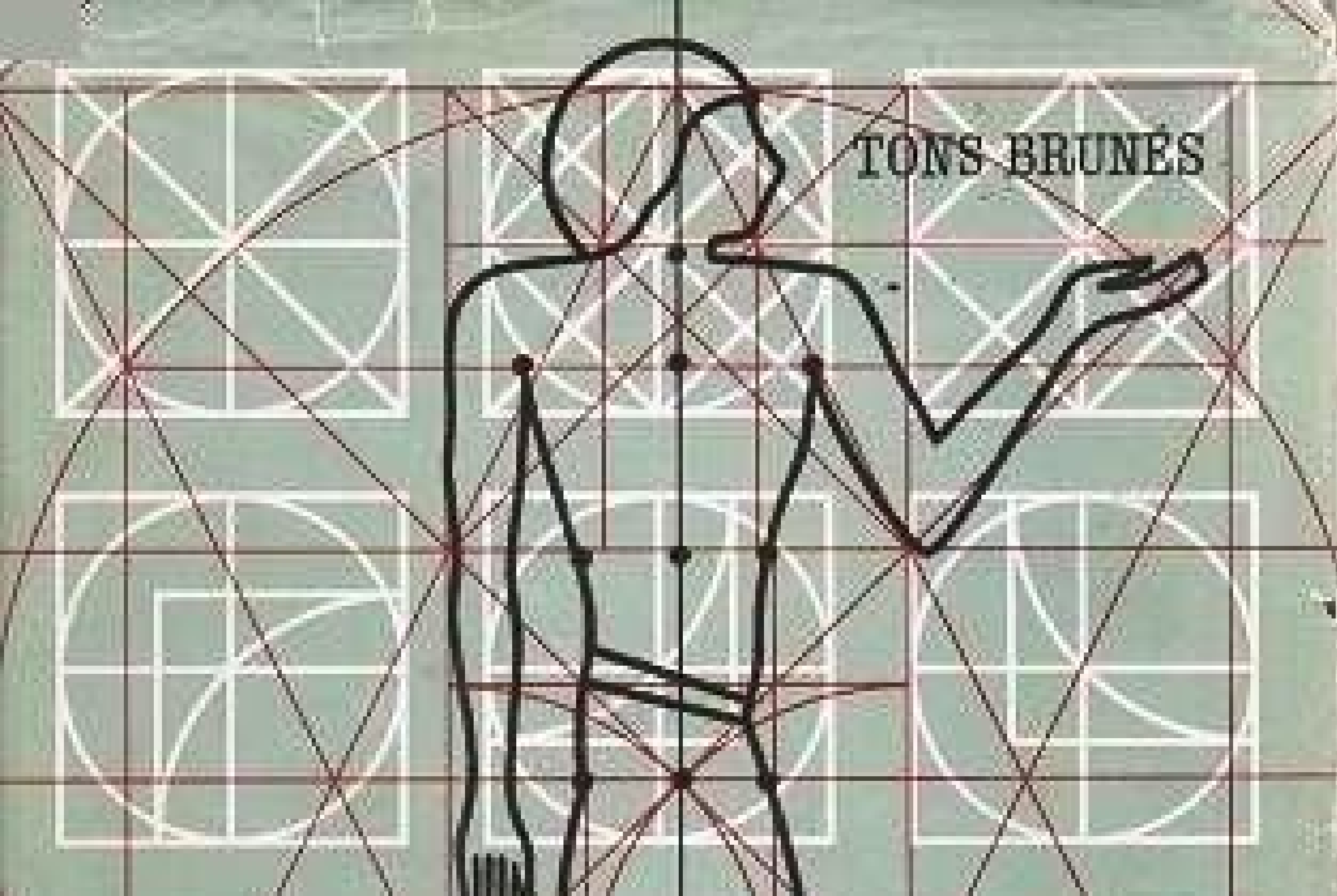
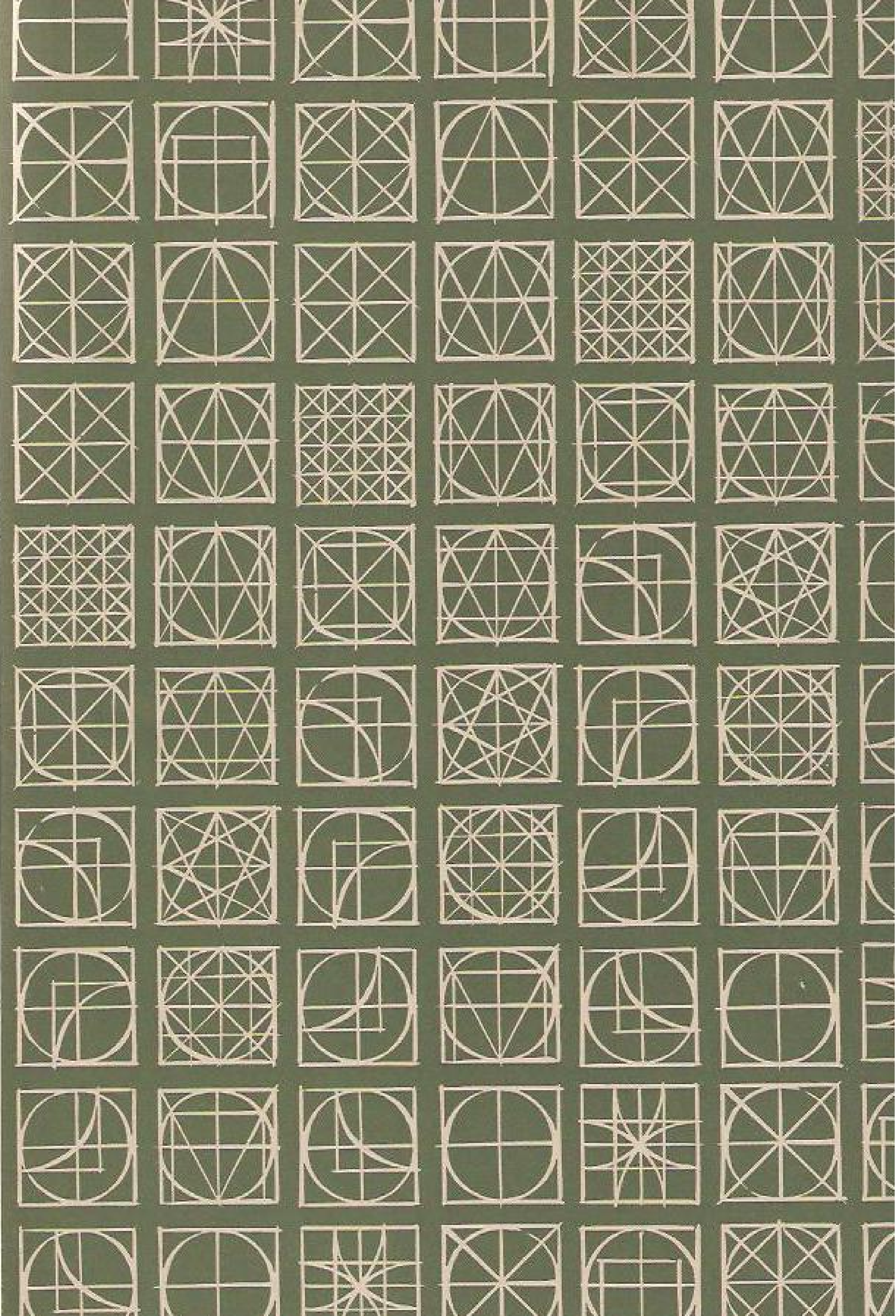
TONS BRUNÉS

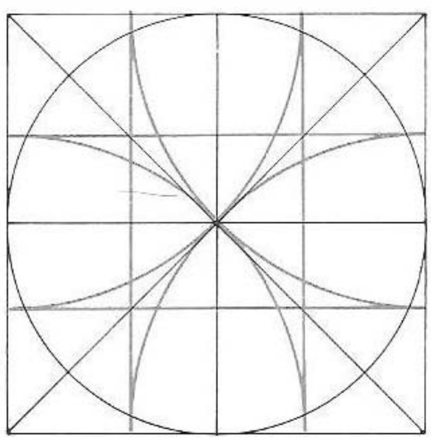


The Secrets of ANCIENT GEOMETRY -and its use



TONS BRUNÉS

THE SECRETS OF Ancient Geometry -AND ITS USE



VOLUME I

RHODOS

INTERNATIONAL SCIENCE PUBLISHERS, COPENHAGEN

THE SECRETS OF ANCIENT GEOMETRY
was translated by *Charles M. Napier* from the original Danish manuscript
Den Hemmelige Oldtidsgeometri og dens Anvendelse.
All drawings and analyses have been made by the author.

★

The Greek quotation at the beginning of the book can be rendered thus:
"Only he who is familiar with geometry shall be admitted here".

★

Jacket design by Erik Bork. Blocks by Romanus Klichéanstalt.
Printed in Denmark by C. Hamburgers Bogtrykkeri A/S, Copenhagen.

★

Sole agency for Scandinavia
RHODOS, Strandgade 36, Copenhagen K.
All other countries
"THE ANCIENT GEOMETRY", Nygaardsvej 41, Copenhagen Ø.

To the worldwide
fraternity of Freemasons—
for its inspiration.

And to my wife—
for her complete understanding and
patience during the period
of writing.

CONTENTS

VOLUME I

PAGE

Foreword 5

Introductory chapter 9

CHAPTER ONE:

The first experiences 19

CHAPTER TWO:

Speculation on time and numbers 31

CHAPTER THREE:

Appearance of figure 7 as a symbolic factor 42

CHAPTER FOUR:

Birth and development of geometric speculation 54

CHAPTER FIVE:

Circle's rectangle and triangle 81
Dividing up the Circle's circumference 97

CHAPTER SIX:

The application of Ancient Geometry 109

CHAPTER SEVEN:

Ancient Geometry and its place in Ancient Egypt 121
Thy Pyramids—the Great Pyramid of Cheops 123, Pyramid of Cheops in Relation to Ancient Geometry 133

CHAPTER EIGHT:

Book of Exodus: Ritual Significance of Moses' Tabernacle 148

CHAPTER NINE:

Egyptian Mathematics and the Influence of Ancient Geometry 177
Measures of Capacity 180, Measures of Area 201, Triangular calculation 215

CHAPTER TEN:

Pythagoras—and a Geometric Analysis of Plato's Timaeus 233
Pythagoras 236, Plato 241

CHAPTER ELEVEN: PAGE
Temples of Antiquity 259
Temple of Ceres 261, The Theseum 274, Temple of Poseidon 285, The Parthenon 301, Plans and proportions of Temple columns 311

CHAPTER TWELVE:
Geometry applied on the Ancient Building Site 316

CHAPTER THIRTEEN:
Triumphal Arches of the Early Christian Era 325

VOLUME II

CHAPTER FOURTEEN:
Ancient Geometry reaches the Middle Ages 7
Cologne Cathedral: the Old 9, Cologne Cathedral: the New 19, The Pantheon 38

CHAPTER FIFTEEN:
The Golden Section v. the Sacred Cut 57

CHAPTER SIXTEEN:
Ancient Geometry and Figurative Art 88
Art in Egypt 93, Sculpture in Greece 122, Vases, jars......and geometry 145

CHAPTER SEVENTEEN:
Cuneiform and Numerals: The shape of things to count 165

CHAPTER EIGHTEEN:
ABC of the Modern Alphabet 195

CHAPTER NINETEEN:
Trelleborg: A Viking Stronghold 229

CHAPTER TWENTY:
The Origin of Chess? 235

CHAPTER TWENTY-ONE:
Ancient Geometry and Modern Times .. [illegible]

FOREWORD

WRITERS, researchers, archaeologists, geologists (and a host of other specialists) have published thousands of books on "The Past", on our forefather, their customs, homes, dress, way of life, wars and alliances. The reader may therefore doubt that anything new can be written on the subject. Indeed there can. With full acknowledgment to the assistance provided by the works of earlier writers, I have tried to draw together many factors hitherto overlooked by researchers, and have shaped a "new" tool which may prove of immeasurable importance in a finely detailed look at historical remains. The tool is the rediscovery of a geometric system popular with our ancient forefathers, but kept secret within certain (religious) circles.

Secret geometry was protected from the blasphamous gaze of outsiders by a wall of silence. Every new adept, on admission to the ranks of geometric planners and philosophers, swore never to reveal any of the group's secrets to non-initiates. He could not therefore write directly about the subject with which he had become familiar. But outsiders could. Or rather, aware that some form of secret geometry existed but unable to fathom it out for themselves, they could guess in many directions regarding its properties.

Thus when we today in literature seek evidence of ancient geometry we find only the work of the ignorant outsider. He alone was able to speak openly.

I hope that the present book can throw sufficient light on ancient geometry to allow the 20th century to appreciate and follow geometry's former path of power.

It is often the case that non-mathematical minds are apprehensive at the thought of attacking a geometric subject. Have no fear! Ancient geometry was originally built up by simple minds for everyday use, for example, on a temple building site. I hope I have succeeded in making the subject attainable to all.

I should like to express my appreciation of the work of hundreds of other writers on historical and related subjects to whom I have referred. Without their valuable reports and thoughts my book would have been impossible. Our theories have not always concurred. But the facts from which we draw our conclusions are the same.

Ancient Geometry has taken me fifteen years to compile and write. And it has led me on a fascinating chase through books, libraries and museums the length and breadth of Europe. Their help, too, was invaluable. Finally, a special note of gratitude to the staff of my local library in Søllerød, Denmark. They requisitioned shelves of books for me from near and far—never once showing signs of exasperation!

Tons Brunés.

Ἀγεωμέτρητος μηδεὶς εἰσίτω.

Plato

Introductory chapter

THIS BOOK is the result of many years' research into the ancient civilisations of the past and their religious ceremonies. It is the compilation of fact and theory, thought and deduction. The end product is an examination of something which radically influenced the cultural development of major ancient civilisations for many thousands of years.

That "something" is an occult geometric system which existed as a hidden and sacred factor in the mystery-shrouded temples of those days; a system so well guarded through the ages that none of its secrets has been revealed until now.

To state blandly that one has uncovered a geometric system which has remained through time unknown to all save the brethren of the inner Temple sounds on the face of it an unexciting discovery, void of interest except to devoted geometricians, but later chapters will demonstrate that the system itself has in practical application had a wide range of outward functions which have left obvious traces in a developing society—traces which a knowledge of the system can determine even today.

Geometry in ancient times was simply the culture-creating tool with which most things were made, and—as I shall illustrate in my book—the importance of this particular tool to the unfolding of culture itself was vital.

The system has its roots in the very beginning of time, further back in Man's history even than the ancient Egyptian dynasties, and the existence and application of the system can be shown over an incalculable length of time right up until the Middle Ages, when it finally died out still swathed in a mantle of secrecy and silence.

The system was attributed immeasurable significance as late as the period of speculation in Greece 2,500 years before our own time, and one can read of it in Plato's eighth book, in which he describes the explanation by Timaeus the philosopher on the universe and its composition.

This chapter describes in detail portions of the old geometric system, geometry being employed as a picture of the univers and the power of the deity, not as we use geometry today as an explanatory symbol between the speaker and the subject, but as identification of the very subject with the geometric pictures and vice versa, uniting these as one. The text is so cunningly composed that a thorough knowledge of the system is required in order to appreciate the full significance of the subject matter. This attempt to conceal was, of course, intentional as both Timaeus's speech and Plato's subsequent written report were meant for the initiated and had therefore to be comprehensible only to them.

The actual system and its formation have been secret from the beginning, a secret kept within certain, clearly defined circles. The holders of this knowledge, of course, made extensive use of the system and its principles in solving a number of practical and well-known projects, but merely making use of the system is not quite the same as making it public.

A solid understanding of it permits one to demonstrate its application in many spheres, and the purpose of my book is precisely to show how the system is built upon logical meditation on the path of the Sun and the Moon across the heavens, how these reflections were indicated by the circle in which a cross was drawn, and how this geometric figure with the help of the number seven and the results of logical conclusions has produced a geometric system which has fully satisfied geometric needs right up almost to our own time.

The explanation of the system is followed by an examination of its uses in various fields from its inception up through a number of civilisations to the Middle Ages.

Ancient geometry—the term I have given to the system—was employed over a tremendous area, and we are able to follow its path from Egypt and her neighbouring countries out across Europe, and its effects can be noted in both Near and Far East.

The spread throughout such an extensive area took an extremely long time, and the apparent delay is perhaps more readily appreciated when one considers the method of communication employed.

In those ancient times the accepted procedure was for Temple brethren from one country to seek out temples in another country or region in order to receive information and teaching there.

Once such a brother had spent some time—perhaps many years—in a foreign temple he would journey back to his mother temple with his treasures: the knowledge and learning he had attained.

This was the manner in which secret knowledge was transferred from one temple to another and from one country to another.

Egyptian temples had gained for themselves from earliest days the reputation of storing the richest knowledge. These ancient temples were the most sought after among outside brethren and it was to be understood that particularly influential letters of introduction of special abilities were needed to gain entry to the Egyptian temple circle.

A part of the mysticism which was in this manner handed from temple to temple was ancient geometry, and this knowledge (remaining in the hands of the same groups of initiated brethren) slowly, gradually spread over wider and wider areas. And bearing in mind the many strongholds, the numerous temples within which its principles were practised, one can understand how the system has survived the decline of certain nations and the birth of others.

Seldom were the secrets of the temples entrusted to print. Communication was by word of mouth. But before a Temple brother received any teaching from the inner circle he had to pledge solemnly never to reveal the secrets to any outside party.

Over an endless period of time these secrets remained intact and it was not until a century before the real period of geometric reflection began in Greece that a murmur of the geometric terminology escaped from the inner Temple circle, when Pythagoras returned to his homeland after many years in Egyptian temples and started up a school in which he offered, among other studies, the teaching of a new geometric theory to non-initiated Greek students.

He never revealed or taught the geometry which he had learned from the Egyptian masters since he remained true to his promises never to do so, but his new teachings were on associated lines and this was sufficient to set the ball rolling.

Greek thinkers, thirsting for knowledge, grasped at any refreshment, and geometric theory and speculation spread like wildfire until, in an amazingly short space of time in relation to previous development, a new geometry emerged, a system which in many ways differed from the ancient variety, only the foundation remaining the same. And whereas the old system built upon traditionally accepted knowledge, the new form of geometry demanded exclusively the satisfaction of proof.

The price of Pythagoras's crime of apparently having profaned the secret knowledge of the Temple was high: at the age of eighty he was forcel to flee his country, leaving his material wealth and school to his enemies.

But in fact Pythagoras had not revealed any of the mysteries; these were still sacred, and two distinct geometric schools of thought now existed in Greece.

One was the ancient, traditional form of geometry whose outlines were still guarded by the temples as a part of their mystic knowledge, and this was the system of which Pythagoras and his contemporaries had full details.

The second arose from the teachings of Pythagoras in his school, where he instructed non-initiated in associated theories as a preliminary to abstract geometric thinking.

These theories were not actually secret, but from this training sprang groups of scholars who tried independently to solve the riddles of ancient geometry, without success.

Through the Temple society the old system had a close contact with the creative intelligentsia who were generally recruited from the temples, and it was therefore the ancient system which was employed in any practical problems facing the temple brethren or their cronies.

As we have seen, the ancient system of geometry was protected by a pledge of utter secrecy, and since it was known only within the confines of the Temple and its immediate sphere of influence the system was not subjected to outside criticism.

The new geometric line of thinking was the rebel released to the public, and this was the system openly discussed by all and sundry outside temple society.

There were experiments, arguments, discussions, assumptions; in short, the new thoughts were transformed into a new and developing science.

Temple society had no comment to make on the new schools of thought. For one thing, it felt a certain superiority in this direction, and for another there could be no question of joining in the discussions since this would have been impossible without revealing the Temple's own system. This would have been tantamount to a breach of promise.

Moreover, there was no need for a revision of thought in the temples for two thousand years were to go by before the emergent system definitely won over the ancient, traditional form. And the Middle Ages were well under way before the ancient system died out entirely, presumably with the passing of the last monastic brothers to have a knowledge of it.

Right from the earliest days the erection of churches and monasteries has been the prerogative of the Temple and, later, the Church. When, well through the Middle Ages, this last privilege was taken from them the remains of the ancient geometrical knowledge was passed on to various guilds of craftsmen, and of the knowledge which had at one time been the bloom of all wisdom only the sad remains were left in the form of craftsmen's guiding rules.

The existing orders of craftsmen to whom the knowledge was passed formed, true to tradition, small societies based on the temple pattern, and these societies were divided into degrees with an initiation ritual for each new step up the ladder, just as their forerunners had.

But apart from the friendly atmosphere in these new secret societies, they could almost be regarded as a type of trade union bound to protect the trade from outside imposters by keeping secret the various procedures used in their work.

We can find traces of these craftsmen's guilds right up to the present time all over Europe.

Thus ancient geometry survived time in a cloak of secrecy, surrounded by threats of severe punishment if this secrecy was penetrated, and the system died the way it had lived: in secrecy.

In those periods of importance enjoyed by the system it was handed on by word of mouth, never put in writing. But if it should occasionally happen that mention was made of it in writing, the text was composed of esoteric terms and the phrasing of these was such that knowledge of the system itself was essential if the reader was to understand what the text was all about. Only the initiated could make sense from such a text, while for others it remained more or less unintelligible nonsense, and this is why students of ancient writing have been unable to trace any useful references to the old geometric system.

In this way ancient geometry ruled supreme up until about 100 years after the Pythagoras era. From this point onwards a new school of geometric thought developed, differing from the old although having its roots there.

The new system did not originally present any competition to the old. In actual fact it had no practical use and was regarded as an amusing game with figures and proportions, but as time mellowed the new line of thought gained more and more in popularity and in many spheres replaced the ancient, traditional theories and methods of working.

But the change-over was slow. The power of the Temple was immense and its brethren were in many practical and speculative spheres unassailable, and it was here that ancient geometry held sway.

Thus it is that in the surviving material from those days we find the ancient and the new thoughts used side by side, initially with the emphasis on the former and much later with the supremacy of the new system and the subsequent dying out of the ancient form, the new system was to develop into the mathematical and geometric knowledge which is ours today.

The ancient system of geometry is actually a related form to the one we know today, but the differences are yet so numerous and profound that it is impossible to substitute the one system for the other. Any attempt in this direction could be compared with using a measuring stick with an unknown graduation. The result finally obtained cannot be transformed straight into a form with which we are familiar.

For a complete comprehension of the old system it is therefore essential that one spends the necessary time to study and reflect upon it and the thoughts which form its foundation. Any omission in this respect and a short cut to the geometric analysis of certain buildings produces nothing but a scanty appraisal of the logic of the system's application. For although one employs—inevitably—one's knowledge of geometric principles, it is not sufficient in this instance.

A familiarity with an alphabet does not give a person the ability to read every language in which the letters are used. He requires to learn the language before the significance of the letters becomes apparent.

The same applies here. We must get to know the ins and outs of the theory before we appreciate the impulse of its course.

Ancient geometry is not a difficult subject. It arose among people who possessed virtually no knowledge of figures, and was based on the logical consideration of primitive drawings, each drawing containing in itself the answer to the immediate problem.

A square can be divided into four smaller squares by entering a cross. That is something everyone can understand, and this is the kind of observation of which the entire system is composed.

Once a person has picked up this knowledge and appreciated the line of thinking which has prompted it, he is able to follow the system and its application from one region to another and from one period in time to another.

As far as possible the book is compiled in chronological order, beginning with an infinitely distant stage in Man's history when the system had its origin, with experience being added to experience until a complete system emerges.

Naturally experiences of this nature had to take on some written form otherwise they would be forgotten, and this form of expression was a number of geometric symbols which in their construction and development illustrate the particular geometrical aspect which they characterise.

This habit of expressing an idea or a situation by means of a single symbol is a natural course of action, and we see the same type of thing used today in chemistry, mathematics, physics and many other spheres.

This procedure has furthermore the advantage that it maintains the already mentioned principle of secrecy. To the knowledgeable the symbol contained a distinct significance, while any person ignorant of the secret saw merely an undecipherable mystic sign which remained so unless he subsequently received information about its content.

A piece of text would have been a different proposition. It would have required a special composition in order to remain unintelligible to the outsider, and the possibility exists that the written language of those days simply did not posses the terms used to cover a given set of circumstances.

The book delineates the origin of the symbols, and the information each symbol contains.

After this examination of the system and its composition the reader sees its application in the construction of one of the ancient pyramids and, from the same period, how the ancient geometrician simply and untroubled worked out all the problems of a geometric nature as quoted in the old parchments known as *Rhind Mathematical Papyrus*. Problems which have hitherto defied solution because it was sought by way of the new system of geometry.

The book also shows how Moses, with his background in the temples of Egypt, had a knowledge of ancient geometry which he used in his instructions for the building of the Tabernacle. All the dimensions are contained in the 2nd Book of the Pentateuch, and a description is given of the drawing of the temple line by line.

We then jump from the earliest period forward to the ancient civilisations, where we can trace the use of ancient geometry in the dimensions of a number of columnar temples.

It may be noted that the system does not merely reveal one or two of the dimensions, but unfolds the complete structural plan of the temple from the lay-out to the facade, with roof pitches, column spacing, etc.

As an intermediate stage we also work on one or two triumphal arches simply to illustrate the basis from which they take their dimensions, and finally there is a section on the system's application within the magnificent edifices of the Church in the Middle Ages, many of which even today stand as memorials to the achievements of the past.

For many years theorists and investigators have pondered on the rules used in ancient days as a basis for dimensions. There were traces of a distinct procedure or set of rules but these rules have continually defied the efforts of research.

This rule, this procedure which has created a common harmony throughout the ages, is ancient geometry. And the reason that science and research have always been unable to break down the code is that a common denominator has been sought within the framework of our own geometric system without seriously considering the possibility of the existence of quite a different system from ours.

But this was the case, and if ancient geometry and its symbols are traced from their origin, one can go from temple to temple, from church to church, fitting them into the system known in ancient literature as the harmony of harmonies.

An equally interesting chapter for anyone not interested in mathematics is that which demonstrates how the symbols of ancient geometry have been the epigraphic root of almost all the written languages of the world since the Egyptian hieroglyphics.

Just as we did with the buildings, we go back in time and trace the remarkable cuneiform writing from its source.

Like everything else of importance in this period, the written language also derived from the temples, and it is fascinating to study the way cuneiform script developed from the ancient geometric diagrams, with hard and fast rules governing the positions of the symbols.

The book contains samples of Assyrian, Chaldean, Sumerian and Babylonian scripts, ending with the last of the cuneiform languages, ancient Persian.

We also discover how the same thoughts are repeated in a later era, symbolised this time by another geometric sign representing a wider knowledge.

From this symbol emerges the complete Phoenician written language, letter for letter, and the symbol contains every line of the alphabet's 22 letters.

The development of the written word then proceeds anew until we see the same symbol used in ancient Greek, modern Greek, Etruscan and finally the Roman writing. The parity of the symbols and the letters is so striking as to leave no doubt concerning their epigraphic relationship.

To complete the picture we see how the numerals which we use today, Arabic, but which we have borrowed from the past are in fact of the same family.

A cultural component with the influence of a geometric system which has laid down a universal system of dimensions must naturally have a similar effect in a related field, i.e. figurative art, and the book points out how ancient geometry has also entered this domain, starting with early Egyptian art.

A study of the material which remains of the past can carry the observer from subject to subject and from one period in time to another, and the reader is perhaps of the opinion that it is not possible for one book to cover such wide-ranging subjects as the laying down of the pyramids and the birth of a written language several thousand years later. But the simple fact is that ancient geometry and its symbols are the keys with which we can release the answers to many puzzling questions concerning the past and its peoples. And in relation to this mass of material the book should merely be regarded as an introduction.

If one wishes to discover a few of the factors which have governed Man's development through time, one has to select a period from which to draw up a list of known experiences and, from this point, work either forward or backward in time.

Almost all progress is built upon experience. Experience is added to experience and their total value can be seen in the stage of development at any one place at any one time.

Naturally the first source to which Man turns in order to discover something of his past is the literature which has been amassed in the libraries of the world. Here he can read of almost any subject from different viewpoints.

The library holds captive the history of Time itself from the earliest period up to present-day problems, and it would appear that there are few chapters in this historic epic which have not at some time or other been illuminated either by historian or the indomitable archeologists who, by their excavations, have established an unending catalogue of facts about the past and its peoples.

A general view of this literary mass permits us to split it into several main headings, but with a heavy dividing line around 1400 A.D. when printing, with Gutenberg's new invention, began its great boom period.

Prior to ca. 1400 written material in relation to the present total volume of literature is extremely sparse, and quite rightly these pre-1400 items are cherished in their respective libraries as priceless cultural treasures.

Circumstances prevent much of this material from coming into the hands of ordinary students and research people, mainly because it is written in dead languages and requires therefore a special study of these ancient tongues before the scholar can interpret the significance of the written word. Considerable portions, however, of the material have been translated into modern languages and published subsequent to 1400, and we can learn that when printing started in Venice in 1464 the first books published by the brothers Johan and Wendelin da Spira (who were the first printers in Italy) were translations into Italian of the Greek masters, among them Aristotle, Aristophanes, Sophocles, Herodotus and Plutarch. This type of translation has steadily developed until today we find many of the writers of the past sharing bookshelves with more modern writers in easily accessible languages and versions.

There is, however, a vast difference between writers of the past and the present, which must be emphasised in order to appreciate the interrelationship of the two groups.

Modern writers and indeed most writers of post-1400 years have been recruited from miscellaneous groups of the population, and we find books written by scientists and university professors side by side with that of ordinary citizens who perhaps in their particular field or among their interests have discovered something of general interest.

But the learned and the ordinary groups have one thing in common: they are free to write of their experiences and observations without the restriction of a bond of silence which forbids them to touch on certain subjects. Public criticism decides whether the resultant literature is of any value.

A university education or similar advanced training is no secret today. Textbooks are available to anyone with the inclination and interest, and it is only a matter of qualifications and ability whether a person is able to utilise the contents of the individual volumes, and this is the distinction between writers before the

15th century and writers after this period.

A study of education before the 15th century shows that it was mainly confined to Church circles, i.e. that the Church ran the existing schools and a higher education inevitably involved an even more intimate association with the Church and its domain.

The further back we look into time, the more pronounced this tendency becomes and in the very earliest days it was only possible—outside the Temple—to receive training as a craftsman or a warrior. The art of reading and writing could only be achieved within the Temple walls and a man had to be accepted into the respective temple order as a brother before he could share in this knowledge. The mere ability to read and write placed a man in the same class as a wise sage, and not unnaturally this group of learned brethren was tiny in proportion to a country's population.

The roots of all monastic orders twine stubbornly back through time, to the Egyptian era which was roughly 8,000 years ago, and one can well understand the colossal build-up of tradition behind the Temple. In their respective countries or regions the temples were independent of each other and yet linked in a form of brotherhood in which knowledge and age had priority.

Egypt possessed both these qualifications and her temples consequently provided the main source of inspiration for counterparts elsewhere. History tells us that around 3000 B. C. it was a common thing to undertake a journey to the Egyptian temples with the aim of gathering wisdom and knowledge.

Literature carries reports of men who having gained their immediate fame sought the teachings of Egypt, but it was not exclusively to men with noteworthy ability that this education was offered. The normal set-up was for a man who had spent a specific number of years in his mother temple to be urged by the temple hierarchy to travel to other temples with the object of learning from them. And in those days —as is often the case today—it was men with the finest qualifications or the best connections who won their way to the main source, namely the Egyptian temples.

One of the assurances a brother, once admitted, had to give was a pledge of impenetrable silence concerning the information and knowledge he would learn in the temple, and as far as I can discern this blanket of secrecy was kept for thousands of years until a completely new development broke out around the year 1400 and shattered the old monopoly.

If we regard the ancient writers on this basis, we find two main groups.

One was the initiated temple brethren who had been in the earliest times the only existing writers.

We cannot expect to find in any of their written remains the slightest open reference to any concrete point in the mysteries or knowledge of the Temple since their pledge bound both mouth and pen all the way to the grave, and if they had entrusted any scrap of information to writing which might remotely be associated with the forbidden subjects, it would be composed in such a way as to make it unintelligible to anyone but their fellow brethren—and therefore would not be understood by the translators of later years.

Their pledge of silence, however, did not prevent them penning thoughts on other subjects. There was no shortage of subject matter in advancing societies, but Temple wisdom was taboo.

Gradually, as civilisation progressed, the art of reading and writing became an accomplishment achieved by expanding groups of the population and thus writers began appearing outside the Temple walls.

Nor from this group writers can we expect any information regarding the temples and their secrets, for the simple reason that they had no information to reveal. Certainly their talk was perhaps more open and less restricted than the Temple group's, and they were willing to discuss eagerly the Temple secrets, but their talk was based on conjecture and half truths. They were not bound by any promises but it did not really matter since they had no actual information to dispense.

Later observers would find it very difficult to differentiate between the writer who, sworn to silence, avoids certain subjects known to him and about which he must neither speak nor write, and the writer who pens screeds and screeds on something of which he has no genuine knowledge.

Later observers might well interpret circumstances incorrectly, concluding that the one who never writes of life in the Temple knows nothing of it, whereas he who does write about it must be doing so from a knowledge of the Temple. And in fact the opposite is probably true.

The result would be that the information with which research students of later years worked would be wrong in many vital respects, and consequently many of the tracks which they followed would lead nowhere, end blindly. There would be no cohesion among the various loose bricks with which the student worked.

From this standpoint I think one must treat with a certain amount of caution much of the surviving literature stemming from before 1400 A. D. as well as later comment and translations. All the information contained therein ought not to be accepted on face value as fact, even though it does have its roots so far distant from our own period.

Then the question arises: Is there no way of penetrating in some small manner this veritable wall of silence in order to gain a foothold on the other side from which to proceed further?

Have these Temple societies left nothing in their wake which one could use, independent of ancient literature, in one's reflections and then progress from there?

Yes, they have indeed and the evidence is convincing, but in order to be able to decipher these traces one must hold the key to their significance. And these keys lie hidden in the secrets of ancient geometry.

Evidence and remains of Temple society can be found almost all over the world in the form of temple structures dedicated to the glorification of high idols or ideologies, and these buildings followed a distinct plan of construction in their measurements and dimensions—based on the geometric system which is no longer with us.

This age-old system of geometry begins at such an early stage that it is one of the first series of experiences Man collected, together with speculation on time itself—a period of time which coincided with the epoch of Temple mystics.

The basis of this speculation is a study of the Sun and Moon, both of which of course leave the image of a circle in the mind.

This circle, combined with the figure "seven", and a concept I have termed "The Sacred Cut", are some of the ingredients of this ancient geometric system.

The principle of secrecy, mentioned previously, was a central feature of knowledge in ancient days. It was simply a part of the structure of society, although it may seem strange to the modern mind. But with a little reflection we must admit that it still exists today as a common part of the pattern of our own society, and the idea of secrecy is just as ancient as Man himself.

We all know the term "industrial secret". It covers processes or procedures which cannot be patented or protected by any other means than by secrecy. The object of keeping the secret is self-evident: it is essential if one is to maintain a particular commercial position.

Almost every society of craftsmen has its own small trade secrets which are passed from master to apprentice, almost every sphere of industry has a screen of mild secrecy.

Today we operate an international patent organisation which affords a certain degree of protection against the exploitation of the ideas and knowledge of other people, but if we had no such protection secrecy would be an even more widely practised principle than in fact it is.

The primitive witch doctor guards the precious secret of the manufacture of his medicines, for if he taught everyone in the tribe how to prepare his healing herbs how long would his position of power last? Indeed he would soon be out of work.

Thus we see that the principle of secrecy is founded in primitive society and has been present in civilisation right up to our own time.

The old temples enjoyed a dominating sense of power which was based on the knowledge and learning which they kept secret. All knowledge and experience was assembled over thousands of years in the Temple, and by permitting educated groups to share to a greater or lesser extent in this pool of knowledge the Temple brethren wielded—through these groups—infinite power.

The only way they could maintain their position of superority was by a consistent preservation of secrecy within the groups.

Their situation was similar to that of the primitive witch doctor: if they distributed their knowledge to all and sundry their power was gone. So one understands the tenacity with which they held on to their secrets, and can appreciate that anyone breaking his pledge had to pay the penalty of death.

This is how I think the principle of secrecy should be viewed, thus sweeping away the apparent mysticism which cloaks occult religions. They become in this light more logical and natural.

The religious spectacle and all of the symbolic ritual was of less importance to the initiated than to outsiders. It was conducted in honour of the band of outsiders who—more or less without option—had to pay taxes or tithes towards the maintenance of the initiated clan who on account of their application of knowledge were immensely wealthy compared with the population at large.

Wars and riots might deprive the temples of their material treasures, but their real wealth—knowledge—could never be stolen or captured. It could only be achieved on a voluntary basis and therein lay one of the Temple's main pillars of an era of power stretching over thousands of years.

If we acknowledge as possible this build-up of society we have a chance of uncovering the Temple's geometric knowledge by beginning our research infinitely far back in time with the first geometric observations Man must have made.

We can then try to find the path taken by later development based on these experiences, and compare our findings with the remains left behind by Temple society. If these things coincide, it is possible that we may be on the right track.

As a starting point, therefore, my book begins with the early experiences.

CHAPTER ONE

The first experiences

ALL THOUGHT and action is prompted by a broad base of experience handed down from the past.

These experiences are expanded gradually and inherited by each new generation, with of course the injection of new ideas which either supply a completely fresh thought to the existing pool, or which amend slightly or radically some previously registered experience. But we continue building on the past.

Written language and numerals are two of the essential conveyors of experience and ideas, and without the use of figures —which we have adopted from Arabic sources—or the help of a written language which has also been built up steadily over an infinitely long time, modern Man would be incapable of performing a fraction of the active life he leads today.

These aids, and countless others, are the tools with which research workers are armed and with which they obtain their splendid results.

The sheer mass of knowledge today is so immense that it demands, in addition to a rational form of education, the division of many studies into special compartments. Medical training is split into a number of different categories, so is the training of engineers—and so it is with most branches of education.

This specialisation is naturally necessary because the total amount of knowledge available on any one subject has with time attained such proportions as to make it impossible for a student to soak it up in one course of study, if the length of the course is to be held within reasonable bounds.

If we consider all Man's knowledge as a whole, it must be granted that it would be incorrect to assume that every experience or the conclusion it has produced is absolutely accurate and a true ascertainment.

Experience is one thing, but the conclusion resulting from the experience is another. If the experience has been properly understood, then the conclusion is presumably correct provided one has been able to allow for all the factors which may have had a bearing on the incident. But if it has not been possible to establish the full account of the incident with complete certainly, taking into consideration cause and effect, a conclusion can only be an approximation of the actual result. This approximation, however, may not only be at variance with the correct result, but may be completely wrong. But correct and incorrect conclusions are of equal value to the total store of Man's knowledge, since it would be unbelievable that every considered solution was accurate at its first appearance.

Later experiences and discoveries often dislodge what were previously regarded as solid fact, but the earlier experiences form a resistance to the new and thus encourage the mind to produce new and perhaps more accurate conclusions within the same field of study, since the discovery of new factors in some incident may prove that former conclusions were wrong. In this way the mind brings to light a fresh conclusion—which may again fall before still newer theories.

This whole process of interaction between correct and incorrect conclusions is the very nucleus on which our knowledge is based.

If we imagine all of our present knowledge as a gigantic pyramid turned upsides down, with the wide base representing the current level of knowledge and with the tip projecting backwards and downwards into the past ending theoretically in a sheer point with which the whole giant structure began, we have a fairly good picture of Man's development over thousands of years, starting with a tiny store of knowledge at the tip. In the beginning only a few experiences and snatches of knowledge were needed to make the pyramid increase its height, but gradually as the reservoir of knowledge filled it needed more and more experiences to raise the level.

If we are to establish the initial thoughts on which the whole of this pyramid rests, in other words the first experiences and reflections made by Man, it is vital that we ignore any experiences presumed to have been gathered at some later stage. We must start by considering Man's needs, since development and progress have always been governed by this factor.

We must try to think back to the time when Man's development started, a stage at which he was unshackled from all learning, all known things, with a very scanty basis of experiences on which to proceed, existing on the strength of his instinct, and with need as his incentive.

The first need Man would try to satisfy must be that of hunger, and the early means of settling his hunger may possibly have been to eat the berries and plants of the forest and similar readily obtainable things. In so doing, he made his first contact with knowledge in discovering which plants and berries he liked and which he did not, which ones gave him a feeling of satisfaction and which of pain. But at the same time he learned something else. Every now and then the berries disappeared just like other plants in his larder, and Man learned to hunt the beasts of the forest to satisfy his hunger.

The next thing he learned was that it was difficult hunting with bare hands. He could certainly catch up with his prey whether antelope or other game, but once he got within killing distance Man found the victim better armed for defence than the hunter. To meet his most elementary need, therefore, he was forced to find a solution: the construction of some form of hunting weapon.

The first weapon Man adopted for his armoury was probably a convenient stone of suitable size and a reasonably handy grip—since he soon realised that a blow with a bare fist did not get him very far with any of the larger animals.

This was not, however, enough. This club was poor competition for many of his victims' means of defence. Their long, pointed fangs and claws in particular left their mark on the hunter's body and taught him to find a tooth or a claw for himself. Man's teeth were of little assistance to him in such an uneven fight, and he armed himself with a dagger. But this jump was not quite so straight-forward as it may sound. It is no easy matter finding just the right shape of sharp branch in the forest. The fallen branches were fairly easy to break but had no strength, the green branches—strong and resilient—were difficult to snap from the tree and left no point, just a frayed end which proved hopeless as a weapon. Man had been faced with the difficulty of making weapons of wood without having any cutting tool, and I believe the first pointed weapon he used was the remains of a broken bone, which can produce quite a fine point. Later he had learned how, by rubbing it against a stone, he could keep the dagger sharp and even produce a keener, more effective point. The next advance had been to assume that if the stone can wear away a bone it must be harder and more suited than bone for making weapons. Man the hunter then began his search among the local rocks until he found the best available type the area could produce, and we note that this search usually ended with the strongest he could find. In extreme northern areas such as Greenland and Alaska where there were no suitable types of stone and where most of the rocks were covered anyway much of the year by ice and snow the principle source of weapon material was the bones of dead animals. Whatever form the material took, the early weapons of this time must have been the dagger or, as mentioned, a tooth or a claw clutched in the hand.

The further improvement of this weapon was most probably the spear, or the dagger fixed on the end of a short stick. The spear had not been intended as a throwing weapon, but simply as a means of extending the use of the dagger by reducing the risk of injury from slashing claws and fangs—of which the hunter had many unhappy experiences.

With the spear in his hand Man quickly recorded his next experience—the usefulness of his new weapon. It had come to him one day as he flung it in a rage after some animal which had suceeded in running out of range. Thus we see the appearance of a weapon that has lasted up to the present day.

Going back to the dagger on the end of a stick, this idea was also adapted at a much later stage in time: by fastening a bayonet on the end of a rifle barrel the army provided itself with a stabbing weapon which allowed a certain distance from the enemy.

The next need which registers with Man—or more correctly another need, which will make itself felt at the same time as hunger—is sleep. The need to sleep generally comes at the close of the day, and in areas where the climate is warm and dry the sleeper requires only to consider the matter of safety in his choice of resting place. In areas where the climate on the other hand is cold with variable temperatures, changing with the seasons of the year, the cold and rain had certainly to be taken into account together with the safety factor. And this combination played a part in making the wandering hunter a man of more fixed abode. He extended his stay a little longer each time in one particular place, perhaps finding a dry cave to which he returned each night.

If at the same time he had found a woman and consequently a satisfaction for his sexual urge, his need to roam endlessly would decline somewhat and soon the appearance of family would make constant rambling more difficult. He would more and more be inclined to stay for long periods in the same place until eventually he found a suitable spot for a permanent dwelling, with ideal conditions of water, food and game, and with a cave for protection against the ravages of the elements and the attacks of wild animals.

He would hesitate leaving such a place, and would establish his domain there—remaining until circumstances drove him out. And the wandering hunter would continue his lonesome way, either alone or in a crowd from place to place looking for a permanent home.

These roving bands of hunters represented an ever-present threat to the permanent dwellers, since a woman was seen as fair game, and the time-honoured rules and rights of ownership we know today simply had not been established. The rule was merely the survival of the fittest and strongest. The hunter could experience arriving home after a day's toil only to find that another hunter had carried off his woman and perhaps stolen his spare supply of weapons into the bargain!

This turn of events resulted in the aggrieved hunter setting out in pursuit of the thief. If he was able to track him down a battle was the only means of settling ownership of the stolen property.

This annoying factor of insecurity must have been an irritating problem, and with Man's sense of the functional this experience had encouraged individual families to group together in small communities whose first law was a man's right of ownership of his woman. For without this law it is hardly possible for society to have developed. Admittedly certain societies practise polygamy while others are run on polyandrous lines but no doubt these conditions have arisen because of a large surplus of women and men respectively. But in these latter forms of society, too, sexual relations are confined within a hard and fast framework, and we can generally assume that if and when the early hunting people assembled in small communities they did so for reasons of convenience, and these reasons must first and foremost have been of a protective nature in respect of the family. And when people unite to avoid attacks from without, it must be understood—if unwritten—that the same attacks do not come from within the community.

The fact that people are people and that in those days there had been—as we have today—a certain law-breaking element in society is quite another matter, but one of the first unwritten laws Man must have had was his right of ownership over his woman.

Time must have been a vital factor in the hunter's life, just as it is today. An examination of the ordinary, everyday chores that were necessary to maintain a family shows that the primitive hunter's existence was no life for an idler. The man's job was to hunt and produce a food supply. When he was home there was the constant work of making weapons or repairing any defects in the existing supply of daggers, clubs and spears.

His armoury would probably have been extensive, and the more inefficient he was as a hunter the more weapons he had to make since presumably he often threw a spear or shot an arrow at some animal and wounded it, only to watch the beast flee with the weapon in its side. If he followed a wounded animal it was not on humane grounds for humanity was unknown. It was first of all to recover the possible kill and secondly to regain his lost weapon.

The work of the woman was to process the prey to provide food and clothing, to collect fuel for any fire the home may have boasted, the fashioning of kitchen utensils and later the herding of domestic animals, as well as growing grain or other foodstuffs.

Considering the few aids at their disposal early men and their women certainly had their hands full merely to exist. These first communities, living in direct contact with Nature and its forces, had few experiences on which to reflect. Their needs were primitive and their minds small since there were—apart from the problems of food and shelter—few thoughts to engage their thinking mechanism. And their language was probably limited to the expression of a few simple ideas.

Whether their intelligence was less than that of present-day Man must depend to some extent on the concept of intelligence. If the term means the ability to observe and draw conclusions from these observations, then intelligence was probably as it is today distributed proportionally, some being born with an insatiable curiosity and thirst for knowledge while others take in nothing apart from events within their direct sphere of interest.

The need must soon have registered with these early people for some form of calendar. Not for its own sake but as a guide to show the approach of winter and an indication of its duration. It must have been a very unpleasant surprise to find a seemingly endless stretch of summer days suddenly change to blustery autumn storms which tore the foliage from the trees, halted the growth of plants and brought darkness and cold to the tiny communities.

Game became more and more sparse, and the wet forest wood would scarcely burn.

Anyone who has been obliged in winter to gather an armful of wood from the forest, and has had to burn wood which has not been stacked and dried, knows only too well the problem of heating by this means. Not to mention the huge quantities required.

Added to the fact that their dwellings were open to the elements, cold, hunger and inconvenience must have been tremendous.

The end of summer must have been seen as disaster for these primitives, knowing winter had arrived and never sure whether the warm days would return.

We know today from experience that there is a strict continuity in the changing seasons, but if one has no means of judging time it must be difficult to understand these changes.

It is so simple for us. We have split the year into spring, summer, autumn and winter, and each of these four seasons has three months. Simply therefore by naming a particular month we have fixed our position in any one year to within one-twelfth. We have further divided this twelfth into roughly thirty days. The days are divided into hours, the hour into minutes, the minutes down into seconds, so that we can fix our position during the year with an accuracy of $^1/_{31536000}$ simply by stating the month, the date and the time. But without these aids we are in a bad way.

How many of us on holiday have suddenly discovered that we have forgotten not only the date, but also the day, whether it is Tuesday or Friday or some other day of the week. We have, however, so many ways of checking the time and date in daily life that the problem presents no hazard. We simply know the answer. Not only do we know the day, but we know its position in the month, and the month's relationship to the year.

The early people whose experiences we are tracing and who depended so much on Nature, were not only in direct contact with it but a part of Nature itself.

They naturally saw the progress of the Sun across the sky, and the forest scenery change as spring's creative energy bloomed lush and green to reach a peak in summer when the Sun was at its zenith. They noticed the arrival of autumn as leaves and greenery withered, followed by the cold and darkness of winter; but the continuity of the seasons must have been too slow for them to work out the order of change. Numerals did not exist, and without some form of arithmetic how does one keep track of time? The massive majority had probably let time and the seasons come and go in an unending procession—but there had been among them individuals with a more finely developed power of observation than the average.

As their lives passed, they had naturally observed changes in the seasons of the year without being able to pinpoint any basis for calculation. But one day they must have realised that the Sun not only moved across the sky each day but also that its points of rising and setting swung gradually across the skyline with a sideways movement.

This observation seems to us rather simple because we know it to be a fact, but every discovery of this kind is simple—once it has been made. It takes a keen power of observation to discover and establish the fact that the Sun had more than one movement, they watched it every day with increasing curiosity. Research was born. It would be many years yet before they realised that the Sun followed one path in summer and a different path in winter, but already they were able to make simple predictions according to the Sun's position. They had noticed that *when the Sun's arc rose in the sky it was* moving towards summer, and when it fell lower and lower it was moving towards winter. These early observers had probably begun advising their people when to begin collecting fuel and other stores for the approaching winter, when fishing conditions were best, and had been able to give an assurance, in a spell of bad weather in summer, that winter was still a long way off and that no one need be anxious.

The leaders of these small communities were usually the physically strongest in the group, and they must have regarded these early observers as a threat to their own position of power, these men who could predict the changing seasons and who enjoyed a certain power in the society.

As the observers were generally old men, however, it is probable that they were tolerated on account of their physical weakness, even to the extent of protection by the leader, who saw in their abilities an opportunity to exploit them to his own advantage and to strengthen his position. These men were probably exempt from hunting and other work and encouraged to carry on their mystical pursuits.

With these early observers we have the origin of tribal witch-doctors and later, as society expanded, the tribe's religious and spiritual leaders, who educated a succession of disciples in practical magic and mystical knowledge—to be replaced later, but still an extremely long time before our day, by the high priests and hierarchy of the Temple.

To discover that the Sun moves it was necessary for our observers first of all to regard the firmament with the warming, circular disc of the Sun as the life-giving factor of existence. Although not initially having any real concept of a god or superior being, merely an anxiety or fear of the unknown, they had begun to keep a *close watch on the Sun, having discovered* that when the Sun is not in the heavens it generally means bad weather.

They of course soon plotted the Sun's daily path as it rose in the morning in one direction low on the horizon and in the course of the day wandered across the sky, to disappear in the evening in the opposite direction.

Their day probably started at sunrise and each morning our observer would watch the Sun, possibly from the door of his hut. From this vantage point he has perhaps had some trees or other lofty points to mark the place where the Sun first appeared. Since its daily lateral movement is minute, this was an exacting observation. To see any movement at all can only be achieved by noting accurately the rising point of the Sun and awaiting its appearance each morning—to be surprised after some time that it has moved from the original position.

Some time must pass before our observer notes that after a certain period the

Sun again returns to the original point of rising, only to pass it and move to the other side. These discoveries had to be done over a period of years to establish that the movement depended on the seasons or vice versa, and it was necessary to remember over a long period that when the Sun rose *there* it was summer, and when it rose *there* it was winter.

Obviously storing this material in one's head was inconvenient and this is where we encounter Man's next need: to be able to portray his thoughts and observations. In other words, Man wants to be able to write.

We are a long way yet, however, from the stage where he begins to illustrate his thoughts in various forms of script. That requires a more developed mind, a wider basis of experience and a long training of the imagination. But what then can an observer do?

It is an immense task for him to keep observations of the Sun in his head. It is perhaps just possible when the tribe lives in the same place over a number of years. But the job is absolutely impossible if flooding, fire, war or other causes force the little community from their dwelling place and they have to find another; maybe it faces a different direction from the previous camp-site which would make the Sun, as far as our observer is concerned, rise in the wrong direction. And moreover he loses the landmarks which formed the basis of his observations and must begin all over again. To solve this problem with existing materials there is possibly only one way out: to try to make some markings on the ground which will record his observations from month to month so that he is released from his task of remembering the Sun's position during the various seasons.

Our observer now finds a suitable spot not too far from the camp, preferably an open space with a clear view on all sides particularly of the Sun's points of rising and setting.

He selects two suitable stones, laying them in line with the rising Sun on a particular day, and in so doing makes a discovery or invention: that without two fixed points it is impossible to take a bearing on a third.

Using the westernmost stone as point of origin for his bearings, he now visits his hill each morning to watch the sunrise and every time it passes the easternmost stone he lays another stone alongside in the direction the Sun is moving, and roughly the same distance from the west stone.

At some stage the Sun stops wandering sideways and begins moving back along the row of stones. Our observer now waits until it is back at the first stone and then completes his picture of the Sun's lateral movement by adding stone to stone in the other direction (see Fig. 1).

He now finds himself with part of a circle with an angle of approx. 90 degrees. This presents a fairly good picture of the year with A as the Sun's lowest point in the sky and B as the summer solstice or the Sun's highest point. With this construction our observer will at sunrise be able to calculate how far the year has progressed. He will know whether the Sun is moving progressively or retrogressively and therefore whether it is heading for midsummer or whether summer is on the wane.

His construction can to a certain extent be compared with a modern wrist-watch or clock which, instead of numerals, is equipped with strokes.

The stone lay-out is a time-gauge which indicates a time of year, and the modern clock is a time-gauge for measuring a twelve-hour period in night or day. Common to both is that a glance will indicate your whereabouts in the year or day without the use of numerals. The position of the hands within the circle

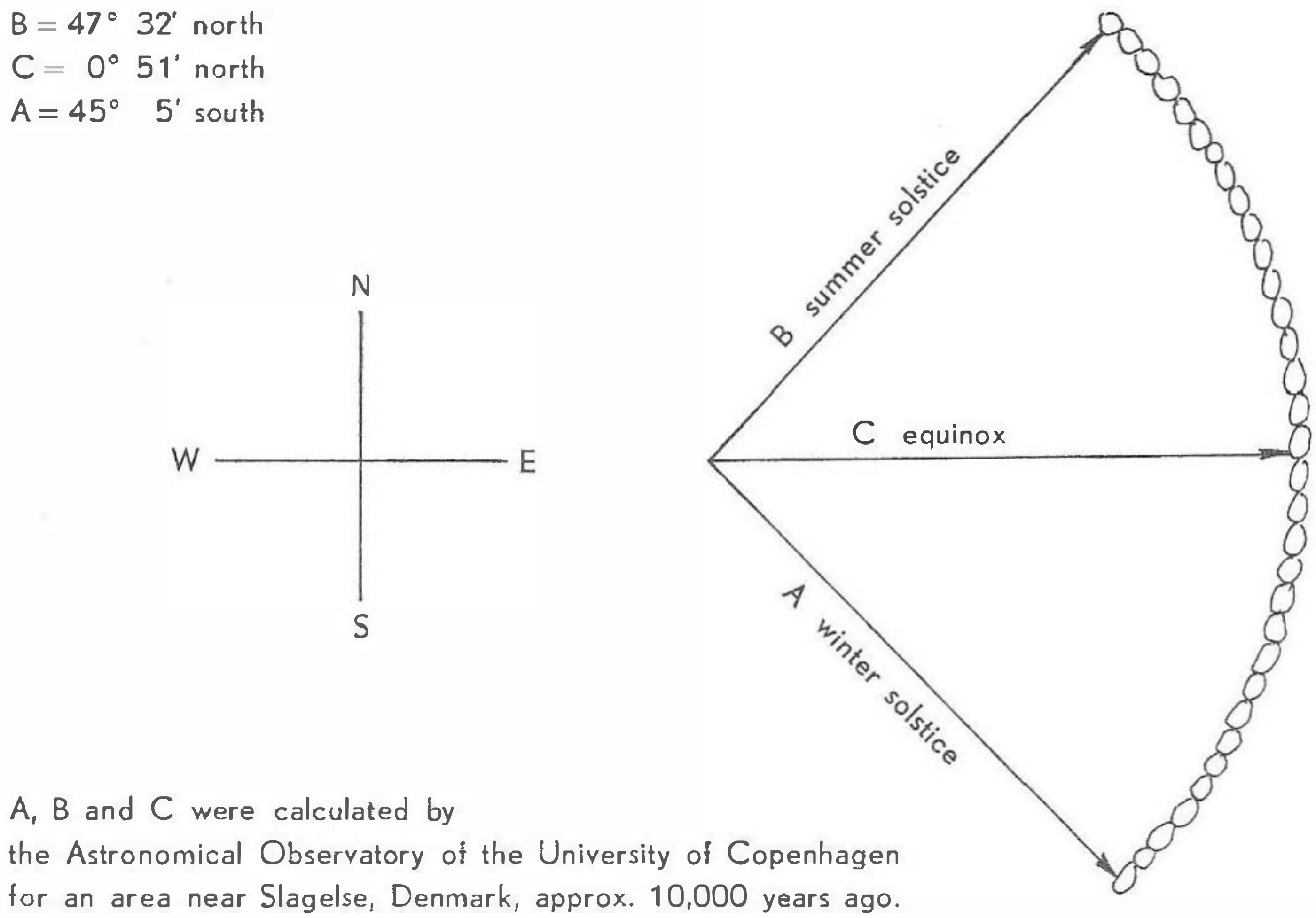


Fig. 1.

tells us what time it is without the necessity for numerals, an accomplishment we have achieved with practice. In fact we could probably manage quite well with the hands alone, without the strokes, merely with an indication of the point 12.

The same ability to read the time of year from a quarter circle could be achieved with practice, as long as our observer remembered which direction the Sun was moving along the line of stones—since, of course, this indicates whether it is approaching summer or winter.

The Danish countryside contains a large number of stone circles. Some are filled in with earth, forming a small hill surrounded by stones. Others have been used as burial places with one or more graves within the same stone circle.

The fact that burial places have been found in some of these stone circles does not necessarily allow one to assume the primary and sole purpose of the stone circle was as a burial place. How can we explain the existence of stone circles which have no burial mounds or graves? The presumption must be that the body and all signs of burial have been removed.

There is every possibility that the stone circle has had a function as described earlier, and the explanation for some of the circles being used as burial places may well be that it would be a natural procedure to bury in the stone circle the man who had spent most of his life working with it.

He has doubtless been a man of some prominence in the local community, and through the ages such men have always received the final honour and respect of

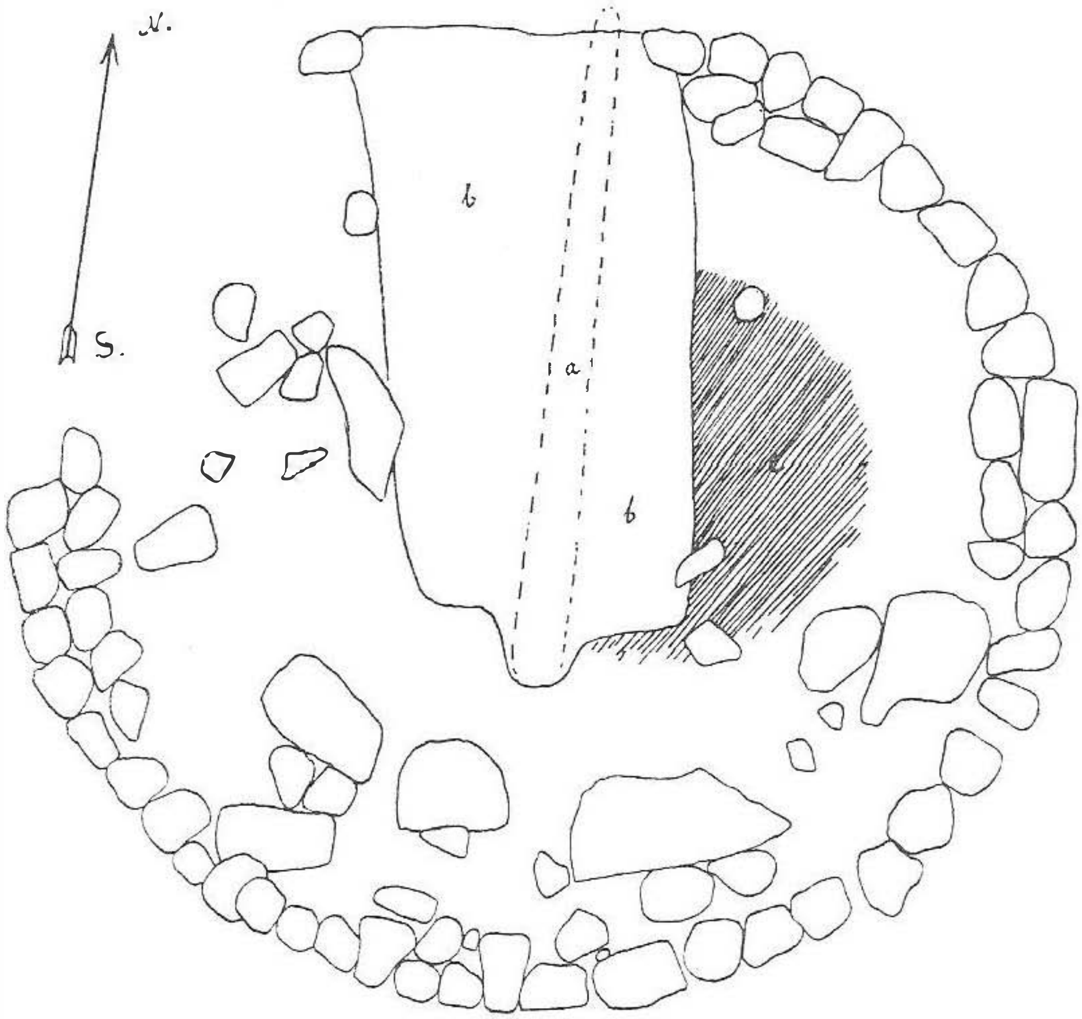


Fig. 2.

Fig. 2 a.

a suitable burial place. What better spot than the circle on the ground where Nature had given up her secrets. In fact this was the only sacred place and the only piece of consecrated ground.

This theory may also explain why some circles contain several graves dating from different times; it would be natural that, having buried the original leader in this manner, his successors received the same degree of respect.

While our stone calendar comprises only a half-circle or even merely a piece of the circumference amounting to just over a quarter and while most stone circles have stones going almost all the way round, the answer may be that our observer has not only laid down a stone for the sunrise but also stones to mark the sunsets. In this way he manages to fill in the arc on the opposite side of the circle, achieving almost a complete ring.

Another possible application of the calendar is by some means of shadow-effect. Instead of having a stone at its centre the circle might have a wooden staff or stone column whose shadow, both in the morning and evening, indicated the seasons of the year but at the same time threw a continual shadow on the stone circle precisely as a sun dial shows the passage of the day.

I think I can give an example of such a stone calendar with a central shadow-throwing column. It is the stone ring found at Erdsvaag in the South Bergenhus county of western Norway.

The stone circle and shadow stone can be seen in *Figs. 2 and 2 a.*

The excavation was made about 3½

kilometers from Bergen, roughly 33 meters from the beach.

The stone circle has a diameter of approx. 4.5 meters up to 4.8 meters, and the stone shown in Fig. 2 has at one time been erected at its centre. The height of the stone is 3.2 meters. When the remains of the circle were uncovered the stone lay as indicated in the drawing and impressions in the ground indicated clearly that the stone had been standing in the centre of the circle.

As mentioned earlier, the stone circle was laid out near the beach with a clear view on all sides. The stones themselves had at the time been placed on clean sandy ground and the circle's position was approx. 4 meters above high-water mark.

As with most other stone circles this one too is not quite circular, having an opening which stretches from a point due west to a point north, leaving a gap of 90 degrees in the circle's circumference.

In his book, *Norges Indskriftor med de ældre Runor*, published in 1891 by A. W. Brøggers, Sophus Bugge wrote:

"Since this place like the surrounding area was covered by earth and grass, the opening must date from ancient times."

This can be established with certainty because the stones themselves poke their tops up through the grass and rest on the actual sandy bottom, the soil and grass have grown up around the stones with time.

The west-facing opening could indicate that this stone calendar has been used in connection with the evening Sun which set in the sea to the west, and the eastern side of the circle was permanent and unbroken. The period of summer solstice is marked by a long stone, the largest in the ring.

Thus it is possible that both early morning and late evening sun was used for calendar observations, the only stipulation being that the Sun must be low in the sky whether the adopted system was to "aim" at the Sun from a central stone or to use a shadow effect.

Other methods than a stone circle can be imagined. For example, one could use a large stone with a more or less even surface, carving out a number of marks to indicate the progress of the Sun.

A calendar of this type is obviously easier to handle, since it could be installed in a suitable open area near the camp, perhaps sited immediately outside the hut-door.

Danish archaeology has produced two stones which might suggest that they have been made and used as Sun calendars, the equivalent of the stone circle being engraved in the actual stone.

The stones in question are not classed as rock engravings and their age is not known, but the decoration of the stones might indicate their use as calendar stones as described above.

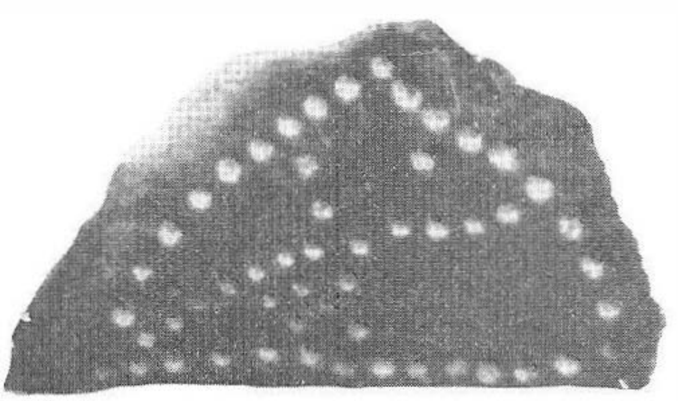


Fig. 3.

We see in *Fig. 3* a stone set up in the courtyard of Ringkøbing Museum. It was found in the Ringkøbing district, near the sea with an excellent view towards the west and the evening Sun. The stone is set up in the yard, as shown in the photograph, with the decoration perpendicular to the ground.

When in use the stone has probably been laid flat on the ground with the engraved and otherwise fairly even surface facing the sky. We can see that three lines run across the stone's decorated surface consisting of small holes or hollows. If we measure the angle between the two outer lines we find that it is very close to 45 degrees, which is of course the same angle as on the stone calendar during the period from winter solstice to equinox or from equinox to summer solstice; this stone, therefore, could well be half a calendar.

If we examine more closely the actual external dimensions of the stone, we see that it is quite flat on the surface which rests on the ground and shaped generally like the eighth sector of a circle. If we imagine that this stone has been used for measuring the setting Sun, we need a matching stone to be able to measure the whole year. If we picture this second stone laid along the side of the stone resting on the ground in our photograph, we get a complete stone calendar, with the east-west line running through the calendar at the point where the two stones touch. The stone in our photograph would then be the southern portion of the calendar, and the missing stone is the one that should have lain on the northern side.

The holes in the circumference can mark the progress of the Sun by the user either placing a small pebble in the hole once the Sun had passed or, better still, filling the hole with a paint or coloured substance gradually as the Sun passes over the stone, perhaps animal's blood was used for the purpose.

The story that the stone was a sacrificial altar for offerings to the Sun, grease being spread in the holes and "eaten" up by the Sun as it shone, is a local interpretation and is related by the museum staff as a legend to anyone who asks about the stone's use. In fact the story may well be correct and at the same time fit the theory set out above. For if the story goes so far back in time that it has become a legend it would fit the context if it was passed on by people who did not know the real purpose of the apparent ritual.

If the history of the stone lies in a later era and research-workers have found animal traces in the holes and on this basis have pieced together their theory of the Sun melting the grease, this would also match in with my own theory since there would also be animal traces if blood was used to marked the holes.

Another stone which presents the same picture but in a slightly different way can be seen in *Fig. 4*.

This stone is in the yard of the National Museum in Copenhagen and was found at Øster Velling in the county district of Viborg.

It will be noted that this stone has been turned through 180 degrees in relation to the Ringkøbing stone, and this may therefore have been the other half of a similar calendar.

Since our observer was a human being, with the human instinct of self-preservation, it is unlikely that he enlightened anyone on the observations he made with the stones. He delivered the results of his observations but not the method by which he achieved them. The area around the stone has perhaps been closed to the other members of the tribe, partly to avoid their seeing how the observations were made and partly to prevent their moving the stones.

By keeping his knowledge secret our observer assured himself of a secure position as the community could use his information, indeed simply could not dispense with it; but as he alone held the key to this information the tribe not only had to leave him in peace but also had to ensure that he was well looked after and received an ample share of the communal food supplies.

When old age began to tell, he probably took on a disciple and taught him the secrets of his art, extracting a promise not to pass them on to anyone but his successor.

Man may in this way have achieved a little control of the seasons of the year without any knowledge of numerals or other written language.

The same observations may have been carried out at different places all over the globe independent of each other and at vastly differing stages of time.

It was not of course essential to use stones or rocks as observation materials. Piles or mounds of earth, or pieces of stick, indeed anything Nature can provide, may have been employed, but since this was a construction intended to last for some time and should be impervious to wandering animals or storms, it was probably most natural to use stone if this material was on hand.

Fig. 4.

CHAPTER TWO

Speculation on time and numbers

At some very early stage in the process of Man's development numbers and counting were introduced, not forced upon him but simply brought into being or invented to meet a need and a necessity.

It would be well-nigh impossible to list the number of things we in modern society compare with the aid of numbers; things and values otherwise indistinguishable are rationally catalogued. In fact without numbers we are helpless. For every form of development depends on the use of this tool.

It is difficult to imagine a society that has no numbers and no way of counting. No way of counting the days that have gone, no way of checking and registering how many animals have been let into or out of enclosures, no way of counting its weapons, yes—and no way of counting its children.

And yet it is a fact these societies which had no form of numbers have existed and in one or two instance possibly exist today, but it is hard to imagine any development or expansion without numbers.

It has of course been possible in times before the advent of counting to obtain a fair idea of small quantities by remembering the different characteristics of the component items, but this is an ability that animals also possess and which has nothing to do with development.

But think, for example, of days. It is not easy over any length of time to recall the special characteristics of different days, and there is no opportunity here for "recognising" them as with other, tangible things.

Many things had required checking even in earliest times. A man had to have some idea, for example, how many arrowheads he had in stock and how many he used on a hunting trip, so that he could judge whether his supplies were adequate. He could by all means estimate according to the visible quantity or pile, but in the long run this guide was unsatisfactory.

He might also be interested in knowing how many children he had, to know how many mouths he needed to feed. But the most powerful impulse that had registered from the beginning was probably the desire and need to keep a check on time, that unending quantity of days and nights that streamed past with no means of identification.

With no means of establishing or defining a particular day or period of days for calculation, Man found that the present simply blended into the past like a drop in the ocean.

Events and impressions naturally left their mark on these early people in the same way as they do today. But whereas we can denote annual red-letter days, e.g. birthdays and wedding anniversaries to mention the more personal kind which involve a single person or a small group, or Christmas, Easter and Whitsun which embrace whole nations, early Man had no means of doing so. Even big events—both personal and communal—simply vanished from their range of remembrance very quickly. It was impossible to recall a particular day or period of time even after a short interval if one had no check on time itself nor any way of using time as a factor of measurement.

Now and again animals may react in some way which tempts us to believe they have the ability to count; for example a duck may miss a duckling from her huge flock. Whether this is achieved by counting or by memory is perhaps elucidated by the following observations of humans.

In his book, *Historisk Matematik*, Poul la Cour writes on the subject:

"In one tribe in South Africa few of its members can count to more than 10, and yet the owner of a herd of 400—500 cattle just by standing at the gate when they are driven home in the evening can observe them and say immediately whether any are missing. But he can also tell which of the animals is missing which proves that he is not in fact counting, but exercising a tremendous memory, because if one counts a herd of cattle one ignores their individual characteristics. Be it black or mottled a cow just counts as one; and in studying and noticing the characteristics of 400—500 head of cattle to such an extent that he knows which of the animals is missing, the owner is carrying out the opposite process to counting. He has noted the special markings of all the animals, whereas in normal counting no characteristics are taken into account. A cow counts as one, whether it is black, brown or mottled."

This degree of trained memory, however, was uncommon, limited to a few. But since Man has for everyday purposes needed to be able to count and keep an account of things in a simple form, his urge has finally developed into a need to find a substitute for his memory act.

To set up an abstract method of counting, without a foundation of development from which to work, would have been impossible. The most logical and obvious conclusion is that the first aid in this direction that Man used was the hand with its five fingers. There are many traces in several languages of this. For example, in the Malayan tongue the word for five and hand is the same.

As five is not a very large unit for use in counting and as it was probably awkward with small numbers to count, for example, one hand $+ 2 = 7$ and so on, the other hand was no doubt brought into use. Words were found for the numbers from 1 to 10 and eventually the toes were incorporated. Thus Man developed a system called $4 \times 5 = 20$, or from 1 to 20.

It then became possible, with a combination of memory and finger-counting, for Man to tot up even large numbers. A primitive tribe in South Africa, in which few if any of their members can count to more than 10, are reported able to check over a herd of 1000 cattle when three men are put on the job.

They do it in this way: The cattle are herded into an enclosure and driven by a helper one by one through a gateway near which the three men have taken up their stance. For each animal that passes through the gate the first man folds 1 finger, starting with the little finger of his left hand and finishing with his right small finger. When 10 animals have gone through he shouts to the second man who also uses his left little finger to record the event, continuing with his other fingers each time he gets a shout from the first man. In this way he folds 1 finger for every 10 head of cattle. Once he has folded all his fingers on the orders of the first man, he calls to the third man who proceeds to fold 1 finger, just as the others did. Each of this man's fingers represents 100 animals.

When the third man has closed both fists, the team have reached 1000—without in fact knowing the number as such and without really having used any form of abstract counting. This is a splendid illustration of how primitive people, even today, use the finger method for counting.

The Aztecs, the inhabitants of ancient Mexico, had the same word for 20 as they used for the human body, which would signify that their forefathers had carried counting from the hands to the feet, originating a 20-figure system.

They transferred this system to their calendar which runs as follow:

1 day = 1 kin
1 month = 1 uinals = 20 kin or days
1 year = 1 tun = 360 days (18 × 20)
20 years = 1 katun = 7200 days
20 × 20 years = 1 cycle = 144,000 days
20 × 20 × 20 years = 1 great cycle = 2,880,000 days.

The Aztec system had apparently been so far developed that they were able to operate with these massive numbers, and they must also have possessed a form of numerical writing to be able to express these quantities.

The Danish language has also clear traces of the former use of a vigesimal system. Not only in the expressions "snes" (score) and "½ snes" (half a score) which of course mean 20 and 10 respectively, but also in the strictly numerical forms which have been evolved and used in the present decimal system, namely:

Halvtredsindstyve = 50 = 2½ times 20
Tresindstyve = 60 = 3 times 20
Halvfjerdsindstyve = 70 = 3½ times 20
Firsindstyve = 80 = 4 times 20

The ordinary numbers for these values are used in everyday Danish as part of the decimal system, but the make-up of the words themselves is perhaps a sign that they were once used in a 20-figure system.

The fact that a 20-figure system was born in South America under the Aztecs and later in Europe does not necessarily indicate contact between the continents, but merely suggests that the most logical way to produce a system of fingers was to begin with the hands and create a 10-figure or decimal system. Some societies would stick to and develop this system, while others would include the feet and produce a 20-figure system which they would then expand.

Other combinations of numbers are of course feasible but they would be short-lived since they are difficult to develop and operate, and the great majority of people would probably take the easy and quick way.

As Denmark has always been subject to strong religious and evolutionary currents from the south, it is hard to prove whether we ourselves created our 20-figure system. But if we look to the south we find that France has also in her numerals relics of a 20-figure system still in use among decimals:

Quatre vingt = 80 = 4 times 20
Quatre vingt dix = 90 = 4 × 20 + 10

If we continue the search for proof—apart from traces in our languages—to support the theory that the hand has been the basis of the origin of numerals, I think we could turn our attention to one or two rock carvings found in Denmark.

When Man carves signs and shapes in stone it can be for one of several reasons. But if we suppose that the human mind of those ancient days resembled our own, we can consider why we write today:

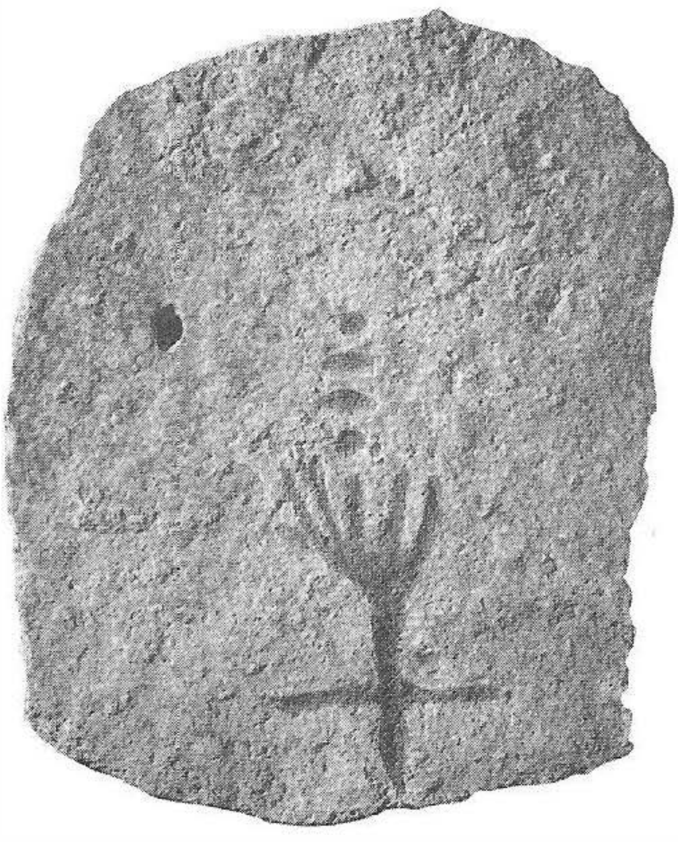


Fig. 5.

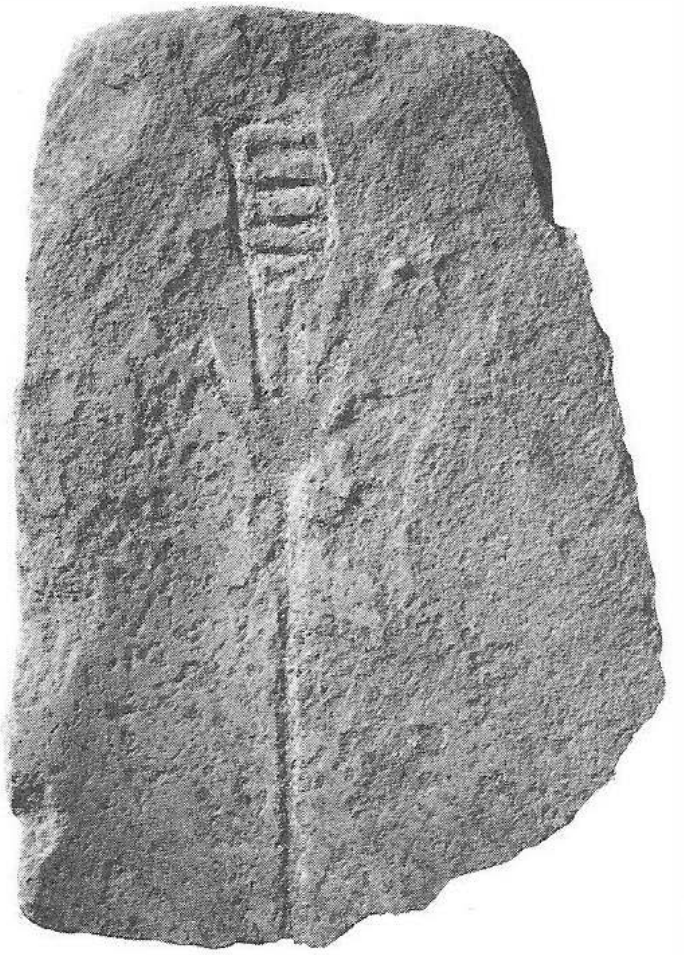


Fig. 6.

1) either to record experiences for the benefit of our successors.
2) or to establish certain technical or speculative lines, of importance to the community.
3) or out of the joy of creating.

If ancient dwellers were to choose one of the three alternatives for indicating that they used a 20-figure system, it would have to be the second. How could they express this in picture form, since it may be assumed that although they were aware of the concept of counting, they had no means of expressing numbers in writing? The most obvious supposition would be that they drew four hands, or one hand and marked how many times it had to be counted.

This is precisely what has been found on several Danish rock engravings, and they are known under the term "Hand sign".

These engravings show the forearm as a deep furrow in the stone. The palm of the hand is not detailed whereas the five fingers are shown in an extended position. Four horizontal strokes can be seen above the fingers.

If we ourselves knew how to count but had no knowledge of numerals as script, and wanted to explain the basis of the finger system to other people, we would be in exactly the same predicament as those ancient pedagogues.

Their solution to the problem, to indicate four times one hand, is excellent and with the means at their disposal is unlikely to be bettered. They were able in

Fig. 7.

Fig. 8.

this manner to present a popular picture to their pupils of what the system was about, and how they could remember their lesson.

The above-mentioned rock engravings can be seen in *Figs. 5, 6, 7 and 8* and were found at the following places in Denmark (see *Helleristninger, Danmarks Oldtid,* page 133):

a. Torup Mensalgård Farm in North Funen
b. Kyndby-Horns Herred area of north-west Zealand
c. Lille Havelse in north Zealand
d. Rungstedlund (Danske oldtidsminder, P. V. Glob).

Research workers and archaeologists who have studied these rock engravings have described them as a sacred symbol or ritual. This assumption was based mainly on the fact that one of the stones was discovered atop a burial site and another found near a second burial site. It was therefore thought that these signs had been of a protective nature, guarding the dead.

Of course, this assumption may well be correct but it would perhaps be more logical to suppose that the man whose remains lay under the stone was the master who taught that particular society the intricate and mystic arts of recognising the year and its seasons, and knowing something of numbers and counting.

On the death of this venerable master he is extended the final honour by having the carved stone placed on his grave. It has possibly but not necessarily been engraved by his own hand.

If it had been a common form of protection, is it not likely to have been found more frequently and in many other places? As I see it, the theory is weak on one important point, namely it is not possible for it to conform in any way with the religious systems of those days.

The numerical interpretation has the advantage of providing a logical explanation of the actual sign, and a possible explanation for the presence of the stone in connection with at least two burial places.

Two of the stones were—as far as can be ascertained by a thorough search of the surrounding area—independent of any form of burial place. At any rate an examination of a wide radius has failed to produce any.

We are now progressing in our theory of development to the stage where our observer has now added a primitive system of counting to his store of knowledge.

He can count to 10 or 20 but has no written means of expressing himself apart from the hand sign, which can express quantities.

So far he knows nothing of the year and the number of days which comprise it. He has a general picture of the year through his Sun calendar and realises there is a continuity of year following year. He is also aware that the long period of time which we call a year is made up of so many days but the number is so great that it is beyond his power to count them.

On the other hand he has taken an interest in the Sun's successor, the Moon. He has noticed its changing appearance from a large, round, light disc which gradually decreases in size until finally all that remains is a small sickle shape, and just as it is on the point of disappearing completely it begins swelling again until it shines in the sky once more in its full, original glory.

He has realised that this is an interval and period of time that he can grasp. And this is a subject which his counting ability can master.

He now begins an experiment to check his guess. He takes a bowl and a pile of pebbles and starts his test one night of a full moon. Every evening he puts a pebble in the bowl and continues throughout the Moon's period of transition, until it reaches full moon again.

Counting the stones in the bowl he finds there are $2 \times 10 + 8$, which is 28. Since days and nights follow each other he can count the days by simply counting the number of nights, and now establishes that a moon-period has 28 days and nights. A finding which is fairly accurate as a moon-period has in fact 27.54 days and nights.

This is his first division of time into numerical sections, and he adopts it as a factor of graduation, i.e. 1 moon or moon-year.

This hypothesis naturally depends for support on theory evolved from the possibilities of that time, but the word "moon" indicating a period of time appears in various languages, lending strength to the theory laid down above.

English Moon:
1 month = 1 "moon"
Danish Måne:
1 måned = 1 "moon"
German Mond:
1 monard = 1 "moon"
Icelandic Mani:
1 manudur = 1 "moon"

When we examine just what the period 1 month in the various languages covers today, the fact registers that the term does not denote a specific length of time since one month may have 30 days, another 31 days and of course February changes its 28 days to 29 every fourth year.

This is because later astronomical observations have split the year up into twelve months instead of the original thirteen, each month being allotted an individually tailored length to make it fit into the chronological year.

The months or "moons" no longer coincide with the Moon's phases, but the name has remained.

Several months = several moons.

The likeness is remarkable.

That the possibility exists that this division into "moons" or moon-years still applies in certain parts of the world is indicated by the following story.

I can recall seeing a television documentary on some aspect of Africa during which an interpreter conversed with the local native population.

The interpreter asked one native mother, who stood with a child of about three years in her arms, a number of different questions which received extremely intelligent replies.

Finally he asked how old the child was, and got the surprising response that the child was 29 years old. The interpreter's question was translated before it was asked, and the answer was translated immediately it was given. At this reply the interpreter laughed and gave the answer, saying the mother of course must have misunderstood his question.

I do not think she did. For if that community still counts time in moons or moon-years, 29 moon-years would fit perfectly, and any mother would be likely to understand a simple query as to her child's age. It should also be borne in mind that there was little chance of the mother having taken the question to be a query on her own age. In the first place, the interpreter was making a show of the youngster and even without dialogue one could understand that he referred to the child. Secondly it would have been unlikely that the stated number of years, as we understand the term, was the mother's age as she was certainly no more than 18.

If we look in the Old Testament, in the book of Genesis, we find many of Abram's forebears named and their ages given. But the remarkable thing is that most of them reached the age of 700 years or more, which presents a baffling picture.

On the other hand if we regard these ages as given in moon-years or moons, we get a more credible assessment of their ages which, by our reckoning, would be around 50 or 60 of our years. And the very old who reached the age of 900 years would in fact be 70 which is a more likely age, knowing what we do of the living conditions of these people.

Our observer thus has a moon of 28 days, and the figure 28 takes a prominent place in the mind—as will be proved later. This number has been supplied by the Great Unknown, the mystical universe, and will play a key role in subsequent development as a symbolic source.

It is impossible to establish how long it took for our observer to apply his knowledge of the lunar year to the solar or sun-year, but he eventually got around to it.

His knowledge of numbers required to be no more profound than when he discovered the moon-year, since he can work on the same principle as the cattle counters we read of earlier, using instead of men two bowls and a pile of pebbles. As before, he counts his pebbles in the bowl which contains a stone for each night, and probably starts his long count at the first full moon following either the Sun's highest point in the sky or its lowest, i.e. at the summer solstice or winter solstice.

When he has put 28 pebbles in bowl number 1 the first moon-period has passed, and he empties this bowl—placing one pebble in bowl number 2. He continues each night counting into the first bowl and every time a moon-period ends he repeats the above process.

When the solar year is at an end and the Sun is back at the summer or winter solstice, he finds he has 13 pebbles in bowl number 2, telling him there are 13 moon-years in one sun-year.

This discovery did not necessarily involve any change in the system of counting time if a community or tribe had become accustomed to dividing time into moon-years, but it is most probable that some people deserted the accepted mode of counting for the new, larger unit of measurement. Others retained both and used the moon-year for indicating short periods and the sun-year for longer periods—pretty much as we today use months and years.

Let us look at the arrangement, mentioned earlier, used by the Aztecs in counting years and time. They had both a very small and a very large unit of measurement:

1 day = 1 kin
1 month = 1 uinals = 20 kin
1 year = 1 tun = 18 uinals
20 years = 1 katun = 20 tuns
400 years = 1 cycle = 20 katuns
8000 years = 1 great cycle = 20 cycles.

The continuity of development that we are studying cannot be attributed to any particular time or place, since Man's development did not simply start in different places at the same time.

Primitive knowledge and highly refined culture lived side by side separated by very short distances in terms of present-day transport. It is worthy of mention in this connection that when Greece 2,500 years ago was reaping benefit of centuries of progress, when mathematics in its speculative form was a subject of discussion by the man in the street, when theatres and plays were not reserved for the upper classes but were patronised by all, and when teachers of philosophy and logic set up schools all over the country, in short when Greece was enjoying an era of culture which was in many respects equal to that which we boast today if we ignore the material things which technology has produced, Northern Europe was at that stage in its development only approaching the close of the Bronze Age.

Stones appear a natural choice of material for counting purposes and were used as such by our ancient people. Like the fingers they were readily available and could be gathered and used quite easily. Different coloured pebbles might also be allotted different values, and as Man as far as possible took up his abode near the sea or a waterway there was probably a constant supply of counting material on hand.

Language also provides some evidence in this direction.

To count in Greek is "psæfizein", probably derived from psæfos (a pebble), and the Latin "calculare" also to count, came from calculus meaning a pebble. The Latin word has been accepted into other European tongues.

English: Calculate
Danish: Kalkulere
Italian: Calcolve.

The stages we have examined so far in Man's development may have taken place in different places all over the world, among different peoples, and at different times, but every time and place had these things in common: a primitive form of counting, the Sun, the Moon and a small collection of pebbles.

By dint of his observations Man raised himself from being merely a two-legged

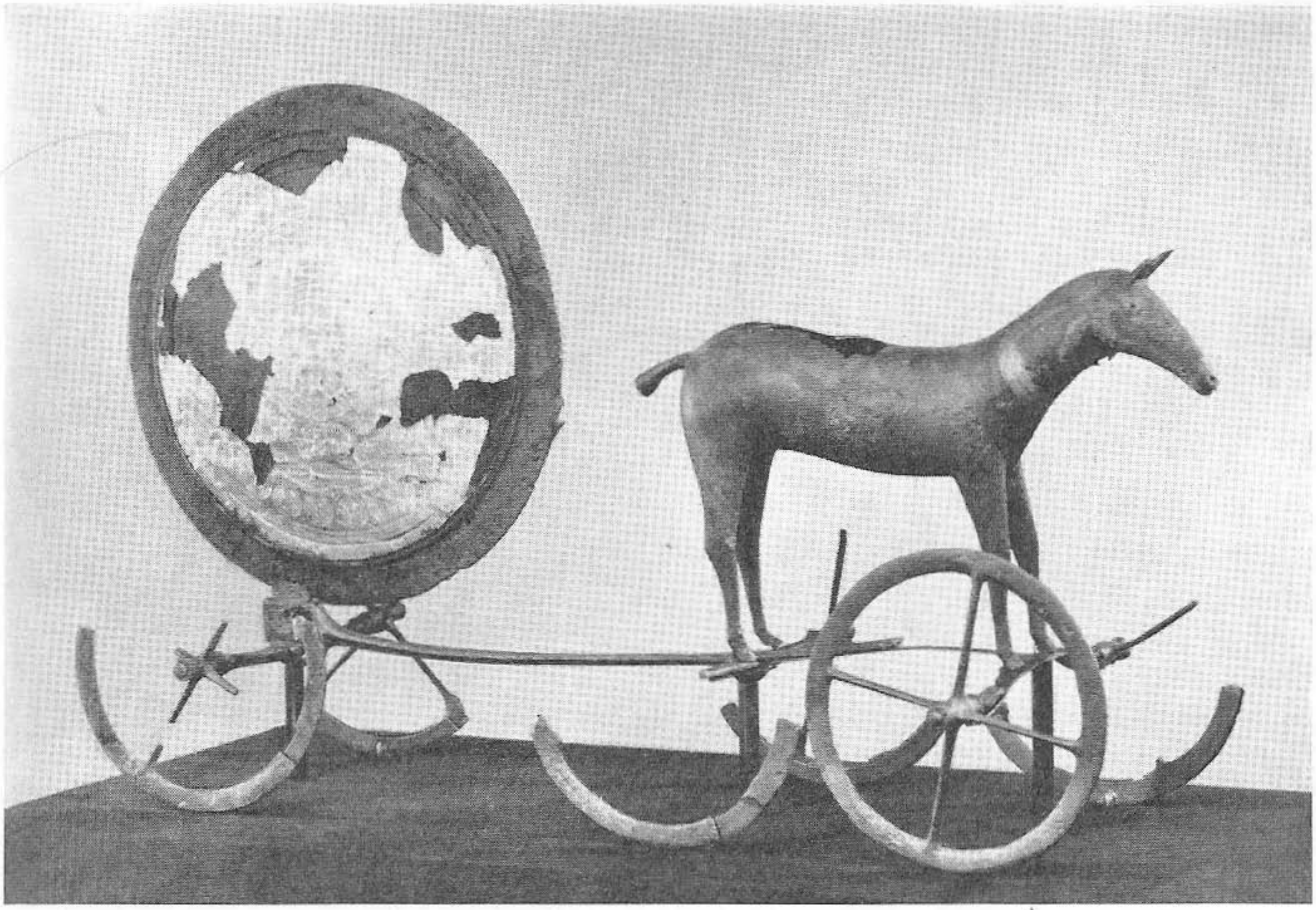


Fig. 9.

animal to becoming a thinking being, who started a chain reaction which has so far ended with our civilisation.

At about the same time as these observations Man began his first attempts at depicting the round disc of the Sun or the Moon, the first writing materials probably being a stick and wet sand or earth. Only after some practice with these materials has he begun inscribing on more permanent materials such as stone or clay tablets.

My assumption that the first thing Man tried to depict was the round disc of the Sun is based on the fact that to these early people the Sun must have held a vital importance, and had become a fundamental part of their world of thought and imagination. When Man's thoughts began toying with the idea of "creation" they must have centred on the Sun as the creative force.

There is evidence, too, in most parts of the world that Sun-worship was the forerunner of more complicated systems of religion, and Denmark has also traces of Sun-worship, e.g. the Sun Chariot recovered from Trundholm Marshes at Odsherred.

It is highly probable that this beautiful item was made in Denmark. But it is also possible that it was brought into the country from some other region, since only one sample has ever been found. But in any event there can be little doubt that it was of importance in the rituals of Sun-worship or idolatry.

The chariot would seem to symbolise the movement of the Sun across the sky, since both the Sun-disc and horse are

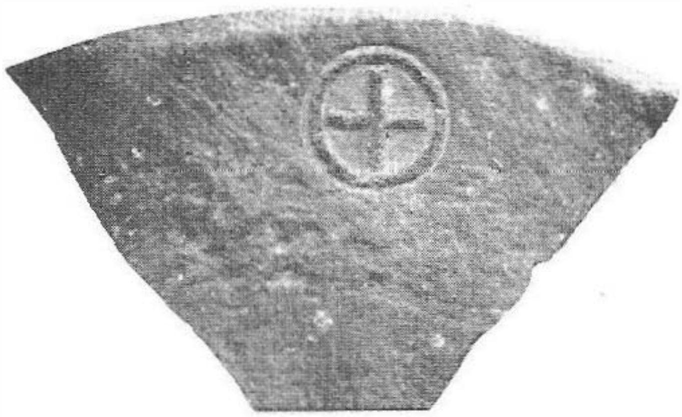


Fig. 10.

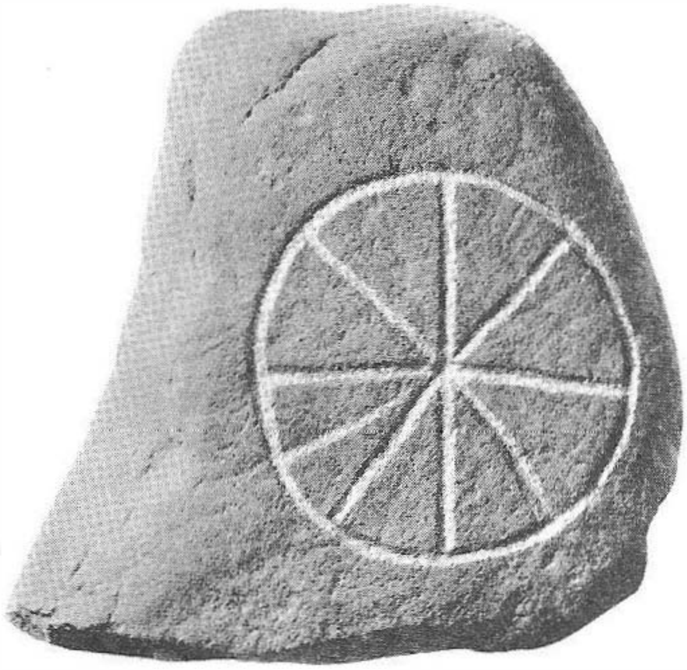


Fig. 11.

poised above the actual chariot. These two must be regarded as a separate unit, independent of the chariot, for together they represent the Sun and movement. The fact that they are mounted on a chariot for transportation purposes has actually nothing to do with the symbol.

The round disc of the Sun is golden on one side and black on the other, and there are no signs that the gold has been eaten away. Thus the disc would symbolise night and day, the golden side being visible when the Sun begins its journey in the morning to the east, lighting and glinting all day; but when the Sun turns in the west and runs all the way back across the sky it turns its dark side towards the Earth.

If these theories have a ring of truth the assumption is that the western and eastern horizons had been the extreme bounds for everything, including the Sun. And when it reached the horizon it had to turn about and gallop back across the heavens the same way it had come, and would thus display its front and back in its journeys to and fro.

Martin A. Hansen in his book *Orm og Tyr* describes how circles and circles with crosses drawn inside them often appear in Danish rock engravings, and it is his opinion that this symbolic figure has been introduce from the East where the symbol is very widely known.

This may be correct because cultural societies existed in the East and were much more advanced than the European at this stage, when the symbolic circle first appeared in Denmark; and tribes wondered constantly from country to country bringing in their wake fresh thought. It is, however, just as probable that these symbols developed from observations made by people themselves in the North.

In *Fig. 10* we have a fragment of stone with a circle and cross engraved upon it. The stone is in Ringkøbing Museum and was found near Fjand, where the Nissum fjord had in early times provided excellent conditions for the growth of a community.

And in *Fig. 11* we find a stone with the Sun symbol. It was discovered at Rygård (*Danmarks Oldtid,* p. 136). Both of these stones show the Sun's symbol, one having only a simple cross drawn inside, the other in addition having a diagonal cross.

It is generally agreed that Sun-worship was the leading form of early religions in most parts of the world.

To people who lived in direct contact with Nature, the Sun would obviously be one of the things that stood out above all else. It is the source of light and a primary factor in our universe, and Man's pleasure and comfort have greatly depended on the passage of this ball of fire across the sky. As a rule, inconvenience resulted when the Sun was not in its heaven; there were periods of rain and cold. And it was logical for primitive people, early in their development, to regard this unknown factor as the ruling power in their lives—and instinctively they sent their prayers to the Sun when they were in trouble.

The fact that we today must admit that without the warming rays from the Sun no organic life can exist on Earth serves only to emphasise that this discovery or assumption would be one of the first that primitive minds, living close to Nature, would form.

If the Sun was regarded as sacred, its picture would consequently be looked upon in the same way; so would anything deriving from the Sun's movements, whether it was its daily passage across the sky or its slower swing from north to south and back. The circle was therefore seen as a sacred sign, and anything deriving from it in Man's subsequent meditation would also be sacred.

I make this point at the present stage as one of the primary causes of our ignorance of existing knowledge of numbers and geometry in the period prior to the age of Greek supremacy roughly 2,500 years before our time. This base ignorance is rooted in the fact that everything associated with numbers in their more abstract philosphical form and with knowledge generally rested behind a curtain of secrecy within those circles engaged in the worship of gods, namely the Temples.

The principle of secrecy is infinitely ancient and was possibly established—as suggested in Chapter One—strictly for reasons of protection as the first thinkers were obliged to keep their knowledge to themselves. If they gave it to everyone intelligent enough to appreciate it, this knowledge could no longer be used to their own ends.

In this day and age we operate a patent system to protect any new invention which may have a commercial possibility. The patent ensures financial profit for its holder. The only form of protection in those ancient times was secrecy.

Secrecy was extended to the temples and was so powerful that a great deal of the real knowledge was not even written down; it was passed on verbally from one initiated brother to another bound by an unbreakable oath of silence. As a form of reminder, many things were incorporated in geometric figures and other symbolic ciphers, but this development belongs to a much later epoch than that with which we are dealing at present.

CHAPTER THREE

Appearance of figure 7 as a symbolic factor

A RÉSUMÉ of Man's experiences in the world of numbers recalls that the ancient observer has some idea of how the year is made up through his stone calendar and observations of the Sun and Moon. He has evolved a primitive system of counting from 1 to 20, and can count so-and-so many times 20 since he has made 20 a major unit. He has not so far discovered any means of writing numerals and has to content himself with remembering their different values, but he has begun drawing circles on the ground—probably the forerunner of script.

For the moment his knowledge extends no further than this, and it is from this point that we must progress.

If, over a length of time, he has been drawing circles of varying sizes on the ground with a stick——from small circles he can sketch from the sitting position to large circles he marks off by pacing round the perimeter and perhaps laying stones along the edge to retain the shape——he will one day become curious about measuring their sizes. He will want to make two circles of the same size and will notice in fact that he cannot. He has also noticed that his circles, compared with the shape of the Sun and Moon, are most unsatisfactory, leaning to one side, and oval.

He has probably realised that he can draw a circle fairly uniformly by standing in one spot with a stick and turning all the way round, scraping the point of the stick on the ground at a constant distance from his body.

This realisation makes him reflect that a circle has a central point, from which he can measure out a circle if he wishes to make one bigger than that he can reach with his stick.

He wants to measure out a line, and thus a need arises.

It is my submission that the process of measuring grew for strictly speculative reasons at a time when there was no practical need for measures.

They had no need, for example, to know how far it was to the next community. Selection of a particular size of hut or living area was made according to the size of the family, and was probably done without any form of measuring. Boats—whether made from a hollow tree-trunk or a pile of rushes—were built according to requirement and the length or quantity of materials. And clothes were made to fit a particular body, whether they were styled in skins or other materials.

But the calendar-man or witch-doctor, the community's budding source of intelligence and the founder of god-worship, came across the problem of measurement in his experiments with circles and had the time at his disposal in which to solve it.

What would he do, not having any

standard of measure to guide him? Probably the same as any practising engineer or surveyor would to today when he finds he has to measure out a short distance but has forgotten his measuring tape or ruler. He simply paces the distance out, and arrives at a certain number of steps.

The surveyor knows from experience, however, that the length of his step is 80, 85 or 100 cm, if it is the metric system he uses. But no matter what system of measurement he has, it is essential for him to convert his paces to the standard of measure used in his particular society for he would otherwise be unable to assess the length of the piece he had just measured. To say that a piece of ground is, for example, 25 paces long tells him nothing about the actual length if he does not know how long his paces are, and without converting the paces to a standard measure he can never compare his measurements with those of anyone else.

If something is measured in this manner it obviously cannot be compared or considered with other measurements without first conversion to a standard unit of measure. It is important to know whether one man has a pace of 70 cm and another man of 100 cm. If they each begin measuring a distance from their respect ends and, when they meet, add their two totals of paces together, the result cannot possibly give a correct answer. But if one man is given the job of dividing a plot of land into, for example, four equal pieces, it can be done using his pace alone.

Since our primitive observer had no need to compare his results with others', but used them purely for his own ends, it is likely that his large unit of measure was the pace.

When he needed to measure something shorter (perhaps a distance rather less than one step) he probably used his foot as a unit of measure. It is always readily available and even today is used on the building site if a man has to compare the lengths of two pieces of work, and a measuring tape or rule is not handy.

There were of course many ways of fixing an individual standard of measure. For example, the arms. But what an uncomfortable position it must have been with one's arms spread out, nose on the ground, crawling fathom after fathom.

A study of the literature dealing with weights and measures reveals that the unit of measure in nearly every country through the Middle Ages was: pace (yard), foot and inch, until in many places this system was replaced by the more rational metric system.

But such a conversion took a tremendously long time, and even countries which switched to the metric system years and years ago still have reminders of the old units, which stubbornly resist dismissal.

Buying timber, for example, in the Scandinavian countries is done in alen (a unit of two feet), feet and inches in spite of the fact that it is more than fifty years since the old norms of measure were officially and finally discarded in favour of the newer metric system.

Most English-speaking countries retain the old units of measure, and even these differ from their Nordic counterparts of the same name. One British foot has 12 inches, just as the Nordic, but compared to the meter the British foot is 0.3048 in length, while the Danish foot is 0.3138 of a meter in length.

Thus when a Dane imports a length of timber it is not enough to know that it measures so many feet, he also has to know which foot is being used as the standard of measure.

This whole problem of maintaining a certain standard of measure has been one which provided our ancestors with many a headache, and even within a relatively confined area there have been several different lengths of the same name.

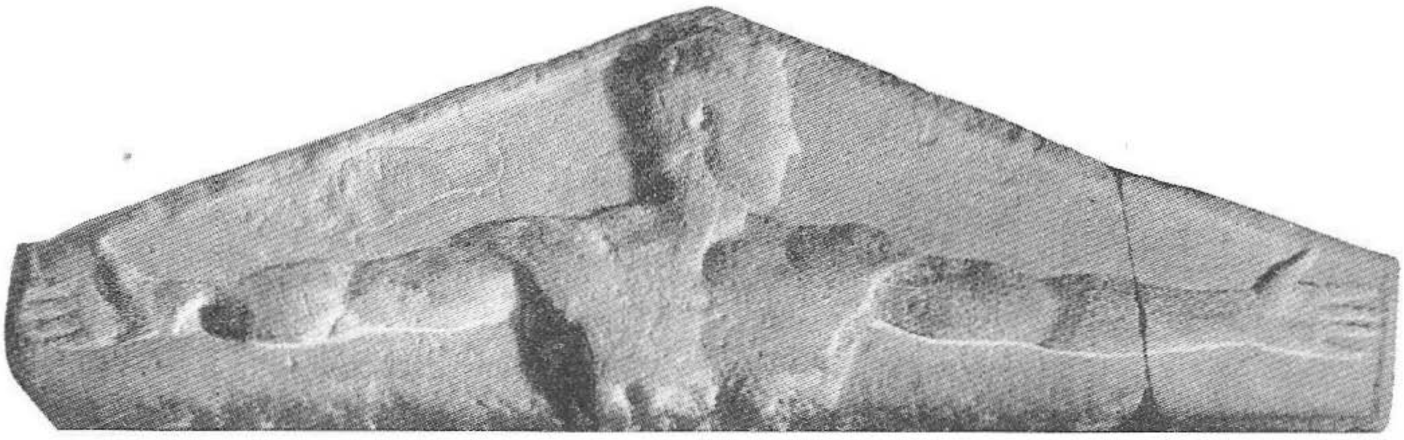


Fig. 12.

Danish scientist Ole Rømer tried in 1687 to have this muddle sorted out in Denmark, where there were almost as many standards of measure as there were tradesmen.

By royal decree the standard length of 1 alen was laid down and samples were made in iron to be nailed to church doors all over Denmark so that anyone could come and check the length of their particular version.

But the day the standard measure disintegrated in rust or was removed, chaos prevailed once more. A closer look at this problem will reveal that maintaining a certain standard measure is something nearly every country has tried to do through their histories.

We see in ***Fig. 12*** a relief which is housed in the Ashmolean Museum of Art and Architecture in Oxford.

It illustrates how the Greeks got around the problem in 450 B.C. It shows the chest, arms and head of a man—with the arms fully outstretched.

According to the accompanying description of the relief the arm-span is 1 fathom. Above the right arm is the imprint of a foot, and if we measure carefully we find that this foot divides exactly 7 times into the total arm-span.

So the Greeks portray here both the small unit of 1 foot, and the larger unit of 1 arm-span or fathom which comprised 7 feet.

A stone, dating from about 1000 B.C., has been discovered in Denmark illustrating, I believe, that problems of unit measure existed at an early stage in growing society.

By Greek standards Denmark was very much an undeveloped country 1000 years before Christ, and yet indications are that an effort was even then made to fix a standard unit of measure.

The stone is shown in ***Fig. 13*** and plainly carries the mark of two foot-prints.

In the same way as with the hand signs (discussed earlier) it is maintained that this stone had some mystical connection with worship of the gods. But my theory is rather that the stone represents one of the earliest standard units of measure for the world's first measuring norm, the foot.

The stone was found in the north of Jutland near the Lim Fjord, and is now in the National Museum in Copenhagen.

The standard of measure favoured by our observer, however, is immaterial to our further study since all measures are made in relation to each other. Drawing a circle with a radius of one foot will produce a diameter of 2 feet, whether the radius is measured in British or in Danish feet.

This in turn means that irrespective

Fig. 13.

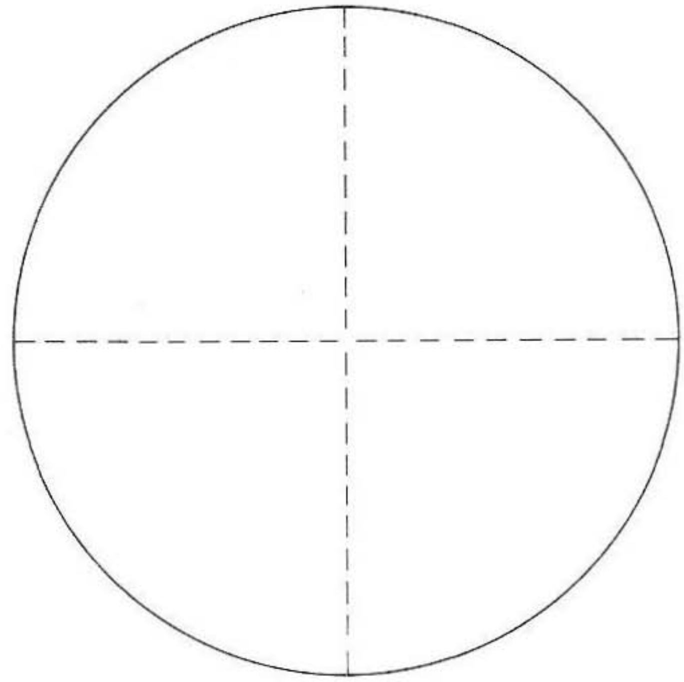


Fig. 14.

of the standard of measurement decided upon by the various communities, the relationships of the individual dimensions would be the same, as indeed would be the results achieved.

