumn of 1991, the Fund bought another such machine and this is now in use in the south of England. Just about the same time my friend, Graham Calderwood, of Camphill, Aberdeen, completed the construction of another similar machine, with which he has been doing most useful work. So since the beginning of 1992, we can hope to have results from at least three independent observers separated by five hundred miles of countryside.

Lawrence Edwards
Strontian, Argyll

Introduction

As we stand in life so we find ourselves immersed in a world which seems too wonderful for our comprehension. We find ourselves surrounded by questions to which we may, perhaps, come to despair of finding adequate answers. It was Rudolf Steiner (philosopher and scientist, 1861–1925) who pointed out to us that this difficulty in comprehending our world stems from a peculiarity of our own human organization. The world confronting us is essentially a unity, an undivided whole; but we, as human beings, are condemned to apprehend it from two opposite directions; on the one hand there is that which comes to us through our senses, through seeing, hearing, touching and so on, the percept: on the other is that part of reality which we approach through our thinking, the concept. As long as these two things remain separate in our consciousness the world presents nothing but incomprehensible riddles to us. The act of knowing must be the activity of bringing these two aspects meaningfully together again.

He pointed out to us the remarkable progress achieved in this direction by the great German natural philosopher, Johann Wolfgang von Goethe. Goethe sought to immerse himself in the world of pure percept, with such devotion, and so selflessly, that out of these perceptions the concepts would speak themselves to him. He sought as it were to perceive his ideas in the living texture of nature itself. And the wonderful achievements which he inaugurated have been developed by others in our day into a whole discipline of what is sometimes called Goethean thinking. And if I now suggest that this is not the only way of approach which we may take to the realities of the world, it is not because I wish in any way to denigrate it. May I come, in time, to practise it more perfectly than I can now!

But I believe that there is another path. It was Rudolf Steiner again who called upon us to develop 'sense-free thinking.' To do this we must plunge ourselves into the world of pure concept. And if we do this, sufficiently concretely, and sufficiently imaginatively, perhaps we can allow the concepts themselves to lead us to their relevant percepts. To do so is to take our start from an exactly opposite standpoint from that adopted by Goethe. He wished, as it were, that he should perceive his ideas, working as living forces of nature; on the other hand it might be possible to make one's conceiving so concrete, and vivid, that it would encompass and illumine our perceiving. Perhaps we might be allowed to characterize such activity as being polar-Goethean thinking, in which case, the whole of this book is simply an essay in polar-Goethean thinking.

In using this expression there can be no suggestion of trying to put forward some kind of antithesis of Goethean thinking. It is indeed just within that thinking, and also within our geometry, that we come to appreciate the peculiarly intimate relationship that exists between true polar opposites. They become for us twin aspects of the one essential reality. Speaking in this sense, to say that a person is practising polar-Goethean thinking would mean that he is being as Goethean as it is possible for him to be!

But Goethean or polar-Goethean, the end-result which is sought is the same: the truthful bringing together of inward thought and outward perception, the healing of the split which the limitations of our organism have imposed on our apprehension of an undivided world. (We call it a *uni*verse, do we not?) And when that which seemed to be two becomes one, then indeed we stand at the threshold of the act of knowing.

But we speak of plunging into a world of pure concept. Where can this be found more truly and more directly than in the study of mathematics? Here indeed we enter a realm of self-existent conceptual reality. The danger in doing so is that we get carried away into regions of pure abstraction; bemused by our own cleverness we allow the symbols to become mathematical playthings, devoid of essential reality. But in the study of projective geometry, carried out properly, we have a discipline which can in a remarkable way imbue our thinking with imaginative reality, and this in its turn can cast much light on other realms of mathematical thought. And this thinking can then become, I believe, a true illuminator of the way our perceiving should take.

But I would like to try to express the matter somewhat differently. The question is sometimes asked: These invariant tetrahedra, these in-curving and out-swelling logarithmic spirals, these twisting vortices, which we can conceive and describe through our studies of projective geometry, are they to be thought of as being really *there* in living nature? Now this is a question which I would rather not answer; in fact it is one which I would really rather not ask. I do not think it leads in the direction of truth. I think that things are otherwise. I believe that reality is something spiritual-physical, and that it is more wonderful by far than anything the mind can conceive or the eye ever see. But when this reality comes to expression in the mind of a mathematician it does so in the beautiful forms, some of which I have tried to describe in the following pages: and when it comes to expression in the outer world it does so in the manifold forms of living nature. And when we bring these two things together, being expressions of the same reality, we find that they fit. And when we bring them together, provided that we can summon the wit and the will to proceed properly further, we can find that we stand at the gateway (the gateway only, be it noted) of knowing.

But on such a path we must proceed with the utmost circumspection. No hint of wishful thinking or fantastical prejudice must be allowed to cloud our judgement. Again and again, and ever again, we must put our questions to Nature. And when she answers 'No' this answer must be accepted unequivocally and absolutely. Balaam's ass was a wise teacher. And a negative answer, if it is heeded carefully, may sometimes be found to give a hint for further progress. Two instances of this are described in Chapters 10 and 14. Other instances of straight-forward negatives I have not detailed; they would make dull reading.

Now does all this mean that anyone aspiring to knowledge needs to become a mathematician? By no means! This is only one of several paths; and even on this, it is possible to let the results achieved illumine one's imagination, without mastering the details by which they have been achieved. Indeed, the non-mathematician may at times fare better; his imagination is free to roam, while that of his more technically trained brother runs the danger of getting bogged down in the mechanical details.

1. Concerning 'Whole' and 'Straight'

One of the principal problems confronting biology today concerns the fundamental nature of the living organism when considered as a whole thing. We have been accustomed so long to regarding the living being as a great conglomeration of cells, and each cell as an admixture of its constituent molecules, that it easily becomes almost second nature with us to see the living organism simply as an aggregate of its parts. Such a viewpoint, often labelled (sometimes one feels with a certain degree of denigration) reductionism, has undoubtedly been the source of many wonderful advances in knowledge, but it is necessary that we should realize its inherent, and very serious, limitations. One reduces the experience of the living whole to the concepts of its various parts; but when one tries to put these parts together again, concept upon concept, the idea of the total living being all too easily evades one; one is left with simply a physical mechanism, knit together in ways which are still totally incomprehensible.

Many of our most percipient biologists have been coming to see, in recent years especially, that the living organism as a whole thing must be considered as something greater than the sum of its component parts. The separate parts must be there of course; but something else is needed, some overriding reality which takes these parts and organizes them into the close-knit unity which is the real living thing, and which has 'reality,' in its own right, just as truly as any of the parts which we can feel and handle. We may decide to speak of a 'network of relationships' or an 'organizing principle' or a 'body of formative forces' but the important thing is to see whether, by careful observation and unprejudiced thought, it is possible to approach some idea of this overriding reality. I am not suggesting that the work which I am describing here has done more than open up the beginning of one possible way which one day may lead to a real vision of these things, but if it does only that, I should consider it to have been worth the doing.

It was Rudolf Steiner who, many years ago, suggested that people who were concerning themselves with such matters would do well to study projective geometry; and this indication was taken up by just a few of his followers and friends (notably George Adams, to whom I am so much indebted) with definitely fruitful results. And as this present work is based almost wholly on projective methods it would perhaps be advisable to say a few words about this geometry here.

What is projective geometry, and how does it differ from other geometrical forms of thought? This is a question about which a whole book could well be written, and what I am about to say cannot be more than an incomplete and perhaps rather naïve description of just one aspect of a wide-ranging matter.

Right at the start of our geometrical studies we normally define a point as that which has position, but no magnitude. And if we consider an infinite set of such points, lying side by side, we have a line. Over two thousand years ago Euclid told us that if this line lies 'evenly' between its end points, then it is 'straight.' Now immediately we notice two things; firstly that the line is defined in terms of its constituent parts, and secondly that the quality of straightness is not defined at all. What is 'straight'? Well, it is 'evenly.' But if we had gone to Euclid and asked him, 'What is "evenly"?' I think he would perforce have had to agree with us that it means 'straight.' The thing is little more than a tautology, and it would give us no idea whatever of what straightness is, unless we had already experienced it as a concept before we ever started on the study of geometry.

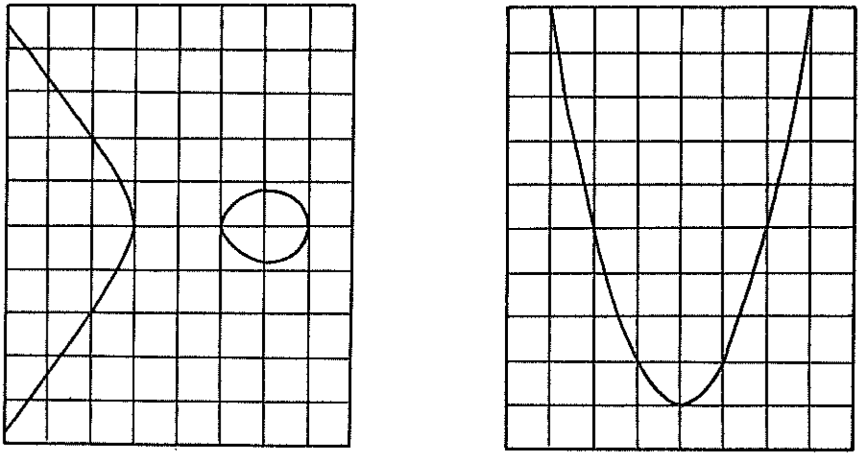
Now in projective geometry we take our start from a rather different standpoint. We say that the primary elements of space are three: point, line (meaning always 'straight line,' otherwise we call it a 'curve'), and plane (the plane having in its 'flatness' a quality intimately connected with the 'straightness' of the line). Each of these elements is considered to be equally primary with the others; for instance a line can be considered as 'containing' an infinitude of points, but a point can also be considered as 'containing' an infinitude of lines (all those which pass through it). And straightness (and with it, flatness) are not normally attempted to be defined. They are qualities which it is assumed that we have already intuitively experienced. We must already know (or think we know) what the straight line and the flat plane are (and are like) before we start our studies. And strangely this, which in reality is the most difficult of all matters, normally causes the student least trouble. For we all 'know,' deeply and intuitively what straightness is, and whether a thing is straight or crooked. This was borne in upon me very incisively recently when I was speaking on just this matter to a roomful of mathematicians. I suddenly became aware that in the whole city in which we were living, we were probably the only people who were not confidently and absolutely sure about this question! Had I gone to any man in the street and asked him whether he knew what straightness was, he would have considered me a fool even to have asked such a question. In a world filled with doubts and fears, everyone knows this! Yet in spite of such inner sureness, it is a concept which grows more and more elusive, the more one thinks about it. Those who hold that it is an elementary experience, incapable of being defined in terms more primary than itself, surely have a good case.

At this point it would be useful to consider a characterization of geometrical study from a rather different point of view. During the past century the study of almost all geometry has been seen to be intimately connected with that of 'transformation'; this is true almost as much of the classical Euclidean geometry as of the many newer ones which have been developed. In modern topology for instance, many kinds of sophisticated and very powerful transformations are studied.

This  can turn into this , and this  into this  without any sort of trouble at all. But if we go to the textbooks and ask what it is that distinguishes a projective transformation from all the others, we are told in book after book that the projective transformation is 'algebraic.' And since this little word seems to be a vitally distinguishing one for our geometry it is well that we should look at it rather more closely.

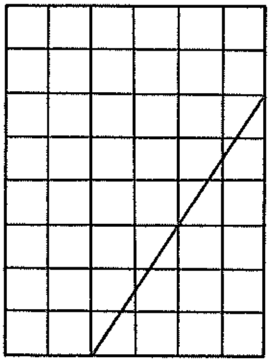
What do we mean when we say that a transformation is algebraic? It simply means that it is one which can be expressed by an algebraic equation. But this is another tautology! What does it *mean* to be algebraic? Let us consider the simple equations of our school days: these *xs* and *ys*, what *are* they? They are certainly not numbers, because normally they bear no fixed quantity. But we can say of them that they are quantityless elements, which nevertheless *behave* according to all the laws of number. And when we say that our projective transformations are algebraic, we are in fact stating that our geometry is one which works according to the laws of number. If we are looking for reasons to support our believe that the truths of projective geometry are specially significant we could at this point hark back to the ancient Pythagorean doctrine that the secret of number is the secret of all things, and that the universe peals with a cosmic music which at all times sounds forth as the nature of number.

But it will perhaps be more useful for us to recall a little of our school-day algebra. We will remember that if we have any simple equation with



$x^3 + y^2 - 4x = 0$

$x^2 - y - 1 = 0$



$3x - 2y - 5 = 0$

Figure 1. Only those equations which have the indices of both x *and* y *as unity (the indices are then usually not written) are able to produce a straight line as their graph.*

two variables, say x and y, we can express it as a simple graph. In Figure 1 we see three such simple graphs, with their relevant equations. As we look at them we become aware that the third has a unique quality which sets it immediately apart from the other two: it is straight whereas they are curved. And when we look to the equations to see where the difference lies, we see that in the two curved ones, at least one of the variables, x or y, carries an index (the little number up in the air), whereas in the straight-line equation, no index appears. But this is something of an illusion; in reality every x and y bears an index, but where that index is unity (1) mathematicians do not generally trouble to write it; they allow it to be understood. (The art of mathematics is bound up with that of simplification; no mathematician worthy of his salt would like to be found guilty of making an unnecessary stroke on his page!)

Now we can enunciate a general rule: whenever an equation has the indices of all its variables unity, and only then, will its graph be straight. And here, I am beginning to feel, we have the possibility of making our way towards a definition of straightness which is perhaps even more primary than the concept of straightness itself. We begin to see straightness as that quality which is associated with oneness, indeed which may be looked on as the picture, so to speak, of oneness in our spatial consciousness. And in the deepest depth of my being, at the most primary level of all, I do know the difference between unity and plurality. And, fundamentally, I know this difference because I am an ego-conscious being. I know what it is to be One!

Surely the significance of these things shows in the life around us. It is my ego-consciousness that separates me in the realm of soul from the beasts; and in the realm of space I am distinguished by my upright posture, that I can stand erect. It enters into our speech. When a man is honest, when what he says and what he means are one, then we say of him, that he is 'straight.' And when he means one thing and says another, we say that he is practising duplicity, and we call him crooked. We are bound up here with the whole question of integrity. When Christ cast the devil out of the man of the Gadarenes he asked him his name, and the devil replied: 'My name is Legion; for we are many' (Mark 5:9). This man was sick because he was many; to be healed he had to become one. How often in the Old Testament does one read that such-and-such a person was an 'upright man'; even as a child when this was read to me I never supposed that it referred to his posture, although today I can believe that he would also have held his head high. When Christ asked of the suppliant: 'What would you that I should do for you?' we are told again and again that the answer was not: 'Lord, that I should be healed' but: 'Lord, that I be made whole,' made one.

This set of interrelated concepts, wholeness, oneness, straightness, we come upon as soon as we follow the consequences of allowing our world of thought to be 'algebraic,' that is permeated with the essence of the laws of number, and it is this same quality which characterizes the nature of our projective transformations. Is it any great wonder then that at the very heart of our projective geometry we find the intuitive notions of straightness and flatness? The projective geometer cannot take a single step without them.

The fact that these relations hold good is borne out by the fact that for many generations of mathematicians any process or transformation which relies solely on the first powers of its variables has been called 'linear'; and mathematicians have always realized that when they are dealing with linear functions or processes, they are working with something of peculiarly fundamental significance. Now at the heart of projective geometry there stand just two kinds of transformation which are linear; they are called *collineation* and *correlation;* they stand so near to the heart of the geometry that some purists would wish to regard as truly projective geometry only those things which arise from these two linear processes. In fact, by using projective methods, we can extend our studies to include many other kinds of transformation of higher order (and therefore less fundamental) that are still algebraic, and most of us will wish to retain the freedom to include these things also in our projective studies. But the heart of the matter lies with the linear processes, and, for our purposes, especially with collineation.

Collineation

In Figure 2a we see a simple diagram, an equilateral triangle with its inscribed circle. In Figure 2b we see it transformed by a quadratic transformation, one which, if it were to be expressed in algebraic form would contain at least one second (or squared) power of the variables. We notice the very dramatic nature of the change; the straight sides of the triangle have changed into a sort of 'cocked hat' form, and the circle has changed into another rather similar one within it. In this particular case something of the symmetry of the original figure has been preserved, but that is only because of the particular way in which the transformation was applied. In a more general case the symmetry would also have completely vanished. In Figure 2c we see the result of submitting the original diagram to a collineation. Again the changes are marked and important, although not quite so dramatic. The circle has changed into a curve of quite different shape (in this particular case, an ellipse); the fact that it used to touch the sides of the triangle at their midpoints no longer holds; all traces of the original symmetry have vanished; the equality of the lengths of the sides of the triangle has gone; but what was straight in

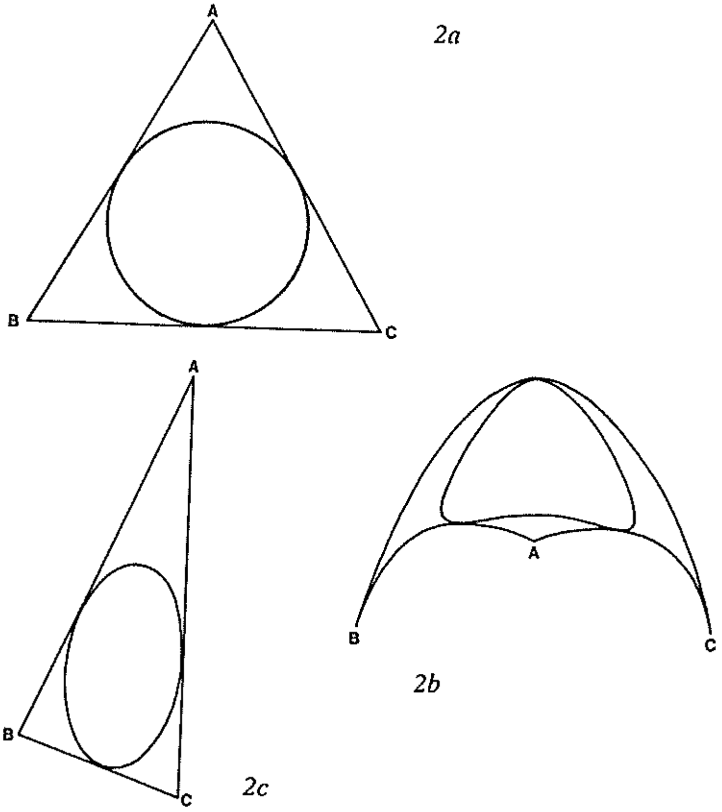


Figure 2. Figure 2a has been transformed into 2c by means of a collineation (a linear transformation) and the straightness of all straight lines has been preserved. A more sophisticated transformation has been used to change 2a into 2b, and everything has become curved.

the beginning inevitably remains so in the sequel. Collineation is a linear transformation and it will not surprise us that of all transformations it is this one which always recognizes and preserves straightness.

All this places the collineation in a special position. It arises from the most fundamental and elementary processes possible in the whole realm of number relations, and it embodies these qualities in spatial change and movement. At the same time it is intimately involved with the quality of straightness which we experience so deeply and intuitively out of the integrity of our own self-conscious being. Indeed I think we can say that it is a process which is very near to the foundation of being itself.

All the work described in this book, up to and including Chapter 8, has arisen completely from the study of collineation. Only in Chapters 9, 10, 12 and 13, where we come to consider the ovary and seed-bearing activities, and after them the embryonic development, has it been necessary to have recourse to a more sophisticated process, the pivot transformation. And even this arises as a direct consequence of the collineation itself.

At this point the reader, after a casual glance through the book, might ask how it comes about that of the forms and surfaces pictured, so many are curved, and so comparatively few are straight or flat. Here we confront one of the remarkable aspects of this realm of thought. It is a fact that we have only to set the most simple and primary processes into action, to find, without our having to import any further complications, that more sophisticated considerations are already implied, inherent within them. And these first curves and curved surfaces, that thus arise, are probably the most elementary things of their kind which it is possible to find.

We meet here a strange parallel with one of the troublesome problems of current biology. Darwinian evolution is often pictured as Nature's struggle towards achieving that quality which has been described as 'fitness to survive.' But many of the primordial few-celled organisms have the greatest survival value of all. If the grand goal is really 'fitness to survive' one is driven to ask why Nature should ever have embarked on the perilous journey towards more complex organisms, which face much greater hazards from their environment. One finds in Nature a continuous urge towards ever increasing complexity which is completely unexplained by the law of the survival of the fittest.

And here in the realm of pure thought it is not possible to set the simplest linear processes in action without having an increasingly sophisticated set of complications arise, as it were of itself. And these, arising as they do with a certain inevitability out of the most elementary linear processes, express themselves in the first instance as the path curves and surfaces with which this work is concerned.

Before going any further we must spend a short time considering the polar opposite qualities of space. George Adams' work on what he sometimes called positive and negative Euclidean space, or extensive and intensive space, was one of the outstanding contributions to the thought of our century. These things were worked out by him in very great detail but can only be touched upon lightly here.

As was pointed out earlier in this chapter, projective geometry sees the primary elements of space as being threefold — point, line and plane — and it has frequently been shown that the first and last of these stand in polar

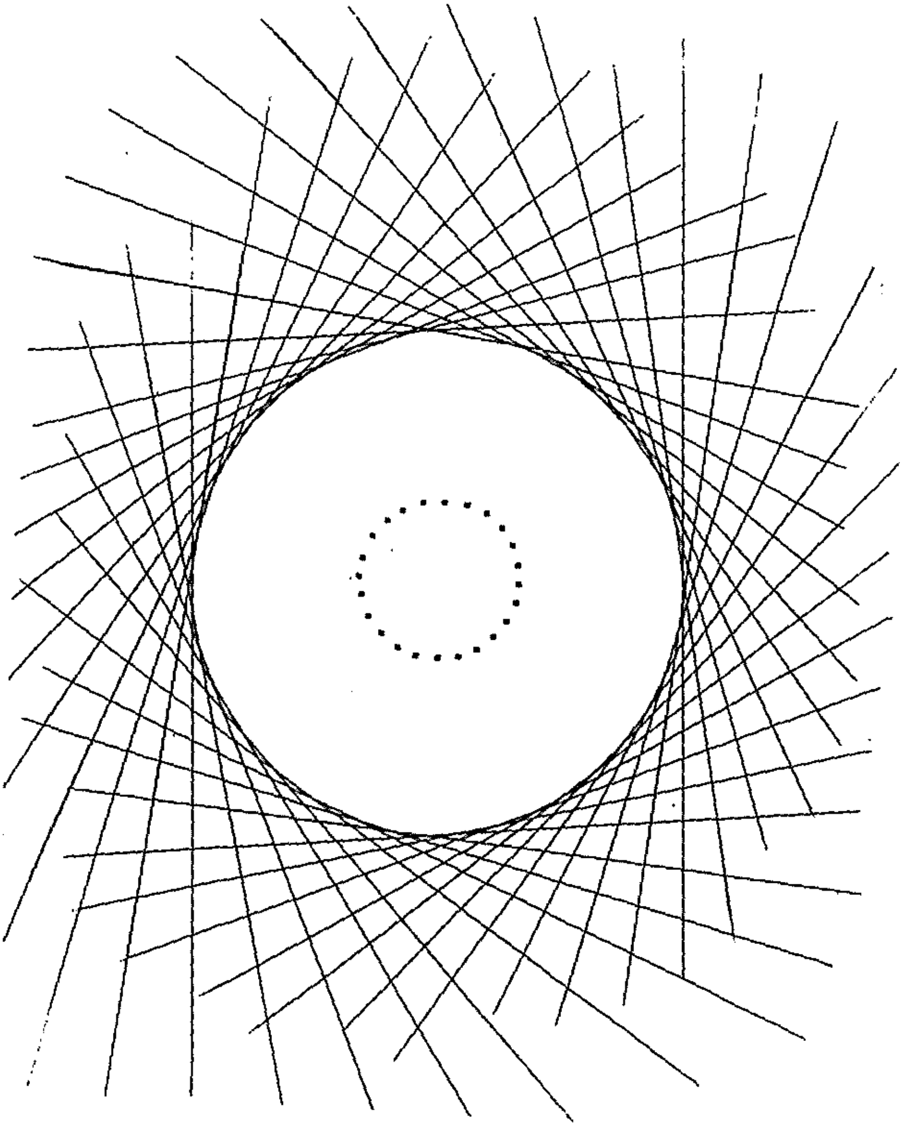


Figure 3. The circle of points can picture for us an extensive sphere; whereas if each line be taken to represent a plane, the totality of these lines will represent for us an intensive sphere.

opposite relation to one another. If in any figure made up of points, lines and planes, we interchange the functions of point and plane, we come upon a new figure which is in every respect the polar opposite of the first one, the lines playing a similar role in each of them. And we find that the new figure is an inside-out equivalent of the first one.

It became an important task for George Adams to apply this process not to any particular figure, but to our concept of the whole of space itself. An observer in extensive space feels himself point-centred in the middle of his universe, and he looks out through the extent of space to an infinitude without-the so-called plane at infinity of our ordinary projective text books. In intensive space he has to regard himself as a planar entity, in some sense a peripheral being, gazing inwards to a point-centred infinitude, the infinitude within. In extensive space, we find it convenient, and natural, to see our objects as being made up from points (atoms if you like) while the planes are formed from, and implied by, myriads of points. Whereas in intensive space the fundamental building blocks for our figures will be enveloping planes.

These two spaces stand in polar relationship to one another, and as we study them we become increasingly convinced that extensive space, with its space-filling figures made up fundamentally of points, applies principally to the physical world of objects; and intensive space, with its inward-looking and enveloping forms, made up in the first place from tangent planes, is more suited to what we may call the ethereal world of life.

To take a simple example of such forms, an extensive sphere will consist of all the points contained on its surface and within it (Figure 3). Its presence will imply a whole set of planes — those which are tangent to its surface. Its inside is the space where its points are, and the outside is where they are not. On the other hand an intensive sphere is made up of all its tangent planes. It is a hollow and enveloping form. Its presence implies a whole set of points, the points at which each plane touches its surface, their points of contact. Its inside will be where the planes are (that is, what we, with ordinary consciousness, would regard as its outside) while its outside will be where its planes are not (what we would normally consider its inside). As the extensive sphere grows larger, its points will cover more and more space, until at last it will merge into the plane at infinity. As the intensive sphere grows 'larger' it will seem to ordinary consciousness to be shrinking, until it merges into the point at infinity within.

With these thoughts in mind, let us consider an extensive line, that is, an infinite set of points, all bound together by the quality of straightness (Figure 4). As against this we can put the intensive line, that is, an infinite

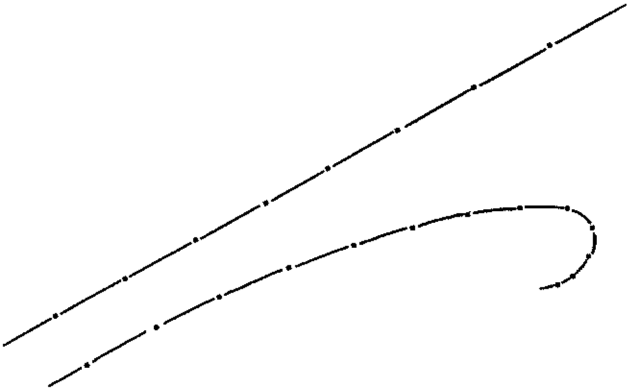


Figure 4. An extensive line is made up of an infinitude of points. When this set of points loses all its straightness, it becomes a two-dimensional extensive curve, all the points still lying in one plane.

set of planes all bound together by the straightness of the line which they all share in common (Figure 5). Notice that no matter from which point of space your eye views the intensive line, it will always lie in just one of its planes. The extensive line is infinitely big because it extends to infinity in both directions. The intensive line is infinitely big, not for this reason, although of course it does extend infinitely in each direction, but because

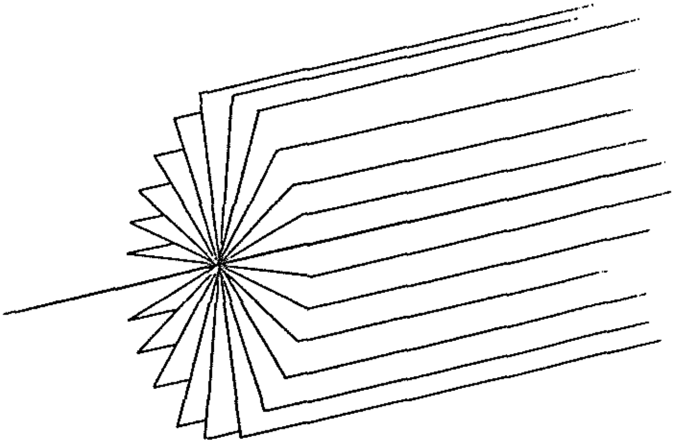


Figure 5. An intensive line is made up of an infinitude of planes.

it contains an infinitude of planes. The extensive line is in a sense a space-filling entity; it is as though its points are saying: 'You cannot be here, because we are filling this space'; while the intensive line gives one much more a hollow feeling — it is like a receiver into which the qualities of all space flow, along its component planes.

Now suppose we have an extensive line, but some transformation comes to bear upon it which causes its points to break free from the straightness which bound them together, maybe into some spiral curve. What would be the equivalent picture for an intensive line? Its component planes would unwrap into a spiral cone (Figure 6). This cone must be imagined entirely planewise; it consists only of its tangent planes, each lying in the central common point, just as the extensive curve consists of points each lying in the common plane of the page.

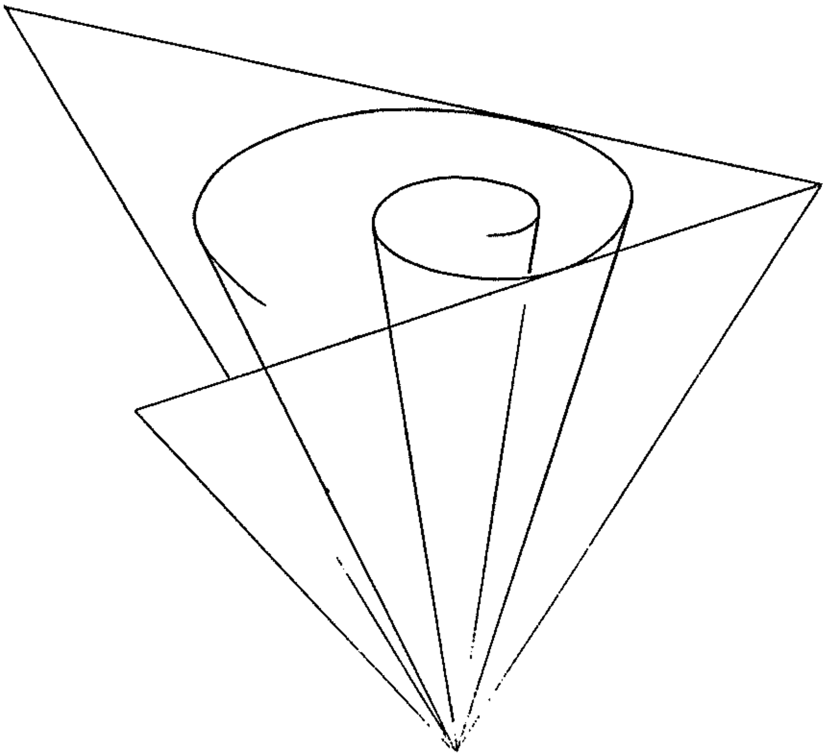


Figure 6. When the set of planes forming an intensive line loses its straightness it becomes a two-dimensional intensive cone made up entirely of its tangent planes, two of which are drawn here. All these planes lie in one point-the apex of the cone.

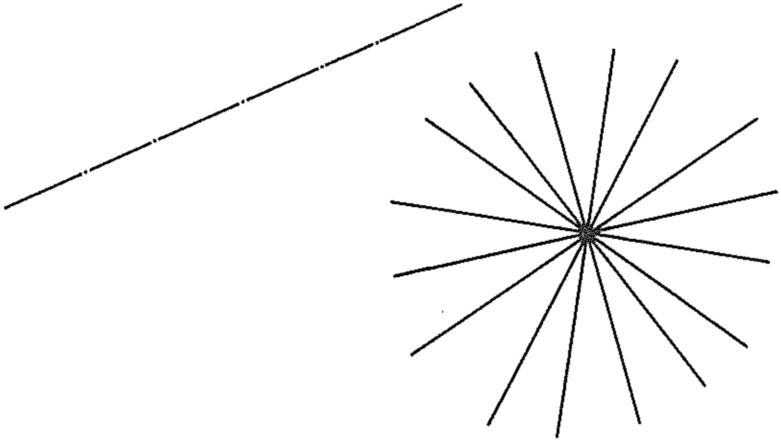


Figure 7. Straightness: *An extensive expression of the principal of oneness. The radiant line: An intensive expression of the principle of oneness.*

In plane geometry these things have a two-dimensional equivalent. Here the polar forms are simply point and line, and these have to be seen as each being equally primary as the other. In as far as I see a line as an aggregate of points, so I must be prepared to see a point as an aggregate (a meeting place) of lines. This latter picture was named by Leonardo da Vinci, with something that seems almost like a cry of joy in his notebook on perspective, the radiant line.

I set a series of points 'in line,' that is, I link them by the principle of oneness. Then I say that they are straight. Straightness becomes for me an *extensive* picture of this unity principle.

I set a series of lines 'in point.' Again I have linked them by the principle of oneness; and I can now speak of the radiant line. And in this I have an *intensive* picture of this same unity principle (Figure 7).

2. Straightness in Action

In the previous chapter we have developed a whole set of ideas about straightness, but it was all done in concept. How about our perceptual world? Where do we see straightness, or its approximation, in our surroundings? We see it in a multitude of places: the edges of our tables and desks, the posts and lintels of our doors, in wall, floor and ceiling, in mast, tower and spire. In fact it might be argued that if we had not experienced at least some of these things, quite naïvely, a large part of the preceding chapter would have been completely incomprehensible to us. But all these things are artefacts of man. Can we find it in nature? Now this is not so easy. Living nature hardly ever exhibits it exactly, although in the stems of plants we are probably justified in seeing a real striving towards the straight. Occasionally we find it in the mineral, for instance the edges and faces of a crystal. But in one phenomenon nature expresses this quality for us with great exactness over and over again. It is in the edges of a beam of light. In the beam of light I believe that we get an archetypal experience of straightness.

In Figure 8 we have a diagram which could have been taken from the early pages of almost any textbook of elementary projective geometry. We have two lines, m_1 and m_2, and a raying point O.* And we have a transformation of the points of m_1 into those of m_2; A_1 transforms into A_2, B_1 into B_2, and so on. This elementary transformation is called perspective. It is related only loosely to the artist's laws of perspective, but the use of the word is evidence of the early projective geometricians' clear perception that we are dealing here with the fundamental laws of light. And the study of projective geometry starts with examining the changes in distance relationships which take place (and more especially those which do *not* take place) between the points of m_1, before transformation and those of m_2, afterwards. And now we can see that this process is wholly linear; the points of m_1 are in line; so are those of m_2. They are straight, extensively. The lines joining A_1 to A_2, B_1 to B_2, and so on are 'in point'; we could say, if we wish, that they also are 'straight,' intensively. The thing is permeated with this principle of oneness; nothing else is allowed to work into it.

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* We shall follow the convention that points are always labelled with capital letters, and lines with small ones.

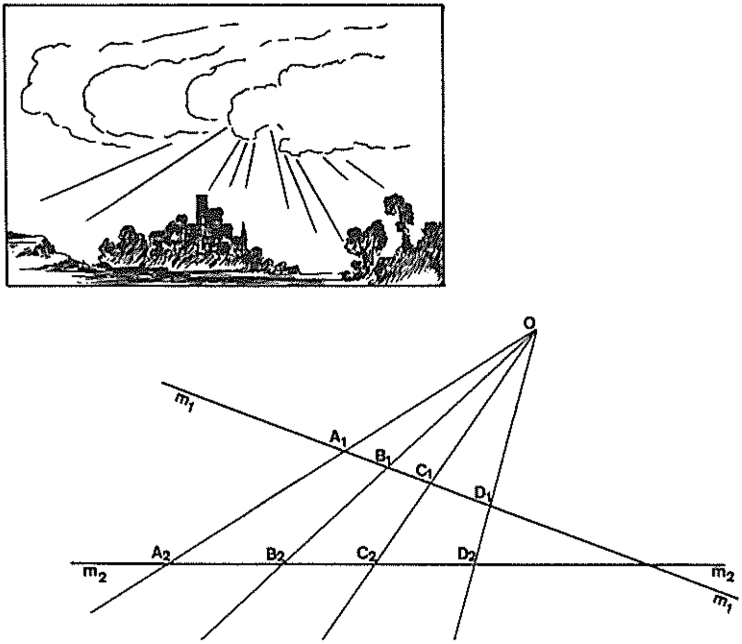

Figure 8. The fundamental projective transformation works by the raying of lines from a point. It is similar to the working of light in space.

An ordinary slide show is a three-dimensional equivalent of this. The line m_1 represents our transparency, m_2 our screen, and point O our projector. Our science is not called 'projective' geometry for nothing; again we see how truly we are working in the realm of light. If the slide and the screen are parallel the transformation is one of pure enlargement; all the shapes and proportions are unchanged; the image on the screen is simply bigger; and this is how we like it! But if we set the screen at various other angles with the slide we can produce all sorts of distortions of the image. Well, this is not strictly true; no matter how wildly we distort the image, one distortion will never be achieved: a line which was straight on the slide will always be straight on the screen. It is a linear process; straightness is recognized, and preserved; it is sacrosanct.

But let us return to our two-dimensional example. It would be perfectly possible to repeat the process and perspect the points of m_2 from a second source of light, on to a third line or, more interesting for our present purposes, back on to the line m_1 from which they came.

3. Path Curves in One and Two Dimensions

Growth measure

We are now going to take some of the questions posed at the end of Chapter 2 and subject them to much closer attention. Let us look back to Figure 9. There we found a simple method of transforming the points of line *m;* A_1 transforms into A_2, and obviously any other general point of the line can similarly be transformed into another point using exactly the same process. We considered the possibility, after we have transformed A_1 into A_2, of transforming A_2 into another point A_3, and then transforming that into yet another, which we call A_4, and continuing this process indefinitely until we arrive at a series of points which would show successive positions of point *A* under repeated applications of the same transformation. What would such a series of points look like? And would it have any special mathematical qualities?

In Figure 10 we show a line *m,* with one of its points treated in just such a way. We find that we have a series of points which are quite widely spread in the middle part of the line, and more narrowly spaced to the right and to the left of this middle section. On more careful examination this proves to be a very important geometrical form and one possessing some remarkable properties. George Adams called such a series of points a growth measure, and we shall continue to do so after him. The appropriateness of the name will become increasingly apparent as our studies proceed.

This one-dimensional collineation is entirely linear in character and it is the simplest and most elementary way possible of transforming a line into itself; it has only to be applied over and over again to set every point of that line moving in growth measure.

Now when a mathematician comes upon any transformation which causes the elements of some form to change, one of the first questions he asks himself is whether there will be any elements of that form which will *not* change, which will be invariant, under that transformation. The

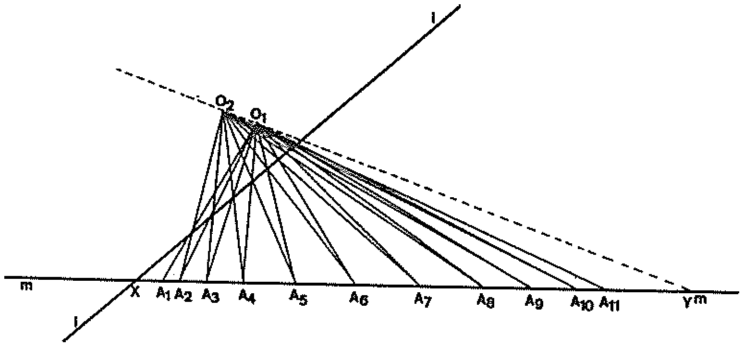

Figure 10. We find the 'history' of point A *under repeated applications of the same transformation.*

case of the collineation of a line is such an important and basic one that it has been minutely examined, and the answer to our question is to be found in all the elementary textbooks of projective geometry. There is not space here to go into all the reasoning which is involved; we will take it as a given fact: In any projective transformation of a line into itself (that is collineation) there will always be — must always be — just two points which do not move, which are invariant. In Figure 10 we find these points at X and Y, where the intermediate line i, meets the base line m, and where the line joining O_1 to O_2 meets the base line.

Geometric series

Now it is a remarkable fact that if we take a projection of such a growth measure from its line m on to some other line, according to the process shown in Figure 8, in such a way that either one of the fixed points X or Y is projected to infinity along that line, then all the points of the measure take up positions along a perfect geometric series, their distances being measured from the other fixed point as zero. To do this, makes a somewhat complicated diagram, but there is an easier way to proceed. We can redraw Figure 10, taking care that we make the line joining O_1 to O_2 parallel to the base line m. This diagram is so easy to draw, and so fundamental in its implications, that you are strongly urged to make it for yourself (Figure 11). If, having done this, you take the distance XA_4, and divide it by the distance XA_3, you will have found the number by which you must multiply XA_3 to transform it into XA_4. And if you do the

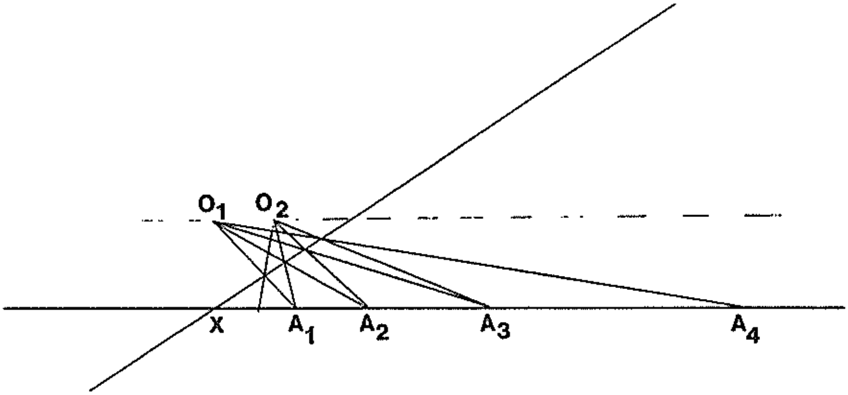

Figure 11. A simple growth measure produces a perfect multiplicative sequence as soon as the line joining O_1 *to* O_2 *is made parallel with the base line of the measure.*

same thing to *any* other pair of consecutive points along the line, you will be led, within the limits of the accuracy of your drawing, to exactly the same number. Your points A_1 A_2 A_3 etc. have fallen into a perfect multiplicative sequence, and no matter how accustomed we become, in the course of time and experience, to this wonderful fact, we should never allow familiarity to blunt the edge of our wonder. For notice that, apart from making the line O_1 O_2 parallel with line *m*, Figure 11 was produced without any regard for number or measure; it could all have been done with an ungraduated straight edge alone. Yet the end result is a series of points which is permeated with number relationships, and these relationships are of the nature of the process of multiplication. When we placed points O_1 and O_2 and line *i* on our page we were, consciously or unconsciously, choosing the number by which our transformation was going to multiply. The actual process of multiplication, as we know it in elementary arithmetic, does not appear until one of the fixed points stands at infinity. But the *quality* of this process is already there in any general growth measure.

Notice that any growth measure contains an endless number of points. Point *A* will have to take an infinite number of steps before it reaches to *Y*, and it has already taken an infinite number in emerging from *X*. Such a measure is called, in some of the textbooks, a hyperbolic metric.

Step measure

It is perfectly possible to perform the above construction in such a way that we ensure that the two fixed points become coincident. We then have just the one double point, $X = Y$, and the line O_1 O_2 must pass through it; so, from another direction, must the line i. Making such a construction we find that we have a measure of points which streams out from the fixed point $X = Y$ in one direction, passes right through infinity and crowds in again towards $X = Y$ from the other side (Figure 12).

Following George Adams we shall call this a step measure. For, if we now project the fixed point to infinity, as before, we shall find that all the other points take up exactly equally spaced positions along the line. The step measure contains the *qualities* of addition (Figure 13).

When we were at school we were taught to add first, and then multiplication was shown to us as the result of repeated addition; we saw multiplication as a higher, or potentized, form of addition. And this is a perfectly valid viewpoint in its own right. But in dealing with

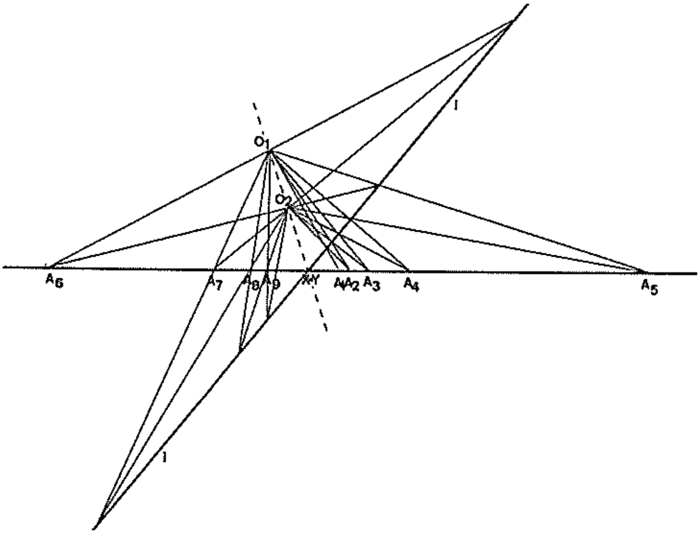

Figure 12. We make the invariant points coincide, and we find that point A *moves in step measure.*

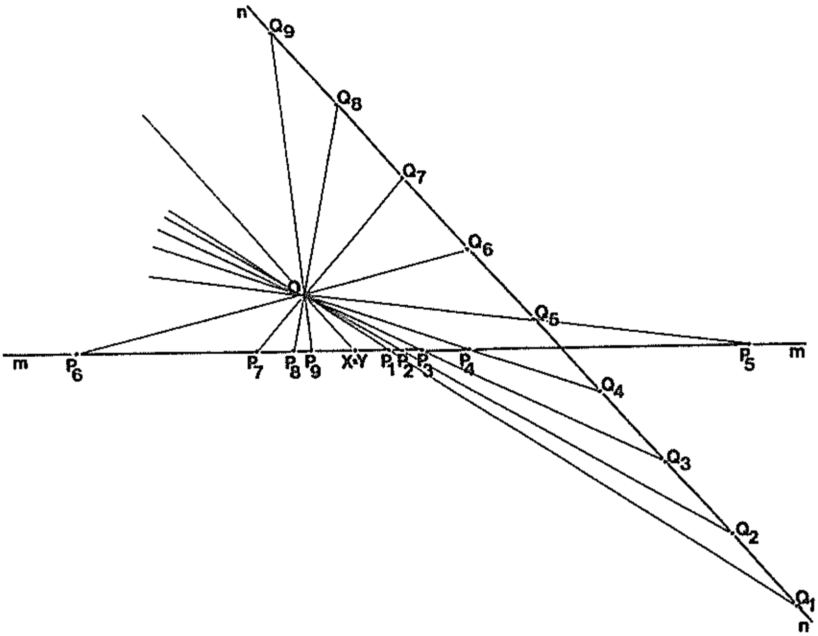

Figure 13. We construct on line m *a step measure using the method of Figure 12. This step measure has its invariant point,* X = Y, *and goes from* P_1 *to* P_5 *and thence through infinity to* P_6 *and so on to* P_9*. Next we project this measure on to line* n *in such a way that the invariant point goes to infinity along line* n. *This is easily done; we put the projecting point* O *anywhere on the page provided that the line joining it to* X = Y *is parallel to line* n. *The resulting projections, points* Q_1 *to* Q_9*, are then found to be exactly equally spaced along line* n.

these archetypal scales we are coming near to the bedrock of thought, and here things appear differently. We say to a line 'Transform!' Left to itself, it will *multiply.* Only when we impose our will upon it, and force it to do it in some special way, will it *add.* We can begin to see multiplication as the archetypal process, and addition as a 'fallen' aspect of it.

Perhaps the fact that all living nature grows by the laws of multiplication, and that only when we come to the mineral world and to the products of man-made machines do we find the additive element in the world around us, may link itself with these facts in our minds.

Two-dimensional path curves

Next we must ask what is the two-dimensional equivalent of this. What happens if we let a whole plane transform on to itself, and we go on applying this same transformation again and again?

Firstly the elementary textbooks of projective geometry show that there must be just three points which are invariant, and these stand at the corners of any triangle, which has been chosen by the particular transformation which is at work. We can call this the invariant triangle *XYZ.* This triangle will be found to have each of its three lines covered with growth measure, moving between the invariant points of that line. And if we join each invariant point to the points of the growth measure which stands opposite to it, we find that each invariant point contains a growth measure of lines.

Considering Figure 14, since *X* remains invariant, and *A* moves to *B,* the line *XA* (= *a*) moves to *XB* (= *b*). Similarly the line *YS* moves to *YT.*

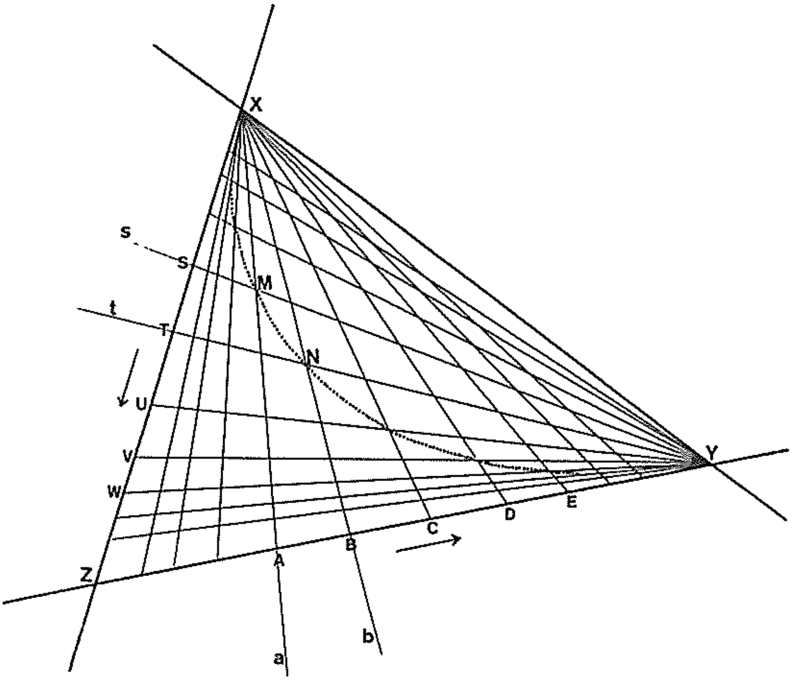

Figure 14. Growth measures moving along the sides of the invariant triangle generate path curves across the rest of the plane.

It follows that the point *M* must move to the point *N*. By following the diagonals across the little quadrilaterals formed by the net of lines, we can trace the curve which shows the successive positions of the point *M* under repeated applications of this transformation. Obviously a whole lot of other points can similarly have their histories traced from this one diagram; in fact the whole plane becomes covered with such curves. These are the path curves discovered and published by Felix Klein and Sophus Lie in the nineteenth century.

Before we go on to study them in greater detail we must note how wonderfully this process is balanced between the extensive and the intensive qualities of space. We have three growth measures of points set 'in line' — extensively 'straight' — and three growth measures of lines, set 'in point' — intensively 'straight' — and out of their weaving there arises a whole set of beautiful *curves*. Figure 15 shows a typical set. This diagram represents the simplest, most elementary, set of curves which it is possible to envisage, that is those showing the movements caused by the simple one-to-one transformation of any plane onto itself. We can think of it as a two-dimensional growth measure. But whereas the growth measure treats its invariant points symmetrically, moving in to one, and out from the other, in exactly the same sort of measure, this one treats its invariant triangle asymmetrically. All the curves pass through two of the points, and none through the third; all of them are tangent to two sides of the triangle and none to the third.

We must remember that whereas every general point of the plane is moving, under repeated applications of the transformation, around the curve on which it finds itself, the points on the lines of the invariant triangle, *XYZ*, are each moving in growth measure along their lines. In fact, these lines may be considered as special 'curves' of the family. And along each of these lines the movement is an expression of some 'multiplier'; this will only appear to our naive consciousness as ordinary multiplication, of course, when one of the points of the triangle is at infinity; nevertheless the spirit of the process is present in every case, and the multiplier in question can be found by ways which are described in the Appendices.

Further study shows us that the *shape* of the entire family of curves is determined by the multipliers which are working along any two sides of the triangle. Details of this are somewhat complicated and will be found in Appendix 2, but putting it very briefly we can say that if we take the ratio of the logarithms of these two numbers, this leads us to a number (or parameter) to which we give the name lambda λ (the Greek letter L);

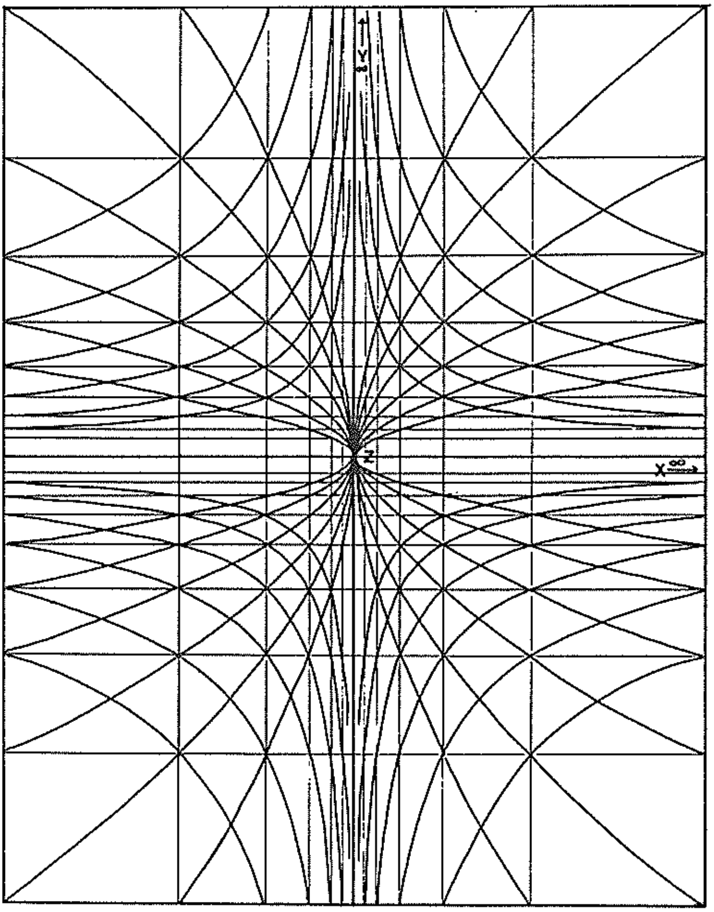

Figure 16. Two fields of path curves having the Cartesian axes and the line at infinity as their invariant triangle.

organism; the substance of which it is made was not in it yesterday, and will not be in it tomorrow; as far as its matter is concerned it is in a state of continual flux; the substance flows in and flows out; if the organism was simply its substance we would not be able to recognize it from one day to another. Yet its being and largely its form are invariant from one moment to another, and from one day to another. The *form* can live within the flux. Something greater than the substance takes it up, moulds it, uses it, and then casts it away.

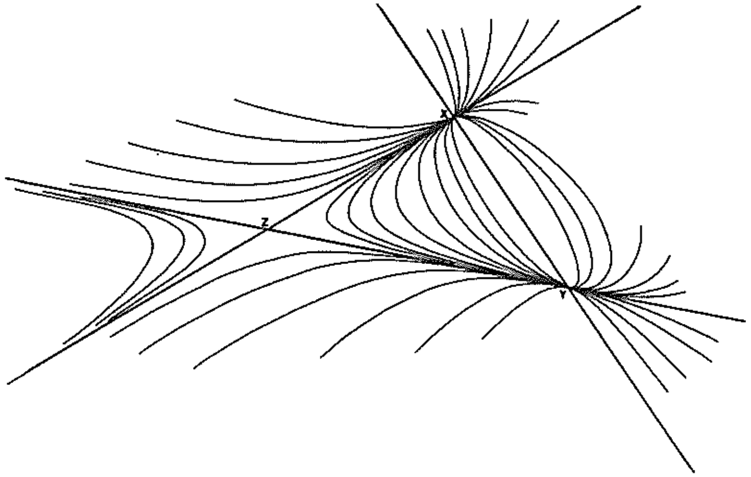

Figure 15. A set of path curves.

and that the value of λ informs us of the form of the complete family of curves. For instance, if the multipliers along two of the sides happen to be equal to one another, that is $\lambda = 1$, then the curves become conic sections — ellipses, hyperbolas and an occasional parabola thrown in for good measure. The farther λ departs from unity the more sharply do these curves leave the form of the conic section. In the ordinary way we are accustomed to regard the conic as the most elementary and fundamental kind of curve which it is possible to find, but now we learn to see it as simply a special case of the much more general path curve.

Now, looking at Figure 15 we have to ask ourselves: What actually is invariant here? The answer is that not only the three points and lines of the triangle, but the whole set of curves, in its totality, is invariant. For notice that whereas, apart from the triangle itself, every point and line of the plane is in movement, all the points (and tangent lines) of each curve transform into fresh points (and tangent lines) of the same curve. Each curve, taken as a whole thing, remains invariant, in the same sense as the invariant lines of the triangle, taken as whole things, remain invariant. In fact the lines of the triangle can be taken as special, degenerate, cases of the curves.

We have a plane in which everything is moving. What can live, can hold itself intact, within the flux? It is the whole set of path curves, and nothing else! Qualitatively we have a similar situation in any living

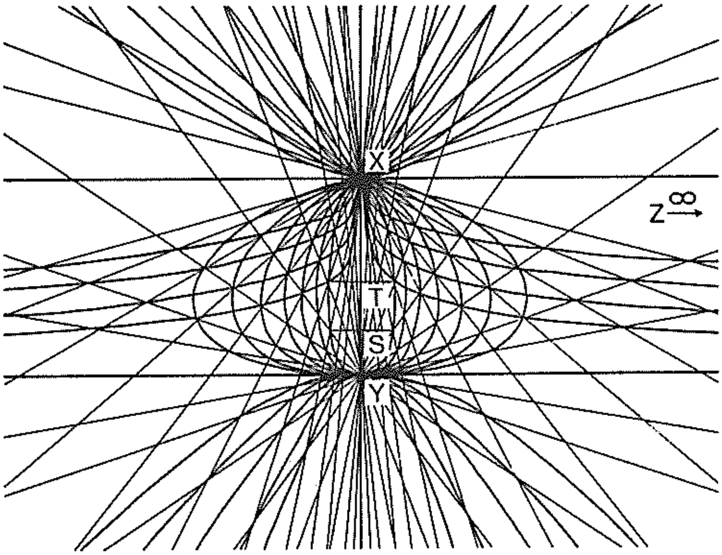

Figure 17. Two more fields of path curves, this time having two of their invariant points finitely placed, and one at infinity.

roots of the negative numbers, and they have, rather unfortunately, been named imaginary numbers; unfortunately, because of all things in the realm of thought these are probably least able to be visualized with the pictorial powers of true imagination. More unfortunately still, the numbers we have known from our childhood were then called real, which gives one the very untrue impression that the imaginary numbers are somehow fake. However, this terminology is so firmly fixed by tradition now, that we must continue to follow it.

Once the possibility of imaginary numbers had been realized, and the rules for working them had been discovered, all in the algebraic realm be it noted, the search was on for their spatial representatives, the imaginary points, lines and planes. Today these are well-known, and can be manipulated at will. They are, by their very nature, for ever invisible, and beyond the range of visual imagination, yet their presence is very real, working from behind the scenes as it were, to influence the forms which appear on our diagram.

We will not go into the details of their working except to point out that they always come as the result of taking square roots, and square roots

Although in the normal way a set of path curves will resemble Figure 15, there are cases where the appearance can be quite different. One such case arises when both points *X* and *Y* move off to infinity forming an infinitely large invariant triangle. Figure 16 represents a weaving of two such sets of curves, formed by two different transformations working through the same invariant triangle. The growth measures along the *x*- and *y*-axes are here true geometric series, and it is thus a very easy diagram to draw, and the reader is encouraged to try it. Great variation is possible. If the multipliers along the axes are equal to one another then we get families of hyperbolas and parabolas; in all other cases we get curves which look superficially similar but have quite different mathematical qualities. It is not necessary that *Y* should go off to infinity in a direction at right angles to that in which *X* has gone; in other words the axes may be inclined to one another at all sorts of angles, and this introduces most interesting variants.

Another important case comes when we keep both points *X* and *Y* on the page, and let *Z* move off to infinity. Here we get a family of egg-like curves which will prove to be of great importance in our further studies. The sides of the invariant triangle are now parallel (meeting *Z* at infinity) and the growth measures along these sides are true geometric series, starting at *X* and *Y*. If their multipliers are equal then the 'eggs' will be elliptic — neither blunt nor sharp at either end. But this is a special case. Normally one will be larger than the other, and the sharp end of the 'egg' will always point towards that side which has the larger multiplier. In Figure 17 this is the side passing through *X*. Here again it is not necessary for the line *XY* to be at right angles to the parallel lines through *X* and *Y* and many interesting variants can be drawn.

We have seen from our studies so far that these beautiful families of flowing path curves have a wide range of variations in the appearance they can present to us; nevertheless their possibilities are strictly limited. The things they cannot do, and the kinds of form which they cannot assume, are far more numerous than those which they can. It may sound paradoxical, but it is true, to say that they are infinitely variable within a strictly limited range of possibilities which we have not yet mentioned.

To describe these we will have to touch upon a matter which non-mathematicians often find hard to visualize. We have seen in the first chapter how our projective geometry, through its algebraic quality, is a spatial expression of number relationships. During the past few centuries mathematicians have discovered the possibility of quite other kinds of number than those with which we ordinarily count. These are the square always come in pairs. The square roots of 9 are +3 and –3, and so on. So it comes about that in the normal way, imaginary points, lines and planes come in pairs. Thus it can happen that a path curve transformation is such that it has a so-called imaginary triangle for its invariant organism; that is to say, a triangle consisting of one real point, and one imaginary pair. In such a case the form of the visible path curves become very much changed. Figure 18 shows a typical field of such curves. The real invariant point is at *Z*, and the imaginary pair, *X* and *Y*, must be envisaged as floating insubstantially in cyclic movements along the line where they are marked. The rest of the plane is filled with sweeping vortex-like curves of great beauty.

Further variation is then possible. If the real invariant line of the triangle, that is the one containing *X* and *Y*, moves out to merge into the line at infinity, and if *X* and *Y* become a special pair of imaginary points, well-known to mathematicians as the Absolute Circling Points at Infinity (called *I* and *J*), then the path curves become logarithmic spirals (Figure 19). This is the first place where we see these curves showing themselves in the realm of living nature, for they are similar to the patterns which we see in the seedhead of the sun flower, and, rather less exactly, in the centres of many daisy-like flowers.

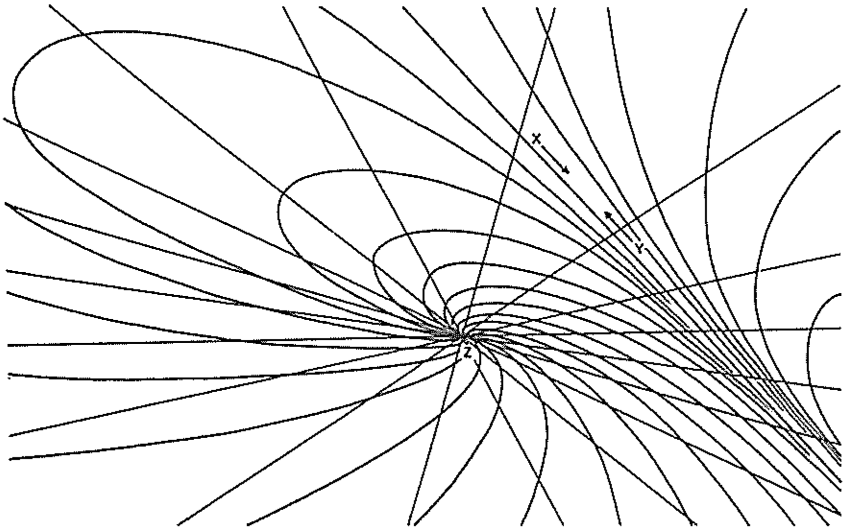

Figure 18. A general field of path curves with respect to an imaginary triangle.

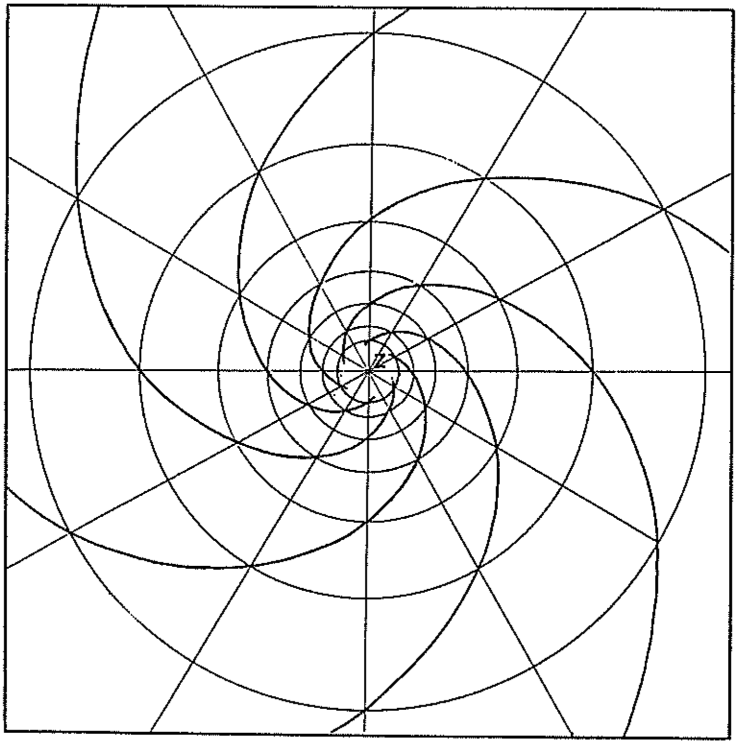

Figure 19. A family of path curves with respect to an invariant triangle of which two points have become imaginary. In this particular case the imaginary points are I *and* J *on the line at infinity and the curves are logarithmic spirals.*

These spirals may curve outwards, away from their centre, more or less steeply, according to the particular transformation which is at work. A possible limiting case is found when the outward striving becomes zero, and the curves become circular. So we see that our old friend, a family of concentric circles, is in fact a set of path curves with respect to an imaginary triangle.

4. Path Curves in Three Dimensions

When we come to transform the whole of space by this simplest possible projective transformation we find, from purely geometrical considerations, that there must always be just four points which remain invariant, and these stand at the corners of some tetrahedron (a triangular-sided pyramid) which is picked out for us by the particular transformation which we are using. This tetrahedron is made up of four points, six connecting lines, and four planes; it is thus self-dual, that is, the points and planes play similar roles, and therefore the invariant organism is perfectly balanced between extensive and intensive space, as we have seen is true also with the two-dimensional cases.

We next find that all the points of these six invariant lines will be moving in growth measure along their lines between the invariant points of those lines, and all the points situated in the planes (sides) of the tetrahedron will be moving in two-dimensional path curves within their planes. All the other points of space will be moving in a set of beautiful path curves which weave through all of space, all such curves passing through two of the points of the tetrahedron, and all avoiding the other two. Figure 20 is an attempt to portray a simplified version of this very complex and beautiful configuration. Only one space curve, of the infinitudes present, is shown (in dark) passing into, through and out of, the tetrahedron, with a cusp at *Y.* Just two of the four families of plane curves are indicated (in light), belonging to planes *XYZ* and *YZW.*

I have never seen anything in the realm of nature which seems to me to resemble such families of curves and we will not spend further time in describing them here.

But when we come to the semi-imaginary case things stand very differently. Here we have two of the invariant points become imaginary while the other two remain real, and therefore visible. This can happen in various ways; we will describe one of the most fundamental and elementary of them. Starting from Figure 20, we will keep points *X* and *Y* real, and here, and imagine the line *XY,* which of course will also remain real, to be vertical. Now let us imagine the line *WZ* to recede further and further into the distance until it merges into the horizontal line at infinity. In the course of this we can see how the planes *XZW* and *YZW* will be sloping toward one another less and less steeply, until finally they

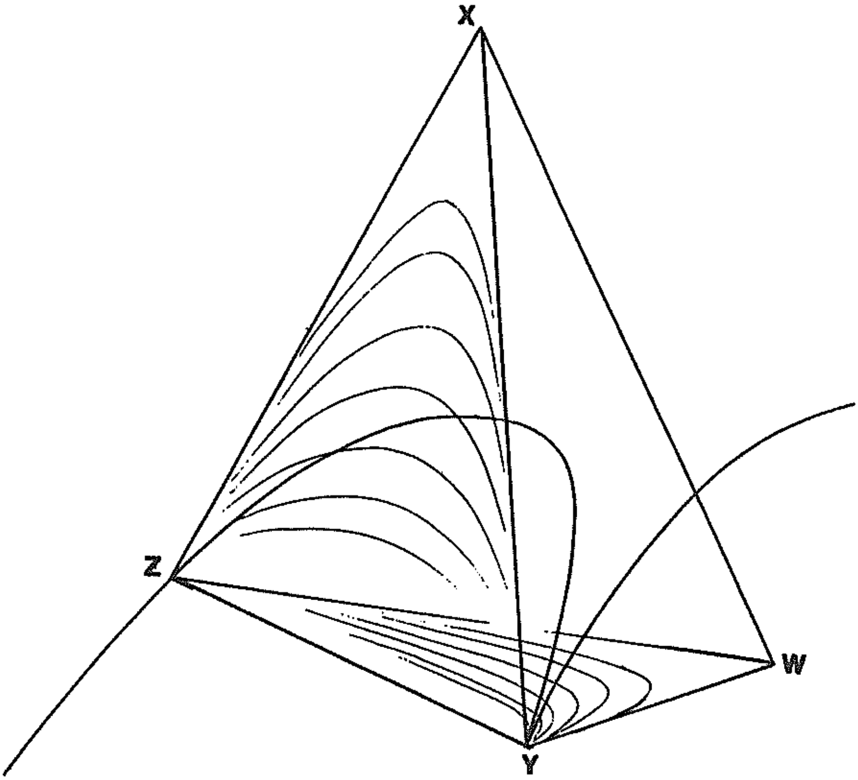

Figure 20. One of a family of three-dimensional path curves set within its invariant tetrahedron.

become parallel horizontal planes through *X* and *Y* (Figure 21). *Z* and *W*, becoming imaginary, and merging into *I* and *J* (see p. 45) now function as groups of cyclic transformations, circling in the horizontal line at infinity, which line remains as a real line of the invariant tetrahedron. The other four lines and two planes disappear into the imaginary. The two real invariant planes will be filled with path curves having the imaginary invariant triangles *XIJ* and *YIJ*. We have already seen what these would look like — logarithmic spirals as shown in Figure 19. Figure 21 pictures what this invariant tetrahedron would look like, in as far as we have so far described it; for the sake of simplicity only one spiral of each family has been drawn in.

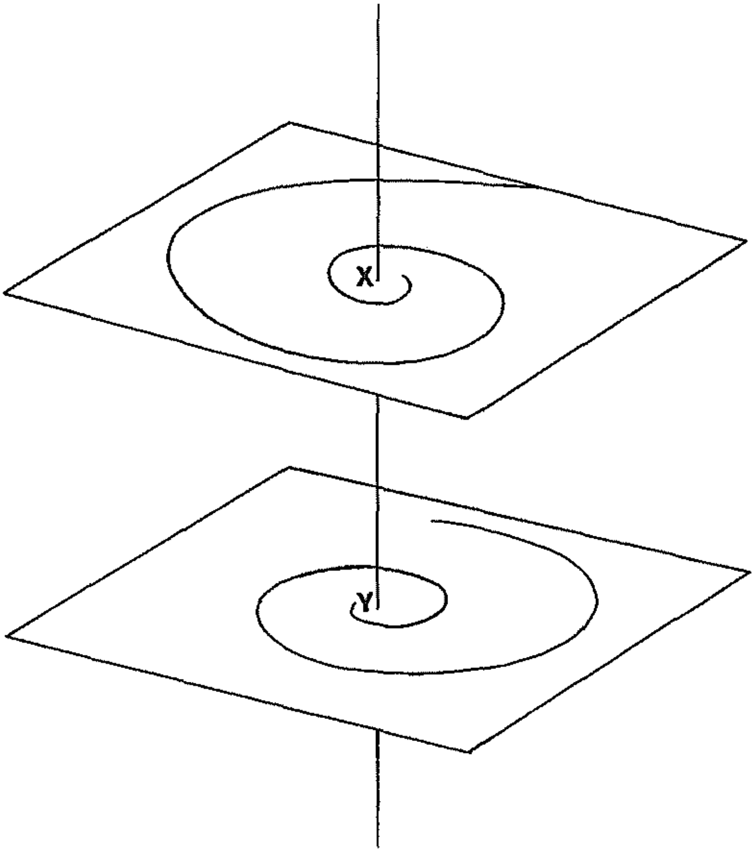

Figure 21. The semi-imaginary case. The invariant planes XZW *and* YZW *of Figure 20 have become parallel horizontal planes through the invariant points* X *and* Y.

But this is only half the picture — the extensive half. We have seen in two dimensions how the invariant organism is always self-dual, that is, perfectly balanced between its extensive and intensive aspects, and the same thing holds here. If we look back to Figure 6 we see how a point can be regarded as a little two-dimensional world, holding a spiral cone of planes in the same way that a plane holds a spiral curve of points. And here we see it in action; just as the parallel invariant planes of the tetrahedron hold spiral curves of points, so the invariant points, *X* and *Y*, hold spiral cones of planes; and these cones are incident with the curves.

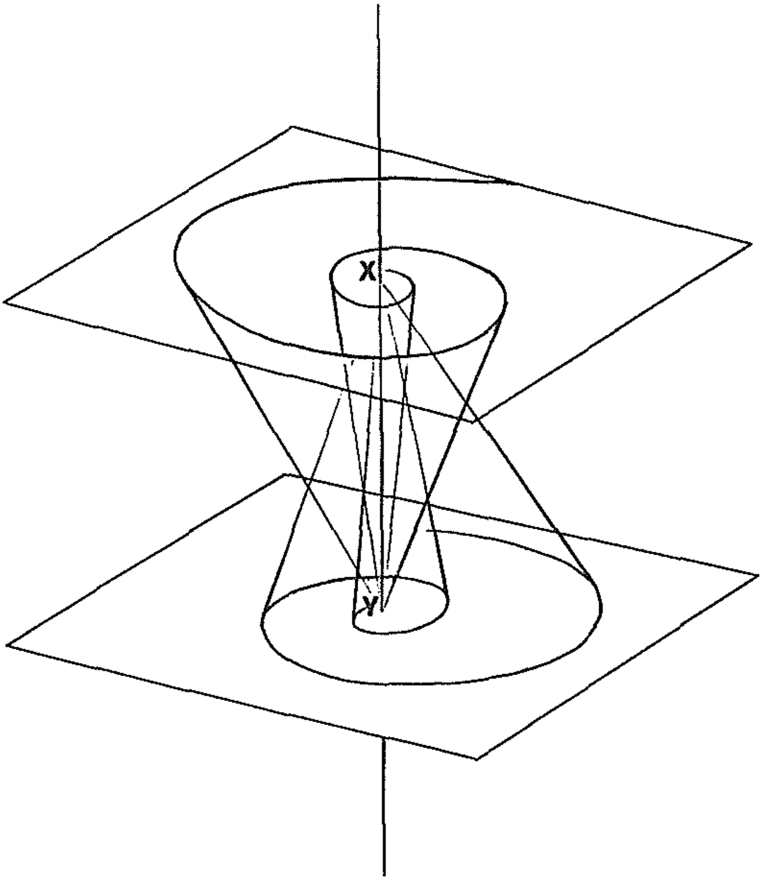

Figure 22. Looked at intensively, the extensive picture of Figure 21 reveals two families of interpenetrating spiral cones held in the invariant points X *and* Y.

Figure 22 pictures this in a partial way. To complete the picture one would have to see in one's imagination not just one spiral cone radiating from *X* and from *Y*, but a whole infinite family of such from each.

This tells us how the points of the invariant planes, and the planes of the invariant points, move within the transformation, but how about all the other points and planes of space? They will be moving, each in its own path curve, which will assume a corkscrew form, winding endlessly out of one of the invariant points, around and around the central axis, and endlessly into the other one. And if one asks how these windings are controlled, the answer is that they take place on the intersections of

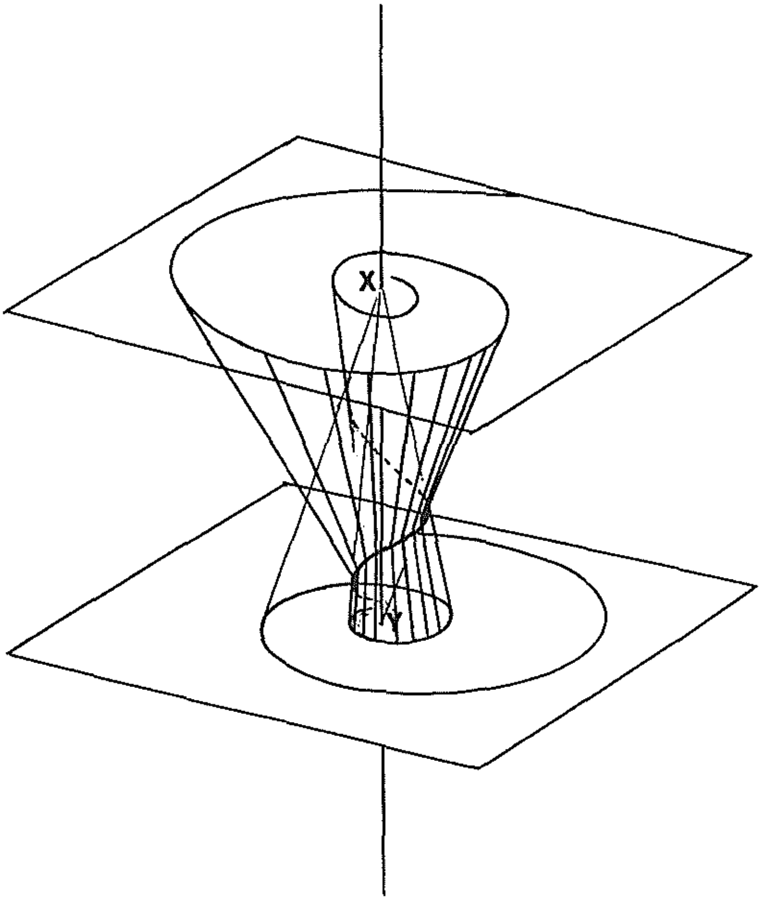

Figure 23. The path curve followed by any general point of space lies on the intersection of two spiral cones.

these interpenetrating spiral cones. Spatially the matter is very complex and not easy to hold concretely in one's imagination. Figure 23 is an attempt to picture just two of such cones, meeting on such a spiral curve, winding in front of, and then behind, the central axis. And now one must enlarge one's picture to include all the points of space, both between the invariant planes and outside them, to be behaving similarly.

But here we must enlarge the range of our ideas still further. Every time we move into a new dimension the range of possibilities increases. We have seen how in two dimensions the set of path curves forms a kind of invariant organism which can live within the moving flux of the transformation — they are the invariant curves of the transformation — so now in three dimensions we can have not only invariant curves, but also invariant surfaces. These surfaces are of great beauty, and can be immensely complicated, but always they are completely covered with path curves. All their points, and their tangent lines and planes, are in constant movement, but as they move they remain on their surface which, itself, remains unchanged. The simplest, and probably the most fundamental of these surfaces is fairly easily envisaged. Imagine a plane somewhere between the two invariant planes and parallel with them, and in this plane a circle whose centre lies on the point in which the central axis cuts the plane. And now consider the set of path curves which go through all the points of this circle. Imagine the infinitude of them, all there, lying side by side. They will form a surface in space, and when one plots the form of it, it turns out to be, in the general case, egg-shaped, rather sharper at one end and blunter at the other; and of course, by the very way in which it has been made, completely covered by the path curves which have generated it (Figure 24). All the elements which go to make this surface (points, lines and planes) are continually moving, but they move in such a way that the *form* of the egg remains invariant within the movement. There are other invariant surfaces possible within this transformation, but many of them are little more than variants on this one; the egg, with the infinite possibilities of spiral families upon it, is probably the simplest and most fundamental of them all, and must be considered as an archetypal form of all space.

Next we must consider the families of logarithmic spirals which appear on the top and bottom planes of the invariant tetrahedron, in more detail. These curves appear by projective processes which are similar in nature to those which produce the growth measure. Looking back to Figure 19 we see curves which turn about their centre, Z, in such a way that their distance outwards from Z increases by a constant multiplicative factor, for any given angle turned. The size of this factor naturally varies, for any given curve, with the size of the angle turned, by means of which one has decided to measure it, but it will then remain constant for measurements on all other parts of that curve. We can thus speak of a logarithmic spiral as 'multiplying' by a certain number. It is the number of times by which its distance outwards from the centre increases for every radian* turned, and it is the same for all the members of any given family of path curves. When our

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* A standard measure of angular turning which is found to be the most fundamental and 'natural' unit of angular measure which one can apply to such a case. It is slightly more than 57°.

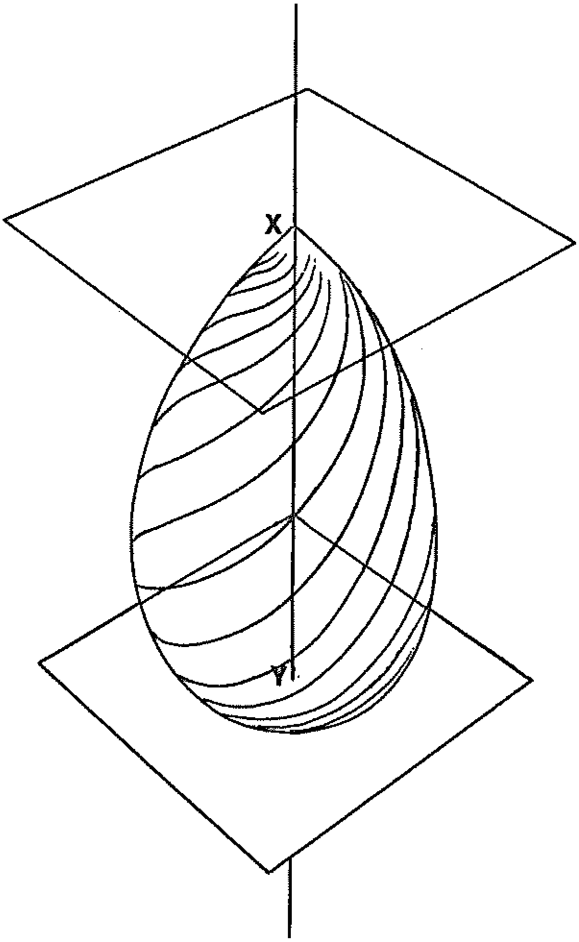


Figure 24. One of the most archetypal and primary forms of space is the egg, having its sharp and blunt ends on the invariant points of X *and* Y, *and covered with an infinite family of path curves.*

transformation, according to its nature, fixes a multiplier for its logarithmic spirals, the form of the whole family of curves is completely determined.

Now there is nothing in the path curve process which ensures that the multipliers on the top and bottom invariant planes must be the same as one another. In fact, unless we have done something very definite about it, they will be different. And this difference completely determines the whole form of the path curves and invariant surfaces with which that transformation fills space.

The situation here is closely parallel with that which we met when studying Figure 14. If we make a fraction, putting the logarithm of the multiplier for the top plane over that for the bottom, we get a parameter, λ, which informs us of the complete shape of the resulting forms (see p. 41). If we ask 'What *is* λ?,' then we may say that it is a number which represents the relative speeds of winding inwards and outwards of the spirals in the top and bottom invariant planes, or, what comes to the same thing, the relative speeds of unwinding of the interpenetrating spiral cones of Figure 22.

The above is a rather simplistic and partial explanation of what is a considerably more complicated matter, but it should be enough for the reader to get a true feeling for the meaning and significance of what follows in the rest of the book. Those who want greater detail are referred to the appendices.

We must next examine the significance of λ in the field of visible form. To do so we will cease to picture the forms in full perspective as in the last few figures. Instead we will depict them in orthogonal perspective, that is, in elevation. This is to see them as they would appear from an infinite distance seen through an infinitely powerful telescope, with our eye in the meeting line of the two horizontal invariant planes. These planes now appear as two horizontal lines passing through the top and bottom poles of the egg. The spirals in these planes now cut these lines in points in growth measure, that is, in geometric series. We are now back to Figure 17; set the forms which are portrayed there into rotation about the central axis, *XY,* and you have the true egg form which we are dealing with here.

When the multipliers on the top and bottom planes are equal to one another, that is, when $\lambda = 1$, then the egg-form is neither blunt nor sharp at either end; it is elliptical in cross-section (Figure 25). As λ increases above 1, the top end gradually sharpens and the bottom end correspondingly becomes blunt. When λ is high (say over 3), the form becomes quite tense-looking — very sharp at one end and very blunt at the other. As λ sinks below 1, the same set of forms is repeated, but now upside down. It works inversely; $\lambda = {}^1\!/_2$ gives the same shape as $\lambda = 2$, $\lambda = {}^1\!/_3$, the same as $\lambda = 3$, and so on. As λ approaches zero, the form becomes exceedingly sharp below and blunt above. When $\lambda = 0$ (which can here be taken as the inverse of infinity), the thing straightens into a cone. Then as λ passes the zero mark and becomes a negative number, the cone curves itself into a beautiful vortex form. So we see that in this context the negative of an egg is a vortex, a fact which proves to be of tremendous significance in the further development of the work.