Having progressed to the apparently simple process of measuring and possessing a standard of measurement, our observer again turns his attention to his circle and now measures from the centre out towards the point at which he wants the circumference to run, and he can immediately see the benefit of his constructions in the way of an improved shape—since he probably marked out from his starting point a specific number of paces in four directions and on this basis constructed his circle, *Fig. 14.*

In this manner he has drawn a cross inside his circle at a very early stage. We had evidence in the illustrations of the previous chapter of how the cross appears at an extremely early date on the stone at Ringkøbing Museum and on the stone from Rygård—indeed almost all rock engravings on which the Sun appears either independently or as part of a Sun-ship show a cross inside the circle.

As we have seen, the circle was regarded as a sacred symbol and, since everything it produced or was associated with was also regarded as having sacred connections, the cross had been accepted as such at a point somewhere near the beginning of Man's religious speculation.

The story is told, too, of one of the oldest cultural races in South America, the Incas—who succeeded the ancient Mayas. When the Spaniard, Captain Francisco Pizarro, and his vandal followers invaded Inca territory in 1532 these rather remarkable events occurred:

When the invaders, using cunning and treachery, had captured the ruler of the kingdom, Atahualpa, and killed his bodyguard, their insatiable greed for treasure brought them to the temples where they proceeded to strip the gold and silver from the building.

Captain Pizarro was—believe it or not—a God-fearing man and was accompanied on his path of plunder by a priest named Valverde. Crucifix in hand, Valverde followed on behind, blessing every attack and misdeed in the name of God.

The supreme symbol for his own religion was the crucifix, and when he discovered that this cross was also the supreme symbol of the Incas' heathen Sun-worship he could scarcely contain his horror. Thinking the appearance of his sacred crucifix in a heathen temple to be some form of witchcraft, he ordered that all written matter and everything connected with Temple ritual should be burned. The Spanish vandals did their work well, and temple upon temple, town after town were denuded of all such material which was destroyed in huge bonfires. Masses of priceless cultural treasure were in this way irredeemably lost to future generations.

Since the Inca religion must be presumed to be infinitely older than Catholicism, this is merely one example of two entirely different societies with no apparent mutual contact arriving at the same sacred symbol: the cross, or the circle with cross inscribed.

We can trace the practice of Sun-worship to almost every part of the world, from the ancient civilisations of South America to the religions of Egypt before direct worship of the Sun was tempered by the mystic laud of Osiris and Isis, from the Druids to China and Japan—where even today the country's ruler is termed Son of Heaven.

Sun-worship would obviously entail portrayal of the Sun, i.e. a circle, and the symbolism arising from this figure would almost invariably require a cross as an integral part of its structure, since further development of the circle as a subject of speculation must needs have a cross drawn in to permit any progress whatsoever in this direction.

As the cross emerges therefore as the first key to ancient geometry and the secret of numerals, it is not really surprising that it appears at an early stage in many areas of the ancient world as a factor within the web of symbolism.

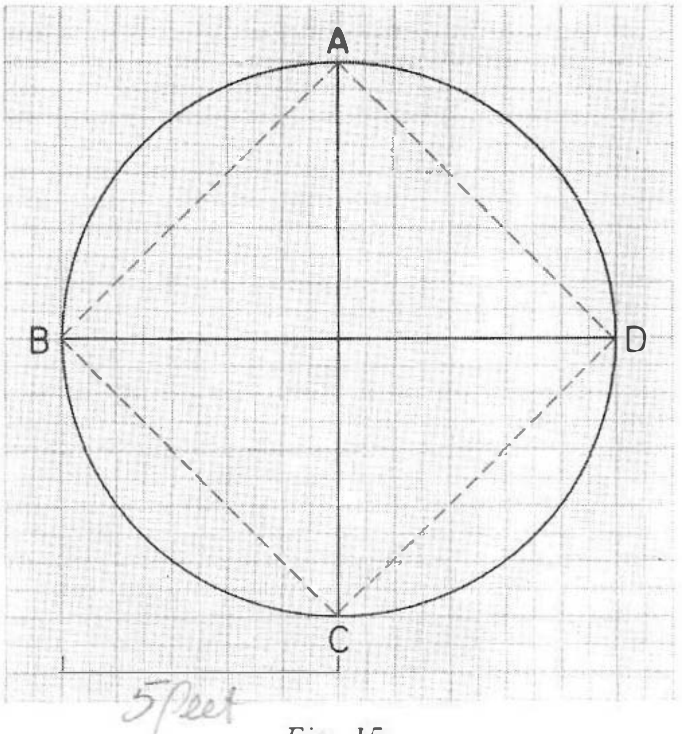


Fig. 15.

◇ = 28 feet

But we must push on with the further development of the circle. We find our observer looking for a break-through which might bring him one step further along the road of knowledge.

Employing his recently evolved system of measurement, he has produced lots of circles of every conceivable size, and has probably tried measuring various points within the circle without obtaining much satisfaction apart from information about some lines.

Then comes the day he makes a circle with a radius of 5 feet.

Perhaps he selected this dimension because he himself stands at the centre and counts out 1 foot for each finger, first to the right and left, then backwards and forwards—producing a circle with a diameter of 10 feet.

He draws in his cross in the circle and sits back to consider his work. With his established ability for measuring, he first of all measures the circumference and arrives at a figure between 31 and 32 feet which in itself is of no immediate interest, apart from the fact that it is the inspiration for his next measurement—which is to mark off the distance from one cross-tip to another all the way round the circle. He measures as we see in *Fig. 15* from A to B, from B to C, from C to D and from D back to A. This provides him with a total measurement of 28 feet, or the same number of days he has in his moon-year.

We can just imagine his surprise at coming across a dimension which—for the first time—fits in with other factors in his knowledge, with something other than the subject in which he is presently engaged. Surprise, too, at the fact that drawing his sacred sun-disc in a certain proportion produces a number which is the same as the days in his moon-year. This discovery of course is a tremendous triumph and goes a long way to emphasising the magical properties of the circle.

*

Of course we know from a present-day mathematical viewpoint that this measurement was not accurate, as lines AB, BC, CD and DA are all hypotenuses in right-angled isosceles triangles.

Since the perpendiculars are equal, line AB for example will be $a^2 + b^2 = c^2 = 25 + 25 = 50 = C^2$

$$C^2 = \sqrt{50} = 7.07$$

or, more easily, $5 \times \sqrt{2}$.

But naturally our observer is unable to discover an error in measurement of 1 % with the materials he uses, and even if he was capable of measuring to this degree of accuracy he would not be able to appreciate or express it.

His drawing would have one or two other drawbacks. He has not yet encountered the term right-angled, in fact he has not seen the significance of an angle as an independent unit, and therefore it is possible that his cross is not completely accurate, as is probably the case with his measurement of the four radial arms.

We may therefore assume that if his procedure has been as described above, the measurement must seem to him absolutely sound. A greater degree of accuracy is at the moment outwith his conception.

Apart from his noteworthy decision to measure the distance from tip to tip on the cross, he is able by filling in the lines which he measured out to create a separate construction, namely a square.

My reason for calling a simple square a construction is purely that I consider it must be regarded as such. No naturally occurring figure can provide him with this inspiration. Everywhere he turns he sees only curved, rounded lines. Stones on the beach are round, the trees, bushes, leaves, berries, bones; everything in his world has curved lines. Thus the square must be termed a construction, and this would probably be the most natural way to achieve it.

This new phenomenon—the square—will of course give rise to a mushroom of speculation, but let us first take a look at the conclusion reached in the observation of the 28 feet.

Whereas our observer previously saw in his symbol of the circle and cross a picture of the Supreme, a primitive picture created by his own hand, he now faces a practical factor: his moon-year is symbolised by the figure of 28 feet and is divided in an obvious manner into four parts by the four lines. Thus each fourth part of the moon-year contains seven days—and acting upon this sign from the Almighty Power, he introduces another division between the small unit of one day and the larger unit of one moon-year, namely the week with its seven days.

*

And on the seventh day God ended his work which he had made: and he rested on the seventh day from all his work which he had made. And God blessed the seventh day and sanctified it: because that in it he had rested from all his work which God created and made.

This is what the Book of Genesis, chapter 2, has to say of the Creation, and at this stage the month was already divided into seven days. But it would actually be difficult to prove this assumption. We must be satisfied with the hard fact that the splitting up of the month into weeks occurred so far beyond our reckoning of time that—like so many other things that happened in antiquity—we can only hazard a guess at its origin.

If we study the earlier observations taking first the Sun as our subject, we discover an indication of the sun-year stretching over a long period.

Observing the Moon's phases we succeed in splitting this long period into 13 shorter intervals, i.e. the more readily appreciated moon-year (month), and finally this moon-year is divided up into 28 days.

The application of these units of measure as a basis for meditation upon time itself seems natural and obvious. The year is measured as 364 days, i.e. 28 × 13, surprisingly accurate when one considers the stage and development of Man's knowledge at this point, and it is true of all three of these divisions that they are laid down by Nature, simply observed by Man in the course of development.

The year is indicated and illustrated by the path of the Sun across the sky, the moon-year is similarly set down by the waxing and waning of the Moon, and the day is shown by the regular change from night to day.

The observer has thus measured each of these factors in relation to each other, gradually as his knowledge permitted. And he has been able to note his discoveries without himself having to make any adjustments. The material was there for the picking, so to speak. He simply had to observe.

But the week—the division of the month into four parts—is presumably Man's invention. Our universe possesses nothing, no division to indicate a reason for so breaking up the month. There is nothing in the movement of the Sun or the Moon, and yet the Sun and Moon were probably the source of inspiration. The Sun symbol, as the sacred sign, with an inscribed cross probably provided this inspiration, since it divided the Moon's period naturally into four equal parts.

Our observer has discovered that the lines joining the cross-tips measure seven feet when the circle diameter is 10 feet. He has worked out that the four lines make a total of 28, consisting of four parts each seven feet, 4 × 7. Whether Sunday began as a sacred day is probably doubtful, but it could well be imagined that our observer (who had possibly won an influential position for himself within his society) encouraged people to implore the Sun to assist in the respective affairs of the community, and as a moon-period seemed too long, he split it by means of the cross into four intervals according to which the Sun is honoured every seventh day, i.e. every time the days of the week rounded another spoke on his original symbol, the circle and the cross.

Initially it had probably been a morning ritual, but the populace had been quick to grab the opportunity for organised rest to break up the week's toil, and this day of rest had soon caught on.

The introduction of this day of rest really signals the start of religious contemplation. When Man takes time off for a short or lengthy period of time solely to dedicate himself to religious ritual or worship, he must have something he can appreciate. The human being always enjoys a spectacle, and the imagination had set to work to produce something he could watch or participate in.

Sun-worship was probably the first form, and many societies have retained its rites through centuries. Others turned, in addition to the Sun-god, to other deities and portrayed their gods in various forms or pictures so that the people would have something material to concentrate upon.

But common to almost all of these systems of worship was the belief that the god in question was something or someone with bad intentions and had always to be appeased by offerings and gifts for the local priests—which our former observer had now become—and for the rulers of the society who, through this fear of the wrath of the gods, had achieved a mastery over his people he did not previously possess.

The means of dividing the year into days, weeks and months and a certain amount of religious ritual are the gifts now passed on to the up-and-coming generations of mystic religions which for centuries controlled all knowledge and art, stretching in terms of time relatively close to the present day.

*

Naturally it would be impossible to prove categorically that the acceptance of the figure 7 occurred as decribed above, but the existence as a ritual factor of the figure in almost all forms of religion is a fact that would be difficult to explain. The figure today is bound up in ethics as a religious factor, and so matter how far back in time we search we are continually stumbling over this mysterious figure 7. Why has this figure meant more to us than any other figure? With our 10-digit system it seems somewhat awkward; we prefer complete figures which can be divided.

The simple reason must be that the first speculative figure arrived at by Man was 28, and the next was a quarter of this, or the figure 7. Spurred by his natural urge to build upon established factors, Man developed these two figures—which were to form the basis of the mathematics which existed before the period of Greek supremacy.

That the figure 7 was such a major factor in the sacred learning which dealt with numbers and mathematics must be contemplated on recognition of the fact that the study of numbers, the creation of religious symbols, the writing of legends, and the recording of events for future generations, all this was carried out by the same group of people: the brethren of the Temple who were for thousands of years indisputably the ruling upper-class of ancient Egypt, Assyria, Babylon and many other cultural societies of antiquity.

Evidence of similar observations in Denmark may be found from stone carvings in the same way as we read of the 20-figure system on the stones bearing the hand sign.

In *Fig. 16* we see a stone found at Sønder Bjært near Kolding.

In his book *Danmarks Oldtid,* vol. 2, page 128, Johannes Brøndsted writes of this stone:

> "A stone found near Sønder Bjært bears the figure of a ship, the style of which differs somewhat from the normal Bronze Age type, possibly belonging to a later period ..."

In other words, it does not seem to fit the Bronze Age and efforts are made to push backwards or forwards in time, without successfully managing to locate its place in history.

Its appearance is quite different from other rock engravings of Sun-ships, and it takes a fair degree of imagination to recognise this figure as a ship.

Fig. 16.

A tally of the vertical strips on the stone reveals that there are 7. This could be coincidental. But since the stone cannot be attributed with accuracy to any other place in time and no explanation can be provided as to its peculiar design, it is just as likely that this was one of the earliest attempts to produce a kind of weekday calendar which could keep a convenient check on days and nights, or perhaps used to help teach the difficult art of recognising the stages of the year.

*

The figure 7 has from time immemorial been very much the dominating figure in ethics, and it would be an almost impossible labour—even with an acceptable means of selection—to quote the many contexts in which the figure 7 appears. Space alone forbids it, and a simple lengthy enumeration of the ritual of various religions fails to provide a proper impression of its copious use.

I have therefore confined myself to one or two passages from one of the world's best-known books—the Bible. The examples I have selected are from the first and second books of the Old Testament.

Genesis: The Creation

And on the 7th day God ended his work which he had made; and he rested on the 7th day from all his work which he had made. And God blessed the 7th day and sanctified it.

Creation of Man

And a river went out of Eden to water the garden; and from thence it was parted, and became into four heads. (The origin of the cross in the centre of the circle).

Man's oldest story

And the Lord said unto him, Therefore whosoever slayeth Cain, vengeance shall be taken on him sevenfold.

If Cain shall be avenged sevenfold, truly Lamech seventy and sevenfold.

And all the days of Lamech were 777 years: and he died.

The Flood

Of every clean beast thou shalt take to thee by sevens, the male and his female. Of fowls also of the air by sevens, the male and the female.

For yet 7 days, and I will cause it to rain upon the earth.

And it came to pass after 7 days, that the waters of the flood were upon the earth.

And the ark rested in the seventh month on the seventeenth day of the month upon the mountains of Ararat.

And he stayed yet other 7 days; and again he sent forth the dove out of the ark.

And he stayed yet other 7 days; and sent forth the dove; which returned not again unto him any more.

Abraham and Abimelech make a covenant

And Abraham took sheep and oxen, and gave them unto Abimelech; and both of them made a covenant. And Abraham set 7 ewe lambs of the flock by themselves. And Abimelech said unto Abraham, What mean these 7 ewe lambs of the flock by themselves? And he said, For these 7 ewe lambs shalt thou take of my hand that they may be a witness unto me, that I have digged this well.

Jacob weds Rachel

And Jacob loved Rachel, and said, I will serve thee 7 years for Rachel thy younger daughter. And Jacob served 7 years for Rachel; and they seemed unto him but a few days, for the love he had to her.

Fulfil her week, and we will give thee this also for the service which thou shalt serve with me yet 7 other years. And he ... served with him yet 7 other years.

And it was told Laban on the third day that Jacob was fled. And he took his brethren with him, and pursued after him 7 days' journey.

Joseph interprets Pharaoh's dream

And, behold, there came up out of the river 7 well favoured kine and fat-fleshed, and they fed in a meadow. And, behold, seven other kine came up after them out of the river, ill favoured and lean-fleshed, and stood by the other kine upon the brink of the river. And the ill favoured and lean-fleshed kine did eat up the 7 well favoured and fat kine. So Pharaoh awoke. And he slept and dreamed the second time: and, behold, 7 ears of corn came up upon one stalk, rank and good. And, behold, 7 thin ears and blasted with the east wind sprung up after them. And the 7 thin ears devoured the 7 rank and full ears.

And Joseph said unto Pharaoh, The dream of Pharaoh is one: God hath shewed Pharaoh what he is about to do.

The 7 good kine are 7 years; and the 7 good ears are 7 years: the dream is one. And the 7 thin and ill favoured kine that came up after them are 7 years; and the

7 empty ears blasted with the east wind shall be 7 years of famine.

Behold, there come 7 years of great plenty throughout the land of Egypt. And there shall arise after them 7 years of famine, and all the plenty shall be forgotten in the land of Egypt.

And take up the fifth part of the land of Egypt in the 7 plenteous years. And that food shall be for store to the land against the 7 years of famine.

And in the 7 plenteous years the earth brought forth by handfuls. And he gathered up all the food of the 7 years which were in the land of Egypt.

And the 7 years of dearth began to come, according as Joseph had said.

Jacob's death and burial

And the Egyptians mourned for him 10 × 7 days. And he made a mourning for his father 7 days.

Exodus: Moses in Egypt

And all the souls that came out of the loins of Jacob were 10 × 7 souls.

Now the priest of Midian had 7 daughters: and they came and drew water, and filled the troughs to water their father's flock.

Water turned to blood

And 7 days were fulfilled, after that the Lord had smitten the river.

Passover is instituted

Seven days shall ye eat unleavened bread. For whosoever eateth leavened bread from the first day until the seventh, that soul shall be cut off from Israel.

In the first month, on the fourteenth day of the month at even, ye shall eat unleavened bread, until the one and twentieth day of the month at even. Seven days shall there be no leaven found in your houses.

Seven days thou shalt eat unleavened bread, and in the seventh day shall be a feast to the Lord.

Wandering in Sinai (Quails and manna)

Six days ye shall gather it, but on the seventh day, which is the sabbath, in it there shall be none. And it came to pass that there went out some of the people on the seventh day for to gather, and they found none.

Abide ye every man in his place, let no man go out of his place on the seventh day. So the people rested on the seventh day.

The laws of the Covenant

If thou buy an Hebrew servant, six years he shall serve, and in the seventh he shall go out free for nothing.

Seven days it shall be with his dam; on the eighth day thou shalt give it me.

And six years thou shalt sow thy land, and shalt gather in the fruit thereof: but the seventh year thou shalt let it rest and lie still.

Six days thou shalt do thy work, and on the seventh day thou shalt rest, that thine ox and thine ass may rest, and the son of thine handmaid, and the stranger, may be refreshed.

Thou shalt eat unleavened bread 7 days, as I commanded thee.

Consecration of the priests

And that son that is priest in his stead shall put them on 7 days.

Seven days shalt thou consecrate them.

Seven days thou shalt make an atonement for the altar and sanctify it.

Sabbath Commandment

Six days may work be done; but in the seventh is the sabbath of rest, holy to the Lord.

*

This completely unrestrained use of the figure 7 in the first and second books of the Old Testament would have been unusual even if other numbers between 1 and 10 had been mentioned in quantity, too. If, for example, fifty per cent of the figures mentioned in Genesis and Exodus had been from 1 to 10 (excluding 7) and the remaining fifty per cent had been 7. This alone would have aroused in the reader a curiosity to know why more interest had been displayed in the figure 7 than in other figures. But it must be even more amazing when number 7 is repeated in passage after passage, almost to the exclusion of other digits. Number 4 is mentioned here and there, and elsewhere mention is made of figures of 100 and more, but generally speaking the figure 7 reappears throughout the tale as a form of ritual, a thought-provoking factor.

The 7-armed candlestick is a well-known feature in the Jewish synagogue, and appears also in the Danish Lutheran church.

The Tibetans have divided their nobility into 7 groups, to mention only two extremes in which the figure 7 is used in two societies with virtually no mutual contact, culturally speaking.

The figure 7 is a factor of ritual within modern freemasonry.

In our everyday language we make use of the number 7 almost without reflection. For example, he was in his 7th heaven. He would need 7-league boots. The 7 Wonders of the World. And so on.

The constant appearance of the figure 7 in all esotericism and symbolism since ancient times is so striking and so much a dominant feature in literature that it cannot avoid giving rise to meditation. Why should the philosophers of the past who were accepted, rightly, as creative and intellectual giants stick to one specific figure when there were several to choose from? One is free to assume that they had their reasons. And we shall later prove this.

CHAPTER FOUR

Birth and development of geometric speculation

In the previous chapter development had reached the stage where Man's first geometric drawing or construction produced a certain result.

By joining the extremities of the cross (in a given circle with a diameter of 10 units), he constructed a square the sides of which were discovered by simple measurement to be 7 units, and our observer was then able to establish that the perimeter or total base-length of the square was $7 + 7 + 7 + 7 = 28$.

This measurement or number fitted exactly into the earlier observations of Sun and Moon, which had encouraged Man to split the month into 28 days and the year into 13 months of 28 days each.

The application of the circle as a symbol of time and its passing made it a natural step for our ancestors to regard this division of the moon-year as a revelation of the gods, a necessity. And the month was therefore split into 4×7 days, to suit the geometric construction they had arrived at.

Astronomers since those ancient times have introduced several amendments to the calendar-year, having discovered that the year has not in fact 364 days ($7 \times 4 \times 13$), but instead a little more than 365 days.

Over a short period of years this difference is of minor importance, but over a number of centuries not to mention several thousand years it assumes unquestionably vast significance.

The year as we know it is today divided into twelve months, the number of days in each month varying between 30 and 31, excepting February which normally has 28 days, but which each leap-year gets an extra day added to at the end, which we term the intercalary day. This is done to compensate for the period by which the chronological solar year exceeds 365 days, the margin of error between the solar and calendar years being regulated by the intercalary day; thus in the next 4 years there is an annual error of approximately ¼ of a day, which is rectified at the next leap-year.

But in spite of this adjustment we have retained the name of the major unit of division of the year, i.e. the month, even though it no longer follows the moon-year, or "moon".

As far as the week is concerned no change has been made in its composition of seven days, even though it is no longer a natural fraction of the month, i.e. ¼.

It is generally reckoned that a year has 52 weeks and 52×7 produces 364, which means that the splitting of time into weeks has been maintained unaltered through the ages, despite the discovery that the year is longer than originally anticipated, that the months have been changed from 13 to 12, and that the division into weeks no longer tallies with either the year or the month, and is made up of 7 days—a very inconvenient number in the realms of division.

The fact that the days of the week have survived this turmoil for thousands of years is probably because the division into weeks was something presented extremely early in Man's history as a divine manifestation, and the established day of rest was a welcome institution.

Although belief in a Supreme Being has taken many forms, each has had its own ritual, and the easiest way to introduce a new creed is to retain as much of the existing ritual as possible as long as it does not interfere with the new idea and thoughts.

Since moreover there was no better substitute, the 7-day week was retained. We have to admit that if today we wanted to alter this state of affairs, for example, to change the week to 5 days with 73 weeks in a year, it would meet a resistance which would probably prove insurmountable.

We now move forward from the earliest days when Man's world was just beginning, when one or two individuals started a primitive form of research, research dictated solely by material needs, into a period when the intelligentsia originated Temple culture which developed with each new generation and grew to be the decisive factor guiding the culture of nations.

In these surroundings there was time and a place for speculation not dictated by Man's immediate needs but instigated by a desire to know something thoroughly. And it was in this encouraging atmosphere of development and growth that mathematical knowledge flourished as a combination of pure numerical meditation, ritual and mysticism.

We can only guess as to who were the first people to nurture the development of mysterious temples of religion. Later European temples were of course merely a garnished imitation and echo of their forerunners in Eastern countries. In this connection it is worth noting that when one of Greece's best-known mathematical philosophers, Pythagoras, was admitted around 500 B.C. to an Egyptian temple for learning and initiation, temples in Egypt had been developing gradually over a period of 6000 years. A 6000-year stretch of relative peace, with war and other border troubles held to a minimum. A period of development by a civilisation which, like the majority section of present-day Man, cultivated the arts and sciences, and in which a Temple training lasted 22 years.

There was, however, one major difference between that ancient education and our own. Every aspect of education in those days was governed by the Temple and closely connected with the existing religious ideal. All learning was a revelation by the gods, the veil of Isis being lifted little by little for the pupil as his training progressed. But these manifestations were not intended for the eyes or ears of the unworthy, and every pupil had to swear under oath that the wrath of the gods would visit them if they revealed their knowledge to the uninitiated.

Once his 22 years of training in the Temple were at an end the initiated brother was free to leave, as opposed to the period of education itself during which he was confined to the Temple precincts. His oath of silence was ubiquitous. He himself could use his knowledge but was prevented from teaching others its secrets. He could teach general subjects and was permitted to reveal his knowledge through symbolic ciphers, stories, ritual, etc., but must never share his knowledge with outsiders.

Thus if one can interpret these things correctly, one can draw back a little of the veil which hides the knowledge of ancient times. But another course is open:

we can try to find our way round to the back of the system and work on the results achieved, using the same tools and methods at the disposal of ancient observers, and thus attempt to uncover the foundation of existing ritual, mysticism and, in fact, knowledge. By comparing the results with the information we have on the ritual of that time, we should be able to check whether we are on the right track. We shall choose the latter of the two courses open to us, and continue the speculation we can imagine to have emanated from the first Temple mysteries. Our location will be that stage in Time at which mathematical or geometrical speculation itself began. We shall carry on building upon material passed on from the previous generation, i.e. the circle with the cross drawn inside, and the resultant square. This amplification of existing ideas must be considered a natural step, as new theories must undoubtedly be the result of speculation upon factors already in existence.

The nature of areas

Our observer first draws up a summary of the factors and values he already knows:

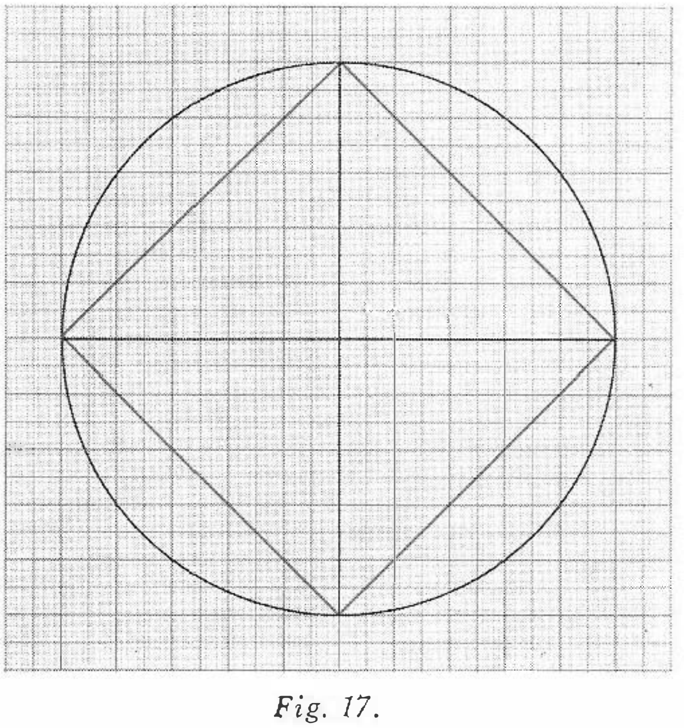


Fig. 17.

1. The cross divides the circle first of all into 4 equally large portions, and of these 4 parts he knows the length of the 2 straight lines, which are 5 feet: half the circle's section or diameter.
2. The same cross divides the square into 4 equally large parts, i.e. 4 triangles whose 2 short sides are 5 feet and the long side has been measured at 7 feet.
3. The value of the triangle is 17, since the sides are worked out at 5 + 5 + 7 = 17.
4. The square has 4 equally long sides and its value is 28, i.e. 7 + 7 + 7 + 7, and is consistent with the number of days in the moon-year.
5. The arms of the cross measure 5 feet from the centre and have a value of 20, 4 × 5 or 2 × 10.

Thus we lack a value for the circle.

The observer can establish that its circumference is greater than that of the square, i.e. greater than 28, and the simplest way would probably be to try to measure the circle with a string.

Let us assume that in order to make the tricky task easier he tries measuring the quarter-circle, placing sticks along the outside edge. His measure shows that it is greater than 7 feet, but shorter than 8 feet.

Now he hits a problem because he is not accustomed to operating with anything other than whole numbers.

His mind—naturally—has already given consideration to the subject of division. He saw, for example, his circle being split into 4 portions, thus where once there had been a whole he could now see 4/4. But he has not yet learned how to work with these parts of a whole, and in any event each of these 4 parts has a curved side which he is unable to measure or check accurately. Thus he discovers that in the

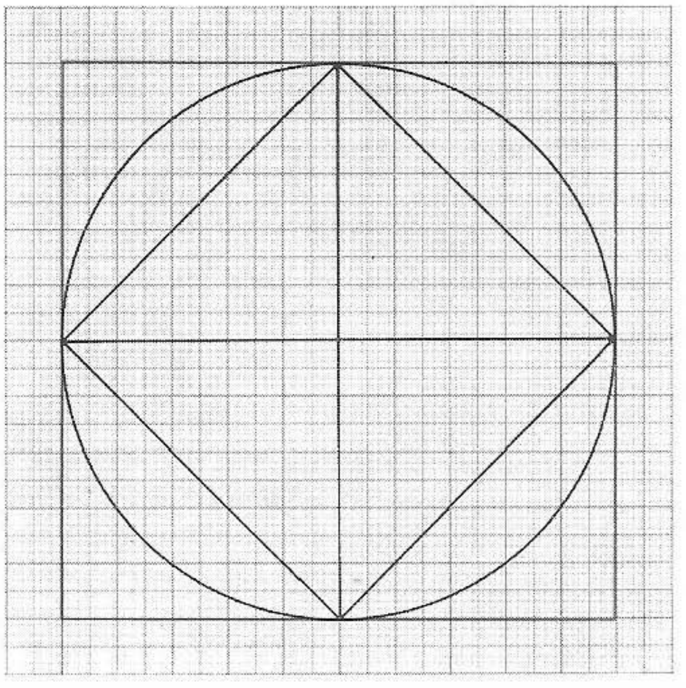


Fig. 18.

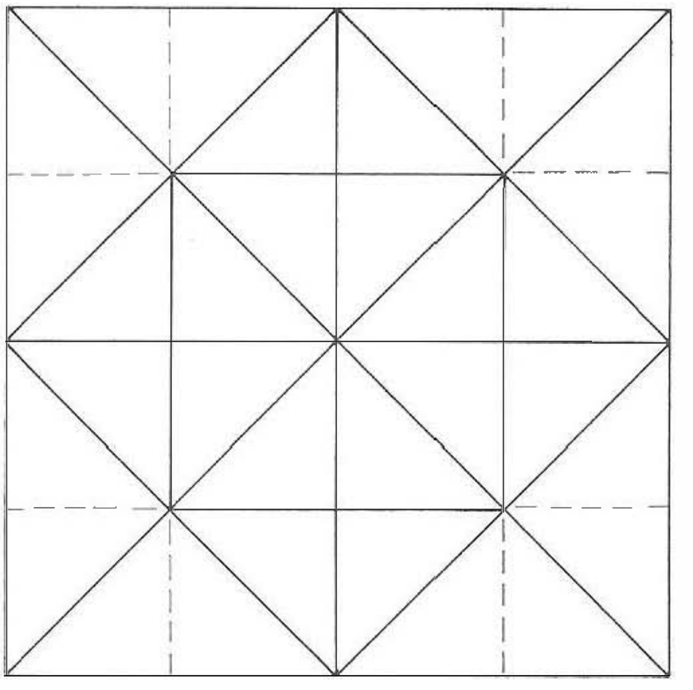


Fig. 19.

case of the circle he must be satisfied for the moment with an approximate value somewhere between 28 and 32.

His experiment continues, and he constructs a square which lies not inside the circle, but outwith the complete symbol, the 4 sides of this new square forming tangents to the arc of the circle, *Fig. 18.*

In so doing he discovers that his symbol certainly contains 2 squares, one inside the other, but he also realises that the whole thing has merged into 8 triangles.

He has already noted the presence of triangles during his experiments with Fig. 17, but it is not until he studies Fig. 18 that he can see that this natural division repeats itself in the diagram in Fig. 18. This revelation gives him his first desire to see surfaces divided into triangles instead of—as in our day—seeing them divided into squares.

He proceeds with his experiment to see whether this triangulation will follow, and draws a square with a sidelength of 10 feet. He measures 5 feet out from a central point in four directions and constructs the outer square. In the normal manner, he constructs square no. 2 by joining the arms of the cross. But in constructing the third and inner square he lacks a guiding line on which to mark the corners, and therefore draws in the diagonals of the outer (or largest) square. Here again he discovers a division into smaller units: by extending all lines to their logical maximum he can count 32 identical triangles within his complete figure. See *Fig. 19.*

It is now within his power quite simply to measure the squares in relation to each other, since the inner square contains 8 triangles, the second square 16 triangles, and the outer square 32 triangles.

Without knowing the size of the squares, he is thus able to measure them in relation to each other. Without having any real knowledge of measuring areas, he is able to take a part of a square, as long as this part lies within the factor of division with which he is operating. His assumption with regard to the triangulation of areas and surfaces has been confirmed, and his subsequent thought will undoubtedly be guided in this direction until other factors come on the scene.

Our later study will show that this assumption is apparently correct, and if we consider the circumstances from a purely logical standpoint we must admit that a division of a square area into triangles would be much easier at an early stage of geometrical development than a division into squares.

If we want to split up a square surface, we have first of all to mark off the vertical and horizontal sides in established and approved units of measure such as centimeters, inches or some other measure, and to draw in horizontal and vertical lines. This provides the initial division, but if we want to divide further we must repeat the measuring process.

We could also perhaps choose to divide the sides into halves, then into half halves, and continue down to the size we want. But this still needs 2 measurements for each line, and the smaller the unit becomes the more difficult is the procedure.

This method of division seems obvious to us today but only when used in association with other factors which have been proved and learned. On the other hand, dividing a square into triangles is much easier to accomplish in practice.

We see in *Fig. 20* a square divided by triangulation. The task is decidedly simpler. We see here that we can accomplish it almost without measurement, merely requiring to draw in the cross and the diagonal cross in a square and proceeding to divide the square into just as many small units as technique permits.

The dividing process is carried out free of measurement, the lines themselves dictating the points where a further dividing line must go.

Quite plainly this method is much more manageable than a division into squares, and we see—still in fig. 20—how the lower right corner is divided by triangulation and although we are far from the limit of the achievable we have already succeeded in dividing the area into 1024 small triangles. Simply by making one further division we can obtain 2048, and so on.

To make the same division into squares

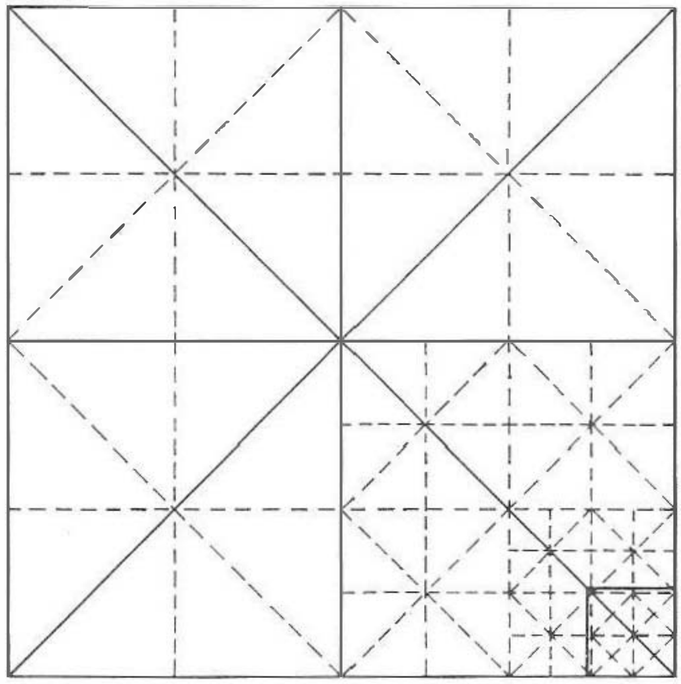


Fig. 20.

would be a fairly complicated task requiring extremely accurate measurement, and without modern technical aids such as drawing equipment it would be difficult, if not impossible, to complete an accurate division into such small units.

Thus our observer, by working on the three squares within each other, finds himself able to measure their mutual relationship, since he has discovered that the inner square comprises 8 triangles, the second square 16, and the outer and largest 32 triangles.

If we call the squares (starting with the inner one) A, B and C, our observer finds that $2 \times A = B$, $2 \times B = C$, and $4 \times A = C$. His findings are perfectly correct.

He is, of course, unfamiliar with the term $4 \times A = C$, but for one thing he can read it plainly in figures by counting the triangles, and in the second place he can see this readily from his diagram. And at this point he comes across another interesting point, *the relation of the sides of the squares.*

He knows the length of the side of the outer square because he has measured it to be 10, i.e. a total base-length of 40. He thinks he knows the length of side B which he has earlier fixed, by measurement, at 7, base-length 28, and he can see from his diagram that square A must be 5, base-length 20. He sees then that the sides of a square do not double their length in proportion to the doubling in area, but miss out one step. This he can see from the fact that square A has a base-length of 20 and square C a base-length of 40, while square B apparently has a base-length of 28, and this phenomenon naturally gives him more food for thought.

His next step in experimentation is to make a diagram as in *Fig. 21* in which we see the 3 squares with their respective circles, to see whether this can tell him anything.

The first obvious piece of information he can read from this diagram is supplied by studying the relationship of the circles and squares to each other.

Circle A is the smallest of both circles and squares, and outside this is square A, but as well as being the external square of circle A it is also the internal square of circle B.

As he has already worked out that square B is twice as big as square A, he can conclude that if he draws a given circle and places a square inside and a square outside, as he has done earlier, the areas of the two squares are in the ratio 1 to 2. By studying square B, which of course lies outside circle B, he comes to the conclusion that when the areas of the squares are in the ratio 1 to 2, the areas of the circle must be the same.

So far he is unable to divide the circle into triangles or any other measurable units, but he can check the circle in relation either to the outside or the inside square and is thus now able—with the assistance of his squares—to construct a circle which is either twice or half the area of a given circle.

He knows neither the area nor the circumference of the circle accurately, but his observations have nevertheless given him a little foothold on the circle.

How do his assumptions stand up to modern knowledge?

The area of course is found by complying with the rule $R^2 \times \pi$, which in the case of

circle A is $(\frac{D}{2})^2 \times \pi = 19.6349$

circle B is $(\frac{5 \times \sqrt{2}}{2}) \times \pi = 39.269875$

Since we must test to see whether circle B is twice as great in area as circle A, we must state:

$$2 \times A = 2 \times 19.6349 = 39.2698$$

Thus we see that his conclusion regarding the doubling of a circle's area outside and inside a square is correct.

With regard to the sum of the sides of a square, we assume that our observer—as shown earlier—has realised that this does not double in keeping with the area, but misses out one step, square A having half the side-length of square C not—as one might expect—square B.

A look at the drawing in *Fig. 21* shows

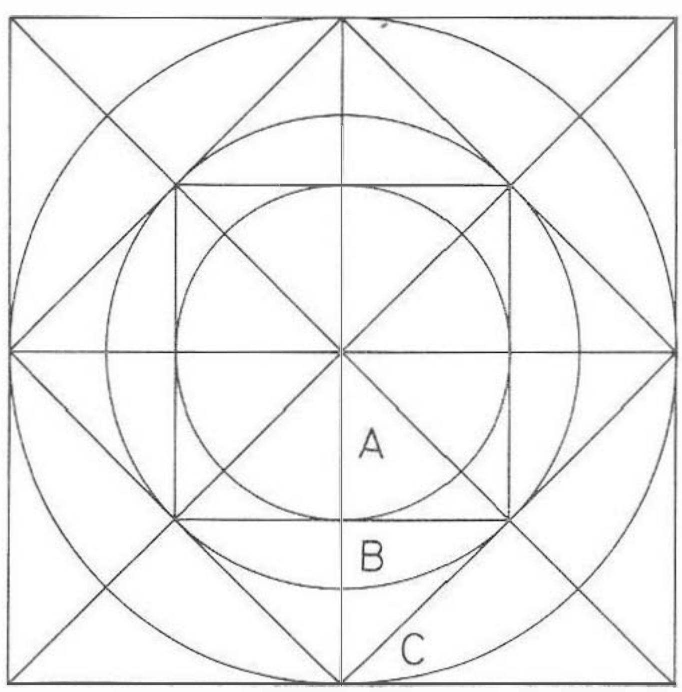


Fig. 21.

that he can simply measure this out to prove it to himself.

Not so with the circle. He cannot make a direct, accurate measurement, but as a logical conclusion from the previous exercise he can reason that the ratio of the circle must be the same as the square that surrounds it.

So he must be satisfied with reasoning. With our more advanced system of calculation we can discover that his reasoning is correct, for if circle A has a diameter of 5, the area of

$$\text{Circle A} = 5 \times \pi = 15.70795$$
$$\text{Circle B} = 10 \times \pi = 31.41590$$

which proves his assumption correct.

Accordingly his diagram permits him to establish that in the case of their respective areas, circle and square A are in the ratio 1 to 2 to circle and square B.

And in the case of their perimeters and side-lengths, circle and square A are in the ratio of 1 to 2 to circle and square C.

If these discoveries had been made at a later date, they would undoubtedly have given rise to theorems stating how a circle was related to its surrounding square and following circles. But our mathematical observer was not conversant with setting out his findings in words. He quite frankly did not have words to express these discoveries, and was therefore reduced to stating them in the form of geometric drawings and diagrams.

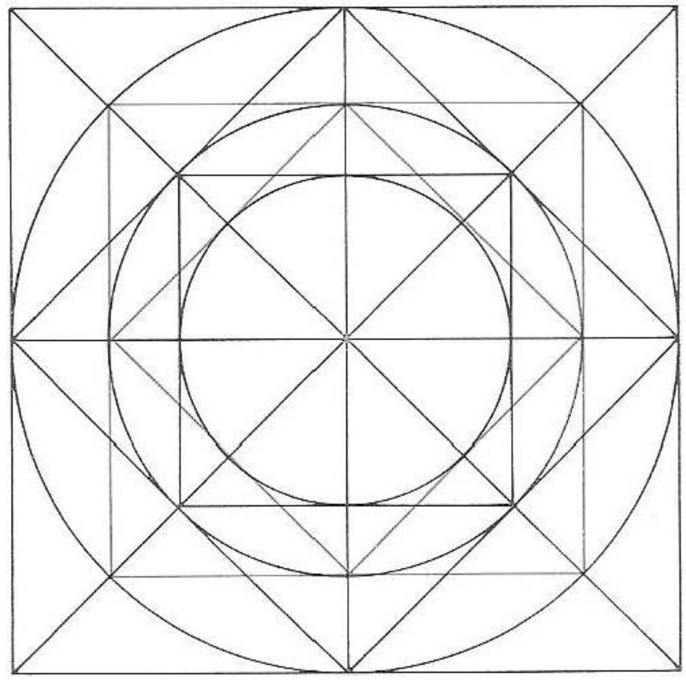


Fig. 22.

His subsequent discovery from the same diagram, which he would probably make as a natural step from the last, can be seen in *Fig. 22* in which he has swung circles A and B through 45 degrees (marked in red in the diagram). In so doing he notices that in their movement both square A and square B have followed respectively the perimeter lines of circles B and C. Thus he observes that, in addition to having its point of origin in the central cross, square B can also originate from square C's diagonal, i.e. at the point where circle C cuts this diagonal; now he can produce his diagram without the cross, with the assistance only of diagonals and circles. This means that he requires no awkward measurement, which he is still finding difficult to perform accurately even though it is as simple as dividing a line into two equal lengths, and he now produces the diagram shown in *Fig. 23*.

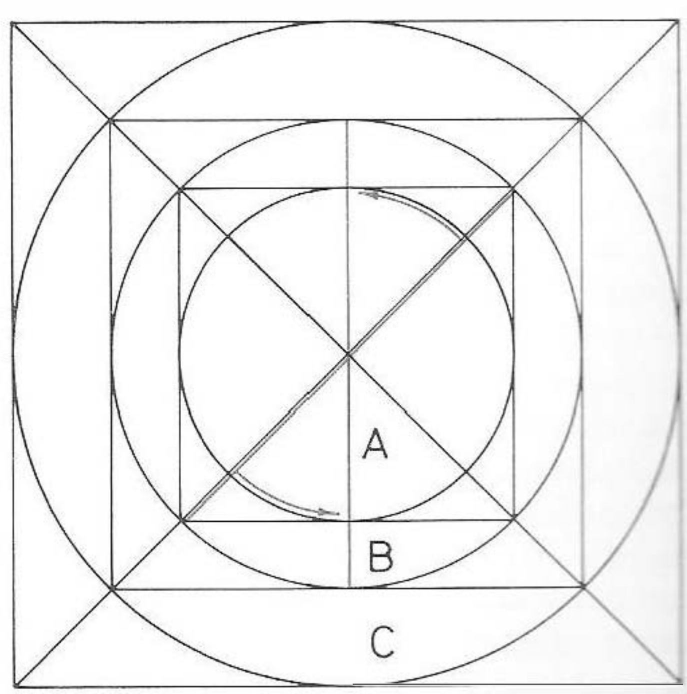


Fig. 23.

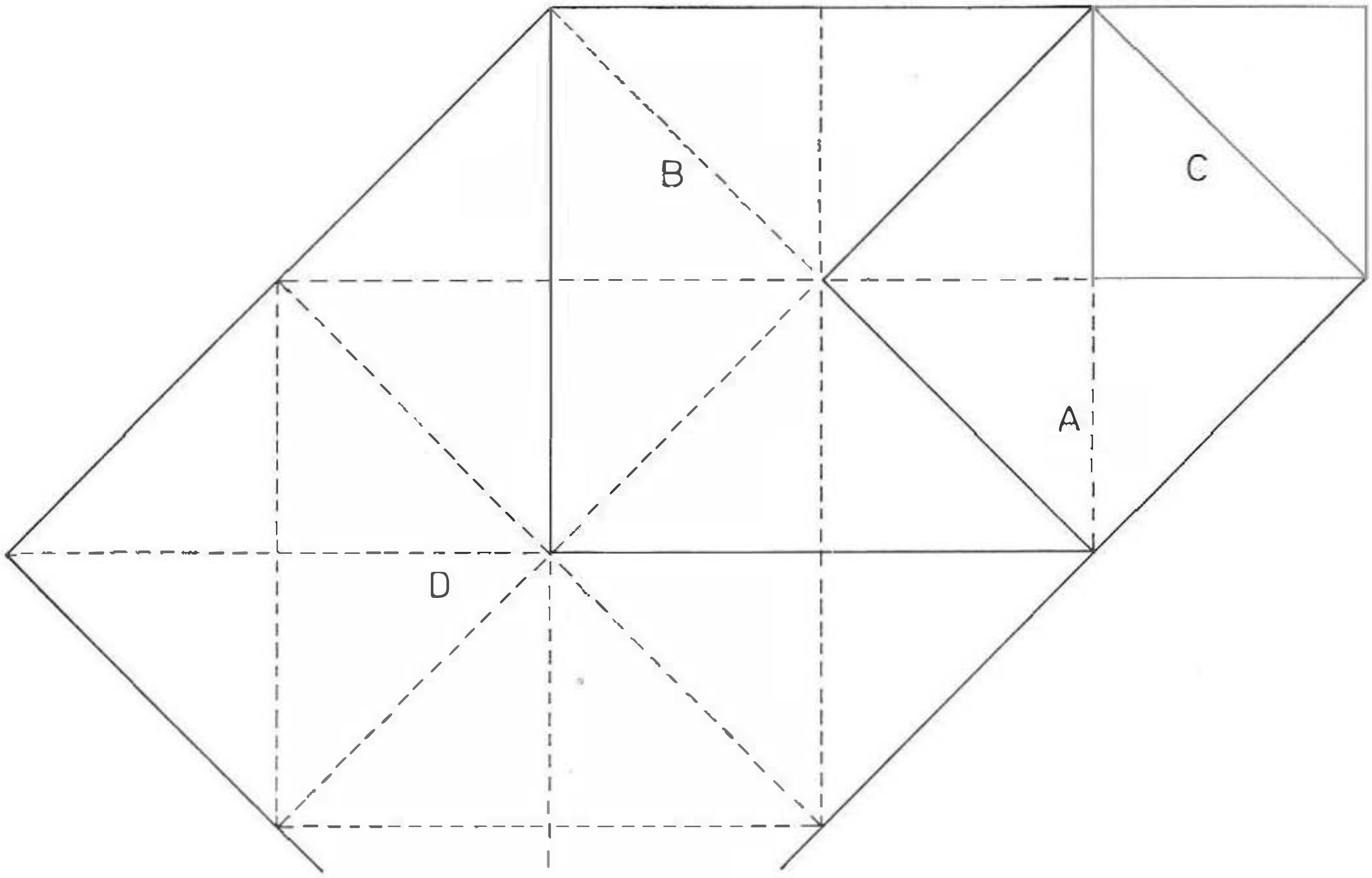


Fig. 24.

The lines of the three squares are now parallel with each other and he studies the figure to see whether he can obtain any information on the relationship between the sides of the various squares. And he realises that the diagonal in square A is identical to the side of square B, and that similarly the diagonal in square B is identical to the side of square C.

Of course, this is a discovery he could have made at an earlier stage, but he has produced many different diagrams and has learned a little from each. As diagram 23, or rather the information it contains, is moreover applied extensively in future development it must be accepted as probable that his discovery of the relationship between the side of a square and the diagonal of the preceding square was made at this point, since this diagram first of all is a logical continuation of the previous and secondly most clearly reveals this geometric fact.

The fact, too, that he has started revolving the square within the circle teaches him to follow the positioning of the lines, and—experimentally—by swinging the A diagonal to the vertical position he can immediately see that it is equal to the side of square B.

Thus he has learned that the diagonal of any given square provides the side of a square with twice the area of the original. And again he encounters the phenomenon that if he is to progress with squares they must be divided into triangles—because drawing in the diagonal of a square is of course the same as transforming the square into two triangles.

To prove and illustrate more clearly this theory we can imagine him drawing a diagram as in *Fig. 24* where he constructs four squares each of which doubles in area successively. Dividing them into triangles of the same size, he can see that his discovery is in fact correct, the smallest

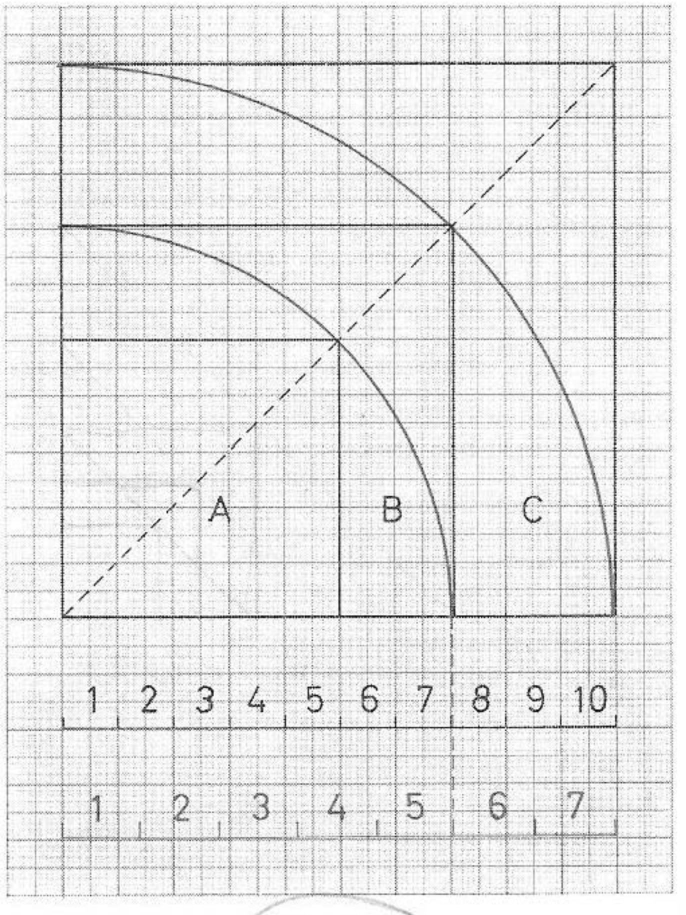


Fig. 25.

square C being divided into 2 triangles by its diagonal, the next square A into 4, the next square B into 8 and the largest square D into 16. Thus he ascertains or is able to ascertain and prove that his first conclusion was correct. From his observations of Fig. 24 he has also found a way to closer study of the side-length of squares, since he now knows that the diagonal of a given square is identical to the side of a square with double the surface area of the original. He is now able to position his lines from the same point of origin, making the diagram clearer to read. He now produces the diagram seen in *Fig. 25* in which he begins by constructing square A, giving it a side of 5 units. The diagonal is swung down to its horizontal axis, and square B is drawn. The diagonal of square B is swung down to its horizontal axis and square C is constructed.

Now he has a diagram in which—his previous experience tells him—he has 3 squares which double in area from A to B and from B to C. He knows their sizes as his construction, which began originally with the external square of a circle with a diameter of 10 (i.e. a square whose sides are 10), had allowed him to measure square B's side to be 7, and square A's side to be 5.

In this instance, however, he starts from the other end of the scale with the smallest square and constructs the 2 larger squares. Measurement then brings him to the same conclusion: that the lines are respectively 5, 7 and 10. It is quite a simple matter for us today to discover that this measurement is incorrect, the side of square B being not 7, but $5 \times \sqrt{2} = 7.07$, whereas both A and C are correct, the original base A being presumed correct, and C being calculated as $5 \times \sqrt{2} \times \sqrt{2} = 10$. We have discovered an error in measurement in square B of 0.07 or approx. 1 %. This was an inaccuracy our observer was probably unable to ascertain, and if in fact he could measure it he had no means of expressing it. But the former belief that he was probably unable to measure it is the more likely.

During the course of these recent speculations he has doubtless reduced the size of his diagrams to something he could survey readily. His drawing materials were probably wet sand and a stick; with this equipment an error of the degree in question would scarcely be spotted even in a drawing in which the side of the largest square was ½ meter, producing an error of 3.5 mm.

He sits back now and studies his new diagram, making the following discoveries —after selecting to use as his point of origin the square which is dearest to him, i.e. that containing the sacred figure 7: Square B has a side of 7 and square A a side of 5. He knows that square B is twice the area of square A and can conclude that a square can be halved in area if you divide its side into 7 equal parts and take 5 of these as the base-line for a new square; and if you wish to double the area of a square, you divide its side into 5 equal parts, add $\frac{2}{5}$ and this gives you the new base-line. In other words he discovers that the total side-length of a square is related to that of a square twice its size in the ratio 5 to 7, so that the total side-length or perimeter of square A is $\frac{5}{7}$ that of square B, and the perimeter of square B is $\frac{5}{7}$ that of square C. To prove this theory he divides the base of square C into 7 equal parts (see bottom graduation in fig. 25) and by counting along his scale he sees that the measurement is correct, since square B's side is 5 parts of the 7 into which he divided square C.

He could have chosen another approach. He could have said that the side of square B is $\frac{7}{10}$ that of square C, and gone on to say that the side of a square becomes $\frac{3}{10}$ longer when the square doubles in area; similarly that the side of the half square is $\frac{7}{10}$ that of the large.

With our advanced decimal system, we would probably have chosen the latter procedure if faced with the problem today. But our ancient forefathers had no decimal system. The figure 7 was the familiar digit and they therefore chose to use it in their division.

This figure had of course a sacred significance and had early become a familiar concept in Man's everyday. It should furthermore be borne in mind that—in any choice of starting from a familiar or an unfamiliar point of origin—Man will generally select the factor to which he is accustomed.

The conclusion reached at the end of his meditation upon Fig. 25 must be that to a certain extent he has achieved a degree of control over the perimeter of a given square in relation to its half and double sizes in area, and that this has been possible by use of the sacred figure 7.

His figures contain a minor error of approx. 1 %, which we today are able to illustrate, whereas his geometric constructions are perfectly correct.

On the basis of these conclusions concerning the relative sizes of the sides of squares, he goes on in his usual manner to reason that the same relationship must apply to the circumferences of circles, an assumption which can be proved only to a certain degree by measurement but which cannot be ascertained precisely.

How does this assumption stand up to present-day calculations?

We observed earlier that the circumference of circle A was 15.70795, and we must now state:

$$\frac{15.70795 \times 7}{5} = 21.98113$$

which should be the same as the circumference of circle B. Circle B is
$5 \times \sqrt{2} \times \pi = 7.07 \times \pi = 22.2110413.$

Here therefore we can calculate an error of 0.2299113 in comparing the ancient assumption of the circumference of a circle with the figure we have today—which is worked out according to π to the fifth decimal place.

The error is obvious to present-day scholars since the ratio 5 to 7 is incorrect since the sides of square B measure 7.07 or $5 \times \sqrt{2}$, but we must still concede that neither simple fractions nor decimal fractions were known. The above conclusions were based on whole numbers, the only possible unit at this stage, and when we see the problem solved by whole numbers and can today with our advanced system of mathematics pinpoint an error of slightly more than 1 %, it must be admitted that the solution at that time was as close to the truth as at all possible.

Further, even if they could appreciate this infinitesimal inaccuracy they simply had to accept it in order to proceed: precisely as we in our time have been obliged to do, since it is a well-known fact that the π which we use today and the value which has been internationally accepted are not absolutely accurate. The error is so far to the right of the decimal point hat it has no practical importance; but it does not explain away the inexorable fact that we, even though the amount is almost beyond comprehension, work today with an approximate value which we are unable with our system of counting to amend.

Only the passage of time will reveal whether we in future will require a more accurate π than we have today. As happened in the past, need will prompt progress.

So we see that through his studies our observer has reached the stage where he can:

1. construct squares inside each other and outside each other in the ratio 1 to 2.
2. construct squares of the same ratio from a horizontal or vertical axis by using one square's diagonal as the next square's side.
3. check the common ratio of squares in relation to their base-lengths, with the sacred universal figure 7 as his point of origin.
4. make a check on circles, both in area and in circumference, by comparing his results with the square and transferring the same procedure to the circle.

He has certainly made amazing progress when we consider that it has all been achieved by the use of whole numbers and without any knowledge of π, involution, square roots, or other speculative mathematics, but founded exclusively on logical consideration of certain geometric drawings based on the circle. And all this with

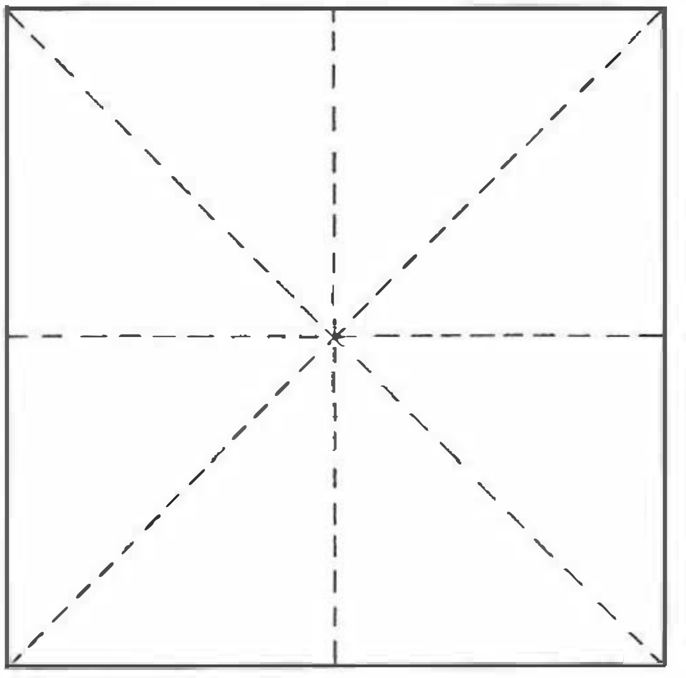


Fig. 26.

virtually no expertise in the handling of figures or the art of arithmetic, and solely conducted in whole numbers, based and supported by the universal figure 7.

During his study of the square our observer has recorded a peculiar fact: that an area can be expressed in several ways and yet amount to the same value.

For example, if he works with square B as his basis, he knows that this square is twice as great as square A. In other words, if he cuts B in two each half will be equal in area to square A.

If he takes one of the lines in the vertical cross as his dividing line, he finds himself with two rectangles each identical in area with square A. If he uses a diagonal as his dividing line he again succeeds in splitting square B in two halves, two triangles, but these halves or triangles must also equal square A in area.

To get this subject clear in his mind, our observer makes 8 three-sided bricks, cut out of a square of suitable proportions, as in *Fig. 26.*

He probably chose this method for reasons of convenience, freeing himself of the risk of mismeasurement. It is also possible that the bricks were not produced until the discovery was made, and were later used as a means of teaching other initiated brethren. I shall shortly be coming to instances of where I believe this method of procedure—at any rate in certain places—has at some time been employed to illustrate the variable nature of areas.

We start, however, with Fig. 26 where we have a square divided into 8 triangles. These triangles are so cut as to provide our observer with 8 three-sided bricks. The first geometric figure he makes is to divide the square into two rectangles: *Fig. 27.*

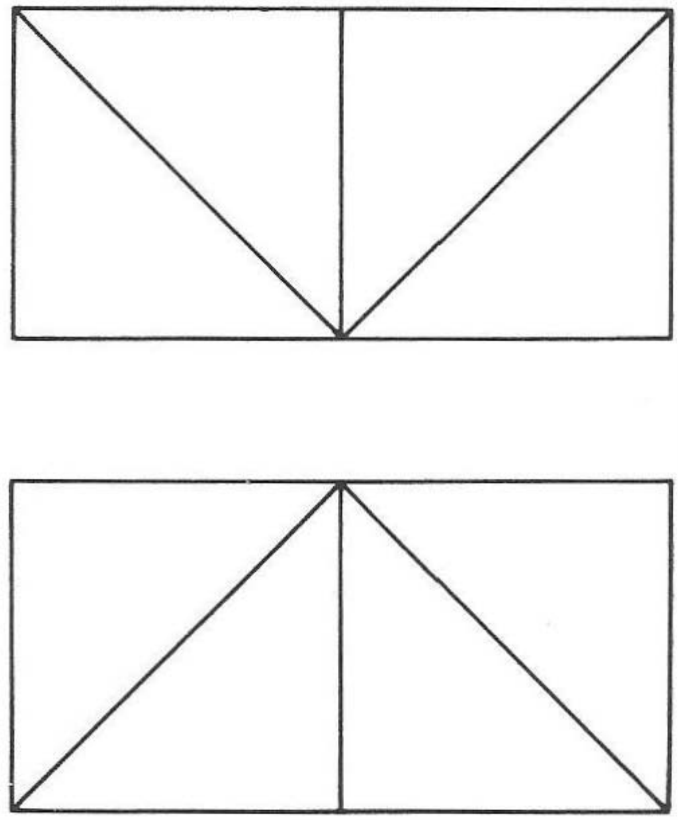


Fig. 27.

The next figure he makes is to divide the square along one diagonal, producing 2 triangles: *Fig. 28.*

The third shape he makes—which is not immediately obvious from a drawing on the ground but which the bricks reveal—is the large triangle: *Fig. 29.*

He can at once see from the bricks that this triangle represents half of the double square, obvious since all 8 bricks are used. He knows from experience that the adjacent square to the original is twice as large, but it gives him a thrill to be able to check this in practice.

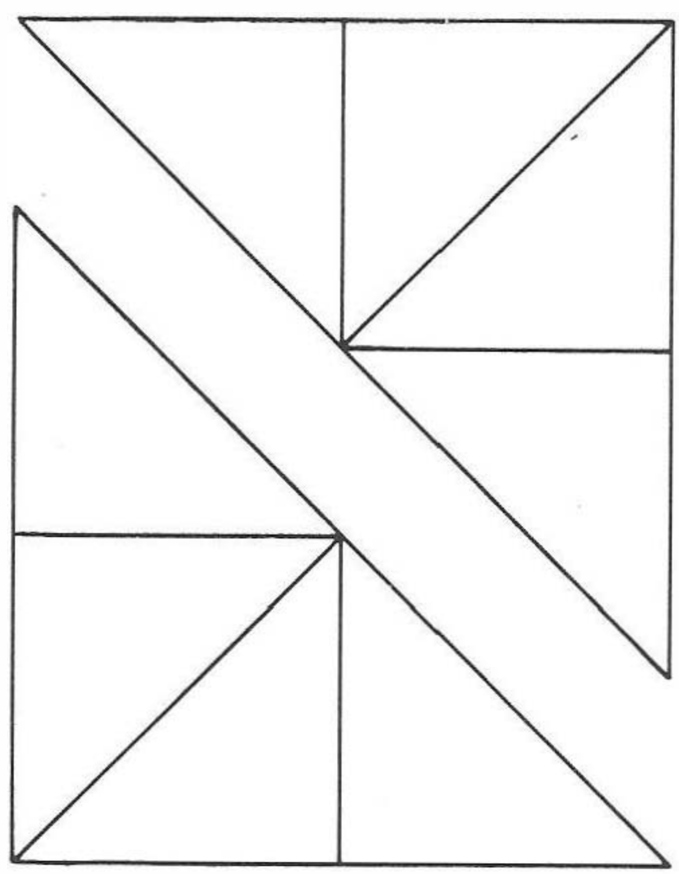


Fig. 28.

The next geometric shape he assembles is the single rectangle containing all 8 bricks, *Fig. 30* and we have now mentioned the most important combinations he can achieve with his 8 triangular bricks, namely:

1. the large square
2. 2 small rectangles
3. 2 triangles
4. the large triangle
5. the large rectangle.

Are we to view his results as total planes, then he has succeeded in placing his bricks in a form of trinity, i.e. a large

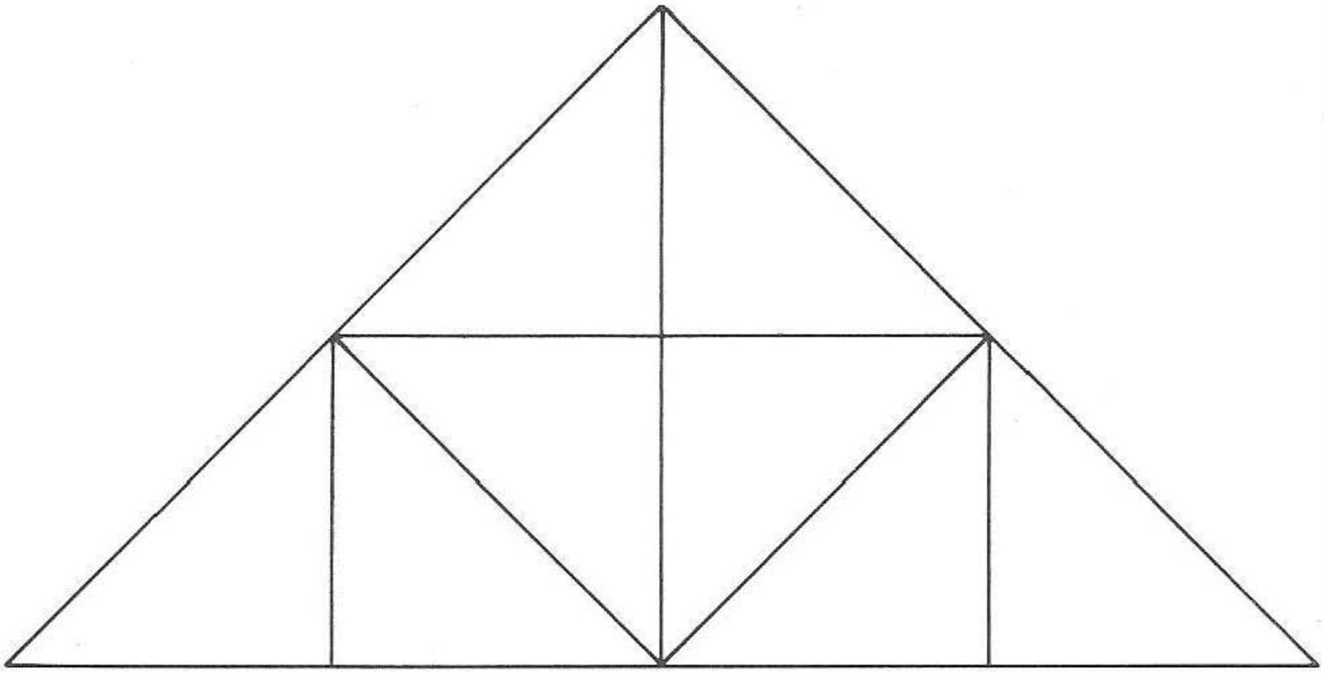


Fig. 29.

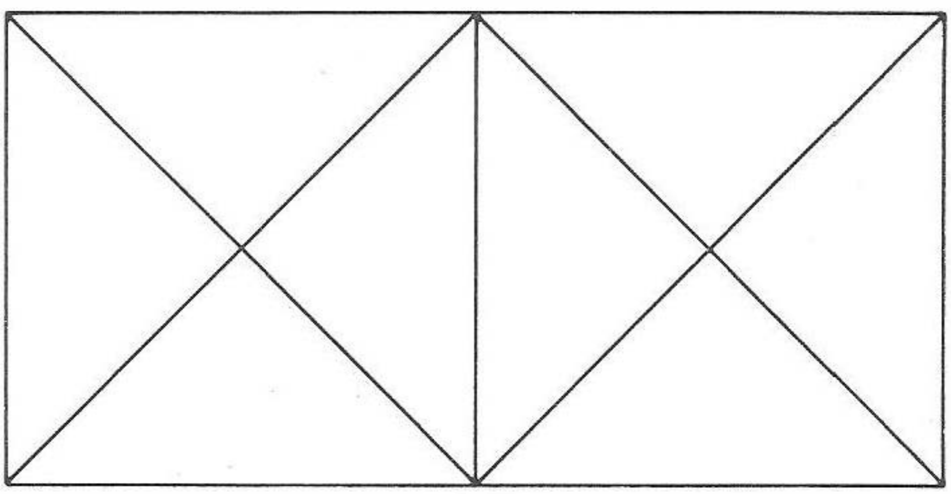


Fig. 30.

square, a large triangle, and a large rectangle.

There are, of course, numerous possible combinations with these bricks, each arrangement representing the same area as long as every brick is used. But the three shapes mentioned above are the most important and these are the ones on which further study is concentrated.

This age-old method of working with geometry would of course be kept a strict secret within the Temple, out of reach of outsiders, just as long as it was important to training or for any other purpose. Not until a later date would the material be released for general use, albeit in a somewhat amended form. I have in mind here the ancient Chinese puzzle game of Hao Wan, which consists of 7 bricks, *Fig. 31*.

Confucius recommended this game, it is said, as a noble exercise of the mind which everyone ought to indulge in to enrich their spirit.

The Chinese original was made of richly engraved and polished ivory, while our photograph shows a Swedish copy of the game in plastic. The game is supplied with a silhouette plan of 49 different shapes (*Fig. 32*) that can be produced with the bricks, but the 3 main shapes are omitted, i.e. square, rectangle and large triangle.

Whether the 49 different silhouettes

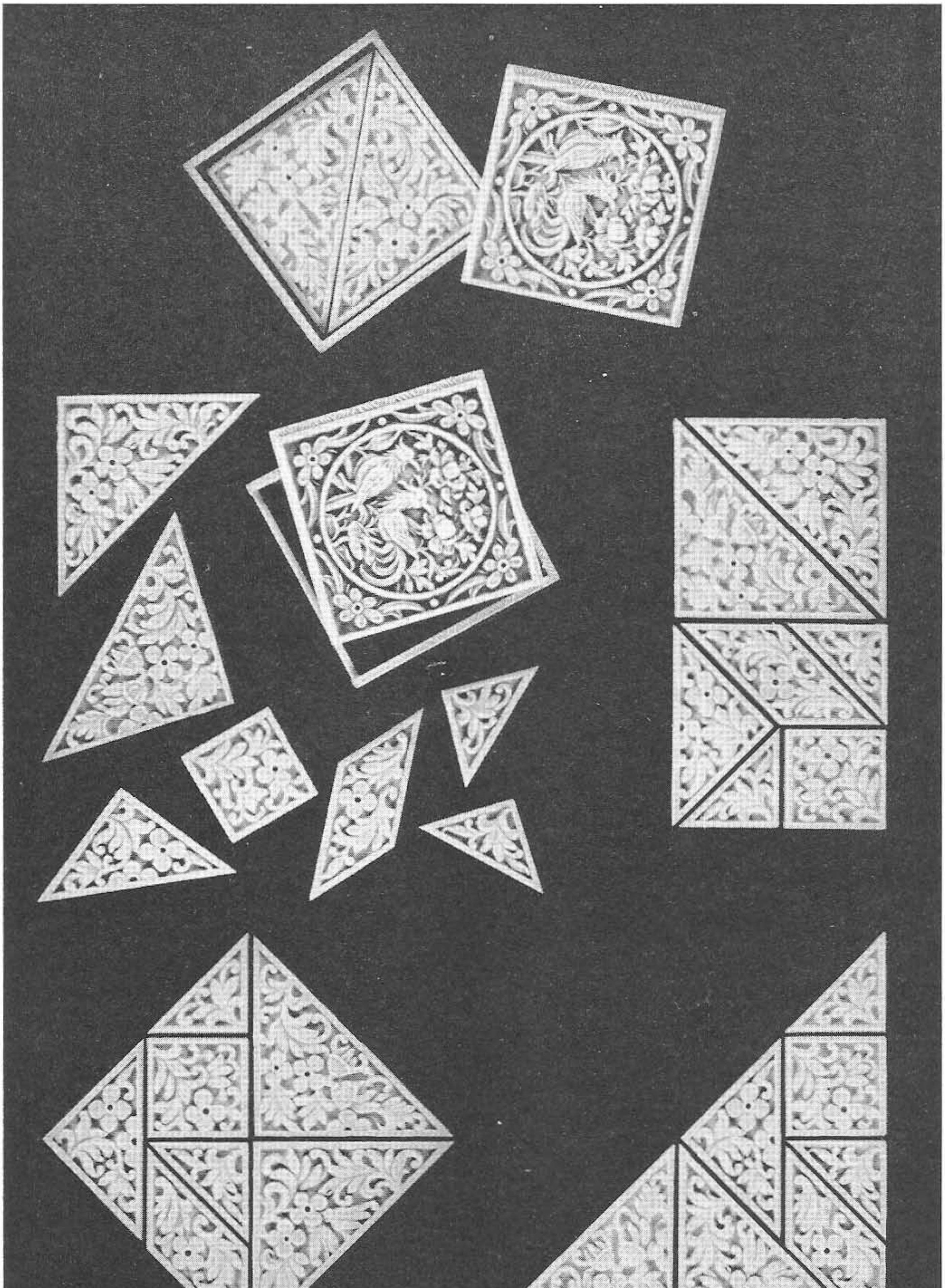


Fig. 31.

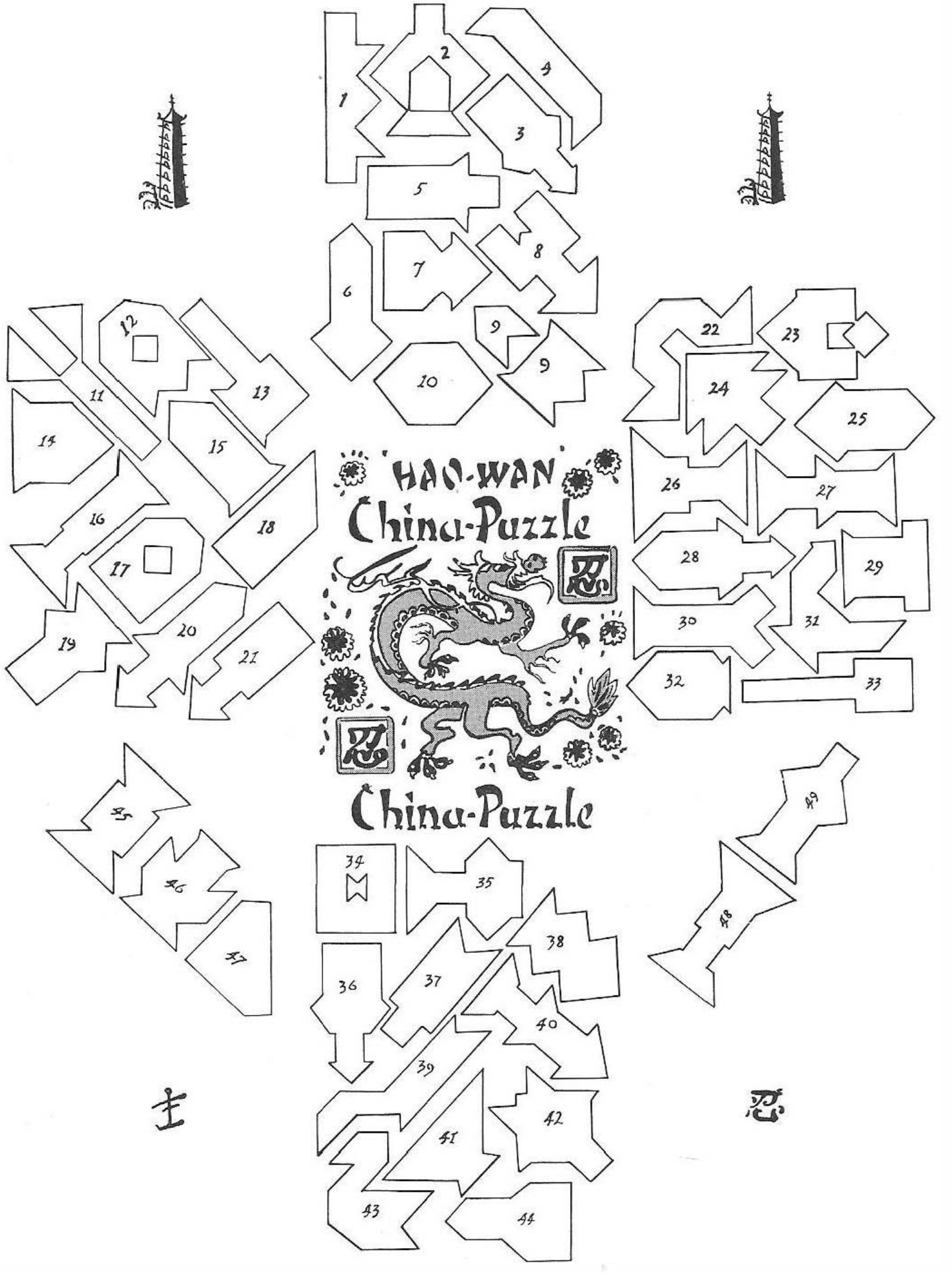


Fig. 32.

have accompanied the game from China or whether the 7 × 7 shapes are purely the result of coincidence I do not know, but as the photograph shows, the game comprises now 7 bricks and not 8 as it must have done originally. A closer look, with the smallest brick as the basic unit, reveals that the game actually comprises 16 small triangles, in other words 2 × 8, some of the triangles merely having been stuck together to produce the sacred and universal figure 7.

The bricks have, however, been so designed that even with the 7 units one is able to assemble the main shapes, and Fig. 31 shows photographs of the bricks laid out in the form of a square, rectangle and triangle.

Toying around with the 8 bricks, our observer comes repeatedly across a new geometric shape he has not paid much previous notice to: the rectangle. This shape has a short side of 1 and a long side of 2.

Naturally, as with other discoveries, he has made contact with this shape before but it was not until he began dividing up the square that the rectangle with these proportions stood apart from the main stream of thought, and appeared as an individual factor—which he now takes on one side as a subject of special study.

He has already seen how two squares together give one rectangle, and he has seen how this rectangle can be converted into one large square.

He has discovered that by transposing his bricks he can produce innumerable combinations of geometric shapes, all of which have the same area and some of which reveal certain information he did not previously possess.

Now he divides his square into two rectangles by drawing a centre line down through the square, and he again divides the resultant rectangles in two by drawing in their diagonals, *Fig. 33.*

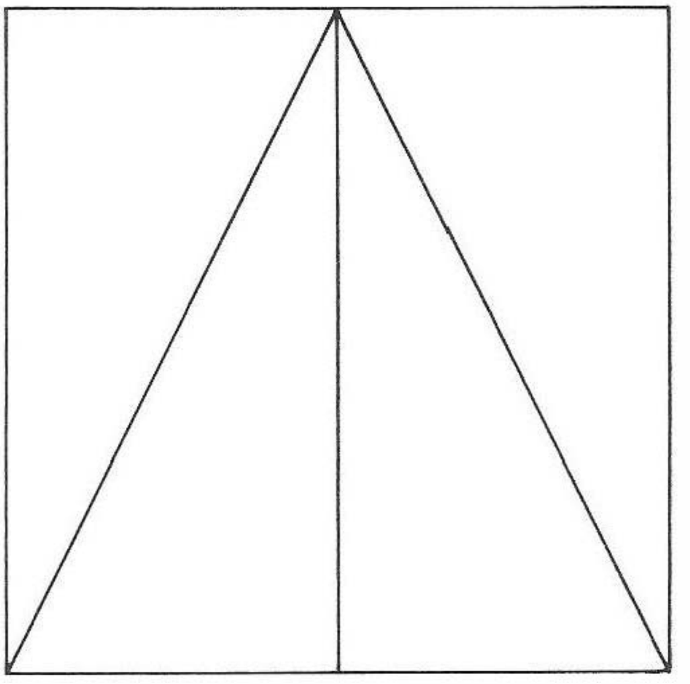


Fig. 33.

Studying this diagram he realises that he has divided his square into 4 equal parts in the same way as he achieved with the cross. But whereas he previously obtained 4 squares, he now finds he has 4 triangles of a completely new shape, this triangle having 3 uneven angles whereas the earlier triangles had 2 equal angles. He knows nothing of the concept of angles as such apart from realising the difference between the new and the old shape, but the form of this newcomer catches his attention and—benefiting from his earlier experience—he fashions a new set of 4 bricks, 4 acute-angled triangles, to see whether they can teach him anything.

The first combination he lays out is *Fig. 34* which really does not tell him much or provide any inspiration.

He then proceeds to place the bricks in the form of a square, the hypotenuses bounding an inner square, the short perpendicular of one triangle and the long perpendicular of another forming the side of the outer square, *Fig. 35.*

This shape, too, holds little information for him, apart from the fact that by turning the bricks through 180 degrees he can change the inner square to the outside

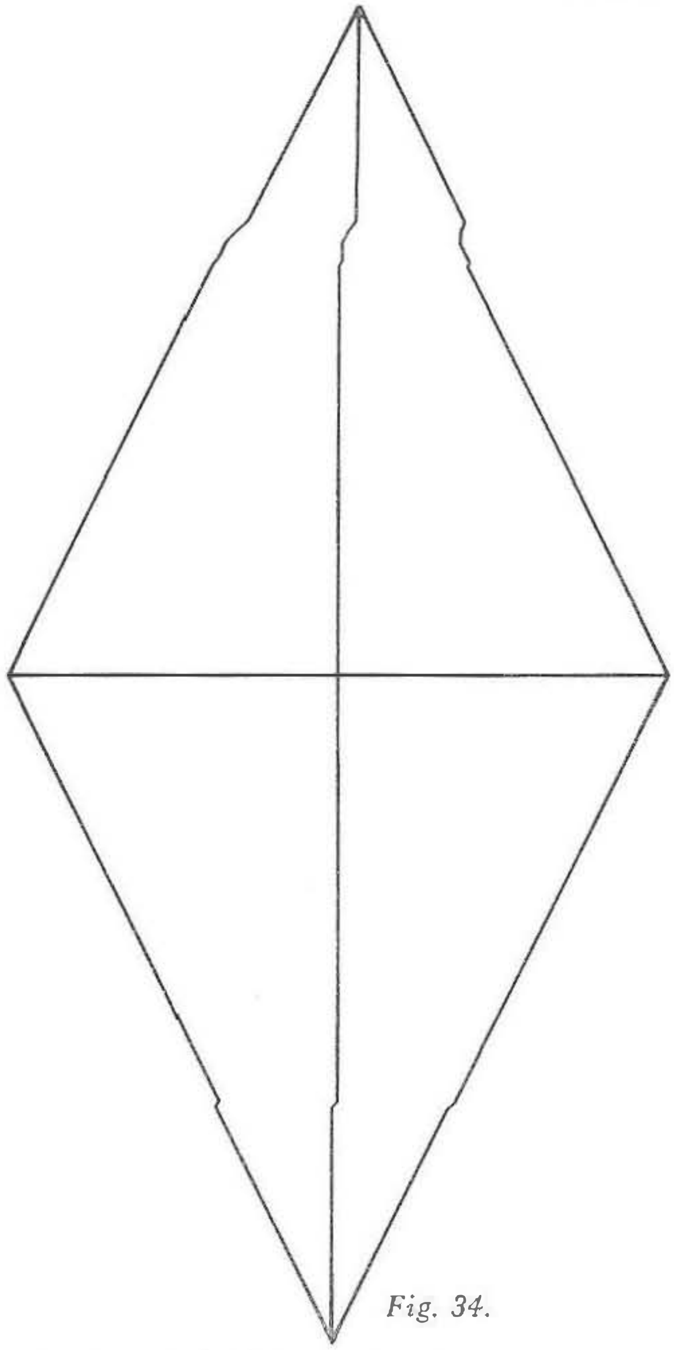


Fig. 34.

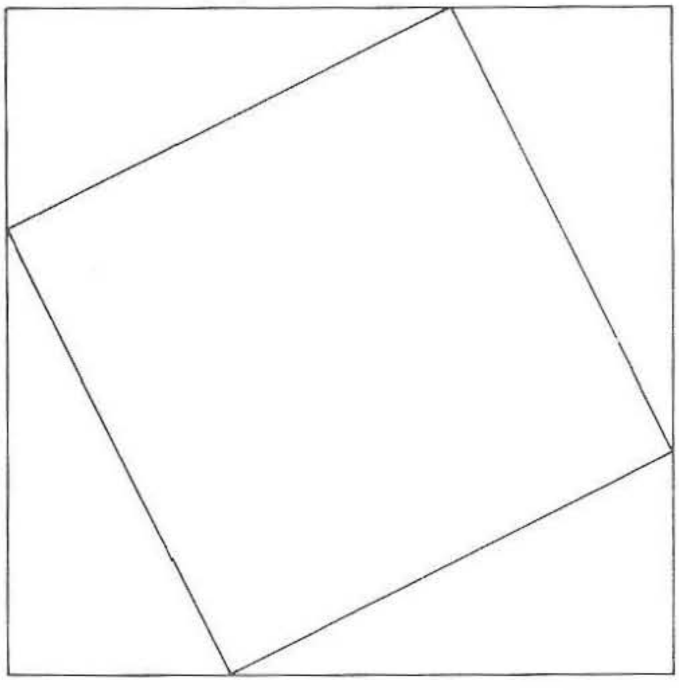


Fig. 35.

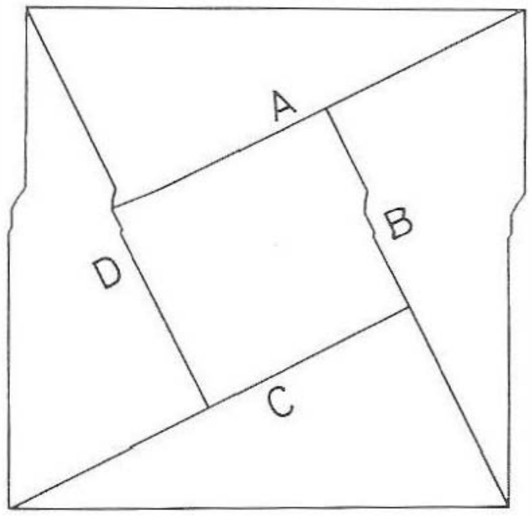


Fig. 36.

and obtain an even smaller square in the centre, *Fig. 36.*

But he does not—as with his other bricks—arrive at any regular geometric shape that can tell him anything new.

As he seems unable to make much progress on these lines, he now decides as he did earlier to try making a few measurements, and therefore turns his attention back to his sketches on the ground. He has at least learned one fact from his triangles with the acute angle.

He has been able to see how the triangle for the smaller square can be constructed from a given square without any awkward measurement.

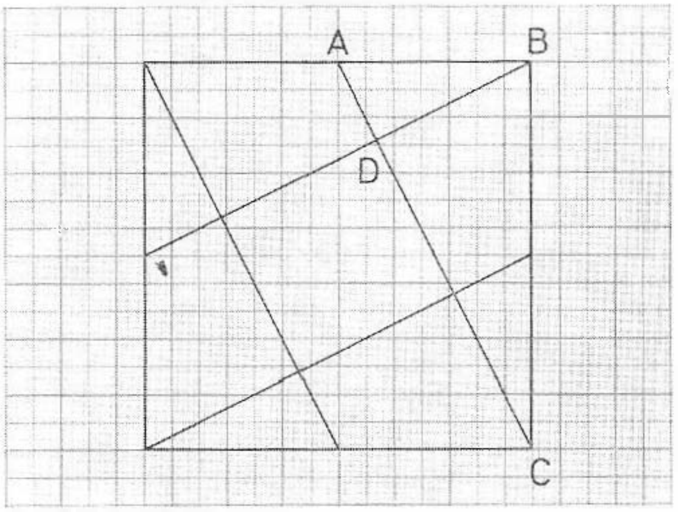


Fig. 37.

In *Fig. 37* we see this done, with an oblique line drawn from the middle of each side-line in the square to one of the

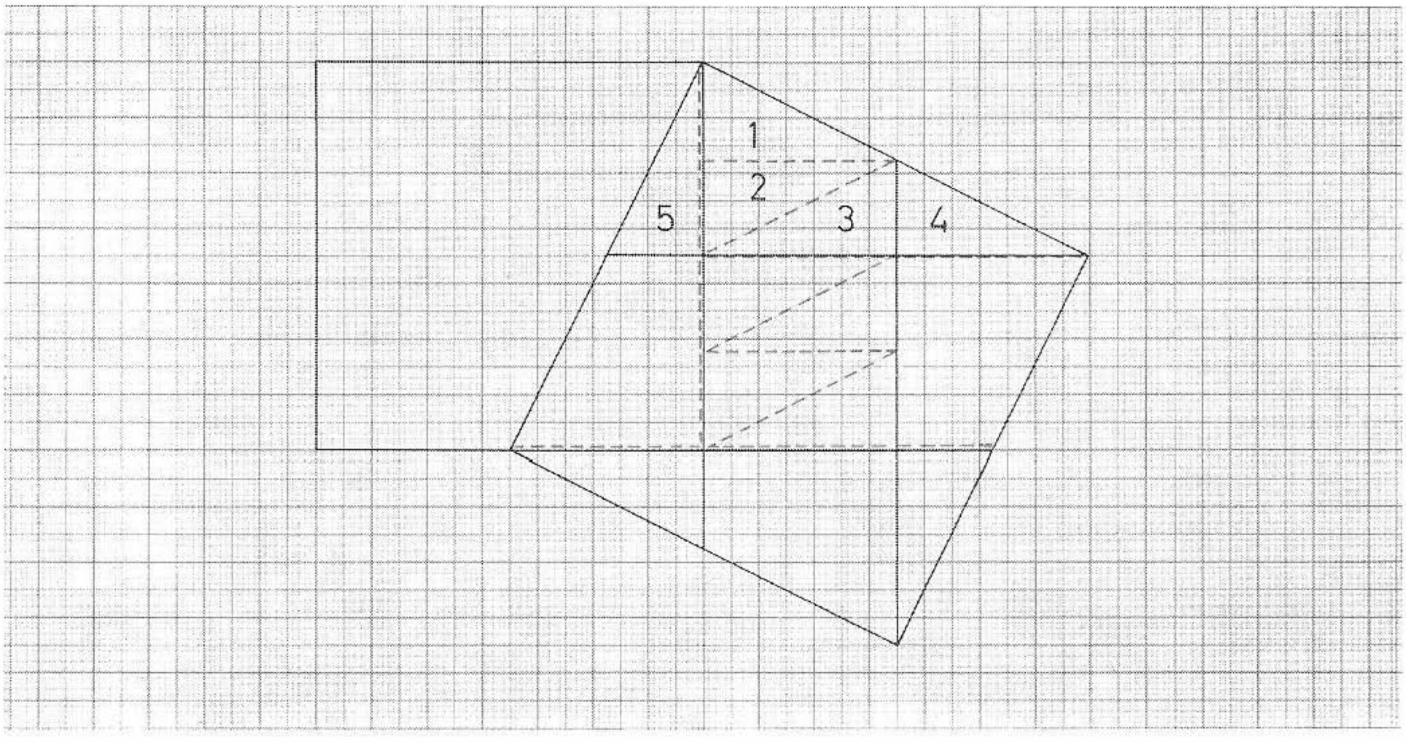


Fig. 38.

opposite corners. This divides the original acute-angled triangle ABC in which B is 90 degrees into a new acute-angled triangle BCD, which has the same shape as the first but in which D is now the angle of 90 degrees.

Line BC, which was previously the long perpendicular, now becomes the hypotenuse of the new triangle, and angle ABD becomes the tip of the adjacent acute-angled triangle ABD.

We see in Fig. 35 that it is possible for him, using as his base the hypotenuse of the acute-angled triangle in a given square, to construct a new square which is larger than the first. But at the same time he can see by how much the new square is greater than the first, since all 4 acute-angled triangles have been drawn within the new square—forming a central square which is thus the unit of increase on the original square.

Taking this as his basis, he now constructs square B with a base of 7 and uses the hypotenuse of its acute-angled triangle as the side of a new square, drawing his latest discovery upon this diagram, the original perpendicular of 7 coming to rest inside the diagram, *Fig. 38*.

If he wishes to measure the area of this diagram he finds he cannot use his normal triangles as he did earlier. He can by all means use them to discover the proportion of the inner square but he cannot apply them to his acute-angled triangles. On the other hand, he is quick to see that the acute-angled triangles can be split up into smaller acute-angled triangles, and he discovers that the original acute-angled triangle contains 4 similar-sized triangles, namely 1, 2, 3 and 4.

When—as shown—it is turned so that the hypotenuse becomes the outer line instead of the perpendicular, it increases in size by one triangle and now comprises 5 triangles, meaning that the triangle has become ⅕ larger than before. If the triangle has increased by ⅕ it follows that the square has also increased by the same amount, a fact which he can readily check since the central small square can easily be split up into 4 triangles of the size in question.

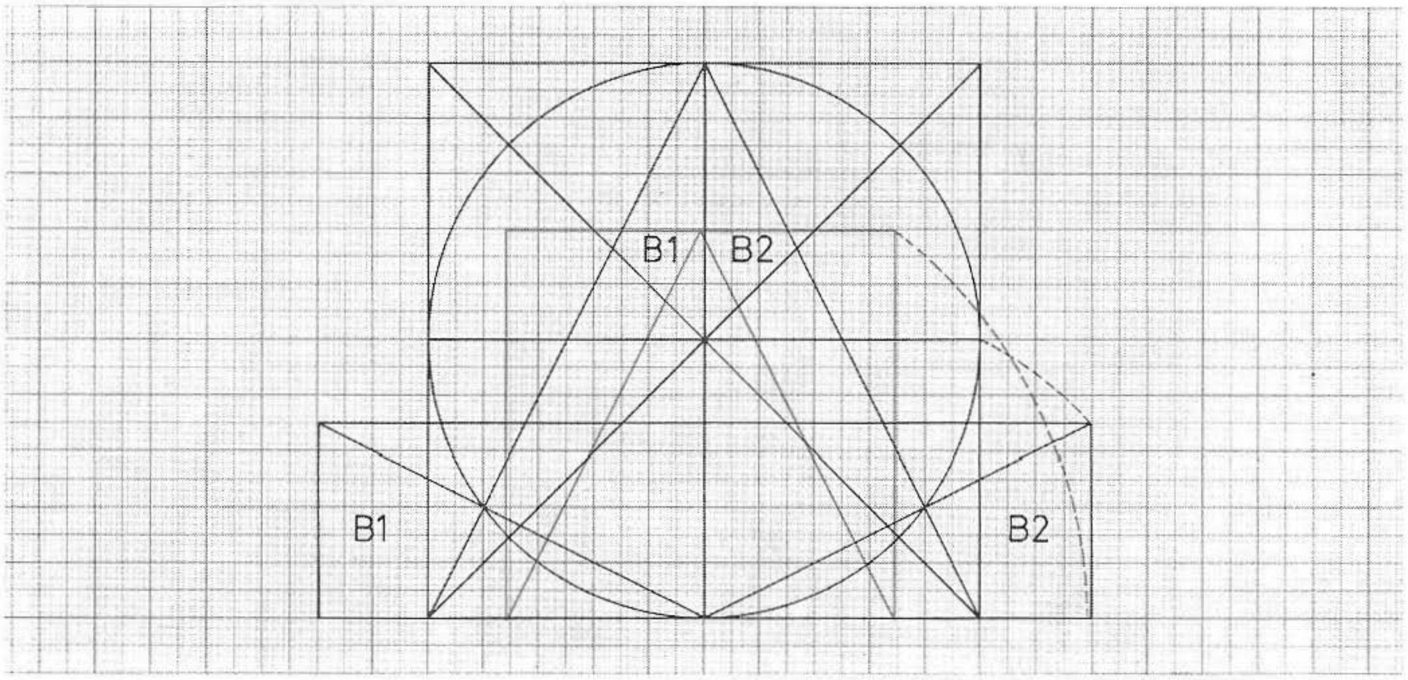


Fig. 39.

He has thus been able to ascertain using whole numbers that a square increases in size by $\frac{1}{5}$ when it is constructed on the hypotenuse of the acute-angled triangle of the given square. This applies only where the perpendiculars are in the ratio 1 to 2 but there is scarcely any doubt that these ancient speculations formed the basis for the theorem much later expounded by Pythagoras that $a^2 + b^2 = c^2$. I do not intend at this stage to go in detail into the mathematical basis from which Pythagoras worked as this will be the subject of later study, but I shall confine myself to indicating that the first speculations concerned only the relationship between the 2 squares, namely that on the long perpendicular and that on the hypotenuse. At some later date Pythagoras included the square on the short perpendicular in his considerations, and discovered that in any right-angled triangle the sum of the squares on the short and long perpendiculars was in the ratio of 1 to 1 to the square on the hypotenuse. But the universally known theorem of our day probably began its development many centuries before Pythagoras's era, and could be deduced by the exclusive use of whole numbers and with no knowledge whatever of involution. In order to try and break new ground our observer uses his standard of measure to measure the sides of the new, larger square with the object of discovering whether they bear any relation to the numbers he already has at his fingertips.

He finds that the side of the new square is longer than 7 but slightly shorter than 8, giving him a perimeter length for the whole square of slightly less than 4×8.

He now recalls from his earlier experiences described in Figs. 17 and 18 how he found that a circle inscribed in a square with a side-length of 10 measured in area somewhere between the circle's outer square and the circle's inner square, i.e. a total base-length or perimeter of between 28 and 32, or in the case of the quarter-circle a size somewhere in between the squares' 2 side-lengths of between 7 and 8.

This needs examination in more detail and he now produces a diagram in which he can measure the side-length of the new square in relation to $\frac{1}{4}$ of the circle's circumference in his old square, *Fig. 39.*

First he draws square C and circle C. The square has a side-length of 10, which is also the diameter of the circle. He then constructs square B, but instead of giving it the same centre as square C, he lets it sink to the bottom of the diagram since he must get as close as possible to the circle's circumference with his diagonals. As the diagram shows, he achieves this best by splitting square B down the centre line and laying B_1 and B_2 out along the base-line, obtaining the closest possible contact between the diagonals of B_1 and B_2 and the arc of the circle.

Now—on his diagram—he begins a minute measuring of these two diagonals, reducing to the smallest unit of measure he possesses. He perhaps lays a series of tiny square tacks along the lines and along the quarter arc of the circle (or he possibly uses some other method of measuring), and finally makes the realisation that the diagonal and the arc are identical in length. He is overwhelmed with delight, for finally after innumerable years' slaving he has succeeded in pinning the circle's circumference down to something he can understand.

The matter goes before the Temple Superior who, after careful experiment, eventually acknowledges the result and our observer is made the subject of much praise and recognition. Banquets are held to mark the great achievement.

Subsequent chapters will show that this in all probability was the manner in which the result was obtained. But how does the result stand up to present-day scrutiny, for our observer's claim is nothing less of course than the squaring of the circle. Using only a ruler and compasses and sticking strictly to geometric diagrams our observer has apparently reached the stage where he can draw a square with the same perimeter length as a given circle, and we know of course that this is a problem which has not been solved and which probably cannot be solved. But let us see how close to perfection they have come with their geometry. We know that the circle has a diameter of 10, and the circumference is $d \times \pi$. As we want to measure the quarter arc of the circle, our calculation must be as follows:

$$\frac{d \times \pi}{4} = \frac{10 \times 3.1459}{4} = 7.8539$$

Now we must find the diagonal or hypotenuse of square B. But to do this we must establish the length of the long perpendicular or square B's side, which is:

$$\frac{10}{\sqrt{2}} = 7.071068$$

The short perpendicular will thus be:

$$\frac{5}{\sqrt{2}} = 3.535534$$

If we call the perpendiculars A and B respectively, and the hypotenuse c, we get:

$$a^2 + b^2 = (\frac{10}{\sqrt{2}})^2 + (\frac{5}{\sqrt{2}})^2 = 62.5 = c^2$$

$$c = \sqrt{62.5} = 7.9056$$

which means that in a present-day calculation using a modern mathematics system a difference can be found of 7.9056 minus 7.8539 = 0.0517 or approx. 0.6 % on the results achieved by mathematicians of antiquity, using a relatively primitive means of measurement. If we assume that the circle has a diameter of 1 meter and thus a circumference of 3.14159 meters, the quarter circle has a length of 0.78539 meter or 785.39 millimeters. The hypotenuse in the acute-angled triangle has a calculated length of 790.56 mm, and the difference between the two lines is thus 5.17 mm.

I wonder if it would be possible by measurement to achieve a closer result even today? This after all was a case of measuring a line of 1 m on the ground, and an error of 5 mm would scarcely have been recordable; it would perhaps have been possible on the straight line, but not on the curved line without tremendous trouble.

It was thus possible for those ancient philosophers to solve the riddle of squaring the circle—dealing meanwhile only with the circumference—to within 0.6 % accuracy, calculating the circle to be 0.6 % larger than we reckon today.

The problem has surrendered to pure geometry, with ruler and compasses and comparative measurement, with no necessity for either a greatly advanced system of counting or any real standard of measure.

Yes, there was every reason for rejoicing and pride within the Temple walls.

An outstanding discovery of this calibre, of ascertaining the circumference of a given circle by following the principle of comparative measure—permitting the mathematician to construct a circle with the same perimeter as a given square, or a square with the same perimeter length as a given circle—had naturally opened up a well of speculation: how to express the new discovery most conveniently in a geometric diagram? It must take every associated factor into account and fulfil the following demands:

1. it must be possible to construct the diagram without resort to unwieldy measurement;
2. the diagram must express the facts as clearly as possible;
3. at the same time the diagram must be capable of informing only initiated brethren, since of course not everyone should possess this information, and the diagram must reveal nothing if it should fall into the hands of outsiders.

The diagram probably looked like the one in *Fig. 40* in which we see our familiar squares A, B and C, B's side being determined by A's diagonal, and C's side determined by B's diagonal, the result being 2 × A = B and 2 × B = C.

In his previous measurement (Fig. 39) our observer established that the hypotenuse of the right-angled triangle in square C, which in this diagram is from point 7 to 5, is identical in length to ¼ of the circle drawn in square B. In other words line 7—5 is the same length as the arc of the circle which goes from corner to corner in square B with point 6 as its centre. In fact, the diagram is identical to Fig. 39.

Our observer has thus fulfilled his aim precisely. He can establish the lengths of the lines within the framework of familiar factors, i.e. his three squares. And since the diagram depends heavily on the relationship between square B and square C he is delighted to be able to establish this fact.

Therefore he terms the point at which square B cuts the base-line of square C "The Sacred Cut" in any given line. This point is thus position 8 on square C's base 5—6.

The importance of the point is obvious. For one thing, it marks out the length of base on which to construct a square exactly half the area of square C; secondly, the length of this base and therefore the point itself will be 7, if square C is 10; and thirdly, it indicates the radius for arc 1—8 which is identical in length to line 7—5 in square C.

The point is truly of immense value. We have often come across it previously without its being specially emphasised, and we shall be seeing it more and more in the future.

We see in *Fig. 41* the possible procedure for the geometric measurement of a given circle.

We have the given circle A, outside which we draw the surrounding square B. Square B is divided by the cross into 4 smaller squares B_1, B_2, B_3 and B_4.

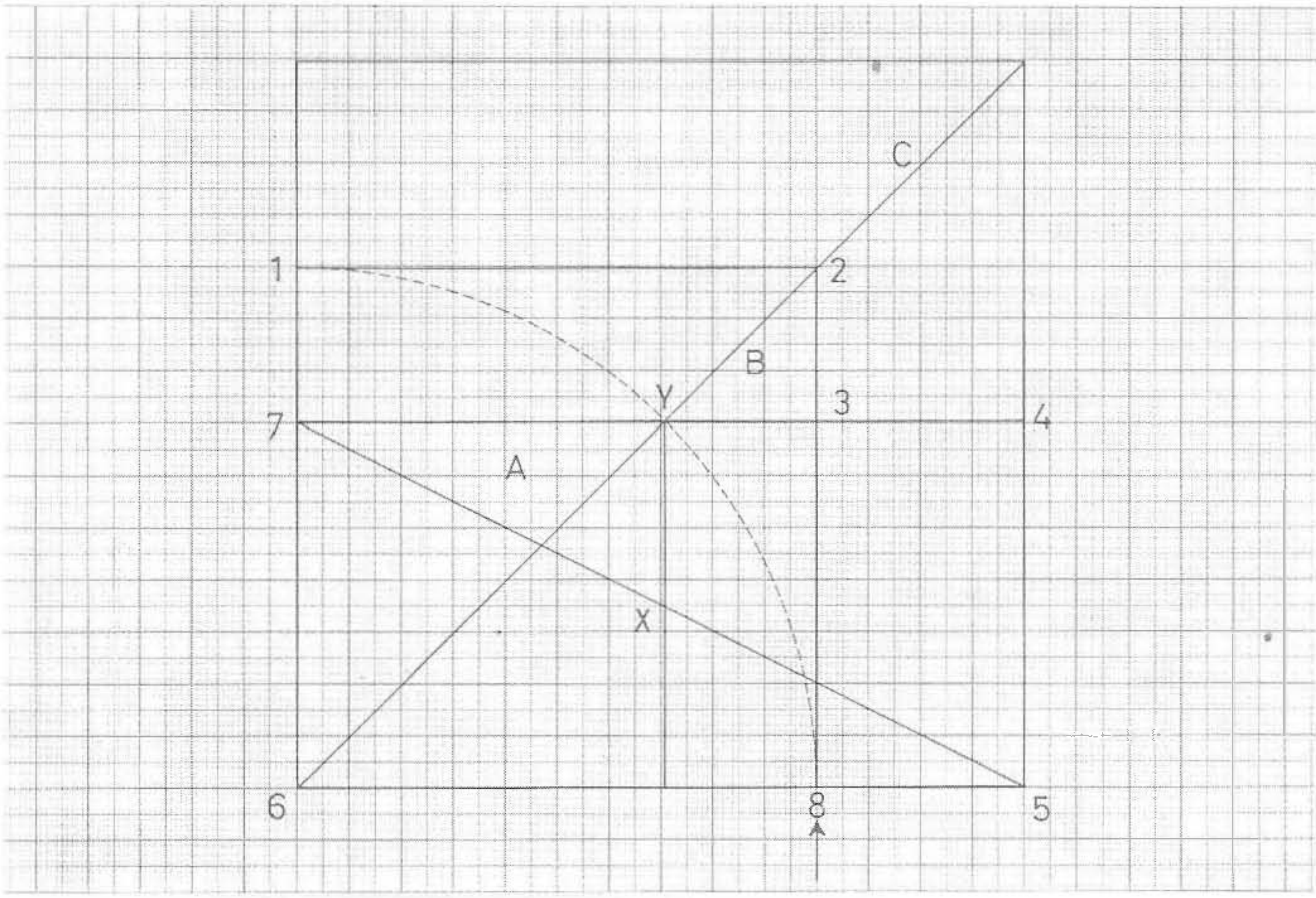


Fig. 40.

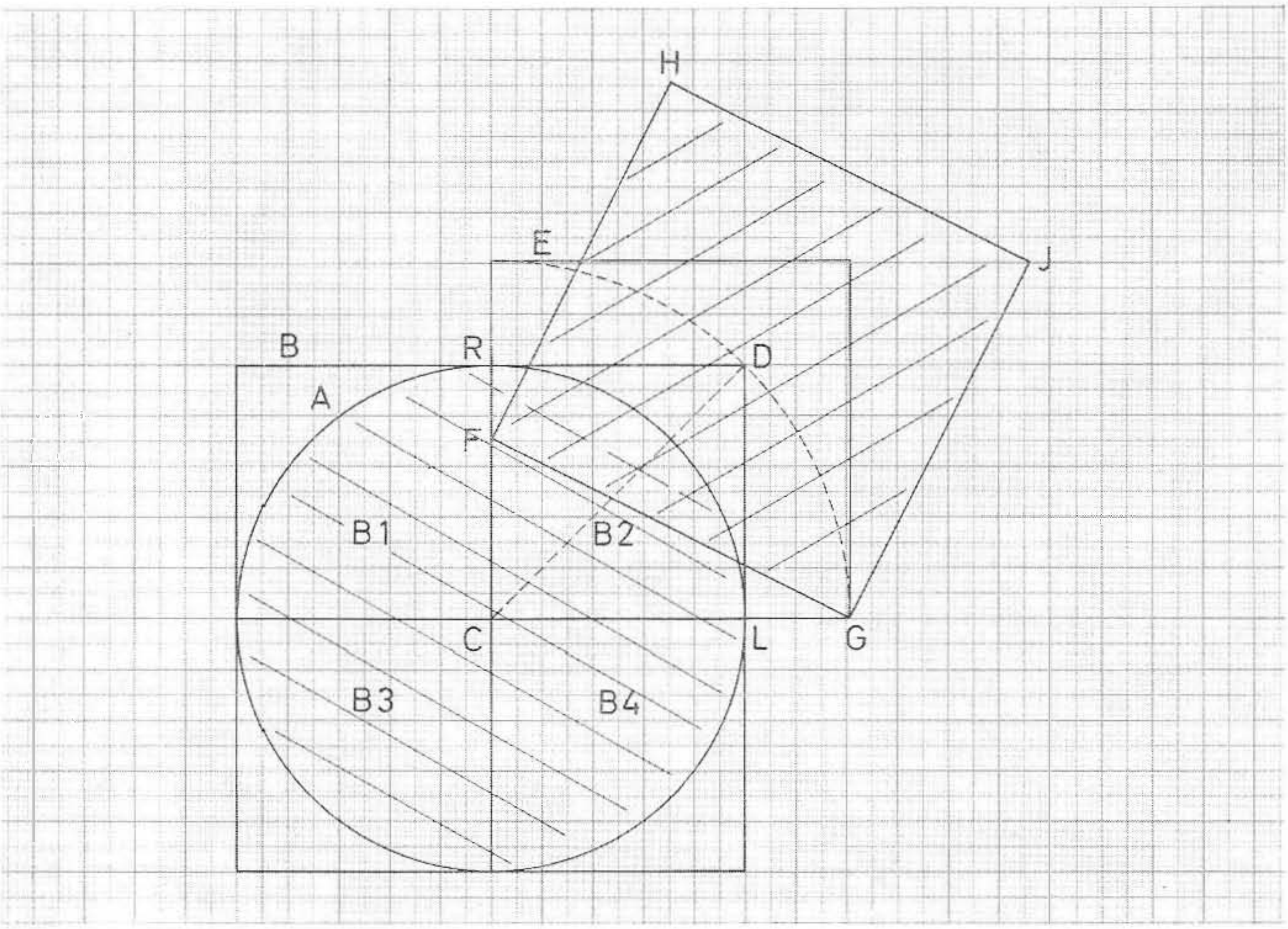


Fig. 41.