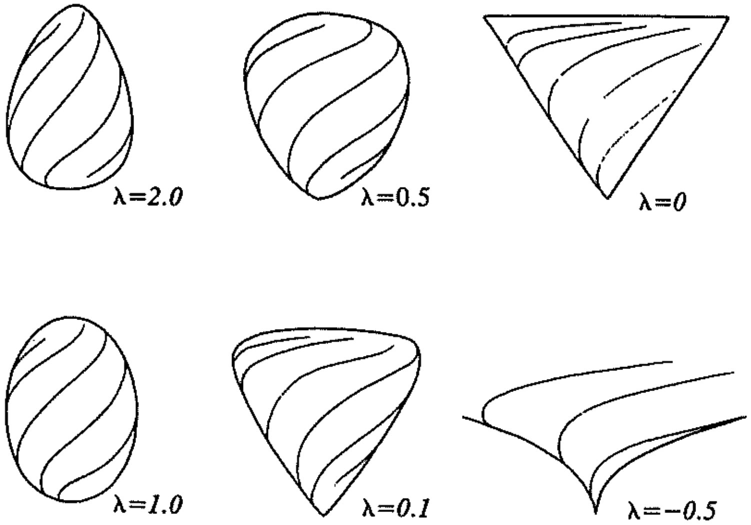


Figure 25. The simple process of diminished λ causes the form to pass through an 'opening' gesture.

Here again it is important to stress that although with the variation of λ from −∞ to +∞ we have a spectrum of forms, from vortices to eggs, which are infinitely variable, this variability is available within strictly limited confines. For instance the degree of sharpening and bluntening at the two ends of the egg is always exactly balanced. One can never find one which is very sharp at the top and slightly blunt at the bottom, or slightly sharp and very blunt, or sharp at both ends, or blunt at both. The things they cannot do are far more numerous than those which they can. And similar restrictions apply to the vortices.

We must be aware that a transformation of the kind we are studying never confronts one with a single egg, or vortex, in isolation; always there is a whole field of them, filling all space, and lying one within the other rather like the Russian dolls of our childhood. Figure 26 depicts such a field as far as it is possible to do so in a two-dimensional diagram. To imagine the thing in its three-dimensional reality one would have to see the whole thing rotated about the vertical central axis. In this particular drawing λ has a value of 3, so the forms are moderately sharp and blunt. Egg-forms proper, as we know them in ordinary life, are found between the invariant planes; outside these planes lie forms which

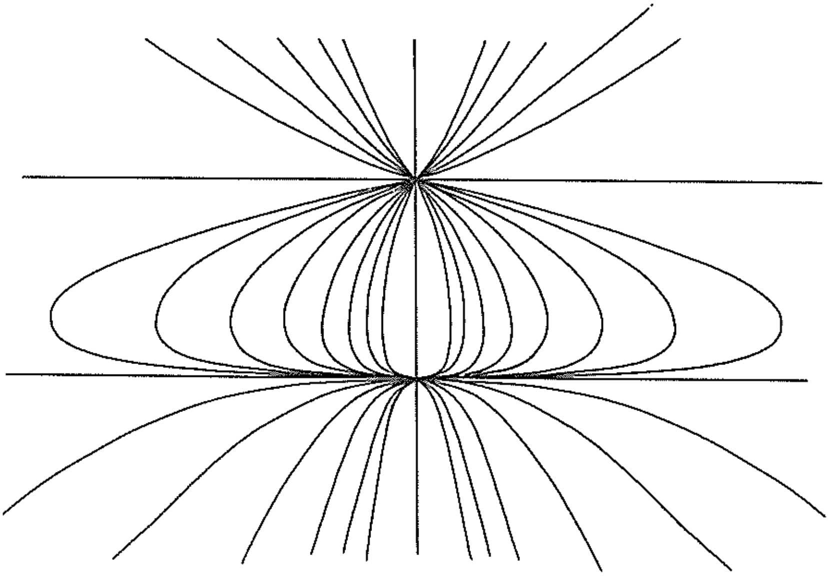
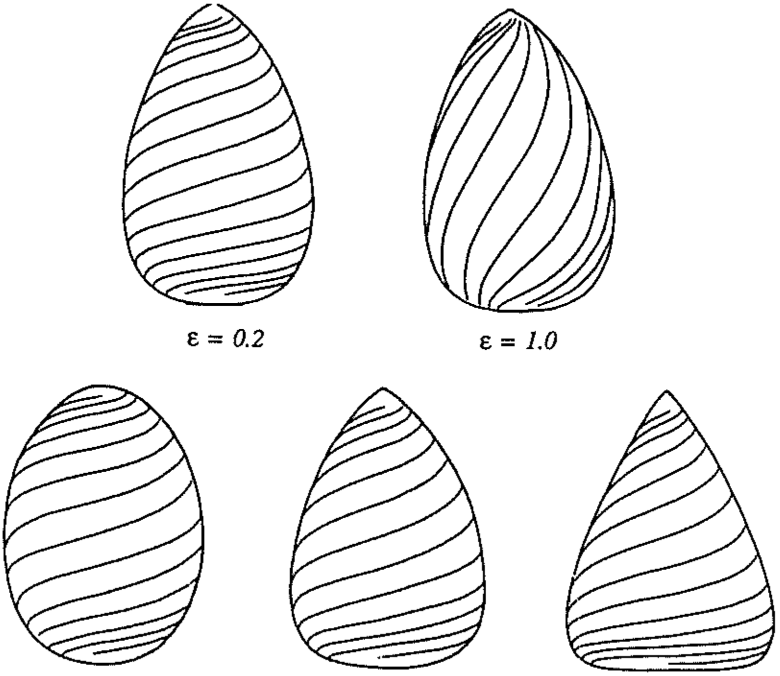


Figure 26. A field of egg-forms seen in orthogonal perspective.

I sometimes call 'hyperbolic eggs'; their upper and lower parts join up with themselves at infinity rather in the manner of hyperbolas, but they retain the same amount of sharpening and blunting at their poles as the ordinary eggs. We can see from this that λ definitely does not control the *shape* of these forms, as shape is ordinarily understood. They all have the same λ but the inner forms are long and thin, while the outer ones are wide and fat. λ controls the *quality* of the form; they are all equally sharp and blunt. It is true to say that whenever λ becomes very large, or very near to zero, the forms tend to become tense and straightened; but when λ is near to unity the forms become relaxed and gently rounded.

Next we must think in a little more detail about the spiral curves which lie upon these vortex- and egg-forms. The quality of the curves, in particular the steepness with which they spiral round their form, is given by another parameter, which we usually call ε (epsilon). When ε is zero, the spirals degenerate into horizontal circles girdling their egg- or vortex-like lines of latitude. As ε increases the curves spiral upwards with ever increasing steepness, until as ε approaches ∞ they tend to become upright, like lines of longitude. The curves spiral infinitely out



$\varepsilon = 0.2$ $\varepsilon = 1.0$

$\lambda = 1.2$ $\lambda = 2.5$ $\lambda = 5.0$

Figure 27. The top eggs both have $\lambda = 2.5$ but ε*s of 0.2 and 1.0. The lower eggs all have* $\varepsilon = 0.2$ *but* λ*s of 1.2, 2.5 and 5.0.*

of one pole and infinitely into the other, never reaching either in a finite number of steps. Each particular egg or vortex is so formed that it can be covered by an infinite family of curves in infinitely different ways — one for each value of ε between 0 and ∞ (Figure 27).

We will close this short description of the three-dimensional path curves and surfaces with a few words about what happens when one's transformation is such that the invariant tetrahedron becomes all-imaginary — as far as that is possible. In such case, all four points and planes become imaginary, and also four of their six connecting lines, but the other two, which are a skew pair (not meeting one another) remain real. We can place these two lines in any relationship to one another which we wish, provided we keep them apart, but we will describe here only the case which is simplest to visualize, and probably most fundamental, when one of them is a vertical central axis, and the other is the horizontal line at infinity.

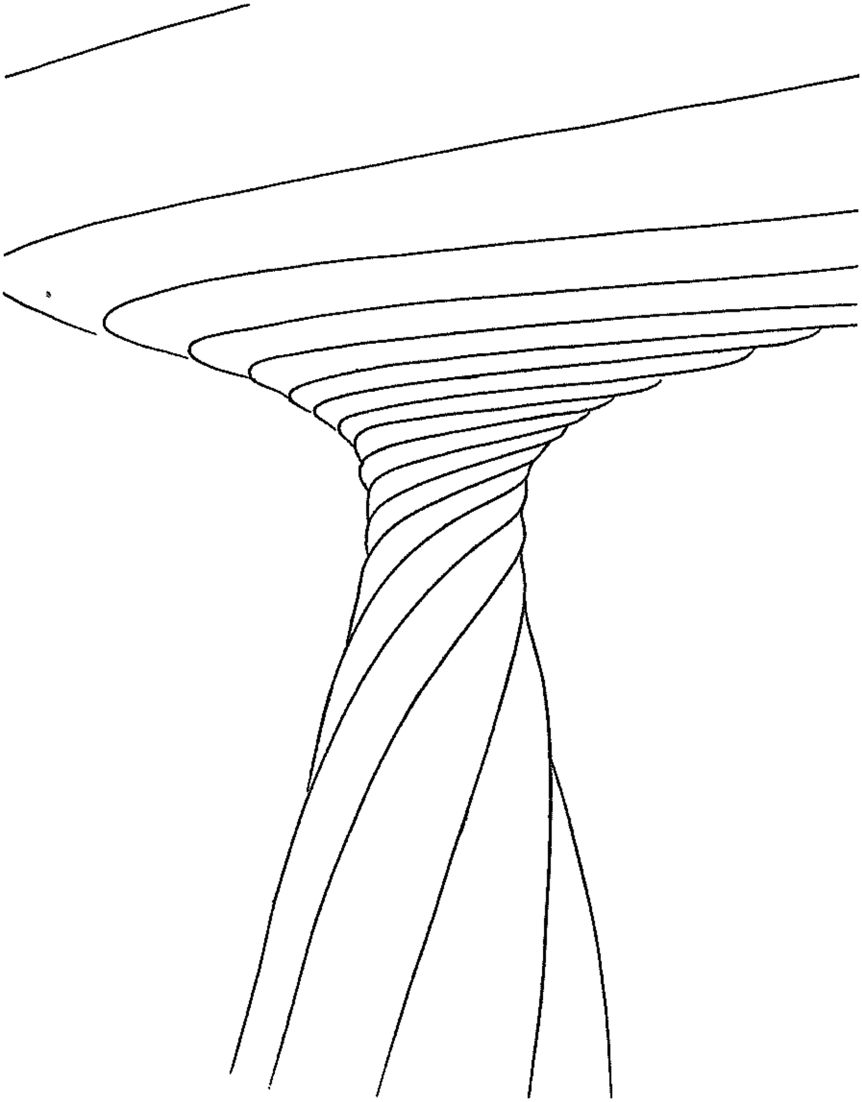


Figure 28. The Chalice Surface, generated from path curves with respect to an all-imaginary tetrahedron.

One can ask what happens to the egg if the points of accumulation at its poles melt into the imaginary. Well, the curves and the surfaces on which they lie will flow right through. Eggs can no longer be in question. In the field of such forms the vortex is paramount. But it will always be a vortex without beginning or end, spanning all space from infinity to infinity. Figure 28 shows what is perhaps the simplest form of such a case. When I first came upon it in my exploration of this field I was so struck by its beauty and suggestiveness that I named it The Chalice Surface. What is drawn here is only a small part of it. One must imagine it flowing upwards and outwards to infinity, passing through infinity, and coming back from below in the shape of a much wider cup-like form which repeats the general form of this one in wider reaches of space. Or alternatively we can let our imagination follow the printed form downwards to, and through, infinity, and see it returning from above as a much thinner cup, passing through the centre of the printed one. Such a surface will have an infinitude of branches threading within and without one another. And the same transformation which generates this surface will fill the spaces between its branches with a field of similar surfaces. The whole of space becomes one mighty chalice-like gesture. But the whole thing floats. Without real invariant points, nowhere can it come to earth or to rest.

Perhaps for this reason we shall find that forms generated by the all-imaginary case are not to be found so easily where nature is coming to expression in sense-perceptible substance.

5. Path Curves in Eggs and in the Plant Kingdom

The main work on the foundations of the path curves and surfaces was achieved by Felix Klein and Sophus Lie in the later decades of the nineteenth century. Seeing its importance it is strange that the geometrical aspect of it should have made so little impact on the culture of their day; and the work lay almost forgotten for over half a century. It was in 1950 that George Adams brought his rediscovery of this work to a small group of us at a mathematical conference in Forest Row, and I was immediately fascinated. To produce such beautiful forms by purely inward contemplation, and then to find them 'speak' — it seemed to me — so clearly and eloquently of forms in living nature, was something which I thought must be significant. In the ensuing years I spent much time studying and marvelling at the whole field of form which was thus opened up for me; but it was not till fourteen years later, in 1964, that the impulse came to me to investigate with the most precise measurement possible whether, and to what extent, the forms of nature really follow such mathematical models. When out of pure thinking the form of the egg covered with its spiralling curves appears to my imagination, and then later I find the wood littered with pine cones, to what extent am I justified in seeing this as a significant linkage of concept and percept, or may it perhaps just be a chance resemblance which is beguiling me?

At the outset one is faced by two, related, problems. Firstly, given some form of nature, how can one devise a system of measurement and analysis which will show the path-curve parameters (especially λ and ε) which must have been at work in producing that form — if indeed they *have* been at work. And secondly one would need to find a way of calculating and denoting exactly how far the given form deviates from the mathematical one.

These problems are easily handled, and full details of the method of working are given in Appendix 3. Here it is necessary to describe the main outline of the steps taken, in order to understand clearly what follows. The long axis of the given form is divided into a number of equal steps; eight is found to be a convenient number, and is the one which I have used for most of this work. This gives us seven levels on the form,

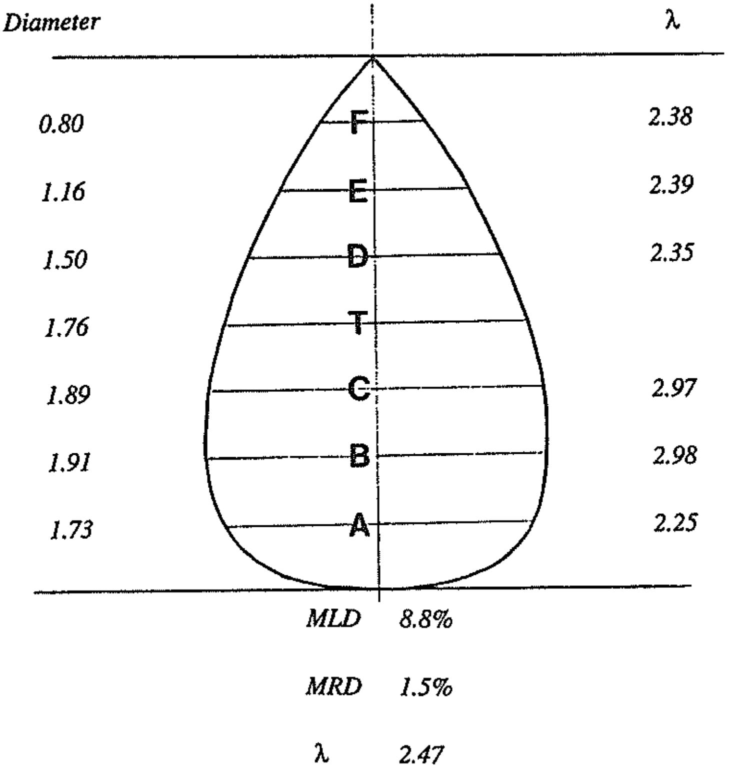


Figure 29. This illustrates the basic method for calculating λ. It is found that the values of those levels which are nearer the poles, e.g. A *and* F, *are more reliable and more significant than those nearer the middle of the form. Therefore the final figure for λ is a weighted mean of those at the various levels. Full details of this will be found in Appendix 3.*

which are labelled A B C T D E and F from below upwards, as shown on Figure 29. Next the diameter at each level is measured; since in the subsequent calculation only ratios are significant, the units used for the measurement are irrelevant. Then it is easy to calculate that unique value of λ which would put a path curve accurately through the levels at A and at T; then we find other λ-values for levels B and T, C and T, and so on. When this is completed we have six λ-values, one for each of the levels A to F. If these six values are all the same, then the given form is a perfect path curve as far as those seven levels are concerned and, unless there are outstanding irregularities in the outline, the whole form is almost certain to be a very good fit indeed along the whole of its length. And in as far as these values vary as one moves from one level to another, so far does the form depart from the true mathematical ideal. It is as simple as that!

And when measuring a form from living nature these numbers are never the same all the way down the list — of course! One could never expect that a living thing, struggling for existence in a rude world, and subject to all the accidents of wind and weather, would ever be able to grow exactly to its archetype. What we can do is to take the mean of these six values and accept that as the best λ which can be found for the form we are measuring. And a consideration of the values at the separate levels can tell us just where, and how, the thing deviates from the ideal form.

Next we can calculate by how much the λ at each level differs from the mean value (which we are now accepting as the true value for the form as a whole) and express this as a percentage. The mean of these percentage differences for the six levels gives us the Mean Lambda Deviation (MLD) for the form we are measuring; and this proves to be a good measure of how accurately the form is following a true path curve shape. And this leads to the important question: 'When can we say that we have good evidence that a given form is really of path curve origin?' It is difficult here to avoid seeming somewhat arbitrary. Much experimenting has shown that if the MLD is less than 20% the fit between a given form and its path curve ideal is remarkably close, and if the MLD is less than 10% it is very exact indeed. We have to draw the line somewhere! And in all the early years of the work I have accepted an MLD of under 20% as showing a 'Satisfactory' fit, and one of under 10% as being 'Very Good Indeed.' Between 20% and 30% the case is doubtful; the path curve forces *may* have been at work, but clearly other things have also intervened; or maybe it is just a chance resemblance.

Twenty per cent may seem at first sight a high figure to set as a dividing line; but experience shows that the MLD is a very sensitive measure of deviation indeed, and 20% in λ-deviation means very little in terms of the visible form. This shows a definite disadvantage in using the MLD; it does not easily convey any real idea of the actual deviation involved. For this reason, in recent years, I have taken to working rather differently. Having once found the best λ possible for a given form, it is easy to calculate the exact radius which the form ought to have if it is to be true to the mathematics, at each level. These figures can then be compared with the measured radii at those levels, and the difference expressed as a percentage. From this we can get the Mean Radius Deviation (MRD). Experience shows that an MRD of less than 4% represents a moderately good fit to the mathematical ideal, under 3% shows a good fit, under 2% a very good, and under 1% an almost perfect, fit. In the latter case the deviations are so minute as to be barely measurable except with a very high degree of enlargement and under a powerful lens.

At this point in the writing it occurred to me to conduct a simple experiment. After working with these forms for nearly forty years I think that I am able to draw a pretty good path curve form free-hand. I therefore did so, with all the care I could summon. The result is the egg-form of Figure 29, which I then measured and analysed. How well did I do? Not so badly, one might say: an MRD of 1.5% represents a Very Good Fit. This form had originated from a background — my mind — where the path curve laws had been actively at work; and the mathematics acknowledges it. But as you read on in this book you will meet numerous cases of tiny organisms of one sort or another which do considerably better in spite of their apparent fragility or vulnerability to the uncertain forces of an erratic environment.

When considering deviations we must remember that these can be of two kinds. There is what I would call 'accidental deviation' — lambda values above and below the mean, occurring irregularly along the length of the form, showing, as it were, as 'noise' in one's figures. This I think is due to the growing organism being unable to cope sufficiently accurately with the disturbing influence in its environment. But there is another possibility. I think we must grant to any *living* thing the privilege of being able to take its archetype and mould it, adapt it, to the special circumstances of its environment, or its role in life. Such adaptation would show normally as a small but systematic variation along the length of the form, and we shall meet several cases later in this book. If such variation grows large enough one would reach the point where one would have to say that we have no longer firm evidence that this is really a path curve form at all, but the decision when this point is reached is one which has to be left to our human judgment.

Eggs

Now, having developed our basic method of measurement and analysis, on what should we employ it? When, nearly a quarter of a century ago, I started on the practical application of this work, my first move was with the ordinary hen's egg. Why not? I had been calling these forms 'eggs' for years, and they were immediately to hand. I could not know what the result would be. Maybe the deviations would be such as to deny any real connection between the path curves and actual eggs. I was prepared for this possibility and would then probably have abandoned the whole investigation. I could not foresee that I was starting a task which, some twenty-five years later, I would have to regard as barely begun. Below are the full results for two eggs selected at random from the larder, and then photographed.

Level	λ value
F	1.18
E	1.22
D	1.20
C	1.21
B	1.20
A	1.15
MLD = 1.9% MRD = 0.5% $\lambda = 1.18$	

Level	λ value
F	1.05
E	1.07
D	1.11
C	1.09
B	1.06
A	1.03
MLD = 1.7% MRD = 0.4% $\lambda = 1.06$	

It will be seen that the humble hen, crouched on her nest, producing forms with an MRD of one half of one percent, was working three times more precisely than I was able to do with all those years of experience behind me. The actual deviation on the egg itself lies in the neighbourhood of one hundredth part of an inch — at the very limits of measurement even using a lens. Notice that the two forms are quite distinct; the lowest value of the first one being higher than the highest of the second. The one thing they share in common is that they are both almost perfect path curve forms. A third egg studied at the same time had $\lambda = 1.12$ with an MRD of 0.7% — not quite as precise as the two previous ones, but still of almost uncanny accuracy, and twice as good as my own effort.

This made an auspicious beginning; nevertheless I did not take these first results very seriously. Had I been able to descry good-fitting path curves on the surface of these eggs it would have made the case more convincing! And just three cases *might* conceivably have been coincidental. But later developments in the work caused me to take the eggs very seriously indeed. And I was very happy, years later, when my friend Douglas Baker, mathematics teacher in Connecticut, offered to undertake a more thorough research into this whole field of the eggs. I am indebted to him for the following results.

He analysed the eggs of 250 different kinds of bird, and found that all but nine of them were good path curves, with MRDs of less than 4%. No less than 77% had MRDs of less than 2% (Very Good to Almost Perfect) and a further 19½% had MRDs between 2% and 4% (Satisfactory to Good). Only 3½% had MRDs greater than 4% and none of them were above 5%.

This is a really remarkable result and suggests that the eggs of a great part of the bird kingdom follow the path curve form with amazing precision. We may remember that, in ancient mythologies, this form of the egg was the first form as such to appear in the history of creation. Clearly it is still with us in a remarkably pure and exact manifestation.

The birds whose eggs had an MRD greater than 4% were:

Arctic Tern
Curlew
Phalarope
Plover
Ruff
Sandpiper (two kinds)
Snipe
Turnstone

It is interesting, and maybe significant, to note that *all* these are shore- or water-loving birds, in spite of the fact that such kinds of bird took up only 41% of the original list.

Baker also analysed the eggs of eleven turtles, a garter snake and two lizards. The average MRD for the turtles was 1.6% and for the others 2.3%, all of them thus being good path curves.

Some time after the above words were written I received from my friend Dr Stuart Brown a box of six duck's eggs, which had been beautifully 'blown' by him; and these, when held at a suitable angle to the light reveal spiral markings, or striations, of just the kind which are suggested by the path curve transformations. These spiral curves are seen very clearly in the double white membrane just underneath the shell, but are also membered into the very calcium of the shell itself, showing themselves there as subtle but quite distinct differences of translucency. These spirals have proved difficult to photograph with clarity, and computer images are produced from measurements of the eggs (Figure 30). When analysed in the way described in Appendix 3, the parameters came out thus:

Left-hand egg	$\lambda = 1.08$	$\varepsilon = 1.3$	MRD = 0.6%
Right-hand egg	$\lambda = 1.16$	$\varepsilon = 0.3$	MRD = 1.05%

With MRDs of around 1% they were both exceedingly accurate path curves, being virtually perfect. When these parameters were put into the

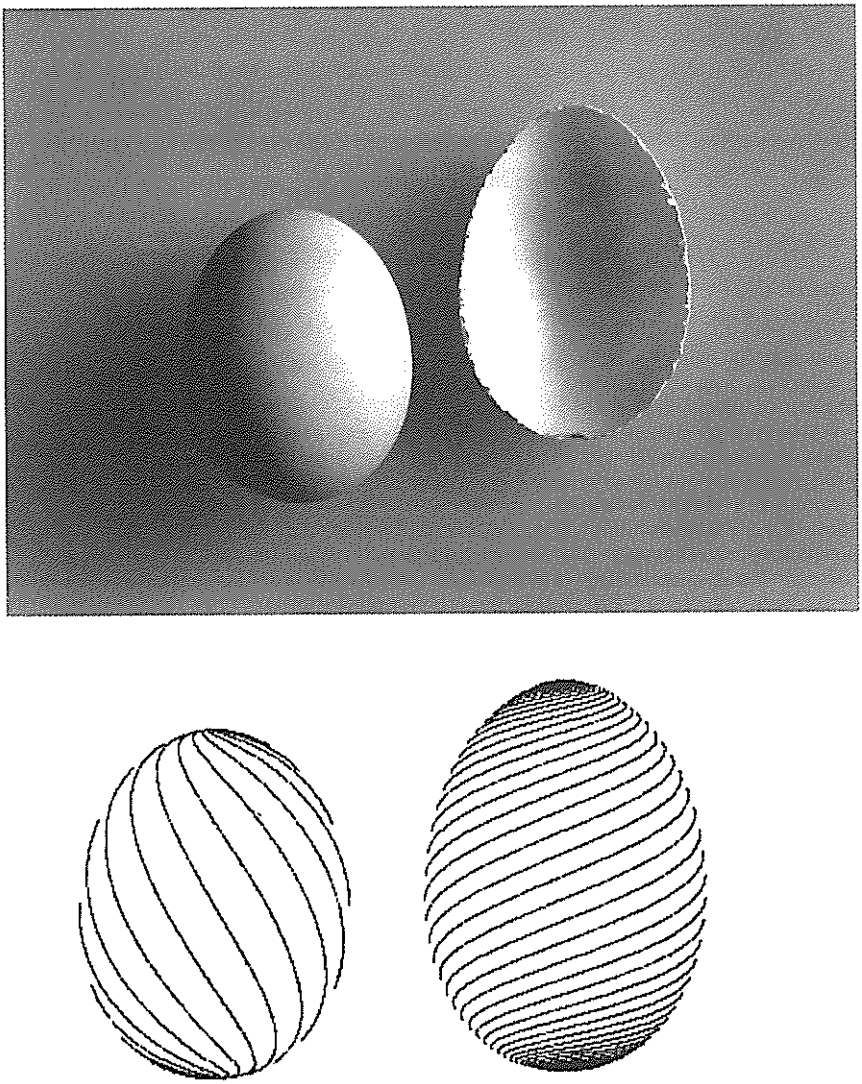


Figure 30. Above: ducks' eggs. The right-hand egg has been cut in half and photographed from the inside. Below: computer images produced from measurements of the eggs.

egg-program printed in Appendix 5, together with suitable width factors (0.415 for the left egg, and 0.38 for the right one) the computer produced the two images printed here (see Figure 30). One can see what a very close resemblance these path curves bear to the actual forms of nature.

Clearly much more research is needed here. How widely are these spiral markings to be found in the realm of the eggs? I have never found them in the shell of the hen's egg, but a suggestion of them can sometimes be found, not nearly as marked or continuous as with these duck's eggs, on the double membrane underneath the shell; this is most easily seen with very brown eggs, the pigmentation in the shell seeming to help in making it visible. The whole range of other species' eggs is in this respect yet unexplored.

As for the genesis of these spirals I think one would have to look to the spiral structure of the mucous glands in the oviduct of the bird, and these are described in Chapter 12 of this book. As far as my actual experience goes these spirals, both in the oviduct and in the actual eggs have always been left-handed ones.

Thus it would appear that this form, which arises from a consideration of some of the most basic (i.e. linear) mathematical thinking, lies also very near to the heart of the life processes in the world around us. And in this respect seems to be very widely disseminated in that world. But are eggs the only place where it is found?

Cones

Obviously the next thing to try was the pine cone, since here the resemblance is perhaps at its greatest. The Scots pines grew on the hill only a quarter of a mile away and offered a ready field of study. I picked a group of them and made large-scale photographs. Here one meets a difficulty which is not to be found with the egg; the outline of the cone is irregular and not amenable to exact measurement in the same way. What I did was to draw a smooth curve through the irregularities, to do this for a set of cones from the same tree, and then to take the mean of the measurements from all these. This, it seemed to me, would give the nearest I could get to the archetypal form which this species was striving to produce in this particular set of circumstances. Another difference from the work with the eggs was that here, not only was the form covered with well-marked spirals, but these were marked by the separate scales on its surface, so that one would have the possibility of calculating not only λ (for the form of the outline) and ε (for the steepness of the spirals) but also a form of the latter to show the discrete steps of the transformation as it moves from one scale to the next. Having done this one could construct a picture of the form, using simply the parameters which one had obtained, and

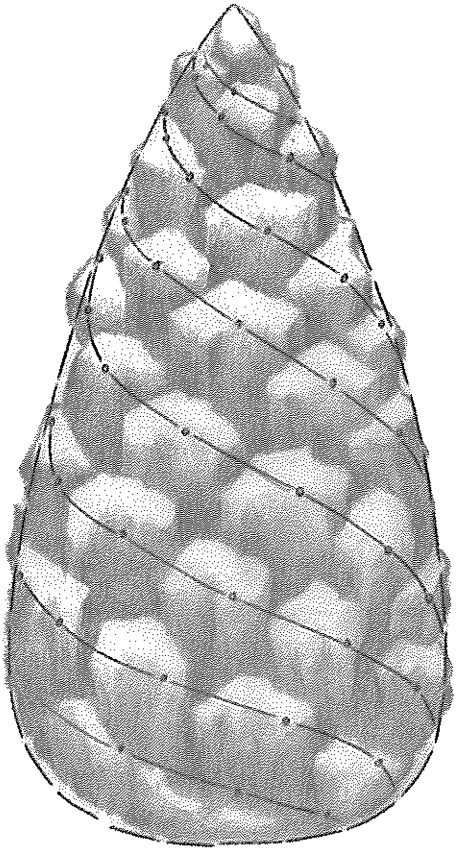


Figure 31. The cone of the Scots pine. After the mean dimensions of the cones had been measured from the photographs, and the resulting parameters had been calculated, these parameters were used, from the pure mathematics and without further reference to nature, to produce the black outlines shown here. A tracing from one of the photographs was then superimposed, in grey, so that we can see how close the correspondence is. Here $\lambda = 3.03$ and $\varepsilon = 0.22$, MRD = 1.3%.

then superimpose on this a tracing from a photograph of one of the cones. Figure 31 shows the result of one of my first efforts in this direction and it gave me a strong feeling of confidence that these cones are really, and with great exactitude, a manifestation of these path curve processes. We notice that λ here is slightly greater than three, and therefore we must expect to see the form much more pointed and blunt than with the egg, where λ is commonly about one and a quarter. We note also that the spirals fit the curves on the cone very closely, even to the discrete placing of the separate scales, along the greater part of the length of the cone, but *not* near the lower and upper poles. These are mathematical infinitudes, into and out of which the curves wind endlessly; substance cannot reach to infinity, and we could not expect the correspondence to hold here; but along most of the length of the cone — quite four-fifths of it — the fit is remarkably good.

Figure 32 shows the result of a similar experiment, done about the same time-very early on in the work, with the cone of the larch. Here λ is lower — only about 1.7 — so we have a form which is rounder (less pointed and blunt) than with the Scots pine. The form also is looser and less well-defined, but again I think it gives the impression of a very real correspondence with the mathematics.

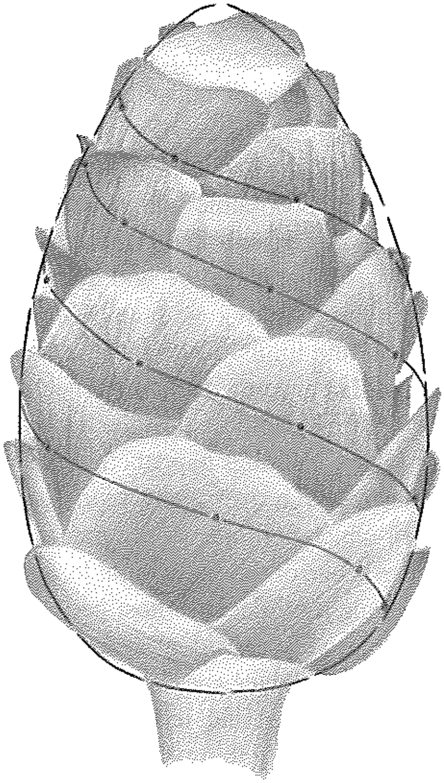


Figure 32. The cone of the Larch. Here $\lambda = 1.7$ *and* $\varepsilon = 0.23$, *MRD = 0.9%.*

Buds

This whole realm of the cones is quite a wide one and it has not been studied, from the point of view of the path curves, with the kind of detail which it deserves. From many passing observations I believe that it will one day prove to be a fruitful one, but before I could find the time to settle down to it, my attention was drawn to another aspect of this study which then absorbed all my energies for a long time. It was late spring, and I was passing a rose bush which was covered with buds, and as I looked at them I saw that not only did they appear to have this same egg-shaped form, but that the petal-edges curved upwards on them in very beautiful spirals. This opened up the possibility that the flower buds, as they open themselves to the light and air, are also manifesting this same archetypal form. If this should prove to be the case, then it would be a vast field of study indeed!

But this was only rather meaningless speculation until it had been tested by exact experiment. I gathered and photographed a set of ten buds and averaged their measurements. In the event it turned out that, with an MRD of only 1.2%, this bush was producing buds that were almost perfect path curve surfaces. This bush produced single (five-petalled) blossoms similar to the wild roses, but no doubt had some degree of artificial breeding in it. I prefer to work as much as possible with wild flowers, which I feel are nearer to nature. So a week or so later I gathered some real wild roses — they were white ones as it happened — and treated them in the same way. The result was even more striking; with an MRD of 0.6% these were perfect path curves within the limits of the most accurate measurements which I could make; and this gave me the confidence to go ahead keenly with an extended study of this whole field of the buds.

Looking back on it I feel that things happened in this respect in a rather strange way. For further research over many years has shown that of all the flowers, so many of which conform so accurately and reliably to the path curve ideal, the rose is one of the most variable, and indeed wayward. Although from time to time it produces almost perfect path curves, more often than not its forms are very considerably removed from the simple mathematical one. Of all the flower species this one most deserves the adjective 'deviant.' In Chapter 7 we shall try to penetrate into something of the deeper meaning of this. Had I with my first efforts chanced upon some of the more extreme and wayward types of rose I might well have been led to conclude that flower buds are not in fact true path curves, in which case I might have abandoned this whole line of research.

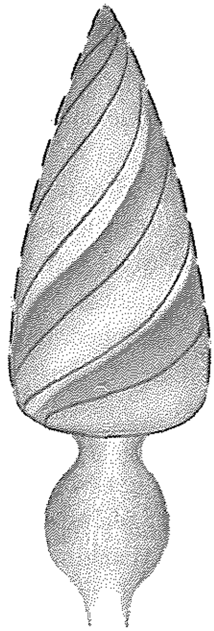


Figure 33. This Rose bud, with a λ of over 4.5 is sharper and blunter than most flower buds.

The result of this first encounter with the buds is shown in Figure 33. The grey shaded form shows the mean shape of the ten buds, and on this is a tracing of the petal edges taken from one individual bud of the set. Superimposed on this, in black, is the pure mathematical form, worked out from the parameters which had been deduced. With an MRD of 1.2% the observed and the calculated outlines are virtually indistinguishable, as we would expect. The fact that the petal edges do not coincide with the spiral curves is irrelevant. The mathematical form is covered with an infinite family of such curves, forming a field of form on this surface. What are shown are only those which I chose, or had time, to draw. The fact that concerns us is whether the gesture of the petal edges conforms to that of the field. Here $\lambda = 4.58$, considerably higher than with most flower buds, and we note the corresponding sharpness and bluntness of the form. High numbers like this are characteristic of the rose.

After this I spent years studying a very great number of species and found a high proportion of them to have buds which are good or almost perfect path curve forms. Lack of space makes it impossible to show more than a very few here; for the moment we will confine ourselves to two more examples.

Buttercup

The buttercup is one of the commonest and most beautiful of our wild flowers. On this occasion (it was in 1969) I picked eight buds, found the average of their measurements, and in this way got as close as was possible to the ideal form which this species was manifesting at that time and place. λ came out to be 1.97 with an MRD of 2.1% thus showing that it was a very good fit to its mathematical archetype (Figure 34). The figure shows the mean shape as photographed, in grey shading, and the mathematical form in black curves. One can thus see just what kind of deviation in form is implied by an MRD of about 2%. The petals on these buds are extremely delicate and will be shifted by the slightest touch. It is clear that one of them has moved slightly during mounting for photography, but the others can be seen to follow our calculated spirals closely, although not quite as closely as with the firmer and more compact bud of the rose.

The λ-values in this case turned out to be rather lower than the mean in the lower part of the bud, and rather higher in the upper part. These tendencies make the bud rather too narrow both in its lower and its upper parts. But reference to the figure will show how very minute these deviations are. The actual bud of course is very much smaller than its portrayal in our figure, its real mean deviation being in the neighbourhood

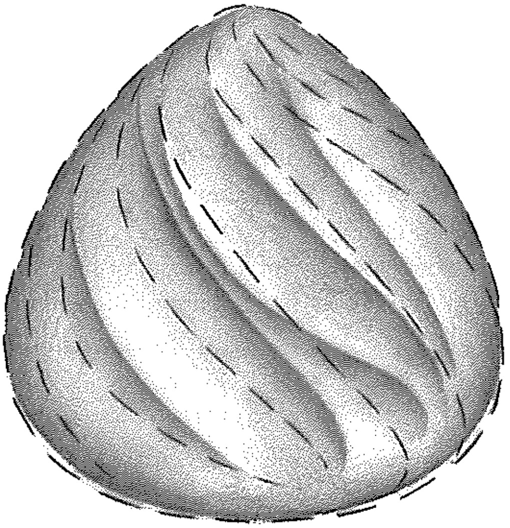


Figure 34. A Buttercup bud with λ = 1.97 and MRD = 2.1%

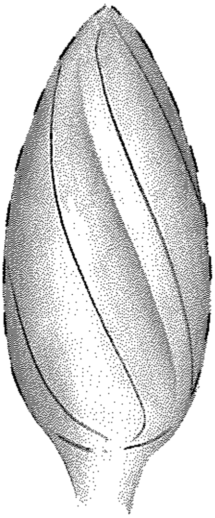


Figure 35. The Wood Sorrel is one of the most graceful of our wild flower buds.

of $^1/_{500}$ of an inch, or approximately $^1/_{20}$ of a millimetre, such deviation only being measurable under a high degree of enlargement. One can only marvel that such a delicate organism as this little bud can grow with such an amazing degree of precision; and when one finds species after species growing in a similar way one begins to feel that even these minute details of form are a matter of significance.

Wood sorrel

The other case we will illustrate is that of the wood sorrel. This delicate little plant has a bud which follows the path curve form with quite remarkable fidelity and constancy from year to year. The mean measurements of the set of buds which was picked on the occasion we are illustrating gave a λ-value of 1.90 with a mean λ-deviation of 6.4% and a mean radius deviation of only 1.1%. The grey shading in Figure 35 shows the mean outline of the set, and a petal-edge from one of the individual buds. Superimposed in black, as in the previous cases, we see the form which arises from pure calculation, the spiral curves which are shown being just a random selection from the infinite family which is mathematically present. We see that the bud follows the mathematical form with hardly any visible deviation.

Leaf buds

Another realm where similar things are found is that of the leaf buds of our great deciduous trees, the oak and beech, and so on. This has since proved to be a field of specially fruitful study, and much more will be said about it later in the book. Here we will confine ourselves to picturing a leaf bud of the oak (Figure 36). These buds had a mean λ of 1.7, and an MRD of only just over 1%. They were therefore almost perfect path curve forms. It would not have been useful to have added to the drawing the calculated outline in black; it would simply have obscured the edge as drawn. And most of the leaf buds of our great trees follow the path curve form with a similar degree of accuracy.

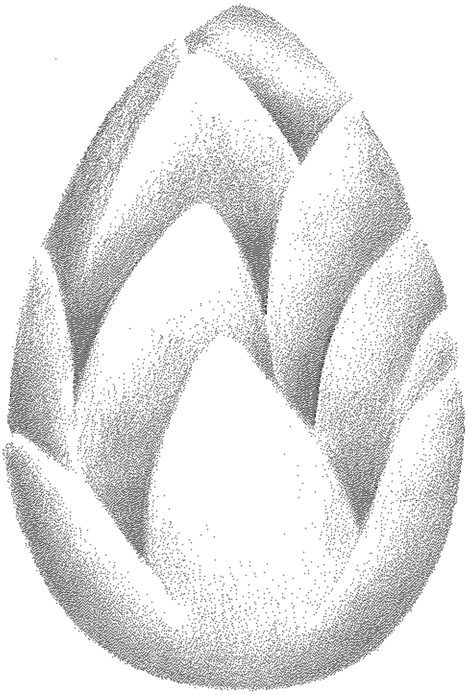


Figure 36. The leaf bud of an Oak.

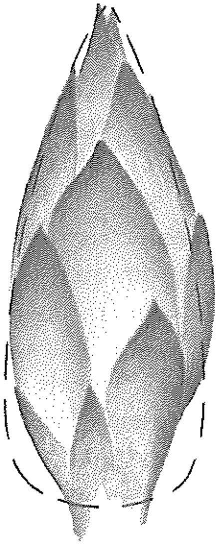


Figure 37. The outer form of the Rhododendron quickly revealed itself as not being a good path curve form.

Rhododendron

However, it was not all plain sailing. Soon I came up against the rhododendron. This is more an inflorescence than an individual bud, but its appearance left me in little doubt that here indeed the path curve form was to be seen. However, on measuring it the results came out differently from what I had expected (Figure 37). Here are the λ-values for levels *A* to *F.*

Level	λ-value
A	1.3
B	1.2
C	2.1
D	3.8
E	6.0
F	7.0
Mean λ	3.6
Mean λ-deviation	56%

The very high values of E and F show that the bud is too narrow and pointed at the top, and the low values at A and B show that it is not blunt enough at the bottom or, phrasing it a little differently, too narrow also there.

Alternatively one can say that the bud is too wide in the middle; but however one chooses to look at it, the fact is inescapable that in no way can the path curve form be made to adapt itself to this shape. Figure 37 shows, in the grey shading, the tracing from the photo of the actual bud, and superimposed on it, in black outline, the nearest path curve form which can attempt to fit it. The MLD of 56% means here an MRD of just over 12% — a long way above the limit of 4% which we set ourselves for a satisfactory fit.

This was a disappointment. However much I might think that I could descry the path-curve form in this rhododendron bud, the mathematics told me that I was wrong. But in this it was also a reassurance. It was clear that the mathematics would quickly spot a case in which the path curve appearance is simply a resemblance, and not strictly true.

In this particular case there was more that one could do. I started with some forceps carefully picking away the little green sepals, one by one, and continued until I reached the actual flower inflorescence which nestled within. This was January, several months before the flower was due to see the light, but already the little buds were delicately coloured.

This form, pictured in Figure 38, subjected to a similar analysis gave the following results:

Level	λ-value
A	1.65
B	1.8
C	1.9
D	1.85
E	2.0
F	1.9
Mean λ	1.85
MLD	8%
MRD	1.2%

The MLD came out at 8% and this indicates a good fit indeed with the path-curve form.

Hollyhock

This result was later confirmed by the study of the Hollyhock. This bud is covered with thick green sepals, and photographed in this form gave a mean λ-deviation well over 20%. In this case it was found impossible to remove the sepals without completely deforming the coloured part of the bud within. However, this bud is compact and firmly built, and it was found possible

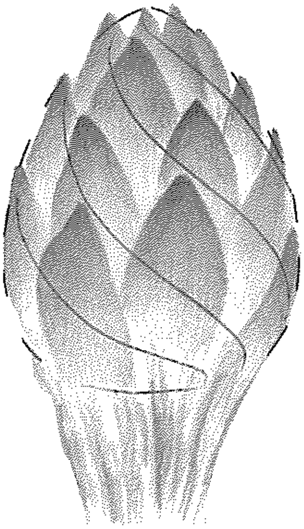


Figure 38. Inside the Rhododendron bud one comes upon the little coloured inflorescence of buds, which, taken as a whole thing, with an MLD of 8% and an MRD of 1.2, is a very accurate path curve form.

to cut it in vertical cross-section with a sharp knife, without deforming its shape. The coloured part of the bud thus revealed turned out to be an almost perfect path curve, with an MLD of only 6% and an MRD of 1.0%.

These results showed me that the path-curve form proper is to be found chiefly in the actual petals, the coloured part of the bud. And one of the problems met in this work on the buds is that of removing the outer layer of green sepals so carefully that the form of the petals within is not interfered with. When it is found impossible to remove the outer sepals without injuring the bud within, one is forced to measure and work with the form of the sepals. When one does this one often finds nevertheless that the form is still a good path curve, but it may be deformed, in other cases, to a greater or lesser degree.

Random tests

Results such as these might be considered striking enough, but the question still remained as to how widely this quality is to be found in the plant world. And so, after working for two or three years along these lines I felt that I must have a 'stocktaking' of results so far. I found at that date that I had analysed the buds of fifty-five different kinds of plant, and that the λ-deviations were disposed as follows:

Mean λ-deviation %	Percentage of buds
0–10	54.5
10–20	36.5
over 20	9.0

Over half of them were of the same order of accuracy as the Wood Sorrel, and less than one tenth of them fell outside the 20% limit.

The doubt still remained as to how flexible this path-curve transformation really is; how easily can it accommodate itself to all sorts of random ovals? Sets of random ovals are not easily obtainable! I solved the problem by asking each of twenty-five young people to draw me an oval.

Before doing this I had to consider another possible cause of doubt. It is obvious to any observer of the plant kingdom that not *all* kinds of plant produce buds of this general form. To take just one instance, the leguminosae have buds in which there is clearly some other factor at work. If I find a bud that is obviously not a path curve I do not go to the trouble of photographing it in order to prove the fact to myself. It seemed to me to be a significant fact that I could find so easily in my immediate environment something like fifty different species which follow the path form so minutely, but I could not hide from myself that there had been, more or less unconsciously, a certain (I hope, small) factor of selectivity in my choosing of them. If my experiment with the random ovals were to be valid something would need to be incorporated into it to balance this small element of selectivity. I therefore asked the pupils to make their ovals symmetrical about a central axis, and to let them tend to be rather more blunt at one end, and sharp at the other. When doing this I felt I was in effect simply ordering a set of path curves.

In the event the result was far otherwise. On analysis the mean λ-deviations of these ovals were as follows:

Mean λ-deviation %	Percentage of ovals
0–10	0
10–20	16
over 20	84

A further experiment carried out some time later, with a different set of pupils, this time, asking them simply, ‘Draw me a bud’ gave a generally similar result.

These figures show that the chances that a randomly produced oval, even if it bears superficial resemblances to the bud form, should be in fact a good path-curve form are remote, and the figures actually obtained for the growing buds become highly significant.

The question as to what percentage of forms which are good path curves can be expected in a set which is not specifically connected with the path curve realm is such an important one that it deserves some more attention. There are two further ways in which the plant world continuously presents us with oval forms more or less of this nature — these are in the ovaries, and the leaves. Treating the ovaries as though they were path-curve forms, the mean deviations came out as follows:

Mean λ-deviation %	Percentage of ovaries
0–10	15
10–20	25
over 20	60

I then analysed leaves from thirty-five different species of plant. Here again a certain selectivity had to be employed. Some leaves, for instance ivy, holly and maple, are so shaped that they are hardly susceptible to the kinds of measurement which we are making here. I chose the first thirty-five species which came to hand, whose leaves were smoothly oval, which in fact, could reasonably be expected to fit a path curve sort of form. Their mean deviations were disposed as follows:

Mean λ-deviation %	Percentage of leaves
0–10	9
10–20	14
over 20	77

Of all leaves 43% had mean deviations greater than 50%.

Now I think it is obvious that we have no right to consider either the ovaries or the leaves as random ovals. They must have organizing principles at work in them which would prevent them from being in any sense random. But as we look at these figures I think it is equally obvious that these principles are not straightforward path-curve ones. And I find it reassuring that the methods which we are using here are such as to make it abundantly clear to us when the path-curve process is *not* at work. The figures for the leaves and the ovaries are, in general, quite similar to those deriving from the ‘random ovals’ drawn for me by my classes of helpers, and I am reasonably convinced that they do show what is to be expected when the forms we are analysing are not really connected with the path curve realm. And the comparison of these with what we find, again and again, by measuring the actual buds, is most revealing.

After twelve years of working practically, in this way, with the forms of nature, I found that I had analysed the buds of 150 different species of plant, chiefly in Scotland, but also many in New Zealand and Australia. A full examination of results showed that no less than 92% of these were producing buds which were either a Good, or a Very Good, fit with the path curve form, and only 8% could be classed as 'deviant.' This is really a very important matter, and I do not think that so much time spent on it could be regarded as wasted. Unless we have evidence, gathered with sobriety and care, that these buds are really manifesting this form, we cannot continue our research with any confidence that we are not following some will-o'-the-wisp of fantasy or wishful thinking.

At this point it is useful to mention that Professor Clopper Almon of Maryland University has shown interest in this work. Investigating the path curves from the point of view of modern linear algebra he arrived at a series of equations which employ rather different parameters from the ones I have described, but which are in every essential respect the equivalent of the results which I have used. His excellent monograph (see bibliography) is to be recommended to anyone wishing to approach the realm of the path curves from the point of view of linear algebra. At the time, I sent Professor Almon the data of the measurements which I had made on a long series of buds, and he fitted these to the path curves using his equations; and then made a fairly full statistical analysis of the results, using greater computer power than I possessed. Since he was using different parameters he did not come upon mean deviations for λ, but in his published results he gave the mean radius deviation (MRD). It is interesting to take notice of these because they have been arrived at by computational methods which are independent of mine.

Out of a total of one hundred and fifty species, only four had a mean radius deviation of more than 4% (all of these from the southern hemisphere), and a further thirteen had deviations between 3% and 4%. No less than one hundred and sixteen of them (77% of the total) had deviations of less than 2%. Consideration of these figures gives one a heightened appreciation of the accuracy with which these little buds grow according to their ideal model. And they show what a sensitive measure the λ-deviation really is.

When I was first working on these matters I was doing so on the assumption that each species would be found to have its characteristic λ-value, and that this number would, I hoped, then help to throw some light on the nature or quality of its species. Further work has shown me that things are not quite like this. We find that a plant will have different λ-values from day to day as it passes through different stages of its development; and two plants of the same species may have different λ-values even at approximately the same stage of development. And there is a suggestion that the same species, growing in a different environment, will differ in its λ-values. The matter is infinitely complex. The one thing that nearly all have in common is that their forms are good path curves. The absolute value of λ found at any one moment is not the important thing. It nevertheless remains a fact that some species have buds with consistently low λ-values, and some with generally high ones. A list of these is instructive.

Buds with low λ-values:

Daffodil
Bluebell (wild hyacinth)
Hawthorn
Comfrey
May
Blackberry
Peach
Cabbage
Viburnum
Ginger
Apple
Bush Clematis (New Zealand)
Poroporo (New Zealand)
Cress
Cherry

It is interesting to note how many of these provide edible fruits or other food.

Buds with high λ-values:

Rose
Campion
Fuchsia
Convolvulus
Garlic

Clematis (Scotland)
Columbine
Periwinkle
Fremontia
Flax
Mallow

Most of the others which I have measured come into the middle region, where λ = 1.5 to 3.

Individual buds

All the work described so far concerns average measurements taken from sets of buds. The individual bud, growing amidst all the accidents of its environment, on the whole, fares remarkably well. Here are some typical sets of ten buds each.

Firstly, the ash leaf bud. This very remarkable little bud seems to consist of two path curve profiles set at right angles to one another, and strangely intertwined. The figures given are for the average measurements of the two profiles for each bud. The buds are very compact and firm, and not liable to damage.

λ-value	Mean λ-deviation %
1.95	4.5
2.41	4.6
2.04	4.4
1.83	14.8
2.47	4.3
1.92	5.0
2.70	16.4
1.99	3.5
1.81	16.5
2.55	18.3

Six of them are almost perfect, and none of the other four strays far from the ideal form.

A big contrast is the buttercup. These buds are so fragile that one has to be very careful indeed when picking them. The slightest clumsiness can disturb the petals and change the shape of the bud, either in gathering them or mounting them for photography.

λ-value	Mean λ-deviation%
1.41	15.7
2.13	10.1
1.22	6.7
1.18	16.0
1.34	5.5
1.18	6.7
1.14	17.0
1.90	36.4
1.14	9.7
1.09	14.7

We note the comparatively wide range of λ-values; but also the fact that no less than half of them have mean deviations ranging from about 10% downwards, and only one, at 36%, has failed to survive unscathed. With a variation in λ from just above 1 to just above 2, there must have been a considerable change, from one bud to another, in the sharpness of the form in the top part of the bud, yet in every case except one, this was quite precisely counter-balanced by a corresponding change in the form of the blunt part of the bud below, and this, in a bud which is remarkable for its delicacy and fragility. I think the buttercup is to be congratulated!

Many other cases could be cited. I will give only one more here: the little flower bud of the honesty. When taken early, as these buds were, the petals are tightly packed within their sepals, and one has to be very careful indeed that the petals do not press outwards as soon as the restraining sepals are removed, thus changing the shape of the bud. Often the petals look a little crushed, making the outline of the bud rather uneven, and hard to measure. Nevertheless the path curve form is preserved with remarkable fidelity, only two of the separate buds coming above the 20% level.

λ-value	Mean λ-deviation %
1.43	16.7
1.43	16.8
1.68	12.6
1.86	9.0
1.35	24.8
1.58	15.7
1.83	27.4
1.58	11.8
1.89	7.9
1.76	10.5

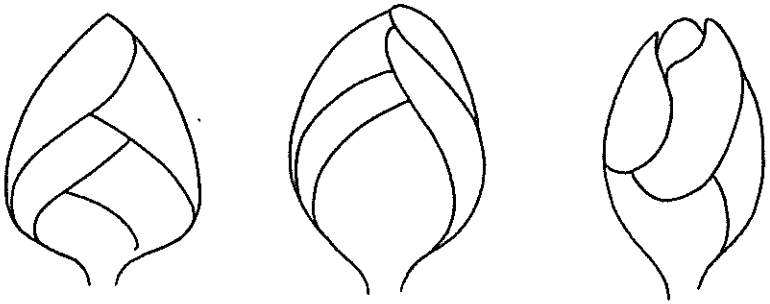
These figures are fairly typical and show that on the whole the individual buds grow with remarkable constancy and accuracy. But we will close this section with details of one of the deviant buds, the snowdrop, about which much more has to be said later.

λ-value	Mean λ-deviation %
2.18	19.5
2.08	18.3
2.01	19.2
2.26	19.7
2.02	20.6
2.10	28.7
2.04	20.3
1.86	21.7
1.95	13.1
2.05	22.4

These buds are firm and compact. Notice how extraordinarily constant the λ-values are; these buds are almost identical within the limits of what one is able to measure, even under a high degree of enlargement. And they are constant also with their deviation. This deviation is no accident of growth, but something which is deeply significant for the life and quality of the plant. This is a comparatively mild case; at certain moments of their development the snowdrops will reach a deviation of 30% or even more. If we wish to see what such a deviation means in terms of actual form we should turn to Figure 44; this should give us a new respect for the accuracy of those buds whose deviations remain below the 20% level.

Spirals

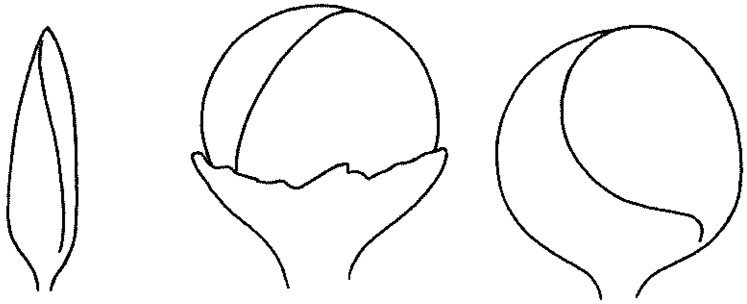
Having dealt with the form of the bud, and its λ-value, we next have to consider the curves taken by the petal edges as they spiral around the bud. This is a matter of greater difficulty. In the first place we must remember that these path curves are logarithmic in character, that is they spiral infinitely into and away from their poles. Neither the petal edges, nor any other curves in nature, can be expected to do this; substance cannot reach infinity! But in the middle part of the bud in many cases the petal edges will be seen to take a truly spiral path, closely resembling the kind of path curves which cover this sort of surface. The parameter which measures these spirals is the one written as ε. If ε is zero the path curves become horizontal 'circles of latitude' and as ε increases towards infinity they tend to 'lines of longitude.'



The wild rose, a beautifully regular interweaving.

Celandine tries to do the same thing, but is much more fragile.

Wild strawberry — very small, very delicate, very beautiful — and almost impossible to measure!



In convolvulus ε becomes larger.

In the almost spherical bud of red may the spiral element is not so easy to see.

In the hollyhock it is almost obscured; one can have almost any value for ε that one wishes, according to where the points M and N are placed (Figure 164); just say 'ε very large.'

Figure 39. Some examples of ε.

When ε is low (say 0.1 to 0.3) the petal edges wind round the bud at a fairly small angle to the horizontal, and their spiral character is then usually quite easily seen. This remains so on the rose (ε ≈ 0.5) and even with higher values of ε. However, when ε becomes very high the spiralling character is not so easily evident, and the buds merge into those in which one would hardly have suspected spiralling unless one had already seen it in the other cases. Figure 39 shows some typical examples of ε.

Summary

I think that we can now really say that these path curves and their related surfaces, arising as they do from the simplest, most elementary mathematical thought-forms, are significantly at work in the realm of these opening buds. It must be a matter of some moment to the plant whether the λ of its buds is high or low. But the evidence goes to show that plants, even within the same species, have considerable freedom to 'play' with this value of λ, although they seldom depart far from the general path curve form itself. It would seem that the absolute value of λ attained at any moment is less important than the way that this value varies during the course of development, and less also than the disposition of the deviations along the central axis of the bud. More will be said about this important matter later in the book.

Further considerations

Before going on to enquire about other kingdoms of nature it would be well to ask ourselves whether this path curve form is confined, within the plants, to that of the buds alone. How about the plant as a whole thing? Obviously the difficulties of investigation here are much greater than with the bud, which fills its space so compactly and so easily-to-be-measured. The ideal form within which the branches and leaves of the plant develop is not so easy to descry. But there is a related problem; if the whole plant is growing on a path curve plan, it would be natural to expect the separate branches to leave the main stem at intervals of a growth measure.

On this I have done a small amount of work and give this account in the hope of its being a stimulus for other people's work rather than for any definite results achieved.

When I was in Sydney, my friend Erwin Berney produced two very fine photographs of fern fronds of the type which grow so profusely there. Together we measured the space of the leaves and applied the formulae of p. 303. I think we were expecting growth measure, but in each case we actually got step measure (see pp. 34, 37). In the first case the invariant point came about 7 to 10 cm beyond the end of the frond and in the second one about 18 to 20 cm beyond the end. Step measure was well established.

But it is not always so. I have tried with eleven stems of stinging nettles taken in the autumn when they were fully grown. Quite good growth measures were found, with a multiplier of 1.46 (the mean result of all the stems) and mean deviations ranging from 5% to 13%. On nine green-pea stems the results were even better: $m = 1.46$ with the average mean deviation only 4.5%. However in both cases there was a small but distinct tendency for the multiplier to increase towards the base of the stem. I had fixed the invariant points as being at the tip of the stem, and at the point where the stem passes into root. If I allowed the lower invariant point to be between 5 and 8 cm lower, in fact about the middle of the roots, then the growth measure became almost perfect, the mean deviation in the case of the green pea shrinking to only 2.4%.

I have an idea that this tendency for the multiplier to grow too large just at the lower end of the stem may turn out to be a fairly general one; I don't know. I have also measured the spacing of side-stems on the dead nettle, and on carrot leaves; in both cases the growth measure was almost perfect, within the limits of the probable error of the measurement. With the dead nettle, $m = 2.49$, mean deviation 2%; with the carrot, $m = 1.7$, and mean deviations ranged from 4% to 8%. But in both cases there were no side-stems anywhere near the root end of the main stem.

This brings us back to our first, and very important, question. We know that a large proportion of buds grow on a path-curve plan, but what about the plant as a whole? We see the branches and leaves of a plant growing outwards from the main stem, and then at a certain point the further outward growth ceases; does it cease on the boundary of an otherwise invisible path-curve form belonging to the plant?

With certain trees (particularly the conifers) one feels that one can see the path-curve form in front of one's eyes; with other trees and bushes it is not so easily discerned and with the flowering plants not at all. But if one were to measure the outward growth of the tip of each side-stem of such a plant, and were to plot it on a diagram against the height above

Figure 40. Dead nettle. A first, rather primitive, attempt to record the mean form of a whole plant.

the ground which it had attained, and were to do this for very many side-stems of many similar plants of the same species, would one then find a path curve form appearing on one's page? Obviously such an investigation would be attended with many difficulties, and a very accurate form could hardly be expected. I have tried it once or twice, with not very convincing results, but I have not had time to give the problem the care it deserves, and a great deal more work needs to be done on this matter. While doing the dead nettles I measured the positions of the tips of the side-stems: how far out they were and how high above the ground. Figure 40 shows these positions. Clearly the 'scatter' is too great to give a very convincing result. I have drawn a smooth curve through them as well as I could. It is in fact a good path curve, with λ = 0.38 and mean deviation 10%. I give it for what it is worth.

6. Path Curves in Other Kingdoms of Nature

Once our imagination is opened to them, we begin to see the possibilities of path-curve forms in a wide variety of places in the surrounding world. The temptation to become fanatical and in a facile way to see them 'everywhere' must be resisted. Fortunately the mathematics itself is a fairly strong corrective for this; we know that the chances of any random oval being a good path curve form are, in fact, slight; if we are being fanciful, the mathematics will soon tell us; but if we find that such ovals, again and again, are accurate path curves, then we have at least good preliminary evidence that the path-curve process is significantly at work.

Douglas Baker found that a series of turtle plastrons (undershells) were all good path curves, and some further work which he was able to do suggested that the whole field of the insect carapace might prove a very fruitful one to study in this respect. Obviously it would be a very large task. I have not been able to follow it up, and must leave this to some enterprising reader.

A further, and obvious, field of application for these path curve forms would be that of the sea shells. I have done no practical work in this direction, but I have the strong expectation that the monovalves will one day be found to be almost perfect path-curve forms. The bivalves may present a more difficult problem, but I wonder whether some of the special cases, where two, three, or even four, of the points of the invariant tetrahedron become coincident, will be found to apply.

I have done some work with the sea urchin. The shells of these beautiful little creatures have an exactly circular horizontal cross-section, and in vertical profile they are strongly reminiscent of the path-curve form. Analysed thus, five specimens gave the following results:

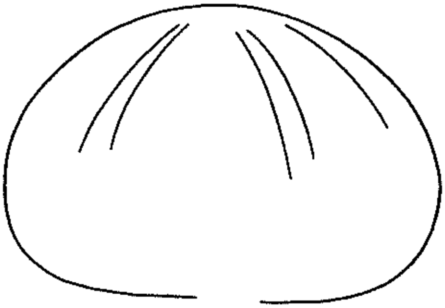


Figure 41. The vertical profile of a sea urchin is an almost perfect path curve form.

Specimen No.	λ-value	Mean λ-deviation %
1	1.44	11
2	1.48	10
3	1.43	12
4	1.37	9
5	1.56	11

The constancy of the λ-value in specimens which varied in size by a factor of more than 2 is remarkable; and the fidelity with which they follow the mathematical model is only slightly less than that of the eggs. The distribution of the deviations was also remarkably constant, in every case λ being rather too high near the base. This merits further consideration. Figure 41 shows the outline of specimen No. 5.

The distribution of the deviations strongly suggested that the 'true' pole of the form should be very slightly below the centre of its base. It was found that if the lower pole is taken as being 5 mm below the observed base then the resultant λ-value is 1.38 and the mean deviation 3.7%.

Figure 42. The teeth of the sea urchin project through the hole in the bottom of the shell.

Now the gap in the lower part of our outline represents a small circular hole in the base of the shell, and this is the mouth of the creature. In life its teeth project through this, and it walks across the seabed, literally, on its teeth. Some years after making the observations which are described here, I found on the seashore a fragment of a sea urchin shell with the teeth still in place (Figure 42). They formed a perfect little five-membered cone, projecting through the mouth.

I measured as accurately as I could just how far the point of this little cone projected below the base of the shell. It came to between 5.6 mm and 6.1 mm, a figure very close indeed to that to which the adjustment process has pointed.

I believe that with further investigation we shall find that this general form is much more widely spread in the world around us than we at the moment can realize. Stephen Eberhart has pointed to two cases where creatures of microscopic size have assumed this form with really remarkable accuracy: the Euglina and a plant chromosome.*

Also Barry Christian has written an exceedingly interesting article† on the megalithic stone circles of the western and northern parts of Britain. Many of these are not circular, but egg-shaped, and follow the path-curve form with remarkable fidelity. In all these, and probably in other realms, it is clear that a great deal of further research is still needed.

I would refer the reader also to an excellent article by Stephen Eberhart‡ on 'Grecian amphorae as path-curve shapes,' but as these are artefacts rather than living forms of nature further details perhaps hardly belong in this book.

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* *Mathematical-Physical Correspondence,* No. 19 and No. 20.

† *Mathematical-Physical Correspondence,* No. 30.

‡ *Mathematical-Physical Correspondence,* No. 27.

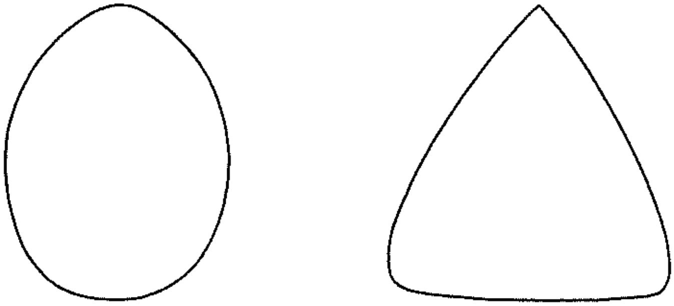
7. Further Considerations About Flower Buds

In the early stages of the work with the plant buds I had assumed that each species would have its typical λ-parameter, and that this value would be characteristic of the species. Further work did not support this assumption. It later became clear that different plants of the same species may produce buds with considerably different λ-values according to the environments in which the plants are growing, the stage of development which the buds have reached when they are picked, and probably a number of other factors. Nature, I think, works like a musician playing variations on a theme. The basic theme is the whole range of path-curve possibilities; and each plant, according to its species, its environmental conditions and its stage of growth, picks a value of λ which is 'right' for it at that moment.

A qualitative consideration of the λ-value

I think that it is important that we should now consider these matters from a more qualitative point of view. Through many years of considering and dealing with these forms I am beginning to have a growing feeling that a high λ is to be associated with a certain situation of tension. A high λ implies a great difference between the multiplicative factors of the logarithmic spirals in the top and bottom invariant planes of the transformation; but a λ near to unity arises when the spirals in those planes run easily and naturally, in harness with one another, as it were. I would not wish to draw any close comparison with the field of electricity, but one thinks of the high tension current induced by a large potential difference in the ends of a wire. And if one contemplates the forms implied by a low and a high λ, I think this feeling can be strengthened.

The lower λ, that is, near to unity, gives a gentle, rounded, easy-going form, one which, we feel, could easily open itself to the world around it. But the higher λ gives a form much more angular, much more tightly woven, more tightly closed upon itself. The left-hand form of Figure 43 is in some way relaxed, out-breathing; and the right-hand one is



$\lambda = 1.2$ $\lambda = 6.0$

Figure 43. On the left the form is relaxed and out-breathing in quality. On the right it is more tense; one is reminded of in-breathing.

more tensed and in-breathing in character. One is reminded of Rudolf Steiner's words about out-breathing and in-breathing, pleasure and pain, joy and sorrow, laughter and weeping.* These feelings may seem fanciful on my part, but such fancies may also contain truth; and they have been supported by other facts which have come to light.

Variations on a theme

In considering the matter of variations on the theme, it is important to ask whether some plants not only choose their particular numerical value for λ but may also be playing some sort of variation on the path curve theme itself. It is fairly easy to see when such is the case. If, looking down the λ-values calculated for the levels *A* to *F*, we find the deviations from the mean disposed randomly, first plus and then minus, and so on, we are probably dealing with a bud which has grown true to the path-curve form, excepting that accidental variations have occurred along its length. But when the variations show some systematic trend then we know that this particular bud is departing, perhaps only slightly, from the path-curve form itself. In the majority of buds so far studied (probably as many as 85%) such systematic deviation is so slight as to be negligible,

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* See Paths of Experience, Chapter V.

but there is a residue of buds which do show small but definite systematic deviations; and these it is important to study.

Looking through my notebooks of many years of observation and measurement of well over a hundred species it becomes clear, to just a preliminary view, that these deviant buds fall into two quite distinct groups. The first group I should like to name after one of its prominent members, the snowdrop. (There are other members of the group which could equally well have been taken to name it, but the snowdrop is perhaps specially characteristic; it grows wild in Scotland through the woodlands and in the hedgerows, and I have done more work on it in one way and another than perhaps on any other kind of flower.)

The characteristics of this group are that each bud maintains an almost perfect invariance in λ through all the lower levels, from *A* to *D,* but then suffers a sharp decrease in λ near the tip, at levels *E* and *F* And almost all the members of this group have a fairly low λ-value, below about 2.2. When one looks at such a bud, one's first impression is, what a perfect form! But as soon as one compares it with the mathematical model one sees immediately that it is just slightly too broad, too wide open as it were, near the tip (Figure 44).

Other strong and very characteristic members of this group are the primrose, daffodil and bluebell (wild hyacinth). In a more moderate

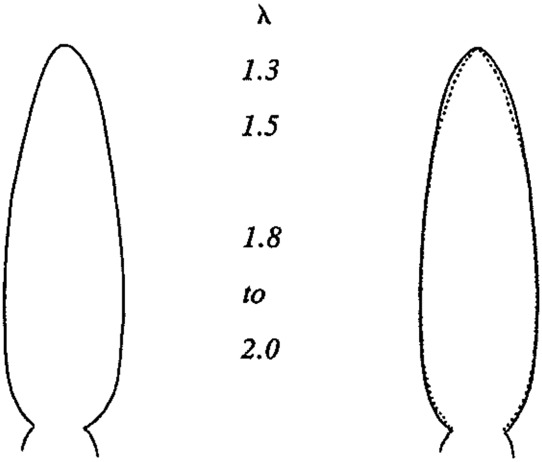

Figure 44. Comparison of a Snowdrop bud with the ideal mathematical form reveals a quality in the former which might not otherwise have been suspected.

form we have the winter jasmine and the little yellow flower of the cabbage. The narcissus shows the decrease in λ near the tip, but differs from the rest of the group in having a higher λ-value.

Now if we review the characteristics of this group of plants we see that, with the exception of the cabbage, they all have one thing in common: they are early spring flowers. They bloom at a time when all nature is streaming into manifestation; one can get the impression that, carried on this springtide, they are almost over-ready to open themselves to the light and air. I begin to see this form as one which carries the gesture of the low-λ forms a stage further; it is the gesture in a very special way of opening, relaxation, out-breathing, of the purely vegetative life forces.

The other group shows the opposite tendency: λ becomes to high in the upper part of the bud. One can have no hesitation or difficulty in naming this group. One plant is so pre-eminently characteristic of it that it has no rival. This is the wild rose.

The rose bud deserves some paragraphs all to itself. I cannot think of another bud which has these very special and peculiar characteristics developed to such a degree. The λ-values vary greatly from plant to plant, but are always high, from about 3 upwards, even sometimes to ∞. In order to understand the matter clearly it is necessary to realize that the method of appraising forms by their λ-deviation is a vitally important one, in that it leads us directly to a view of what is really going on in the transformation processes which have led to these forms; but also it is necessary to realize that when applied to the 'closeness of fit' of the actual physical form which is visible to us in space, this method has definite limitations which must be borne in mind.

λ-*deviation*

To get a full picture we need to find also the radius-deviation at each level. This gives us a view of the physical form in the same way as the λ-deviation gives us a view of the transformation process which is involved. The λ-deviation is a very sensitive measure. In a general way we can say that a deviation of 10% in λ is equivalent to one of only about 3% in radius. But this varies with the size of λ. When λ is near to unity, a form, if it is to be a good fit, needs to have a λ-deviation of not more than 10% to 12%. When λ gets to the neighbourhood of 2 or 2.5, the λ-deviation becomes much more sensitive and mean deviations of 20% become fairly acceptable; and with very high λ this deviation measure becomes so sensitive that it has to be used with great circumspection. (In the early work I fixed 20% mean deviation as a general limit for a good-fitting bud; one had to draw the line somewhere. But it was always clear that these ideas must be refined and elaborated considerably in due course.)

The next thing to realize is that the sensitiveness of the λ-deviation varies considerably from one part of the bud to another. In fact it varies with the gradient of the curves shown in the nomographic method described in the appendix. This means that the sensitiveness of the λ-deviations is not only greater for the higher λs, but it is also much greater at levels *C* and *D* than at levels *A* and *F*. Thus it comes about that with high λs one can have very high λ-deviations indeed, more especially in the middle part of the bud, without the actual radius-deviations becoming alarmingly large.

And this is what happens, quite often, in the case of the rose. Two varieties of rose were the first flower buds I measured; and those first two plants were ones whose buds were moderate forms. And this gave me the encouragement to continue with the study. Later, the rose became a source of some embarrassment, when so many more extreme forms were found; but by then I had done enough other kinds of plants to realize that the rose is in some way a special case. Had I come upon the more extreme specimen of rose right at the start I might well have concluded that buds are not really path curves at all, and might never have proceeded with their study.

Here are the figures for a rose bud:

Level	λ-value
A	4.02
B	4.75
C	6.29
D	11.5
E	10.6
F	4.72

This is a fairly moderate, and typical, rose bud. It has a mean λ (weighted* × 4, 2, 1, 1, 2, 4) of almost exactly 6 and a mean deviation (also weighted) of 36%. In an ordinary bud this would mean that it is not a good path curve, but because of the size of λ here, and of the special disposition of the values at the various levels, this particular bud has a mean radius deviation of under 4% as is shown in Chapter 5. Notice that

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* See Appendix 3 for weighting procedure which is used from now on.

the upper part of the bud (*D* to *F*) has a mean λ of 7.4 and the lower part (*A* to *C*) has λ equal to only 4.6. The fact that the highest values occur not right at the tip but just below it, is typical of this kind of form.

Now let us look at a more extreme rose bud form. This was a specimen of the ordinary pink wild rose that grows so freely in our hedgerows. (The first two rose buds which I measured were both white ones; I have the impression that the more extreme forms come more often with the deeper colours, but I have not at the moment enough evidence to support this firmly.) Here are the figures:

Level	λ-value
A	5.28
B	18.3
C	∞
D	∞
E	∞
F	7.65

Wherever ∞ is noted it means that the value has run up to ∞, or beyond, coming out as a negative number. Let us just call it, for the moment, 'very large indeed.' Notice that these numbers, although they look at first sight to be so different from those of the previous example, are in fact disposed in a similar way. It is a typical form.

One cannot of course calculate a mean λ, or a mean λ-deviation for such a form, but one can endeavour to see what sort of fit is possible. We could for instance calculate a form for, say, $\lambda = 15$. We then find that this bud has a mean radius-deviation of 9%, which means a very perceptible deviation from the calculated form, but not nearly as wild as one might have expected from the above figures. Figure 45 shows what it looks like.

On the left is shown the form of the bud itself, and on the right, the bud compared with the path-curve form (dotted curve) calculated for $\lambda = 15$. The more moderate form of the rose bud quoted above would have an outline which would come somewhere intermediate between the two superimposed forms on the right-hand diagram; in fact we can see that its form would not be greatly different from the more extreme form we are now dealing with.

At this point, if we are to make further progress we must have the courage to contemplate these forms qualitatively. Firstly I think we can agree that although this more extreme form shows perceptible deviation, it is still nevertheless recognizably of the path-curve quality. The plant

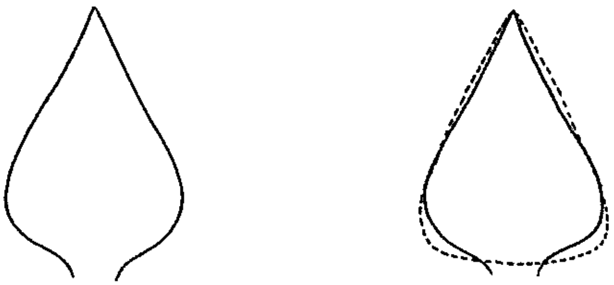

Figure 45. The Rose shows an opposite gesture from the Snowdrop; notice how pinched and narrow it becomes in its middle and upper part.

has obviously taken the liberty of making a considerably free variation, but it is still a variation of the original theme. Next we look at the middle and upper part of the bud; we see how closed it has become, how tightly woven upon itself. We compare it with the form of the snowdrop. These two surely show opposite gestures in the language of form. If the snowdrop is out-breathing, opening to the light and the world, the rose is in-breathing, tightly closed, a little withdrawn. The snowdrop is like an eager child; the rose is virginal. The one is innocent; the other chaste.

I would refer again to Steiner's words on out-breathing and in-breathing (see p. 93); I am sure he does not mean simply the taking in and out of air in the lungs; an organism can breathe in and out the spiritual essence of its being, can *in*carnate and *ex*carnate; and he shows how, in the higher kingdoms, with in-breathing a heightening of consciousness can take place, a heightening which in the last resort can take the form of pain. In-breathing in just such a way does not take place with the plants, but I believe that a shadow, a reflection, of it does. If the plants could share a similar consciousness with us, then I believe that, of all plants, it is the rose which would be able to feel pain.

Down through many ages the rose has had a special connection with Man and his consciousness; its thorny nature, just as much as its beauty and fragrance, has permeated literature and poetry, even to the point, in the Middle Ages, where it was taken as the symbol and picture of the purified blood.

Such considerations go beyond the numbers, and I would not wish to put them forward in any dogmatic sense; but I believe that the parameters can guide us to them.

The rose plays extreme variations on the theme, so much so that sometimes, in the outline of its form, there appear two inflexions on each side, one usually just above the middle (level *T*) and the other near the top. Such behaviour is most unusual among the buds; in fact it is hard to find, in all my collection of measurements, another plant which behaves just like this. Nevertheless there is a small group of plants which follows the rose in a more moderate way, most of them never coming to the point of having flexes in their outline, but definitely showing a systematic increase in λ in the top half of their buds. They also have high λs, from about 2.5 upwards. It is interesting to consider which plants come into this small group; at first sight they make a rather strange-seeming set of bed-fellows.

Firstly comes the wild garlic. The garlic bud usually has a λ of between 4 and 5, and shares many of the qualities of the rose bud form. Botanically the plant is quite different from the rose; what can it have in common with it? Firstly we are struck with the powerful and pungent taste and smell of the garlic. Of all plants this is one which, in a way, calls us to consciousness! It is a diuretic, acting on the kidneys and stimulating the flow of urine, that process which, of all those working below the abdomen, is most intimately connected with our consciousness (as for instance in cases of fear and anxiety). And it has been pointed out to me that the garlic and the rose are 'friendly' plants: they assist one another when planted together.

Then in my notebooks I find two separate varieties of poppy. The figures for these must be taken with reserve. There are many kinds of poppy; the ones I measured were taken from the hedgerow, growing wild; the buds were covered with thick green sepals which were very difficult to remove without changing the shape of the bud. Nevertheless, when I had done my best with them, the results were definitely in the rose group of forms. I think I need not comment further than just to remind ourselves of the traditional connection of the poppy with drugs, and their powerful effect on consciousness.

Next on the list, and much more moderately, comes the geranium. This was rather a puzzle to me until a friend pointed out that the geranium has a powerful scent, and one moreover that is so similar to the rose that perfume manufacturers often substitute it when rose petals become scarce.

This rose group is not a large one. I have not many more examples to give. In a mild way the buttercup, the wild iris that grows in the Scottish Highlands and also the type of water-lily which grows wild on our lochs, come into this group. (It would be interesting to know whether water lilies in general share this characteristic; would there be any connection with the lotus?)

How does a bud open?

The next stage of the investigation obviously must be to find out what changes of form a bud goes through in the course of its development as it opens. From the foregoing I think it becomes clear that the general gesture of opening will be connected with a decrease in λ. In order to test how true this would be, I made the set of drawings shown in Figure 25. In these six drawings ε has been kept constant, at 0.75, but λ has been gradually reduced from +2 to –0.5. We see the form passing from a typical bud shape, through the elliptical cross-section ($\lambda = 1$), through the conical form ($\lambda = 0$) until it opens, with the negative values of λ, into a vortex form. The question then was: How closely does an actual bud follow such forms during its opening? As sometimes happens, this primary question became somewhat over-shadowed by other considerations as soon as the investigation got really under way.

Obviously it will vary with the type of bud; equally obviously there will be many buds which will not pass through the conical form during their development; nevertheless, it is surprising how many buds are to be found which do pass through a very approximate conical stage. As a bud opens, its parts become so loosely connected that one could not expect anything but approximate results at that stage.

What I wanted to do was to photograph an individual bud day by day in the course of its development. This posed difficulties; the part of the bud in which I was interested was the coloured petals, and most buds grow surrounded by their sepals. However, it was midwinter and the winter jasmine was in bud. These buds emerge from their sepals about halfway through their development and they seemed therefore a possible species on which to start. The photography proved difficult; it was hard to get the camera into just the right position from which to photograph the bud on the living stem; the light was dim; so it happened that I had only poor photos from which to work. However, I persevered, day by day, waiting for the decrease in λ which would signify the start of the opening process and which I thought must inevitably come.

For nearly a week the λ-values remained almost constant, and then, to my surprise, I took a photo which registered a slight increase. It was so slight — and unexpected — and the photos were so poor, that I found it hard to believe that it was significant. However the next day the rise seemed to be confirmed, and after that the value of λ began to plunge quickly; the opening process had started at last.

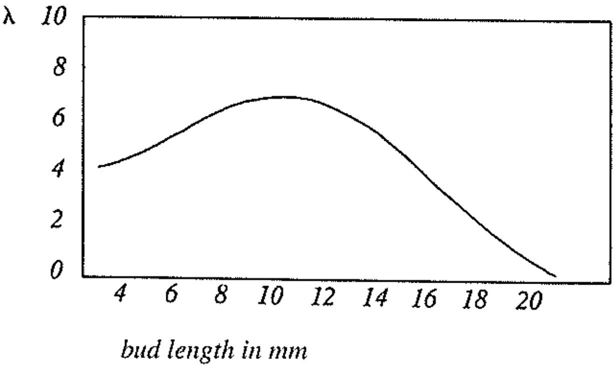

Figure 46. This shows approximately the variation in λ in the course of the development of the Rose-bud.

I had the distinct impression that this little bud had been waiting (eagerly, can we say?) for the great moment when it should open itself to the light, and to the world, and then, just before the actual opening, it had had a moment of hesitation, of withdrawal, of tension, before the final relaxation of opening. It was as though it passed through a little moment of climax, a little climactic rhythm. But the whole thing was so slight that I could not be sure that it had really happened. Observations on two further buds were no more conclusive; the whole thing was so subtle; the further observations were *consistent* with such a thing happening, but could not furnish evidence for its reality.

And in this unsatisfactory state I left the problem, my attention being diverted to some other phenomena. However, six months later, in the course of these further investigations, I was standing in front of a wild rose bush; it was covered with buds, in all stages of development from the smallest to those which were almost open; and as I looked at them it seemed to me that these buds were doing, in quite an obvious and even dramatic way, just what the little winter jasmine had been hinting at so subtly six months earlier. It was not possible to follow the progress of an individual bud since each was firmly encased in its sepals throughout most of its development. I therefore decided that I would take bud length to be a measure of 'stage of development.' I picked some two dozen buds, from the smallest which could be divested of their sepals, right up to some which were nearly open. Their λs, when plotted on a graph, showed a dramatic rise just before the final fall which signified the actual opening. Figure 46 shows the graph which resulted.

The buds have been divided into class-lengths and the mean of each class taken. But length is plotted along the x-axis, which may stand for 'stage of development'; and this of course may be thought of as a sort of approximate time-axis.

Within a couple of months I found that the buttercup, the hypericum and the cabbage flower all gave similar graphs, although rather less markedly. And the next winter I returned to the winter jasmine, with improved photographic technique, and was able to convince myself that my suppositions of the previous season were indeed true: the winter jasmine, in a very subtle and slight way, was behaving similarly. And these further measurements confirmed something else which I had suspected in the previous year; during the climactic moment the two halves of the bud, the top and bottom, tend to behave a little independently of one another. I then went back to my photographs of the rose, the hypericum, the buttercup and the cabbage flower; on remeasuring, with the exception of the buttercup, all of these showed this same peculiarity. This means of course that if the bud is a good path curve during its early development, it will probably become a less good one during the climactic rhythm, and this in general seems to be the case, the bud often returning to its better fit at the actual start of opening.

Since then I have studied fourteen different species from this point of view, and the climactic rhythm has shown itself in every case. It seems to be a rule of plant growth that just before the moment of opening the bud goes through a little climax of closing, of withdrawal, of in-breathing before out-breathing, of tension before relaxation. But the ways in which the different kinds of plant do it are considerably varied. Here again the manifold variety of the living world manifests; and I have been at pains to try to find whether there is any general pattern of behaviour, and what it can be. And as one works, a general pattern does seem to emerge, although individual species vary this according to their nature.

In the first place we have the quality that the bud is a good path curve form in its early stages, and then becomes less good, as it were, under the stress of the climax, returning to a better fit just before, or during, the actual opening. In eight cases out of the fourteen this seemed to be definitely true, and rather more questionably and slightly in two further cases.