We now take diagonal CD of the square B_2 and place it horizontally and vertically, thus constructing square E. This latter square is therefore twice the size of B_2 and half as large as the original square B, and is also the same size of square as can be constructed from tip to tip of the cross in the large square.

We now enter the diagonal FG in square E and use this as the base for a new square, indicated as FGJH. The perimeter length of this square should then be the same as the circumference of circle A, and the line FG which is a quarter of the total FGJH is the same as the quarter circle in square B_2 joined at R and L.

By producing this construction on a given circle our observer is able to measure out the quarter circle on the line FG with the same degree of accuracy as indicated earlier.

We shall later be coming across the diagram in Fig. 40 more and more often and in various guises, as it represented one of the main factors in the practical application of ancient mathematics, and we shall see evidence of the sacred cut and its proportions in many spheres of building. But the proportions revealed by the sacred cut can be traced elsewhere than merely in the strictly practical world. It is discussed in Greek speculative philosophy, a period very much later on in time—granted—from that which we are studying at present, but the fact that Greek thinkers busied themselves with this diagram illustrates graphically its ability to endure over thousands of years.

Plato discusses it in his *Timaeus,* saying:

"Now anything that has come to be must be corporeal, visible, and tangible: but nothing can be visible without fire, nor tangible without solidarity, and nothing can be solid without earth. So god, when he began to put together the body of the universe, made it of fire and earth. But it is not possible to combine two things properly without a third to act as a bond to hold them together. And the best bond is one that effects the closest unity between itself and the terms it is combining; and this is best done by a *continued geometrical proportion.* For whenever you have three *cube* or *square* numbers with a middle term such that the first term is to it as it is to the third term, and conversely what the third term is to the mean the mean is to the first term, then since the middle becomes first and last and similarly the first and last become middle, it will follow necessarily that all can stand in the same relation to each other, and in so doing achieve unity together."

In attempting to analyse this text, we must realise that it has been subjected to a number of translations over the centuries, and if the respective translators have not possessed a comprehensive understanding of the material, difficulties may have crept into the finished versions in the way of wrongly positioned words.

But if we start at the beginning of the quotation, Plato says first that anything that has come to be must be corporeal, visible, and tangible. Thus he establishes straight away that he is referring to something concrete and not something in the realms of abstract thought.

He then says that the universe was created from three parts, i.e. two main parts which, however, cannot be satisfactorily bound together without a third part which, by continued geometrical proportion, combines the initial two parts such that they through the middle part form a united whole.

He continues: "for whenever you have three cube or square numbers with a middle term ...". I think this is an error in translation which has been perpetrated by most students on the subject, resulting in philosophers and mathematicians speculating furiously over this passage since they conclude that Plato is speaking of numbers, and they have consequently tried to fit his text into a strictly numerical background which has to a certain extent proved successful.

But I think the translation is wrong in this respect. In my opinion this passage should read, "For whenever you have three cubes or squares with a middle term ...".

The sentence has not undergone a radical change, indeed one might say there has been no change whatever, but the mind now is put on the track of cubes and squares instead of numbers. I think the big mistake with this particular passage is to speculate exclusively on numbers. And to ignore the main allusion in the quotation—that it concerns cubes and squares.

The text continues with: "... such that the first term is to it as it is to the third term, and conversely what the third term is to the mean the mean is to the first term". If we now examine the passage in the light of Fig. 40, which indicated the sacred cut in a square, we find three squares: the small, the middle and the large.

We shall call the large square C the first, the second square B the middle, and the small square A the third.

The first square C is to the middle square B as this is to the third square A, as

$$2 \times A = B \quad \text{and } 2 \times B = C\text{, then}$$

$$\frac{C}{2} = B \quad \text{and } \frac{B}{2} = A$$

and the terms of Plato's text are fully met, both as regards proportion and the fact that they concern cubes or squares.

We have dealt with squares. The text continues: "... then since the middle becomes first and last and similarly the first and last become middle, it will follow necessarily that all can stand in the same relation to each other". This part should be viewed on a background of the geometric knowledge prevalent at the time.

The thing here of course is a certain relationship of size between each of the three squares, but with no particular size for the individual squares. If you work down the scale, both B and A can become relatively speaking the largest square, and if you work up the scale, C can become the smallest or middle square.

The sentence is intended as yet another reminder of the mutual relationship of the three squares, without mentioning any actual size.

This particular passage we have quoted and analysed has been plucked from a wider context, but the whole text indeed illustrates clearly that the subject matter is not outwith the grasp of ordinary mortals, but is taken as read by initiated brethren.

The complete Plato text discusses some of the elementary and basic rules of geometry, and is also touched upon earlier in the book as an introduction to the story.

Plato speaks of Timaeus as being a famous philosopher from the city of Locris in the Greek area of southern Italy, and he gives a lecture on the universe for the initiated brethren Socrates, Critias and Hermocrates, and a fifth brother, not named but probably Plato himself.

Timaeus is described as an astronomer of the philosophical school, with strong links with the Pythagorean line and a sweep of the learning of Empedocles, but it is a fact that all associates had been connected with mystic schools and were thus initiated brethren.

All initiated students learned geometry according to the methods evolved over thousands of years, and I shall be showing later how the same Timaeus ridicules the new thoughts which were on the point of breaking into mathematics, having slipped out of the Temple and mystic schools of learning and become common knowledge.

Innovations, however, take time to establish themselves particularly when they have—as in ancient Greece—to fight against a powerful clique of initiates from mystic schools of learning; opponents backed by high reputation, splendid grasp of rhetoric and the tradition of centuries.

Although temples and schools followed certain well-beaten paths in their teaching of geometry and numbers, it is obvious that the trained philosophers who graduated from these institutions applied their knowledge to other philosophical pursuits in the same way as we today employ mathematics and figures in subjects like astronomy, chemistry, physics, etc.

Although Timaeus is described as an astronomer, he has also of course had an extensive knowledge of numbers and geometry, but he is not regarded as a mathematical philosopher and though he introduces mathematics into his speech he does so not to illustrate his knowledge of figures but to illustrate his speech with terms known to all the brothers who heard him.

We know that Socrates was present during Timaeus's discourse, and a most important listener, too; but we are also aware that he was not there as a mathematical expert, and we must therefore assume that the subject under discussion was not an extremely profound and abstract form of mathematical thinking understood only by a handful of specialists, but something accessible to any thinking person not particularly well-versed in mathematics.

If we regard the Plato epic from this angle, then his mathematical postulations fit perfectly. The reader can also see that the discussion is not a mathematical or geometrical speech or lesson. Timaeus's tale is a history of creation, perhaps not the history we recognise today, but a creation story nevertheless—which contains numerous references to geometric situations. Situations it is essential to know in order to make any sense whatever of the actual story.

It was therefore unnecessary for Timaeus to illustrate his mathematical explanations in detail. After all, his listeners were initiates and as such ought to have sufficient insight of the geometric proportions under discussion.

Space alone prevents repetition of the whole story since of course lots of material is of minimum interest as regards geometry. We shall look briefly however at the main theme since it has a certain association with squares and their ability to change into triangles, acute-angled triangles, rectangles, small squares, etc. This history of creation begins by explaining how the four basic substances, fire, water, earth and air, by means of common connections can fuse together and become twice as great as another material, or split when two meet and transform to three or more.

If, for example, we call the acute-angled triangle fire, and the square is earth, there will be a split in which earth is divided into two elements, e.g. water and air, water being the right-angled triangle, and air the rectangle.

Conversely the same elements can also fuse together to form the square, called of course earth, and similar situations of fusion and fission can be described within this framework.

If we consider this tale in relation to the diagram, we can also understand why this picture in particular and the function of the squares are well-chosen materials for illustration purposes and for portraying the thoughts and postulations on creation and a picture of the world. We see moreover how geometry is used as a purely philosophical material, a comprehension of mathematics being essential to an understanding of the story but mathematics themselves being secondary to the actual story of Creation.

Thus we see how the introduction to Timaeus emerges as a clear-cut and fully illustrated whole when regarded with the knowledge of the initiates. But if the reader does not possess this key, the text and its subject matter are at once incomprehensible—which of course was precisely the desired effect. The knowledge which these initiated brethren had assembled had naturally to be kept out of the hands of outsiders, and this text has through the ages demonstrated its ingenuity in that countless heads have been scratched, numerous able brains racked in an effort to decipher and analyse it—in vain.

Without the key the text remains a closed book, and the theories expounded regarding its meaning become mere guesses which may well succeed in fitting a few details of the passage into things which have in fact nothing to do with the actual text. But there is no other way of matching up the complete text, nor can one make sense of Plato's story in general.

There is but one solution to this text, and its secrets surrender only to the key of geometry.

Down through the ages this piece of text has been the subject of many attempts at interpretation and efforts to uncover the real significance of the wording. But all failed because the passage was removed from its context and subjected to a separate analysis.

In such analyses most investigators stuck blindly to speculation on specific numbers, with mean proportionals, etc. Such speculation in fact belonged to a much later period of Greek development.

Others concluded that the passage in fact discusses special Pythagorean speculation on numbers from which one cannot expect to derive a logical theme, and still others draw parallels between this passage and the section of Euclid's 8th Book in which he discusses the division of a given line in such a way that the shortest piece is to the longest, as the longest is to the whole line.

Regarding the latter comparison it should be borne in mind that when Euclid put his propositions and theories in writing, Plato had been dead for 250 years. And the problem set by Euclid is a straightforward geometric and mathematical arrangement which is comprehensible to anyone, while Plato's text does not apparently yield to logical consideration.

Common to all interpretations of this passage is that their executers have assumed that Plato was referring to actual things; no one seemingly has realised that the concepts under discussion are in fact the symbols or pictures of these things.

The text is part of a larger whole, and a most important part, being the introduction to a lecture on the creation of the World. In this introduction Timaeus relates how the Universe is constructed and of which materials.

Just as we must resort to numbers when we discuss, for example, astronomy, so the philosophers of ancient days turned to geometry when they wanted to expound their theories on such an abstract theme as a picture of the World's creation. But Timaeus was in the unfortunate situation of not being permitted to describe the occult form of geometry practised by himself and his confederates, instead being forced to attribute special names and terms to the geometric symbols he wished to employ.

This procedure was doubtless familiar to all initiates who, at the mention of the various symbolic names, could nod appreciatively at the terms and descriptions they knew so well—whereas the speech to outsiders must have sounded, to say the least, mysterious.

The passage therefore deals with the

symbols *of symbols*, geometric shapes being the speculative basic element of the Creation. But since the geometric symbols themselves were taboo to all outsiders, they were disguised by words which represented the symbols and terms of geometry.

Before Timaeus sets out on his actual introduction to the symbols he states expressly in a foreword that he will later be referring to likenesses and their pattern.

Of the World he says:

"It must have been constructed on the pattern of what is apprehensible by reason and understanding and eternally unchanging; from which again it follows that the world is a likeness of something else. Now it is always most important to begin at the proper place; and therefore we must lay it down that the words in which likeness and pattern are described will be of the same order as that which they describe. Thus a description of what is changeless, fixed and clearly intelligible will be changeless and fixed—will be, that is, as irrefutable and uncontrovertible as a description in words can be; but analogously a description of a likeness of the changeless, being a description of a mere likeness will be merely likely; for being has to becoming the same relation as truth to belief. Don't therefore be surprised, Socrates, if on many matters concerning the gods and the whole world of change we are unable in every respect and on every occasion to render a consistent and accurate account. You must be satisfied if our account is as likely as any, remembering that both I and you who are sitting in judgement on it are merely human, and should not look for anything more than a likely story in such matters."

In this prelude Timaeus is indicating quite clearly that the descriptions will relate to likenesses and not to actual objects. The inaccuracies to which he refers appear later in his lengthy account, and will be illuminated in Chapter Ten.

But at this stage our conclusion must be that the subject matter is likenesses not numerical speculation; when the subject switches to squares and cubes this is not a reference to a mysterious Pythagorean theory but purely and simply to squares, triangles and rectangles, the latter two being termed cubes.

And Timaeus's introduction itself indicates not that what he will say is in a specifically mathematical field, accessible only to specialists, but that he will talk of likenesses and that these—used as descriptions—are common knowledge among his listeners.

CHAPTER FIVE

The Circle's rectangle and triangle

The previous chapters have followed a series of development originating in the most primitive observations, discovery being added to discovery, building up a gradual understanding of geometry and numbers until our observer pauses at a distinct milestone—his ability to find the circumference of a circle.

This knowledge is contained in the diagram showing the three interrelated squares with the sacred cut on the base line (Fig. 40).

The trail followed has invariably been that of logic, and at no time has our observer operated with numbers or values outside his field of comprehension, at no time has he employed methods of calculation which belong to a later era, and yet his primitive system has brought him a long way from his point of origin. He has been able to achieve a number of exact deductions with complete accuracy; and a number of deductions so near perfection that their accuracy must be regarded as remarkable bearing in mind the stage of development of his numbers, a development which has not yet broken into the field of fractions or decimals, nor has it provided any other means of achieving complete accuracy. And yet it has been possible to establish a number of facts absolutely correctly and others to within 1 % accuracy, facts which we today might tend to assume could not be expressed in anything other than fractional values. His successes have been founded on the principle of comparative measure, based on the sacred circle and the number 7, so it is not really surprising that both of these factors are to play a vital role in esoteric speculation over the next few thousand years.

But we can still follow in the footsteps of our ancient observer for some time yet, for the prime object—to produce a true picture of the circle's area—beckons ahead. The problem still has not been solved and naturally stirs him on to better things. His experiments and experiences with squares A, B and C tell him that if A is divided up by the vertical and diagonal cross it splits into eight triangles. Square B has twice as many triangles and must therefore be twice as big since the triangles are the same size, and the same ratio applies from square B to C.

He has thus learned how to measure the squares in relation to each other by triangulation.

On the other hand he has observed that it is not feasible to construct the squares inside each other on the same base and then to split them into triangles, since his dividing line does not match in with the sides of his squares, instead cutting large and small portions off the triangles and providing a division which he cannot accurately establish.

One thing he has really got the hang of, however, is the principle of comparative measure, and if he now wants to pinpoint the area of the circle he can choose either to find a relationship between the circle and its inner square or a relationship between the circle and its outer square.

I am convinced that he selected the latter, partly because later evidence will support the belief, and partly because it is the easier of the two.

He has probably tried innumerable variations with his construction, but to make any headway at all he must have some idea of how great the area of the circle is in relation to a constant, whether that constant is the outer or the inner square.

He therefore produces a diagram such as *Fig. 42* in which he draws a quarter-circle inside a square. Guided by previous experience, he divides this square into triangles of some suitable size.

We have chosen to divide the square up into 1024 triangles in the interests of clarity, but a coarser or finer graduation is possible. And still the whole diagram has been assembled without any form of measure, other than the cross in the large square.

Our observer now tries counting up the triangles which lie outside the arc of the circle, having divided this outer area up —to make things easier—into three parts, A which is the corner, and B and C which are two identical areas running along the arc. He finally reaches this total:

A	contains	162	triangles
B	,,	21.5	triangles
C	,,	21.5	triangles
		205.0	triangles

Compared with the numbers he has been accustomed to working with, these figures are of course huge. But let us assume that he has been able to count them up, and that in counting he has been able to reason out the complicated part-triangles, joining them in complete units.

For the first time he has in this job had to work with half sizes, the final result, however, being whole units, namely 205 small triangles lying outside the arc of the circle.

His next step now is to find out how many times these 205 triangles are to be found in the whole square.

He can obtain this information simply by striking out the triangles one by one, of course, but perhaps he has progressed far enough in his arithmetical ability to be able to count it out. This probably was not the case, but with a little imagination the job can be done relying only on the ability to count. Whichever method he uses, however, he achieves the result that to within one triangle, the square is five times as great as the area lying outside the arc, which means that our observer can establish to within one triangle (i.e. $\frac{1}{1024}$) that $\frac{1}{5}$ of the square's area lies outside the arc of the circle.

What applies to a quarter-circle must of course also apply to the remainder of the circle and, since he has been open to reason in calculating the intersected triangles along the arc of the circle thus raising the possibility of a minor error of judgment, it is laid down and accepted that the circle is $\frac{1}{5}$ less in area than the surrounding square.

How well does this assumption tally with the results obtainable by modern mathematics?

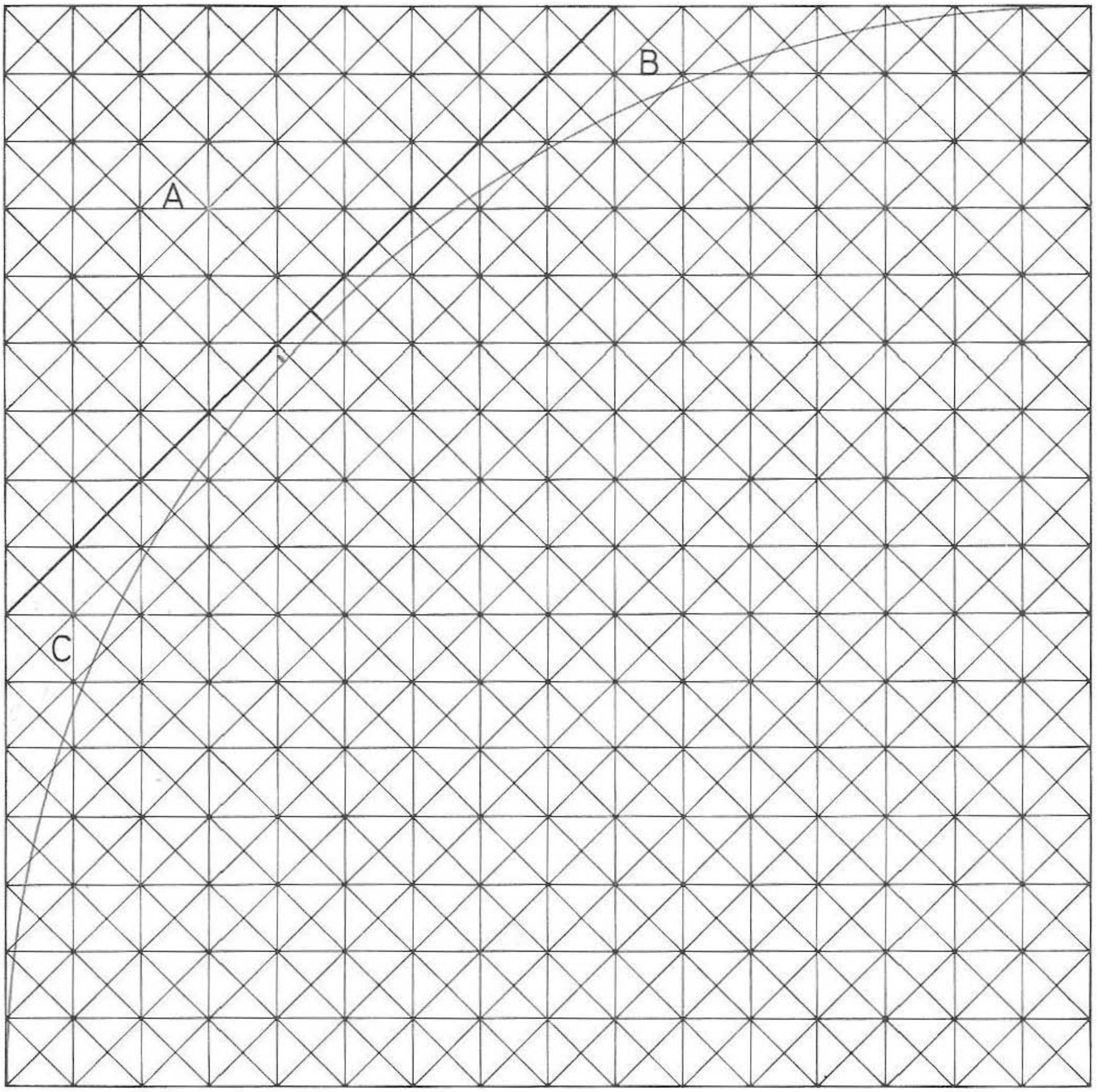
If we take a given square with a side of 10 cm, we get a surface area of $10 \times 10 = 100$. A circle drawn inside the square has a radius of 5, and its area is thus:

$$R^2 \times \pi = 5^2 \times 3.1416 = 78.54$$

By his method, our observer arrives at the following area:

100 for the square,

and for the circle: $\frac{100 \times 4}{5} = 80$



A = 162 B = 21,5 C = 21,5

Fig. 42.

Thus we see an error of a similar degree to some of his other assumptions, namely 1.46 sq.cm., whereby he has judged the area of the circle to be 1.86 % greater than we reckon today.

Against this error of course he has the little triangle that was left over and thrown away after his last experiment. In a whole circle this amounts to 4 triangles, but the effect is negligible in an area of more than four thousand triangles.

What our observer has succeeded in doing is to establish what proportion of the area of the surrounding square is covered by the circle.

Perhaps we think today that this was an odd procedure, but in fact this is precisely how we ourselves calculate the area

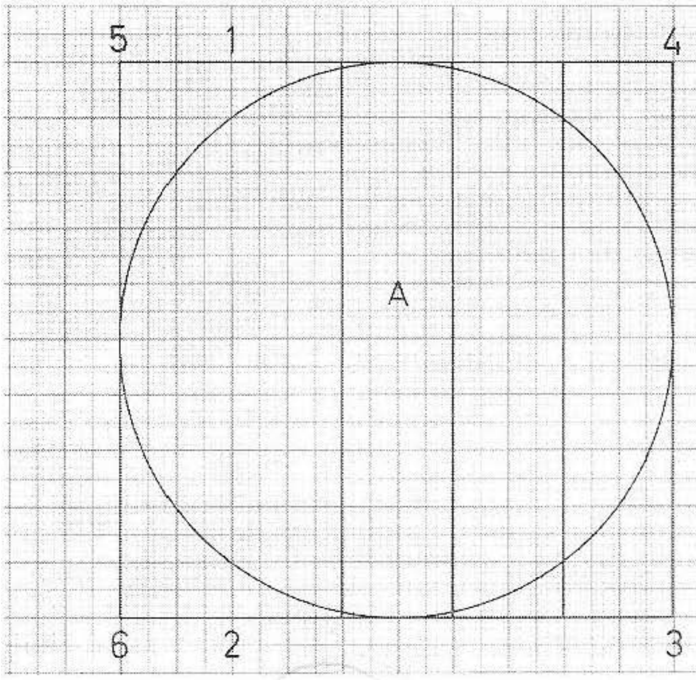


Fig. 43.

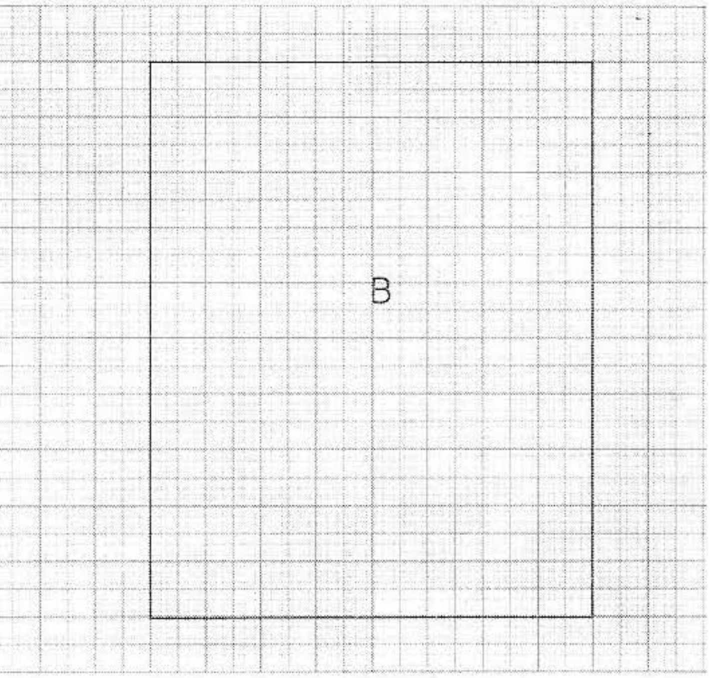


Fig. 44.

of the circle in modern times. We have simply come to accept the term π so readily that we do not pause to consider what it covers. If we split the formula $R^2 \times \pi$ up into two separate units, we see that when we say R^2 it is exactly the same as calculating $\frac{1}{4}$ of the circle's outer square. If we said $R^2 \times 4$ we would arrive at the area of the circle's outer square, but we know that the circle is only a part of the square, and we therefore say

$$\tfrac{1}{4} \text{ square} \times 3.14159 \text{ (or } \pi)$$

by which means we manage to calculate how great the circle's area is in relation to the surrounding square. We save ourselves the trouble of working out the area of the circle, but it is in fact hidden in the term R^2.

The only difference really between the method used in ancient times and that we apply today is that our observer has worked out how much of the square lies outside the circumference of the circle and subtracted this from the whole square, while we calculate how much of the total square is covered by the circle. The procedures are slightly but not radically different since both are based on the ratio of the square to its inner circle. The difference is that the old method maintained that the circle was $\frac{4}{5}$ af the square, whereas we have a more accurate figure which claims the circle to be $\frac{3.14159}{4}$ of the square, a difference of opinion of 1.86 %.

The approximation is in fact admirable when we bear in mind the stage of our observer's development, for despite his veritable advance he still has no real mathematical knowledge.

The logical step now for our observer is to express this discovery in a diagram, in keeping with his earlier practice, and his approach will be as in *Fig. 43* in which we see the square and the circle.

The square is divided into five horizontal strips, each representing $\frac{1}{5}$ of the square, which means that when rectangle 1-2-6-5 is removed we are left with the area 1-2-3-4, which is thus equal to the area of the circle.

In *Fig. 44* we see this area removed from the main diagram, and our observer has now succeeded in expressing his observation in the form of a rectangle.

Let us call this the circle's rectangle. It is an important figure, one with which we shall be working much in the future.

The next stage in the development of our geometric diagram is that the circle's rectangle again takes its place in the square, *Fig. 45.* But not as in Fig. 43 in which it was at either one side or the other, depending on which strip was cut away, but placed now in the middle of the square. And so for the first time we come across a uniform division of 10, since each of the two equal areas of the square on either side of the rectangle is $\frac{1}{10}$ of the square's area.

Our observer once more has a complete diagram, and studies it in detail bringing to bear all his previous experiences. It will probably not take him long to do as he has done with his earlier geometric areas, i.e. split this important new rectangle up into four triangles, *Fig. 46.*

This division presents fruit for much thought, since his experiences declare that when a square is divided up by its two diagonals, it becomes four triangles equal in shape and area.

Now he is faced with the phenomenon of a rectangle certainly being divided into four triangles but having two of these triangles completely different from the other two.

If he divides the rectangle only by one diagonal, the area is split into two halves, providing two acute-angled triangles that are equal in shape and size, but it is when he enters the second diagonal that the discord arises. He probably solves this problem by entering the vertical and horizontal axes, *Fig. 47.*

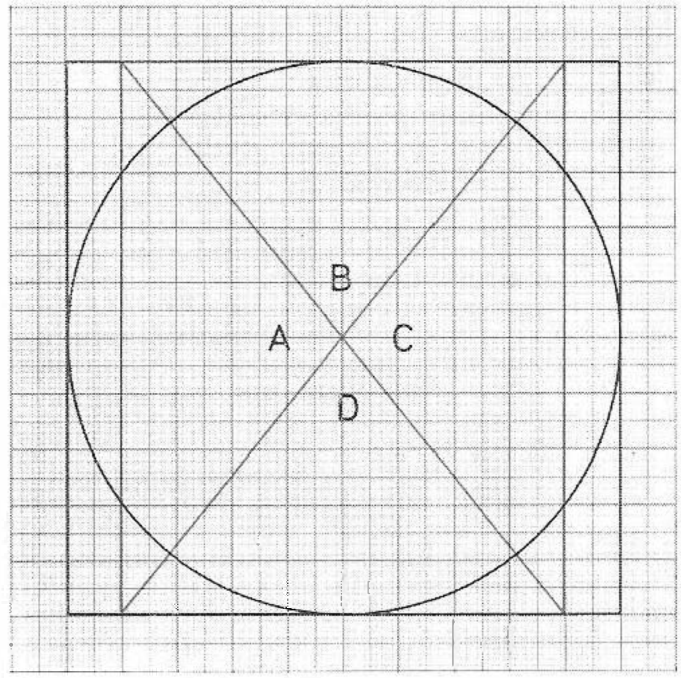


Fig. 46.

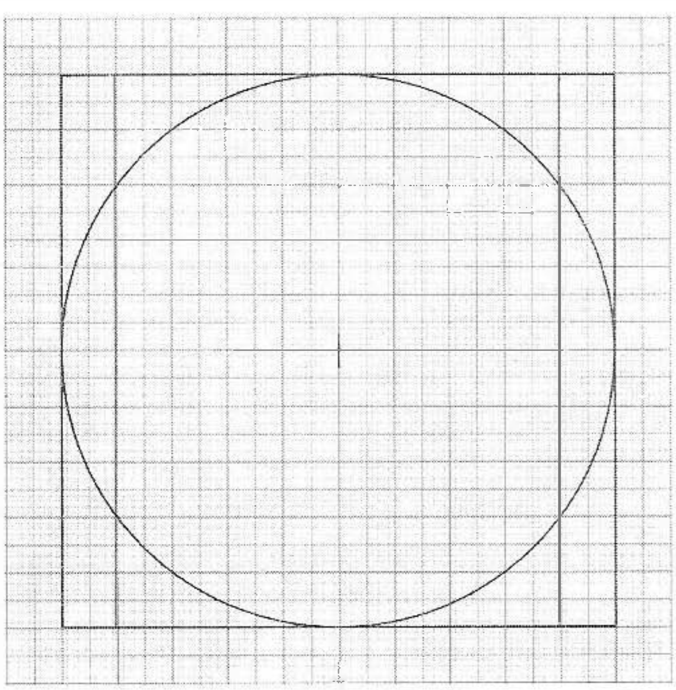


Fig. 45.

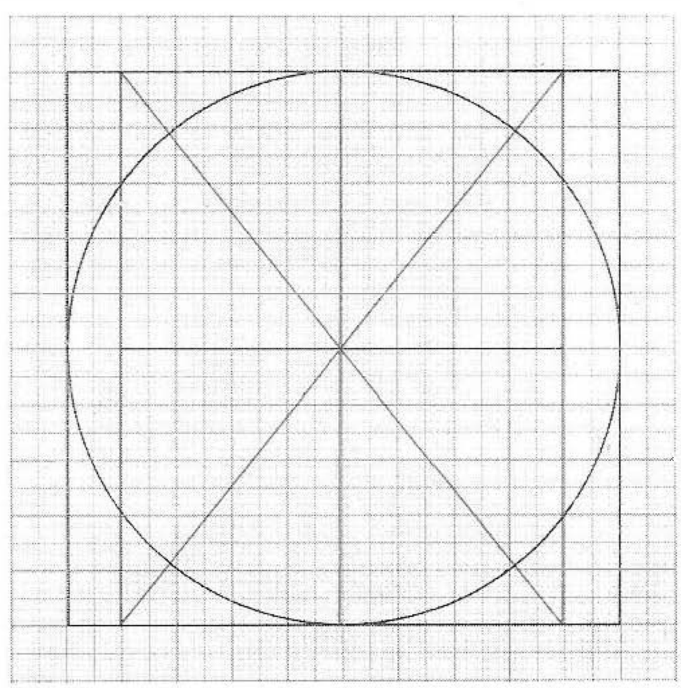


Fig. 47.

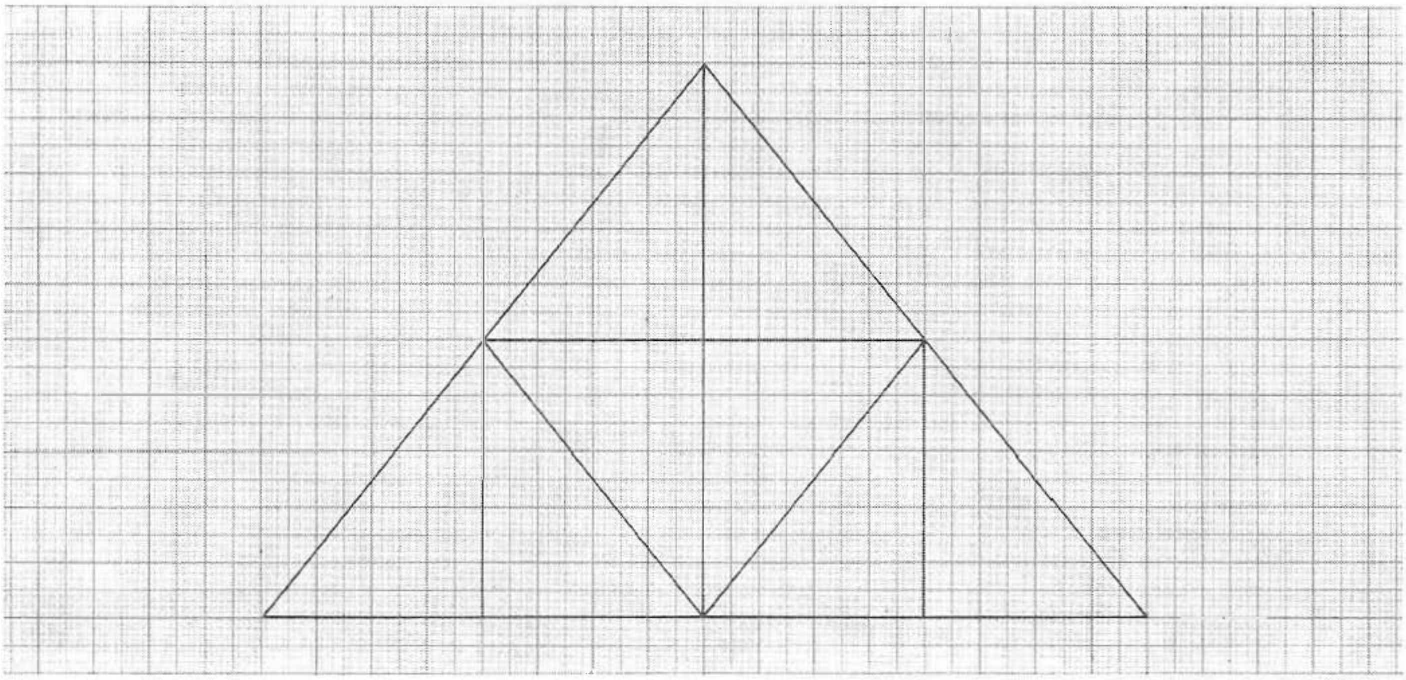


Fig. 48.

He discovers in this way which triangle is the main shape in the diagram since the addition of the vertical cross provides a uniform picture, the whole rectangle dissolving into eight identical triangles.

He is now able to see that in Fig. 46 triangles A and C are in fact the same as B and D, only they are composed in a slightly different manner.

Following his normal procedure, he fashions a set of eight bricks so that he can experiment with different shapes, and he first of all produces *Fig. 48*, a triangle which in angles is the same as triangles B and D in Fig. 43, only four times larger and therefore the same area as the circle.

Fig. 49 is a different triangle, being in angles the same as triangles A and C in Fig. 46 but still four times larger.

Thus our observer has managed to express the area of the circle in three different ways, namely by a rectangle and by two different triangles.

He now enters in his diagram the triangle he made in Fig. 48, and we see the result in *Fig. 50*.

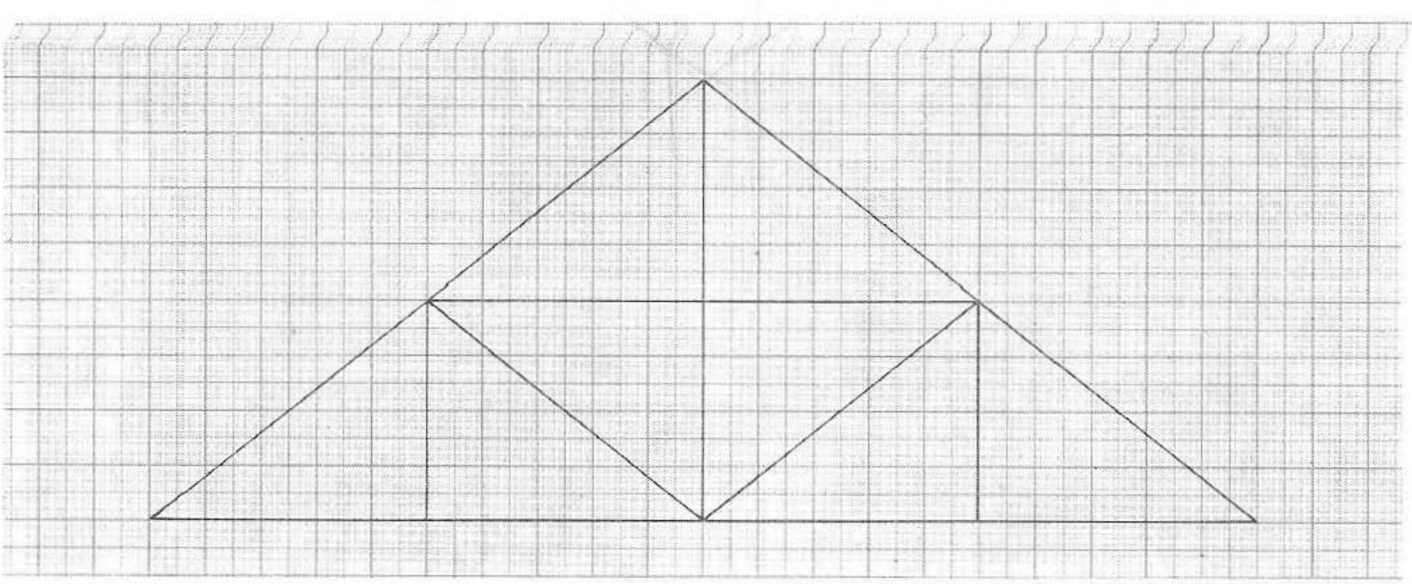


Fig. 49.

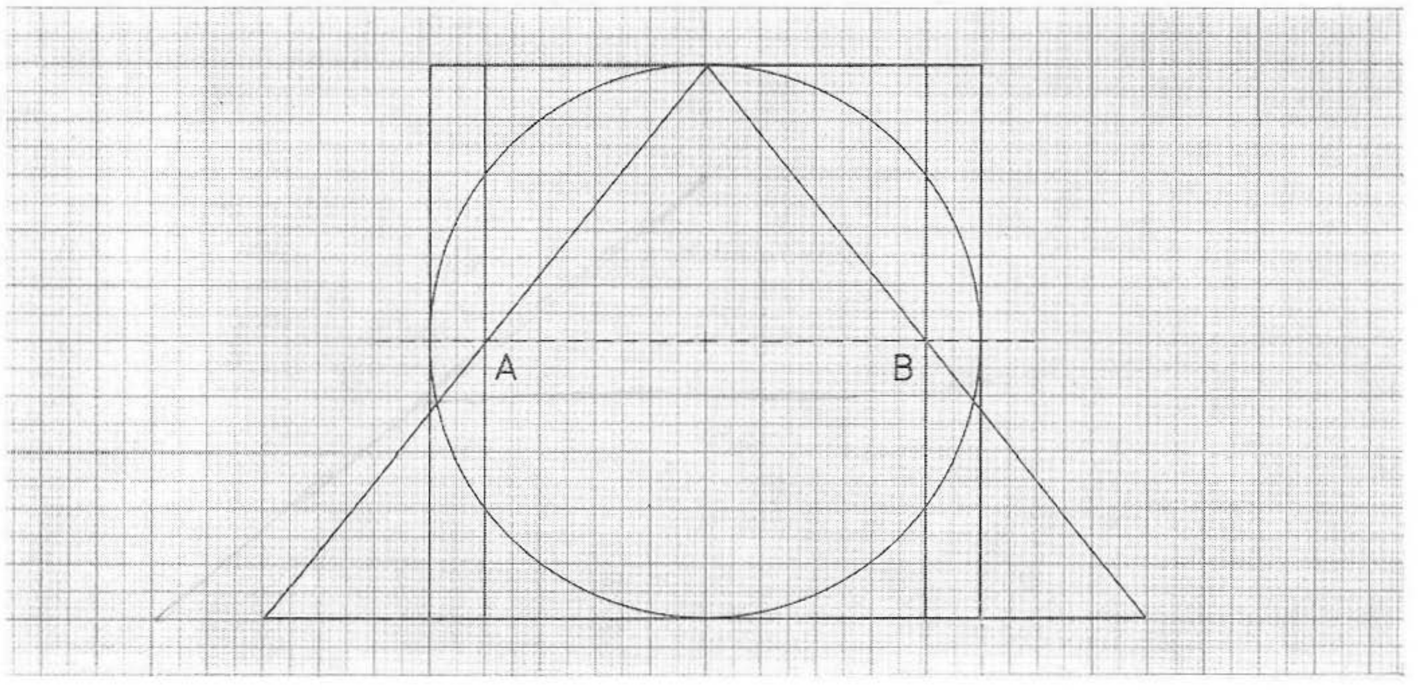


Fig. 50.

He has probably selected this triangle as opposed to Fig. 49 because it fits better into the diagram, its height being the same as that of the square—which of course is the basis of the whole construction.

The actual construction of the triangle is quite straight-forward. He enters the rectangle and then measures the length of the rectangle's shorter side along the base-line, on each side of its centre. The oblique sides of the triangle can now be drawn in.

This drawing thus contains the square and the circle as well as the circle symbolised by the rectangle and by the triangle, since the area of the circle is—according to his recent observations—the same as the rectangle and the rectangle's other shape, the triangle.

In his further study of this diagram he discovers a new fact which is actually obvious and could probably have been discovered earlier, but I think a lot of work preceded its appearance.

The square's central horizontal axis cuts precisely ¼ off the triangle, ¼ being above this line and ¾ being beneath it. The upper quarter of the triangle is thus identical to triangles B and D in Fig. 46 which resulted when the diagonals were entered in the rectangle.

He notices, too, that this central axis has a peculiar point of intersection, cutting the sides of the circle's rectangle at the same point as the sides of the triangle cross these lines. A look at Fig. 50 shows this to be a natural occurrence, but one really needs to have it drawn to appreciate it fully, and from a purely geometric viewpoint it is both a pleasing and a clean intersection in the diagram.

Our observer turns again to experiment. He takes this upper quarter of the triangle and lowers it in the diagram until it rests on the arc of the circle. He now works only with the quarter of the triangle which was cut off by the above-mentioned intersection since it is apparent to him that this small triangle in its present form symbolises both the circle and, in relation to the rectangle with which he has been working, the quarter-circle. The small triangle then, as mentioned above, comes to rest with its base touching the arc of the circle, *Fig. 51.*

At this point in the development of his symbol he pauses to summarise once more results achieved so far.

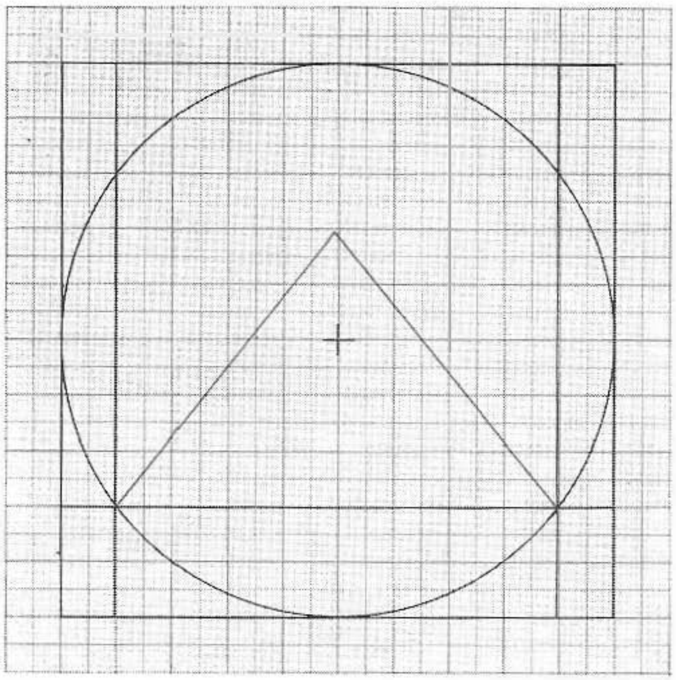
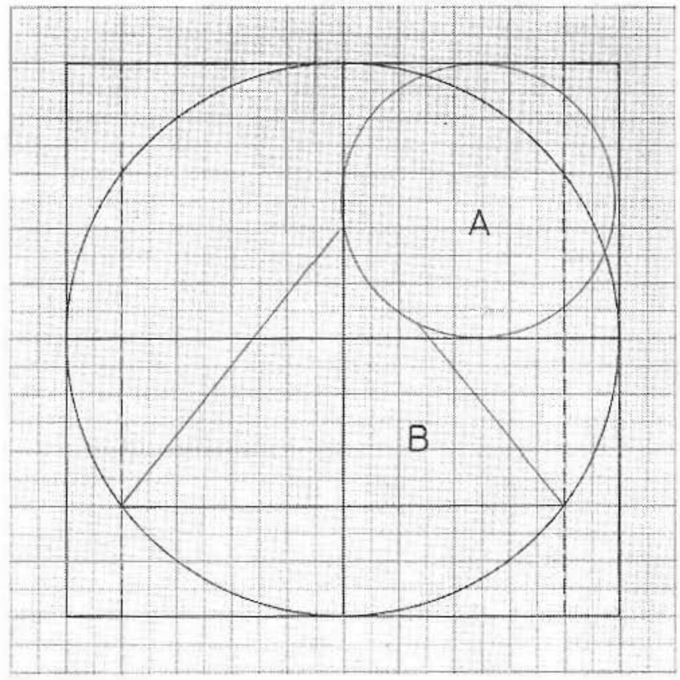


Fig. 51.



← The circle's rectangle →

Fig. 52.

His diagram comprises the basic shape, the circle, and its partner, the outer square. Both of these familiar figures are made without resorting to measurement, but with a knowledge of how to construct a right angle.

The circle's rectangle has been entered in the diagram but its construction is unsatisfactory in his eyes since it can at present only be done by measurement, which procedure he finds strange and unaccustomed apart from the fact that from a geometric standpoint it may give rise to unfortunate uncertainty. He perhaps possesses a measuring stick or rope for practical surveying of land, but at this stage he has no real measure or unit of measure for indicating the small sizes with which he works in his diagrams.

Having entered his rectangle in the diagram, he obtains at the same time the base-line for the small triangle, i.e. the points at which the rectangle's two vertical sides intersect the lower half of the circle.

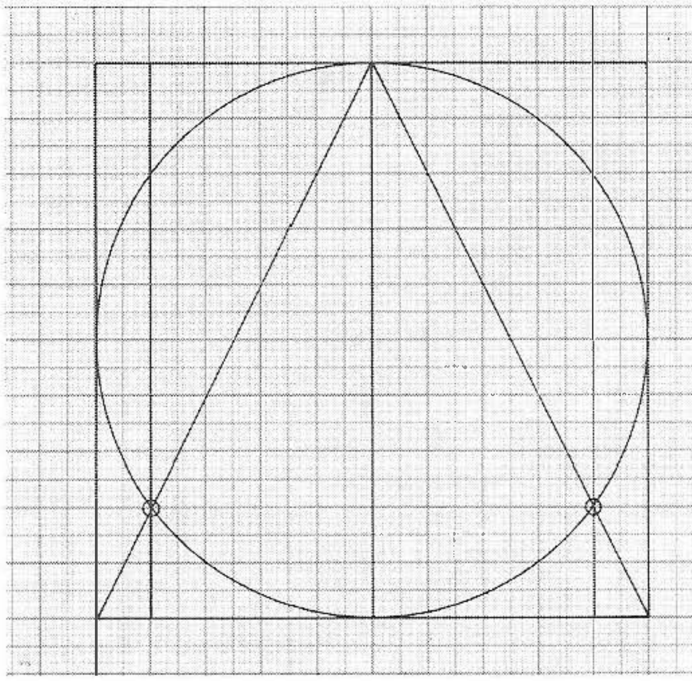
In order to complete the triangle he needs an indication of its height.

This height is in fact indicated in the diagram but not in its proper place, since it is the same as the distance from the diagram's central horizontal axis to either the top or the base. But it must be moved into position on the base-line of the triangle which he has just obtained. This course of action is however also unsatisfactory to our observer since it does not provide a clean-cut geometric construction in which one line can create the basis and point of origin of the next.

The reason for his busying himself with this special triangle is that he sees this particular shape as symbolising a circle, not any old circle but one which he knows to have a certain area.

He is able to construct a given circle, transform this circle to a rectangle, and in turn transform this to particular form of triangle.

But his experience with Fig. 50 told him that this triangle could not be fitted inside his diagram. He reasons, however, that the symbol provides him with certain information as to the area of a circle in the form of a triangle, and although the



|← The circle's rectangle →|

Fig. 53.

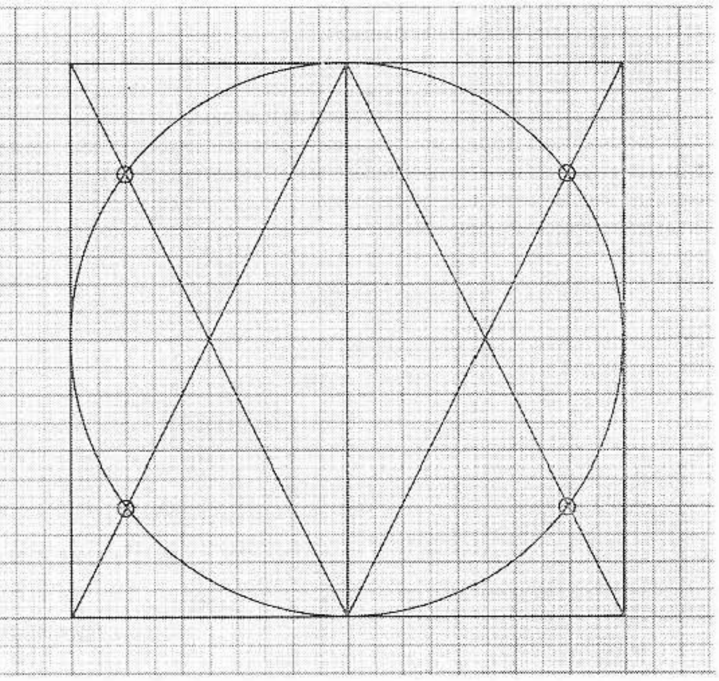


Fig. 54.

symbol represents only a shape which covers ¼ of the circle in his diagram it is nevertheless correct—and he can quite easily construct in ¼ of the square a circle which has the same area as the triangle. We see this in *Fig. 52.*

Thus although the symbol would appear to be correct the geometric purity seems somewhat suspect, so our observer continues the development of his diagrams in the hope of finding the missing link.

The primary and no doubt most important link he discovers is a purely geometric construction in which the circle's rectangle is marked off. He finds just such a diagram by resorting to the now familiar acute-angled triangle.

We can see in *Fig. 53* how the point at which the acute-angled triangle intersects the circumference of the circle is precisely the same point at which the circumference is cut by its rectangle. By entering the acute-angled triangle (pointing both upwards and downwards) in his diagram he gets four points on the circumference of the circle between which to connect his lines. This is shown in *Fig. 54.*

The upper limit of his triangle is obtained by resorting to other familiar factors.

By entering the half-size square at the base of the diagram, he notices that the upper edge of this square coincides exactly with the apex of his triangle. In other words, the triangle's apex is the same as the upper sacred cut in the main square. This can be seen in *Fig. 55.*

His search has ended most satisfactorily, and his diagram now comprises only factors with which he is familiar.

It contains the three squares within each other which earlier played such an important part in his research, partly by giving him an inroad to the very substance of geometry, and partly by helping with such a major discovery as the circumference of the circle. Now the same shapes have been reintroduced, and provide him with the perfect symbol for the area of the circle.

His complete diagram now comprises the basic square and its inner circle, the three squares, and the acute-angled triangle.

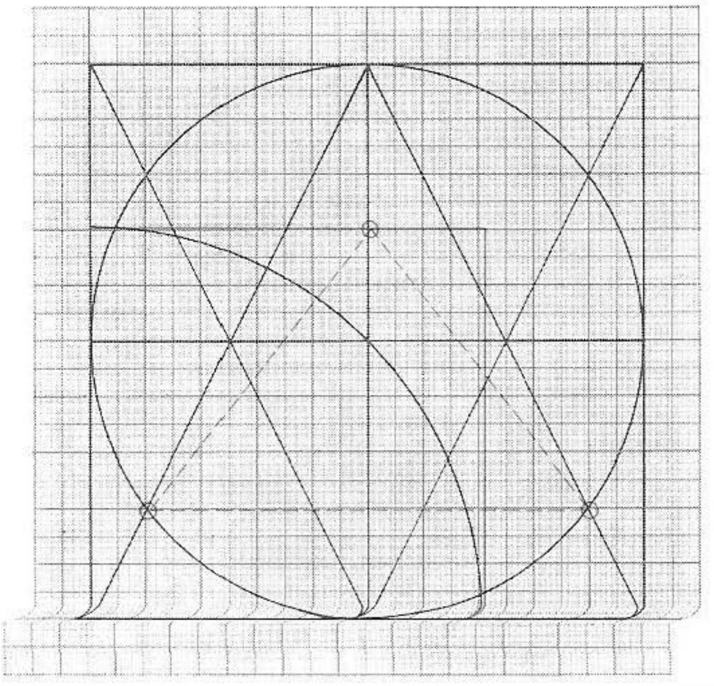


Fig. 55.

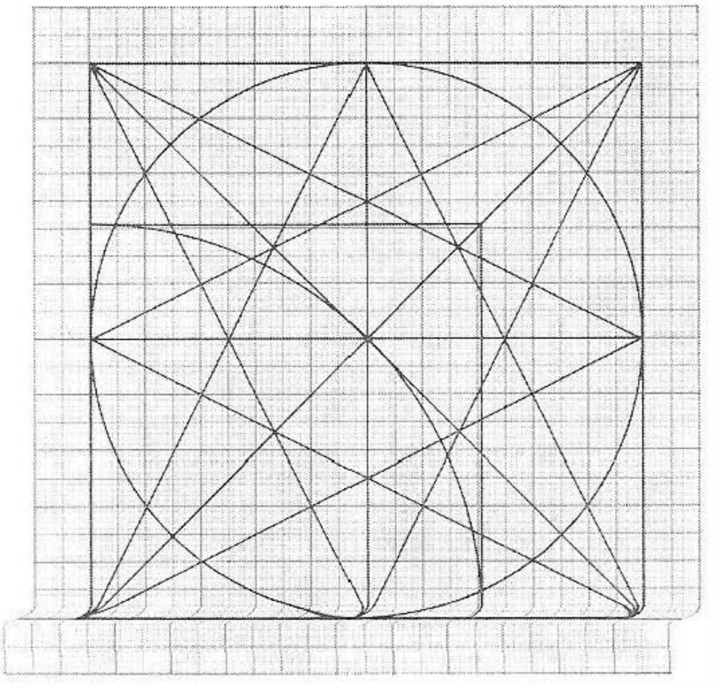


Fig. 56.

He goes on to finalise his diagram as the perfect symbol, expressing his entire depth of geometric knowledge, by combining Figs. 40 and 55 in one unit since they contain the same shapes.

This grand diagram is seen in *Fig. 56* which—with the presence of all his guide-lines such as the cross and the diagonal cross—looks rather complicated. But if we examine it line by line we see that the figure contains: the circle and square. The square has the normal guide-lines, the vertical and diagonal crosses.

The half-size square is entered, and indicates vertically and horizontally the sacred cut in the main square. To find the circle's rectangle the two vertical acute-angled triangles are entered, pointing upwards and downwards.

The horizontal acute-angled triangle was entered to establish the circle's circumference, and to complete the symbol he includes the horizontal acute-angled triangle's counterpart in the same way as with the circle's rectangle.

The symbol is extremely well-known and is seen in *Fig. 57* as the eight-pointed star, of which we can find numerous examples in Temple symbolism, e.g. on floor mosaics, stained glass windows, and many other places.

Our observer has thus succeeded in capturing his entire knowledge of geometry in the perfect symbol, namely the eight-pointed star.

This symbol itself lacks direct indication of many of the well-known factors such as the three squares within each other, the

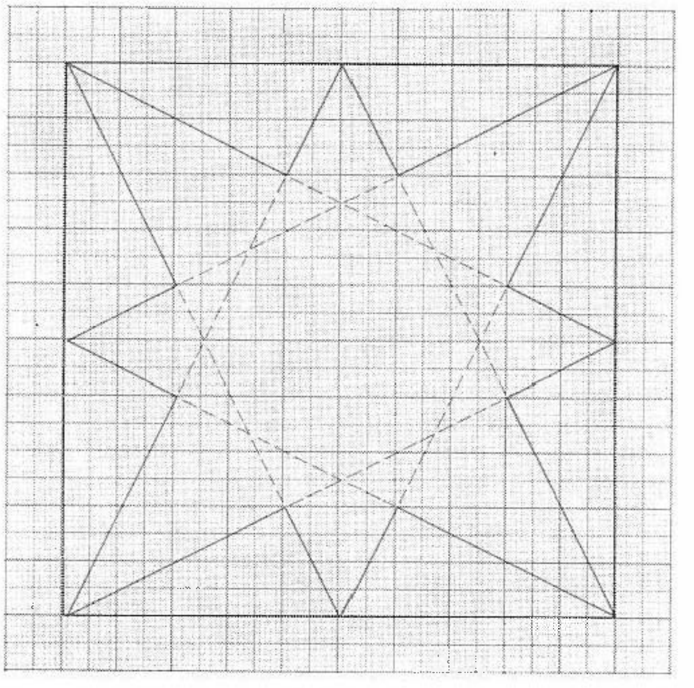


Fig. 57.

sacred cut and the circle's rectangle, but they are to be found in the guide-lines of the actual structure once the circle is entered. The symbol is perfect, since it hides or veils these factors.

To the outsider the eight-pointed star has virtually no importance; it is a geometric shape, a decorative figure or a sacred symbol whose background is unknown.

But to the initiate the picture changes; to him it represents the Beginning, and with the introduction of the Supreme symbol, i.e. the circle, light is thrown over the diagram in both an esoteric and a practical sense.

*

The process of development described in the foregoing pages extended over thousands of years, and our observer of course is not a single mystic living in his own little world and working away with squares and circles. He is himself a symbol of the thinking class in a growing society.

But the symbol he has evolved through the ages was not merely of speculative and geometrical interest: it was employed extensively in practical matters, and from earliest times right up through the centuries to the late Middle Ages it formed the basis of construction for temples, churches and other edifices. It became the background of dimensions in sculpture, and generally filled a tremendously important role in the development of culture.

We must not, however, simply take the view that the eight-pointed star itself was the basis of these structures; just as we in our system of numbers and counting adopt various patterns and standards (and can, for example, use 1, 2, 4, 8, 16 ... or 1, 3, 6, 12, 24 ... or a myriad of other combinations) the eight-pointed symbol was also divided up in patterns. Some planners may have used as their dimensioning factor for a particular building the circle's rectangle in association with the basic square, others may have used the half-size square—again with the basic square as the starting point. And the variety of possibilities for fixing the dimensions of a certain area within the framework of the diagram was so wide, that its application was almost unbounded. We shall see in the following pages one or two examples of this planning work.

The diagram thus was not a mere object of speculation, but was a practical tool, occupying a place of honour in the Temple. And it was to the eight-pointed star and its manifold facets that the Temple brethren turned for the answers to their problems.

We have sensed an unwillingness in the development so far of geometry to trust to measurement. This unwillingness was caused not so much by a stubborn streak in Man's make-up, as recognition of the uncertainty it involved.

The absence of a predetermined and uniform standard of measure presents immediate problems if more than one persons is engaged in measuring a particular area of ground. And in surveying and measuring a building site, for instance, it is to say the least inconvenient if the master builder himself has to witness every single measurement.

Oh, certainly, it can be done! He can make a stick of a certain length and get his men to measure according to his instructions, but if wants to check the work he has to get down to it and measure it himself.

To get around this inconvenient arrangement, the ancient builder of yore employed a system of strings, illustrations of which have been seen on old Egyptian pictures. We today know that one of the elementary though invaluable abilities of the system was to provide its user with a right angle. But apart from this we know very little about the string technique.

In order to mark off a square—be it large or small—on an area of ground, you require to lay out a right angle; if you can manage this, you can proceed with your square.

If we imagine a huge square laid out on an area of land, then we can see it would be possible to use strings inside this square and construct the whole of our symbol, line by line—since every line apart from the circle is straight.

The different points of intersection of the various strings—once we have studied our diagram—indicate the respective areas within the square.

But this diagram or symbol in its practical sense is not content merely with marking off geometric figures: it can also provide the means of splitting the square into any number of portions you wish. For example, if we stick to whole numbers and divide the side of the square into three, the area of the square can then be split into nine.

Dividing the side of the square in four, provides a division of the whole square into 16 equal parts, and so on.

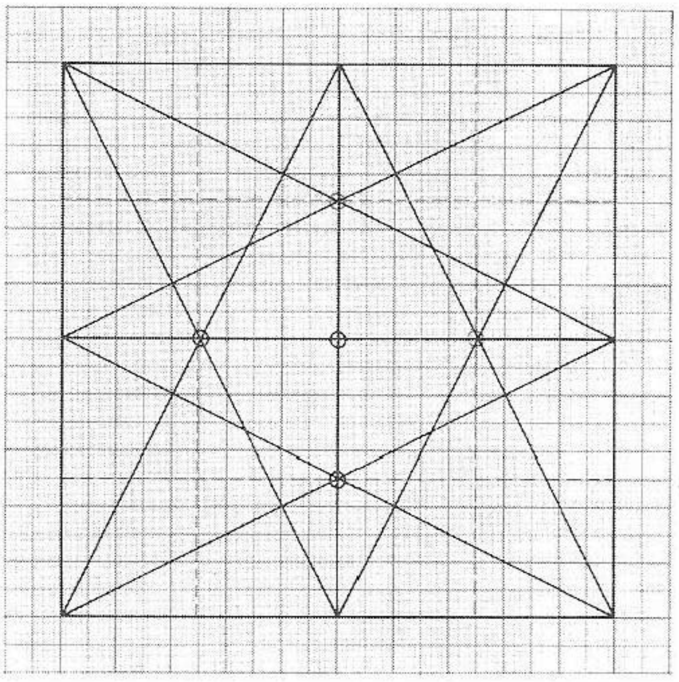


Fig. 58.

We see in *Fig. 58* first of all the most elementary division, i.e. into 2 or 4 parts.

We note here how the vertical and horizontal central axes, at the points where

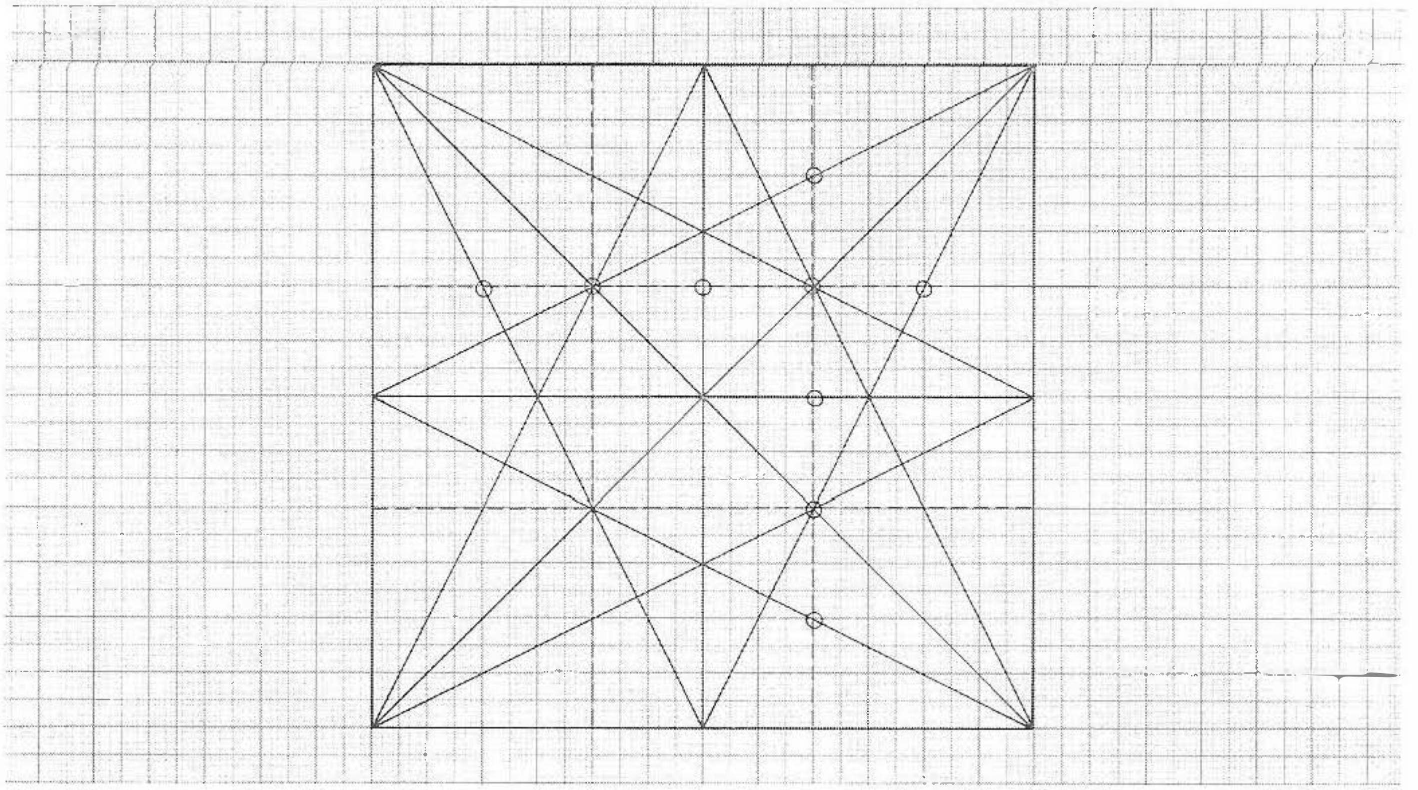


Fig. 59.

they intersect the acute-angled triangles, divide the diagram into four. Producing vertical and horizontal lines through these points allows us to divide the square into 4 × 4 squares, total 16.

In *Fig. 59* we see the 3 or 6 division. Whereas in the preceding example we started at the centre of the diagram and worked out along the vertical/horizontal cross in order to obtain our points of intersection, on this occasion we make use of the diagonal cross. Moving out from the centre along the diagonal cross we find that the first intersection we meet gives us the central point for the division into 3 or 6 lines. If we draw a vertical and a horizontal axis on the diagram each is divided by the diagram into either six or three parts whichever is required. And we are thus able to divide the whole square into 9 or 36 smaller squares.

Fig. 60 shows the division of a line into 5, the point of division being at the intersection of the hypotenuses of the acute-angled triangles on the outside of the diagram. We have a choice here either of drawing in the vertical or horizontal line, the other being indicated by the inter-

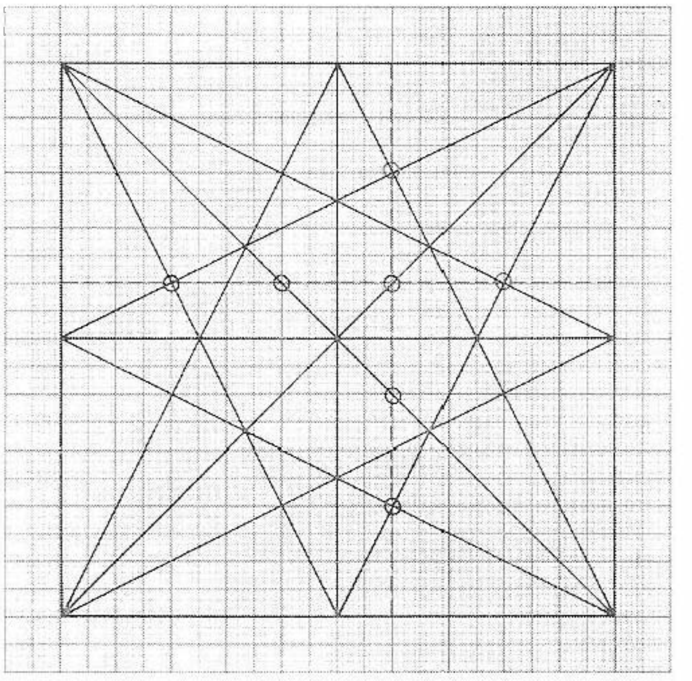


Fig. 60.

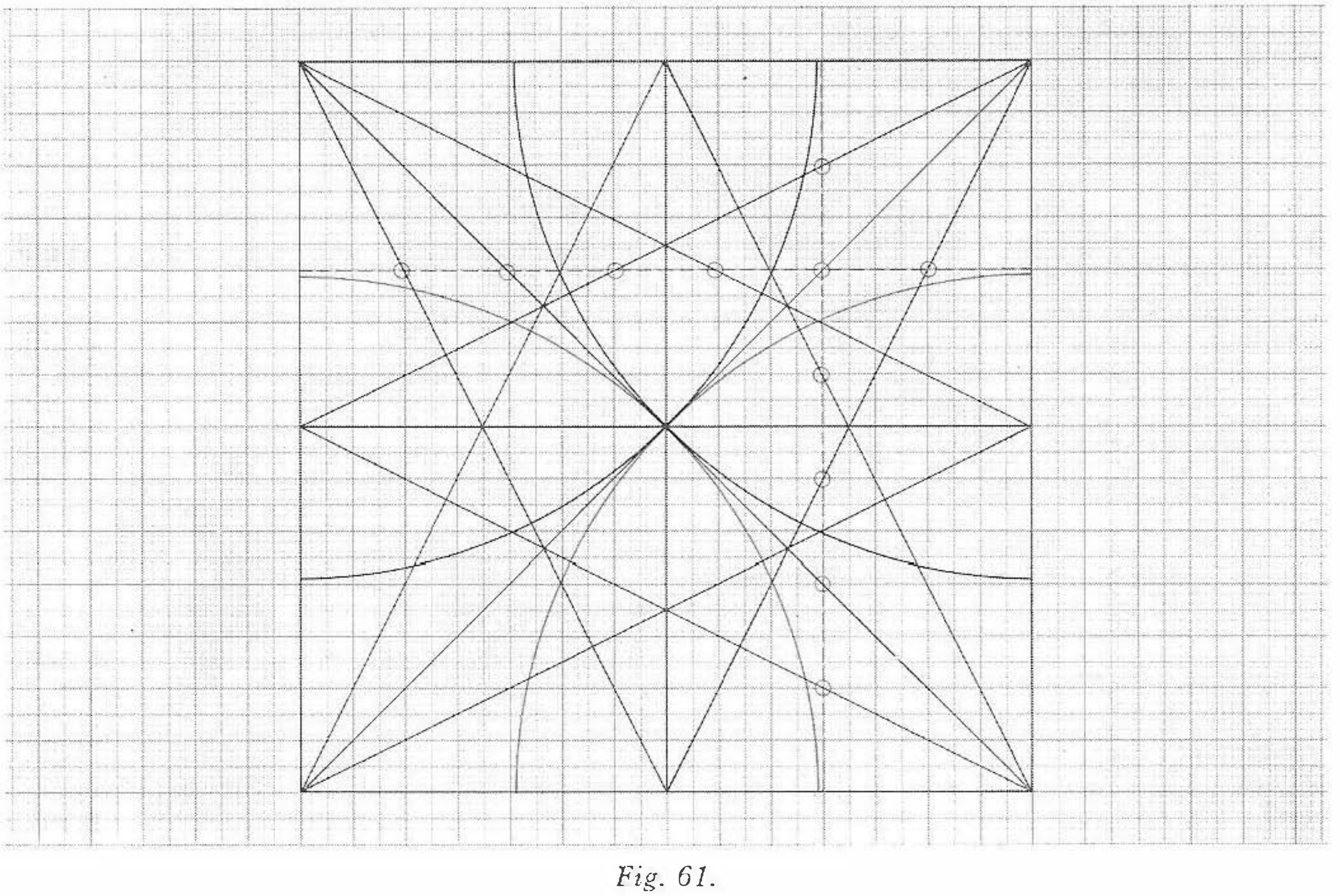


Fig. 61.

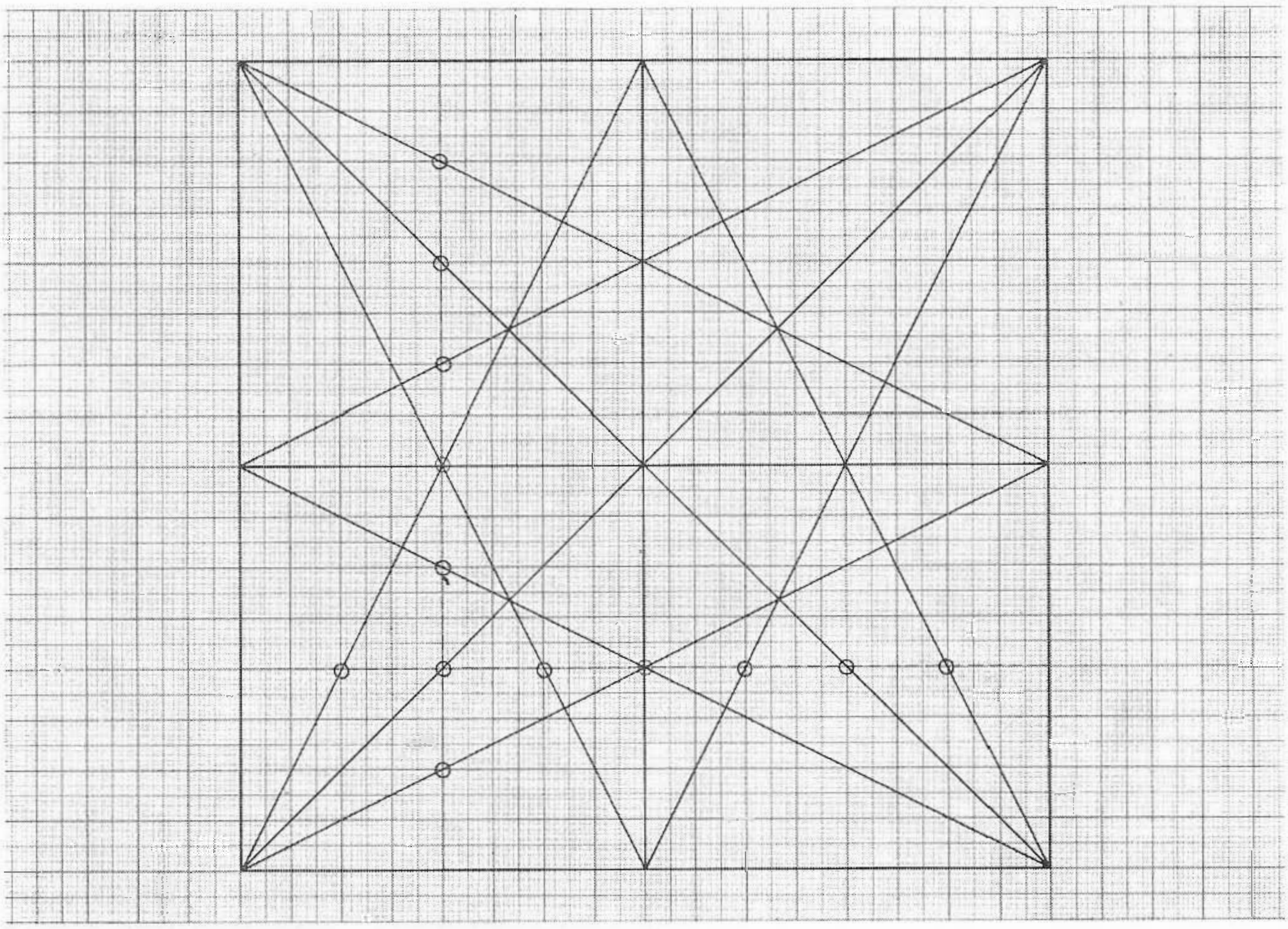


Fig. 62.

section of this line with the diagonal cross of the diagram, or of selecting two points of origin in the diagram, one for the vertical line and the other for the horizontal. But the final result in both cases is a perfect division of the line into five, which of course permits a splitting of the square into 25 equal portions.

Fig. 61 illustrates the division of the diagram into seven parts, the only instance where we do not as with the others find our point of division within the diagram itself. We must find the solution in the introduction of the sacred cut, vertically and horizontally. These lines, at their points of intersection with the diagram, provide a division into seven parts.

Accurate marking of the sacred cut—as in the diagram—reveals a small error in the construction, the same error of 1 % which we discovered in our earlier experiments. But if we compensate for this error by making in this instance the side of the square 14 units (and the sacred cut therefore 10) the construction is true.

The sacred cut in any proportion of the diagram thus divides the square almost correctly into seven parts, with a division of the area into 49 squares, but compensating for the 1 % error permits an absolutely accurate division of the square.

Fig. 62 shows the division of the diagram into 8, which is more or less the same as for the 2 and 4 division. We arrive at the point of division by moving along the vertical cross to the point where we had the division into 4. If we draw a line through this point it is split by the diagram into 8 equal parts, and if we take its opposite counterpart on the diagram, we have a regular division of any given square into 64 smaller squares.

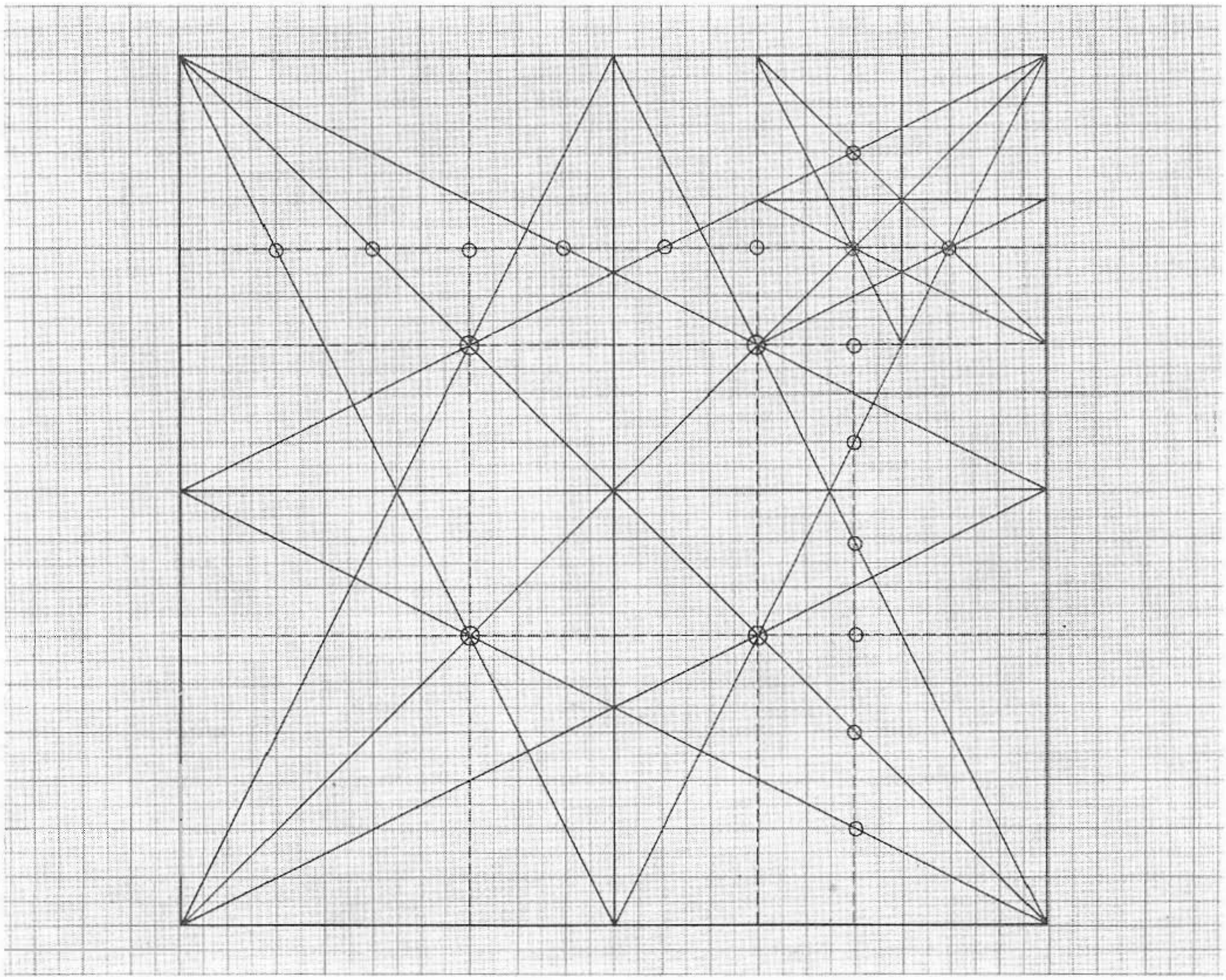


Fig. 63.

Fig. 63 shows the square divided into 9 portions or 81 smaller squares, and is perhaps the best concealed of all the methods of division, since a double construction is necessary.

First, we make a division into three, and draw in the relative dividing lines. Then we split one of the resultant corner squares into three, achieving not only a division into three of the small square but the point of division for the whole of the large square. We can see on the illustration how both the vertical and horizontal axes through this point are divided into nine parts, splitting the main square therefore into 81 minor squares.

Fig. 64 shows the diagram divided by the 10-part line. This can of course be obtained by repeating the process for division into five and simply halving the resultant sections of the intersecting line. But this is not however necessary. By drawing in the circle's rectangle, the process of dividing into 10 has already been started.

We see the lines of the circle's rectangle indicated by AB and CD. If we first produce the division into 5 and then enter the circle's rectangle, the same point of origin as for the 5-part line then gives us a division into 10. We are now able therefore to divide our square into 100 smaller squares.

Using the points of division indicated above ancient architects and planners were thus able to crossrule any given square

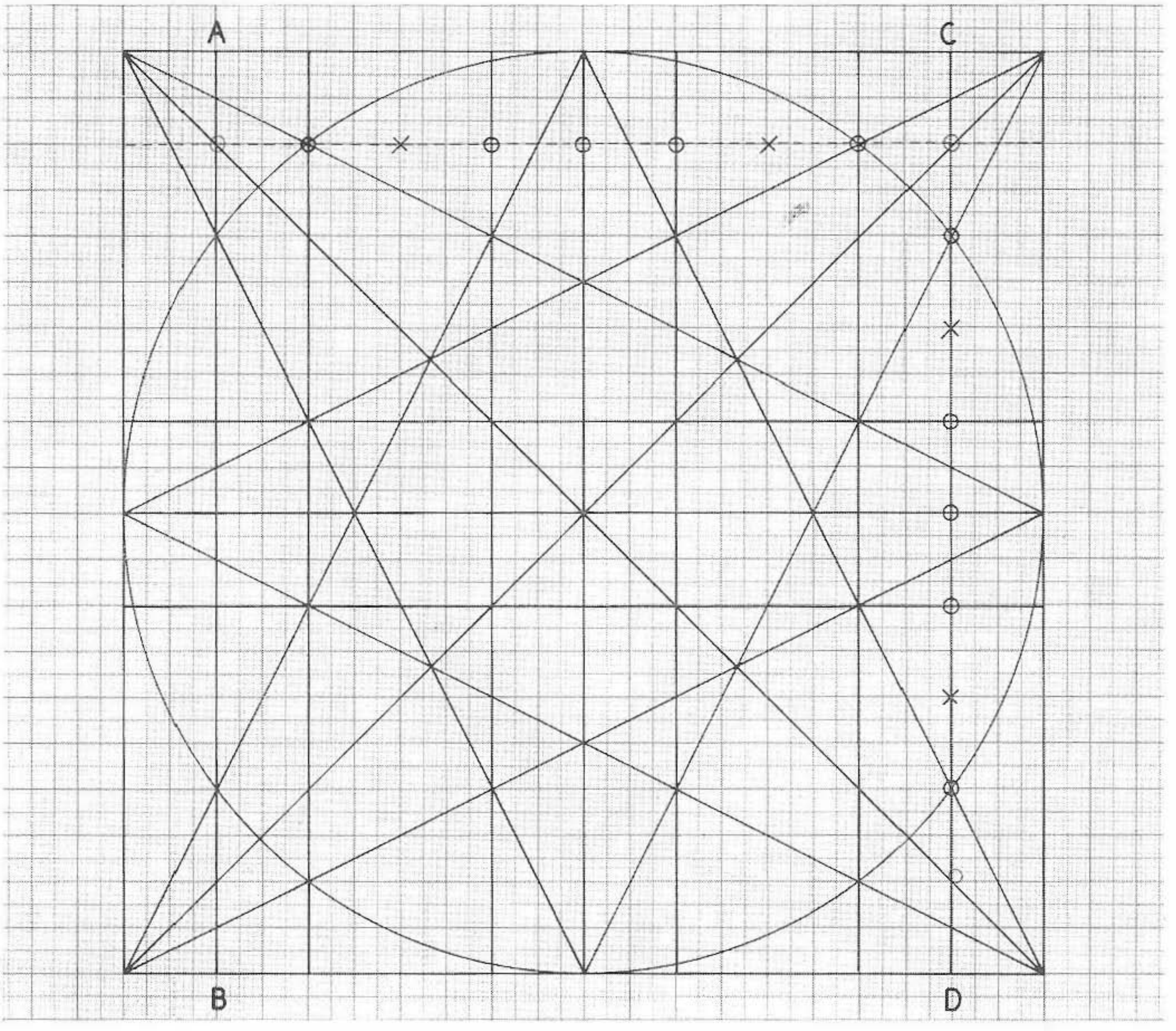


Fig. 64.

into smaller squares numbering from 4 to 100, within the limits of the 2nd power of numbers from 1 to 10. And this almost without any need or knowledge of measurement. With the exception of the division into 10, the user of this method would be able on any building site to obtain any of the aforementioned divisions simply by stretching strings at strategic points within the square he wished to split up.

We are familiar with this technique of measuring by means of lengths of string from the Egyptian era, and there is no reason to believe that other civilisations which leant for support on the Egyptians did not in their turn adopt the system and —with the years—improve and supplement it with new methods, until finally it was replaced entirely by precise measurements with yard-sticks, foot-rules or whatever standard succeeded it.

In any event we are able to establish that the string method was a known feature of building in the Egyptian period, and we can also take it for granted it was possible to execute the above-mentioned divisions of squares of practically any size, provided the user had sufficient string and a comprehensive knowledge of the points at which to make his divisions.

I contend that this was the technique

employed in laying out the majority of the buildings of ancient times. It had been a matter of selecting a suitable area of ground for a building site, and using the string system and ancient geometry to plan the temples. This is why it is so difficult to pinpoint any real standard of measure in such buildings. There simply was none. All the dimensions were contained in the geometric diagrams—and nowhere else.

The constructional technique itself—the actual setting out of the string—must needs remain a theory which will probably never be proved. But comparison of the buildings with the diagrams we now have within our grasp, and geometry's esoteric status as a factor in ancient temple building, is quite another matter.

A number of these structures are intact today, and others lie in ruins but with sufficient detail to permit fairly accurate reconstruction, and it is with these that we can illustrate the buildings' relationship with ancient geometry.

Dividing up the Circle's circumference

Our study so far has shown that the ancient geometrician was able—by judicious application of his subject—to delve deeply into the mathematical sphere. His experiences and results are still the fruit of the principle of comparative measurement and the relationship of the circle to the square.

The circle was the natural starting point. It inspired the square, and divisions of the square provided related experiences with the circle, just as divisions of the circle led to similar discoveries about the square.

This exchange effect added to the growing mound of experience until a complete geometric system resulted. A system which in many ways burst the bonds of pure geometric thought, and touched occasionally upon real mathematical knowledge.

We left our observer at the point where, with the aid of the eight-pointed star, he had succeeded in dividing the sides of the square into 2, 3, 4, 5, 6, 7, 8, 9 or 10 parts. Enabling him therefore to divide any given square into as many small squares as the second power of these numbers provides.

The ancients had naturally tested their accomplishments on the circle's circumference, trying by the same procedure to break it into several uniform pieces.

We can see a number of these divisions quite easily obtained by some of the lines in our diagrams. In *Fig. 65* we have a diagram, comprising the circle, square and the guide-lines made up of the vertical and diagonal crosses. The half-size square, too, is indicated by 1-2-3-4.

This diagram illustrates a natural division of the circumference into 4 and 8 parts. We can plainly see that the line 1-2 is $\frac{1}{4}$ of the circumference, while line 1-5 is $\frac{1}{8}$ of the same arc. And each point of division is indicated quite naturally all the way round the perimeter.

This division would appear to be a straight-forward and self-evident observation. But in establishing this, we must acknowledge that it is not the arrangement within the circle which provides this natural division. It is the guide-lines of the

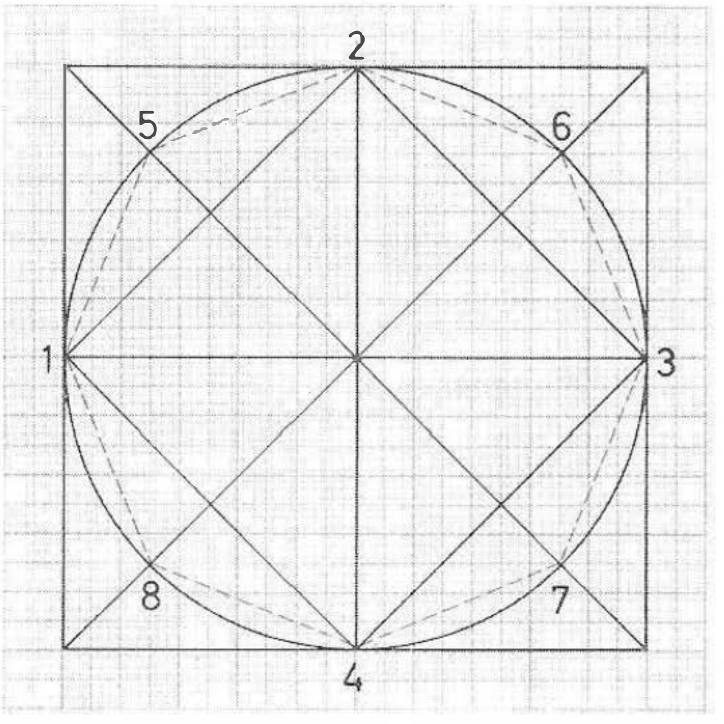


Fig. 65.

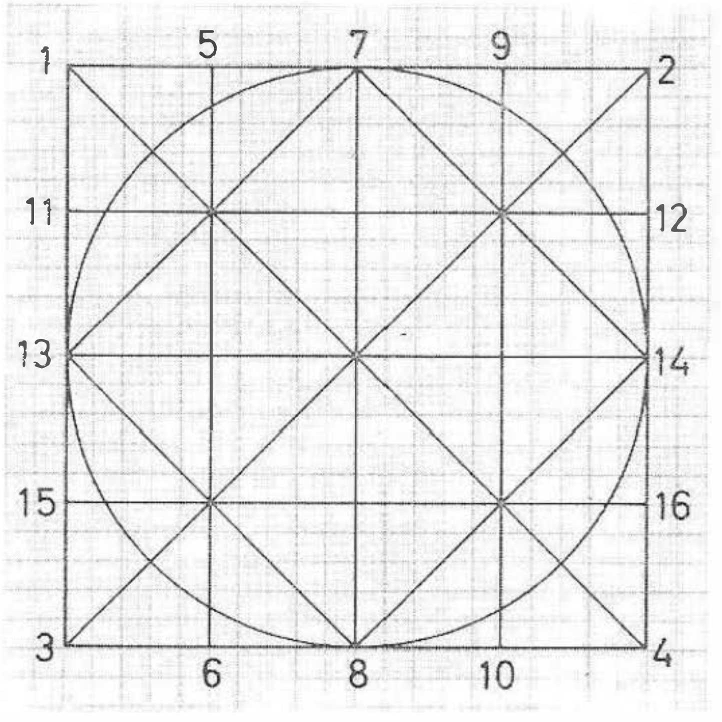


Fig. 66.

square, with their points of intersection upon the circle's circumference, that mark out the division of the circle. We immediately hit trouble the minute we try dividing the circle up without these guidelines.

Now we take the same diagram a step further and, following recognised procedure, we enter two vertical and two horizontal lines—which was one of the first steps we took in breaking a square down into small triangles without any form of measurement.

In *Fig. 66* we see these vertical lines 5-6 and 9-10 and the horizontals 11-12 and 15-16.

Whereas the diagram was previously split up into 16 triangles, these lines now divide it into 32, and we have the same geometric construction we saw in Figs. 19 and 20, where we examined the problem of finding the areas of different squares.

In *Fig. 67* we can observe the division of the circumference into 3 equal arcs, by the lowest horizontal 15-16. We can see that this line intersects the circumference twice, and these points cut precisely ⅓ off of the whole perimeter.

If we produce two lines from these points of intersection to the middle of the top lines (7), we succeed in constructing a triangle inside the circle with three equal angles and three equal sides, and the circumference is therefore divided correctly into three equally long arcs. If we draw radii into the centre from these three points of intersection on the circumference we naturally split the area of the circle into three parts in the same way as the cross splits the same area into four parts.

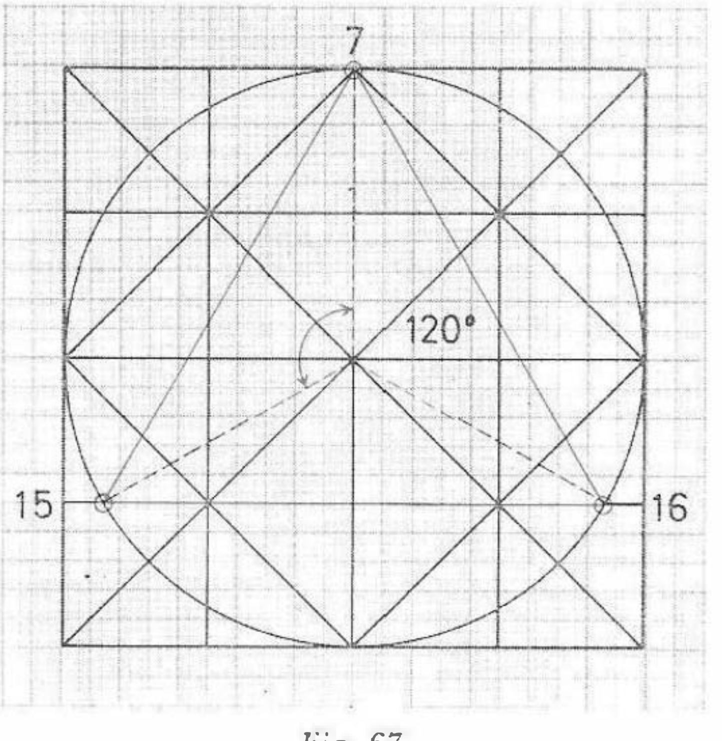


Fig. 67.

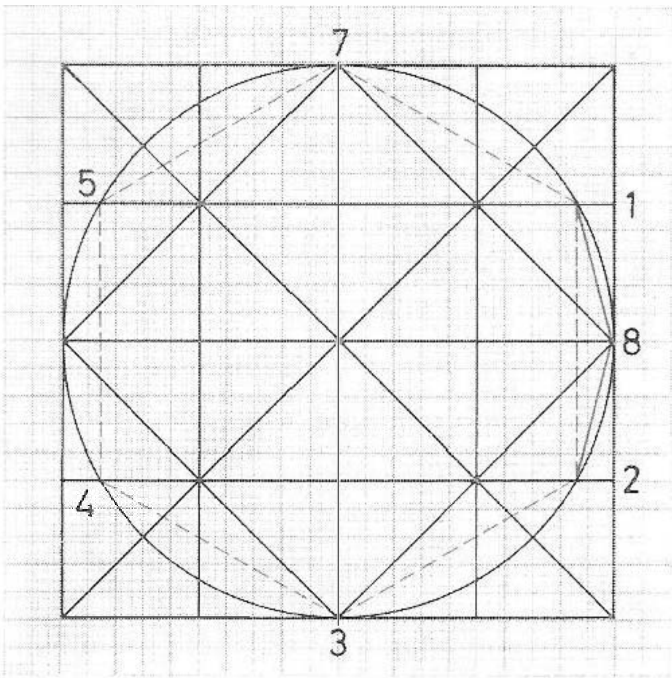


Fig. 68.

In *Fig. 68* we see the same diagram applied to the division of the circumference into 6 and 12 parts: as in the previous constructions these are shown quite clearly and easily by the points at which the square's component lines intersect the circumference.

The 6-division is shown by 1-2-3-4-5-7 and the 12-division by 1-8 and 8-2.

We can now divide the circumference with no difficulty into 2, 3, 4, 6, 8 and 12 parts. Each of these divisions is unquestionably accurate and can be read from the diagram without any need for measurement. Each is obtained by a comparison of certain factors in the circle and the square, and in reality the same diagram is employed for all.

It is worthwhile emphasising that all the points for these divisions are shown on the diagram, and that all of these points are yielded by the intersection of the square and its structural lines with the circumference of the circle.

In none of these instances do we require to measure a given line and mark it off along the curved arc of the circle—which on a large-scale diagram would lead inevitably to undesirable mismeasurement.

I refer here to the generally known and accepted division of the circle into six, whereby the radius is divided six times into the circumference. The procedure we have followed ensures a greater degree of accuracy since the question does not arise of measuring a particular line and marking it off on the curve of the circumference. We simply read the points of division on the diagram itself.

Let us summarise our findings so far:

We are able to half a square, not only into two rectangles but also into two smaller squares together equal in area to our original.

We can convert our square into two triangles, and are able also to split it into just as many small, numerable triangles as existing technique will permit. This triangulation provides a division of the side of the square into 2, 4, 8, 16, 32, 64 etc. And the lines resulting from triangulation gives us at the same time a picture of the division of the square with the same numerical values.

Apart from these divisions, we are able —with the aid of the eight-pointed star— to divide the side of the square into an even or uneven number of equal parts.

Using the sacred cut, we have obtained fairly accurate values for both the circle's circumference and its area, approximations which are seemingly accurate when the error cannot be traced. And we have also been able to divide the circumference into a number of equal parts, which simultaneously split the circle's area into a similar number of portions.

The entire range of our knowledge has been laid out in the respective geometric symbols shown in the study of the ancient system, and we now possess a number of symbols from the circle and square with the various crosses up to the eight-pointed star.

Before going on to illustrate the practical application of these symbols in a

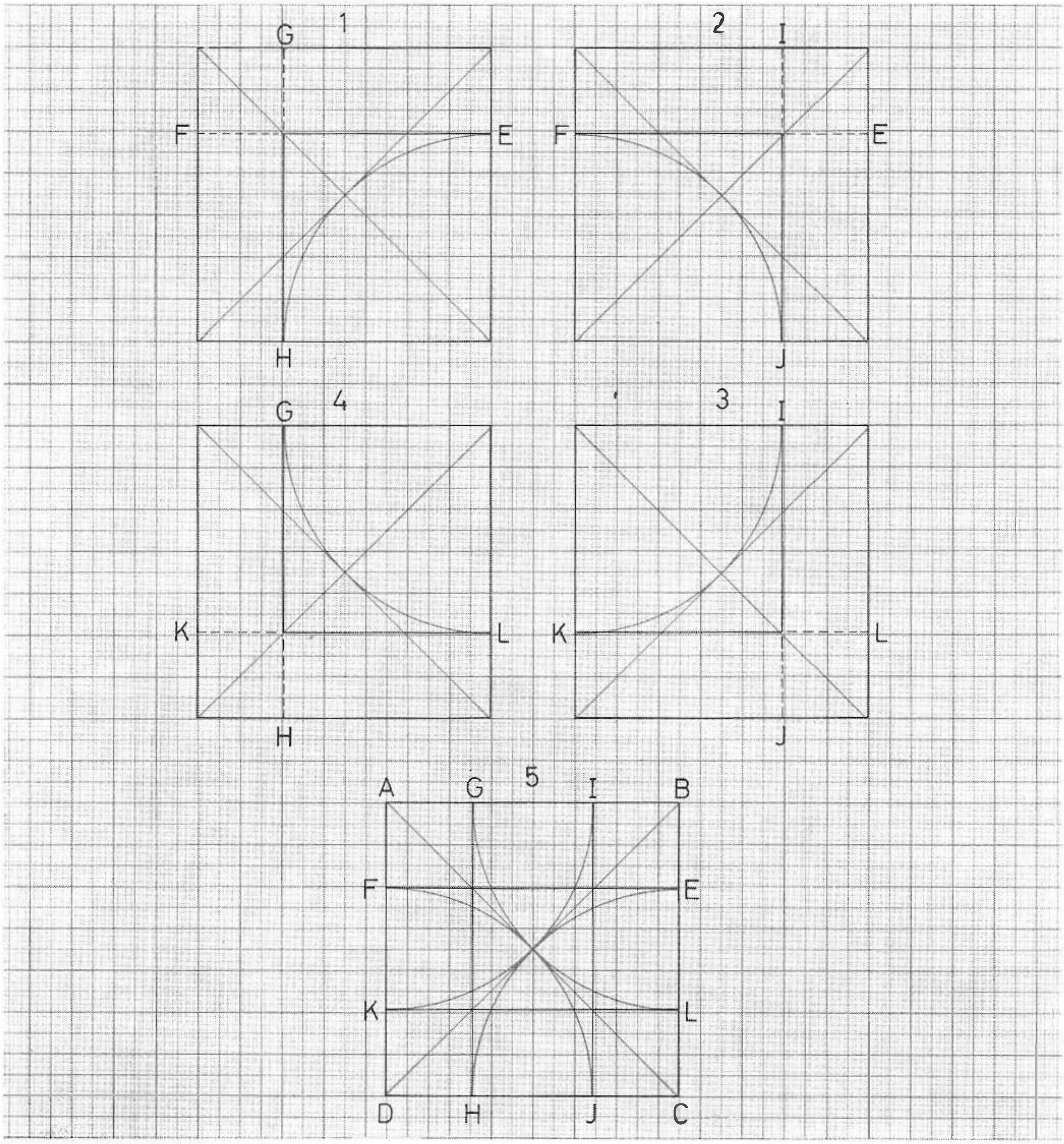


Fig. 69.

number of fields, it would perhaps be advisable to recapitulate one of the factors with which we shall be dealing increasingly in future study. My reason for ploughing what might on the face of it be familiar ground is that this factor has several possible varieties which have not yet been brought to the reader's notice.

This factor is the sacred cut and its many facets, some of which were discussed in Chapter Four, Fig. 40.

In *Fig. 69* we have five squares. In Square 1 the half-size square is constructed in the lower right corner, which provides two sacred cuts in the main square, i.e. the vertical GH and the horizontal cut FE.

In Square 2 the same construction is

produced in the lower left corner and introduces another vertical sacred cut, namely IJ, while the horizontal cut is the same as before.

We repeat the construction in Square 3, upper right corner, and produce a new horizontal cut in KL, while line GH is the same as in Square 1.

In Square 4 the construction is again repeated, this time in the upper left corner of the diagram, but produces no lines not already introduced by the three previous constructions.

In actual fact these four diagrams should be regarded as one, the half-size square merely being repeated in the four corners of the main square. Together—as can be seen in the fifth square of Fig. 69—they provide two vertical sacred cuts and two horizontal sacred cuts, i.e. verticals GH and IJ and horizontals FE and KL.

This particular version of the sacred cut is one which will be seen many times in determining various dimensions, and it is therefore important to understand how the diagram is achieved.

Another variation on the same theme is seen in *Fig. 70* where we have three squares, A, B and C.

In A we see the half-size square constructed as in the previous illustration in the lower left corner. Thus we have the horizontal sacred cut marked by the line 1-2 and the vertical by line 2-3.

In B the half-size square is moved to a central position on the base of the main square. Here it indicates only the upper horizontal sacred cut at 5-6, and finally in C it is shifted over into the bottom right corner and reveals the horizontal sacred cut at 1-2 and the vertical at 9-11.

Thus we see that by moving the small square along the base-line into three positions—left, central and right—we obtain three variations on the same theme, operating only with the large square's upper horizontal sacred cut, while the ex-

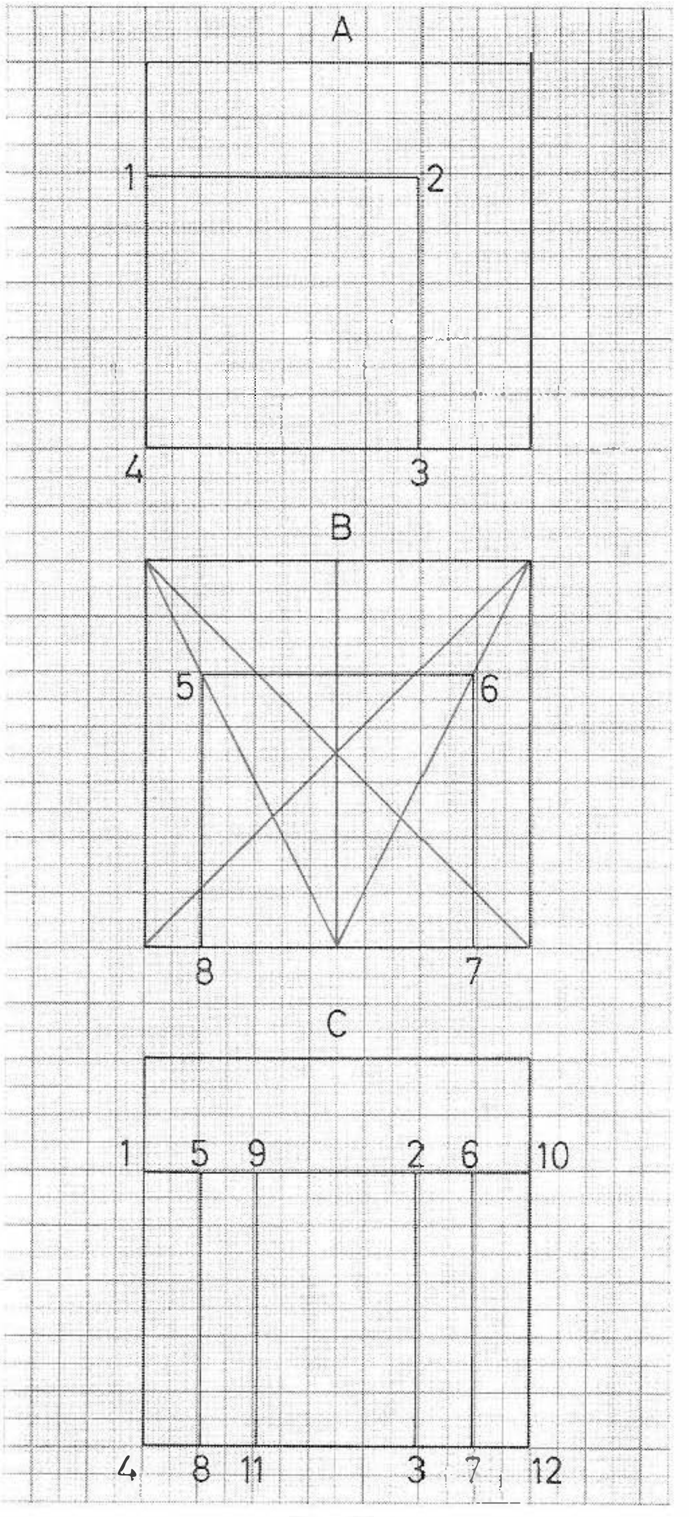


Fig. 70.

tremities of this variation show both the vertical cuts.

Diagram C has all three squares, the left square 1-2-3-4, the central square 5-6-7-8, and the right-hand square 9-10-11-12. Lines 5-8 and 6-7 do not indicate the sacred cut. They are simply included as a guide.

The same procedure—as one can well imagine—is equally applicable, for example, to the right or left vertical side of the main square, or along the inside of the upper side.

In other words, the construction combining the main and half-size squares can be executed in eight different ways. In fact it can be taken for granted that we shall see all of these lay-outs used in practice in future study.

All of the symbols we have examined so far, and the entire development we have followed, have been based exclusively upon geometry—with almost a complete lack of genuine speculative knowledge of numbers.

To achieve any progress at all in geometry it is of course necessary to have some form of numbers, to be able to weigh the respective geometrical information against the observer's previous experience and to obtain a result. But still the emphasis has been on comparative arithmetic and the principle of comparative measure. This was the basic element of geometric development.

Of course the day arrives when our observer's experiences and knowledge are to be applied to practical tasks such as surveying land. He must therefore try to transfer his knowledge in numbers alone to paper, to calculate the result instead of drawing geometric symbols.

This does not entail the outright abolition of symbols as such nor the revelation of their secrets. On the contrary, symbols remain a fountain of the knowledge applied but indicate nothing of the source of the knowledge.

Even at a very early stage Man had begun calculating certain things, and this calculation can be traced back as far as ancient Egypt. The people of old Egypt wrote down some of their knowledge in hieroglyphics on scrolls of papyrus, a number of which were discovered during the excavation of Egyptian temples. A study of these ancient texts can be read in *Rhind Mathematical Papyrus*, written in 1923 by T. Eric Peet.