Secondly, we have the quality that the climax hits the top of the bud first, and affects the bottom part shortly afterwards; the climax passes from the tip towards the base. This was found to be true also in eight cases out of the fourteen; in two cases the top and bottom halves seemed to be affected simultaneously, and in only one case was the bottom half

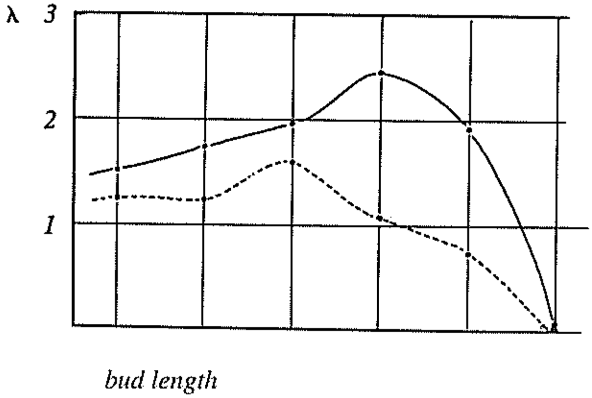

Figure 47. Honesty.

apparently affected first. In two cases the top part only was affected, and in one the bottom half only.

A further rule seemed to be suggested, although much more work will need to be done to substantiate it firmly. That is that during the climax the bud tends to accentuate its type. If it inclines to the snowdrop type, that is if it tends to have lower λs in its top part (even if this tendency is so slight that one does not feel inclined to put it definitely into the snowdrop group) then this tendency becomes exaggerated during the climax. Similarly, if it inclines to the rose type, it accentuates its type.

It seems therefore to be a general rule that during the climactic moment the bud endures a state of tension which draws it somewhat away from the true path-curve form; but that it returns towards this form immediately after. And that this state of tension moves generally from the tip to the base.

Here follow some typical graphs showing this in particular cases. The λ for the top half of the bud is shown in dotted curves, and that for the lower half in continuous curves. A perfect path-curve form would demand that these curves should coincide exactly; their separation therefore shows some measure of the degree to which the form diverges from the perfect mathematical model. The horizontal axis, representing in an approximate way the time element, is usually plotted in stages of bud length. In a few cases however it is plotted in actual stages of development; for instance the snowdrop increases its length very little during most of its development, but it passes through definite, and visible, stages of development, when it bursts out of one sheath after another, and when it starts to hang its head, and so on.

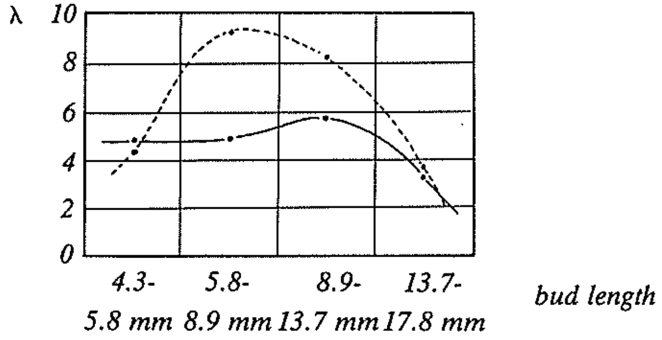

Figure 48. Wild Rose.

The honesty (Figure 47) starts off as very mildly of snowdrop type. The top half of the bud (dotted curve) has λ a little lower than the bottom half (continuous curve). Notice that during the climax this becomes accentuated, but is 'healed' later.

The wild rose (Figure 48). This particular rose started life as an almost perfect path curve, but the climax took it immediately and dramatically into the 'rose gesture' (note the scale of the vertical axis). In the actual opening phase it was again a very good path curve.

The primrose (Figure 49) shows a typical climax except that one cannot see any clear indication of the top half of the bud being affected earlier than the bottom. The stages of development are:

A. Bud very low down in its sepals.
B. Bud fairly low in its sepals.
C. Tip nearly reached the top of the sepals.
D. Tip just at the top of the sepals.
E. Tip just above the top of the sepals.
F. Bud just starting to open.
G. Bud half open.
H. Almost fully open.

The wild garlic (Figure 50) is an interesting case compared with the rose. I started work on this very early in the season, when only the very smallest buds were to be found. Then work was interrupted. When I was able to resume, the plants had become much more mature, and there were no small buds to be had, thus the break in the graphs. In the meantime the forms had become much less extreme, but the same general

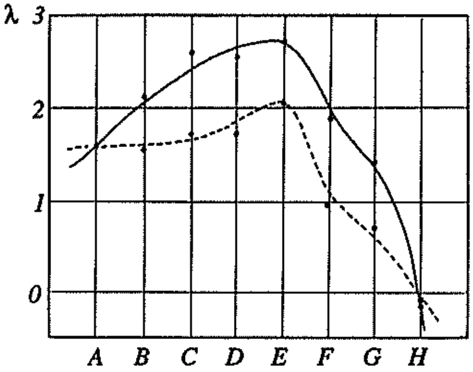

Figure 49. Primrose. A typical climax except that one cannot see any clear indication of the top half of the bud being affected earlier than the bottom. However, primroses gathered a year or so later yielded a graph which showed this feature also.

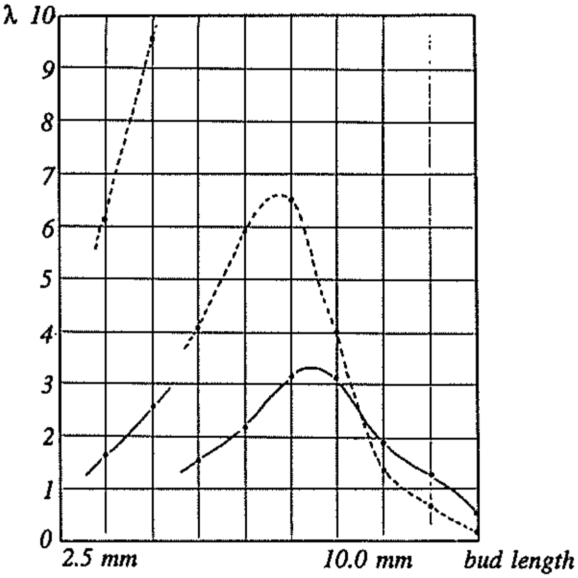

Figure 50. Wild Garlic.

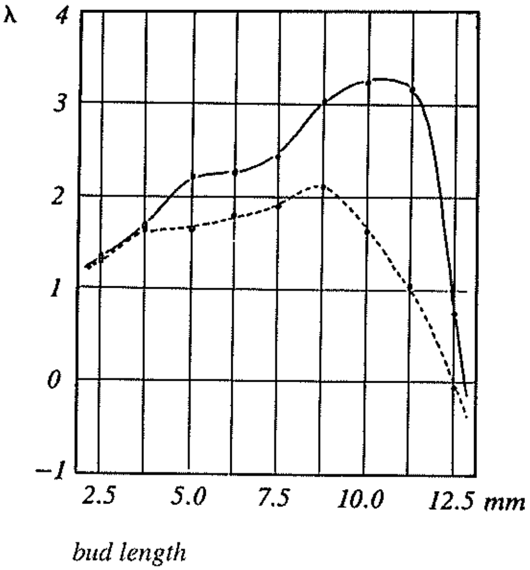

Figure 51. Forsythia.

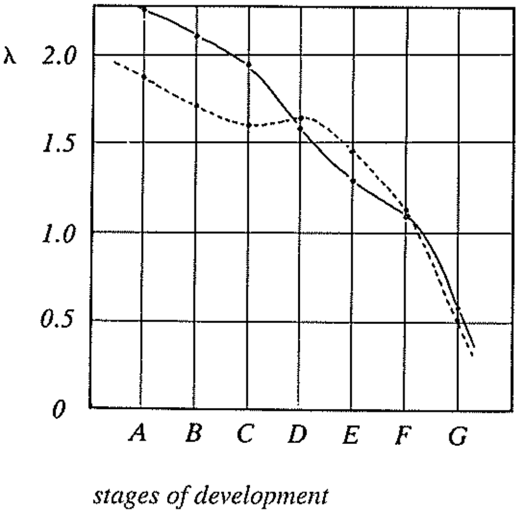

Figure 52. Snowdrop.

upward tendency for λ was preserved. After the buds had reached a size of about 10 mm there was no further increase in size observable, but there were still clear stages of development.

Forsythia (Figure 51) shows a typical climax. The bud starts at an early stage as an almost perfect path curve, and thereafter for quite a time it is a very good fit but with a slight tendency to the snowdrop type. This becomes greatly accentuated just at the climax. The λ is rather high for a normal member of the snowdrop group.

As leader of a group the snowdrop (Figure 52) maybe has the right to be a little eccentric! The lower part of the bud is completely untouched by the climax. The uninterrupted plunge of the curve is the indication of this little plant's steady surge toward manifestation. Only the tip suffers an increase in λ, and that so slightly and subtly that it almost escapes measurement. In fact it was not until I had done two further sets of buds and had got similar results that I was convinced of the climax here. The stages of development chosen were:

A. Bud is low down in its outer sheath.
B. Bud stands high in the outer sheath.
C. Bud is upstanding, clear of outer sheath but still within inner sheath.
D. Bud has *just* burst free from inner sheath.
E. It hangs its head.
F. Starting to open.
G. Well advanced in opening.

Stages A to D take several weeks to accomplish, but stages E to G are over in a few hours, so it remains a fact that the snowdrop gesture (dotted curve is below the continuous one on the graph) does predominate for nearly the whole time.

8. The Heart

The investigations described in the previous chapter were the first into which I was able to introduce a certain element of time and it seemed to me significant that these led immediately to a consideration of rhythm, of in-breathing and out-breathing. I think that perhaps the essence of time is only to be truly approached in the realm of such rhythms and rhythmic elements. And this led my thoughts back to some work I had done many years before on the form of the heart.

I had noticed, whenever I had occasion to consult an anatomical atlas, that forms of the general path-curve quality seemed again and again to leap to the eye. This was partly so when looking at pictures of the bones, but especially so when studying the forms of the muscles and muscle fibres. It was tantalizing; over and over again I sensed possibilities, but when I tried to find the exact point at which concrete research could start — just where would the invariant tetrahedron be? — the essential elements eluded me. To this there was one exception: the heart. The heart is surely the perfect, the archetypal, muscle of the whole body. After a time we can come to the feeling that all the other muscles are really variants of the heart, transformed and adapted to the needs of their various situations. And just to superficial observation it seemed that the heart could be as perfect a path-curve form as any muscle could attain.

So I decided to start with the heart. There were initial difficulties. I was interested not only in the outward form of the organ, but also in the detailed configuration of the muscle fibres which compose it, and the books, although agreeing that this is a matter of great complexity, were not always clear, or in agreement with one another, about the details. There was also the problem of getting good specimens to work with, and to photograph; human ones were of course not available to me.

The work of J. Bell Pettigrew

At this point I received great help from the work of the Scottish anatomist, J. Bell Pettigrew, who worked during the last half of the nineteenth century. Towards the end of his life he published his great work, *Design in Nature,* three volumes filled with drawings and observations taken from a lifetime of research, and enlivened with many flashes of truly Goethean insight. Pettigrew had made a specially intensive study of the heart, and his observations are of particular interest to us. In describing this four-chambered organ Pettigrew remarks that the auricles are to be regarded as little more than an extension of the whole vascular system turned inwards, as though at this point the veins have simply turned in, involuted, upon themselves. He suggests that in studying the essence of the heart one should, from a certain point of view anyway, remove the auricles, and deal just with the ventricles. Further, he points out that if one cuts very lightly along the shallow groove which marks the septum dividing the ventricles, one can peel off the right ventricle, leaving only the left in one's hand. And then one finds that what had formed the right ventricle is no more than a pathetic little bundle of spiralling fibres which it is difficult to imagine had once formed a chamber; and one finds that in the left ventricle one has something like 95% of the original substance of the heart, what he calls the 'heart of the heart,' and this, he says, is what he proposes to describe in detail.

In this paragraph I am trying to give just a précis of some of the things which Pettigrew says about the left ventricle. It takes the approximate form of an inverted asymmetric cone, the outer wall of which consists of fibres lying in a left-handed vortex passing steeply from the base to the apex. (It must be remembered that, as it is situated in the human, the apex of the heart points downwards, and the so-called base therefore forms the top part of the heart.) Pettigrew then goes on to say that the wall of this chamber is formed of seven distinct layers; what we see when we hold it in our hand is simply the first, outer layer. Inside is another layer, the second, the fibres of which spiral, still left-handedly downwards, but at a less steep pitch; the third layer is similar, but with the spirals even more horizontally pitched. The fourth layer is unique, in that the fibres do not really spiral at all; they run horizontally round the ventricle. The fifth layer is similar to the third, with the fibres running at about the same pitch or steepness, but now the vortex is right-handed, going 'upwards' against the 'downwards' of the third layer. Similarly he found that the sixth layer is a kind of right-handed reflection of the second, and the seventh of the first.

Figure 53 is Pettigrew's drawings which I have simplified in Figure 54 to illustrate the general 'idea' which is at work; my diagram does not pretend to follow exact forms and dimensions.

Now we must consider a little more closely just what is implied by this remarkable description of Pettigrew. If I draw a spiral curve on a

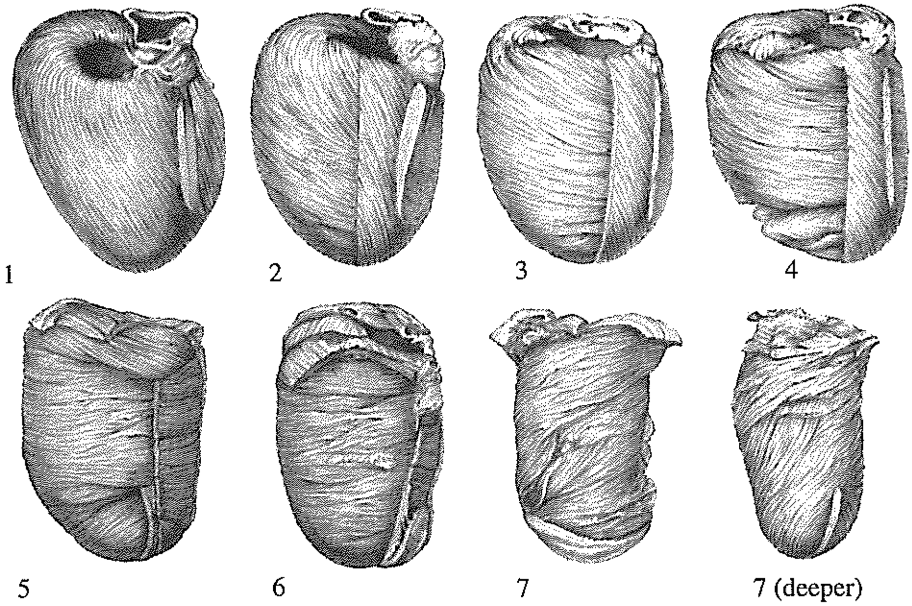

Figure 53. The sevenfold structure of the wall of the left ventricle according to Pettigrew, from his dissections and photographs.

sheet of transparent paper, I can draw it in such a way that it turns clockwise as it grows outwards (that is when viewed from its centre, it moves outwards from left to right); I could call this, if I wished, a right-handed spiral. But if I were to turn the paper over and view it from the other side, this 'right-handed' spiral would immediately be seen as a 'left-handed' one. In fact it is not possible to label a plane spiral as either right-handed or left-handed; it depends on the direction from which one views it. But as soon as my spiral becomes a space curve, that is to say, it leaves the plane and starts to move through the three dimensions of space, the case is different.

If the reader has any doubts about this let him take a piece of wire and bend it into a rough coil; he will find that as he moves from one end of the coil to the other, the curve is either turning clockwise or anti-clockwise; in the first case we call it a right-handed helix, and in the second a left-handed one. If he now goes to the other end of his coil and retraces

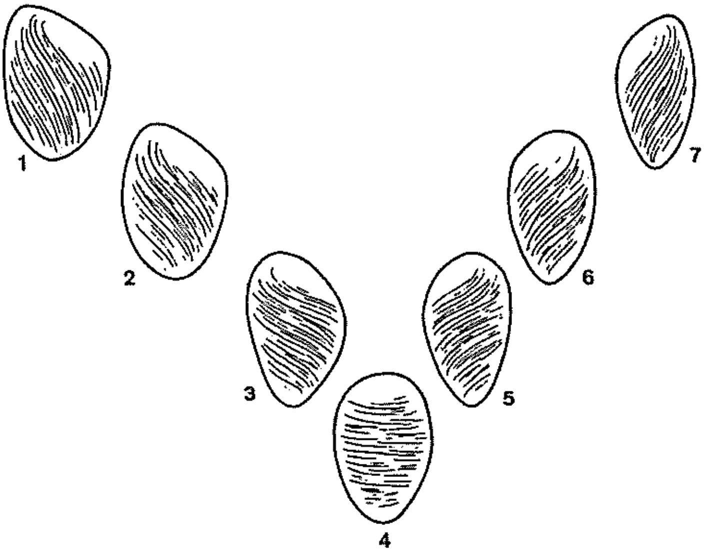

Figure 54. Simplification of Pettigrew's drawings, showing the principle at work.

his movement, going in the opposite direction, he will find inevitably that his coil still has the same handedness; it is intrinsically either right-handed or left-handed, and it will remain so from whatever direction, or however he views it. This last statement is not absolutely true; if he views his helical coil of wire in a mirror, behold, its handedness will have changed! The mathematical transformation of mirror-reflection is the one which changes left-handedness into right-handedness. And in effect Pettigrew is saying that, as far as the spiralling fibres are concerned, this central, unique, fourth layer in the wall of the ventricle is acting as a mirror which reflects the outer into the inner parts of the heart. And it is interesting to note that he says of this central layer, whose fibres run more or less horizontally, that 'the fourth layer is composed of loops of very oblique figure-of-eight spiral fibres.'

This remarkable picture was discovered by Pettigrew in 1858, yet I have not found it paralleled in any other book. Working with sheep's hearts (which Pettigrew recommends as being very near in size and form to the human) I have tried to confirm or deny it by my own experience. But my dissection-skill has not been sufficient. Pettigrew appears to have developed many special techniques, for instance, setting the specimen in wax, dissecting under a jet of hot steam, and maybe he used one of these in order to separate his seven layers. I do not know. But I have, from my own experience, been able to confirm quite definitely that the general change in direction of the fibres, as one cuts deeper and deeper into the ventricle wall, is very much as Pettigrew describes it.

Be that as it may, it was the outer, first, layer with which I would be interested in the first place. I decided to work with sheeps' hearts, and in the course of the work I analysed some seven or eight of these, as well as several pigs'. The work is much more difficult than with the buds, chiefly owing to the asymmetry which is involved. One has to start with a number of assumptions, particularly with regard to the size and shape of the invariant tetrahedron; and one has to be constantly on the alert, as the work continues, for indications which suggest needful modifications or changes to these assumptions.

In the first place I was unwilling to separate the two ventricles. I felt that I wanted to work with the whole thing! And this I attempted. But after many hours of work I became convinced that in no way would I find it possible to represent the two ventricles taken as a single organism, by any path-curve system which I knew or could devise. I was driven by my experience to follow Pettigrew's advice, and separate the ventricles.

The sheep's heart

As soon as I started working with the left ventricle alone, things were different. First I had to decide on the most probable disposition of the invariant tetrahedron. To do this one has to consider how the heart stands in the human frame. It is displaced rather to the left side of the chest cavity, and it faces towards the left front, with its apex pointing downwards. Its top part, the so-called base, is like a platform sloping downwards towards the back right, that is, in the general direction of the spine. It seemed to me that the top plane of this tetrahedron would be marked by this sloping base, and the bottom plane would touch the apex more or less horizontally. These two planes would then meet, maybe in the neighbourhood of the spine or just beyond it, in an invariant line which would run from back-left to front-right. This line would be the equivalent for the heart of the horizontal line at infinity for the bud. The direction from which to photograph the left ventricle would then be from back-left. I therefore set it up in such a position and photographed it, getting the profile which is illustrated in Figure 55.

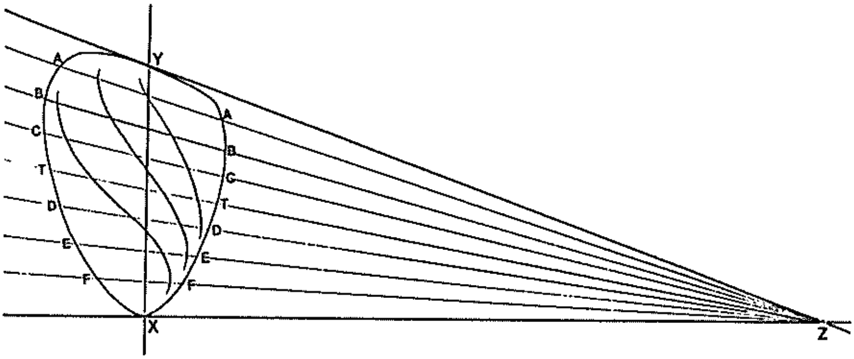

Figure 55. The left ventricle, seen from left-back, in relation to its probable invariant tetrahedron.

The line *YZ* represents the top invariant plane of the tetrahedron, tangent to the sloping base of the heart; it must be pictured as being roughly at right angles to the plane of the paper on which it is drawn. I felt that the shape of the profile of the ventricle enabled me to fix this line, *YZ,* with some degree of confidence. The other invariant plane, represented by line *XZ,* is also to be pictured as being approximately perpendicular to the page; but this line, *XZ,* cannot be so confidently placed, simply by looking at the profile; there is quite a range of positions which it could hold, tangent to the profile in the general neighbourhood of the apex. After considerable experimenting I came to the conclusion that its most probable positions are round about when the angle *YXZ* is a right angle, therefore in all the succeeding work I have taken care to choose the tetrahedron so that this angle is exactly 90°. It is not that I believe that there is any special virtue in the precise 90° — plus or minus five, or perhaps even ten, degrees might well be just as good — but in the course of the work many occasions have arisen in which one needs to compare the shape of one heart with another, or the form of a heart at one moment with its form a moment later, and one needs a definite rule like this in order to make such comparisons as valid as possible.

Now we have to ask ourselves in how far this is a good path-curve form, and if it differs, where exactly are the deviations to be found? The general method to be employed is the same as that for the buds, except that instead of taking ratios along the two lines *YZ* and *XZ* one has to take cross-ratios. The details of the method are given in Appendix 3. The results for this profile are as follows:

Level	λ-value	
	Left side	Right side
A	1.74	3.33
B	1.74	4.23
C	1.91	4.39
D	1.83	2.49
E	1.84	2.10
F	1.58	1.45
Mean λ (weighted)	1.73	2.76
Mean λ-deviation	5%	35%

We see that the left-hand side of the profile is a perfect path curve; a mean deviation of 5% signifies that any deviations present are smaller than the probable error of the finest measurements one is able to make; and results such as this were common to almost all the hearts I analysed. On the right side however, there is obviously appreciable deviation; but when we come to examine the figures we see that this is almost wholly confined to levels *A*, *B* and *C*. Now this is very near to the region where the mitral valve gives connection to the left auricle; it is a place where the ordinary muscular development of the heart is impinged upon by quite other forces, and where one might well expect deviations from the general form. If we disregard just these three points we find that we have as our result for the ventricle as a whole:

Mean λ = 1.75
Mean λ-deviation (MLD) 10%

This result compares for accuracy favourably with the best of the flower buds. Thus we can sum up by saying that, apart from this region around the mitral valve, this profile of the left ventricle is a remarkably good path curve (Figure 56). And this kind of result was general for all the hearts of sheep and pigs which I analysed.

I have sketched in three of the muscle fibres from the photograph; it is much harder to be precise over these, but analysis of these in a similar way to that which we used for the flower buds, showed that these are quite good path curves, at any rate during their middle parts, and that they all have an ε-value of almost exactly unity.

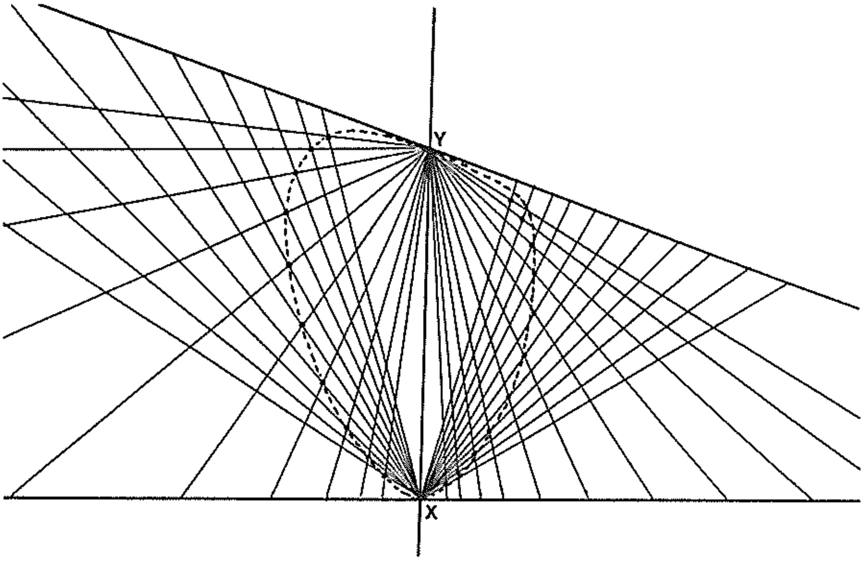

Figure 56. This shows just how closely the left ventricle follows the ideal path curve form. The black dots at the intersections of the lines show the calculated form; the dotted curve shows the tracing of the actual outline from the photograph.

At this point a further step became possible in attempting to test whether this ventricle is a really good path curve form. Having found that it appears so from the back-left view, and having found the parameters which apply to that view, we can calculate, and construct, a picture of what it ought to look like from the front-left view. And we can then compare this with the photograph of the front-left view. In doing this a further assumption had to be made. We have assumed that the axis *YZ* is at right angles to the line *XZ* representing the lower invariant plane when seen from the back-left view; will it also be at right angles to this plane when seen from the front-left? Seeing that this seemed to be a reasonable assumption, and that I had no evidence to suggest that it ought to be otherwise, I assumed a right angle here also. Having decided on this I produced Figure 57.

The left one shows the form which ought to have come if my path curve measurements and assumptions were correct, and the right one is the tracing from the actual photograph. I was not wholly satisfied with the relationship of these two drawings, but at the time I was not able to see any way of improving it, and I left the matter there for the time being.

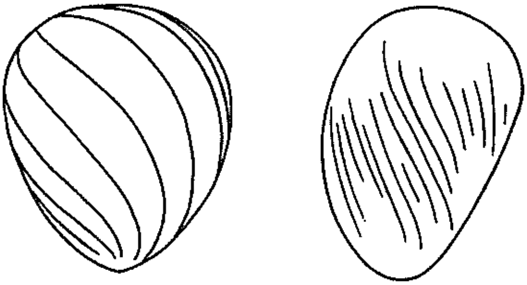

Figure 57. This shows the first attempt at calculating the left-front view of the ventricle. The left-hand drawing shows what the ventricle ought to look like, starting from the parameters obtained from the left-back view, and assuming that the organ is a good path-curve form. On the right is the tracing from the relevant photograph.

I next went on to consider the relationship of the two ventricles with one another. Holding a complete organ in my hand it seemed to me that the probable and reasonable solution to the problem was that the ventricles should be represented by two separate interpenetrating path-curve systems. And that in order to do this one would have to represent the axis of the left ventricle, when seen from the left-front, not as shown in the left drawing of Figure 58, which is the way I had pictured it till then, but rather the two axes as shown in the right drawing.

Since I had now changed the angle of the axis of the left ventricle it meant that the previous drawing had to be recalculated. When I had done so it looked like the left-hand drawing of Figure 59. I give beside it the

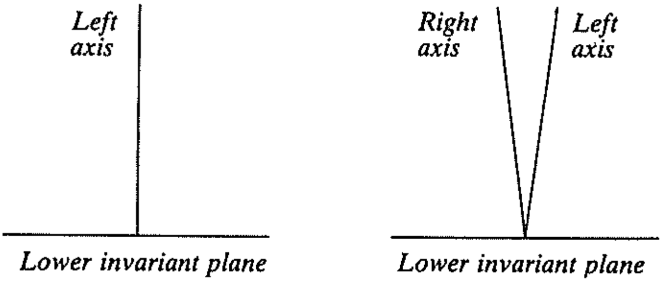

Figure 58. By considering the interrelationship of the two ventricles, a change in the slope of their axes was suggested.

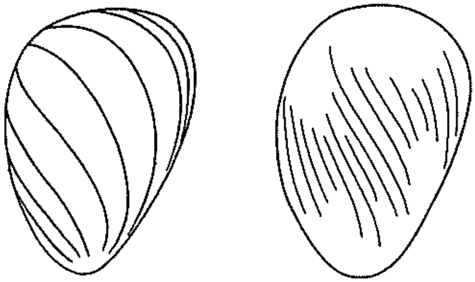

Figure 59. The view from left-front. The calculated form, using the new axis, is shown on the left, and the tracing from the photograph on the right.

actual tracing from the photograph, for the purpose of easy comparison. It will be noted that most of the discrepancies which had been there previously were now ironed out. The thing had become a remarkably good fit.

According to this theory the right ventricle would not be easy to deal with, since a large part of its ideal form would be hidden where it interpenetrates the left one. However, working with that part of it which could be seen, I was able to get approximate figures. It came out to be a fairly good path-curve form, with mean deviation of 14%, but for the particular heart shown its λ-value was higher than that for the left ventricle, 2.9 instead of 1.75. That is for the particular heart shown; other hearts had parameters which varied a little above and below these figures. The ε-value, denoting the angle at which the fibres run on the right ventricle, was not easy to measure but was obviously very high, the fibres running almost longitudinally.

Taking all these parameters which I had arrived at, and with the axes which I had now assumed, I was able to draw perspective pictures of what these intersecting path-curve forms would look like, and to compare them with the actual heart photographs. Figure 60 shows the front-left view.

The left-hand picture is of the two intersecting path-curve forms, and the right-hand one is the tracing of the actual ventricles from the photograph.

Next we show the back-right view of the same thing (Figure 61).

Had I spent more time with the manipulation of parameters and axes I am sure that I could have got the agreement to be even closer than

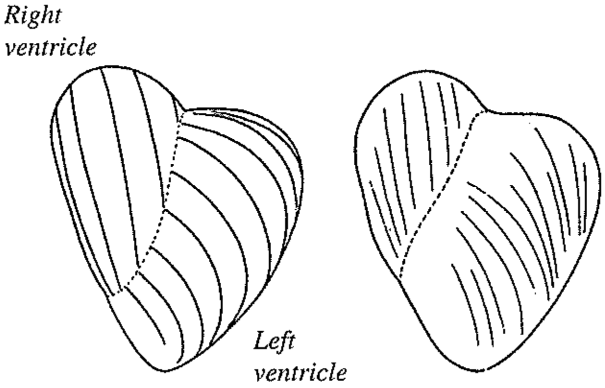

Figure 60. The two path-curve forms interpenetrating, viewed from the left-front: the calculated form on the left, and the tracing from the photograph on the right.

this. But I was carrying a full school timetable at the time, besides other responsibilities, and in addition other aspects of this research work were crying out to be tackled, so I did not think that this would be the most fruitful way to spend my time. Nevertheless what I had done was enough to convince me that the ventricular part of the heart can be represented very closely indeed by these two intersecting path-curve forms, and that the heart is really an expression of those forces which show themselves in the linear process which leads to such a transformation as collineation.

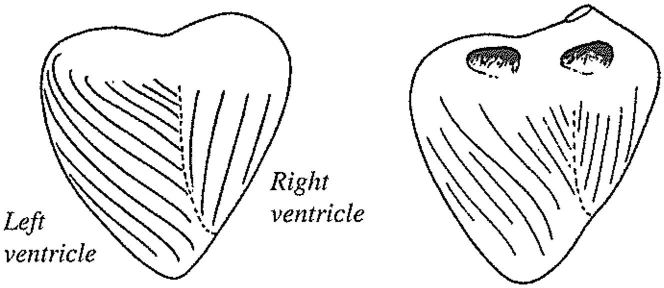

Figure 61. This is similar to Figure 60 but seen from the right-back.

This was satisfactory as far as it went, but I was still dissatisfied on two counts. Firstly, I was able to deal with only animal hearts, and I would dearly have liked to measure some human ones. Secondly, and more importantly, the thing I was working with was dead. The heart is a living organ, perhaps more living than any other. It is continually moving, continually changing, never static. But the thing I was photographing and measuring was inert and still: the dead ash only, of a living process.

The human heart

The hearts I had studied were considerably different from one another in shape; was this due to some inherent difference in the animal or in the circumstances of its death? In the moment of death, does the form of the heart 'freeze' in that phase of the beat in which it happens to be? Or does it fall back into some relaxed condition? I had no means of knowing. And the buds were crying out to be studied! So I laid this aspect of the work aside for many years.

But when, some time later, I found the little climactic rhythm which the buds go through as they open, the desire to know just how the heart beats arose in me so strongly that I was driven to further action. I wrote to the Royal Infirmary in Edinburgh to ask whether, with modern X-ray technique, they had anything which could help me in this problem, and there I was introduced to the study of the angiogram, an X-ray moving picture of the beating heart. Shorn of technicalities the method can be described thus: an artery is opened and a long rubber catheter is inserted; this catheter is pushed right through the arterial system until the end of it passes through the aortic valve and into the left ventricle. Once it is there, some opaque substance is squeezed through the catheter into the ventricle, and for about three or four heart beats the ventricle can be filmed by X-ray. Once developed the film can be run through a machine which throws the whole process on a screen; and by pressing a button the sequence can be stopped at any desired moment so that one can take a tracing. The machine in use in Edinburgh takes 50 frames per second, so one can follow the beating of the heart each 0.02 second.

I had so often tried to imagine what it would be like; and it was a thrill now to be able actually to watch it, even in terms of the dim shadows of an X-ray screen. What a wonderful organ this is, so mobile, so ever-changing; sometimes little ripples of contraction seem to pass from one end to the other, always rhythmic, always nevertiring. For a few minutes,

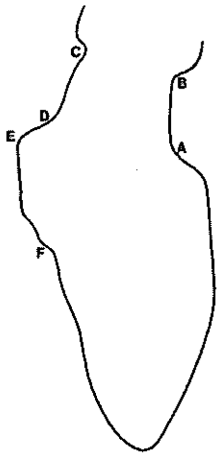

Figure 62. A typical X-ray picture of the left ventricle, shortly after the moment of full diastole (greatest relaxation).

I watched it at normal speed; then I slowed it down and studied it in more detail; and finally I started taking tracings of the separate pictures.

I was excited because here, at last, was the human heart. Now I would be able to find the difference between human and animal! The quickly I realized that this was not so. My sheep's heart drawings are of the outside, Pettigrew's first layer; now I was dealing with the inside, his seventh layer. The two things were not comparable.

But the first thing was to find out whether the angiogram was presenting me with a true path curve form. Figure 62 is a tracing of a ventricle taken a short time after the moment of extreme diastole, that is, at a moment when the muscle has started to contract, but is still comparatively large and relaxed, and filled with blood.

The first thing to notice is that this, as it stands, is certainly not a path-curve form. But we should not pass judgment on it until we have learned a little 'geography.' The line from *B* to *C* marks the position of the aortic valve; this is the top of the ventricle, and from a medical point of view marks its termination; all above there is artery. The little approximately quadrilateral space *ABCD* is called the aortic vestibule, and it is here that the blood gathers before it is expelled into the aorta. The rather strange protuberance *DEF* appears on the photographs of all the hearts which I have studied, more or less strongly marked. It marks the position of the mitral valve, a region where the ordinary muscular activity of the ventricular wall is very definitely interfered with. It is this same region which gave the biggest deviations on the profile of the outside of the sheep's heart (see Figure 55, levels *A*, *B* and *C* on the right hand side).

Now we must bear keenly in mind that the heart is, fundamentally, 'muscle'; I believe that it is the archetype of all the muscles of the body; its being is ceaseless activity and rhythmic change; and the wonders of its rhythmic activity and its muscular development are probably to be seen in their purest essence in the form and the life of the left ventricle. Yet this ventricle in at least two places is non-muscular. These two places are the aortic vestibule, and the immediate region of the mitral valve. In an organ which is in a state of continual flux, this part of the profile, from *A* through *B* and *C* to *F*, is largely invariant. It does not take part in the beating of the heart. And the significant thing is that if we ignore just this part of the profile, the remainder is found to fit the path curve form quite closely.

In order to measure the left-hand side of this profile one has to draw a smooth continuation of the curve above level *T*, and this renders the measures for this top left-hand corner of the profile rather questionable, but the whole of the rest of the outline can be measured just as it stands. The λ-values for the various levels come out as follows:

Level	λ-value	
	Left-hand	Right-hand
A	3.07	2.11
B	2.55	1.54
C	2.22	1.61
D	3.11	3.75
E	2.43	2.31
F	1.75	2.35
Mean λ (weighted) 2.34 Mean λ-deviation 18%		

This is a result which is comparable with quite a number of the flower buds. What this 18% deviation means in real terms can be seen by looking at the dotted curve of Figure 63, which is the actual calculated result. The only appreciably visible deviations are at level *F* on the left-hand side, and level *B* on the right, and neither of these is larger than the little ripples of expansion and contraction which seem to be moving constantly across the surface of this very mobile organ. This is a fairly typical result for a photograph taken at this stage of the heart's cycle.

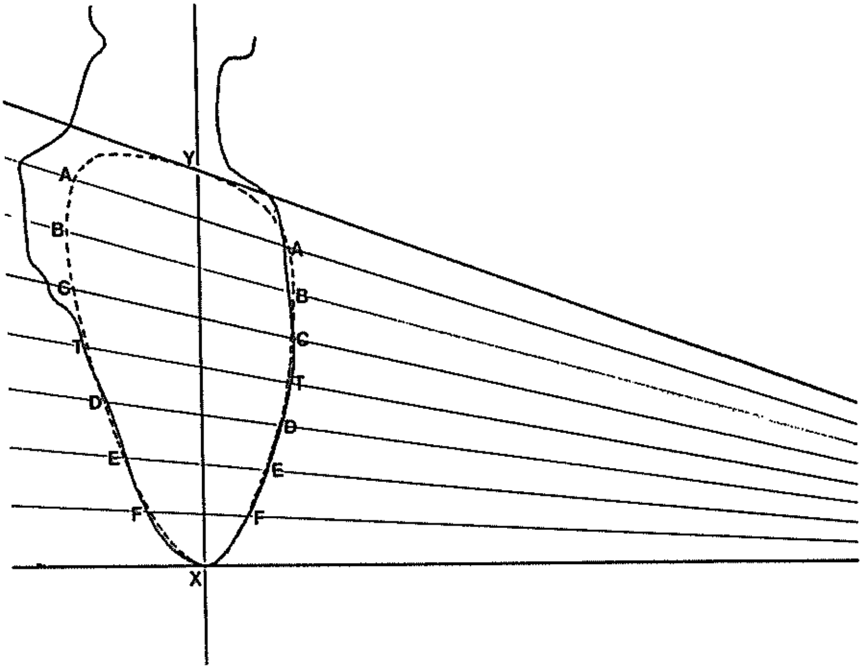

Figure 63. The human left ventricle, placed within its probable invariant tetrahedron, shown by triangle, XYZ. *The dotted curve shows the calculated form, with* $\lambda = 2.34$*, the continuous curve being the tracing from the photograph. The value of* λ *calculated for levels* A, B *and* C *on the left-hand side can, of course, only be very approximate.*

Earlier, at full diastole, when the organ is totally relaxed, the mean deviation will often be less than this; later, as systole approaches, it becomes very much greater.

The next interesting thing to notice is the form of the invariant tetrahedron, which is indicated in section by the triangle *XYZ.* The line *YZ* has had to be drawn using one's judgment. In all the work which follows I have made it lie along the bottom level of the aortic vestibule, and I think that this is the most probable and correct place for it. Line *XZ* has been drawn following the right-angle rule ($YXZ = 90°$). Further experience may lead us to vary the details of this procedure; but in studying the way the heart beats I think the most important thing is that the procedure should be strictly constant through all stages of the heart's cycle.

One must realize that the invariant lines of the tetrahedron are two, the axis *XY* and a line through *Z* approximately perpendicular to the page of the diagram. Now it is a general rule, in all the various hearts which I have studied in this way, that the size of this tetrahedron varies during the heart-beat. In full diastole, *Z* is far away, right off the page of Figure 63, and occasionally almost infinitely distant. As systole proceeds, *Z* comes swinging in; in our picture we have caught it part way in; a few fractions of a second later it will occupy a position only a half, or a third of this distance from *X*. Then as soon as the moment of full systole is past, and the organ starts to relax, *Z* retreats again into the distance. Thus we must see the contraction of the heart as being associated with the swinging in of this invariant line from the far depths of space to a close position, and then back again. This immediately gives us the impression of systole as an in-breathing, and diastole as out-breathing.

The next interesting thing to notice is the similarity of this picture with that of the sheep's heart which we have already seen (Figure 55). This becomes more than interesting, definitely intriguing, when one realizes that these two pictures were taken from diametrically opposite directions. I photographed the sheep's heart from the back-left; in taking their angiograms the doctors work from front-right. The fact that these two pictures look so much alike means that the tetrahedron which one is led to from studying Pettigrew's inner, seventh, layer, is an almost exact mirror image of that which one finds in the outer, first, layer. And this, coupled with Pettigrew's assertion that as far as their fibre-spiralling is concerned these two layers are mirror-images of one another, must I think be taken as a highly significant fact.

The beating of the heart

When one comes to study what occurs during the course of a heartbeat, further significant things come to light. In the first place my experience in studying these X-ray photographs has impressed again upon me the extraordinary variety of living nature. Not only does the actual shape of the heart vary considerably from one person to another, but the details of the actual way of beating are quite individual. These individual differences may become important in further study, but in this preliminary work I have been concerned to find the general rules which are common to all, or at any rate, the majority of hearts. One thing that can be said, I think, absolutely generally, is that although in diastole λ is quite low, usually between about 1.8 and 2.8, with the onset of systole it suffers a gradual rise, which then usually becomes dramatic; by the moment of full systole λ will be very high, sometimes reaching to infinity or beyond; and this gradually falls to a low level as diastole proceeds. I think we see here a further confirmation of what I had earlier supposed, that high λs are connected with in-breathing, and a certain state of tension.

However there is more to it than this. In nearly every case I have found that this increase in λ affects the apex half of the organ (levels *D, E* and *F*) much more strongly than the base half. In fact, as my studies have continued, it has become increasingly and strikingly clear to me that the dispositions of the λ-values at the various levels, as the moment of full systole approaches, take on a special and characteristic form. It is a form which in all my studies of the living world I have met in only one other place: the bud of the wild rose.

The actual, absolute, λ-values yielded by the buds of different rose bushes vary considerably from one bush to another, but the way they are disposed is generally characteristic, and fairly constant. Let us take the values for a moderate type of rose bud, already quoted in Chapter 7.

Level	λ-value
A	4.02
B	4.75
C	6.29
D	11.5
E	10.6
F	4.72

We notice that λ is a little higher at *F* than at *A,* but that it reaches its maximum value at *D* and *E,* with *C* also fairly high.

Now we compare this with the values for one of the heart sequences, a picture taken 7 frames (0.14 second) before the moment of full systole.

Level	λ-value
A	4.78
B	5.74
C	4.28
D	8.34
E	6.41
F	7.84

Again the value of *F* is higher than that at *A,* and the maximum value is at *D,* although the increase in the region *D* and *E* is only just starting.

Two frames (0.04 second) later, we have the following figures:

Level	λ-value
A	7.53
B	7.04
C	4.39
D	∞
E	17.3
F	6.59

Three frames (0.06 second) later still, we find these figures:

Level	λ-value
A	6.08
B	8.99
C	∞
D	∞
E	∞
F	13.2

Now let us compare this with the figures given earlier for a rose of the more extreme kind.

Level	λ-value
A	5.28
B	18.3
C	∞
D	∞
E	∞
F	7.65

Where we are dealing with such a mobile field of form we must expect considerable variations in the actual figures, but the general trend and similarity between the two forms is unmistakable.

Now we must ask ourselves what these figures mean in terms of actual visible form. Figures 65–67 show the three moments in the heart cycle which we have discussed here. The dotted curves show the calculated form. Of course it is not possible to calculate a mean λ for the second and third cases; I have adopted $\lambda = 12$ for the second, and $\lambda = 20$ for the third; with such high λs a considerable change in λ makes very little difference to the visible form, so I think these dotted curves do give a fair impression of how good a fit it is possible to get.

First we show this same heart at about the moment of full diastole, for the sake of comparison (Figure 64). This particular heart had higher mean deviations than are usual at this part of the cycle, having already,

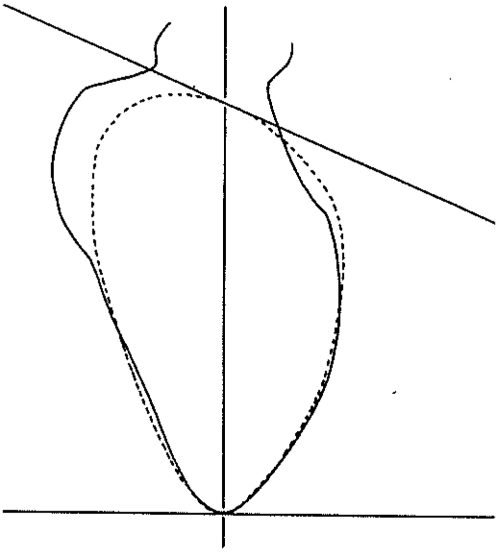

Figure 64. The left ventricle at the moment of full diastole. The form is gently rounded, less so in this case than often, as this particular heart had a higher λ than usual.

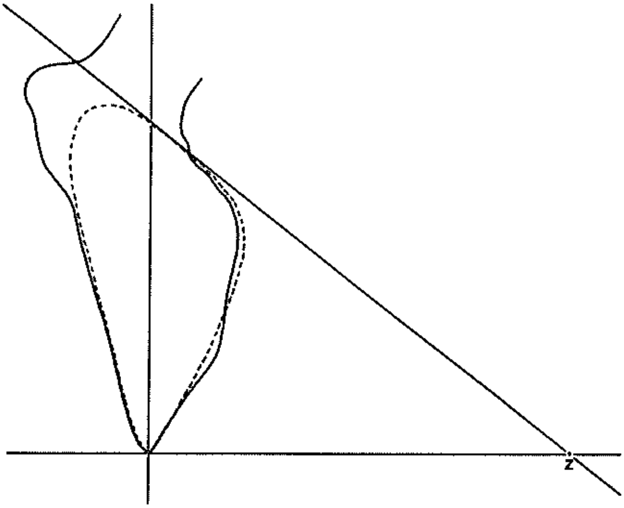

Figure 65. The same ventricle 0.14 seconds before full systole. The form is becoming more straight-sided; and the point Z is already swinging in on to the page.

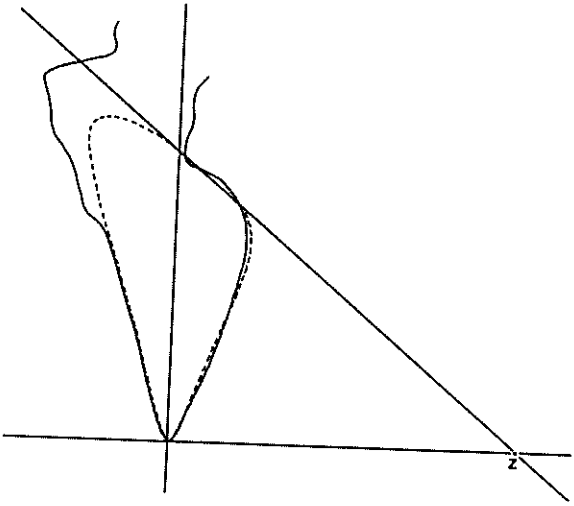

Figure 66. The same, 0.10 seconds before full systole.

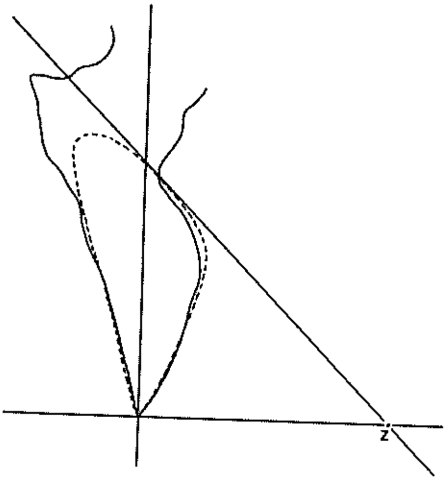

Figure 67. 0.04 seconds before full systole. Point Z has reached its most inward position; and the organ has assumed almost the form of a thorn.

even at that time, a slight tendency to the rose gesture, that is, higher λs at the apex end than the base. Could one speak of this as being an unusually 'ardent' heart? It would be interesting to know something about the people whose hearts I had studied!

In these pictures one can sense more than just the physical contraction of the form, that is, its decrease in volume; there is a change in the *quality* of the form. The rounded sides and the gentle curve of the apex, seen in the diastolic form, give way to the straight sides and a sharp apex almost like the point of a cone. Everything bespeaks a state of growing tension; the thing has taken on the form almost of a thorn. And if one speaks of 'pain' now in the sense in which I used the word in a previous chapter when referring to the rose, meaning not 'what hurts' but rather that condition in which spirit sinks itself into matter, immerses itself in the bond of substance, then I think we are entitled to say that the heart in this moment, just for a fraction of a second, suffers what we may speak of as an ecstasy of pain.

But now we must go further and ask ourselves what this ventricle undergoes in the course of transforming itself from the rounded forms of diastole to the sharp ones of systole. To do this we will study our λ-values on a graph; along the x-axis we put 'time,' each unit representing one frame (0.02 second); on the y-axis we put the λ-values; we plot the mean λ for the apex half of the organ (levels *D, E* and *F*) in a dotted curve, and the mean λ for the base half in a continuous curve. Figure 68 is the graph for the particular heart we have been studying.

Along the time axis we mark our zero ordinate at the moment, as far as we can determine it, of full systole. Five frames (0.1 second) before full systole is shown as –5; +5 means 5 frames after systole.

Now let us study this extremely interesting graph. Firstly we notice that almost the whole time the dotted curve (apex half) is above the continuous one; it has the gesture of the rose, and that this is much more strongly marked between –10 and +10, while the ventricle is under contraction. On the whole the continuous curve (base half) echoes the movement of the dotted one, but in a much milder way (in this particular graph this tendency is broken between about –12 and –7, but this is exceptional; other similar graphs do not show this). This ensures that as λ increases, the form of the organ changes thus strongly into the extreme rose form.

Pulsation during heartbeat

But now we see that this extreme rose form seems to be achieved not smoothly, or once, during each heart beat, but several times, in a series of leaps and bounds. The dotted curve goes very high at –10 and to infinity thereafter no less than three times. And even the smooth curve follows it twice, during the moments of extreme tension. In fact the whole graph gives one a strongly cyclic feeling, as though the heart is pulsating during the course of the single beat. But when I took the tracings for this and a number of other similar graphs I had not expected any phenomenon like this, and therefore I had not thought it necessary to take tracings of every picture; I had contented myself with every second, or, in the case of diastole, every third or even every fourth, frame. With so few observations to work with, and a degree of probable error in the figures which is not easy to determine, one is not entitled just to join the points of one's graph

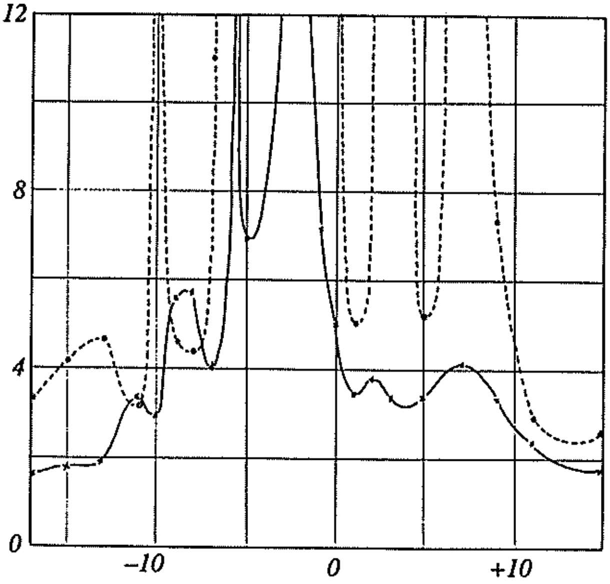

Figure 68. An early chart showing the variation of λ during the complete heart beat. The dotted curve shows the λ value for the apex half of the heart (the lower portion) and the continuous curve shows that for the base half (the upper part).

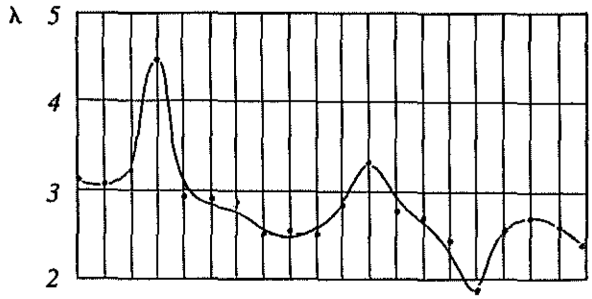

Figure 69. Pulsations within the heart beat. λ value falling with diastole.

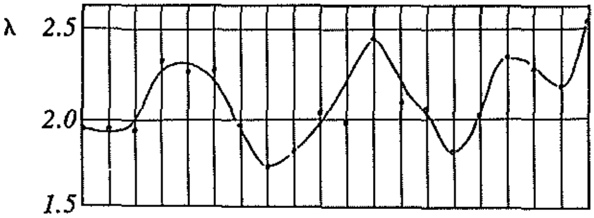

Figure 70. Pulsations within the heart beat. λ value starting to rise as the organ goes into systole.

into a nice smooth curve, and then announce to the world that one has discovered a cyclic pulsation! How far are these 'pulses' just the result of random errors asserting themselves? What was happening to the ventricle during those spaces between these rather widely spaced observations?

Clearly I needed a set of tracings which should include *every* frame. Now I had several such to hand, which I had taken for a rather different purpose, but they included only that part of the heart beat from a little before the moment of full diastole to a little after it, that is the time when the ventricle is comparatively relaxed and large. This is the time when the ventricle shows up most clearly on the X-ray film and one can get the most accurate tracings; as the organ goes into systole the photos become more confused and difficult to follow, and sometimes it is almost impossible to get a reliable tracing.

I therefore set to work analysing and making graphs for these sets of tracings. I found that I had seven such sets, and in the result I found that one of them did not appear to show such a quick pulsation, two of

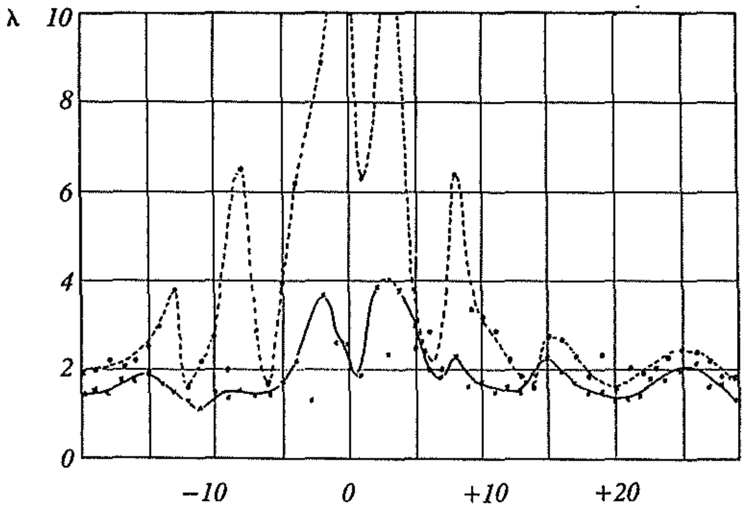

Figure 71. A complete heart beat, using every available picture. The consistency of the observations is remarkable. In a situation where a slight slip of the pencil can throw the result seriously out we have, out of the fifty observations, only two for the apex half (at −11 and +18) and two for the base half (at −3 and +3) which do not lie closely on the smooth curve which has been drawn.

them seemed to show it, but not with a very great degree of certainty, and the remaining four seemed to me to show it with a satisfactory degree of certainty. I show here two of the graphs so that the reader can see for himself what they are like. Figure 69 shows λ plunging into the trough of diastole, and Figure 70 shows it as it is just about to start climbing out of it. We see that in nearly every case each maximum and minimum of the curve is supported by at least two observations and usually more.

Looking at this whole set of graphs I found that I had thirteen places where I could arrive at a measure for the wave length of this little pulsation, that is the time from one maximum to another or from one minimum to another. The wave lengths varied from 3 to 8 frames, but their mean value came out at 5.8 frames, nearly six. Now the average rate of heart beat is usually considered to be 72 per minute, which means that the average heart beat will take just 42 frames. So if these little pulsations are really there, and if these were on the whole average hearts I was measuring, and if the pulsation continues through systole at the same rate that I had observed during the diastolic part of the rhythm, then we could expect to find just seven little pulsations to every single beat of the heart. Quite a lot of 'ifs' but an interesting result!

At this point I came in touch with Dr George Aitken, an experienced radiologist, who showed interest in this work; and he offered to make for me a set of tracings for every frame, for a complete beat of a heart. This was an offer which I was very happy to accept. The interpretation of these rather shadowy X-ray pictures is not always an easy matter, especially to a layman like me, and I have not always been quite sure where my pencil should go. To have an expert hand on the job was a great relief. As soon as the tracings came to hand I measured and analysed them, and produced the graph of Figure 71.

The first thing to notice about this diagram is its essential similarity with most of the others which I had already got from my own tracings. We see the λ of the apex half of the organ (the dotted line) rising in a series of bounds, two of which shoot right off the page; we see the curve for the base half echoing this rhythm, but in a much more modest way. The great gap between the curves around times –5 to +5 indicates the extreme form of the 'rose parameters' which the organ assumes just at the time of full systole. But beyond all this the graph shows more clearly than I had dared to expect the sevenfold pulse within the single heart beat which I had been led to look for.

Now can we say that in every beat of every heart we shall always find this sevenfold pulse? I doubt that we are in a position to make such a forthright statement. The matter is infinitely subtle, and individual. One thing I have learnt during this study is that each heart beats differently. The thing is further complicated by the conditions in which the photographs are taken. All the hospitals which have helped me in this investigation have assured me that they are showing me only photographs of healthy hearts. Nevertheless I am sure that it is a fact that many of the angiograms which I have worked with are those of patients, people who have come in with symptoms which have led the doctors to suspect heart disease. Does the fact that they have been subsequently cleared of actual heart disease guarantee that in these moments their hearts are really beating normally, especially in such a subtle respect as this? Some of the hearts have been beating at a rate of up to 160 beats per minute, over twice the normal rate. Such rates are found only in three kinds of circumstance: great physical exertion (unlikely on the X-ray couch), disease, or extreme mental and emotional stress. These photographs are not normally taken under general

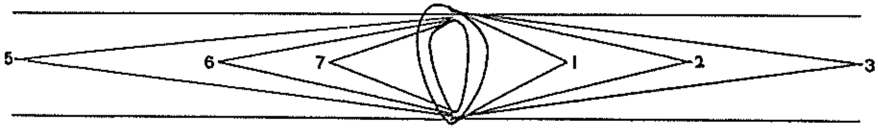

Figure 72. An attempted visualization of the invisible part of the heart. Through the points labelled 1, 2, 3, 5, 6 and 7 one must imagine lines, approximately perpendicular to the plane of our page; during systole these lines would come sweeping in towards the physical form of the organ, and in diastole they would retreat once again into the depths of space.

anaesthesia but under fairly heavy sedation; people's tolerance for such treatment, while remaining reasonably imperturbable, varies greatly from one individual to another; can a heartbeat photographed under conditions of fear and anxiety really be taken as a normal one? The matter is beset with so many difficulties that it would be surprising if I had not found some hearts which do not show such a subtle rhythm very clearly.

Nevertheless I have found this little sevenfold pulse, or traces of it, in so many cases that I am becoming convinced that it is in fact a feature of the normal heart beat; and that this wonderful organ which Pettigrew assures us is seven-walled, is also seven-pulsed within each single beat.

It is important to realize that these little pulses do not represent changes in size, but rather changes in form. The organ becomes more rounded, or less; in other words, less straight-sided in its shape, or more so.

At this point it is tempting to speculate what the complete form of the invisible part of this wonderful organ could possibly be like. Let us assume for the moment the truth of Pettigrew's assertion about the seven walls. We have some knowledge from actual measurement of what the invariant tetrahedra must be like for the first and for the seventh. In Figure 72 I give a picture of what I suppose the whole thing might look like, as seen from the back-left.

Through each of the points labelled 1, 2, 3, 5, 6 and 7 one would need to envisage a real invariant line, more or less at right angles to the plane of the page. The spirals sweeping inwards from behind, from lines 1, 2 and 3, would fashion the outer part of the ventricle, and those sweeping inwards from the front, from lines 5, 6 and 7, would form the inner part of the organ. The generating line for the middle, fourth layer would be the line at infinity, and under the aegis of this, the spirals would merge into grand circling movements whose intricacies we can hardly begin to guess at this stage, but which would, in some way, form the balancing, mirroring, agency which would link 1 and 7, 2 and 6, 3 and 5. And now imagine, in the course of the heart-beat, from front and from behind, these lines breathing inwards, in tension, for systole, and then relaxing outwards, for diastole, into the far reaches of space. Each heart beat would fill the cosmos.

Of course such a picture must be treated with circumspection. We must be ready to modify it with each new piece of evidence which arises. But I do not think it is too fanciful to hold in one's imagination, until further insight comes to us.

Since the first appearance of the above account I have measured and analysed a series of further angiograms. The images appear on the film with varying degrees of clarity, but in every case, without exception, my measurements have yielded evidence, with more or less certainty, for the existence of this little seven-phased rhythm within each single heart beat. And it has been universally the case that the clearer the pictures, the more clearly does the rhythm show itself. This further experience has been such as to dispel any lingering doubts I may have had as to the reality of this little rhythm. In view of what I have to say in Chapter 16 I think this an important matter, involving man's one-ness with the whole of nature.

Some time after I had done the main work of this chapter, a friend drew my attention to a lecture given by Rudolf Steiner in the spring of 1920, and with which, until that time, I was unfamiliar. In the course of this he says:

> . . . imagine a wind whirling with a certain velocity from above downwards, and another from below upwards, and they whirl into one another . . . One whirls downwards, and because the other whirls upwards with a greater velocity . . . [there arises] through the collision, a condensation, a certain figure. This figure, disregarding imperfections, is a silhouette of the human heart.*

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* *Man, hieroglyph of the Universe,* Lecture 14, p. 193f.

I find this a remarkable description coming from a man who, as far as we can ascertain, did not possess the details of this particular branch of geometry. If we compare what he says with Figures 21–23, and what was said about them, I think we can have no doubt that he is speaking about the same things. By what particular means of cognition he reached a perception of these things I do not know, but I do know that if I were to attempt to give a purely descriptive, non-mathematical, account, I could not do better.

He, as a seer, insisted that it is the difference between the speeds of whirling of the two vortices which imparts to the heart its characteristic form; and I, as a geometrician, know that the contrast between these two speeds (technically speaking it is a quotient rather than a difference) gives me that parameter which we designate as lambda (λ).

Rudolf Steiner had a keen appreciation of the importance of keeping a clear distinction between those things in our world which can be calculated, and those which lie more within the chaotic ambit of the incalculable. And he told his audience that the form of the heart definitely comes within the sphere of the calculable; 'but,' he said, 'the calculation cannot be followed up here, first because it would take too long and secondly because you would not be able to follow it.'* The moment passed, and this crucial calculation was not given. I say 'crucial' because from it there follows the possibility of discovering the seven-fold beat in the pulse, the fourteen-day rhythms of the buds described in Chapter 15, the seven year rhythm of the phase-shift chart, also in the same chapter, and indeed almost everything which is in this book.

The pineal gland

At this point, where we are describing how this egg-like form appears in the breast of man, it is perhaps most fitting to include a note of another place in the human organism where something very like it appears, that is, the pineal gland. This mysterious little organ has not received the attention that it deserves from medical science, and not a great deal is known about it. Its name derives from its close resemblance to a pine cone — a fact which alone should be enough to excite our interest! It is very small — scarcely more than one third of an inch in length.

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* Ibid. p. 193.

With its long axis approximately horizontal and the sharp end pointing posteriorly, it is buried deep in the heart of the brain. The rather scanty literature which has gathered around it (improving somewhat in very recent years) largely ignores its gross morphology, and I have found it a matter of very great difficulty to obtain good pictures of it. The best one I could find, which claims to be an actual photograph, shows a moderately clear outline but gives no indication whatever of the spiral forms which I think we must suppose lie on its surface, because of its name, if nothing else. The outline, when measured and analysed, gave the following result:

$\lambda = 2.05$
MRD 1.8%

Thus in the upper part of our organism, this form is present, with a quality and an accuracy fully comparable with the world of the plant buds and the ventricles of the heart.

In certain primitive creatures, the pineal gland can appear in two apparent capacities, one of which comes near the surface and shows strongly optical qualities, sometimes possessing a clearly developed cornea, lens and retina. It is hard to reject the idea that this was once something of a single eye. One thinks of the Cyclops. But in the higher mammals and Man it has completely lost this quality and has sunk down into the deepest and darkest regions of the brain.

Even down there however it has not lost its relationship with the light. Strong light falling on our bodies stimulates, in ways that are not fully understood, its activity of secreting melatonin, which among other things, is involved in the tanning of our skin. Various procreative cycles are probably also affected.

I am sure that this little organ has been in the past, and will again be in the future, of vital importance to our whole evolution and development. But what chiefly concerns us at the moment is that here again, and now in the upper part of man, we find this egg-like form which is intimately connected with the most ancient aspects of our history.

More on the heart

by Graham Calderwood

As we read in Lawrence Edward's own account of his work with the heart, measurements of a ventricle were done by hand and eye, using a ruler and a pencil, and were set out on his estimate of the most likely position and disposition of an invariant triangle. Relatively few estimates and corresponding sets of measurements were practicable because of the considerable, time-consuming labour entailed by them — too few, in fact, for complete confidence in the results obtained from them. A computer program was developed to tackle the problem.

Using a random-trial method, the program converges on the invariant triangle yielding the best match of a path curve to the outline of the ventricle. It does this for each successive frame of the 'movie,' X-ray ventriculogram, and supplies the λ-value corresponding to that fit. As it runs, the program plots these values against the corresponding frame numbers on a chart, so that eventually most-probable-values of λ for the whole beat of the heart are presented, along with the corresponding quality-of-fit numbers. For nearly all the samples available, the fit is excellent throughout the beat (the correlation coefficient stays above 99%), except close to the moment of systole, when the ventricles contract to minimum volume and — in Edward's description — assume almost thorn-like form. The coefficient drops just then, though not below 0.8 even in the worst case. The program thus confirms that the ventricle is indeed a good path curve, and is very probably formed according to path principles.

The program may be directed to do its work on either the left or the right side of the ventricle, or on both sides taken together. But, as one side (the left as the ventricle is presented) is the *interventricular septum,* or 'party wall' between left and right ventricles, and has an embedded valve which interferes with its form, a match to a path curve is usually sought only for the right side. Figure 72a is an example.

As may be seen, the seven λ-peaks corresponding to the sevenfold 'beat-within-a-beat' detected by Edwards emerge to view distinctly, and confirm its existence with high probability. Shown below (Figs 72b–h, p. 139) are the ventricle outlines corresponding to these peaks, with their matching invariant triangles and fitted path curves.

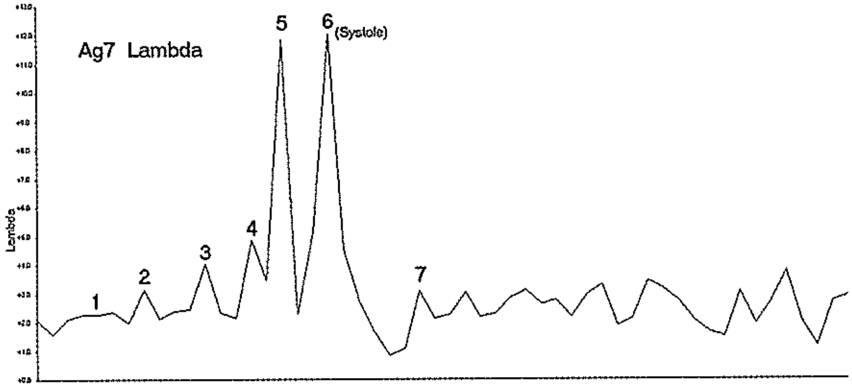


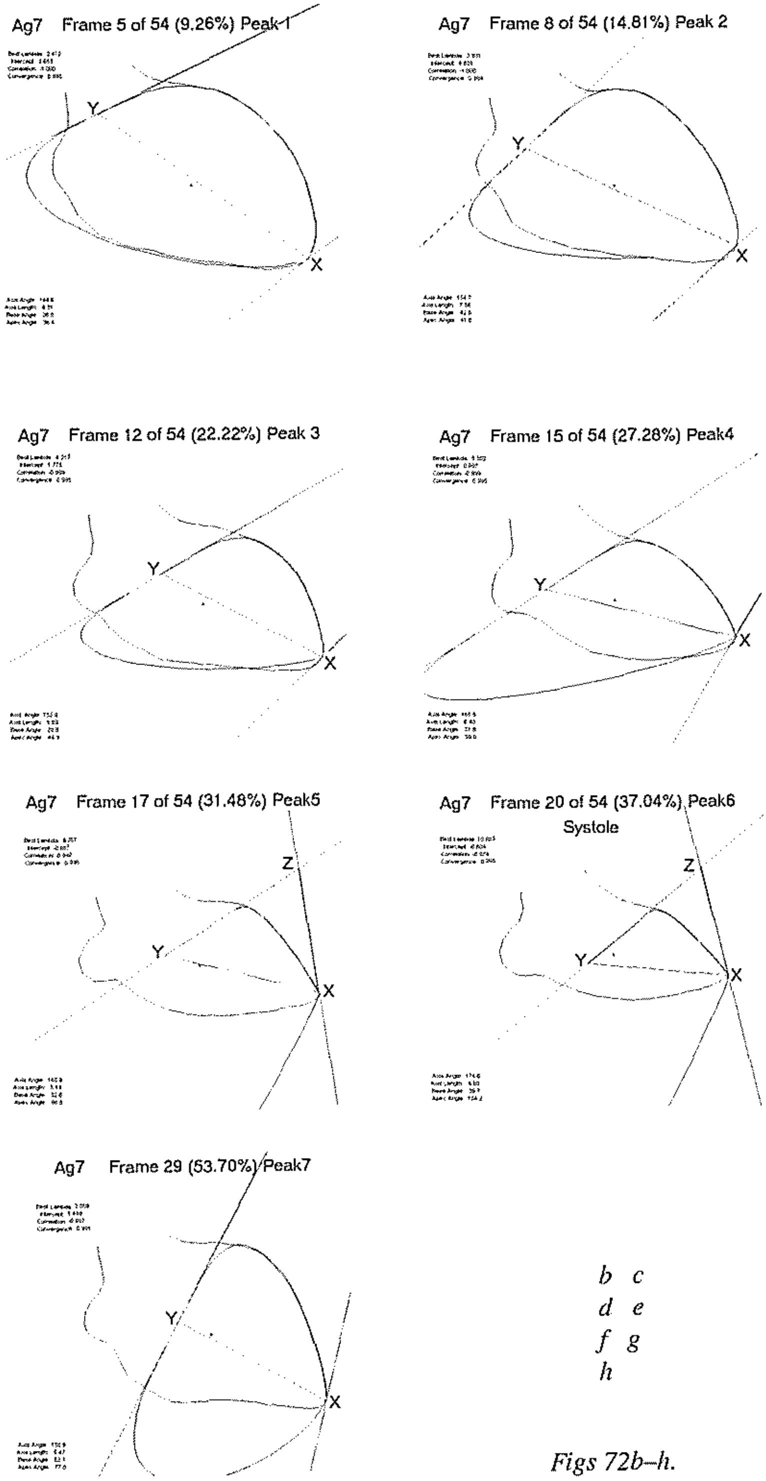
Fig 72a λ-chart for ventriculogram Ag7, containing 54 outlines.

In Figure 72 on page 133, Edward attempts to provide a view of (in his words) 'the invisible part of the heart' — by which, of course, he means the invariant triangle — as the heart goes through a beat. If one casts one's eye over the sequence of pictures opposite, one must surely conclude that his attempt comes very close to the actuality.

The variations of λ to the right of peak 7 on the λ-chart for this heart, coming as the ventricle relaxes into diastole and refills with blood, then prepares for the next contraction by going into full expansion, are untypical; most ventriculograms exhibit just a gentle λ-wobble. Indeed, when watching a ventricle in action using the program's 'movie mode,' one usually sees no more than a slight quivering at this phase of the beat. This one is unusual. It is also unusual in respect of how well it preserves its path form right in to systole; most do not.

These unusual features may be symptoms of some pathological condition present in this particular heart. No doubt a clinical diagnostician could tell us whether they are or are not, and indeed it is not beyond the bounds of possibility that, in the hands of such an expert, Edward's investigations generally into the form of the heart, along with the means to make them, could be a clinically useful tool.

Now I am no sort of expert on the heart, but I suppose it was inevitable that, as I was developing the program mentioned above, I would, in passing as it were, notice several things about the ventricles besides their form that would seriously pique my interest. I beg leave briefly to present and discuss one of these.



b c
d e
f g
h

Figs 72b–h.

An estimate of the rate of ingress and egress of blood at various stages of the beat can be obtained from comparison of the areas of the ventricles, frame by frame. Here is a graph of the sequence of these areas for Ag7.

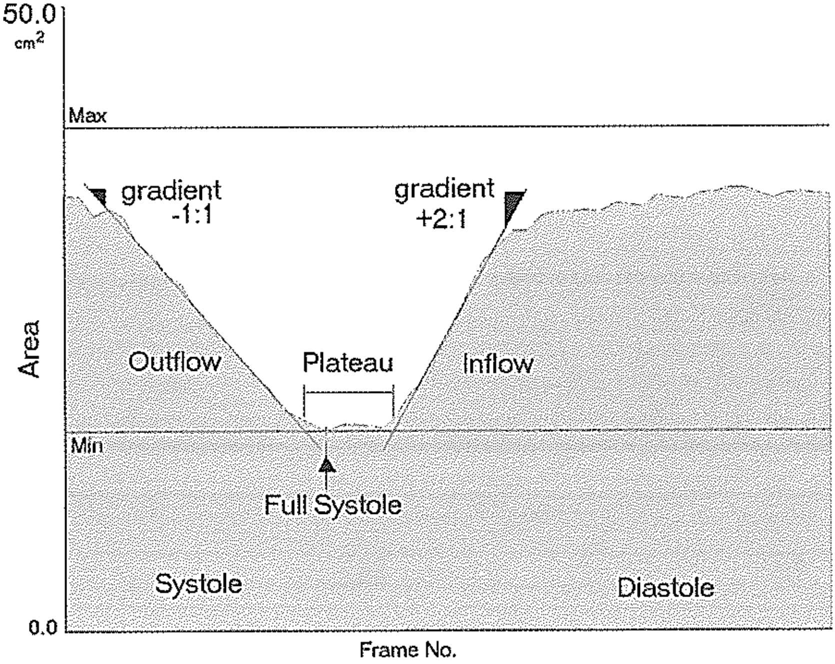


Figure 72i Ag7 ventricle area versus frame number.

The gradient of this area-trace is, roughly, the rate of flow. We notice at once that there are three main phases: first there is a period of reducing area (negative gradient) indicating outflow from the ventricle, then a very curious period during which there is no flow (the plateau of constant area, zero gradient), and finally a period of rising area and positive gradient, indicating inflow.

The first surprising thing to note from this graph (which is, incidentally, a typical one), is that the heart never quite empties. If one thinks that the heart is a pump, surely this failure to empty itself completely means it is pumping inefficiently.

It is also striking that inflow, showing about twice the gradient of the outflow, is around twice as fast as outflow. One might have expected the reverse to be the case; the heart, one would have thought, would seek to deliver blood as urgently as possible, and then retrieve it either at the same rate, or possibly some more leisurely rate. Instead, against expectation, the ventricle fills faster than it empties.

Now, the plateau signifying *absence* of flow occurs at a most significant moment in the beat; it begins precisely at the instant of full systole, when the ventricle is just reaching its limit of contraction, and when, until it is abruptly halted at this point, blood is leaving the ventricle at its greatest rate for this phase. Flow stops because both of the valves serving the ventricle and controlling blood flow shut down together. Why do they *both* close, and why just then? What utility is there in halting delivery exactly at its height?

Physically, the heart is quantity of relatively soft tissue with a mass I would guess to be on the order of a kilogram. It is anchored firmly at its base, so cannot move much there; all movement of the heart must occur roughly on a line between this anchorage and the heart's apex — and so it appears in movie view. One sees a mass of flesh and blood launch itself towards the anchor point; one sees it plucking itself into motion as it embarks upon its discharge phase. Then, just when it is moving at it fastest, and before it completely empties, it is brought to an abrupt and literally shuddering halt (the shudders sometimes show up on the λ-chart as 'noise'). Why?

I have done simple experiments with very thin, soft-plastic tubes, and liquids with viscosities similar to that of blood, and these showed that a very considerable expenditure of energy (in the order of kilowatts) would be needed to drive blood *continuously* through a vascular system like that in the human body — far more than could possibly be available from a pump with the dimensions of the heart. The simplest conclusion to draw from this is that the heart is not a pump of this kind. So, since it seems clear that it is a pump of *some* kind (for blood is delivered and delivery stops if the heart stops), what kind is it?

Well, the blood flow is *not* continuous; it comes in pulses, with a certain duty-cycle. If we estimate this cycle to be around 50%, then we have at a stroke reduced the energy expenditure by half, but still not by far enough for the heart to cope with the burden of delivering it. Another means of accounting for the energies in the system is needed.

I suspect that the body and the heart together form a *partially conservative system;* I mean by this that the total energy shared between them remains substantially constant. How could this work? If the vascular system and the tissue in which it is embedded are elastic to some degree and are inflated, then they hold potential energy in the tensions of this inflation — which they may release back to the mechanism that presumably produced these tensions; the heart. There may be small losses to the external world in the exchange. There probably are; the body is not an energy-closed system, as anyone who has either shivered or perspired may testify.

I wish to suggest that the heart is called upon only to replace these losses as it otherwise mediates the energy exchange; it is *not* called upon to supply the *entire* energy involved in blood transport on each and every beat. But how precisely does it do this?

Consider only the discharge phase for the moment. A *small* volume of blood is to be sent to augment a *large* volume, by way of a portal of small area. This is classical 'hydraulic ram' territory! This is how enormous tyres may be inflated to huge pressures by small pumps; the pumps are rams. It works because the force needed at the portal to effect the transfer of fluid is proportional to the area of the portal, and that area is generally small. The required force, being small, may be supplied by relatively modest apparatus.

Of course, there must be some way of preventing back-flow after delivery, such as one-way-only valves, and both the heart and the arterial system have plenty of these. I consider their presence to be good evidence in favour of what I am now proposing, for why would they be there if no back-pressure were expected? They *are* there, so there *is* back-pressure. From what does it come? Not from the heart; it delivers *forward* pressure. It must come from the vascular system.

But all this does not account immediately for the sudden, strangely illogical-seeming cessation of flow at its height. Nevertheless I think we may understand it in terms of the energy exchange and the need to bring it about with the least strain on the heart. A very simple demonstration will show what I have in mind: place three coins in line and touching each other on a flat, smooth surface. Place a finger on the middle of the three, to pin it firmly in position, then with the other hand draw aside a flanking coin, and use it to deliver a smart blow to the pinned coin. The third coin will be seen to fly away *as if it had received the blow directly,* not as it in fact does, via the pinned coin, which will move hardly at all.

Now think of the pinned coin as a closed heart valve, and of the mass of the heart and still-contained blood together crashing into it as the coin delivering the blow, and it will be seen that what is sent flying on its way like the third coin is the blood delivered up to the point of valve-closure. The closed door may stop the blood flow, but is no impediment at all to the transmission of momentum, and the corresponding energy.

I think that for just a moment in the course of a beat, the heart converts from ram-pump to hammer. It does this in order to use *inertia* and *conservation of momentum in elastic collision* to supply just-delivered blood with energy enough to re-inflate the system — energy the heart had recovered in large measure from the system during the preceding diastole.

This may also help to explain why the ventricle fills faster than it empties: the back pressure from it during its re-inflation is probably smaller than the by-now forward pressure of the system, simply because the heart has the smaller volume of the two. The system has no need of a hammer to help it return blood and energy.

But how do the body and heart first come by the total quantity of energy they together conserve and must continually top up? The obvious answer, and possibly the correct one, is that they do so slowly, using the very same mechanism that maintains the conservation, as both body and heart develop from embryo to adulthood.

This, for what it is worth, is my thesis, and my contribution to the resolution of a debate that sometimes rages quite hotly in some circles on the topic of whether the heart is, or is not, a pump. I did not find an opportunity to discuss it with Lawrence Edwards before he died. I would have been most interested in his opinion.

9. The Pivot Transformation

The thoughts which I am going to detail in this chapter are ones which the non-mathematical reader may not find easy. Nevertheless the first several pages are descriptive and I believe can stir the imagination to a new feeling for the wonderful ways in which things can move in three-dimensional space; and at the end of the chapter I will put a paragraph detailing, as far as it is possible in non-mathematical terms, the general results reached, so that we can go on to the very important ideas of Chapter 10, with some understanding of the things involved.

Positive and negative Euclidean space

These ideas arose from the work which I did with the late George Adams during the fifties and sixties. He, in England, and Dr Louis Locher in Switzerland, had been simultaneously and quite independently developing the concepts of what Adams called positive and negative Euclidean space, and of what Locher called *Raum* and *Gegenraum*. The details of their work have been published elsewhere (see bibliography) and it would be superfluous to repeat them here, but it will be helpful to remind ourselves of the basic principles because unless we experience intensively the fundamental ideas, the relevance of what follows cannot be apparent to us.

In positive Euclidean space we have a metric imposed on pure projective space by a given absolute plane (the plane at infinity) and an absolute imaginary circle contained within it. Dualizing this concept we have a space governed by an absolute point containing an absolute imaginary cone, and within the metric of this space we have to see that this point is as unreachable (infinitely distant) as the plane at infinity is for positive space. George Adams often called this the 'infinite centre' of his negative space.

It is important for us to realize as vividly as possible the qualitative difference between these two concepts of space. Positive space is essentially *extensive* in quality. It is governed by an extensive absolute and it is the space in which our minds and imaginations, which are normally extensive in nature, move easily and naturally. When we wish to simplify matters by restricting ourselves to two dimensions, we write books on plane geometry, that is two-dimensional configurations spread out over the plane. On the other hand the natural basic element for such a space is the point; normally, and easily, we think 'pointwise' and we readily picture lines and planes as infinite manifolds of points lying 'evenly.'

Negative space is much more difficult for us to picture to ourselves. Here we have an *intensive* space, governed by an intensive absolute. Working with the consciousness of negative space we should be led to write books on point geometry. It is not easy for us to see the point as a two-dimensional element (that is as containing ∞^2 of lines and planes) and to realize that within it there is just as great a richness of configuration between its lines and planes as there is between the lines and points of the plane. In negative space the basic element, the one which is most easily and naturally handled, is the plane, considered not as an infinite set of extensively placed points, but as a thing in itself.

And just as in positive space a plane is normally pictured in just this extensive way, so in negative space we have to learn to think of a point as being the meeting ground for infinitudes of planes (all those which it 'contains').

Now those who have studied the work of Rudolf Steiner will be familiar with the descriptions which he gives of the hidden world of life forces (which he terms the etheric world) and with the strange and often seemingly paradoxical statements which he makes about them; and it is a strange fact that as soon as one moves into the realm of negative space one meets as naturally-occurring phenomena just those things which otherwise seem strangest and hardest to believe in Steiner's statements. This led Adams and Locher, and other anthroposophical mathematicians, to the conviction that in the metric of negative space we have a mathematical instrument for the detailed study of Steiner's etheric world.

In particular Steiner contrasts physical consciousness, within the body, with that which a seer experiences when he is outside it. Here we feel ourselves point-centred while looking outwards to an extensive outer world; but there, we would have to imagine our consciousness as being spread out through all the periphery while we gaze inwardly into an intensive not-self. He speaks at one point of being turned inside out like a glove.

Now let us consider a pointful of planes and lines as it could be imagined to be 'seen' by such a peripheral consciousness. Imagine this peripheral consciousness gazing in and in, from all directions, towards its central point. It would be immersed into a two-dimensional cosmos, so small that it is contained in a single point, yet as vast in the ramifications of its possible configurations as all the possibilities of an infinite plane. Within it the peripheral consciousness would have the possibility of 'seeing' all sorts of things!

Let us suppose that at some particular moment it 'sees' a cone. This cone would be formed of a single infinitude of generating lines and a single infinitude of tangent planes. But these planes would be seen as things in themselves, not as extensive manifolds of points; the extensively-placed points of each plane simply would not enter the consciousness; each plane would be 'seen' (I cannot find words to express it otherwise) as the essence of flatness in a certain direction *as it passes through the central point.* And the cone itself would be 'seen' as the quintessence of cone concentrated into the point where all its tangent planes and generating lines pass through that point. Points placed on what we, with our normal extensive consciousness, would regard as the surface of the cone, would simply not exist for this peripheral consciousness.

In this way we can build up a picture of what such a peripheral consciousness could 'see' in any point of a negative space, but if it were to gaze into the three-dimensionality of a negative space it could 'see' multitudes of other two-dimensional forms, as surfaces (spheres, perhaps, or hyperboloids) all enveloped by their tangent planes, each plane being apprehended as a thing-in-itself and not as an aggregate of points.

George Adams continually insisted that although one could have what one might call pseudo-positive-Euclidean spaces, that is, metrics in which some randomly-chosen, finitely-placed, plane is chosen as one's absolute, with a random absolute conic placed within it, these could not be thought of as having any more relevance to reality than just interesting exercises in thought. The plane at infinity stands as such an obvious and necessary absolute that one cannot imagine any other which would have any bearing on reality.