The Rhind Papyrus illustrated that the Egyptians had developed a rather special method of calculating the area of a circle in simple whole figures. The practical problems mentioned in the book show that to solve the puzzle of the circle's area, the ancient Egyptian mathematician measured the diameter of the circle and subtracted $\frac{1}{9}$ of its length. He then arrived at the area of the circle by multiplying the remaining $\frac{8}{9}$ by itself.

From a geometrical standpoint this procedure of taking $\frac{8}{9}$ of the diameter to the second power is the same as constructing a square the sides of which are $\frac{8}{9}$ of the circle diameter. In other words the circle is converted into a square based on $\frac{8}{9}$ of the diameter, and this square can then be calculated in area.

Peet's book touches only three times on this calculation, and each example shows a diameter of nine. This meant a simple, straight-forward conversion since $9 - \frac{1}{9} = 8$, and $8^2 = 64$. In other words a circle with a diameter of 9 has an area of 64, which is a very creditable approximation since our present-day calculation of the same circle would of course be: $R^2 \times \pi = 4.5^2 \times \pi = 63.615$.

On the face of it this tempts us to believe that the ancients possessed a numerical knowledge of the circle's area which we thought belonged to a much later period in history. And this has naturally prompted many minds to search for a source of such knowledge.

The answer, however, is not far distant, since speculation on one of the symbols at our disposal provides a clear-cut solution. This is the symbol containing the circle's rectangle, discussed in detail in Figs. 53 and 54.

This symbol is repeated in *Fig. 71* with the addition of one or two new lines.

We have first of all the main square

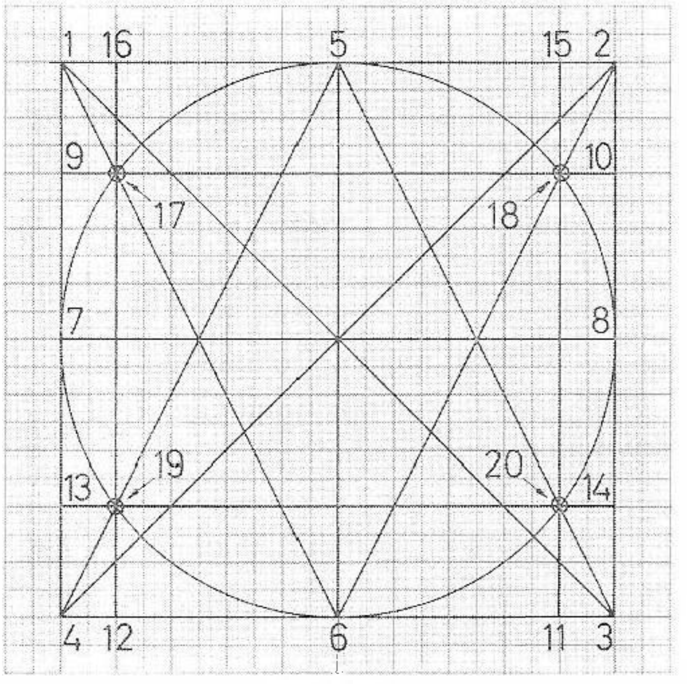


Fig. 71.

1-2-3-4, with the inscribed circle. Then the vertical cross 5-6 and 7-8. The two acute-angled triangles are seen at 4-5-3 and 2-6-1.

The intersection of these two triangles with the circumference of the circle indicate (as shown in Fig. 54) the circle's rectangle, which is entered at 16-12-15-11.

So far we recognise the diagram, and the new lines are the horizontals 9-10 and 13-14.

These lines share with the verticals of the circle's rectangle the same points of intersection on the circle's circumference. And reflection on their individual relationship to the complete diagram reveals that each horizontal indicates the circle's rectangle—suspended horizontally across the square.

The diagram shows that the sides of the circle's rectangle are in the ratio of 8 to 10.

We have this ratio again when we examine line 9-10 and the rectangle 9-10-3-4, and similarly in the rectangle 13-14-1-2.

This means in effect that with a horizontal line we can produce the circle's rectangle and place it either at the top or bottom of the main square, lying horizontally.

If, as in Fig. 71, we have drawn both the vertical and one of the horizontal rectangles, this combination gives rise to a new peculiar factor in our diagram, since it creates the largest square it is possible to draw inside the circle's rectangle. We can see this square 17-18-11-12, if we ignore in this instance the line 19-20.

This new square has an interesting geometric property in relation to the main square in that, when the small square is divided up in accordance with our established practice, its lines are such that they match in precisely with those of the main square. All these areas can then be divided into uniform squares or triangles, allowing us to calculate the values of the individual areas and compare these with the main square. Thus we can see in numbers how large these different areas are in relation to the primary square and of course how large they are compared with each other.

We find this division of the diagram in *Fig. 72* where we have exactly the same symbol as in Fig. 71. We begin the division of the square on the circle's rectangle in

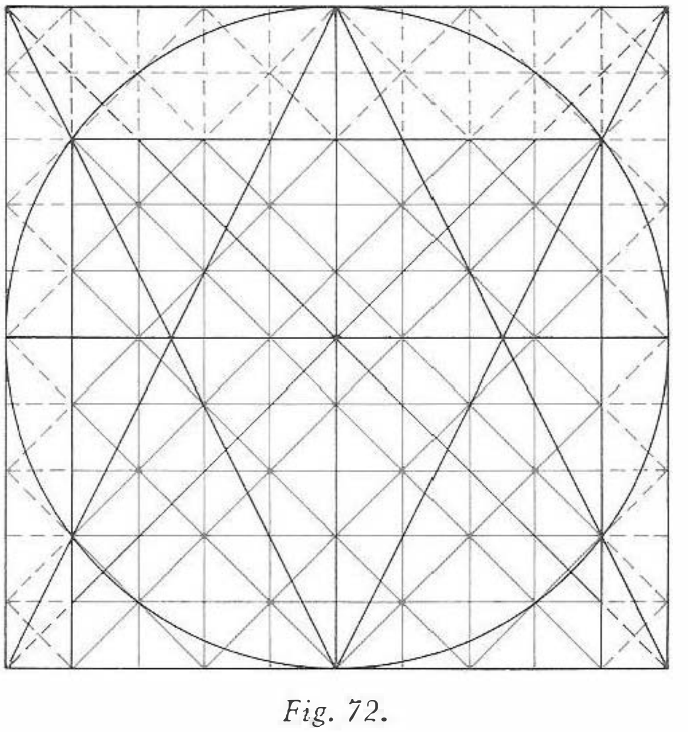


Fig. 72.

our normal manner with the vertical and diagonal crosses, after which without any form of measure we can continue the dividing process as far down as technique allows.

In this instance we have split the small square into 128 small triangles or 64 small squares.

If we produce the same division into the large square, we find that the same lines divide this into 200 small triangles or 100 small squares. Thus we have a regular splitting of the main square 10 × 10 = 100 small squares.

We discover with this division that—in whole numbers—the main square contains 5 × 20 small squares and the circle's rectangle contains 4 × 20 small squares. Thus we see how ancient observers/mathematicians were able without a profound knowledge of numbers to establish that the rectangle is ⅕ smaller than the square. At the same time they ascertained that the circle's rectangle consists of 80 squares or 5 × 16, and the square on the circle's rectangle comprises 64 or 4 × 16, and they could thus observe that this smaller square was ⅕ less in area than the circle's rectangle.

The fact that these two assessments are based on a division into 5 is because the upper side of the square on the circle's rectangle cuts ⅕ off both the main square and the circle's rectangle. It is also an easy task to count these 20 and 16 squares respectively and to consider these in relation to the remaining areas. Further, this particular division is fortunate in that the problem with both areas works out evenly.

As our ancient observer ponders over his material, he will only naturally search for a square equal in area to the circle's rectangle. And since he has his small squares to count as a guide it is simple for him to approach this problem, as all the small squares are equal in size whether inside or outside the rectangle. The nearest he can get to such a square, however, is one that is 9 × 9 = 81.

This gives him a square which he can readily see is slightly greater than the rectangle. Perhaps during his minute measurement he has earlier discovered that the circle's rectangle is in fact a tiny bit bigger than the circle itself—a fact we saw earlier. But at that stage he was unable to achieve a greater degree of accuracy.

If he appreciates this error then he also realises that to accept the resultant square as being identical to the circle would merely increase the degree of inaccuracy.

Since he knows that the ratio of the main square to the circle's rectangle is the same as the ratio of the circle's rectangle to its smaller square, i.e. ⅘ to 5/5, he turns to another way around the problem. He says to himself that the square which is nearest in area to the circle's rectangle is the square with the 9 × 9 small squares; if we describe a circle within this the result will be that this circle's square will be slightly under ⅕ less than the circle. Thus he has compensated for the tiny variation he faced earlier, without being able to establish precisely how close he has in fact come to the correct result. But by logic he can work out that he has obtained a result nearer perfection than he would have achieved simply by accepting the obvious difference.

The result then is that he says that one calculates the area of a circle by subtracting ⅑ from the diameter and constructing a square on the remaining line.

We see this geometric construction in *Fig. 73* where the circle is described in a square which is 9 × 9 = 81, and the counterpart of this circle is the square which we see to be 8 × 8 = 64.

We must here remind ourselves that calculation as such was an innovation in our observer's study, still very much in its infancy. Indeed it was of very much minor importance than the age-old tradition of

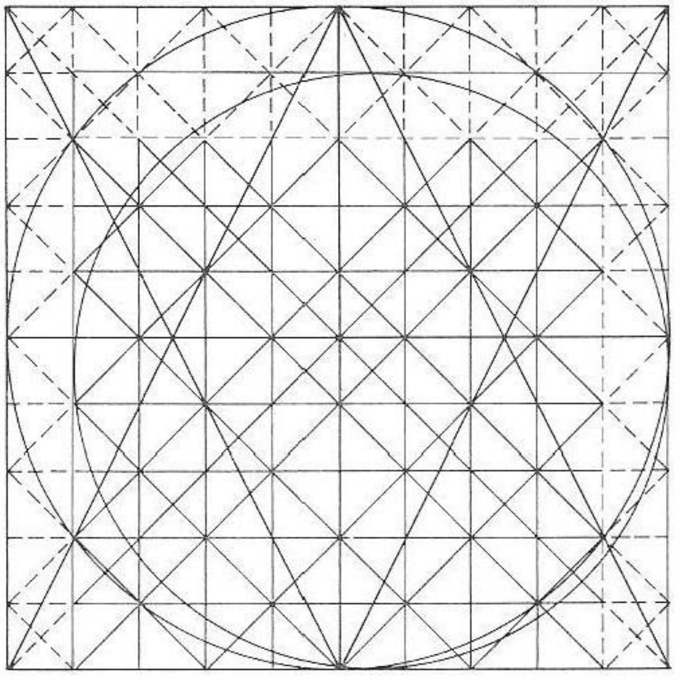


Fig. 73.

geometric symbols. And an innovation of this type had had virtually no effect on the symbols or their future use. In fact even approaching our own era we find traces of the use of the symbols, including the circle's rectangle and the square on the circle's rectangle.

Moreover there is no denying the possibility that the discovery of the calculation of the circle's area in the manner described above remained more or less a secret within the immediate vicinity of the temple where the papyrus was found. This possibility is based on the fact that excavation work has produced only one or two samples of this method of calculation, and there is much to indicate that these and similar theories met vehement resistance. As late as the main period of Grecian greatness we come across a diagram resembling that in Fig. 71 complete with a detailed account of numbers required in finding the area of the circle.

This account is to be found in Plato's *Timaeus* and will be discussed more fully later.

Of the numerous symbols and diagrams we have seen described in the foregoing pages, we can select 21 which we shall term from now on, the primary symbols.

I sincerely trust I have been successful in describing for the reader how and in which sequence they arose, and—not the least important—precisely what information each symbol contains. All the symbols are in fact individually complete in themselves and as a whole, since each arises out of the previous. Each new line represents a discovery in geometry or mathematics. The absence of words to describe these discoveries is in my opinion an ingenious way of depicting one's knowledge, a method which reveals no information to a party not intended to have it. But for the observer who has been initiated in the mysteries of mathematics the symbols would remain fresh in the memory as indeed would their meaning.

We see the complete list of symbols in *Fig. 74* where to clarify the discussion we have supplied each symbol with a letter of our own alphabet, so that later we may simply refer to each symbol by the relevant letter.

The list begins with:

A) This symbol was the first and the point at which all mathematical speculations began, namely the *circle* and its vertical cross.

B) In this Man produces his first geometric construction by drawing two vertical and two horizontal lines outside the circle.

C) The same diagram, with the addition of the vertical cross dividing the *square* into four smaller squares.

D) Entering the diagonal cross splits the square into four large triangles and when the vertical cross is reintroduced the square is divided into eight triangles.

E) The *half-size square* is entered from tip to tip of the vertical cross, and the quadratic area is divided in this way into 16 small triangles—

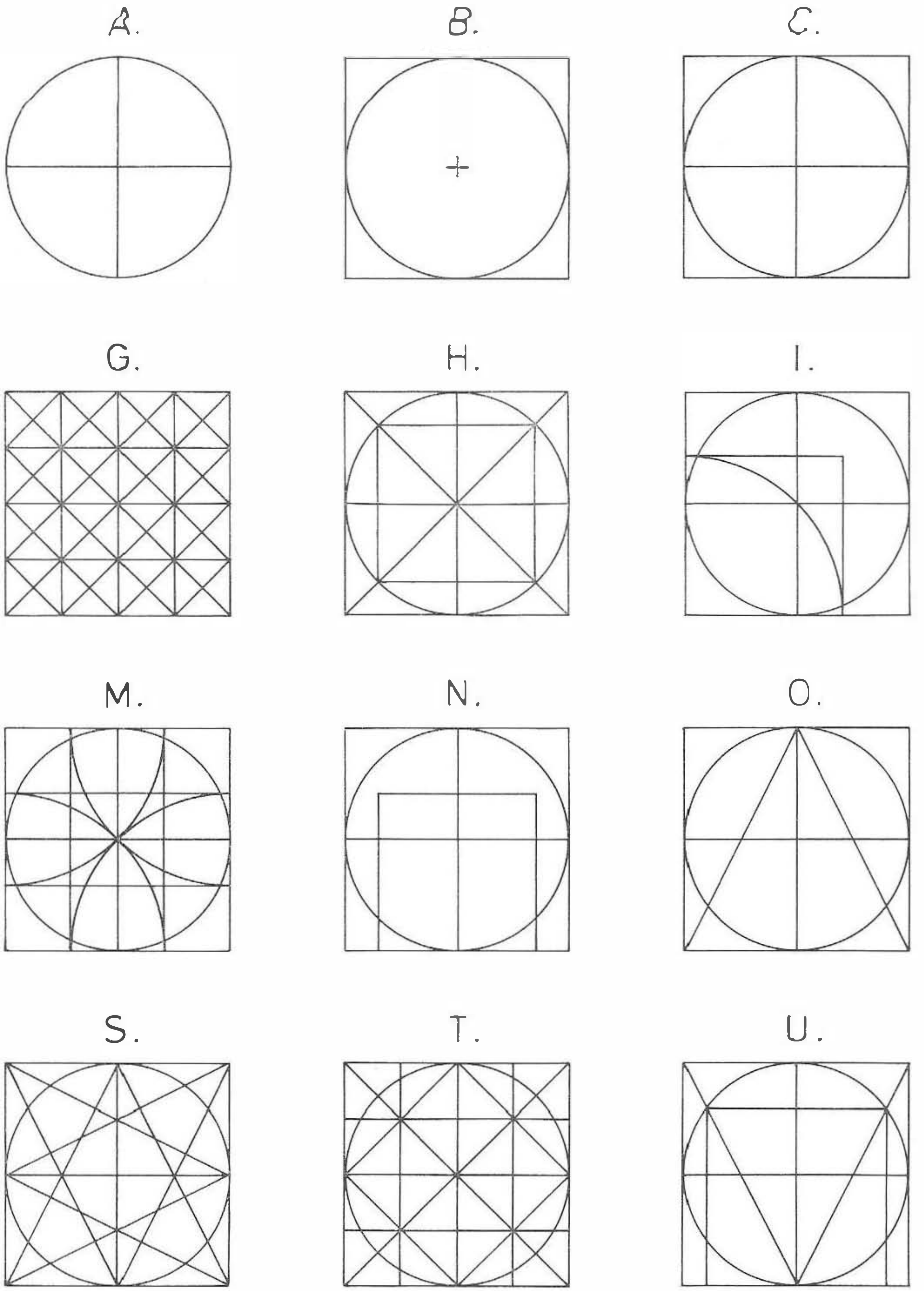
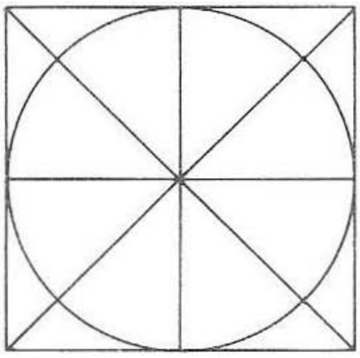
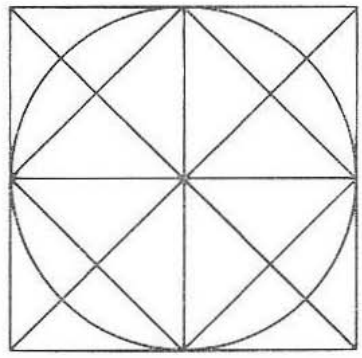


Fig. 74.

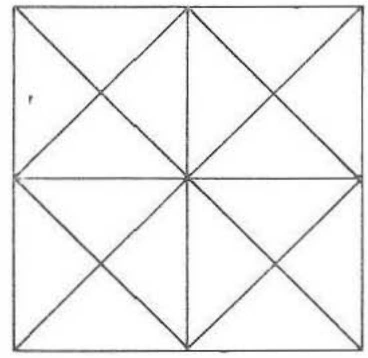
D.



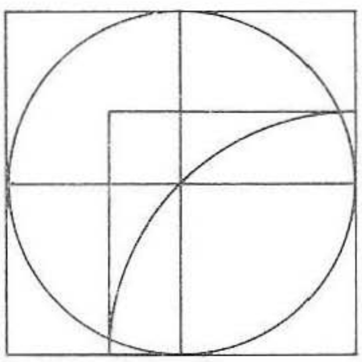
E.



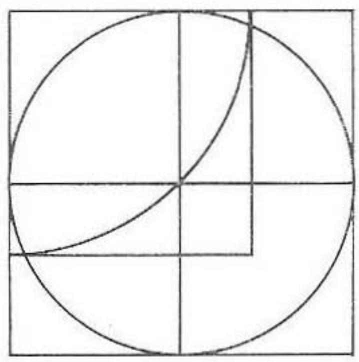
F.



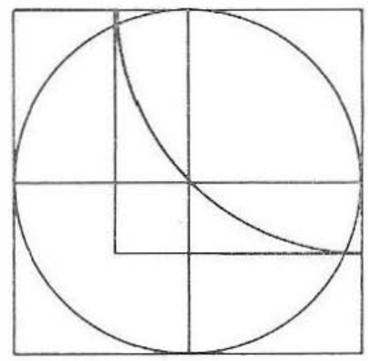
J.



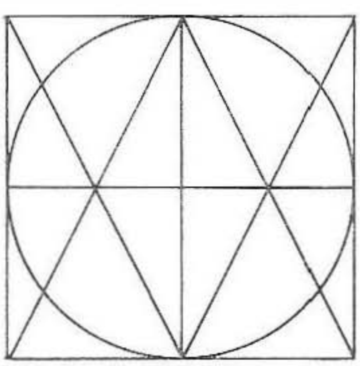
K.



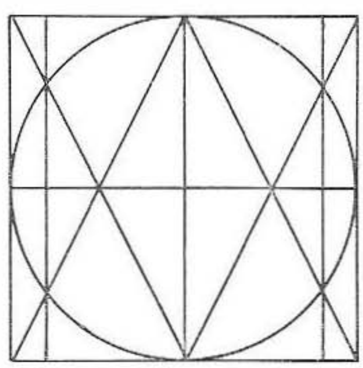
L.



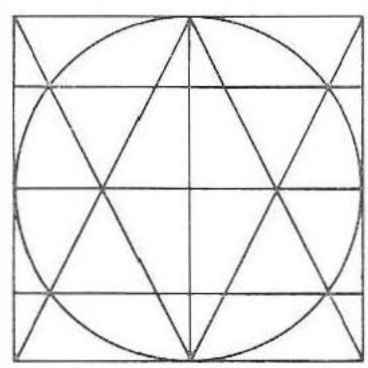
P.



Q.



R.



forming the foundation of the subsequent procedure of dividing surfaces into triangles and not into smaller squares. This construction emphasises the place of the figure 7 as one of importance.

F) Previous experience is applied to dividing the square into triangles in the order 2, 4, 8, 16, 32 and 64.

G) This shows the division into 64.

H) Shows how the half-size square can be constructed on the points at which the diagonal cross intersects the circumference of the circle, and the first step towards calculating the length of the circumference has been taken.

I) The half-size square is constructed in the lower left corner of the square. This construction indicates the upper horizontal and the right-hand vertical *sacred cuts.* The symbol also contains the successive doubling in area of the three squares inside each other.

J) The same construction executed in the lower right of the main square, showing the left-hand vertical and the upper horizontal cuts.

K) Shows the lower horizontal sacred cut and the same vertical as I.

L) Also shows the lower horizontal cut and the same vertical as J.

M) This is a combination of the four previously indicated sacred cuts, two vertical and two horizontal. We recall that the sacred cut held the key to the circle's circumference, and we have already on several occasions during the development of our geometric system seen this factor assume considerable importance.

N) Shows an arrangement with the half-size square in which it occupies the centre of the base-line on the original square.

O) This shows the important *acute-angled triangle* which is to prove the first step towards calculating the area of the circle. The area of the triangle is also half the area of the square.

P) Its opposite partner is entered, their intersections with the circumference marking off the *circle's rectangle.*

Q) This is the same diagram as in P, only with the circle's rectangle entered.

R) The same as Q, but showing the two horizontal lines which indicate the circle's rectangle horizontally at the foot and at the top of the original square.

S) Diagram with the four acute-angled triangles, showing the eight-pointed star.

It was this star that allowed the side of the square to be divided from 2 to 10 by means of mutual points of intersection of the diagram's component lines.

T) The diagram which provided us with a number of accurate divisions of the circle's circumference.

U) This diagram is in effect part of R, the *square on the circle's rectangle* appearing more as an independent unit.

This concludes the summarised description of the 21 primary symbols which we shall see repeated time and again from building to building as the dominating factor in planning elevations, lay-out plans and decorations of such buildings. We shall find the symbols described in ancient literature. We shall find them in the realms of figurative art. We shall find them, too, as the basic element and constructive factor in the relatively later form of Egyptian mathematics which we of today know from inscriptions and papyri.

Indeed it is true to say that we shall see evidence of these symbols in innumerable fields right up through the years to the Middle Ages, and even at the conclusion of the book only a fraction of the geometric system's application will have been revealed.

CHAPTER SIX

The application of Ancient Geometry

IN PREVIOUS chapters we have engaged only in the formation of ancient geometry, sticking to a purely speculative and theoretic theme, without illustrating the application of the system in practice.

We shall make a point therefore in the following part of our study of demonstrating the existence of this system and its utilisation in a number of spheres in which it played a key role in the cultural development of the respective eras. And one of the first domains in which we can trace the system is the allotting of dimensions in the planning of a number of old temples.

Before, however, delving straight into the practical application of our list of symbols it would perhaps be useful to outline generally the manner in which those ancient symbols found their way into architectural and other planning. This will naturally be dealt with more fully in subsequent analyses but an advance explanation would probably facilitate understanding of their use. It would moreover be important to provide a mental picture of the status of the symbols as sacred factors within Society, factors whose innermost secrets were known only to a handful of initiates.

As we have seen, I am convinced that the entire field of geometric knowledge in those days had its roots in meditation on the Sun and Moon. These have both from earliest times been acknowledged as gods and hallowed as such. Human history teems with references and proof to this effect.

Depiction of these two gods can only be met by the circle. The circle thus became, not only a geometric shape (as we would term it today), but also a picture of god, in this way became the first sacred symbol Man possessed.

In time the circle became an object of reverence and prayer, but simultaneously its geometric form sparked off in certain quarters a glow of speculation which fanned into a blaze of geometric thinking. The circle being sacred, knowledge derived from it was not made the property of the common man.

These discoveries were made by the temples or temple brethren. The results were regarded as divine revelations, but revelations intended for tight circles of initiates. The reason, as much as anything, was possible that outsiders not acquaint with the Temple and its workings simply did not possess the knowledge necessary to follow these thoughts.

Gradually—as discoveries multiplied—they were streamlined into symbols of which 21 primary symbols emerged. Since each originates in the picture of a god, the circle, each is sacred. The symbols were a sign from god and their formation evolved through countless years.

Major discoveries were entered on existing diagrams, or perhaps warranted a brand-new symbol, but in relation to time itself development was an extremely slow process.

Thus the application and importance of the symbols as geometric shapes was appreciated only by initiated Temple brethren, and even though a particular symbol might be noticed by an outsider it would probably be considered a decorative figure or pattern.

There need in fact be a minimum of secrecy concerning the actual appearance of the symbols, but the reverse would be the case regarding their practical application. This blanket was so dense that nothing in fact leaked out of the temples over thousands of years—and without any shadow of doubt the penalty for profaning the sacred knowledge would have been death.

We must therefore consider these symbols to be the sacred property of the Temple. Work based on this knowledge can derive only from the Temple, and we cannot expect outsiders to have access either to the application or geometric significance of the symbols.

As Fig. 74 shows, the whole range of symbols is surrounded by the square. And it is this square which we shall see has been used—with its various divisions and dividing lines—as a basis for dimensions.

In fact it is surprising yet at the same time obvious that the square should be used for this purpose. One might have expected to see the circle employed as the basic point of origin since geometric knowledge stems from this shape, and it is possible that all the symbols should be regarded as having an outside circle, with the square placed inside.

Symbolically then we have the circle as the original shape. The inscribed square is inspired by the gods but created by Man.

Irrespective how it should be regarded, however, the following pages will illustrate that every dimension is determined by an external square which is larger than the subject planned, since this almost invariably lies completely within the square, filling the whole square or a part of it. It is both possible and probable that the square is *in its turn fixed by the dimensions of an* external circle. But as it is the square and its division which lay down the actual dimensions, and as we in our proof in fact are working the system in reverse by starting with the item under examination and finding our way back to the square and its divisions, we are really also determining the square's external circle. But as this circle has no separate effect on the dimensions of the temple (or whatever we may be studying), we shall omit it.

The simplest method of demonstrating the application of the circle is to imagine ourselves as the builders of an ancient temple. We shall not at this stage deal with any one particular temple, but merely select a period round about 500 B.C. when a number of characteristic columned temples of that time were created. Furthermore we shall restrict ourselves to the main lines of the temple, not going into the detail, since the object for the time being is to show how to use the system and not to construct a complete temple.

Our first step is to choose one of the 21 primary symbols as a starting point for our planning, and we pick symbol "M", which contains two vertical and two horizontal sacred cuts, *Fig. 75.*

The width of the square is selected as the width of the temple, and the base of the square is therefore the width of the front of the temple.

The diagram has two horizontal lines which could be employed as the height of the roof, namely line 1-2 which is the diagram's horizontal central axis, and 3-4 which is the upper sacred cut in the

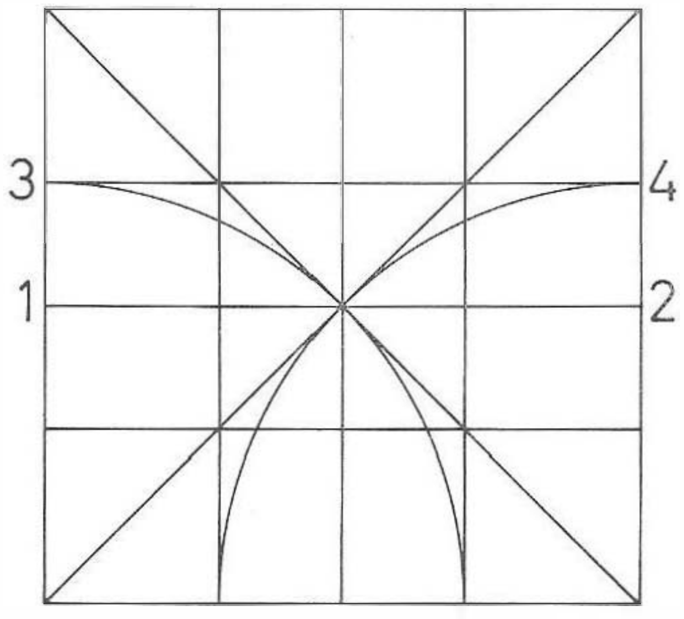


Fig. 75.

square. One of these two lines is to be the temple's horizontal roof line, and will thus give us the height at which to build the roof.

If we select the horizontal central axis as the line determining the under-side of the roof, we are faced in the erection of our temple with four possibilities for deciding the angle of pitch on the roof.

In Fig. 76 A we have drawn the roof from the horizontal central axis up to the line of the sacred cut. In Fig. 76 B we enter a diagonal cross in the rectangle formed between the horizontal central axis and the upper sacred cut, and produce a roof-pitch which is half as steep as the preceding.

In Fig. 76 C we have drawn the diagonal cross in the upper half of the main square, which provides a total height equal to $\frac{3}{4}$ of the selected width of our temple. And in Fig. 76 D we have the fourth possibility for the roof-pitch, the pediment and main temple wall being equal in height.

If we choose the upper sacred cut as the horizontal line of the roof, we are left with only two possible roof-pitches. Either that in Fig. 76 E which shows lines produced to the top of the central vertical axis of the main square, or as in Fig. 76 F half of the upper rectangle by entering a diagonal cross.

Thus this diagram has given us six possible versions of the front elevation of our temple, and we select in this instance Fig. 76 A because we think its proportions are the most beautiful of all six.

The first symbol then has provided us with a number of important dimensions for the front of our temple, i.e. the width, the vertical height of the roof, the total height of the temple, and thus the roof-pitch.

We must now find a new symbol for some of the remaining dimensions since we require certain points of construction not contained in our present symbol. For this purpose we select symbol "U".

This was the symbol in which we constructed the circle's rectangle and also the square on the circle's rectangle. The construction is described in detail in Fig. 71.

The selection of this particular symbol is perhaps a little deceitful on my part, since it was not chosen by accident. But it is simply that this is the symbol I have discovered used in so many temples for the dimensions we now require, i.e. the detailed proportions and plan of the front elevation apart from the general dimensions laid down by symbol "M".

Fig. 77 illustrates which dimensions are indicated by this diagram in the front of the temple. In this Fig. the preceding symbol "M" as well as the actual outer dimensons are shown in black, while the new symbol "U" is drawn in red.

On the face of it the symbol itself reveals nothing in particular and it is not until we toy around with its guide-lines and divisions that we see the various dimensions emerge.

We see in the analytical drawing that the central horizontal axis of the main square delineates the horizontal level of

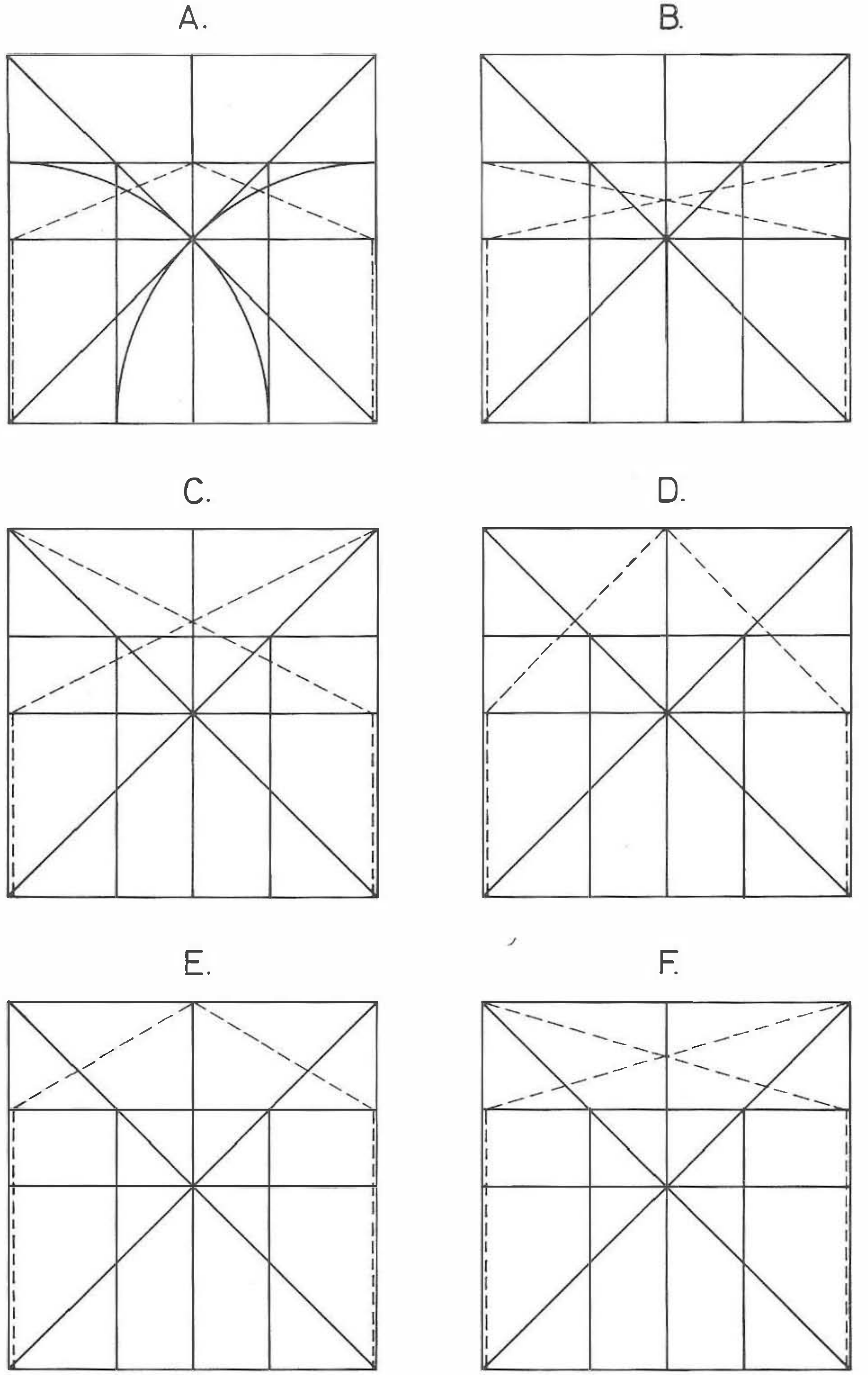


Fig. 76.

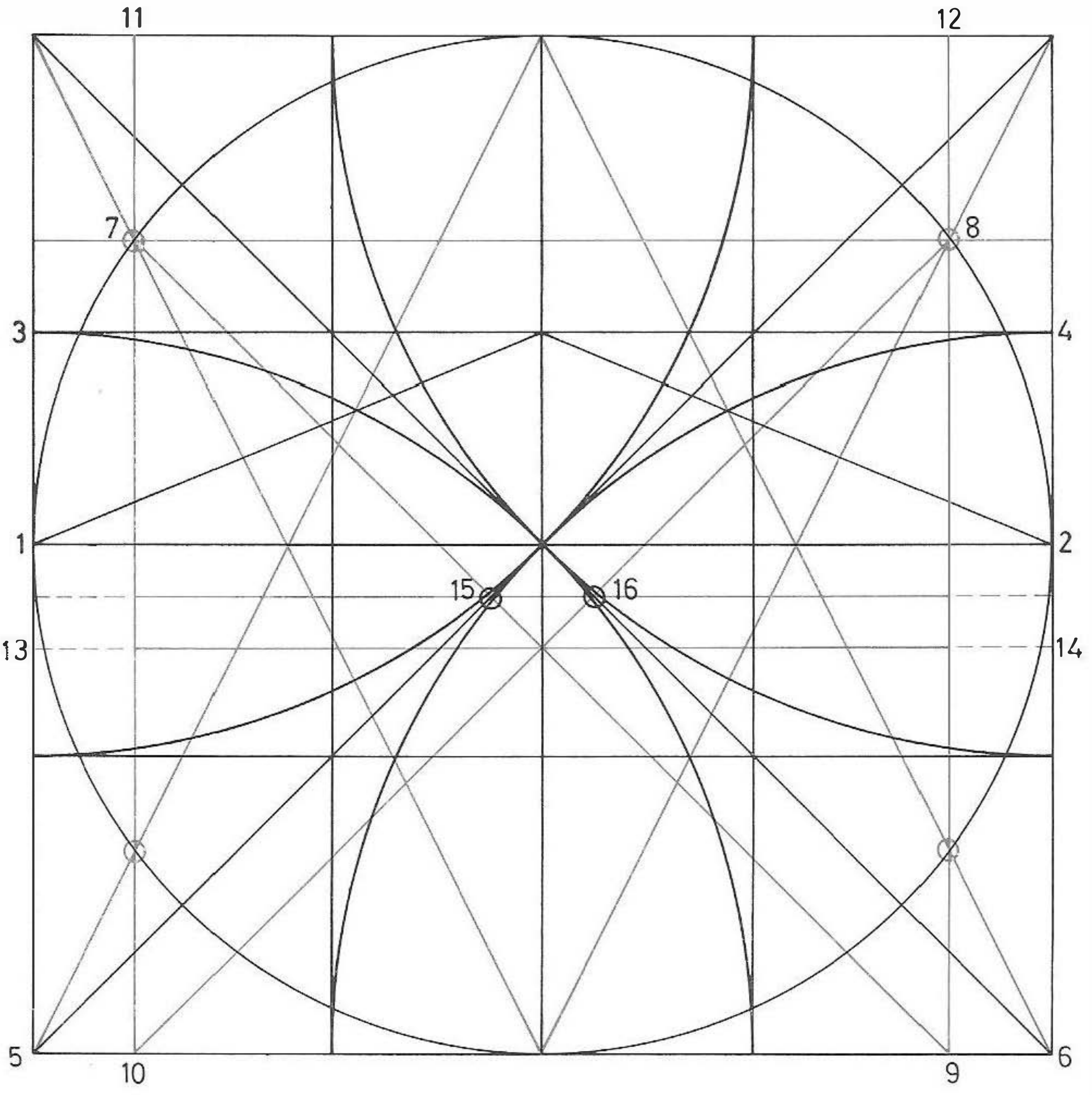


Fig. 77.

the roof at 1-2. The upper extremity of the roof is on the line 3-4, which is the upper horizontal sacred cut in the main square. The base of the same square (5-6) is identical to the base of the front of the temple, and all of this information is provided by the original symbol "M".

We now enter the vertical and diagonal crosses in the smaller square in symbol U, i.e. the square at 7-8-9-10. This, we recall, is termed the square on the circle's rectangle.

The circle's rectangle is described by the lines 11-10 and 12-9, and the abovementioned square is therefore the largest square that can be constructed inside the rectangle. And this is the square which, in its subdivision into triangles, divides up the main square in such a way as to allow the two areas to be compared.

By entering the vertical and diagonal crosses in this square, we produce the next line in our plan, i.e. 13-14, which is the central horizontal axis of the smaller square. We decide to make this line the underside of the supporting architrave (or lintel) on which the whole of the roof structure will rest. These beams are normally supported in their turn by the temple columns, and are usually decorated with a frieze immediately under the roof, the lower part of the architrave remaining clean and smooth above the columns.

The whole of this entablature is indicated in our diagram. The diagonals of the large square and those of the smaller square combine to produce a tiny square of which the two vertical points (in reality the centres respectively of the main square and the rectangle's square) indicate the thickness of the beams; the two horizontally placed corners divide the entablature in two equal layers.

We follow these lines and select the upper half for the decorative frieze and the lower half as the smooth lintel.

The corners are shown at points 15 and 16, and the dividing line is shown as a broken line.

Apart from the width of the temple, we have so far obtained only the building's height. But this of course has an important effect on horizontal dimensions, including those of the columns. We shall now mark these in on our plan, but to avoid having too many distracting new lines, we move over to a fresh diagram: *Fig. 78*, in which all of the previously discussed lines are shown in black, the new lines being indicated in red.

We saw earlier, in Fig. 59, how the relationship between the acute-angled triangle and the square's diagonal cross provided at the points of intersection a division of the square into 3 × 3.

We now enter this 3-part division in our diagram in the square on the circle's rectangle, i.e. square 7-8-9-10. This division is already indicated in part by existing guide-lines, and we see two of the intersections at points 17 and 18. The vertical dividing lines for the 3-part division are entered at 19-20 and 21-22, but we content ourselves with only the lower of the two horizontal lines 23-24, and we now have in the lower third of our square three smaller adjacent squares.

The 3 × 3 division is again executed in these three small squares, the result being that the base-line of the square on the circle's rectangle is divided into nine parts. Only the vertical dividing lines are entered. The dividing points are ringed in the diagram.

The vertical dividing lines are produced (broken lines) to meet line 13-14 which we have made the underside of the supporting lintel, and therefore the top of the bearing columns.

We see that the whole of the base-line 5-6 on the original square is divided into eleven pieces. We know from our drawing that nine of these pieces are equal in length, while we do not know the length of the outer two.

The whole of the dividing process has been based upon the circle's rectangle or its square, and we saw earlier that the short side of the rectangle extend along $\frac{8}{10}$ of the base of the square in which it was constructed. We are thus able to say that lines 5-10 and 9-6 each represent $\frac{1}{10}$ of the base.

Line 9-10 must therefore be $\frac{8}{10}$ of the same base-line.

If we assume that the width of the temple has been fixed at 30 meters, obviously lines 5-10 ond 9-6 will each be 3 meters long, while line 9-10 will be 30 minus 6 = 24.

The nine inner parts must therefore each be $\frac{24}{9} = 2.66$ m., and we have thus proved that the two outer parts are somewhat longer than the nine inner—a fact that was of course obvious to the ancient planners.

This division of the front elevation indicates the placing of the temple columns as follows:

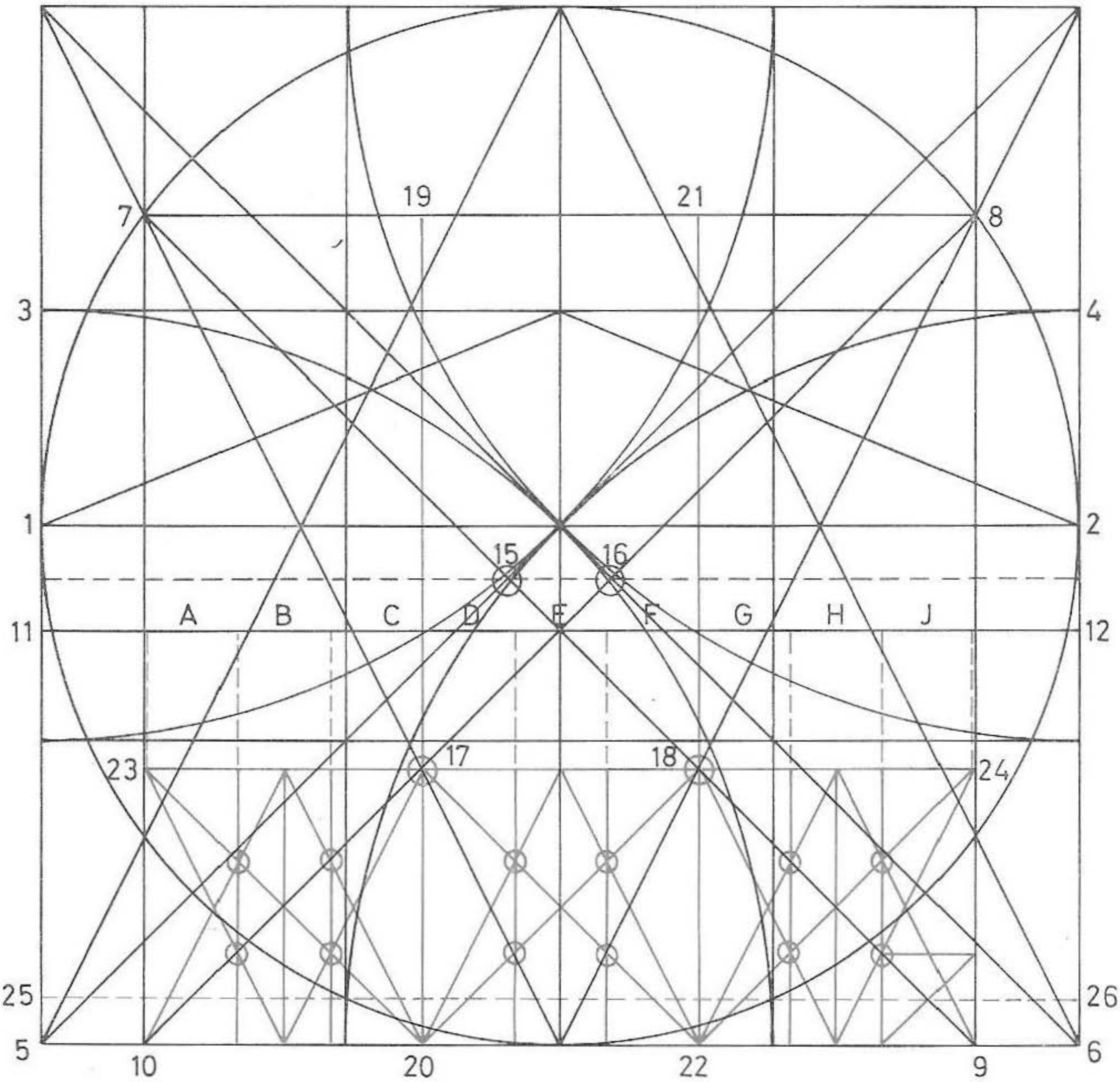


Fig. 78.

Each of the nine inner areas has been designated a letter from A to J.

We begin on the left of the diagram at area A. This is left empty to form a gap and we place our first column in B. We proceed to alternate between columns and spaces, until we have used up all nine areas. We thus have columns in B, D, F and H, and spaces in A, C, E, G and J.

Our front now has four columns and five spaces. But our temple is of course wider than line 9-10, and in the placing of columns so far we have started and finished with a space.

We therefore place another column at each side, immediately after the space, in the areas which we discovered were slightly wider than our nine inner sections.

These two final columns are positioned against lines 23-10 and 24-9 respectively and we thus obtain a perfectly uniform facade with equal distance between all 6 columns. This provides a division into 11, with 6 columns and 5 spaces, a most characteristic arrangement in ancient temples.

We could also have decided to move the final two columns out in line with the vertical sides of the main square, which would have given a somewhat wider space between the outer columns and their neighbours. This arrangement was also a popular one among temple-builders of antiquity, and has the practical advantage of making this outer space, which would form the cloister, as wide as possible.

But our choice is the most traditional, with the columns placed as compactly as geometric construction allows.

We shall not at this point go into detail regarding the choice of columnar thickness, and the different thickness at the top and base of the column, since this is discussed fully later. But it may be noted here that if we make the columns slimmer than the dimensions above, the spaces between the columns become correspondingly wider, and the optical effect is altered radically even by relatively small changes in dimensions—without our requiring to change any of our divisions or diagrams.

For our particular temple we shall stick to the dimensions indicated in the diagram, the column and the space being equal, a typical arrangement.

As our plan is at present, the columns go all the way to the ground, but our temple cannot, of course, stand on the bare earth. It needs a base to raise it above the surrounding terrain, so that the cloisters and inner temple are suitably elevated.

We therefore turn again to our diagram to find a suitable indication of the height of such a base, and study the plan in Fig. 78. The columns are placed in the three small squares which resulted when we divided the square on the circle's rectangle into three.

These small squares are again divided vertically into three. The same marking shows the horizontal division. If we carry this structural line through to its conclusion we obtain a line of minute squares along the base of the diagram, one of which we see in the lower right corner at point 9.

We have several possibilities at our finger-tips. We can either select the upper horizontal side of this small square as the top of the temple base, and by executing yet another 3-part division within this square obtain the dimensions of the three steps we want to have up to the temple.

Still assuming the temple to be 30 m. in width, this makes the base 2.66 m. high which in turn makes each step approx. 88 cm. which is of course an awkward height.

We choose therefore first to divide the square into two, and then to divide the lower half into three—which provides a step-height of 44 cm.

The top of the temple base is shown by the broken line 25-26.

If we had desired a lower step, we could perhaps have chosen a 3-part division followed by another 3-part division. This illustrates just one of the many examples where the architect has a certain freedom of choice in selecting his dimensions even though the system is confined to a given number of symbols.

Practical requirements naturally play their part in fixing the dimensions, and when the planner is designing steps he must bear in mind that they are intended for use by human feet. No good making them impossibly high.

The smaller the temple, the higher we can afford to rise in our diagram; the larger the temple, the more we must take care to fit our plan to human dimensions.

We are a long way from exhausting the possibilities of this symbol. We could, for example, apply the 9-part division of the square to the placing of the decorations in the frieze along the underside of the roof.

We could use the small square in the bottom right corner for fixing the dimensions of the columns, both top and bottom,

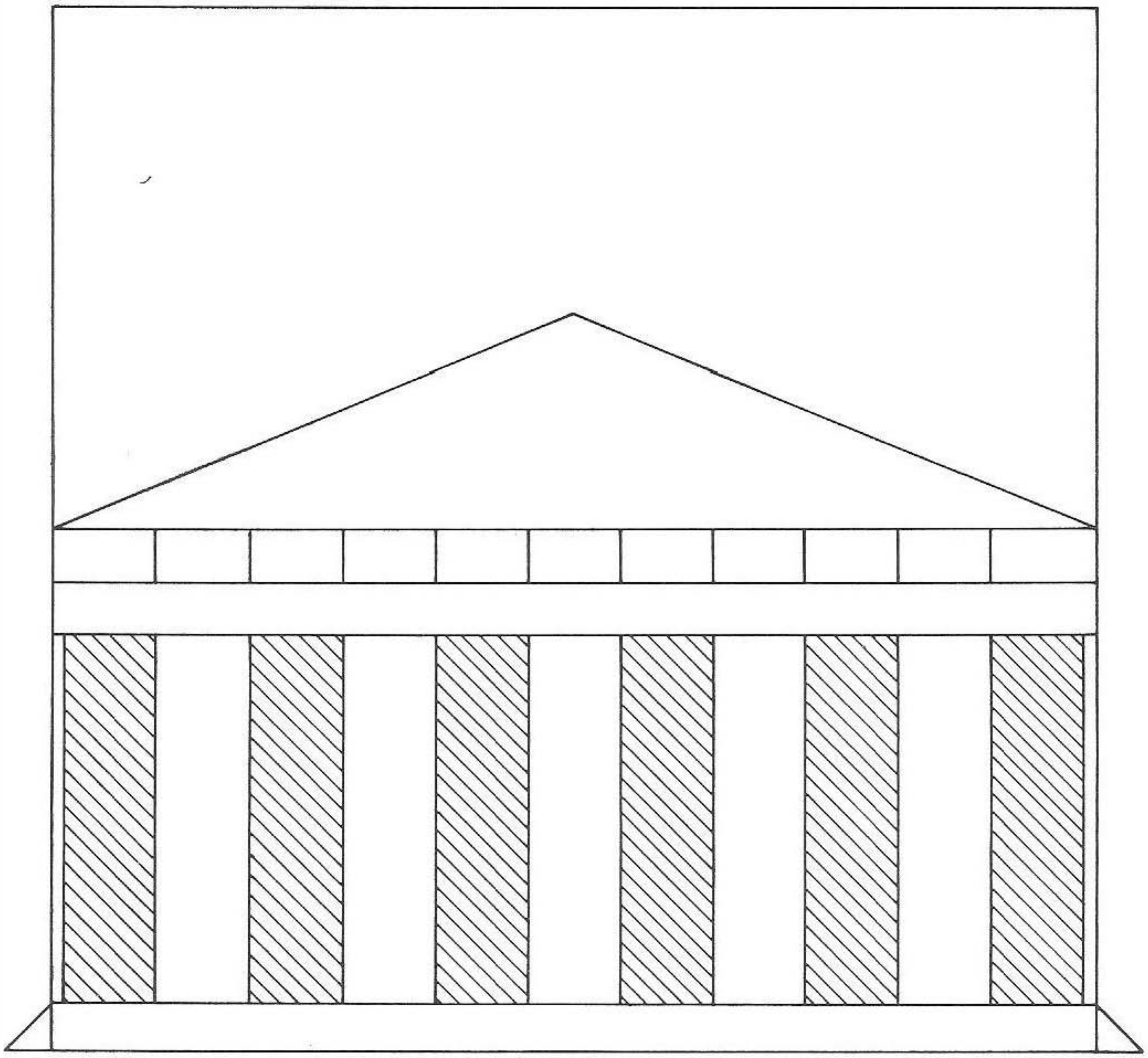


Fig. 78 A.

and we could from our guide-lines position the figures or other ornamental work in the triangle which represents the pediment of the temple.

But we need not engage in such detail here as we are not planning the final temple, but merely showing generally how ancient geometry was applied to dimensions.

The facade is now more or less complete and we see it once more in *Fig. 78 A* in which all the guide-lines have been removed, leaving behind only the original square and the dimensions which provided the front elevation. We have without doubt obtained a construction sharing many points of similarity with several antique columned temples. Now we move on to the ground plan.

In laying out our ground plan we must observe two things. First, it must be closely related to the front elevation. We cannot simply say that the temple will be such-and-such a length. The second point is that the ground plan must also originate in a square, for there are lots of details for which we shall require dimensions in the ground plan, and without the surrounding square we are unable to construct our symbols—which are of course the all-important factors in our design.

We see in *Fig. 79* the completed plan of the front elevation, and observe that square 7-8-9-10 with its 3 × 3-part division indicated the positioning of the columns.

We now take this square and go the opposite way. We *multiply* it 3 × 3 times, arriving at a large square which we see indicated by 23-24-25-26. Our motive in multiplying the square in this way is to project to the ground plan the same 3 × 3 division we had in planning the facade. The first of the ground plan lines we shall examine are the two verticals which split the square into three parts. These are 8-31 and 6-32. These lines have been projected direct from the front elevation and of course indicate the inner edge of the outer rows of columns, and similarly the outer edge of the cloister.

We now enter symbol "M" in the large ground plan square, this being the symbol which provided dimensions of the roof-pitch in the front elevation. It gives us four sacred cuts in the large square, i.e. the two vertical and the two horizontal.

In laying out the front elevation, we employed the upper horizontal to determine the total height of the temple. Here we shall use the two verticals to determine its gross width. These lines are 27-28 and 29-30.

At this point we enter symbol "H" in our diagram, by putting in the vertical and horizontal crosses, the circle, and the half-size square from diagonal to diagonal. This square is shown at 33-34-35-36.

In this square we already have the diagonal cross, and we now divide the square 3 × 3 by entering the acute-angled triangle. Since we shall require only the vertical dividing lines, we enter only these. The points of division are ringed in the diagram for clarity, and the verticals are 37-38 and 39-40.

Thus we have two lines passing down through the temple which we take as the gross width of the inner temple. We take line 33-34 as marking the entrance to the temple, so that we obtain a front hall between the temple's frontal colonnade and its actual entrance. In the same way we use line 35-36 as an indication of the rear wall of the inner temple, having an equally large hall at the back as we have at the front of the temple.

Once again we inscribe a circle in our diagram, this time in the inner, half-size square. This provides the dimensions of the third and smallest square which we have not entered in full. It is sufficient for our purposes to produce the vertical sides of this square to meet the main square at 41-42 and 43-44. This determines for us the placing of the three steps leading up to the temple colonnade. The height of these steps was fixed in the planning of the front elevation.

With regard to the distribution of columns along the length of the temple, this long side of course consists of three times the width of the square which—in the front elevation—gave a distribution of four columns and five spaces.

We have marked the three squares on the left of the diagram A, B and C. First, we place the column distribution in the central square A, starting with four columns and five spaces. In squares B and C we reverse the situation, entering five columns and four spaces.

This provides us with fourteen columns on the long side of our temple. Thus we have achieved six columns along the front and fourteen columns along the side, which is a common combination in several Greek temples.

We have now established all the major dimensions of our ground plan. We have the length and width of both the outer

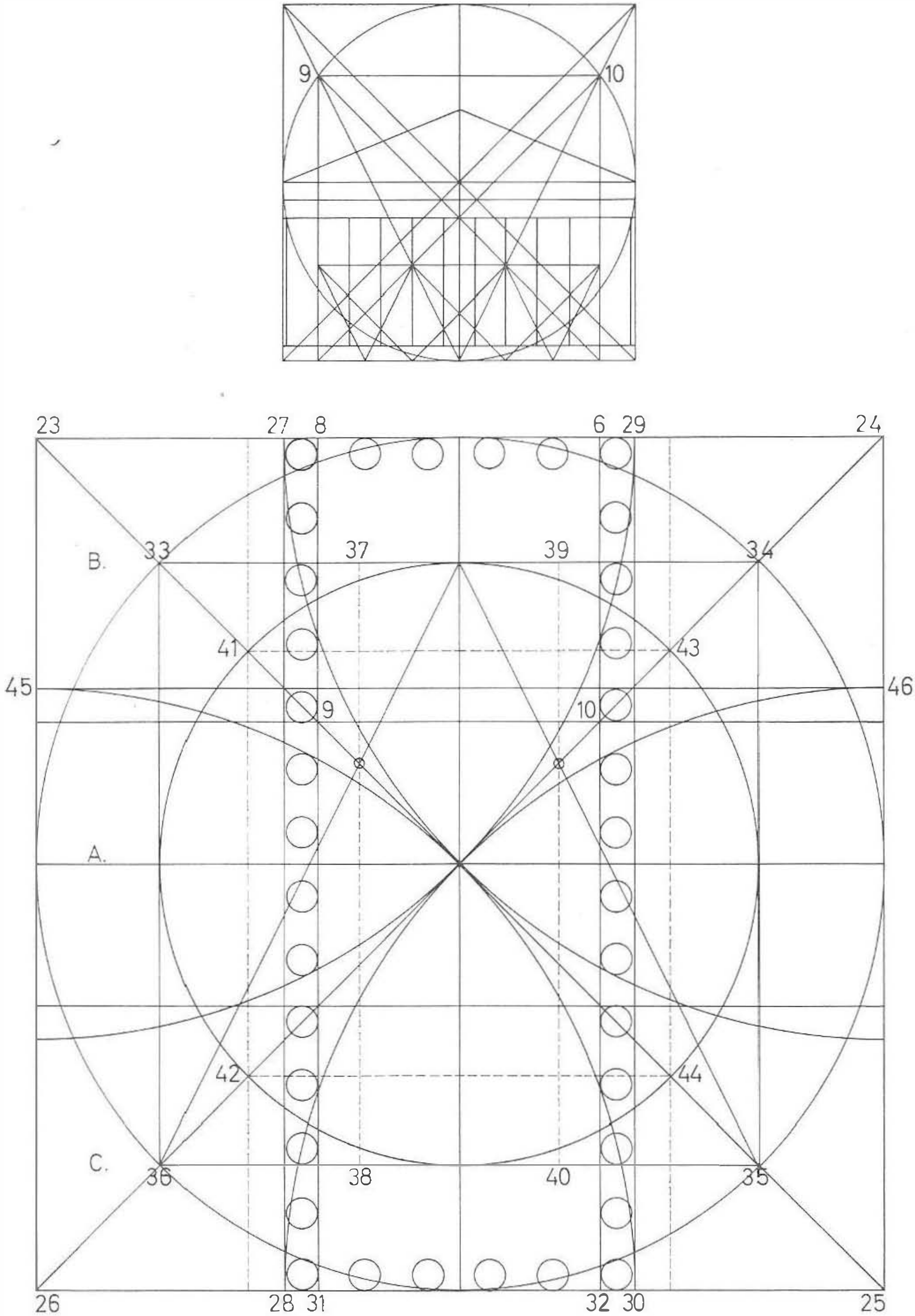

9
10
23
27
8
6
29
24
B.
33
37
39
34
41
43
45
46
9
10
A.
42
44
C.
36
38
40
35
26
28
31
32
30
25

Fig. 79.

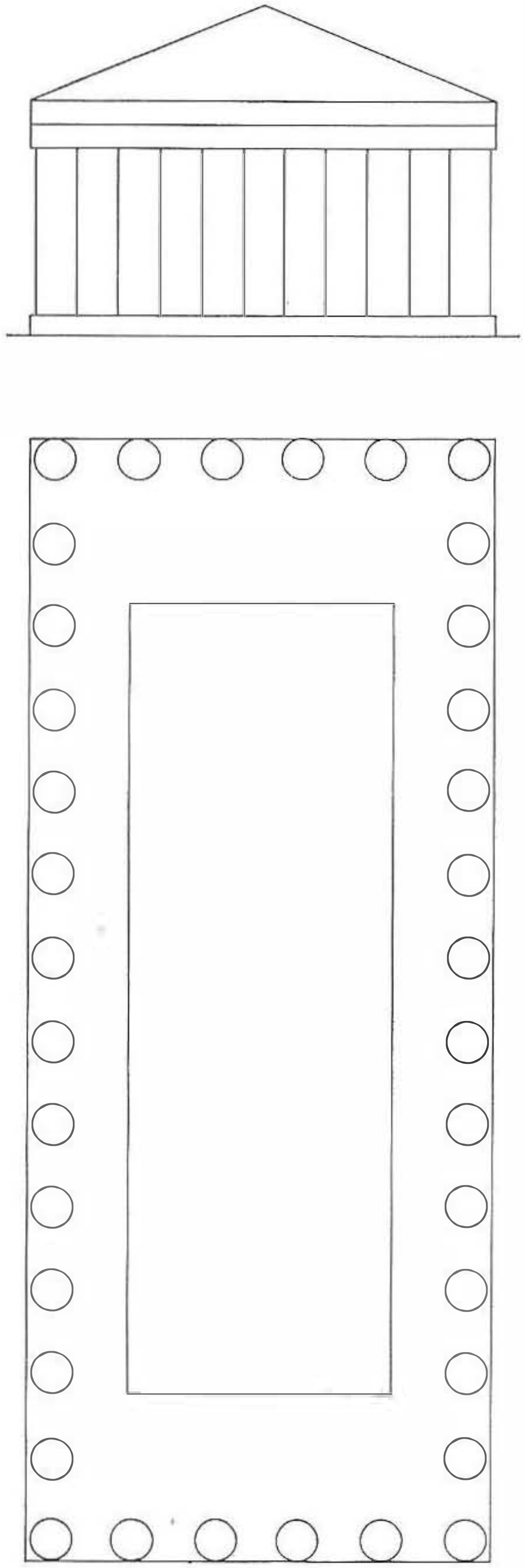


Fig. 80.

and the inner temple, and have found the appropriate distribution of columns.

If we were to go ahead and build our temple there would naturally be numerous details to attend to and calculate, but they would be mere details nevertheless. The principal dimensions of both the front elevation and the ground plan have been established from our symbols, of which we have in fact used only three, namely "M", "U" and "H". Combined with the 3 × 3 division of the square, these symbols have formed the basis of our whole temple construction.

In *Fig. 80* we see the plans of the temple, withdrawn from the relative obscurity of the diagram, and it must be admitted that in shape and proportions between ground plan and elevation it bears an unmistakeable likeness to the well-known traditional Greek temples, even though we cannot identify our particular building with any other special temple.

In illustrating the application of the system by planning a Grecian temple, I could well have taken as my example one of the well-known buildings and planned it from scratch on the same basis. But I rejected this possibility since later analysis will show that most of the temples receive their dimensions from a more complicated plan employing several more symbols than we used in our simplified version.

Greece's period of architectural greatness occurred of course infinitely later than the origin of the system, and many years before Grecian temples raised their portals to the skies the system had been applied in a similar manner by civilisations which—compared with Greece—had a considerably earlier start in history.

But the Greeks, with their sense of proportion, knew how to exploit the system and its manifold aspects in their building. And to avoid making the introduction to the system's application any more complicated than strictly necessary, I chose to produce a new temple requiring the use of only three symbols.

CHAPTER SEVEN

Ancient Geometry and its place in Ancient Egypt

OVER A COUNTLESS period of years numerous independent attempts have been made to breach the secret of the design which formed the basis of the splendid buildings of Antiquity and of the Middle Ages. All over the world edifices and ruins of former splendour have been discovered and examined by archaeologists; modern Man has been astounded by the magnificent ornamentation of these buildings, and by their wonderful craftsmanship and finish. But particularly we have marvelled at the builders' incredible ability to achieve such variety within the same general theme.

Despite this seemingly limitless variation in design there has nevertheless existed a suspicion that these buildings bore a mutual relation one to the other since they possessed a vague harmony which reflected from building to building. It was this constructive common denominator which modern Man has sought.

A number of research workers have expounded a variety of systems and theories of both a numerical and geometric nature in the hope of throwing light on the buildings' mutual factors, but most such theories have the unfortunate drawback that they fit only the individual building under inspection. And even within this particular building obvious deviations occur so relentlessly and so many points are left unanswered that the system appears rather forced.

No material has so far been forthcoming that can detail a system which—uniformly or with certain distinctly set-out variations—can be applied to a broad cross-section of these buildings we refer to. And even less has any writer or theorist put forward for examination a system which could with relatively few changes be employed as a basis for planning over a period of several thousand years.

On this point the theories set out in this book differ radically from similar material in the same field.

The system I refer to as the secret geometry of the Ancients appears not only to have been applied from the earliest times up until the Middle Ages as a factor in assigning dimensions to almost every monumental building of that period, but—as will be shown later—the system has also provided an answer to many practical problems in successive civilisations.

The system does not have its roots in profound speculative mathematics and therefore reserved for a handful of specialists. Instead its very structure and substance rests on the numerical facts the eye can see in a given geometric shape.

The ability to pick out these facts naturally demanded a logical sense of observation and the desire to see the truth that lay behind the lines, but nevertheless it was the visible, the concrete, that formed the bearing elements.

In other words, ancient geometry is simply that form of mathematics which springs from the facts the eye can observe.

Since we shall be spending much time in illustrating the practical application of the symbols of ancient geometry the most obvious course might be to jump straight over to the epoch of the Greek temple-builders who—as we shall see presently—stuck closely to the symbols in their planning. But to retain the chronological order of events I think it best that we begin at the time when a number of the symbols possibly originated, namely in Ancient Egypt, where our geometry was employed in the planning of, among other things, the Pyramids.

It is of course a well-known fact that when one observes a pyramid it shows itself to be constructed of triangles.

This is perfectly in agreement with buildings and geometric thought as it existed in that era, when every surface was split up into a series of triangles. It is thus quite understandable that these early buildings were constructed in this style, with the symbol that had provided the solution to so many geometric problems. In symbolic terms, the triangle was second in importance only to the sacred circle.

The first structure in which we shall illustrate the application of the geometric symbols is the Great Pyramid, the Pyramid of Cheops, the gargantuan monument standing aloof in the desert south of Cairo. We shall see the symbols applied to this building, to determine both the exterior dimensions including the ground-plan, and the Pyramid's remarkable interior.

This whole complex of dimensions in the Great Pyramid has a definite geometric thread, a thread which can invariably be traced back to the sacred symbols. And the complete plan rests on the same basis as our future study, namely the ability to select the correct square of origin.

The particular manner in which the Egyptians employed the symbols doubtless veiled a special significance, and here they had several possibilities.

As they chose to make the triangle the visible exterior of the pyramid, we must expect to find that they picked on one of the more important triangles in the geometric system. And this in fact was the case. For the very triangle they selected as the shape of their pyramid was none other than the triangle we saw earlier as a symbol for the circle.

Thus in its outward appearance the Great Pyramid of Cheops symbolises the principal geometric shape, i.e. the circle. The interior of the building is on the other hand planned according to practical requirements, its dimensions having no real symbolic significance other than the fact that they conform to the rules of ancient geometry.

It is by all means possible that in planning the interior a certain symbolism was adhered to, as with the exterior, but at this juncture it has not been possible for me to pick it out. Such symbolism could either be of a strictly geometric or ritualistic nature, but as I say this question remains for the time being unanswered.

Before setting out on the real analysis of this time-worn structure, it might be worthwhile to recall some of the story of the Great Pyramid for readers not fully acquaint with its background.

THE PYRAMIDS

The Great Pyramid of Cheops

A FEW MILES south of Cairo, on the edge of the Libyan or Western Desert, lies a collection of pyramids known to most of the peoples of today, and according to the literature of the Middle Ages these edifices were also known to large sections of the Earth's population in the past.

The complex comprises three large and six small pyramids and, close-by, the Great Sphinx.

The largest of the pyramids has been called the Great Pyramid or the Pyramid of Cheops, the name arising from the widely accepted assumption that it was built by the Egyptian king Cheops of the IV Dynasty as his burial chamber.

This particular pyramid has through the ages been the subject of considerable discussion, more so than other like buildings, partly because of its colossal size and partly because it is built in accordance with certain dimensions which lead the observer to believe that the pyramid-builders boasted a mathematical knowledge that had hitherto been considered part of modern mathematical development.

Paul Brunton writes in his book, *A Search in Secret Egypt* (London, 1936):

"When the scientists and experts whom Napoleon took with him on his invasion of Egypt were commissioned to make a survey of the country, they fixed the Great Pyramid as the central meridian from which they would mark out the longitudes. After they had mapped out Lower Egypt, they were surprised at the apparent coincidence of this meridian exactly cutting the Delta region, formed by the mouth of the Nile and practically constituting the whole of Lower Egypt, into two portions. They were still more surprised when they found that two diagonal lines drawn through the Pyramid at right-angles to each other would completely enclose the entire Delta area. And they were profoundly astonished when reflection revealed to them that the Great Pyramid's position was not only suitable as a central meridian for Egypt, but also for the entire globe, for the Great Pyramid stands exactly on the middle dividing line of the world! This amazing fact results from its position: If a vertical line is drawn through it, the land area to the east will be found equal to the land area lying to the west of the line. The meridian of the Great Pyramid is thus the natural zero of longitude for the whole globe. Its position on the land surface of our earth is therefore unique."

Thoughts and theories such as these, which attribute to the Pyramid both astronomical accuracy and mathematical perfection, have surrounded it since its rediscovery by Napoleon's army for the present era.

As related earlier, the Pyramid has huge dimensions, which are perfectly positioned north-south and east-west. Some idea may be obtained of the size when one hears that the height of the Pyramid measured vertically from its foundations up through its central axis is 485 British feet or approximately 148 meters.

Each of its four sides measured from one sunken cornerstone to another is 761 feet in length or approx. 233 m. In other words the Pyramid covers an area of approx. 13 acres.

Not until recent times have buildings been erected which exceed the Great Pyra-

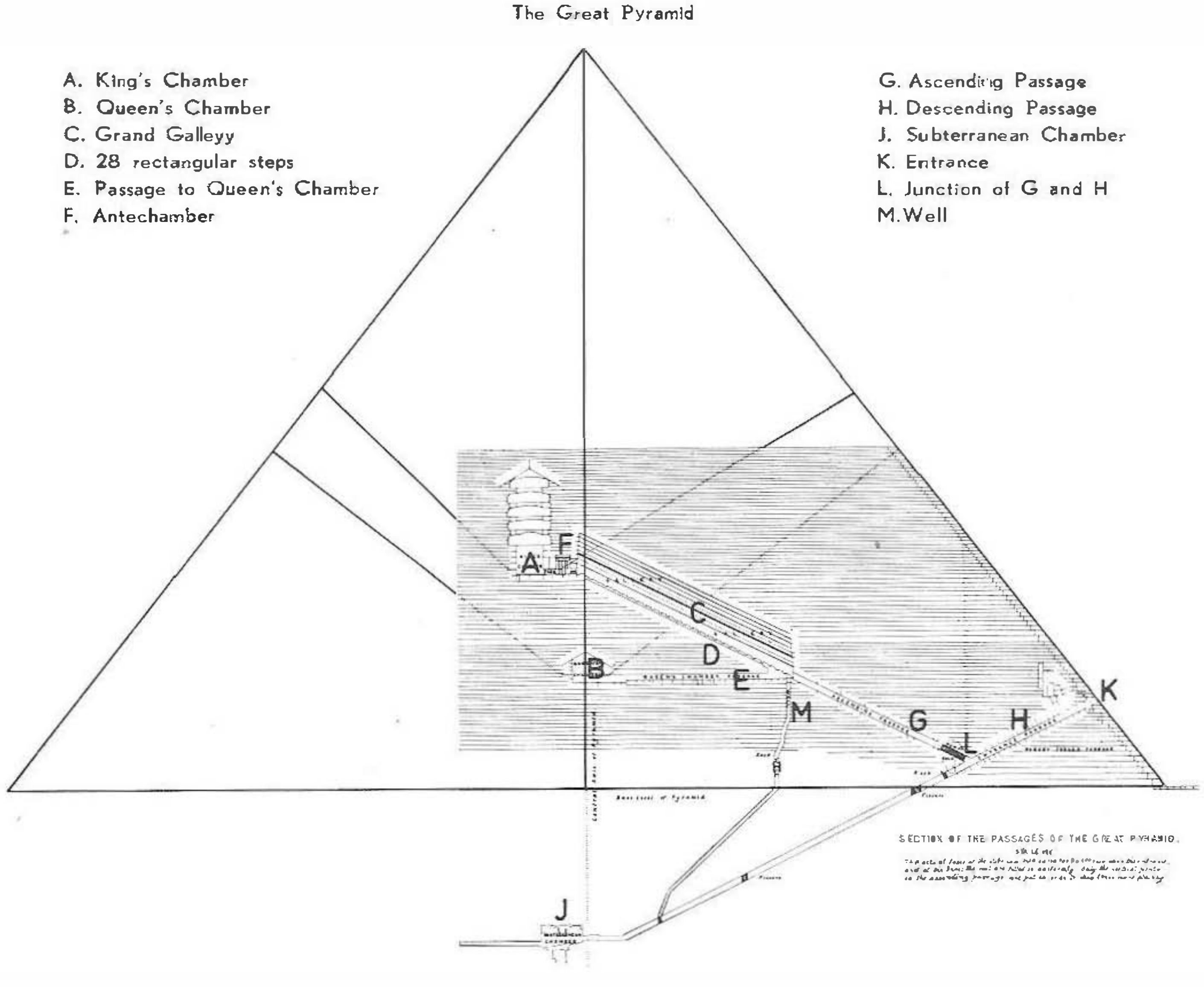


Fig. 81.

mid in height, but probably none of these exceeds it in base/surface area and in height, and it is thus no surprise that for thousands of years the Pyramid has been one of Man's Seven Wonders of the World.

Fig. 81 is a sectional view of the Pyramid of Cheops, revealing its interior. As in the other pyramids the entrance lies on the north side approx. 15 meters from ground level (K).

The entrance is approached up a form of ramp constructed of sand and stone. The entrance itself is 1 m. high, slightly less than 1 m. wide and roofed by huge blocks of limestone, placed at a slope probably to deflect the pressure from the stone mass above.

The passage one enters by (H) is of the same dimensions as the entrance and leads down into the Pyramid at an angle of 26° 18″ and has a very smooth floor.

A hundred meters down it opens out into a large Subterranean Chamber (J) which has been hewn out of the rocky ground on which the whole Pyramid rests. The ceiling and walls of this cavern have been carefully formed, whereas the floor is rough and untreated, with large irregularities, making it difficult to walk on the floor. Immediately opposite the point in the chamber where the passage emerges it continues for another 17 m. horizontally into the rock but comes to a dead-end.

If we climb back up the passage down which we came, we notice—about 60 m.

from the underground chamber—a large black stone (L) set in the ceiling which stands out against the surrounding lighter limestone. It acts as a giant plug, sealing off the ascending passage of the same dimensions as the one in which we are standing (or rather, crouching).

Early investigators of the Pyramid discovered this black stone plug, but as they were unable to move it they tunnelled a way through the softer adjacent limestone past the granite block. We can crawl through this crack into the ascending passage (G) and follow it up into the heart of the Pyramid.

The passage opens on to a small platform which forms the start of the Grand Gallery, and the impression one receives at this point has been described by every visitor as overwhelmingly imposing.

Having crawled, bent double, along narrow passages we have now burst into a gallery (C) which in height is seven times as great as the passage from which we emerge. The gallery continues at the same slope as the ascending passage, which in its turn was at the same angle as the initial descending passage, namely 26° 18″.

As we stand and reflect on this huge gallery which fades away in the darkness with its upward sloping ceiling, we are astounded at the genius of its builders.

Immediately ahead we can see the start of a low, dark passageway (E), and on each side a narrow, steep ramp climbing up into the great gallery. These unite above the entrance to E in an upward path.

The gallery is only slightly more than 2 m. wide and becomes narrower towards its ceiling, the walls slopping inward in seven stages each overlapping the preceding. The effect is such that the central ceiling is actually the same width as the central passageway up through the gallery.

On the right one of the ramps has been breached and a very low passage, almost at floor level, leads off into the wall. It takes one to the opening of a restricted, square shaft known to us as the Well (M) which leads almost in a vertical drop to a point on the initial descending passage just before the Subterranean Chamber we discussed earlier.

The low, dark passage (E) straight ahead leads horizontally to a room normally referred to as the Queen's Chamber (B). This room is square and the ceiling is formed by a number of stone beams sloped at an angle to each other.

The wall on the left has a strange niche built up in five sections in the same manner as the walls of the Grand Gallery. The last seventh part of the passage leading to this chamber is one step lower than the first six-sevenths, permitting a person of average height to stand upright.

To move from the platform at the top of the ascending passage up into the Grand Gallery, one must balance against the wall on one of the side ramps to the point, above the entrance to the Queen's Chamber passage, where the ramps unit to make a very smooth upward path. One can alternatively remain on one of the ramps, which run up the whole length of the gallery, or use the 28 rectangular recesses which are hewn in them to climb up the steep floor of the passage itself.

At the upper end of the gallery we are faced by a huge step, 1 m. in height, which extends 8 m. to the end wall in the form of a horizontal platform. From this platform leads yet another passage which opens out within a few paces into a small room (F). This room is divided into two irregular parts by a granite block fitted into small projections in the wall, the underside of the block being the same level as the passage through which we have just entered. The walls of this anteroom are fitted with several peculiar details.

On each side there are one or two projecting panels which reach to about 1 m. from the ceiling. The western panel is approximately 24 cm. higher.

The panels have cut into them three vertical notches on each side, about 50 cm. wide and separated by narrow projections. On the west wall they end in a semicircular engraving. The notches are produced a couple of centimeters below the level of the floor. The end wall, where we can see a new low passageway looming, is divided by engravings into five vertical projecting strips which reached from the ceiling to the edge of the passage.

Up as far as this room the Pyramid is built exclusively of limestone apart from the black granite rock we crawled past at the junction of the descending and ascending passages. But as we pass now from the ante-room into the main chamber (A) of the Pyramid we notice that everything here is built of granite.

This room—which has come to be known as the King's Chamber—is 10.5 m. long and half as wide, a rectangle familiar to us with the proportions 1 : 2. The walls of the chamber are made of giant blocks of granite polished so finely that the surface is almost a mirror, and the joints are so accurate they are difficult to trace. In the ceiling are nine enormous granite beams, and it has been estimated that they must weigh at least 70 tons apiece.

Along the west wall stands a granite coffin carved out of one solid piece of rock, having no lid and nothing in the way of inscription or decoration. The coffin is about the same size as a small bath.

From both the King's and Queen's Chambers air ducts have been cut through the otherwise solid walls of the Pyramid to the outside wall, as shown in the sketch in Fig. 81.

The foregoing description represents the essence of several researchers' visits to the Pyramid and their accounts of its interior.

Seldom has a building or ancient memorial been subjected to so many and such differing theories and opinions as the Great Pyramid. The theories range from the attempt by the German writer Witte to prove that the Pyramid was not of human origin but the result of volcanic eruptions, to theories concerning every conceivable age for the structure in which one party was of the opinion that the Pyramid was exclusively a burial chamber for King Cheops of the IV Dynasty, and others, for example, the Scottish brothers John and Morton Edgar, claimed that in the Pyramid's internal measurements they could prove that its builders possessed a knowledge of the world's complete development and had laid out this knowledge in Pyramid inches. The Edgar brothers in fact produced a two-volume dissertation in 1910 in which they gave an account of their theories.

According to ancient legend the first to break into the Great Pyramid was Caliph Al Mamun, son of Caliph Harun al Raschid, in 820 A.D. He arrived at the Pyramid with a team of workers intent on forcing an entry and carrying off the riches which fable held to be stored there.

It took him several months to gain access even though true to tradition he began his search in the centre of the northern face. But the entrance had been cunningly hidden off-centre and slightly higher than anticipated. Mamun's men started with hammer and chisel, but later resorted to heating the stones to make them crack.

Just as they were on the point of giving up their seemingly hopeless task, the workers heard the echo of a stone fall inside the Pyramid. This gave them a renewed hope and they pressed on, finally cutting into the descending passageway.

There is scarcely much doubt about the truth of this story as we can still—1200 years after the event—see the blackened stones which resulted from Mamun's attack. The same tale tells how the Caliph returned to Baghdad empty-handed. He found neither treasure nor mummies in the whole building, and it has never been shown that the Pyramid was opened at any earlier date, nor that any sign was ever recorded of a mummy or body or burial accoutrements.

The real present-day series of research projects on the Great Pyramid can probably be reckoned from 1789 when Napoleon's soldiers gaped at these giant structures the like of which they had never before seen.

When Napoleon heard of these remarkable buildings of tremendous proportions and of the various archaeological treasures found nearby, he equipped an expedition of scientists and archaeologists to study the area closely, and this expedition returned to France with ancient treasures of inestimable value which were deposited in the Louvre where a special Egyptian section was opened.

The resultant material naturally gave rise to a mass of guesses and theories which were voiced in various publications. These found their way from France all over Europe and roused in researchers of other countries the thirst to investigate this country of many treasures.

One of the best-known researchers in the new field of Egyptology was a Frenchman, Mariette, who from 1850 to 1880 led several excavations in Egypt. Auguste Mariette was a school teacher from Boulogne, but moved to Paris and became an assistant in the Egyptian department of the Louvre from which he was dispatched to Egypt.

One of the activities Mariette engaged in was the removal of the sand from around the Sphinx. This figure, with its colossal size and majestic beauty, made a great impression on him. An idea of its dimensions may be obtained when one reads that from the foundation to its top the Sphinx measures 20 meters, and is 56 m. long. The face alone is more than 4 m. wide.

Mariette regarded this figure, which lies in the immediate vicinity of the pyramids, as the oldest existing work of Man, older even than the pyramids themselves.

On the death of Mariette the sole right, so to speak, of the French in Egypt to carry on research disappeared. But they had established such a strong position for themselves over a long period of years and had assembled so much relevant material that their influence prevailed. Mariette's French successor was the clear-sighted linguist and expert in hieroglyphs, Gaston Maspero, born in Paris in 1846.

The French had a museum in Cairo of which Mariette had been director and leader. This position was now filled by Maspero and any nation wishing to carry on excavation work or investigations in the area was placed under the control of this museum. The French therefore kept a constant check on proceedings at the pyramids.

Under the leadership of Maspero and his successors every effort was made to uncover burial objects, burial chambers and the like. And excavations, from the archaeological viewpoint, were aimed at discovering anything which would reveal the secret of this infinitely ancient Egyptian cultural empire. If the objects themselves could not speak, then the thousands of hieroglyphs could tell their story for them. Linguistic experts learned much of the former empire from the strange writings.

But in the midst of all this activity the Great Pyramid itself became, if not the outcast, then certainly the passed-over. As a monumental structure it was naturally tremendously impressive, but from the point of view of the archaeologists it was of minor interest for it contained nothing for their museums.

From a linguistic angle it was also fairly dull, since both the outer and inner surfaces were completely devoid of script. The hieroglyphic students therefore made no progress with this giant.

The accurate measurement of the Great Pyramid was due principally to the British. Colonel Howard Vyse undertook the task of measuring it in detail in 1837. He surveyed the exterior and every accessible interior surface, and when he and his colleague John S. Perring published an account of their work, *Operations at the Great Pyramid,* London, 1840, it aroused great interest in Britain.

This was a completely new approach, to survey a place with the aid of a yardstick and form theories afterwards.

The British publisher and mathematician, John Taylor, then wrote another book on the basis of Vyse's, which was published in London in 1859 under the title *The Great Pyramid. Why was it built? And who built it?*

He points out in his book that the proportions of the Pyramid conceal a number of geometric and astronomical laws expressed in a simple and easily comprehended style, and concludes that the purpose of the Pyramid must have been to preserve and pass on this knowledge to future generations.

This volume, too, created considerable interest, and a renowned Scots astronomer, Professor C. Piazzi Smyth, became profoundly interested in the possibilities which lay in the observations of an astronomical nature which were claimed existed in the Pyramid. A year or two after publication of Taylor's book Smyth journeyed to Egypt partly to check the measurements and information it contained and partly to search for other factors which might fit into the general picture. Smyth, who lived by the pyramids throughout the winter of 1864—65, succeeded not only in confirming the measurements dealt with by Taylor, but also made a number of similar discoveries for himself.

Thoroughly delighted with the success of his expedition, he returned to Scotland and wrote an account of his own observations in a work of three volumes: *Life and Work at the Great Pyramid,* Edinburgh, 1867. In these volumes he reported his precise measurements of the inner pyramid with the exception of the part situated underground, and gave an account of the Pyramid's relative position in astronomical speculation.

The measurements made by Vyse and Smyth were in their turn later tested and confirmed by W. M. Flinders Petrie who in 1880 went out from England to Egypt to undertake an exhaustive examination of the Great Pyramid.

Flinders Petrie was born in Charlton, England, in 1853 and trained as an engineer. He ran his own engineering firm but his interest in the natural sciences brought him into wider spheres, and precisely because of his engineer's training and his familiarity with surveying and accurate measurement he was regarded by The Egyptian Fund as better qualified than any other person to conduct a minutely correct survey of the Pyramid.

He and his assistant spent a year on the project, and went methodically to work. Petrie is the only researcher to provide a detailed list of the instruments applied in his survey, and his book, *Pyramids and Temples of Gizeh,* London, 1883, contains long accounts of the actual measurement process. It certainly carries the art of such surveying to a fine pitch. His are the dimensions used in the analysis of the Pyramid in the next section.

Petrie found that the length of each of the Pyramid's four sides at ground-level was 761 feet (232.8 m.), and its vertical height 485 feet (148.2 m.).

There was a slight difference in the length of each of the four sides, but those stated here are the mean dimensions which Petrie considered were the intended ones.

The total area of ground occupied by the Pyramid is approximately 13 acres, enough to make a small farm!

Surveying a structure of this stature was naturally a mammoth job, even with fine equipment. After all, each side of the Pyramid measures about twice the length of a soccer pitch.

The building had been partially covered several times by drifting sand, and Petrie had to dig the foundations clear before taking his measurements. But even the clearance of sand could not guarantee that the Englishman had reached rock bottom, i.e. that he had found the original dimensions of the Pyramid. The building's top-coat of limestone, which presumably represented the finished dimensions, had long since eroded and fallen away. Some of it had been stolen. Today—and in Petrie's time—the Pyramid is like a mountainous four-sided staircase, shorn of the limestone and layer of plaster it once wore.

Petrie puts up the theory that the Pyramid was built by a series of building masters, basing his suggestion on the fact that the lower reaches of the building are executed with a greater respect for accuracy than the upper part. He was able to determine this because the limestone layer is no longer in position.

The slope of the Pyramid's sides, Petrie found, is 51° 51′ 14″. This he calculated from one or two pieces of the original lime caking still stuck to the foundation of the Pyramid. It was completely covered by sand and had thus escaped the attack of wind, weather and vandals.

If Petrie's theory is correct, that the Pyramid's later builders were less careful in their work than the men who began the work, then it was presumably the final series of builders who completed the building and fitted the limestone top-coat, and therefore may well have been slightly careless with this part, too. We might expect a small difference between the intended angle of slope and the actual finished job, and it is also unlikely that *every* facing stone in a building of this size was placed at precisely the required slope.

Petrie tells us furthermore that the Pyramid had lost a small portion of its tip, and that the height he quotes is based not on measurement but on calculations of angle of the handful of limestone blocks found at the foundation.

In other words, he was obliged to take as his basis the slightly less accurate handiwork of the later builders in order to establish the Pyramid's slope and height. These were average figures, since the limestone blocks differed minutely in elevation.

The oldest report of the pyramids we find in historical writings of the Middle Ages is probably a translation of Herodotus in which he gives a number of details as being absolutely accurate. He discusses most carefully the Great Pyramid and provides one or two of its dimensions.

Subsequent research, by the men mentioned above, proved that Herodotus's statements were incorrect. But factors such as measurements and their accuracy are not vitally important if the writer himself has not placed any emphasis or importance upon them, and has only used them in a text to provide a mental picture of the structure's size. In that case an estimation would be quite sufficient.

He mentions the subterranean tomb of King Cheops in the Pyramid. According to his account, it lay on a kind of island, surrounded by a canal which, by means of another canal, was linked with the Nile so that the tomb was ringed with water when the Nile was running high.

This is one of the more decisive errors in the descriptions put forward by Herodotus. Both upper chambers, the subterranean chamber and the so-called well lie within Cheops' Pyramid, much higher than the Nile's high water mark. Even if we try to cover up the error by assuming that the Nile perhaps rose higher in the old days than it does today, we come to a dead-end. Because none of the previously mentioned investigators of the Pyramid ever found any suggestion of a canal connected to the Nile, or of a burial place on an island.

On the other hand, at a spot some distance to the south-east of the Pyramid, just behind the Sphinx, there is a burial place—now called Campbell's Tomb after the man who discovered it. It consists of a deep rectangular shaft at the bottom of which is a burial chamber.

The chamber is surrounded by a narrow moat, the bottom of which is lower than the level of the Nile. Thus this burial place can be filled with water from the Nile, and fits into Herodotus's story better than does the Pyramid. So if Herodotus himself did not visit the Pyramid, but received his information second-hand, a mistake must either have been made by Herodotus's informant or the historian himself.

But we lose faith perhaps most of all in the Herodotus version when we hear that the Pyramid was covered with strange script which—his Egyptian guide is reputed to have explained—was a calculation of the amount of sugar-beet and garlic the workers had eaten and how much it had cost.

This guide was obviously never at a loss for a reply to any traveller's question—if in fact the man ever existed. For after this tale we must doubt that Herodotus was ever near the Pyramids. Because it has not been at all possible for early-modern investigators to trace any form of hieroglyphics or other scripts either inside or outside the Pyramid.

The Pyramid was formerly covered by a layer of smooth white limestone which has, however, crumbled away or been stolen over the years. But some of it still lies in lumps around the base of the Pyramid. And neither on these lumps which lie on the north side where they were previously buried by sand and therefore secure from the ravages of vandals, nor on those pieces of stone discovered in Cairo as part of other buildings and proved to be removed from the Pyramid's outer layer, has any trace been found of script. So even this part of the story assumes the mantle, slightly, of fairy tale.

Other early writers such as Strabo, Pliny and Diodorus Siculus who also mention the pyramids tell similar stories to that of Herodotus, except that they disagree as to which king had been buried there. But the fact that no mummy or other burial equipment has been found is strange. If the Pyramid was built as a burial place for Cheops, why was the normal burial ritual not adhered to, and the king interred with pomp and circumstance? Why were the walls of the burial chamber not covered from top to bottom with inscriptions?

Because of course the custom of those times was: the greater the man the more inscriptions on the wall of his tomb. Even a simple labourer in the fields would never have dared cross the threshold of Death without his name and magical formula on his simple grave. How much more remarkable then is it that the room claimed to be the burial chamber has completely smooth granite walls, finished so finely that their like has not been seen from that same period.

The stone faces are polished to a mirror-like degree, and the joints so finely executed that it is impossible to force a razor blade between the blocks of granite. Can we afford to ignore this vital difference between normal, recognised burial places and this chamber which is so unlike the accepted tomb that the only common point they share is the presence of a coffin?

With regard to the solitary coffin, let us examine its dimensions. According to the figures given by Flinders Petrie in his book, *Pyramids and Temples of Gizeh,* page 86, its measurements are:

		(British standard inch)
External	length	89.62″
,,	width	38.50″
,,	height	41.31″

and the thickness of the coffin is stated to vary between 5.67″ and 5.82″.

He describes an extremely accurate measurement of all the passages in the Pyramid all the way from the entrance corridor to the King's Chamber. On page 63 he describes the ascending passage, the one leading from the descending passage up into the Grand Gallery.

We are particularly interested in the measurement of this passageway before it opens into the Gallery.

He writes:

"The result therefore is that from the intersections of entrance and ascending passage floors to the floor point at the east side of the Grand Gallery doorway is 1546.8″ on the slope. The granite plugs are kept back from slipping down by the narrowing of the lower end of the passage to which contraction they fit. Thus at the lower or north end the plug is but 38.2″ wide in place of 41.6″ at the upper end. The height, however is unaltered, being at lower end 47.3″, east 47.15″, mid 47.26″ west and at upper or south end 47.3″. In the trial passages the breadth is contracted from 41.6″ to 38″ and 37.5″, but the height is also contracted there from 47.3″ to 42.3″. These granite plugs are cut out of boulder stones of red granite and have not the faces cut sufficiently to remove the rounded outer surfaces of the corners."

Thus we see that the width of this passage varies from 41.6″ to 37.5″.

If we now try to bring the coffin into the King's Chamber from the outside, we could in theory get it down the descending passage and into the mouth of the passage leading to the Grand Gallery, but there it would be stuck. The width of the coffin is 38.5″ and would thus be ½″ to 1″ too wide to pass through.

I think the whole burial theory collapses on this half or whole inch, even ignoring the absence of burial accoutrements and mummies.

There is no doubt whatever that the Pyramid has been constructed with infinite care and attention to detail.

It is also obvious to any builder that a structure of this magnitude would be impossible without a detailed plan. In this connection, a rough engraving of the interior of the Pyramid has been found in a rock face to the east. This of course had not been the actual plan for construction purposes, but had probably been made to assist the builders in charge of the project.

The overall planning of the building and its ingenious structure make it impossible to believe that during the building they forgot the purpose of the Pyramid, and indeed it vindicates the builder's reputation that the coffin cannot be passed along the corridors of the building. In fact, there is only one possible explanation: that the coffin was placed in the Chamber during the actual building stage, just before the nine 70-ton beams were laid in position on the roof of the Chamber. But in that case what was the coffin used for?

If we cast our thoughts back for a time to the various civilisations which have come and gone we have to admit that the principal factor behind the culture of these peoples was their religious customs and beliefs. It is on a religious background, in fact, that we find most of our information about these dead civilisations, whether excavated from a temple or a burial mound.

When we wander back to the present time, studying the religions which govern today, Christianity, Catholicism, Mohammedanism, Hinduism, Judaism, etc., we find they all have their origin in religious systems of bygone days, namely the religions of mystery, different countries having their own indeed several systems. The Romans, Celts, Druids, Greeks, Cretans, Syrians, Hindus, Persians, Mayas, Egyptians, and many others—they all had their different religious systems.

Common, however, to all was the fact that the Temple admitted brethren to their particular order, and that brethren underwent various rituals prior to receiving the education and learning of the system.

The whole association with the Temple was a graduated affair, by which they slowly—as their knowledge increased—attained a higher place in the ranks of the brethren, until finally they were "initiates", this initiation being the supreme honour. Aristotle was quite open in declaring that he considered the welfare of Greece to be secure as long as the Eleusinian Mysteries existed. Socrates in his time said that anyone becoming acquaint with the Mysteries assured himself of comfort at the point of death, and individuals who directly admitted or hinted that they were initiates in the Mysteries included the orators Aristides, Menippus and Sophocles, the writer Aeschylus, and legislators Solon, Cicero, Heraclitus of Ephesus, Pindar and Pythagoras.

Although the actual ritual itself varied from society to society, the model was nevertheless the same: the Egyptian initiations of Isis and Osiris. The fact that these mystery religions attracted such a high proportion of intellectuals must be considered on the background of knowledge. Not only did they give participants religious knowledge, but they also revealed a practical knowledge clothed in religious ritual and symbolism.

The climax in all of these mystery religions was the initiation ceremony.

During the ceremony the candidate for initiation was placed by the hierophant, the Temple leader, in a deathlike trance symbolising Death itself. On awakening from this condition, having wandered alone in the world of the gods, he was regarded as having been reborn. Indeed in early times they took initiation so deeply to heart that after the final ceremony they frequently assumed another name.

For this process of initiation all mystery religions introduced the coffin, in societies of course where coffins were the custom for the dead. Where timber coffins were used the ceremony involved these; in societies where the dead were placed in caves, then this method was employed.

In the Tibetan mysteries, where normal burial custom is to break up the body and lay it out for the vultures, it would of course have no psychological effect to be placed in a coffin, which might very easily be regarded as a bed. In this case the initiate was laid on a stone block similar to that used by the monks in their normal burial technique.

Since the Great Pyramid is equipped with a coffin, I think therefore it is plain that this in fact was the coffin of initiation. And the three chambers:

1) Subterranean Chamber
2) Queen's Chamber
3) King's Chamber, with coffin,

are in my view the places of initiation

for the three grades of admission to the order, the highest degree of initiation being raised well above the others both physically and symbolically. Fighting one's way towards this initiation through dark, narrow passages—which were probably equipped with various obstacles—only served to emphasise the symbolism of achieving knowledge by struggling against adversity.

Pyramid of Cheops in relation to Ancient Geometry

After that short glimpse of the historical background of the Pyramid of Cheops we turn our attention now to our real task, namely a geometric analysis of the plan of construction for this great building.

What in fact do we mean when we speak of an analysis, and what can this tell us of the finished building that we cannot see for ourselves?

An analysis of a building—quite briefly—is an attempt using available illustrations and drawings to discover which of our symbols was chosen as the constructive point of origin for the building. If we can trace this, then we try to pick out which symbols were used together or in connection with this original construction in order to plan the remainder of the building.

An analysis is thus the reverse of a construction. We must break down the very structure of the building in order to arrive at the symbol on which planning was based.

This reconstruction can of course often be extremely difficult to perform as the possible computation of symbols is virtually countless, and one of the main difficulties is the fact that the constructive symbol of origin is frequently slightly larger than the subject being planned.

It is impossible in the initial stages to say just how much larger, since this must be established on an extensive section of the building.

The architect can place his main subject within the circle's rectangle and use the circle to indicate the projected details, or he can apply to his subject the small square created by the combination of sacred cuts and let the details extend to the half-size square, or of course he could use the whole of the outer main square for his construction. There are few bounds to his planning possibilities.

Although these possibilities, however, are manifold the system itself possesses a tremendous degree of certainty because if you cannot pin down the proper point or symbol of origin, it will prove impossible to fit any of the other symbols into the complete picture, and the analysis will in turn prove abortive since none of the lines will coincide.

But on the other hand once the investigator has found the correct point of origin, the main subject fits relatively easily and smoothly into the diagram, and since all following symbols bear a predetermined geometric relation to the original symbol, it is possible with a little dexterity to discover the remaining symbols, and thus follow the construction af the building in detail.

But the analysis demands strict accuracy. A properly executed analysis reveals —with absolute certainty—the slightest deviation between the symbol and the subject, and if the basic square has been drawn a mere millimeter or two too large or small none of the symbol's lines will fit the plan.

The chance of fitting an analysis to a building that has not been built according to geometric symbols is just as small as solving a large crossword puzzle with other words than those intended, and yet end up with some kind of an answer.

The greatest difficulty in executing analyses is the accuracy or lack of it in existing illustrations and drawings.

It is a regrettable but unavoidable fact that many of the buildings on which analyses are to be performed are in ruins or entirely in a state of collapse, and existing drawings therefore rely on measurements and reconstructions.

Where absolutely accurate measurement has proved difficult on account, for example, of deterioration or collapse of the building in question, the resultant measurements are based on estimation, and this may not of course always be a correct estimation.

Once measurements have been made, the investigator then takes his notes and figures to a drawing office or studio where the material is processed. But this office may be 100 or 1000 miles from the building in question, and if he has forgotten to take a particular measurement or if there are any other obscure points in his picture the investigator will probably compensate for these in his drawing, and this may often mean that two dimensional drawings of the same building do not show exactly the same result.

In the case of buildings erected at a more recent period of history than the classics of antiquity, a number of these are still standing, apparently in their original state. But a close study of these buildings and their history will often reveal that they have been rebuilt or restored perhaps several times, the restoration being so severe that the original structure of the building may be entirely lost.

If a builder at some late date in history had to rebuild or restore one of the classic edifices of a former time and was not acquaint with the rules of ancient geometry, it is inconceivable that he would stick to these rules. And it was often the case, too, that builders engaged in restoration work preferred to leave their personal mark on a building rather than follow slavishly in the tracks of their predecessors.

Thus there are one or two factors of possible uncertainty to take into account in our geometric analyses, points we must continually bear in mind while assessing the following material—for obviously if a subject building has changed its appearance from the original to its present state then no analysis on earth will apply.

A final small factor of doubt which may creep in while we perform our analyses is the actual execution of the reconstructed drawings. I have often been faced with a situation of a particular piece of text mentioning one figure, although a check on the drawing reveals that this figure was not maintained by the draughtsman.

I sincerely trust that the reader will not view these past few paragraphs as a deliberate attempt to explain away a number of gross inaccuracies—for there are none. On the contrary, despite these possible errors there is an amazing degree of similarity between the analytical symbols and the subject matter, but a relentless geometric analysis—as mentioned earlier —uncovers the slightest inaccuracy. The comments made in the foregoing must therefore be regarded as an explanation of why we cannot always expect perfection on every point, and of how the intended plan is one thing, while the manner in

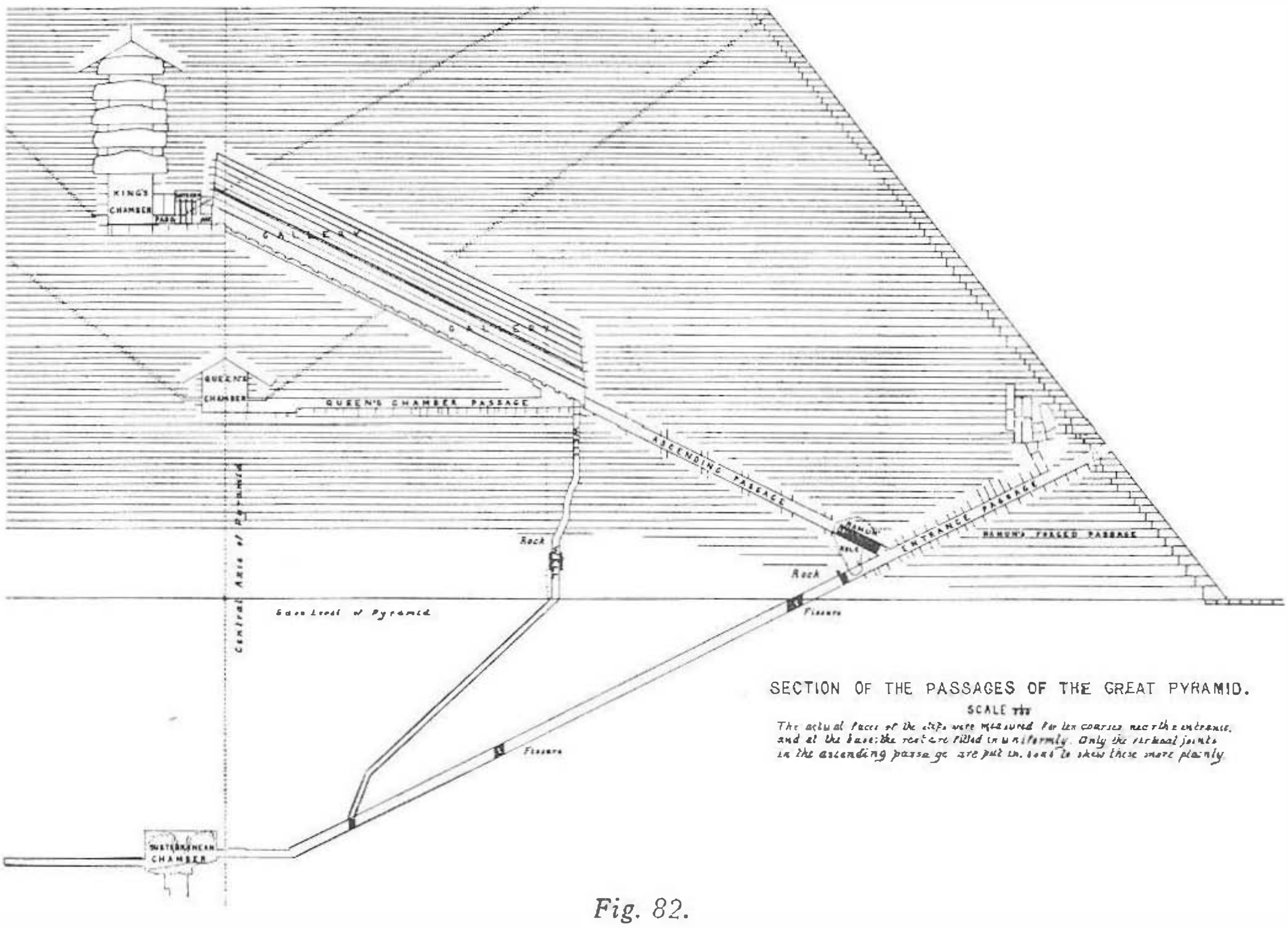


Fig. 82.

which it is executed in practice by the builder may be another.

We can surely see that differences may arise between the drawing which represents the origin of a building, and the measurements made on the completed structure.

Good workmanship can reduce the chance of this error to the infinitesimal, but poor work on the part of the craftsman can mean that errors are exaggerated, and the correction of these errors may produce minor alterations of otherwise correctly laid-out sections elsewhere in the building.

We can read in Petrie's book of measurements he made of the pyramid's ground plan. These show tiny differences from side to side, and the dimensions Petrie gives were an average of the four sides of the building. The differences in length of the sides are so small that we may assume that the ground plan of the pyramid was square in the original drawing, and this immediately provides an example of a deviation between the intended and the executed.

Our analysis of the Great Pyramid will be based on a drawing in Petrie's book, *Pyramids and Temples of Gizeh,* which shows only a small section of the building.

We see this portion of the Pyramid in *Fig. 82* and from this drawing we are able to construct the outer shell of the Pyramid, since we have the base, side and central axis indicated on the drawing.

We saw the completed sectional drawing of the Pyramid in Fig. 81.

We must now use this drawing to find the original square on which the further dimensions of this remarkable structure were based.

After a considerable number of experi-

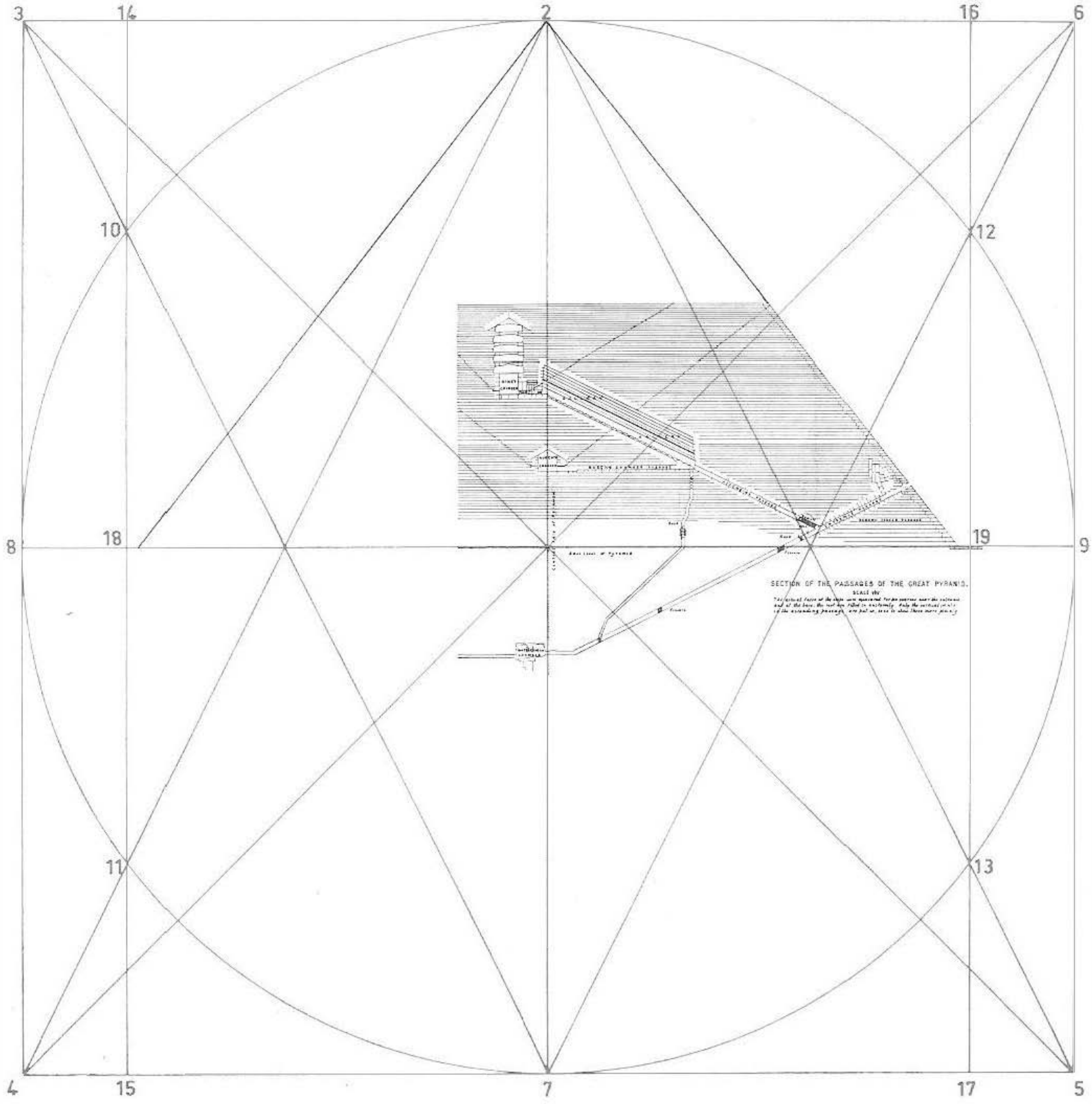


Fig. 83.

mental attempts I finally achieved the analytical diagram seen in *Fig. 83.*

I have taken here as my point of origin the central axis of the Pyramid and applied it as the radius of a circle the centre of which is the middle of the Pyramid's base-line. Outside this circle is drawn what I call the main square of construction.

In the diagram we see the Pyramid axis marked 1-2 and the square 3-4-5-6.