But when we come to consider the infinite centre of a negative space no point has superior merits to any other. Whereas there can only be considered to be one positive space, there are multitudes of negative ones, all interpenetrating one another. And it became a life-quest for George Adams to seek for what places, in the sense-perceptible world around him, he could truly conceive that infinite centres of negative spaces would be at work.

It was shortly after the Second World War, and just before I started work with him that, in a moment of what I believe to have been true inspiration, he apprehended one such place. It was spring, and he was observing the unfolding of the new leaf buds. One can find, as one penetrates inwards towards the centre of such a bud, leaf upon leaf upon leaf, as though the plant is there sheltering its most precious possession. But when one actually reaches the centre, what is it that one finds? Physically there is nothing; it is a little hollow space! And there, where substance is not, George Adams saw that one must conceive the infinite centre of a negative space. Towards this point, as towards an unreachable absolute, the life forces stream peripherally.

One thinks of gravity forces as working concentrically, outwards, from a centre of mass. One can picture a set of spheres stepping outwards towards the infinite rim of our world, with equally increasing radius. What would be the polar of this? In negative space one would see a set of spheres, 'increasing' in negative size, as they shrink towards their infinite centre, in *step measure* (see Figure 12). There would be an infinite number of spheres before one could reach the centre; this latter is truly the unreachable place.* And all these spheres would have to be thought of planewise, with the sheltering quality of their tangent planes.

Such a conception, developed only thus far, is still quite primitive, but as a primitive beginning it contains, I believe, the possibility of studying the polar qualities of the force of gravity, and that force which George Adams, amongst others, would call levity. And in the course of his life he did much detailed work to follow up these ideas. But for our present purposes I want just to consider the primitive notions involved, to see and to feel, the *qualitative* similarity of the two pictures: the planar, sheltering quality of the leaves as they crowd in towards their centre which they never reach (or alternatively one can think of them as growing outwards from this centre, as from an infinite well of life), and the planewise spheres growing inwards towards their unreachable infinite (Figure 73).

Again at this point I would like to stress that we are not concerned so far with exact comparisons (all sorts of detailed work must follow afterwards) but simply with feeling the qualitative similarity of the two pictures. And working for many years with George Adams on the more detailed concepts of negative space I became imbued with the conviction that these considerations do provide a true basis for a detailed

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* In such a dual drawing the former arithmetic means of positive step measure would appear as harmonic means to our positive eyesight, though they would satisfy the algebraic definition of arithmetic means in their own negative space.

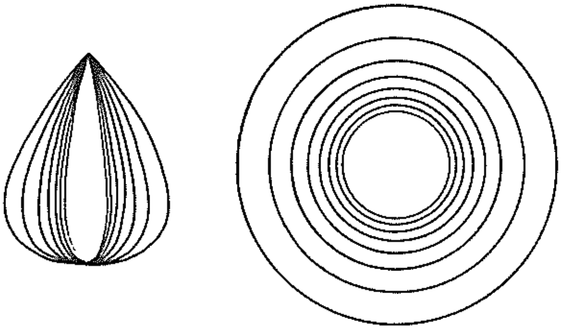

Figure 73. A set of circles equally spaced in negative space bears a certain qualitative similarity to the situation of the leaflets in a leaf bud.

study of the life forces of nature. And when, many years later, I was led to study the connections of the path curves with the bud forms of the plant world, I very much wanted to bring these two aspects of thought together. There grew up in me the conviction that it should be possible one day to find what peripheral, planewise, forms one would have to conceive as working inward towards the centre of the bud which, when they were appropriately transformed, would give rise to what physically follows the budding process, the forms of the seed-bearing part of the plant.

Clearly there are two separate questions involved here. Firstly what kind of negative, planewise forms we must see as working into the negative space of the bud, and secondly what sort of transformation would govern their relationship with the physically-following forms of the seed-bearing part of the plant. And it is with this second question that this chapter is concerned.

The relevant transformation process

What qualities would such a transformation need to have? We are thinking of our generating forms as being essentially planewise but the seed-forms must surely be seen as pointwise (the great advances made by molecular biology, especially as regards the genetic forces, can be seen as an indication of this). We need a transformation which will turn negative space into positive, plane into point. And clearly it must be basically *simple.*

There is no bound to the scope of human ingenuity! One could devise artificial transformations to do almost *any* task. If one allowed oneself the use of as many parameters as one wanted (and one had a large enough computer to do all the necessary calculations) I suppose that one could turn anything into absolutely anything. But no-one would believe that anything significant had been achieved, except perhaps the computer salesman, who would have demonstrated the power of his machine.

Now so far there is an obvious candidate for the post: the transformation of pole and polar. It changes planes into points, and one cannot have anything more basic or fundamental than this first degree process. But strangely enough this very virtue turns out in a way to be a disadvantage. This transformation changes the quality of a form, but not any of its logical interconnections. It adds nothing to, and subtracts nothing from, the woven texture of inter-relations which forms a given figure. Of course it would always be theoretically possible to take any seed-bearing form whatever, as a pointwise figure, and to transform it into a planewise equivalent, by pole and polar. Whether the resulting planewise form would be possible to visualize with one's imagination is questionable; what is certain is that basically nothing new would have arisen; one would simply have taken an incomprehensible complexity in one realm and have transferred it into an equally incomprehensible complexity in another. What would one have achieved?

It is clear that our transformation, if it is to be of any real service to us, should have the power of being able to produce something like the manifold complexity of the observed organic world, out of forms which are comparatively more simple.

The pole and polar transformation suffers from another disadvantage in that, in order to work with it, one must somewhere have a polarizing quadric surface. What should it be? Sphere, spheroid, ellipsoid, paraboloid, hyperboloid; of what eccentricity and what radius, where centred? The possibilities of choice are bewildering, and embarrassing! And our observation of the plant gives us no clue as to the answer. In fact it becomes uneasily clear that the plant gives us no hint that any such quadric exists. And in fact we have seriously to ask ourselves what right we have to impose on any living organism such a process, just to produce a planewise precedent for the pointwise form which we see there.

Then one day, quite suddenly — I do not know how the thought had been hidden from me for so long — I realized that I did not any more need to search for such a transformation; it was there before my eyes in the plant itself! *The path-curve form of the bud is just such a*

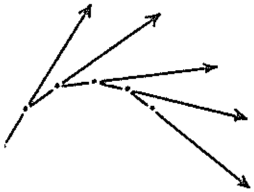

Figure 74. A point moving along a line, while that line turns about the point.

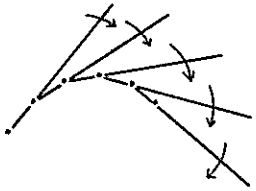

Figure 75. A line turning around a point, while that point moves along the line.

transforming agent. It is there whether I recognize the fact or not; and it turns planes into points; it produces, as I found after further study, complexity out of comparative simplicity; and its basic concepts are as simple and fundamental as one could wish.

In order to see how this can come about I think it is useful to consider a little of what George Adams sometimes called 'movement geometry.' I wish to speak here descriptively, and maybe a little loosely, leaving a more closely-reasoned treatment to those who have finer logical sensibilities than I. I wish to speak of the 'momentary movement' of an element, its movement within an infinitesimal moment. And I wish to speak of 'neighbouring positions' of that element within its movement, positions which are distinct yet are infinitesimally close. (Logicians please allow me!) And now we must ask: Within the plane, how can a point momentarily move? And the answer is: Only in one way, along a line. This follows immediately from the axiom that two neighbouring positions of the point must have just one line in common. Dually: How can a line momentarily move within a plane? Answer: Only in one way, around one of its points. (Even if it is moving parallel to itself it is moving around its point at infinity.)

But now suppose that a point is moving along a line, while that line is simultaneously turning around that point. We make a picture of it, letting neighbouring positions of the elements have a finite distance between them, so that we can see what is happening. Let the gaps between the elements get smaller and smaller, and in the limit we have a smooth curve (Figure 74).

But if we now imagine a line turning about one of its points, while that point moves along the line, and if we draw it in a similar way, we come to identically the same picture (Figure 75).

In the first case the line of movement is the tangent implied by a pointwise curve, and in the second the point of turning is the point of contact implied by a linewise curve. The curve is in this sense a completely self-dual organism. I call this the anatomy of a plane curve.

In three dimensions the case is more complicated. In the first place two lines may have a point in common (in which case they also have a plane in common) or they may be skew (with nothing in common). Therefore there are two distinct and different ways in which a line may momentarily move. It may move skew to itself; then two neighbouring positions of the line will have neither point nor plane in common, and the line will generate a ruled surface (a line-woven single-sheeted hyperboloid is a typical example). Or it may move around one of its points, in which case it follows a twisted (or space) curve.

Now let us consider a twisted curve. We have a point moving, momentarily, along one of its lines while that line is turning around the point. But this means that two neighbouring positions of the line have a point in common, and they therefore also have a plane in common. This is the plane in which both the point and the line are momentarily moving, the so-called osculating plane of the curve at that point. Now we can describe the complicated organism which is a twisted curve: A point moving along a line and in a plane, while that line is turning around the point and in the plane, while that plane is turning around that line and in that point.

Any twisted curve is a close-knit threefold organism, and the points, lines and planes which form it are each of them as basic and fundamental as each of the others. It is only our extensive consciousness which sees the curve so easily as a collection of points, less easily as a set of implied tangent lines, and less easily still with the implied osculating planes. If we had negative space consciousness we would see the curve as a manifold of planes, each one seen not extensively as spread out evenly in space, but as the essence of flatness just in that place where it is turning around its curve. The points of the curve would be pictured only with difficulty and as they are implied by the moving plane.

We must live as intensively with this thought as possible. I have only to move my finger tip in any arbitrary curve which I like, to have determined a linkage between an infinitude of points with an infinitude of their related planes, each point of the movement being related to its osculating plane in the curve, and, reciprocally, each plane of the movement being related to the point around which it is pivoting, its pivot point — to say nothing of the tangent lines as well. And movement is of the essence of life, whether it be the slow movement of the growth and development of a plant, or the quicker movements of an animal or human.

Now if we have a path-curve transformation, this sets every point, line and plane of space moving, excepting those which are in the invariant tetrahedron. It thus links every point to its osculating plane, and every plane to its pivot point. In general the relationship, or transformation, is one-to-one, but owing to the presence of the invariant elements it is not universally so, and the transformation is not of first degree. (The case of the lines of space is a little more complicated, and due to lack of space a fuller treatment of this is perhaps best left to another place.)

Now I would like to return to our concept of the peripheral consciousness gazing in and in, to the vast little two-dimensional world contained within its central point. Let us vividly imagine what it would 'see' there, as it is enwrapped in its own little cosmos. Apart from the central point itself, it sees no points whatever, only lines and planes, each seen intensively as the essence of direction or the essence of flatness concentrated within the central point. And within this inward-looking world it would 'see' some form, a cone perhaps.

And now this little world comes within the field of a path-curve transformation. What will happen? Each of the planes of this cone will start to move, each around its pivot point, determined for it by the path-curve transformation, and these pivot points will be extensively placed throughout space. The peripheral consciousness would feel its hitherto closed universe rocking about points which till then it had not known to exist! It would have to become conscious of an extensive, pointwise form in what, to it, would be a new world.

In this way I pictured the path-curve transformation as a force which would take any planewise form and would produce from it a new, pointwise, form, the form around which the planewise form is set moving. I feel that this description is hardly adequate; perhaps some may consider what I have written as naïve. I do not know any other way to describe things which are so far removed from our normal ideas but which are basic to the fundamental rules of movement and space.

Once this pivot transformation has been envisaged it must be clear to us that, conceptually, it has to be present wherever the path-curve transformation is seen to be at work; and this we know is the fact for a very large number of plant species. It therefore immediately became a matter of importance to find out just how such a transformation works. The resulting mathematics is given in Appendix 5; here we will confine ourselves to giving a description of how this transformation looks to our imagination.

Let us look back to Figure 5. There we saw a planewise, we might say an ethereal, form, a line made up of all the planes which pass through it; or, stating it otherwise, a little one-dimensional world (that is, a single infinitude) of planes all bound together by the straightness of their common line. There we saw how, under the influence of some transformation the planes could be induced to new positions, maybe taking up these new positions as tangent planes to a planewise cone, as pictured in Figure 6. But now we are asking a different question: Supposing this planewise line is approached, not by a transformation which seeks to move its planes, but by one which will transform each plane into a point, where will all these points be found to lie? The answer is that if the transforming agent is of the simplest kind, a linear transformation, of first degree (the well-known theorem of pole and polar would be such a case), then if the original planes all lay in a straight line, their transformed points would also lie in a straight line. Any linear transformation preserves straightness.

But when I came to apply the first rules for the pivot transformation just to this case — a straight line of planes — I found to my intense interest, that the transformed points do not lie on a line, but on a beautiful spiralling curve called a Twisted Cubic. This was the first indication I had that this transformation which I had found is a more potent one than the ordinary linear projective transformations such as correlation, or its special case, pole and polar. More potent in that it effects more dramatic changes — it raises forms of first order into ones of third order, for instance — having more sophistication, but at the same time, for this very reason, being less near to the elementary heart of things.

The twisted cubic is such an interesting, and important form that it deserves a mention to itself.

If we look back to Figure 1, we see there three curves with their associated equations. (The first curve consists of a wing-like branch and also an apparently separate egg-shaped curve; but from the point of view of the mathematics these two have to be seen as parts of one and the same curve.) The first equation has as its highest index a '3' (x cubed); the second has as its highest index a '2' (x squared) and the third one has only '1's, assumed for its variables, but not written. These facts are intimately connected with the fact that it is possible to find lines which cut the first curve in three points, whereby it is said to be of third order (or a cubic), whereas it is only possible to find lines which cut the second curve in two points. It is therefore of second order; and clearly the straight line of the third diagram is a curve of first order. It can be proved that, no matter how far we seek, we shall never be able to find curves of second order outside three main types: ellipses (amongst which we must include the circle), parabolas and hyperbolas. These three types of curve are all projectively equivalent; they can be transformed one into the other with the greatest of ease; and they are known by the collective name of 'conics' (because they can all be generated as sections of a cone). When we consider the cubics we enter a realm of much greater variety.

Thus we see that the conic is the simplest and most fundamental kind of curve which it is possible to find. It is the aristocrat in the realm of curves. But this is only the case as long as we remain within the plane. As soon as we let our curve leave the plane and start moving freely through three-dimensional space it becomes known as a 'space curve' or more frequently a 'twisted curve,' and it can be shown that it is not possible ever to get a twisted curve of second order. The simplest and most fundamental space curve possible, the doyen of all space curves as it were, is the twisted cubic, and it is into just such a form, I found, that the pivot transformation changes every straight line of space. When we start working with the pivot transformation we move into a realm where the twisted cubic plays in a way a similar role to that played by the straight line in ordinary space.

We are all familiar with ellipses and parabolas, but what does the twisted cubic *look* like? It can of course assume an infinitude of particular forms, but they all share certain distinguishing features in common. It is always infinitely long, moving from infinity to infinity, and joining up with itself there. It may move through space in one strand, or in three, running side by side, but never in two. The curve is always spiralling in quality, but not endlessly turning like a helix; in the whole of its journey through space it makes only one turn. Coming in from vast distances it is almost straight for a large part of the way, then as it approaches its central part, it makes a graceful spiralling curve of nearly a whole turn, before settling down for an almost straight journey on to the infinite distance again. The cubic quality of this curve is seen in the fact that it cuts any plane of space in just three points, although it may happen in any particular case that two of these cutting points are imaginary and therefore not visible to the imagination. Figure 76 can only give a very inadequate idea of the richness of concept and relationship which is to be found associated with this wonderful curve.

All sorts of interesting constructions can be made. Figure 77 for instance, is an elevation view of the pivot form of a thin cylinder passing close, but skew, to the central axis of a path curve system. It is of course woven all over with twisted cubics, one for each of the generating lines of the cylinder.

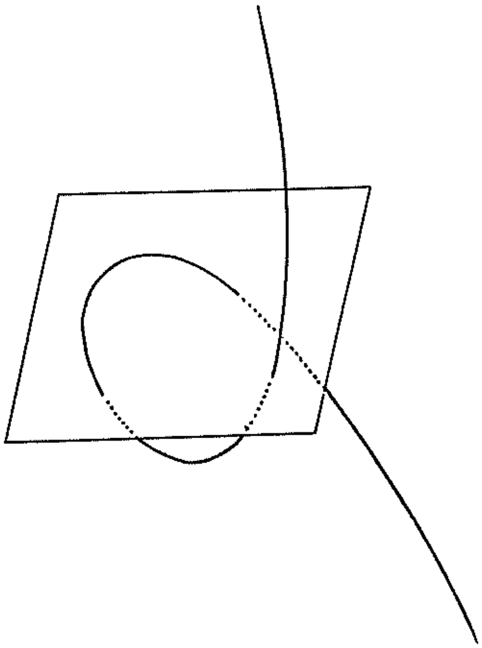

Figure 76. A twisted cubic cuts any arbitrary plane of space in just three points.

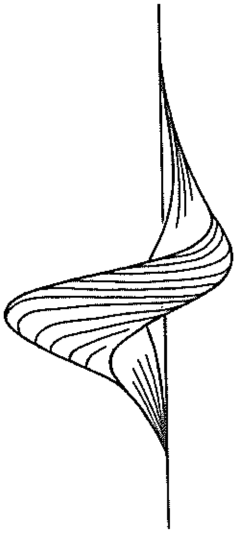

Figure 77. We take a thin cylinder passing skew to the central axis of the transformation. Each line of the cylinder transforms into a twisted cubic. Thus we get a beautiful surface entirely woven over with twisted cubics.

Summary

To sum up the results of this chapter, we can say that if we have a path-curve field, then every general plane of space is set moving by this field, each one about one of its points, around which it is momentarily pivoting. Thus if we have any form composed of tangent planes, and we place it within a path curve field, then immediately we can find within this form another hidden, pointwise form, composed of all the points around which the planes of the original form are pivoting. We can speak of the new form as representing the momentarily stationary element in the movement which has been imparted to the original one. In a rather free and easy way, we can speak of the original form as pivoting around the elements of the new one. Thus we find that the path-curve form, which we can see so clearly and measure so accurately in the plants, is itself a transforming agent which turns planewise encompassing forms into pointwise, atomic ones.

10. Ethereal and Physical Spaces in Flower and Fruit Forms

We have seen how wonderful egg-like form appears out of the most elementary geometrical thinking, and how by observation and exact measurement one can test just where this form is really manifesting itself in nature — eggs, opening buds, plant inflorescences, pine cones, the ventricle of the heart, and so on. But equally, and by the same methods, we are shown realms where, despite apparent, superficial resemblances, it becomes clear that this form is not in fact showing itself with a significant degree of precision. Something else, and maybe more sophisticated, must be at work. And this is markedly the case when we come to consider the seed-bearing part of the plant: the gynoecium and the ovaries. And for many years it was my wish to find the relation between this central mystery of the plant world, this capacity for self-reproduction, and the manifest path-curve forms to be seen in the growth of the plant itself.

As a first step to this, we have considered the bud, with its path-curve form, in relation to the metric of negative- or counter-space. This metric sees a finitely-placed central point as its absolute, its 'infinite centre,' and its fundamental forms are made up of enveloping planes, in contradistinction to our ordinary Euclidean space which has an extensive absolute, in the plane at infinity, and whose most easily-handled forms are made up fundamentally of points. We saw how in positive Euclidean space we could envisage an evenly-moving expansion, outwards towards the plane-at-infinity; and how, in negative space, this would be countered by an inward 'expansion' shrinking ever closer and closer to, but never reaching, the infinite centre. And some questions were posed. Can we see the infinite centre of a negative space as a 'life centre' towards which the ethereal forces of life stream from the outermost periphery? Can such a centre be functioning in the hollow space at the heart of a living bud? If it does, what sort of planar, enveloping forms must be envisaged as working towards or in this centre? And can we conceive a transformation which would take these planewise forms and produce from them the forms of the more pointwise, seed-bearing part of the plant?

The pivot transformation could be just such a transformation. It is present, mathematically at any rate, wherever a path-curve system is present — path curves can be seen, and measured, in so many of our buds — and the laws of this transformation stem from the most fundamental laws of movement which, one might think, must be very near to the principle of life itself.

Now we must describe a search to see whether the pivot transformation could provide any consistent and convincing answers to these questions, and to find what sort of answers these would need to be.

The question was: Could one find some fundamental planewise form working in or towards some centre within the bud which, when transformed by the movement-principles of the bud itself, would produce the more pointwise, seed-bearing part of the plant?

The most obvious way to answer this was to try and see. And so I tried. I tried firstly cones, of varying vertical angles, set at varying inclinations and centred at various heights within the bud. When these failed I tried hyperboloids, I tried spheres and segments of spheres, and then various other types of surfaces. The resulting transforms were always intriguing, often beautiful, and many of them had a distinctly organic appearance; but none of them bore any resemblance to the seed-bearing part of the plant.

At this time I was working with the transformation of the bud of the wild rose ($\lambda = 3.1$, $\varepsilon = 0.5$) and I had the photographs of the buds, and under each bud, of course, was the curved swelling of the rose hip. Seeing that a frontal attack on the problem had had no results, I decided to reverse my method of working. Using the inverse process I would start from the points on the outline of the hip, and ask what planes would be necessary to produce them. And as I worked there came on my drawing, shadowy and rather approximate but distinctly to be seen, a form which I thought I recognized. It looked rather like Figure 78, the dotted curves showing the bud and hip, and the lines representing sections of the planewise form, the planes being at right angles to that of the page. It seemed to me that it was like a vortex outline. Now the vortex is just as much a fundamental path-curve form (λ negative) as is the egg (λ positive). Furthermore I thought I could guess the invariant tetrahedron which would be needed to give such a form.

This tetrahedron is so important that it deserves a paragraph to itself. It is the semi-imaginary case, where two points and planes are real and two imaginary, two lines are real and four imaginary (see p. 44).

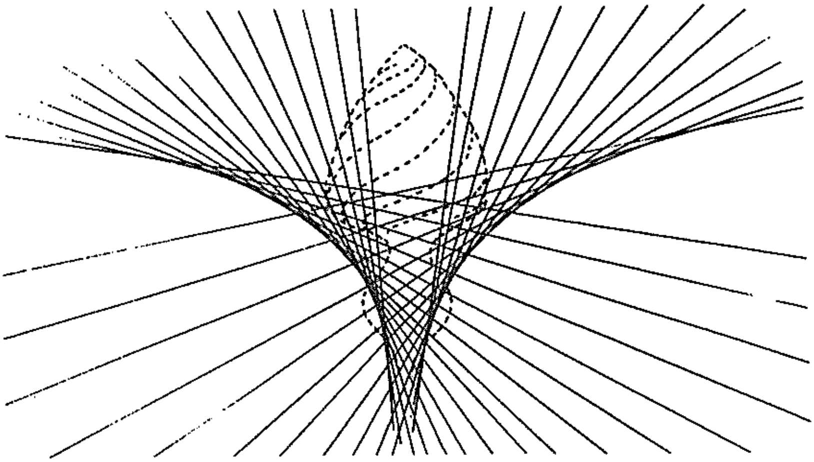

Figure 78. The form of the rose led to the concept that the bud would mediate between a planewise vortex and the form of the hip.

Imagine our ordinary egg transformation, with invariant point X at the top pole, and Y at the bottom; now take X to infinity in a direction at right angles to the invariant plane through Y. The two real points are X and Y and the two imaginary are I and J determining equal-angled circling measures along the horizontal line at infinity. The two real planes are the plane at infinity itself and the horizontal plane through Y, and the two imaginary planes are represented by circling measures of planes contained in the vertical axis through Y. The two real lines are the horizontal line at infinity and the vertical axis through Y, and the imaginaries are represented by circling measures of lines centred in Y and in the horizontal plane through Y, and by circling measures in the plane at infinity centred in X. If we now let the infinite centre of a negative space be at Y we find that this tetrahedron bears remarkable relationships with the absolutes of both positive and negative space. One of its planes *is* the absolute plane of positive space, and one of its points *is* the absolute point of negative space. Two of its points are points of the absolute imaginary circle of positive space, and two of its planes are planes of the absolute imaginary cone of negative space. It has a point and a line which are pole and polar with respect to the absolute circle of positive space, and it has a plane and a line which are pole and polar with respect to the absolute cone of negative space. It is interwoven, through and through, with the absolutes of positive and negative space. Also it is the largest tetrahedron which it is possible to have: they don't come any larger. It spans all space. In one of my less prosaic moments I named it the cosmic tetrahedron.

And this was the tetrahedron which, it seemed to me, would be needed to produce the vortex-like form which had appeared on my drawing. And it seemed to me appropriate, and intellectually satisfying, that this tetrahedron, which mediates in such a wonderful way between the outer periphery and the inward centre, should be the one in question. It was certainly interesting, and possibly instructive, but, so far, largely guesswork.

The crucial question now was: Given the measured parameters of the rose-bud, and given the cosmic tetrahedron with its centre somewhere within the rose-bud, would it be *possible* to find a path-curve vortex within that tetrahedron which, when transformed by the bud, would give a form exactly like that which had been measured for the rose-hip? The possibilities of variation within the path-curve forms are strictly limited and it was not at all certain that such a vortex could be found. But if it were found to be possible one would be interested to know just where the infinite centre would have to be placed within the bud, and what (negative) λ the vortex would need to have.

The details of the somewhat tedious system of measurement and calculation by which these questions can be answered are given in Appendix 5. Here we shall just give an account of their results. At the end of one's working one is able to draw what I call the b-λ chart. This is a graph of which the upright axis shows the λ of the vortex (always, of course, a negative number) and the horizontal axis shows b, which is the height of the centre of the vortex above the bottom pole of the bud. Now as the result of one's calculations one can plot three curves on this b-λ chart and know for certain that unless one's three curves all meet at one point on the graph, there can be no path-curve vortex which will fit the case. If they do all meet on the graph then we have a very high degree of probability that a vortex with the given λ, and its centre situated at the height indicated by b, will give an exceedingly good fit along all the length of the ovary.

As soon as this method was developed I applied it with some eagerness to the case of the rose. The resulting b-λ chart is shown in Figure 79. This gave some certainty, and added precision to my first intuitive guess. A vortex with a λ of -2.6 with its centre situated 0.77 of the bud height above its base, when transformed by the movement qualities of the bud itself would indeed give a form which would be very close to that of the rose hip.

To picture what this means we have to remember that path-curve forms, whether they are egg-shaped or vortex-like, never come singly. They are always part of a whole field of similar forms, one within another (see Figure 26). Thus when we speak of a planewise vortex of some particular λ, and in such and such a situation, we mean a whole field of such vortices, and on transformation they will yield a veritable field of form within which the ovary-form will be developing. Figure 80 shows four selected forms from this field, calculated for the wild rose with which I was working at the time, shaded in grey, and the black dots show positions on the outline of the ovary as they were actually measured from the photograph. We notice that any discrepancies between the calculated and the measured form are so slight as to be almost invisible to the unaided eye. Note also that no ordinary path curve could possibly accommodate itself to this form. The amount of sharpness at either end would demand a corresponding degree of bluntness at the other; whereas this form tends to be sharp at both ends.

This was a promising beginning, but taken by itself it did not prove anything except that in the case of this particular rose there did happen to be a possible path-curve vortex which would transform exactly into the form of the hip. It did not prove that this was a matter of significance; it might have been a coincidence. It was needful to test species after species to see whether what had been found for the rose was in fact a general phenomenon. And as species after species was found to respond in a similar way, so the probability that this was a matter of real significance began to grow.

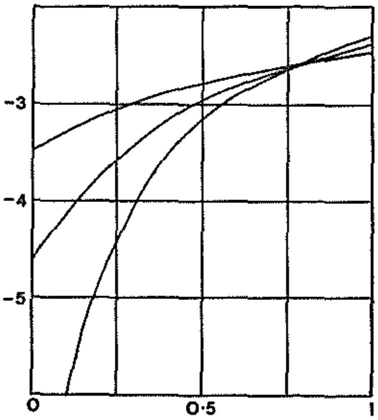

Figure 79. There is no mathematical necessity that the three curves on the b-λ *chart must meet somewhere on the chart, but if they do then we know that there is a possible path-curve vortex which will transform closely into the ovary form which we are seeking.*

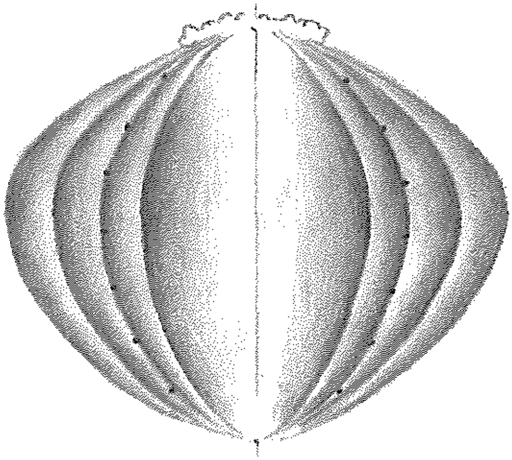

Figure 80. The field of form for the hip of the Wild Rose, compared with the actual measurements of one such hip.

Figure 81 shows, in a similar way to Figure 80, the field of form calculated for the ovary of the Campanula, with the dotted outline of the actual ovary, taken from the photograph, superimposed. It will be seen that although the fit is not quite as exact as with the rose hip, it is remarkably close. Notice that although this form is indeed rather blunter

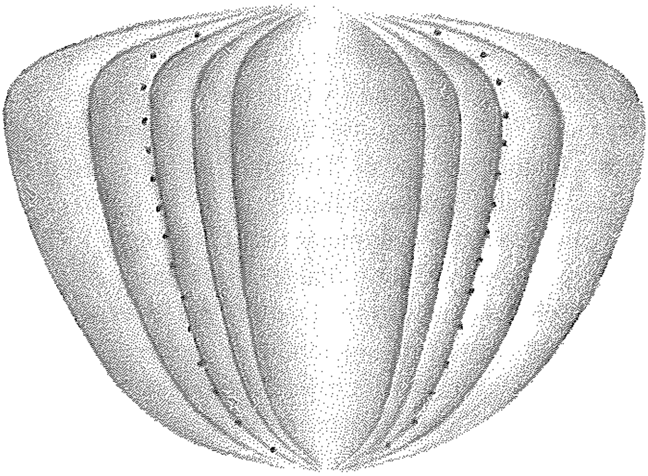

Figure 81. The field of form for the ovary of Campanula.

at one end and sharper at the other, no path-curve form could be found to fit it; it is as though, during their middle part, these curves are almost trying to be straight, and then, above and below this, there is a tendency for them to have something of a 'shoulder' as they turn sharply in towards their poles. This is something no path-curve form can do, but is a phenomenon which is seen fairly frequently in the world of vegetable, fruit and ovary forms. It will be seen from Figure 81 that the only discrepancy is that the actual ovary displayed a rather sharper shoulder in its upper part than the calculated field. I believe that had I taken point P, of Figure 174, (see Appendix 5, pp. 361f) rather higher on the form, the resulting parameters could have been refined, and a better fit still have been obtained.

This degree of accuracy is typical of the many cases that I have investigated, and shows that the pivot transformation of these vortices has a remarkable capacity for producing just those kinds of form which one finds in the seed-bearing parts of the plant. In fact when I first began to sense the importance of the vortex in this respect, my immediate reaction was to produce a series of drawings, using random bud and vortex parameters, just to see what sort of forms would be forthcoming; and I was struck by their general similarity to the seed-bearing parts of the plant kingdom; ovaries, fruits, nuts, pumpkins, seemed to multiply on the page.

Test for significance

Next we have to ask ourselves a similar question to that asked when we were considering the evidence that the buds are true path-curve forms. How adaptable is this process? Could the transformations of these vortices be made to fit almost *anything*? The question is not as easily answered as with the path curves. The latter, as far as just their profile is concerned, form a one-parameter family. (λ only), but the curves we are considering now are a two-parameter family (b and λ_c)* and they must be expected to be more flexible than the ordinary path curves. I have not a finally convincing answer to this, but I did an experiment in which I took a set of typical bud parameters, and confronted them with a set of six ovary-forms, each of which was, I thought, a perfectly possible kind of shape which a plant *might* produce. In only three of the six cases did the curves meet on the b-λ chart. I repeated the experiment with six

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* λ_c denoting λ of the cosmic vortex, not that of the relevant bud.

further — as I thought — *possible* forms, and the result was the same. This leaves me with the impression that if this pivot transformation of the vortex is not of significance, meetings of the b-λ chart should not come, in the long run, on more than about 50% of the cases.

In actual fact, out of thirty-one different kinds of plants studied in this way, twenty-eight of them respond perfectly. The other three all have the curves meeting, but in one case, the wild iris, the centre of the vortex would have to be minutely below the bottom of the bud (about 0.7% of the total bud height) and the other two, syringa and sibirica, would have to have it about the same amount above the top of the bud. Many of these ovary forms are small and not easy to measure accurately, and probable errors in the measurements could easily account for these cases. These figures are subject to one important reservation, which will be discussed at the end of the chapter.

Having reached the idea of a vortex for the planewise form working into the infinite centre within the bud, the next question which arises is as to what pitch the spirals will have upon it, that is, the ε of the vortex transformation. The only way to answer this, as with all such questions, is to go to the plant and observe what we see there.

Now it is an interesting, and I think highly significant, fact that the plant, in almost all its members, is intimately concerned with spiralling movements; we see this continually, more or less explicitly, in the phyllotactic arrangements of the leaves and branch stems, in the spiralling petal-edges of the buds, and in many other places, but when we come to the form of the ovary itself, and the whole seed-bearing part of the plant surrounding it, all hint of spiralling ceases; or so I have usually found it. No matter how carefully we examine the seed chamber, and indeed the ovaries themselves, for any evidence of spiralling construction, either in the general form or in the fibrous constitution, it is in vain. The whole form, and its fibrous construction, seems to speak only of the direction 'up and down'; everything lies along lines of longitude.

It seems as though the plant, in the act of preparing for seed-production, finds the wonderful spiralling movements in which it has been growing and developing, freeze into a certain immobility. It seems that we have to envisage this vortex transformation with $\varepsilon = \infty$, when the path curves on it would be like lines of longitude, each one remaining all the time in one of the planes of the central axis.

We can visualize the movement of the planes of our form, all of them being held at the outset in the horizontal line at infinity, then each one sliding inwards, always tangent to the profile of the vortex until they all merge together into the central axis; a grand vortical procession of planes moving simultaneously inwards from every direction of the horizontal infinite, into the central upright, but, as far as our evidence goes, without actually spiralling in the process. We hear much of the complementary qualities of expansion and contraction between the centre and the periphery, and as a rule I think we are inclined to picture this as a spherical process, movement between a central point and a peripheral plane, but here we are led to see it in a rather different form, as movement from line to line, and ending in the upright Staff of Life. And it is this whole inward and outward movement-organism which, when taken hold of and transformed by the spiralling bud transformation, produces the form of the ovaries or the ovary-bearing part of the plant.

Some years after I had been led to these ideas my attention was drawn by a friend to Steiner's description of the two contrasting parts of the plant, that which manifests in the spiralling metamorphosis from leaf to sepal to petal to stamen (which he mentions as being connected with the planetary movements) and that which is seen in the upright stalk, which then swells out ('elongates' is the word used in the English translation) into the ovaries and gynoecium, and which he connects with the influence of the sun. And he points out that the direction of this stalk is found by connecting the centre point of the earth with the sun:

> Thus we have to compose the whole plant out of that which grows towards the sun or towards the centre of the earth; and that which winds itself round and copies in the stipules the movements of the planets.

He then goes on to say that:

> both these forces are brought together . . . at that time of the year when the plant progress towards its fructification. The spiral principle of movement is then united with the principle which works in the stalk . . . [as he later says] . . . in a sort of marriage.*

On reading this I asked myself whether I dare believe that in the pivot transformation, by which the spiralling path curves of the bud take hold of, and transform, the forms of the vortex (which do not

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* *The Spiritual Beings in the Heavenly Bodies and in the Kingdoms of Nature,* p. 171ff.

spiral but are always striving towards the upright of the central axis) we have found a mathematical expression of at least something which is working in this 'marriage.'

The field of form

I have chosen the name carefully — field of form — because I think we should be in serious error if we were to confuse this with being anything like the physicist's field of force. Here we are dealing with form only — just pure form. A field of force is anchored in space; it has position and size; we can point, and say, just here the force has such-and-such a direction and such-and-such magnitude, and here at some other point it is different. I think the field of form must be envisaged more like the medieval philosopher's IDEA; there is only one idea 'rose,' but it can manifest in a thousand different places and times. Would it make sense to ask of the idea, where it is, or how big it is, or even in what direction it faces? Surely, in a way, it fills the cosmos. But high in importance in its list of qualities will be its form; this is one of the foremost ways in which we recognize a rose when we see one. I believe that something is at work in the plant that has the qualities of pure form, divested of any spatial limitations (apart of course from form itself); that this can be brought to visible manifestation by the organism of the plant, when it immediately and necessarily accepts limitations of place and size; and I dare to hope that in the field of form this same entity comes to expression in the mind of the thinker, whereupon on his drawing-board it takes visible form, but again, and this time by mathematical necessity, limited by place and size. But when one compares these two things, the plant reality, and the mathematical drawing, they are found to be identical in form, because this is the thing which gave birth to them both.

Watery and airy vortex

Now we must deal with the reservation which I referred to earlier in this chapter. We must be clear that the form of vortex pictured in Figure 78 is not the only kind possible. Such a vortex has a λ which is less than -1 and on transformation by the pivot transformation this always gives us a seed-chamber form which stretches downwards from the bottom of the bud, what a botanist calls an inferior ovary. But it is perfectly possible to

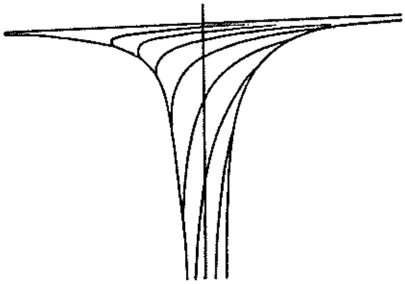

Figure 82. The watery vortex, in this case with $\lambda = -2.7$.

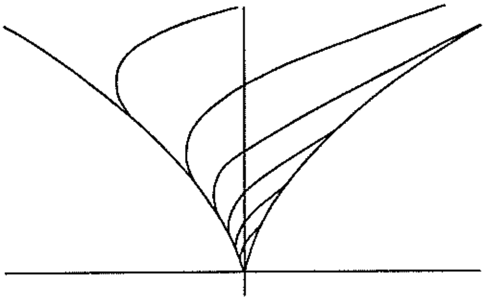

Figure 83. The airy vortex, with $\lambda = -0.4$.

envisage a vortex whose λ, while remaining negative, is greater than -1, that is, somewhere between -1 and zero.

These two vortices are so important in the further development of this work that we illustrate examples of each in Figures 82 and 83. In making these general drawings ε has been given a finite value, so they are covered with spiralling curves, but it need not necessarily be so. It has been pointed out to me by a friend that these two types of spiral give one a very different qualitative feel; the first, where $\lambda_c < -1$, could be a picture of water disappearing down a hole, or alternatively water welling up, spreading out, and finding its level. It has a quiet, tranquillizing effect. The second, where $-1 < \lambda_c < 0$, has nothing of this tranquil feel; it is cyclonic in appearance; it gives one an airy feel. Now I felt that this very unmathematical observation is also in some sense a significant one; and since then I have always thought and spoken of them as the watery, and the airy, vortices. But I do not mean by this to suggest that I am trying to link them in any technical sense with the ancient Greek elements of these names. I am using them simply as convenient, and descriptive, names for these forms.

Now it is an interesting fact that the airy vortex, placed with its centre somewhere within the bud, gives superior seed-chamber forms, growing up into the interior of the bud, like the superior ovaries. It can be treated for a *b*-λ chart, using the same equations as are given in this article; the method is similar but has one necessary difference, which makes the working much more tedious, and less neat in the way it comes out. And the first plant I tried with it, the hypericum, came out with a good meeting of the three curves, and a near-perfect fit for the form of the ovary. I thought that I had the secret: airy vortex for the superior case, and watery for the inferior. However, as the work continued I met many cases where the three curves stubbornly refused to meet. The matter became so depressing that I decided to have a review of everything I had done. I found that, whereas *every* case of the inferior ovaries, using the watery vortex, had worked perfectly (I had not at that time done the wild iris), a great many of the superior ones, using the airy vortex, had no possible vortex which would fit the observed form. It became obvious that the airy vortex could not any longer be considered as a generator for these ovary forms. Figure 84 is the *b*-λ chart for the geranium, typical of many others like it.

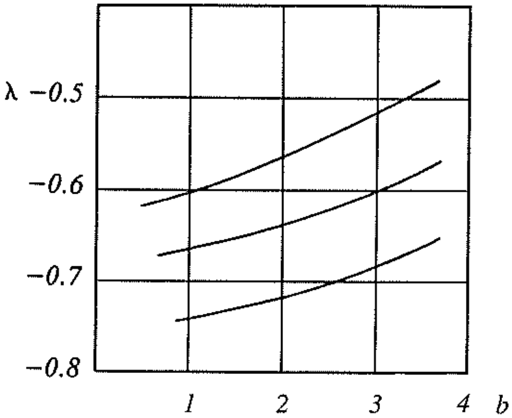

Figure 84. The b-λ *chart for the Geranium ovary.*

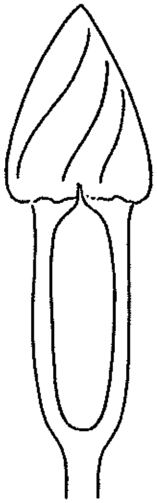

Figure 85. The Red Campion bud, with ovary.

This was disappointing but it did give point to the fact that there is no mathematical necessity for these curves to meet, and that it really is remarkable that the watery vortex should be so consistently successful in the inferior cases.

While casting around for what could be the solution of this problem, I tried one day, rather idly, inverting one of the superior ovary-forms, and letting it reach downward from the lower pole of the bud. Using the watery vortex, I immediately got a b-λ chart with a good meeting of the three curves, and subsequently an ovary-form with a near-perfect fit. This was all right for one case, but there were nineteen others. One by one I tried them, and found that they all gave perfect or near-perfect results, except two, and even these had good meeting points, but with the centre of the vortex very slightly above the top of the bud.

Now what are we to say about this? That the field of form includes its mirror reflection in the bottom plane of the bud transformation? Or that the field shows the *form* that the swelling of the stalk (which becomes the ovaries) will take, regardless of its orientation, up or down? Can we find some justification for this latter view, in Rudolf Steiner's words already quoted (see p. 166), where he is speaking of the sun forces working in the stalk and he says: 'Thus we have to connect the whole plant world with that which grows towards the sun or towards the centre of the earth . . .' as though the vital thing here is the orientation of the line between sun and earth, rather than the direction, up or down, taken along it?

When one deals with individual plants one comes upon anomalous situations, for instance the campion (Figure 85). This bud consists of two parts, a long, tubular, neutrally-coloured part, surmounted by a coloured spiralling part. The ovary is contained within the tubular part, and I suppose a botanist would say that it is superior. The top spiralling part is a good path curve and its lower pole is at the top of the ovary case. Is it this, or should we regard it as a superior ovary which has somehow slipped down, or the bottom pole of whose bud has somehow grown above it? I have consulted several botanists, but none would venture an authoritative answer in view of the geometrical considerations. And this is not the only case where one finds it had to decide whether the ovary is growing downwards or upwards. Another for instance, is the primrose, and in Australia I met cases where the ovary seems to be almost bisected by what seemed like the lower plane of the bud (but then the eucalypts are strange things). No-one would say whether it was superior or inferior.

I don't think the last word has been said on this matter; but this I know: that in over thirty cases the pivot transformation of the watery vortex has given exactly the form that the swelling of the stem will take in forming the ovaries or the ovary-bearing part of the plant. And the airy vortex, which we have had to reject in our studies of the plant, can be shown in further work to have a remarkable bearing on many aspects of animal and human embryonic development.

I close with a list of the λ-values for the vortices of the plants which I have calculated by the foregoing methods. The probable error for these figures must be considered as rather larger than that for the bud-λ and it must vary considerably from bud to bud, according to the size of the ovaries (sometimes very small), the ease with which they could be measured, and the reliability of the ε-value found for the bud, also a variable matter. The probable error ranges from about ±0.1 to about ±0.4 in the hardest cases. It is interesting to note the complete absence of λ in the range –1 to –2, and the great concentration between –2.6 and –2.8.

It should be remembered that the λ-value is not just determined by the shape of the ovary, but by this shape in combination with the bud-parameters. Thus the buds, for instance, included in the range –2.6 to –2.8 have, many of them, ovaries of quite different shape from one another.

Plant	λ_c-value
Syringa	–2.0
Hypericum	–2.05
Potentilla	–2.2
Geranium	–2.2
Convolvulus	–2.25
Wild Cherry	–2.4
Bluebell	–2.4
Stitchwort	–2.5
Sibirica	–2.5
Chickweed	–2.6
Forsythia	–2.7
Snowdrop	–2.7
Daffodil	–2.7
Wild Rose	–2.7
Primrose	–2.7
Foxglove	–2.8
Buttercup	–2.9
Wood Sorrel	–3.0
Narcissus	–3.1
Campanula	–3.2
Lady's Smock	–3.4
Red Campion	–3.6
Wild Iris	–3.6
Honesty	–3.6
Aubretia	–3.75
Speedwell	–3.8
Summer Snow	–3.9
Winter Jasmine	–3.9
Star of Bethlehem	–4.3
Wild Garlic	–4.8
White Campion	–5.0

11. The Watery Vortex

In the previous chapter I showed how a planewise path-curve vortex, working within what I there called the cosmic tetrahedron (consisting of the two absolute imaginary points *I* and *J* in the horizontal line at infinity, and two real points, one at the centre and the other at infinity along the vertical line), when transformed by the pivot transformation could give an almost perfect pointwise delineation of the form of the seed-bearing part of the plant. The type of vortex under consideration I named, for purely qualitative and imaginative reasons, the watery vortex. On a number of occasions, when speaking about this, I have been asked whether this is in fact the same type of vortex which water assumes when it runs down a plug-hole; and until recently I have always had to answer simply that I do not know.

However, after I had left Australia some years ago, my friend John Blackwood of Sydney had the initiative to try to answer this question. He designed and built an apparatus to generate and photograph vortices in water. It consists of a transparent-walled tank from which water can flow at a strictly controlled rate through a narrow pipe in the centre of its base, while a second pipe delivers water at the same rate into the top of the tank, so that the water level remains constant throughout the experiment. The resulting vortex can be photographed through the side of the tank.

As a result I received some really beautiful colour photos of three vortices, made by letting the water run out at three different speeds. At first glance the three vortices seem very differently proportioned, that made by letting the water run out at a high rate being long and thin, and that which had the water running out slowly being short and dumpy in form. The outline of the vortex was in each case somewhat uneven, almost rope-like in appearance, but I thought one could confidently draw a smooth curve through the slightly crinkled edge to arrive at the basic form which the vortex had assumed.

One of the first, and often greatest, difficulties which one encounters when one wishes to analyse a form of this kind is that of fixing with certainty the top and bottom poles of the transformation, that is of identifying the real part of the invariant tetrahedron. In this case I thought there would be no trouble; the photos showed the water level and also

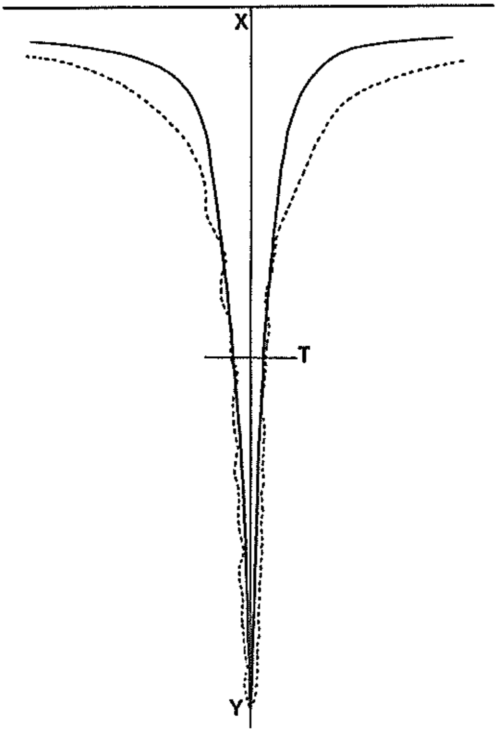

Figure 86. If we let the lower invariant point of our tetrahedron be at the visible bottom point of the vortex, then it is not possible to find a path curve form which will be a good fit all along the length of the vortex.

the bottom tip of the vortex with wonderful clarity. In the first instance at any rate there seemed no excuse for doing other than placing the top pole X in the centre of the vortex at water level, and the lower pole Y at the bottom tip of the vortex. It was then easy to divide the distance from X to Y into the usual eight equal parts, and to calculate the λ-values for the six levels, A, B, C, D, E and F. Doing this, however, I was letting myself in for a disappointment. Working with the largest of the three vortices the λ-values came out thus:

Level	λ-value
A	+0.43
B	+0.115
C	–0.07
D	–0.17
E	–0.32
F	–0.37
Mean λ (weighted)	–0.03
Mean λ-deviation	787%

The alarming figure for the mean deviation need not startle us too much. Obviously, to express it as a percentage of so small a λ-number is no valid way of carrying on; just as obviously however, when we look at the figures themselves, it is clear that their deviation is unacceptably large. This is not a good path curve form; not, at any rate, within this tetrahedron. Measurements for the two smaller vortices gave similar results.

It is interesting to see what these deviations imply in terms of the actual form. Figure 86 shows, with the dotted curve, the tracing of the actual outline of the vortex from the photograph, and with the continuous curve, the mathematical form calculated so that it should be a good fit at the level *T*, halfway between the poles *X* and *Y*. It will be seen that the mathematical form is distinctly too narrow both above and below this level. Of course by calculating the mathematical form for a much wider radius one could contrive to get a rather better fit at the top end of the diagram, but only at the expense of having it much too wide in the middle part. In fact it is clear that the mathematical form is just the wrong shape, being much too flat-sided.

There remained however another possibility: that the lower pole *Y* should be at infinity, that the ideal form should be infinitely long, and that the actual water vortex should be seeking to reach this form as far as the limitations of substance rendered it possible. At any rate it seemed necessary to find how closely, or otherwise, the actual form approximated to such an ideal one.

The method of working had to be somewhat different since obviously one cannot divide the infinite distance between *X* and *Y* into eight equal parts! The details of the revised method are given in Appendix 3.

Applying this idea to our largest vortex we immediately get much more satisfactory results:

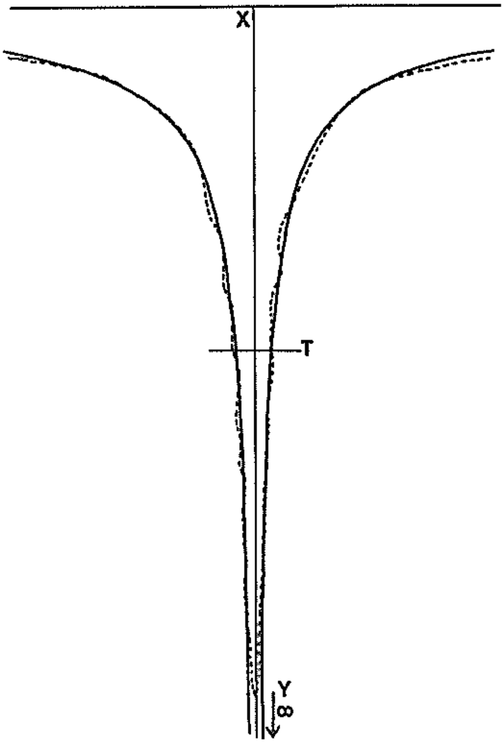

Figure 87. As soon as we let our tetrahedron be the same as the one which we used for the plant ovaries, that is with point Y at infinity, then we get an almost perfect fit with the form of the vortex. The dotted curve is the tracing from the photograph.

Level	λ-value
A	–1.66
B	–1.70
C	–2.00
D	–1.82
E	–1.71
F	–1.82
Mean λ (weighted)	–1.75
Mean λ-deviation	5.4%

The results for the two smaller vortices were λ = –1.71 with mean deviation 4.0%, and λ = –1.74 with mean deviation 5.5%. Figure 87 shows how close a fit these figures imply.

Thus we see that the vortex which seems to play such an important role in the reproduction of plants, is in fact of exactly the same type as that assumed by water, at any rate to within the limits of the finest measurements I have been able to make. And the fact that these three vortices, whose photographs John Blackwood sent me, and which had such different apparent proportions, yield almost identical projective parameters (meaning that the different vortices for different rates of flow are different contours of essentially one and the same field of form) gives confidence that this projective approach is a significant one.

Later John Blackwood photographed and measured three further examples, with substantially the same results. He wrote to me:

> If you had seen the vortex actually running as, of course, I did you would probably have jumped straight for the cosmic tetrahedron! It is a wonderfully sensitive form, like a mobile jewel . . . The smallest change in flow-rate alters the depth where the 'molecular' or 'surface-tension' effects supervene.

12. From Seeds to Embryos

The next stage of the work came, it would seem, almost by accident, and quite unexpectedly to me. On page 77 it was pointed out that not *all* types of bud are amenable to fitting by a straightforward path curve form; what further things may be working in these more sophisticated forms awaits future research. Until now I have confined myself to those species which are symmetrical about a central axis, and in such cases the symmetry almost always extends to the ovary and seed formation also. Thus it was a very natural thing to envisage the watery vortex as having its central axis coincident with that of the bud itself; and this proceeding has accounted satisfactorily for all the cases which have been studied. This left an unanswered question: What kind of form would be produced if the axis of the vortex were to be asymmetric to that of the transforming bud form?

This question lay for quite a long time unanswered. I did not think it was a matter of great importance; I felt it was a question of intellectual curiosity and I was too busy with more important matters to spend much time on it. However this 'tag-end' of a question began to nag at me and the time came when I felt that I must try to find an answer.

I therefore selected six vortices from a field of them, and set their central axis parallel to, but to one side of, that of the transforming bud-form. This was just prior to the time when I came to realize that the watery vortex was the only one which was giving me effective results, and it just happened that I decided to work with an airy vortex field. The situation I was envisaging is pictured in Figure 88. The dotted curve represents the transforming bud form. This transformation also contains a whole field of forms, of course, but since as a transforming agent it is working as a whole thing, it has only been represented by a single one of its forms. The vortices on the other hand, although all belonging to a single field, are going to be transformed one by one, so they have been drawn in separately.

Now the question was this: Each of the vortices must be imagined as being made up of all its tangent planes; when these planes are taken up by the transforming bud field, and made to start moving, around which of its points would each plane be pivoting? We would start with six planewise forms and end with six pointwise ones.

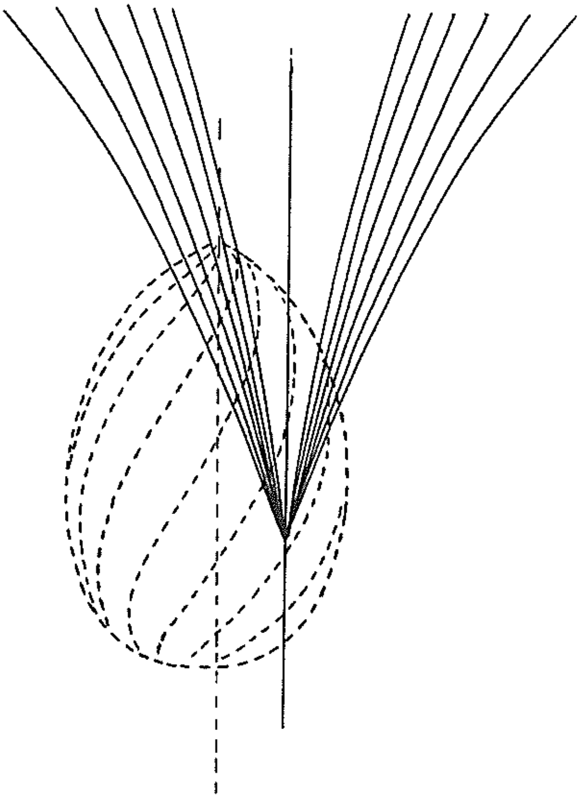

Figure 88. This is the situation which was envisaged for that first experiment to see what would happen when some asymmetrically placed vortices are subjected to the pivot transformation.

Right at the start it was obvious that the asymmetry would make the problem far more difficult and complicated than had been the case with the ovary forms which have just been described. With them the resulting form was a solid of rotation. All one had to do was to calculate its outline in one of the planes of the central axis, and then in imagination set this outline in rotation about that axis. Every horizontal section of the resulting form would be a circle. With the asymmetric case it was clear that the horizontal sections would not be circular, and furthermore their shape would vary as one passed up the length of the form. The only way to proceed was to construct the horizontal cross-section at a low level of the form, and then do the same for successively higher levels; then one would be able to put these contours together in one's imagination to obtain a picture of the whole form.

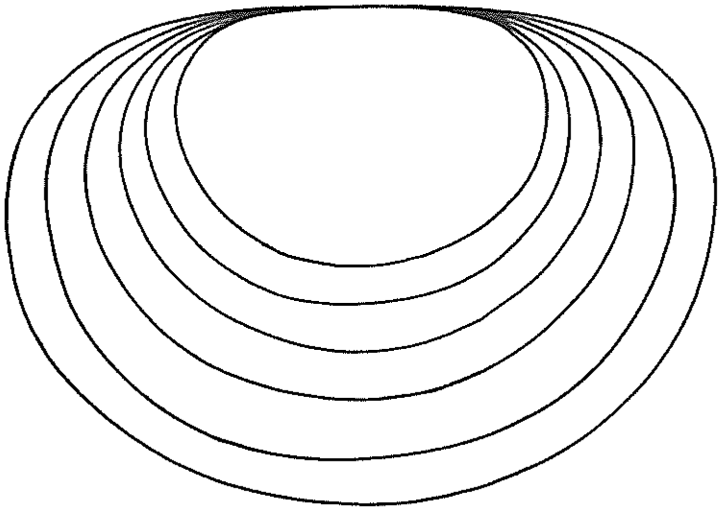

Figure 89. We let the forms resulting from the experiment of Figure 88 be cut by a horizontal plane at a fairly low level in the forms. Their cross-sections become a series of flat-sided ovals.

Since there were six vortices being transformed, each cross-section would contain six forms. The method which was devised for doing this is shown in Appendix 5; here we just show the result. Figure 89 shows the horizontal cross-section at a low level of the resulting forms. The inside oval results from the narrowest of the vortices, and the others in order outwards. These ovals are not ellipses, being too flat on the side which faces towards the top of the page. We notice that one axis of symmetry has been retained, that which gives balance from left to right. In the sequel it became clear that this is because the axis of the vortices, although asymmetric to that of the transforming bud, is parallel, and therefore co-planar, with it.

The next step was to move the level of the cutting plane a short way up the height of the form I was constructing, and see what the new cross-sections would look like. As I worked I had no idea what the result would be. In the event the new forms proved to be as illustrated in Figure 90. What had been the flat 'top' to the ovals began to heap up into two bulges on either side of the central axis, the heaping being largest in the form arising from the largest vortex.

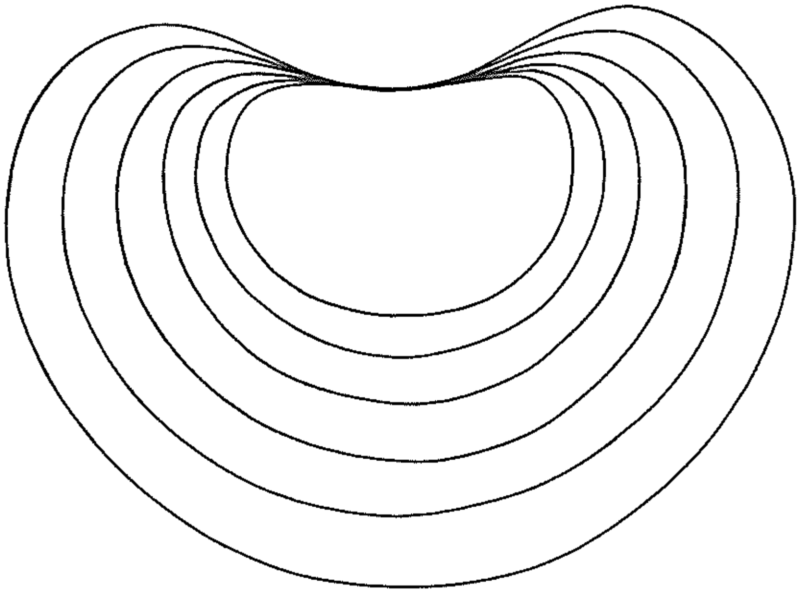

Figure 90. We raise the level of the cutting plane a little, and the cross-sections begin to bulge slightly on either side of their axis of symmetry.

By moving the cutting plane a step further up the height of the bud, this process was accentuated as shown in Figure 91.

The next stage gave more trouble in the construction. After I had sorted the matter out I found that the bulges in the outer form had, in the meantime, become so large that they had met one another at the top, had there fused to form the largest enclosing form, and at the same time dropped a little 'egg' into the middle of the form, as shown in Figure 92. At this stage one can see that the next form is shortly about to do a similar thing.

Figure 93 shows a further stage in the process. Three of the forms have gone through this 'egg' producing metamorphosis, while three more are still waiting their turn to do so.

Now this whole remarkable metamorphosis came as a surprise to me. At this time I did not know much about embryology, neither had I expected this area of inexperience to be broached; but I did know enough to recognize the general gesture of invagination when I met it. And it seemed to me a matter of significance that the process which I had been led to by considering the formation of the seeds and ovaries should now,

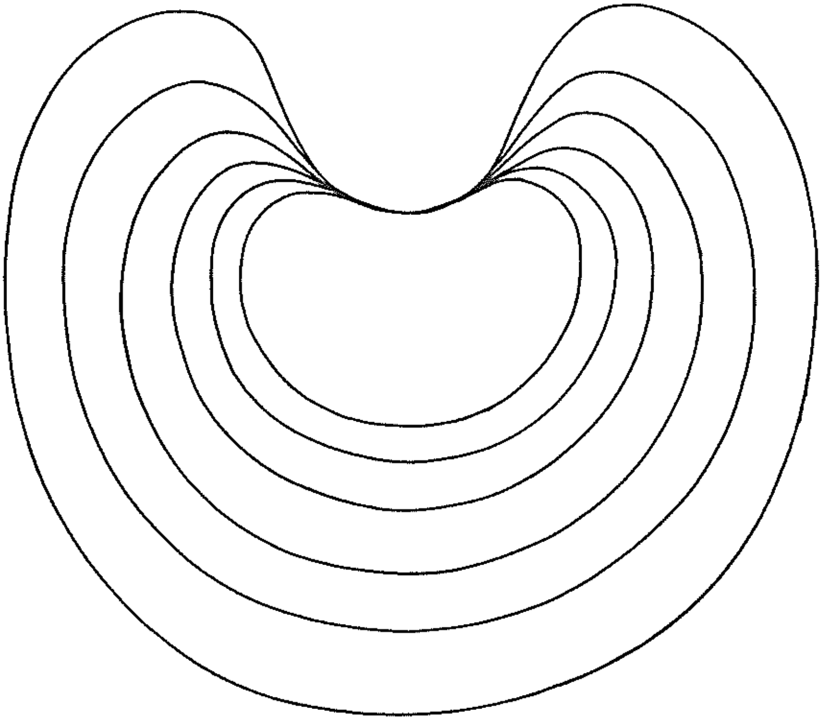

Figure 91. As the cutting plane is raised further the bulging becomes more pronounced.

with these small changes, be presenting me with embryological forms. And with this my attention was turned very keenly to this field.

Notice that the 'egg' dropped by the outermost of the six forms becomes, in the final result, the innermost, surrounded in turn by the 'eggs' dropped by successively smaller forms. The figure undergoes an inversion, outside into inside.

The next question to ask was: What happens if we do away with the last trace of symmetry in the generating forms? We let the axis of the vortex be not parallel with that of the transforming bud transformation, but skew to it, passing it by with complete asymmetry. In such a case the cross-section we are constructing loses all trace of symmetry; one bulge increases in size while the other decreases; the form becomes more suggestive still of embryonic gesture, a clear indication of 'head' and 'tail' appearing.

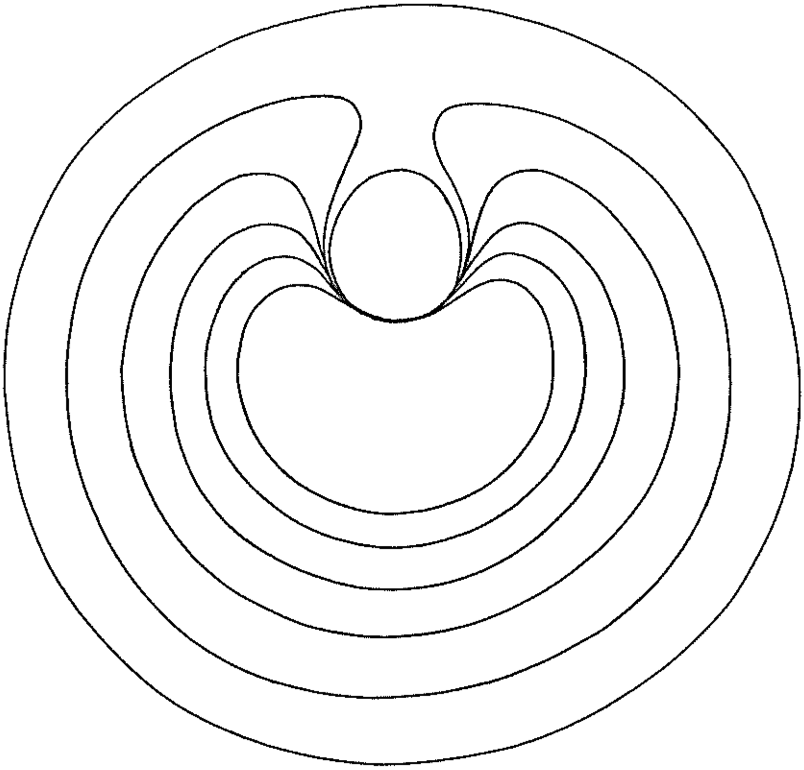

Figure 92. A further rise in the cutting plane produces a critical situation: the largest bulges join at the top, dropping a little egg-like form into the middle.

At this point I set myself to rove freely for a time in this new realm of form. I would set a vortex axis in some position freely chosen with regard to that of the transforming bud, then move it in some methodical way, and watch what metamorphosis of forms was produced in the cross-section form. From such an exercise I discovered that this embryo-like form is in such circumstances fairly ubiquitous; in one variant or another it appears again and again.

For instance, one day I set a vortex with its axis cutting asymmetrically through the bud transformation, kept its apex fixed, and then allowed its axis to move in a vertical plane of that apex, making a 'cut' right through the bud transformation. I let it move in six stages and constructed the resulting cross-sections in a horizontal plane of the apex point, that is, one which is at right angles to the axis of the bud transformation. Figure 94 shows the resulting six forms, passing through shapes

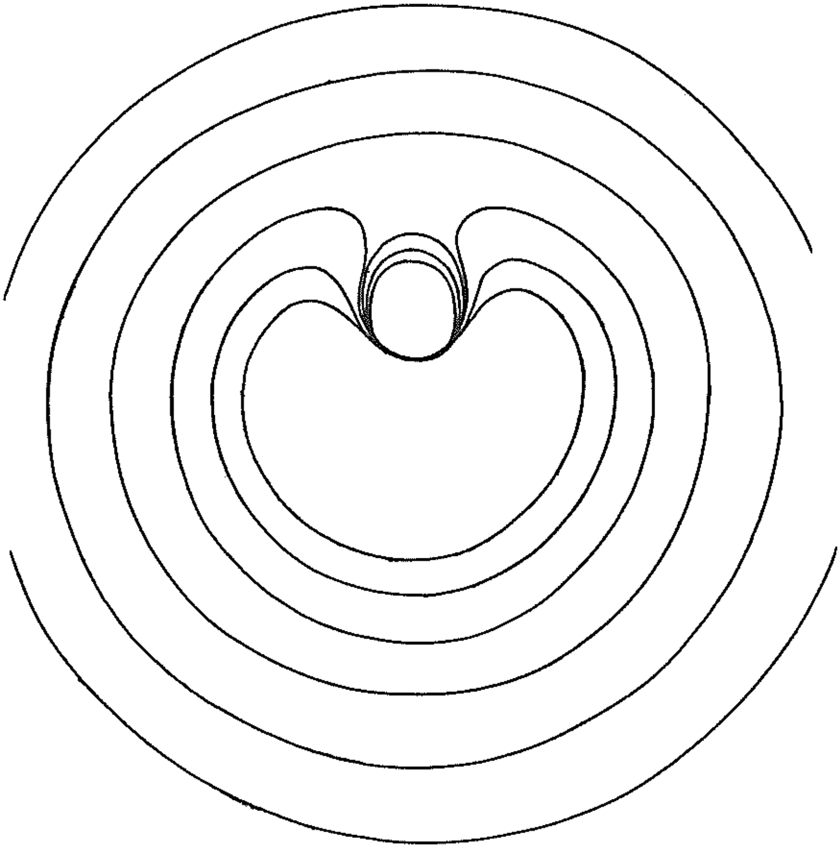

Figure 93. The process pictured in the previous figures continues for vortex after vortex as the level of the cutting plane continues to rise.

that are strangely reminiscent of embryonic development, until at the last stage the 'head' and 'tail' link to tie into a queer little knot. If one carries the process further this knot becomes more and more elaborate and the form as a whole less and less like what one normally associates with an embryonic gesture, but even there one can still trace those parts of the form which have metamorphosed out of the more recognizable features of the ordinary embryonic gesture; it becomes like a deeply involuted embryo form.

This is only one of many experiments which I did at that time. One can do various things. One can keep the apex of the vortex fixed and allow the axis to move stage by stage round a cone of that apex. The cone can have all kinds of orientations and vertex angles. Again and again one finds metamorphoses similar in general character to that of Figure 94, but different of course in many details.

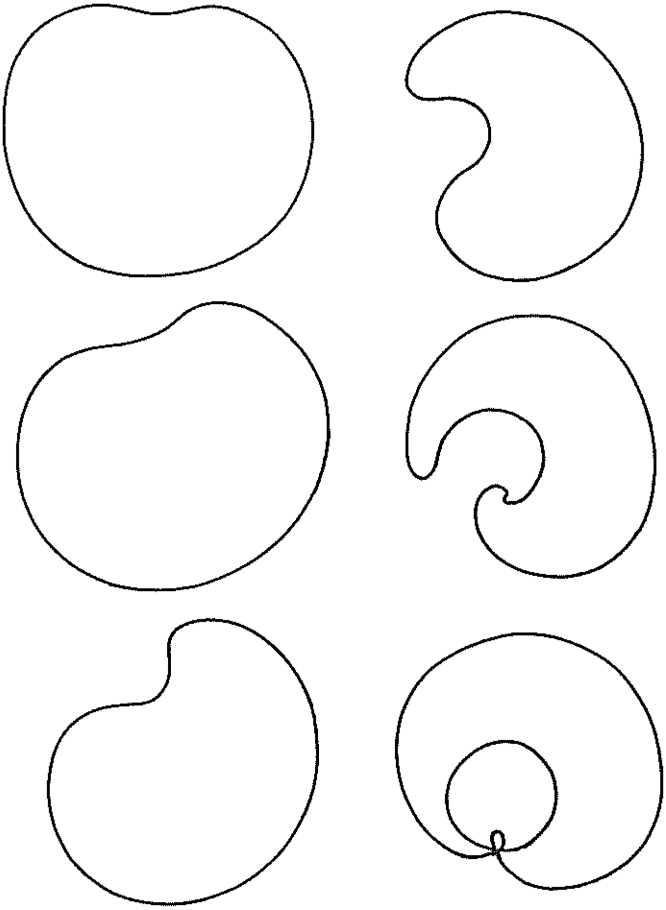

Figure 94. This shows the general kind of metamorphosis which is obtained when one allows the axis of an airy vortex to move in some consistent way through the field of the transforming egg-transformation.

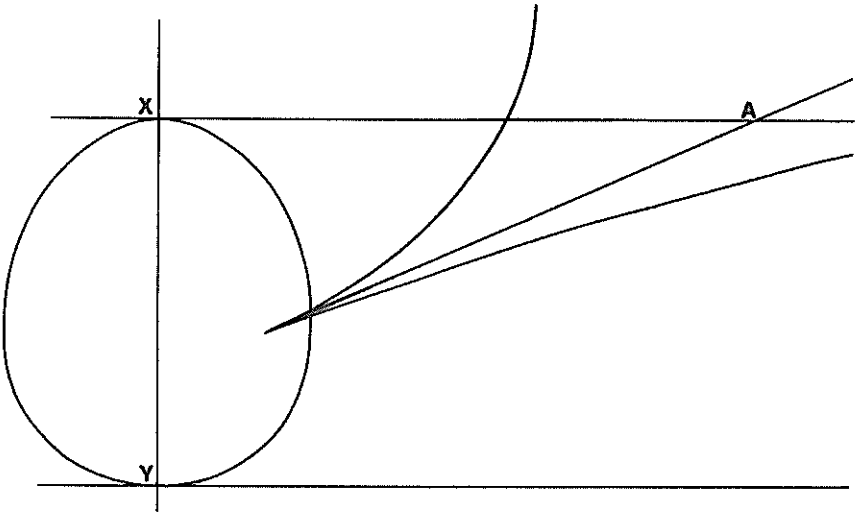

Figure 95. The general situation which we can envisage for the formation of the chick embryo.

All this led me to take my work on the eggs, some years previously, much more seriously than I had then done. Clearly we are moving into a realm here, where the activity of the bud transformation, as transforming agent, is to be taken over by that of the egg. The general situation is pictured in Figure 95. Here we have the egg, and within it the fertilized germ, lying somewhere on the border between albumen (the white) and yolk. To begin with we are not seeking to 'prove' anything tremendously dramatic, but rather to ask and, if possible answer, a significant question: Would it be possible to find some path-curve vortex which with some *simple* change or movement, when taken up and transformed by the egg transformation, would portray for us something of the general gesture which the evolving chick is going to make within that egg? And if so, what kind of change would have to be envisaged in the vortex?

We remember from Chapter 4 that these path-curve forms, although they appear within a certain infinite variability, nevertheless occur within strictly defined limitations. The things which they cannot do are more numerous than those which they can. It was by no means a foregone conclusion that the answer to the first of these questions would be in the affirmative. And before an answer could be even attempted there was another formidable problem to be overcome.

Anyone who has worked with the details of the pivot transformation, as given in Appendix 5, must be well aware that these constructions cannot be made unless one has full knowledge of the parameters of the transforming bud-, or as the case may be, egg-transformation which is involved. This means determining not only λ but also ε, and the latter has to be found by measuring the steepness of the spirals. But there are in the ordinary way no spirals to be seen on the surface of the egg!* This problem held the work up for a considerable time and further progress was not possible until I had learnt more of the biology of hens and of egg-laying.

When a hen is dissected, one comes across the ovaries, looking like a brightly coloured bunch of yellow grapes, varying in size from about a small pea to that of a golf ball. These are the ova, each of which is due, in the course of time, to become the yolk of an egg. When an ovum is ready to leave the ovary it is already a fully grown and constituted yolk and little further development is needed to it. Connecting the ovaries with the uterus, which is the chamber from which the egg will ultimately be laid, is a long tubular organ called the oviduct. The oviduct is coiled closely upon itself and held by a membrane called the messentery. If we remove an oviduct and straighten it out it reveals itself as a tube, some 12 to 15 inches (30 to 40 cm) long. When an ovum is completely developed some little hairs, or cilia, at the top of the oviduct reach out, grip the ovum, and draw it down into the oviduct to start its journey downwards towards the uterus.

Now the inner wall of the oviduct is thickly covered by mucous glands, and these secrete albumen (the white of the egg) around the yolk as it passes by. When the egg reaches a lower part of the oviduct, called the isthmus, the shell membranes are laid down upon it, and lower still, in the uterus, it receives the calcium, which then hardens into the shell as it is laid.

The mucous glands which secrete the albumen, and which line a very large part of the oviduct, are all situated in spiral curves, and it seems very probable that the egg twists slowly in these spirals as it makes its downward journey. Once, when dissecting a hen, we found an egg halfway down the oviduct, and I was able to photograph it in this critical phase of its development. Although the spirally placed glands are situated on the inner wall of the oviduct, the position and direction of the spirals were clearly visible from without. Figure 96 is a diagrammatic tracing from this photograph. We see the bulge in the wall of the oviduct caused by the presence of the rapidly growing egg, and we cannot help

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* This was before I knew about the spirals pictured in Figure 30.

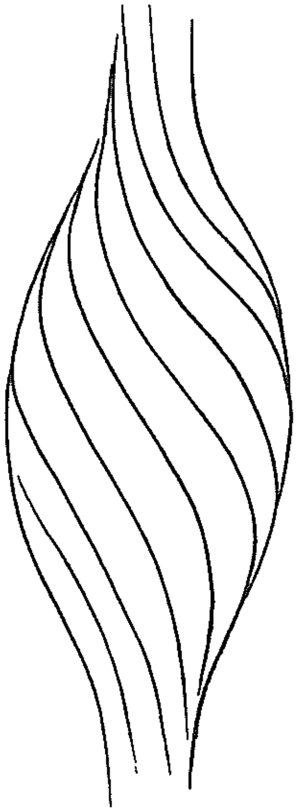


Figure 96. A tracing from the photograph of the partially-formed egg part of the way down the oviduct.

being struck by the resemblance of the whole thing to the egg path-curve form which is now so familiar to us from our studies.

I measured and analysed this, and found that for a very large part of its length this is an almost perfect path-curve form. I could find no perceptible sharpening or blunting at either end: $\lambda = 1$; and the value for ε also came out to be just about unity. From the study of the curves in other oviducts it seems probable that this value may vary in individual cases but is usually somewhere between 0.7 and 1.3. Unity seemed as good a value as one could get, and this parameter was used in the subsequent calculations. I then made the construction of the purely mathematical form, using unity for these two parameters, and this is shown by the grey shading of Figure 97 with the tracing from the photograph shown in black lines over it. We see that the fit is almost perfect, apart from

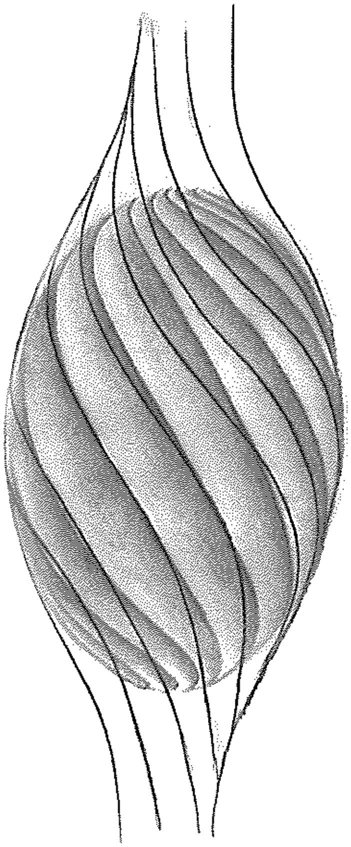


Figure 97. This shows Figure 96 compared with the nearest path curve equivalent which could be found.

the small regions close to the poles of the form. These are the places where the mathematics runs to infinity; substance could not in any case follow it there, as we have seen, both in the case of the buds and the ventricles.

We see here, I believe, in its true genesis, the egg cradled and nurtured in a little path-curve cosmos of form. Let us be quite clear that it is form, not substance, which is important here; as the egg moves on its way the substance around it changes from moment to moment, but the form remains, largely at any rate, invariant, for the whole of its journey. And this form, I believe, is the true active element in its further development. What later appears in the shape of the shell is a last frozen manifestation of the more living processes which went before it. So it was with these parameters from the oviduct that I went on to do my further work.

And now we can return to the question of a few paragraphs back: Would it be possible that the general gesture of the developing embryo could be shown by some simple movement of a vortex, being transformed by the egg transformation which has just been described? Obviously the matter is one of immense complexity and difficulty, and we are surrounded by 'unknowns.' At the moment we are able to make only a preliminary and tentative study, just to explore *possibilities*. And we have to confine ourselves to drawing just the 'shadow' of gestures, as it were, on some horizontal cutting plane.

Where should the apex of the vortex be placed? The obvious place would be in the fructified germ, nestling there in the surface between white and yolk. And the cutting plane which shall receive our drawings could well be the horizontal plane (that is, at right angles to the long axis of the egg) through that point. This is the only real finite plane of the cosmic tetrahedron described on page 161; and this we shall use; it seems the obvious and 'right' one for our purpose.

The question we are asking ourselves is one that can never be answered on purely theoretic grounds. If we could answer it in such a way there would be no need to ask it! In the last resort we must go to nature and see what she says. The theory will enable us to construct pictures of the kinds of form possible. They cover a large range. Many scores of pictures had to be drawn. And as one's eye ranged over the field of possibilities thus brought to view, always comparing them with the things which nature actually shows us, gradually a structured form for the movement of the vortex axis began to reveal itself.

I found that if we let the apex of the vortex be in the fructified germ, a typical position for which we can take from the textbooks, and let the axis of the vortex move in part of a simple cone, while the radius of the vortex gradually decreases somewhat, then the cross-section in the horizontal plane through that point goes through a succession of forms very close to the general gesture of the developing embryo. Figure 98 shows this in elevation, the axis of the egg, and the point, G, of the germ, lying in the plane of our page. GM_1 is the first position of the axis of the vortex, also lying in the plane of our page, and cutting the top invariant plane of the egg transformation at M_1. Notice that this means that the axis of the vortex and that of the egg have a common point; they meet on the plane of our page, and, by what has gone before, this means that the first cross-section will have an axis of symmetry. Thereafter, while the axis of the vortex is still centred in point G, in its conical movement it moves away from the plane of our page, and all symmetry is lost in

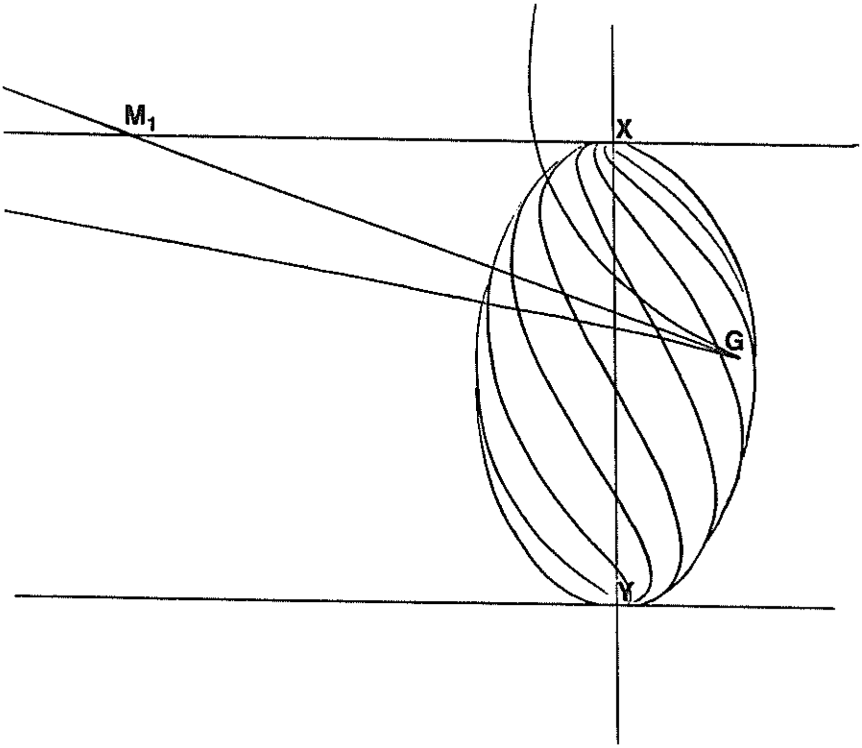


Figure 98. This is the situation we are led to, by comparing the geometrical possibilities with the actual phenomena displayed to us by nature.

the cross-sections. Line XM_1 represents the top invariant plane of the egg transformation, and in Figure 99 we see it in plan, that is, as seen from above. G is here the orthogonal projection of G on the top plane, M_1, M_2, M_3 and M_4 show successive positions of the points in which the vortex axis cuts the top plane, each surrounded by its circle showing the radius of the vortex at that moment.

In Figure 100, on the right are shown the cross-section forms which are produced when the vortices whose axes cut the top invariant plane in the points M are transformed by the egg transformation. These all lie in the horizontal plane of point G. Opposite them, on the left are shown sketches of the relevant stages in the development of the embryo, taken from an ordinary text book of biology. At stage 1 we see the embryo, wider at top and bottom, with a comparatively narrow 'waist' in between, where the earliest somites are beginning to show. In stage 2 we see the narrow waisted part of the geometry rising to the top of the form, and tying itself into a strange little knot there, making the first recognizable differentiation of 'head' and 'tail.' The thinly drawn part just below the 'head' marks the form of the incoming heart. Thereafter stages 3 and 4 show the continuous enlargement and development of the head as compared with that of the trunk.

I do not claim that this is anything more than a preliminary and incomplete approach to a subject of immense complexity and difficulty. It cannot be looked upon as anything but a first tentative and imperfect effort. A large amount of work was called for. It was done just before the advent of the personal computer; each drawing, of many scores, had to be made by hand; had I had the time to do much more I believe that more satisfactory results might have been possible.