This gives us a square in which to enter one of our symbols, and the first one we select is "Q", which contains the circle's rectangle.

To produce this symbol we extend the Pyramid axis to the base of the square (line 2-7) and enter the square's central horizontal axis 8-9. As can be seen, this line forms the base of the Pyramid.

We then enter the diagonal cross 3-5 and 4-6, and finally the two acute-angled triangles 4-2-5 and 3-7-6. We recall that the points of intersection by these triangles of the circle's circumference indicate the position of the circle's rectangle, these points being 11, 13, 10 and 12.

Now we can complete the diagram by entering the rectangle itself, i.e. 14-15-17-16.

We see that the Pyramid is situated in the upper half of the circle's rectangle.

If we now make a critical inspection of our analytical drawing (Fig. 83) we can see that this is not absolutely accurate, since the base of the Pyramid does not extend all the way to the sides of the circle's rectangle. But this difference has its explanation.

We will recall that the sides of the Pyramid were constructed, not measured directly. As the Great Pyramid lacked both its peak and its outer shell of white limestone which indicated the angle of its sides and as only a few of the stones were discovered buried under sand to the north of the Pyramid (which Flinders Petrie later used as the basis of his measurements), these stones represented the difference between the Pyramid's present slope and the estimated slope of its original limestone crust. Petrie actually recorded the mean of these two angles.

We see therefore even in our first analysis an example of the assessment we discussed earlier, whereby absolute accuracy cannot be achieved by measurement, and approximation must suffice.

Apart from this variation, Petrie himself in his book states a slightly different angle as being the intended from that which he shows in his drawing.

I have chosen however not to compensate for this error in the drawing, since analyses must be conducted, I believe, on existing material. Indication of a relationship between ancient geometry and the building in question would otherwise be based on a faulty foundation.

Thus the Pyramid occupies in fact ¼ of the circle's rectangle, and consequently represents the circle which can be drawn inside ¼ of our main square.

The reader may be a little surprised that this analysis is based on a square of such proportions that the Pyramid itself occupies only ⅕ of its area, but here I should refer to Fig. 45 in which we used the selfsame basic square as a means of finding the circle's rectangle. On this point therefore we have a precedent. A further factor of importance which may have had some bearing in the selection of this square —although this cannot be proved with any degree of certainty—is that if we deduct the area of the Pyramid from the main square of construction, the remaining area is equal to the circle formed inside that square (with radius 1-2).

If we visualise the main square split into four smaller squares by its vertical cross and these smaller squares each divided into five rectangular areas, then our main square is made up of 20 of these rectangles.

We know from previous experience that the circle's rectangle is ⅕ less in area than the main square, which means that here the circle's rectangle consists of 16 rectangular pieces, which in turn represents the large circle.

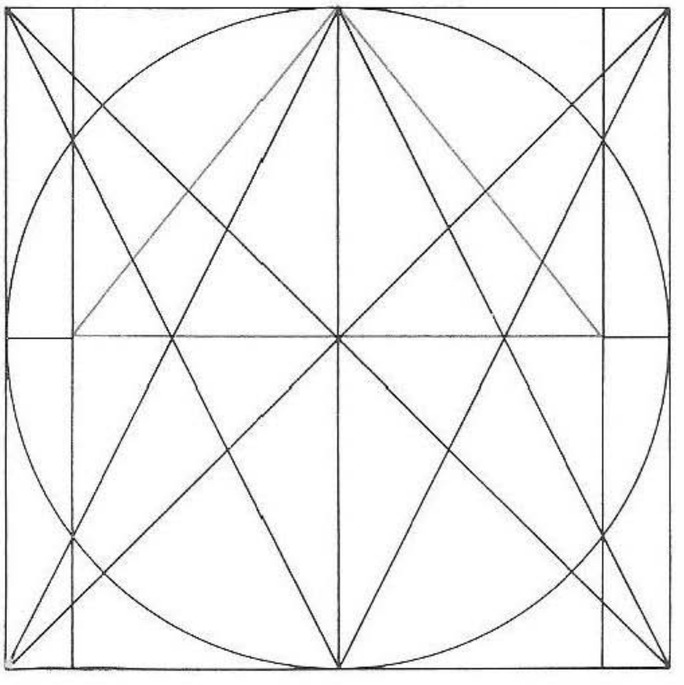
The Pyramid is drawn inside the circle's rectangle and occupies ¼ of its area, i.e. four rectangular pieces.

If we deduct these four from the main square's 20, we get 16 rectangles, i.e. the main circle again.

Thus on the one hand the Pyramid represents the circle that can be described in ¼ of the main square, and on the other hand it equals that area of the main square situated outside the main circle.

It is the former of these two observations which will in particular be applied to the further analysis of the Pyramid.

A Pyramid Side elevation



B Pyramid Ground-plan

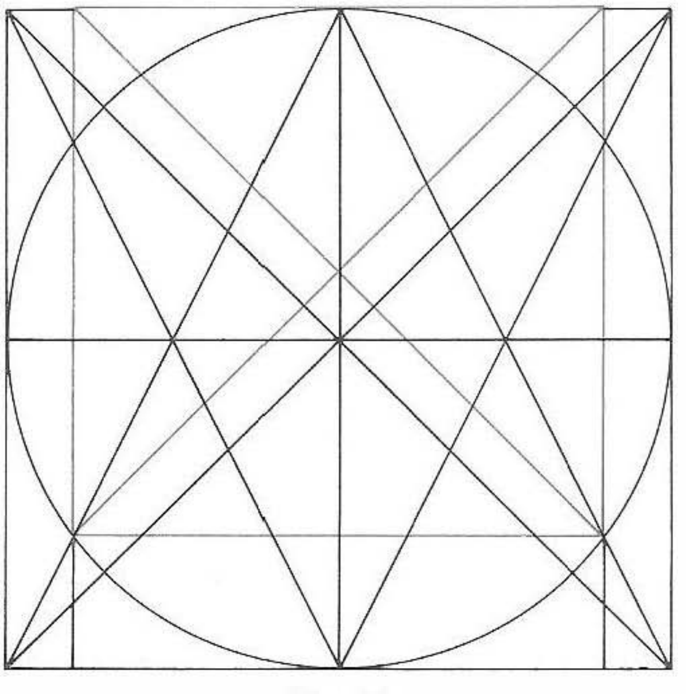


Fig. 84.

The diagram in Fig. 83 permits us to assume that it was indeed symbol "Q" with its symbolic reference to the all-powerful circle that inspired the remarkable exterior of the Great Pyramid. This symbol must have possessed, we presume, extraordinary significance—and there is perhaps still much to be read into its geometric form, much that I have not been able to define.

Now that we have established the starting point from which the Pyramid was planned, we must remember that all subsequent diagrams must bear a predetermined geometric relation to the basic square.

If the Pyramid—or rather its ground surface—had been rectangular in shape, it would have been a natural step to select the circle's rectangle as the ground plan. But the Pyramid is not rectangular, it is square. And it is thus the square on the circle's rectangle that forms the inspiration for the building's ground plan.

The symbol that held this square was "U", and we went fully into its geometric significance at an earlier stage. It should thus be sufficient to observe that the symbol in this instance has no additional lines drawn upon it; it is the lines of the symbol itself that are applied to the Pyramid.

We see the area presented in its symbolic surroundings in *Fig. 84* in which A shows the Pyramid side, and B its ground plan. When the diagonal cross is entered in the square on the circle's rectangle, we see the Pyramid as viewed from a point directly above the apex, the lines of the cross indicating where the sides of the building meet.

We have now followed the planning of the outer dimensions of the Pyramid and have seen a side and ground-plan view. But the building has of course a number of passages and chambers—and these, too, have been positioned according to the dictates of the basic square.

In the constructive diagram for the Pyramid's side view, there was a reference to ¼ of our main square—the side of the building in the diagram representing a circle drawn inside ¼ of the main square. This can be interpreted as an indication that our attention should now turn to this ¼ square. And this ¼ must of course lie in the same area of the diagram as that occupied by the passages and chambers in

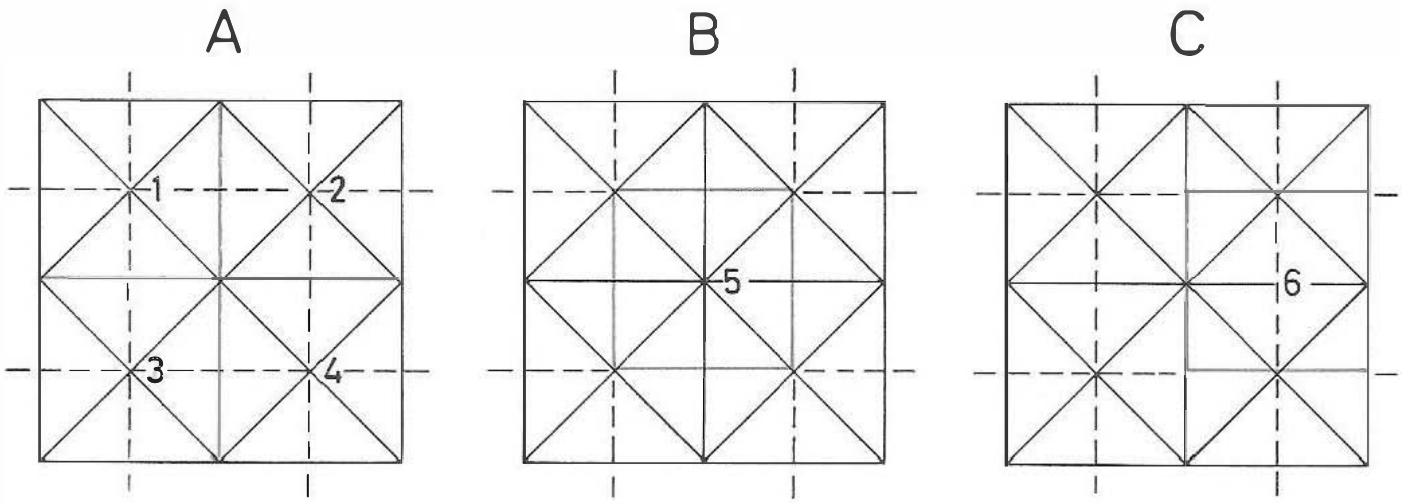


Fig. 85.

the Pyramid. It cannot—just like that—be plucked from thin air as an entirely independent figure.

In *Fig. 85* we have one or two possible methods of dividing our main square into 4 × ¼ squares. (N.B. The triangle in the upper half of the main square in the diagram does not, of course, represent the Pyramid, but is merely part of the dividing guide-lines.)

In A we see the square split up in the normal manner by the vertical cross, symbol "F" being applied here. The four resultant squares are numbered 1 to 4. If we can imagine the Pyramid in place in this diagram we see that square 2 is the only one that comes in contact with the passages (which of course are on the right of the building's vertical axis) but we shall see that this is not the square we require for future analysis.

In B we have drawn in a square numbered 5, and this is also ¼ of the main square with contact with the passages and chambers. But although this is a possibility, study and experiment show it is not the square we are looking for.

In alternative C we have a square drawn on one side of the diagram. We can see that it is ¼ of the main square and that it contains the whole of the area occupied by the passages. Work on this square provides more success than with the previous attempts, and it becomes apparent that this is the quarter square used by our architect in planning the next stage of his construction.

In *Fig. 86* we see this diagram repeated in its proper context, the quarter square being indicated as 20-21-22-23.

This is the stage at which one symbol is constructed inside another.

We have here a square of a certain ratio to the basic square, and within this square we now construct symbol "K" which is the half-size square in the upper left corner of our new basic square.

We see the resultant square at 20-24-25-26.

The first thing we notice is that the base of this square 25-26 lands directly on the floor of the Subterranean Chamber, deep under the foundations of the Pyramid. And if we enter in this square the acute-angled triangle 20-27-26, we notice that line 27-26 follows exactly the descending passage and has probably been the deciding factor in fixing the angle of this passageway.

We further observe that the horizontal axis of this square 27-32 cuts across the junction of the ascending passage and the Grand Gallery; simultaneously at point 32 it forms the floor of the intermediate room in the Pyramid, known as the Queen's Chamber.

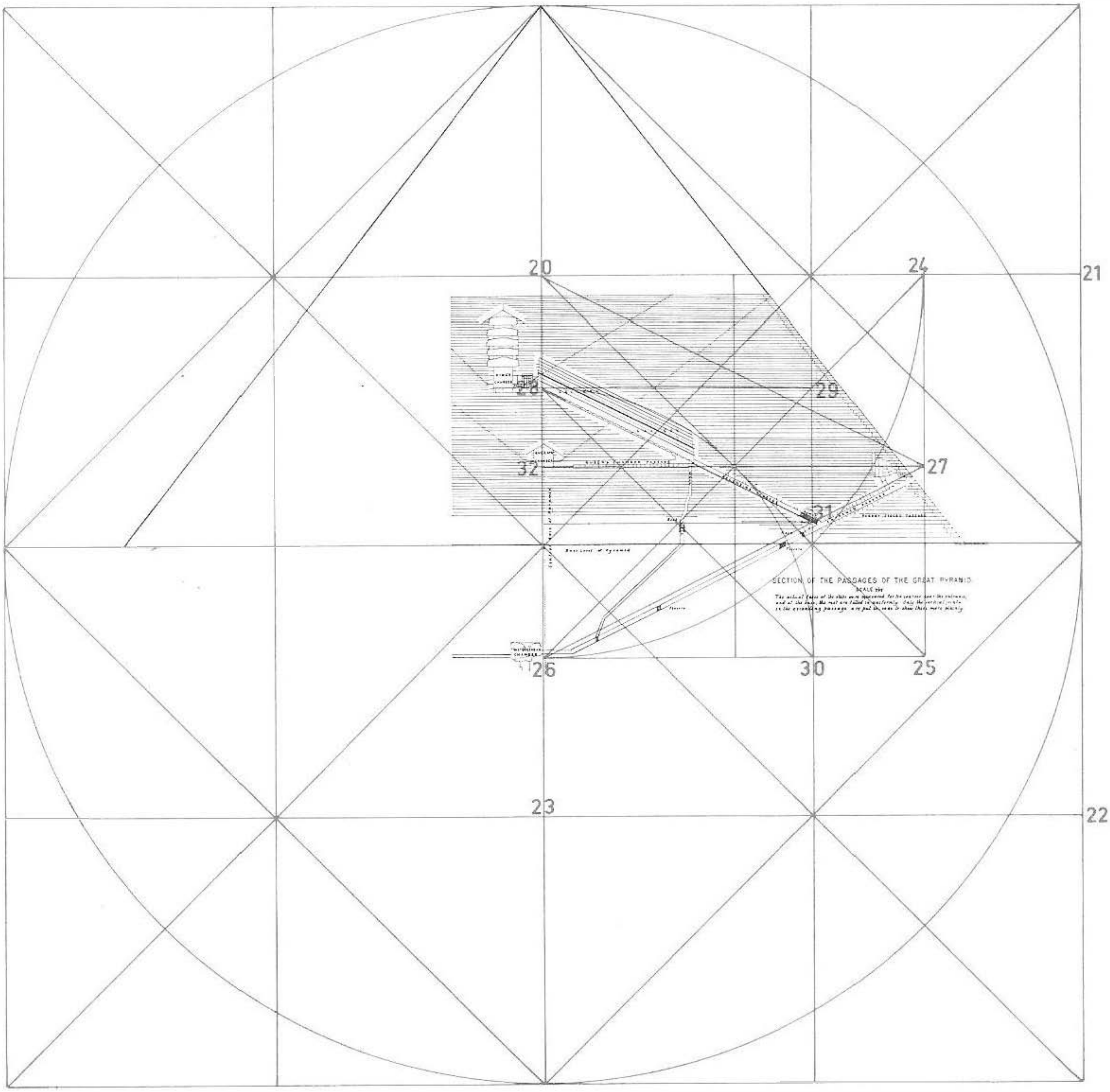


Fig. 86.

We now take as our new basic square, the one we have just discussed 20-24-25-26 and construct within it symbol "J". From a geometric standpoint this is the same symbol as "K", having the half-size square and therefore the sacred cut, but as opposed to "K" the half-size square in this case is placed in the lower left corner. This was probably done to consolidate the plan, but geometrically we recognise the basic and significant diagram of three squares inside each other, each in the ratio to its neighbour of 1 : 2.

Our new square is 28-29-30-26.

We observe now that the upper line 28-29 of this square marks the floor of the most exalted room in the Pyramid, i.e. the King's Chamber, while the base *of this square—just as the base of the* preceding square—forms the floor of the Subterranean Chamber.

Geometry tells us that line 26-28, i.e. the distance from the floor of the underground grotto to the floor of the King's Chamber is identical to ¼ of the vertical axis of the main, original outer square, but this quarter length does not lie at the centre of the main square.

If we now in our last square enter the acute-angled triangle 28-31-26, we notice that we already have line 31-26 as part of the acute-angled triangle in the preceding square that marked off the descending passage.

Line 28-31 provides us with the Pyramid's ascending passage; it follows precisely the floor of the ascending passage and the Grand Gallery, and ends just outside the King's Chamber.

By studying square 20-24-25-26 we see that the vertical sacred cut in this square is an extension of line 30-29. This line indicates the point at which the descending passage is intersected by the ascending passage.

In the actual Pyramid this point is marked by a huge black granite rock.

The upper sacred cut of the same square lands, as shown by line 28-29, in the bottom of the King's Chamber.

We can also see that the centre of square 28-29-30-26 indicates, at the intersection of the two diagonals, the tiny chamber in the Pyramid known as the Well.

We recognise here in the lay-out of the Great Pyramid the ratio—discussed and illustrated earlier—of the three diminishing squares inside each other. The large square is 20-24-25-26, the intermediate square is 28-29-30-26 and the small square is marked by line 32-26. If we call the large square A, the intermediate B and the small C, we note that the floor of the King's Chamber is level with the line of *the upper horizontal sacred cut in square* A. The floor of the Queen's Chamber is marked by the same sacred cut in square B, while the floor of the Subterranean Chamber is indicated by base-line 25-26 of the whole complex of squares.

Before passing on to the next analytical drawing there are one or two small details worthy of note.

We observe that diagonal 25-20—at its intersection with the descending passage—marks the place where, as shown in Flinders Petrie's drawing, several special stones have been fitted into the passageway.

We observe, too, that square 28-29-30-26 was constructed by applying the half diagonal (26-24) of the preceding square as the radius of an arc with point 26 as its centre. This circle arc, too, is marked in the descending passage. It will be noted that the point at which the arc intersects this passage is precisely the point at which Petrie reports the existence of a deep fissure in the rock.

The two marks are interesting for although they have no apparent significance in the building itself they provide—in the context of the diagram—an indication of important structural guide lines which although having no independent purpose in the planning are essential to the construction of the diagram.

Now let us move on to the analytical diagram in *Fig. 87*.

The basis of this diagram is the same as that in Fig. 86, and we shall continue working with the same part of the main square, namely square 20-21-22-23.

It is also the half-size square we use here, but whereas in the preceding diagram we saw this square in the upper left corner it is now placed in the centre left of its main square.

In other words, we have here symbol

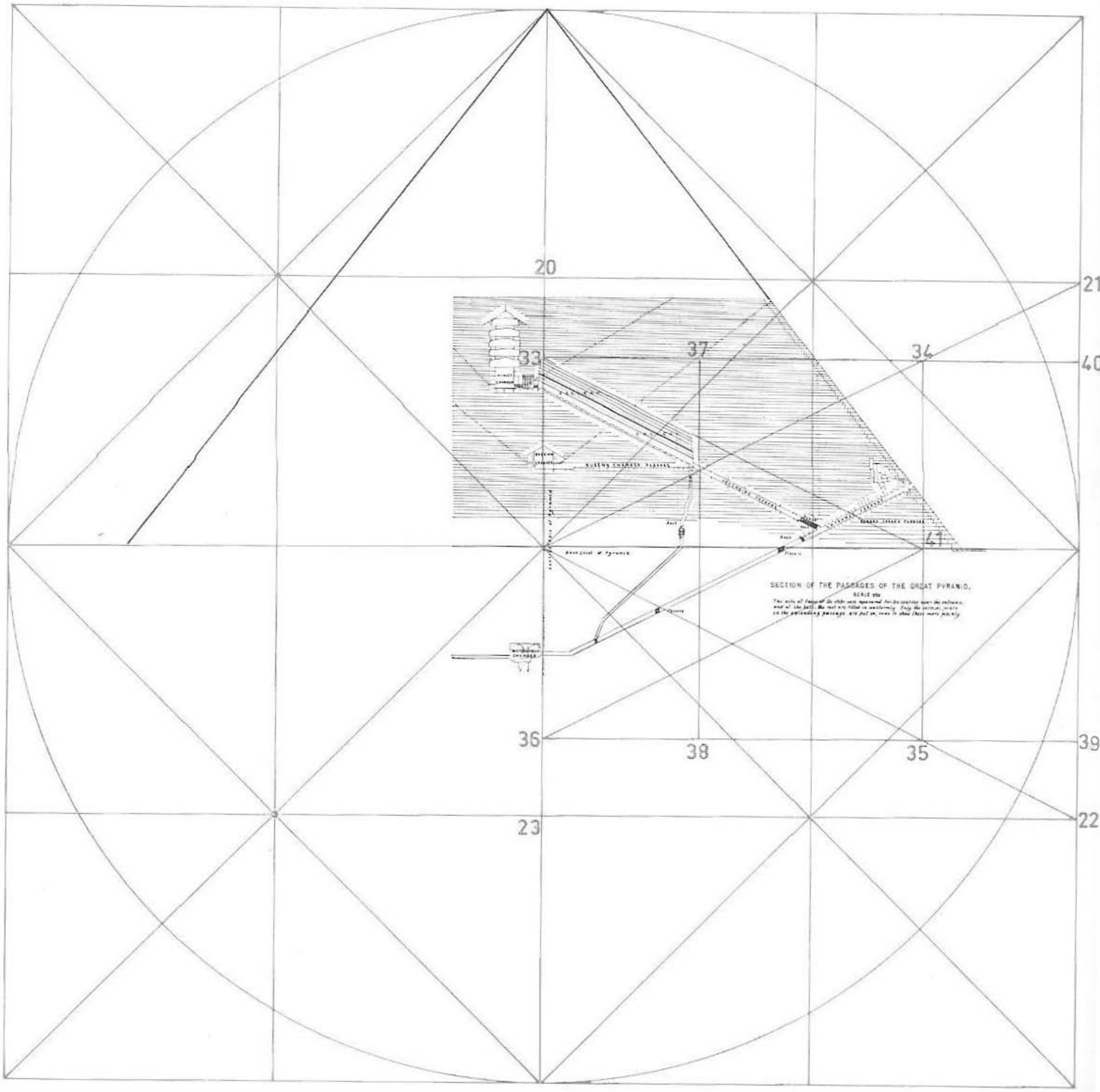


Fig. 87.

"N", *but the symbol has been given a 90°* turn to the right and its inside circle has been omitted for reasons of clarity.

As ¼ of the main square, therefore, we have 20-21-22-23, and as the half-size of this square we have 33-34-35-36 on the left.

If we enter in this small square the acute-angled triangle 33-41-36 we can see how line 41-33 follows closely the line of the Grand Gallery's ceiling.

In Fig. 86 line 28-31 indicated the floor of this gallery. We have thus an accurate indication of its height.

The same square on the right of the diagram can be seen as 37-38-39-40. Line 37-38 indicates approximately the lower end of the gallery, but this observation is not quite as accurate as the others.

We can, however, see that the upper end of the Grand Gallery extends slightly over the line entered as the Pyramid's vertical axis. In other words, the actual length of the gallery is accurate as shown in the diagram, but the gallery itself has shifted somewhat to the left of its likely position in our theory.

Again we face possible difficulty in assessing available drawings. It is not the object itself but a reconstruction with which we are comparing.

In reflecting on this, we must also acknowledge that the task of reconstructing the Pyramid on paper was doubtless a tremendous undertaking. It is extremely difficult to take measurements of the inside of a building and to transfer these to its exterior when one has only one point from which to reckon, i.e. the entrance doorway. The only other guides are the angle of ascent and descent of the passages, and their lengths. It is thus possible that an error in measurement was made here. It is likewise possible that the artist erred in committing the dimensions to paper. But compared with the precise nature of our other observations this placing of the Grand Gallery is a very small error. And of course we cannot ignore the possibility that the drawing is accurate and that the gallery's length was determined by quite another set of lines.

The architect, in planning the Great Pyramid, had no doubt armed himself with a mass of supplementary diagrams for indication of passage widths, chamber sizes, positioning of chamber entrances, etc. But we can satisfy our minds with the thought that our analysis has established the principal lines of construction in this intriguing monument. We have traced all heights and angles and are now able to reconstruct in any given square the main proportions of the Pyramid without the aid either of the building itself or its plan.

We see this complete drawing in *Fig. 88* in which all the symbols we employed have been entered, and the Great Pyramid as well as its strange interior emerge quite clearly.

The actual analysis proved, of course, that this reconstruction—line by line—corresponds absolutely with available material concerning the dimensions of the building.

The basis of construction lies, as we have seen, well beyond the actual external dimensions of the Pyramid, and all subsequent lines in the analysis bear a distinct geometric relation to the original square of construction.

It would be inconceivable that these factors, this entire plan, should fit the details of the Great Pyramid so neatly unless the pyramid was in fact created in accordance with the symbols. A fact which I consider lends strong support to the theory that the Great Pyramid was planned according to these symbols is that they are not something artificially produced to illustrate certain predetermined factors about a building.

The symbols arose as essential stages of a geometric and mathematical development in which the individual symbols form part of a whole structure, a complete development which from the start had nothing to do with either buildings or pyramids. A development in which the symbols represent the written expression of certain experiences made in a clearly defined field, namely geometry and mathematics.

We are now left only with the question of motive for the choice of these particular symbols. And in this connection it is not so much the basic symbol of con-

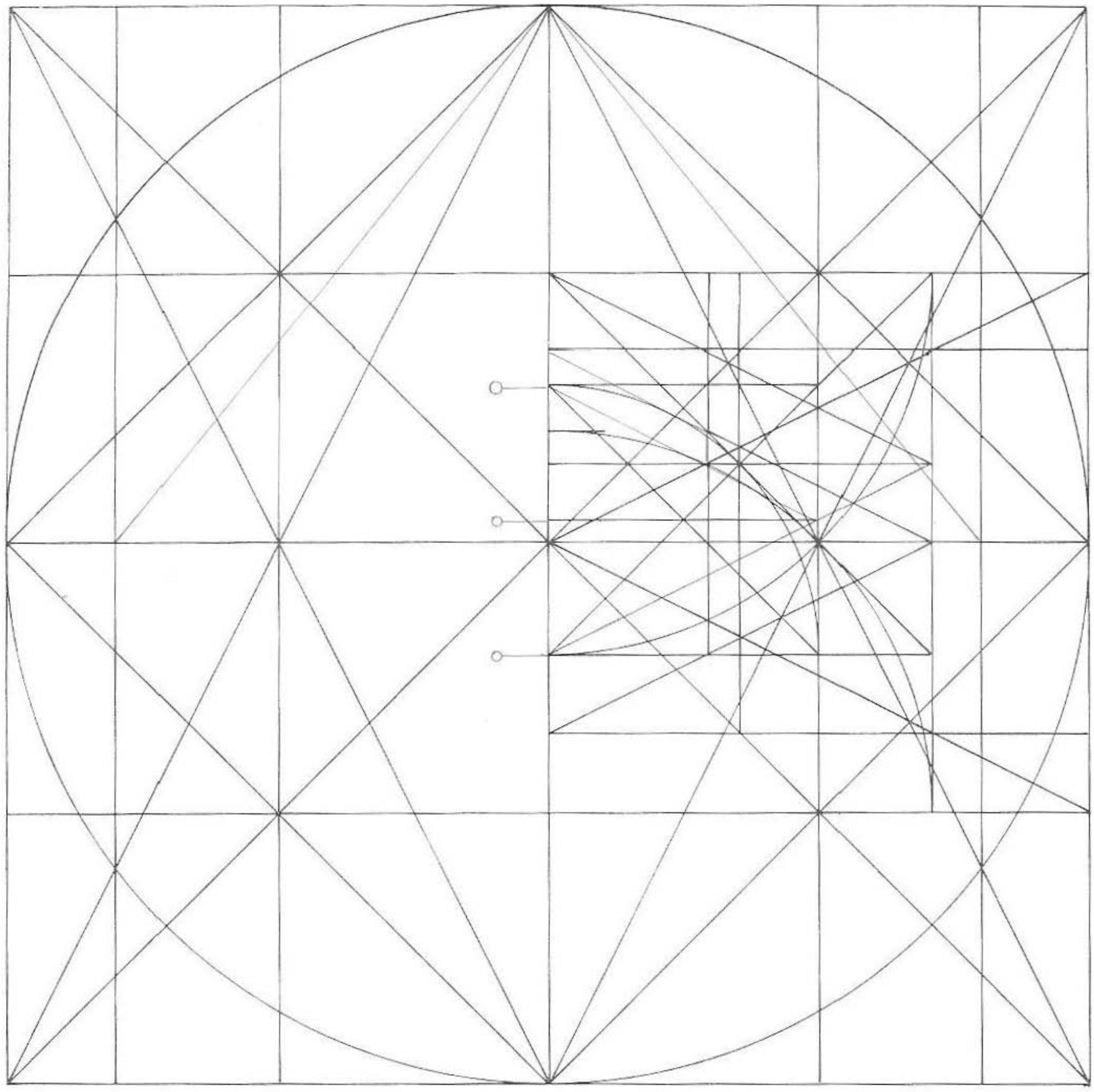


Fig. 88.

struction we require to consider as the symbols placed within it.

With regard to the choice of the basic symbol, it of course contained what the Ancients considered immense knowledge, since with its rectangle it indicated the area of the circle that could be drawn inside the main square of construction, and it is not surprising that the choice of starting point should fall on this particular symbol.

The significance of the symbol is thrown into even sharper relief when viewed on the basis of an analysis on the Pyramid itself. We recall that the first thing we established (in the first analytical drawing Fig. 83) was the side of the Pyramid.

We recall also that this side occupied ¼ of the circle's rectangle.

If we now pictured in our mind's eye the Pyramid itself and note that it has four equal sides, we realise that with its four sides the Great Pyramid represents the total area of the circle's rectangle, and

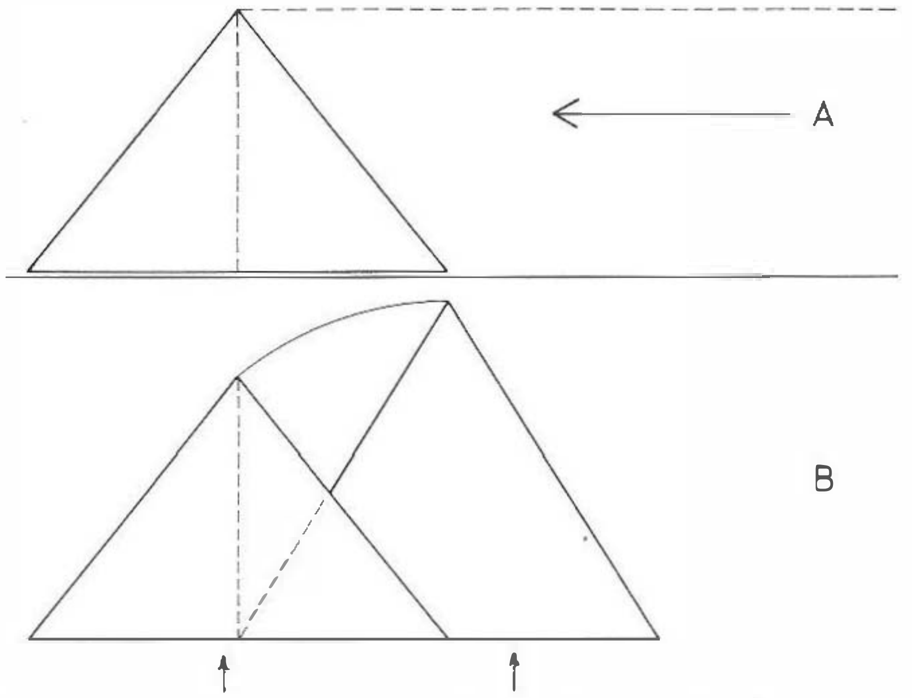


Fig. 89.

consequently *of the sacred circle itself*. In other words, the viewer must walk round the whole Pyramid in order to see the complete area of the circle.

This symbol contains yet another point for consideration, namely the difference between the visible and the measurable, for the visible surface of the Pyramid is the one the construction is based on, but this surface is not the same as the measurable area.

In *Fig. 89* we can see the difference.

In A we see the Pyramid shown in perspective, the broken lines indicating the line of vision and the triangle the eye can see.

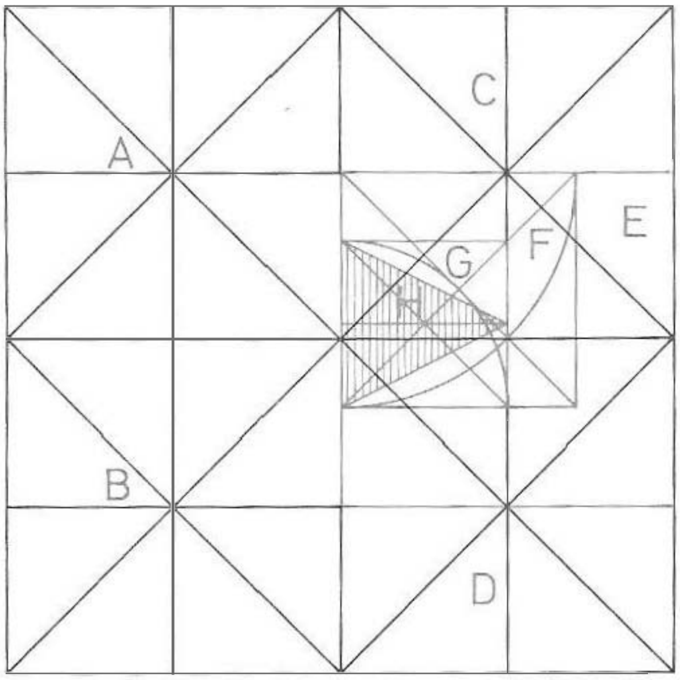
In B we see the side of the Pyramid measured and raised perpendicular to the ground showing its real area, and we can immediately see how this area is greater than the section of the Pyramid on which construction is based.

The area of the circle is thus in the actual structure concealed in such a way that it can be viewed by anyone but not measured, and if one believes that the early Egyptians had developed a school of philosophy beyond the purely primitive stage then this structure is a fine symbol of the fact that the truth can be seen but not measured; it is quite another matter that the circle's rectangle and the area of the circle itself are proved to be slightly different, and that truth must be regarded as relative to its surroundings.

The symbols within the basic square must presumably have possessed a structural significance as well as an association with ritual.

With regard to Fig. 85 which presented three possible variations for selection of a square within the symbol, we must consider the chosen square from the point of view that a room was required under the Pyramid for ritualistic reasons. Bearing this in mind and at the same time requiring a square in contact with the exterior of the Pyramid (in order to fix the position of the entrance and the descending passage) there is only one possible answer, i.e. the square shown later to be the correct choice, Fig. 85 C, square 6.

This would tend to emphasise that once the outer dimensions of the Pyramid were



A = 1/4 main square = 8 triangles
B = 1/4 main square = 8 triangles
C = 1/8 main square = 4 triangles
D = 1/8 main square = 4 triangles
E (red square) = A = 8 triangles
F = 1/2-size E = 4 triangles
G = 1/2-size F = 2 triangles
H = 1/2 G = 1 triangle

Fig. 90.

fixed the architect's next task was to combine the functional with the ritual so that these two factors formed one, and if this supposition is correct we must admit the attempt was completely successful.

The internal lay-out of the Pyramid, too, possesses a peculiar geometric property, the placing of the symbols in relation to the main square also providing a picture of the circle's area, but in this case a picture that is less than the circle's rectangle and rather less than the π we use today.

Omitting the outline and details of the Pyramid we see in *Fig. 90* the diagram that was used in planning the interior. Fig. 90 is in effect identical to Fig. 86.

The main square is split up by the vertical and diagonal crosses, and as before we divide the whole area into triangles, the four quarter-squares each containing 8 triangles, making a total of 32 in the main square.

In addition, we have the previously executed division in which ¼ of the diagram was split in two stages, each square inside the other, the final half-size square being intersected by the acute-angled triangle, the sloping sides of which indicated the ascending and descending passages.

From experience we know that the circle's rectangle is ⅘ of the area of the main square, which in this instance means that, since the square is split into 32 triangles, ⅘ of 32 = 25⅗ triangles, a calculation which was probably beyond their capabilities.

If we now look once more at Fig. 90, we can count the triangles. There are four small squares, two of which are again subdivided.

A	=	1 square	=	8 triangles
B	=	1 square	=	8 triangles
C	=	½ square	=	4 triangles
D	=	½ square	=	4 triangles
H	=	⅛ square	=	1 triangle
		Total	=	25 triangles

The component triangles in A, B, C and D are equal to each other in shape and size, whereas H differs in shape from the others. (H represents of course triangle 28-31-26 in Fig. 86.)

But a little calculation on the drawing proves that this triangle, although varying in shape, still makes up 1/32 of the main square.

Thus we see that the area occupied by A, B, C, D and H is 25/32 of the main square, i.e. in terms of the four quarter-size squares it contains 3⅛ of these, or 3.125, which is slightly less than our π with its 3.142.

The circle's rectangle contains an area termed 3.2, and we see therefore that it is slightly more than π, while the figure shown in the interior of the Pyramid is slightly less.

It is just a possibility that the intention here was to demonstrate the two different methods of solving the same problem, one by the Pyramid's outer structure, the other by its interior. But it would be sheer guesswork to try to prove the architect's thoughts in this direction, whereas it is an undeniable fact that details outlined above can be found in both the outside and inside planning of the Pyramid.

Some concluding facts about the practical background of the Great Pyramid are worth recording.

At one time, of course, the Pyramid has had a number of more conventional temple buildings around it, and was itself part of a huge complex of buildings and courtyards which together were probably a restricted temple area, open only to initiated brethren.

The Temple Order itself was based on a gradual process of initiation and admission, in which the Great Pyramid was probably used for the initiation of the highest degree or perhaps the three highest degrees in the Order. Throughout the graduated admission, which lasted 22 years, the prospective initiate was taught the various sciences of which geometry and numbers were among the most important, and in this context it is not really surprising that they should work this knowledge into the structure of their initiation temple. Perhaps they actually applied the Pyramid as symbolic teaching material, being a symbol of supreme wisdom.

This procedure is perhaps foreign to our minds, and yet there are traces even today of the same symbolism in a sphere which represents possibly the last slender connection with the civilisations of mystery of the past, namely freemasonry, where an effort is made to build *freemasons' lodges according to certain* dimensions bearing—within the lodge—a symbolic significance.

The Pharaohs, the rulers of ancient Egypt, enjoyed tremendous power and influence both in and outside their country, and it was apparently in their hands that all the power lay—but only apparently. Behind these rulers of Egypt were the Temples, and *their* power and influence were even greater than those of the Pharaohs. They were strong enough to dethrone a ruler who proved himself unworthy of the name. All officials and functionaries of importance were recruited from the ranks of the Temple, which naturally strengthened the latter's power immensely.

It should not therefore be surprising to learn that this colossal structure, the Great Pyramid of Egypt, was built as an integral part of the ritual that surrounded these men. The pyramids were used for the initiation of the highest degrees of learning, from which circles both the Temple and the State recruited their leaders, and we can in fact trace the same tendency up through the ages, with temples and churches outdoing each other in audacity, dimensions and splendid decoration. And the procedure with knowledge has been identical. Through the centuries practically all education lay within the confines of the temple or temples who thus constructed a network of solid power over the intelligentsia of a country, and in this manner became—behind the scenes—the real rulers of the people.

And the portrayal of their concealed strength was the magnificent buildings they occupied, which in their sumptuousness and splendour almost invariably exceeded the castles and palaces that housed the nation's official rulers.

CHAPTER EIGHT

Book of Exodus: Ritual Significance of Moses' Tabernacle

CHAPTER SIX demonstrated how geometric knowledge was built into the very structure of the largest of the pyramids in the assembly of temples at Gizeh, involving both the exterior, and the inner passages and chambers of the building.

This massive pyramid was not of course built solely for the purpose of passing on its architects' knowledge to an infinitely distant future generation, for it is highly improbable that any people would be so unselfish as to construct such a giant monument exclusively for their successors without in some way deriving benefit or pleasure from it themselves.

As described, the Great Pyramid was part and parcel of a large complex which as such was an area open only to temple initiates, and the Pyramid itself was probably used as a stage in the admission of brethren of various degrees.

This temple area was in fact a complete world of its own, with homes for the innumerable brethren and priests who to a certain extent spent a secluded existence during their period of admission which in toto lasted 22 years.

During this training there were various branches in which the student could specialise after he had been at the temple for seven years. He could perhaps choose to follow the path of medicine or architecture, or he could train for a post in State government. All persons of standing were recruited from the Temple, and the power of the Temple was consequently great enough to control the comings and goings of the Pharaoh—even dismiss him should he be an unsuitable ruler of his country.

It was not essential to undergo the complete course of training. In gaining admission to the Temple the student required only to give an assurance that he would stay for the statutory seven years. At the end of this period he was free to quit if he so desired, and was then capable of supervising general subordinate Osiris priests or scribes.

If his abilities permitted, he could go on to complete the training of 3 × 7 years, and remain for a further period of twelve months, bringing his total training to 22 years. At this stage the initiate had achieved the full training of the Temple and become an adept or hierophant.

The man who passed through the various stages of Egyptian Temple initiation and who is best known in our time is no doubt Moses. History certainly acknowledges that he had received training in the Egyptian Mysteries but maintains that Moses was of Jewish stock.

This assertion should be regarded on the basis of the fact that this section of biblical history was written by the Jewish or Israelite people who naturally had a positive interest in claiming a personality such as Moses.

In an explanatory note to his chapter on Moses, the Frenchman Edouard Schuré writes in his book, *The Great Initiates:*

"The Biblical account makes of Moses a Jew of the tribe of Levi, found by Pharaoh's daughter among the reeds of the Nile, placed there by the mother's cunning to touch the heart of the princess, and save the child from a persecution similar to that of Herod. On the other hand, Manethon, the Egyptian priest, to whom we are indebted for the most authentic information regarding the dynasties of the Pharaohs, information now confirmed by the inscriptions on the monuments, affirms that Moses was a priest of Osiris. Strabo, who obtained his information from the same source, i.e. from the Egyptian priests, also bears witness to this fact. Here the Egyptian origin has more value than the Jewish, for the priests of Egypt had not the slightest interest in making Greeks or Romans believe that Moses belonged to their race, whilst the national amour-propre of the Jews compelled them to regard the founder of their nation as a man of the same blood with themselves. The Biblical narrative also recognises that Moses was brought up in Egypt and sent by his government as inspector of the Jews of Goshen. This is the important fact establishing the secret filiation between the Mosaic religion and Egyptian initiation. Clement of Alexandria believed that Moses was a profound initiate of the science of Egypt; indeed, the work of the creator of Israel would otherwise be incomprehensible."

Edouard Schuré, following an intensive study of his subject, goes on to describe how Moses came as a fairly young man to the Temple for training and demonstrated his extraordinary character. His name was Osarsiph, and he was a cousin of Menephtah, son of Rameses II.

As Osarsiph possessed quite unusual abilities and outshone his cousin Menephtah, he was often the subject of intrigue, since his cousin feared that he would challenge his own right as future Pharaoh.

Since Rameses II himself bore considerable sympathy for Osarsiph but naturally preferred to see his own son on the throne, he decreed that Osarsiph should take up the pen of a Temple scribe, a post demanding insight into the symbols, cosmography and astronomy of the Temple, but which by tradition excluded its holder (who had then taken the first step towards the position of high priest) from occupying the throne of the Pharaoh.

Osarsiph who himself had few ambitions in this direction but who regarded the royal decree as a mark of honour delved even deeper than before into his studies, and one of his practical duties was an inspection of the provinces as state inspector.

On one tour of Goshen he was so overcome by the treatment to which the Jews were subjected that in a clash with one of the Pharaoh's labour overseers he killed the man.

This was the reason he was forced to flee Egypt.

On the other side of the Sinai peninsula in the area known as Midian, there was a temple which did not come under the sphere of influence of the Egyptian priests. The area was made up of a broad belt of land between the Gulf of Elam and the Arabian Desert. It was wedged between the Red Sea and the desert, protected by a range of vulcanic mountains and thus almost inaccessible from the outside world.

The temple was dedicated to Osiris but prayer was also offered to the supreme god under the name of Elohim. The temple, which had been founded by an Ethiopian brotherhood, was the religious oasis of Arabs, Semites and those members of black nations who sought initiation.

For centuries past Sinai and Horeb had fostered a monotheistic cult. Semitic pilgrims journeyed in flocks to worship Elohim here, and they used to spend days of fasting and prayer in the caves and subterranean passages that riddled the slopes of Sinai, but first they attended the Temple of Midian to purify themselves and receive instructions.

It was to this temple—outwith the control of the Egyptians—that Osarsiph fled and was welcomed with open arms by Jethro, the high priest, who was a man of great wisdom but slow to act. He recognised in Osarsiph a powerful personality who could benefit and strengthen the ancient Temple of Midian.

It was Osarsiph's first wish to undergo the punishment prescribed by Temple legislation for any priest who, intentionally or otherwise, is guilty of murder, and he had to pass through an initiation which proved more awful and more dangerous than anything he had previously experienced. But his iron-will took him safely through the dangers, and after this initiation Osarsiph assumed the name of Moses, which means he who is saved.

That is a brief account of the story related by the French researcher and based on Egyptian material. The reference in the Jewish legend to Jethro's seven daughters probably bears a symbolic meaning since Jethro (also known as Reuel) is mentioned as a shepherd, as is Christ; and the seven daughters tending the sheep are possibly symbols of the seven virtues to be mastered in order to open the Well of Truth.

After a period of years in Jethro's temple Moses returned to Egypt to lead the Jews to freedom and to become their overlord.

It would of course have been impossible for a man with the knowledge and experience of Moses to remember everything he knew without having it in writing—just as any doctor, engineer, chemist and other professional member of the sciences must today resort to books in which they can look up the information they require.

I mention this to illustrate that through the ages knowledge and wisdom have always been identified with power, and when Moses set out with the Jewish nation to wander in the wilderness he could only have led them if he possessed the necessary power to do so, a power few could share; and that power was knowledge.

An important factor in the account of the Jewish wandering from Egypt was the Ark of the Covenant, a chest possessing universal power that only Moses might approach. We have never been told what the Ark contained, but if Moses had written down his knowledge and his learning, then he would have been forced to make this chest taboo to everyone but himself—and he himself would decide who should be trained to share in the knowledge.

In a temple in a regular society it is quite a simple matter to keep one's secrets hidden from prying eyes—by the obvious expedient of forbidding outsiders from entering the temple.

But to select a handful of people suitable for training and to begin such training in the open air in front of thousands of spectators would be impossible, and one of the first things therefore that Moses did after leaving Egypt was to announce a decree from God, commanding the children of Israel to build a temple for His worship.

The temple or tabernacle had apparently three functions:

1) to serve as a resting place for the Ark of the Covenant during longer pauses in the Jews' wanderings;
2) to replace a regular temple for training suitable leaders;
3) to serve as a central point of focus for the people of Israel in respect of the worship of God which Moses in his departure from Egypt would introduce and foster in them.

Since Moses' people were destined to live a nomadic life it would have been pointless to build the usual form of permanent temple since of course they would have been obliged to leave it behind each time they moved, so the tabernacle was made of portable materials.

The tabernacle is dealt with in considerable detail in the Book of Exodus, the Second Book of Moses. The version reproduced below is based on a translation from the original Hebrew canon by Danish scholar, Dr. Frants P. W. Buhl, of the Universities of Copenhagen and Leipzig, towards the end of last century. In preparing this passage—as with several others—Dr. Buhl reported numerous discrepancies between the Hebrew and existing Danish/English translations, which had been taken mainly from Latin. I believe therefore that in adapting Dr. Buhl's translation to the authorised version we come closest to the original text and its meaning.

Voluntary Offerings for the Tabernacle: And the Lord spake unto Moses saying, Speak unto the Israelites that they bring me an offering: of every man that giveth it willingly with his heart ye shall take my offering. And this is the offering which ye shall take of them: gold, silver, bronze, and blue, purple, scarlet and crimson wool, and fine linen, and goats' hair, and rams' skins dyed red, and badgers' skins, and shittim (acacia) wood, oil for the candlestick, spices for anointing oil, and for sweet incense, onyx stones, and precious stones to be set in the ephod and in the breastplate. And let them make me a sanctuary; that I may dwell among them. According to all that I shew thee after the pattern of the tabernacle and all the instruments thereof, even so shall ye make it.

Ark of the Covenant: And thou shalt make a chest of shittim wood: two cubits and a half shall be the length thereof, a cubit and a half the width, and a cubit and a half the height. And thou shalt overlay it within and without with pure gold, and shalt place about it an edging of gold. And thou shalt cast four rings of gold, and put them in the four feet thereof; and two rings shall be on each side of it; And thou shalt make staves of shittim wood, and overlay them with gold. And thou shalt put the staves into the rings by the sides of the ark, that the ark may thus be borne. The staves shall remain in the rings of the ark; they shall not be taken from it. And thou shalt put into the ark the testimony which I shall give thee. And thou shalt make a mercy seat of pure gold: two cubits and a half shall be the length thereof, and a cubit and a half the width. And thou shalt make two cherubims of gold of beaten work in the two ends of the mercy seat. And make one cherub on the one end, and the other cherub on the other end: even of the mercy seat shall ye make the cherubims on the two ends thereof. And the cherubims shall stretch forth their wings on high, covering the mercy seat with their wings, and their faces shall look one to another; towards the mercy seat shall the faces of the cherubims be. And thou shalt put the mercy seat upon the ark; and in the ark thou shalt put the testimony that I shall give thee. And there I will meet with thee, and I will commune with thee from above the mercy seat, from between the two cherubims which are upon the ark of the testimony, of all things which I will give thee in commandment unto the children of Israel.

The Shewbread Table: Thou shalt also make a table of shittim wood: two cubits shall be the length thereof and a cubit the width, and a cubit and a half the height. And thou shalt overlay it with pure gold, and make thereto an edging of gold. And thou shalt make a border of an handbreadth round about its legs, and thou shalt make a golden edging to the border. And thou shalt make for it four rings of gold and put the rings in the four corners that are on the four feet. Over against the border shall the rings make places for the staves to bear the table. And thou shalt make the staves of shittim wood, and overlay them with gold, that the table may be borne with them. And thou shalt make the dishes and bowls and jars and jugs thereof of pure gold. And thou shalt set upon the table shewbread before me always.

The Candlestick: And thou shalt make a candlestick of pure gold: of beaten work shall the candlestick be made: its base and stem shalt thou make, and its flowers with cups and corollae shall be parts of it. And six branches shall come out of the sides of it; three branches from the one side, and three branches out of the other side. On each branch of the candlestick there shall be three flowers resembling the almond-flower with cups and corollae. But on the stem of the candlestick shall be four flowers resembling the almond-flower with their cups and corollae. And there shall be a cup under each pair of branches of the six branches of the candlestick. Their cups and their branches shall be parts of it: so that it shall be one beaten work of pure gold. And thou shalt make seven lamps for it: and they shall place the lamps thereof that they may give light upon the other side. And the tongs thereof and the snuff-dishes thereof shall be of pure gold. A talent of pure gold shall be used for this and all these vessels. And look that thou make them after the pattern, which will be shown thee in the mount.

The Tabernacle: Moreover thou shalt make the tabernacle with ten curtains of fine twined linen, and blue, purple and scarlet wool. With cherubims of cunning work shalt thou make them. The length of one curtain shall be eight and twenty cubits, and the width of one curtain four cubits: and every one of the curtains shall have one measure. Five curtains shall be coupled together with the other five; and along the coupling of one of the last curtains at its joining place thou shalt make loops of blue wool; and in the same manner thou shalt make loops along the coupling of the last curtain at the other joining place. Fifty loops shalt thou make in the one curtain, and fifty loops shalt thou make in the joining place of the other curtain; the loops shall be placed exactly opposite one another. And thou shalt make fifty gold hooks and use them to join the curtains together; and it shall be one tabernacle. And thou shalt make curtains of goats' hair to be a tent covering the tabernacle: eleven curtains shalt thou make. The length of one curtain shall be thirty cubits, and the width four cubits: and the eleven curtains shall all be of one measure. And thou shalt couple five curtains by themselves, and six by themselves, and shalt lay the sixth curtain double over the front side af the tent. And thou shalt make fifty loops on the edge of one of the last curtains at its joining place, and fifty loops on the edge of the last curtain at the other joining place. And thou shalt make fifty bronze hooks, and put them into the loops, and couple the tent together that it may be one. And the remnant that remains of the curtains of the tent, the half curtain that remains, shall hang over the back of the tabernacle. And the remaining cubit on each of the long sides of the tent curtains shall hang over the sides of the tabernacle to cover it. And thou shalt make a covering for the tent of ram's skins dyed red and above this another covering of badgers' skin. And thou shalt make boards for the tabernacle of shittim wood standing upright. Each board shall be ten cubits high and a cubit and a half wide. Two tenons shall there be in each board, set in order one against the other: thus shalt thou make for all the boards of the tabernacle. And thou shalt make the boards for the tabernacle, twenty boards on the south side. And thou shalt make forty sockets of silver under the twenty boards, two sockets under one board for its two tenons. And on the north side of the tabernacle there shall be twenty boards: and their forty sockets of silver: two sockets under each board. And for the sides of the tabernacle on the west thou shalt make six boards. But for the corners of the tabernacle at the back thou shalt make two boards. And they shall be coupled together twin-like at the bottom .... unto the first ring: thus shall it be for them both, they shall be for the two corners. Thus for the back there shall be eight boards, and their sixteen sockets of silver, two sockets under each board. And thou shalt make crossbars of shittim wood; five for the boards of the one side of the tabernacle, and five for the boards of the other side, and five for the westward side of the tabernacle at the back. And the middle crossbar in the midst of the boards shall reach from end to end (of the wall). And thou shalt overlay the boards with gold, and make the rings (for the bars) of silver: and thou shalt overlay the crossbars with gold. And thou shalt erect the tabernacle according to the fashion thereof which was shown thee in the mount. And thou shalt make a veil of blue and purple and scarlet wool, and fine twined linen; with cherubims shall it be made. And thou shalt hang it upon four pillars of shittim wood overlaid with gold: their hooks shall be of gold upon the four sockets of silver. And thou shalt hang up the veil under the hooks, that thou may bring in thither within the veil the ark of the testimony: and the veil shall be unto you a division between the holy place and the most holy. And thou shalt put the mercy seat upon the ark of the testimony in the most holy place. But set the table outwith the veil, and the candlestick opposite the table by the south wall of the tabernacle: and thou shalt put the table by the north wall. And thou shalt make an hanging for the door of the tent, of blue and purple and scarlet wool, and fine twined linen of a many-coloured weave. And thou shalt make for the hanging curtain five pillars of shittim wood, and overlay them with gold, and their hooks shall be of gold. And thou shalt cast five sockets of bronze for them.

The Altar of Burned Offerings: And thou shalt make an altar of shittim wood, five cubits long and five broad; the altar shall thus be square; and the height thereof shall be three cubits. And thou shalt make the horns of it upon the four corners thereof: its horns shall be a part of it: and thou shalt overlay it with bronze. The pans to receive the ashes, and its shovels, basins, forks and firepans and all the vessels thereof thou shalt make of bronze. And thou shalt make for it a grate of woven bronze; and upon this corner place four bronze rings. And thou shalt put the grate under the central beading of the altar; that the grate may reach half-way up the height of the altar. And thou shalt make staves for the altar of shittim wood, and overlay them with bronze. And the staves shall be put into the rings, and the staves shall be upon the two sides of the altar, to bear it. It shall be made as a hollow, timber chest: as it was shown thee in the mount, so shall they make it.

The Surrounding Forecourt: And thou shalt make the court of the tabernacle: for the south side there shall be one hundred cubits of curtain of fine twined linen for one side: and the twenty pillars thereof and their twenty sockets shall be of bronze; the hooks of the pillars and their fillets shall be of silver. And likewise for the long north side there shall be hangings of one hundred cubits, and twenty pillars with twenty sockets of bronze;

the hooks and fillets of silver. And for the width of the court on the west side shall be hangings of fifty cubits, their pillars ten, and their sockets ten. And the width of the court on the east side shall total fifty cubits, such that one side shall have fifteen cubits of curtain: their pillars three, and their sockets three. And for the gate of the court shall be an hanging of twenty cubits, of blue and purple and scarlet wool, and fine twined linen, of many colours: and their pillars shall be four, and their sockets four. All the pillars round about the court shall be filleted with silver; their hooks shall be of silver, and their sockets of bronze. The length of the curtain around the court shall be one hundred cubits, the width fifty, and the height five cubits of fine twined linen, and their sockets of bronze. All the tools of the tabernacle for the work therein, and all the tent staves, and all the staves of the court shall be of bronze.

*

So much for the description. We have here an account of the instructions Moses received from his god to build a temple, and as opposed to many other legends this one is full of concrete figures and instructions combined with a number of more obscure concepts.

It is hardly credible that a god should indicate at such length and in such detail the construction and size of temple to be built in his honour. And yet it would be equally inconceivable that such a list of precise figures and instructions should appear in a text and yet contain no meaning or significance.

Since we know that Moses received his training in the temples of Egypt and that these were the centre of the then form of ancient geometry, we should be at liberty to assume that his dimensions for the tabernacle are bound up with this geometric system. There is scarcely any doubt that the figures in fact stem from Moses himself, if one accepts the authenticity of the tale. For according to the legend Moses spent a considerable time on the slopes of Sinai in meditation, and on returning to his people he had completed the plans for the tabernacle. In other words, quite understandably, Moses had like any other person faced with a mental problem wanted to be alone while he concentrated on his creative work—which, tradition lays down, was a revelation by God.

With regard to the actual sizes and figures mentioned in the legend there is a fairly firm guarantee that these have been conveyed accurately in translation from the ancient text since figures normally cannot be interpreted in more ways than one, as is the case with words and ideas. Numbers are an exact and clear-cut factor in any language, and we may therefore safely assume that all those mentioned in the above quotation have existed in their original form through the ages, whereas the text may have altered slightly in its numerous translations.

We shall begin our examination of the passage with the section on the tabernacle itself, looking first at the areas quoted by Moses for the covering of the structure. The text reads:

> "Moreover thou shalt make the tabernacle with ten curtains of fine twined linen, and blue, purple and scarlet wool. With cherubims of cunning work shalt thou make them. The length of one curtain shall be eight and twenty cubits, and the width of one curtain four cubits: and every one of the curtains shall have one measure."

There is really nothing about this first piece of text that can be misinterpreted: we must make ten curtains each of which is 4 × 28 in proportions.

We see this curtain in *Fig. 91* the sides of the curtain being in the ratio 1 to 7,

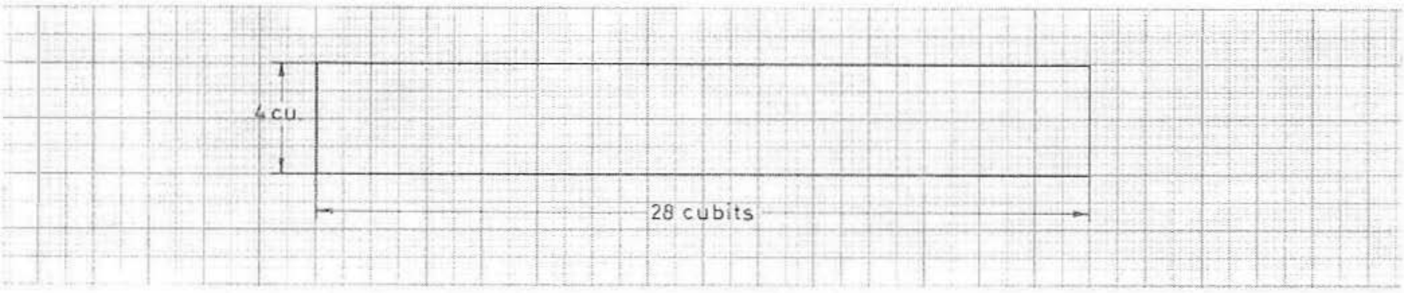


Fig. 91.

and already we sense the presence of ancient geometry and the preoccupation with the sacred number 7.

We have ten of these curtains, and the text goes on:

> "Five curtains shall be coupled together with the other five."

As the curtains are to be used to cover the building this coupling of the five curtains must undoubtedly refer to the long sides of the curtains. Otherwise we should end up with two long strips of curtain each 4 × 140 cubits in size, and a shape of that kind cannot be used as a cover for a building or small temple.

We see the two pieces of coupled curtain in *Fig. 92* and call a halt for the moment to examine their areas.

They are 28 long and 20 wide. If we reduce these figures we find that the sides of the curtains are in the ratio 7 to 5.

We know this ratio from our earlier speculation with the three squares and the sacred cut, the inner square of the trio having a side of 5, the intermediate a side of 7 and the outer square a side of 10.

In *Fig. 93* we have taken the two curtains and laid them across each other, the long side of one coinciding with the short side of the other.

One of the curtains is seen as 1-2-3-4 and the other as 4-5-6-7.

We now see how—apart from one corner—these two curtains create a square.

The large square is 1-8-7-4 and line 2-3 is one of the vertical sacred cuts while 5-6 is one of the horizontal sacred cuts. The smaller square 5-9-3-4 is exactly half as big as the main square, and the curtains

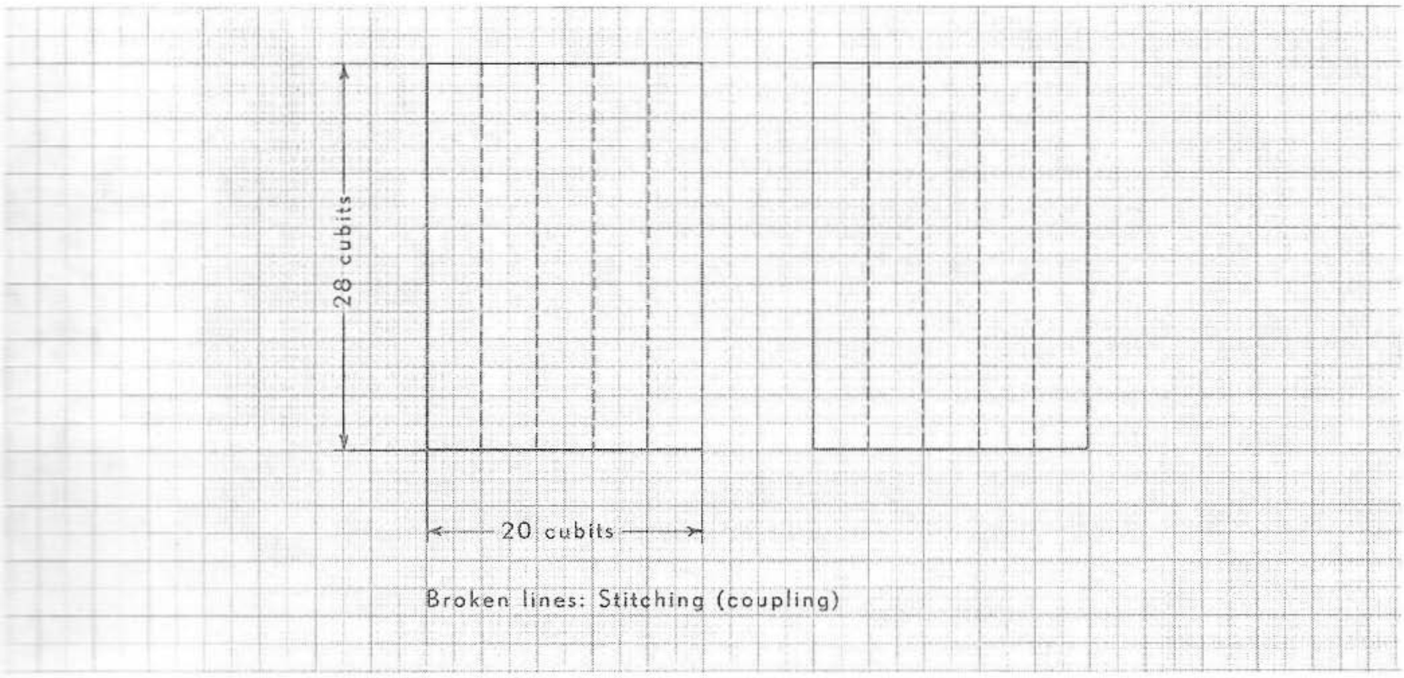


Fig. 92.

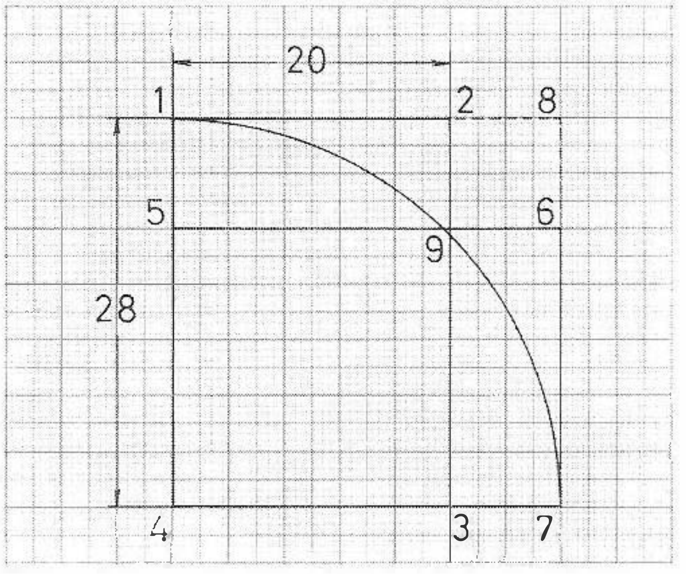


Fig. 93.

placed in this way thus provide all the facets of symbol "J".

So Moses was able here to demonstrate this principle to his followers during the erection and dismantling of his portable temple, and the curtains may have had two purposes: as instructional aids and as purely practical roofing for the temple.

If we now place the two curtains side by side, long side against long side, we produce a rectangle which (in cubit units) provides an area of 40 × 28.

Reducing these figures we again arrive at a well-known ratio, i.e. 10 × 7, and we see this executed in *Fig. 94.*

The area of the two curtains is contained by the corners A, B, C and D.

The base-line is raised to the perpendicular CE and we now construct the large square CEFD.

The upper edge of the curtain AB has now become the upper horizontal sacred cut in the new square, and square AGHC is the half-size square and in turn identical to the large square in the preceding illustration.

It is of course a natural sequence that these areas should double in size since the curtains have been given these special dimensions, and the astounding fact is that they should have precisely this length and width—as no other proportions would illustrate the sacred cut and the doubling of the squares than the very sizes selected by Moses.

My reason for illustrating this point, taking two stages, is partly to show how easily and clearly Moses was able with his curtains to demonstrate the same point in several stages, and partly that in the passage that follows Moses works on the areas of the two curtains, and for this we need the latter diagram as a basis.

The text continues:

> "And along the coupling of one of the last curtains at its joining place thou shalt make loops of blue wool; and in the same manner thou shalt make loops along the coupling of the last curtain at the other joining place. Fifty loops shalt thou make in the one curtain, and fifty loops shalt thou make in the joining place of the other curtain; the loops shall be placed exactly opposite one another. And thou shalt make fifty gold hooks and use them to join the curtains together; and it shall be one tabernacle."

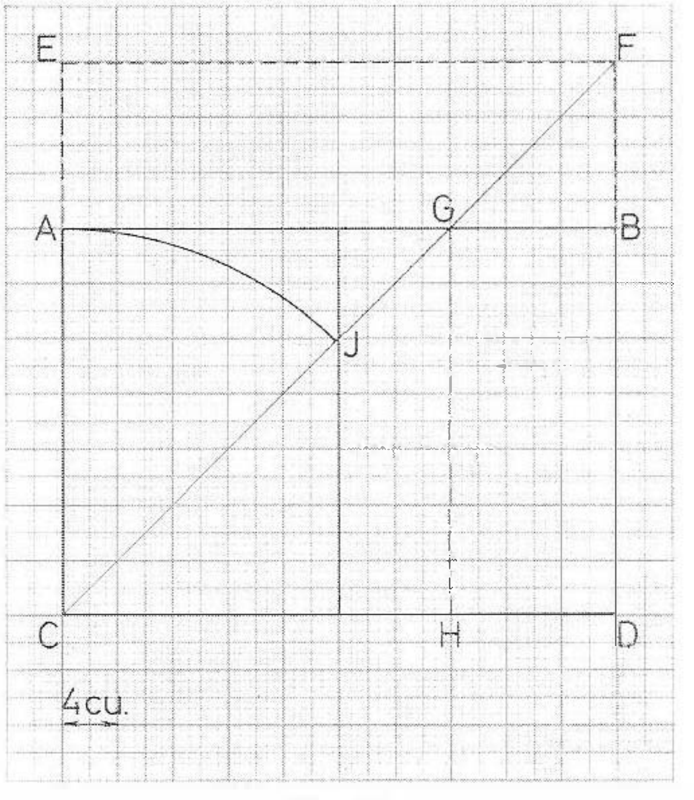


Fig. 94.

The beginning of this piece may be a little difficult to follow since there can be some doubt as to which of the curtains the instructions apply, whether in fact it is the last of the two large curtains or the last curtain in each of the two composite curtains.

If reference was being made to the latter of the two large curtains, the text would have to make it clear that one was made before the other; but since in fact the text mentions that loops should be placed at certain joints on both curtains, and concludes by saying that both curtains should be joined to make one, the correct assumption must be that it is the last strip in each large curtain that is meant.

Since the curtains are stitched together side by side, each large curtain of five strips has four long seams or joining places, and it must be these that are mentioned in the text. A curtain made up of five equal strips has two last or outer strips, namely the right and the left. If we follow the instructions, we must sew 50 loops on the large curtain at the joining place of the last or outer curtain, i.e. the place where the fifth or last curtain is stitched to the other four.

We have to do this on both large curtains and then join them by means of gold hooks.

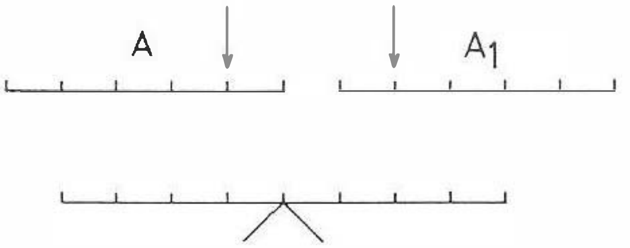
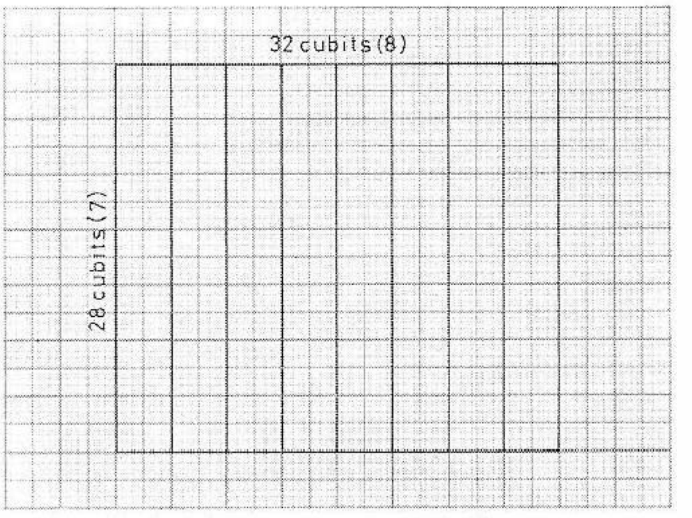


Fig. 95.

In *Fig. 95* we have a view of the two curtains end-on, 2 × 5 strips. And we also see how the two are to be joined. The two curtains now form one area which is 28 cubits in length and 32 cubits wide, seen in *Fig. 96*.



Length of each curtain-strip = 28 cubits
8 strips of curtain each 4 cubits wide = 32 cubits

Fig. 96.

Perfectly legitimately, the reader may be rather puzzled by all this juggling about with the same pieces of curtain. First, 10 curtains were woven to a particular size, then they were altered or stitched into two large curtains and finally they were fastened together in a most peculiar manner to form one curtain. He may well ask why a man of Moses' ability takes this long way round the problem if there is no particular reason for doing so. If he merely wanted a piece of fancy canvas for his tent would he not instruct his people simply to make one of this size and a framework of that size and throw the canvas over?

But in fact his roundabout instructions contain certain concrete geometric knowledge, which is seen in *Fig. 97*.

We recognise the area occupied by the two curtains placed side by side (ABCD) and recall that line AB is the sacred cut in the main square EFDC.

The coupling or junction of the two curtains is line GR but this coupling has made the curtain 8 cubits narrower, because 2 strips of curtain each of 4 cubits were—as demonstrated in the previous diagram—folded back. And the complete curtain is now seen as square KLMJ.

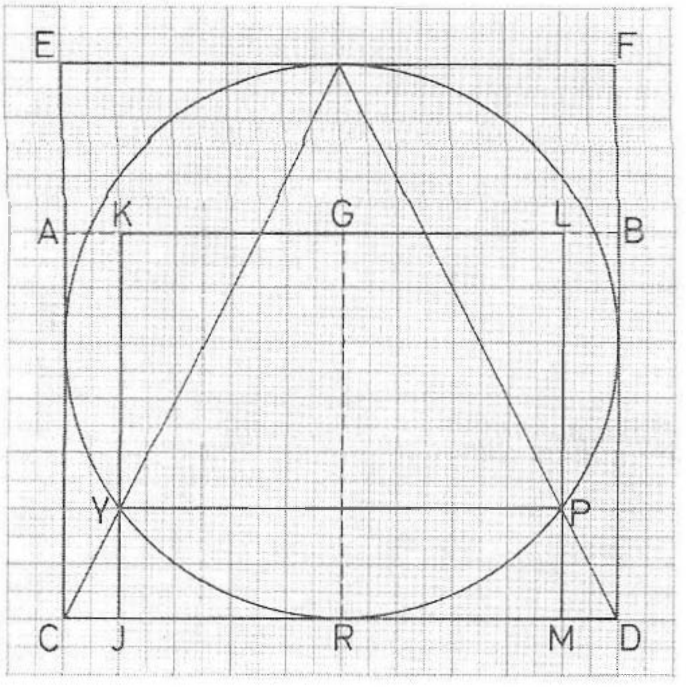


Fig. 97.

The acute-angled triangle and large circle have been entered in this diagram to demonstrate that what Moses has achieved by this very special stitching together of the curtains is in fact the lines of the circle's rectangle in the main square.

We note how the outer edge of the curtain KJ intersects the side of the acute-angled triangle at the same point as the circumference of the circle and we remember from earlier experience that this is precisely the point through which one side of the circle's rectangle passes. The same is true of line LM on the opposite side of the diagram.

If we now enter a line from G to Y and from G to P we produce the same symbol we saw in Fig. 51 in which we arrived at a triangle which symbolised the circle. The same triangle we saw used as a factor in planning the exterior of the Great Pyramid.

Thus Moses with this construction achieves not only the circle's rectangle within his main square, but also indicates all three points of his symbolic triangle.

In so doing I think Moses has demonstrated that he is a true master of geometry for this is really an ingenious means of illustrating such a baffling geometric factor, with only two curtains placed in one particular manner. Stitch them together in a certain way and, assisted by geometric knowledge, you have the solution to a problem that has been the subject of thousands of years of research.

Demonstrated in this manner it would be easy for Moses to show his disciples, with a little training, how to remember the procedure for themselves in preparation for the day the master was no longer in their midst.

Apart from the strictly geometric picture, it is also possible that Moses appreciated the not unimportant sphere of sacred rites and symbolism.

It is acknowledged that Moses was trained in the temples of Egypt and, if we accept the theory that the Great Pyramid was not in fact a burial place for King Cheops but instead a part of the temple complex reserved for initiation of the higher degrees, then we may also reasonably assume that Moses had been associated with this pyramid, too.

If we now turn our study to Moses' text concerning the actual hanging of the curtain in the tabernacle, the gold hooks (taches) and stitching hang across the rectangular temple building, and appear to divide it in two halves.

Moses calls this line the division between the holy and the most holy.

It was in the most holy that the Ark of the Covenant was placed.

If we look once more at Fig. 97, we see that the stitching and gold hooks are on line GR and thus identical to the central axis of the pyramid-shaped triangle, which has not been entered but which would be formed by lines GY, GP and YP.

A glance back at *Fig. 81* which shows a section of the Great Pyramid, reveals that the so-called Queen's Chamber is situated exactly on the Pyramid's vertical axis, while both the King's Chamber and the Subterranean Chamber lie to the left of the axis, and the entrance and passageways are on the right.

If in fact the Pyramid was used for the purpose I have outlined, the vertical axis formed a dividing line to the left of which all the important rites were conducted, while the less important part of the building lay on the right of the axis.

Here again a dividing line between the holy and the most holy.

Precisely the same line Moses uses in his tabernacle.

Although the pointed shape of the Pyramid and Moses' rectangular temple of fabric seem to have nothing in common, they resemble each other nevertheless to this extent: both symbolise the same vital factor in ancient geometry, namely the symbol of perfection, the circle.

Since this is indeed the most eminent of all the system's many facets, Moses throws it into relief by making the curtains which form the areas in question of the finest fabric, and the joints of the curtains of gold hooks.

Similarly, the next curtain—which does not reveal knowledge of the same degree of importance—is made from goats' hair and assembled with mere bronze hooks.

For the text goes on:

> "And thou shalt make curtains of goats' hair to be a tent covering the tabernacle: eleven curtains shalt thou make. The length of one curtain shall be thirty cubits, and the width four cubits: and the eleven curtains shall all be of one measure. And thou shalt couple five curtains by themselves, and six by themselves, and shalt lay the sixth curtain double over the front side of the tent. And thou shalt make fifty loops on the edge of one of the last curtains at its joining place, and fifty loops on the edge of the last curtain at the other joining place. And thou shalt make fifty bronze hooks, and put them into the loops, and couple the tent together that it may be one. And the remnant that remains of the curtains of the tent, the half curtain that remains, shall hang over the back of the tabernacle. And the remaining cubit on each of the long sides of the tent curtains shall hang over the sides of the tabernacle to cover it. And thou shalt make a covering for the tent of rams' skins dyed red and above this another covering of badgers' skins."

The principal point in this text is the important fact concerning the practical form of the building, i.e. that the new curtain is to be used as a cover for the inner tabernacle.

Since this is expressly laid down here—as opposed to the preceding curtain for which no instructions were given about its use—the conclusion must be that the preceding curtain of wool must be used inside the temple, presumably to line the ceiling and walls.

This is to a certain extent in keeping with esoteric tradition, since the information revealed by the inner curtain was intended for a high degree of initiation, indicating as it did how to construct the circle's rectangle, i.e. a means of calculating the area of the sacred circle.

But let us turn now to the new curtain, the one of goats' hair, to see what its area can tell us. We are presumably right in assuming that it conceals information of some sort—otherwise there would be little point in mentioning exact dimensions.

The text we are quoting of course mentions other curtains or covers, in addition to the two main curtains, but the sizes of the others are not indicated at all:

> "And thou shalt make a covering for the tent of rams' skins dyed red and above this another covering of badgers' skins."

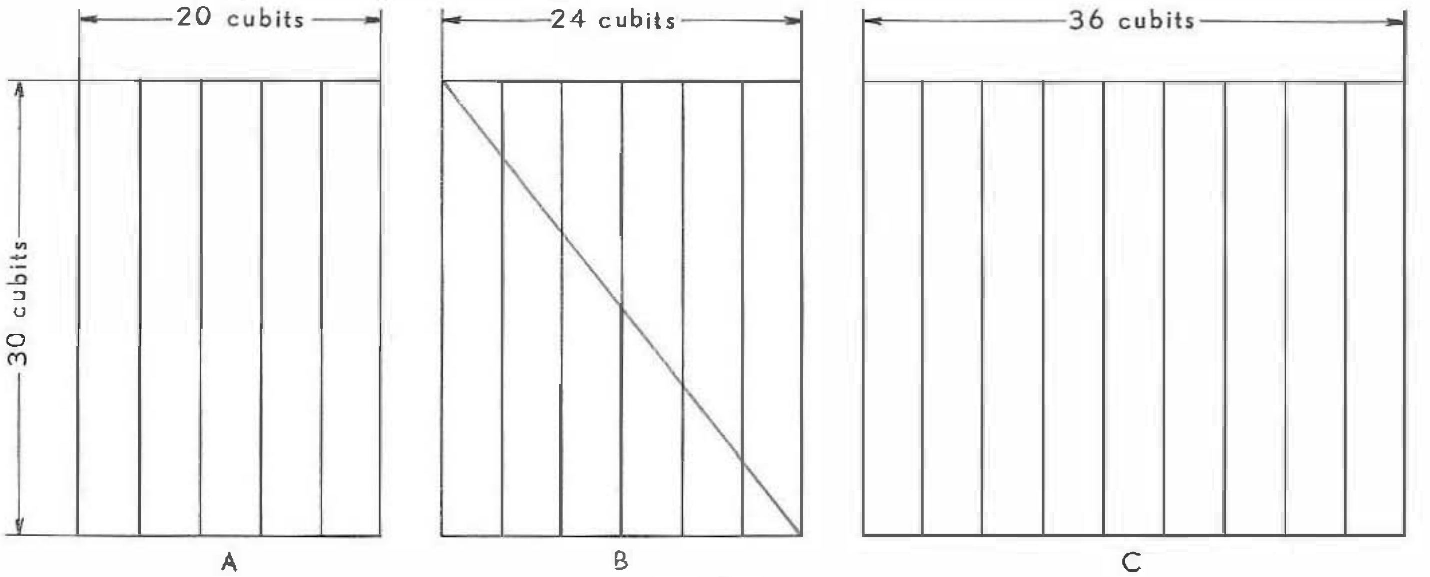


Fig. 98.

Note the significant difference. The passage mentions four curtains all of which appear to be of equal importance in the construction of the building. Two of them are gone over in detail, with precise dimensions and a peculiar mode of manufacture, the curtains changing continually from one size to another. But of the two remaining curtains no dimensions are given whatever.

Why should Moses go to all this bother if there was no particular meaning in the whole thing? There must have been a meaning. Moses knew well what he was doing when he planned the temple according to his geometric training—true to tradition.

If we now examine the two new curtains and the combination of these two in our search for information, we come to the curtain with the six strips.

This curtain is 24 × 30, and if we draw a diagonal through it we discover a familiar factor: the diagonal reproduces the angle of pitch of the side of the Great Pyramid.

We find this shown geometrically in *Fig. 98.*

But what about the other two areas: the curtain with the five strips and the combined curtain. It would appear to be difficult to get these to fit into the diagram and to understand the area properly.

We saw how one of the curtains, i.e. the one with the six strips, indicated the pitch of the Great Pyramid by its diagonal, which might lead one to imagine that in his symbolism Moses tried with the outer curtain to achieve the outer appearance of the Pyramid. But if we are to advance along this line of thinking we must examine the manner in which the curtain is used in the tabernacle.

We have some indication of this in the piece which reads:

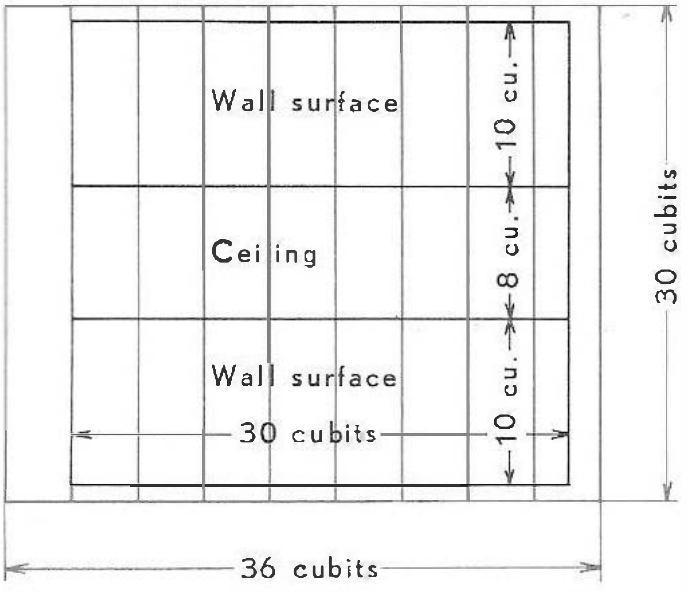
> "And the remnant that remains of the curtains of the tent, the half curtain that remains, shall hang over the back of the tabernacle. And the remaining cubit on each of the long sides of the tent curtains shall hang over the sides of the tabernacle to cover it."

And we read earlier in the passage:

> "Thou ... shalt lay the sixth curtain double over the front side of the tent."

In other words, over the entrance.

This then is a curtain which is not perfectly suited to its apparent purpose, being both too wide and too long. The text



Position of the second curtain on the portable tabernacle

Fig. 99.

states that one of the curtains, i.e. one of the nine strips that make up the complete goats' hair carpet, must be laid double over the entrance, and that half of the last or ninth strip of curtain must hang loose over the back.

Similarly the width of the curtain is two cubits too much for the job it has to do, the instructions being that the remaining cubit at each side must hang over the sides, and in fact lie in folds along the ground.

We can see this arrangement by considering the dimensions of the building. The text says of these later (and we shall be turning to them in detail shortly) that it shall have a length of 30 cubits, height of 10 cubits and a width of 8 cubits.

In *Fig. 99* we have the building laid out flat, covered by the curtain and in *Fig. 100* a diagram showing the placing of the curtain.

Our curtain measures 36 cubits by 30 cubits and as the building is 30 cubits long we are at first tempted to assume that the strips of curtain should lie along the length of the building parallel with its sides, but this does not meet the requirements of the text as we would then be without the strip of curtain to double over the entrance and the half strip to hang down the back.

Furthermore, there can be no alternative to laying the curtain with its strips across the building, since the instructions expressly state that one single strip is doubled at the front and a half strip hangs at the back, whereas as far as the sides are concerned only one measurement is indicated, namely 1 cubit on each side. This pinpoints the fact that at the sides one cannot pick out any particular strip of curtain since they all of course hang down on each side and thus can be indicated by only one measure.

Thus across the strips of curtain we have a length of 36 cubits. The building is 30 cubits long, and the instructions say that one strip must be doubled over the

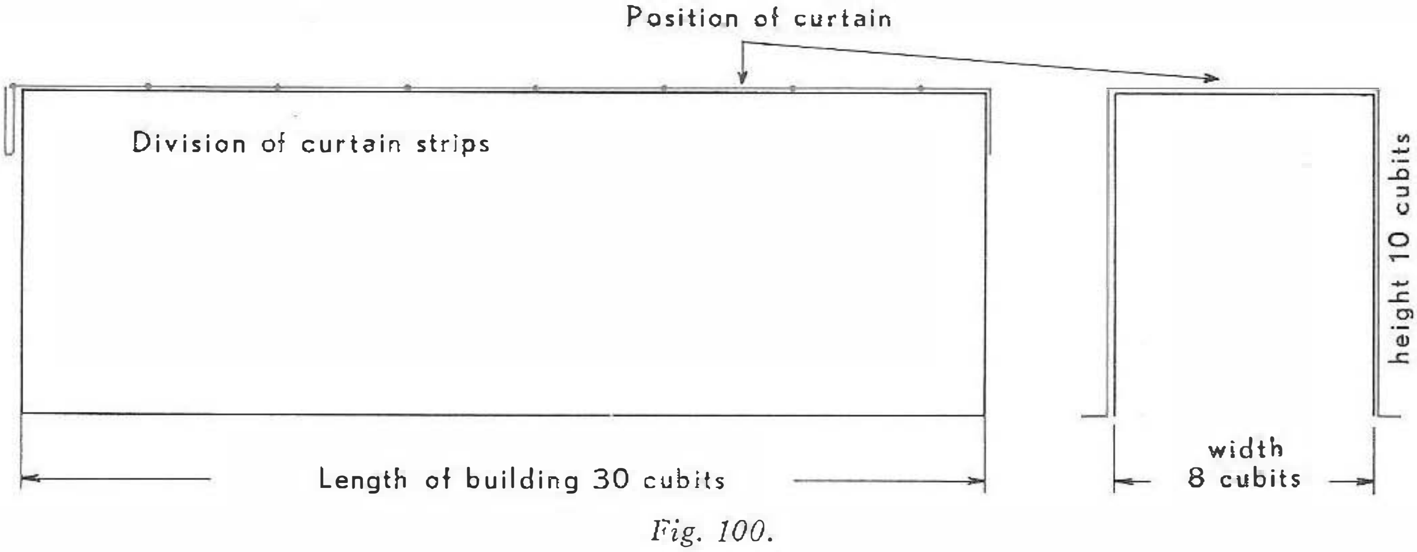


Fig. 100.

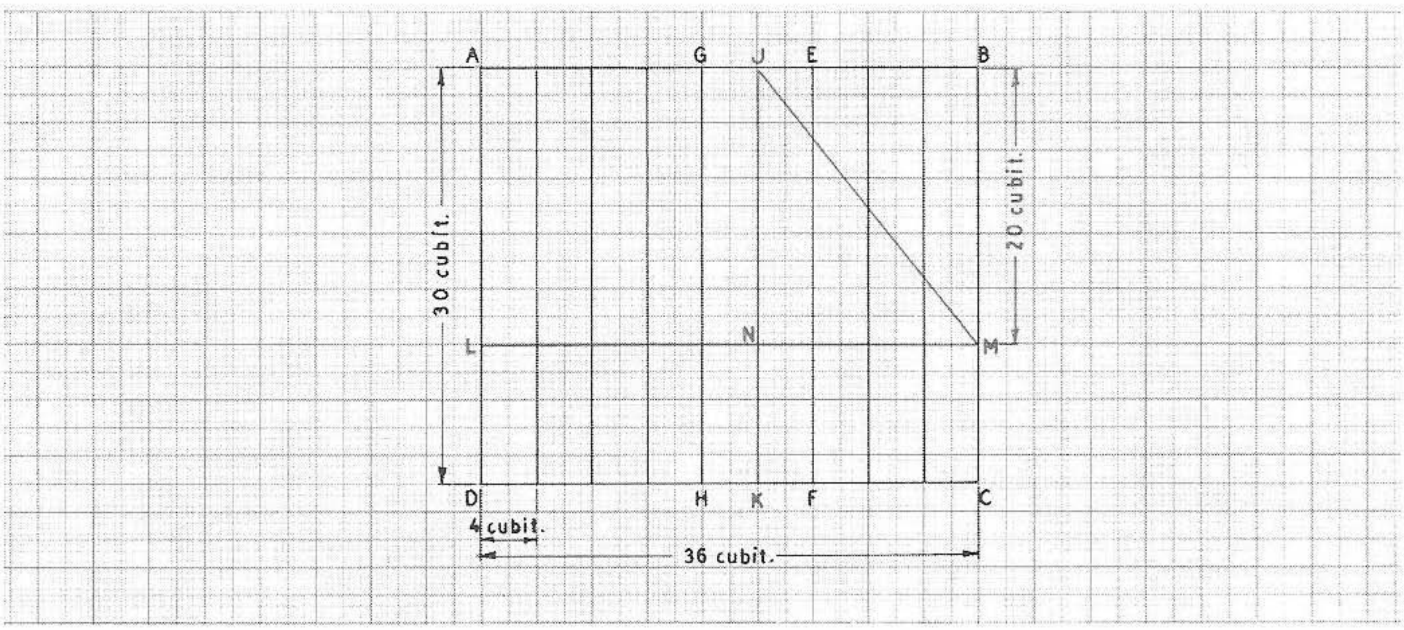


Fig. 101.

front and a half strip must hang over the back.

We will recall that the width of each strip is four cubits, which means that we use four cubits more than the building's length at the front and two cubits at the back. Thus the measurements fit this instruction, since the building is 30 cubits to which is added 4 cubits at the front and 2 at the back, making a total of 36 cubits.

The width of the material (being the length of the individual strips) is 30 cubits and it is said of the building that the height is 10 cubits and the width 8. We have therefore to cover two walls and a horizontal roof with the width of this curtain, a total distance of 28 cubits. This leaves two cubits over, and on this point the passage merely says that these should be allowed to hang down the sides of the building, one cubit on each side.

After that examination there should be no doubt about the manner in which this goats' hair curtain is used in the building —and at the same time we have established that the curtain, both in length and width, is larger than strictly necessary for the tabernacle.

If we now look at *Fig. 101* we see the finished curtain as area ABCD. It is stitched together in the same way as the preceding (see Fig. 96) with one strip of curtain on each half doubled at the line of joining. Thus a curtain of 6 + 5 strips is transformed into a total length of nine strips.

From a strictly symbolic standpoint the 6-strip curtain is on the left at AEFD, and the 5-strip curtain on the right GBCH, with the seam at JK.

Line AD is thus 30 cubits long and line DC 36 cubits.

We recall from the study just completed that this curtain was to be placed with the nine strips transversely across the building, i.e. the 36 cubits running along the length of the tabernacle.

If Moses' temple had been shaped like an inverted "V" with sloping sides, with the spine running down the centre of the curtain and lines AB and DC lying on the ground on each side, the curtain would have been used to indicate the tent's height. But such was not the shape. Only the two vertical sides give the building its elevation. The eight cubits of width across the roof and the two excess cubits (one on each side) do not affect the height.

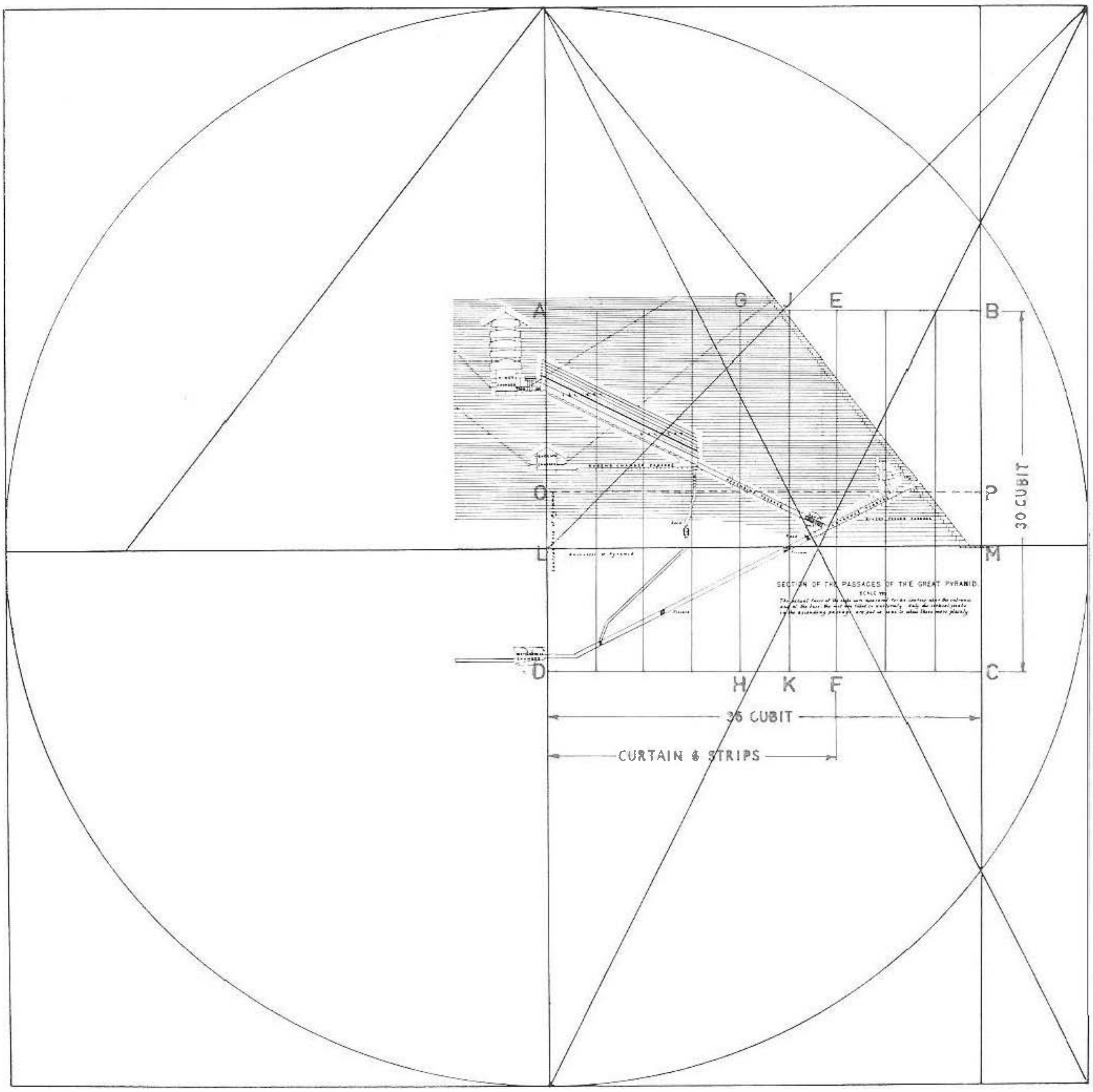


Fig. 102.

In Fig. 101 we enter the geometric construction that illustrates which section of the curtain forms the actual height. The division is line LM.

Line AL = 2 × 10 = 20 cubits.
Line LD = 8 + 1 + 1 = 10 cubits.

The curtain is now divided into two rectangles across the "grain" of its component strips, rectangles ABML and LMCD. Both are of course split by the seam JK.

If we study rectangle ABML we can probably recognise an arrangement seen earlier in Figs. 43 and 44, i.e. the square and the circle's rectangle, the latter being removed from the square and placed alongside it. The stitching marks the junction of the two areas, square AJNL and rectangle JBMN.

If we now enter the diagonal JM we arrive at the same angle as the sloping side of the Great Pyramid of Cheops. The curtain symbolises the secret of the circle's rectangle, and whereas in the preceding inner curtain we discovered the direct symbol for the circle, we have here in the outer curtain considerably inferior knowledge, the circle's rectangle. This fits very well the symbolic reasoning behind the placing of the two curtains: the greater the wisdom the more it has to be concealed.

So far with this curtain we have applied only a part of the combined area to discover the circle's rectangle. It is possible that Moses vested further symbolism in this curtain. Let us speculate.

He had received his training at the very heart of the Egyptian Temple movement and naturally was strongly influenced by Egyptian thought. The area of this outer curtain might well indicate that Moses was trying to reproduce certain dimensions of his shining image, namely the Great Pyramid of Cheops, the centre of inner initiation. The reason for this assumption can be seen in *Fig. 102* where we see the curtain drawn in its appropriate symbolic context.

The complete curtain has been sketched into symbol "Q" which, as we saw earlier, indicated the bulk of the Pyramid's main dimensions.

Take the horizontal distance from the Pyramid's vertical axis to the outer extreme of the building at its base, i.e. to the line coinciding with BC which forms the side of the circle's rectangle in the large symbol "Q". Imagine that distance to be 36 cubits and identical with line LM in Fig. 101. It is now divided into 9 strips each of 4 cubits. On the vertical axis of the Pyramid we measure 20 cubits upwards and 10 cubits downwards from ground level. Our diagram can now be entered in correct proportion.

If we consider rectangle LABM we see the same situation as described above with the square and rectangle, divided by seam JN. A most remarkable fact is that line LA reaches exactly from the base to a point level with the upper limit of the King's Chamber.

If we look now at the portion of the curtain not previously "required" symbolically, i.e. rectangle LMCD, we note that it stretches over those passages and chambers which lie under the base-line of the Pyramid, and we see that DC equals precisely the distance from the base of the Pyramid to the bottom of the Subterranean Chamber.

Naturally the curtain reaches exactly from the Pyramid's vertical axis to its outside base; we proportioned it so. But the fact that the curtain upwards from LM ends level with the upper limit of the King's Chamber and downwards marks the lowest point of the underground grotto can scarcely be attributed to coincidence. That is how Moses planned it.

If we count six curtain strips from the right, we come to the start of the Grand Gallery and the almost vertical drop of the Well.

Six strips from the left bring us to the point on the Descending Passage floor where the Ascending Passage begins.

As mentioned earlier, the curtain with its nine strips was placed transversely across the tabernacle, i.e. as you entered, the individual strips were at right-angles to your path and you were at their middle point. If we enter this centre-line in Fig. 102 as OP we see that it cuts through the Pyramid entrance. Here again Moses has hit the symbolic nail on the head.

Thus this particular placing of the curtain upon a side elevation drawing of the Great Pyramid reveals several interesting resemblances. These may by some be dismissed as coincidence—but mere chance could not have accounted for them. The subsequent study of the tabernacle and its fittings will, I think, prove that whenever precise figures are mentioned in Moses' text they fit perfectly into the framework of ancient geometry. Almost every geometric area discussed so far is described and constructed in detail in the dimensions of the temple and fittings. I really believe we can rule out coincidence.

Moses fully appreciated the art of converting complicated symbols into simple, comprehensible figures. With this material he was able to pass on some of his mathematical knowledge to his close followers—who boasted no previous experience at all.

His aim was to create a circle of initiated brethren in a relatively short time to provide the Jews with a core of able leaders. For of all the mass of people who wandered with him in the wilderness of Sinai Moses alone was educated in the mysteries of geometry and mathematics. He knew that knowledge was power, and he wanted to share this power in as simple a manner as possible with those among the Jews whom he found worthy.

The text continues with instructions for the actual supporting structure of the tabernacle:

> "And thou shalt make boards for the tabernacle of shittim wood standing upright. Each board shall be ten cubits high and a cubit and a half wide. Two tenons shall there be in each board, set in order one against the other: thus shalt thou make for all the boards of the tabernacle. And thou shalt make the boards for the tabernacle, twenty boards on the south side. And thou shalt make forty sockets of silver under the twenty boards, two sockets under one board for its two tenons. And on the north side of the tabernacle there shall be twenty boards: and their forty sockets of silver: two sockets under each board. And for the sides of the tabernacle on the west thou shalt make six boards. (But for the corners of the tabernacle at the back thou shalt make two boards. And they shall be coupled together twin-like at the bottom ... unto the first ring: thus shall it be for them both, they shall be for the two corners. Thus for the back there shall be eight boards, and their sixteen sockets of silver, two sockets under each board.)"

The latter piece of this passage (in brackets) concerning the end and the corners must have presented a bit of a headache to the translator since some words would appear to be missing, or else the original text included phrases which defied translation.

In any event, eight boards and sixteen sockets are needed for the back, two silver sockets under each board.

> "And thou shalt make crossbars of shittim wood; five for the boards of the one side of the tabernacle, and five for the boards of the other side, and five for the westward side of the tabernacle at the back. And the middle crossbar in the midst of the boards shall reach from end to end (of the wall). And thou shalt overlay the boards with gold, and make the ring (for the bars) of silver: and thou shalt overlay the crossbars with gold. And thou shalt erect the tabernacle according to the fashion thereof which was shown thee in the mount."

This passage finally gives some indication and concrete information of the actual structure of the building—and "building" it most certainly is. The boards are after all massive lumps of timber.

We are told today that the ancient Egyptian royal cubit corresponded roughly to the modern measure of 523 mm (20.6 inches). If we use this measure as the basis for a mental reconstruction of Moses' temple we arrive at a height of 5230 mm (17 ft); since each plank is 1.5 cubits wide and there are 20 of them we find the total length to be 20 × 784.5 mm (or 31 ins) = 1569.00 cm or 51.75 ft. Because the respective units of cubit, millimeter and

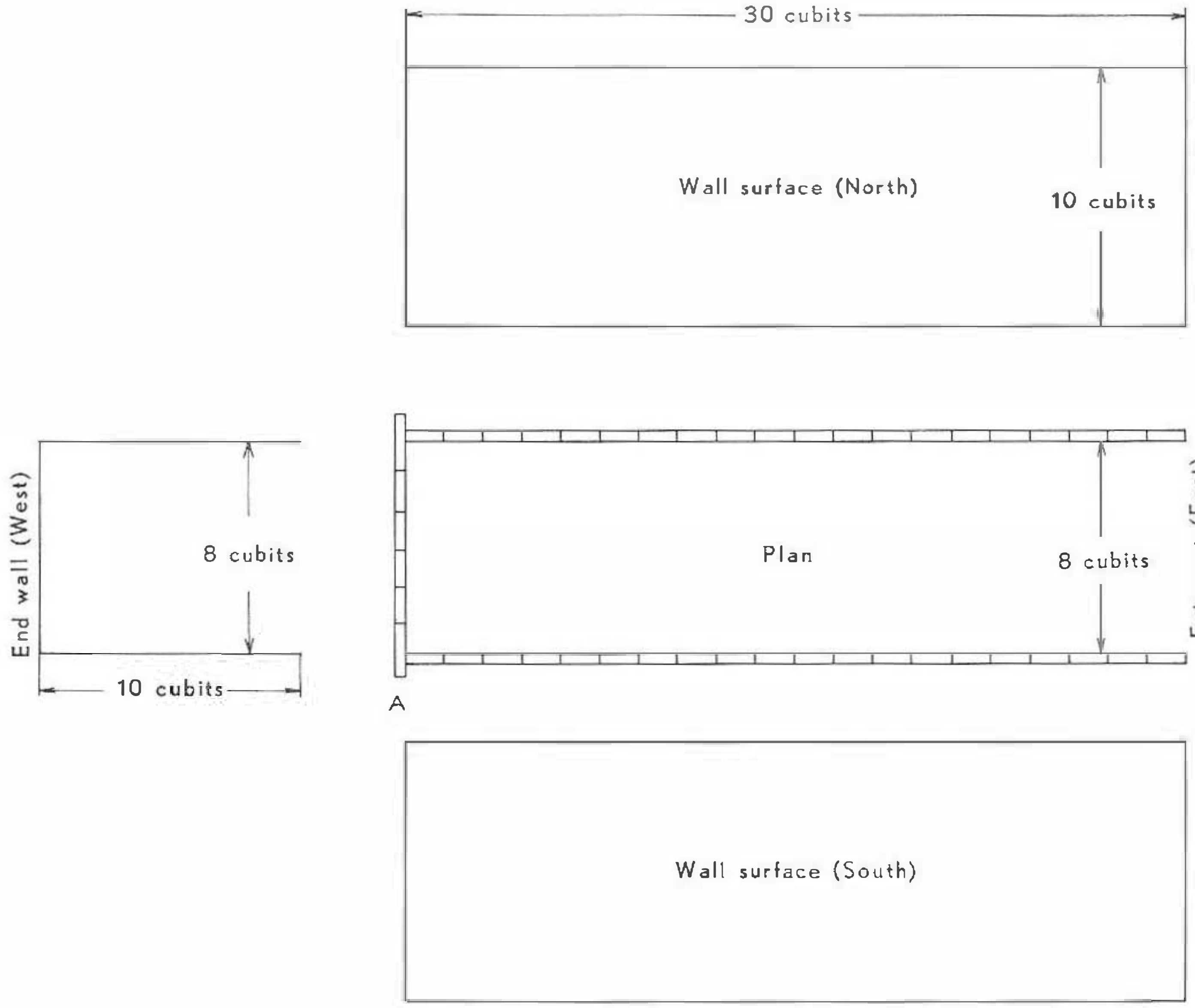


Fig. 103.

inch do not correspond exactly the above dimensions are approximate.

To continue: twenty of these heavy beams are to be erected for each side wall, and six at the end for the west wall.

The mention in one portion of the text of six boards for the west end-wall and later, as a kind of résumé, mention of eight boards for the back is perfectly straightforward. The intention is that six boards are to be made for this wall and instructions are given to make the corner beams "twin-like". In modern terminology we should probably call them T-shaped, the two end boards of each of the long sides forming a "T" with the corner beams at the back wall.

It had perhaps been difficult to express this formation in the text, and the writer therefore simply left out any mention of the last board on the side-walls and chose to explain the construction by giving two separate orders.

There are also instructions for giving each board two bases or sockets of silver, which is naturally an essential for a building of this size—not that they need be of silver but that there must be a foundation to hold the boards in place.

The foundation was possibly a means of securing the individual board by giving it a wider bearing surface against the ground (probably loose desert sand) on which the boards were to rest. It was possibly also intended as a device to minimise any outward or inward leaning of the walls. Here again we come across a typical "failing" in the text where something of vital importance to the actual structure is glossed over quickly, while other seemingly trivial factors are described in full.

We saw how the height and width of the boards were given, while the third dimension, the thickness, was omitted completely.

The thickness of the boards, the foundation, the two outer coverings of rams' and badgers' skins have no symbolic or geometric significance, and it is left to the discretion of the workmen to calculate sizes for these. They are mentioned only in the passing—while those areas of consequence in geometry and esoteric tradition are given comprehensive coverage.

Despite its proportions the complete building is, as we have seen, nevertheless a portable arrangement inasmuch as it can be dismantled and moved from place to place in the course of the wanderings of the Israelites. And since this tallies with the historic account of their nomadic movements, it would be worthwhile to examine for a moment this detail of the text, namely the erection of the tabernacle.

The outer dimensions of the building are laid out in their respective areas in *Fig. 103* where, by referring to existing information, we are able to draw these outer proportions exactly.

The sides. Of these we read that both the north and the south sides comprise 20 boards and that each board is $1\frac{1}{2}$ cubits wide, providing a total length for the building of 30 cubits—assuming the boards stand shoulder to shoulder with no gaps.

The transverse wall. The west wall, we are told, consists of six boards plus the corner board and one is tempted to imagine the width of the temple to be $6 \times 1.5 = 9$ cubits.

The curtains, however, assist us on this point since we recall that the length of the individual strips of material (and therefore the width of the curtain) was 28 cubits.

If we make the interior dimensions of the temple a height of 10 cubits and a width across the ceiling of 9, this makes a total of 29 cubits, and the curtain would not therefore be able to cover both sides. We are thus encouraged to asume that the sides of the temple were brought slightly in from the ends of the transverse (or west wall) making a total internal width of 8 cubits.

We see this adjustment at "A" in Fig. 103.

Thus the *inner* curtain fits perfectly, and as far as the *outer* curtain of goats' hair is concerned we recall that there was a cubit extra on each side of the building which was meant simply to hang loosely down the outside. The adjustment described above does not therefore matter, nor does the thickness of the boards.

With regard to the geometric aspect of these basic dimensions, the sides contain the scholar's early encounter with geometric knowledge: namely the doubling of the square and the sacred cut.

The proportions of the side of the building have been based on any given square, in this case a square of 10×10 cubits.

This square is laid out and repeated the number of times corresponding to the information the designer is prepared to pass on in his measurements, suited to a certain extent to practical requirements.