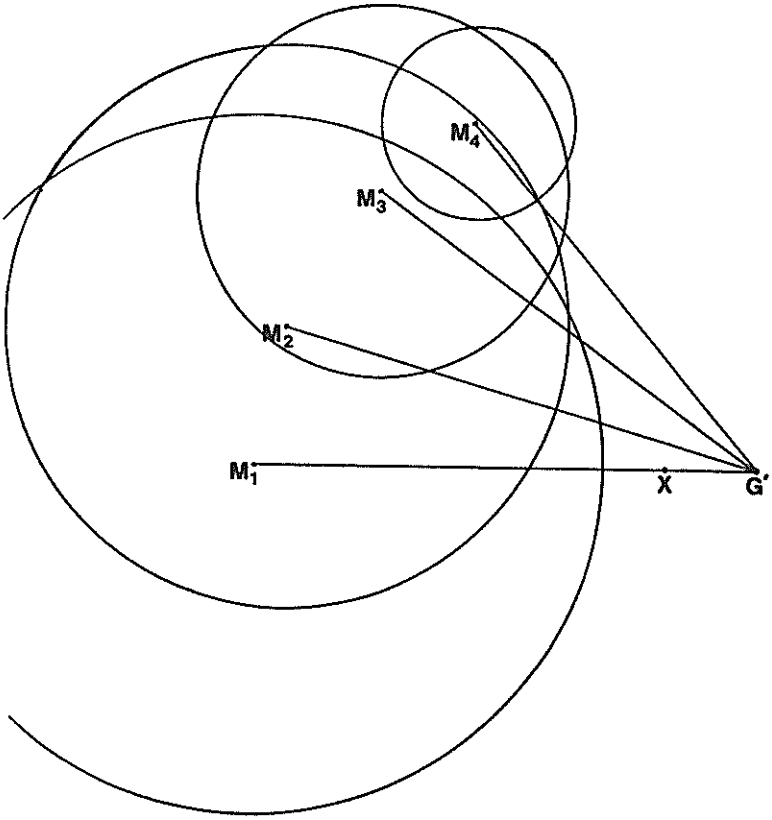


Figure 99. This is looking down, from above, on to the situation of Figure 98. It shows four successive positions of the vortex, where they cut the top invariant plane.

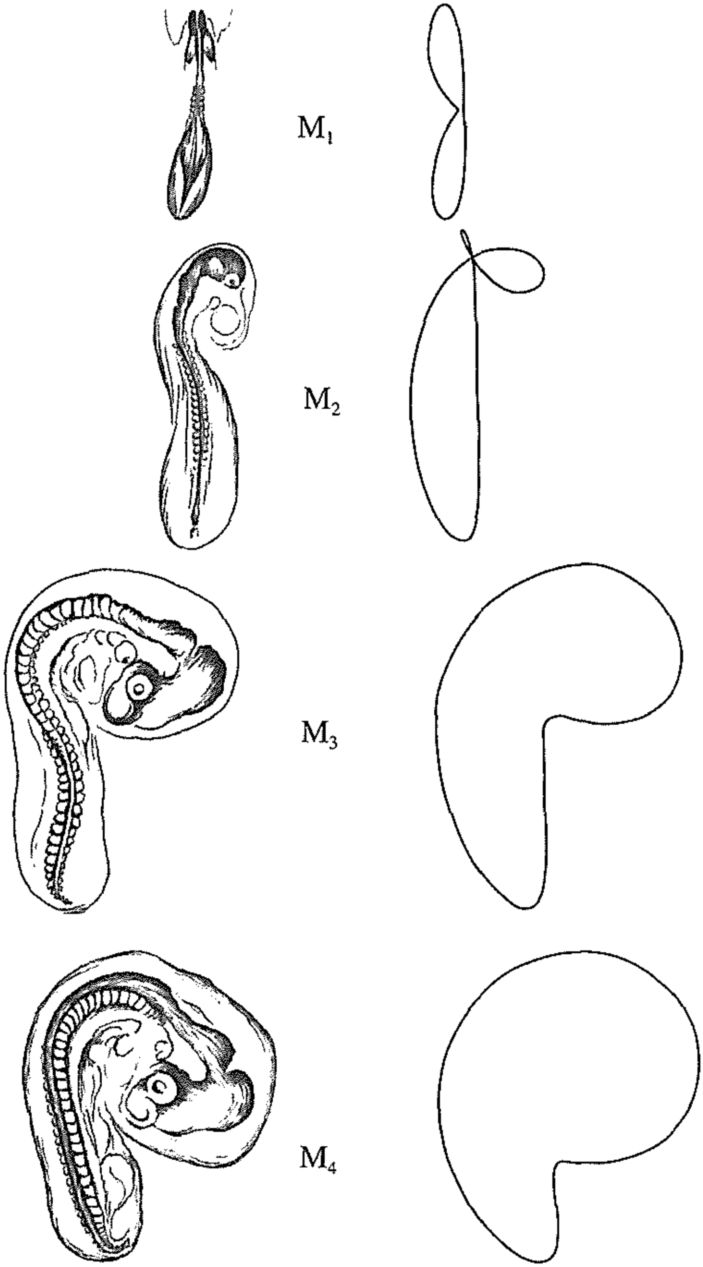


Figure 100. On the left: sketches of the relevant stages of the development of an embryo. On the right: cross-sections produced by transforming the vortices shown in Figure 99.

However the work left me with the conviction that the general gestures of embryonic development are indeed to be found by these transformation processes.

Next I was anxious to come to the human case and see whether similar things might be possible there. It seemed to me obvious at the time that the role taken by the bud in the plant and by the egg in the bird creation would here be taken by the uterus. So the first question was to see whether the uterus is a recognizable path curve form; and it was immediately obvious that in normal life it is manifestly not; it is a small shrunken T-shaped organ quite unlike any path-curve form which I know. However in the first weeks and months of pregnancy it swells out to a shape which is certainly reminiscent of the path curves. The difficulty was to get a really accurate picture of it, at such a time. Eventually I settled for a pencil sketch from one of the standard textbooks of the subject, made by a surgeon, as being probably the most careful and authentic; it is illustrated in Figure 101. Except that it is not closed at the bottom, the inner cavity certainly looks like a path curve. During pregnancy the lower end is filled by a plug of mucous, and here we have to 'dot in' the completion of the form as well as we are able; the measurement of level F must be treated with reserve.

It is highly improbable that the surgeon who made this drawing had any knowledge of projective geometry, yet he has in fact produced a very passable path curve form. On analysis it turns out to have a λ-value of 2.8 with MRD equal to 1.5%, a result comparable with many of the buds and the eggs.

However, when we come to consider the spiralling element things are not so easy. We have seen in the case of the heart how the twisting muscle fibres show very clearly the spiralling element of the path curves. In the uterus the muscle fibres also spiral, but not in any manner which I can find to be a path curve. It was not for want of trying! I spent many hours on the problem. If we wish to understand this I think we must look back to the embryonic development of the uterus itself. There we find that it takes its start from the fusion of two little tubes, and I believe that its final form, including the spiralling of the muscle fibres, can be explained from this genesis. But it is not a path curve. However, wonderfully, it produces a form which is capable of bearing the *imprint* of the path-curve form when this is needed — in the first weeks of pregnancy. Probably a somewhat similar thing is seen in the case of the heart. In its first embryonic appearance the heart is not a path-curve form at all; but it receives the stamp of this form very early on, while still in the embryo,

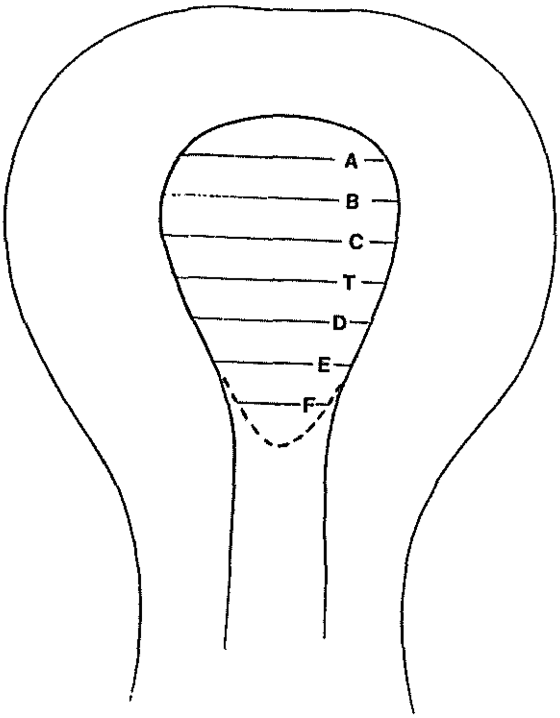


Figure 101. A surgeon's sketch of a uterus in the early months of pregnancy. On analysis the inner part of this reveals itself as quite an accurate path curve form.

and it retains it for the rest of life. In the uterus this happens much later — only when the organ is needed for use — and the change does not extend to the directions of the muscle fibres.

This leads to an important consideration. From what has gone before in this book it might seem that I am suggesting that the natures of the development of seeds and bird embryos are *caused* by the forms of bud and egg. Well, maybe — but not by the substantial nature of these organs. When I speak of the form of bud or egg I mean the active forces at work there. These are essentially invisible and non-sensible. The substance involved flows into the field of these forces and is deposited there and forms an accurate picture of these forces. Then I have something which I can see and measure.

I have said that it seems that during the first weeks of pregnancy this form is imprinted on the uterus. Whence does it come? I believe that it comes from the forces of the fertilized ovum itself. It is a notable fact that in the very first stage of its journey from the ovary into life the ovum

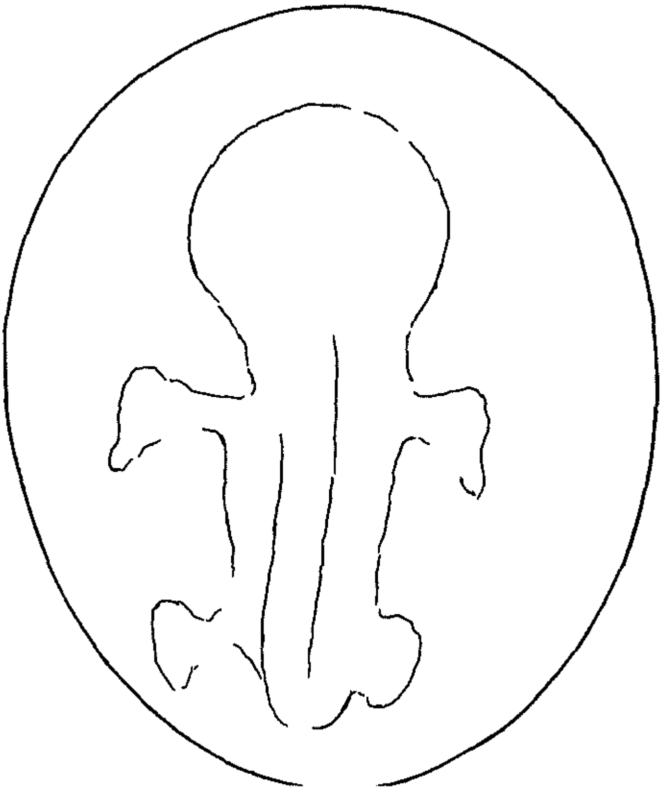


Figure 102. Tracing of a scan of a six-week embryo in its amniotic sac.

has to negotiate a hazard. Between the ovary and the top of the fallopian tube there is a gap, and this has to be safely crossed. Normally the hair-like cilia at the top of the tube draw the ovum safely down into the tube. But it can occasionally happen that the ovum fails to make this crossing and falls into the peritoneal tract of the mother; and it can *very* occasionally happen that in the course of this the ovum becomes impregnated. In such case it will implant itself in the wall of the peritoneum, and start developing there. Normally such a pregnancy is terminated fairly quickly, but it can happen, very rarely indeed, that it goes full term, and the child is delivered by Caesarian section not from its mother's womb but from her peritoneal cavity. This shows us that although the environment of the womb is highly desirable for the development of the embryo, it is not strictly essential. The form-producing forces seem to be carried by the impregnated ovum itself; when nature seeks to provide the ideal environment for the subsequent development, something of these form-producing forces is impressed into the shape of the uterus, and there we can see and measure it.

This must all be seen in connection with something else. From its very earliest day the embryo undergoes its development within a series of watery sheaths. There is the chorionic sac, within this the amniotic sac, and within that again the embryo itself. With the latest scanning techniques these can now be photographed with great accuracy and clarity, and the latest observations show that these sacs are in fact path-curve egg-forms of quite remarkable accuracy. Their λs are not the same as that of the uterus, being much more in line with those of the bird creation. We see the embryo, right from its inception, immersed in a highly complex path-curve environment, of which the actual form of the uterus is only one part. This path-curve form, which comes from the most elementary mathematical thoughts we can find, seems to be almost ubiquitously present right from the very start of life itself, manifesting wherever the fertilized ovum finds itself.

Figure 102 shows a tracing of a scan of a six-week embryo in its amniotic sac. The latter has a λ of 1.11 and is such a perfect path curve form that its MRD is only 0.3%. Figure 103 is a tracing of a scan of a seven-week embryo in its amniotic sac, and that, within the chorionic sac. The former has λ equal as nearly as measurement can make it to unity, again with the remarkable MRD of 0.3%. The chorion has λ equal to 1.16; it is not such a perfect mathematical form but with MRD equal to about 3.8% it is still recognizably path-curve in form.

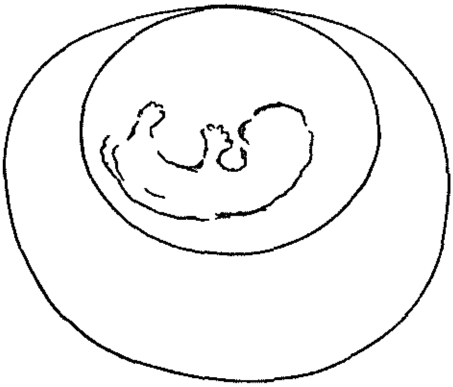


Figure 103. Tracing of a seven-week embryo in its chorionic and amniotic sacs.

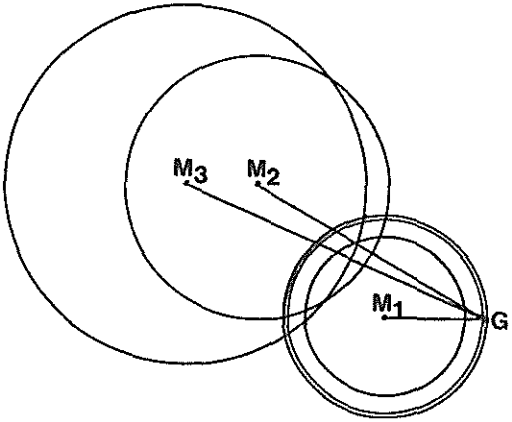


Figure 104. This shows the bottom invariant plane of the uterus transformation, and how the vortex needs to cut this, in order to generate the main gesture of the early, evolving, embryo.

This all goes to underline to us how deeply the beginnings of life are immersed in these path-curve fields but I have yet to find a place where the spiralling element manifests itself in a substantial measurable form. For the following calculations I therefore took unity as a likely value for ε, the same as for the hen's egg, and worked with that. If with further experience we find cause to change this value of ε, it would make changes of detail but not of the general quality of the final results.

The question now confronting us is the same as we faced with the hen's egg and the chick. Would it be possible to find a path-curve vortex such that, with its apex in the point of implantation of the fertilized ovum in the wall of the uterus, and with some simple movement, it would describe for us the general gesture of the developing human embryo? And the method of trying to answer this question must be the same. We will catch our cross-sections on the horizontal plane through *G*, the position of the ovum, and having made many scores of such drawings we will be able to make for ourselves a picture of the whole field of possible form involved. Then by comparing this with the actual forms which the embryo goes through we can arrive at a suggestion of an answer. And having done this the answer came out similarly with that of the chick, but with some notable differences.

Firstly it became clear that we must envisage the vortex striking upwards from below, and that its axis must then turn in part of a simple cone centred on point *G*. However it must turn in the opposite direction from the case of the chick and while doing this must slowly increase in radius rather than decrease. Figure 104 pictures the situation, as seen in plan, from above. The double circle represents the widest part of the uterus, at approximately level *B* of Figure 101. This is a typical point of implantation for the ovum, so we have put point *G* there, in the wall of the uterus. The rest of the diagram must be visualized as being on the horizontal plane through the pointed end of the uterus, below level *F* of Figure 101. M_1, M_2 and M_3 show successive points in which the vortex axis will cut this bottom invariant plane of the uterus transformation, and the circles around them show the cross-sections which the vortex will make with the invariant plane as it increases its radius. Note that the first position of the vortex, needed to show the form of the very early embryo, will have its axis passing through, or at any rate very close to, the pointed end of the uterus itself.

Figure 105 shows, on the right, the cross-sections which result from this simple change of the vortex, and on the left are some tracings of the relevant stages from a textbook of embryology. In the course of this both the actual embryo and the geometrical cross-section are growing rapidly, but for ease of comparison they have all been drawn to the same size. I think it will be agreed that the similarity of the geometry to the biological reality is very striking, perhaps even more so than in the case of the chick.

On the left, at M_1 we see a drawing of a very early embryo, sketched with its amniotic sac; at M_2 we have a 24-somite embryo, at about 28 days, and at M_3 we see a tracing from a photograph of a 37-day embryo.

It must be stressed that by the very nature of things a diagram such as that shown in Figure 104 has to be considered as approximate. When one has many fairly similar forms to compare, it must be a matter of judgment which particular ones are considered to be the best fit. By slightly changing the λ of the vortex, or its radius, one could arrive at conical movements for the vortex axis which would be a little different in detail from the one shown here. But the general, overall form of the movement of the axis is not in doubt.

This work was done making all the geometric drawings by hand — a very long and tedious process — and only a few score drawings were made. Shortly afterwards there came the arrival of the computer and it was possible to program it, as described in Appendix 5. This gave rise to nearly five hundred drawings, and with these it was possible to cover the whole field much more thoroughly. We shall see that the final answer did not differ greatly from that which had been obtained by hand work.

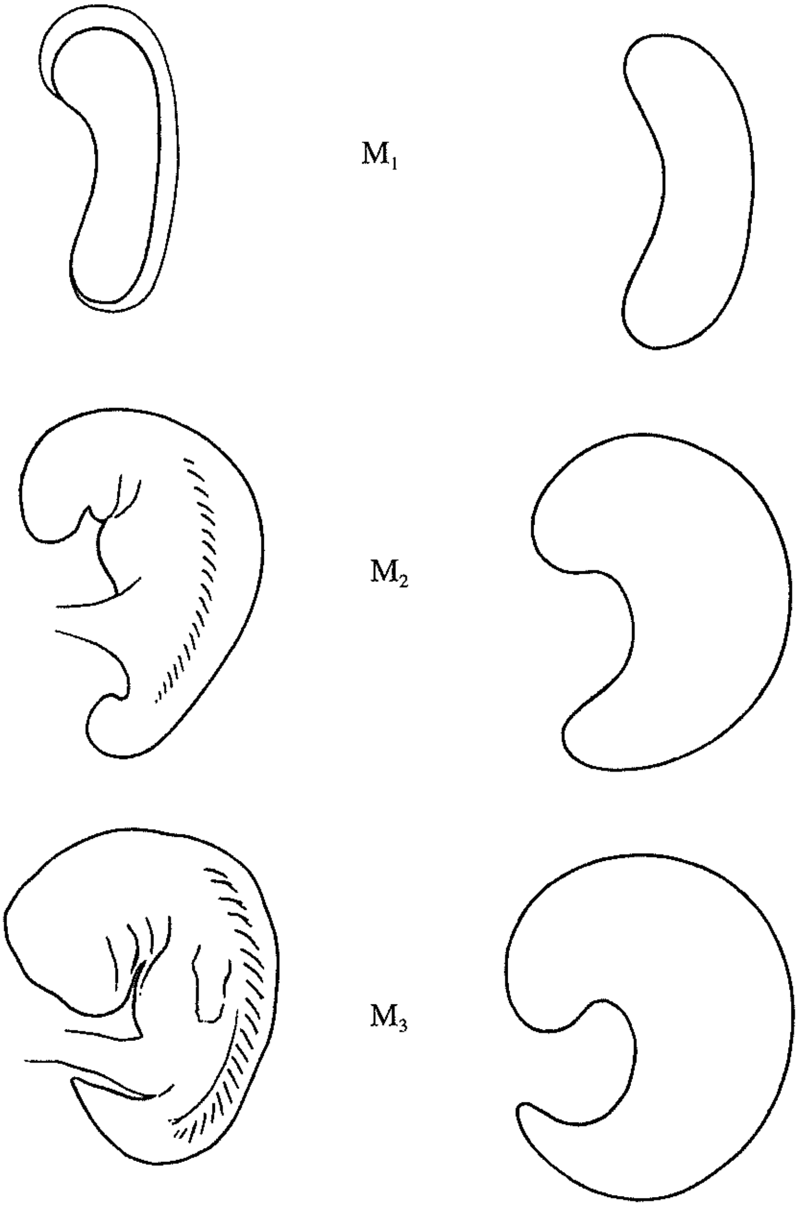


Figure 105. On the right we see the cross-sections arrived at by pure geometry from the dispositions of the vortex illustrated in Figure 104; and on the left, relevant tracings from a textbook of embryology.

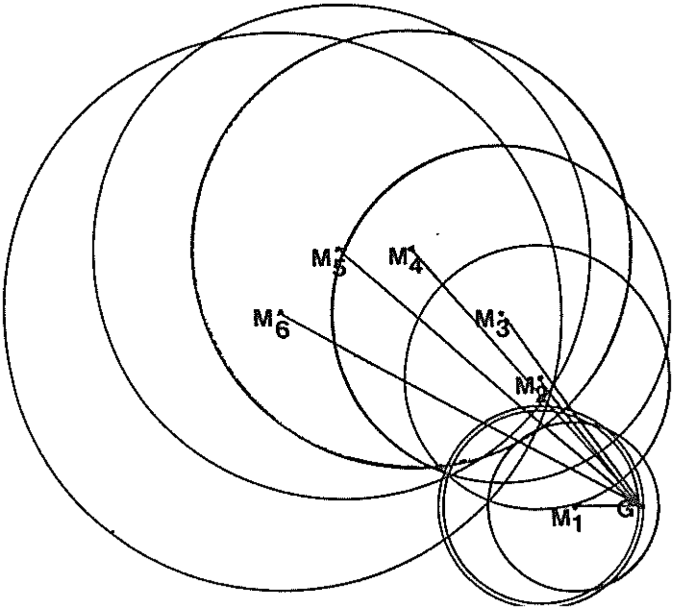


Figure 106. This is the same exercise as Figure 104, but done more thoroughly by the computer.

It is interesting to compare what the machine was able to do with that which had been achieved by hand (see Figures 105 and 107, also Figures 104 and 106). Similar to Figure 104, Figure 106 shows the points in which the vortex axis will need to cut the invariant plane through the pointed pole of the uterus transformation (that is, the lower end as it is placed in the organism). Since we now have so many more drawings to guide us this must be considered a more thoroughly researched result than that of Figure 104, and it has been possible to carry the metamorphosis to a further stage. However the main result is almost identical. The vortex axis has to move in part of a cone, while its radius slowly increases.

Figure 107 shows, on the right hand side, the six geometrical forms resulting from the positions of the vortex axis pictured in Figure 106, and on the left a selection of tracings from an embryology textbook for comparison.

Modern molecular biology has had many triumphs. It is possible today to explain many of the details of any particular organism from purely genetic considerations, but our best genetic science has not

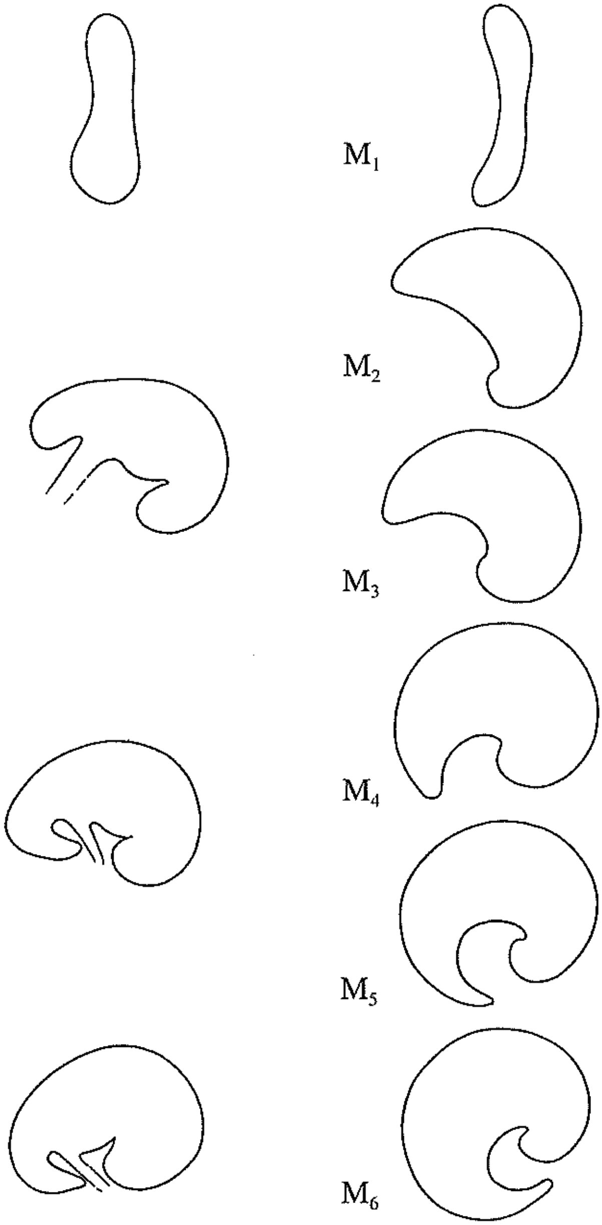


Figure 107. On the left: a selection of tracings from an embryology textbook. On the right: the six geometrical forms resulting from the positions of the vortex axis pictured in Figure 106.

even begun to understand the great formative gestures which shape the organism as a whole. It has been said that we can find encoded in our DNA the information as to whether we shall have blue, brown or squint eyes. But nowhere, and never, shall we find encoded there the wisdom — how to build an eye! I am daring to hope that in the work which has been described here we see the first slight beginning of an approach towards a view of the great shaping gestures which impart the wonders of the human form. And if this is the case we see the formative forces, not arising from the molecular intricacies of the genes, but streaming, vortex-like, from the bounds of cosmic space which must be recognized as giving birth to it.

But also, if this is the case, we must see how imperfect and tentative this first attempt has to be. All we have been able to do is to catch as it were a two-dimensional shadow of the thing we are seeking on a suitably chosen plane. Nevertheless we may hope that this will show the way forward for future work.

However, further consideration can show us that the form described at the start of this chapter does in fact contain significant three-dimensional truth and this we must next go on to investigate.

13. The Human Embryo: A Three-Dimensional Picture

In the previous chapter we considered the cross-sections through the form which arises when an airy vortex is transformed by the uterus transformation; and we found that in a remarkable way these two-dimensional cross-sections show us the general gestures of embryonic development. Now we must unite these cross-sections in our imagination to achieve a picture of the whole three-dimensional form. Imagine Figures 89 to 93 placed one behind the other, Figure 89 being closest to us and 93 furthest away. This is depicted by hand in Figure 108 and by the computer in Figure 113. We see a rather plate-like form, with a flat top in that part which is nearest to us, but as our eye travels along its length we see the form heaping up on either side, leaving an ever deepening groove along the middle. At a certain point the heapings up on either side become so high that they would meet one another at the top, and thereafter leave a little tube which extends along the rest of the length of the form.

It is a remarkable picture that is presented to us. Notice that the widest vortex produces a form which achieves the closing of the tube by Figure 92; that the two next widest only achieve this by Figure 93, and that the remainder have yet to manage this. It follows that if we think of a single rather narrow vortex steadily increasing in width, we shall see this plate-like form on which the sides will heap up until at a certain moment they will meet one another to form the beginning of a tube at one end, and thereafter, as the vortex continues just to grow in width, the closing of the tube will run along the length of the form until the tube is closed all along. And to obtain this remarkable metamorphosis all we need is simply to let a vortex grow in width.

Figure 109 is an early attempt to picture this, as seen in plan, looking down from above on to the plate. In the first picture the heaping up has reached the stage at which the two sides are about to meet just at the bottom end of the form. In the succeeding pictures we see the closing up of the tube travelling steadily across the length of the form, as the diameter of the vortex increases. These pictures were drawn by hand, using the constructions and equations given in Appendix 5. Later these

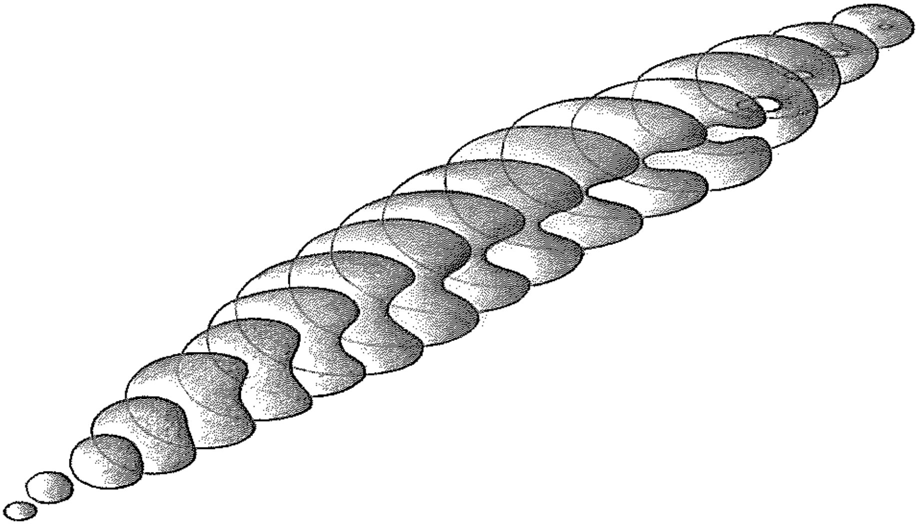


Figure 108. An image of the cross-sections in Figures 89–93 placed one behind the other. Freehand sketch of the computer delineation of the program 'EMB 1' given in Appendix 4.

same equations were fed into a computer and pictures like Figures 108 and 113 resulted. Here we see the same form, but in a different perspective, and it has been copied free-hand in Figure 108 to give a greater distance between the different cross-sections so that the eye can pick out with greater clarity just what is happening from one end of the form to the other.

At this point it is well to remind ourselves of the early stages of embryonic development. From the moment of conception onwards, for a period of rather more than two weeks, the process of cell division produces a series of wonderful metamorphoses of form, but within them, to begin with, nothing appears which one could really identify as a true beginning of the embryo which is to come. It is as though an artist were preparing his canvas with infinite care before making a single brush-stroke of the picture which is to come; or a dramatist setting his stage with utmost attention to every detail before allowing a single character to come on and act his part. And by the fifteenth or sixteenth day the stage is indeed set for the enactment of the greatest miracle which we can find in the whole world-order of events.

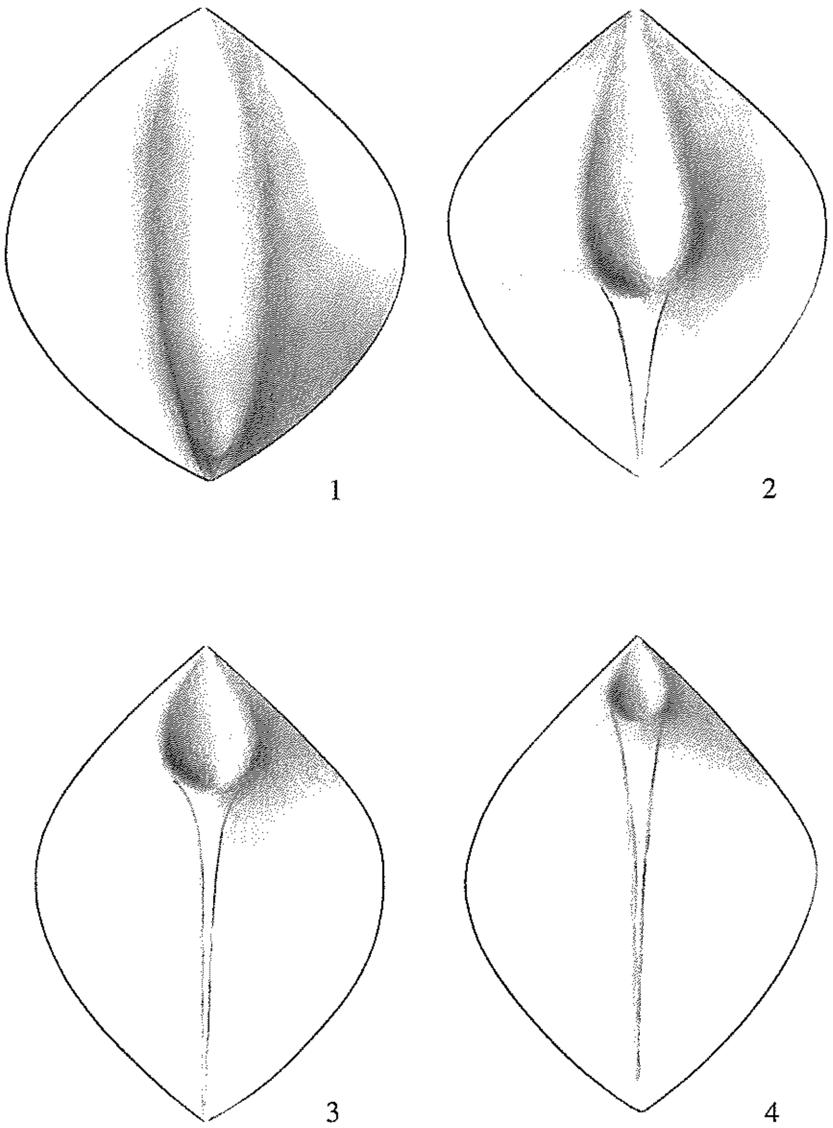


Figure 109. Succession of images produced by the widening of an airy vortex through a uterus transformation.

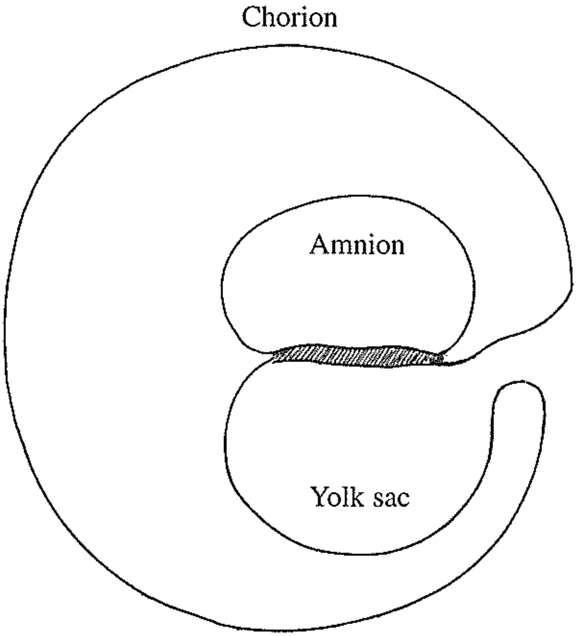


Figure 110. A very approximate diagrammatic sketch of the environment of the embryonic disc in the very early weeks of development.

At this time the whole thing is still very minute, but out of the complex and remarkable changes taking place there gradually emerges a small, flat, plate-like form, the embryonic disc, comparatively round in shape to begin with, and then elongating into a long oval-like form. Over it there arches the membrane of the amniotic sac, filled with fluid, and under it, also fluid-filled, lies the yolk sac. Surrounding all is another membrane, the chorionic sac, within whose embrace the multitudinous processes which are to take place will be welded into a marvellous organic unity. Figure 110 is a diagrammatic representation of what we should see if our eye were to be in the plane of the disc, looking at it so that we see its long side. It is a wonderful picture. Looking at it we can see the disc itself (shaded) as the ground on which we walk and act, with the curve of the amniotic sac like the arch of the starry heavens above, and the yolk sac like the bowels of the earth below-truly a stage set for human activity and effort. And we have seen the significant fact that as development progresses both the amniotic and the chorionic sacs show themselves as being not hemispherical but definitely egg-shaped in form.

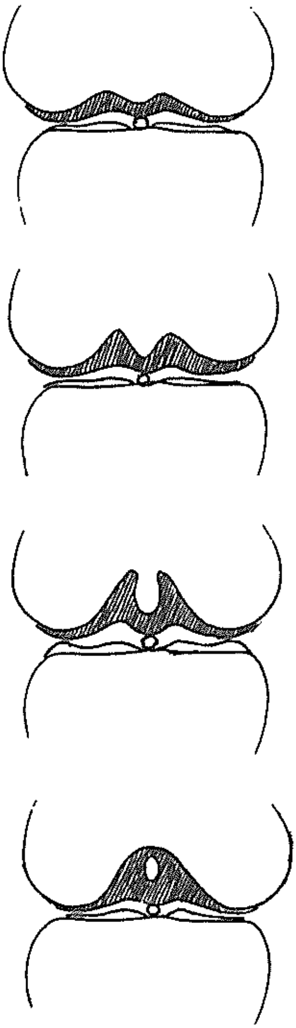


Figure 111. Diagrammatic representation of the formation of the neural canal, shown in four stages.

Next we must consider the remarkable events which are about to take place on the upper surface of the disc itself. Only a few days after the disc is prepared the substance on its upper surface begins to heap up into two parallel ridges, with a little valley between them — the neural groove — along the length of the disc. This groove to begin with is open to the amniotic fluid above, but at a certain point the two ridges rise to such a height that they join above and form the start of a little tube. The closing of this little tube then runs along the length of the disc, and later will form the spinal column of a being who is going to stand upright. Figure 111 shows several stages in the formation of this little tube, quite diagrammatically as it would be seen if one were looking towards the narrow end of the disc.

As has been pointed out, in order to produce this very remarkable transformation all we need to do, geometrically, is to let an airy vortex, transformed by the uterus transformation, simply grow in radius. But once the thing is programmed on to a computer, and we can therefore make as many drawings as we wish, in a short time, it becomes clear that the matter is much more general than this. We find that we can keep the radius of the vortex unchanged and just let the vortex move upwards from below, as at A in Figure 112, or move downwards from above, as at B, or inwards from either side, as at C and D; in each case, although the exact proportions of the resulting figure vary somewhat, its general gesture remains that which we have already shown here. This surprising fact can be confirmed by anyone who programmes this construction as it is given in Appendix 5. Figure 113 shows three actual

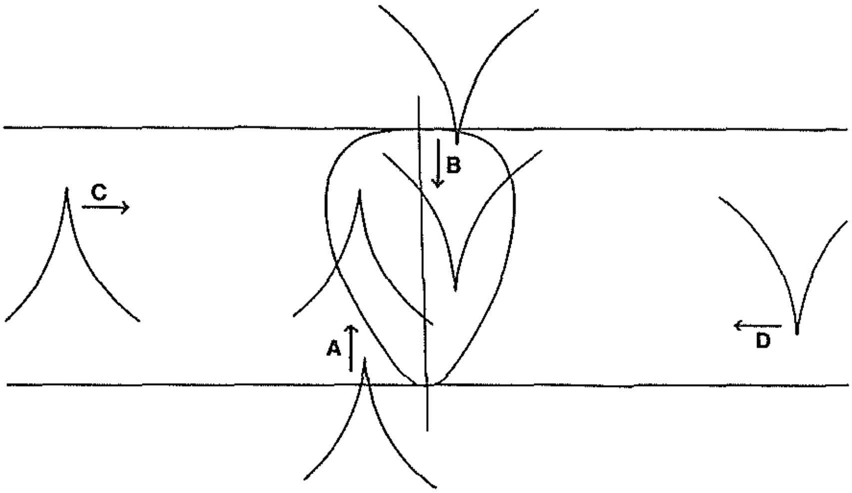


Figure 112. Here are four different ways in which the incoming vortex may move, in order to produce the general gesture of the forming of the neural canal.

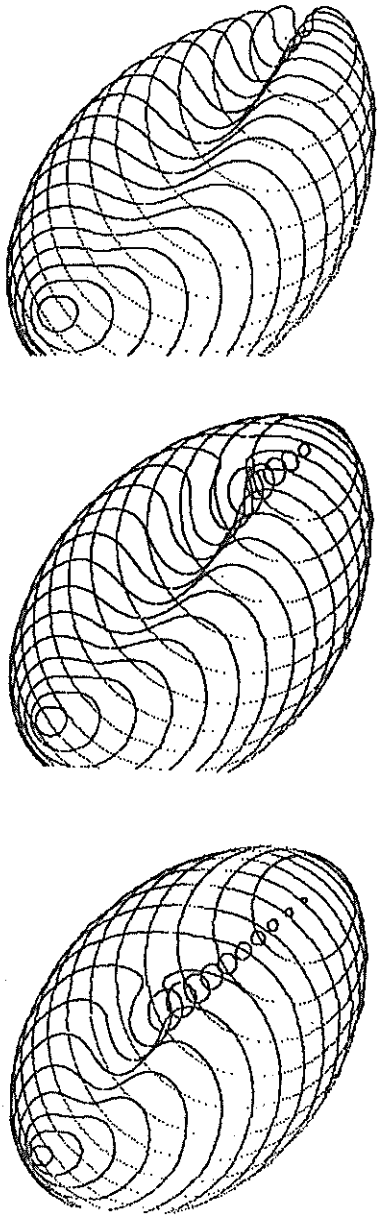


Figure 113. Three computer drawings made with all parameters, except one, unchanged. The λ of the vortex is –0.5 and its axis is approximately parallel with that of the uterus. Its apex is 0.2 of the uterus height from the blunt end to the sharp, but its distance outwards is 0.9, 0.6 and 0.3 of the uterus height from the uterus axis.

computer drawings illustrating what happens to the final form when the vortex is made to move inwards from one side, as at C in Figure 112. The formation, and gradual closing of the neural canal can be clearly seen. It seems that the general gesture of this first really decisive act in the great drama of embryonic development can follow from the approach of such a vortex from almost any direction of space. This is a fact which perhaps allows for the very great variety in unity we display. We are all human beings but how much we differ from one another!

The foregoing covers the case of the general gesture of the forming of the neural canal fairly well, but when we go further into details we find that the matter is more complex than this. If we ask, in which direction does the closing up of the tube take place, from the head towards the tail? — or the reverse? — we get the rather surprising answer that it is neither of these things. The closing up starts rather in the middle of the neural plate, somewhat in the region of what will later become the shoulders or the base of the neck, and it then proceeds simultaneously, forwards towards the head and backwards down the presumptive spine. And after many hours of work I have convinced myself that I cannot find a single vortex situated in such a way as to produce these two opposite movements. And I am beginning to believe that I never shall. My human nature is poised between two opposite poles of my being. My head is the seat of consciousness and knowledge; here is 'wisdom,' the ultimate crystallization of past experience, inert, shorn now of life, but invested with clarity. In trunk and limbs I have the region of unconsciousness, warm, thrilling and alive with the strength of my will, which can now lead to future action.

I think it is significant that the geometry, at any rate so far, has refused to link two such opposite things in a single transformation. To get the whole gesture which we see in the embryo it is necessary to envisage two vortices, coming in from opposite directions, it could be simultaneously as at A and B in Figure 112, or as at C and D. You can think of these as one coming in from the past, and the other stretching into the future, if you like, or as one mediating to us the conscious part of our being, and the other the unconscious part, but two vortices, it would seem, it has to be. When one comes to make the actual drawings of this, one finds that the two movements, headwards and tailwards, tend to overlap one another, and interpenetrate somewhat along the length of the neural plate, but apart from this the vortices as situated in Figure 112 produce a remarkably true and complete picture of the whole gesture of the forming of the neural tube.

Once the neural tube is complete, along the whole length of the embryonic disc, the next act in the drama is a remarkable one. In order to appreciate it we must forsake our view of the narrow aspect of the embryonic disc, as represented in Figure 111, and view it at right angles to this, in its long aspect, as shown in Figure 110. The tube by now extends all the way along the disc, from extreme left to right. The whole thing now begins to bend downwards at its two ends, towards the yolk sac, the head gradually forming at the left end (as we see it in Figure 110) and the tail at the right. This metamorphosis we have already seen; it is that pictured in Figures 105 and 107, and with it the whole wonder of the human form begins to show itself. I have never seen it suggested that these two great acts in the early generation of the human form are fundamentally the same gesture, repeating itself the second time in a direction at right angles to the first, and with just one further element, of asymmetry, added to it, but the geometry reveals that this is so. And our understanding of this adds an element of unification to our appreciation of what is in so many other ways such a vastly complex and complicated process.

14. The Relationship of Form to Life

The next aspect of the work was prompted by yet another question which I found myself putting to Nature. If we look back to Chapter 5 we find there the results of considering not only the mean form of the buds of some species, but the forms of a series of individual buds; and it became apparent how remarkably accurately most of these delicate little buds achieve the ideal form, amidst all the accidents of a rigorous physical world in which they have had to grow. In a set of ten buds, picked at random, it is usual to find that five or six have mean deviations so low as to indicate that they are exceedingly accurate path-curve forms, and commonly, of the remainder, only one or two will deviate at all seriously from this form. Nevertheless these deviants are there, and they have to be considered. And the question then arises: Is it a matter of concern to the plant whether its buds are good path-curve forms or not? Or, to put it a little differently: If we have two buds, one a perfect path-curve form, and the other deviating sensibly from it, can we find that the life forces of the plant are working more strongly in the first than in the second?

In order to find an answer to this question it is clear that we must find some way of estimating how strongly the life forces are working in any particular bud. At first glance this does not seem an easy matter, but, on consideration, more than one possible ways suggest themselves.

For my first attempts I used the method of capillary dynamolysis developed by L. Kolisko in collaboration with Rudolf Steiner. The details of the method have been described in quite an extensive literature (see bibliography); here we will confine ourselves to a description of the main outlines of the procedure employed.

The substance to be analysed, in this case a plant sap, is put into a shallow dish and a cylinder of filter paper is stood in the dish until the sap is completely absorbed into the paper by capillary action. When it has been allowed to dry, this sap forms a narrow, and often almost invisible, ring round the bottom of the cylinder. The cylinder is next stood in a dish containing a measured amount of metal reagent, commonly silver nitrate, until the reagent has risen through the sap up into the filter paper. When this is dry, and in the course of being submitted to light, the paper begins to 'print out' into a series of interesting and striking patterns.

In the course of nearly half a century since this method was first developed, many scientific workers have come to recognize that the sap pictures, both in their quality and their strength, represent in some way an expression of the qualities of the life forces which were working in the plant to produce that sap. If no sap is present in the paper, then no pictures arise, and it has been established that the pictures are produced by the presence of various metaboloids, that is, the results of the metabolic, life, forces which were working in the plant. And I think it a reasonable assumption that a strongly developed picture indicates a greater presence of metaboloids, and stronger life forces in the plant, than a weaker picture.

The first thing was to test how such a situation works in practice. Suppose I take ten buds from the same branch of a tree, or ten flower buds from the same patch of ground, and make a picture from each, how will these compare with one another? Before we go on to answer this, we must be very careful of the validity of what we are doing. Since we are testing for a possible correlation between form and life forces, we must take every possible precaution to ensure that all other factors remain invariant. In the case of flower buds we must try to see that the plants from which they are gathered are all growing in the same kind of soil and atmospheric conditions and, above all, that each bud is at exactly the same stage in its development. With many species that latter is difficult or almost impossible to ensure. The best species to work with I have found are those in which the bud starts its life totally enclosed in green sepals but finishes its development as a bud with its coloured petals fully bared to view. In such a bud there is a moment when the sepals are just beginning to part, and the coloured petals within are just starting to show. This forms a quite exact and easily identified moment of development in the life of the bud, and with many of the species with which I have done a lot of my work, this is the moment I have confined myself to.

With the leaf buds of the deciduous trees the matter is easier. One gathers the buds at the same moment, from the same part of the same branch, taking care to distinguish between terminal and lateral ones.

If one proceeds thus, and takes say ten buds which are as identical as possible, both in their size, stage of development and the conditions of their growth, and makes a separate picture from each, one normally finds that these pictures will all be of the same general quality (depending largely on species and season) but will differ markedly in strength. Some of them will be strongly etched and striking in appearance, and others will be so weak as to be almost invisible. It would seem that the strength of the life forces, as expressed in these capillary-dynamic pictures, varies considerably from one bud to another.

And we also know that these buds differ from one another in the exactitude with which they adhere to the mathematical archetype to which they are growing. The question before us is whether there can be a correlation between these two things. Are the more exact buds growing more 'successfully' than the others?

With a set of such pictures in front of one it is possible to grade them with a fair degree of confidence into stronger and weaker, and thus to produce a graded set. I do it in this way. Each bud, and of course its picture, has been assigned, quite arbitrarily, a number from one to ten. A graded set would then look something like this:

5 3 8 1 9 4 10 6 2 7

This would mean that bud number 5 produced the strongest picture, number 3 the next strongest, and so on all the way down to number 7 which would have the weakest picture of all. But before these buds were crushed to obtain their sap, they will have been photographed; and when their MRDs are calculated one can make another graded set having the buds with the smallest MRDs, that is, those with the most mathematically accurate form, on the left, down to the least accurate ones on the right.

If one makes two such sets in this way, will one find that they correlate? Will those buds which are found in the left, or the right, ends of the first set be found also, significantly in the same ends of the second?

There is a well-tried method of testing two ranges for such a correlation. It is called the Spearman's Rank Correlation Index. It is not difficult to calculate but the details must be left to the textbooks of statistics. It is sufficient for us to understand that if the Spearman Index comes to zero, then there is no indication at all of a correlation. A positive index indicates evidence in favour of correlation, the correlation being perfect when the index becomes +1. A negative index indicates what one could call anti-correlation; that is, there is some degree of correlation, but it is in the opposite direction from what one expected, the opposite correlation being perfect when the index becomes –1.

In the kind of case which we are considering one could not expect to find anything near to a perfect correlation. The growth of a living organism is too subtle, and is affected by too many diverse influences for any one of them to be strongly dominant. Therefore it would not be possible to draw any firm conclusion from doing just a few sets of buds;

but if over a long series of observations one were to find that the Index consistently comes positive much more often than negative, then one has evidence to show that something of a correlation is actually at work.

Proceeding in this way I set to work. But after photographing and analysing many hundred buds I was forced to the conclusion that no evidence could be found for a correlation of this sort. As far as the life forces are concerned it seemed to be a matter of indifference whether the bud was a perfect path curve or not. The question had been put to Nature, and she had answered: 'No.' And I was about to give up this line of investigation altogether, and turn to other apparently more fruitful directions when, on having a last look through my figures, I noticed, or thought that I noticed, something else.

It is clear that if we have any set of, say, ten buds, it will present us with a range of deviations from the perfect mathematical form, some being more perfect and others less. But it will also present us with a range of λs. There will be a mean λ of the set, and each bud will deviate from this, above or below, more or less widely. And now as I looked through my figures it seemed to me that those buds which deviated widely from the mean, both above it and below, were consistently appearing more often in the right-hand part of the range which showed picture strength. In other words these deviant buds were producing weaker pictures and, I had to assume, were less vitally alive than those which were near to the mean of their group.

Could it be that I had been putting the wrong question to Nature? As a mathematician I had expected her to favour those buds which *I* deemed the best, those which followed my mathematical ideal most closely. But she answered with something different. It was as though she were saying: 'Each plant, at any moment of its development, has an ideal form toward which it is striving; and those buds which achieve just this form are the ones into which my life forces will more strongly flow.'

My method of working now had to change. For each set of buds I would have to find the mean λ, and take this as the ideal towards which the plant at that moment was striving. Then my range of deviations would not show deviation from the pure mathematical form but deviation from this ideal λ. And as soon as I started working in this way, the evidence for a correlation between the two ranges began to accumulate.

That year I analysed thirty-five sets of buds and found that twenty-seven of them (77%) gave a positive Spearman index, the mean index being +0.222. On the face of it, this ought to be considered a significant result. This is a straightforward Yes/No question. If I toss a fair coin thirty-five times, the odds against getting twenty-seven heads can easily be calculated, and they are very heavily against. Nevertheless I hesitated to be wholly convinced. This is a matter of great difficulty, and complete certainty is far to reach. I had tried it with a fairly wide variety of species: leaf buds of our great deciduous trees and quite a number of kinds of flower buds. Not all species seem to react positively, notably the horse chestnut, dandelion and, often, the sycamore. Probably the outer form of these buds, which is what I was measuring, is not the significant thing, as we know is the case with the rhododendron. Then we must remember that the correctness of this calculation depends wholly on having achieved a true value of the mean λ for the day; each bud is measured twice, from two directions at right angles to one another; but the mean of only twenty measurements is bound to have uncertainties. The decision as to which pictures are stronger and weaker is, I think, usually fairly clear, but here human judgment necessarily comes in, and there are sometimes uncertainties in this. It is not always easy to be absolutely certain that one has picked the buds at exactly the same stage of development. Nevertheless after working with the greatest care possible, I was left with the impression that we have here at any rate preliminary evidence for the reality of a correlation between the finer form of these buds and the life forces working within them.

The following year I returned to the matter, and measured a further thirty-three sets of buds. This time only 67% of them gave a positive Spearman index with a mean value of +0.121; but these included a number of dandelion and sycamore, which had already shown themselves as not responding to this treatment. Ignoring these I found that I had 80% of sets with a positive index, its mean value being +0.242, a result completely consistent with that of the previous year.

At this point I thought I saw a rather different way of working. An outstanding characteristic of a living organism as compared with the mineral, is the flow of liquid which we find in the former: sap in the plant, blood and lymph in animal and man. A bud which is filled with sap is surely more fully alive than one which is dry and shrivelled. If I could find a way of measuring the sap content of each bud this would be an indication of the strength of the life forces which had been working in it. I therefore had made for me an exceedingly sensitive torsion balance; it would weigh, literally, a hair from one's head. The method of working then was to take one's bud, shred it finely, weigh the shredded pieces, subject them to a stream of hot air until they were completely dessicated, and to weigh them again. The difference in weight represented the amount of moisture which was driven off and a simple calculation would then give the percentage water content of the bud. My graded set would now have the most watery, that is, living, buds at the left end, and the driest at the right.

I worked for two years along these lines, measuring seventy-seven sets of buds; of these, fifty-nine sets had a positive index (77%) with a mean value of +0.22. The following table gives some particulars of these results. These figures are also typical of those coming from the capillary dynamic pictures of the two previous years:

Species	Total number of sets	Number of sets with positive index	Mean Spearman Index
Beech leaf buds	18	13 (72%)	+0.17
Oak leaf buds	17	14 (82%)	+0.26
Wild rose	14	12 (86%)	+0.32
Primrose	13	9 (69%)	+0.12
Buttercup	9	8 (89%)	+0.26
Wild Iris	6	3 (50%)	+0.13
Total	77	59 (77%)	+0.22

The probability of such a result happening by chance comes out at 0.0003%. This, combined with the consistency with which these results echoed those of previous years, did much to resolve the doubts that I still harboured.

However I considered this such an important matter that some four years later I took up this branch of the research again, this time returning to the use of the capillary dynamic filter paper pictures. In that year I measured fifty-one sets of buds, recording that 80% of them had a positive index, with a mean value of +0.244. Two years later I did another thirty-eight sets, of which 78% came out with a positive index, with a mean value of +0.249.

As a further test of the validity of what I had been doing, I took three of the years' work and recalculated the indices 'marrying' the sets wrongly; that is to say I compared one day's set of graded sap pictures with the following day's set of graded λs. The results of this exercise are shown in the following table:

Year	Percentage of Sets having a Positive Index	Mean Spearman Index
1978	46%	–0.016
1986	51%	–0.04
1988	41%	–0.12

With a random exercise like this one ought to expect that results would be roughly half-and-half, and we see that this is just what they are — approximately 50% positive and 50% negative, and with the mean index near to zero. If there is a slight bias in the working it seems to be leaning slightly toward the negative, and this makes our consistently positive results over six years of working all the more convincing. And after many years of working I have been slowly convinced.

Now this is a very remarkable matter. In most cases the difference between the λ-values of the various buds in a set has not been great, representing the very finest shades of difference in the visible form, sometimes only to be detected by exact measurement. Yet we find, again and again, that these differences correlate with the life forces working in the bud.

What can this mean? In the first place, I think, one of two things. It could be that the plant brings stronger or weaker life forces to bear on each of its buds, and according to this is able to form the bud more or less perfectly to its ideal form for that moment of its development. For different plants growing on the same bank I think that this is a tenable idea. But each set of leaf buds from beech and oak was gathered not only from the one tree, but from the same branch, even almost the same twig. Can we imagine that the closely integrated life-organism of a tree would differentiate thus between its various buds growing side by side on the same twig? Possibly; but I feel that it is unlikely.

The other possibility is that the *form* of the buds — just the pure form itself — acts as a vehicle for the life force. It could be that the plant, working in the intractable world of substance, to build its buds as closely to the ideal form as the accidents of that world will allow, has greater success with one bud than another; and that those buds which achieve the ideal form most closely are those which are able to receive the life forces most strongly. In this view the bud would have to be seen rather as a chalice, but its capacity to bear the life forces into the substantial world would rest not alone with its substance (although that is doubtless of importance) but also with its pure form. Form becomes a vehicle for spirit.

We have here two possible ways of looking at the world. One sees form as a necessary adjunct of matter. The substance is there, therefore it must have a form of some sort or another. The form is there because of the substance. The other sees form as the primary reality, which can only then become visible to our eyes when it takes up substance and moulds it to its purpose. The substance is there because of the form.

To me it seems possible that the full reality might take account of both aspects. A bud, at its first appearance, finds itself close to the ideal form; because of this it is able, from the start, to take up strong life forces, which then in their turn steer its progress vigorously further towards the ideal form. Another bud, also at its first appearance, maybe due to some accident of wind and weather, finds itself ill-formed, and therefore finds itself inhibited from taking up the life forces strongly, which then in their turn are unable to mould it so well in the way it should go. It has been said, has it not: 'To him that has, shall be given; and from him that has not, shall be taken even that which he seems to have.' I wonder sometimes whether, from certain aspects, this is a rather wide-ranging law of life.

Such ideas, with their possible implications, if really established could, I believe, be described as revolutionary for science. All I have been able to do these past years is to adduce just a little evidence which may point in this direction; but I think it explains why I think this is a very important chapter of the work.

However, one more point of exceptional interest needs to be dealt with. It is clear that once our interest is diverted from the deviation from the true path-curve form to the divergence from the mean form, there is no longer a necessity to calculate λ, or indeed to use projective geometry at all. It is the easiest thing possible simply to find the mean of all one's measurements, and denote this as the ideal form for that moment, for that plant. Each individual bud could then be compared with this, and its divergence from it calculated; no more than schoolboy arithmetic would be needed. And from this one could make a graded range, ranking the buds by their divergence from the mean shape, just as the range described above ranks them by their divergence from the mean λ. What would be the difference, in meaning, between these two ranges? The answer is that the new range would rank the buds according to the difference in their physical *shape,* as they stand before us in space, and as they are photographed; whereas the previous range ranks them according to the difference in the geometrical *process* which generates the form they have assumed: the winding and unwinding of the logarithmic spirals on the top and bottom planes of the invariant tetrahedron, the process which generates the form of the bud in the physical world, and is at the same time, for our calculations, implied by it.

When we do our calculations by these two methods, that of the shape and that of the λ-process, we must expect our answers to come very nearly alike. Were they not to do so we would be led to suspect that something had gone wrong with our sums! Nevertheless, owing to the subtle way in which the geometrical processes work into the world of three-dimensional form there will inevitably be small differences in the final results. If a correlation shows when we calculate the λ-divergencies, it will also show when we calculate the shape-divergencies; but not necessarily to the same extent.

Let us assume for the moment that a correlation has firmly been established. Then it becomes possible to put yet another question to Nature: 'Which *really* concerns you, the finished form which you put into the physical world, or the living process by which you put it there?' The factors which are the most significant in causing the correlation may well be expected to be the ones which will show it the most clearly in the calculations. Thus it becomes a matter of great interest to know which divergencies — those of simple shape, or those of the λ-process — show the correlation most strongly.

Those who study Appendix 3 will find that there is more than one way of calculating λ; the answers one gets by the various methods are usually very close together, and it is not easy to judge on purely theoretical grounds that one method is 'right' and another 'wrong'; it depends on the criteria one uses in making the calculation. But I had hoped that the above considerations might, just possibly, throw some light on which method is the more significant for the life of the plant.

Here are the results from 235 sets (2350 buds) using two of the methods for calculating λ which are described in Appendix 3, and also using just the divergencies from the simple shape as calculated from elementary arithmetic:

	Projective Method		Regreesion Method		Shape	
Year	% of sets positive	Mean Spearman Index	% of sets positive	Mean Spearman Index	% of sets positive	Mean Spearman Index
1978	77%	+0.222	70%	+0.219	78%	+0.189
1979	80%	+0.242	72%	+0.237	68%	+0.179
1980	80%	+0.200	76%	+0.219	72%	+0.158
1981						
1986	80%	+0.244	78%	+0.205	78%	+0.180
1988	78%	+0.249	77%	+0.244	74%	+0.182
Mean	79%	+0.231	75%	+0.225	74%	+0.178

The first thing to notice is the quite remarkable consistency in all the results, from year to year. Secondly, although the results calculated according to the simple physical shape of the buds are uniformly positive, they are consistently, and significantly, less strongly so than with either of the two λ-methods. I think this is clear experimental evidence that the true projective quality of these curves is something of significance for the life of the plant. Thirdly, although the projective method appears to have done marginally better than that based on regression, I do not think the difference is sufficiently marked for us to draw any firm conclusions about their respective merits from the point of view of the plant.

15. Plants and Stars

The validity of the work described in the previous chapter depended very strongly on my having achieved true values for the mean λ of the buds day by day; and by the beginning of 1982 I had amassed many hundred such figures for various species including amongst them several of our great deciduous trees, oak, beech, ash, and so on. Earlier on in the work I had imagined that each species would have its characteristic value which it would adhere to pretty strictly. However experience has shown that nature is more mobile than this, and in that spring of 1982, while looking through my figures it became apparent that these buds, even those growing on the same branch of the same tree, seemed to be going through subtle variations from week to week. Although I had many observations to guide me, they had not been taken with a view to studying such a variation, and I did not have a consistent, day-to-day account of any tree or species. And I resolved that the following autumn, when the new leaf buds of the deciduous trees should appear, I must make it my business to find out exactly what these little buds are doing during their long winter wait on the branch.

These leaf buds of our great trees are a somewhat strange phenomenon. Next year's buds appear on the branch somewhere about midsummer; by the end of September they are fully fashioned, and thereafter they hang on their branch all the winter, dormant as the biologist says, till they are ready to open in the following spring. Dormant they might be, but it now seemed that they were twisting and turning just a little in their sleep!

I decided that I needed to have daily measurements of each bud studied, over the period of a whole winter. This presented difficulties. Normally I pick the bud and measure it in my specially designed machine at home, getting very precise results. To photograph it while it is still on the tree is not so easy. A slight change in the point of view from which it is photographed will make a small change in the calculated λ; but it was only small changes which I was looking for anyway. How could I be certain that the bud had been photographed from exactly the same point of view day after day? Eventually I devised a method. I would take a small piece of transparent Perspex, say about 3 × 1 inches (75 × 25 mm), and after drilling needle-sized holes in it in suitable places, would strap the bud down on to it, with fine fuse wire. The strip of Perspex would then be left hanging on the tree. Next I made a wooden framework into which my camera would fit, having a slot the size of the Perspex strip, so arranged that when the strip was placed in the slot I knew that the bud was in the centre of the field of view, was in focus, and, of course, was being seen from day to day from exactly the same point of view.

The next problem was to decide which species to study. The leaf buds of the deciduous trees were obvious first candidates; they are mostly very good path curves and therefore easy to measure for really accurate λ-values; and they are present in a fairly constant state for many months at a time. But there was another aspect which, I must confess, swayed my choice. The previous spring, the figures which I already had, although not numerous or consistent enough to tell me anything definite, had given me hints that there might possibly be astronomical connections here. Now Rudolf Steiner, whose anthroposophy gave me the basic inspiration for the work I have been doing, suggested the following correlations between trees and planets:

Sun	Ash
Moon	Cherry
Mercury	Elm
Venus	Birch
Mars	Oak
Jupiter	Maple, Sycamore
Saturn	Hornbeam, Beech and the Conifers

At the time I had no knowledge of what the detailed connection could be, or that any connection would be found at all; but the fact that such suggestions had been made certainly heightened the interest of the search.

I therefore chose five buds — ash, cherry, elm, oak and beech — strapped them down on their pieces of Perspex and in the autumn of 1982 started work.

Leaf buds

There were difficulties. Often traces of the night's dew were still on the strip making the obtaining of a sharp image difficult, and it had to be dried with blotting paper. Sometimes the strip was encased with frost or ice, which had to be melted away by breathing on it, and then, of course,

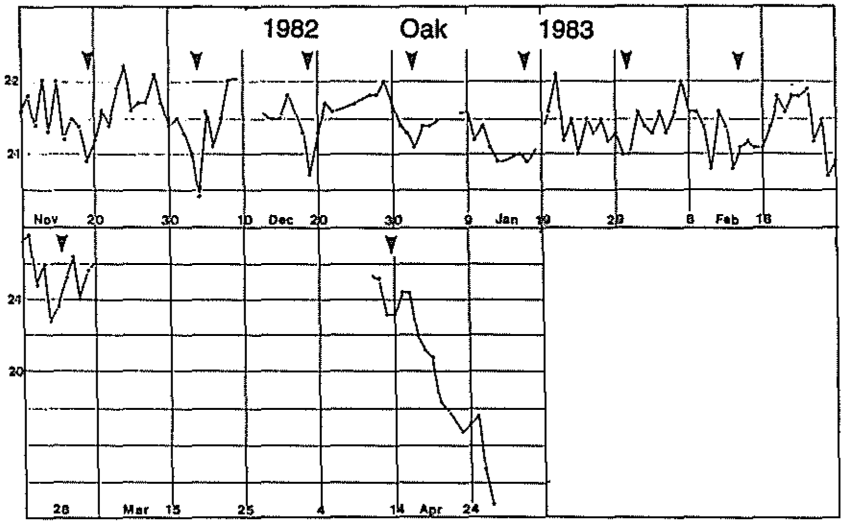


Figure 114. Mean λ values for Oak leaf buds, winter 1982–83, with corresponding alignments of Moon and Mars.

dried. It was not clear how hard one could bind the bud down without killing it; sometimes after a windy night I suspected that it had moved in its bonds. And always one was plagued by the very dull midwinter light in the Scottish Highlands. The enlarged images which I got were not always as clear and sharp as I needed for really exact measurement. But I persisted through that winter.

It was the oak which proved most amenable to exact measurement in this case. Figure 114 shows the graph of its λ-values from November to the end of April. It shows a clear fortnightly rhythm, the λ-value dropping below 2.1 at two-weekly intervals, and, except for February 11, at no other time. On consulting an ephemeris one finds that these were the times, to the day, when the Moon and Mars had drawn into straight line with the Earth, that is, when they were to be seen in conjunction or opposition in the sky. We notice that the graph does not seem to make any distinction between these two aspects, and until such a distinction makes itself apparent I am calling them by the same name: 'alignment.' The moments of such alignments are shown on this, and all succeeding graphs, by little downward or upward pointing arrows.

The general form of the graph is worth noting. It set a pattern which was to be repeated in one form or another many times over in succeeding years. We see the rhythm clearly and strongly during the autumn, but as mid-December approaches the amplitude of the variation grows less, until in early January the bud becomes almost inert. In fact at that time I thought that I must have strapped it down too tightly, and had killed it. I went so far as to strap another bud down on to a new piece of Perspex and start with that. Hardly had I done so when the original bud 'awoke' and continued with its variation, although not quite as strongly as it had done so in the autumn. I cannot state that this midwinter sleep always takes place with the leaf buds, but it does seem to be a common feature with them.

The steep descent at the end of the graph marks the true opening of the bud; the start of this was apparent to my exact measurements some ten days before ordinary observation would have noticed it on the tree.

Now what do we understand by a result like this? Every time the Moon and Mars come into alignment with the Earth, this little oak bud responds with a subtle gesture which takes it a little way towards opening and almost immediately closing again. Can we believe that this is true for all oak buds all over the world? Certainly not on the evidence of this one little bud alone. What about the beech and the elm? Owing to the imperfections of these midwinter photographs the evidence here was far from conclusive, but their graphs could certainly be interpreted as being consistent with the beech behaving similarly on alignments of Moon and Saturn, and the elm on alignments of Moon and Mercury. From the ash during that first winter I could get no evidence of such response at all.

The cherry, if it was to be judged in a similar way to these others, posed something of a problem; one can hardly imagine the Moon coming into alignment with itself! In the event this leaf bud of the cherry proved to have a well-marked fourteen-day cycle of its own, the λ-value coming, not to a dip, but to a peak, every fortnight. And throughout that winter these peaks coincided every time with alignments of Moon and Sun, that is, full moon and new moon. Figure 115 shows the λ-graph for the cherry that winter. Notice that the variation was vigorous during the autumn but that as the new year approached the amplitude grew gradually smaller until the alignment of January 13 was hardly measurable; and strong variation did not resume until the middle of February. Although the actual variation was inverted the general pattern bore a distinct resemblance to that of the oak.

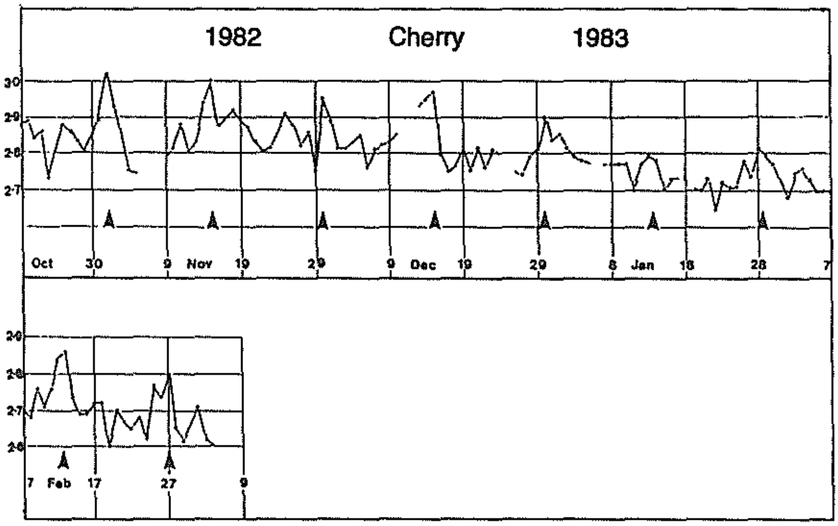


Figure 115. Mean λ values for cherry leaf buds, winter 1982–83, with corresponding alignments of Sun and Moon.

The results described here represented a whole winter's work with the buds, but they could not be described as more than a very meagre beginning to an important aspect of the work. The variation was on the whole small and subtle, and I was aware that, in some cases especially, I had been working near to the limits of possible error in my measurements. At the end of the season I made a careful reappraisal of all results, going to the length of getting out old negatives, re-enlarging and re-measuring, in random order, so that while measuring I was not aware of the dates written on the back of each picture. The most sober re-assessment convinced me that although the results for the elm and the beech could be seen as consistent with such variation they could not be accepted as firm evidence to prove it; but that those for the oak and the cherry could be seen as some real, positive, evidence. Obviously much more work needed to be done, but it would have to wait some six months until the new buds were ready for examination the next autumn.

Primrose

Meanwhile the spring was upon us and the primroses were already coming into bud; and the question arose whether, just possibly, these flower buds might be behaving in a similar way. Here the question being asked of nature had to be phrased a little differently. A leaf bud hangs on its branch, in a more or less invariant state, all the winter, and if one deciphers what that single bud was doing all that time one can hope to have a picture of what the whole tree, maybe the whole species, was doing. And this is the thing we are aiming for. With the flower bud it is different; it is too delicate to be strapped down on to a piece of Perspex without killing it; and we have already studied, and described on pages 100–7, what the single flower bud is doing in the course of its development. Here we are concerned with what the whole species is doing in the weeks of its budding season. The method followed must be to pick, say, ten buds each day, gathered under the most identical conditions possible, find their mean λ, take this to be the λ of the species for that day, and to investigate how this value varies from week to week. Obviously the validity of such work depends on the complete constancy of the conditions in which the buds are picked, and especially that they are all taken exactly at the same stage in their development. This is not easy to ensure with all species, but the primrose is a very good subject in this respect. The moment when the coloured petals first begin to push through the green covering sepals

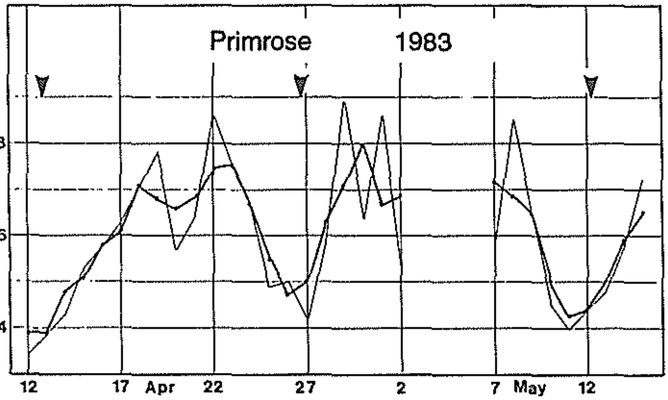


Figure 116. Mean λ values for Primrose buds, spring 1983, with corresponding alignments of Sun and Moon.

is clearly identifiable, and all buds for this experiment were picked at exactly that moment.

Figure 116 shows the results of these first observations of this sort on a flower bud — the primrose 1983. When examining this, one must remember that what is plotted on one's graph are mean values of λ, and mean values, by the very nature of things, can never be exact; according to the random selection of the buds each day, more, or less, high, or low, values may creep into each mean; thus the mean value will fluctuate, from day to day, above and below the real trend which we are seeking, forming a certain amount of 'noise' in the graph. A useful way of trying to eliminate some of this noise is to plot three-day means — that is, for each day we plot the mean of the preceding day, the day itself and the following day. Such a curve shows the general trend of the observations better, and in Figure 116 it is drawn in heavy lines. The light lines weaving above and below it show the actual daily values so that the interested reader can judge for himself the consistency of the observations. And all graphs following this one will be made in a similar way.

Looking at Figure 116 we see that the λ-value fell below 2.5 just three times, at fortnightly intervals. We have no theoretical reason for assigning any astronomical aspect to these dips in the curve, but working quite empirically we find that they coincide in each case with alignments of Sun and Moon, that is, new moon and full moon. With observations extending over scarcely six weeks and only two dips fully plotted this can by itself not make a very convincing case — it might have been

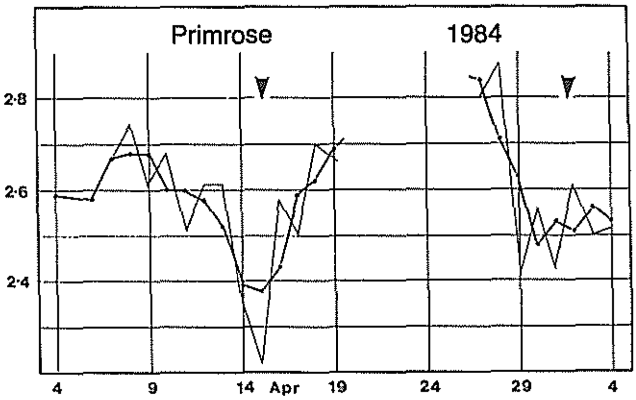


Figure 117. Mean λ values for Primrose buds, spring 1984, with corresponding alignments of Sun and Moon.

coincidence! It is therefore worthwhile to jump a year and to show the corresponding graph for 1984 (Figure 117). Again we see a curve which dips below 2.5 just twice, at fortnightly intervals, and again these coincide with alignments of Sun and Moon. These two seasons' observations taken together do give us, I believe, a good indication that something significant is at work here.

If we ask what it is, we find ourselves on more debatable ground. *If* we are to judge the flower buds in the same way as the leaf buds of the trees then we would say that the primrose probably comes under the influence of the Sun. I say 'probably' because lunar alignments with Mercury and Venus tend to come close to those with the Sun, and they are therefore hard to disentangle; further work has shown that the Sun in this case is more likely than the other two.

Can we envisage the primrose as a Sun plant? If we gaze into the heart of the plant, we will see there upwards of half a dozen or more little buds, radiating from the central stem, each encased in its sheath of green sepals, of a glowing yellow; one can get the impression of a source of radiating light, bravely growing there in the cold days of early spring. The poet had an intuition that the primrose growing by the river's brim was something more than just a primrose; can we see it as a ray of the spiritual sun shining forth from our winter-frozen earth? Well . . . let us get back to our facts and figures.

Stitchwort

Just as the primrose buds are finishing, the little white-starred flower of the stitchwort is beginning to show itself in our hedgerows. This little bud also grows ensheathed in green sepals until at a certain moment the white of the petals begins to show through and it is therefore suitable for similar treatment. Figure 118 shows the graph for the stitchwort in 1983. We see two dips, at approximately a fortnightly interval; after May 24 I had to be away from home, but returned just in time to catch λ, maybe, recovering from another dip in the first week of June. Like the primrose, taken by itself, this graph cannot prove very much, but taken in conjunction with the work of succeeding years I think it is highly significant. I could find no lunar alignments to coincide with these dips but they both come just one day before those of Moon and Saturn, and as we shall see from the work done since then, this appears to be a Saturn phenomenon.

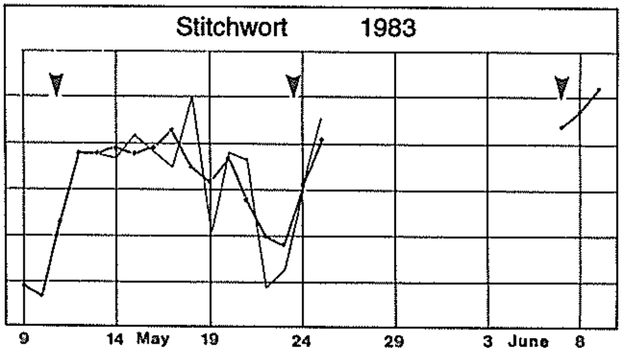


Figure 118. Mean λ values for Stitchwort, spring 1983, with corresponding alignments of Moon and Saturn.

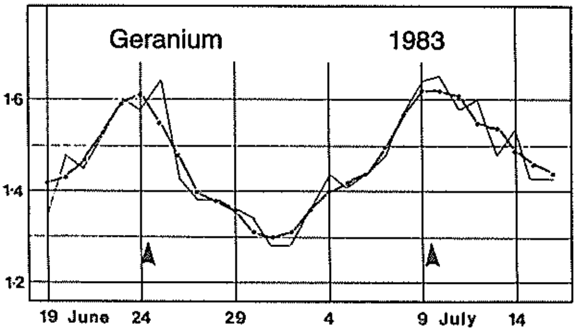


Figure 119. Mean λ values for Geranium, summer 1983, with corresponding alignments of Moon and Mars.

Geranium

That first summer's work ended with a series on a small pink garden geranium, again always picking the buds just as the first signs of red were showing through the green sepals. This plant produced beautifully consistent results from day to day and a strongly marked fortnightly rhythm (Figure 119). I was not able to identify any lunar alignments with the dips in the curve, but the peaks in the curve were almost absolutely coincident with alignments of Moon with both Sun and Mars. The work of later years soon ruled out the Sun, but Mars continued to be relevant, and I think there is little doubt that this is indeed a Mars plant.

Further observations

So there were the results of that first year's work on this aspect of the research. In no less than five species — two leaf buds of trees and three flower buds — there seemed to be consistent evidence of approximately fortnightly rhythms, relating themselves to astronomical configurations which were valid, not only for the west coast of Scotland where I was working, but for the whole world. The picture which was beginning to build itself up in my mind was of a series of mighty 'heart beats' of nature, expressing themselves in these subtle variations of form in the great plant garment which girdles our planet. However beautiful one might feel such an idea to be, it had to stand or fall by one thing only: Was it true? I began to realize that this was a very big idea indeed and it would need far more evidence before it could be put forward with the kind of confidence which I wanted.

I therefore decided that I would spend the following winter, 1983/84, getting definite evidence — for, or against — in the simplest way possible. I would pare the problem down to its simplest elements. Take the hypothesis, for the beech tree, that when the Moon is in alignment with Saturn, λ is lower than when it is 90° from it; and test this week by week throughout the winter. This is a straightforward Yes/No question and could surely be answered!

The method of strapping the buds down on to a piece of Perspex had proved to have serious disadvantages and I decided to work with the leaf buds the same way as I had with the flower buds; ten buds were taken each day, in as identical conditions as possible — from the same part of the same branch — and their mean λ was taken as the λ for the day. The following table shows the results for that first experiment in the autumn of 1983. On October 15, when the Moon was approximately 90° from Saturn, the λ of the buds was 2.54; a week later, on October 22, when the Moon was aligned with Saturn, it had fallen to 2.37; and a week later again, it had risen to 2.65. Column A shows all the observations when the Moon was approximately 90° from Saturn, and column B when it was in alignment with it. The third column, difference, is always calculated A – B, so that whenever the figure here is positive it is evidence in favour of the hypothesis, and whenever it is zero or negative, it is evidence against. If there is no truth in the hypothesis, then the figures in the third column should be approximately equally plus and minus.

Beech 1983–84

	A	B	(A–B)
Date	Moon 90° from Sat.	Moon align with Saturn	Difference
Oct 15	2.54		+0.17
Oct 22		2.37	+0.28
Oct 27	2.65		+0.24
Nov 4		2.41	+0.27
Nov 11	2.68		+0.10
Nov 18		2.58	+0.14
Nov 25	2.72		+0.10
Dec 2		2.62	+0.19
Dec 9	2.81		+0.29
Dec 16		2.52	
Dec 23			
Dec 29			

	A	B	(A–B)
Date	Moon 90° from Sat.	Moon align with Saturn	Difference
Jan 5	2.60		+0.08
Jan 13		2.52	+0.18
Jan 20	2.70		+0.03
Jan 26		2.67	+0.11
Feb 2	2.78		+0.30
Feb 9		2.48	+0.20
Feb 16	2.68		+0.11
Feb 22		2.57	+0.16
Mar 1	2.73		+0.20
Mar 8		2.53	+0.09
Mar 13	2.62		+0.16
Mar 20		2.46	
Mean	2.68	2.52	+0.16

In the event we see that throughout that winter the 'difference' results were positive on every occasion. The odds against such a result coming by chance are almost impossibly against. For good measure I did the same thing with another beech growing some fifty yards from the first one. This time out of twenty 'difference' results, nineteen were positive, a very small negative (–0.01) coming in the second week of January, when we know this rhythm is at its smallest anyway; so this second tree had almost as strikingly a positive general result as the first one.

The next table shows the results of a similar experiment conducted with an oak, this time the relevant configurations being between Moon and Mars. Here we see, out of seventeen differences, that no less than sixteen are positive, again a result which cannot possibly be reconciled with chance.

Oak 1983–84

	A	B	(A–B)
Date	Moon 90° from Mars	Moon align with Mars	Difference
Oct 25	2.17		+0.12
Nov 1		2.05	+0.12
Nov 8	2.17		+0.19
Nov 15		1.98	+0.08
Nov 24	2.06		+0.13
Nov 29		1.93	+0.29
Dec 5	2.22		+0.17
Dec 14		2.05	
Dec 21			
Dec 28			+0.10
Jan 6	2.15		

	A	B	(A–B)
Date	Moon 90° from Mars	Moon align with Mars	Difference
Jan 12		2.05	+0.19
Jan 19	2.24		+0.27
Jan 25		1.97	+0.14
Jan 31	2.11		0.0
Feb 9		2.11	+0.19
Feb 16	2.30		+0.25
Feb 22		2.05	+0.24
Mar 1	2.29		+0.20
Mar 8		2.09	+0.20
Mar 15	2.29		+0.29
Mar 21		2.00	
Mean	2.20	2.03	+0.17

That same winter I did shorter series on the birch, in relation to Venus (eight differences with seven positive); the ash, in relation to the Sun (eight differences, all positive); and the elm, in relation to Mercury (six differences, five of them positive); so considering these further three species we have definite, if preliminary, evidence for the truth of the hypothesis.

At this time a new and unexpected factor inserted itself into the research. Owing to the way of working that winter, I only needed to visit each tree once a week, and this left me time for some further work; so I decided to start a new *daily* series on a beech, taking the mean λ of ten buds a day, all gathered from the same branch, as the λ for the day. This method has proved so successful that it is the one I have followed ever since (except that in recent years the number of buds taken each day has been greater than ten in many cases). However in this case I was due for a surprise. The day of the alignment came, and passed, and the buds gave no acknowledgment of it at all. Apart altogether from the other species, the beech tree of the previous winter had suggested such variation, and the two beeches of that same winter were showing it strongly — why was this one failing to do so?

Even while I was photographing and measuring the buds of this latest tree I had had a slightly uneasy feeling that they were in some way different from the others. Usually the beech buds are long and thin, and almost completely straight; however most of them show a very slight bend in their long axis. This bend is often so slight as to be almost invisible. I have always taken it to be a phyllotactic effect, imposed on an organism whose archetypal form is fundamentally straight. The light plays more strongly on the growing buds from above than from below, and they reach ever so slightly towards the light. But in the case of these buds the curvature was much more severe; some of them looked more like claws. Then most beech buds are fairly shiny in appearance and of a warm brown colour. These had a rather dessicated look; it was as though they had been dusted with cement powder. I had had the feeling that I was working with something that had the qualities of old age. Could it possibly be that I had selected an old and tired tree that had lost its capacity to respond to these subtle rhythms? I went to have a closer look. It was less than half the size of a normal mature beech and I could find no evidence that it was very old or in any other way different from the others.

I was turning away in some bewilderment when I suddenly saw what of course I had really known all the time, but not had the wit to appreciate, that this tree was growing only a few yards from an electricity substation; the great transformer was humming away just the other side of a fence labelled DANGER — 33,000 VOLTS, and the high tension cable came into the sub-station only a few feet above the topmost branches of my tree. It was obviously growing in a powerful artificial magnetic and electric field; and the question naturally arose whether such conditions formed a shield which would cut a plant off from the cosmic connections in which it is otherwise enmeshed.

As soon as this situation became clear I put the tree down on my list of those which I would follow week by week through the winter. The measurements which resulted from this are given in the table overleaf.