In this instance three squares have been laid side by side, as we see in *Fig. 104.* Squares A, B and C.

We see also how the diagonal of square A has been entered, enabling us to construct square A's double-sized relative 1-2-4-3.

We may then notice that the side of the new square marked by 3-4 runs exactly

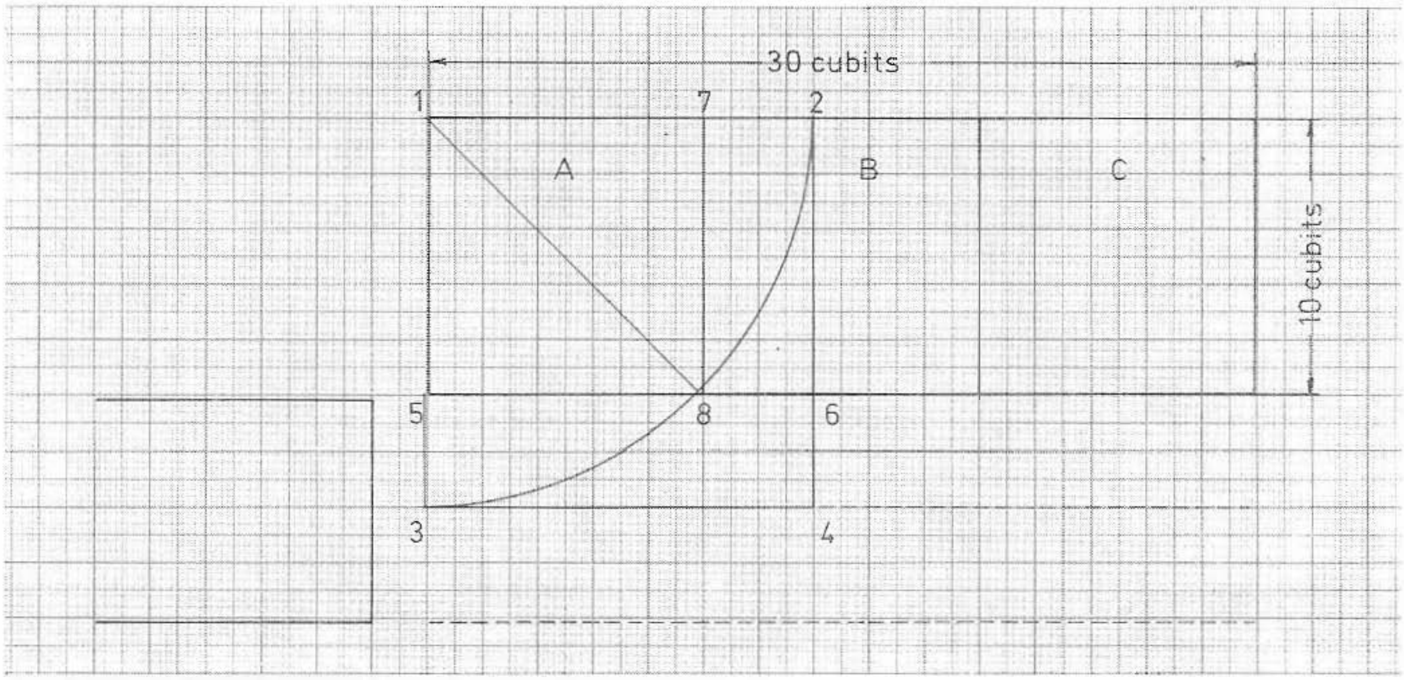


Fig. 104.

through the middle of the ceiling lengthwise. Thus we have the main dimensions, 1-7-8-5 being the original square and 1-2-4-3 being the new square. Rectangle 4-3-5-6 forms the folded-over part of the curtain from the wall to the centre of the ceiling. This is evident from the diagram, which also shows the entrance drawn in the appropriate place.

If we now try to trace the path Moses followed in his planning of the structure, we naturally require a point of origin from which to establish the other measurements.

In *Fig. 105* we find Moses' original or basic diagram which gave rise to all the other structural dimensions in his tabernacle. We have here a large square of 30 × 30 cubits divided into nine small squares, each 10 × 10 cubits. The side-length of these smaller squares (10 cubits) is fixed as the height of the tabernacle, the two wall areas being indicated in the diagram by points AB and EF.

The double-size square is then entered from each side, producing points C and D.

In deciding the dimensions of the building, it is agreed that one of the sides shall be equal to AB, half the width of the ceiling BC, the other half ceiling DE and the height of the other side EF.

Thus we arrive at

AB (height of first side)	10 cubits
BC (half width of ceiling)	4 cubits
DE (half width of ceiling)	4 cubits
EF (height of other side)	10 cubits
	28 cubits

Area CHJD is omitted from the construction at this point, providing a total width of 28 cubits, being 2 × 10 for the walls and 1 × 8 for the ceiling.

The actual length of the building remains at 30 cubits. Thus by starting with a particular height for our building we have a point of origin from which to work. This in turn provides a structural diagram which on one hand fixes the width of the first curtain at 28 cubits, and moreover represents one of the most elementary constructions in geometry, doubling the size of a square.

We see how each factor has its rightful place and significance and how the different bits and pieces combine to make a whole, originating from the fixing of the tabernacle's height.

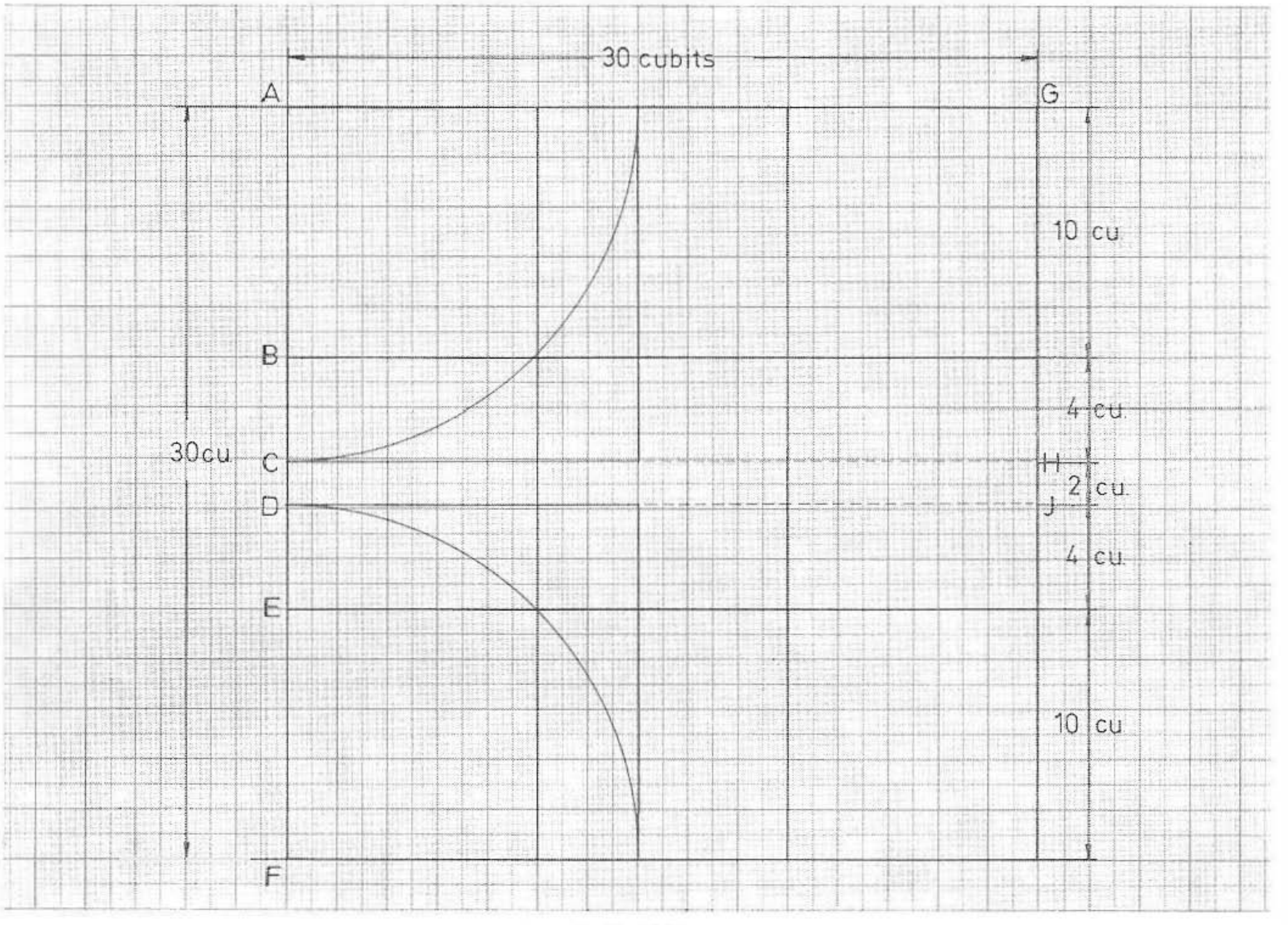


Fig. 105.

We have now gone over the main dimensions of the temple and have analysed the various areas mentioned in the text. We have seen how all the figures have a particular meaning and that this meaning is of a strictly geometric nature; and—most important of all—that all of the areas mentioned bear a mutual harmony one to the other through the intricacies of ancient geometry; and that every measurement is in fact derived from a common denominator, i.e. the predetermined height of the tabernacle.

In a later chapter, on classical temple design, we shall again meet two factors which have proved important in the design of Moses' tabernacle, i.e. *that* a length/width ratio of 3 : 1 (represented in this case by the trebling of a basic square in Fig. 105) was a common feature in temples planned according to ancient geometric principles, and *that* a temple's height was a natural source from which to proportion the remainder of the building.

But there are other factors in the text which have geometric and symbolic connotation, namely the various ceremonial instruments and tools.

The text begins by laying down how these should be made and later turns to the actual tabernacle.

My reason for leaving them to the end, for apparently altering the order of things, is simply that I considered it easiest to do so, and in any event the information revealed by the ceremonial accoutrements is not of such import as that concealed in the building itself. This is presumably why they are placed at the outset of the passage. But to the reader who, in his study of the foregoing chapters, now has at his fingertips the initiate's knowledge of ancient geometry it is immaterial where in the text we choose to start.

The passage begins:

"*Voluntary Offerings for the Tabernacle:* And the Lord spake unto Moses saying, Speak unto the Israelites that they bring me an offering: of every man that giveth it willingly with his heart ye shall take my offering. And this is the offering which ye shall take of them: Gold, silver, bronze, and blue, purple, scarlet and crimson wool, and fine linen, and goats' hair, and rams' skins dyed red, and badgers' skins, and shittim (acacia) wood, oil for the candlestick, spices for anointing oil, and for sweet incense, onyx stones, and precious stones to be set in the ephod and in the breastplate. And let them make me a santuary; that I may dwell among them. According to all that I shew thee after the pattern of the tabernacle and all the instruments thereof, even so shall ye make it."

Moses had to repeat what God told him on the mount for he was alone and there were no witnesses to relate the tale for him: thus everything must have come from Moses' lips.

The fact that he has to obtain the materials he requires for building purposes by voluntary contribution is of little consequence to our study; but when he says that God will dwell in the centre of the tabernacle this must be regarded as a ritualistic truth if one views a knowledge of numbers and counting as the pursuit of the Sons of God (Initiates); and by applying themselves to these subjects they will be close to God, and since they must enter the temple or tabernacle to receive initiation in the mysteries of mathematics, among other things, they will—in the temple—be near to God: which can be interpreted as meaning that God is present in the temple.

"*Ark of the Covenant:* And thou shalt make a chest of shittim wood: two cubits and a half shall be the length thereof, a cubit and a half the width, and a cubit and a half the height. And thou shalt overlay it within and without with pure gold, and shalt place about it an edging of gold. And thou shalt cast four rings of gold, and put them in the four feet thereof; and two rings shall be on each side of it. And thou shalt make staves of shittim wood, and overlay them with gold. And thou shalt put the staves into the rings by the sides of the ark, that the ark may thus be borne. The staves shall remain in the rings of the ark; they shall not be taken from it. And thou shalt put into the ark the testimony which I shall give thee. And thou shalt make a mercy seat of pure gold: two cubits and a half shall be the length thereof, and a cubit and a half the width. And thou shalt make two cherubims of gold of beaten work in the two ends of the mercy seat. And make one cherub on the one end, and the other cherub on the other end: even of the mercy seat shall ye make the cherubims on the two ends thereof. And the cherubims shall stretch forth their wings on high, covering the mercy seat with their wings, and their faces shall look one to another; towards the mercy seat shall the faces of the cherubims be. And thou shalt put the mercy seat upon the ark; and in the ark thou shalt put the testimony that I shall give thee. And there I will meet with thee, and I will commune with thee from above the mercy seat, from between the two cherubims which are upon the ark of the testimony, of all things which I will give thee in commandment unto the children of Israel."

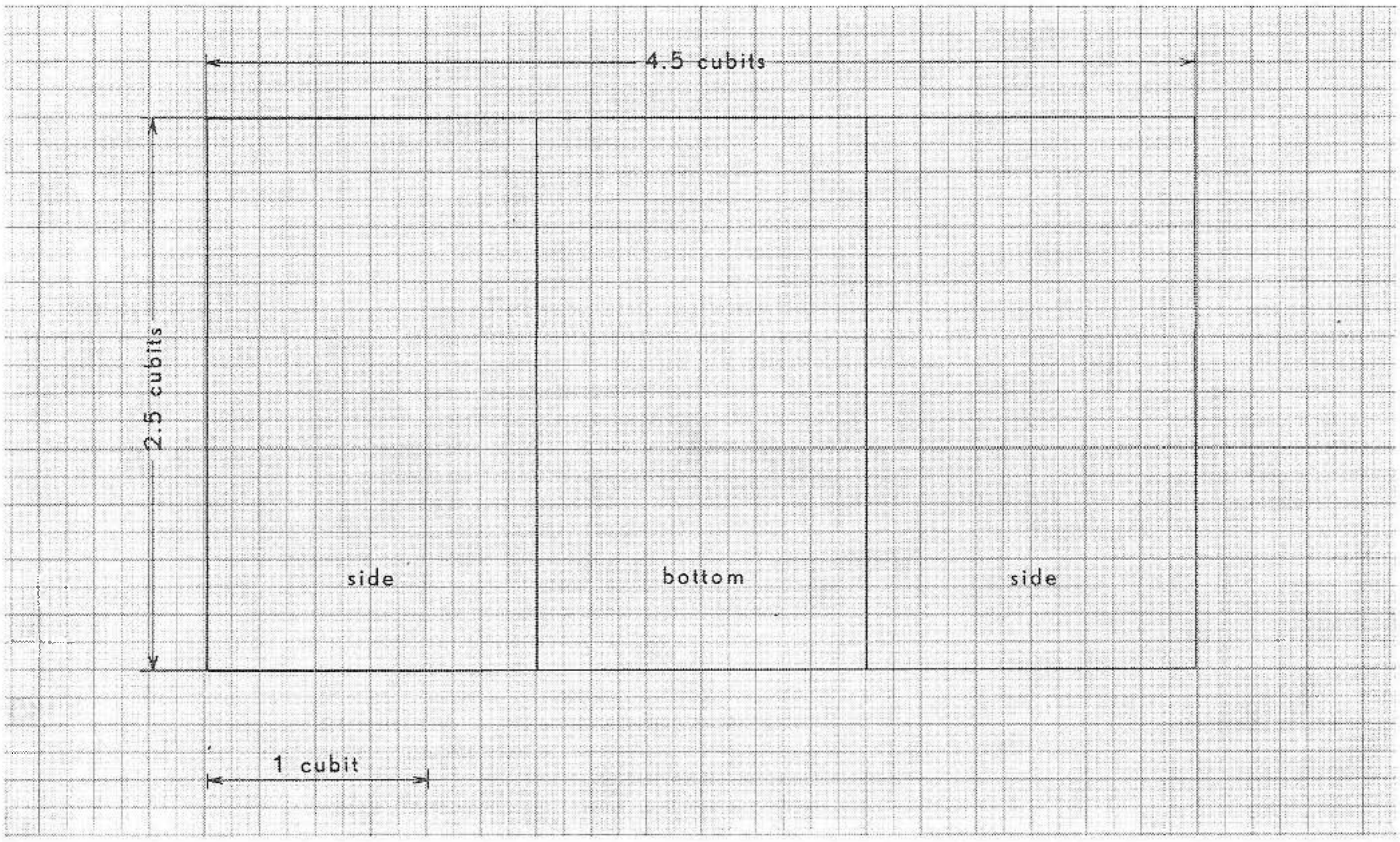


Fig. 106.

In reading the description of how the Ark of the Covenant must be constructed, one will notice that certain aspects are laid down in hard-and-fast figures, while others —equally important—are passed over with a minimum of comment.

This is a procedure we have met with several times in the course of this text: in the designation of rams' and badgers' skins for the outer covering, in the size of sockets for the wall boards, and in the description of the boards themselves where we received no instructions about thickness.

In this instance we are told only of the ark's outer dimensions, height, width and length—and nothing else.

The rings are merely mentioned in passing, no size is stipulated. It is stated that they must be affixed to the feet of the ark, but since there is otherwise no mention of feet in the construction there must either be an error in translation or an inaccurate interpretation of the text, the sides of the chest being termed as legs.

Had this been a building directive and only a building directive, it would have been essential to indicate the placing of the rings on the ark. Their size, too, would probably also have been mentioned, as would the thickness of the carrying staves.

But the intention behind the text was not in fact to act as a general code of building regulations, as detailed instructions for the carpenter or builder. The text was meant as a guide to ensure that certain—very special—ritualistic measurements were maintained in the general plan. The remaining details were left to the master builder or workmen who carried out the job—as in the construction of the framework of the temple.

In analysing the new set of measurements, we must first decide which parts are to be included in the study.

The chest, which was described as a complete unit, comprises in fact two sides, a bottom and two end pieces.

We saw in the building of the tabernacle how the end pieces or gables con-

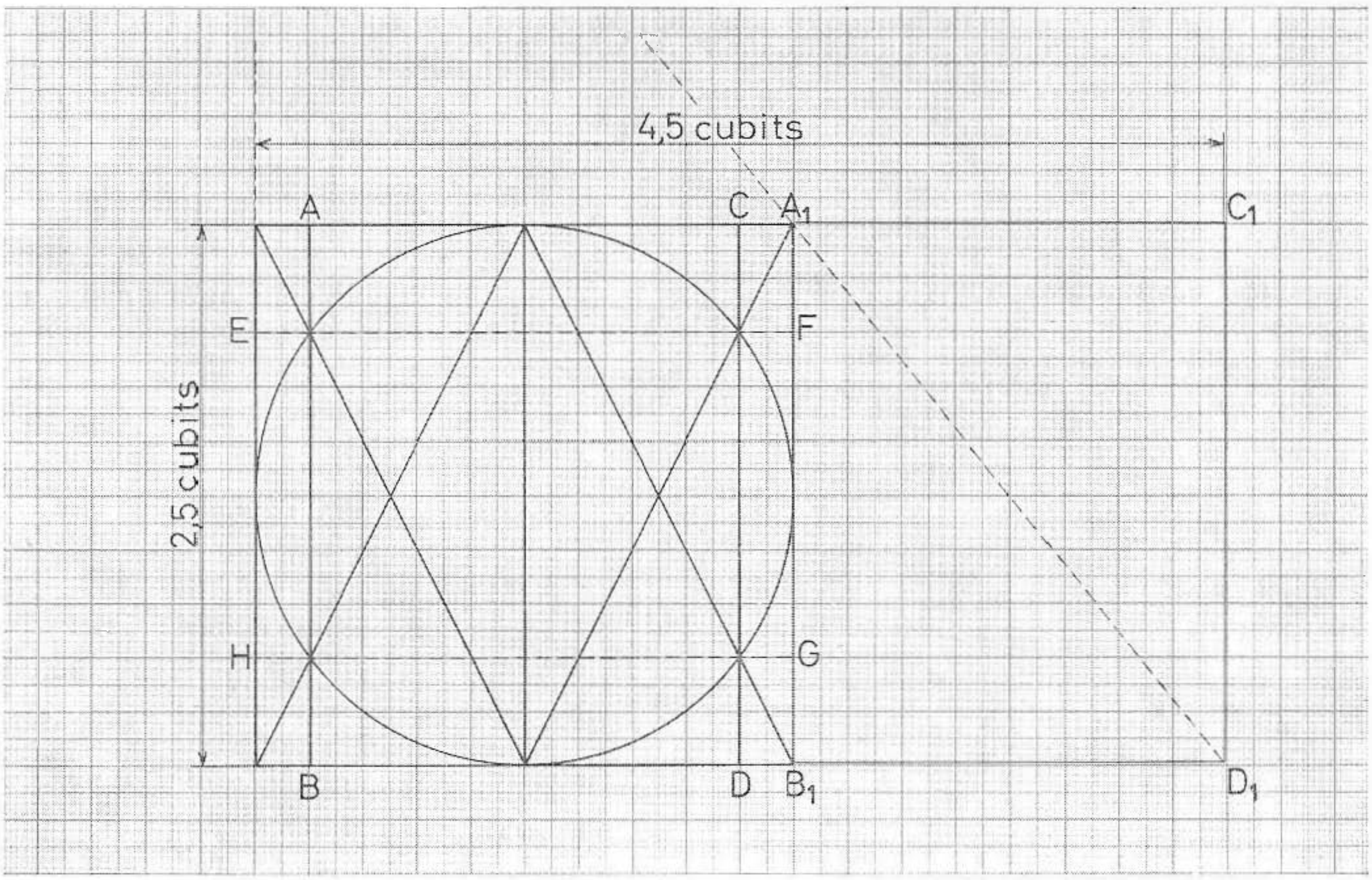


Fig. 107.

tained no apparent significance in relation to the other areas of the building, and in the case of the ark therefore we shall work only with the sides and the bottom.

In *Fig. 106* we have laid these out flat, obtaining an area measuring 4.5 cubits long by 2.5 cubits wide.

The lid or mercy seat which has to be placed on top of the chest is described separately and must be included in the analysis as a separate area. It is the same size as the bottom, i.e. 2.5 cubits long and 1.5 cubits wide.

In Fig. 106 therefore we have a rectangle measuring 4.5 cubits by 2.5 cubits.

In *Fig. 107* we are able to appreciate the geometric significance of this area, having drawn a square containing symbol "R" (i.e. the circle, the two acute-angled triangles and the circle's rectangle).

The circle's rectangle, marked by AB CD, is removed from the diagram and placed alongside the square, thus providing exactly the area occupied by the chest.

Thus in the ark we find the same information concealed as we had in the goats' hair curtain of the tabernacle itself, but in this instance revealed in a simpler manner.

If we look again at Fig. 107 we notice that the circle's rectangle is drawn vertically into our square, and that these two lines are indicated by the intersections of the acute-angled triangles and the circle's circumference.

These same points of intersection can also be joined horizontally, as shown in Fig. 71. The horizontals are EF and HG.

When we examine rectangle EFGH we notice that, if the side of the square is 2.5 cubits, this area measures 2.5 × 1.5 cubits.

This area is identical to the mercy seat placed above the Ark of the Covenant.

It was therefore these two intersections,

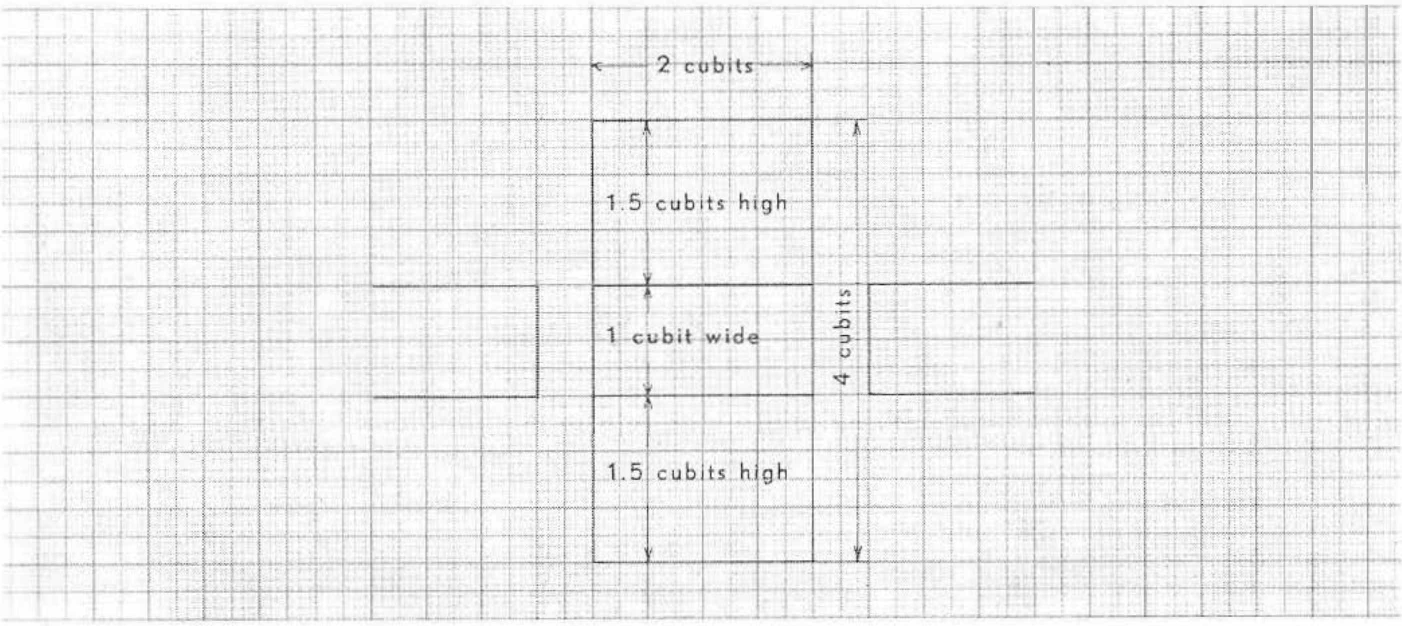


Fig. 108.

the vertical and horizontal divisions of symbol "R", that decided the dimensions of the chest which, according to the story related by Moses, was intended as a depository for the testimony. By its apparently simple proportions the ark reveals some of the most sacred information in ancient geometry, namely directions on how to construct the circle's rectangle. It was not therefore surprising that it had to be placed well inside the temple, in the home of the Most Holy, behind the dividing curtain.

The text proceeds:

"But set the table outwith the veil, and the candlestick opposite the table by the south wall of the tabernacle: and thou shalt put the table by the north wall."

The candlestick does not then stand on the table but by the wall on the opposite side of the tabernacle, so that it can throw its light on the table. The text says of the table:

"*The Shewbread Table.* Thou shalt also make a table of shittim wood: two cubits shall be the length thereof and a cubit the width, and a cubit and a half the height. And thou shalt overlay it with pure gold, and make thereto an edging of gold. And thou shalt make a border of an hand-breadth round about its legs, and thou shalt make a golden edging to the border. And thou shalt make for it four rings of gold and put the rings in the four corners that are on the four feet. Over against the border shall the rings make places for the staves to bear the table. And thou shalt make the staves of shittim wood, and overlay them with gold, that the table may be borne with them. And thou shalt make the dishes and bowls and jars and jugs thereof of pure gold. And thou shalt set upon the table shewbread before me always."

If we consider the principle details of the above description, we find the only concrete figures mentioned are the dimensions of the table: 1.5 cubits high, 1 cubit wide, and 2 cubits long.

In our established manner, we examine these areas, starting with the table-top which is 1 cubit wide and 2 cubits long. This is a rectangle with its sides in the ratio 2 to 1.

We lay out the whole table (sides and top, omitting as usual the end pieces) and

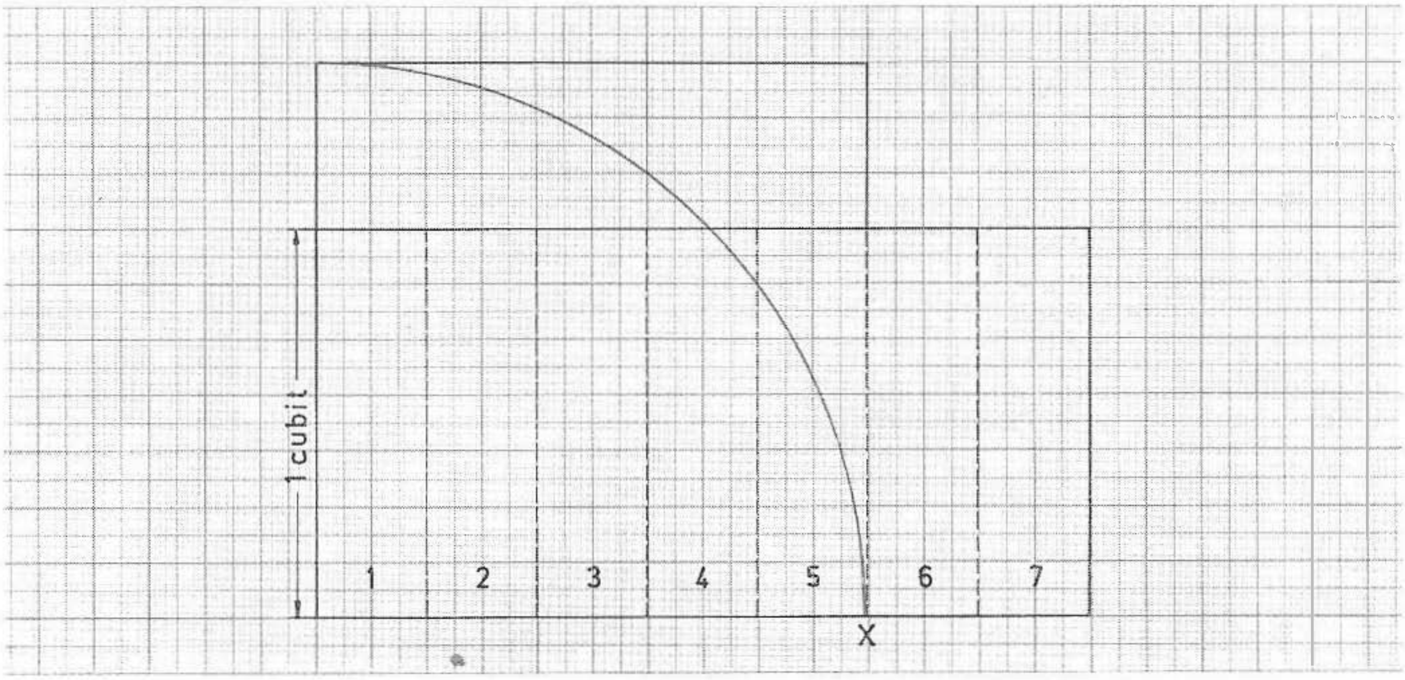


Fig. 109.

find in *Fig. 108* that we again have a rectangle in the ratio 2 to 1, the total length of the "flattened" table being 4 cubits and its width 2.

Here, in this table, we find one of the earliest and most elementary aspects of the geometry of those times, i.e. directions on the use of the sacred number 7 as a factor in division.

The symbolism, once observed, is perfectly plain: the seven-armed candlestick is standing on the ground by the south wall, throwing its revealing light upon the rectangle against the north wall.

Accepting this symbolism in its literal sense, we divide the rectangle into seven strips or parts, in *Fig. 109*.

At the edge of the fifth strip we have the sacred cut marked X, and by drawing the total width of these five strips vertically in our diagram we can construct the square on this rectangle. This reflection on the relationship of 5, 7 and 10 was of course one of the first successes Man registered in ancient geometry, and complete understanding of it was necessary in order to proceed. It is therefore a brilliant piece of symbolic planning that this should be one of the first things one sees on entering the temple, and thus one of the first lessons learned by the person sufficiently favoured to join the ranks of the temple brotherhood.

Notice the placing of the candlestick, not upon the table as an integral part of it, but by the opposite wall on the floor. Thus the figure 7 is presented as an individual and basic instrument, casting its light on the very essentials of geometry—highlighted in the dimensions of the shewbread table.

The details of the candlestick are given as follows:

> "And thou shalt make a candlestick of pure gold: of beaten work shall the candlestick be made: its base and stem shalt thou make, and its flowers with cups and corollae shall be parts of it. And six branches shall come out of the sides of it; three branches from the one side, and three branches out of the other side. On each branch of the candlestick there shall be three flowers resembling the almond-flower with cups and corollae. But on the stem of the candlestick shall be four flowers resembling the almond-flower with their cups and corollae. And there shall be a cup

> under each pair of branches of the six branches of the candlestick. Their cups and their branches shall be parts of it: so that it shall be one beaten work of pure gold. And thou shalt make seven lamps for it: and they shall place the lamps thereof that they may give light upon the other side."

As we have seen, the essence of symbolism in the candlestick rests in the tremendous emphasis on the figure 7 as an integral part of the understanding of the whole geometric system. The role of the candlestick itself is to draw attention, by the pure and simple symbolism of its seven-armed form, to the area occupied by the shewbread table and to indicate the division of this area into seven, thus revealing the geometric knowledge hidden in the very structure of the tabernacle itself; but the same candlestick may also have concealed yet another aspect of the occult.

We read that each of the three arms of the candlestick on either side of the stem should bear three almond-flowers, and that there should be four such flowers on the central stem or arm. This makes a total on the whole candlestick of 22 almond-flowers.

In Egypt the almond-flower was the symbol of wisdom. By decorating the candlestick with 22 of these emblems, Moses might well be hinting at the 22 years occupied by temple-initiation; an almond-flower for each complete year of knowledge. Thus we would find in the candlestick the elementary significance of the number 7, and the complete range of wisdom represented by the flowers.

Moses had undoubtedly introduced some form of graduated admittance of his temple brethren, but whether it was possible for him to have one year or so between each stage of training before advancing to a higher degree is less certain.

After all, he was out in the wilderness at the head of thousands of Israelites and was himself probably the only initiate in the Egyptian Mysteries. His main problem was to train suitable leaders as rapidly as possible—for without them he could not control such a mass of people.

There is scarcely any question that—in keeping with ancient tradition—the material he used in training these men lay in the structure of the temple. From the candlestick to the shewbread table, from the dimensions of the various curtains to the size of the Ark of the Covenant, not to forget the planning of the structure itself, he presented the key to the mysteries of ancient geometry. It was a question of applying the key in the proper manner in order to unlock, one by one, the various doors.

The beauty of the Great Pyramid of Egypt lies in the fact that its dimensions hold the essentials of esoteric ancient geometry.

When Moses therefore attempts to achieve certain goals in the proportions of his tabernacle, which are apparently related to those of the Pyramid, he is not so much trying to reproduce a symbolic image of that building as to pass on to the Jewish people the same geometric knowledge as contained in the Pyramid.

When Moses' temple was dismantled and packed for transport, the gold and bronze hooks in the curtains were removed and the curtains reassumed their original sizes, the basic areas. When the temple was erected at its new site, there was ample opportunity for giving the Chosen Few an object lesson in ancient geometry and the secrets of its component areas.

The Ancients who at one time captured Moses' story in print certainly were familiar with these secrets, for it would otherwise be totally inconceivable that all the sizes and descriptions should fit as neatly into the framework of geometry as in fact they do. These specifications are really ingeniously compiled. They unfold in stages information on geometric factors, using simple ideas, figures and measurements, which we in modern terminology would require to express by frequent use of fractions and mathematical formulae.

CHAPTER NINE

Egyptian Mathematics and the Influence of Ancient Geometry

WE SAW in the first part of our study how it was possible to work with geometry and to derive information from this work—with only a scanty appreciation of numbers and their application to counting.

Indeed such knowledge need only initially extend to a list of numbers from 1 to 28 and the ability to count the number of units in the list.

Geometry and this number grouping may well have developed simultaneously since in drawing a line or diagonal through a square we face our first need to count: the unit is split into two parts. Subsequent lines produce a further visible division of the unit, so the wish to count and the need for expression of numbers becomes apparent.

But the need to count may possibly have arisen at an earlier stage, and a primitive system of numbering may have existed at the outset of geometric speculation. There is no doubt however that the course of geometric development as described in the foregoing chapters encouraged a familiarity with numbers and gave early researchers experience in handling them.

Gradually, as this experience widened, the elementary knowledge of numbers had been removed at some stage from the geometric context and established as an independent subject; Man had calculated his problems in numbers alone instead of drawing geometric shapes and reading off the results.

This form of calculation had developed for purely practical reasons, applied to many spheres of daily life—but never to the field of planning or laying down proportions. The sacred symbols were retained for this purpose.

We know the ancient Egyptians were familiar with numbers and counting. We know, too, that they divided areas up into large and small units, and that in the same way they calculated capacity and volume.

We have obtained much of this information from several parchments discovered in excavation of Egyptian temples. The best-known of these mathematical and geometric papyri are preserved in Britain and the Soviet Union respectively.

One is the Rhind Papyrus in the British Museum, London, the other being the Moscow Papyrus and kept in the Museum of Fine Arts in Moscow.

The definitive work on the Rhind Mathematical Papyrus was written (complete with translation and comment) by T. Eric Peet in 1923 and published by the University Press of Liverpool.

This book discusses the papyri in considerable detail, reproducing photographic copies of the hieroglyphs and commenting on their meaning. It takes into account all existing theories at the time of writing.

The fact that Eric Peet's *Rhind Mathematic Papyrus* has been the subject of heated comment and opinion is due probably equally as much to the intricate translation from the heavy language of hieroglyphics as to the fact that the translation of the purely mathematical material does not seem to reveal any coherent purpose. There appear to be a number of loose ends which have defied the efforts of many experts to explain their ulterior significance.

Failure in these attempts until now has been caused by modern man's determined effort to fit the old Egyptian geometric theories into our own present day methods of working and thinking. Such attempts are doomed inevitably to founder.

The two worlds—the ancient and the modern—had a different basic approach. We use numbers as the primary factor and geometry as the subsidiary—regarded merely as a sphere of mathematics illuminated by numbers, but also by letters. The Egyptians reversed the order of importance. Their numbers and system of counting were directly governed by geometric factors, and their ideas and theories were bound up in geometric rules. To the Egyptian a square was always two or more triangles; these were their primary subdivisions, and it was from these that they drew their knowledge of numbers.

It is essential that one adopts this view in judging existing Egyptian material. The ancient remains otherwise are without rhyme or reason.

View the ancient Egyptian mathematics with the mind of the early geometrician and behold! Suddenly the loose ends tie neatly into place, the whole picture takes on a logical appearance. Not only do almost all the problems contained in the Rhind papyrus yield to our calculation, but we are also able—accurately—to follow the working of the Egyptian arithmetic, much of which is detailed in the book.

The Rhind mathematical papyrus can be divided into two principle groups. One deals with purely arithmetical problems: multiplications, subtraction and the like. The other deals with the measurement of areas, capacity and volume, i.e. fields associated with geometric reckoning. And since geometry is our principal field of study in this book we shall confine ourselves to the latter of these two categories.

The first requirement in measurement, whether of area or capacity, is some form of agreed standard size. The object to be gauged is then placed in relation to this standard, the answer being a comparison with the latter.

Without a standard unit of measure no measurement is possible, and we can therefore establish on this basis that the Egyptians must have possessed such standards in order to calculate area and capacity.

If the technique of measurement in any civilisation is at all developed, it is essential that the standard unit of measure can be divided uniformly into various sizes. This permits the user to apply his system to small as well as large objects, enabling him in fact to measure anything within his particular society in relatively convenient figures.

Thus we acknowledge the need first for a standard unit of measure; second, a system of division.

Naturally the units of measure for area and capacity need not be identical, as in the metric system. Metric measure in fact is of relatively recent invention. Only 250 years ago—compared with the meticulous order that exists today—the continent of Europe boasted hundreds of different units of measure for capacity, and perhaps an even greater number of units of length. The names were often the same from district to district—but the measurement depended entirely on local practice.

Surviving Egyptian material does provide evidence of such divisions as we discuss above, and Eric Peet broaches these problems at the outset of his book, with a résumé of knowledge on the subject to date.

The introduction contains a section entitled *Measures of Capacity.* This occupies a full page of the book and is mainly concerned with comments on the actual translation.

This section mentions that the Egyptians employed a unit of capacity known as the *hekat.* The hekat was divided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, and $\frac{1}{64}$. There was also, says Peet, a division at a later stage in Egyptian mathematics of the hekat into 10 *henu* and 320 *ro.*

Another unit of measure is mentioned, the *quadruple hekat,* although it is not associated with the others given above.

A theory is gone into in great detail of how the hieroglyphs for the above sizes can be placed together to form a picture of an eye, but there is otherwise no explanation of the actual sizes or their relation to each other.

Another section is *Measures of Area.* This occupies only 21 lines in the book and says little on the subject, the same being true of the section headed *Measures of Length,* which takes up 10 lines.

What I consider to be a tremendously important section, *Geometry,* is confined to 16 lines.

These extremely brief comments should be regarded on the background of the comprehensive material presented by the book, the main part of the work being translations of Egyptian calculations in precisely the category these comments should cover.

Moreover it is obvious from a close study of the comments on the respective mathematical problems that nothing is in fact known of the actual Egyptian methods. It has been possible for Peet to trace the Egyptian arithmetic only in the few places of the papyri where the procedure and knowledge resembles our own.

The whole basis of the comments (which are intended to assist understanding of the problems in the papyrus) is sketchy; in fact it can scarcely be otherwise, since the material has been considered from the wrong viewpoint entirely.

A logical explanation of the material in question can be obtained only when it is regarded on the basis from which it is derived, i.e. ancient geometry.

In the following pages the subjects discussed above will be seen through the eyes of the ancient geometrician. In order better to understand the difference between the theories set out in my book and those which have applied hitherto, each section of problems has been preceded by Eric Peet's comments on the same material.

When the material has been examined and shown in its geometric context, the new views will be applied to the calculation of some of the old Egyptian problems, these being reproduced exactly as they appear in Peet's book.

Let us look first at the Egyptian measures of capacity.

Measures of Capacity

IN *Rhind Mathematical Papyrus* Peet says that the papyri indicate several different units of measure of capacity operated by the early Egyptians.

One of the larger units was the *hekat*, this being subjected to two different methods of division.

By one method the hekat was divided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$ hekat, and as far as can be ascertained division stopped there. This method had no smaller units.

The hekat was divided by the second method into 10 *henu* and again into *ro:* 32 ro in 1 henu, which in turn meant that 1 hekat comprised 320 ro.

These two apparently independent systems of division must indubitably have borne some relation to each other, for one reads several instances of one of the former divisions ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ or $\frac{1}{64}$) written and converted into henu and ro.

The smallest unit in the first method of division is $\frac{1}{64}$ hekat, the smallest unit in the second system is 1 ro, which is $\frac{1}{320}$ hekat.

This therefore means that 5 ro = $\frac{1}{64}$ hekat. Since the two smallest units are divisible by each other, it follows that doubling their size ($\frac{1}{32}$, $\frac{1}{16}$, etc.) can also be described in an equal number of ro.

In considering these remarkable sub-divisions, the query immediately springs to the curious mind: How did they originate? What has produced this form of division, which differs peculiarly from our own present-day procedure?

I regard it as obvious that they originated in a system or procedure in vogue at the time of the early Egyptians. And I intend trying to illustrate the connection between these measures of capacity and the geometric form I refer to as ancient geometry.

To achieve a complete understanding of the methods of the ancient mathematician, it is helpful to examine briefly the most common measure of capacity in use today, the litre (or liter).

A mental picture of the litre is of course a cube, the sides of which measure 10 cm. The cube thus contains $10 \times 10 \times 10 = 1000$ cu.cm.

The litre can be divided again into 10 parts, each (known as a decilitre) containing 100 cm^3, each decilitre being divided again into cubic centimeters. These latter units are the most commonly used by metric countries.

(In non-metric countries the system is considerably more involved, with units of gills, pints, quarts, gallons, pecks, bushels and chaldrons. Not to mention cubic inches and feet. But it is not with this form of capacity/volume measure that we shall deal. The metric system resembles better the Egyptian form of measure.)

Of course, the cube will only occupy/contain 1 l if its sides really are 10 cm; measuring the cube depends on a fixed norm in the metric system.

If we take as this norm the meter and construct a cube 1 meter on all sides, then it will contain 1000 smaller cubes each holding 1 l.

Thus we can see that the metric system of measure has the two essential factors: an established norm, and a system of division.

The divisional system can easily be demonstrated without resorting to any particular standard of measure since the unit of measure decides only the dimensions of our particular cube. The system of division can be applied to any cube whatever, large or small. The unit of measure therefore decides and reveals whether the cube in question really contains the volume accepted as one litre.

I thought it necessary to emphasise these truisms concerning the metric system in order to let the reader appreciate fully the procedure I believe was used in ancient Egypt. I shall shortly be delving into the Egyptian system and its composition as well as the relation of the various sizes to each other, and I consider that although this relation can be illustrated by means of geometry, the precise size of the smallest unit (and thus the larger units) cannot be expressed accurately in modern terms.

The actual system employed by the Egyptians shares many points of similarity with the Western European, i.e. metric, measure of capacity. But whereas ours is geometrically based on the quadrature or squaring of an area as a factor of division, the early dwellers of the Nile operated, I believe, a different system of geometry by which they were equally strictly bound.

Their system was based on the triangle as a factor of division, a procedure we have already come across in this book. Triangulation was discussed in Chapter Four.

Any given square can be divided in two halves simply by entering one of its diagonals. No measurement is required. *Fig. 110.*

If we enter the other diagonal, the square is split into four equal parts, i.e. four triangles. *Fig. 111.*

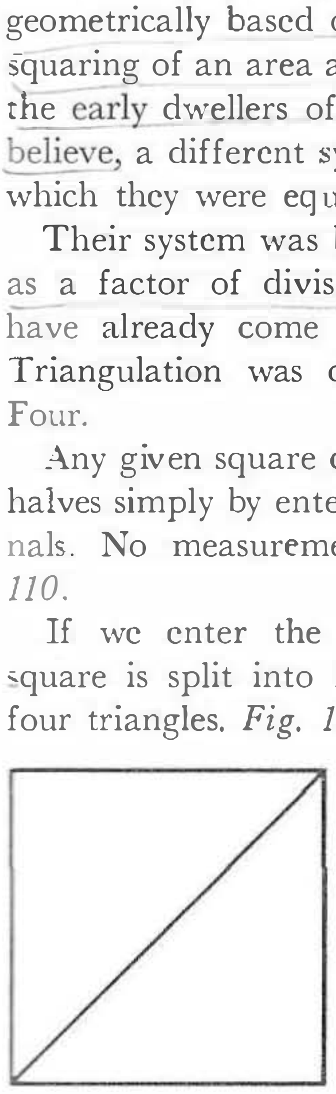


Fig. 110.

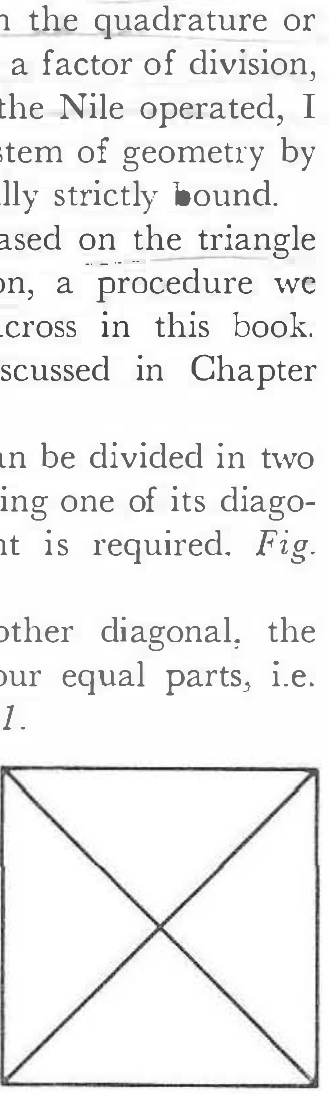


Fig. 111.

The shapes and angles of the triangles in Figs. 110 and 111 are identical—the triangles are in fact identical, apart from size.

The ability to divide up an area without the aid of measurement was from the very earliest times necessary, especially in mathematical speculation.

The next step is to enter the vertical cross along with the two diagonals in the square. This, too, can be done without measurement, since it is simply a matter of transferring to the side of the square the distance of the diagonals' intersection from the side.

Once this has been done we are able—still without requiring a standard of measure—to split our square into as many triangular units as technique will permit. We see this done in *Fig. 112* where the basic square is sub-divided in seven stages without by any means having exhausted the numerical possibilities.

A the basic square is 1
B the basic square is split into 2 triangles
C the basic square is split into 4 triangles
D the basic square is split into 8 triangles
E the basic square is split into 16 triangles
F the basic square is split into 32 triangles
G the basic square is split into 64 triangles

A study of the individual partitions shows that we can read from these geometric shapes:

1 - $\frac{1}{2}$ - $\frac{1}{4}$ - $\frac{1}{8}$ - $\frac{1}{16}$ - $\frac{1}{32}$ and $\frac{1}{64}$ of the square's area. Only the whole unit is quadratic, the rest are triangles.

These are all of the same shape but of varying sizes, and the small triangles can be grouped together in order to express a larger unit, e.g. we can see in diagram F that two $\frac{1}{64}$ triangles equal one $\frac{1}{32}$, and two $\frac{1}{32}$ equal one $\frac{1}{16}$.

Thus it would appear that triangulation produced precisely the figures shown in the old papyri of Egypt, and some con-

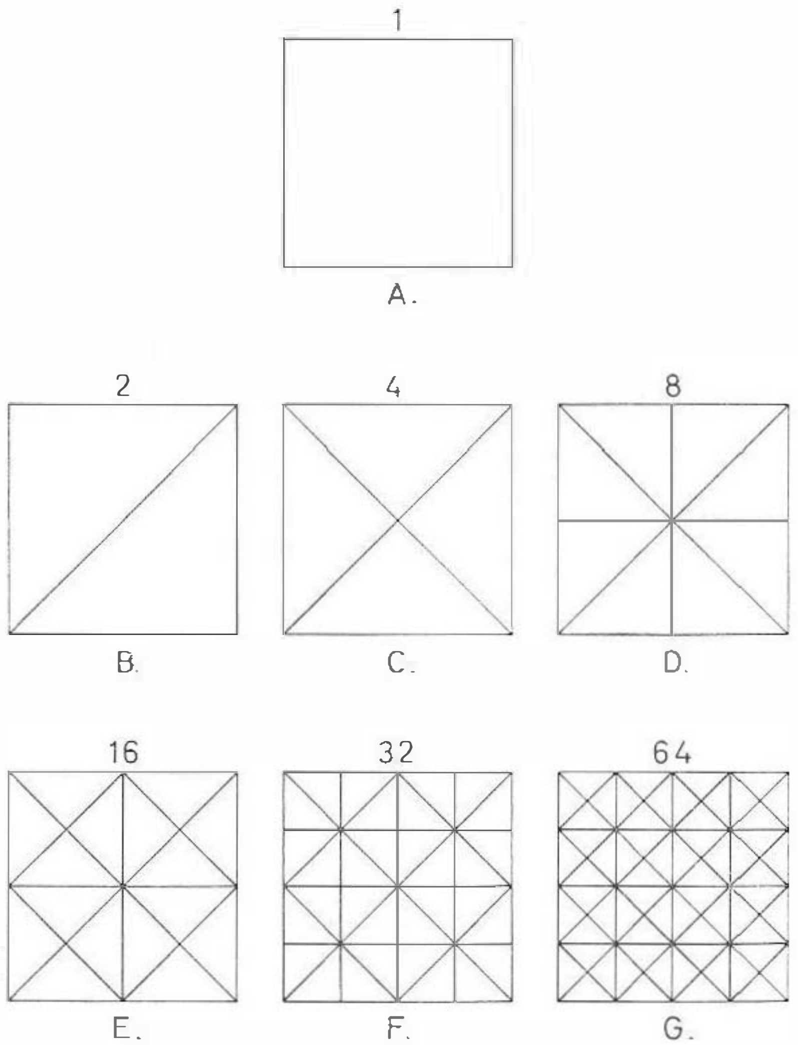


Fig. 112.

nection is apparent since this special order of division is produced geometrically only by triangulation. I believe the connection can be proved conclusively.

Let us imagine this sub-divided square placed on the base of a cube. The cube represents a unit of capacity or volume which the Egyptians called one hekat. The lines of triangulation are produced upwards through the cube, splitting it into triangular bars. A cube so divided is shown in *Fig. 113* where the cube has been placed on its side to show the process more clearly.

We have divided the cube into a number of the smallest unit of capacity/volume measure used by the ancient Egyptians, namely $\frac{1}{64}$. And beneath the cube we have taken to one side the portion that represents $\frac{1}{64}$.

This unit is, as we can see, a triangular bar and it may be that the Egyptians converted this shape into a series of triangular measuring containers.

A measuring container or beaker of this shape would be easy to gauge, and a craftsman would find it simple to construct. The user would be more certain with this shape of container that it held a certain measure than he would with any other shape—apart from a cube or box, which is two fused triangular containers.

It would be intriguing to discover whether excavations in the ancient world have produced anything in this direction.

This was the earlier means of dividing up the hekat. But surviving material has shown that this was later replaced by another form of division, the hekat being split into 10 henu, and the henu into 32 ro.

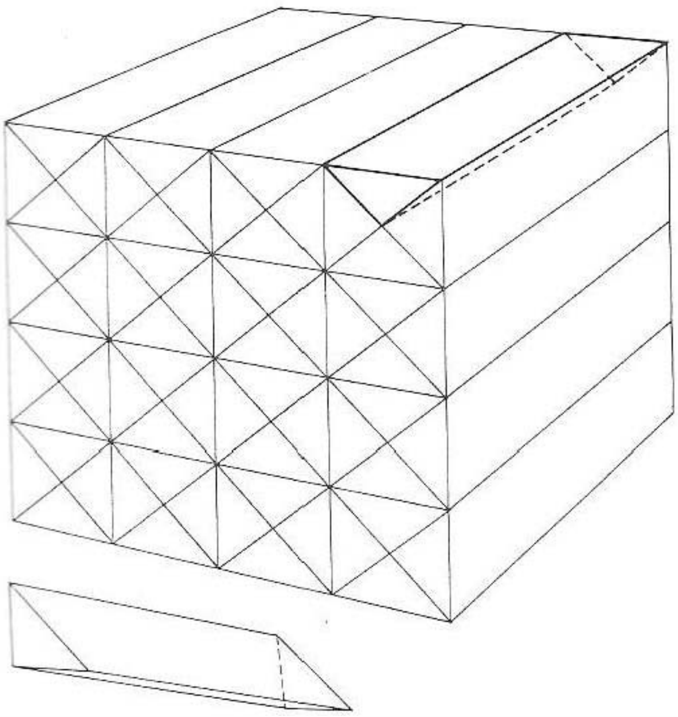


Fig. 113.

This process, too, is obtained by means of ancient geometry, adapted to suit the Egyptians' requirements.

In *Fig. 114* we again have the hekat cube. And again it is placed on its side with its base towards us.

In this case the base is divided into 32 triangles and not, as before, 64. The cube is further divided up into 10 slices by executing 9 horizontal cuts (drawn vertically in Fig. 114 since the cube is on its side).

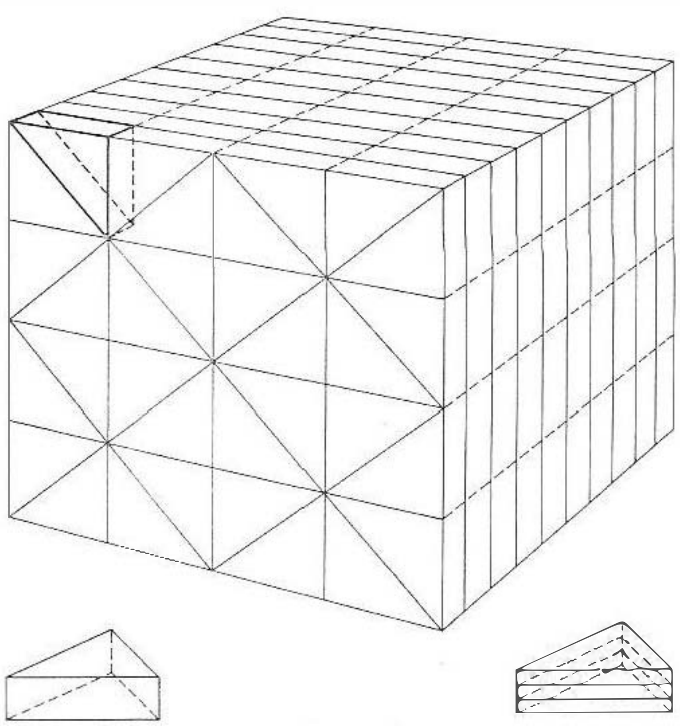


Fig. 114.

The latest division thus gives us a cube of 10 equal square parts or slices. Each of these represents 1 henu = $\frac{1}{10}$ hekat.

Each of these slices is further divided up—by the process of ancient geometric triangulation—into 32 triangular blocks. One of these triangles is 1 ro, there being thus 32 ro in 1 henu, and 320 ro in 1 hekat. We see therefore that by following the above procedure we have achieved complete geometric and numerical agreement between the diagram and the information contained in the old papyri on the old and newer division of the hekat cube. We are thus at liberty to assume that we are on the right track. Our assumption is further supported by the fact that only by triangulation can a quadratic surface be divided into 32 smaller uniform parts. Efforts to solve the same problem by our current method of squaring are unsuccessful.

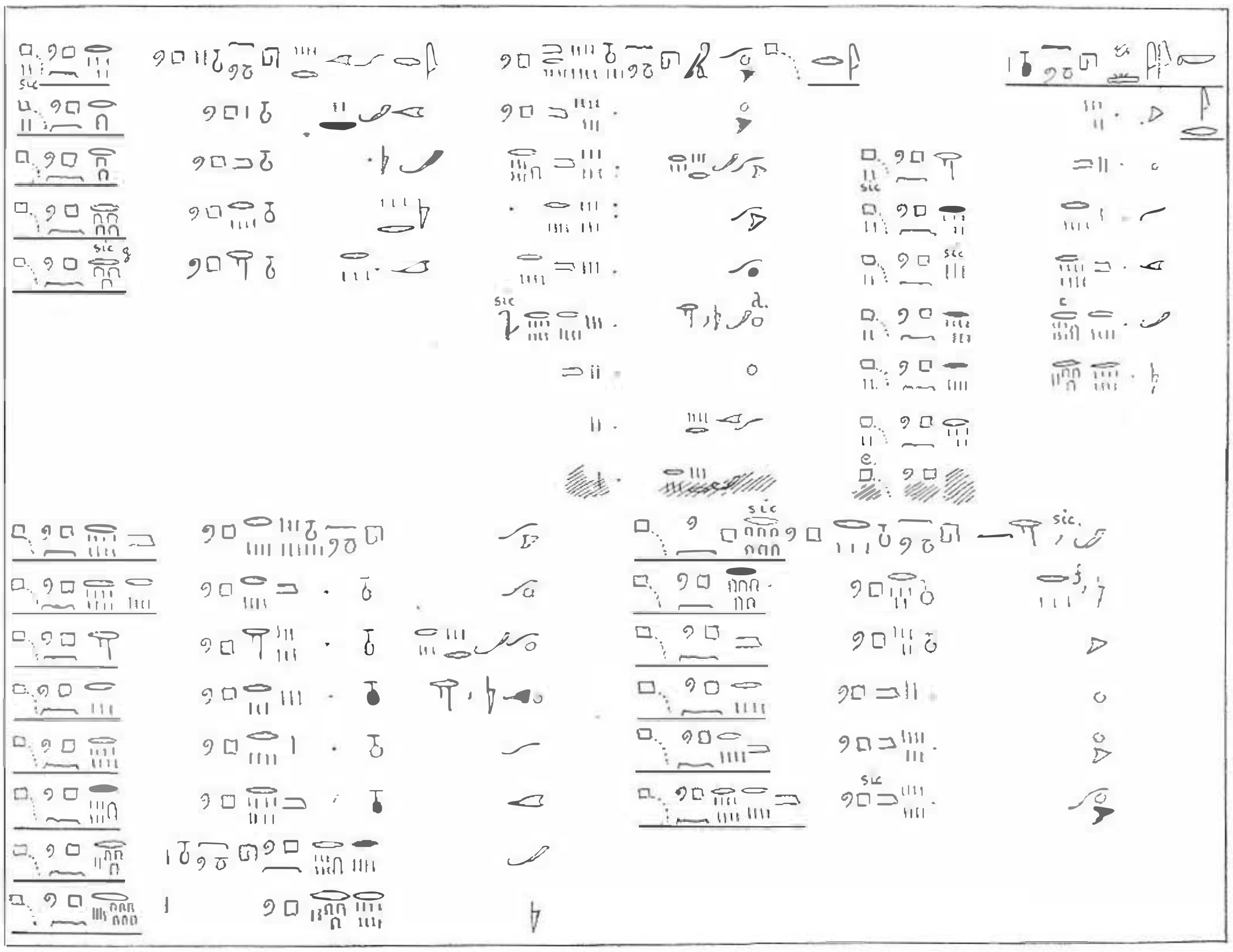
We have now established a really logical association of the three terms, hekat, henu and ro. We can see the limitless possibilities for expressing capacity, and we shall test our theory on surviving Egyptian material, but first it may be practical to sum up our results so far.

One hekat is represented by a cube which by vertical sub-division is split into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$.

One hekat moreover comprises 10 henu by horizontal division, and a combination of the vertical and horizontal divisions cuts 1 henu into 32 ro, hence 1 hekat = 10 henu = 320 ro.

$\frac{1}{2}$ hekat =	160 ro	or	5 henu
$\frac{1}{4}$ hekat =	80 ro	or	2.5 henu
$\frac{1}{8}$ hekat =	40 ro	or	1 henu + 8 ro
$\frac{1}{16}$ hekat =	20 ro	or	$\frac{1}{2}$ henu + 4 ro
$\frac{1}{32}$ hekat =	10 ro	or	$\frac{1}{4}$ henu + 2 ro
$\frac{1}{64}$ hekat =	5 ro	or	$\frac{1}{8}$ henu + 1 ro

We can also achieve these combinations



B. M. Facs. Plates XVIII and XIX.

Fig. 115.

by modern methods of calculation, the basis being 320 ro to a hekat.

$\frac{1}{4}$ hekat is thus $\frac{320}{4} = 80$ ro
$\frac{80}{32} = 2 + \frac{16}{32} = 2$ henu and 16 ro = 2.5 henu.

Let us now try the system on some of the problems posed in the old papyri. In *Fig. 115* we have a reprint from Peet's *Rhind Mathematical Papyrus,* showing a mathematical text with speculation on the hekat and henu.

Fig. 116 shows the translation of the text in the same book. For reasons of clarity I have given the individual problems consecutive numbers.

As an introduction to these calculations of the henu, hekat and ro, we read:

$\frac{1}{2}$ hekat = 5 henu
$\frac{1}{4}$ hekat = $2\frac{1}{2}$ henu
$\frac{1}{8}$ hekat = $1\frac{1}{4}$ henu
$\frac{1}{16}$ hekat = $\frac{1}{2} + \frac{1}{8}$ henu
$\frac{1}{32}$ hekat = $\frac{1}{4} + \frac{1}{16}$ henu
$\frac{1}{64}$ hekat = $\frac{1}{8} + \frac{1}{32}$ henu

These are numbered 1 to 6 in Fig. 116. This introduction shows the triangular division of the hekat compared with the henu.

This may be a sign that the triangulation of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$ is

No. 81. (Pl. W.)

"Another reckoning of the *henu*.

1.	Now $\frac{1}{2}$ *hekat*	is	5 (*henu*)	
2.	$\frac{1}{4}$	,,	,,	$2\frac{1}{2}$
3.	$\frac{1}{8}$	,,	,,	$1\frac{1}{4}$
4.	$\frac{1}{16}$	,,	,,	$\frac{1}{2} + \frac{1}{8}$
5.	$\frac{1}{32}$	,,	,,	$\frac{1}{4} + \frac{1}{16}$ [1]
6.	$\frac{1}{64}$	,,	,,	$\frac{1}{8} + \frac{1}{32}$

7.	Now $(\frac{1}{2} + \frac{1}{4} + \frac{1}{8})$ *hekat*			in *henu* is	$8\frac{1}{2} + \frac{1}{4}$	
8.	$(\frac{1}{2} + \frac{1}{4})$	,,		,, ,, ,,	$7\frac{1}{2}$	
9.	$(\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$	,,	$+ 3\frac{1}{3}$ *ro*	,, ,, ,,	$6\frac{1}{2} + \frac{1}{16}$ (*sic*)	That is $\frac{2}{3}$ of a *hekat*
10.	$(\frac{1}{2} + \frac{1}{8})$	,,		,, ,, ,,	$6\frac{1}{4}$	That is $\frac{1}{5}$ (? $\frac{5}{8}$) ,, ,,
11.	$(\frac{1}{4} + \frac{1}{8})$	,,		,, ,, ,,	$3\frac{1}{2} + \frac{1}{4}$	That is 3 (? $\frac{3}{8}$) ,, ,,
12.	$(\frac{1}{4} + \frac{1}{32} + \frac{1}{64})$	,,	$+ 1\frac{2}{3}$ *ro*	,, ,, ,,	$(3\frac{1}{4} + \frac{1}{8}) + \frac{2}{3}$ (*sic*)	That is $\frac{1}{7}$ (*sic*) ,, ,,
13.	$\frac{1}{4}$	,,		,, ,, ,,	$2\frac{1}{2}$	That is $\frac{1}{4}$,, ,,
14.	$(\frac{1}{8} + \frac{1}{16})$	,,	$+ 4$ *ro*	,, ,, ,,	2	That is $\frac{1}{5}$,, ,,
15.	$([\frac{1}{8}] + \frac{1}{32})$	,,	$+ 3\frac{1}{3}$ *ro*	,, ,, ,,	$1 + [\frac{2}{3}]$	That is $[\frac{1}{6}]$,, ,,
16.	Now $(\frac{1}{8} + \frac{1}{16})$ *hekat*		$+ 4$ *ro*	is	2 *henu*	That is $\frac{1}{5}$ of a *hekat*
17.	$(\frac{1}{16} + \frac{1}{32})$	,,	$+ 2$ *ro*	,,	1 ,,	That is $\frac{1}{10}$,, ,,
18.	$(\frac{1}{32} + \frac{1}{64})$	,,	$+ 1$ *ro*	,,	$\frac{1}{2}$,,	That is $\frac{1}{20}$,, ,,
19.	$\frac{1}{64}$	,,	$+ 3$ *ro*	,,	$\frac{1}{4}$,,	That is $\frac{1}{40}$,, ,,
20.	$\frac{1}{16}$	,,	$+ 1\frac{1}{3}$ *ro* [2]	,,	$\frac{2}{3}$,,	That is $\frac{1}{30}$ (? *sic*) of a *hekat*
21.	$\frac{1}{32}$ *hekat*		$+ 1\frac{2}{3}$ *ro* (*sic*) [3]	,,	$\frac{1}{3}$,,	That is $\frac{1}{60}$ (*sic*) of a *hekat*
22.	$\frac{1}{64}$	,,	$+ 1\frac{1}{3}$ *ro* (*sic*) [4]	,,	$\frac{1}{5}$,,	That is $\frac{1}{50}$,, ,,
23.	$\frac{1}{2}$	,,		,,	5 ,,	That is $\frac{1}{2}$,, ,,
24.	$\frac{1}{4}$	,,		,,	$2\frac{1}{2}$,,	That is $\frac{1}{4}$,, ,,
25.	$(\frac{1}{2} + \frac{1}{4})$	*hekat*		,,	$7\frac{1}{2}$,,	That is $(\frac{1}{2} + \frac{1}{4})$,, ,,
26.	$(\frac{1}{2} + \frac{1}{4} + \frac{1}{8})$	,,		,,	$8\frac{1}{2}$ (*sic*) *henu*	That is $(\frac{1}{2} + \frac{1}{4} + \frac{1}{8})$ of a *hekat*
27.	$(\frac{1}{2} + \frac{1}{8})$	,,		,,	$6\frac{1}{4}$ *henu*	That is $(\frac{1}{2} + \frac{1}{8})$ *hekat*
28.	$(\frac{1}{4} + \frac{1}{8})$	,,		,,	$(\frac{1}{2} + \frac{1}{4})$ (*sic*) *henu*	That is $(\frac{1}{4} + \frac{1}{8})$,,
29.	$(\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$	,,	$+ 3\frac{1}{3}$ *ro*	,,	$6\frac{2}{3}$ *henu*	That is $\frac{2}{3}$ *hekat*
30.	$(\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$	,,	$+ 1\frac{2}{3}$ *ro*	,,	$3\frac{1}{3}$,,	That is $\frac{1}{3}$,,
31.	$\frac{1}{8}$ *hekat*			,,	$1\frac{1}{4}$,,	That is $\frac{1}{8}$,,
32.	$\frac{1}{16}$	,,		,,	$(\frac{1}{2} + \frac{1}{8})$ *henu*	That is $\frac{1}{16}$,,
33.	$\frac{1}{32}$	,,		,,	$(\frac{1}{4} + \frac{1}{16})$,,	That is $\frac{1}{32}$,,
34.	$\frac{1}{64}$	,,		,,	$(\frac{1}{8} + \frac{1}{32})$,,	That is $\frac{1}{64}$,,

[1] The B. M. Facs. gives a vertical stroke instead of the ligature for 6.

[2] Below this, on the bottom edge of the papyrus, the tops of the figures $1/16$ hekat (?) and $1/3$. Not shown in B. M. Facs.

[3] The 1 is certain, though wrong.

[4] The scribe wrote $1\,1/3$ and then crossed it out. Read $1\,2/3$.

Fig. 116.

the older of the two forms of division, the the henu/ro coming later.

It should be unnecessary to break down these first six problems. The examination of the cube in the past few pages produced the same results exactly. The only difference is that in the earlier study of the cube we split it into hekat, henu and ro, whereas here there is no mention of the latter.

In the following solution of the various problems, the problem itself will be given in brackets. I shall then convert the quantities involved into the appropriate number of small units, i.e. ro, and this will be written underneath, followed by a reconversion into whole numbers.

Problem 7.
($\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ hekat $= 8\frac{1}{2} + \frac{1}{4}$ henu)
$160 + 80 + 40 = 280$ ro $= 8$ henu $+$ 24 ro $= 8\frac{1}{2} + \frac{1}{4}$.

The reckoning of this problem is simple. 1 hekat comprises as we have seen 320 ro, therefore $\frac{1}{2}$ hekat is 160 ro, $\frac{1}{4}$ hekat is 80 ro and $\frac{1}{8}$ hekat is 40 ro. Addition provides the answer 280 ro. Since 1 henu contains 32 ro, we find therefore

$\frac{280}{32} = 8$ henu and a remainder of 24 ro.

$\frac{1}{2}$ henu contains 16 ro, therefore we have 8 henu $+ \frac{1}{2}$ henu plus a remainder of 8 ro. Since 8 ro is the same as $\frac{1}{4}$ henu, we find the result $8 + \frac{1}{2} + \frac{1}{4}$, which is perfectly correct.

Problem 8.
($\frac{1}{2} + \frac{1}{4}$ hekat $= 7\frac{1}{2}$ henu)
$160 + 80$ ro $= 240$ ro.
$\frac{240}{32} = 7$ henu $+ 16$ ro $= 7\frac{1}{2}$ henu.

Problem 9.
($\frac{1}{2} + \frac{1}{8} + \frac{1}{32}$ hekat $+ 3\frac{1}{3}$ ro $=$ $6\frac{1}{2} + \frac{1}{16}$ henu that is $\frac{2}{3}$ of a hekat)

It is interesting here to see that the problem in this form is really two separate problems. The first three sizes should be taken on their own as follows:
$\frac{1}{2} = 160$ ro; $\frac{1}{8} = 40$ ro; $\frac{1}{32} = 10$ ro; a total of 210 ro.
$\frac{210}{32} = 6$ henu $+ 18$ ro $= 6\frac{1}{2}$ henu $+$ 2 ro $= 6\frac{1}{2} + \frac{1}{16}$ henu.

This provides the answer to the problem, our calculation and the original translation matching exactly.

With regard to the statement that it equals $\frac{2}{3}$ of a hekat, we come to the second part of the problem and introduce the fourth figure, namely $3\frac{1}{3}$ ro.
210 ro $+ 3\frac{1}{3}$ ro $= 213\frac{1}{3}$ ro
$\frac{1}{3}$ hekat $= \frac{320}{3} = 106\frac{2}{3}$
$\frac{2}{3}$ hekat $= 106\frac{2}{3} \times 2 = 213\frac{1}{3}$.

Thus we see how the first three figures in the problem provide the result shown, after which the fourth figure is added, fulfilling the requirements of statement no. 2.

We notice that in this problem we introduced a size smaller than our smallest unit, namely $\frac{1}{3}$ ro. We saw this illustrated in Fig. 114.

The Egyptians obviously faced the same setbacks as we ourselves, for if we try to divide, for example, 1 litre into three parts we must necessarily split even the smallest part into three pieces to solve the problem.

Problem 10.
($\frac{1}{2} + \frac{1}{8}$ hekat $= 6\frac{1}{4}$ henu, that is $\frac{1}{5}$ or $\frac{5}{8}$ hekat)

The hieroglyphs must have presented the translator with some difficulty since for some reason he was unable to say whether the answer should be $\frac{1}{5}$ or $\frac{5}{8}$.

With our grip of the ancient Egyptian method of calculation, however, we can clear aside this uncertainty and state quite categorically—after working out the problem—that the answer is in fact $\frac{5}{8}$ of a hekat:
$\frac{1}{2} = 160$
$\frac{1}{8} = 40$
total $\overline{200}$ ro. This is 6 henu $+$ 8 ro $=$ $6\frac{1}{4}$ henu.

$\frac{1}{8}$ hekat $= 40$ ro
$\frac{5}{8}$ hekat $= 200$ ro.

Since 200 ro was the answer to the problem and 200 ro is the same as $\frac{5}{8}$ hekat, it is obvious that the text should read $\frac{5}{8}$ and not $\frac{1}{5}$, as $\frac{1}{5}$ hekat is 2 henu or 64 ro.

Problem 11.
($\frac{1}{4} + \frac{1}{8}$ hekat $= 3\frac{1}{2} + \frac{1}{4}$ henu $= 3$ or $\frac{3}{8}$ of a hekat)

Here, too, the translator has been in some difficulty, not knowing whether to write 3 or $\frac{3}{8}$.

As with the preceding problem we can see what the correct answer should be:

$\frac{1}{4} = 80$
$\frac{1}{8} = 40$
total $\overline{120}$ ro

120 ro $= \frac{120}{32}$ henu $= 3$ henu $+ 24$ ro $=$

$3 + \frac{1}{2} + \frac{1}{4}$ henu which tallies exactly with the answer in the papyrus.

$\frac{1}{8}$ hekat $= \frac{320}{8}$ ro $= 40$ ro
$\frac{3}{8}$ hekat $= 40 \times 3 = 120$ ro

Thus we can clearly see the translation should read $\frac{3}{8}$ and not 3.

Problem 12.
($\frac{1}{4} + \frac{1}{32} + \frac{1}{64}$ hekat $+ 1\frac{2}{3}$ ro $=$ $3\frac{1}{4} + \frac{1}{8} + \frac{2}{3}$ henu, that is $\frac{1}{7}$ of a hekat)

Here for the first time we meet a serious disagreement between the set problem and its solution. Neither the result in henu nor in fractions of henu is correct, nor is the final answer, $\frac{1}{7}$ of a hekat.

As the translation shows in Fig. 116, some difficulty was encountered in deciphering this piece of text, and if the system with which we are working is correct (which seems to be borne out by the solution of the other problems) we have two possible explanations. Either the problem was wrongly copied by the Egyptian scribe, or there has been an error in translating the hieroglyphs into English.

Problem 13.
($\frac{1}{4}$ hekat $= 2\frac{1}{2}$ henu $= \frac{1}{4}$ of a hekat)

This problem was included in the first six we discussed (no. 2) so we need not say any more about it, except that both the problem and the solution are correct.

Problem 14.
($\frac{1}{8} + \frac{1}{16}$ hekat $+ 4$ ro $= 2$ henu $=$ $\frac{1}{5}$ of a hekat)
$40 + 20 + 4$ ro $= 64$ ro $= 2$ henu $=$ $\frac{1}{5}$ of a hekat.

Problem and solution are correct here, too.

Problem 15.
($\frac{1}{8} + \frac{1}{32}$ hekat $= 3\frac{1}{3}$ ro $=$ $1 + \frac{2}{3}$ henu $= \frac{1}{6}$ of a hekat)
$40 + 10 + 3\frac{1}{3}$ ro $= 53\frac{1}{3}$ ro $=$ 1 henu $+ 21\frac{1}{3}$ ro $= 1\frac{2}{3}$ henu.

Problem and answer correspond exactly, as does the final statement that $1\frac{2}{3}$ henu $= \frac{1}{6}$ hekat, since
$1\frac{2}{3}$ henu $= 53\frac{1}{3}$ ro and $6 \times 53\frac{1}{3}$ ro $=$ 320 ro $= 1$ hekat.

Problem 16.
($\frac{1}{8} + \frac{1}{16}$ hekat $+ 4$ ro $= 2$ henu $=$ $\frac{1}{5}$ hekat)

We need not go over this problem since it is identical to no. 14.

Problem 17.
($\frac{1}{16} + \frac{1}{32}$ hekat $+ 2$ ro $= 1$ henu $=$ $\frac{1}{10}$ hekat)
$20 + 10 + 2$ ro $= 32$ ro $= 1$ henu $=$ $\frac{1}{10}$ hekat.

No comment required, this problem is correct.

Problem 18.

($\frac{1}{32}$ + $\frac{1}{16}$ hekat + 1 ro = $\frac{1}{2}$ henu = $\frac{1}{20}$ hekat)

10 + 5 + 1 ro = 16 ro = $\frac{1}{2}$ henu = $\frac{1}{20}$ hekat.

Problem 19.

($\frac{1}{64}$ + 3 ro = $\frac{1}{4}$ henu = $\frac{1}{40}$ hekat)

5 + 3 ro = 8 ro = $\frac{1}{4}$ henu = $\frac{1}{40}$ hekat.

Problem 20.

($\frac{1}{16}$ hekat + $1\frac{1}{3}$ ro = $\frac{2}{3}$ henu = $\frac{1}{30}$ hekat)

20 + $1\frac{1}{3}$ ro = $21\frac{1}{3}$ = $\frac{2}{3}$ henu

at this point the problem and answer are correct, since

1 henu is 32 ro and $\frac{32 \times 2}{3} = 21\frac{1}{3}$.

But the final statement—$\frac{1}{30}$ hekat—is wrong, since

$\frac{1}{30}$ hekat $= \frac{320}{30} = 10\frac{2}{3}$.

The answer should be $\frac{1}{15}$. But this piece, too, as can be seen in Fig. 116, presented its difficulties in translation. The footnote to this problem indicates that the translator had to work with figures written close to the edge of the papyrus.

Problem 21.

($\frac{1}{32}$ hekat + $1\frac{2}{3}$ ro = $\frac{1}{3}$ henu = $\frac{1}{60}$ hekat)

10 + $1\frac{2}{3}$ ro = $11\frac{2}{3}$ ro.

Already we can detect the first error in this problem since $\frac{1}{3}$ henu is in fact $10\frac{2}{3}$ ro not $11\frac{2}{3}$. This must definitely be a mistake in calculation or in writing on the part of the Egyptians since the translator states quite categorically in his footnote that the figure 1 is given in the text. The final statement, that this is $\frac{1}{60}$ of a hekat, is also incorrect since $\frac{1}{60}$ hekat = $5\frac{1}{3}$ ro.

Problem 22.

($\frac{1}{64}$ hekat + $1\frac{1}{3}$ ro = $\frac{1}{1}$ henu = $\frac{1}{50}$ hekat)

This one has been wrongly copied by the Egyptian scribe, as can be seen in Peet's footnote: "The scribe wrote $1\frac{1}{3}$ and then crossed it out. Read $1\frac{2}{5}$."

Thus it should be:

($\frac{1}{64}$ hekat + $1\frac{2}{5}$ ro = $\frac{1}{5}$ henu = $\frac{1}{50}$ hekat)

5 + $1\frac{2}{5}$ ro = $6\frac{2}{5}$ ro = $\frac{1}{5}$ henu = $\frac{1}{50}$ hekat.

With this correction in text both the problem and the answer are perfectly correct.

Problem 23.

No need to go over this problem since it is a repeat of no. 1.

Problem 24.

A repeat of no. 2.

Problem 25.

A repeat of no. 8.

Problem 26.

This is a repeat of no. 7, but whereas there was no doubt before about the translation of the answer, $8\frac{1}{2} + \frac{1}{4}$ henu, some uncertainty has arisen in the repetition. The answer is given as $8\frac{1}{2}$ but is accompanied by a mark of doubt on the part of the translator. The figure $\frac{1}{4}$ should in fact have been included for the result obtained in no. 7 was perfectly correct.

Problem 27.

This is a repetition of no. 10 in which we saw by comment and calculation that there was doubt about the answer, whether $\frac{1}{5}$ or $\frac{5}{8}$.

Calculation demonstrated that the correct result should have been $\frac{5}{8}$, and the answer to this present version confirms our earlier working, since the answer here is $\frac{1}{2} + \frac{1}{8}$ hekat, which of course is the same as $\frac{5}{8}$ hekat.

Problem 28.

This one, too, is a repeat. It was shown and completed correctly in no. 11. In this instance the correct result of $3\frac{1}{2} + \frac{1}{4}$ henu is shown wrongly as $\frac{1}{2} + \frac{1}{4}$ henu.

In no. 28 doubt has been expressed about the translation (Fig. 116). Either the figure 3 was missing from the original, or it proved illegible.

Problem 29.

Another repeat, this time of no. 9, but the procedure is slightly different here.

The problem states:

$(\frac{1}{2} + \frac{1}{8} + \frac{1}{32}) + 3\frac{1}{3}$ ro $= 6\frac{2}{3}$ henu $=$ $\frac{2}{3}$ hekat.

In no. 9 the first part (in brackets) was calculated separately and correctly to $6\frac{1}{2} + \frac{1}{16}$ henu. When the $3\frac{1}{3}$ ro was added, we arrived at the second part of the answer, namely a total of $\frac{2}{3}$ hekat.

In this case we work out the full sum at one move, arriving correctly at $6\frac{2}{3}$ henu $= \frac{2}{3}$ hekat, since

$\frac{1}{2} = 160$, $\frac{1}{8} = 40$, $\frac{1}{32} = 10$, total $210 = 3\frac{1}{2} = 213\frac{1}{3}$ ro.

And $\frac{2}{3}$ henu $= 21\frac{1}{3}$ ro, 6 henu $= 192$ ro, total $213\frac{1}{3}$ ro.

Thus we see that this solution to the problem is correct, too.

Problem 30.

$(\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$ hekat $+ 1\frac{2}{3}$ ro $=$ $3\frac{1}{3}$ henu $= \frac{1}{3}$ hekat)

$80 + 20 + 5 + 1\frac{2}{3}$ ro $= 106\frac{2}{3}$ ro $=$ 3 henu $+ 10\frac{2}{3}$ ro $= 3\frac{1}{3}$ henu $=$ $\frac{1}{3}$ hekat.

We prove that the statement and problem match exactly.

Problem 31.

This is simply a repeat of no. 3, the difference being that the result is shown in hekat, whereas previously it was in henu.

$1\frac{1}{4}$ henu $= \frac{1}{8}$ hekat,

which of course in the light of our knowledge is an obvious statement of fact.

Problems 32, 33 and 34.

These are also repeats, of nos. 4, 5 and 6 respectively.

*

Out of the above 34 problems we have seen how it is possible with very few exceptions to solve the calculations correctly.

Our knowledge of the system of calculation permits us to clarify one or two points of doubt in the text as far as answers are concerned. And, most important of all, we have succeeded in fitting the problems into a unified picture, for the object apparently of this part of the papyrus had been to show how to convert the earliest form of hekat division $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., into henu and ro.

The problems of course can easily be solved without a geometric knowledge or understanding of henu and ro, since a study of the material and figures would soon reveal the fact that a hekat consists of 320 ro. But we should not forget that a unit of capacity measure is an extremely practical thing, something with which or in which to measure.

A measure of capacity (or cubic unit) is not an abstract concept to be expressed only in figures, but something people must work with. I think that in the foregoing I have illustrated a system which would be comprehensible to early practitioners who, in order to understand capacity, must have known the mutual relation and connection of hekat, henu and ro.

As far as I can discover by experiment,

the division of the cube shown at the beginning of this chapter is the only one which produces this relationship, and since this method of division furthermore is one of the basic components of ancient geometry, to which the book is devoted, the question of whether the division is correct seems to demand a positive answer.

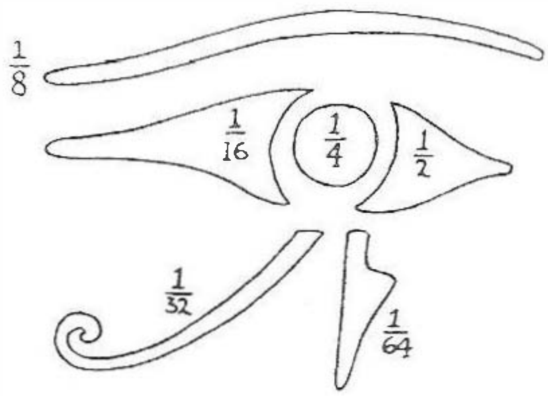


Fig. 117.

It is not outwith the bounds of possibility that the relationship to which we refer was in fact, for non-initiates and outsiders, set out symbolically to resemble the Sacred Horus eye, as shown by Möller *Fig. 117* since this type of symbolism would fit very well into the esoteric teachings of those days.

I have illustrated how the hekat was divided into a number of smaller units. But at the same time, of course, the Egyptians had larger units into which they divided a certain number of hekat. We can read in *Fig. 118* what Peet had to say on the subject (underlined passage).

He maintains that one larger term is *khar* which comprises, according to him, five quadruple hekat.

I think this apparent statement of fact has been wrongly interpreted, since several of the problems in the Rhind papyrus show clearly that there are in fact 20 khar in a quadruple hekat.

In *Fig. 119* we have a facsimile of problem no. 44 Pl. N.

After a remarkable jump in the calculation of the cubic content (a point we shall return to later) we read:

> "1500: this is its content in khar. You are to take a twentieth of 1500, it becomes 75. This is the amount that will go into it in quadruple-hekat."

This demonstrates clearly that by calculation a given cube which contains 1000 units of one size can be described as having 1500 units of another size, and that this latter unit is called a khar.

We can also see that the figure of 1500

The unit used in the papyrus is the cubit (*mḥ*). Griffith has very ingeniously shown that this is the "royal cubit" of 20·6 inches, from the fact, evident from Nos. 41 ff. of this papyrus, that a *khar*, which is 5 quadruple *hekat* or 50 quadruple *henu*, is two-thirds of the cubic cubit. Now the capacity of various inscribed *henu*-measures which have survived is on the average just over 29 cubic inches, which demands that the cubit used in the above equation should be roughly 20·6 inches or about 523 mm. The "short cubit" marked on the Egyptian cubit measures would not give this result.[2]

The royal cubit is in this papyrus divided into 7 palms (Nos. 56 ff.), a palm being the breadth of the four fingers, and the palm again into 4 fingers (Nos. 58 and 59).

In the measurement of land the unit of length was the *khet* (*ḫt*) of 100 cubits, the full name of which was *ḫt nt nwḥ*, or "reel (?) of cord," a measure which may well be compared with our chain. For the use of this unit see below.

Fig. 118.

No. 44. (Pl. N.)

"Example of reckoning out a square container of 10 in its length, 10 in its breadth and 10 in its height. What is the amount that will go into it in corn?

Multiply 10 by 10, it becomes 100.

Multiply 100 by 10, it becomes 1000.

Take a half of 1000, that is 500, it becomes 1500: this is its content in *khar*.

You are to take a twentieth of 1500, it becomes 75. This is the amount that will go into it in quadruple-*hekat*, viz. 75 hundreds of quadruple-*hekat* of corn.

Working out:

1	10	1	100	10
10	100	10	1000	[10] 10

1	1000
$\frac{1}{2}$	500
1	1500
$\frac{1}{10}$	150
$\frac{1}{20}$	75
1	75
10	750
\20	1500

$\frac{1}{10}$th	150
$\frac{1}{10}$ of $\frac{1}{10}$th	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$th of it	10"

We have now come to the determination of the volume of a rectangular parallelopiped, which in this particular case happens to be a cube. It is described as a *šꜣʿ ỉfd*. The word *ỉfd* is generally translated "square" or "rectangular," but it is quite possible that we ought to extend its use to three dimensions to cover parallelopipedal. It is clearly a derivative of *fdw*, the numeral 4, and its original meaning must therefore have been four-sided, perhaps with the tacit addition of rectangular. In the present instance the meaning parallelopipedal[1] is conveyed by implication if not directly, for a *šꜣʿ* is a figure in three dimensions, and the adjective accompanying it need only describe its base. Thus a *šꜣʿ dbn* is a cylinder, and a *šꜣʿ ỉfd* is a parallelopiped, or in special cases a cube.

This cube has a side of 10 cubits, and its volume is correctly calculated as 1000. This is then multiplied by $1\frac{1}{2}$ to obtain the number of *khar* contained, and the division by 20 reduces this last to 75 hundreds of quadruple-*hekat*. The three last lines constitute a proof by continued division, if indeed they have not merely strayed hither from No. 45.

There is nothing to note in the text except a slight confusion in the first line. The word 10 occurs four times, whereas we only need it three times. This is due to the fact that there is a confusion between two methods of statement.

šꜣʿ n mḥ 10 *m ꜣw·f mḥ* 10 *m šḫw·f mḥ* 10 *m ḳꜣi·f*

šꜣʿ ꜣw·f 10 *šḫw·f* 10 *ḳꜣi·f* 10

We must delete the words *n* 10 *m* which follow *ỉfd*, or else insert an *m* between the second 10 and *šḫw·f* and another between the third 10 and *ḳꜣi·f*, finally deleting the fourth 10.

Fig. 119.

is divided by 20 in order to find the cube's content in units of quadruple-hekat. Thus we have established that a khar is smaller than a quadruple-hekat—equal in fact to $\frac{1}{20}$ of the latter unit.

The same procedure is implemented in:

Problem no. 41 Pl. N
Problem no. 43 Pl. N
Problem no. 44 Pl. N
Problem no. 45 Pl. N

and it may therefore be safe to assume that our conclusion is correct, at any rate as far as it concerns the above extracts from *Rhind Mathematical Papyrus.*

To relate the hekat and the larger units of khar and quadruple-hekat we require a connecting unit. This appears quite logically on the scene.

We have not yet broached the subject of Egyptian units of length but, without going too deeply into detail, we must at this stage examine the main unit since it has a certain relation to capacity measure. In the modern system of capacity/volume and length measurement we normally apply the same unit to both fields. Whether we are to measure out a piece of land or a cubic container, we express their respective values in meters (or feet). The area is said to be so many *square meters (feet)* and the capacity/volume a certain number of *cubic meters (or feet).*

Although an obvious means of expression to our modern minds, it is by no means very old, relatively speaking. Only a century ago we would have boggled at the maze of different terms in use. Indeed, even to this day certain sections of English-speaking society measure capacity in gills, pints, quarts, gallons, pecks, bushels, quarters, loads and chaldrons. And shippers speak of volume in barrels, tons and loads.

By contrast I think it true to say the Egyptians succeeded in putting their house in order rather well. From their early civilisation they appear to have had a connection between their measures of length and capacity/volume.

We learn from, among other places, the surviving papyri that one particular unit of length in general usage was the *cubit.* The precise length of the cubit has been impossible to ascertain but archaeologists are fairly sure from their researches that it was approx. 523 mm or 20.6 inches. It was divided into 7 *palms,* each of which was in turn divided into 4 *fingers.*

In Problem no. 56 Pl. Q we read:

"A cubit being 7 palms, you are to multiply by 7 ..."

Thus we have a cubit of 7 palms or 28 fingers in all.

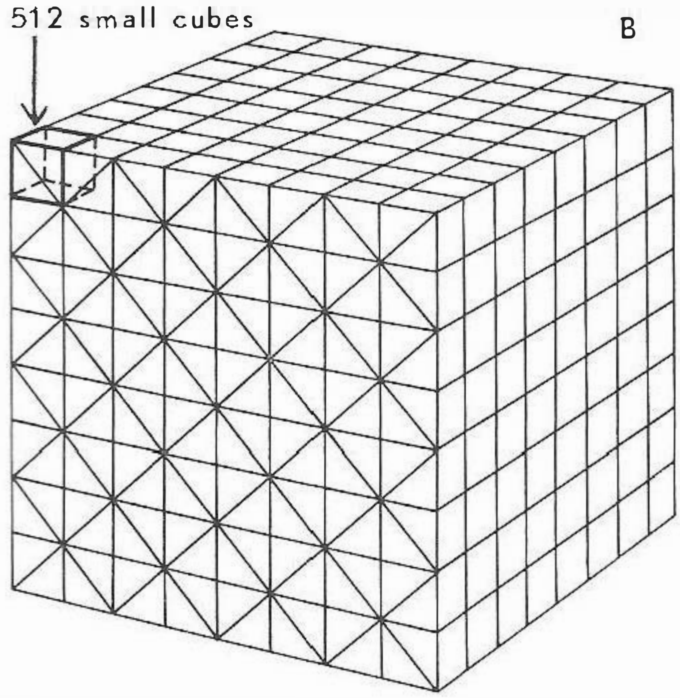
We shall read elsewhere of cubic cubits, and this would seem to indicate that the Egyptian had taken the basic measure of length of 1 cubit and constructed a cube.

As a larger (and for greater volumes, more wieldy) unit they had a larger cube, measuring 7 cubits in each of its three dimensions. It thus contained $7 \times 7 \times 7 = 343$ cubic cubits.

Although, in stating the use of this larger unit of capacity/volume measure, I cannot give it the Egyptian name nor indicate its existence in the problems of the papyri, I firmly believe in its use by the Egyptians. I believe it was essential to the conversion of the cubic cubit into the new system of measure which—as I shall demonstrate—replaced the early system.

This necessity can perhaps best be illustrated by a parallel from our own sphere of numbers.

If we are faced with the problem of dividing the side of a cube measuring 1 meter into three parts, the task is insurmountable without a repeating decimal fraction. But if we make the line instead 1.5 meters, the job can be accomplished without difficulty.



8-part division by triangulation

Fig. 120.

The method and procedure of division which I am about to describe appears slightly more complicated as the Egyptians had fewer sub-divisions than we have.

The simple cubit and its associated cubic cubit, comprising 7 palms or 28 fingers, could not be sub-divided into as many degrees as one might wish, while a division of the 7 × 7 × 7 cubit cube was more satisfactory. We are therefore permitted to assume that this latter unit existed.

Thus we have the large unit, consisting of 343 cubic cubits. And the small unit of 1 cubic cubit.

This small unit was further sub-divided into perhaps cubic palms. And it may have been that yet another division into cubic fingers was known.

The above scale must have been the oldest form of division—which was passed over in favour of a new method at some time in the history of Egyptian mathematics.

The new system of division was inspired by triangulation, while still retaining contact with the cubit as a measure of length.

If on a square we execute our now familiar division into triangles ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.), we carry on until we reach $\frac{1}{128}$ instead of stopping at $\frac{1}{32}$ as previously.

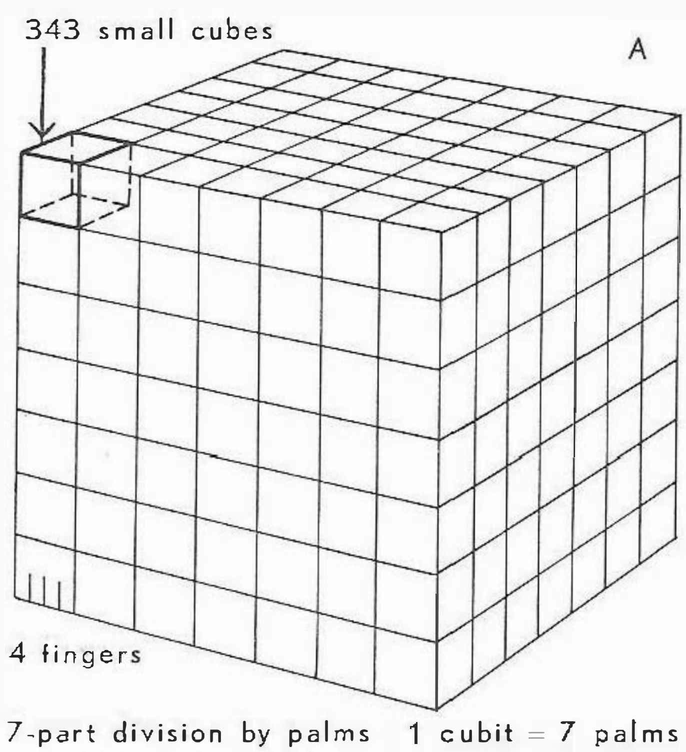
This is what we have done in *Fig. 120* where we can check off $\frac{1}{128}$ of the cube's side, represented by one of the small triangles.

But at the same time we see that the side of the cube has been divided into small squares—a natural observation although perhaps not readily noticeable at first.

This quadrature produces a division of the cube side into 64 small squares, horizontal and vertical edges each being divided into 8 pieces.

Fig. 121 shows the division of 7 × 7 × 7 used hitherto and our task now is to discover how the unit content of the large cube divided 7 × 7 × 7 is related to the unit content of the same large cube divided 8 × 8 × 8.

The diagram actually shows 1 cubic cubit, but we must imagine that we are in fact looking at the larger unit of 343 cubic cubits.



7-part division by palms 1 cubit = 7 palms

Fig. 121.

Our cube has a side-length of 7 cubits and we try first to divide this into 8 parts in order to compare the old and the new divisions.

Without resorting to sub-divisions it is impossible to divide 7 by 8. Since 7 cubits each contain 7 palms, however, this provides a sub-division of the cube side into 49 (palms).

But here, too, it is difficult to form a clear picture of the relation of the two sides to each other. So we further divide the cube side into fingers, $4 \times 49 = 196$.

Dividing 8 into 196 we obtain $24\frac{1}{2}$ fingers. Now we have a workable figure. We can divide the smallest unit in half. This division of the large cube by 8 thus produces a small cube the sides of which measure $24\frac{1}{2}$ fingers, in relation to the 28 fingers of the cube produced by division into 7.

If we divide the $24\frac{1}{2}$ fingers obtained by the "8" division by 7, we get the answer $3\frac{1}{2}$ fingers. And since $8 \times 3\frac{1}{2} = 28$ it is obvious that the side of the small component cube in the old manner of division is $\frac{1}{7}$ greater than the cube in the new division.

Thus by dividing the side of the large unit of $7 \times 7 \times 7$ cubic cubits into 196 fingers we see the ratio of new and old divisions in terms and sizes familiar to the Egyptians.

But simply knowing the length of side of the cube does not tell us the difference in cubic content between the two forms of division. A little arithmetic and logic, however, supply the answer.

We have our basic cube. By the old method, it is divided into $7 \times 7 \times 7 = 343$ small cubes. The later method, $8 \times 8 \times 8$, divides the same cube into 512 smaller cubes.

Obviously—since the basic cube is the same in both cases—the unit cube in the $7 \times 7 \times 7$ method is larger than its later counterpart. And an examination of the figures reveals that the "8" division produces almost exactly 50 % more cubes than the "7"; 343 + 171.5 (half) gives us 514.5, which is extremely close to the 512 cubes of the "8" division.

Thus we find a difference of $\frac{1}{200}$ or $\frac{1}{2}$ % between the number of cubes by the new "8" division and the number of cubes of the old "7" division + 50 %.

If we look back at the problem set in the Rhind papyrus (No. 44 pl. N) in Fig. 119 we see that the result of the problem is calculated to be 1000, then it says:

> "Take a half of 1000, that is 500, it becomes 1500: this is its content in khar."

Thus, once the result is achieved, the mathematician adds 50 % and calls his new unit of measure khar. This might be taken to mean that the first calculation is in cubic cubits, after which he adds 50 % in order to express the same volume in khar.

The new unit of measure is thus smaller than the old, and $\frac{2}{3}$ of the cubic cubit. We recall that this is not exactly accurate, there being an error of $\frac{1}{2}$ %.

It is most likely that the Egyptians were aware of this difference but in practice forced to ignore it in order to be able to convert from one system to the other.

By so ignoring the small error and adopting such a simple conversion process the Egyptians gained the practical advantage of keeping all the old measuring vessels in use.

The Egyptian system of volume measure was almost certainly standard throughout most of the country. And the problem of replacing every measuring vessel—particularly at one and the same time—must have been tremendous.

What better solution therefore than to retain the old vessels, while supplementing them with new? To measure out a quantity by the new system with old vessels, the Nile-dweller simply added $\frac{1}{2}$; conversely, to measure something by the old standard with new vessels, he deducted $\frac{1}{3}$.

We see this procedure applied to several of the arithmetical problems in the Rhind papyrus, and it is worth noting that while most of the problems carry an explanation as to the stages of execution, this particular conversion process is introduced without comment. The instruction is simply: add another half, and this is the answer in khar.

The inference here may be that the Egyptians were so familiar with the process that they did not regard it worthy of comment.

We saw earlier the division by triangulation of an area of 1 hekat, and we saw how this was related to what I refer to as the later division into henu and ro.

We read, too, how the khar was associated with the quadruple-hekat, there being 20 of the former in one of the latter. In fact, we now lack a thread of connection between the quadruple-hekat and the hekat in order to have a complete picture of all the sizes and their mutual relation.

In the passage reproduced in Fig. 118 Eric Peet mentions a size of 5 quadruple-hekat as being identical to 1 khar. He also refers to something called quadruple-henu, and maintains that there are 50 of these in 1 khar.

As far as I am aware the Rhind papyrus makes no mention whatever of *quadruple*-henu.

I am absolutely convinced that the wrong approach has been taken in establishing the relationship of the khar and quadruple-hekat.

The problems in the Rhind papyrus demonstrate rather clearly that there are 20 khar in a quadruple-hekat, and (as we shall see presently) that 5 hekat make up 1 khar. Thus it follows that 100 hekat make up 1 quadruple-hekat. We can now compose the Egyptian table of volume/capacity:

1 quadruple-hekat	=	20 khar
1 khar	=	5 hekat
1 hekat	=	10 henu
1 henu	=	32 ro.

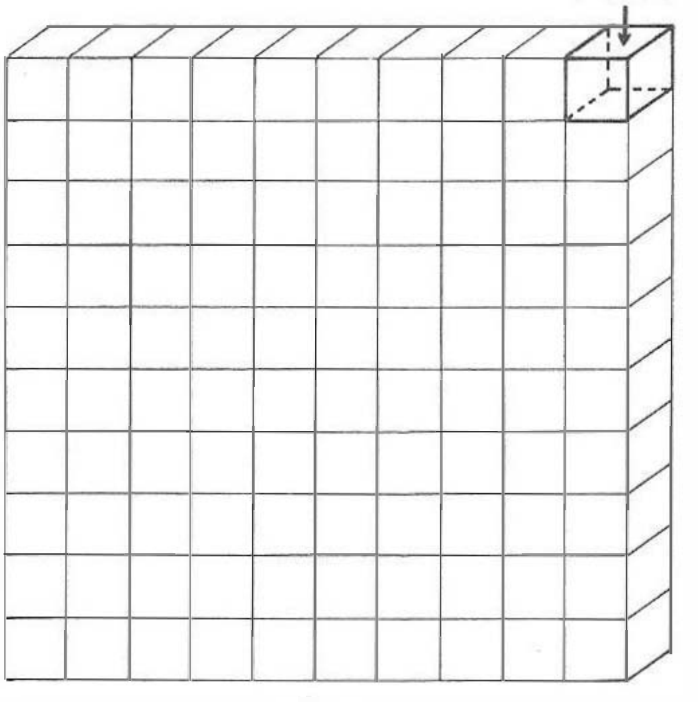


Fig. 122.

Fig. 122 illustrates how I visualise the quadruple-hekat, i.e. as a "slice" 1 hekat thick and 10×10 hekat in area, containing a total of 100 hekat cubes.

We observe in the respective problems that the final result is indicated in hundreds of quadruple-hekat: "namely 48 hundreds of quadruple-hekat of corn", "namely 75 hundreds of quadruple-hekat of corn", etc.

The form of this result would appear to indicate that the Egyptian's largest unit of capacity was the quadruple-hekat and that final quantities were given in multiples thereof. No larger unit apparently existed. One would in the context be tempted to imagine a cube made up of ten quadruple-hekat "slices", i.e. $10 \times 10 \times 10$ hekat $= 1000$ hekat. But the texts available in the Rhind papyrus

1 quadruple-hekat = 100 hekat

20 khar = 1 quadruple-hekat

1 khar = 5 hekat = $\frac{1}{20}$ quadruple-hekat

1 khar = $\frac{2}{3}$ cubic cubit

1 cubic cubit = 7 × 7 × 7 palms

1 cubit = 7 palms

1 palm = 4 fingers

1 hekat = $\frac{1}{5}$ khar = 10 henu

1 henu = 32 ro

$\frac{1}{2}$	hekat	=	160	ro
$\frac{1}{4}$	,,	=	80	,,
$\frac{1}{8}$	,,	=	40	,,
$\frac{1}{16}$	,,	=	20	,,
$\frac{1}{32}$	,,	=	10	,,
$\frac{1}{64}$	,,	=	5	,,

Fig. 123.

do not support this theory. The largest unit mentioned in any of the problems is the quadruple-hekat (and multiples of it).

In actual fact the unit to which Peet ascribes the name "quadruple-hekat" may not have existed as such. It may simply have been a particular multiple (100) of the hekat. This would better explain the term "hundreds of quadruple-hekat". What the Egyptian hieroglyph may in effect be depicting is "a hundred hekat in the form of a square", i.e. 10 × 10 hekat. Thus Peet's rendering, "75 hundreds of quadruple-hekat", might better be interpreted: "75 of the square slices each containing a hundred hekat". But to avoid unnecessary confusion I have retained the term "quadruple-hekat" throughout.

Fig. 123 shows a table of the complete Egyptian system of capacity measure and the mutual relation of the component units.

Starting at the lower end of the scale, we have in the bottom left-hand corner the *finger*. This alters at the third stage to a cubic unit: *cubic cubit*. In the interests of clarity I have omitted the large unit of 343 cubic cubits, showing directly the conversion to *khar*. The relation of the latter is then shown (upwards) to the new divisions of *hekat* and *quadruple-hekat*, and (downwards) to *henu* and *ro*.

My conviction that this was indeed the arrangement of Egyptian units of capacity measure is based on the fact that if these are accepted as demonstrated, all respective problems in the Rhind papyrus form a solvable, intelligible whole. Not only can we produce the same results as those achieved by the Egyptians, but we can also follow their intermediate working step by step. This would be impossible unless the units and values given in Fig. 123 were correct.

Information in the Rhind papyrus has indicated—although the precise source is not quoted—that the royal cubit had a length of 523 mm or about 20.6 inches. If this is accepted, it permits us to compare the Egyptian units with those in use today.

One cubic cubit measured approximately 523 mm or 20.6 inches along each side.

1 cubic cubit contained 523 mm^3 = 143055667 mm^3 = 143 litres

1 khar = 143 × ⅔ = 95.33 l

1 hekat = $\frac{95.33}{5}$ = 19.066 l

1 quadruple-hekat = 19.066 × 100 = 1906.6 l

1 henu = $\frac{1}{10}$ hekat = $\frac{19.066}{10}$ = 1.9066 l

1 ro = $\frac{1}{32}$ henu = $\frac{1.9066}{32}$ = 59.58 cm^3

For comparison with the British imperial system of measure it should be noted that 1 litre equals approx. 1¾ pints, 1 hektoliter (or 100 litres) equals 22 gallons.

Following this examination of the composition of capacity measure, let us try the system on a few of the papyrus problems.

Fig. 124 is a facsimile of Problem no. 44 Pl. N in which we are given a container measuring 10 × 10 × 10.

No indication is given in the text as to the unit of measure in question but naturally one must have been intended in order to solve the problem.

The most likely is the cubit, providing us with a container of 1000 cubic cubits.

Now we are to convert this figure of cubic cubits into khar. We have seen how 1 cubic cubit contains (to within ½ %) 1.5 khar. Understandably therefore the text continues:

"Take a half of 1000, that is 500, it becomes 1500; this is its content in khar."

No. 44. (Pl. N.)

"Example of reckoning out a square container of 10 in its length, 10 in its breadth and 10 in its height. What is the amount that will go into it in corn?

Multiply 10 by 10, it becomes 100.

Multiply 100 by 10, it becomes 1000.

Take a half of 1000, that is 500, it becomes 1500; this is its content in *khar*.

You are to take a twentieth of 1500, it becomes 75. This is the amount that will go into it in quadruple-*hekat*, viz. 75 hundreds of quadruple-*hekat* of corn.

Working out:

1	10
10	100

1	100
10	1000

10
[10] 10

1	*1000*
½	500
1	1500
$\frac{1}{10}$	150
$\frac{1}{20}$	75

Fig. 124 (contd. overleaf)

1	75
10	750
⁄20	1500

$\frac{1}{10}$th	150
$\frac{1}{10}$ of $\frac{1}{10}$th	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$th of it	10 ”

We have now come to the determination of the volume of a rectangular parallelopiped, which in this particular case happens to be a cube. It is described as a *šꜣʿ ỉfd*. The word *ỉfd* is generally translated "square" or "rectangular," but it is quite possible that we ought to extend its use to three dimensions to cover parallelopipedal. It is clearly a derivative of *fdw*, the numeral 4, and its original meaning must therefore have been four-sided, perhaps with the tacit addition of rectangular. In the present instance the meaning parallelopipedal[1] is conveyed by implication if not directly, for a *šꜣʿ* is a figure in three dimensions, and the adjective accompanying it need only describe its base. Thus a *šꜣʿ dbn* is a cylinder, and a *šꜣʿ ỉfd* is a parallelopiped, or in special cases a cube.

This cube has a side of 10 cubits, and its volume is correctly calculated as 1000. This is then multiplied by $1\frac{1}{2}$ to obtain the number of *khar* contained, and the division by 20 reduces this last to 75 hundreds of quadruple-*hekat*. The three last lines constitute a proof by continued division, if indeed they have not merely strayed hither from No. 45.

There is nothing to note in the text except a slight confusion in the first line. The word 10 occurs four times, whereas we only need it three times. This is due to the fact that there is a confusion between two methods of statement.