Beech (1983–84) growing under High Tension Cables

	A	B	(A–B)
Date	Moon 90° from Sat.	Moon align with Saturn	Difference
Nov 25	2.46		
			–0.06
Dec 2		2.52	
			+0.11
Dec 9	2.63		
			–0.04
Dec 16		2.67	
Dec 23			
Dec 29			
Jan 5	2.44		
			–0.27
Jan 13		2.71	
			–0.25
Jan 20	2.46		
			+0.08
Jan 26		2.38	
			+0.08
Feb 2	2.46		
			+0.15
Feb 9		2.31	
			+0.35
Feb 16	2.66		
			–0.19
Feb 22		2.85	
			–0.10
Mar 1	2.75		
			+0.40
Mar 8		2.35	
			+0.02
Mar 13	2.37		
			–0.20
Mar 20		2.57	
Mean	2.53	2.55	–0.02

As it turned out that winter, for this tree we had fourteen differences, seven of them positive and seven negative; and the mean difference was exceedingly close to zero. Now this is exactly the sort of result which we could confidently expect if the growth of this tree had no correlation with the astronomical events which were being considered; and we see that the methods being used here clearly show this when it is the case.

This being the case I immediately set about seeing whether I could get further evidence to support, or deny, the idea that a strong artificial electric or magnetic field would prevent a plant from responding to these subtle rhythms. It was not easy. The electric company do not like trees growing under their power lines. From time to time they cut a swathe through all the vegetation growing under the cables, and trees of any size are particularly destined to destruction. I do not know why the one I had studied had been left so long intact. Finally, after much searching, I found a small oak sapling growing under the cable out on the moorland some five miles from home (it has since been felled). These buds, it seemed to me, also had the rather dessicated and grey look of the beech buds just described. Was I 'seeing' what I expected (or wanted?) to see? It is a subtle matter; I could not tell for sure. But the measurements which followed during the next two months could not be anything but objective. The final table proved to be very similar to the one for the beech shown above. Out of eight differences, three were positive and five were negative; and the mean difference was –0.05. The thing was not quite as evenly balanced as it was with the beech but still very nearly neutral; where it departed from this it was to lean ever so slightly towards the negative. Again I had found a tree which was not responding.

It was in this way that these preliminary observations seemed to indicate that the presence of an artificial electric field could act as a screen which isolates the plants from their cosmic connections; within this screen they continue to do all the 'proper' things, germinating, budding, blossoming, seeding, but now out of time with any heavenly context. But we must beware of making too much of this at this stage. A great deal more work needs to be done on this before we can speak with any sense of assurance. And this work I have not been able to do, partly because, as already mentioned, it is not at all easy to find good examples to work on, but also partly for lack of time. I felt that priority must be given to establishing as thoroughly as possible what happens in the way of normal nature before we can hope to appreciate properly the divergencies which may arise with exceptional circumstances. And the first of these tasks has proved so vast, and so complex, as to take up all my available time.

The fact that things are more complex than the simple rule that I had enunciated became clear during the summer of 1984. During that season the behaviour of both the stitchwort and the geranium were in a general way similar to the previous summer, but greatly reduced in amplitude, that is, in strength of variation. Figure 120 shows the behaviour of the geranium for that year, and is interesting to compare with Figure 119.

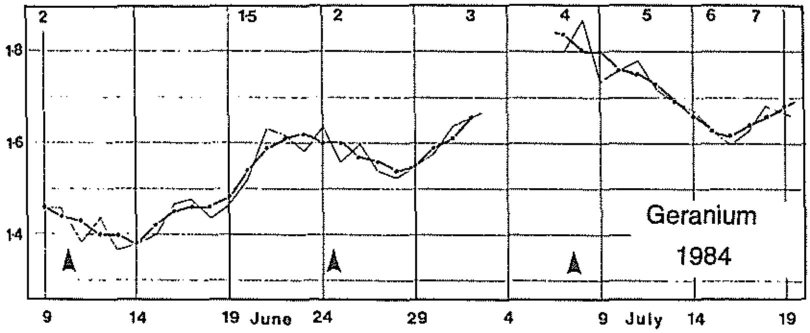


Figure 120. Mean λ values for Geranium, summer 1984, with corresponding alignments of Moon and Mars, but a possible inhibitory effect from Saturn.

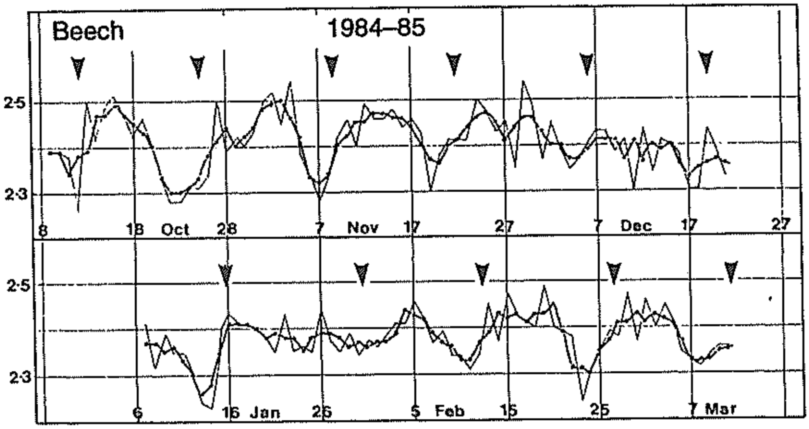


Figure 121. Mean λ values for Beech leaf buds, winter 1984–85, with corresponding alignments of Moon and Saturn.

The peak in the curve towards the end of June has become so slight as to be barely measurable, and a somewhat similar effect was to be seen with the stitchwort. Now it is a fact that during that time Mars and Saturn were in close alignment with one another, and the results seemed to be inhibitory on the part of both of them. The figures along the top of Figure 120 show the number of degrees of longitude separating Mars and Saturn during that time, and it will be seen that variation of the kind of strength which was observed in the previous year, and which has been seen since, did not show itself until these two planets were at least 5° to 6° apart. Further work in more recent years has gone to confirm this relationship between these two planets which traditionally have such opposite natures. Further relationships of this kind — either of inhibition or enhancement — between various planets may well come to light in due course, but I have no specific examples of this yet which I can report.

The phase-shift

However, the fact that things were going to prove more complex, and stranger, than I had imagined was really born in upon me by the work of the winter of 1984–85. I decided to pursue a consistent and long-sustained study of the buds of a mature beech tree, taking ten buds per day throughout the winter. This aspect of the work was still new to me at that time and

I was not at all confident that significant results would show themselves. However, in the event it was clear that I need not have feared. Figure 121 shows the resulting λ-values. Apart from one small particular I could hardly have hoped for a more consistent corroboration of previous years' results. We see, in the autumn, a strong and clear fortnightly rhythm; as mid-December approaches we see the amplitude of this rhythm getting smaller and smaller until by January it is hardly measurable. Not until the beginning of February do the buds wake up, and resume their two-weekly variation, but now not quite as strongly as they had done in the early autumn.

There was just one unexpected feature. The arrows on the graph, as with all those so far, show the moments of the relevant alignments, in this case those of Moon and Saturn. The first dip in the curve, on October 11, seemed to come something like a day before the actual moment of the alignment. This did not cause me much anxiety; working with mean values as I was, the exact timing of a dip in the curve is not easy, and, working from past experience I was fairly confident that with the next dip things would be found to be back on time again. However when it came, as a glance at the graph will show, it was fully two days before the astronomical alignment. And all through that winter this old tree obstinately insisted on reacting to the alignments about two days before they actually occurred. And far from this tendency diminishing we see, in February and March, that this tree was pre-empting by almost three rather than two days.

At this time I had no means of knowing whether this was a peculiarity of this particular tree, or perhaps of that particular season; but a few weeks later, when I came to study the primroses, I had my first intimation that this was a phenomenon that affected a much wider range of organisms than the old beech I had selected for study during that winter. Figure 122 shows the λ-values for the primrose during that spring of 1985. We see the curve dipping below 2.5 three times, at fortnightly intervals. It is very similar indeed to previous years, except that the whole thing has been translated some two to three days to the left. The first dip comes some two-and-a-half days early, the second one about one day, and the third one fully three days, before the moments of the Sun/Moon alignments.

At this point, in order to make a very strange situation as clear as possible, it should perhaps be pointed out straightaway that this tendency for the buds to pre-empt the astronomical aspects has, in the months and years since 1985, proved to be a slowly but consistently increasing phenomenon. It is an idea which I resisted as long as possible. The precision and punctuality with which the buds had responded to the celestial configurations in 1982 and 1983 was something which had caused me

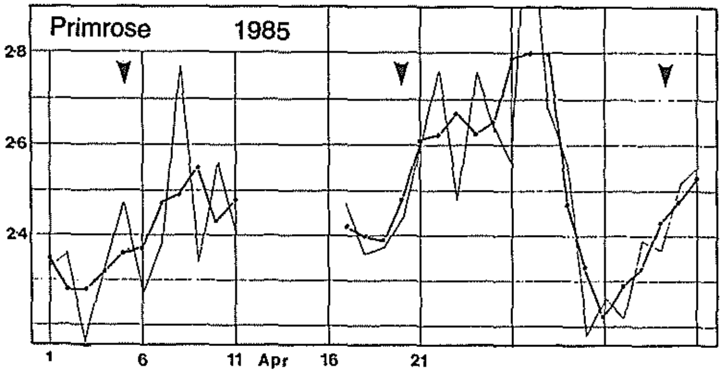


Figure 122. Mean λ values for Primrose buds, spring 1985, with corresponding alignments of Sun and Moon.

much satisfaction and I would have liked to keep it this way. But by the beginning of 1985 I could no longer close my eyes to the fact that things were proving more complex than this. And in the following years I have seen the amount of pre-emption grow gradually larger and larger. It was as though this primary fortnightly rhythm, which one can observe from month to month, is overlaid by another of much longer period. By early 1985 this phase-shift amounted to about three days; as this gradually increases, and if it continues this way long enough, a time will come when the dips in the curves are coming fourteen days early, then they will be on time again, and the overlying rhythm will have completed just one (or maybe possibly, a half) cycle. And it became a matter of importance to me to find just how long such a cycle would take.

At this point I felt it needful to develop a terminology to deal with this phenomenon. When I speak of the buds, in any particular season, having a phase-shift of –3, I mean that the turning points in the λ-curves (usually dips, but sometimes peaks) are coming three days *before* the relevant alignment. But with a phase-shift of +3 they would be coming three days *after* the alignment. Clearly then, as it gradually increases the phase-shift will pass from –6 to –7. But seven days early will be approximately the same thing as seven days late; we can call it ±7. And thereafter it would go on to +6, +5 and so on until it becomes zero again, and the secondary cycle is completed.

In the realm of the stars, rhythms and cycles are always of this general nature. Never does one find a rhythm working on its own; always it is overlaid by other rhythms, and they in their turn by others again. And

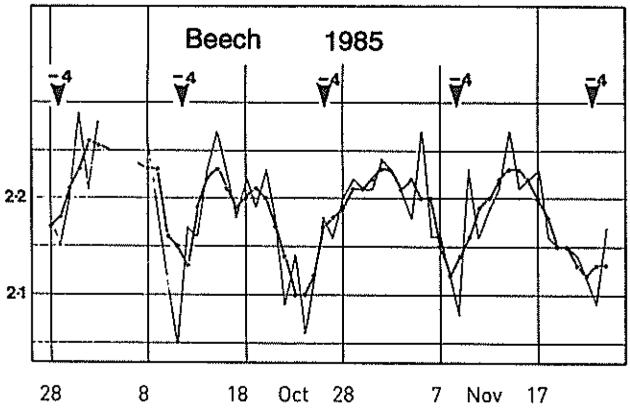


Figure 123. Mean λ values for Beech leaf buds, autumn 1985, with phase-shift from corresponding alignments of Moon and Saturn.

if we find celestial correlations working in the realm of the plants we should expect to find patterns of this general nature working there also. Nevertheless to find a variable phase-shift of just such a pattern is, as far as I know, something without precedent. Such a finding should not be put forward without careful consideration, and powerful evidence to support it. It is not possible within the limits of a book like this, to show all the evidence which has come to hand in recent years, but I think it important to print enough typical results that the reader can judge what sort of weight this evidence has, and just where it is leading.

Up to this point the arrows on the graphs have marked the actual moments of the alignments, but from now on an arrow conjoined with a number will mark the moment of the alignment altered by the corresponding phase-shift: that is, an arrow marked –4 will show the moment four days *before* the alignment, and one marked +5 will show the moment nine days ahead of the alignment, or five days *after* the preceding one.

With this in mind, let us move from the spring of 1985 to the autumn of the same year. Figure 123 shows the graph given by the buds of a beech tree at that time. We see that this is characteristic of those given by the beech in previous years, except that all the dips, with the exception of that of October 24, come between four and five days earlier than their alignments, the exceptional dip coming nearer six days early. It is safe to say that the beech was running at a phase-shift of –4½ to –5 days at that time. Simultaneously with this, observations on an oak showed a clear phase-shift of –5 days.

At this point two questions presented themselves. In the course of the three years from autumn 1982 to 1985 the size of the phase-shift had grown from zero to about $4^3/_4$ days. Provided that it was to continue in this way, it was easy to calculate that the complete phase-shift of fourteen days, marking the end of a whole cycle of the secondary rhythm, would be achieved in almost exactly nine years. This is very close to the time that the line of the lunar nodes takes to make a half-circle of the zodiac and to lie once again upon the position it first had; and this led me at that time to speculate that this strange phenomenon of the increasing phase-shift might be in some way connected with the changing relationships of the lunar nodes. I was aware of course that I was here extrapolating on very insufficient evidence, and when later observations forced me to abandon this idea it came as no great surprise. We shall see later at least one of the underlying factors which caused this misapprehension at that time, and it is a matter of great significance I believe for our future understanding of this work.

The other question was why, if this tendency had been going on for three years, it had taken me rather over two years to notice it. In this connection it is interesting to look back to Figures 117 and 120, the primroses and geraniums of 1984, and even to Figures 116 and 118, the primroses and stitchworts of 1983, to see, with hindsight, that this tendency was already showing itself, although so slightly that I had not been alert enough to spot it. The reason why the indications were so slight, even as late as the summer of 1984, is again something that we shall see later.

But first let us go forward from the autumn of 1985 to the late spring of 1986. Figure 124 shows the results for the stitchwort in May and June. We see a characteristic curve, with three dips at phase-shifts of about $-5^3/_4$, $-5^1/_2$ and $-6^1/_4$, a mean shift of about –6 days. The primrose, in the preceding month, had had a phase-shift of about $-5^1/_2$ days.

And some four or five months later, in October of 1986 we find the oak (Figure 125) running just about seven days early. It is interesting to notice in this graph of the oak that the fortnightly rhythm in which we are chiefly interested seems to be superimposed on another slower change, probably of a rhythmic character, but with a much longer period. Through all that autumn the general level of λ was undergoing a slow but consistent rise. A phenomenon of this sort is not always seen, but it is by no means unusual; sometimes the general level is found to be slowly rising, sometimes falling, and sometimes even falling to a minimum and starting to rise again. Much more research is needed here.

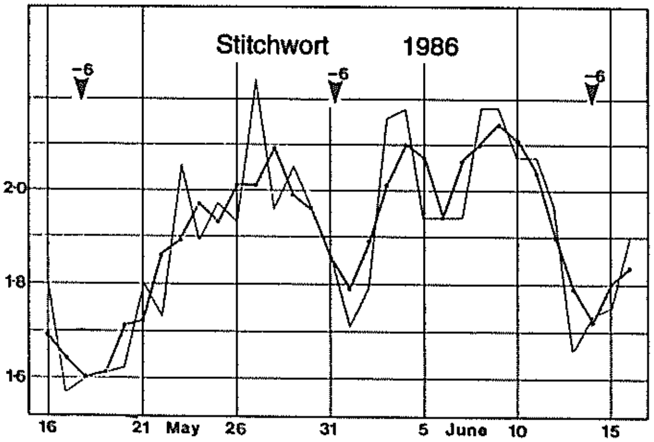


Figure 124. Mean λ values for Stitchwort, summer 1986, with phase-shift from corresponding alignments of Moon and Saturn.

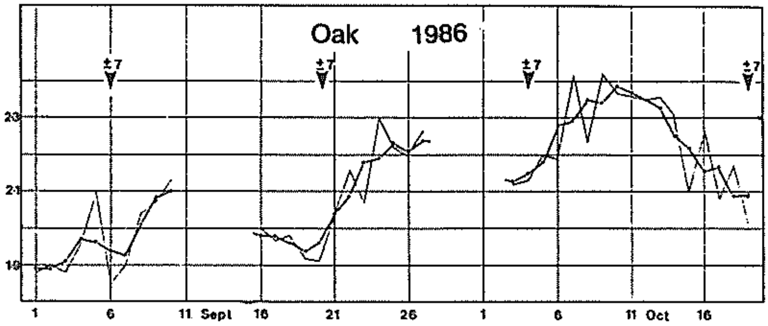


Figure 125. Mean λ values for Oak leaf buds, autumn 1986, with phase-shift from corresponding alignments of Moon and Mars.

Meanwhile we note that a phase-shift of −7 is the same thing as one of +7. At the same time a Beech tree was also running at a phase-shift of ±7. One half of the secondary cycle was completed, and if things were to continue like this one would expect the phase-shift to move into positive numbers. And this is what actually happened, the numbers denoting the phase-shift gradually and consistently decreasing until in due course zero was again reached.

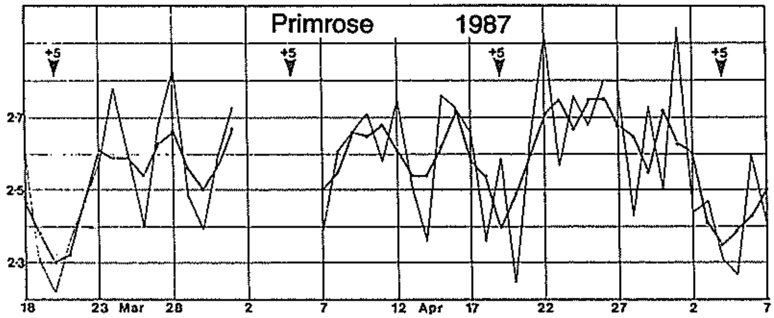


Figure 126. Mean λ values for Primrose buds, spring 1987, with corresponding phase-shift.

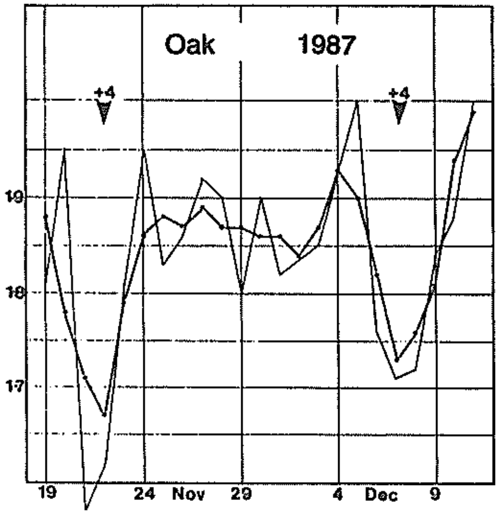


Figure 127. Mean λ values for Oak leaf buds, autumn 1987, with corresponding phase-shift.

In a book of this sort it would seem superfluous to go on printing graphs *ad nauseam;* and only a few typical ones will be shown, of many which have been recorded.

Let us go on to the primrose in the spring of 1987 (Figure 126). Here we see a characteristic graph for the primrose. The heavy curve, showing the main trend, falls below the level of 2.5 just three times, at fortnightly intervals, and on each occasion these coincide exactly with a phase-shift of +5 days.

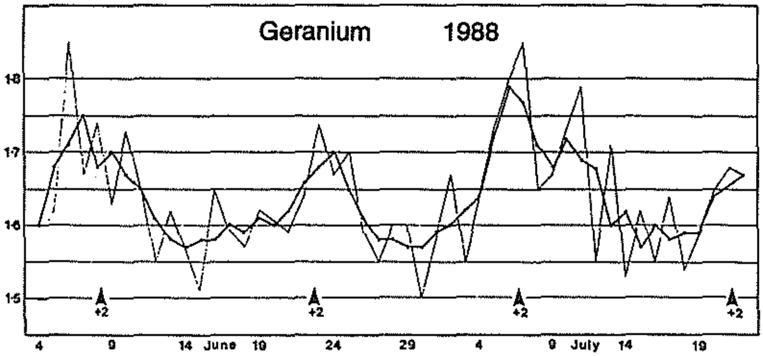


Figure 128. Mean λ values for Geranium buds, summer 1988, with corresponding phase-shift.

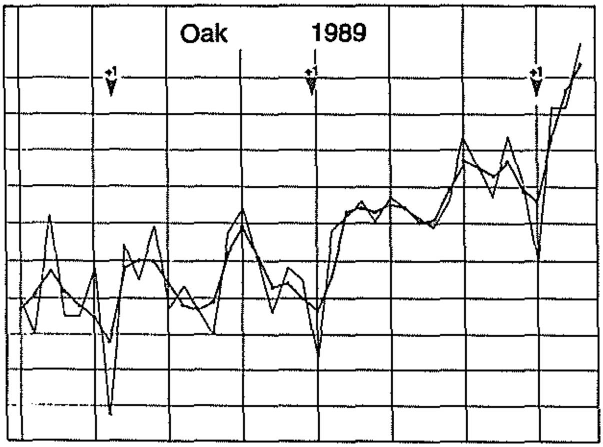


Figure 129. Mean λ values for Oak leaf buds, spring 1989, with corresponding phase-shift.

Six months later, an oak (Figure 127) in Autumn 1987, was running at a phase-shift of +4 days, or maybe, in the case of the first dip, very slightly under.

The following summer, in June and July 1988, (Figure 128) the geraniums gave a most interesting graph, with a series of peaks, the first and third of which came at a phase-shift of +1 day, the third at +3 and the fourth probably at about +2$^1/_2$ to +3 days — a mean phase-shift of about +2 days.

Figure 129 shows the graph for an oak tree in the following spring, February 1989. The phase-shift here has reduced to just +1 day. And two months later, in April and May, the primroses registered a phase-shift of zero. During all that summer of 1989, and on into the autumn, of six different species studied, five had a phase-shift of zero, and one of +$^1/_2$ day. We had come, in the fullness of time, 'full circle.' Once again, the turning points in the curves were coming exactly 'on time' with the celestial alignments, as they had when this aspect of the work started in 1982. The period of the overriding rhythm was not 9$^3/_4$ years as I had at one time supposed, but just seven years. It was now clear that this over-riding rhythm has nothing to do with the lunar nodes, but must have quite other connections, the possibilities of which will be discussed in the next chapter.

Meanwhile we should notice a subtle change in quality which the graphs had shown during the course of the years. If we look back to Figure 114, for the oak in 1982, we see that the dips in the curve are quite short, each lasting not much more than a couple of days. Thereafter, as the phase-shift increased, the graphs showed a change which came on so gradually as to be almost unnoticed at the time. The dips became progressively broader until, in Figures 124 and 125, when the phase-shift had become numerically about a maximum, the curves became almost similar to sine waves. But if we look on to Figure 129, when the phase-shift is only +1 day, we see the dips so narrow that they only show for one day at a time. This means that they are almost obscured in the heavy trend-curve showing the three-day means, and one has to look to the lighter curve showing the daily values to appreciate that they are there. And this quality was seen in nearly all the graphs for that summer and autumn of 1989. Without further investigation it is not possible to be sure that it is always the case, but there does seem to be a tendency that the smaller the numerical value of the phase-shift, the shorter and sharper are the dips in the curve.

Our next task is to examine this varying value of the phase-shift in greater detail; and for this purpose we make a Phase-Shift Chart (Figure 130). Along the vertical axis we mark the phase-shift, going all the way from +14 (fourteen days late) through zero to −14 (fourteen days early); and we have seen that these three phase-shifts are in fact equivalent; thus the 'two' curves which appear on our chart are in fact one and the same, appearing in two of its repetitions. Along the horizontal axis we have 'time' starting with 1982 when this aspect of the work began. Each point on the chart marks the mean of a whole species, for one season. For instance the leftmost point represents the oak for the autumn of 1982; the third from the left shows the primroses for the spring of 1983; the fourth

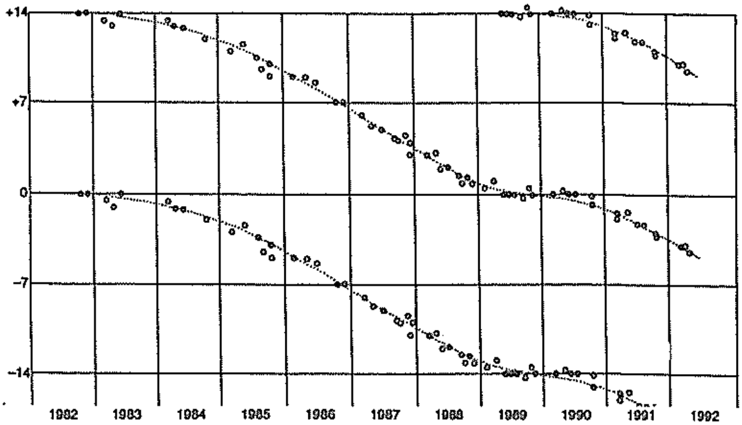


Figure 130. Overall phase-shift chart, 1982–92.

from the left, the stitchwort for the late spring of that year; and so on. Thus each point represents the results of measuring some 500 to 2000 buds.

That early extrapolation to find the periodicity of the overriding rhythm which led to an answer of between nine and ten years was based on the assumption that the rate of change of the phase-shift was constant from month to month and year to year; that is, that the points on the phase-shift chart would lie on a straight line. When later I came actually to plot these points it soon became apparent that their form is anything but straight. And the form of the curve explains fully why its periodicity had been so difficult to predict, and also why during those months of 1983 and the early part of 1984 the phenomenon had remained unnoticed. But even up to the end of 1989 it was not possible to guess what the curve would do next. After levelling off on the zero line during the summer and autumn of 1989 would the curve plunge downward again, as it had in 1983? Or would it decide to turn upwards again?

I had no means of predicting, and for the first eight months of 1990 the buds continued to keep me guessing. However with the coming of autumn the trend for the future really seemed to be showing itself. During that season, of three beech trees studied, two seemed to be showing a very slight and rather uncertain tendency to respond maybe $^1/_4$ to $^1/_2$ a day early, and three oaks were united in showing a phase-shift of fully minus a whole day. By the spring of 1991 the trend seemed to be fully established. A long period of measurements on a beech tree then showed a clear mean phase-shift of about –1$^1/_2$, and an oak at the same time showed one of –2. And so it has continued up to the moment of publication of this edition.

Of course no-one can predict what will happen next. I can only record the phenomenon as I have measured it, with the greatest care. The phase-shift chart shows a consistency of rhythm and symmetry which leaves me rather bewildered but at the same time somewhat reassured. I know of no other phenomenon in the world of nature quite like it. Maybe it is something which can only show itself just in this very subtle field of form which we are studying here. But if we cannot understand it, at least we must take note of it.

We see that the phase-shift was zero about January 1983 and then again about January 1990 so we have to say that the periodicity of this variation is just about seven years. But this is almost certainly an oversimplification of a much more complex and subtle reality. Seldom does nature present us with any rhythm which works with clockwork regularity. Always there are other overlying rhythms and also sometimes incidental irregularities as well. Particularly is this the case in the astronomical realm. For instance we read below (see page 252) that the periodicity for the alignments of Moon and Saturn is 13.7 days and that this figure has a constant value. Considered as a *mean* this is true, but the actual figure varies month by month by several hours, above and below this value. A careful examination of the phase-shift chart reveals that something of the kind seems to be happening here. There is slight but distinct evidence to show that the rate of change of the phase-shift has been quickening somewhat in the most recent years. The curve is more sharply curved in 1990/91 than it was in 1983/84. In autumn 1984 the phase-shift was –2 to –2$^1/_2$ days, but in autumn 1991 it was –3 to –3$^1/_2$ days. An extrapolation on these figures would give a value for the periodicity of about 6$^1/_2$ years. On the other hand the curve between January 1988 and January 1992 appears to be exactly symmetrical about the January 1990 line. Is this an indication that the quickening of the rate of change has now reached its maximum, and that it will begin to revert to a slower rate of change henceforth? This is a new phenomenon and we are filled with questions. Only time, and much more research, will tell.

Having surveyed the strange business of this varying phase-shift on the phase-shift chart, there are a number of further questions which may be asked. The first of these is: How widely spread is this phenomenon in the field of nature? Obviously this cannot be properly answered without a great deal of further work being done. The work is exceedingly time-consuming, and I have not had time to do more than start upon it. In my description in this chapter I have deliberately confined myself to five species, three of flower buds and two of tree buds, in order that comparisons from year to year can be the more easily made. But I have many graphs for the leaf-buds of birch (varying with alignments of Moon and Venus), the sycamore (Moon and Jupiter) and the flower heads of knapweed (Moon and Jupiter, the alignment being marked with a peak in the curve, as with the geranium). In each of these cases the current phase-shift has been the same as with the various species which feature on the phase-shift chart of Figure 130. Further than this nothing can be said except on the basis of more research but on the available evidence it would seem probable that this is a widely-spread phenomenon. In this connection we should always consider the possibility that some species might be found to be sensitive to the alignments of more than one planet, and in such a case it would of course be much more difficult to establish.

The knapweed proved an interesting case. It grows in wild profusion all over our lower hillsides and meadows here in the north-west of Scotland. Being one of the Compositae this species does not produce a bud in the normal sense, but rather a bud-head of little flowerlets contained in a small brown case. In the course of their long budding season (over two months) these cases undergo considerable changes of shape, but remain the whole time as almost perfect path curve forms. They are covered with little hairs which are set in perfect path curve spirals, but which pose a problem for the researcher, making it difficult to get a clear measurable image. However by soaking the buds for a short time before measuring, the hairs are laid and a good clear outline can be obtained.

When the buds first appear they have very low λ-values but in the course of the next few weeks these values steadily increase. Just before the casing opens, to reveal the lovely inflorescence of little purple flowerlets within, a small white spot appears at its tip, and the first appearance of this spot is a firmly identifiable moment of development at which the buds may be picked day by day. However these spots do not begin to appear until the budding season is halfway finished and if one confines oneself to them one loses half of one's data. The policy is therefore followed of starting work when the first buds appear, choosing each day twenty of the most fully developed buds that can be found, and continuing like this until the time when the first white spots are beginning to show. Following such a policy one would expect to see a steadily climbing curve on the λ-chart, until near the end of the budding season.

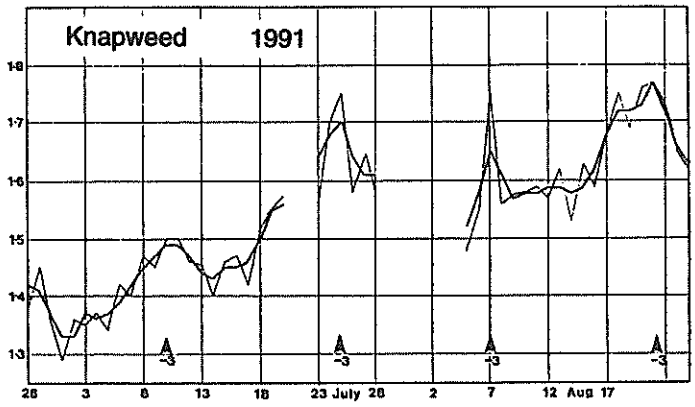


Figure 131. Mean λ values for Knapweed buds, summer 1991, with phase-shift from corresponding alignments of Moon and Jupiter.

What in fact eventuates is a curve increasing in a series of fourteen-day leaps. The first year I discovered this, the maximum points of these leaps coincided fairly closely with alignments of Sun and Moon (new moon, full moon, etc) but also with alignments of Moon and Jupiter.

Experience in subsequent years went to show that the soli-lunar relationships are irrelevant to this phenomenon, but the peaks in the curve continued to coincide with the Moon-Jupiter alignments very closely, always consonant with the current phase-shift as shown by the other species being studied, and as can be read off the phase-shift chart of Figure 130. Figure 131 shows the results from the latest series of measurements to be taken, completed in August 1991. The arrows show the moments of Moon-Jupiter alignments, altered by the indicated amount of phase-shift. This graph is typical of the results found in each of the five seasons in which this species has been studied.

The knapweed proves to be of special interest because some of these plants are to be found growing underneath the high tension cables. In my immediate neighbourhood the number of buds in such a situation is not very great, and it is often not easy to find a sufficient number of buds at the requisite stage of development, each day. However on two occasions I have managed to follow such buds through a large part of the budding season. Figure 132 shows the result of this in 1985. The continuous curve is derived from buds growing on open hillsides, and the dotted curve shows those which were gathered from under the cables. Both these curves are

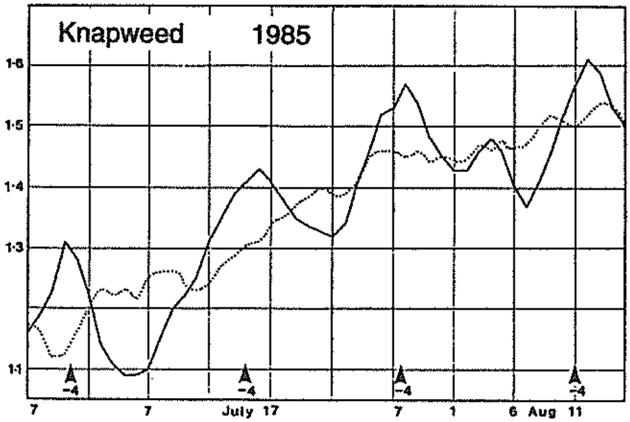


Figure 132. Mean λ values for Knapweed buds, summer 1985, with corresponding phase-shift. The dotted curve shows buds gathered under high-tension cables.

for three-day means. On the only other occasion on which I have so far carried such an exercise through, the results were similar to this, although rather less striking. Much more research is needed here.

Another question often arises when I am describing this work to someone, who then says: 'Well, weren't you *lucky* to have started this phase of the work just at the time when the phase-shift was zero. Suppose you had started it at some other time. . . .' Well, yes, indeed. Suppose I had. For instance let us consider Figure 123 once again. If I had started the work in November 1985 I would have found that the dip in the curve for the beech on November 9 coincided with an alignment of Moon and Mars, and furthermore the dip on November 27 also fell on the same day as the next alignment of Moon and Mars. If I had been quicker at drawing conclusions than I am, I could then have gone around saying that I had evidence to show that the beech buds vary in the rhythms of the Moon/Mars alignments. However, if I had continued on such lines I would soon have found the Moon/Mars alignments rapidly, and in the course of time erratically, growing out of step with the behaviour of the buds, very much more rapidly than the exceedingly slow, and steady, movement of the phase-shift. And if I examine several years of observations, armed with an ephemeris I can find dips in the beech curves to coincide with alignments of Moon with almost any planet which I wish. What right have I then to say that the Saturn alignments are the significant ones, rather than any of the others?

The answer to this is, that when synchronicity lets us down, as it clearly does as soon as the phase-shift is acknowledged, periodicity comes to our aid. The period between one alignment of the Moon with any particular planet, and the next such alignment, varies somewhat from month to month, but the *mean* period is quite constant, is different for each planet, and can be easily and accurately calculated. The following table shows these mean periods for each planet. The lefthand column of figures shows the actual period, and the righthand column shows how much this period would be if one takes into account the seven-year period of the changing phase-shift.

Sun Mercury Venus	14.77 days	14.68 days
Mars	14.23 days	14.15 days
Jupiter	13.75 days	13.67 days
Saturn	13.70 days	13.61 days

The beech is the species for which I have by far the longest and best sets of observations. Going through my figures I find that the mean period between dips in the curve comes out at 13.60 days. This is from many thousands of observations and must be very close indeed to the correct figure. We see in fact that it matches the Saturn period almost exactly and it could not possibly apply to any of the others.

A similarly calculated figure for the oak came to 14.29 days. This comes very close indeed to the figure for Mars, and again could not possibly apply to any of the other planets.

The periods between turning points in the curves for stitchwort and geranium, for which I have much fewer observations, come to 13.45 and 14.38 days respectively. These do not tally with Saturn and Mars with quite the same precision as the two preceding examples, but they are very close, and again could not possibly apply to any of the other planets.

Similar remarks apply to the knapweed and Jupiter with a period between turning points of 13.87 days.

When we come to the cases of primrose and birch the situation is rather more difficult. With periods between dips of 14.63 and 14.69 days respectively they equate with quite remarkable precision with the periods for Sun, Mercury and Venus, but considerations of periodicity alone do not make it possible to distinguish between these three heavenly bodies. Sun and Mercury stay so close together in the sky that it will always be difficult to differentiate between them. Venus on the other hand moves quite widely apart from the Sun, from time to time, and there is fairly strong evidence, on two or three occasions, that the Venus alignments do not always fit with the primrose dips, but that they always do with those of the Birch. This, combined with Rudolf Steiner's intuition on the matter, gives me fair confidence that the birch and Venus are truly linked. With regard to the primrose I can only say that it is my belief that this plant varies with the alignments of Moon and Sun, but that as far as the observations go at the moment it *could* equally be Moon and Mercury.

These considerations give us good evidence that the correlations between planets and plants which were occurring on time in 1982 and again in 1989 are in fact the correct ones.

It is important that work such as is being described here should be treated with all sobriety and caution, and in this respect another question needs to be confronted. Apart from those of 1982, all the graphs shown in this chapter are depicting mean values. Now mean values are never exact; they always contain an element of uncertainty owing to the random picking of the buds; on any particular day an extra number of high, or low, values may have coincided. It is important to be sure that the size of the dips is larger than this degree of uncertainty; if it is not so one can hardly consider the dip as being something of significance. By analysing our measurements we can find what is called their standard deviation, and from this we can calculate, by statistical theory, the probability that the mean values will stay within certain limits. For instance we can calculate the limits within which our mean values can be expected to lie with a probability of 95%, and any mean value which reaches this limit is said, with the most sober statistical judgment, to be *probably significant* in its difference from the Mean of all the means. Further we can calculate the limits between which all values may be expected to lie with a probability of 99$^1/_2$% and any result reaching this value is said to have a difference from the Mean which is *definitely significant.* In practice any value which comes between these two limits — the 95% and the 99$^1/_2$% — must be considered as being really significant in its difference from the mean.

In this connection Figure 133 is interesting to study. It is one of a number of similar diagrams to be found in my notebooks. It represents the results of an extended series of observations on a Beech tree during the winter of 1989/90. The arrows show the actual moments of the Moon/Saturn alignments, the general phase-shift at that time being as near zero as it was possible to ascertain. By taking nine-day means one can smooth almost all the short-term variation from the curve, and the

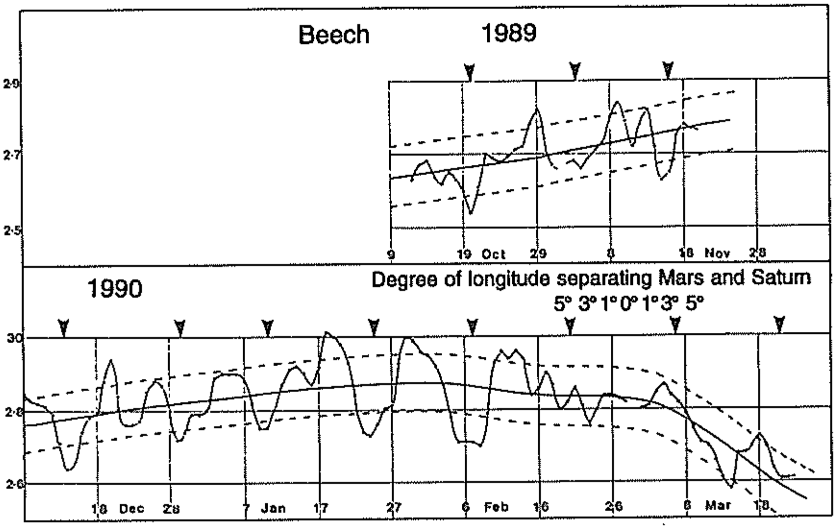


Figure 133. Mean λ values for Beech leaf buds, winter 1989–90, with corresponding alignments of Moon and Saturn, apparently influenced by the conjunction of Mars with Saturn in February/March.

central, continuous curve, showing the general trend of the variation during the whole of the winter, was obtained in this way. We see a slow and steady rise in λ until the early days of February, followed by an increasingly steep drop towards the end of March. This latter is probably due to the bud starting very slightly to swell, prior to its opening a few weeks later. The dotted curves above and below it show the two standard-deviation confidence limits. Any variation beyond these limits has a probability of significance of more than 95%. Between the middle of October and the middle of February, during the time of actual observations, we see that there were eight alignments of Moon and Saturn, and all of these are acknowledged by dips in the λ-curve; and in no less than seven of them the dips come well below the 95% limit; and they do so on no other occasion.

However, on February 28, Mars moved into conjunction with Saturn, and since on a number of occasions in the past these two planets have seemed to inhibit one another when they are in alignment, it was interesting to see, once again, what would happen. The numbers on the graph during the last half of February and the first week of March, show the numbers of degrees of longitude separating Mars and

Saturn. We see that the fortnightly dips in the curve, coinciding with alignments of Moon and Saturn, which had been regularly present during all the preceding months, cease from the middle of February onwards. To my disappointment they were not re-instated even by the end of March, when observations had to be discontinued. Possibly this was due to the fact that we had come so near to the end of the season. Apart from this, this general result was one which might have been expected from past experience.

What was not expected was that, once the influence of Saturn had been annulled, there appeared to be a smaller, residual variation, involving two peaks, on March 5 and 18. If this were all the evidence which I had to go upon, I would have been inclined to treat this simply as 'noise' in the graph. However, it happened that I was at the time doing a series on another beech tree growing on the other side of the river, and I found that this was behaving in almost exactly the same way. The behaviour of these two trees was so similar that I was forced to consider the possibility that this residual variation was a significant reality. These little peaks in the curve coincide very closely with alignments of Moon and Jupiter. Can it be that while the beech responds principally to Saturn, it is also, in a much slighter way, influenced by Jupiter? Much more evidence would be needed before such a statement could be made, and in the normal way it would be very difficult to get. I simply record the suggestion here, in the hope that one day someone may be able to make observations which either support, or reject, it.

The main purpose of Figure 133 was to examine the degree of significance which we can attribute to the dips in these λ-curves, and to show that it is nearly always greater than 95% probability, and usually nearer to 98% or 99%. Almost all the similar graphs which I have in my notes are of similar nature in this respect.

Just one year later, in the spring of 1991 there was an interesting sequel to this. A further beech tree studied then, yielded the graph which is shown in Figure 134. We have here a tree whose behaviour, for the first part of the period under study, was almost identical with that of its fellow a year earlier; however, on March 16, Jupiter moved into alignment with Saturn; already in the first week of March the two planets had come close to opposition, and again, as in the previous year, we see significant variation ceasing from this moment on. It would seem that we have to posit a rule that rhythmic variation of the sort which is being described here is often strongly inhibited when the ruling planet of the species is encumbered by alignments with other planetary bodies.

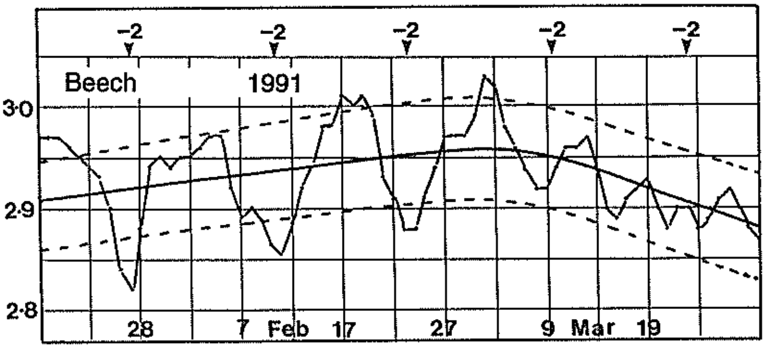


Figure 134. Mean λ values for Beech leaf buds, spring 1991, with corresponding alignments of Moon and Saturn, apparently influenced by the conjunction of Jupiter with Saturn during March.

Just as this edition was due to go to press an important new phase of the research was opened when I was joined in the work by my friend Graham Calderwood of Aberdeen. For the first time it became possible to observe two trees simultaneously, growing in different parts of the country, the work being done by two completely independent observers. We had very short time, from the first appearance of the buds near the end of August, until the middle of October, so only the merest hint of a beginning can be reported here.

We decided to start with two beech trees. They would be growing in quite different environments and almost opposite climatic conditions, the one in Strontian in the moist and mild air of the west coast, and the other in the cold and bracing climate of Aberdeen. They would be separated by some 160 miles, and observed by quite independent observers.

Before we can assess the results of this particular piece of work we must be aware of the special conditions imposed by the limitations of time. The beech buds are not properly fully developed until the first or second weeks of October. If one starts work before that time, the experience of many years' work shows that one must expect a quite typical pattern to show itself in one's results. Starting near the beginning of September one finds λ to be abnormally high, and during the next six weeks one measures a steady fall, the fall however being punctuated by the fortnightly dips in the curve which correspond to the Moon/Saturn alignments, always with due allowance being made for the current phase-shift, of course. Each dip in the curve is strongly marked but

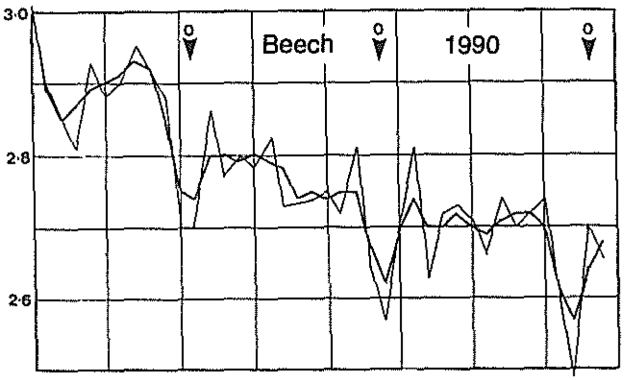


Figure 135. A typical mean λ curve for Beech Leaf buds, autumn 1990.

followed by a comparatively small rise, followed by a very strong dip again. Figure 135 is a typical curve from previous years; of the three dips shown, two have zero phase-shift and the other fully –1; we could say a mean phase-shift for Autumn 1990 of $-{}^{1}/_{3}$.

Figure 136 shows the observations made in 1991 in Strontian, and pictures an almost identical result to 1990, except that the phase-shift has in the meantime advanced to about –3. The last dip, on October 13 was obviously present, but illness prevented me from recording it in detail.

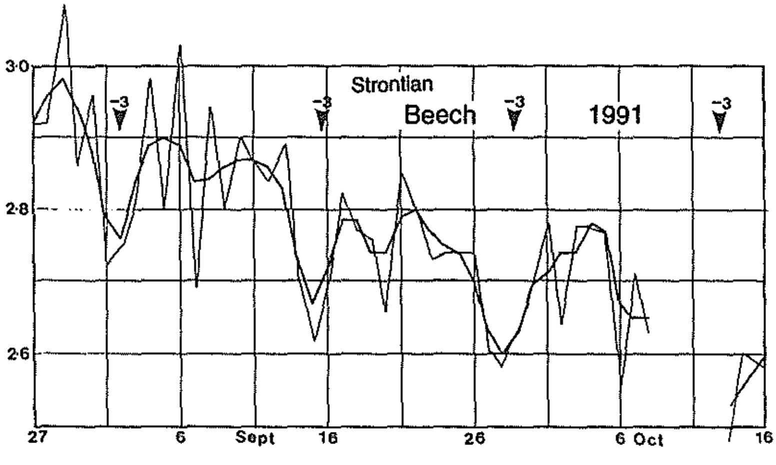


Figure 136. Mean λ values for Beech leaf buds observed in Strontian, autumn 1991.

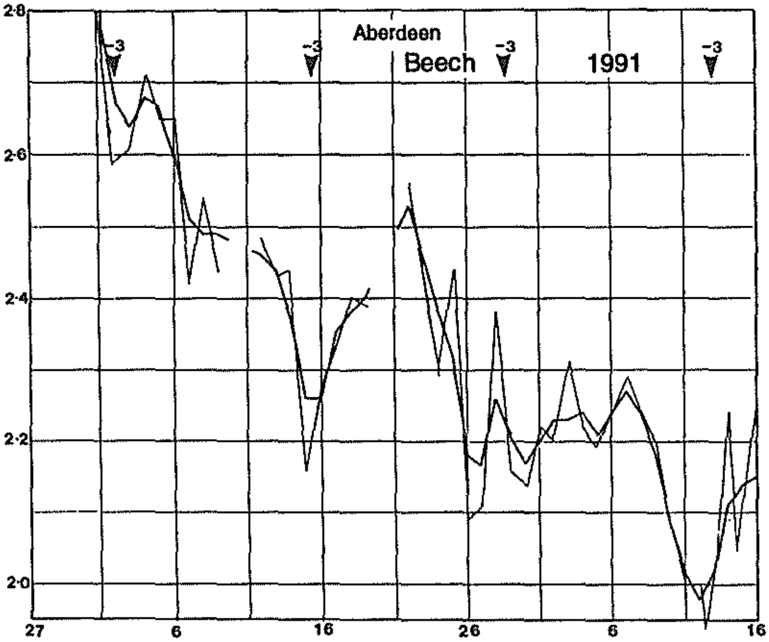


Figure 137. Mean λ values for Beech leaf buds observed in Aberdeen during the same period as in Figure 136.

When we look to the Aberdeen result (Figure 137), seeing that so many variables are present, we could not expect that the curves would be exactly the same. Nevertheless the resemblances are substantial and reassuring. The same general downward trend is seen, punctuated by the same dips on the same days. One could hardly have expected better corroboration. What was not expected is the much greater liveliness of the Aberdeen tree, its greater range of variation. Whether this will prove to be characteristic of the two places must remain a question for further research to answer. But what is of importance is that we have here the first piece of objective evidence that the phenomena described in this chapter are indeed not confined just to the west coast of Scotland, but are to be observed by other people, in other places.

16. From Space to Time

This concluding chapter has a somewhat different purpose from those that preceded it, and it must be rather differently written. Up to this point it has been my aim to remain as far as possible with the normal processes of thinking and observation which are available to the ordinary man or woman of our day, and to see just how far these can carry us in the direction of such investigations. Anyone reading thus far must be aware that that far-seeing philosopher of the early years of our century, Rudolf Steiner, has provided inspiration for my efforts; yet his name appears but rarely in this book, and his words hardly at all. The things which I have written here do not stand under the authority of Rudolf Steiner. The only authority I can claim, or would wish to claim, is that which derives from the nature of thinking and the characteristics of ordinary sense observation. Study the geometry, I say; go out into the field; measure the forms of heart, and egg, and cone; gather the buds, and see whether they are behaving as I have described. This is the only authority which I can give.

When I started on this line of research, over forty years ago, I had no idea of where it would lead. And it has led into realms of experience which I could not have guessed. And some of them, to the ordinary thinking of our day, must seem strange indeed. What I have written so far is, for the most part, simply a factual description of what I have found. If, by following the indications given by a man like Rudolf Steiner, who has shown himself in many respects to have insights which go beyond the normal consciousness of his day, we can see these facts in a new and comprehensive perspective, the attempt is surely worth making.

We consider his vision of the living organism, seen as a whole thing — a single, unitary entity, gathering, subsisting on, and dominating, its parts, imposing on them a web of relationships which spring from its own essential nature. It is perhaps a matter of temperament whether one prefers to see the organism simply as the sum total of its separate parts, or as a thing-in-itself, which is the real cause and reason for those parts being there at all. Both views can be defended and may be of use, but surely it is with the second one that we come near to the heart of reality. Rudolf Steiner was speaking against reductionism many years before the word itself had been coined.

An extraordinarily intimate interdependence between its various parts is one of the distinguishing marks of the living organism. No single part can change, or be changed, without all the others being affected, and responding. Another typical mark of the living organism is that it stamps something of the quality of its being, considered as a whole entity, in one way or another, on each of its component parts. Each part becomes, in its own way, a little image of the whole thing whose life it helps to support.

A form which is such that each of its parts resembles the whole thing is sometimes said to be self-similar, and this quality of self-similarity has been discovered in remarkable ways by the recent computer-aided work which has come to be known as Chaos Research. Here mathematicians, by repeatedly iterating quite simple processes, in ways which are somewhat similar to those by which we produce our path curves, have discovered forms of quite amazing complexity and interest. Not only does each part mirror the form of the whole, but each part of each part, and again each part of each smaller part. The modern computer is able to carry this process to great lengths; there seem to be no limits, theoretically, to where one could go; it is possible to say that such forms are almost literally of infinite complexity; and yet they are, in a way, of great simplicity, in that the one motif informs all their parts.

Such forms are said to be fractal in quality. Chaos researchers have come to see this fractal quality, in a more general way in the forms of much that surrounds us, of mountains and their component boulders, of coastlines and clouds and trees. Moreover they have detected fractal qualities in many processes occurring in time, mysteriously in recurring weather patterns, in the fertility of crops year by year, and so in the changes of prices on the corn and other world markets. It is as though organisms exist in the realm of time whereby variations during short periods are found remarkably to mirror those of longer duration.

Artists have long known how to impart a sense of organic unity to their work by ensuring that each separate part of a picture, each in its own way, mirrors forth the essential theme which pervades the whole; and the beholder, perhaps unconscious of the device being used, nevertheless greets this feeling of wholeness with satisfaction and joy. Great works of music, especially in their more fugal moments, where some simple theme appears and re-appears again and again, in one variation after another, fall into the same class. In a burst of the two processes go hand in hand, every big outbreathing being preceded by a little moment of inbreathing.

At this point the reader might object that while it is interesting to find such rhythms at work so widely in the plant kingdom, if one reads Chapter 7 critically one will realize that the actual changes of shape recorded there are very slight in magnitude. Can we really believe that such slight changes are of significance? In this connection we should bear in mind the work which is described in Chapter 14. This work is some of the hardest to verify with certainty of all that is contained in this book. The results of any single season's work could not be considered as more than suggestive. But over no less than six years' work the results have been so consistent that I think they must be given credence. And, taken over all these years, they show that there really is a correlation between subtle and slight differences in form and the actual way in which the buds grow, in which the life forces manifest in them. It would seem that just this form which has come down to us from such ancient days is especially sensitive regarding the minute proportions of its shape. And there is more than a little evidence that just the pure FORM of the bud, considered as a thing in itself, can act in some way as a medium, a bearer, of life force. And when we consider that all the winter, and in the spring, we are surrounded by countless millions of such little buds, all acting in concert, the work described in Chapter 15 begins to assume great importance in our minds.

The phenomena which began to reveal themselves as soon as I persisted with a consistent daily record of the λs of the buds can be described I think, without exaggeration, as revolutionary. We begin to see this world in which we live as a vast organism, in which the roles of planet and plant are intimately interwoven, one with the other. No star can move, but a plant responds. The glory of the heavens is not only spread out above our heads; its rhythms come to meet us from the ground under our feet.

These little approximately fourteen-day rhythms — what I once referred to as the Dance of the Buds — like a great heartbeat of nature, have they really been going on, all these ages, all around us, un-noticed? I was slow to allow myself to be convinced. I said: 'This once, this twice, these three times, these buds have responded to the astronomical alignment; it may have been coincidence; what of tomorrow? next month? next year?' The next phases of the work started as a determined effort to test the truth of the matter, with all objectivity possible, over a long period. But its character soon changed. There were surprises in store. Things did not always go the way I had expected. This group of phenomena proved more complex and wide-ranging than I could have foreseen. The search for a proof of the reality of one phenomenon changed into a quest for a clear overview of a whole field of related phenomena.

The first thing, as I first saw it, concerned the leaf buds of our great deciduous trees. All the long winter they lie on their branch, awaiting the great day when, in the spring, they will open to the light and air of the world around them. This opening will be denoted in the geometry when their λ plunges away down into the negative numbers. But all that long winter they are not idle. Roughly once in every fourteen days their λ makes a little fall. All the buds of any particular species make a little gesture, all in concert. It is rather like an opening of the hands, a gesture; of what? Of asking? Of receiving? Of gratitude? Who can say? But almost immediately it becomes clear that it is too early; the time is not yet . . . not yet. λ returns to its normal value; the bud closes tight again, for another fourteen days of waiting.

This 'breathing' rhythm of the deciduous trees is, I believe, the basic, the archetypal one in this realm, and having watched and measured it for nearly nine years now, continuing unfailingly, I can have no doubt of its reality.

But it soon became apparent that these things go far beyond just this rhythm of the deciduous trees. There is convincing evidence that some species of flower buds, opening in spring and summer, follow somewhat analogous rhythms. The work is much more difficult here. One cannot follow any species for longer than the period of its budding season — some four to six weeks in a favourable case — and any results which one may find cannot be relied upon unless they have been confirmed in several different years. Many species are not suitable for this kind of observation because they do not present us with a clearly identifiable moment of development in the course of their opening. Here the great diversity of the plant kingdom shows itself; sometimes a species will show, not a decrease, but an *increase* in λ, to coincide with some particular alignment, year after year. Why some species should work the one way, and others the opposite way, I do not know. How widely this phenomenon is spread through the plant kingdom I do not know. There is a whole world of discovery waiting to be made here.

But meanwhile we should return to the main phenomenon, the breathing rhythm of the deciduous trees, and try to see it in relation to the breathing rhythm of the year. During the autumn this great plant garment of the earth is busily withdrawing its essence within itself, until at last its virtue is totally concentrated into the outwardly insignificant forms of the seed; also in the case of the great trees, into the forms of the tightly closed leafbuds. By midwinter the earth is filled with these little 'fire-points' of the seeds, and the great trees stand gaunt, and bare of their foliage. Rudolf Steiner points out that then the earth is fully inbreathed, fully *awake,* and waiting.

Following this the opposite process begins. Week by week, and month by month, in sprouting green, in opening bud and unfolding leaf, form by form and colour by colour, the plants move on to the open manifestation of their glory and beauty in mid-summer. And then one can see the Earth, in the long hot days, amidst the droning of the myriad insect life, given over to its midsummer dreams. The Earth is fully outbreathed.

But half way during this time, at mid-spring, the outbreathing process is at its strongest. This is the time when, each year, we experience anew the Easter miracle: the transformation of Egg into Cup. For these are the two ancient symbols of Easter and the Easter time of year, are they not? We have the enclosed, and enclosing, form of the egg, and the bud, enshrining within them the forces of our most elemental beginnings; and we see them, before our very eyes, changing into the form of the cup, the chalice, open to whatever the future may bring. This Easter mystery is enacted before us each year, and only custom, and the dullness of our spirit, blunt the full sense of wonder which we should always feel for it.

And in the geometry, all we need, that this miraculous transformation should be fulfilled, is that λ should move from the positive to the negative numbers. These two types of form are really one and the same thing. We cannot have the one without the possibility of the other being present (Figure 138).

And now we think of those long winter months. Approximately every fourteen days the tree experiences a 'little Easter,' all in consonance with the music of moon and planet. Just for a few hours λ falls, the buds take a little step, just a little one, in the direction of their Easter apotheosis and then — the time is not ripe; and another fourteen days of waiting commences.

And one may feel urged to ask the question: 'Do our great trees, twice every month, live once again the glories of a past Easter? Or are they living, in anticipation as it were, that which is to come?' Who can

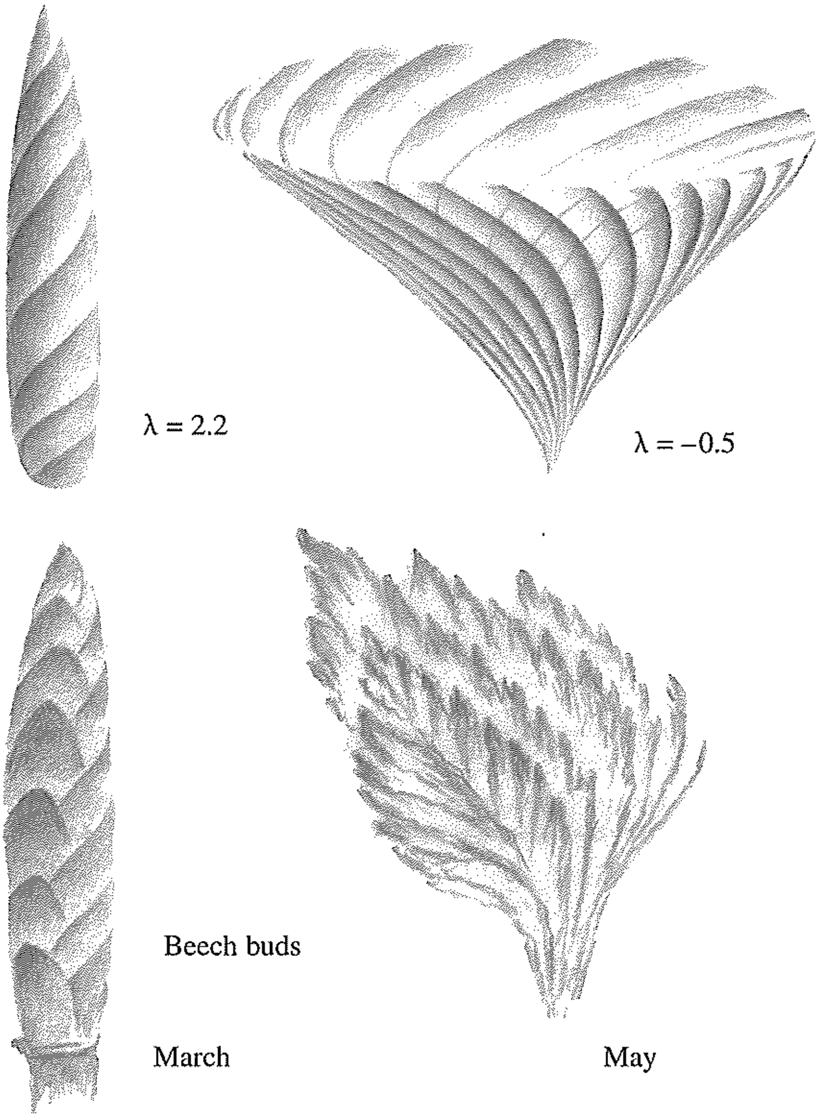


Figure 138. The Easter Miracle.
Above: Archetype: a simple fall in λ changes the enclosed form of the bud into the open chalice of the vortex.
Below: Phenomenon: the passage of time transforms the tightly-closed bud into the open whorl of new leaves.

say? Perhaps the question is unnecessary, even irrelevant. The important thing to realize is that this great organism of transformation in which, as Earth dwellers, we are all immersed, is fractal throughout. It is fractal in space, stretching from the heavens above, to the ground under our feet: as Above, so Below. And it is fractal in Time; each little portion of the winter mirrors the culmination of the year: as in the Great, so in the Small.

But we are not able to see this picture whole as long as we exclude ourselves from it. The antithesis between human and plant is remarkable, and complete; the one has to be recognized as an inverted replica of the other. The plant obtains its nourishment from below upwards, by means of its roots, and stem. Man obtains his from above downwards, through mouth and gullet. The plant holds its reproductive organs — the blossom containing ovum and seed, and so on — upwards and opens them to the light and the air. In the human, all the generative organs, both male and female, point downwards, towards the darkness. Reproduction in the plant takes place in a realm of beauty and innocence; in the human, over long ages, it has become associated with feelings of guilt and shame. The one can be seen pictured in the steady flow of the green sap, and the other in the hot pulse of the thick red blood. Red and green are complementary colours.

In the middle realm, that of the spreading leaves, we find the breathing organism of the plant; so also in Man, as we have already described. Draw a picture of bronchi and lungs, and show it to any class of schoolchildren. The cry goes up straight away: 'Why, it's a *tree!*' The recognition is immediate and spontaneous. And then: 'But it's upside down!' Even the chemistry is back to front. The plant breathes in carbon dioxide, and oxygen out; with the human it is the other way round. Plant and man are the perfect companions to live side by side on earth; what is life to one is death to the other.

All this being so, we should count it as no surprise that in the middle sphere of man, in the breast, this ancient form of our evolution should appear again, in the ventricles of the heart. Just as in the middle sphere of the plants, that of the leaves, the buds appear, characteristically pointing upwards, and finding their final metamorphosis in the blossom which opens in the expanses of the heavens, so in the human we find this form primarily represented in the ventricles in the breast, but coming to a kind of metamorphosis below in the downward-pointing form of the uterus.

And these phenomena are oppositely-related also in their behaviour. All the long winter the tree buds are waiting, tightly closed, for the moment of their apotheosis in the spring, when with a plunge of their λ they will open to the light and the air of their surroundings. When we come to consider the cycle of the heartbeat we find that most of the time the ventricle lies there, relaxed and rounded in form, with low λ, in a condition of diastole, waiting for the moment of culmination, when with the coming of systole, λ shoots up high and the form becomes tight and tense. This we could call 'the inner Easter,' the magic moment, lasting only the twinkling of an eye, which we have to associate with something like an ecstasy of 'pain,' using that word in the sense of the descriptions of Chapter 8.

And just as, approximately fourteen times during the long winter, the trees experience their little Easter, so in the course of each single heartbeat, the ventricle suffers *its* little Easter, just seven times. Is it at this moment living anew the ecstasy of the last systole, or savouring in advance the one which is to come? Who can say? But one thing is clear: just as in Figure 72 we endeavoured to picture the heart as a sevenfold organism in space, here we are dealing with it as a sevenfold organism in time, and as such we find it to be fractal throughout. Each seventh part of the cycle mirrors the form of the whole.

And now we can begin to see the world as organism, as an organic unity, in which Man and Nature are immersed in one another, both parts of the same whole. And the ancient Hermetic teaching appears again: as Without, so Within. But the inner picture is inverted, and introverted: inside out.

The changing phase-shift

The picture which has emerged up to this point is one with which I would willingly have been content. It seemed to me that it was one which was, in a way, complete and satisfying. The further work which I next proposed to do was with the purpose of verifying that this wonderful rhythmic interweaving of star and plant, and its reflection in the pulse really represented the truth. It was something which needed to be tested, and tested, and tested yet again. And it was in the course of this next phase of the work that the strange business of the changing phase-shift began to make itself apparent. I did not expect it, nor did I seek it. You may say that I did not want it. I was happy with things as they were. The phenomenon asserted itself so gradually that at first I did not notice it. Only later, when I came to look over my earlier figures did I see that the first signs of it had already been there. When I could no longer ignore it, I resisted it. But it continued, and grew through the years, with such consistency that at last I had to acknowledge it.

It was years before its true form became apparent, and even now we must be prepared to modify our view of it in the light of further observations. Nevertheless I believe that Figure 130 really does give a true idea of the way it is working. To begin with I toyed with the idea that it might have been due to some sort of atmospheric or electrical pollution, but the consistency and, above all, the symmetry, with which it was seen to work through many years, has convinced me that it is indeed a natural phenomenon. For a time I believed that the rate of change would be constant until observations over a long period proved the opposite.

And now, what can one say about a thing like this? I do not know any other place in the natural world where quite such a thing has been observed. At the moment it shows me only one thing with certainty: that this world is a more complex, and wonderful, place than the present range of my concepts can embrace. And I think it will be many years before the last word can be said about this matter.

Perhaps it is premature to speculate about such a thing at this moment, but I cannot bring myself to close this book without expressing a few thoughts which may indicate a possible future direction which research may take. And I can find a basis for such thoughts only in the work of Rudolf Steiner. On numerous occasions he stressed the importance of our coming to a true understanding of the element of Time, time seen as the ongoing *activity* of spiritual being. We cannot go here into the complexity of all he had to say about this subject, but we should remember that he distinguished two separate streams of time, the ordinary one we know in this our physical world, which flows from past to future, and another negative, backward-flowing, stream, belonging to a more spiritual world which interpenetrates our physical world. Maybe in certain special circumstances this backward flow of time, streaming from future towards the past, can, in some sense, overflow, as it were, into our physical world. And a symptom that such a thing is occurring would be when we find the Effect coming before the Cause. The whole realm of these egg-like forms, which expresses itself in bud and cone, in blossom and seed, in ventricle and uterus and pineal gland, which then works into the forms of developing embryos, and manifesting in the vortex movements of water and air, has shown itself to be sensitive to the most subtle differentiations of form, and also to be intimately concerned with the genesis and the flow of time: would it be possible that this subtle realm of form is one into which something of this backward-streaming time could sometimes overflow?

I think we have to be very careful indeed in our working with the ideas of Cause and Effect; and in our modern thinking we often apportion these concepts with undue confidence. We believe that everything in life is subject to the laws of cause and effect; but, is it? And especially when dealing with the living organism, the matter is often far from clear. There is a certain centre in the brain which controls the beating of the heart. If this centre fails, the heartbeat ceases. Can we then say that the brain is the cause of the heartbeat? But if for any reason the heart stops beating, the brain dies. Can we then say that the heartbeat is the cause of life of the brain? Rather must we see these two aspects of the organism as being intimately interrelated with one another, but neither having ultimate precedence over the other. And when we come to view this mighty organism of our world, involving the closely interrelated activities of plant and planet, and their complex reflections in the organisms of man and animal, this is I believe a good and fruitful way to proceed.

Nevertheless, the concepts of Cause and Effect are also truly operative in many places and many ways, and should also be taken into account. There are places in the world process where I can say of something: 'Here is a cause, and from it a certain effect can be expected to follow.' In this strange interrelation between star and plant it would not be possible for me to imagine that the Moon comes into alignment with Mars *because* some oak-buds make a certain gesture; the alignment could have been predicted many years before those trees were even in existence. But it is perfectly possible to imagine that those little changes in form were made as an effect, a consequence, of the heavenly alignment. And if we see it this way, we see, again and again, and increasingly in the course of the years, the consequence, the effect, coming *before* the relevant cause.

Figure 130 stands there as the result of observed fact. Nine years' work and the measurement of somewhere in the neighbourhood of forty thousand buds stands behind it. We may not understand it, but it stands there as Phenomenon, and as such it has to be contended with. But it contains as it were an inbuilt ambiguity. Of no single point in it can we say: 'This point marks an effect which is Early, or Late.' Each one is either, or both! Five days early is, approximately at any rate, the same thing as nine days late. But one thing about this figure is definite, and unambiguous, and that is the *gradient* of the curve appearing on it. And through all those nine years, apart from short periods in 1982 and 1989/90, that gradient was consistently negative. If one cannot say at any moment that these effects were early, one can say that through all that time they were obstinately and consistently coming earli*er*, and this in spite of my wishes or expectations!

And now we have to ask the question, and at the moment it cannot be more than that: 'Is this the result of an inflow of negative time from a more spiritual world than ours, into this subtle realm of form which we have been studying?' If this is the case, and if Figure 130 continues the way it has been going these last nine years, then we would have to say that negative time flows into our world in seven year surges. The last such surge started some time in 1982, culminated about the end of 1986 (the moment of steepest gradient) and came to a close in the autumn of 1989; since then a new surge has now started.

It is hardly possible to consider these twin streams of time without being reminded of the work which was described in Chapter 13, culminating particularly in what was shown in Figure 112. There we saw how, if this first subtle gesture of the developing embryo is to be understood, it can only be in terms of *two* vortices coming in from opposite directions. As this closing of the neural canal proceeds forwards towards the head it is as though something is streaming forwards to us from the past, while with the closing downwards towards the trunk something is streaming backwards to us from our future. And the confluence of these two streams of time will lie, in the adult organism, in the general region of the neck. Here, in a certain way, is a nodal point, a meeting of Past and Future. And here we come upon a remarkable parallel with something which we meet in the lower part of our body. Deep in the pelvic region we find the uterus, with the Fallopian tubes rising up on each side of it to the ovaries. And high up, above the cage of the thorax we see the pharynx with its Eustachian tubes rising up on each side of it to the semi-circular canals in the ears. The uterus, at any rate during a large part of the period of pregnancy, assumes a path-curve form, and the muscles of the neck, which enclose the pharynx, bear a decided resemblance to the path curves of the all-imaginary case, such as are pictured in Figure 28.

Now we have seen, in Chapter 13, how a simple path-curve vortex interpenetrating the uterus form will engender the much more complex gesture of the closing of the neural canal in the early embryo. In the case of the neck we cannot, unfortunately, work with such precision. The enclosing form of the pharynx, although not greatly dissimilar from that of the uterus, is only an approximation to a path-curve form. Nevertheless if we suppose it to have, in its whole form and function, some of these path curve qualities, then the appearance of a simple vortex interpenetrating it, would give rise to a similar appearance to those which were pictured in Chapter 13.

Figure 139 shows a tracing from an ordinary anatomical textbook. It shows a posterior view of some of the organs of the neck. Outside we see some of the enveloping muscles of the pharynx, and within, rather like a little 20/22-day-old embryo, nestles the larynx. Compare this with Figure 109; the resemblance is quite striking. We can begin to see the larynx, this wonderful organ of speech, as being still almost embryonic in nature. Most of its growth and development lie still ahead of it.

By the very nature of the case this cannot be supported by precise observation and measurement as have so many of the things in the earlier part of this book. If the reader feels that this is all too vague and fanciful for belief or serious consideration he may close the book here and now. But I was anxious to ascertain just how close to the truth this *could* be.

I procured a cow's larynx, divested it of its surrounding tissues, except the two little arytenoid cartilages which I feared to remove in case of deforming the organ as a whole, and photographed it from what would have been the posterior view in a human. A tracing of the result is seen in the left hand portion of Figure 140. I then asked myself the question: 'If I impute to the surrounding pharyngeal tissues the same path curve qualities which I have been using for the uterus, how closely could a transformed path-curve vortex resemble this?' The right hand portion of Figure 140 shows the best result I was able to achieve. The fit is not perfect; considering the paucity of our knowledge and understanding we could hardly expect it to be so. But I think it must be agreed that it is remarkably close. We are not working here in the realm of ascertained fact, but in that of imagination; and this may lead us farther towards the truth in future than we can now foresee.

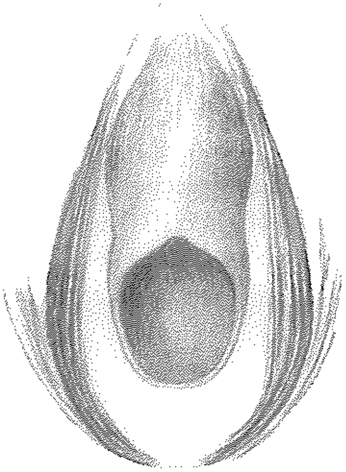


Figure 139. Posterior view of some of the organs of the neck, including the pharynx and larynx.

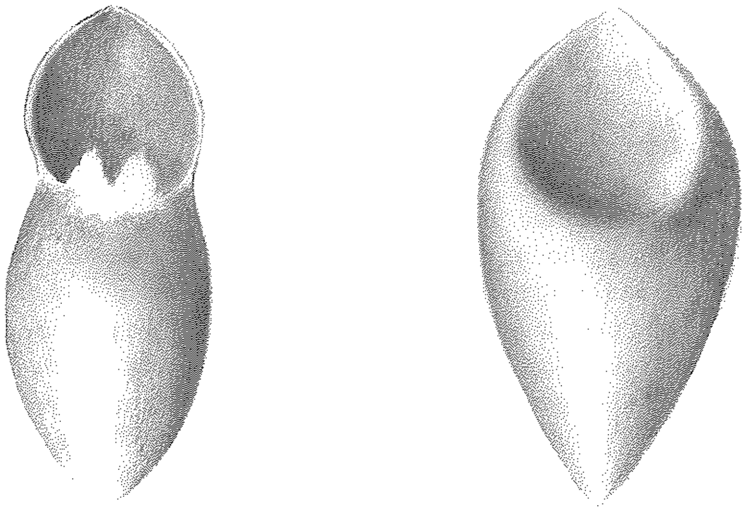


Figure 140. Left: a cow's larynx, with the two arytenoid cartilages left intact. Right: the form which arises when one allows a path curve vortex to interpenetrate the path-curve-like tissues of the pharynx.

More on the phase-shift

by Graham Calderwood

In the first edition of this book, Lawrence Edwards said of the phase shift, by then charted from the turn of 1982/1983 to about mid-1992, '... if we cannot understand it, at least we must make note of it.' He went on making note of it until October 1999, and the complete chart of his observations is shown below.

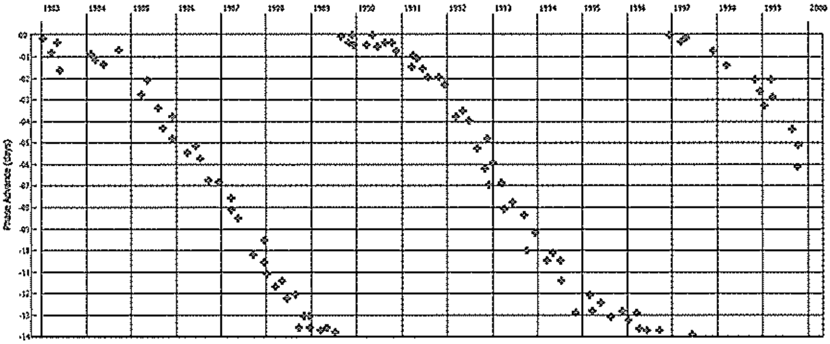


Figure 140a Phase-shift at Strontian, Scotland (1983–2000).

I have attempted to understand this phenomenon since it first came to light.

The fact that the time 'slip' of the phase-shift is substantially the same for all species and bodies has led me to concentrate on the idea that the shift is governed by all the bodies in the solar system acting in some way together. Now, there are literally thousands of ways in which they could do this, and I have searched through a large selection of these ways in the hope of finding one which yields a credible fit to Edward's observed data. I will not provide a detailed history of that search here: it would fill a book, and much of it was fruitless in any case. Instead, I will show my most successful result to date, and discuss what it might imply if it is correct.

But I must first provide a few definitions. I call the mean position of all the bodies, the *mean body:* if all the bodies were in one place, this would be that place. It is very simply found by summing the Cartesian coordinates of the bodies (deriving from a reference frame based on the vernal equinox and either the ecliptic or the equatorial plane — most usually the former), and dividing the sums by the total number of bodies in the system. It is just an ordinary, arithmetic average. I stress that although it is an aggregated position of bodies, it is non-corporeal. It has no 'stuff' in it. It is a geometric point, and nothing else.

I fear that the name, 'mean body,' has by now lodged in my mind, and I persist in using it, but it would have been better to have called this position something like the 'systemic position,' or, 'systemic mean position,' instead, as there is another mean position with perhaps a superior claim to the title of 'body,' for this other mean *is* corporeal: it has to do with mass. It is the place where all the mass of the bodies of the solar system would be were it to be gathered at one place. And having said this, I must at once qualify it: the mass in question is inertial, not gravitic. If we could connect the separate bodies of the system with a framework of stiff but massless rods so as to render the entire ensemble perfectly rigid, a point could be found within it at which to apply a force from any direction whatever that would accelerate the whole thing away without the least hint of rotation. I call this point the *centre of mass,* or *mass centre.* It is *not* a centre of gravity. It would exist even if gravity did not.

These two positions are not the same and are generally well-separated, and on the move, and each has velocity and acceleration.

I now must speak of *directivity.* This is a measure of the *degree* of direction, not of direction itself. The idea of this measure arose from the observed behaviour of the buds in respect of their acknowledged bodies, and from the need to find a measure which best captured the 'spirit' of that behaviour. It seems that buds are neither concerned with inter-body distances, nor with the *order* in which the bodies appear in the relevant line-ups. That is, neither distance nor *sense,* both of which are point-to-point affairs, seems to bear upon what the buds do in respect of their bodies. Only direction — which is a line-to-line affair — appears to affect things. We know this from the fact that in the normal way of things a bud event (a dip or peak of λ) occurs twice per lunar circuit.

Early in his studies, before the phase-shift began to show itself, Edwards said that a bud event could be expected whenever the bud's acknowledged body falls into alignment with Earth and Moon. It follows that a bud event is least to be expected when these bodies are as far out of line as they can be — that is, when they are most misaligned.

When the phase shift came to light, I found it at least conceivable that the buds would all be 'in time' (synchronized) with their bodies when *all* the bodies were *most aligned with each other,* and as 'out of time' as they could be (7 days) when all the bodies taken together were as misaligned as they could be.

On investigation, this hypothesis proved untenable, but to make the investigation at all I needed a measure of alignment that could, while retaining and responding to direction, ignore both distance and sense, and could moreover cope with ensembles. In an ideal universe, all I should need to do with such a measure to find a phase shift, in days, from it is multiply the number it supplies by fourteen. Well, it is not an ideal universe, but I found my measure in cosine squared.

The technical details are in Appendix 4 (pp. 340–45). The essentials are that the measure is always positive, equal to one (1.0) on perfect alignment (zero or 180°), zero (0.0) on maximum misalignment (odd multiples of 90°) and half (0.5) on half-alignment (odd multiples of 45°). These 'directivities' can be added, subtracted and averaged, so apply to ensembles and, unlike pure angle, show no discontinuities, so can be differentiated too. This last property is important in that it enables the finding of what might be termed the 'best direction' of an ensemble of directions: in our case it gives the orientation of the entire system of bodies. It is no doubt obvious that the single direction best representing two directions is the direction of their bisector, half way between them. The best direction of several directions is a *bisectoid*, in that it comes as close as it can to being half way between all of them at once. How well it does this is the bisectoid's (in this case the system's) directivity.

Now that we have the means to interpret it, here is the promised result.

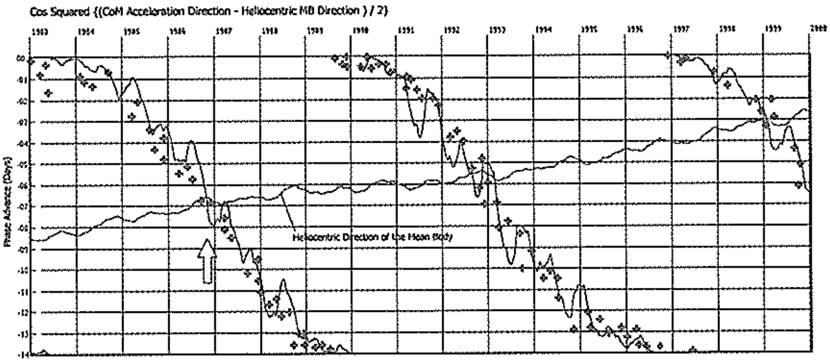


Figure 140b Directivity of the bisector of the angle made by the heliocentric direction of the mean body and the direction of the acceleration of the mass centre, compared to the phase-shift recorded from 1983–2000 by Lawrence Edwards.

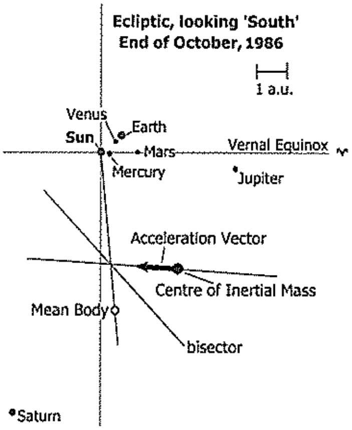


Figure 140c.

The things to which this graph pertains are depicted in Figure 140c (in what an architect would call a ‘plan view’ of the ecliptic) as they stood around the end of October 1986, the moment approximately indicated by the up-arrow on the graph. The diagram is heliocentric (the Sun is at the origin), and shows the disposition of some of the planets on the date of the ‘snapshot.’ It also shows the position of the mean body on that date, and the position of the centre of mass. The line from Sun to mean body and the line of acceleration of the mass centre form an angle, and it is with *half* of this angle (indicated on the diagram by the ‘bisector’) that we are concerned.

The graph (Figure 140b) shows a plot of the square of the cosine of this half angle against time, superimposed on Edward’s phase shift markers, so it is also a plot of its *directivity* relative to both the heliocentric direction of the mean body and the mass centre’s acceleration direction.

I think the correspondence between this directivity and the phase shift markers is good enough for us to venture that the two things match, at least to a first approximation, according to the simple equation,

$$\Delta T = -14.0\cos^2\left(\frac{\phi_{mb} - \phi_{com}}{2}\right) \quad \text{days, with} \quad \{0 \le (\phi_{mb} - \phi_{com}) \le \pi\}$$

Where ϕ_{mb} is the heliocentric longitude of the mean body, ϕ_{com} is the angle of the centre of mass acceleration vector to the vernal equinox, and ΔT is the phase advance in days.

Let us see how well this works for a λ series of a beech bud, B3, from a tree in our garden here in Aberdeen, Scotland, photographed daily from October 1998 to April 1999.

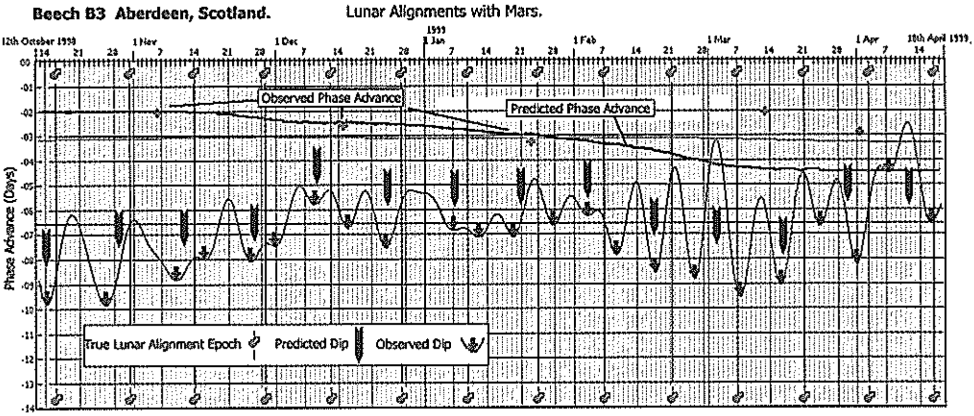


Figure 140d Beech bud λ.

This series was subjected to Fourier analysis, then re-synthesized as depicted above from the resulting spectrum, to help eliminate 'noise' and facilitate accurate location of dips. As the *variation* of λ is of interest to us here, rather than its actual value, the λ scale has been omitted. The cosine-squared curve for the period of observation is superimposed, and bold down-arrows indicate the moments at which, according to that curve, dips should occur, relative to actual epochs of lunar alignment, in this case with Mars.

I think it can be said that the prediction works well on the whole, though there are fairly serious discrepancies in the final quarter or so of the chart: two predicted dips actually fall on λ peaks.

Edward's observations of the phase-shift in the period of this series are on the graph too, and we see that prediction matches the first three of these very well, but the final two less well: only to within about two days.

However, this particular series is anomalous — and therefore very interesting! Beech is usually found to be a Saturn bud, not a Mars bud. But when this series is examined against Saturn, there is almost no correlation, no acknowledgement of Saturn to be found.

This is worth examining in some detail, so I give below two raster views of the series.

The name, 'raster,' is borrowed from the world of television: following a TV signal, a TV picture is rapidly painted in successive, left-to-right horizontal stripes down the screen, in a scan-structure known as a raster. A TV signal must be sent serially, in time, but a picture is seen 'all at once': the raster structure (rather ingeniously, I always think) allows this. If we are to see a coherent picture, the raster must be synchronized with the signal, which accordingly carries embedded 'clock' pulses which a TV receiver can act upon to bring about this synchrony.

In our raster views, the clock pulses that start scan-lines are the lunar 'axial moments,' that is, the moments when the Moon passes through perigee or apogee (the *apsides*), which lie in the lunar orbit at its intersections with its major axis — so, 'axial moments': I could have called them 'apsidal epochs.' This provides a very accurate and relevant clock, and a rock-steady 'picture.' Each scan line falls between two such moments, and markers on the lines show the true epochs of lunar alignment of any planet or body of interest, along with timing markers for any other event of interest — such as the λ-dips. The computer program that generates the raster view also allows the dip markers to be artificially moved as group left or right from their true positions by a controlled amount, and it is then possible to see at a glance whether or not the group falls on and matches well (or badly) with some planet's alignment moments. With luck, we will discover from a raster view what body is acknowledged by the bud, *and* obtain a direct estimate of the phase-shift. Here are the raster views for B3.

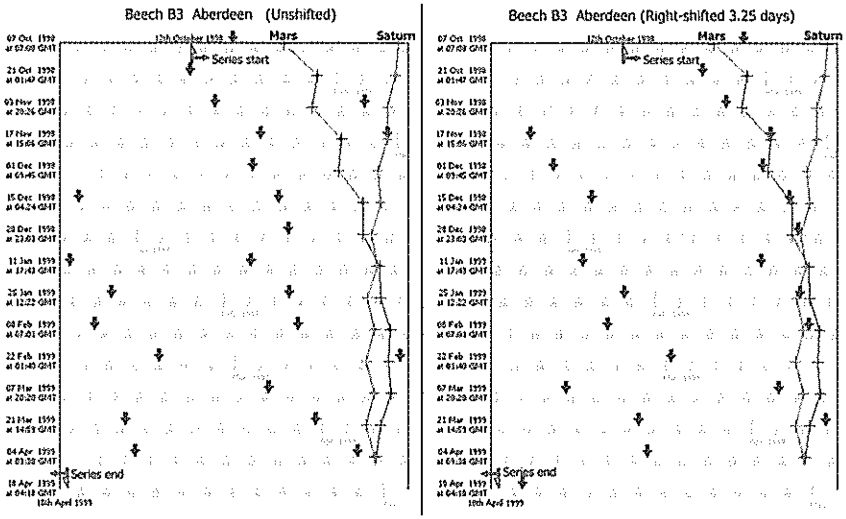


Figure 140e Two raster views of Beech bud B3 with respect to Mars. The left view is unshifted; the right is artificially shifted right by 3.25 days.

Both raster views show the lunar alignment epochs of Mars and Saturn, 'joined up.' The view on the left shows the dip markers for B3, just as they were found, distributed over the semi-lunar, roughly 14 day, line-scans of the raster. The view on the right shows the same dip-markers artificially shifted right by three and a quarter days. It is rather clear from this view that, if this beech bud is correlating with any planet at all, it is correlating with Mars, with a phase shift of just over three days. In neither view are markers matching up with Saturn to any serious extent. In the left-hand, unshifted view, only one, the sixth, does so exactly.

I do not know what to make of the group of dips that accompanies the group that directly correlates with Mars: many come *very* approximately a week away from correlating dips, suggesting that they correlate with misalignments. This needs investigation!

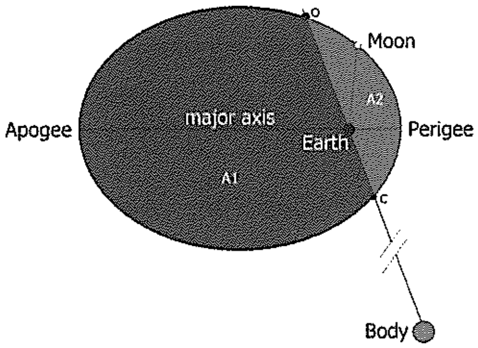


Figure 140f Towards understanding the variation of inter-alignment period (the 'zigzag').

It will be noticed that the alignment epoch markers 'zigzag' down the raster, apparently doing this least when the markers stray closest to the limits of the scan lines, giving the epochs of the Moon's transits of the apsides. This zigzag indicates that intervals between lunar alignments are alternately short and long within varying limits. It may be understood through an appeal to Kepler's second law, which states that the radius of an orbiting body sweeps out equal areas in equal times. The times for the Moon to pass from conjunction with a Body at C to opposition at O via the perigee, then back to C via the apogee, stand in the ratio of the corresponding swept areas A_2 and A_1: these are most different when the Earth-Body line is at right angles to the major lunar axis, and the same when that line is *in* the major axis. Clearly, then, the degree of zigzag is 'body-specific.'

The fact that the dip-markers tend to match and follow this zigzag is excellent evidence that the bud really is following its body's rhythm with respect to the Moon, and confirms directly what Edwards found statistically; that rhythm is preserved in the dips when synchronicity is lost.

Two other beech buds, B1 and B4, from the same tree and photographed over the same period as B3, show similar correlation with Mars — and lack of correlation with Saturn — though not so clearly and definitely. They appear to exhibit the same phase shift, however.

I do not know what general conclusion is to be drawn from this and other, similar anomalies that I have seen, except perhaps that rulership is less firmly established than we had thought it to be.

Edwards mentions (page 254f) that, in spring 1990, buds from no fewer than three beech trees at Strontian, though mainly responding as expected to Saturn, seemed also to be showing a small response to Jupiter. As we have just seen, my raster views have revealed three beech buds, from a single tree, that followed Mars, not Jupiter, and to all seeming actually ignored Saturn. Indeed, my raster views have sometimes shown me buds that appear to have switched allegiance among *several* planets in the course of a season! It is difficult to be sure that they did; such behaviour is hard to distinguish from randomicity.

But it seems to me that there is a growing body of evidence to suggest that the buds can and do pay homage to more than one heavenly master. I have already mentioned Fourier analysis, and synthesis: I have used these mathematical techniques to devise a kind of filter capable of 'tuning in' to specific planetary rhythms present in λ series, and, while using the filter for just this purpose, have detected phenomena very reminiscent of 'heterodyning.' Now, heterodyning is well known to piano tuners. They hear its effects in the 'beats' produced in the sounding together of mutually mistuned notes. Essentially, they hear the changing loudness of a *sum* of rhythms. The λ chart of a single bud seen through the filter seems sometimes to be just such a sum, though not of sounds. But this is a topic for another day. It is part of a growing chapter of new research.

Whatever the rulership, and however it is established or changed, it seems that the phase-shift is unaffected by it, and, for that matter, *vice versa.*

It is as if bud and planet were being carried on the stream of time like revellers dancing on a great cruise liner. The revellers enjoy the intricacies of their measures and delightful interweaving, all oblivious to the location of the ship on which they travel. *Where* they are on the ocean of time is not their business; it is the captain's. All they need to know to be on with their dance is that they are safely somewhere!

This may be more than mere metaphor: I think we may take seriously, and propose as a testable hypothesis, that things not made of 'time-stuff' are linked in a variable way to things that are. What do I mean by this?

It is probably accepted, and certainly experienced, that events pass over a threshold from future to past. Once over the threshold, events cannot return over it to their former places in the future: 'what is done is done, and cannot be undone.' For some events it seems inevitable that they *must* traverse this threshold in just the way they do. For others, it seems there is choice. For yet others, notably subatomic ones, transits appear to occur without rhyme or reason — at random. For these, apparently, anything can happen, until something does. One thing seems sure: that time *itself* always crosses the threshold, one imagines as a stream of moments. The question seems to be whether or not anything else accompanies these moments across it, and if they do how they do it.

How do we measure time? We think of it as a line, and, at least in imagination, set down equidistant markers on the line, calling the intervals between them hours, or seconds, or years, and so define a ruler or metric for time, analogous to the carpenter's rule that measures lengths of, say, wood, in space. But how far does the analogy truly reach?

How many and which of the features of a space ruler carry over and apply to a time ruler? Well, it obviously depends, and the dependency is very interesting.

There needs to be things to measure, for the ticks of a clock do not measure themselves, any more than do the marks on a ruler. These things will, presumably, be events. What is an event?

I wish to suggest that an event is something that is *not* time making an association *with* time in some way. For example, mass is not time, but mass under acceleration is mass *associated* with time, which makes the mass/acceleration an event. If we accept this suggestion we will be speaking of events as *types of linkage* between the non-temporal and the temporal, and at least considering that a number of different types of linkage, ranging from none (to cover the impossible) to random (to cover the unpredictable), might exist.

I wish also to suggest that, if we find a set of non-temporal 'items' linked *strictly one-to-one* with a set of temporal 'items' (which I suppose can only be moments), we have also found an event that is what might be called a 'cause-effect pair.' Until such an event happens, it is only cause. After it happens, it is only effect. *While* it happens, cause converts to effect. This event, I suggest, is *one* thing (made one by linkage) characterized by *two* possible states, between which it can be switched by happening. It is both a unary and a binary thing. If this is thought preposterously strange, I submit that stranger things, such as quanta, and photons, actually exist. So what is this 'happening' that can switch an event from cause to effect?

I think that 'happening' is simply passage over time's threshold from future to past. And if we seek for a place to put the origin of the ordinate by which we measure time (our time ruler), this threshold is rather obviously that place. For to one side of it is the irrecoverable Past, and to the other side of it is the unborn and uncertain Future: the states of events are according to which side of this origin they lie, and how far they are from it. And even if we would, perhaps for reasons of convenience, put the origin elsewhere on the ruler, this is clearly where Nature wants it to be.

I am not done making hypotheses! I say, '. . . where Nature wants it to be.' Nature has many domains, has she not? They divide one from another along many boundaries. Perhaps the greatest and longest of these separates the living from the dead — the animate from the inanimate. And if Nature sets down one origin of time, could she not set down two, or more, as befit her domains? I suggest that she could, and that she does.

Suppose that we are observing events with respect to the origin or zero-threshold that applies to us, and that we notice things that, by the definition of cause and effect ventured above, appear to be coming in the wrong order; that is, things that by our reckoning *ought* to be effects precede things that by that same reckoning *ought* to be their causes. Of course, I have the negative slip that *is* the phase-shift in mind, but I want to find as general an approach to the issue as possible before focussing specifically upon the phase-shift.

Now, if there are *two* origins, one for us, and one for the for-us-anomalous events, and if the origin for the anomalous events lies up-stream of ours, then the anomalies are explained; they have already 'happened' with respect to that origin — that is, the events have switched from cause to effect — but have not yet 'happened' with respect to ours.

Already I hear, in my imagination, the thundering of the physicists. 'This,' I hear them say, 'would make nonsense of conservation of mass and energy!' And they would be right; so it would — if we were not talking of two different types of linkage with time, which I think we probably are. Bud-events are changes of form, just pure form. Neither mass nor energy (at any rate of the ordinary kinds), is significantly involved in them. Whatever it is that causatively associates the apparently lifeless domain of astronomy with the domain of living form, it is likely to be something quite new to us — and is the object of our search, or if you like, the subject of our *re*search.

Going back now to the correlation with which this article began, if it is correct, then it furnishes clues to the nature of this new . . . well, what? Shall we call it an 'agency?'

Direction, or rather directivity, matters to it, very much, and motion, specifically change of motion with time. Distance and sense do not matter. It concerns aggregates, one corporeal, one not, and separation of sources by type and time-origin. It seems heliocentric, Sun-based.

There is a good deal of work ahead!

The two worlds

The problem of the relationship between the two worlds, the inner one of thought and imagination, which comes to expression so beautifully in our geometry, and the outer one of sense perceptible phenomena, is one which has been with mankind through the ages. It has been the aim of this book to explore, with all mathematical precision, just how far we are justified in seeing the one as an expression of the other. And it is found that, in one perhaps rather restricted but nevertheless fundamentally important sector of our experience — that of the egg-like and cone-like forms — we are so justified to a quite remarkable extent. But in the first place this is so only as long as we confine ourselves to the manifest, and what we may in a certain sense regard as the finished, forms of nature: the eggs, pine cones, buds, ventricles, and so on.

As soon as we come to study the mysterious *processes* by which these come into being, the realm of the ovum and the gynoecium, and that of embryo-genesis in general, then we have to allow further, purely ideal, forms to intrude themselves into our considerations: the vortices whose workings are described in Chapters 9 to 13. And the fact that these turn out to be of exactly the same mathematical nature as the forms which they are helping to generate, may be taken as a tribute to the unitary quality of this field of form.

Inevitably the question arises as to what these vortices really *are*. Obviously something very potent and real is at work, but should we envisage them as working in and around the *space* of the plant and the womb? Increasingly I am coming to feel that we should not. I believe that we are dealing here with forms not of Space, but of Time. And I feel strongly, but alas, only so far dimly, that the truths behind the undulating curve on the phase-shift chart are connected in some way with the flow of some vast, seven-year vortex in time.

What would a 'vortex in time' be like? Well, if I could see it, that is to say if it could express itself spatially, it would be what I envisage and what I draw on my page. Within its own spatial context this would be a true picture of it; and I could learn from it. But to approach its *reality* I believe we would have to rise to something more like a musical experience; and this is something that would lie outside the scope of this book.

If all this is accepted, then a book which started with considerations of Space, has ended before the portals of Time. In which case everything that has been written here has to be considered as merely a preparation for the beginning of what needs to come; and I am content that this should be so.

Appendices

The full evidence

The phenomena described in this book, if established in general belief, would lead to a very different picture of our world from the one which is usually held in our civilization — a world in which the activities of plant and planet are inextricably bound together in intermingling rhythms, a world behaving like a great living organism. However beautiful, or philosophically satisfying, one may, or may not, find such a conception, in the last resort it will stand or fall by one thing and one thing only — in sober reality, is it *true*? But no-one can form a balanced opinion about this unless all the available evidence is to hand. And it is not possible for a book of this kind to present the great mass of material which this would entail.

The author has put together a number of supplements and sequels containing all the evidence which has accrued over twenty years' concentrated activity, as well as much that has been found subsequent to the original publication of this book. With it the reader will be in as good a position as possible to assess the worth of this work, and its credibility.

The supplements also give full details of the methods of working, and the precautions which need to be taken, for the help of those who would like to pursue the work further on their own behalf — and it is hoped there will be not a few.

The supplements can be downloaded from the Floris Books website:

www.florisbooks.co.uk

Appendix 1

Path Curves in Two Dimensions

Number or multiplier

Firstly we must have a closer look at the growth measure. We know, from elementary projective geometry, that a transformation of a line onto itself is completely determined by the giving of three different pairs of points. Thus we can see that, having chosen our two fixed points wherever we wish, we can still move a third point A arbitrarily to any point A' that we like. Our transformation, and thereby our growth measure, is then completely determined. In choosing our intermediate line i, and our raying points O_1 and O_2, arbitrarily, we are in effect doing just this. And in doing this we have, consciously or unconsciously, chosen some number by which the growth measure will multiply. This number may be said to completely characterize the resulting growth measure, and I will call it the *characteristic number* or *multiplier* of that measure.

If we consider any four consecutive points of the measure, A, B, C, D, we realize that they will transform into the next four B, C, D, E, and so on all along the line. It is thus apparent that a growth measure is such that any four consecutive points have a constant cross-ratio all along the measure. Similarly, consider any consecutive *pair* together with the fixed points, X and Y; X, A, B, Y transform into X, B, C, Y, and so on. Thus we can say that any consecutive pair together with the fixed points have a constant cross-ratio all along the measure. This cross-ratio is not the same as the one mentioned above, but is a simple function of it.

We know that the points X, A, B, Y can be associated with one another in twenty-four ways to give a cross-ratio. Any one of these ways can be selected by us, and as long as we agree to stick to it in all our further considerations, we can ignore all the rest. The way we shall decide to do it is to call our cross-ratio

$$\frac{XB}{BY} : \frac{XA}{AY}$$

and to calculate this as

$$\frac{XB}{BY} \times \frac{YA}{AX}$$

(This is, of course, numerically equal to $(XB/BY) \times (AY/XA)$, but follows the flow more naturally along the line.)

The cross-ratio calculated in this way has the advantage of being identical with the characteristic number (the multiplier) of the growth measure, instead of being some function of it, and all cross-ratios in what follows will be considered as being calculated in this way unless it is stated to the contrary.

Note that a growth measure has one unique characteristic number, but that this number (provided a pair of fixed points has been given) has a single infinity of growth measures associated with it, that is the set of all the growth measures with that multiplier, starting from different points *A*.

Two-dimensional path curves

The general appearance of what happens when we let a whole plane transform into itself has been described in the main body of the book. The fact that in such a case there will always be an invariant triangle is fully described in the ordinary textbooks of projective geometry. This triangle is made of three lines, each of which, considered as a whole line, is invariant, but the points of which are all moving in growth measure between the two fixed points of that line. Similarly the three invariant points will each contain a pencil of lines in growth measure between the two fixed lines of that point. It will be seen that the invariant organism is completely self-dual, and this holds for all path curve situations.

Notice that in order to determine completely a set of path curves all we need to do is to choose our invariant triangle arbitrarily and then to move one point *M* to another point *N*, again arbitrarily. Having done this, the whole thing is fixed, and every other point of the plane now begins to move along its determined curve. The movement from *M* to *N* (see Figure 14) of course fixes that from *S* to *T* and from *A* to *B*, and thereby determines the two growth measures along the sides of the triangle. All the rest follows.

We now have to ask: if movements along two sides of the triangle determine the whole thing, what about the third side? To answer this important question we must refer to a theorem of projective geometry.

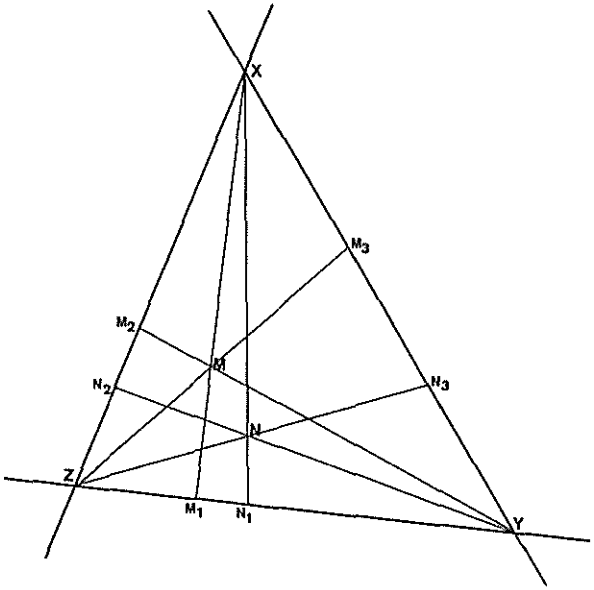


Figure 141. The law of cross-ratios round the sides of the invariant triangle.

Given any triangle XYZ and two points M and N, we project M and N from the points of the triangle onto the opposite sides as M_1, N_1, M_2, N_2, M_3, N_3, as shown in Figure 141. Let cross-ratio XM_3N_3Y equal α, YM_1N_1Z equal β, and ZM_2N_2X equal γ. Then the theorem tells us that

$$\alpha\beta\gamma = 1.$$

Applying the theorem to our actual diagram, and going round the triangle clockwise from X, we find that XM_3N_3Y, that is α, equals approximately 3.41. Next we find YN_1M_1Z to be approximately 1.70. But we notice that we have gone 'backwards' past the points, N to M instead of M to N. This involves the inverse process-division instead of multiplication. We must find the reciprocal of 1.70; $\beta = 0.588$. Similarly the cross-ratio from Z to X comes out at approximately 2.00 and here again we have to take the reciprocal, $\gamma = 0.5$. Simple calculation confirms for us that these have a product of unity. This arithmetic example shows how we must be careful always to take the ordering of the points into consideration when deciding whether to multiply or divide by the cross-ratio that we have found.

Returning now to Figure 141 we see that if the multiplier along XZ is β and along ZY is α then that along YX is already fixed: it is $1/\alpha\beta$. The fact that it is less than 1 is an indication that the movement will be in the opposite direction, that is from X to Y, and reference to the diagram will confirm that this is the case, the actual multiplier from X to Y being, of course, $\alpha\beta$. If one takes some point other than M, and joins its successive positions to Z, the lines so formed will not make the same growth measure along XY, but it will be another measure of the same sort, that is having the same multiplier.

The λ-parameter

Next we must ask ourselves what sort of a curve we have produced. Seeing that, being given a particular invariant triangle, the whole process is determined by the multipliers α and β, the shape of the curve must in some way depend just on them. It transpires that the form of the whole family of curves depends, not on the ratio of α to β, but on what one might describe as their 'exponential ratio,' the ratio of their powers, or in other words the ratio between their logarithms. For this reason we do not normally deal with the parameters α and β, but with their logarithms. Where $a = \ln\alpha$, and $b = \ln\beta$, the form of the family is determined by the parameter

$$\lambda = \frac{a}{b}$$

This is a parameter of great importance in the further study of these curves. (It is usual to take the logs to the base e, although of course for the purpose of evaluating λ the choice of base makes no difference.)

Thus it is clear that if we had a case where $\alpha = 16$ and $\beta = 4$, and a second case where $\alpha = 9$ and $\beta = 3$, the curves produced would be identical in form; the difference would be that, per step of the transformation, the points would move in larger intervals round the first than they would around the second. Given a constant λ we can make α, and therefore β, as small as we wish, obtaining in the limit, the infinitesimal case, smooth, continuous curves.

The general path curve

In Figure 15 we have seen what such a system of path curves looks like, and how the invariant triangle treats the curves asymmetrically, all the curves passing through two of the points and all of them avoiding the third. Reference to Figure 141 shows the reason why this must be so. While passing round the sides of the triangle in one direction, we come on *M* and *N* in that order on two of them, and in the reverse order, *N* and *M*, on the third of them. No matter how we place the original *M* and *N* in the middle of the triangle, there will always be just one of the sides which will be 'odd man out.' This is the side which contains the two points through which all curves pass, and it will also be the side which is tangent to none of them. (The special case where *M* and *N* are collinear with one of the invariant points will be dealt with later.)

The equations of these path curves are as elementary as the primeval nature of the geometry would suggest. If we take a, b, and c to be the natural logarithms of α, β, and γ, and if we take the invariant triangle as the triangle of reference of a system of homogeneous co-ordinates, the equation for the whole set of curves is simply

$$x^a y^b z^c = k.$$

Since $\alpha\beta\gamma = 1$, $a + b + c = 0$, and this equation is of degree zero on both sides, thus preserving its homogeneity. As we run through different values of k we pass from one curve to another of the system.

Looking at Figure 15 we must envisage all the points streaming infinitely outwards, in all directions, from *X*, and infinitely inwards towards *Y*; or of course, with the inverse transformation, the other way round. If we follow a point in its journey towards *Y* we realize that it can never reach it: it has an infinite number of steps to take before it can get there. The question as to what it does after it has passed through *Y* has not, in this sense, any meaning. Nevertheless taking the curve as an entity in itself, as a whole thing, we can ask where it goes after it has passed through *Y*. And the strange thing is that by the very nature of the case, the geometry is unable ever to give us an answer to this question. As far as the *form* of the curve is concerned it is equally possible that it curves back on itself at *Y*, making the line *ZY* one of its tangents, or that it crosses this line while passing through *Y*, making *ZY* a flex-tangent, or that it 'backs out' from *Y*, forming a cusp there.

But the algebra can, in some sense, do what the geometry cannot. Let us suppose that the parameters a, b and c are rational to one another. Since the form of the curves depends only on the ratios $a{:}b{:}c$ we will suppose that these have been cancelled down to the smallest whole numbers possible, that is they have no common factor. They cannot then all be even or they will have the common factor 2; since they add up to zero it is not possible for two of them to be even without the third also being even; no three odd numbers can add up to zero; there is only one possibility left to us: two of them must be odd, and the other one even. Notice also that two of them must be of one sign, either plus or minus, while the third, which will be the greatest numerically, must be of the opposite sign, either minus or plus. Again, out of the pure number relations, we see the asymmetry with regard to the points and sides of the triangle. That parameter, a, b or c, which is greatest numerically, that is, that has the opposite sign from the other two, will lie along that side of the triangle which is tangent to none of the curves, and the point opposite it will be the point of avoidance of all curves. If this greatest parameter (numerically) is odd, the curves will have a cusp at one of the invariant points and a flex at the other. If on the other hand this parameter is even, the curves will not cross the invariant lines, but will just be ordinarily tangent to them.

If, however, a and b are irrational to one another — and this is the most likely thing, unless we have done something very consciously about it — even the algebra cannot say which way they go. The curves themselves do not know; one can say that they float!

We could consider the case where $a = 100$, $b = 201$, and $c = -301$. This would give a set of curves of a certain shape, having cusps and flexes. If on the other hand we were to have $a = 99$, $b = 201$ and $c = -300$ we would have curves of almost exactly the same shape, but without cusp or flex. Obviously, by taking large enough numbers, we could get the shapes of our curves to be as nearly identical as we wished, while still keeping one of them cusped and flexed and the other not. And, no matter how close to one another they come in shape, in between them would be a whole host of irrational possibilities — lost souls who don't even know which way they are going!

Thus we see that although the variation of form which these path curves can assume is rigorously limited, within these limitations there is also an infinite richness of possibility.

Special path curves: two points at infinity

Certain special cases are instructive to examine. In Figure 16 we let Z stay in the centre of our page, and send X and Y to infinity, preferably, though not necessarily, at right angle to one another. The growth measures along ZX and ZY then become ordinary geometric series and the curves assume the forms associated with the Cartesian equation $y = kx^a$.

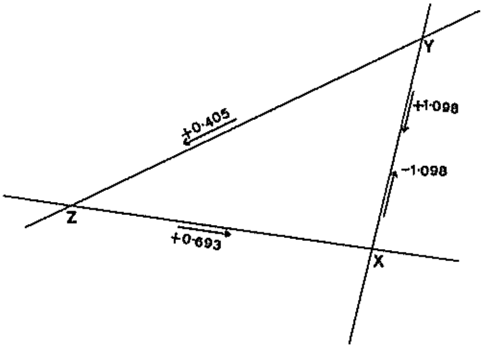


Figure 142.

These can form a field of rectangular hyperbolas when $a = -1$, a pencil of lines when $a = +1$, a field of parabolas when $a = 2$, cubics when $a = 3$, etc. All these are simply special forms of path curves. Two typical such sets are shown, interlacing one another, in Figure 16. The constant ratio along the x-axis has been taken as 2 and along the y-axis as 1.5.

Suppose, to begin with, we let the measures work inwards towards Z along the y-axis, and outwards towards X on the x-axis. This will give us the set of hyperbola-looking curves. Figure 142 shows the projective case of this, line XY representing the line at infinity. Along YZ we have the parameter 0.405 (ln1.5) and along ZX the parameter will be 0.693 (ln2). It follows that the parameter along XY must be –1.098, that is to say +1.098 in the direction YX. This means that the lines joining successive points of the curve to Z will turn rapidly in a clockwise direction as the measure moves from Y towards Z, and reference to Figure 16 confirms that this is the case. The equation of the set will be

$$x^{0.405}\, y^{0.693}\, z^{-1.098} = k.$$

Transforming into Cartesian co-ordinates we have

$$x^{0.405}\, y^{0.693} = k$$

or $\quad y = hx^{-0.405/0.693}$

or $\quad y = hx^{-0.584}$

(where h is a simple function of k).

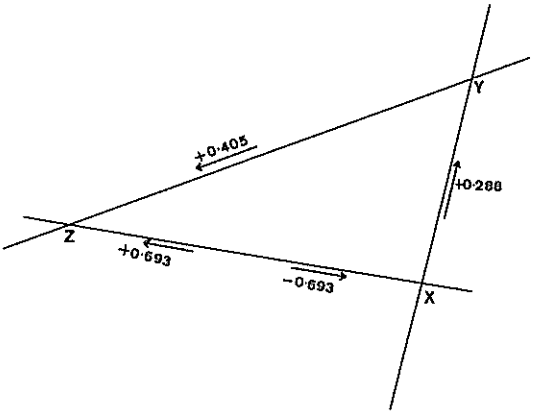


Figure 143.

But suppose the transformation is such that while the measure along the y-axis moves inwards towards Z the one along the x-axis also moves inwards towards Z. We shall now have the parabola-like curves. The projective picture will look like Figure 143. The parameter along YZ will again be 0.405 but the movement from X to Z having a parameter of 0.693, the true parameter along ZX will be –0.693. This gives us a necessary parameter along XY of +0.288.

Thus, as the measure moves inward from Y to Z, and of course also inwards from X to Z, we shall expect to see a slow anticlockwise turning of the line joining the points of the curve to Z. And again, reference to Figure 16 confirms this. The equation of this set will be

$$x^{0.405}\,y^{-0.693}\,z^{0.288} = k,$$

and putting this into Cartesian co-ordinates we have

$$y = kx^{0.584},$$

showing that these curves are in fact very nearly (but not quite) true parabolas.

The important thing to notice about this exercise is that we must first of all choose one direction (normally the anticlockwise) with respect to which all our measurements are taken. Having done this we must remember that every arrow pointing in this direction shows us the true *parameter* of that movement. However every arrow attached to a positive number shows us the true *direction* of the movement along that axis.

We have gone into some detail over this because, in the sequel, a clear understanding of the above paragraph is essential.

Special path curves: one point at infinity

Another case of importance arises when *X* and *Y* are at finite positions on our page, and *Z* goes to infinity, again preferably, but not necessarily, at right angles to the line *XY.* The growth measures along *XZ* and *YZ* will again be ordinary geometric series. In Figure 17 we have again taken their constant ratios (multipliers) to be 2 along *XZ* and 1.5 along *YZ.*

First we consider the case where the measures move inwards from *Z* to *X* and outwards from *Y* to *Z.* We obtain a set of egg-shaped forms, rather blunted at *Y* and rather sharper at *X.* Projectively this is fundamentally the same case as Figure 142, but for ease of comparison with Figure 17 we draw it again (Figure 144).

We see that the parameter from *X* to *Y* is –1.098, thus involving an actual movement, while the measure runs inwards from *Z* to *X,* from *Y* to *X* with a parameter of +1.098. This means that if we take two consecutive pairs of points on one of the eggs, join them by horizontal lines, and let these lines cut the central axis at *S* and *T,* then the cross-ratio *YSTX* will have a natural logarithm of 1.098, a fact which we can easily verify by measurement from the diagram. Notice that if we consider only the arrows with positive numbers, that is the actual directions of the movements involved, then it is the movement *Y* to *X* which runs counter to the other two, giving *Z* as the point of avoidance.

If on the other hand (Figure 145), we take as our transformation one which has the measures running inwards from *Z* to *X* and also from *Z* to *Y* we get the other set of curves, all flowing inwards from *Z* and upwards to *X.* Only four of these have been drawn, and in order that the diagram should not become overloaded their counterparts above and below the lines have been omitted.

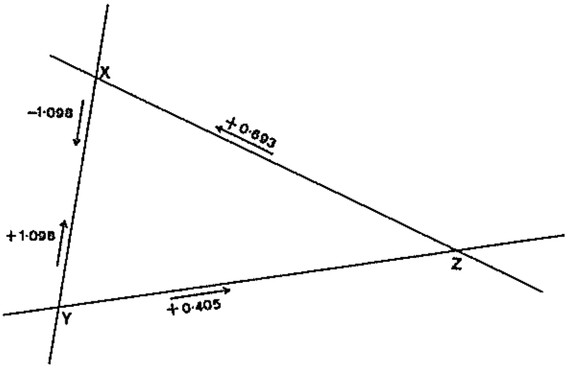


Figure 144.

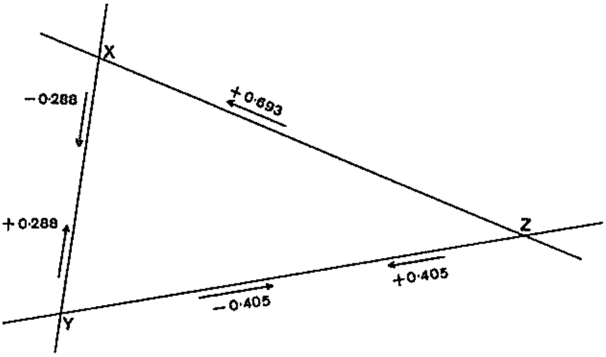


Figure 145.

We see that the true parameters are +0.693 along *ZX* and −0.405 along *YZ*, giving us, by our rule, a true parameter of −0.288 along *XY*. This, of course, means an actual movement from *Y* to *X* with much smaller steps than from *S* to *T* in Figure 17, and just a glance at that diagram verifies this. Notice that if we consider only those arrows with positive numbers we see that the *Z* to *X* movement is the one flowing in an opposite sense from the other two, giving *Y* as the point of avoidance.

We can define λ in this diagram as

$$\lambda = \frac{\text{log multiplier from } Z \text{ to } X}{\text{log multiplier from } Y \text{ to } Z}$$

Here $\lambda = +0.693/+0.405 = +1.71$ for the egg-shapes and $+0.693/-0.405 = -1.71$ for the other set of curves.

If $\lambda = 1$ the egg curves become a set of ellipses, and if it equals −1 the other set of curves degenerates into horizontal straight lines. This latter case needs to be examined more carefully from a projective point of view.

In Figure 146 we have a set of path curves with equal parameters, *a*, working along *XZ* and *ZY*. These two growth measures therefore are projective ranges of points. It follows that *XZ* is a tangent to the system. Similarly with *YZ*. And the curves are conics. This is the case where $\lambda = 1$ and the results follow from elementary projective considerations.

But now suppose that the transformation is such that the growth measure has a parameter of *a* in the direction *XZ* and one of *a* in the direction *YZ*. In the limit, in one direction, the line *YZ* of *Y* will correspond with *XZ* of *X*. Therefore all curves will pass through *Z*. In the limit, in the other direction, the line *YX* of *Y* will correspond with *XY* of *X*. The pencils will thus have a self-corresponding line; they will be in direct perspective and the meets (intersections) of corresponding lines will all lie on lines, which, of course, pass through Z.

In such a case the parameter along *XZ* will be *a*, and that along *ZY* will be $-a$. It follows that the parameter along *XY* will be zero. The transformation along that line will be 'identity' — nothing moves. This is the algebraic equivalent of the geometrical reason why the second set of 'curves' will be straight lines through Z. This is the case where $\lambda = -1$. The transformation is now a simple homology, centre Z and invariant line *XY*. *So we see that homology is a special case of path-curve transformation.*

In this latter transformation all points move along straight lines of Z. But a moment's consideration of the diagram will show us that those lines which we have drawn are arranged in growth measure, cutting *XY* with parameter $2a$. We thus see that we have constructed a net of conics and lines, each of which cuts the others in growth measure. With such a net we can immediately draw any number of other sets of path curves. For instance we could start from any intersection for our first point, then move one conic away and two lines away for our next, then again one

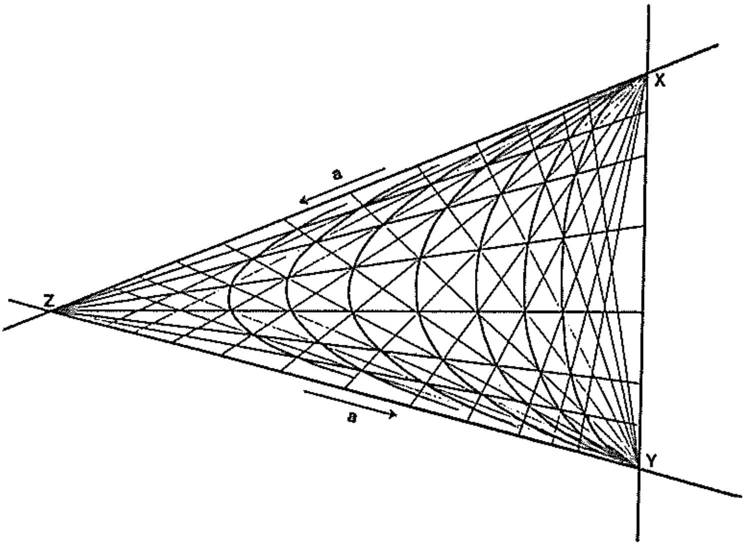


Figure 146. In the special case where the multipliers along two sides of the invariant triangle are equal, the path curves become a set of conics and straight lines.

conic and two lines away; continuing like this we should have a path curve of quite a new set. Or we could have chosen to move, say, three conics and two lines away each time; this would have given us a path curve of yet another set. In fact our net represents growth measure movements which can be compounded in any proportions we like, similar to the way in which we add two vectors to give us a third.

Circling measure

Now we must refer to some known facts of projective geometry. They have been described in full in my book *Projective Geometry* (see bibliography). It would take up too much space to repeat these descriptions here but we summarize the results.

In addition to growth and step measure, there is a third kind of measure, which George Adams, and we after him, called circling measure. (Any equiangular pencil of lines will cut a general line of the plane in a set of points which form a circling measure. The textbooks sometimes call it an elliptic metric.) Any circling measure is the exact projective equivalent of a growth measure, except that the fixed points, which in the growth measure are real, have now become imaginary (conjugate complex). All coplanar circles cut the infinite line of their common plane in one special pair of such imaginary points, the so-called absolute circling points at infinity. We will call them *I* and *J*. The circling measures controlled by the fixed points *I* and *J* are those along the line at infinity whose points are equiangularly spaced. (From this derives the well-known Euclidean theorem that all the angles in a given segment of a circle are equal, and so on.) In fact we could give a projective definition of a circle as any conic which cuts the line at infinity in the points *I* and *J*.

Special path curves: two imaginary points

Now we are ready to construct an imaginary analogue of Figure 146. We will let two of the invariant points, say *X* and *Y*, become conjugate imaginary (hence having a real line in common). To begin with we will let them be *I* and *J*, so that we have sent one line of our triangle (their real joining line) to infinity. *Z* we will keep real (the intersection point of the conjugate imaginary lines of the other two sides of the triangle) and on our page. The conics of our diagram now become circles. Notice that in Figure 146 *Z* and line XY are common pole and polar to the whole system. So they will be in our new diagram. Z will be the common pole of the line at infinity for all the circles. We see that concentric circles are just a special case of path curves. The pencil of lines through the new Z will cut the line at infinity in a 'growth measure' with double points I and J. But I and J are imaginary, so this will in fact be a circling measure. The lines through Z will be an equiangular pencil, another special case of path curves.

Notice that in Figure 146 the path curves cut the lines of Z in growth measure. Therefore in our new diagram the concentric circles will cut the lines of the equiangular pencil in growth measures which all have one of their fixed points on the line at infinity. The circles will have radii in geometric series.

And now if we move across the curves and lines of our new net, say one circle out and one line round, or, perhaps, two circles out and three lines round, or any other combination of movements we like, our new path curves will be logarithmic spirals (Figure 19).

By taking a projective transformation of Figure 19 we can easily produce a more general picture of a set of path curves having an imaginary invariant triangle (Figure 18).

Some further formulae

If we do not know the position of the invariant points we can resort to a different method. Any four consecutive points in a measure will have a constant cross-ratio, R, and it can easily be shown that

$$R = \frac{(m+1)^2}{m}$$

Rearranging for m we have

$$m = \frac{R - 2 \pm \sqrt{(R^2 - 4R)}}{2}$$

This is an interesting little formula; it is so constituted that no matter what value we put in for R, the two answers will be reciprocals of one another. One takes us forward along the measure, and the other backward. If $R < 4$ we have circling measure (an elliptical collineation); if R is > 4 we have growth measure (hyperbolic); and if $R = 4$, $m = 1$, and we have step measure (parabolic, projectible into an arithmetic series). Having found m we can calculate the position of points X and Y.

Let $BX = x$ and $BY = y$, both measured positive, outwards, from *B;* and let $BA = a$, $BC = c$ (also measured positive, outwards, from *B,* where *A, B* and *C* are any three consecutive points of the measure; then

$$x = \frac{ac\,(1 + m)}{mc - a}$$

$$y = \frac{ac\,(1 + m)}{ma - c}$$

Appendix 2

Path Curves in Three Dimensions

General case

If we imagine the whole of three-dimensional space to be transformed onto itself by the most general one-to-one projective transformation, it can be shown that there will necessarily be just four points which will be self-transforming. In the most general case these will stand at four corners of a tetrahedron. Thus we will have four invariant points, four invariant planes, and six invariant lines, grouped into three skew pairs. All the points of each invariant line will be moving in growth measure along their line, and all the points of each invariant plane will be moving in path curves within their plane. Dually all the planes of each invariant line will be turning in growth measure around their line, and all the planes (and lines) of each invariant point will be moving in path cones within their point. Not only the invariant organism, but also the whole system of movement, is always self-dual.

It is a general rule that, the invariant elements being given, in order to determine completely a path curve transformation the same number of independent parameters is needed as the number of dimensions in which the movement takes place. Thus a system of growth measures is completely specified by the giving of one parameter, the multiplier or characteristic number. A plane path curve system is completely specified by two parameters, a and b, along two sides of the triangle (the third parameter c not being independent). Notice however that if we are concerned to know only the *shape* of the curves, and are not interested in the *size* of the stepping with which the transformation moves round them, then one parameter only is needed which is most easily expressed as the ratio $a:b$, or $\lambda = a/b$.

Now it is easy to see that just three parameters, given independently, will completely determine the three-dimensional case, provided that they do not work along three coplanar lines; obviously, from what has gone before, it would not be possible for them to do so.

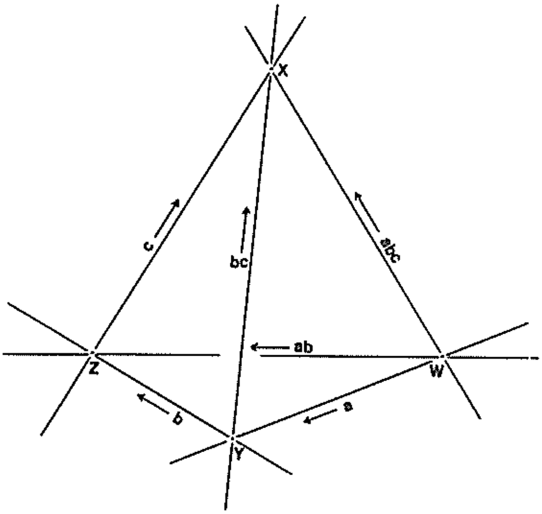


Figure 147. Any three multipliers, a, b, *and* c, *being given, the movements along all six sides of the invariant tetrahedron are already determined.*

Suppose that we fix three arbitrary multipliers with logarithms of *a, b,* and *c,* along the lines *WY, YZ,* and *ZX,* respectively (Figure 147). By our triangle rule, considering triangle *WYZ,* we know that we would then have *–ab* along *ZW* or, perhaps more conveniently, *+ab* along *WZ,* and this we mark in our diagram. Similarly along *XY,* considering triangle *XYZ,* we have *–bc,* or *+bc* along *YX.* Now along *XW,* considering triangle *XZW,* we have *–abc,* or, along *WX, +abc.* But if we consider triangle *XYW* we shall see that we again have *+abc* along *WX;* the thing is consistent.

Thus we see that the giving of three arbitrary parameters has completely determined path curve systems within all four invariant planes.

Now we take any point *P,* not on an invariant plane (Figure 148). We join *XP* and mark the point *X′* where this line cuts plane *WYZ.* We note that *X′* can go anywhere we wish along line *XP,* but in marking *X′* we have used up our last freedom. We let *WX′* meet *ZY* in point *S,* and join *SX.* Where *WP* meets *SX* we mark the point *W′*, the point in which *WP* meets plane *XYZ.* Similarly, by finding points *T* and *U,* we can find points *Y′*, in which *YP* meets plane *WXZ,* and *Z′*, in which *ZP* meets plane *WXY.*

Knowing, from Figure 147, the multipliers along the invariant lines, we can now construct the path curves followed by *X′*, *Y′*, *Z′*, *W′*. Since

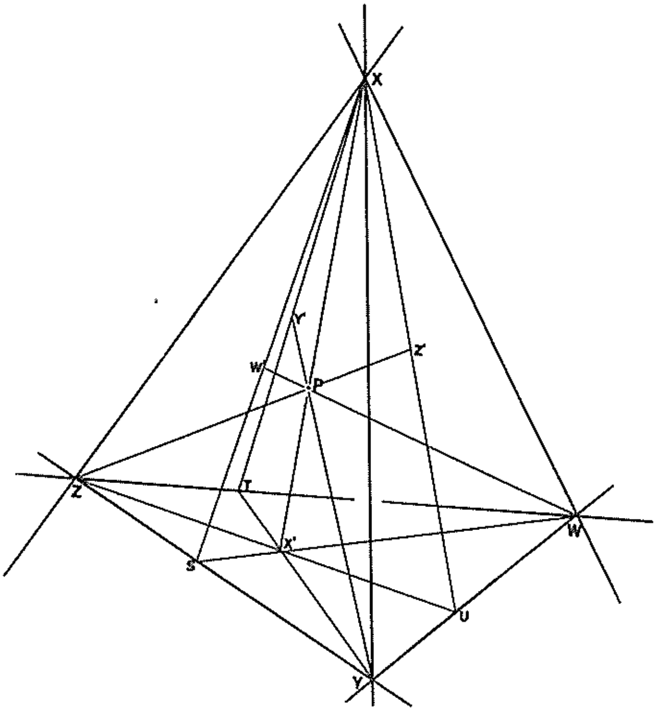


Figure 148. This is how a general point P *of space, is projected from the invariant points* W, X, Y *and* Z *on to the faces of the tetrahedron which are opposite to them.*

the four lines *XP*, *YP*, *ZP* and *WP* are concurrent, and projective transformations leave properties of incidence unchanged, we can be sure that the lines into which these four will transform at the next step of the process will also be concurrent, and their common point will be the next one in the path curve which is being traced out by the point *P*. In this way it is easy to draw three dimensional path curves. Of course, in actual practice, it is only necessary to draw two of the plane path curves, the other two being taken for granted, but even so it will be found to be a considerably toilsome process to get anything like a full drawing of such a set.

Having made such a drawing we shall find that we have a family of invariant curves, weaving through all space, all of them passing through two of the invariant points, and none of them passing through the other two, and all of them osculating two of the invariant planes and none of them osculating the other two.

We note that, being given the invariant tetrahedron, we have only to move any point P arbitrarily to some point P', to have the whole process determined. Or, and this is in effect the same thing, we can specify multipliers along any three non-coplanar invariant lines. However, if we are interested only in the *shape* of the curves, all we need to specify is the ratios $a:b:c$, this involving fundamentally only *two* independent numbers, say a/b and a/c. In later work we shall sometimes have to change the nature of our parameters, but the *number* of them which will be needed will never change.

Now let us consider the infinitesimal case — the limit — where the points are in smooth movement around their curves, the true picture of what we may call a linear flux. Every point of space is moving along its determined curve. At any given moment it is moving along the tangent line to the curve at that point, and again at the given moment the tangent line is moving in the osculating plane of the curve at that point. Dually every plane of space is moving around its tangent line and in its osculating, or pivot, point. The tangent line moves in such a way that it is always tangent to its curve; it traces out a developable surface.

It is clear that our transformation assigns to every point of space a line, along which that point is momentarily moving in the transformation. But we know that whereas the lines of space form a four-dimensional manifold, the points form only a three-dimensional one. Thus it is that if we choose any arbitrary line to cut through our path curve system, it is probable that it will not be a tangent to any of the curves of the system. For every line of space that is tangent to one of the curves, there will be infinitely many lines which are not. Those lines which are tangent to any of the curves we call path lines of the system.

Thus we may say that any given path curve system chooses, out of the ∞^4 lines of space, ∞^3 lines for its set of path lines. That is to say that it determines a complex of lines. It is interesting to consider what sort of complex this must be.

Consider any path line, m, of the system, and the line m' into which it transforms after an infinitesimal step of the transformation. Figure 149 gives a very 'diagrammatic' picture of what we are considering. Since this is an infinitesimal step which we are considering, we may reckon that the two lines meet in the point P whose path line is m, and that therefore they lie in one plane, π, the osculating plane of P. Consider any point A of line m. It will transform into point A' of m', and clearly line AA' will be the path line of the system assigned to A.

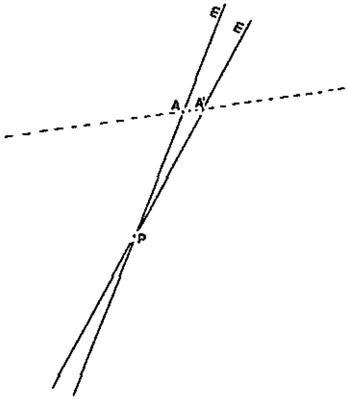


Figure 149. The momentary movement of a path line.

If we think now of the range of points A of m, and the range of points A' of m' into which they transform, since it is a projective transformation, it is clear that they form two projective ranges. Therefore their joins envelop a conic. This conic contains the path lines of all the points of m. Since plane π is turning about line m, it is clear that no other point of π could possibly have its path line in π. Therefore we may say that all the path lines of the system which lie in plane π form a conic envelope in that plane. And this holds for every general plane of space.

We may say therefore that the path lines of a path-curve system form a quadratic, or second order, complex.

If on the other hand we consider any general line of space — not a path line of the system — and watch its progress in the course of the transformation, we see that it will move skew to itself, and doing so will generate a ruled surface. Obviously this surface will be completely covered with path curves, and every point of the surface will always transform into another point of the surface. Thus we see that in three-dimensional space we have the possibility not only of invariant curves, but of a multitude of invariant surfaces. These surfaces are of great beauty and, some of them, of great importance, so we must now study them more closely.

Watching the progress of a general line of space through the moves of the transformation is not the only, or even the most usual, way of generating such a surface. We have only to draw any arbitrary curve whatever through a path curve system and take the path curves passing through every point of this arbitrary curve. The set of path curves so chosen will form a surface, often of immense complexity, and a moment's thought suffices to show us that this must be an invariant surface. (The method described above, watching the movement of a general line of space, is clearly just a particular case of this more general method.)

Obviously this is an extremely arbitrary way of forming an invariant surface, and its very arbitrariness gives us infinite possibilities of variation. By the same token it seems likely that most of these forms are of little interest or significance.

But there are certain surfaces which are of great importance. These are so fundamental to the nature of the path-curve system from which they are derived that we will call them the fundamental surfaces of the system.

Suppose that to generate our surface we start, not with an arbitrary curve through the system, but with a path curve of some other transformation working within the same invariant tetrahedron. Let us think of this path curve as being drawn in blue. Through every point of our blue curve we draw the path curves, say in red, of our first transformation. We now have an invariant surface covered with red curves. Now the property of being a path curve is a projective one, and as the red transformation works upon the blue curve, this property will be retained. Thus we see clearly that our invariant surface will be covered with two sets of invariant curves, belonging to two different transformations within the same invariant tetrahedron. The surface will be covered with a network of path curves. And now we can begin to move across the diagonals of the network, one red and one blue, two red and one blue, two red and three blue, just how we like, to form as many different sets of path curves as we wish. This is the same way in which we moved across the net of path curves in Figure 146.

Thus although we form such a surface from the path curves of one transformation we find that there is an infinite set of transformations, all within the same invariant tetrahedron, each of which can completely cover the surface with its path curves, and from each of which the surface could have been equally easily derived. Whereas an arbitrary surface such as is described above is covered with only one infinite family of path curves, each fundamental surface is covered with an infinite family of such infinite families. The range of such surfaces possible is much more limited, and many of them are of great significance.

Two imaginary points: first case

Our next task is to consider what happens if two of the invariant points become conjugate imaginary. This can of course happen in a multitude of ways, but there are two cases of special importance and these we will discuss in detail.

Firstly we will let *Z* and *W* of Figure 147 become *I* and *J* of the horizontal plane in our new tetrahedron. Our tetrahedron, shown in Figure 150, will now contain two real points, *X* and *Y*, two imaginary points, *I* and *J*; two real (and horizontal) planes, *XIJ* and *YIJ*, two imaginary planes; two real lines, *XY* and the line at infinity (*IJ*), and four imaginary lines. The two imaginary planes are represented by involutions of planes carried by the line *XY*, and the two pairs of imaginary lines are represented by involutions of lines centred in *X* and *Y* in the horizontal planes. For more details about these imaginary elements the reader is referred to my book, *Projective Geometry*.

Along the line at infinity, one of the two real invariant lines of the transformation, we shall have a movement in circling measure between the invariant points *I* and *J*. This will give us constant, equiangular movement. Each of the real invariant planes contains an imaginary triangle, *XIJ* and *YIJ*. The path curves in these planes will therefore be logarithmic spirals, the rate of turning being the same, of course, in each.

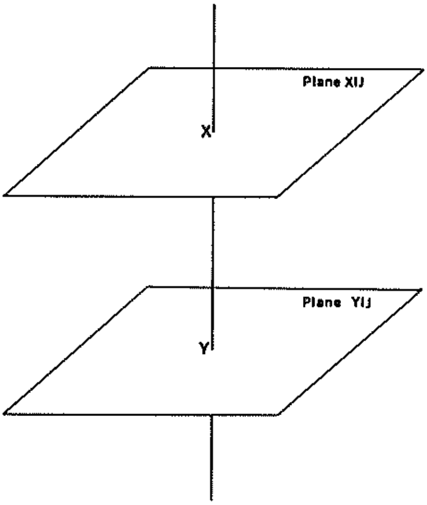


Figure 150. This is the real and visible part of a semi-imaginary tetrahedron.

The spirals will usually be of different constant multipliers, and the general form of the curves will depend to a large extent upon whether, while the spirals turn in, say, an anticlockwise direction, they are both working inwards, or whether one works inwards and the other works outwards. Both possibilities are valid.

How does a general point of space move within such a transformation? The process is quite simple. In Figure 151 we see a point *P* set between the invariant planes of the transformation. We let *XP* and *YP* meet the bottom and top planes in *T* and *S*, respectively. Note that *XS* and *YT* are necessarily parallel. Now *S* and *T* will be moving with equal, and constant, turning speed, say anticlockwise, each in its own logarithmic spiral; we suppose in this case that *S* is working inwards while *T* is working outwards. After one step of the transformation *S* has reached to *S*′, and *T* to *T*′. Since they are both turning in perspective with the circling measure between *I* and *J*, *XS*′ and *YT*′ will still be parallel; *YS*′ and *XT*′ will be concurrent and their common point, *P*′, will be the next point in the path curve followed by *P*.

A fully perspective view such as is seen in Figure 151 is very tedious to draw, as it is necessary to construct perspective views of the spirals in the top and bottom planes. It is easier and quicker to make drawings in orthogonal perspective, seeing the thing from the horizontal direction. The two invariant planes then appear as horizontal lines with points

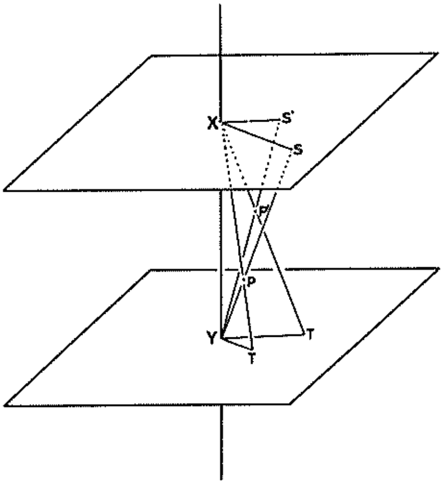


Figure 151. The method of plotting how a point moves in the course of the path-curve transformation.

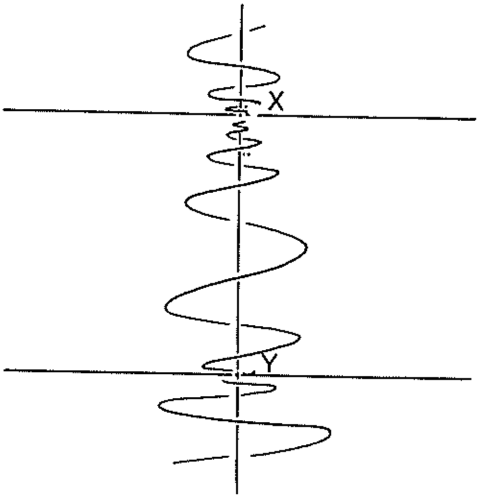


Figure 152. A typical path curve within the semi-imaginary tetrahedron.

moving in geometric series. The details of the construction are perfectly straightforward and need not be described here.

If, as in Figure 151, one spiral winds inward while the other winds outward, the general form of the curves is as in Figure 152. The curves wind infinitely outward from one pole and inward toward the other. If on the other hand both spirals are winding in the same direction, then the curves take on a vortex-like appearance, winding from the infinite periphery inward toward one of the finite poles only, as shown in Figure 153.

If we were to take, let us say, the curve shown in Figure 153 (which we shall call the A-curve) and use this as our generating curve for an invariant surface, and insert this into the complete transformation-field which produces, amongst an infinitude of others, the curve of Figure 152 (the B-curve), and if we were to draw all the B-curves which pass through the points of the A-curve, the totality of these B-curves would form for us one of the fundamental surfaces associated with this particular invariant tetrahedron. Having got our surface we would find that not only would it contain an infinite family of B-curves, and an infinite family of A-curves, but also an infinite family of other infinite families, each associated with one of the other transformations which work within this tetrahedron. The surface would be very difficult to draw (by this means), harder still to imagine, but of great interest and beauty.

Fortunately an easier way presents itself. Amongst all the transformations which are possible within this tetrahedron there is a family of special ones, in which the spirals of the top and bottom planes both wind inward with the same constant multiplier: they are congruent spirals. In such a case, all the path curves become plane logarithmic spirals, each lying in its own horizontal plane. Such curves can, just as well as any of the others, be used as generating curves for our fundamental surfaces, and of course they are much easier to work with.

So we can say now that to form a fundamental surface within this tetrahedron, all we have to do is to place within it any horizontal plane containing a logarithmic spiral centred on the point in which line *XY* cuts this plane. All the path curves of the system which pass through the points of this spiral form the surface we want.

The spirals possible form a spectrum with multipliers (per radian turned) stretching all the way from zero to infinity. In the former case we have a circle, and in the latter a straight line radiating from the central axis. The latter case gives us ruled surfaces of spiralling form, and the former a series of surfaces of revolution. These turn out to be of such importance that we must study them in more detail.

Suppose we have a transformation of a similar kind to that of Figure 152, and we place within it a horizontal circle centred on the axis *XY*, and we now take all the path curves which pass through the points of

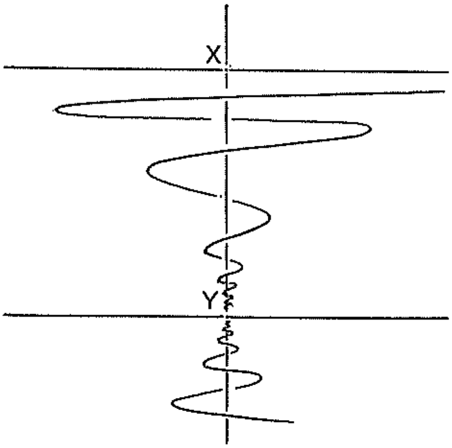


Figure 153. The type of path curve which is produced when λ becomes negative.

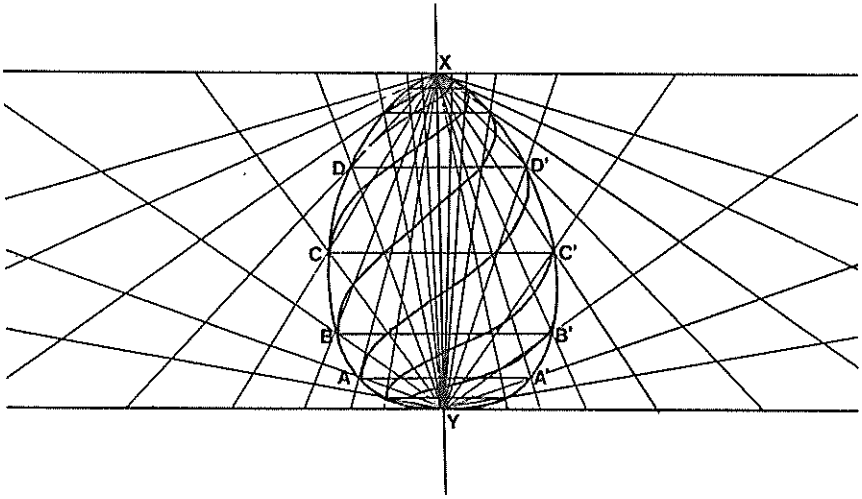


Figure 154. The method for constructing the spiral path curves which lie on an egg-surface.

this circle. What sort of surface will be produced? It will be an egg-shaped surface, rather sharper towards *X* and blunter towards *Y*. It will be completely covered with spiralling curves, winding infinitely out of, and into, the two poles at *X* and *Y*. There will be an infinite family of other transformations which can each cover it similarly, but each with curves that spiral at different pitch. One such surface, with one family of covering curves, is shown in Figure 154.

We suppose that the curve on the top plane is winding inwards with a multiplier of 2 for every 60° turned, and on the bottom plane that it is winding outwards with a multiplier of $1^1/_2$ for 60° turned. These curves are represented by geometric series with these constant ratios along the top and bottom lines of the drawing. The construction now continues in the same way as for the plane curves pictured in Figure 17. Successive points *A, B, C, D,* and so on show the heights which a point will have attained in successive steps of the transformation and its distance outwards from the central axis. We have now only to add the rotation between the imaginary invariant points *I* and *J*. We mark the midpoint between *B* and the axis, the midpoint between *C* and the axis, etc. The curve will go from *A* to the first of these midpoints, to the second of them, and then to *D* before disappearing round the other side of the surface. And so on. Elementary use of trigonometrical tables will easily allow one to cope with transformations which turn at some other angle than 60°.

Proceeding similarly we can make pictures of the surfaces of revolution for transformations of the type shown in Figure 153. These come out to be beautiful funnel-like vortices.

We have now to develop a system of parameters which will enable us exactly to determine, and control, these kinds of curves and surfaces. We cannot do it with three straightforward parameters, *a, b,* and *c,* along edges of the tetrahedron, as we did it with the all-real case, since we now only have two real edges left. One of these is the line at infinity, and here we have rotation with respect to *I* and *J.* This movement we designate θ, and of course it is always measured in radians.

We shall be specially interested in the shapes of the egg surfaces, pictured in Figure 154, and these shapes we know, from what has gone before, are completely determined by the ratios of the logarithms of the multipliers along the horizontal lines through *X* and *Y,* that is to say, in the three-dimensional case, the logarithms of the multipliers of the logarithmic spirals in the planes *XIJ* and *YIJ.* When, as often happens, we are not interested in the size of the discrete stepping of the transformation, but only in the shapes of the forms, we let θ equal unity, and we work out the multipliers of the spirals, per one radian turned.

Now we know that the log multiplier (parameter) along the *X*-plane, plus the parameter along the *Y*-plane, gives the parameter of the growth measure which is then produced working along the central axis, from *Y* to *X.* So we proceed as follows: We let ε be the average of the parameters on the *X*- and *Y*-planes. The *X*-parameter then becomes ε + α and the *Y*-parameter ε – α, where α is half the difference between the *X*-parameter and the *Y*-parameter. It is then clear that the parameter of the growth measure moving from *Y* to *X* will be 2ε. Hereafter, as far as the case of the semi-imaginary tetrahedron is concerned (that is, two real and two imaginary invariant points) we shall not be dealing with what we have called above the *X*-parameter or the *Y*-parameter. We shall be dealing with the three parameters ε, α, and θ.

Our egg and vortex forms will be generated thus: We have a θ-rotation moving with constant speed along the line at infinity (circling measure between *I* and *J*), we have logarithmic spirals moving inwards in the *X*-plane with a log-multiplier of ε + α per one radian of θ turned, and logarithmic spirals moving outwards in the *Y*-plane with a log-multiplier of ε – α per one radian of θ turned. Along the central axis we have a

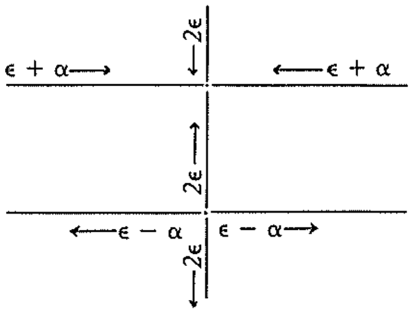


Figure 155. The movements associated with the positive parameters.

growth measure moving from *Y* to *X* with a log-multiplier of 2ε. The shape of the set of egg- or vortex-surfaces is exactly determined by the parameter λ where

$$\lambda = \frac{\varepsilon + \alpha}{\varepsilon - \alpha}$$

Figure 155 shows us the positions and the directions of these parameters. Notice that, by the triangle law which was described in the preceding chapter, since the parameters along the two horizontal lines are $\varepsilon + \alpha$ and $\varepsilon - \alpha$, the one along *XY* must be -2ε. This, of course, implies $+2\varepsilon$ from *Y* to *X*, and this is the reason why this parameter, 2ε, must be considered positive when it is running 'against the tide,' that is to say away from *Y*. Notice also that, although the actual parameters which we are now using are different from the ones we employed in the all-real case, the *number* of independent ones — three — is unchanged.

If we are interested also in the size of the stepping of the transformation, then of course we have to have some other value than unity for θ, according to the size of the step. And then of course there will be consequent changes in ε and α. In such a case, where $\theta \neq 1$, we generally refer to our parameters as ε′, α′, and θ′. Now we know that the size of the angular stepping in a circling measure is a geometrical equivalent of the argument of a complex number, and arguments behave in a similar way to logarithms, that is they add for multiplying and subtract for dividing, and so on. Since ε and α are both logarithms, it means that ε, α, and θ always change in direct proportion to one another.

Thus we may say that the complete transformation is determined by the three parameters ε', α', and θ'. Disregarding the size of stepping, the complete form is determined by the ratios $\varepsilon:\alpha:\theta$. The form of the eggs or vortices is determined by the ratio $\varepsilon:\alpha$, or more conveniently by $\lambda = (\varepsilon + \alpha)/(\varepsilon - \alpha)$. But the pitch of the spirals covering the egg is determined by the ratio $\varepsilon:\theta$.

We are now in a position to picture the whole spectrum of possible forms for the fundamental surfaces which are given by a circular cross-section, as λ varies. The drawings are vertical cross-sections.

When $\lambda = 1$ (Figure 156a), the surfaces are spheroids, with elliptic cross-sections between the invariant planes, and hyperbolic outside them.

As λ increases above unity (Figure 156b), the eggs become pointed above and blunted below. Notice that the amount of sharpening and blunting is exactly comparable and is shown by the size of λ. One can never get an egg which is slightly sharp at one end and very blunt at the other, or vice versa.

As λ approaches ∞ (Figure 156c), the egg shape approaches the cone.

As λ 'passes through infinity' (Figure 156d), the cone bends slightly inward to make a vortex. As λ increases towards -2, the vortex becomes more and more sharply bent.

When $\lambda = -2$ (Figure 156e), the sides of the vortex have assumed a hyperbolic cross-section. (This follows from elementary algebra. If $\lambda = (\varepsilon + \alpha)/(\varepsilon - \alpha) = -2$, then $\alpha = 3\varepsilon$. Substituting this into the formula for λ we have $\lambda = 4\varepsilon/(-2\varepsilon)$. That is to say that the movement along the horizontal line through Y is of equal multiplier with the movement from Y to X, both are 2ε. Thus the pencils of lines in X and Z are projective and must meet on conics.)

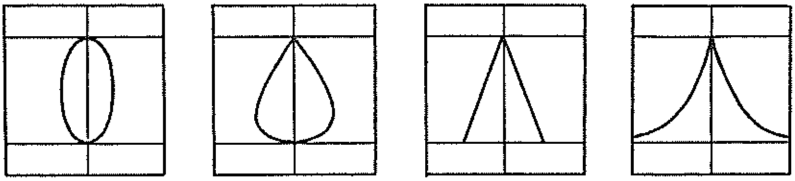
As λ approaches -1 (Figure 156f), the vortices become more and more sharply curved.

When $\lambda = -1$ (Figure 156g), the path curves degenerate into plane spirals, indicated in cross-section as horizontal lines.

As λ increases from -1 to zero (Figures 156h, i and j), the vortices go through exactly the same metamorphoses as from $-\infty$ to -1, but upside down. Congruent forms are given by reciprocal values of λ.

When $\lambda = 0$ (Figure 156k), the surfaces have again become cones.

Between zero and unity (Figure 156l) we have the same metamorphoses as between unity and infinity, but upside down. Again congruent forms are given by reciprocal values of λ.

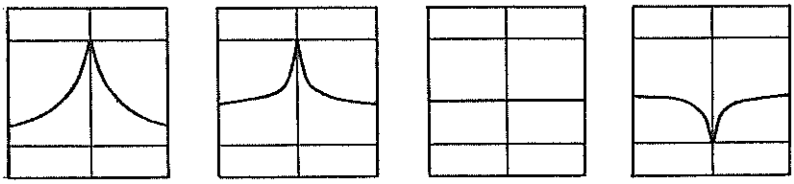


a. Spheroids
$\lambda = 1$

b. Eggs
$1 < \lambda < \infty$

c. Cones
$\lambda = \infty$

d. Vortices
$-\infty < \lambda < -2$

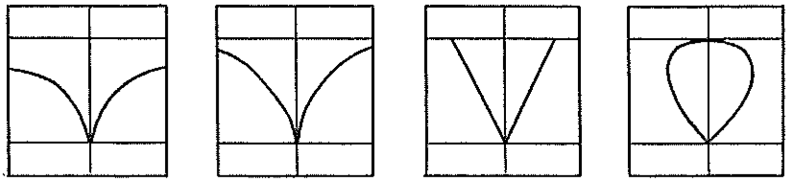


e. Vortices with hyperbolic cross-sections
$\lambda = -2$

f. Vortices
$-2 < \lambda < -1$

g. Plane spirals
$\lambda = -1$

h. Vortices
$-2 < \lambda < -\frac{1}{2}$



i. Vortices with hyperbolic cross-sections
$\lambda = -\frac{1}{2}$

j. Vortices
$-\frac{1}{2} < \lambda < 0$

k. Cones
$\lambda = 0$

l. Eggs
$0 < \lambda < 1$

Figure 156.

It would perhaps be useful to give here a sample of the calculations needed to construct a picture of an egg-form with given parameters. We let $\lambda = 2.10$ and $\varepsilon = 1.50$. It follows that

$$\frac{1.50 + \alpha}{1.50 - \alpha} = 2.10$$

$$\text{whence} \quad \alpha = 0.532.$$

This gives us an expression of 2.032/0.968 for λ. The top and bottom lines of this fraction give us the parameters for the growth measures along the top and bottom lines of our drawing, for one radian turned in the θ-measure. However, we shall draw our form for a measure of 30° turned. These parameters have then to be multiplied by $30\pi/180$, giving 1.06 for the top line and 0.507 for the bottom. Antilogging (base *e*) these we have multipliers for the top and bottom lines of 2.90 and 1.66.

We now make our drawing like Figure 154), but using these multipliers for our geometric series (Figure 157). And of course referring again to Figure 154 we shall have to multiply the distances between *A, B, C* and so on, and the central axis, not only by 0.500 (sin 30°) but also by 0.866 (sin 60°) in order to draw our path curves on the egg.

We next have to investigate a little more closely what is the meaning of the parameter α. To do this we shall need to repeat Figure 154, but this time with the parameters along the *X*-line and the *Y*-line the same as one another. In other words we make pictures of our egg-surfaces with

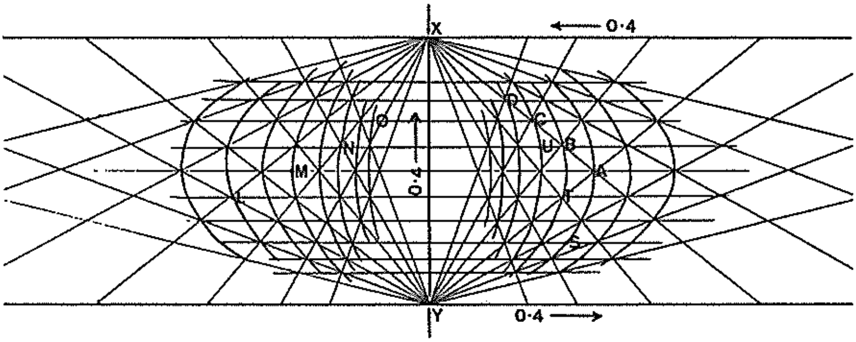


Figure 157. This diagram shows how the parameters ε and λ actually work in practice.

$\lambda = 1$, or, which is the same thing, with $\alpha = 0$. We then find ourselves with a set of ellipses in the plane diagram, representing spheroids in the three-dimensional reality.

This is really just a special configuration of Figure 146, and these conics and straight lines, representing spheroids and horizontal planes through the points of a growth measure along *XY*, form a perfect grid for path-curve metrics. Now α shows the multiplicative factor for the distance of the point from the central axis, *as measured between the spheroids of the system.*

We have made our geometric series along the *X*- and *Y*-lines with a multiplicative factor of 1.492, giving a logarithmic parameter of 0.4. Horizontal lines through consecutive pairs of points on any of the ellipses will cut the line *XY* in points of a growth measure with a parameter of 0.8. Reference to the diagram will show that this would mean every other line of those that have been drawn; therefore we see that those lines which have been drawn must have a growth measure, from *Y* to *X*, with a parameter of 0.4. Remembering Figure 155, we realize that this movement will have a positive ε-value in this direction. It is easy to see that each horizontal line cuts alternate ellipses with a multiplicative factor of 1.492. Therefore to go from one ellipse to the next but one involves a parameter of 0.4 and to go from one ellipse to the next, consecutive, one, a parameter of 0.2. Notice that α works *with* ε along the inwards movement of the top line, and *against* ε along the outward movement of the bottom line; on both counts therefore it is clear that α represents an inward movement in consecutive steps of the transformation — the distance will be divided by α each step.

Now suppose we have a transformation which takes a point from *A* to *B* to *C* to *D* and so on, while θ turns by one radian per step. Horizontal lines through these points cut *XY* with a parameter of 0.4. This we know is 2ε, therefore for this transformation ε must be 0.2. But the points are moving inwards through the spheroids of the system with a parameter of 0.2, that is $\alpha = 0.2$. Therefore

$$\lambda = \frac{\varepsilon + \alpha}{\varepsilon - \alpha}$$

$$= \frac{0.2 + 0.2}{0.2 - 0.2}$$

which in this case we are justified as representing as ∞. This is the conical case of Figure 156c.

However, if the point moves from *S* to *T* to *U* to *D* and so on, the horizontal lines are moving with a parameter of 0.8 along *XY*, giving a value of 0.4 for ε. We are still moving inwards through the spheroids of the system with a parameter of 0.2, so in this case

$$\lambda = \frac{\varepsilon + \alpha}{\varepsilon - \alpha}$$

$$= \frac{0.4 + 0.2}{0.4 - 0.2}$$

$$= 3$$

This is a case like Figure 156b.

But if we let the point move from *L* to *M* to *N* to *O* and so on, we see that $\varepsilon = 0.2$ but $\alpha = 0.4$. In this case

$$\lambda = \frac{\varepsilon + \alpha}{\varepsilon - \alpha}$$

$$= \frac{0.2 + 0.4}{0.2 - 0.4}$$

$$= -3$$

This is the case of Figure 156d, a vortex curve.

To sum up we can say:

ε is half the parameter with which the transformation moves along the central axis, from *Y* to *X;*

α is the parameter with which it moves inwards through the spheroids of the system;

$\varepsilon + \alpha$ is the parameter with which the logarithmic spiral winds inwards on the *X*-plane;

$\varepsilon - \alpha$ is the parameter with which the logarithmic spiral winds outwards on the *Y*-plane;

λ is the parameter which shows the shape of the egg-surface generated by a horizontal circle, while ε is the parameter which shows the pitch of the spirals which lie on it, all these being calculated against a constant θ-value of one radian turned.

When we come to consider the fundamental surfaces which can be generated by putting not a circle but a logarithmic spiral on a horizontal plane through the path-curve system, we enter a realm of extraordinary subtlety and complexity, and of great beauty and interest. The surfaces are of two general kinds: spiral eggs and spiral vortices. If we turn once again to Figure 17 and imagine a surface which passes through the right-hand edge of the inmost egg, winds round the back, reappears on the left-hand edge of the second egg, winds round the front to the right-hand edge of the third egg, and so on, we have an idea of the spiral egg. The spiral vortex would work similarly with the vortex forms of that diagram.

Thus we see that the vertical cross-section of such a form is a family of cross-sections associated with a λ-value of one of the forms generated by a circular horizontal cross-section. In order to gain control of these we need to introduce two new parameters, β which is the log multiplicative factor of the horizontal generating spiral (for θ = one radian turned, of course) and μ which we define as

$$\mu = \frac{\varepsilon + \alpha + \beta}{\varepsilon - \alpha - \beta}$$

Now μ gives us the λ-value of the vertical cross-sections of our surface: in other words, if $\mu = 2$ then the cross-sections of our surface will be the same in shape as those which horizontal circles will generate in a transformation whose λ is 2.

Strange things now become possible. Suppose that $\varepsilon = 0.8$, $\alpha = 0.5$, $\beta = 0.7$. Then we find that $\mu = -5.0$ and $\lambda = +4.3$. We have egg-like curves lying, along all their length, on a spiral vortex.

But if $\varepsilon = 0.2$, $\alpha = 0.3$, $\beta = -0.4$, then $\mu = +0.3$ and $\lambda = -5$. Here we have vortex-like curves lying on an egg-like surface. Such things are hard to draw, and harder still to imagine. Probably the only effective way to tackle them is to make models of them.

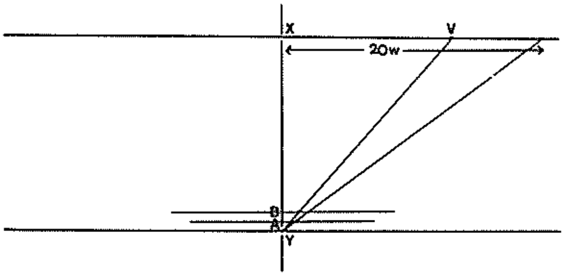


Figure 158. The constructions needed for making a Perspex model of the path curve surfaces.

This system of parameters enables us to have complete control of what we do in the realm of these surfaces. If we keep μ and β constant, we have a simple relation connecting ε and α. If we now vary ε, putting in the consequent values of α, we keep a constant surface but we run through the different kinds of curves with which different transformations will cover it. But if we keep ε and α constant we get a simple relation between μ and β. Now by varying β we run through the gamut of the different surfaces possible within a constant transformation; in each case the consequent value of μ gives us a clue as to the shape of each surface. We may not be able to imagine what it will *look* like, but we know immediately what it will *do.*

Probably the simplest way to make a model is by 'layering.' Suppose our model is to measure 50 mm upwards from Y to X: we take twenty pieces of Perspex (Plexiglas) each 2.5 mm thick and draw on each the horizontal cross-section of the surface at that height. When they are placed one on top of the other, the eye makes a fairly satisfactory integration of the separate curves into the appearance of a surface.

Here is how the calculations for such a surface would start. Let us suppose that $\varepsilon = 0.3$ and $\alpha = 0.1$ and that we have decided on some value of β. Figure 158 shows the thing in elevation.

On the top of our first piece of Perspex, at a height of 2.5 mm above Y (the height of A in Figure 158) we draw a logarithmic spiral with our given parameter β. Now to draw our second layer, at a height of point B, we must calculate by what multiplicative factor each point of the original spiral has been carried outward, and by what θ-value it has been rotated. To do this we must find out how big a step of the transformation is involved by moving along the central axis from A to B. We find the cross-ratio concerned to be

$$\frac{YB}{BX}\cdot\frac{XA}{AY}=\frac{5.0}{45.0}\cdot\frac{47.5}{2.5}$$

$$=2.11.$$

$$\text{Ln } 2.11 = 0.747$$

Now one complete step of the transformation will be 2ε, so the step from A to B represents 0.747/0.6, that is 1.245 times a complete step.

$$\lambda=\frac{0.3+0.1}{0.3-0.1}$$

$$\lambda=\frac{0.4}{0.2}$$

so the multiplicative factor along the top line will be

$$\text{antiln}\,(0.4 \times 1.245) = \text{antiln}\,0.498$$
$$= 1.646$$

Suppose a point of our spiral at the height of *A* cuts the page of our paper at some distance *w* from the central axis. The line from *Y* through this point will cut the top line of the model at a distance of 20*w* from *X*. We know that the parameter $\varepsilon + \alpha$ works inwards along this line, so we must divide this distance by our multiplicative factor 1.646, giving us a distance outwards of 12.2*w* (point *V*). Obviously the distance outwards of the new point at the height of *B* will be one tenth of this, that is 1.22*w*. Hence we see that for the move from the height of *A* to *B* every point on our spiral will have its distance from the centre increased by a factor of 1.22. Also it will have experienced a rotation in the θ-direction of 1.245 radians or about 71°.

This enables us to draw our new spiral for the height of *B*, and further similar calculations give us those for the other pieces of Perspex.

Two imaginary points: second case

The other example of special interest in the semi-imaginary case is when we keep point *Y* on our page and let *X* move to infinity, in a direction at right angles to the plane *XIJ*. The invariant plane through *X* now becomes the plane at infinity itself.

If we now place the infinite centre of a negative-Euclidean space or counter-space at *Y* we find that this tetrahedron is wonderfully knit with the whole interplay between negative and positive space. Two of its elements, the plane at infinity and point *Y*, *are* the positive and negative absolutes themselves. In the positive absolute our tetrahedron has six elements, real point *X* and real line at infinity, two imaginary lines carried by *X* and two imaginary points *I* and *J*, carried by the line at infinity. The imaginary points *I* and *J*, are points of the absolute imaginary circle which presides over Euclidean space, the imaginary lines are tangents to it, and point *X* and the line at infinity are pole and polar with respect to it. In the negative absolute we also have six elements, the real (horizontal) plane through *Y*, and the line *YX*, two imaginary lines carried by the point *Y* (in its horizontal plane) and two imaginary planes carried by the line *YX*. These planes are tangent planes of the imaginary cone which presides over the negative space, the imaginary lines also belong to this cone, and the real line *YX* and the horizontal plane through *Y* are pole and polar with respect to it.

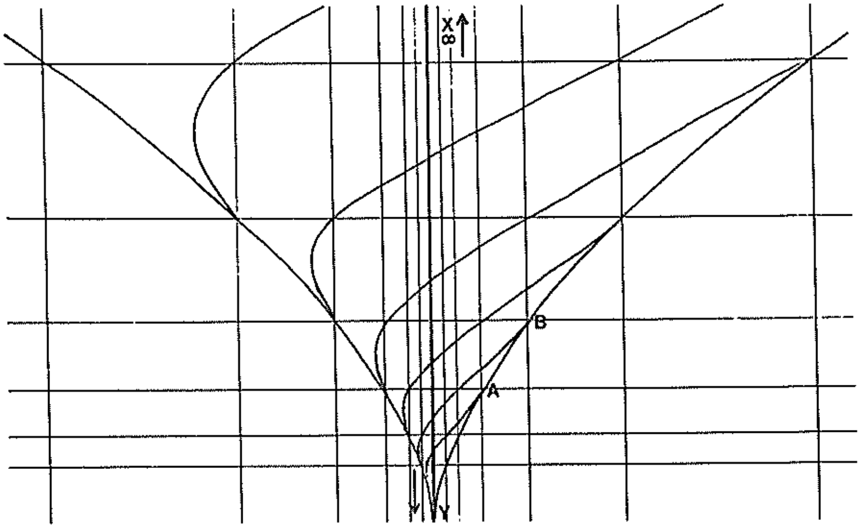


Figure 159. A vortex path curve surface, with a negative λ and one of the invariant points at infinity.

This tetrahedron can be said not only to span all space, but to be interwoven most subtly with the positive and negative interconnections of it, and surely the path curves with respect to it must be of interest to us. It is quite easy to draw pictures of them using the parameters $\varepsilon - \alpha$ along the horizontal plane and 2ε along the central axis. The general form of the fundamental invariant surfaces generated by horizontal circles can be fairly easily imagined by considering Figures 156 and imagining the top invariant point removed to a very great distance. When $\lambda = 1$ we have a system of paraboloids of revolution. When λ is negative we have beautiful, and very important, vortex forms. Figure 159 shows one such, and it seems worth while putting on record the method of determining its λ-value.

We have chosen a multiplier of 2 along the horizontal plane, and one of 1.5 along the vertical axis, and, for the sake of convenience in drawing, a turn of 60° per step of the transformation. For the moment we consider the thing as a plane diagram; the arrows show the directions of movement round the sides of our invariant triangle which we have conventionally adopted as our standard directions; but we remember that 2ε is a parameter which has to be considered positive when working 'against the stream,' that is, away from *Y*. Now in moving from *A* to

B we go horizontally outwards from *Y* with a factor of 2, that is $\varepsilon' - \alpha' = \ln 2 = 0.693$. At the same time we move with a factor of 1.5 from *Y* towards *X;* this gives a positive value of 0.405 for $2\varepsilon'$. It follows that $\alpha' = -0.49$; therefore

$$\lambda = \frac{\varepsilon' + \alpha'}{\varepsilon' - \alpha'}$$

$$= \frac{0.203 - 0.49}{0.203 + 0.49}$$

$$= -0.414.$$

This is a case of Figure 156j.

Notice that since these calculations have been made against a θ-movement of 60° we would have to multiply both ε' and α' by $180/60\pi$ to convert them to values of ε and α balanced against one radian turned.