šꜣʿ n mḥ 10 m ꜣw·f mḥ 10 m šḫw·f mḥ 10 m ḳꜣỉ·f
šꜣʿ ꜣw·f 10 šḫw·f 10 ḳꜣỉ·f 10

We must delete the words *n* 10 *m* which follow *ỉfd*, or else insert an *m* between the second 10 and *šḫw·f* and another between the third 10 and *ḳꜣỉ·f*, finally deleting the fourth 10.

Fig. 124.

We have the quantity in khar, and since 1 khar equals $\frac{1}{20}$ quadruple-hekat, the figure of 1500 is divided by 20, the result being 75 quadruple-hekat.

Basing our calculations on the already detailed relation between cubic cubit, khar and quadruple-hekat, the problem becomes a simple one and completely logical in composition. No doubt should linger in our minds concerning its working.

Fig. 125 shows a facsimile of Problem no. 45 Pl. N, which is in effect the reverse of the preceding.

Here we are told that a certain container holds 75 (hundreds of) quadruple-hekat, and we are then asked:

"How much is it by how much?"

In other words, as Griffith has translated it: What are its dimensions?

The procedure is interesting since it involves a form of cube-root of the content of the container in khar. The 75 quadruple-hekat are first converted to khar (1500) by multiplying by 20. As we now know, there are 20 khar in a quadruple-hekat. The 1500 is divided first by 10 (= 150) and the result is again divided by 10, producing the answer: 15. He knows that 1 khar = $\frac{2}{3}$ cubic cubit; 3 khar must thus equal 2 cubic cubits, and 15 equal 10 cubic cubits.

Thus by dividing the original quantity in khar by 10 and again by 10 he has found expression for $\frac{1}{100}$ of the container

No. 45. (Pl. N.)

"A container into which corn has gone to the amount of 75 ⟨hundreds of⟩ quadruple-*hekat*. How much is it by how much?

Multiply 75 twenty times; it becomes 1500.

Operate on 1500: you are to take one-tenth of it, namely 150;

$\frac{1}{10}$ of $\frac{1}{10}$ of it, namely 15;

$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of it, namely 10.

Therefore it is 10 by 10 by 10.

1	75
10	750
20	1500: behold this is its content.

1	1500
$\frac{1}{10}$th	150
$\frac{1}{10}$ of $\frac{1}{10}$th of it	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$th of it	10 "

This problem is the reverse of the preceding. Here we are given the cubic content of a container in hundreds of quadruple-*hekat*, and we are asked to find its dimensions. It is assumed that the container is parallelopipedal, and even, as the answer shows, that it is a cube.

Fig. 125.

in cubic cubit. He then works back again in cubic cubit multiplying 10 by 10 (= 1 "slice") and again by 10 to express the whole container in cubic cubits, i.e. 1000. He is thus able to state that the container measures, in cubits, 10 × 10 × 10.

The geometric explanation of the problem is shown in *Fig. 126.*

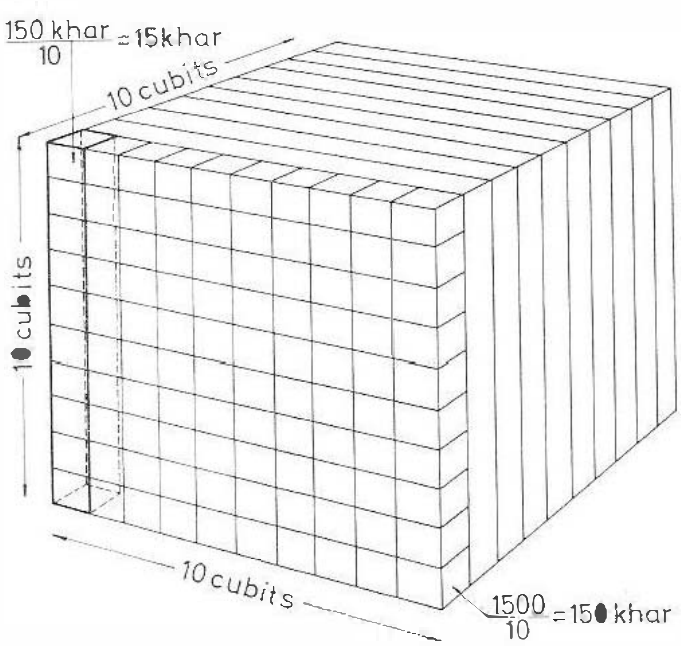


Fig. 126.

We see in *Fig. 127* Problem no. 41 Pl. M in which we are asked to find the capacity of a circular container having a height of 10 and a diameter of 9. Once again we have a problem in which no indication is given of a unit of measure. But again we shall assume this to be cubits.

The Egyptian method of working in this particular type of problem was to convert the circular base of the container into a square, with a side-length equal to 8/9 of the cylinder's diameter.

Their reason and the theory behind their calculation was dealt with in detail in an earlier chapter. Suffice here to say that by this time the Egyptian was able to construct a square very close in area to a given circle by taking as its side-line 8/9 of the circle's diameter.

This 8 × 8 cubit square (and therefore the circular base of the container) has an area of 64. In the normal manner this figure is multiplied by the height to provide a cubic capacity for the cylindrical container of 640 cubic cubits.

No. 41. (Pl. M.)

" Example of working out a circular container of diameter 9 and height 10.

You are to subtract a ninth of 9, namely 1 ; remainder 8.

Multiply 8 eight times, result 64.

You are to multiply 64 ten times ; it becomes 640.

Its half is now added to it ; it becomes 960.

⟨This is⟩[2] *its content in khar. You are to take a twentieth of 960, namely 48.* This is the amount which will go into it in quadruple-*hekat*, namely 48 hundreds of quadruple-*hekat* of corn.

Form of its working :

1	8
2	16
4	32
/ 8	64
1	64
/ 10	640
/ ½	320
Total	960
$\frac{1}{10}$	96
$\frac{1}{20}$	48 "

In this problem the base of the š;ʿ is circular and 9 cubits in diameter, the figure being in fact a cylinder. The diameter is decreased by its ninth part and the result squared, which gives roughly the area of the base in square cubits.[3]

Fig. 127.

The next stage is to convert these to khar, of which there are 1.5 in terms of cubic cubit. Multiply 640 by 1.5.

The Egyptian follows his regular procedure here, adding half of the sum to the original, thus 640 + 320 = 960. This, he reasons, is the container's content in khar.

Finally he divides the 960 khar by 20 in order to obtain the answer in quadruple-hekat, and produces the correct result, 960 khar = 48 quadruple-hekat.

The problem, apart from conversion of the circular base to a square, is the same as preceding problems. And the relation of the cubic cubit, khar and quadruple-hekat is once again vindicated.

Measures of Area

WE TRACED in the past few pages an apparently strong link between the early Egyptian measures of capacity and the geometric system which I believe operated at that time. Application of these geometric theories brings a unity to the entire system of cubic measure, a unity which satisfies perfectly the problems set in the Rhind papyrus; not only are these problems solved readily but in their solution they moreover confirm my theory on the basis of origin of cubic measure.

We may expect to find a similar system and theory applied to the survey of land and other areas; for the mechanical process of surveying is simply the manipulation of geometry, and in planning a suitable system of measure the Egyptians would doubtless have followed the same lines as for their measure of capacity, with variations to meet practical requirements.

In measuring areas they evidently employed two different systems, one older than the other.

In *Fig. 128* we see a facsimile from *Rhind Mathematical Papyrus* in which

2. MEASURES OF AREA.

The commonest unit of area is the *setat* (*sṯ;t*) or square *khet*, which contained 10,000 square cubits. This was divided into dimidiated fractions, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., each of which was distinguished by a special sign and doubtless a special name. Thus :—

$\frac{1}{2}$ *setat* = *rmn*,

$\frac{1}{4}$ *setat* = *ḥsb* (later *ḥsp*), written ×, later

$\frac{1}{8}$ *setat* = *s;* (?), or (late writings)[1]

$\frac{1}{16}$ *setat* = *sw*, (late)

$\frac{1}{32}$ *setat* = *r m;*, (late)

Of these parts only the $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ occur in our papyrus, Nos. 53 and 54. The whole of this system of division of the *setat* is referred by Sethe to a very early stage in Egyptian civilization.[2]

For practical purposes in land-measuring there was a tendency for a unit called the "cubit-of-land" and a 1000-fold multiple of it called the "thousand-of-land" to be used in preference to the *setat*. A cubit-of-land is a narrow strip of land 100 cubits long and 1 cubit broad and is expressed in Rhind by (the ordinary cubit sign, see No. 55): this unit is clearly one-hundredth part of a *setat*. The thousand-of-land was written in early times : in Rhind, however, it is regarded as a unit and indicated simply by a vertical stroke. Thus in No. 53 we find 7 *setat* doubled and the result given as , *i.e.* 1 thousand-of-land and 4 *setat*. Fractions of the *setat* are expressed in the dimidiated fractions described above, so far as possible, and the small remainders in cubits-of-land (*i.e.* hundredths of the *setat*). In No. 54 we have, quite exceptionally, a special hieratic sign for 10 cubits-of-land resembling the numeral 30.

Fig. 128.

Eric Peet supplies an account of the terms used in land-measuring.

We notice first of all that the most common area is the *setat* and that this was divided into halved fractions precisely in the manner of cubic measure, i.e. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$.

This method of sub-division immediately leads one to suspect that the shape of the unit in question is square, and as with the measures of capacity this division was probably the oldest and was supplemented at some later stage by various innovations.

Peet does state that Kurt H. Sethe has placed this system of dividing the setat at an early stage in Egyptian civilisation.

This division of the setat produces sub-divisions of various areas, each being halved to form the next unit, but these sub-divisions have not been named (as was also the case with capacity measures).

It is not until new units are introduced that a division occurs which can be named independently and which can stand as an independent unit, yet remain a part of the larger setat.

One of these new units was a cubit-of-land. Peet tells us that it measured 1 cubit wide and 100 cubits in length; this rectangular shape was a break with the traditional method of division into triangles. We may thus assume that the cubit-of-land was a component of the later system.

Another unit discussed by Peet is the thousand-of-land, containing 1000 cubits-of-land. Since the thousand-of-land is a multiple of and connected with the cubit-of-land, it too must belong to the later and alternative system.

It is maintained furthermore that 100 cubits-of-land make up the setat. It thus follows that the thousand-of-land consists of 10 setat.

At the very outset of his passage on measures of area (Fig. 128) Peet says the setat equals 1 square *khet*, and in a passage on measures of length it is recorded that 1 khet equals 100 cubits.

As far as I am aware nowhere do the problems in the Rhind papyrus mention the unit, square khet. There is frequent use, however, of the unit, khet. I assume Peet must have obtained the term square khet from some other source, but in order to form a clear picture of the system used in land-measurement by the Egyptians, and a firm understanding of the terms used to denote large and small areas we must be extremely careful about determining sizes and their names. We cannot risk including sizes which are not mentioned, and we must moreover bear in mind the strict dividing line between measures of length and measures of area, since in principle these vary vastly.

Linear measure is the numerical value of a measured line and in effect is merely the addition of a number of units, whereas the statement of a particular area invariably infers that a measure of length has been placed in relation to a transverse measure.

The method of calculation may vary but it is essential to calculate before arriving at an area. This calculation produces a result which must be given a new name, for an area cannot be described in plain meters or feet.

A rectangle 3 meters long and 6 meters wide has an area of $3 \times 6 = 18$ *square meters* (or square yards, depending on the system in use).

Granted, our system has the word *meter* (or yard) in the new term "square meter" and that meter and square meter have something in common—but they are two absolutely different concepts, and in no circumstances can one be substituted for the other.

An obvious statement, yes! But we must be extremely wary, in establishing or dissecting the system of Egyptian measures of area, to distinguish between a measure of length and one of area.

At some stage it is necessary to determine which are measures of length and which of area, and I think in this case that khet is a statement of length.

One khet consists of 100 cubits and consequently is the measure of one side of the setat, since the side of a setat is 100 cubits.

Fig. 118 as we noticed earlier is a reproduction of Peet's account of measures of length in the Rhind papyrus. We see that the same two units are mentioned, the cubit and the khet.

The cubit is exactly $\frac{1}{100}$ khet, and is moreover divided into 28 smaller units, namely 7 palms each of 4 fingers.

The ancient Egyptians were thus able to divide the side of a setat into 2800 parts, and since we are informed that 1 cubit = approx. 523 mm, we can also record that the side of a setat was about 52.3 meters.

With the information we now possess we have to attempt to find some base from which to work out the system behind the measurement of area that I have called "the new influence", but obviously this is impossible without extensive support from the calculations contained in *Rhind*. Going on these lines we shall be able to trace the general geometric background, and furthermore to pinpoint some of the system's component units and the geometric proportions used in the calculations, but which have been given no name.

The simplest approach is for me to set out my conclusions regarding the development of the system and then to illustrate its application by working out some of the problems.

We shall base our study on the older method of division, i.e. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$. In measures of capacity we saw that a square had first been split up in this way.

This sub-division is repeated in *Fig. 129*.

In applying this special method of division, we may assume that it relates to a square area and we can thus establish that in its constructive shape the setat is square and will remain so, irrespective which sub-divisions are made subsequently.

We are told that a setat is divided into 100 cubits-of-land, and that a cubit-of-land is a strip 100 cubits long and 1 cubit wide.

We may take it from this that a cubit is a unit of length while the term cubit-of-land is one of area.

If we place 100 of these cubits-of-land side by side, we arrive at a square which is 100 cubits long by 100 cubits wide, containing simultaneously 100 cubits-of-land or 1 setat.

If the Egyptians had operated a quadratic system such as ours, with a unit of

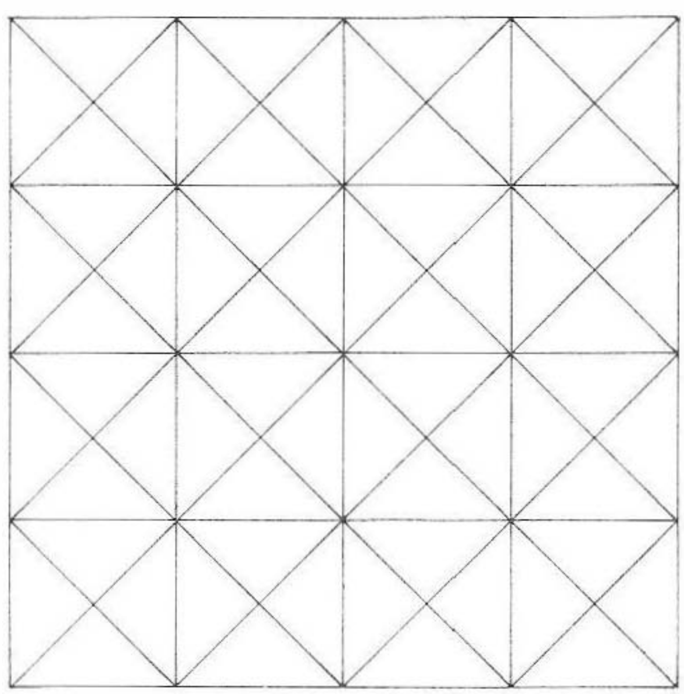


Fig. 129.

area called the square cubit, then 1 setat would have consisted of $100 \times 100 =$ 10,000 square cubits; but apparently theirs was a different method. Division was not into square cubits but in cubits-of-land, and in some instances a cubit-of-land was divided further into ten smaller pieces, each a rectangle 10 cubits long by 1 cubit

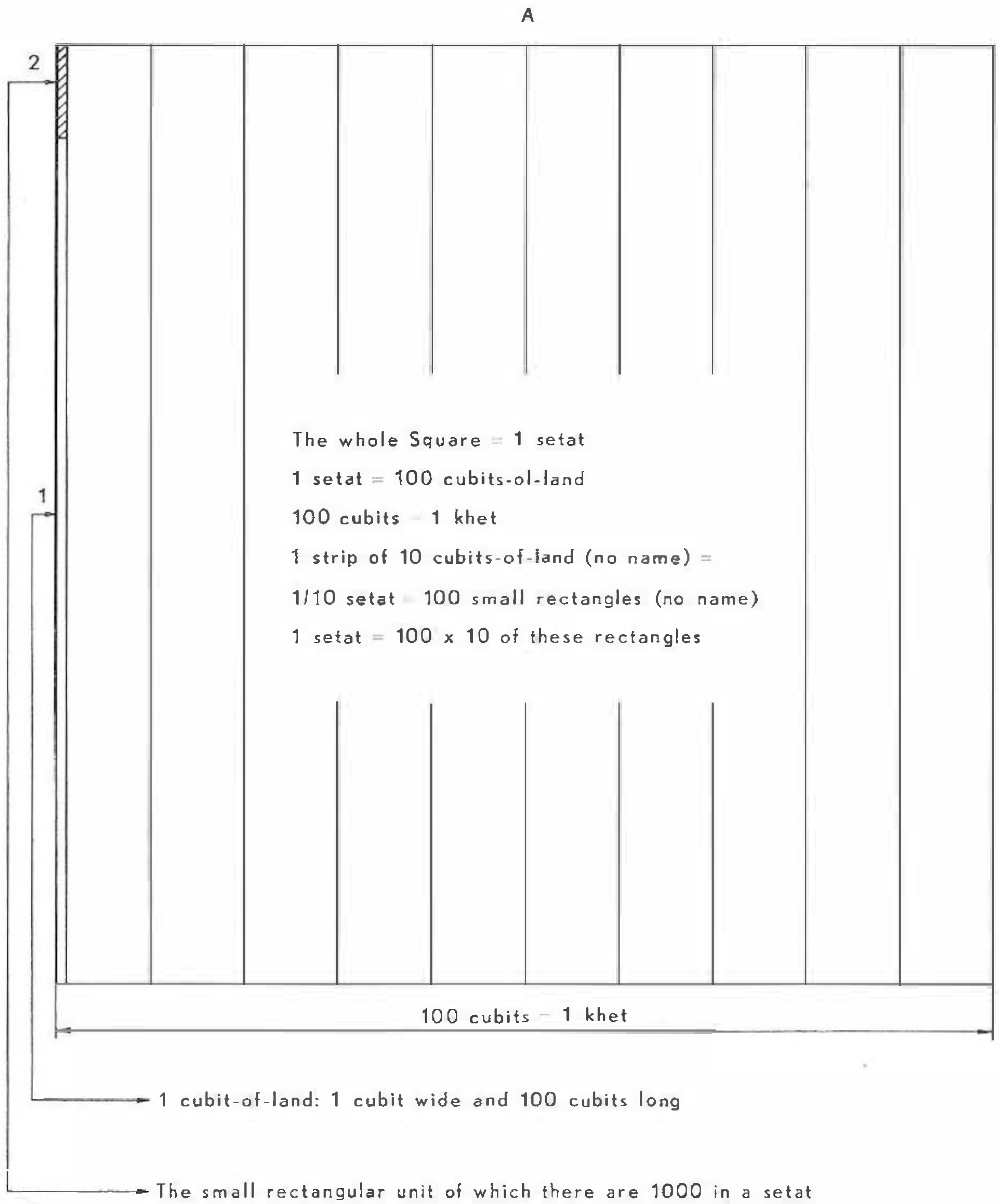


Fig. 130.

in width, but no name has been discovered in modern times for this measure.

Another unit which often appears in the problems we shall examine is a strip of land consisting of ten cubits-of-land, equal therefore to $\frac{1}{10}$ setat.

Peet mentions this size once in his book, stating in connection with one of the problems that this unit is indicated by a certain unidentified sign. But in our further examination we shall see how important in fact this unit is, and it too lacks a name.

In *Fig. 130* we have an attempt to illustrate geometrically the system of area measurement.

This square represents the setat, and its sides therefore measure 1 khet each.

On the extreme left at point 1 we see a strip of land which, in relation to the square, is 1 cubit-of-land. One side of this strip extends along one side of the square and thus equals 1 khet, while the other side of the strip occupies $\frac{1}{100}$ khet, i.e. 1 cubit.

In the upper left corner of the square at point 2 we see $\frac{1}{10}$ of a cubit-of-land. This is of course $\frac{1}{10}$ khet or 10 cubits long and one cubit wide. This small area is $\frac{1}{1000}$ of the whole square and thus represents a unit equal to $\frac{1}{1000}$ setat. It lacks a specific name.

The square is moreover divided into ten larger strips, each containing 10 cubits-of-land, and for these strips, too, we lack a name.

Hence we have the following divisions of the setat:

$\frac{1}{10}$ setat (no name)
$\frac{1}{100}$ setat (1 cubit-of-land)
$\frac{1}{1000}$ setat (no name)

But in land-surveying, of course, the setat was a rather small unit, so it was necessary to have a larger unit of area into which the setat fitted.

This larger unit is shown in *Fig. 131.*

Although it is not proportionally larger than the diagram in Fig. 130, this new square measures 10 khet along each side.

It is sub-divided into 100 squares, each representing 1 setat: a total of 100 setat.

Vertically this square is made up of strips containing 10 setat, and since each setat contains 100 cubits-of-land, each of these new strips holds 1000 cubits-of-land, and is thus the same as the unit termed 1 thousand-of-land.

The whole square therefore contains 10 thousands-of-land, but we have no special name for this new, large unit.

Thus in this material we find a system upon which the Egyptians may have based their measurement of areas. The information we possess so far has been fitted into the emergent system, a khet measur-

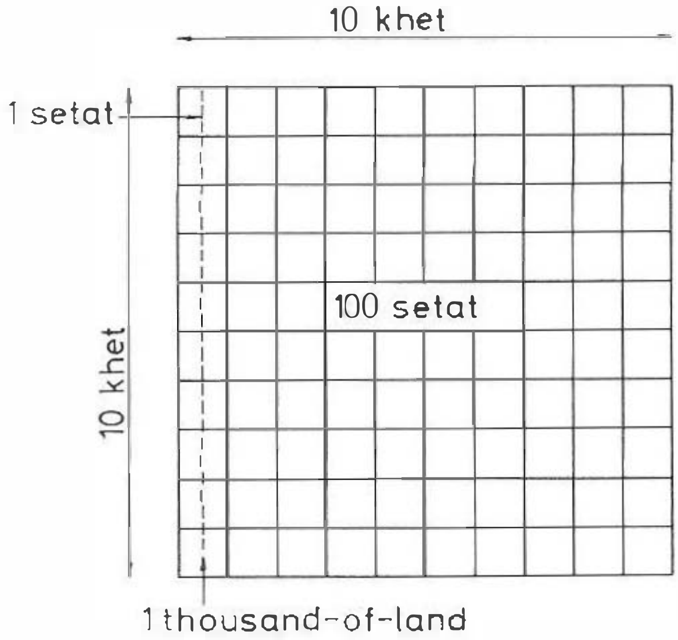


Fig. 131.

ing 100 cubits, 100 cubits-of-land occupying 1 setat, and 1000 cubits-of-land or 10 setat equalling 1 thousand-of-land.

We have not required to tailor the information in any way in order to fit it into the picture, nor have we omitted anything. But the system reveals on the other hand a number of other units which—as we shall see—were used in practice but for which in the text we lack names.

With this system the Egyptians surveyed huge areas of land without a need for astronomical numbers; the large unit of 100 setat, for example, covered an area of approx. 250,000 square meters (or roughly 68 acres).

This area we shall call 10-thousands-of-land (although the Egyptians probably had a special name for it) and simply by stating 20, 50 or 100 of these 10-thousands-of-land they were able to define tremendous land areas in a relatively convenient manner.

On the other hand, the system also encompasses small areas, $\frac{1}{1000}$ setat corresponding roughly to 2.5 sq.m. (or 3.5 square yards), and at this point we are down to areas which scarcely come into the sphere of surveying.

An even smaller division is possible,

No. 54. (Pl. P.)

"To divide ⟨7 *setat*⟩ of land into 10 fields.

1	10
\ $\frac{1}{2}$	5
\ $\frac{1}{5}$	2

1	$(\frac{1}{2} + \frac{1}{8})$ *setat* + $7\frac{1}{2}$ cubits-of-land
\ 2	$(1\frac{1}{4} + \frac{1}{8})$ *setat* + $2\frac{1}{2}$ cubits-of-land
4	$(2\frac{1}{2} + [\frac{1}{4}])$ *setat* + 5 cubits-of-land
\ 8	$5\frac{1}{2}$ *setat* + 10 cubits-of-land."

The problem, as is clear from the working, is to divide 7 *setat* of land (unfortunately omitted by the scribe in line 1) into 10 fields, for "fields" must be the meaning of the second *ꜣḥt*. The method is to divide 7 by 10 (see, however, notes on No. 55), result $\frac{1}{2} + \frac{1}{5}$. This quantity is then expressed in *setat* and cubits-of-land, multiplied by 10, and shown (though the actual addition is missing) to yield 7 *setat*.

Note that in the first step, division of 7 by 10, no units are mentioned: contrast No. 55, and see commentary there.

The notation of the *setat* here used is perfectly regular, see pp. 24–5. The special hieratic signs are used for the $\frac{1}{2}$, the $\frac{1}{4}$ and the $\frac{1}{8}$ of the *setat*, which, with $\frac{1}{16}$ and $\frac{1}{32}$, are the only fractions permitted, odd amounts being entered in cubits-of-land, expressed by placing the number under the arm or cubit-sign. Ten cubits-of-land has, however, a special sign, precisely like the hieratic for 30 (*cf.* No. 53), but perhaps in reality a cursive writing of a 10 under an arm: half a cubit-of-land is shown in hieratic by a sign equivalent to that used for the pure fraction $\frac{1}{3}$.

Fig. 132.

bearing in mind the earlier mention of cubic cubit. There may have been an area of 1 square cubit.

Since $\frac{1}{100}$ setat is a rectangle, 1 cubit wide and 10 cubits long, it is a simple matter to divide this into 10 square cubits, and each of these could be split into palms and fingers.

From one extreme to the other, the system covers just about every area imaginable. It is ingeniously interlocking and in practice was probably just as easy to operate as the metric square meter system today, which emerged thousands of years later.

Now let us try the system on some of the geometric problems posed in the Rhind papyrus, and examine its application in practice.

Fig. 132 is a facsimile of Problem no. 54 Pl. P and of the commentary by Eric Peet.

Using the system we have just discussed, the problem here seems clear, cutting 7 setat into 10 equal pieces or fields.

It is not a question of dividing the figure 7 by 10, but simply the area as a whole.

Before going on to examine the ancient Egyptian method of calculation, we shall try the problem with the old system, but using our own simpler method of counting, "converting" the 7 setat into the appropriate number of cubits-of-land.

1 setat	=	100 cubits-of-land
7 setat	=	700 cubits-of-land
$\frac{1}{10}$ of 700	=	70 cubits-of-land

These 70 cubits-of-land =
½ setat (50 cubits-of-land) plus
⅕ setat (20 cubits-of-land)

Each of the 10 pieces or fields must therefore contain ½ + ⅕ setat. This division is seen geometrically in *Fig. 133*.

Since the problem begins by the Egyptian scribe writing:

1 = 10
½ = 5
⅕ = 2

then it follows that he knew even at this stage that each of the fields had to contain 70 cubits-of-land. Even for the Egyptians this was therefore a piece of mental calculation. It was well within their capabilities to divide 70 strips of land, each containing 10 cubits-of-land, into 10 pieces and to produce the result 7 strips (7 thousands-of-land).

The object of the exercise is to see what proportion of a setat was involved, and it is conceivable that the scribe or teacher produced a drawing similar to that in Fig. 133. He would then be able to read directly the information given above.

In the following calculation, in which he works out the series 1, 2, 4 and 8 in order to achieve the figure 10 by adding the results of the 2 and 8 division together to arrive at 1 setat, he has to reduce to units in which he works with parts of a cubit-of-land; 100 cubits-of-land cannot otherwise be divided into eight portions without the use of fractions.

He writes first:

1 = ½ + ⅛ setat + 7½ cubits-of-land
and we can see that

½ setat = 50 cubits-of-land
⅛ setat = 12½ cubits-of-land
62½ + 7½
= 70 cubits-of-land

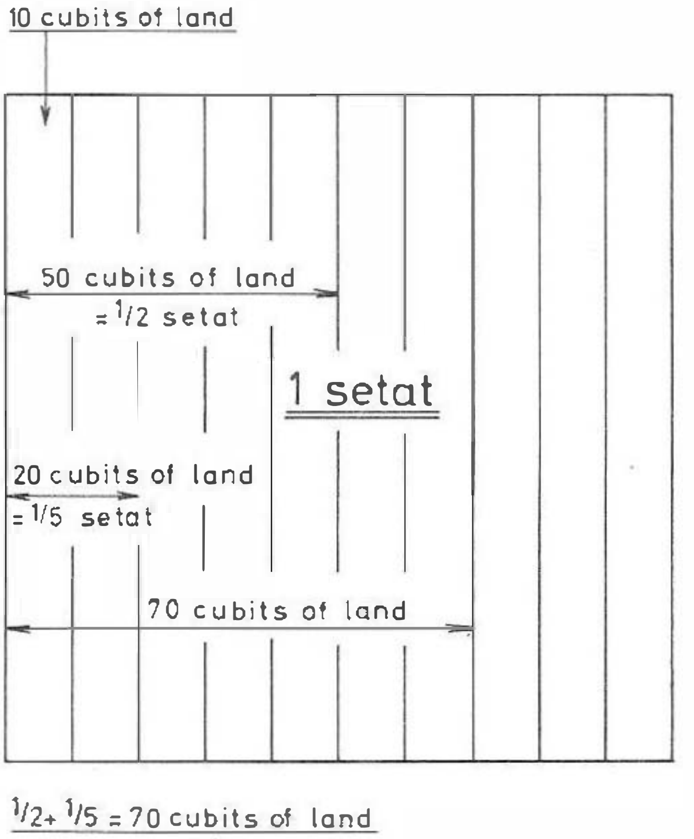


Fig. 133.

2 = 1¼ + ⅛ setat + 2½ cubits-of-land
which is
100 + 25 + 12½ + 2½ = 140 or 2 fields

4 = 2½ + ¼ setat + 5 cubits-of-land
which is
250 + 25 + 5 = 280 or 4 fields

8 = 5½ setat + 10 cubits-of-land
which is
500 + 50 + 10 = 560 or 8 fields.

If we now take the two sums mentioned earlier (2 and 8) we get

2 = 140 cubits-of-land
8 = 560 cubits-of-land
700 cubits-of-land or 7 setat.

The observations the second half of the calculation, i.e. where he works back to 7 setat, could have been read from the square, or calculated numerically. But on the background of geometric theory, the problem is plain and simple, all results

No. 55. (Pl. P.).

"To divide 3 *setat* of land into 5 fields. You are to operate on 5 *setat* (*sic*) to find three *setat* of land.

1	5
sic $\frac{1}{2}$	$2\frac{1}{2}$
sic $\frac{1}{10}$	$\frac{1}{2}$
	$\frac{1}{2} + \frac{1}{10}$ results.

You are to multiply $\frac{1}{2} + \frac{1}{10}$ five times :

/1	$\frac{1}{2}$ *setat* + 10 cubits-of-land
2	$1\frac{1}{8}$ *setat* + $7\frac{1}{2}$ cubits-of-land
/4	$2\frac{1}{4} + \frac{1}{8}$ *setat* + $2\frac{1}{2}$ cubits-of-land

Thus you find the acreage to be 3 *setat*."

The problem is exactly similar to the last. In the setting out the scribe has written "1 *setat*" instead of the simple numeral 5, which it remotely resembles. There is considerable confusion of units and dimensions. A modern worker would divide 3 *setat* by 5 and get his answer direct in *setat*. The Egyptian here quite illogically divides 3 *setat* by 5 *setat*, and gets his answer as a pure fraction. Yet there is a reason for this. The very nature of Egyptian division makes it impossible to obtain the quotient otherwise than in the form of pure number, for it is obtained by adding together certain of the trial multipliers on the left, which can only be pure numbers, not weights or lengths.

Fig. 134.

having an obvious relation to the objective, namely to divide 7 setat into 10 pieces.

In *Fig. 134* we have Problem no. 55 Pl. P which, as with the preceding problem, is purely a question of dividing an area, in this case 3 setat, into 5 fields or equal parts.

The problem presents no difficulty since the setat of course comprises 10 strips, each of 10 cubits-of-land.

The total area thus measures 30 strips to be divided into 5 pieces. Each field must therefore have 6 strips. These 6 strips occupy $\frac{1}{2} + \frac{1}{10}$ of a setat.

The division is seen in *Fig. 135*.

This shows complete agreement between the problem, its solution and the solution arrived at by the Egyptian.

His procedure in the first part of the problem is a sort of reflection on the division of the setat, a piece of quick mental work to see how large a part of the setat was involved.

In this process of consideration he looks first at the central dividing line of the setat and says, one half equals 5 (strips). Thus he writes 1 = 5.

His next conclusion, $\frac{1}{2} = 2\frac{1}{2}$, means that he is thinking of half of $\frac{1}{2}$ setat, since his basis is $\frac{1}{2}$ setat. Naturally half of $\frac{1}{2}$ setat must be $2\frac{1}{2}$ strips.

He concludes by saying that $\frac{1}{10} = \frac{1}{2}$. Here he means $\frac{1}{10}$ of $\frac{1}{2}$ setat, being $\frac{1}{2}$ strip.

He should perhaps have written

1	setat	=	10
$\frac{1}{2}$	setat	=	5
$\frac{1}{4}$	setat	=	$2\frac{1}{2}$
$\frac{1}{10}$	setat	=	1
$\frac{1}{20}$	setat	=	$\frac{1}{2}$

This gives us the complete (for our purposes) break-down of 1 setat, whereas the Egyptian seems to have based his working on $\frac{1}{2}$ setat and worked downwards.

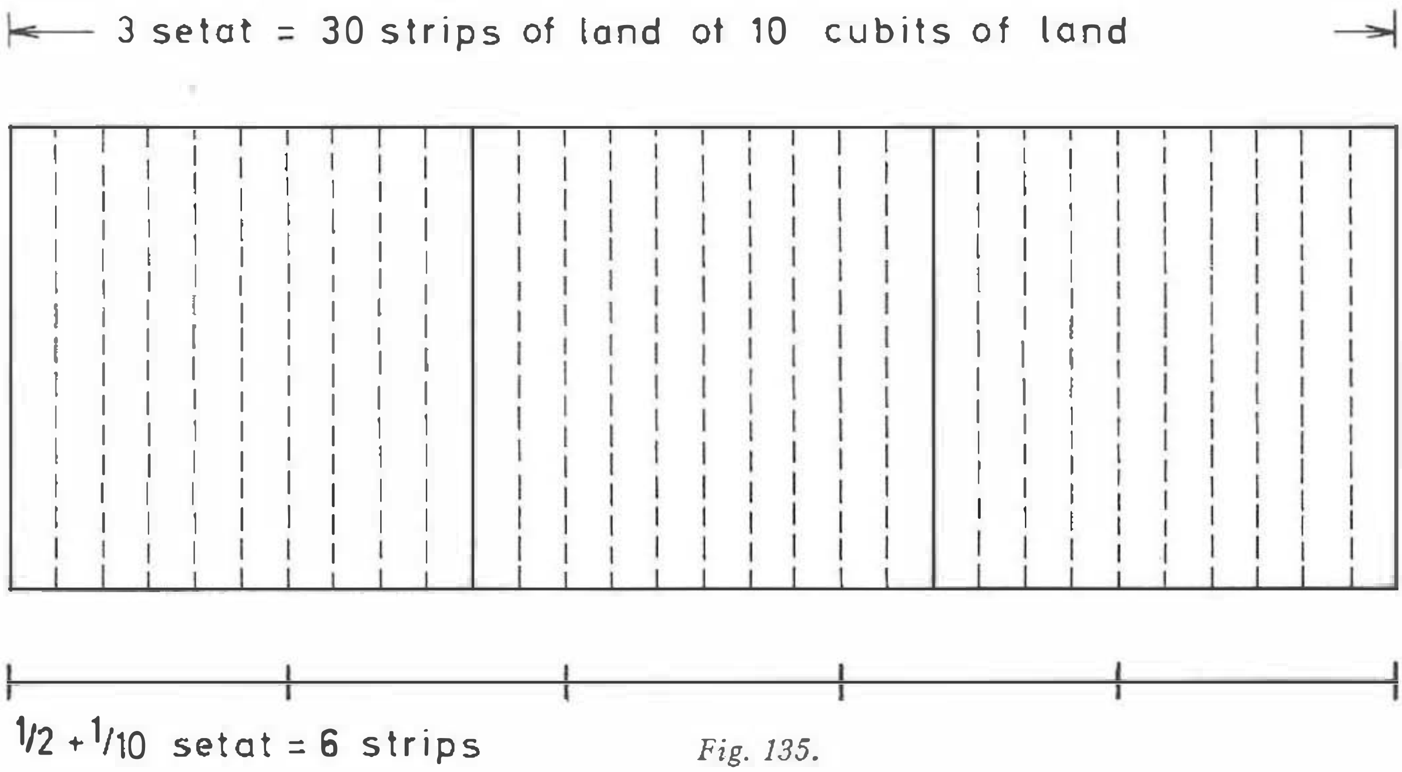


Fig. 135.

Perhaps there should have been a mark opposite ½ + ⅒, since this is in fact the result.

In calculating back, he leaves no doubt whatever.

He writes

½ setat + 10 cubits-of-land = 1
this gives
50 + 10 = 60 cubits-of-land = 1 field

1⅛ setat + 7½ cubits-of-land = 2
this gives
100 + 12½ + 7½ = 120 cubits-of-land = 2 fields

2¼ + ⅛ setat + 2½ cubits-of-land = 4
this gives
200 + 25 + 12½ = 240 cubits-of-land = 4 fields.

His calculations agree entirely with the facts and with his main result. Only in the first part of the sum is there perhaps a little confusion, when he thinks in terms of ½ setat instead of 1, and when he divides down further than strictly necessary to 1/20 of 1 setat instead of stopping at ⅒.

But in spite of this, the problem and its execution are straightforward from the geometric standpoint, and both the result and subsequent checking agree completely.

Fig. 136 is a reproduction of the original version of Problem no. 48 Pl. O, Egyptian-

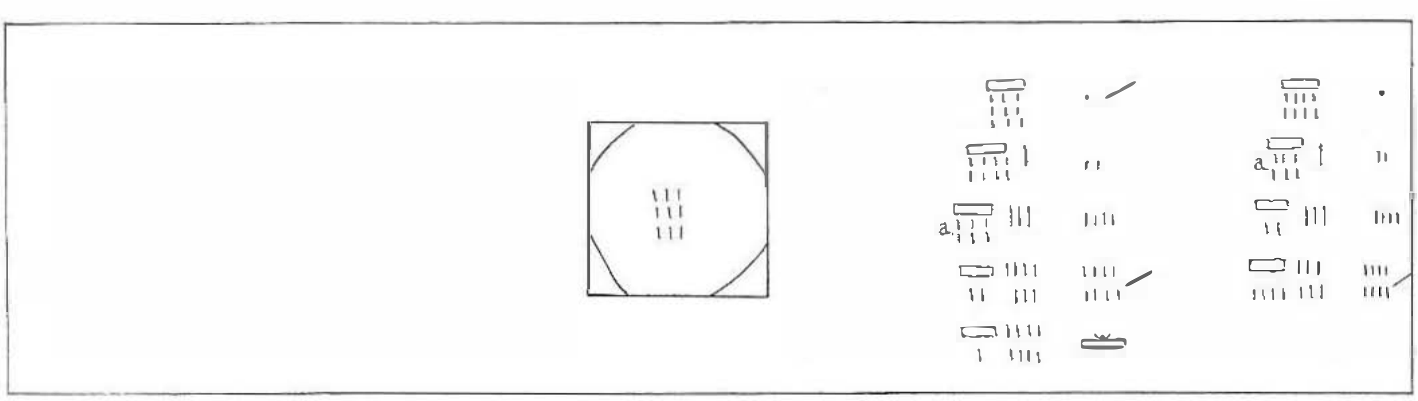
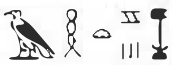


Fig. 136.

No. 48. (Pl. O.)

1	8 *setat*	⁄1	9 *setat*
2	1 thousand-of-land 6 *setat*	2	1 thousand-of-land 8 *setat*
4	3 ,, ,, 2 ,,	4	3 ,, ,, 6 ,,[3]
⁄8	6 ,, ,, 4 ,,	⁄8	7 ,, ,, 2 ,,
		Total	8 ,, ,, 1 ,,

This problem, which has no wording and consists merely of a figure with working out, is clearly the comparison of the area of a square of side 9 *khet* with that of a circle of diameter 9 *khet*. The area of the circle (on the left) is obtained as usual (*cf.* No. 41) by squaring $\frac{8}{9}$ of the diameter,[4] *viz.* 8. The area of the square (on the right) is got of course by squaring the side 9. The nature of the measurements has been cleared up by Griffith. It is plain that the units placed under the sign ▭ are each one-tenth of the units which stand free in front of it. Moreover, since the square of 9 *khet* is written 8 plus ▭ with 1 under it, it is further clear that the free standing units each represent ten *setat* or square *khet*, while the units under the ▭ are each one square *khet*. Now the *khet* measures 100 cubits, and therefore a square *khet* (10,000 sq. cubits) may be regarded as the sum of 100 strips of land each one cubit broad and 100 long. Each of these strips was called a cubit-of-land, because it measured a cubit along one side of the square *khet*, and a thousand of these, known technically as  "a thousand-of-land," would make ten square *khet*, which is precisely what is represented by the free standing units in our example. Each of these then represents a thousand-of-land, *i.e.* a thousand of the narrow strips each 100 cubits by one cubit, and would be represented in early hieroglyphs by the sign for thousand.

The example seems curiously out of place here, and is perhaps an illustration to No. 50.

There is one point in this reckoning which has hardly received the attention it deserves. As a general rule the Egyptian does not trouble in his working out to insert the dimensions of the figures he uses. Thus he does not always openly distinguish between pure numbers and concrete lengths, weights, etc. So in finding the volume of a cylinder he will square $\frac{8}{9}$ths of the diameter of the base and proceed to multiply this by the height without stopping to point out whether the figures used are cubits, square cubits, or cubic cubits until the conclusion of the reckoning.

For this reason it is interesting in No. 48 to find the dimensions inserted throughout. It is still more so to notice that in the first line of all, "1 8 *setat*," the unit is stated as *setat*. To our modern feeling this is wrong. The 8 in question is, strictly speaking, in units of long measure, viz. *khet*, and it is not until we multiply it by another unit of long measure, viz. 8 *khet*, that it can logically be expressed in square units. Similarly in the second half of the sum, the finding of the area of a field 9 *khet* square, the logical process is to multiply 9 *khet* by 9 *khet*; but the working actually written looks like the multiplication of 9 *setat* by the pure number 9. The explanation of this is that, if dimensions are to be stated at all in the working, the peculiar form of the Egyptian system of multiplication demands that the final dimension should be stated immediately in the first line. Thus it is obviously impossible to write 9 *khet* in the first line and 18, 36, and 72 *setat* in the following, for the first and last lines have next to be added together, which is impossible if one is in long and the other in square measure.

In other words, the difficulty lies in the very nature of the Egyptian multiplication process. It is strictly speaking not a multiplication but an addition or counting, and in reality it is impossible to get an area by adding together lengths. The Egyptian solved the logical difficulty in one of two ways. He either put in no dimensions until the working was complete, or he put in the final dimensions straight away. The latter method might be justified by considering the first line of the multiplication as giving the product of say 9 *khet* by 1 *khet, i.e.* 9 square *khet* or *setat*, and not as a mere statement that the multiplicand is 9 *khet*.

There is a somewhat similar example in No. 53. Here, despite the obscurity of detail and meaning, we are clearly dealing with the calculation of certain areas. In the *wꜣḥ tp* process the products (on the left in the original) are given in square measure from the first line to the last. The multipliers (on the right) ought, of course, to be in the pure arithmetical notation, and indeed the integers 1 and 2 are ; but when we come to the fractional multiplier $\frac{1}{2}$ we are surprised to find it expressed in the form peculiar to square measure, the arm (half-*setat*) standing instead of the pure number $\frac{1}{2}$.

Fig. 137.

style. In *Fig. 137* we see the English translation and Peet's commentary.

As can be seen, the problem does not in fact pose any question, and Peet's comment dismisses it as a group of figures to be regarded as pure numerical speculation, without any real comprehension of what quantities are in fact being discussed.

But the problem is a relevant one and in fact discusses facts and sizes very familiar to the Egyptian mathematician and geometrician. It is a summary of the ratio in size of three known factors.

The basis of his reflection is a circle with a diameter of 9 khet. This is converted into a square by the Egyptian method of taking 8/9 of the diameter as the side of the square. This square's area would then be equal to that of the circle.

There is no question here of calculating the area of the circle alone but of considering this area in relation to the unit of measure which we saw earlier, i.e. that containing 100 setat. We recall that 1 khet is a measure of length and that a square, the sides of which measure 1 khet, contains 1 setat.

Our large unit of measure is a square, the sides of which are 10 khet in length and which contains 100 setat.

The square representing the area of the circle has therefore sides of 8 khet and contains 64 setat. The object now is to show how these 64 setat are related to the unit of 100 setat.

The geometric figure containing 100 setat is shown in *Fig. 138* where sides AB and AD each equal 10 khet.

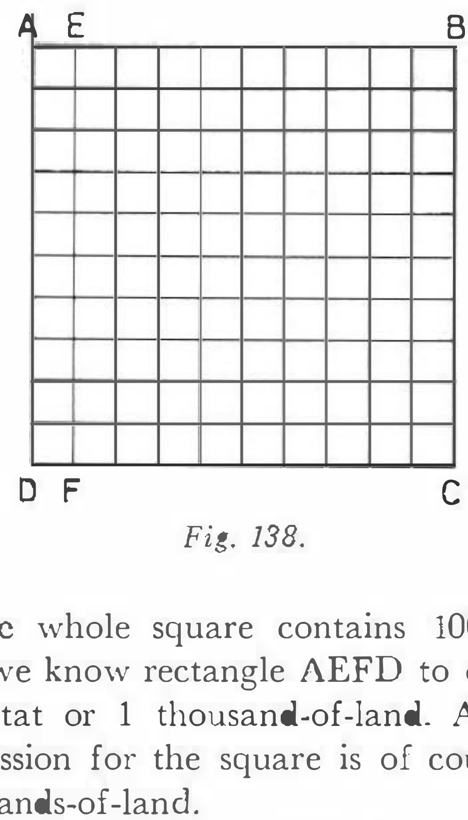


Fig. 138.

The whole square contains 100 setat and we know rectangle AEFD to contain 10 setat or 1 thousand-of-land. Another expression for the square is of course 10 thousands-of-land.

The summary in the Rhind papyrus begins by saying:

$$(1 = 8)$$

To our minds this statement has no basis of truth for 1 cannot be equal to 8, but when the unwritten but understood text is supplied, the statement makes sense:

1 strip of the circle's square is equal to 8 setat. This of course is an obvious fact since the circle's square is 8 × 8 khet, and therefore contains 64 setat.

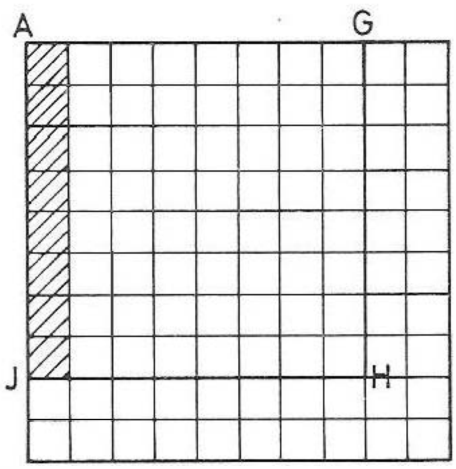


Fig. 139.

This is shown in *Fig. 139* in which the circle's square is drawn into the large unit of 100 setat. The square, containing 64 setat, is seen as AGHJ and has 8 setat shaded in.

The next part of the problem says:

(2 = 1 thousand-of-land + 6 setat)

The object of the problem at this point becomes quite obvious—it is to show the area of the circle measured in thousands-of-land.

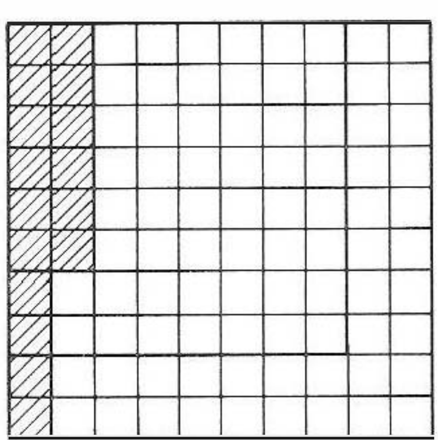


Fig. 140.

The problem states:

2 strips of the circle's square comprise 16 setat or 1 thousand-of-land + 6 setat. This is shown geometrically in Fig. 140 where we see the 16 setat shaded in for clarity.

The statement continues:

(4 = 3 thousands-of-land + 2 setat)

The meaning here is that 4 strips are equal to 32 setat, which is identical to 3 thousands-of-land + 2 setat.

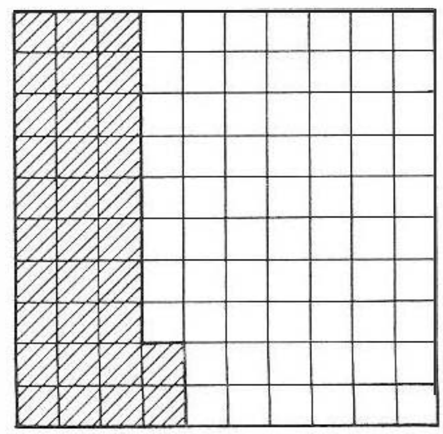


Fig. 141.

The result is perfectly correct as can be seen in *Fig. 141* which shows the geometric picture of this statement.

In the usual Egyptian manner the summary is given as a series of double equations: 2, 4, 8 etc.

The final statement reads:

(8 = 6 thousands-of-land + 4 setat)

This is correct and we with our modern means of calculation have been able to see well in advance that 64 setat is the same as 6 thousands-of-land + 4 setat.

To complete this part of the examination we see this shown geometrically in *Fig. 142* in which the 64 setat are represented by the shaded area. The object of the problem then is to illustrate how great a portion of the large unit of measure is occupied by a circle with a diameter of 9 khet.

When the problem is regarded from this standpoint it becomes so easily com-

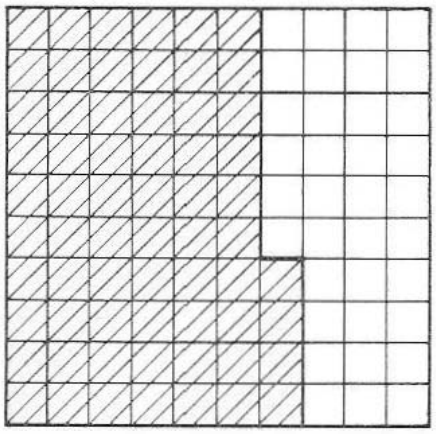


Fig. 142.

prehended as to be explicable and shown with full understanding to a child in elementary school. My own belief is that this material was used to educate mathematical scholars at an early stage in their training. But the simplicity of the problem is naturally dependent on an understanding of Egyptian thinking and the subject to which it refers. I think that until now the subject matter has remained a mystery, this simple problem assuming the air of inaccessibility and peculiarity.

The next step in the same problem is a kind of counterpart to the first: to illustrate the relation of the circle's surroundind square to the same unit of measure (100 setat), the said square having a side-length equal to the circle's diameter, i.e. 9 khet.

In the same way as before we are told:

(1 = 9)

A

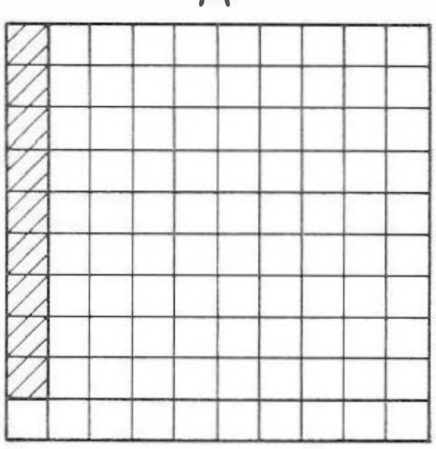


Fig. 143.

which we now recognise as denoting that 1 strip of the square surrounding the circle occupies 9 setat.

This is shown geometrically in *Fig. 143.* The problem continues:

(2 = 1 thousand-of-land + 8 setat)
(4 = 3 thousands-of-land + 6 setat)
(8 = 7 thousands-of-land + 2 setat)

and by adding the first statement (1 = 9 setat) to the last we see the object of this part of the problem, i.e. to show that the circle's surrounding square occupies 8 thousands-of-land + 1 setat.

The various geometric aspects of the problem are shown stage by stage below.

A detailed study of *Rhind Mathematical Papyrus* shows that the same prob-

B

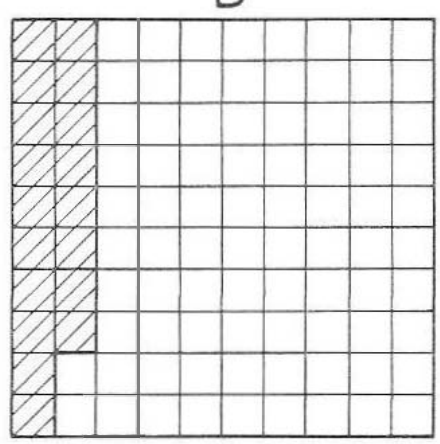


Fig. 144.

C

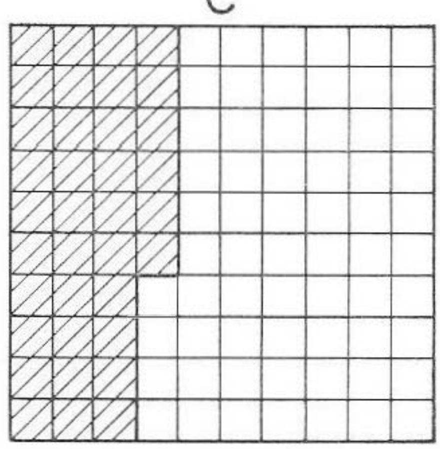


Fig. 145.

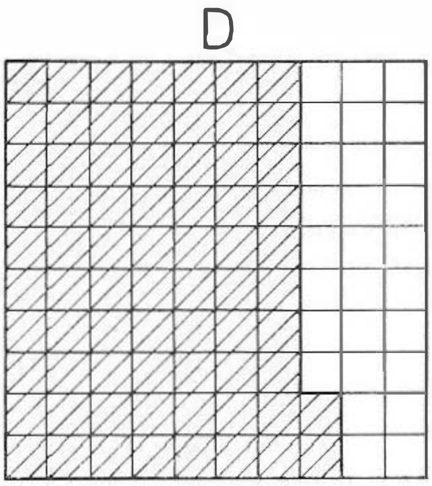


Fig. 146.

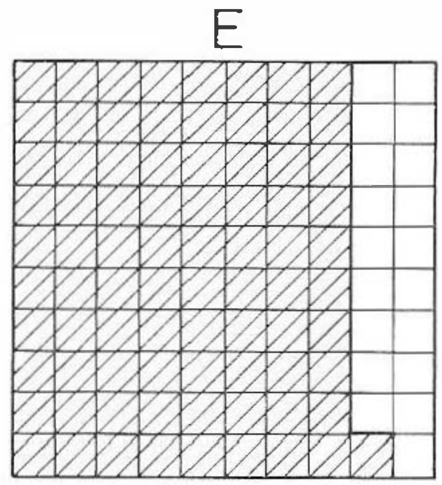


Fig. 147.

lems are often repeated, with only minor amendments. The example we have just examined is also repeated, as problem no. 50 Pl. O. In that instance the difference is simply that the problem deals with the area of the circle alone, and does not include the area of the circle's surrounding square.

The purpose of this study in fact is more to demonstrate the construction of the Egyptian system of area measurement and its practical application than to go into every one of the problems in the Rhind papyrus. I do not intend therefore to include further examples of simple area measurement here, but can give an assurance that with a geometric turn of mind and the system of square measurement detailed in the past few pages the reader can for himself solve and understand all the problems of area given in Peet's *Rhind Mathematical Papyrus,* provided that no doubts are indicated in that book on the accuracy of the actual translated text. In places where the original hieroglyphics proved difficult to read, thus introducing doubt in certain problems, the system of measure we have examined here will put one in a favourable position to reconstruct the missing text. For if one understands precisely how the problem is compiled, it stands to reason the task of supplying missing words or numbers becomes much easier.

We shall leave the calculation of area and move over now to another extremely interesting sphere of Egyptian calculation, namely an examination of the method the ancient masters of Egypt used to make and measure angles. And once the system has been explained thoroughly we shall test our ability—on a few of the problems in the Rhind papyrus. In this particular system we shall find complete explanation for these problems—which have hitherto been an absolute mystery to Egyptologists.

Triangular calculation

THE SECTION OF *Rhind Mathematical Papyrus* dealing with triangles is apparently the department of Egyptian mathematics that has presented the greatest difficulty to modern investigators. The reason must be that the problems themselves give no quarter to partial solution or part truth; they demand precision. If approached from the wrong viewpoint they admit no solution whatever and therefore appear a meaningless statement of numbers.

Since, as I see it, consideration of these problems has been wrongly based from the start, we will notice that few comments are available on the actual working of problems. There is certainly a commentary of sorts, indeed these are the problems on which most comments are made; but they give no explanation of the substance of these problems, and the reason for this complete absence of understanding must surely be the attempt to force the material into the same mode of thinking as that based on our modern mathematical processes. Any such attempt to equate ancient methods with modern theory is doomed in advance. We end in a cul-de-sac, with neither the problem nor its result showing signs of logic, and the working of the problem by Egyptian methods remains lost and incomprehensible.

Understanding of the Egyptian's calculation of triangles demands complete appreciation of and familiarity with the theory on which are based not only the problems but also the system that lies behind these. For without this comprehension and familiarity the scholar does not fully understand the question posed by the problem; and without understanding the question, it is to say the least difficult to follow the working of the problem through to the final result. It would therefore be helpful to emphasise one or two of the factors that form the background to the problems.

The first thing one should remember in considering the problems is that a basic principle in ancient geometry and maths is that of comparative measure, of comparing two shapes, two lengths or two sizes in order to ascertain their mutual relationship.

We saw in our study of capacity and area measure the principal of comparative measure, since the divisions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$ are in fact achieved by comparative means; none of these sizes or subdivisions is predetermined by a particular standard of measure. All are measured in relation to each other. Geometrically, for example, we can read quite simply that $\frac{1}{8}$ consists of $\frac{2}{16}$, and that 1 whole square comprises 2 halves; these are elementary observations, and yet in the actual theory and procedure there lies a fundamental deviation from modern methods and views of mathematical calculation.

The only sphere in modern mathematics in which we find anything approaching this ancient procedure is trigonometry, where an angle is invariably measured in relation to a circle of 360 degrees, irrespective of the area of the triangle.

We saw, too, the existence of comparative measure in calculating ordinary surface area; the ancient Egyptians instead of concluding that an area was a certain number of setat went on to convert this into such-and-such a proportion of the large unit containing 100 setat.

And I have elsewhere in this volume described a number of examples of the principle of comparative measure. We have seen how this method of procedure was not merely practical but essential, an essential factor which formed the very core of geometric and mathemati-

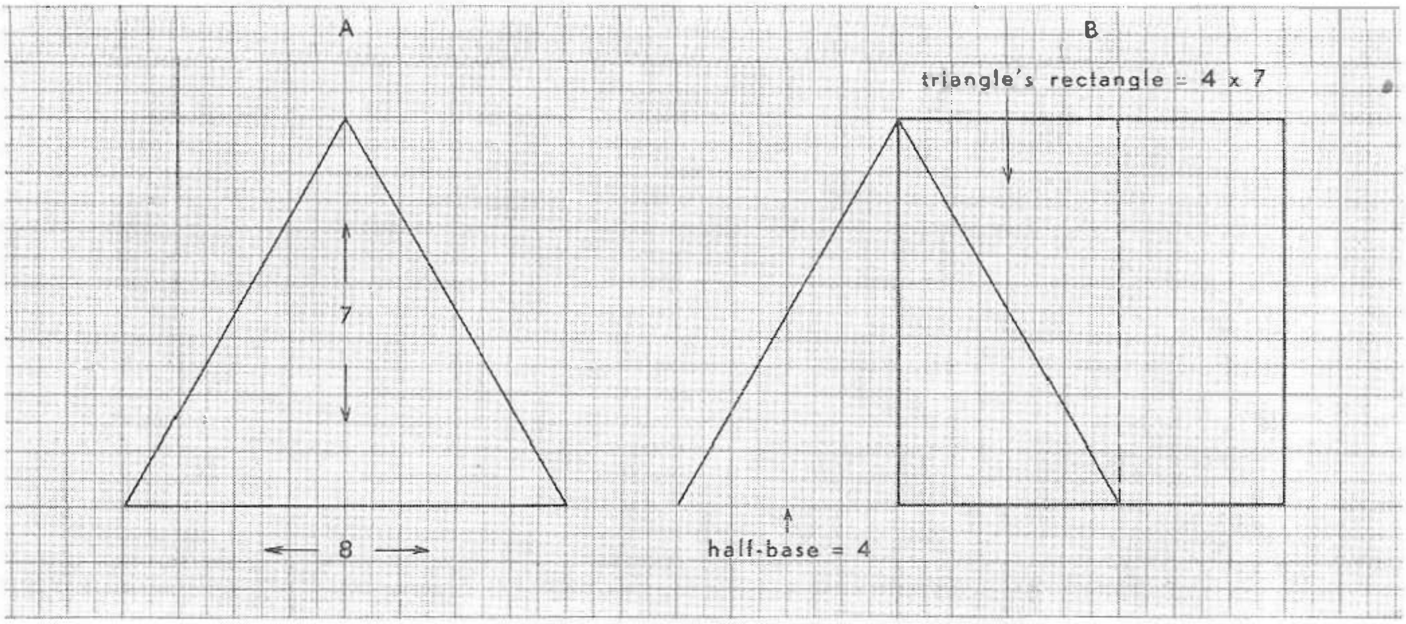


Fig. 148.

cal speculation prior to the development and popularity of our present system. The principle of comparative measure virtually made up the foundation on which speculative mathematics was based.

Geometric speculation itself and a number of different divisions of the square are well beyond the establishment of a standard measure; not until one of the units of division was compared with and measured out according to some form of measuring norm (e.g. a thumb joint, the length of a foot, an arm-span) did a firm system of actual measure emerge. But it would be inconceivable that a measuring norm was chosen first, and the system of comparison developed later. There is no question that the opposite order of events was true, experiences (the divisions of the square, etc.) being matched to requirements by the introduction of a norm.

If we today must calculate the area of a triangle but have not been given details of all its sides, we discover the information we require by means of trigonometry and can then calculate the area. Often we are given or arrive at only one of the triangle's sides but at least two of its angles. By the same trigonometric means we can supply the missing links.

The essence of this fact must be that we consider the triangle as an independent geometric object which we can—without referring to any related or surrounding area—apply as part of our calculations.

This is a vital and fundamental point of difference between modern mathematics and those of ancient times.

To the Egyptian the triangle was admittedly an independent object but when it was applied geometrically it became invariably a part of a given square. The two were inseparable.

The square on which he based his triangle was that which had as its side the perpendicular distance from the base of the triangle to its apex. This is why in all the Rhind papyrus problems we see that the first step is to divide the base-line in half and enter the vertical axis. With this axis as its side a square is then constructed which becomes the basis of comparison for the triangle.

We see this done in *Fig. 148* in which at A we have an isosceles triangle with a height of 7 units and a base or maximum width of 8.

At B this triangle is shown in its proper element according to the dictates of comparative measure, i.e. the square.

The next step is to construct the triangle's rectangle. If we have a triangle with two equal sides the operation is simple, for the rectangle's short side is half the base-line and its long side is the vertical axis of the triangle.

When the problems indicate that the base-line of the triangle must be halved in order to construct the rectangle this infers that the problem deals with an isosceles triangle. Subsequent examination of these problems will demonstrate that the supposition is correct.

The next development in the problem must be recognised as a procedure of vast interest since it reveals a completely new aspect of Egyptian speculative geometry: it is an ability to express and calculate angles within a square (as we today calculate angles within a circle of 360 degrees) although perhaps they did not recognise the concept of angles as we do.

Yet another curious point about their system of angles is that to achieve their purpose the Egyptians applied two of the previously introduced measures of length, namely palms and fingers.

We recall that 1 palm equals $\frac{1}{7}$ of 1 cubit and that the palm is in turn divided into 4 fingers. This provides a unit length of 1 cubit divided into 7 sections of 1 palm or 28 sections of 1 finger.

We also recall that 1 square cubit is a square with sides measuring 1 cubit and is therefore also divided into 7 palms or 28 fingers.

Here, in the measurement of angles, we come across the phenomenon of the Egyptians dividing the sides of *any* given square into 7 palms each of 4 fingers. In this instance palms and fingers denote not a particular linear measure fixed by norms, but simply a means of describing the division of the sides of any square into 28 pieces.

No notice whatever is taken of the normal standard of measure by which 1 palm equals $\frac{1}{7}$ part of the cubit; by their remarkable system the Egyptians divided, for example, a line of 140 cubits into 7 palms, each of these thus containing 20 cubits.

This system of division is based on the same speculative background as the process of dividing any given square into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., whereby we—independent of any standard measure—simply divide the square, with the original whole square itself as the only standard unit.

The use to which they put this special means of division was to illustrate the factor which in the papyrus is referred to as the "batter", this being the ratio of half the triangle's base to the scale of 7 palms graduated along the base of the square constructed on the triangle's vertical axis. Difficult perhaps to visualise, but which I shall try to explain.

If in a given square we enter one of its diagonals, we produce a slope or angle of 45°, and if the base of the square is graduated into 28 fingers as discussed above and lines drawn from these 28 points to the same opposite corner as occupied by the diagonal, we produce a range of 28 angles.

We see this done in *Fig. 149*.

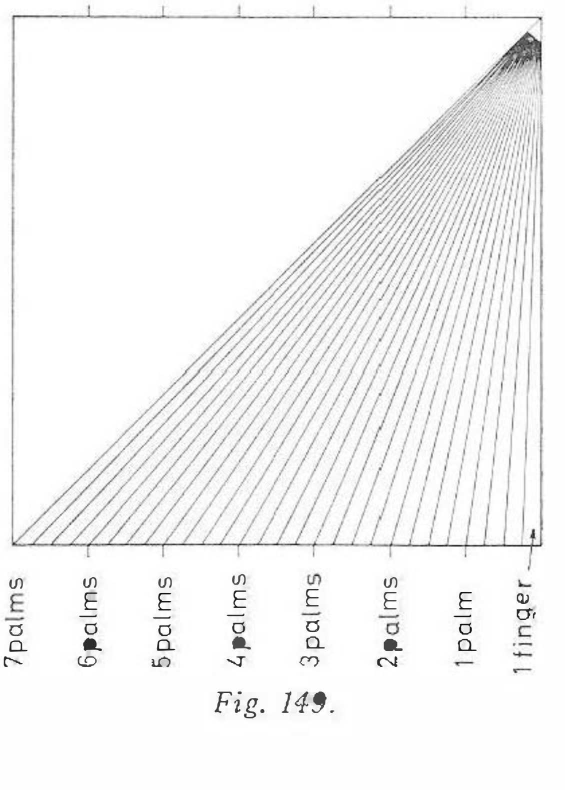


Fig. 149.

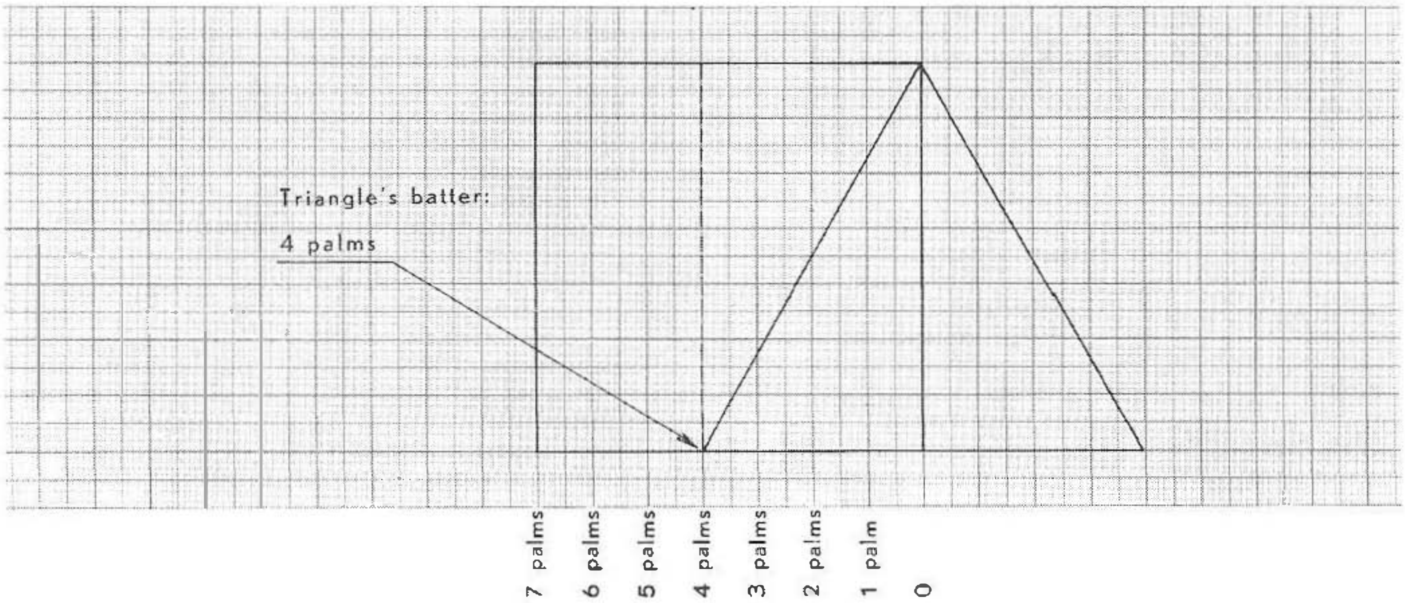


Fig. 150.

We see here how it was possible for the Egyptians to work with and calculate on a kind of graduated scale in palms and fingers, which provided 28 different divisions (I hesitate to call them "degrees" for fear of confusion with our present-day system) where we today reckon with 45°. And if they furthermore worked with half or quarter fingers, they could increase these 28 divisions to 56 or 112.

It will be appreciated that their division of the square's base-line into 28 units is mathematically accurate, since a triangle's batter would remain constant irrespective of the size of the square as long as the graduated scale remained constant.

The reader may be a little surprised that this system of angular measurement employs the same terms as those used to describe certain fixed units of length. It would be interesting to discover whether the original hieroglyphics contain some slight variation in form to denote the difference between the sign for palms as a linear measurement, and the sign for palms as a means of measuring angles.

To conclude our study of Fig. 149 and to demonstrate (I trust convincingly!) the term "batter", we shall apply the triangle in Fig. 148 to our graduated 7-palm scale. We see this in *Fig. 150* where we find the batter to be the distance on the scale from the centre point of the triangle's base-line to its extremity, i.e. 4 palms.

In this examination of some of the basic elements in the Egyptian method of calculating the proportions of triangles we have assembled sufficient material to allow us to deal with the geometric problems set in the Rhind papyrus. Let us start with tests on Problem no. 51, Pl. O, a facsimile of which we find in *Fig. 151.*

This states that a triangle has a height of 10 khet and a base of 4 khet, and the problem quite simply is to find its area.

By our modern method we would immediately say—that the area equals half the base times the height. The result is 20 and since the unit of measure in this instance is khet and we have already seen that 1 × 1 khet is 1 setat, the result must then be 20 setat.

But the result in the papyrus is given as 2, not 20. Let us examine this.

We read at the outset of the working: "You are to take half of 4, namely 2, in order to give its rectangle."

Here we have the arrangement we saw earlier whereby we halve the base-line in order to find the vertical axis, as shown in Fig. 148.

In *Fig. 152* we have constructed the

No. 51. (Pl. O.)

"Example of reckoning a triangle of land.[1] If it is said to thee, A triangle of 10 *khet* in its height and 4 *khet* in its base. What is its acreage?

The doing as it occurs:

You are to take half of 4, namely 2, in order to give its rectangle. You are to multiply 10 by 2. This is its acreage.

1	400	1	1000
$\frac{1}{2}$	200	2	2000

Its acreage is 2.[2]"

Here we meet for the first time the triangle (*śpdt*, "the pointed") and its area. Is this correctly determined? Eisenlohr thought not, and mathematicians have for the most part accepted his dictum. Yet the matter is hardly so simple as he supposed. There are in reality two interdependent problems to be solved. Firstly does the solution given apply to all triangles or only to those of a special type, and secondly is it correct? The evidence at our disposal consists of the following:—

(1) The names of the triangle and its parts.
(2) The position of the numbers marked in the figure.
(3) The shape of the figure.
(4) The striking phrase "This is its rectangle."

Fig. 151.

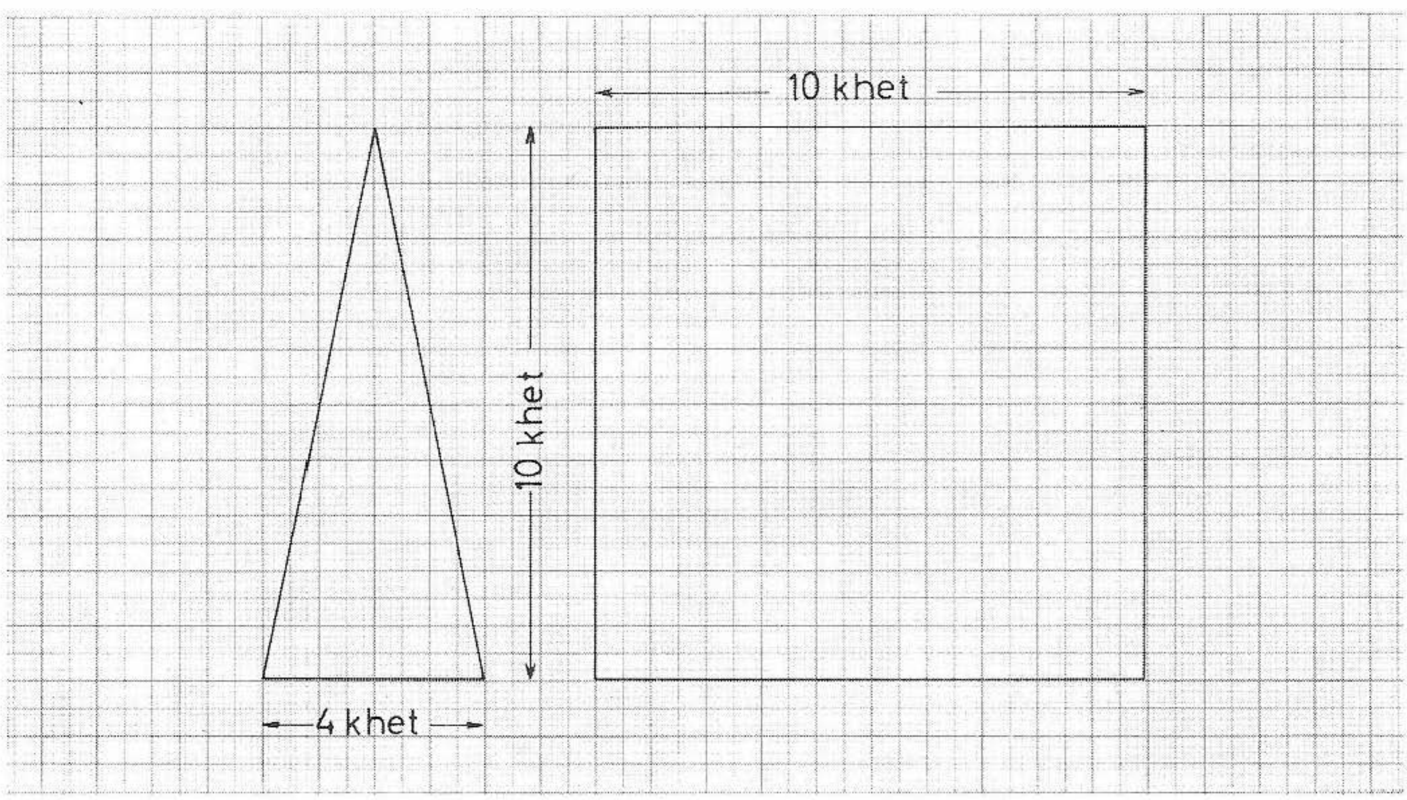


Fig. 152.

triangle mentioned in the above problem and placed it alongside its square (as distinct from its rectangle). The dimensions are as follows: Since the triangle is 10 khet high, the square measures 10 × 10 khet and is thus in fact the large unit of measure we have seen earlier containing 100 setat. Each side of the square is 10 khet in length and since each khet extends across 100 cubits-of-land, the side

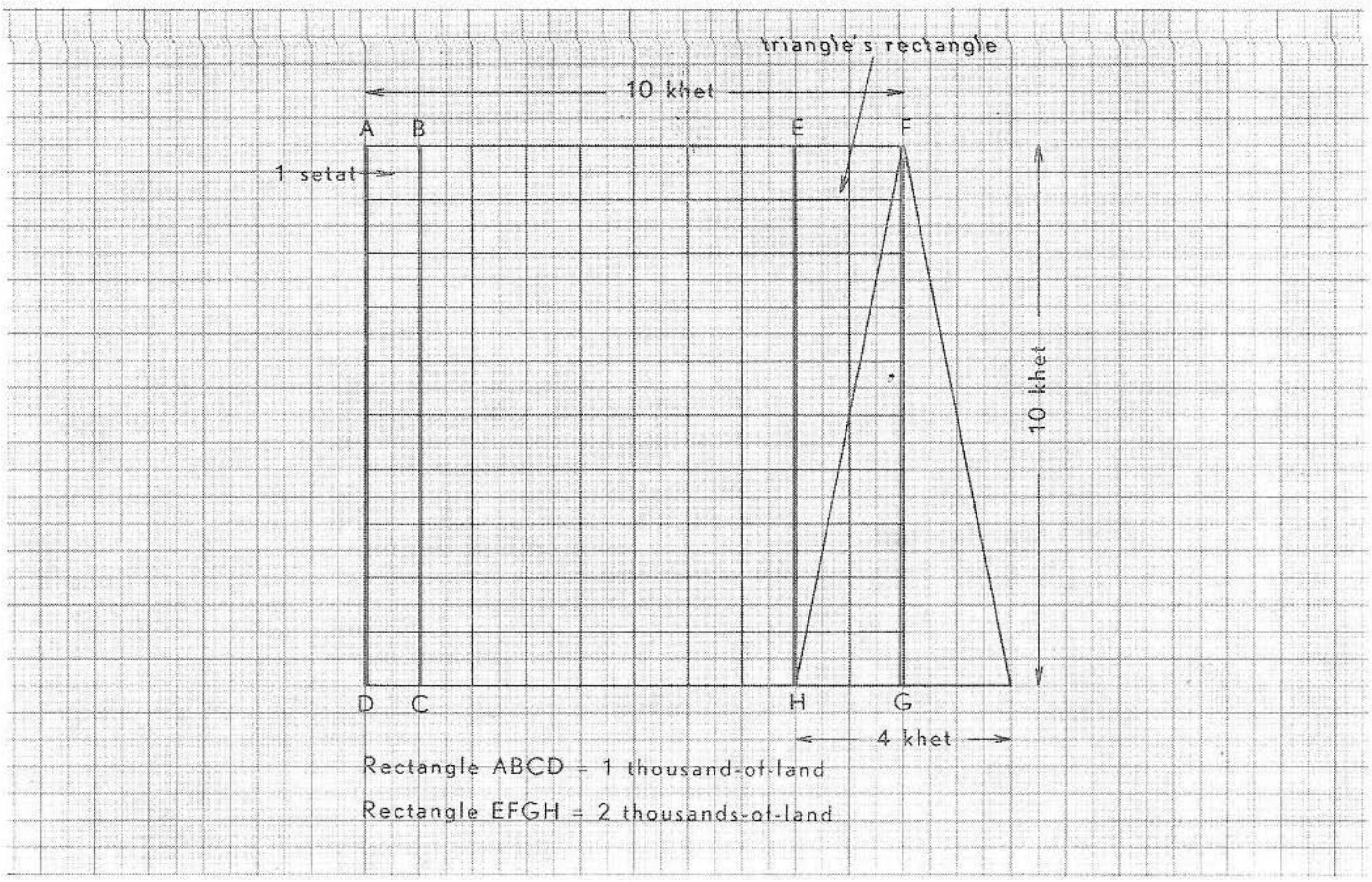


Fig. 153.

of the square measures 1000 cubits. The total length of the triangle's base is therefore 400 cubits, but since we use only half of this in converting to a rectangle the reckoning begins:

total base measures 400 cubits (1 = 400)
half base measures 200 cubits (½ = 200).

This concludes the first part of the reckoning, and the next part states:

whole square contains 100 setat each made up of 100 cubits-of-land.

One strip measuring 1 khet wide and 10 khet long is therefore 10 setat of 100 cubits-of-land:

1 strip = 1 thousand-of-land (1 = 1000)
2 strips = 2 thousands-of-land (2 = 2000).

The result is therefore: 2 thousands-of-land or 2, which is the result stated in the papyrus.

The problem is set out geometrically in *Fig. 153.*

Neither the problem nor its reckoning require, in this geometric form, any comment. Only logical factors and calculations are brought into use, and nothing in this problem remains obscure or unknown. The triangle's rectangle mentioned earlier can quite clearly be seen as equalling the triangle in area.

Our next problem, No. 52 Pl. P, can be seen in facsimile form in *Fig. 154.*

The problem here is to calculate the area of a triangle that remains after the triangle's top is cut off. In fact the area is that of a trapezium with two sloping sides.

It is stated that the trapezium is 20 khet high, 6 khet wide at its base and 4 khet wide at its top (the line of truncation).

It is also stated that the first step is to find the trapezium's rectangle, and the instructions for doing so are absolutely

No. 52. (Pl. P.)

"Example of reckoning a truncated triangle of land. If it is said to thee, A truncated triangle of land of 20 *khet* in its height, 6 *khet* in its base and 4 *khet* in the cut side. What is its acreage?

You are to combine its base with the cut side: result 10. You are to take a half of 10, namely 5, in order to give its rectangle. You are to multiply 20 five times, result 10 (*sic*). This is its area.

The doing as it occurs:

1	1000	✓1	2000
$\frac{1}{2}$	500	2	4000
		✓4	8000
		Total	10000, making in land 20 (read 10)

This is its area in land."

Fig. 154.

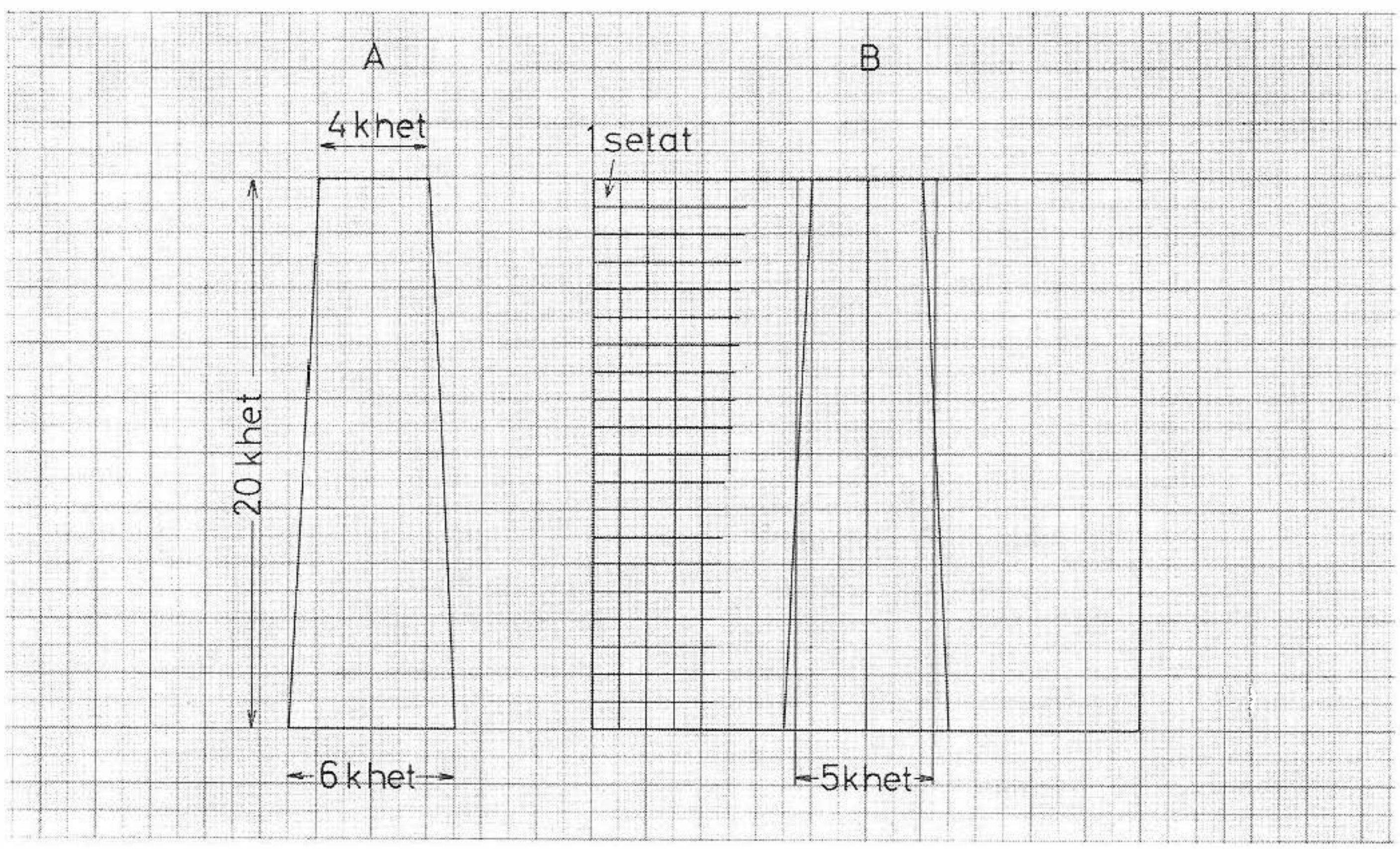


Fig. 155.

sound: adding the base-line and top-line together, then dividing in two.

We see the trapezium and the rectangle executed geometrically in *Fig. 155*.

Since we are dealing here not with a pyramid but with a four-sided figure with sloping sides, it would probably be drawn in the middle of the main square instead of at one side, as in the preceding problem.

Both procedures can of course be applied with equal accuracy, but with a pyramid or triangle it is in practice simplest and most clear to execute it at one side of the square, and with a four-sided figure it is most convenient to construct it around the square's vertical axis. I think therefore that in the main the Egyptians chose the simplest method as long as it conformed to the general system.

No. 56. (Pl. Q.)

"Example of reckoning out a pyramid 360 in length of side and 250 in its vertical height. Let me know its batter.

You are to take half of 360: it becomes 180.

You are to reckon with 250 to find 180.

Result $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$ of a cubit.

A cubit being 7 palms, you are to multiply by 7:

1	7
$\frac{1}{2}$	$3\frac{1}{2}$
$\frac{1}{5}$	$1\frac{1}{3} + \frac{1}{15}$
$\frac{1}{50}$	$\frac{1}{10} + \frac{1}{25}$

Its batter is $5\frac{1}{25}$ palms."

The meaning of this and the following examples depends entirely on the interpretation given to the three terms *wḫꜣ-tbt*, *pr-m-wś* and *śḳd*. It is fairly obvious from the figure that the *wḫꜣ-tbt* is a ground measurement, and if so it can hardly be other than the diagonal or side of the base. Similarly *pr-m-wś* is clearly a measurement of height, not necessarily vertical, and as such it might be either the vertical height, the slant height from a corner to the apex, or the slant height from the mid-point of a side to the apex. The required *śḳd* is clearly the relation of the *pr-m-wś* to half the *wḫꜣ-tbt*. See Fig. 6.

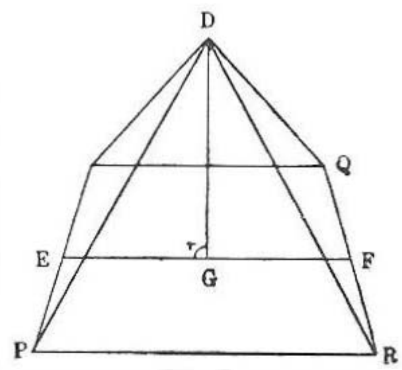


Fig. 6.

Eisenlohr took the *wḫꜣ-tbt* to be the diagonal of the base, PQ, and the *pr-m-wś* to be the slant height from a corner to the apex, DP; the *śḳd* would then be the cosine of the angle DPQ made by the edge with the diagonal of the base. In support of this interpretation he argued that it gives a batter which agrees well with that of certain existing pyramids and secondly that it is unlikely that *wḫꜣ-tbt* and *pr-m-wś* should mean the same as *snti* and *ḳꜣ n ḥrw* of No. 60, which he held to be beyond all doubt the length of the side of the base and the vertical height.

This view is certainly erroneous.[1] In the first place the monument dealt with in No. 60 is not a pyramid but an *iwn*, and there is no reason at all why similar measurements in this and a pyramid should not have had totally different technical names.

Fig. 156.

Thus in Fig. 155 we have at A the area in question and at B we see the area entered at the vertical axis of the square.

The conversion from a trapezium or partial triangle to a rectangle is shown by two vertical broken lines. This shows the rectangle has an area of 20 × 5 = 100 setat, and the answer of 10 given in the papyrus is correct since 100 setat equal *10 thousands-of-land. We can therefore accept the first reckoning without question.*

As a repetition of this calculation we read:

1 = 1000

½ = 500.

This operation involves conversion of the trapezium to a rectangle having two short sides measuring 5 khet, but the rectangle *is of course split by the central* vertical axis of the square so that we have 2½ khet on each side of the axis. We must therefore discover how much each half khet contains.

This calculation takes us back to the normal large unit with sides of 10 khet and containing 100 setat, stating that $\frac{1}{10}$ of this unit (i.e. 10 setat) equals:

1 thousand-of-land (1 = 1000)
half of this is 500 ($\frac{1}{2}$ = 500).

The teacher has thus highlighted the conversion of the area from a partial triangle to a rectangle, and continues:

But a strip of this square measuring 20 khet is divided in the same way at the top as the bottom. Since the strip is twice as long as the normal large unit, however, 1 khet extends in this instance not over the normal 1000 cubits-of-land but over 2000 cubits-of-land.

Therefore:

1 strip = 2000 cubits-of-land (1 = 2000)
2 strips = 4000 cubits-of-land (2 = 4000)
4 strips = 8000 cubits-of-land (4 = 8000)

and since the width of the rectangle is 5 khet (strips), each containing 2000 cubits-of-land, the result is 10,000 cubits-of-land or 10 thousands-of-land (which incidentally is the area of the large unit of measure).

As with the preceding problem, the reckoning and material are clear and concise when seen from a geometric standpoint, and the problem presents no difficulty and leaves no question unanswered.

Problem no. 56 Pl. Q. A copy of this problem is seen in *Fig. 156.*

Here for the first time is a sample of the calculation of angles which we discussed earlier. The problem is definitely designed for advanced students since this particular form of calculation would have been fairly complicated by the Egyptian method. Simpler examples will be shown later but we shall take the problems in the order in which they appear in the papyrus.

Obviously the reckoning of this problem was lengthy and awkward, and for this reason the Egyptians themselves did not include the full details, content merely to state the result.

The information we have is that a pyramid is 250 high and 360 wide at its base; no indication is given of the unit of measure involved, nor in fact is it necessary since we are not asked for the area or volume of the pyramid but for its batter.

The instructions start by halving the base-line, giving two parts each of 180.

We now recognise the procedure and can see the problem illustrated geometrically in *Fig. 157* and we can turn our attention straight away to the triangle's rectangle, seen as ABCD.

We know the height of the pyramid to be 250, and as a reminder we are told, "You are to reckon with 250 to find 180."

The area of the square is thus

$$250 \times 250 = 62500$$

Area of the rectangle is thus

$$250 \times 180 = 45000$$

It is now stated that the result is

$$\frac{1}{2} + \frac{1}{5} + \frac{1}{50} \text{ of a cubit.}$$

As with the wider significance of palms and fingers as units of proportion rather than of length, so here too the cubit must be recognised as being a unit of proportion rather than of length, since the short side of the rectangle (and therefore half the base-line of the pyramid) is said to be $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$ of a cubit, i.e. of the square constructed on the pyramid's vertical axis. Let us see how this works out:

$$\frac{1}{2} = {}^{62500}\!/_{2} = 31250$$
$$\frac{1}{5} = {}^{62500}\!/_{5} = 12500$$
$$\frac{1}{50} = {}^{62500}\!/_{50} = 1250$$
$$\overline{45000}$$

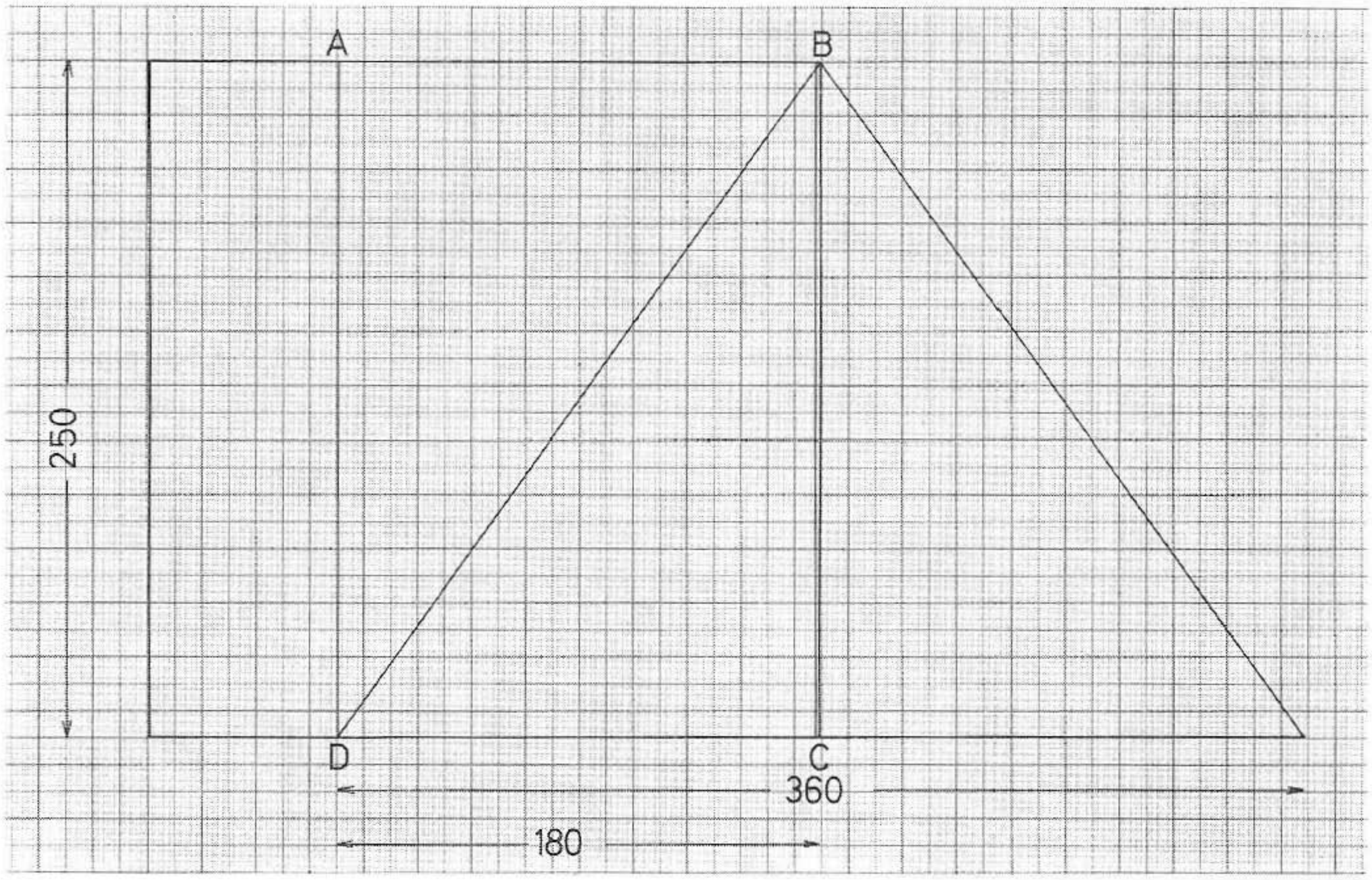


Fig. 157.

Thus we have complete agreement between the assertion and the facts.

This agrees absolutely with the results produced by our method and that of the Egyptians, but whereas we have shown how the result was achieved, the Egyptians thought it necessary to state only the final result.

The next piece of arithmetic is to find just how many units of the square's base-line of 250 are to be found in 1 palm.

This is achieved simply by dividing 7 into 250 and producing the answer 35.71 units in 1 palm. Since we know that the base-line of the triangle's rectangle occupies 180 of the square's 250 units, and since the task is to discover how many of the 7 palms are occupied by 180 units (in order to obtain the batter) we must say:

$^{180}/_{35.71}$ = 5 palms + an excess of 1.45 units, since 5 $\times$ 35.71 = 178.55.

We see therefore there are slightly more than 5 palms in the base of the triangle's rectangle, and it is precisely to determine this difference that we see the next piece of calculation in the papyrus problem.

This starts by saying 1 = 7. This indicates of course that the whole of the square's base-line equals 7 palms.

The reckoning continues: $\frac{1}{2} = 3\frac{1}{2}$, meaning that half the base equals $3\frac{1}{2}$ palms.

At this point in the reckoning there is a definite break with standard Egyptian practice, which as we know normally continues by halving 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. We suddenly find the problem stating: $\frac{1}{5} = 1\frac{1}{3} + \frac{1}{15}$.

By this is meant that $\frac{1}{5}$ of the base-line is equal to 1 palm + $\frac{1}{3}$ palm + $\frac{1}{15}$ palm.

As we know from our modern means of calculation how many units make up 1 palm, i.e. 35.71, we can test this statement:

1	palm	= 35.71	units
$\frac{1}{3}$	palm	= 11.90	units
$\frac{1}{15}$	palm	= 2.38	units
		49.99	repeater

As we know that the base-line is 250 units, then $\frac{1}{5}$ of this is $\frac{250}{5} = 50$, and since 1 palm is in fact 0.0043 larger than 35.71 as this cannot be calculated precisely, and as the same applies to $\frac{1}{3}$ palm and $\frac{1}{15}$ palm, we have to admit that 49.99 comes as close to the result as is otherwise possible.

Why the Egyptian jumps from $\frac{1}{2}$ of the base-line to $\frac{1}{5}$ is difficult to say. He would have little trouble in working out $\frac{1}{5}$ of the 250 units in the base-line, but how he managed to convert 50 units into palms and achieve such a close fraction as $1 + \frac{1}{3} + \frac{1}{15}$ is perhaps best explained by a suspicion that this problem was possibly solved not by reckoning alone but also by measuring on a suitably proportioned drawing. This may in fact be the reason that much of the intermediate reckoning is lacking, and that only the concrete observations were established as facts.

Finally he produces the calculation $\frac{1}{50} = \frac{1}{10} + \frac{1}{25}$. By this he states that $\frac{1}{50}$ of the base-line is the same as $\frac{1}{10} + \frac{1}{25}$ palm.

This, too, we discover is very nearly correct:

$$\frac{1}{10} \text{ of } 35.71 = 3.571$$
$$\frac{1}{25} \text{ of } 35.71 = 1.428$$
$$4.999.$$

The base-line is 250, and $\frac{1}{50}$ is thus $\frac{250}{50} = 5$.

Once again we have employed modern methods of reckoning to check that his assertion was correct, and this latter statement brought us as close as $\frac{1}{25}$ of 1 palm.

The result is 1.428 and we recall from our first piece of working that our aim was to establish what part of 1 palm 1.45 units represented.

We have arrived at a size which is very close indeed ($\frac{1}{25}$ palm = 1.428 units), and the Egyptian then states that the batter of the pyramid (or the proportion of half its base to the length of its vertical axis of 7 palms) was $5 + \frac{1}{25}$ palms.

There is an error here of $\frac{22}{1000}$ palm which we can discover by our modern means of reckoning, but it is most unlikely that the Egyptians were able to spot this difference.

The structure of the problem indicates —as I stated earlier—the possibility that the result was achieved partly by measurement and partly by reckoning, but of course it is not at all impossible that reckoning alone was used. Only, if that was the case, we might have expected to find one or two notes to assist in calculation. As these are completely lacking and only the result is shown, then measurement is the most probable solution.

We have been able to follow the text and instructions at every stage and with our modern means of calculation have checked that the results produced by the Egyptians tally precisely with the theory I have put forward in respect of the problem, and in fact nothing about the problem remains incomprehensible if seen from this viewpoint.

Problem no. 57 Pl. Q is shown in facsimile form in *Fig. 158* where we gather from the commentary that some doubt is evident to the modern way of thinking about the reckoning of this problem. But as with the other problems it depends on how we approach the matter. If we apply the thought process and terminology used by the ancient Egyptians themselves then the problem shines through in complete clarity.

The first fact we are given is that the

No. 57. (Pl. Q.)

"A pyramid 140 in length of side, and 5 palms and a finger in its batter. What is the vertical height thereof?

You are to divide one cubit by the batter doubled, which amounts to $10\frac{1}{2}$.[1] You are to reckon with $10\frac{1}{2}$ to find 7, for this is one cubit.

Reckon with $10\frac{1}{2}$: two-thirds of $10\frac{1}{2}$ is 7.

You are now to reckon with 140, for this is the length of the side:

Make two-thirds of 140, namely $93\frac{1}{3}$. This is the vertical height thereof."

In this example we are given the length of the side and the amount of the batter in palms and fingers (a finger being one-fourth of a palm) per vertical cubit. We are asked to find the vertical height.

The working, though correct, is slightly obscured to our modern way of thinking by the fact that, instead of finding EG (see Fig. 6, p. 97) from the datum EF by halving, and then determining DG by means of the batter, the batter is doubled (twice $5\frac{1}{4}$ palms = $10\frac{1}{2}$) and the proportion used is:—

$$DG : EF :: 7 : 10\tfrac{1}{2}$$

There is an admirable parallel to this illogical style of working in No. 45.

Fig. 158.

problem deals with a pyramid, i.e. geometrically a triangle.

We are told of this triangle that its base is 140 in length and that its batter is 5 palms and 1 finger.

The problem is to find the height of the triangle, which will enable us to draw the triangle in its proper dimensions.

With the information at our disposal the calculation is a simple procedure.

We start off with the base-line, which is stated to be 140. The vertical axis of the triangle must rest in the middle of this base-line since the triangle in question is most certainly isosceles. If not, we would have been given not one batter, but two.

We are now told that the batter is $5\frac{1}{4}$ palms, and we know from our experience that the batter of a triangle can be obtained or read by comparing the base-line of the triangle's rectangle with that of its square.

With half the pyramid's base-line we have the short side of the rectangle. We are told that this is $5\frac{1}{4}$. Since we know that the base-line of the square is 7 palms, the problem remains to calculate this on the basis of the rectangle's base-line; for once we have ascertained the square's base-line we are able to construct the square, and since one of its sides is in fact the vertical height of the triangle, our problem is solved.

We say:

$5\frac{1}{4}$ palms = 70 units (being half triangle's base)

$21/4$ palms = 70 units

$1/4$ palm = $70/21$ = $3\frac{1}{3}$ units

1 palm = $4 \times 3\frac{1}{3}$ = $13\frac{1}{3}$ units

7 palms = $7 \times 13\frac{1}{3}$ = $93\frac{1}{3}$ units.

The base-line of the square is thus $93\frac{1}{3}$, which is also the height of the triangle and therefore the solution to our problem.

The problem is laid out geometrically in *Fig. 159*.

We can see that the answer we have arrived at is the same as that produced by the Egyptians. On this point therefore complete agreement.

We can further see that the problem is extremely pertinent and logical and in-

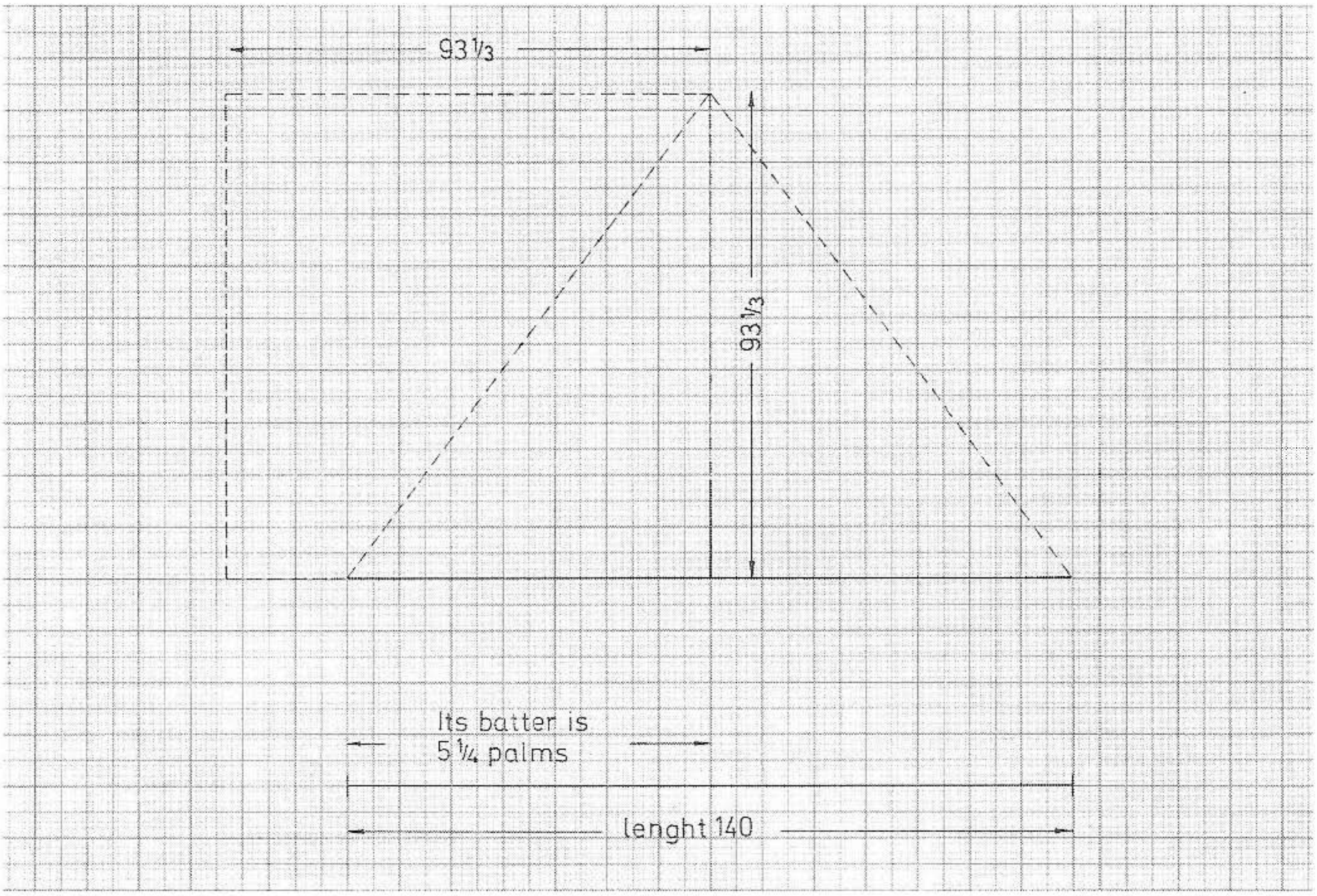


Fig. 159.

volves clear-cut and familiar geometric factors.

But we have achieved the actual solution by modern methods of calculation, multiplying and dividing as we do normally.

The method adopted by the Egyptians is slightly different although basically the same as ours.

The instruction is to "divide one cubit by the batter doubled. You are to reckon with 10½ to find 7".

The teacher chooses to say that when the batter, which is half the triangle's base-line, is 5¼ palms, the whole base-line is 10½ palms. And with this as his starting point he proceeds to reckon out how much 7 palms amounts to.

He then says that 7 palms is ⅔ of 10½ palms, i.e. the cubit which equals the height of the pyramid is ⅔ of 140.

$$\frac{140 \times 2}{3} = 93\tfrac{1}{3}$$

The result he achieves is of course the same but instead of working out the 7 palms in relation to 5¼ palms, he apparently considered it simpler to illustrate the relationship of 7 palms to the base-line's 10½ palms, and from this multiplied the base-line by the resultant fraction.

The result speaks for itself. The Egyptian procedure is both logical and relevant to this problem, it is simply a matter of deciding which size is chosen for gauging the 7 palms—which equal 93⅓ units of unknown proportions.

Problem no. 58 Pl. Q is seen in *Fig. 160.*

This problem concerns the same pyramid (or triangle) as dealt with in Problem 57, but whereas in the latter problem we were told the pyramid's base and batter, with instructions to find its height, here we are given the height and base and told to find its batter.

No. 58. (Pl. Q.)

"A pyramid whose vertical height is $93\frac{1}{3}$. Let me know its batter, 140 being the length of its side.

You are to take half of 140, namely 70. You are now to reckon with $93\frac{1}{3}$ to find 70.
Reckon with $93\frac{1}{3}$: its half is $46\frac{2}{3}$,
its quarter is $23\frac{1}{3}$.

You are to make $\frac{1}{2}+\frac{1}{4}$ of a cubit. Reckon with 7; its half is $3\frac{1}{2}$; its quarter is $1\frac{1}{2}+\frac{1}{4}$; total 5 palms 1 finger. This is its batter.

Working out:

1	$93\frac{1}{3}$
/ $\frac{1}{2}$	$46\frac{2}{3}$
/ $\frac{1}{4}$	$23\frac{1}{3}$

You are to make $\frac{1}{2}+\frac{1}{4}$ of a cubit.
Now a cubit is seven palms.

1	7
$\frac{1}{2}$	$3\frac{1}{2}$
$\frac{1}{4}$	$1\frac{1}{4}$ (read $1\frac{1}{2}+\frac{1}{4}$)
Total	5 palms 1 finger.

This is the batter."