Four imaginary points

The next thing to consider is what happens if all four of our invariant points become imaginary, in two conjugate pairs. The whole of our invariant tetrahedron now passes into the imaginary with the exception

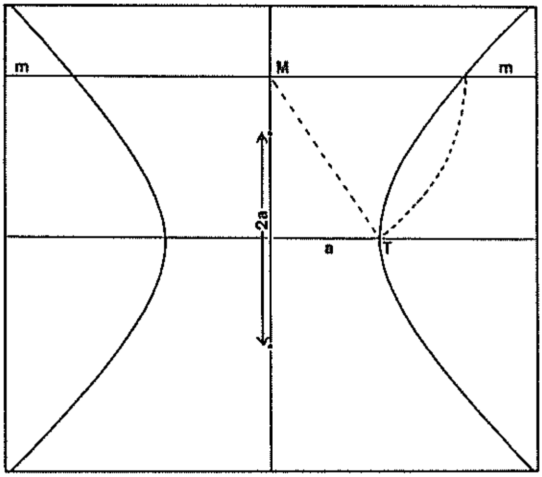


Figure 160. A construction which helps in drawing path curves with an all-imaginary tetrahedron.

of two real lines: the central axis, bearer of the imaginary pair *X* and *Y*, and, let us say, the horizontal line at infinity, bearing *I* and *J*.

Drawings of curves and surfaces now become rather more difficult. It is best to proceed by making an imaginary analogue of Figure 157. The ellipses of this diagram passing through *X* and *Y* become hyperbolae containing the imaginary pair *X* and *Y*, and representing single-sheeted hyperboloids in the three-dimensional reality.

If we have an imaginary pair *X* and *Y* along the central axis, with a given amplitude, $2a$, Figure 160 gives an easy way of constructing a hyperbola which shall contain these points. Take a point *T* distant *a* from the axis. Now draw any horizontal line *m*, to cut the axis in *M*. The distance *MT* measured from *M* along *m* gives two points of the hyperbola. The other hyperbolae of the system are obtained by a simple projective stretch parallel to the x-axis.

Now our path curves will be controlled by the three parameters: *a*, measured in radians along the circling measure of the central axis controlled by the imaginary pair *X* and *Y*; *b*, the logarithm of the multiplicative factor outwards, measured through the hyperboloids of the system; and *c*, measured in radians along the line at infinity between the imaginary pair *I* and *J*.

Fundamental invariant surfaces are obtained in the same way as in the semi-imaginary case, by putting logarithmic spirals through the system, centred on the central axis, and taking path curves through their points. The details of the constructions necessary can be left to the reader.

Appendix 3

The Determination of λ

The projective method

This is the method which was first devised, and is still the one which I mostly use. The main outlines of the method are described on page 60. In essence we need to know any two diameters of the egg-form with their respective heights above the bottom pole, and we can calculate the unique value of λ which will put a path curve through the top and bottom poles and through the ends of those diameters. If d_n is the diameter at a fractional height h_n measured upwards from Y using the distance XY as unity, then λ is given by

$$\lambda = \frac{\log \dfrac{d_2\, h_1}{d_1\, h_2}}{\log \dfrac{d_1\,(1-h_2)}{d_2\,(1-h_1)}}$$

(If one consults Figure 154 and remembers that the intervals along the horizontal lines through X and Y are geometric series representing the in- and out-winding logarithmic spirals of the top and bottom invariant planes, by the use of similar triangles the above formula is easily derived.)

It is not necessary to use just 7 levels (i.e. dividing the total height of the bud by 8) but this is found to be an effective number for a good balanced result.

When doing hand measurement it is convenient to express all diameters at the various levels in terms of the diameter at level T, the midpoint of XY, as unity. This can be done quickly and easily at the time of measuring. We let A, B, C, and so on, represent the diameters at levels A, B, C, expressed in terms of the diameter at level T as unity. The elementary arithmetic shows that the λ-values at the various levels are found as:

Level	λ-value
A	$\frac{\log (4A)}{\log (7/4A)}$
B	$\frac{\log (2B)}{\log (3/2B)}$
C	$\frac{\log (4C/3)}{\log (5/4C)}$
D	$\frac{\log (5/4D)}{\log (4D/3)}$
E	$\frac{\log (3/2E)}{\log (2E)}$
F	$\frac{\log (7/4F)}{\log (4F)}$

Having found all these we find the mean.

This is still a tedious calculation, especially if it has to be done many times over for a large number of buds, and it can be greatly shortened by a further consideration. It is a fact that for any given value of λ, there is a whole field of egg-like forms, one within another, and all of these can be derived from any one of their number by a simple horizontal stretch (a stretch at right angles to the XY axis). This means that for any given value of λ, the ratio of the diameter at any level to that of the diameter at level T is constant, whether the egg-like form being measured is a wide or a narrow one. In other words this ratio is fixed simply by the value of λ, and is unaffected by the absolute diameter of the form. Thus with the help of the little formulae given above one can easily arrive at the graph of Figure 161.

Along the x-axis we have the diameter at any given level, measured in terms of the diameter at level T as unity. Along the y-axis we have the corresponding λ-values. The body of the graph is made up of six curves, one for each of the levels A to F. Having got our diameters at the six levels we simply read off the corresponding λ-values, and then find their mean. It is quite quick and easy. This is how I had done it for years, but with the coming of the computer it is easy to program the formula at the beginning of this appendix so that the machine will deliver λ from the measurements of any number of levels which one wishes to make.

It is quite clear that those levels which are near to the central level, T, will give less reliable results than those which are nearer the poles of

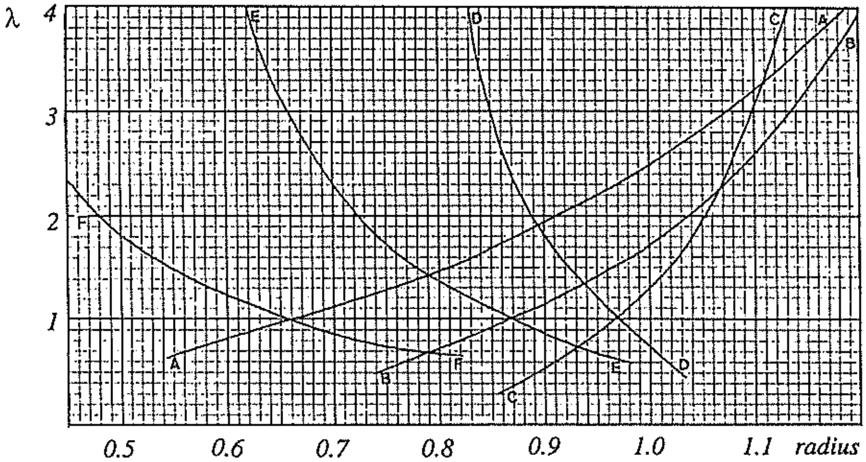


Figure 161. Graph for determining λ.

the form, which suggests that one ought to take a weighted mean of the λs derived from the various levels. In fact the reliability of the results derived from the various levels varies approximately with the inverse of the gradients of the curves in Figure 161. But there is another reason why a weighting procedure ought to be followed. The result obtained from the diameters at levels *A* and *T* gives a value of λ which is applicable to $^3/_8$ of the length of the bud, whereas that obtained from the measurement at level *C* concerns only $^1/_8$ of the bud length. It is therefore clear that the result obtained from level *A* ought to weigh more heavily than that from level *C*. It is found that good consistent results are obtained by weighting each level according to its distance from level *T* as measured in terms of the transformation movement along the axis *XY*. The weighting factor at level h_n is then given by

$$w_n = \left| \log\left(\frac{h_n}{1-h_n} \right) \right|$$

The λ for each level should be multiplied by the *w* for that level, and the total result should be divided by the sum of all the *w*-values. In the case where we take 7 levels (i.e. dividing the bud-length by 8) this amounts approximately, and sufficiently accurately, to multiplying the results from levels *A* and *F* by 4, those from levels *B* and *E* by 2, leaving the values from levels *C* and *D* as they are, and dividing the total of the values so obtained by 14.

The morphographic method

This is one of several possible variants of the preceding. Instead of comparing all diameters at all levels with that at the central level, one compares consecutive levels, *A* with *B, B* with *C, C* with *D* etc. This gives us a series of λ-values along the length of the bud, from base to tip, and the mean of these is taken as the λ for the bud as a whole. There seems no reason for any weighting procedure, and the straightforward mean is accepted. Since the levels being compared with one another are close together one would not expect quite such reliable and consistent results and experience goes to confirm this.

However it seems likely that the way in which the λ-value varies as one passes from the base to the tip might sometimes give useful information about any peculiarities in the form of the bud, and when this has been further researched it may turn out to be a method of some value. It is for this reason that I call it the morphographic method.

The regression method

I owe this to my friend Graham Calderwood, of Camphill, Aberdeen. If we refer back to Figure 154, we see how if we project the ends of the diameters, *A B C* etc. from *Y* onto the invariant line through *X* we get a series of points which represent for us the branches of the in-circling logarithmic spirals which lie in the top invariant plane of the transformation. Thus, if the form which we are measuring is a good path curve, we shall find this series of points marking out a geometric series along this line. Similarly with the projections from *X* of the same set of diameters, onto the invariant line through *Y.* Calderwood points out that if we take any point of the curve, say *A,* and consider its projections, from *Y* onto the horizontal line through *X,* and from *X* onto the horizontal line through *Y,* we can take the logarithms of the distances of the resulting points, from *X* and from *Y* respectively, as 'co-ordinates' of point *A* in a system of what we might call path-curve co-ordinates. And if the curve we are measuring is a good path curve, this set of co-ordinates, when plotted on an ordinary Cartesian grid will lie on a straight line. With our new set of co-ordinates we have moved into a new level of experience, as it were, and at that level the path curve is an expression of an ordinary linear function.

At this point we should notice two things. Firstly the distances measured along the invariant line through X, being projections from Y, are best called our y co-ordinates, and the others our x co-ordinates. Secondly since, when the true movement of the path curve system is inwards along the line through X, it will be outwards along the line through Y, one of these sets of ordinates will be going, as it were, against the grain compared with the other. Therefore if our y co-ordinates are considered positive, it will be logical to consider the others negative.

Now, by the ordinary methods of linear regression, it will be easy the find the best fitting straight line for these points, and if the above conventions are followed, the gradient of this line will be our parameter λ. If we call the distances measured along the line through X our x co-ordinates and the others the y co-ordinates, then λ will be the reciprocal of the gradient given by regression. And if we neglect to change the signs of one set of co-ordinates, then λ will be the negative of the given gradient.

This is a quick and easy method of finding λ, specially when using a computer, and gives reliable results.

Christian's formula

This useful formula, based on the method of least squares, was first given by Barry Christian. In its most convenient form it gives λ very quickly from measurements made at seven levels. The formula is as follows:

$$\lambda = \frac{2[\ln^2(5/3) + \ln^2 3 + \ln^2 7]}{\ln 7[\ln(7F/A)] + \ln 3[\ln(3E/B)] + \ln(5/3)[\ln(5D/3C)]} - 1$$

where A B C etc. are the measured diameters at those levels. The top line is a constant, approximately equal to 10.51.

Goodness of fit

When using either of the first two methods described above, the projective and the morphographic, the process itself provides one with a ready and easy method of measuring how closely the object being measured approximates to a true path curve form. If it is a true path curve form the λ-values at all levels will be identical, and in as far as they vary, the object departs from the true mathematical form. Having found the mean

λ one can quickly calculate the percentage deviation at each level, and the mean of these gives us the MLD (mean lambda deviation).

The MLD is a very sensitive figure. It is found by experience that if the MLD is less than 20% the fit is tolerably good, at less than 10% it is definitely good, and at less than 5% or 6% it is very good indeed. The MLD also suffers from the defect that it is not easy to visualize just what any figure for MLD really implies in terms of visible form.

It is often therefore preferable to use the MRD (mean radius deviation). To calculate this we select one of our levels as our standard level and assume that the measured radius at that level is correct. Given this, and the calculated λ for the form, we can calculate what the radius ought to be at each of the other levels if the form is a perfect path curve. By comparing these radii with those which were actually measured, we can quickly find the percentage deviation at each level, and thus arrive at the mean radius deviation for the whole form. If our standard radius is r_1 at fractional height, h_1, then the radius, r_2, at fractional height h_2, is given by

$$r_2 = r_1 \left(\frac{h_2\,(1-h_2)^{\lambda}}{h_1\,(1-h_1)^{\lambda}}\right)^{\frac{1}{\lambda+1}}$$

If we now re-calculate the whole thing using another level for our standard radius, we shall get an answer which is very nearly the same as the first one, although there will normally be some slight difference. For ordinary purposes once is enough! But since we are looking for the best fit possible, and if we are using the computer, we can easily program the machine to calculate the MRD using each of the levels, to select the least of these answers, and to deliver that.

As has been stated in the body of the book, an MRD of less than 4% can be considered as fairly satisfactory, one of less than 3% as good, less than 2% as very good, and less than 1% as almost perfect.

Which method is best?

If the form which we are measuring is a perfect path curve, all methods will give identically the same answer, and it will be the right one! If our form is not a true path curve, and no forms of living nature are, then there is no 'right' answer. The thing has become an approximation and the best any method can do is to give an approximate answer. The question can then very well be asked which method gives the best approximation; and this is not easily settled.

An obvious criterion is to ask: Which method gives a λ having the smallest MRD? In order to investigate this I took the measurements of 36 buds selected at random from my notebooks and calculated the λs and the MRDs for each of them, by all four methods. The results are given in the following table:

	Projective method	Morphographic method	Regression method	Christian's formula
Number of times that the best result was achieved	15	8	11	8
Mean MRD	2.17%	2.24%	2.19%	2.18%
Mean λ	1.87	1.96	1.78	1.80

(The reason why the numbers in the first row add up to more than 36 is that in several cases two of the methods tied for first place)

As was rather to be expected the morphographic method, with a mean MRD of 2.24% definitely does not do as well as the others, the difference between which is marginal, with a slight bias in favour of the projective method. For ordinary purposes I do not think it matters very greatly which method is chosen, but when comparing one species with another, or one day with another, one should stick to the same method. When one is dealing with buds of the snowdrop/primrose group it is noticeable that the projective method usually does better than the others, while when one is dealing with buds of the rose group the regression method or Christian's formula give definitely lower MRDs. Indeed, with buds of the extreme rose-type, the projective method will frequently give values of ∞ or above (that is, negative numbers) at certain levels, and obviously in such cases the mean value for all the levels is meaningless. Although the projective method in such cases cannot be used to obtain a λ-value for the bud as a whole, it can be very useful indeed by giving information about the peculiarities of the form, from the way the λ-value varies along the length of the bud.

Notice that the two methods employing techniques of 'least squares' (the regression method and Christian's formula) consistently, but not inevitably in every case, give values slightly lower than the projective methods. The difference becomes greater with high MRDs.

I think that the least squares methods, specially Christian's formula, are asking rather a different question from the projective methods. The former are asking: 'What λ will give the best fit to the physical form which we have actually measured?' while the projective methods are

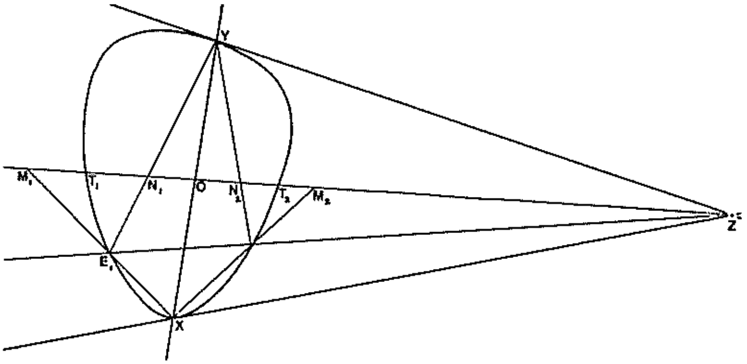


Figure 162. A construction which can be used for calculating the λ-value of an asymmetrical form.

asking: 'What λ will give the best fit with the invisible geometrical factors which generate this form?' It is for this reason that I prefer to use the projective method for my ordinary work, but I have programmed my computer always to give me a 'back-up' value by one of the least squares methods for checking and for use where it is advisable.

Asymmetrical forms

In order to calculate λ for an asymmetric form, such as the heart, we follow fundamentally the same procedure as with the original method already described, but instead of taking ratios along the lines XZ and YZ we have to take cross-ratios. One of the practical difficulties is that when we project the points of the curve from X and Y, on to ZY and ZX respectively, a number of the points arising are far off the edge of our page. It is therefore convenient to project all points on to the central line ZT; since cross-ratios are preserved under projection, this is a valid thing to do. Referring to Figure 162, the λ-value for level E on the left-hand side will be given by

$$\lambda = \frac{\ln\,[(ZM/MO)\cdot(OT/TZ)]}{\ln\,[(ZT/TO)\cdot(ON/NZ)]}$$

The value for the right-hand side will be calculated separately using measurements for M_2 and N_2.

There is an alternative and very useful way of finding this λ. For simplicity's sake Figure 163 shows it for two levels only, T and C. We project point Z to infinity along line XZ, keeping lines XT and XC fixed, and also the points along XY fixed. The new positions of T' and C' now lie on a symmetrical egg-like form, which can be treated in exactly the same way as the ordinary bud forms, with Figure 161, and so on.

The infinitely long vortex

If we have a vortex of the kind described in Chapter 11, and we wish to determine its λ assuming that the 'true' form reaches to infinity, although owing to the limitations of substance only a finite part of it is visible to us, we proceed as follows. We divide the visible part of the form into, say, seven levels, in the usual way, lettered from above downwards as F, E, D, T, C, B, and A. We measure the fraction XT/XF and the natural logarithm of this, which we call a, gives us the 2ε measure along the central axis. We let the radius at F be called f, and that at T, t; then the fraction t/f gives us the $(\varepsilon - \alpha)$ measure along the horizontal invariant line through X.

We let $\ln (t/f) = b$

Then $\lambda = (a-b)/b$

We notice that in the case of this sort of vortex b is negative, and the task of looking up negative logarithms is often inconvenient; it is therefore convenient to write $\lambda = -(a + b)/b$ where $b = \ln(f/t)$, that is, always putting the larger number in the numerator.

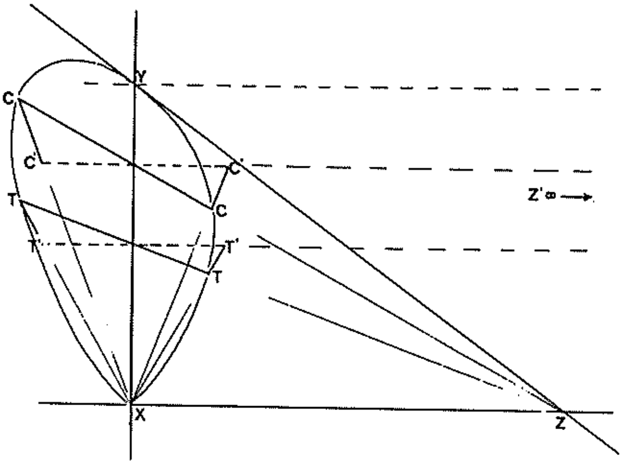


Figure 163. An alternative construction for calculating λ.

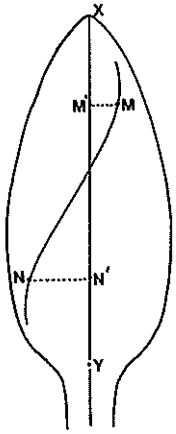


Figure 164.

The determination of ε

2ε is the parameter measuring the upward movement of the transformation from *Y* to *X* in the course of one radian turned in the θ-rotation along the horizontal line at infinity. Thus ε is the parameter which tells us the steepness of the spirals which lie on our egg. When ε is zero the spirals become horizontal circles and when it is infinite they become vertical lines of longitude. Since these curves are logarithmic in character, that is, they wind infinitely into, and out of, their poles at *X* and *Y*, it is clear that no curves of nature can possibly follow them along their whole extent; substance can never reach to infinity! But experience goes to show that in many cases in nature — the curves of the scales on a pine cone, the petal edges on a bud, the muscle fibres in the ventricle, etc. — the path curve spiral is followed very closely over a large part of its length; something like the middle three quarters or four fifths of the bud.

To measure ε we must first select two points on the curve which mark off the middle three quarters of the bud length, approximately, *M* and *N* (see Figure 164). We must find the multiplier for the transformation which carries *N*′ to *M*′. This is the cross-ratio

$$\frac{YN'}{N'X} \cdot \frac{XM'}{M'Y}$$

which was measured, on the original of our diagram, as

$$\frac{58.7}{20.1} \cdot \frac{59.3}{19.5} = 8.88$$

$$\text{Ln } 8.88 = 2.18$$

Next we must find, in the θ-rotation, how far things have turned from *N* to *M*. We measure *NN′* and divide it by the radius at level *N*.

$$\frac{13.9}{18.2} = 0.764$$

$$\text{Arcsin } 0.764 = 49.8°$$

This means that *N* is turned 49.8° to the left of the central axis. Similarly we can find that *M* is turned 30° to the right. Therefore the transformation has turned 79.8° from *N* to *M*. We divide this by 57.3 to find the number of radians turned (=1.39).

Our parameter now is $\frac{2.18}{1.39} = 1.57$

for one radian turned. But this is 2ε. So ε is 0.79.

Use of the parameters

The reader will find that the parameters described here enable one to have complete power over these egg-like path curve forms and surfaces. We give one very simple example: given λ and ε, the problem is to draw an exact picture of the resulting egg form.

First we take the equation

$$\lambda = \frac{\varepsilon + \alpha}{\varepsilon - \alpha}$$

By putting λ and ε into this we can calculate α. By putting this value for α, and the given value of ε, into this equation we get a fraction the top and bottom lines of which give us the log-multipliers for the geometric series of points along the top and bottom horizontal lines of Figure 154. However we must remember that these parameters are worked out for one radian turned of the transformation, whereas the construction of Figure 154 is assuming 60° turned. Therefore these log-multipliers must be multiplied by 60/57.3 before they are used. Working this way we shall get a true elevation picture of the desired egg-form.

Appendix 4

Mean Body and Phase-shift

by Graham Calderwood

The mean body

This is the mean, Cartesian position of the *N* bodies comprising the solar system.

That is, if we have a three-axis Cartesian frame, then the positions of bodies relative to it are given by three numbers: x_n, y_n and z_n, where *n* refers to the nth body of the *N* bodies. The mean body lies at a position, also given by three numbers, which are simply the arithmetic averages of the *N* individual ordinates in the respective *x, y* and *z* axes. Thus,

$$\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n, \quad \bar{y} = \frac{1}{N}\sum_{n=1}^{N} y_n, \quad \bar{z} = \frac{1}{N}\sum_{n=1}^{N} z_n$$

Though not a centre (as there are several, different radii), the mean body has the *character* of a centre: it is a 'centroid.'

The *x* and *y* axes usually lie in the ecliptic, with the *x* axis directed along the vernal equinox: the *z* axis in this case stands at right angles to the ecliptic, positive north. The origin of coordinates is usually taken either to lie in the Sun, or in the Earth, less often in one of the other bodies, and occasionally at some other location, such as the centre of mass.

Using *overdot* notation $\dot{x} = \frac{dx}{dt}$, $\quad \ddot{x} = \frac{d^2x}{dt^2}$, etc.,

Velocity of the mean body

$$\bar{\dot{x}}_B = \frac{1}{N}\sum_{n=1}^{N} \dot{x}_n, \; \bar{\dot{y}}_B = \frac{1}{N}\sum_{n=1}^{N} \dot{y}_n, \; \bar{\dot{\mathrm{z}}}_{\mathrm{B}} = \frac{1}{N}\sum_{n=1}^{N} \dot{z}_n$$

Acceleration of the Mean Body

$$\bar{\ddot{x}}_B = \frac{1}{N}\sum_{n=1}^{N} \ddot{x}_n, \; \bar{\ddot{y}}_B = \frac{1}{N}\sum_{n=1}^{N} \ddot{y}_n, \; \bar{\ddot{\mathrm{z}}}_{\mathrm{B}} = \frac{1}{N}\sum_{n=1}^{N} \ddot{z}_n$$

The mean body is a non-corporeal, massless geometric point.

The centre of mass

This is the centre of *inertial* mass of the N bodies of the solar system. Were the masses to be held rigidly in position relative to each other, the centre of mass would be that point in the system at which a force applied from any direction would accelerate the whole system without rotation — that is, without producing a 'couple.' Alternatively, if the system were supported at this point in a *uniform* gravitational field, it would hold whatever orientation it had: it is always in equilibrium.

It may be found iteratively: the centre is first found for one of the pairs of masses, and taken to be the place at which the *sum* of the two masses stands. That is, the two bodies can be considered combined into one body at that centre. This 'body' can then be paired with a third, actual body. When the centre corresponding to this new pair is then found, the mass of the third body is added to the sum of the first two, and the new sum is taken to be the mass of a single 'body' located at the new centre, ready to be paired with a fourth, actual body — and so on, iteratively, until all N bodies are accounted for.

If d is the distance between the bodies of any such pair, comprised of masses m_1 and m_2, and a is the distance from m_1 to the centre, then

$$a = \frac{d}{1 + \dfrac{m_1}{m_2}} \, .$$

The mass centre may also be more conveniently found from

$$C_M(\bar{x}_M, \bar{y}_M, \bar{z}_M) = \left(\frac{\sum_{n=1}^{N} m_n x_n}{\sum_{1}^{N} m_n}, \frac{\sum_{1}^{N} m_n y_n}{\sum_{1}^{N} m_n}, \frac{\sum_{1}^{N} m_n z_n}{\sum_{1}^{N} m_n} \right)$$

$$\Rightarrow C_M(\bar{x}_M, \bar{y}_M, \bar{z}_M) = \left(\frac{\sum_{n=1}^{N} m_n x_n}{M}, \frac{\sum_{1}^{N} m_n y_n}{M}, \frac{\sum_{1}^{N} m_n z_n}{M} \right)$$

Velocity of mass centre

$$V_M(\overline{\dot{x}}_M, \overline{\dot{y}}_M, \overline{\dot{z}}_M) = \left(\frac{\sum_{n=1}^{N} m_n \dot{x}_n}{M}, \frac{\sum_{1}^{N} m_n \dot{y}_n}{M}, \frac{\sum_{1}^{N} m_n \dot{z}_n}{M} \right)$$

Acceleration of mass centre

$$A_M(\ddot{\bar{x}}_M, \ddot{\bar{y}}_M, \ddot{\bar{z}}_M) = \left(\frac{\sum_{n=1}^{N} m_n \ddot{x}_n}{M}, \frac{\sum_{1}^{N} m_n \ddot{y}_n}{M}, \frac{\sum_{1}^{N} m_n \ddot{z}_n}{M}\right)$$

The mass centre is a corporeal point — it has mass — and may be regarded as the location of all the mass of the system.

The mean body and the mass centre are rarely in the same place.

Systemic alignment (directivity)

We consider two kinds of *systemic alignment, or directivity:* firstly, systemic alignment by angular position alone, and secondly, systemic alignment by both angular position *and* mass.

Systemic alignment by angular position

We have a set of lines all in a plane, each through the position of a body as projected in some manner into the plane and set at an angle ϕ_n to a reference line, which is in our case the vernal equinox. The plane is the ecliptic. Subscript n denotes the nth body of the ensemble of N bodies comprising the solar system.

The mean alignment of the set of directions with respect to the chosen centre and to an arbitrarily chosen direction at angle ϕ is then given by

$$A = \frac{1}{N}\sum_{n=1}^{N}\cos^2(\phi_n - \phi)$$

Because $\cos^2\theta \equiv \frac{1}{2}(1 + \cos 2\theta)$ this may be written as

$$A = \frac{1}{2N}\sum_{n=1}^{N}(1 + \cos 2(\phi_n - \phi))$$

$$\Rightarrow A = \frac{1}{2}\left(1 + \frac{1}{N}\sum_{n=1}^{N}\cos 2(\phi_n - \phi)\right), \text{or}$$

$$A = \frac{1}{2}\left(1 + \sum_{n=1}^{N}\frac{1}{N}\cos 2(\phi_n - \phi)\right) \text{-------------} (1.1)$$

Equation (1.1) is the degree to which the line, or direction, at angle ϕ is aligned to the whole ensemble of directions, and is in fact that direction's *systemic alignment.*

We now find the angle, ϕ_o, for which this systemic alignment is a maximum. This direction may be regarded as the direction with which the whole system of lines is *best* aligned.

To do this, we differentiate equation (1.1) with respect to ϕ, equate this to zero, and solve the resulting equation for ϕ_o. That is, we solve

$$\frac{dA}{d\phi_o} = 0 \quad \text{for } \phi_o$$

As, $\cos(P-Q) \equiv \cos P \cos Q + \sin P \sin Q$, we may write

$$A = \frac{1}{2}\left[1 + \frac{1}{N}\left(\cos 2\phi \sum_{n=1}^{N} \cos 2\phi_n + \sin 2\phi \sum_{n=1}^{N} \sin 2\phi_n\right)\right].$$

We note that the sums in $\cos 2\phi_n$ and $\sin 2\phi_n$ do not change with ϕ, so

$$\frac{dA}{d\phi} = \frac{1}{2N}\left(\frac{d\cos 2\phi}{d\phi}\sum_{n=1}^{N}\cos 2\phi_n + \frac{d\sin 2\phi}{d\phi}\sum_{n=1}^{N}\sin 2\phi_n\right)$$
$$= \frac{1}{N}\left(-\sin 2\phi \sum_{n=1}^{N}\cos 2\phi_n + \cos 2\phi \sum_{n=1}^{N}\sin 2\phi_n\right)$$

Equating this to zero, and because $1/N$ is not zero, we obtain

$$\cos 2\phi_o \sum_{n=1}^{N}\sin 2\phi_n = \sin 2\phi_o \sum_{n=1}^{N}\cos 2\phi_n$$

$$\Rightarrow \frac{\sin 2\phi_o}{\cos 2\phi_o} = \frac{\sum_{n=1}^{N}\sin 2\phi_n}{\sum_{n=1}^{N}\cos 2\phi_n} = \tan 2\phi_o$$

$$\Rightarrow \phi_o = \frac{1}{2}\tan^{-1}\left(\frac{\sum_{n=1}^{N}\sin 2\phi_n}{\sum_{n=1}^{N}\cos 2\phi_n}\right) \text{------------} (1.2)$$

This is the required angle of the line through the chosen centre which is best aligned to the whole system of directions.

This *systemic direction* is not a bisector (since there is more than one angle to bisect), but by virtue of lying, as it does, as closely as possible to all the directions at once, it has the *character* of a bisector: it is a 'bisectoid.'

Systemic alignment and direction by angular position and mass

As above, we have a set of lines all through some point regarded as centre, all in a plane, and each through a body with mass m_n, but now *each direction* is counted m_n times over. That is, the directions are weighted according to the masses through which they pass.

There is thus a total of M effective directions, where $M=\sum_{n=1}^{N} m_n$,

and the mean alignment with respect to the chosen centre and an arbitrary angle ϕ is given by

$$A=\frac{1}{M}\sum_{n=1}^{N} m_n \cos^2(\phi_n-\phi).$$

This is,

$$A=\frac{1}{2M}\sum_{n=1}^{N} m_n\left(1+\cos 2(\phi_n-\phi)\right)$$

$$\Rightarrow A=\frac{1}{2M}\left[\sum_{n=1}^{N} m_n+\sum_{n=1}^{N} m_n \cos 2(\phi_n-\phi)\right]$$

$$\Rightarrow A=\frac{1}{2}\left(1+\frac{1}{M}\sum_{n=1}^{N} m_n \cos 2(\phi_n-\phi)\right)$$

$$\Rightarrow A=\frac{1}{2}\left(1+\sum_{n=1}^{N}\frac{m_n}{M}\cos 2(\phi_n-\phi)\right)\text{---------------}(2.1)$$

We see that the *alignment by mass* of the direction, at angle ϕ with the ensemble of what may be termed 'mass directions' is identical to *alignment by position*, except that, here, the contribution of each direction-group is modified according to the ratio of its target mass to the total mass.

By differentiating and equating to zero in the same way as is done for the angular position case, we find that the *direction of the best systemic alignment by mass* is given by

$$\phi_m = \frac{1}{2}\tan^{-1}\left(\frac{\sum_{n=1}^{N} m_n \sin 2\phi_n}{\sum_{n=1}^{N} m_n \cos 2\phi_n}\right) \text{ ------------------ } (2.2)$$

It is perhaps worth remarking that masses are seldom, if ever, integers. This implies that when using this method we must consider 'fractional directions,' which may give us pause, and should be rigorously justified — we assume validity for the time being.

Appendix 5

The Pivot Transformation

Given a planewise form inserted into a path-curve system, we are looking for the locus of the pivot points of the planes of the form. How do we proceed?

In the first instance we are interested in path curves of the semi-imaginary case. (Two planes of the invariant tetrahedron are taken real, parallel; the other two planes taken as complex conjugate.) The bud or egg transformation which I described in Chapter 4 is such a case.

Here are some basic constructions of egg geometry which we shall need:

Construction 1. Given a plane, to find its path line, the line around which it is turning.

Recapitulating briefly, we have a θ-rotation moving with constant speed along the line at infinity (circling measure between I and J), we have logarithmic spirals moving inwards in the X-plane with a log-multiplier of $(\varepsilon + \alpha)$ per one radian of θ turned, and logarithmic spirals moving outwards in the Y plane with a log-multiplier of $(\varepsilon - \alpha)$ per one radian of θ turned. Along the central axis we have a growth measure moving from Y to X with a log-multiplier of 2ε (see Figure 155). The shape of the set of egg- or vortex-surfaces is exactly determined by the parameter λ where

$$\lambda = \frac{\varepsilon + \alpha}{\varepsilon - \alpha}$$

We remember that the path curves in the top and bottom invariant planes are sets of logarithmic spirals, normally with different constant ratios, and that if the invariant surfaces are to be eggs, and not vortices, they must unwind in opposite directions. Since the top and bottom invariant planes are parallel they will be cut by a plane π in a pair of parallel lines l_1 and l_2, which lines we can take as defining plane π on our diagram (see Figure 165). Now we mark on each of lines l_1 and l_2 the point in which the line touches its spiral of the

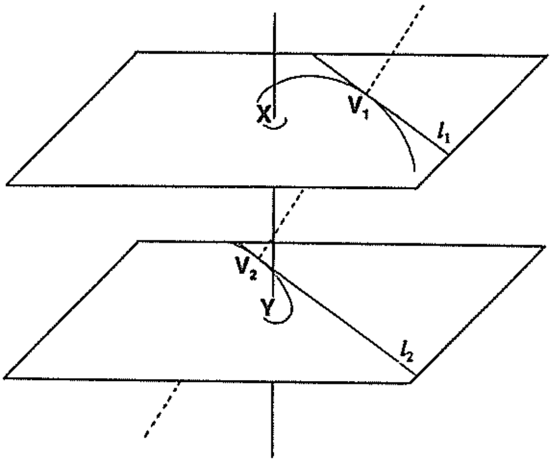


Figure 165.

system, points V_1 and V_2. This is easily done, using the equiangular property of the spirals. The line XV_1 will be at an angle of θ_1 with line l_1 given by

$$\theta_1 = \cot^{-1}(\varepsilon + \alpha)$$

and for the bottom plane by

$$\theta_2 = \cot^{-1}(\varepsilon - \alpha).$$

Now line l_1 is turning momentarily about V_1 and line l_2 about V_2 so plane π is turning momentarily about line V_1V_2. This line is the path line we are seeking.

Construction 2. To find the path line of a given point P, that is the tangent line along which P is moving in its path curve.

Remembering that the invariant organism of a set of path curves is always self-dual, we simply dualize Figure 165 and get Figure 166. The new diagram looks surprisingly more complicated. First we find lines l_1 and l_2, being common lines of P and X, and of P and Y. Now in Construction 1 we had to find the point in which l_1 was turning in the spiral curve of the top invariant plane. In this construction we have to find the

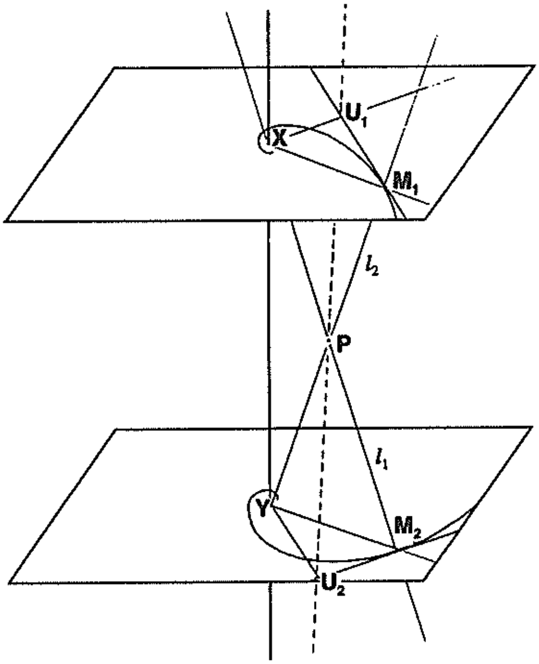


Figure 166.

plane in which l_1 is turning in its spiral cone centred in X. To do this we let l_1 meet the bottom invariant plane in point M_2, and through M_2 we draw the tangent line to the spiral, again using the constant angle property. The tangent plane we are looking for, dual to V_1, is determined by this tangent line and the point X and it will cut the top invariant plane in a line through X parallel to the tangent through M_2, and we let this meet the tangent at M_1 in U_1. Similarly we find the plane dual to V_2 by letting line l_2 meet the top invariant plane in point M_1, by drawing through M_1 the tangent to its spiral, and by drawing through Y a line in the bottom plane parallel to this tangent, meeting the tangent at M_2 in U_2. Clearly the line we are seeking is that which is common to the planes which are dual to V_1 and V_2; this is the line U_1U_2 in our diagram, whose points will be found to be collinear with point P. (A not altogether easy piece of dualizing!)

Construction 3. To find the osculating plane of this point P. This is a simple extension of Construction 2. Through the points U_1 and U_2 we put the tangent lines to their spirals, again using the constant angles θ_1 and θ_2. These lines will be found to be parallel, and they determine the osculating plane which we are seeking.

Construction 4. To find the pivot point of a given plane π. (And this is the one we are really wanting!) We simply follow Constructions 3 and 2 backwards. We let plane π be determined by the (parallel) lines in which it cuts the top and bottom invariant planes. First we find its path line, meeting the top and bottom planes in U_1 and U_2. Through U_1 we draw U_1M_1 parallel to YU_2 and on it we construct M_1, where it meets its spiral (using the constant angle θ_1). Line M_1Y meets U_1U_2 at the required pivot point P. (Also of course we could have drawn U_2M_2 parallel to XU_1, meeting its spiral at M_2. Line XM_2 would then have been concurrent with the other two lines at P.)

Now we are in a position to draw our first pivot form. Let us find the pivot form of a line. We shall not consider it as a line of points; it will be a negative-space line, a shaft of planes. What is the locus of the pivot points of each of the planes composing this line?

We make our diagram (Figure 167) as seen from above; all that part of the diagram which is above the dotted line is to be taken as being on the top invariant plane (close to us) and what is below the dotted line as being on the bottom invariant plane (further away from us).

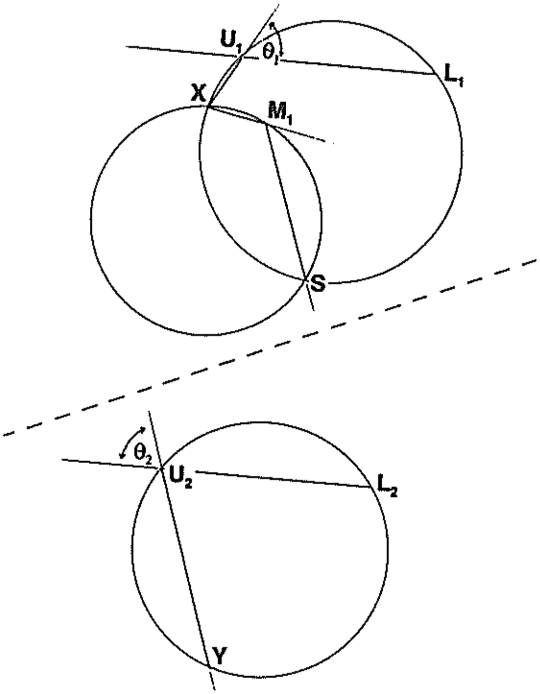


Figure 167.

We let the top and bottom planes be cut by an arbitrary line in points L_1 and L_2 respectively. Now we consider any plane of our line L_1L_2. It will cut the top and bottom planes in a pair of parallel lines through L_1 and L_2, which we draw; and on these lines we mark their points of contact with their spirals, U_1 and U_2, using the constant angles, θ_1 and θ_2. Note here that we are considering the egg transformation as being point upwards therefore

$\varepsilon + \alpha$ (for the top plane) $> \varepsilon - \alpha$ (for the bottom plane)

and $\theta_1 < \theta_2$.

Also that the sense in which the angles are taken must be opposite for the two planes, or else our path curve transformation would be a vortex one and not an egg one.

Now by elementary geometry (angles in a segment of a circle are equal) it is clear that as the line L_1U_1 turns around L_1, U_1 will describe a circle through L_1 and through X, while U_2 will similarly describe a circle through L_2 and Y. And if we let U_1 move in equal steps around its circle, U_2 will do the same around its circle. The point of our locus is some point of the line U_1U_2 and this is a moving generator of a hyperboloid passing through the two circles. So our locus, whatever else it may do, lies on a hyperboloid. Next we draw the line through U_1 parallel to YU_2, marking on it (again by using the constant angle θ_1) its point of contact M_1, with its spiral. The point of our locus will lie on YM_1 and will be where this line meets U_1U_2.

Now notice that as line L_1U_1 turns through some angle ϕ, L_2U_2 turns through ϕ, so does YU_2 (by elementary geometry again), and so does U_1M_1. Therefore the angle $L_1U_1M_1$ is constant and all the lines U_1M_1 must be concurrent at some point of the circle through L_1, U_1 and X. We call this point S. But the angle XM_1S is constant $(\pi - \theta_1)$ radians so M_1 must move round a circle through X and S. Therefore the line YM_1, which contains our locus, describes a cone centred in Y. Therefore our locus is the intersection of two quadric surfaces, a cone and a hyperboloid. In general this would be a twisted quartic; but we must be careful, and not jump to hasty conclusions.

Let us consider the line U_1U_2 a little more carefully. It is a line of the hyperboloid, therefore any plane of this line will cut the hyperboloid in a generator of the other set. But the parallel lines U_1S and U_2Y represent just such a plane. Therefore the line SY is a generator of the other set, of the hyperboloid. It is also a generator of the cone. Therefore our two intersecting quadrics have a common generator and they must intersect on a twisted cubic. Since the cone containing our locus is centred on Y, and there is clearly another one, also containing our locus, centred on X, we can say:

> The pivot form of any arbitrary line of space is a twisted cubic passing through the invariant points X and Y.

This leads to a rather pretty construction. We can repeat Figure 167, making points X and Y coincide on our page. This gives us an exact plan view of the situation. The point of our locus of course is the common point of the lines U_1U_2 and YM_1. If we mark this as P we then have the ratio YP/PM_1 to show us the fractional height by which P is raised from the U_2-plane towards the U_1-plane. Using this we can easily construct an elevation view of the curve. In our diagram, P will be found to move in a plane cubic, which is the orthogonal projection of the twisted cubic of the three-dimensional reality. However, if we have chosen L_1 and L_2 in such a way that their common line meets the central axis, then we shall find that P describes a circle, passing through the point $X = Y$. In this case the twisted cubic has degenerated into a horizontal circle taken together with the central axis of the path curve transformation.

But now we must study the plan view of Figure 166 in greater detail. We let the central axis of the path-curve system cut our plan-plane in the point $X = Y$. Continuous lines, and the suffix$_1$ represent top plane elements, and dotted lines and the suffix$_2$ represent those on the bottom plane; the only exception to this will be where, at one point, the angle θ_2 has to be measured on the top plane, and θ_1 on the bottom. I think the context will show this quite clearly. We let the plane whose pivot point we are finding cut the top and bottom planes in the parallel lines l_1 and l_2 respectively. We let the distance of l_1 from X be d_1. Now we find U_1 on l_1 by using angle θ_1, and U_2 on l_2 by using θ_2. Now we draw a line through U_1 parallel to U_2Y, thus bringing the angle θ_2 onto the top plane as marked on Figure 168. Then along this line we mark point M_1 using the angle θ_1.

Now we need to find the distance XM_1. Noting the fact that

$$XU_1 = d_1 \operatorname{cosec}\theta_1$$

by elementary trigonometry, using the same rule, we find that

$$XM_1 = \frac{d_1\sin(\theta_1 + \theta_2)\mathrm{cosec}\theta_1}{\sin\theta_1}$$

$$= \frac{d_1\sin(\theta_1 + \theta_2)}{\sin^2\theta_1}$$

Doing the same things for the bottom plane, and letting l_2 have a distance from Y of d_2, we find that M_2 necessarily lies on the line XM_1 and that the distance M_2Y is given by

$$M_2Y = \frac{d_1\sin(\theta_1 + \theta_2)}{\sin^2\theta_2}$$

We let
$$M^2k_1 = \frac{\sin(\theta_1 + \theta_2)}{\sin^2\theta_1}$$

for the top plane, and
$$k_2 = \frac{\sin(\theta_1 + \theta_2)}{\sin^2\theta_2}$$

for the bottom plane.

These parameters k_1 and k_2 turn out to be of fundamental importance in these studies and we shall use them again and again.

Now we see the whole thing in elevation (Figure 169), from a direction at right angles to the plane of the lines XM_1 and YM_2.

P is the pivot point of the plane we are considering, and this plane will cut the central axis of the path-curve transformation at some point, say Q. We let the height of P above the bottom invariant plane be y, and

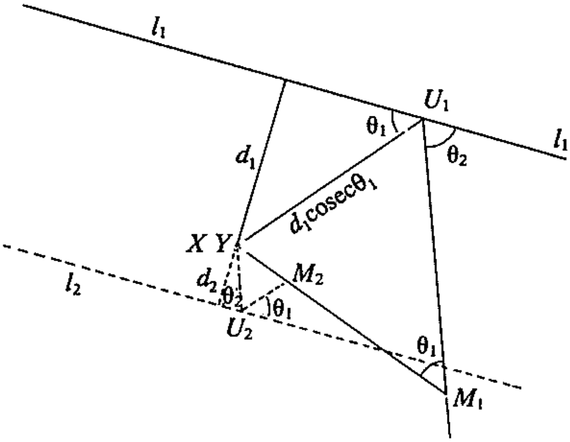


Figure 168.

we let the distance XY be unity. Then we may call y the fractional height of P above the bottom plane. We let the fractional height of Q be y'. We want to find y' in terms of y.

Clearly $$\frac{y'}{1-y'} = \frac{d_2}{d_1}$$

whence $$d_2 = \frac{y'd_1}{1-y'}; \qquad \ldots (*)$$

Also $$\frac{y}{1-y} = \frac{k_2d_2}{k_1d_1}$$

Solving for y we have

$$y = \frac{k_2d_2}{k_1d_1 + k_2d_2},$$

and substituting (*) in this we have

$$\begin{aligned} y &= \frac{(k_2d_1y')/(1-y')}{k_1d_1 + (k_2d_1y')/(1-y')} \\ &= \frac{(k_2d_1y')/(1-y')}{[k_1d_1(1-y') + k_2d_1y']/(1-y')} \\ &= \frac{(k_2y')}{k_2y' + k_1(1-y')} \end{aligned}$$

or, changing the subject

$$y' = \frac{(k_1y)}{k_1y + k_2(1-y)}$$

and the line XM_1 is inclined to the direction of l_1 and l_2 at an angle of $\theta_2 - \theta_1$.

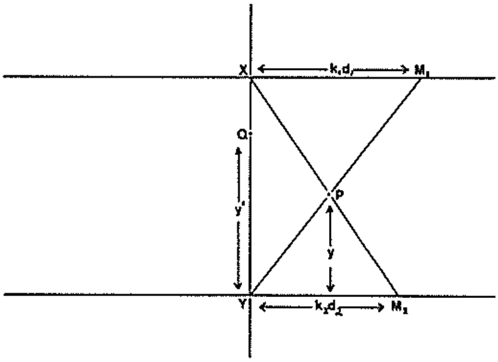


Figure 169.

These two little equations, of y and y' in terms of one another, are basic to pivot geometry and give the key to most of the problems which will face us in the course of further work. We note that d_1 and d_2 have dropped out; this means that all planes cutting the central axis at a height of y' will have their pivot points on a horizontal plane at a height of y; alternatively if we wish to find the horizontal cross-section of some pivot form at a height of y we shall be concerned only with those planes of the original planewise form which cut the main axis of the path curve system at a height of y'.

We are now in a position to construct the pivot form for a whole pointful of planes, the little two-dimensional universe of the peripheral consciousness which we have pictured to ourselves earlier. When such a pointful of planes comes within the influence of a path-curve system, around what locus do all its points start to move? Since there are ∞^2 planes to be considered, the locus will obviously not be a curve, but a surface. In order to be sure that we have really coped with all the lines possible, we have simply to take any plane of our point, to consider all the lines of our point which lie in that plane (an ordinary plane pencil) and now to consider all the planes of each of these lines.

Now our point, together with the main axis of the path-curve system, determines just such a plane, and the lines of the pencil in this plane all meet the central axis. Any such line, meeting the axis at a height of y', we know will have as its pivot form a horizontal circle at a height of y. Therefore our pivot surface is going to be everywhere circular in horizontal cross-section. Clearly, from all the foregoing, if such a line meets the top invariant plane at a distance of d_1 from X, the diameter of its pivot circle-section is going to be yk_1d_1, and it will be at a height of y above the lower invariant plane. The resulting surface is very hard to depict in perspective, and not easy to imagine; it is interesting to make a papier mâché model of it.

It is a cubic surface, being cut by any general plane of space in a cubic curve; but if this cutting plane contains a line of the invariant tetrahedron of the path curve system, then this cross-section degenerates into a conic taken together with that line. The general appearance of this surface is that of what I would call a 'double trumpet' and it is woven over with ∞^2 twisted cubics, set in infinite families, and there are ∞^2 such families to be found. Any cone, or other one-dimensional form, within our point, will have as its pivot form a curve inscribed upon this surface.

It is of interest to consider one or two of the plane two-dimensional possibilities that are inherent in the pivot transformation. Such transformations will turn lines into points.

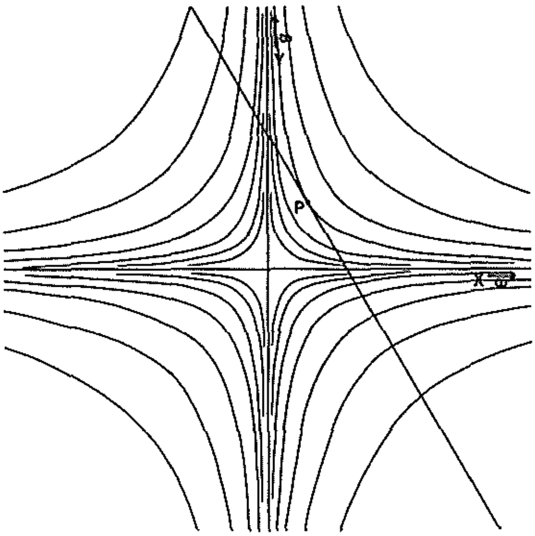


Figure 170.

As an instance we could have our path curve invariant triangle to be *XYZ*, where *X* and *Y* are at ∞ and we have *Z* on our page, *X* and *Y* being at right angles (Figure 170). If we allow the growth measure (a pure multiplicative sequence in this case) along *ZX* to be equal to that along *YZ* our path curves will be rectangular hyperbolae, and each line of the plane will find its pivot point where it touches its curve of the family. Now we know from elementary geometry that this is at the midpoint between the points where it cuts the asymptotes.

We can now forget about our hyperbolae and simply say, our transformation is determined by the putting down of any pair of right-angled base lines. Then any third line transforms into the midpoint between the points in which it cuts the base lines. This gives us an interesting quadratic transformation, very easy and quick to work. We will find that any plane pencil transforms into a pointwise conic, and a linewise conic into a pointwise curve of fourth order.

Now we can generalize. The base lines need not be at right angles. Generalizing further we can say, put down any three base lines, two blue and one red. Any fourth line will transform into the harmonic conjugate of the point in which it cuts the red line with respect to the points in which it cuts the blue. This in fact makes a very interesting quadratic transformation, but one which is not quite so quick and easy to work as the foregoing. Figure 171 shows the transformation of a circle with respect to an equilateral triangle.

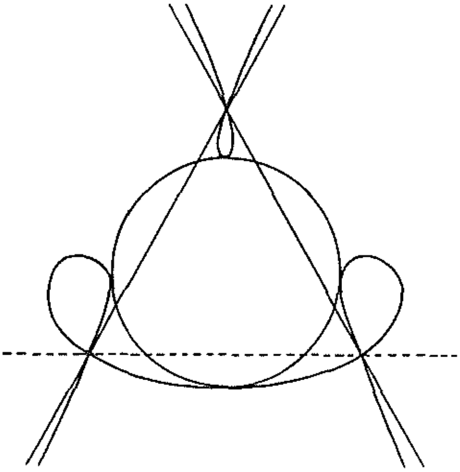


Figure 171.

One could generalize even further and repeat the construction of the last diagram letting the transformed point be such that it makes *any* constant anharmonic ratio with the three cutting points on the base triangle.

Then one could let one's path-curve system have an imaginary triangle for its invariant (one side real, the other two conjugate complex). If this is the well-used triangle *OIJ,* with *O* at the centre of the page, a special case of such path curves is a set of concentric circles. Then any line transforms into the point which is at the foot of the perpendicular onto it from the centre *O*. This is the well-known pedal transformation, which turns out to be just a special case of the much more general pivot transformations. It transforms any pencil of lines into a circle, and conics into cardioids, limaçons, lemniscates, and so on. But the most general case of path curves with respect to this triangle, *OIJ,* is a set of logarithmic spirals. Then one can repeat the pedal transformation, but this time marking not the foot of the perpendicular but of some other constant angle (the constant angle of that particular set of spirals).

So we see that the pivot transformations include a very wide range of possibilities. In three dimensions of course the range is much wider still, and many of the possibilities are, as far as I know, quite unexplored. I have worked on the particular case described earlier in this chapter because I felt that it might have relevance to the formations of plant growth, and I think that subsequent work has produced some evidence that this is so.

Practical application

The question now confronting us is: given the bud parameters, λ and ε, and the measured form of the seed-chamber, could we find, within the existing possibilities of the path curve forms, a vortex which, when transformed by that bud transformation, would accurately fit the seed-chamber form? It is by no means certain that the answer to this will be positive; but if it is, we next need to know just where the centre of the vortex must lie, and what must be its (negative) λ.

First we must construct the picture of the vortex, as seen in elevation. This is easily done (Figure 172). We put a geometric series along the horizontal line through *Y′*, and another, usually but not necessarily with a different constant ratio, along the vertical axis through *Y′*. The transformation is such that while the points along the horizontal axis move in towards *Y′*, those along the vertical axis move outwards away from *Y′*. Thus we connect these points in the way shown in the diagram, and the resulting envelope gives us the profile of the vortex. The actual form, of course, is a surface of rotation about the vertical axis. If the θ-rotation of the transformation, between *I* and *J* on the horizontal line at infinity, is zero, then these lines represent the movement of a plane, staying always perpendicular to the plane

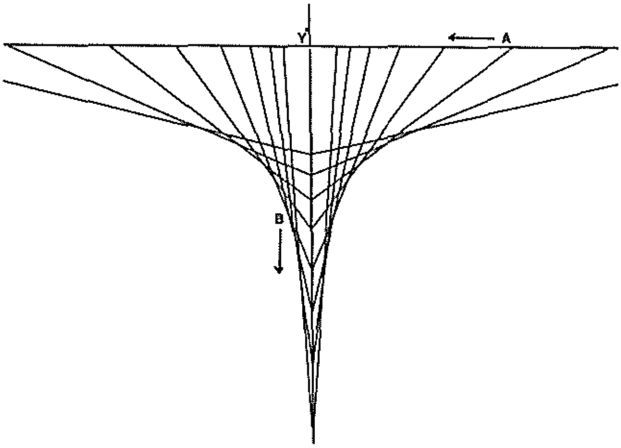


Figure 172. The form of the vortex is generated by two simple multiplicative movements along the horizontal and vertical axes.

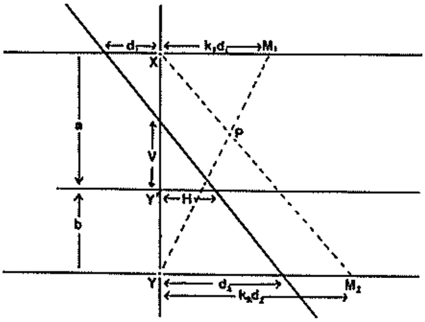


Figure 173.

of the page, and sliding inwards along the outline of the profile. The natural logarithm A of the multiplier along the horizontal axis is $-(\varepsilon - \alpha)$. The natural logarithm B of the multiplier along the vertical axis is 2ε. Then λ_c (cosmic λ) of the vortex is given by

$$\lambda_c = -\frac{A + B}{A}$$

(A and B counted as absolute values, that is, always positive.)

Our method of procedure will be to take a plane of the vortex, determined by its intercepts H with the horizontal plane through Y', and V with the vertical axis through Y', and to find its pivot point in the bud transformation, which will be labelled with the ordinary x and y notation; its third co-ordinate, θ, in the circling measure between I and J, is irrelevant in a surface of rotation such as this.

Now we must refer to Figure 168. This is a plan view, but we must now draw it in elevation (Figure 173). We remember that $XM_1 = k_1d_1$, and $XM_2 = k_2d_2$. Also that since we are dealing with a surface of rotation we are concerned only with the *distances* of M_1, M_2, and so on, outwards from X; the angle at which line XM_1M_2 is situated is not significant for us. Therefore in our elevation drawing we can depict the distances XM_1 and XM_2 along the same line as d_1 and d_2, and this we shall do.

In Figure 173, the continuous sloping line represents the plane whose pivot point we are seeking, and Y' is the centre of the cosmic tetrahedron. Our plane is thus determined by the distances H and V. The points X and Y are the poles of the bud transformation. The pivot point we are seeking is P and its distance upwards from Y is its true y co-ordinate. Its x co-ordinate is its distance outwards from the central axis, but its true position will clearly be in some other plane of that axis; a and b are the distances of the cosmic centre from the poles of the bud transformation.

Now the y co-ordinate of P was worked out in the previous chapter. It is given by

$$\mathrm{y} = \frac{k_2 y'}{k_2 y' + k_1(1 - y')} \quad \text{.............................}(1)$$

We remember that both y and y' are fractional distances, so y' will now be $(V + b)/(a + b)$ in terms of our new diagram, and $(1 - y')$ will be $(a - V)/(a + b)$. Putting these into (1) we have y in terms of H and V, expressed as an actual distance

$$y = (a + b) \cdot \frac{k_2(b + V)}{V(k_2 - k_1) + ak_1 + bk_2} \quad \text{.......}(2)$$

Next, working with similar triangles YPV and YM_1X, we see that

$$\frac{x}{k_1 d_1} = \frac{y}{a + b}$$

whence

$$x = \frac{k_1 d_1 y}{\mathrm{a + b}} \quad \text{..}(3)$$

But also working with similar triangles we see that

$$\frac{d_1}{H} = \frac{a - V}{V}$$

$$d_1 = \frac{H(a - V)}{V} \quad \text{...}(4)$$

Putting (2) and (4) into (3) we have

$$x = \frac{k_1 k_2 H(a - V)(b + V)}{V[V(k_2 - k_1) + ak_1 + bk_2]} \quad \text{.................}(5)$$

Given first the parameters of the bud transformation, second the position of the cosmic centre, Y', and third the cosmic λ of the vortex, we can now quite easily construct the profile of the resulting pivot form. First gives us k_1 and k_2; second gives a and b; third gives us the possibility of calculating two corresponding series for H and V. These can all be put into equations (2) and (5) and the co-ordinates for the profile worked out. (Note: in these equations b and V follow the usual sign-conventions, b measuring positive upwards from Y, and V measuring positive upwards from Y'; but a measures positive *downwards* from X, and H and x must always be considered as absolute quantities.)

Equations (2) and (5) enable us to find the seed chamber form given the position and the λ of the vortex. The problem we set ourselves is really the inverse of this. Given the form of the seed chamber, to find whether there is a possible vortex which will produce it, and if so, to find its position and λ. One of our variables will now be b (giving the possible position of the vortex centre) so we will change the subject of equation (2) to give V in terms of b. We let $a + b = t$ (the total length of the bud) and we omit the elementary algebra. We now have

$$V = \frac{-yk_1t}{y(k_2 - k_1) - tk_2} - b \quad \text{........................(6)}$$

The next thing we must do is to take this value of V put it into equation (5) and rearrange it to give H in terms of b. To simplify matters we let $V = m - b$. Then we finish with the equation

$$H = \frac{x[m(k_1 - k_2) - k_1t]}{k_1k_2(m - t)} \cdot \left(1 - \frac{b}{m}\right) \quad \text{........(7)}$$

We now have all the tools we need to solve our problem: Is there a possible vortex which will transform exactly into a given seed-chamber form, and if so, what is the height, b, of its infinite centre, and what is its λ? The method is best shown by applying it to an actual case.

Campanula. We will take the case of the campanula. This longish white bud has λ = 1.36 with MLD 9%, according to the measurements I made on it some years ago. Its ε is high, being about 5. These figures lead us to calculate that $\theta_1 = 9.9°$, $\theta_2 = 13.3°$, giving $k_1 = 13.4$ and $k_2 = 7.45$. We are working with a 10 cm bud as far as our measurements are concerned (the actual bud was of course much smaller) so $tk_1 = 134$, $tk_2 = 74.5$, and so on.

The first thing we must realize is that, as far as the geometry is concerned, the length of the seed-chamber form is solely determined by the parameters of the bud; it is quite independent of either the position or the λ of the vortex transformation. The position of the lower end of the form is given by putting $V = \infty$ into equation (2). I call this the limit length of the seed-form and it is given by

$$L = \frac{tk_2}{k_2 - k_1}$$

The question may immediately arise as to whether this length is always the right one when checked against the observations of the actual plant. I think the answer to this question is that it really has no answer; it is a non-question. Most buds are accompanied by the ovaries, sometimes with the little chamber which encloses them, at a very early stage indeed. Thereafter the growths of the two — the bud itself, and the ovaries — continue at very different speeds, the bud usually growing much faster than the seed-bearing part. Thus it is useless to ask what is the 'right' ratio of bud to seed-chamber length; it varies considerably during the development of the bud. Anyway our projective geometry, being intimately connected with *form,* is not a geometry of size or magnitude. It is *form* which we are seeking.

The limit length of the seed-bearing form for the campanula comes out at 12.5 cm, considerably larger than what I actually measured. It is interesting to note that a large part of the 9% deviation in the λ-values for the bud was due to the values being rather too small at the top end of the bud; it was slightly too wide there. And this suggests that perhaps I picked the buds a little too late; they were already just beginning to open. If I had caught them a little earlier I might have found a smaller mean deviation, and the ratio of bud to seed-chamber length would almost certainly have been much nearer to what the mathematical limit predicts.

It is necessary to enlarge the measurements for the seed-chamber, in proportion, so that it has a length equal to the limit length, otherwise our calculations cannot proceed consistently. Now we take three points, *P, Q* and *R,* on the profile of the seed-chamber, one high up, one in the middle and one low down. We take the coordinates of these, x and y, and put them first into equation (6) and then into equation (7). This gives us V and H, in terms of b, for each of the points. For the case in question we get these results:

For point P ($x = 4.42$ cm, $y = -1.565$ cm)

$$V = -3.22 - b,\ H = 0.514 + 0.159b$$

For point Q ($x = 4.05$ cm, $y = -6.23$ cm)

$$V = -22.45 - b,\ H = 0.336 + 0.0149b.$$

The calculations for point *R* must be done similarly, but are omitted here for the sake of brevity.

Next we make out a table for point *P,* giving a series of values for *V* and *H* to correspond with various values of *b:*

b	V_{p}	H_{p}
0	–3.22	0.514
2	–5.22	0.834
4	–7.22	1.154
6	–9.22	1.474
. . .	. . .	. . .

And another similar table for point *Q*

b	V_{q}	H_{q}
0	–22.45	0.336
2	–24.45	0.366
4	–26.45	0.396
6	–28.45	0.426
. . .	. . .	. . .

Now we refer back to the little formula for finding the λ of the vortex:

$$\lambda_c = -\frac{A + B}{A}$$

In this formula B is of course $\ln(V_q/V_p)$, and A is $\ln(H_p/H_q)$. So having two values of V and H, we can calculate very easily the λ which corresponds to $b = 0$. This means that if the centre of the vortex is at height of 0 above point *Y*, and the vortex has this λ, then the resulting transform would be a profile which would go exactly through both points *P* and *Q*. By putting various

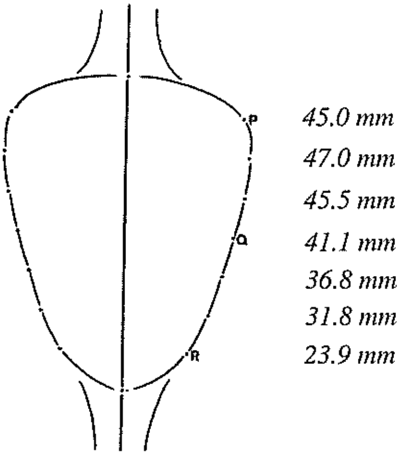


Figure 174. The form of the Campanula ovary, the measurements having been adjusted to fit the limit length of the bud.

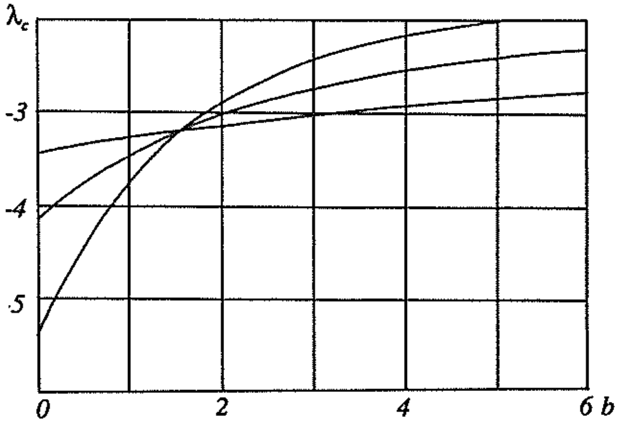


Figure 175. The b-λ *chart.*

values for *b,* we can get a curve, on what I call the *b*-λ chart (Figure 175); every point on this curve represents a position of *b* with its corresponding λ, which would put a profile through *P* and *Q*. Now if we do the same thing for points *P* and *R* we get a second curve on the *b*-λ chart, and *if* these curves meet, then their common point represents the only *b* and λ which would put a profile through all three points, *P, Q* and *R.* If they do not meet then no vortex is possible to fit the case. (As a check we can repeat the whole thing for points *Q* and *R;* if the first two curves meet, it is necessary, mathematically, that the third one will be concurrent at the same point.)

Of course the fact that the three curves meet does not give a proof that the curve will be a perfect fit all the way round the profile, but as a matter of experience I have found that in every one of very many cases the fit is quite remarkably exact.

We see that there is a meeting at $b = 1.5$ and $\lambda = -3.22$ (the second decimal place was got by calculating several values near to the meeting point, and interpolating; it may not be completely reliable).

In order to find how accurately this vortex will transform into the ovary-form, we can take the *y*-values of the seven measured points on the ovary, put them into equation (6) and get their corresponding *V*-values. Now, starting from the *H*-value which we have already found for the middle point of the form (when $y = -6.25$ cm) and using the value of $\lambda = -3.22$ we can find the corresponding *H*-values. We put these into equation (5) and this will give us the 'predicted' *x*-values, which we can compare with our measurements. Having done this the results come out as follows, calculated for the 12.52 cm form, from above downwards:

x (calculated) cm	x (observed) cm	error %
4.38	4.42	0.9
4.52	4.62	2.2
4.35	4.48	3.0
4.05	4.05	0
3.68	3.62	1.6
3.12	3.12	0
2.35	2.38	1.3

The mean error in the radius is 1.3%, considerably less than the probable error of the measurements.

Airy and watery vortices

We have seen above, (page 357) an easy way of constructing the envelope of a watery vortex, and how to calculate its λ. But it could also happen that as the points move outwards along the vertical axis the ones on the horizontal axis are also moving outwards. This changes the sign of either A or B, and it can easily be verified that λ_c is now given by

$$\lambda_c = \frac{B - A}{A}$$

where A and B are again to be considered as positive quantities. If $B > A$, then λ is positive and the vortex is cup-like-really the lower end of an infinitely long egg. If $B < A$ then we have the airy vortex, and this form we must next study.

The asymmetric case

Suppose an airy vortex penetrates into the field of an egg- or bud-transformation in such a way that the axis of the vortex is to one side of the egg-axis — what sort of form arises when the vortex is transformed by the egg-transformation? We will start with the simplest case, where the vortex axis has a common point with that of the transforming egg.

If the axes are coincident, the resulting transform has a symmetrical profile, and at every level its horizontal cross-section is circular.

However, when the axes are separate the case is more complicated: all-round rotational symmetry is lost, and the form of the horizontal cross-section differs from one level to another. All one can do is to draw horizontal cross-sections at various levels — contours as it were — and build up the picture of the finished form by putting these together.

In order to do this, we remember the rule, that planes which meet the central axis of the bud transformation at some point, say Q, will have their pivot points all in one horizontal plane which will cut the central axis at some point P. If Q had the fractional height of y' between the invariant points X and Y of the bud transformation, and P has the fractional height of y, then

$$y' = \frac{k_1 y}{k_1 y + k_2(1 - y)}$$

Thus we can fix the height of our cutting or contour plane (the plane in which we are going to draw the contour of our pivot form), which is the plane that must contain the pivot points which we are going to find, anywhere we wish. We call this fractional height y, and from this we can calculate the fractional height y' of the point Q through which all the planes pivoting on these points must pass. While we are drawing this particular contour we can ignore all planes except those which pass through Q.

Now we must remember the basic rule of the pivot transformation. This diagram represents the top invariant plane of the bud transformation, seen in plan from above. We have a plane which cuts this top plane in line l and which cuts the central axis of the bud transformation in point Q, at our calculated fractional height of y'. The process of finding the pivot point of this plane has several steps (see Figure 176). Briefly summarized, they are:

1) Drop a perpendicular, XL, onto l from X,
2) extend XL to L' so that $XL' = k_1 XL$,
3) rotate L' around X to L'' by an amount of $90° + \theta_2 - \theta_1$,
4) join L'' to Y, the bottom invariant point of the bud transformation;
5) this line will intersect our horizontal contour plane through P in the pivot point we are seeking.

If we are not interested in the orientation of the final pivot form we may ignore the third step, the rotation; and this we will do in the work which follows.

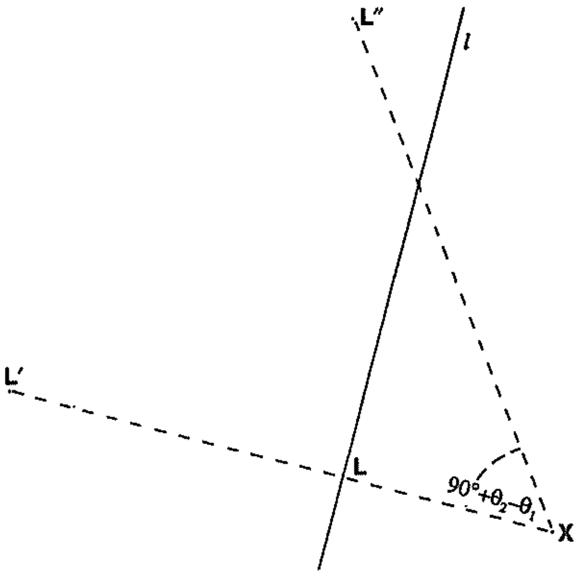


Figure 176.

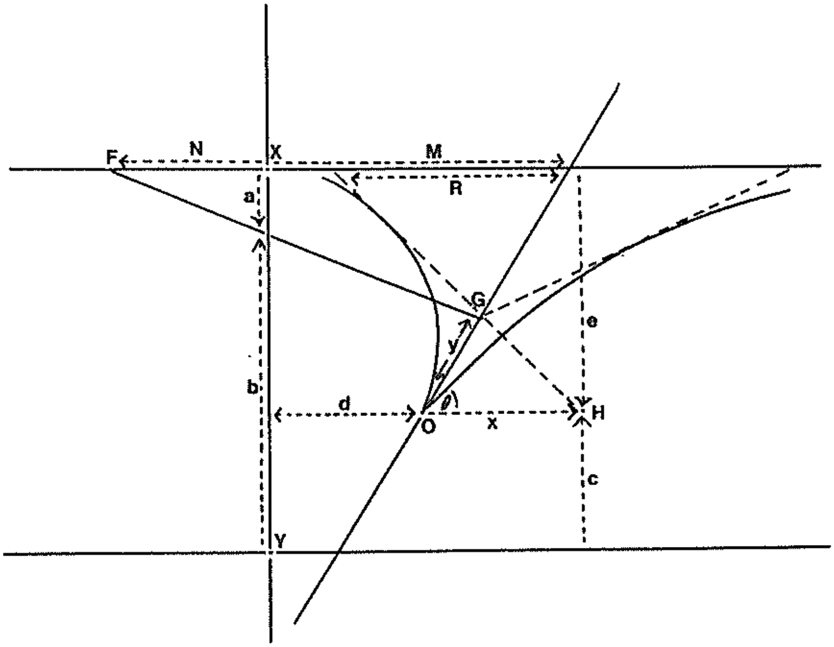


Figure 177.

We must now see the thing in elevation (see Figure 177). The elevation is seen from a direction that is orthogonal to the common plane of the two axes. *XY* is the central axis of the bud transformation, and the axis of the vortex passes through *O* and is inclined to the horizontal at an angle of θ. (Note: Owing to the nature of the bud and seed-bearing process, θ in the watery vortex has always so far been found to be 90°, but this has been found purely empirically, by observing the forms of nature. Geometrically θ can assume any value, and in the following work we shall need to examine also what happens when it is not a right angle.) *Q* is the point on the central axis through which all planes must pass if their pivot points are to lie on our chosen contour plane; it is *b* units measured upwards from *Y*, and *a* units measured positive downwards from *X*. The centre of the vortex transformation is at point *O*, *d* units from the central bud axis, *c* units upward from the lower invariant plane of the bud transformation, and *e* units measured positive downwards from the upper invariant plane. Our vortex profile is generated by joining successive pairs of points in the geometrical series along the lines *OH* (which represents the finitely-placed real invariant plane of the vortex transformation) and *OG* which is its central axis. Let *H* and *G* be one such related pair in these geometric series. Then the line *HG* will be tangent to the vortex profile, and the two dotted lines through *G* will represent, in the full three-dimensional reality, a cone which will be tangent to the vortex surface, and which will cut the top invariant plane in a circle; we call the radius of this circle *R*. Just two of the tangent planes of this cone will pass through *Q* and these are the planes with which we are concerned. Clearly they will pass through *F* where the line *GQ* cuts the top invariant plane. We let the distance *XF* be called *N*, and the distance from *X* to the centre of the circle (where the vortex axis cuts the top invariant plane) be called *M*.

Now we must see the top invariant plane again in plan, from above (Figure 178). To find where the two planes of the cone which pass through *Q* cut the top invariant plane, we simply draw the two tangents from *F* to the circle. Next we must drop perpendiculars *XL* onto them from *X*, and produce them out to *L′* so that $XL' = k_1 XL$. The two points *L′* are points on the preliminary curve which we are seeking. By now taking a series of points *G* along the line *OG* in their geometric series, and coupling them with their corresponding points in the series along *OH*, we can get a series of points *L′* in our curve on the top invariant plane. This curve simply has to be projected from point *Y* onto our contour plane, and we have our desired contour.

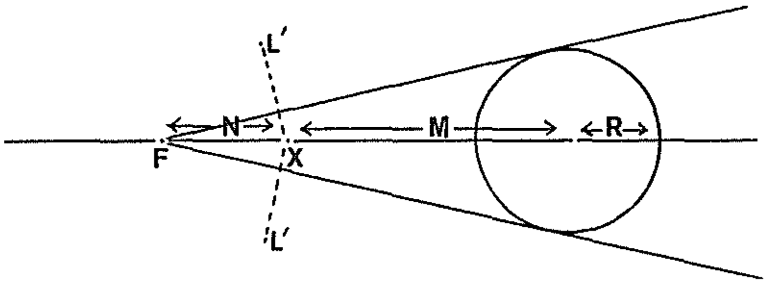


Figure 178.

This may sound a long and rather tedious process, but I have not found a short cut. However, with a pocket calculator, especially a programmable one, it can in fact be cut down to quite a quick calculation. The following little formulae have been worked out from elementary mathematics, similar triangles, etc.; it seems superfluous to give their complete derivation as surely any interested reader who wishes can work them for themselves. (Here x and y are the distances OH and OG, respectively.)

First to find the distance XF. This is given by

$$N = \frac{a(y\cos\theta + d)}{y\sin\theta + c - b}$$

The radius of the circle in which the cone cuts the top plane is given by

$$R = \left| \frac{x(e - y\sin\theta)}{y\sin\theta} \right|$$

And the distance of X from the centre of the circle is given by

$$M = e\cot\theta + d.$$

Finally we work in polar co-ordinates (r, ϕ) centred on X, which are given by

$$r = \left| \frac{k_1 NR}{M - N} \right| \qquad \phi = \cos^{-1}\left| \frac{R}{M - N} \right|$$

the angles to be measured onto the diagram being ± ϕ when N is positive, and 180° ± ϕ when N is negative.

In order to draw a contour, we prepare two geometric series — one for x (the distance OH) and the other for y (the distance OG) — with multipliers determined by the λ of the vortex. We decide which term in the x-series is to be paired with which in the y-series (that is, we decide on the width of one's vortex) and then put these pairs (x, y), together with the other parameters we have chosen, into the above little formulae; the resulting co-ordinates (r, ϕ) give us the locus of L', which is the projection from the bottom pole of the bud transformation (Y), of the contour we are seeking, on the top invariant plane.

When we come to work with the case in which the axis of the vortex is skew to that of the bud transformation the general method of construction is the same, but owing to the greater freedom which has been introduced it is convenient to work slightly differently. We will have parameters (x_1, y_1) to mark the position of the centre of the circles, i.e. the point at which the vortex axis cuts the top invariant plane of the bud transformation, and (x_2, y_2) which shall be the orthogonal projection of point O, the centre of the vortex, on the top plane. Let a still be the distance downwards of our point Q and e the distance downwards of the point O, the centre of the vortex.

Next we consider the distance from the centre of the circles (x_1, y_1) to point O, the centre of the vortex, to be divided into N equal parts (the larger the number we choose for N the closer together will be the points on our final curve). We now introduce a parameter H which will fix for us the size of our vortex. We can consider the distance from point O to the point (x_1, y_1) divided by N to be a sort of unit measurement along the axis of the vortex. When the centre of our cone, point G, is one such unit above point O, then its corresponding point in the horizontal plane through O, point H, will be H units from O. We next introduce a counting number n which shall tell us how many such units the point G is from O. Then by similar triangles we can easily calculate that the radius of our circle in the top invariant plane is given by

$$r = \frac{N-n}{n} \cdot Hn^{\mu}$$

where $\mu = 1/(\lambda + 1)$. For the same number n, the co-ordinates of point F (x_3, y_3) are given by

$$x_3 = \frac{a[x_2(N-n) + nx_1]}{Na - e(N-n)}$$

and

$$y_3 = \frac{a[y_2(N-n) + ny_1]}{Na - e(N-n)}$$

Thus we see that each time we assign a value to our counting number *n*, we get a position for our point F (x_3, y_3) and a corresponding circle centred on (x_1, y_1).

We next consider, for each value of *n*, the tangent from point *F* to its corresponding circle, drop the perpendicular to it from *X* (which is at the origin of course), and then proceed as before.

Computer programs

The computer programs below were written in Basic. If they are put on to any other computer system they will probably need more or less adaptation, especially in the instructions for input and output. Therefore only the executive commands are included, the reader being left to settle the details of input and output according to the system being used.

Egg

For this program we need four parameters for the input:

R, controlling the width of the egg;
L, giving the λ of the egg;
E, giving the ε-parameter which controls the steepness of the spirals on the egg;
M, a magnitude factor which controls the size of the picture.

The executive lines are then as follows:

```
 60  FOR S = 0 TO 2 STEP 0.25
 70  FOR T = -7 TO 7 STEP 0.05
 80  Y = M*EXP(2*E*T)/(1+EXP(2*E*T))
 90  X = 2*Y/EXP((2*E*L*T)/(L+1))*SIN(T+S*π)*R
100  IF COS(T+S*π) < 0 THEN 120
110  PLOT (X, -Y)
120  NEXT T
130  NEXT S
```

Notes

R is expressed in terms of the total height of the form as unity.

If one puts a negative number for *L* the egg transforms into a vortex.

Line 60. By reducing the size of the step one puts a greater number of spirals on the egg.

Line 70. By reducing the size of the step one packs the points more closely along their spirals.

Line 100. By reversing the inequality sign one changes the spirals from left-handed to right-handed. By deleting this line altogether, the egg becomes transparent!

Suggested parameters would be:

$R = 0.6$
$L = 3$ or, say, -0.5
$E = 0.3$
$M = 200$

EMB

The EMB (embryo) is programmed with $k_1 = 3$ and $k_2 = 1$.

For this we need size parameters for our input, taken from Figure 177.

L is the λ of the vortex, anywhere between 0 and −1;

W shows the width of the vortex, effectively the ratio of x to y in Figure 177;

C and *D,* as show in Figure 177, denote the position of the centre of the vortex;

S, the slope of the vortex-axis is marked in Figure 177 as θ, measured in radians anticlockwise from the horizontal;

J, a magnitude parameter fixing the size of the picture on the screen.

The executive lines then are:

```
 80  FOR P = 0.05 TO 0.95 STEP 0.05
 90  FOR T = -50 TO 300 STEP 2
100  Y = 0.1*EXP(T*0.01)
110  X = W*EXP(T*0.01/(1+L))
120  B = 3*P/(2*P+1)
130  N = (1-B)*(Y*COS(S)+D)/(Y*SIN(S)+C-B)
140  R = ABS(X*(1-C-Y*SIN(S))/(Y*SIN(S))
```

```
150  M = (1-C)/TAN(S)+D
160  Z = R/ABS(M-N)
170  IF Z > 1 THEN 260
180  Q = -ATN(Z/SQR(-Z*Z+1))+1.5708
190  U = ABS(3*N*R/(M-N))
200  G = P*J*U*COS(Q)
210  H = P*J*U*SIN(Q)
220  IF N > 0 THEN 230
230  G = -G
240  PLOT (250*P+H),(G-250*P)
250  PLOT (250*P-H),(G-250*P)
260  NEXT T
270  NEXT P
```

Suggested parameters would be:

$L = 0.5$
$W = 0.03$
$C = 0.2$
$D = 0.3$
$S = 1.7$
$J = 200$

It is an interesting exercise to make a series of pictures keeping all these constant except one, varying either *W*, *C* or *D*.

The Embryo (Asymmetric) Program
We need input of nine parameters:

$X(1)$ and $Y(1)$, the co-ordinates of the point in which the vortex axis cuts the top invariant plane;
$X(2)$ and $Y(2)$, the co-ordinates of the projection of the vortex centre on the top invariant plane;
P, the fractional height of the picture plane measured downwards from the top pole;
J, the fractional height of the vortex centre measured downwards from the top pole;
H, a size factor of the vortex;
L, the λ of the vortex (somewhere between 0 and -1);
S, the size factor for the picture which is required.

The executive commands are then like this:

```
110   K = 1/(L+1)
120   E = P/(3-2*P)
130   FOR N = 0.01 TO 20 STEP 0.05
140   X(3) = (E*(X(2)*(20-N)+N*X(1)))/(20*E-J*(20-N))
150   Y(3) = (E*(X(2)*(20-N)+N*X(1)))/(20*E-J*(20-N))
160   R = (20-N)*H*N^(K-1)
170   G = X(3)-X(1)
180   F = Y(3)-Y(1)
190   C = G^2+F^2-R^2
200   IF C*(G^2+F^2) < C^2 THEN 290
210   M = SQR(-G*F+(C*(G^2+F^2)-C^2))/(F^2-C)
220   X(4) = M*(M*X(3)-Y(3))/(M^2+1)
222   IF N > 0 THEN 230
225   G = -G
230   Y(4) = -X(4)/M
240   PLOT (S*X(4)),(S*Y(4))
250   M = SQR(-G*F-(C*(G^2+F^2)-C^2))/(F^2-C)
260   X(4) = M*(M*X(3)-Y(3))/(M^2+1)
270   Y(4) = -X(4)/M
280   PLOT (S*X(4)),(S*Y(4))
290   NEXT N
```

Six sets of suggested parameters (plus a suitable size factor, S, according to the machine being used, say 200) would be:

X(1), *Y*(1)	0.06, 0	–0.06, 0.38	–0.25, 0.5	–0.5, 0.55	–0.75, 0.5	–0.94, 0.38
X(2), *Y*(2)	0.3, 0	0.3, 0	0.3, 0	0.3, 0	0.3, 0	0.3, 0
P	0.7	0.7	0.7	0.7	0.7	0.7
J	0.7	0.7	0.7	0.7	0.7	0.7
H	0.007	0.008	0.009	0.010	0.012	0.015
L	–0.3	–0.3	–0.3	–0.3	–0.3	–0.3

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When *The Vortex of Life* was first published in 1993, Lawrence Edwards's pioneering work on bud shapes had already attracted the attention of many scientists around the world. In the book, Edwards gave a fuller account of his research, widening it to include the forms of plants, embryos and organs such as the heart.

His work suggests that there are universal laws, not yet fully understood, which guide an organism's growth into predetermined patterns. His work has profound implications for those working in genetics and stem-cell research.

Edwards died in 2004 at the age of 92 and Graham Calderwood has edited and revised this classic work.

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