This example is concerned with the same numbers as the last, but here we are given the length of the side and the height and are asked to find the batter. The method is logical and consists simply in halving the side and dividing the resulting 70 by the height $93\frac{1}{3}$. The numbers are suitably chosen, for 70 is precisely $\frac{3}{4}$ or $(\frac{1}{2}+\frac{1}{4})$ of $93\frac{1}{3}$. It then only remains to reduce $\frac{1}{2}+\frac{1}{4}$ of a cubit to palms and fingers, which is done by multiplying 7 by it. Result $5\frac{1}{4}$ palms or 5 palms 1 finger.

iw ir mḥ 1 *šsp* 4 *pw*. The position of *iw* in front of *ir* is interesting syntactically. Cf. *iw ir didit ḥr nb dbn* in No. 62. Gunn quotes also *Urk.*, IV, 366, 13.

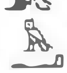
The traces after the numeral do not suit  (for its use in the Nominal Sentence with *pw* cf. Nos. 64, 70 and 71 and notes), nor yet  read by Borchardt.

Fig. 160.

We recall that in Problem 57 the Egyptian calculated the height of the square (and therefore pyramid) on the basis of the base-line's $10\frac{1}{2}$ palms. In this new problem, however, he takes the more direct path via the square to discover the $5\frac{1}{4}$ palms answer.

First he constructs the triangle with a height of $93\frac{1}{3}$, and uses this height to construct the square.

This geometric figure is in fact the same as Fig. 159 and is repeated in *Fig. 161* where we have square ABCD and the point of the triangle's batter at E.

The instructions now read, "You are to reckon with $93\frac{1}{3}$ to find 70."

We know that line CD is $93\frac{1}{3}$ and that its value in palms is 7. We know, too, that line CE is 70. The problem is to reckon this in palms and fingers.

The Egyptian knows from the previous problem that the ratio of the batter to the square's base-line is 3:4, and he states:

total base-line = $93\frac{1}{3}$ (1 = $93\frac{1}{3}$)
half base-line = $46\frac{2}{3}$ ($\frac{1}{2}$ = $46\frac{2}{3}$)

This length is marked on the base-line by point G.

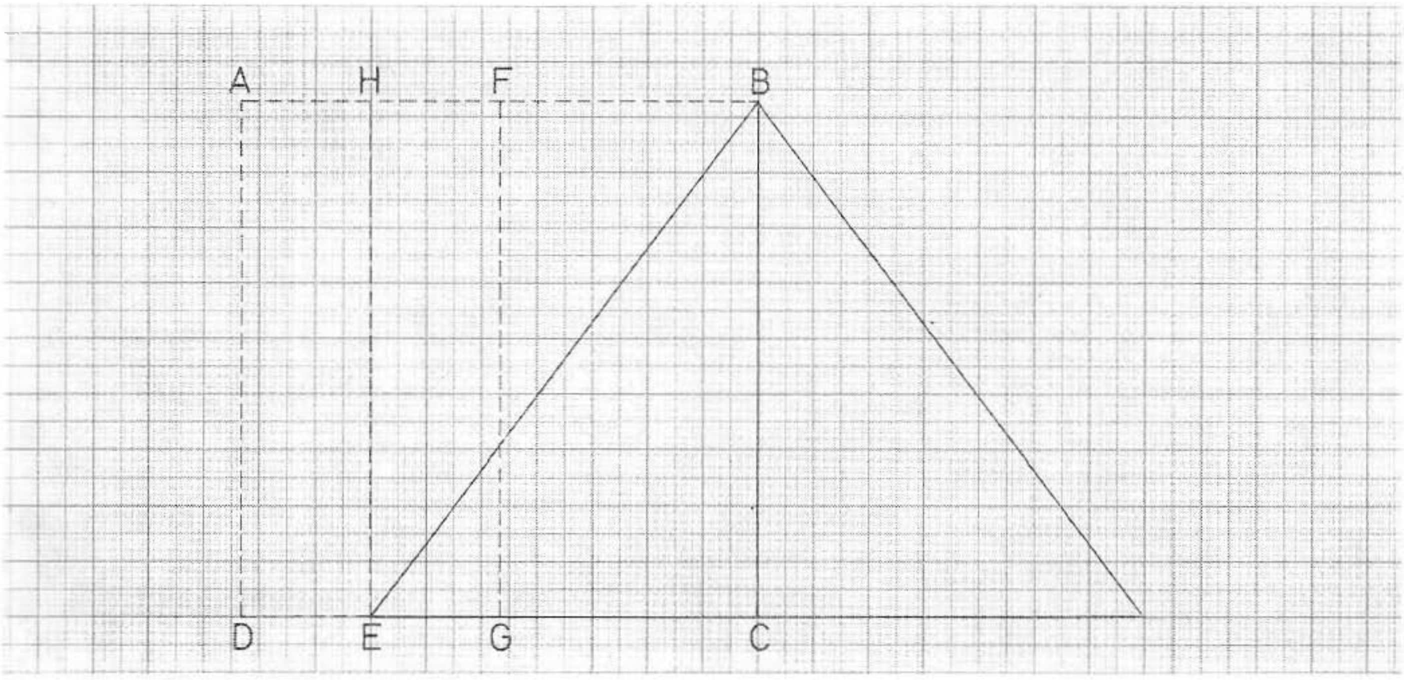


Fig. 161.

He continues:
quarter base-line = 23⅓ (¼ = 23⅓)

Indicated on base-line by point E.

HE is in fact the line which would constitute one side of the triangle's rectangle, and since the point (E) at which this line meets the base-line indicates the point of calculation of the triangle's batter (ratio of CE to CD) the problem is solved geometrically; the Egyptian teacher can say that line CE is ½ + ¼ (CG + GE) of the base-line, thus:

half base-line	=	46⅔
quarter base-line	=	23⅓
		70 units

Since the total base-line represents 7 palms he goes on to complete the problem:

total base-line = 7 palms
(1 = 7)
half base-line = 3½ palms
(½ = 3½)
quarter base-line = 1½ + ¼ palms
(¼ = 1½ + ¼)

No. 59. (Pl. Q.)

"A pyramid the vertical height (*sic*) whereof is 12 and the side (*sic*) 8. ⟨Find its batter.⟩
You are to reckon with 8 to find 6, for this is half the height.

1	8
⁄ ½	4
⁄ ¼	2

You are to take a half and a quarter of 7, for this is 1 cubit.

1	7
⁄ ½	3½
⁄ ¼	1½ + ¼

It comes to 5 palms 1 finger. Behold this is its batter.
What . . . . . ?"

Fig. 162.

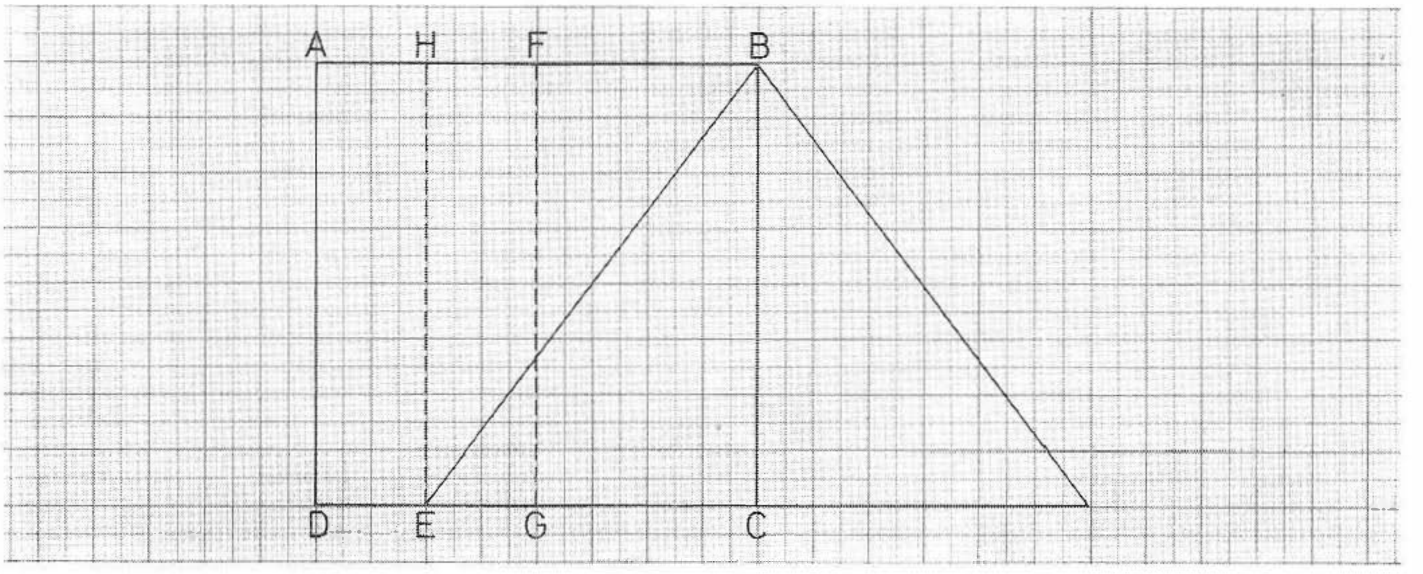


Fig. 163.

This provides him with a picture of the number of palms in line CE. For he knows that this line is ½ + ¼ of the base-line, and can thus calculate that CE contains 5¼ palms or 5 palms + 1 finger. This is its batter.

A facsimile of *Problem no. 59 Pl. Q* is seen in *Fig. 162* and, as Peet's comment indicates, there is apparently an error (on the part of the Egyptian scribe) in the setting out of this problem.

The problem deals with a triangle or pyramid which, we are told, has a height of 12 and a side of 8.

The mistake is obvious, if the person who set the problem meant as we do by "side". If the height or vertical axis of the pyramid is 12 then the side or hypotenuse must without question exceed 12. He cannot therefore be referring to the side.

Since none of the other problems mentions the length of the hypotenuse it is possible that they had no means of reckoning its length. We might therefore suspect that it is in fact the base that is 8. But this does not fit the information and result of the problem, i.e. that its batter is 5 palms and 1 finger.

It is only when we transpose the figures, making the height 8 and the *base* 12 (N.B. Not, as Peet suggests, the *side*) do we equate the problem with the result shown.

We see the geometric explanation of the problem in *Fig. 163* in which the triangle's square is ABCD and the triangle's batter lies at E.

The square's base-line CD is 8 and half the triangle's base-line is CE = 6.

We now have to find the ratio between these two lines. The Egyptian proceeds as follows. First he notes that the base of the square equals 8, and writes:

$$1 = 8.$$

He then splits the square in two halves by its vertical axis (FG) and can then state:

$$\tfrac{1}{2} = 4$$

since each half of the base-line covers 4 of the unnamed units.

This provides two rectangles, and since the triangle's batter lies within rectangle AFGD it in turn is split in two in order to come closer to the point of batter. These two new rectangles each cover ¼ of the square's base.

The working thus states correctly:

$$\tfrac{1}{4} = 2.$$

The geometric line of division is HE.

No. 59B. (Pl. Q.)

"You are to reckon a pyramid of 12 (*sic*), whose batter is 5 palms 1 finger. Let ⟨me⟩ know the height thereof.

You are to reckon with $5\frac{1}{4}$ doubled to find 1 cubit, which is 7 spans: it comes to $10\frac{1}{2}$. Two-thirds of it is 7.

Reckon with 12: two-thirds of it is 4 (read 8). Behold this is the height."

Nos. 59 and 59B are two quite distinct problems, the second of which is the reverse of the first. Unfortunately the scribe did not realize this, and has made No. 59B appear as part of the working of No. 59 by introducing it by the word *ỉr·ḥr·k* and failing to use red ink for the opening word or words. The verbal form *śḏm·ḥr·k* is never used in this papyrus to introduce a problem. In the original from which our scribe copied the problem was probably, like its fellows, introduced directly by *mr* "a pyramid." It is possible that this was in black instead of being, as it should be, in red. We know enough of our scribe's intelligence to assert that this would be quite sufficient to conceal from him the fact that a fresh problem had begun.

In No. 59 there is an unfortunate error in the setting out, for the side and the height have been transposed. In order to give a batter of 5 palms 1 finger it is the height which must be 8 and the side 12. Otherwise the batter would be only $2\frac{1}{3}$ spans.

The phrase which ends the problem is a mystery. It may be corrupt. Is it connected with the scribe's error in running the next problem on to this?

In No. 59B we are given the side and batter and asked to find the height. After the words "a pyramid of 12" we expect *m wḫ₃-ṯbt·f* "in its side"; but it is quite possible that the Egyptian is sufficient as it stands, and that "a pyramid of 12" was the technical expression for "a pyramid built on a base 12 square."

Fig. 164.

As with most problems in any good primary mathematics class this works out exactly!

HE joins the base-line precisely at the same point as the side of the triangle. We can say therefore that line CE occupies ¾ of line CD.

We know that, in addition to comprising 8 units of division, line CD is made up of 7 palms. We must now therefore find the ratio of the 8 units to the 7 palms.

On the same lines as in the above working, we say:

total base-line is 7 palms
(1 = 7)
half base-line is 3½ palms
(½ = 3½)
quarter base-line is 1½ + ¼ palms
(¼ = 1½ + ¼).

The base of the square CD was 8. Half the triangle's base (and therefore its batter) was CE which was ¾ of CD, i.e. ½ + ¼ of the square's base.

Thus we can conclude by saying:

half base-line	=	3½	palms
quarter base-line	=	1½ + ¼	palms
totals		5¼	palms

Our answer is thus 5¼ palms and since ¼ palm equals 1 finger, the correct result—as in the papyrus—is 5 palms + 1 finger.

Fig. 164 is a facsimile of 59 B; it confirms our transposition of figures in the preceding problem. In this instance we are given a triangle/pyramid with a batter of 5 palms 1 finger and a base (again Peet in his comment wrongly says "side") of 12. The object is to calculate the triangle's height.

We are familiar with all the figures from the previous problem and indeed saw the calculation of a similar exercise in No. 58. We need not therefore go through the motions of working out this problem It is included merely to illustrate that our suspicions regarding the height/base were well founded.

*

After this lengthy study of a number of well-known ancient Egyptian mathematical problems we can, I think, establish that this diverse material forms a logical unity only when seen from the standpoint of ancient geometry. The certainty with which we have elucidated the mathematical working of these problems is so evident that there can be no doubt concerning the procedure's accuracy.

It would have been impossible with a doubtful or unfounded system to work out all the examples shown in the preceding pages, to follow closely the intermediate working, and to achieve a correct result.

And of course the system was not something specially invented for Egyptian mathematics. We have seen how the theory behind the Nile mathematics was part (and only part) of a wider subject, i.e. the vast knowledge of geometry that existed in numerous other spheres, too.

The problems are the first known attempts to convert geometry into figures and sums. Hence the initial problems are uncomplicated—as long as we are familiar with the basis on which they are founded.

It must consequently be the case that mathematics sprang from geometry and its many aspects, rather than the reverse.

I regard the problems in *Rhind Mathematical Papyrus* to be proof that this assumption is correct.

CHAPTER TEN

Pythagoras - and a Geometric Analysis of Plato's Timaeus

MOSES' ESCAPE from Egypt at the head of the Jews was discussed in an earlier chapter where we saw how he passed on the geometric aspect of his wisdom through the medium of the building instructions for the Tabernacle. In this way he handed over his knowledge to those among the Israelites whom he found worthy of initiation. It is to be expected that this knowledge spread to a certain extent to countries bordering the land of Midian, finally to reach the temples of Jerusalem where the traditions were adapted to surroundings and incorporated in holy teaching in that city. But we can trace the dissemination of Egyptian tradition as far as our own civilisation through other sources than Moses. For although the Book of Exodus today occupies a place on the book-shelves of many homes as a part of the Book of Life, the key is missing. And the text has retained its hidden significance constantly throughout time.

Egypt for thousands of years remained the inspirational centre for the whole of Africa and Europe, and travellers came from every compass point to soak in the knowledge and wisdom of the Egyptian Temple.

When the time came for these students after years of education to leave the Temple they were obliged to pledge never to enlighten an outsider on the knowledge they had obtained; and these pledges were impeachable: woe to the foolhardy soul who ventured to profane the sacred knowledge which had over a period of uncountable centuries been hidden from the public eye by veiled frontages and symbolism of the Temple, knowledge which had never before leaked through the shield of concealment. Even an initiate of the lower degrees was banned from obtaining the genuine article too early in his training. It was released little by little as he progressed in his Temple instruction.

Greece is generally looked upon as the birth-place of mathematics and the nation responsible for handing this sphere over to modern civilisations. This is based upon the fact that most surviving written material on the subject of mathematics and geometry is of Greek origin. The tendency —admittedly an inviting one—is therefore to assume that the knowledge passed on by the Greeks in fact sprang into being in that country. Little consideration is given to the material that Grecian mathematicians and philosophers themselves received from yet earlier civilisations.

Greece was perhaps the country that first broke with the age-old esoteric tradition, and partially burst the bonds of secrecy. It was Pythagoras of Samos, after living 34 years in Egypt and Babylon, who returned to Greece and opened the world's first school (as we today recognise the term) with education in elementary subjects and the philosophy of life.

This school, in the pattern of the Temple, operated a process of admission by which the pupil's courage and ability were put on trial. But as was perhaps only natural admission to the Pythagorean school was not nearly such a strict procedure as a full-blooded temple initiation.

The school had the same tradition of secrecy as the temples, holding all learning within a tight circle of students, but as with the process of admission this essential secrecy too was loosely observed.

Pupils were educated in geometry and counting from the earliest classes, Pythagoras's opinion being that practice in these spheres was the finest training in independent thought, but of course *all* the secrets of ancient geometry were not released to students.

Numerous other aspects of geometry and numbers could be taught and practised, without directly profaning the real Temple learning.

The position is likely to have been that pupils who attended the school for a suitable period of time and who were regarded by Pythagoras as worthy men were at some stage initiated in the genuine arts with ancient geometry (the principle of divinity) as the major subject. But this only for the few who had demonstrated their reliability and had kept lesser secrets to themselves. Moreover the ancient knowledge had been released on exchange for an unbreakable vow of silence.

There was no obligation on the part of pupils to remain at the Pythagorean school for a predetermined length of time, and the large group of students who frequented the lower classes of the school began—once they had gone as far as they could without being accepted for further training by the old Master—to philosophise on their own about the material now in their possession. And they developed their own theories. In this way a considerable corps of thinkers arose, men with a certain insight into numbers and their handling, men who at the same time realised the significance something called geometric proportion, proportions which the philosopher could develop. This was a completely fresh concept in the cultural development of people devoid of a Temple initiation.

This was how the philosophy of geometric thinking leaked out of the Temple and to a limited degree became public property. Naturally the first flush of secret knowledge sparked off both accurate and erroneous thoughts and theories.

But this independent stream of theorists held the undoubted advantage over the occult that they were free to write about their results. And these written thoughts were hailed by a culture-hungry populace and quickly became widespread, compared with the knowledge of the Temple which was reserved for initiates.

Thus over a certain period two groups developed in Greece, each with its own distinct though related views on geometry. One group was made up of the temples and their brethren, the other of the free thinkers who worked with geometry for its own sake on a numerical basis.

But in fact both groups sprang from the same source: the material handed down to the Greeks by the Egyptians.

This was a period of conflict for mathematics in Greece. It shines through the following piece of dialogue from Plato's *Timaeus* in which Timaeus has launched into his account of the World's creation. As shown earlier, he engages in secret geometry to illustrate his tale:

"The creator's ... purpose ... was, firstly, that it should be as complete a living being as possible, a whole of complete parts, and further, that it should be single and there should be nothing left over out of which another such whole could come into being, and finally that it should be ageless and free from disease. For he knew that heat and cold and other things that have powerful effects attack a composite body from without, so causing untimely dissolution, and make it decay by bringing disease and old age upon it."

We have here Timaeus's excellent defence of the geometric directions used in the rest of his account.

He states that the world's geometric likeness must be a complete being, consisting of complete parts, namely squares, rectangles and triangles. It should be the only one of its kind since from an esoteric standpoint there could not and must not exist any geometric truth other than the ancient accepted form.

The system was not rendered defective by the faults alleged by outsiders, such as old age (which in this context meant old-fashioned influence) or disease (which here meant inaccuracy).

His final remark indicates the will of the creator in respect of the powerful opposition which will face the ancient system, and it is a point of corroboration that the passage concludes by stating that the system is attacked from the outside, i.e. by the group outwith the Temple. This same outside group would bring, he says, disease and old age to the secret system.

By the word "disease" we normally infer sickness in a human being or animal. The application of the term to an inanimate object or system must infer faults or errors.

The term "old age" on the other hand cannot be applied to something which is brought upon a thing, a system or a human being. Old age is something that time alone can produce.

Reference here to disease and old age obviously indicates that the words are used to disguise other concepts, namely inaccuracy and obsolete systems. The whole of the last sentence—in my opinion—builds up and supports this theory.

This defence by Timaeus of his geometric views was due probably to the fact that external opposition had become so great that one could no longer tell whether some of his listeners—all of whom were initiated brethren—had in fact lent an ear to the new theories that had begun to penetrate from every direction with increasing force. In some instances the new thoughts attracted initiates and lured them from the ancient system since many had—blindly—acknowledged geometry to be a birthright from the past, based on faith, without becoming familiar with its defence or proof; whereas the emergent group—with no tradition to build upon—were so much more able to illustrate the proof of their theories.

In fact we are in a rather similar situation today with mathematical education. The maths student is loaded with a pile of books full of formulae he is obliged to accept at face value. Only in one or two instances is it possible to test the veracity of the propositions. It is quite another matter that the formulae have been at one time the result of practical tests. The pupil who is told by authority, i.e. his teacher, that such-and-such a formula is the case must accept this and cannot doubt its accuracy. This is the parallel drawn with the secrets of ancient geometry.

The ancient student was also told certain things as fact and was obliged to take them at face value.

The other group, i.e. the group outside the Temple, attacked these statements and propositions fiercely and in their turn were sternly resisted by the brethren of the Temple. This resistance in fact was one of the main factors that brought about the collapse of Pythagoras's school of learning, which was finally abolished from Greece; the Temple eventually could not tolerate a development which threatened its very foundation.

But the path from a critical murmuring to a victorious overturning of accepted facts is a long one. It is today and it was in ancient days. The Temple had the tremendous advantage of tradition, and encompassed brethren with a splendid gift of speech who lent their able tongues to the defence of established theory. We shall see later how the temples fought continuously and stubbornly to maintain their own points of view in the face of outside developments. We shall also be able to trace the ancient traditions up to the Middle Ages, when the system of ancient geometry eventually dies out.

Possibly the greatest sage of them all and the man who conveyed Egyptian wisdom to Greece was Pythagoras of Samos. His renown is partly due to his immense personality the effects of which are felt even in modern times, and his Pythagorean style of school spread after his death all over Europe, promoted by his pupils and his pupils' pupils.

But part of Pythagoras's fame is also accountable to the fact that he was in the vanguard of Grecian mathematical philosophers.

The following list illustrates how the great majority of these philosophers came after the Pythagorean era. The dates are as accurate as history permits.

Pythagoras of Samos	600	B.C.
Theodorus of Cyrene	570	B.C.
Protagoras of Abdera	485–411	B.C.
Hippocrates of Cos	470–400	B.C.
Socrates of Athens	469–399	B.C.
Archytas of Tarentum	430–365	B.C.
Plato of Athens	428–347	B.C.
Eudoxus of Cnidus	409–356	B.C.
Aristotle of Stagira	384–322	B.C.
Euclid of Alexandria	323–285	B.C.
Archimedes of Syracuse ...	287–212	B.C.

We see from this chronological list that Pythagoras had been dead for well over 200 years by the time Euclid wrote *The Elements of Geometry*, regarded today as one of the foundation works of Greek mathematics.

Pythagoras

THE PYTHAGORAS story makes interesting reading. The precise date of his birth is cloudy but generally reckoned to be somewhere between 600 and 570 B.C. His place of birth, Samos, was one of the most flourishing islands in the Ionian group, just off the coast of present-day Turkey. His father was a rich merchant according to some accounts, and a maker of seals according to other sources.

A handsome lad even from early boyhood, Pythagoras proved to have an extraordinary intelligence and to possess a sense of justice and power of judgment far in advance of his years.

During his youth he frequented the schools of learning run by the Ionian temples and his mentors included, among the finest brains of his era, Pherecydes of Syros and Thales of Miletus (one of the Seven Sages).

But his urgent passion for knowledge was insatiable, and as a scholar of 20 he sought the assistance of the Ionian tyrant king, Polycrates, whose friend and martial ally at that time was the Egyptian king, Amasis. It was through the Egyptian ruler that Pythagoras made contact with the Temple of Memphis and after considerable difficulty the young Samian was admitted to the Temple as a junior brother.

Many years were to pass and innumerable tests and stiff trials to be undergone before he achieved the knowledge dispensed to higher degrees of the Order, but behind his seemingly gentle exterior Pythagoras hid a stubborn and invincible will-power. It saw him through. After the customary 22 years of learning he began turning his thoughts once more in the direction of Samos and Greece.

But war was to delay his return. Cambyses III, warrior son of Cyrus the Great (who had conquered and mastered Media, Persia and Babylonia and founded the Persian Empire), in the battling tradition of his father fell upon the giant kingdom of Egypt. In 527 B.C. this commercial and intellectual giant of the Ancient World, whose influence stretched to all the Mediterranean countries from Pheonicia over Greece to Etruria, submitted to the conqueror.

Pythagoras witnessed the invasion and the terrible aftermath. He saw the temples of Memphis and Thebes burn to the ground. He saw the Pharaoh and his family as well as a horde of young courtiers dragged to the scaffold and executed. He saw the same treatment meted out to some of his fellow students and temple teachers. His education was complete. He had seen Man at the pinnacle of culture—now he watched him at the depths of barbarity.

Pythagoras himself was one of the lucky ones. He was clamped in irons and taken to Babylon as a prisoner of the Persian master.

Cambyses, despite his bestiality, recognised the source of power and strength of his fallen foe: a vigorous army of priests, steeped in tradition and knowledge. This was an excellent opportunity to take a collection of these Egyptian priests back with him to Babylon to inspire his own religious leaders and perhaps to give them the benefit of Egyptian teaching.

The modern equivalent would be to seize machinery, drawings, plans, patents. But in ancient days there was only one course: to carry away the actual people responsible for teaching or planning. For the sacred symbols of the Egyptians were worthless to an outsider without the advice of a handful of men who knew their meaning.

To a certain extent we have an even closer parallel today. In working with new sciences such as atomic power it is essential for major countries to obtain for themselves a human source of inspiration by one means or the other since the mathematical symbols applied in this science retain their secrets unless someone can provide the key.

Pythagoras was well received by the Babylonian priesthood, who harboured a deep respect for their Egyptian brethren, and his life changed from that of a prisoner to that of a relatively free man. He was free to wander within the confines of the Temple and, while teaching and revealing as much or as little as he thought necessary, he soaked up knowledge from the sources around him.

Babylonia presented a mixed appearance to the world. It had been governed by a succession of tyrants and had held victorious campaigns against Chaldea, Assyria, Persia, Judea, Syria and Asia Minor. Captured priests and temple brethren had been transferred from these countries to Babylon.

Excavations at ancient Babylon indicate that the city occupied an area roughly four times the size of present-day London. This monster-city, inhabited by a myriad of nationalities, housed just about every religious system in existence.

Pythagoras seized the opportunity to study all of these religious orders and to delve as deeply as permitted into their mysteries. His superior intellect ensured that he drained every available benefit from his experience.

For twelve years he lived in Babylon, years of partial imprisonment and semi-freedom. Finally help came from an unexpected and welcome quarter. Democedes of Crotona, himself captured by the Babylonians in 522 B.C. during one of their conquests in the North and now Royal Physician at Babylon, put in a word for Pythagoras whom he had befriended. His release was granted. And after an absence of 34 years Pythagoras returned to his native Samos.

To his dismay he found all the island temples closed and their inmates fled. The Babylonians had here, too, been on the warpath and now controlled the Aegean.

But his grief was coloured with pleasure when he found Phartenis, his mother, had survived the years of tyranny and welcomed him back to Samos as the island's intellectual saviour. Her son, she had been certain all along, would lift the burden of oppression from the weary Samians.

But not wishing to entangle himself once more with the Babylonian conquerors, Pythagoras himself fled to Greece, bringing his mother with him. And it was in Greece that he was to further his knowledge of mathematics and apply it to philosophy.

Pythagoras's presence in Greece and his immensely rich knowledge derived from his years of foreign travel gave the Mystery Temples of that country a tremendous stimulus. Within a short time of his arrival he had set up a school and begun the teaching of knowledge originated in Egypt, Babylon and other areas he had visited. Alas, no document of any kind has survived or been heard of in Pythagoras's own hand. Everything we know of him and his teaching was recorded by his disciples and students. This fact fits in well with the picture we form of Pythagoras: an astute man of honour, bound to silence by initiation and an unbreakable vow.

He in his turn demanded a pledge of silence from his pupils and followers but as the selective process of pupils was not as strict and enduring as his own temple initiation, and since to a certain extent the ritual background was lacking at his school, and furthermore since he had no qualified assistants who had faced the same temple training as he himself had, the vows of his pupils were not absolute. A large part of his practical teaching became public property and the subject of discussion with all and sundry.

On his death his school was carried on and spread throughout Europe, branches were set up by his pupils. And at these offshoot centres of learning the vow of secrecy was perhaps maintained to an even lesser degree than previously. The secrecy depended more and more on moral responsibility than the fear of punishment. But we frequently see instances where secrecy was upheld despite the outbursts of individual students, and how the information released by the latter was not always accepted on face value by outsiders.

There is a modern parallel in the lodges of secret societies, whether Knights Templars, Freemasons or other orders, or in guilds and corporations of tradesmen and craftsmen.

Although many books are available to outside parties on these societies, it is not possible to appreciate fully the ritualistic aspect for the vital key is invariably missing and without it the ritual proceedings remain incomprehensible and mysterious.

We have established by research and excavation that the great majority of sources of written mathematics originate after the Pythagoras era in Greek mathematical speculation, but this is insufficient reason to assume that mathematics did not exist in Greece before Pythagoras.

It has been quite firmly established that prior to this period mathematical or geometric knowledge was the province of a select few within the mystery temples of Greece, but not until Pythagoras opened his school did this geometric system and thought burst upon larger sections of the public—who for many years were bound by their vows to a form of secrecy. But the Pythagoras school was not, after all, a despotic mystery temple. It attracted mainly the young and, of course, one of Pythagoras's grand ideals was to spread wisdom and knowledge (on a basis of responsibility) to the people at large. In this way, he reasoned, Greece would be preserved from the fate of sinking into a morass of ignorance and total eclipse. It was with this in mind that he called his mystery society a school and not a temple of Isis or Osiris dedicated to a central god.

Many of the finest, questing brains in Greece assembled at Pythagoras's school and although it was not a large institution it carried tremendous influence and indeed had the support of the State. But gradually, as the school grew in proportion and authority and Pythagorean pupils occupied an increasing number of influential posts in society, a fear grew in administrative circles that this upstart and his young followers would seize the very power of the executive. Consequently an active resistance to Pythagoras and his teachings arose, urged on by the envy and bitterness of orthodox temple society.

Pythagoras was not in effect a mathematician. He was certainly a brilliant geometrician but this subject formed only a minor part of his teaching and knowledge.

He also educated his disciples in the natural sciences, astronomy and the philosophy of life.

A youth or man had to possess special abilities before being admitted to his school. It was hopeless to try without these.

Anyone could apply for membership and be admitted for a trial period during which they had to undergo and pass a series of tests. They were kept under strict observation and at the end of this period were either given assent or turned away as unsuitable.

This procedure was maintained strictly. A youth's social or financial background played no part in Pythagoras's decision. And it was particularly among the sons and parents of the upper classes that the philosopher made his enemies when, after the trial period, a rich young candidate was rejected.

Pythagoras in his legacy left the world several clear mathematical formulae which even in our time find regular expression. For example, the best-known Pythagorean theorem $a^2 + b^2 = c^2$ in which he proved that in any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Another geometric factor, less well-known, which is thought attributable to Pythagoreans is a type of recognition sign, a characteristic of their society, namely the five-pointed star or pentacle, drawn as a signature and indeed used generally as a

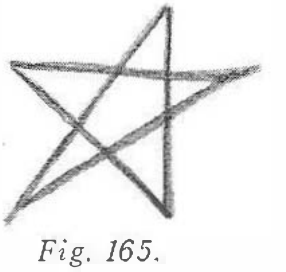


Fig. 165.

form of symbol in the same manner, for example, as Freemasons use the square and compasses. The five-pointed star was drawn as a flowing figure in *Fig. 165.*

The star can be drawn as a shape without the pen having to leave the paper. Naturally a star drawn free hand in this way cannot be uniform, but later theorists have assumed that the original symbol was the five-pointed star or pentacle which could be drawn inside a uniform five-sided or pentagon shape.

Euclid, for example, who lived two or three hundred years after Pythagoras spent considerable time and thought on this geometric shape. He wrote often of the relation of a given circle to an equilateral, equiangular pentagon.

He describes this problem on no less than six occasions in his writings:

Book IV, Propositions 11, 12, 13, 14 and 16.

Book IX, a recapitulation of the foregoing.

This phenomenon, that Euclid speculated and gave instructions on how to construct a pentagon both outside and inside a circle, is no proof whatever that the Pythagorean pentacle was uniform in shape. Since, as far as I can establish, no information has ever been forthcoming to confirm that it was indeed uniform, it is equally logical to assume the contrary—especially if an irregular star may have contained a symbolic significance for the Pythagorean group.

In reality the five-pointed star has no characteristics outstandingly superior to, for example, the six or eight-pointed version. And the fact that its construction has presented more difficulty to geometricians than its more or less sided associates is no real reason for its being recorded as something special. Euclid's attention to the pentagon/pentacle and his philosophising upon this problem must on the other hand indicate that the five-pointed star reigned as a symbol in geometric circles. But Euclid himself was no Pythagorean, and scarcely familiar in fact with the run-of-the-mill schools of mystery—but we shall touch on this later. Suffice at this stage to say that he had no knowledge of esoteric mathematics, and spent much time therefore in trying to ascertain why the pentagon or five-sided plane figure was so special and why it was significant to mathematics. These speculations produced a number of constructions of the pentagon, without Euclid discovering the meaning of the five-pointed star as a symbol.

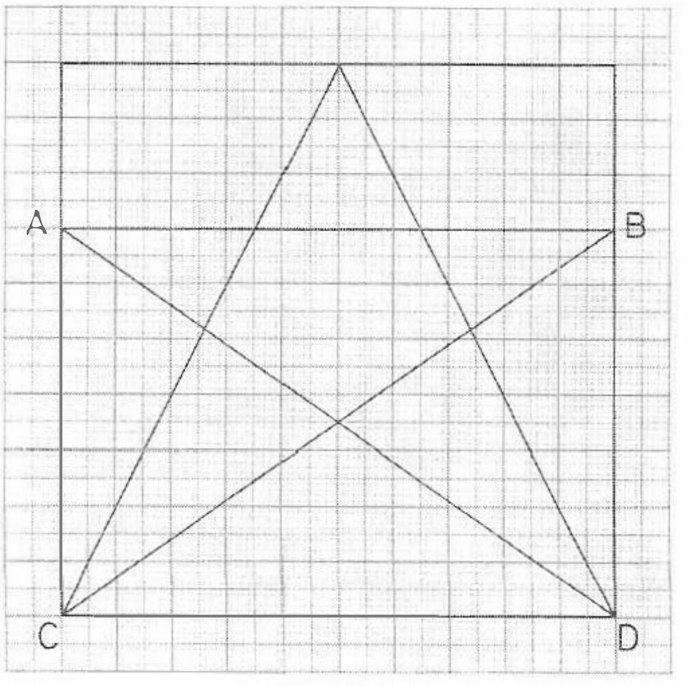


Fig. 166.

Fig. 166 suggests why indeed it was the pentacle and not the pentagon that was important.

The Pythagorean pentacle was not an equiangular, uniform figure. Symbolically it demonstrated the shape (significant to ancient geometry) of the division of the square by the acute-angled triangle and the placing of the sacred cut, and thus the halving of the square.

Line AB indicates the sacred cut in the main square which, as we see, is divided up by the acute-angled triangle.

The diagonal cross AD-BC has no geometric significance apart from the fact that these two lines connect the remaining lines in the diagram and thus create the symbolic five-pointed, irregular star.

This particular construction places the pentacle in the richly symbolic world of the initiate since it illustrates factors so familiar to him in the sphere of ancient geometry. Non-initiates on the other hand see nothing of this. Euclid's fruitless search for the importance of the pentagon, in my opinion, shows this to the full; similarly I believe that it proves that Euclid was not an initiate in the secrets of ancient geometry. For although mathematics had become a subject for public discussion, this applied only to certain aspects. The once mystic subject of numbers and counting had progressed so rapidly in open talk that any nimble mind could juggle with figures and sizes independent of any form of training, but the genuine wisdom of ancient mathematics remained within the Temple and within the mystery societies. It was never released to the public gaze. Not even by Pythagoras.

Pythagoras was undoubtedly a major source of inspiration, an intellectual master, who founded his school with the planning of a genius, giving it through his massive personality strength to survive the difficult times that followed his death.

Plato

One of Pythagoras's supporters was later to be Plato, the most prolific of Greek philosophic writers. Plato was the son of a wealthy aristocratic Athenian and was born about 80 years after the death of Pythagoras. He spent his youth among the upper classes of Athens and received an excellent contemporary education. As a youth he had already begun writing poetry and tragedies, and by the age of 21 had submitted several works to the state theatre of Athens.

At about this point he heard several speeches by Socrates. The fearless utterances of this great philosopher so profoundly impressed the young Plato that he deserted his career as a writer to follow Socrates. During the last three years of the latter's life Plato was one of his most faithful disciples, and even as Socrates awaited execution for heresy and sedition Plato listened to the words of the doomed man in his cell.

Depressed by this unjust treatment of his friend and teacher, Plato later sought out and joined the various Pythagorean societies which maintained an occult existence in the face of public disapproval. He journeyed out to similar Pythagorean groups in Asia Minor, India and elsewhere. He, too, included Egypt in his travels and was initiated in the lower degrees of learning in Egyptian temples—which were slowly regaining the ground lost by their almost total disbandment by the Babylonian-Persian conquerors more than a hundred years earlier.

On his return to Athens, Plato established his own school of learning, naming it the Academy in full recognition of Pythagoras's place as the supreme initiate and the greatest personality Greece had ever seen. Still very much a rich man, Plato spent huge sums buying up as many of the documents and writings directly connected with Pythagoras as he could lay his hands upon.

His initiation forbade Plato from writing in direct language of many things, but since from an early age he had been well practised in the art of writing (and considerably better than most) he was particularly adept at flirting publicly with a subject—without, however, actually revealing anything of consequence. In Chapter Four we saw how part of a Plato dialogue indicates how to construct the sacred cut, without revealing it to other than initiates.

That text has since been read by millions without apparently anyone previously having tumbled to its genuine significance. Yet another proof of his genius as a writer.

He was not as highly initiated as his ideal, Pythagoras, nor was his knowledge —naturally—as extensive, but much of his colossal importance to later scholars lies in the fact that he wrote more than any other of his contemporaries, and has presented us with a rich intellectual feast. But to gain most benefit from that fruit we need some kind of secret code or key to unlock the orchard door. Without the Open Sesame his screed of print remains just that, pages and pages of profound, well-written nonsense! Attributable more to Plato the writer than Plato the philosopher.

In Chapter Four we analysed an extract from Plato's *Timaeus* and saw how, illustrated by the symbols of esoteric geometry, this passage emerges clearly from obscurity and alters character so radically that what previously was considered illogical style and rather incomprehensible verbiage now assumes definite shape. In fact the text never entertains the possibility of mathematical speculation; this belongs to a later period of time and was thus missing from the text right from the start.

I intend now to spotlight yet another passage from the same book, or more correctly a continuation of the same passage.

In its entirety this section of *Timaeus* reads:

"The construction of the world used up the whole of each of these four elements. For the creator constructed it of all the fire and water and air and earth available, leaving over no part or property of any of them, his purpose being, firstly, that it should be as complete a living being as possible, a whole of complete parts, and further, that it should be single and there should be nothing left over out of which another such whole could come into being, and finally that it should be ageless and free from disease. For he knew that heat and cold and other things that have powerful effects attack a composite body from without, so causing untimely dissolution, and make it decay by bringing disease and old age upon it. On this account and for this reason he made this world a single complete whole, consisting of parts that are wholes, and subject neither to age nor to disease. The shape he gave it was suitable to its nature. A suitable shape for a living being that was to contain within itself all living beings would be a figure that contains all possible figures within itself. Therefore he turned it into a rounded spherical shape, with the extremes equidistant in all directions from the centre, a figure that has the greatest degree of completeness and uniformity, as he judged uniformity to be incalculably superior to its opposite. And he gave it a perfectly smooth external finish all round, for many reasons. For it had no need of eyes, as there remained nothing visible outside it, nor of hearing, as there remained nothing audible; there was no surrounding air which it needed to breathe in, nor was it in need of any organ by which to take food into itself and discharge it later after digestion. Nothing was taken from it or added to it, for there was nothing that could be; for it was designed to supply its own nourishment from its own decay and to comprise and cause all processes, as its creator thought that it was better for it to be self-sufficient than dependent on anything else. He did not think there was any purpose in providing it with hands as it had no need to grasp anything or defend itself, nor with feet or any other means of support. For of the seven physical motions he allotted to it the one which most properly belongs to intelligence and reason, and made it move with a uniform circular motion on the same spot; any deviation into movement of the other six kinds he entirely precluded. And because for its revolution it needed no feet he created it without feet or legs.

"This was the plan of the external god when he gave to the god about to come into existence a smooth and unbroken surface, equidistant in every direction from the centre, and made it a physical body whole and complete, whose components were also complete physical bodies. And he put soul in the centre and diffused it through the whole and enclosed the body in it. So he established a single spherical universe in circular motion, alone but because of its excellence needing no company other than itself, and satisfied to be its own acquaintance and friend. His creation, then, for all these reasons, was a blessed god.

"God did not of course contrive the soul later than the body, as it has appeared in the narrative we are giving; for when he put them together he would never have allowed the older to be controlled by the younger. Our narrative is bound to reflect much of our own contingent and accidental state. But god created the soul before the body and gave it precedence both in time and value, and made it the dominating and controlling partner. And he composed it in the following way and out of the following constituents. From the indivisible, eternally unchanging Existence and the divisible, changing Existence of the physical world he mixed a third kind of Existence intermediate between them: again with the Same and the Different he made, in the same way, compounds intermediate between their indivisible element and their physical and divisible element: and taking these three components he mixed them into single unity, forcing the Different, which was by nature allergic to mixture, into union with the Same, and mixing both with Existence. Having thus made a single whole of these three, he went on to make appropriate subdivisions, each containing a mixture of Same and Different and Existence. He began the division as follows. He first marked off a section of the whole, and then another twice the size of the first; next a third, half as much again as the second and three times the first, a fourth twice the size of the second, a fifth three times the third, a sixth eight times the first, a seventh twenty-seven times the first. Next he filled in the double and treble intervals by cutting off further sections and inserting them in the gaps, so that there were two mean terms in each interval, one exceeding one extreme and being exceeded by the other by the same fraction of the extremes, the other exceeding and being exceeded by the same numerical amount. These links produced intervals of $\frac{3}{2}$ and $\frac{4}{3}$ and $\frac{9}{8}$ within the previous intervals, and he went on to fill all intervals of $\frac{4}{3}$ with the interval $\frac{9}{8}$; this left, as a remainder in each, an interval whose terms bore the numerical ratio of 256 to 243. And at that stage the mixture from which these sections were being cut was all used up.

"He then took the whole fabric and cut it down the middle into two strips, which he placed crosswise at their middle points to form a shape like the letter X; he then bent the ends round in a circle and fastened them to each other opposite the point at which the strips crossed, to make two circles, one inner and one outer. And he endowed them with uniform motion in the same place, and named the movement of the outer circle after the nature of the Same, of the inner after the nature of the Different. The circle of the Same he caused to revolve from left to right, and the circle of the Different from right to left on an axis inclined to it as the side of a rectangle to its diagonal; and made the master revolution that of the Same. For he left the circle of the Same whole and undivided, but slit the inner circle six times to make seven unequal circles, whose intervals were double or triple, three of each; and he made these circles revolve in contrary senses relative to each other, three of them at a similar speed, and four at speeds different from each other and from that of the first three but related proportionately."

The passage continues with philosophic reflections on the coming into being of the universe, but we have taken a sufficiently large section of the quotation to provide us with the information we require from the standpoint of ancient geometry.

At first sight the text is rather confused and incoherent, with apparently little hope of fusion. Much has at various times been written about this particular quotation, the critics in their turn selecting one or two pieces for discussion—and regarding such excerpts as complete in themselves.

One of the more typical reviews has been written by the Dane, Professor Carsten Høegh. In a footnote to his book Høegh puts forward the theory that since one or two of the figures mentioned by Plato in his text fit the modern musical scale, and since the word "interval" is used, the indication is that this passage deals with training in harmony.

But he completes his comment on this chapter:

"To conclude I should like to emphasise that this has merely been an elementary introduction to this remarkable text and its structure, and that a number of the explanations and translations must be regarded as purely hypothetical. It should be borne in mind that it is quite impossible to translate this text or to review its underlying theme without continually having to choose between several feasible solutions, and many of the more puzzling aspects must at present be left unanswered. On certain points one is inclined to believe that Plato in this text has set out side by side explanations which cannot and must not be conjoined, and there are details, too, in the illustrative language to which ought not to be attributed any genuine philosophic meaning.

"It is, however, worth noting that in recent years researchers have been successful in uncovering a profound significance in details of the structure which were earlier regarded as strictly "mystical" and irrelevant to philosophy. But of one thing let there be no doubt: even the most penetrating and comprehending interpretation will never succeed in plumbing the full depth and clarity of thought mulled by Plato as he reflected on problems the understanding of which he has illustrated or hinted at in this peculiar piece of dialogue."

We see then that there is fairly general agreement on the incomprehensibility of this passage. No writing, no literature has ever come to light to explain it in its entirety.

We cannot on the other hand ignore the fact that Plato was an intelligent and inspired philosopher, teacher and writer. It would therefore be quite incompatible with the man's make-up that he would jot down—in a work such as *Timaeus*—thoughts which could not find expression in a subsequent readership, especially since the material was written down for others to read. If one considers for a moment that others (albeit an intimate circle of readers) are meant to understand the subject matter, it must be agreed that at least a small number of those readers found the subject intelligible. And if Plato's followers and associates, with less intelligence than he himself for he was an undoubted leader, could understand his writings then it is perhaps logical to assume that men of lesser intellectual stature than Plato might be able to interpret the meaning correctly.

This again would mean that the material under discussion is not the sole property and was not originated by an individual no matter how intelligent; otherwise its significance would be lost to others. I believe therefore success with this text depends on the viewpoint from which the reader regards it.

We saw from the previous quotation how Plato, in preparing for the dialogue, employed a figurative language to illustrate his picture of the world, the picture so formed being his symbol of the world's image and geometric appreciation being essential to an understanding of the picture and thus of the story.

We for our part must realise that Plato is in the position of describing a picture of the world or a story of creation which is both exoteric and esoteric: the former because the story is in itself no secret, the latter because the theories regarding the manner applied by god in his work were secret.

In order to explain to his initiate brethren how god had performed his task of creation, Plato was obliged to resort to geometry and numbers since the story of creation was from ancient times built upon this sacred teaching, the teaching that everything divine resulted from geometry and its associate, numbers.

Had he been an ordinary Greek-in-the-street who knew the story of creation, a simple man boasting no connections with mystery societies, devoid of a Pythagorean initiation, Plato could have come straight to the point and talked openly of geometric proportion. He need not have shrouded his theories in intricate verbal disguise. But he was not an ordinary person. Plato was bound by his pledge of silence, a vow given at his admittance to the circle of brethren and reaffirmed, renewed and strengthened with each successive degree of initiation received. As a man of honour Plato could not and would not breach his pledge. Plato thus found himself in the tricky situation of wishing to write of geometric shapes without mentioning them by name or directly mentioning any of their features.

This was a tremendously demanding task. It required a description of the *symbols' symbols*. As an indication of the intricacy involved, the reader should try to describe, for example, a square—without mentioning the word "square" or "quadrate" or referring to a four-sided plane figure. When these words and phrases are banned the problem immediately becomes acute, and how much more difficult would it have been for Plato in like fashion to describe a complete diagram such as that for the circle's rectangle. One might well imagine it impossible. And yet I believe that Plato has succeeded with this dialogue. I shall demonstrate in the following pages how the text can be read and interpreted to produce—quite clearly— a comprehensible unity. We quoted the complete piece of dialogue in order to show the relevant portions in their proper context, but many of the details naturally concern only the story and will be ignored in the following analysis.

I shall be content with selecting only those portions I consider to harbour direct reference to the geometric analysis. The various sentences have been taken in consecutive order from the quotation.

We begin:

> "The construction of the world used up the whole of each of these four elements. For the creator constructed it of all the fire and water and air and earth available, leaving over no part or property of any of them."

We see here that Plato states the construction of the world to have required four things. We read in Chapter Four in our analysis of the first part of this quotation how Plato used air, fire, water and earth to represent certain pre-determined geometric figures. Here again he refers to the same shapes and his listeners or readers know without further prompting to which shapes he relates.

I shall not at this stage recapitulate the shapes intended since Plato goes into this in more detail later. Suffice to say that to his listeners mention either of the shapes or of the equivalent elements would be synonymous.

> "Therefore he turned it into a rounded spherical shape, with the extremes equidistant in all directions from the centre."

This is clear indication of a sphere, and we may subsequently be surprised to find Plato deserting the sphere as such and describing the circle and its features. This must have been due to the fact that either he and his contemporaries were unable to calculate with a sphere, or the subject proved to be outwith the range of his listeners. He was thus obliged to lower his dialogue to a level that could be understood. In his earlier text, too, Plato writes of cubes and squares and analogises between them, cubes merely being mentioned while the actual geometric analysis is completed with plane figures.

> "For of the seven physical motions he allotted to it the one which most properly belongs to intelligence and reason, and made it move with a uniform circular motion on the same spot; any deviation into movement of the other six kinds he entirely precluded."

This reveals seven main motions, six of which are rejected, leaving the circular motion. The other six motions were up and down, forwards and backwards, right and left.

It is typical that the motions should be numbered seven. If one considers every kind of movement, of course, there is an infinite number of variations of the circular, from horizontal rotation to vertical rotation with numerous angles through the 90°. And there are right and left rotation on the same spot.

There is also an endless variety of movements on the vertical and horizontal planes (i.e. up-down-right and left-forward-backward) but in spite of this only seven are detailed. Obviously Plato and other philosophers were aware of the fact that other motions existed but in their choice between an infinite number and very few they selected—true to ancient tradition—the sacred figure seven.

After another piece of text in which he describes how the sphere was given a smooth and unbroken surface, Plato turns to the real point in his story, namely the geometric explanation, stating:

> "And he composed it in the following way and out of the following constituents. From the indivisible, eternally unchanging Existence and the divisible, changing Existence of the physical world he mixed a third kind of Existence intermediate between them."

He says here that—for the time being—three components were required, the indivisible, the divisible and a third type formed from a mixture of the two.

We may be a little confused about Plato's use of the word "existence". It is possible that the original Greek has a somewhat more closely defined significance, but reflection shows that it is difficult to find another word to express better Plato's intention. If we draw a square, then it *exists* in reality. This is the existence to which is referred here.

Plato begins with the indivisible and eternally unchanging, and if we were to choose from among the familiar geometric shapes one to fit this description, it would have to be the circle.

In mentioning the circle first, Plato is merely following ancient ritual since the entire geometric system had built upon the circle.

Reference to the circle as the indivisible is related to the contrary features between this shape and others of straight lines.

Plato actually mentions at a later stage that "all rectilinear surfaces are composed of triangles".

This illustrates the recognition of triangulation as late as the period of Greek speculation and shows that it was still a major subject for theory.

The circle cannot be split accurately into triangles, squares or other units. If a square is divided laterally we obtain two rectangles, but a circle divided similarly produces merely two part-circles. Any other division of the circle produces the same result.

A square divided from corner to corner is composed of two triangles, but the "same" line through a circle produces two half circles.

These factors are obvious and natural when one thinks of them, but must be emphasised in order to illustrate why the circle is termed the indivisible.

The other shape to which he refers is the divisible Existence "of the physical world".

This is a reference to the circle's opposite number, the square.

The definition of the square as the physically divisible is due to the division, so familiar to geometric brethren, of the square into its half size, the rectangle, the acute-angled triangle, etc. And of course the square can be split into a myriad of small triangles or squares.

When Plato had combined these two shapes, the Indivisible Circle and the Divisible Square, he formed another type of Existence, a mixture of the two. In other words, a new geometric area or shape in which both shared. We see this in *Fig. 167* in which we have the circle and, outside it, the square.

The third version of Existence was the acute-angled triangle, seen as ABC. And we have here the symbol in ancient geometry which we have called "O".

We can see how the lines of this triangle intersect the circumference of the circle at D and E, thus providing a triangle which in area is a mixture of the square and the circle.

Plato then continues his instructions:

> "Again with the Same and the Different he made, in the same way, combounds intermediate between their indivisible element and their physical and divisible element."

Here, "the Same" can be interpreted as the thing we have just made, i.e. the triangle, and when he refers to the Different I think it can mean only one thing: that the triangle is placed within the diagram in a different manner from the first triangle.

We see this in *Fig. 168* in which we have placed the triangle in familiar fashion, i.e. turned through 180° in relation to the first triangle. We have here the "same" and the "different", or rather the same triangle but yet different, FGH. This construction is the same as the symbol we have called "P" (in Fig. 74).

Plato's dialogue on the Same and the Different is in fact not so obscure or strange as one at first supposes, even out of context. Another example may illustrate the point more vividly.

If in one hand you hold *one* variety of apple and in the other hand you take *an-*

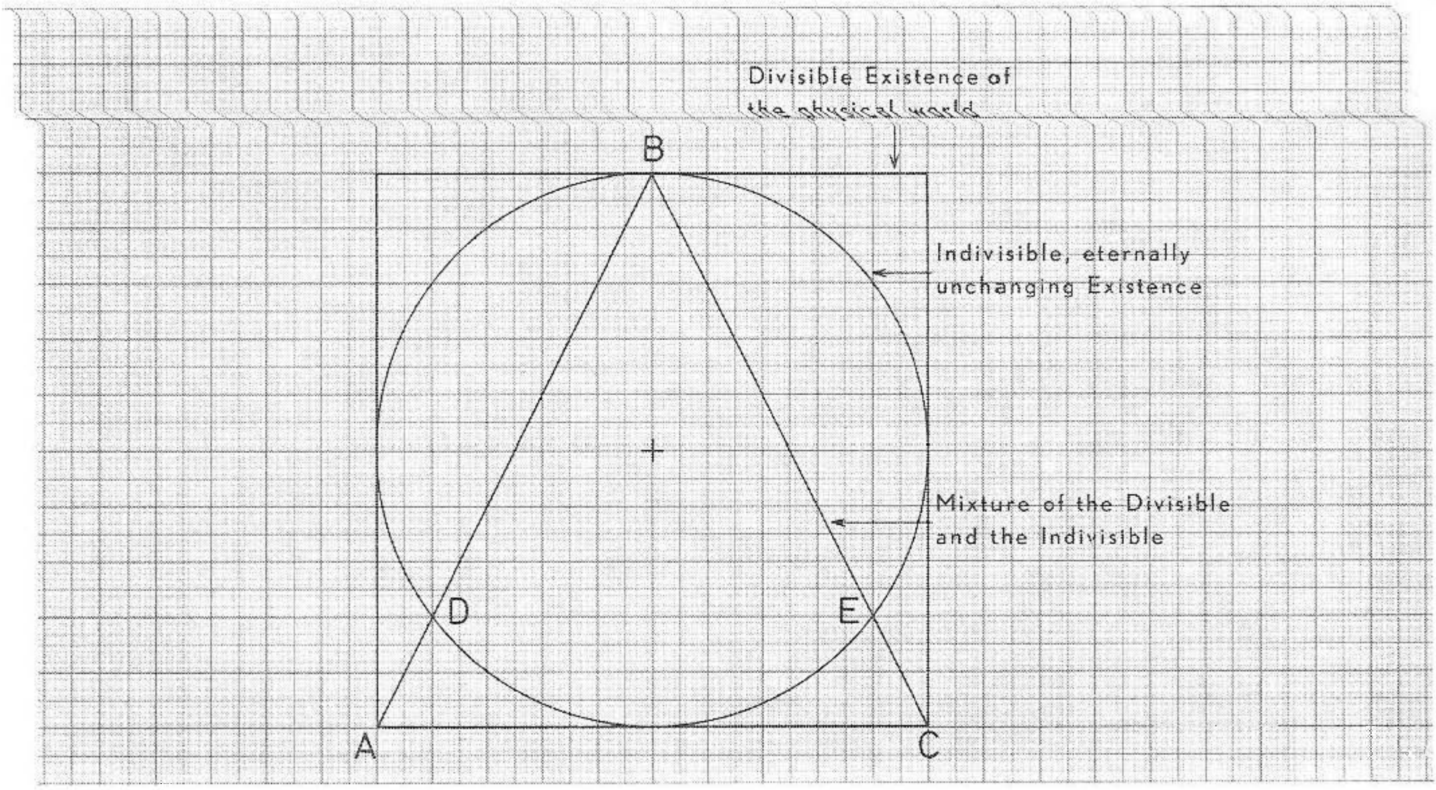


Fig. 167.

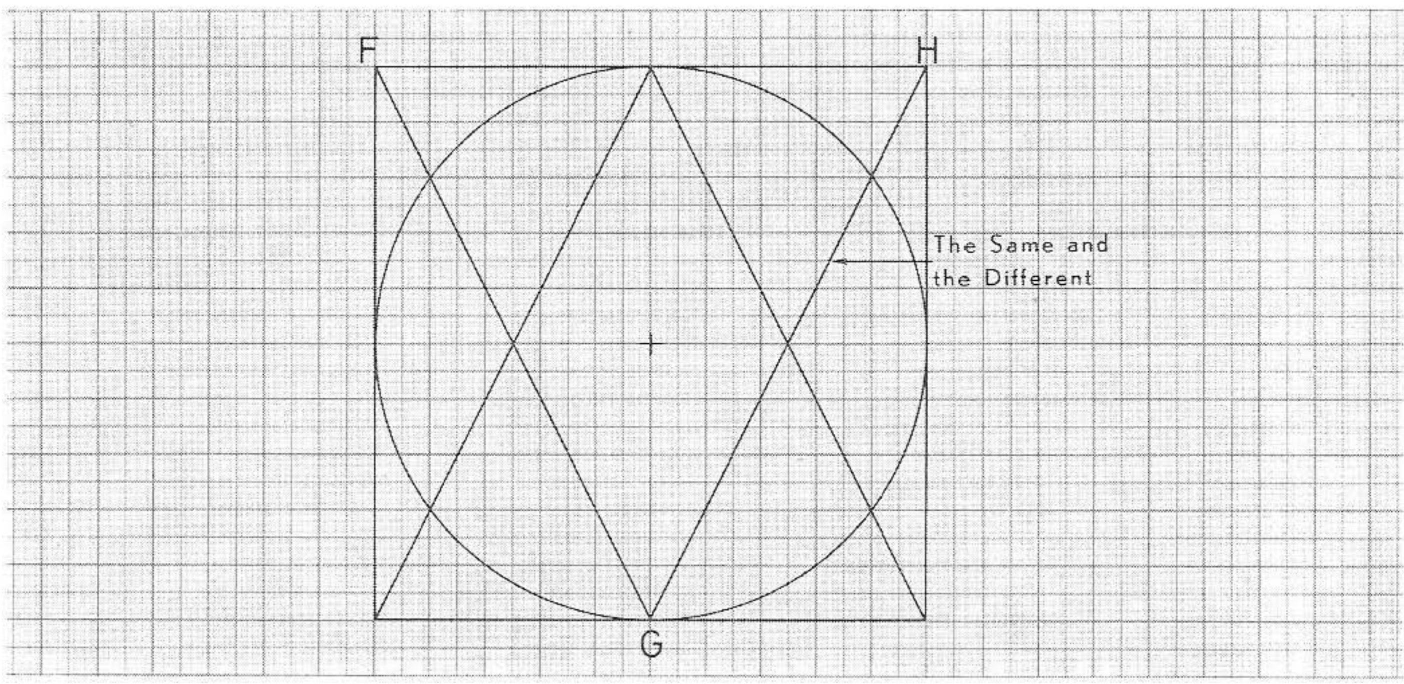


Fig. 168.

other variety, you have done what Plato demands. You have the same thing in each hand, namely an apple, but simultaneously there is a big difference between these two apples so you have fulfilled the demand regarding the Different.

Plato's use of this mode of expression for two triangles which are in reality identical in size and shape, the difference being disguised in the fact that one is turned upsides down, meets the demand of his phraseology perfectly. We are left musing over the mastery of his formulation which—on such a clear subject—has baffled able minds for more than two thousand years.

He continues:

"Taking these three components he mixed them into single unity, forcing the Different, which was by nature allergic to mixture, into union with the Same, and mixing both with Existence."

In this he produces yet another geometric construction, mixing the second triangle with the first triangle and his original circle-square.

There seems to be some discrepancy about the number of components (or mixtures) Plato has here. The text says he has "three components" but by my reckoning he has five. He started with the indivisible Existence (one) and the divisible Existence (two), producing a "third kind of Existence" (three). He goes on to mix the Same and the Different (four), and finally in the passage above he produces a mixture (five) of Same, Different and Existence.

Of course his reasoning may have been that a mixture of two components (divisible and indivisible Existence) produced *one unit;* a mixture of Same and Different a *second unit;* and a mixture of units one and two produced a *third.*

In *Fig. 169* the latter is indicated by the figure JKLM, and we recognise this from earlier experience as the circle's rectangle and symbol "Q".

Thus we have produced the finished diagram, built up completely in accordance with Plato's esoteric instructions. I have tried to clear away the curtain of secrecy which Plato intentionally draped over his text to render it incomprehensible to non-initiates—in which he was successful. But if the piece is read and interpreted in this way and if the reader is familiar with the diagram, the text becomes neither mystifying nor incomprehensible, expressing instead a remarkably clear and logical train of thought brilliantly formulated. The complete diagram is

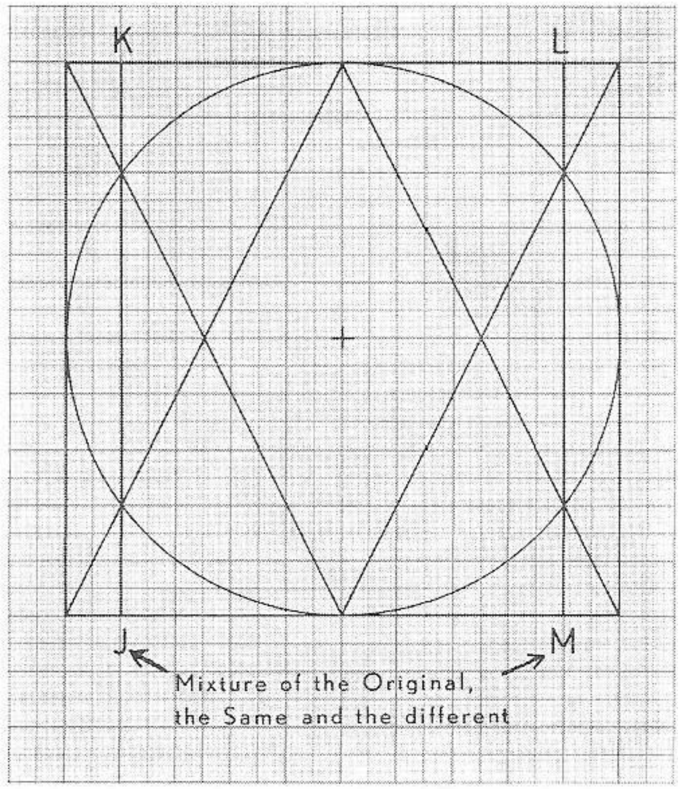


Fig. 169.

indicated in words only, no mention being made of a single line or geometric shape by its proper designation.

Plato then proceeds to prove how the circle and its rectangle really are related in the manner we know to be the case, i.e. that the area of the circle's rectangle is equal to that of the circle itself.

In this part of his tale it would be impossible for him to continue without mentioning certain figures and sizes, but since the preceding section regarding the divisible, the indivisible, the Same and Different, etc., is beyond the understanding of ordinary mortals without prior knowledge of ancient geometry Plato is at ease on this score.

We note also that Plato in his continuation mentions certain details about the diagram quite openly; there is no need to hide them as long as the point of origin is concealed.

He goes on:

"Having thus made a single whole of these three, he went on to make appropriate subdivisions, each containing a

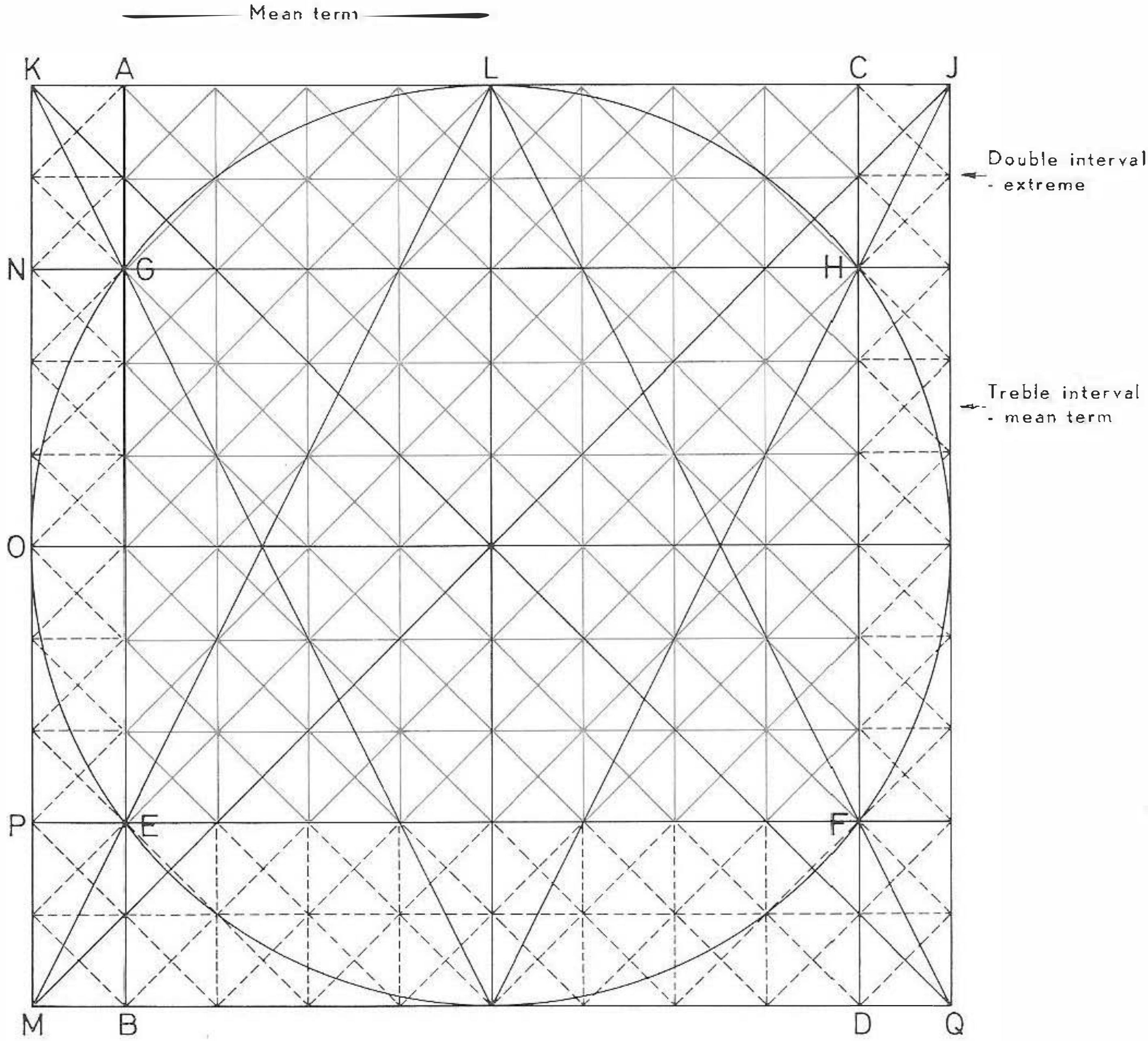


Fig. 170.

mixture of Same and Different and Existence."

He is informing us that the area so far achieved is to be divided into as many portions as deemed suitable. Since no instructions are given on what should be considered "suitable" we must search for a hint in the standard method of sub-division, i.e. triangulation, by which we have seen that a quadratic area can be split into as many small triangles as technique permits.

We must arrive at a size of unit suitably small to be used as factors in measurement, and the text later gives some indication in this direction—allowing us to check whether our selected size is correct.

Since we are to divide up a rectangle and not a square we must follow the special procedure of sub-dividing first of all the part of the rectangle that constitutes a square (i.e. the square on the circle's rectangle, symbol "U") and we see that not only does this division match in with the part of the rectangle outwith the square, but it also fits the whole diagram perfectly. We see this in *Fig. 170* in which the circle's rectangle is bounded by lines ABCD.

The square on the circle's rectangle is AEFC, the base-line of which rests on the circle's circumference at the same points (E and F) as the circumference is intersected by the lines of the circle's rectangle. And it is naturally the case, too, that the circle's rectangle is marked by the sides of the acute-angled triangles as they intersect the circumference, i.e. the acute-angled triangle that can be drawn in the circle's surrounding square. These points of intersection are now crossed by all three lines, the vertical lines representing the sides of the circle's rectangle, the horizontal being the rectangle's square (or rather the part of the rectangle making up a square).

We recall from earlier working with the half-size square how it is placed to the right and left respectively, producing two overlapping squares which mark out a rectangle within the square.

The same technique is applied here, the square on the circle's rectangle being lowered to the base of the diagram and indicated GBDH. This provides a combination of symbols "U" and "Q".

Line GH has an equal status in the diagram to line EF.

We resort to our system of triangulation and divide square AEFC into small triangles, starting with the diagonal and vertical crosses. We call a halt when we have produced 256 small triangles. These in the process have constructed 64 small squares, thus our object square is divided into 8 × 8 smaller squares.

If we examine the upper side of the diagram we see it contains line AC which in turn is made up of eight square-sides and split by a vertical axis with four square-sides to the right and left respectively.

But we have executed our triangulation/quadrature process throughout the diagram and we note that lines KA and CJ also consist of one square-side each. The complete side of the main outer square is thus divided into ten equal parts.

The upper line is now divided into two extremes, i.e. KA and CJ, and two mean terms both contained in line AC, on each side of point L. Thus we have first extreme KA, first mean term AL, second mean term LC and second extreme CJ.

If we similarly examine the diagram's vertical side KM we find it split up in like manner; none of the lines is in the least irregular, all are natural lines of division.

Line GH cuts precisely two rows of squares from the diagram. The diagram's horizontal axis cuts area GHFE in two, each with three rows of squares, and finally we see how line EF is produced to cut the two bottom rows of squares from the diagram.

In this way we have KN as the first double interval, NO as the first treble interval, OP as the second treble interval, and PM as the second double interval.

This division splits up our whole diagram. Vertically, into two mean terms and two extremes, and horizontally, into four areas Plato calls intervals, two double and two treble. The demands of Plato's text are seen here:

> "There were two mean terms in each interval, one exceeding one extreme and being exceeded by the other by the same fraction of the extremes, the other exceeding and being exceeded by the same numerical amount."

He said expressly that we required two mean terms in each interval, using the expression "mean term" in both a horizontal and a vertical sense. And indeed we have two mean terms and two extremes.

In comparing the size of these mean terms with that of the extremes, we discover it is the horizontal division to which Plato refers.

We see that the rectangle bounded by the short side KN and the long side KJ holds 80 triangles. So does the other extreme, bounded by PM and MQ.

The two mean terms, which at the same time are the treble intervals, have short sides NO and OP and long sides all the width of the diagram (as with the extremes). These mean terms each contain 120 small triangles.

The difference between one mean term and two extremes is as follows:

2 extremes	=	160 triangles
1 mean term	=	120 triangles
difference	=	40 triangles

Thus, if we re-examine that last piece of quotation, we find that one (of the mean terms) exceeds one extreme (by 40) and is exceeded by the other extreme by the same fraction (40).

The second requirement we had to meet was "the other exceeding and being exceeded by the same numerical amount". This is also fulfilled if we apply the same procedure to the other mean term, containing 120 small triangles.

From this figure we subtract the sum of the triangles comprising the two vertical extremes (KABM and CJQD), i.e. 40 each = 80.

1 mean term	=	120 triangles
2 extremes	=	80 triangles
difference	=	40 triangles

This demand therefore is also met, and we achieve the "same numerical amount" (40) as mentioned earlier.

This precision may well be regarded as meaningless mental arithmetic, but it is nevertheless well formulated. With a familiarity with the subject matter the writer/reader has a much more accurate check on the relevant postulates than by merely stating a figure, which cannot be compared or checked with anything in practice.

So far so good. By interpreting the text in this way we have fulfilled every requirement in, I trust, a clear and logical manner. The apparently disconnected has been bound up firmly in a diagram of immense import to esoteric geometry. But we have some distance to go yet.

Timaeus (or Plato) continues with the dialogue, turning to the proof that the circle and the circle's rectangle are equal in area:

> "He began the division as follows. He first marked off a section of the whole, and then another twice the size of the first; next a third, half as much again as the second and three times the first, a fourth twice the size of the second, a fifth three times the third, a sixth eight times the first, a seventh twenty-seven times the first. Next he filled in the double and treble intervals by cutting off further sections and inserting them in the gaps, so that there were two mean terms in each interval, one exceeding one extreme and being exceeded by the other by the same fraction of the extremes, the other exceeding and being exceeded by the same numerical amount. These links produced intervals of $3/2$ and $4/3$ and $9/8$ within the previous intervals, and he went on to fill all intervals of $4/3$ with the interval $9/8$; this left, as a remainder in each, an interval whose terms bore the numerical ratio of 256 to 243. And at that stage the mixture from which these sections were being cut was all used up."

Plato's disclosure in this passage is that he used the sub-division of the whole diagram as a sub-division of the internal circle. He divided the circle in accordance with the above directions and replaced the pieces in the circle's rectangle to prove that the two areas are in fact the same.

I am not inclined to believe that the fraction with which he finished, i.e. $^{256}/_{243}$, was evident from this particular diagram. It is not possible by the method at his disposal to achieve it. The numerator itself is self-evident. Square ACEF, the basic square of triangulation, comprises 256 triangles, and if he concludes by having slightly more than one triangle left it is possible that he arrived at this figure. But in order to gauge and express this tiny error he must turn to another and a finer standard of measure than indicated in his text.

But the application of the fraction ought not to be regarded in the same way as we today know fractions, for our treatment of numbers is quite different from the method applied in Plato's time. In an instance such as this we would, by $^{256}/_{243}$, recognise a particular portion in square centimeters or square inches. The standard of measure would be necessary, however, before we could appreciate the portion that was left over.

Timaeus had no experience or knowledge of this method and thus expressed the remainder in terms of his original unit-triangle $^{1}/_{256}$.

But let us revert to the actual sub-division. We are told that the circle is divided into seven pieces, each larger than the preceding. If we start with the first piece, we cannot discover how many triangles it comprises. We therefore assign to it the value "x".

The first part is thus $^{1}/_{x}$, the second twice as great $^{2}/_{x}$; the third $1\frac{1}{2}$ times as large as the second and three times the first = $^{3}/_{x}$; the fourth twice as large as the second = $^{4}/_{x}$; the fifth three times as large as the third = $^{9}/_{x}$; the sixth eight times the first = $^{8}/_{x}$; the seventh and last piece 27 times as large as the first = $^{27}/_{x}$.

Thus we have:

$$\frac{1}{x}+\frac{2}{x}+\frac{3}{x}+\frac{4}{x}+\frac{9}{x}+\frac{8}{x}+\frac{27}{x}=\frac{54}{x}$$

We see from Plato's text that the triangles comprising the circle are to be divided into 7 portions of differing sizes. We see, too, that the seventh piece is equal to the sum of the other six, i.e. that it occupies one half of the circle.

This reveals a little of the procedure. They had measured out the half-circle, and simply took the other equal half for the purpose of division.

We see this in *Fig. 171.*

If we count the small triangles in the circle's rectangle we find these total 320, and—working on the factor $^{54}/_{x}$—we see that $^{1}/_{54}$ of the rectangle equals 5.926 triangles.

Since it is impossible to perform a geometric division of an area with such a fraction, we take the nearest whole number, i.e. 6. We shall assume that speculation was confined to the circumference of the circle and concentrate our sub-division on this. We are also aware that we need only work upon one half of the circle, the other half being equal to the seventh and final piece.

1) We marked out area A as a unit of 6 triangles.
2) Area B has twice as many as A, i.e. 12 triangles.
3) Area C has half again as much as B, i.e. 18 triangles.
4) Area D has four times as many as A, i.e. 24 triangles.
5) Area E has nine times as many as A, i.e. 54 triangles.
6) Area F has eight times as many as A, i.e. 48 triangles.

Thus we have used up one complete half of the circle and achieved a total of 162 triangles.

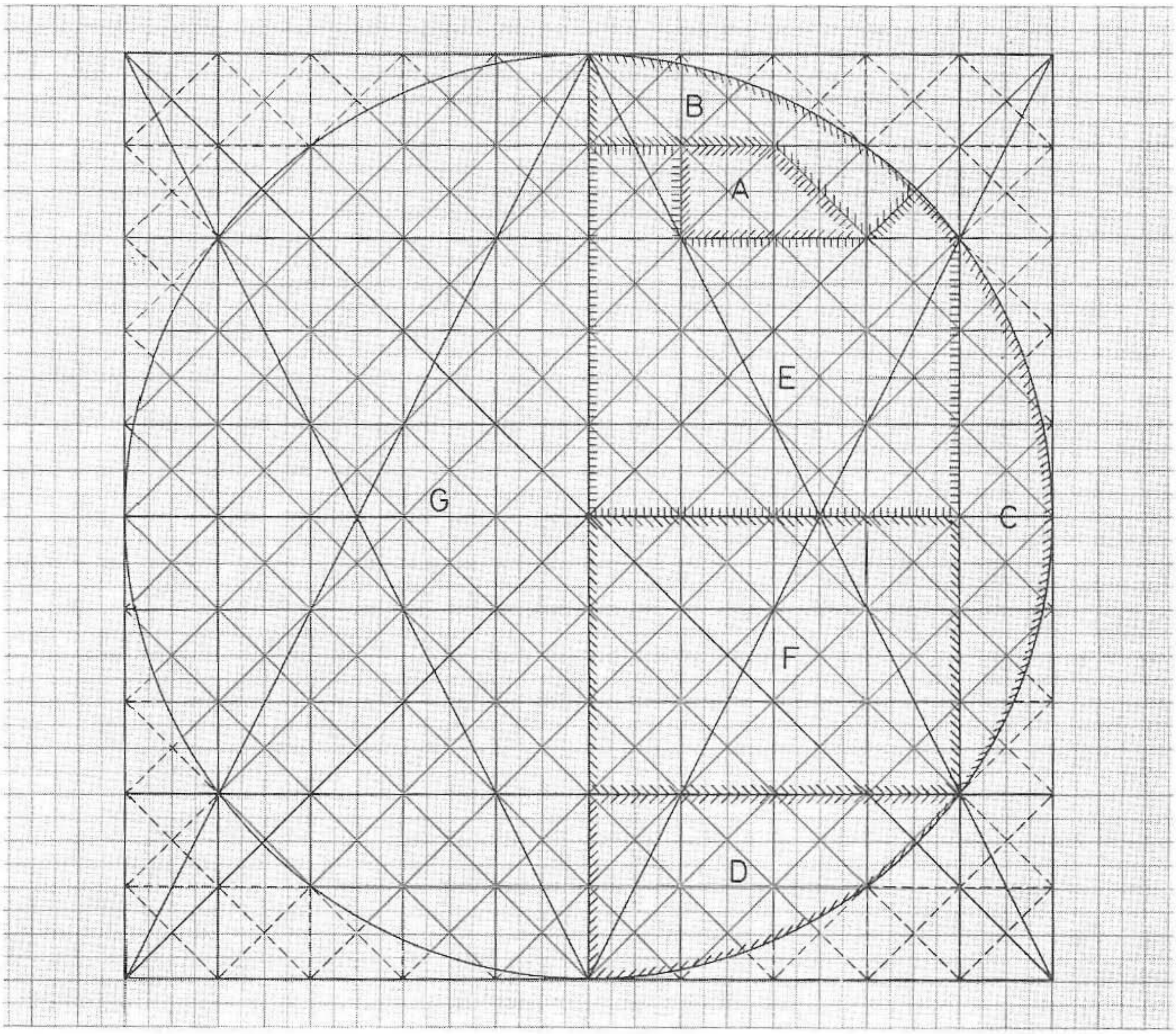


Fig. 171.

7) Area G, which is the seventh piece, must equal the sum of the other six areas, and must therefore have 162 triangles, the whole circle thus having or apparently having 324 triangles.

Division of the circle brought us to 324 and calculation of its rectangle to 320, making a difference of four triangles. But on the other hand we recall that we actually increased 5.926 to 6 and thus took a tiny fraction more each time.

Timaeus, who is telling Plato's story in the book, is fully aware of this and says so in the introduction to his speech:

"It is always most important to begin at the proper place; and therefore we must lay it down that the words in which likeness and pattern are described will be of the same order as that which they describe. Thus a description of what is changeless, fixed and clearly intelligible will be changeless and fixed—will be, that is, as irrefutable and uncontrovertible as a description in words can be; but analogously a description of a likeness of the changeless, being a description of a mere likeness will be merely likely; for being has to becoming the same relation as truth to belief. Don't therefore be surprised, Socrates, if on many matters concerning the gods and the whole world of change we are unable in every respect and on every occasion to render a consistent and accurate account. You must be satisfied if our account is as likely as any, remembering that both I and you who are sitting in judgment on it are merely human, and should not look for anything more than a likely story in such matters."

We have an obvious demonstration here of the reservations he made in advance regarding the forthcoming postulates, the geometric truth of which the ancients knew well was suspect. But they were placed in the situation of acknowledging the error although unable to replace it by something more accurate—particularly in an oral text. They speculated considerably over the circle and its features, of that there is no doubt. We see that when Timaeus places the circle into its rectangle he finishes with a fraction which I think should be $^{243}/_{256}$, i.e. slightly less than one triangle.

We came across this observation in Chap. Five when the ancient mathematician, counting the triangles outside the arc of the circle, finished with a small remainder of which he could make no use.

After this division of the circle Timaeus suddenly turns aside from his original means of sub-division and considers the diagram as a whole, divided by intervals and links, stating that "these links produced intervals of $^{3}/_{2}$ and $^{4}/_{3}$ and $^{9}/_{8}$ within the previous intervals". This indicates that he began distinguishing between various sizes within the diagram. When he says that he then went on to fill "all intervals of $^{4}/_{3}$ with the interval $^{9}/_{8}$", we must assume that he refers to areas A, B and C in *Fig. 172*.

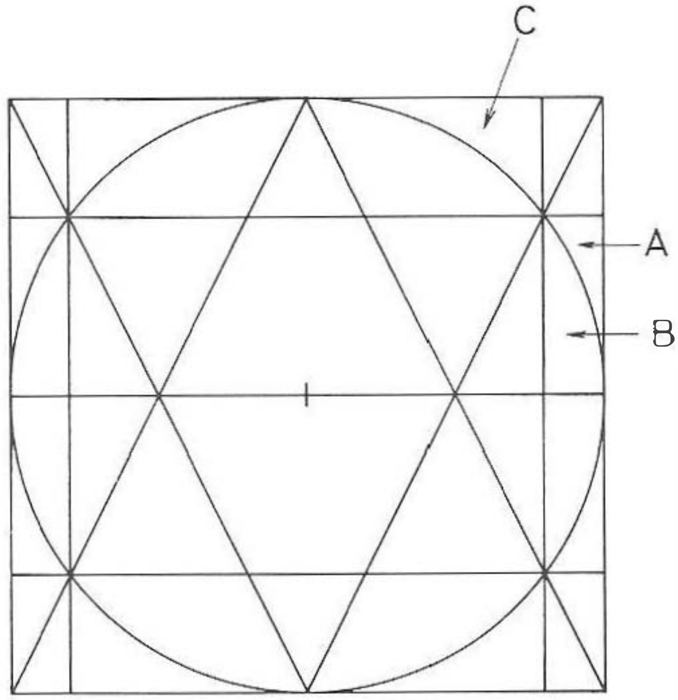


Fig. 172.

Here he estimates how to fit area C into area B in order to place the whole circle "between the links", i.e. that in fact in his proof he employs a more accurate assessment than his sub-divisions have indicated. This shows that he is familiar with a more accurate form of division than he is apparently able to disclose in his speech. He therefore goes straight to the point without detailing his method and without showing how he knows that area C is $^{4}/_{3}$ and area B $^{9}/_{8}$. But in both instances these areas are slightly in excess of $^{1}/_{1}$, converted to fractions C is $^{32}/_{24}$ and B $^{27}/_{24}$. By studying the areas we can by certainly appreciate that C is somewhat larger than B, but since there is nothing further in the text to help us we are left to guess about the sub-divisions applied by the ancient mathematician to achieve an accurate answer.

We know today that the circle is slightly less in area than the circle's rectangle and if we are to express their relationship in the ancient standard, i.e. triangulation, we find that the main square comprises 400 small triangles, its circle 314.159 small triangles, and the circle's rectangle of 320 triangles.

Whether those countries from whom Greece inherited her esoteric geometry came closer to 320 triangles, as shown in the diagram, cannot be proved. But there are some grounds for supposing that

Greek mathematicians arrived at the conclusion that the circle is slightly larger, for Timaeus says later, in a passage which seems to hold no other interest than precisely this point, that the circle is slightly larger than the rectangle. In fact he says, after using up the whole of the circle's area and placing it in the rectangle, that he finishes with a remainder which is expressed by the fraction $^{256}/_{243}$. It is not too clear whether this remainder is in the rectangle which is not completely filled up, or part of the circle which is left over, but he writes:

> "He turned again to the same bowl in which he had mixed the soul of the universe and poured into it what was left of the former ingredients, mixing them in much the same fashion as before, only not quite so pure."

The creator thus carries on his work from what was left over of the diagram. Whether it was a remainder from the rectangle or from the circle must be judged from an assessment of the text. But since the creator, after "cutting off further sections and inserting them in the gaps", produces a rectangle, it is probably correct to assume that Greeks at the time of Timaeus, or at any rate Timaeus himself, supposed the circle to be slightly larger than its rectangle.

That he concludes with a rectangle is stated directly in the text. It says:

> "He then took the whole fabric and cut it down the middle into two strips, which he placed crosswise at their middle points to form a shape like the letter X; he then bent the ends round in a circle and fastened them to each other opposite the point at which the strips crossed, to make two circles, one inner and one outer. And he endowed them with uniform motion in the same place, and named the movement of the outer circle after the nature of the Same, of the inner after the nature of the Different. The circle of the Same he caused to revolve from left to right, and the circle of the Different from right to left on an axis inclined to it as the side of a rectangle to its diagonal."

Timaeus is saying here that he splits the fabric (or construction), which he has formed, into two halves. This construction of course is the circle's rectangle.

The immediate supposition might be that he divides this rectangle in two equal parts by drawing a line down the middle, parallel to the long sides, but he goes on to say that by placing them crosswise they form the letter X. Thus they must be divided by a diagonal if the two halves are not to be placed at an angle. Moreover the text indicates more clearly later that it is in fact a diagonal division. When he has constructed the two circles he actually says that they are "inclined" or proportioned as the side of a rectangle to its diagonal. In other words, he splits the circle's rectangle (the fabric) in two by a diagonal line. The two resulting triangles are placed together so that the hypotenuse of one intersects the long perpendicular of the other. He constructs from these two lines two circles, one having a circumference equal to the triangle's hypotenuse, i.e. the *diagonal*, the other having a circumference equal to the triangle's long perpendicular, i.e. rectangle's *side*. We see this in *Fig. 173* in which we see one of the triangles as ABC and the other as DEF. A circle is now constructed of line AB and one of line ED, and we shall see how the areas of the two circle cover each other.

Thus again we have followed the directions of the text and produced a geometric construction from the details.

The story relates that this construction is an image of the creation of the whole world, not the earth but the entire universe. We shall shortly see how the struc-

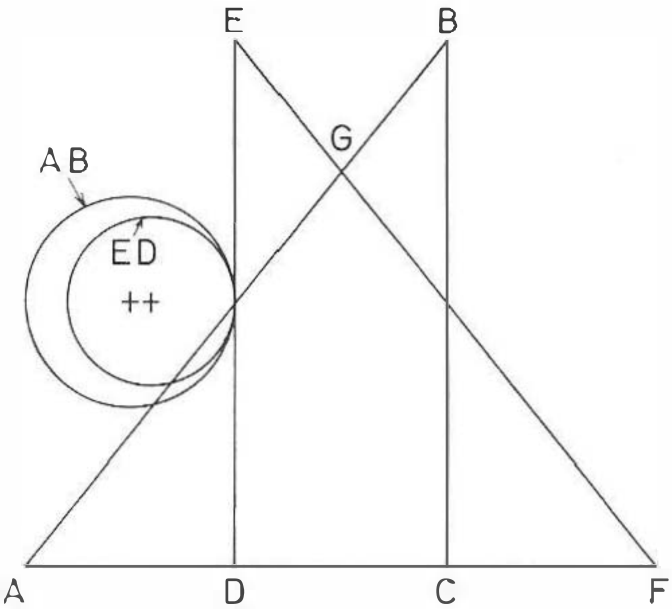


Fig. 173.

ture is further divided, but first let us remark on one point about the construction as it stands.

We will recall from our earlier reading that when we enter the diagonal in the circle's rectangle we produce two triangles which, placed long perpendiculars together, indicate precisely the proportions and angles of the Great Pyramid of Egypt. If we thus pull these two triangles in Fig. 173 slight apart so that points E and B, and D and C coincide, we again have the Pyramid angles in AFG.

We see therefore that Plato's directions require the side of the pyramid (the diagonal) and the pyramid's vertical height (the side of the rectangle) as the ideal dimensions for creation of the universe. This proves the colossal significance vested in these geometric speculations and results.

The text continues:

"He ... made the master revolution that of the Same. For he left the circle of the Same whole and undivided, but slit the inner circle six times to make seven unequal circles, whose intervals were double or triple, three of each; and he made these circles revolve in contrary senses relative to each other, three of them at a similar speed, and four at speeds different from each other and from that of the first three but related proportionately."

In this passage Timaeus visualises the smaller of the two circles (the one made with the perpendicular) in the diagram of sub-division, Figs. 170 and 171.

We note in these two figures that the natural division by triangulation has itself produced a six-pointed star within the circle, and it is to this star that Timaeus refers when he says that the circle was "slit" into seven unequal smaller circles, corresponding to the double and treble links and intervals. We see this diagram in *Fig. 174.*

Timaeus places his six moon/planets in the tips of this star and rotates them around the seventh body, presumably the sun, each with a different revolution and speed.

Here we have a clear illustration of the prevailing belief regarding the structure of the universe, the circle created from the diagonal (the outer of the two in Fig. 173) being considered the outer extreme of the heavens. Within the circle lay the six most familiar planets revolving at varying speeds around the central point of the universe, the sun.

The text then switches to a more nar-

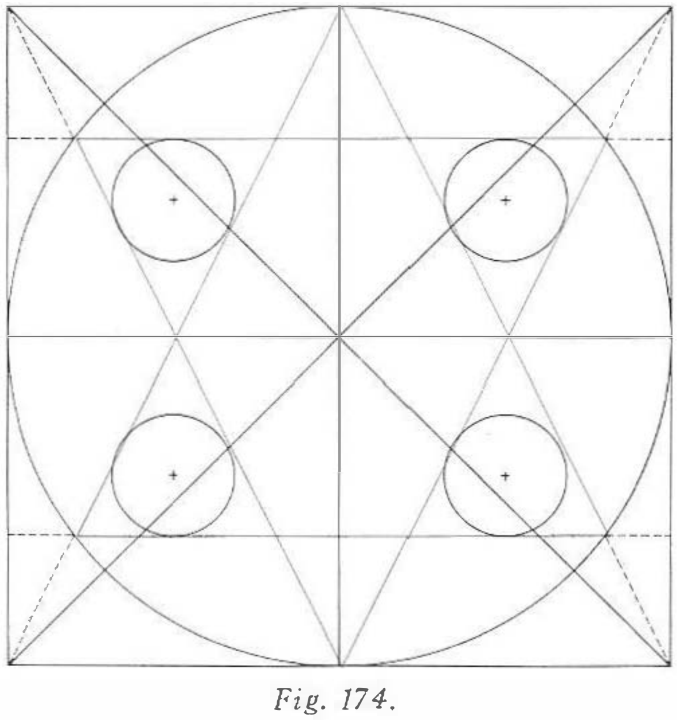


Fig. 174.

rative vein, but to complete the picture we should perhaps include a piece of dialogue which appears one or two pages later. Timaeus is summing up the main points of his long story:

> "As a result of this plan and purpose of god for the birth of time, the sun and moon and the five planets as they are called came into being to define and preserve the measure of time. And when he had made a physical body for each of them, god set the seven of them in the seven orbits of the circle of the Different. The moon he set in the orbit nearest the earth, the sun in the next and the morning star and the one called sacred to Hermes in orbits which they complete in the same time as the sun does his, but with a power of motion in a contrary sense to him; consequently the sun, Hermes and the morning star all alike overtake and are overtaken by each other."

We should observe one thing in particular about this quotation. Timaeus says that seven planets, namely the sun, moon and five other planets, have been placed in the universe to "preserve the measure of time". Measure in this respect can only be a reference to the number of these planets, i.e. seven. But this figure was so sacred that it could not even be mentioned directly at this stage, but in the fashion 2 + 5 = the measure (or numbers) of time.

With this final quotation I believe the most inaccessible portions of Plato's story through the lips of Timaeus have been analysed; and we are satisfied that when the text is explained from the point of view of ancient, secret geometry it opens up and loses its mystical, impenetrable initial facade.

It is a matter for discussion that the text does not contain as great a mathematical and philosophical insight as many researchers have tried to prove. But the explanation detailed here, combined with the actual story, provides in any event a logical continuity of thought. The story can be absorbed as a whole tale.

It must be granted that when all the more difficult passages can be revealed in content by the key of ancient geometry, and when each section builds on the previous piece of text, this explanation must be judged more fitting than any which deals with pieces of dialogue individually, without matching them to any cohesive pattern. And better than theories which end in a cul-de-sac of hypothesis with little or no connection with the story behind the text, the story—as seen by the ancients—of creation.

CHAPTER ELEVEN

Temples of Antiquity

PRECEDING CHAPTERS have traced a geometric development from Man's initial observations of surrounding nature, the Sun and Moon, and we discovered how his observations led to a geometric expression of the first circle and how reflection on this shape produced wider and deeper experience until finally a positive system of geometric calculation evolved, the principal factors being the sacred cut and the sacred number seven.

We saw how the ancient pyramid-builders applied the system to their projects, and how Moses conveyed the same knowledge from Egypt, transplanting it among the initiates of the Israelites via his instructions for the Tabernacle.

The trail has led us to the period of Greek supremacy, with Pythagoras handing the Egyptian geometry to his Greek contemporaries, and we saw through Plato's text how he wrote intimately of the ancient geometric tradition.

What then became of the established tradition? Did it die with Plato and his era, vanishing from Man's mind? Or did the public success of the new, emerging systems supersede traditional methods by dint of their greater accuracy with numbers?

There is no doubt that in many spheres the up-and-coming systems took over from the old, but in one field the old method retained its position: the art of building. Here, everything depended upon the established means of working with of course adaptations of design and materials. But new theory simply could not sweep aside the well-founded rhythm of construction—in spite of their more precise geometric speculation.

The power of the mystics and their temples had by this stage been partially broken. Public schools were set up, offering a training more or less independent of the religious orders. Temples were no longer the sole source of learning and training.

Fresh forms of religious belief arose. Their adherents had a different approach from established mystery communities. The emphasis switched from blind religious belief and faith to a demand for pure knowledge—backed by proof. Despite strong opposition from the Inner Temple these new groups became more and more popular. They suited better the needs of the people. Anyone so desiring could satisfy their religious needs within one of the new communities without—at the same time—having to undertake a course of practical training they did not really want. And those wishing knowledge for its own sake could fill their minds to the brim without blending it with unwanted religious ritual—and their studies were completed over a shorter period than in a regular temple.

Solid screeds of Temple wisdom gradually seeped out to a waiting populace, especially concerning spheres reckoned to be exclusive to individual training.

A legal training, then as now, was definitely one for the individual, for the specialist. So was that of a physician, artist, sculptor, and others.

It was more difficult in their case for the Temple hierarchy to keep the operative in check. More difficult than with other branches of temple industry where training constituted a part of the whole rather than the end in itself. The abilities of the ordinary brother were distributed widely, and made up a motley whole. The importance of these latter brethren to the unity of a project might be immense, but as individual workers their value dropped drastically.

A worker in modern times, for example, may be an expert train driver. It is essential to public transport to have a capable locomotive driver who knows his duties thoroughly. As long as he fulfils them faithfully he remains an integral part of the complex. But the moment he loses the desire to drive trains and leaves the railway company his personal position changes. The company merely put another man in his seat up front. But what about the ex-driver? His ability and knowledge alone are insufficient to allow him to drive locomotives on his own account.

He has by all means his expertise as a driver, but cannot apply it as an individual outside the company.

The process of building springs particularly to mind. Whether it concerns a temple, a road, bridge or other large project.

Here of all places teamwork takes priority. The architect or engineer prepares the plan from a mental picture without it being essential for him to train as a sculptor or other craftsman engaged in the building work. In fact he may have the same training as one of the site foremen, but not necessarily. His training, his ability lies rather on a higher level, knowing the possibilities of each trade to knit these into a united whole: the finished building.

If the builder knows his business and can draw an attractive structure on paper, then he can take the first creative step towards erecting the building. But that same builder requires a complete staff of efficient workers to be able to carry out the practical part of the job. And there may be cases where the efficiency or otherwise of his men can decide whether the job is well done.

Experienced craftsmen are needed to do the sculpting. Artists (or inspired artisans) must take care of the ornamental facings. Trained men must prepare the foundations and level off the site. In short, the whole process of construction is linked in a unity one could term "a graduated joint training".

The ancient mystery temples boasted this entire organisation: the Temple brotherhood.

Some of the brethren were sculptors. This was their vocation and they would advance no further within the Temple, nor did they harbour ambition in this direction. Other groups spent their time placing brick upon brick, stone upon stone. Still others were concerned with the ornamental part of the edifice. And thus by a system of cell units the building was completed.

Each special trade had its apprentices and masters, and it was the latter who co-ordinated and executed work on the actual project, headed by overseers or master builders.

We have echoes of this set-up in our various craftsmen's guilds, federations, etc., which like their forebears have a secret ritual which must on no account be passed to anyone whom the master has not initiated in the appropriate degree.

But the new geometric systems and their

followers could not draw upon the benefits of such an organisation since none but the closed clans of the Temple had the experience and practice to erect a building or project of any magnitude. It was immaterial if an individual initiate should break with tradition and escape the control of the Temple. Without a trained staff he could not hope to build. And even if a master or site foreman chose to "desert" to the outside world he did not know enough of the geometric aspect of structure to reveal any substantial information. Only the upper strata of initiated master builders had knowledge of the geometric side of planning, and these were normally such staunch supporters of Temple tradition that they formed the bulwark of defence from outside attack and inside treachery.

Almost every temple and cathedral from Antiquity to the Middle Ages was planned on the basis of traditional geometric proportion, and I shall be showing one or two selected examples from the ancient world, analysed through the eyes of the traditional geometrician.

The examples illustrated will demonstrate the manifold variations possible on a single theme. In spite of their restriction to a single given system the planners of old knew how to give each construction an outwardly different appearance from anything previously built and yet retain a general geometric harmony from structure to structure.

Temple of Ceres

THE FIRST Greek temple we shall examine by means of the ancient geometric symbols lies, however, not in Greece but in southern Italy.

During the 6th century B.C. Troezenian and Achaean colonists from Sybaris founded by the Gulf of Salerno a city named variously Poseidon and Paestum. The citizens devoted themselves to maritime trading and consequently assembled great wealth. As a worthy prize it attracted the attacks of neighbouring Lucanians but held out for nearly a century. Finally it wilted, and the decline of Paestum was under way.

Three temples, however, still stand witness to the Greek influence. Two of these are particularly well-preserved. One is the temple of Ceres, the other dedicated to Neptune or Poseidon. Both are attributed to the 550—450 B.C. period.

We shall look first at the Temple of Ceres.

Our drawings are copies of accurately dimensioned drawings in the Academy of Fine Arts (Kunstakademiet) in Copenhagen.

In *Fig. 175* we have a photograph of the temple from which are obvious the building's main proportions.

On the basis of this material we must discover the constructive point of origin. Experience and experiment have revealed that in the Greek temple this is invariably to be found in the facade. The proportions of the basic facade square determine not only the facade itself but also the dimensions of the ground-plan.

We recall from our work in Chapter V that the constructive point of origin is a square which as a rule is larger than the project being planned, since all detail must be placed within the square.

In the majority of Greek temples I have analysed I have found that the basic square generally coincides with the trans-

Fig. 175.

verse extremity of the building's roof. But we cannot follow this course with the Ceres temple as the vital parts have disappeared with the years.

It is often the case that the outer tip of the roof line is flush with the length of the supporting beam which generally rests immediately above the column capital.

But none such is to be found on the Ceres temple. Instead each column has upon it a huge square slab of stone, these acting as a bed for the beam above which in turn provides the foundation for the portion of the facade bearing the frieze.

We see in Fig. 175 how the two outer slabs extend further out from the width of the facade than the upper beam. If we take their outer edges as the two extremes of the facade and draw in our basic square accordingly, we discover this constructive point of origin to be correct. This fairly convincingly assures us that the roof of the Ceres temple has at one time stretched out to this point.

Our first analytical drawing on this basis is seen as *Fig. 176*.

The symbol applied in the large square is recognisable as "M", with the two horizontal and vertical sacred cuts (four lines in all).

This construction provides a smaller square in the centre. In this we again describe symbol "M".

We discover the placing of the symbol by the following means:

The outer or widest part of the building is shown by a broken line (AB) exactly at the point chosen as the base of our analysis, i.e. the square slabs above the columns. We transfer this width to the foundation of the temple under the lowest outside step. The base-line is CD.

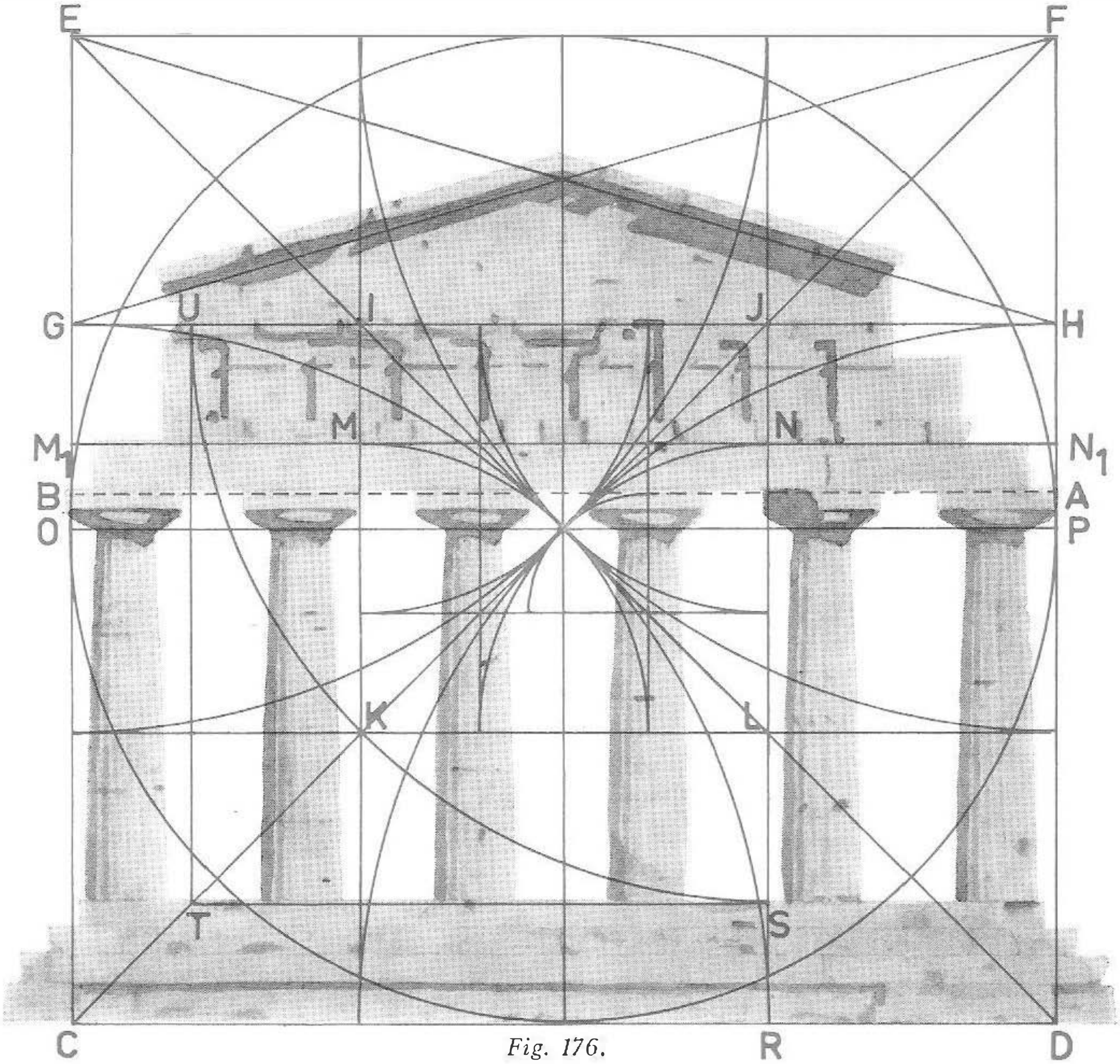


Fig. 176.

This line is measured vertically at CE and DF, providing our basic square CDEF.

Enter the vertical and diagonal crosses. And the "M" symbol is readily completed. The vertical cross, in marking the vertical and horizontal axes, gives us line OP, which cuts through the temple precisely flush with the top of the columns and under the capital.

We shall call this marking

Observation 1: Indication of column height.

Construction of symbol "M" gave us the four sacred cuts as follows. The line from point C to the centre of the diagram is marked off along the base-line CD, giving us CR. The same marking is made along CE, providing us with CG.

This allows us to construct the half-size square CGJR.

The same construction is made from the other three corners, D, F and E. So we have symbol "M".

Of the four sacred cuts in our square we see that the upper horizontal GH quite clearly marks the horizontal line of the roof.

In planning the Temple of Ceres the architect has thus employed the alternative shown in Fig. 76 F.

This is

Observation 2: Indication of the roof's horizontal line.

In the same manner as in Fig. 76 F we have a shallow rectangle EFHG in which the roof pitch is to be determined.

Here again the architect has applied Fig. 76 F, entering a diagonal cross, and we see on the analytical drawing how the angle of the cross and that of the roof pitch are identical.

He has apparently used the cross to mark the underside of the roof. The centre of the cross is thus the height of the temple under the roofing tiles.

This gives us two further observations:

Observation 3: Roof pitch.

Observation 4: Total height of temple to apex of the pediment.

As we have already noted, the combination of sacred cuts in this diagram has created a smaller square IJLK.

Here, too, we enter symbol "M" with two horizontal and two vertical sacred cuts.

In the large square only the upper horizontal sacred cut was used as a factor in planning. The others served only to provide the smaller inside square.

The same applies again in this new square: the operative line is the upper sacred cut MN, extended to M_1 and N_1.

Line GH in the large square indicated the roof's horizontal line and of course marked the upper edge of the frieze. Here the small square indicates by line M_1-N_1 the bottom edge of the frieze.

Observation 5: Indication of frieze depth.

The small inner square creates, like the large square, a new square in its centre. The upper vertical sacred cut in this third square indicates the under edge of the supporting beam as it rests on the square slabs of stone. This cut is in fact our old friend, the original AB.

Since the previous line (M_1-N_1) marked the bottom of the frieze—and thus the top of the beam—we now have the thickness of the supporting lintel.

Observation 6: Indication of thickness of supporting lintel.

Thus in the initial analysis of the facade we can trace the architect's course. He constructed the horizontal sacred cut in symbol "M" three times within each other and each time used the line for the various elevations of his facade structure.

Now he changes course. His next step is an adaptation of his symbol.

He still works with the sacred cut but in a different way. Instead of constructing the half-size square, he takes square IJKL and constructs the double size. Diagonal JK is measured out along line JR and gives us point S. Along the horizontal JG it provides point U. The double square is thus JSTU.

This construction provides our architect with the floor level of his temple, line ST. This line corresponds of course with the base of the colonnade and, since we already in OP have the height of the columns, we now have their complete length.

Observation 7: Indication of column length and floor level.

Another detail becomes evident from the diagram at this stage. We recall that line AB was the original line in our diagram and was dictated purely and simply by the temple's widest point. Later analysis showed that in fact this line was the upper horizontal sacred cut in the small central square, and marked the junction of the upper lintel and the square slabs of stone above the columns.

The horizontal axis of the main square (OP) sliced through the facade just level with the bottom of the capital. Between them therefore AB and OP contain the head of the column and the upper stone slabs.

As far as can be ascertained the capital and the slab are equally deep. It would seem that the architect simple halved the distance between AB and OP in order to fix these two dimensions. But material at my disposal was inadequate to determine this fact precisely. We shall therefore merely note it in passing.

The first analytical drawing has revealed seven clear, irrefutable clues to the structural planning of the facade—which does differ in several aspects from more usual Greek facades. We have the width, height and roof pitch of the temple, depth of frieze, depth of the supporting lintel, thickness of the capital and of the stone slabs above.

We have also been given the length of column, the floor line and the base line.

We can safely say that the first diagram has provided the basic information necessary to erect the temple, all of it coming from the original main square and having a fixed geometric relation to it.

To avoid having more lines on our facade drawing than we can handle, we transfer our analysis to another diagram *Fig. 177* where of course the basic square is the same as in the preceding figure.

Here, instead of the sacred cut, we enter symbol "Q" which contained the circle's rectangle, and extend the diagram to symbol "U" containing the square on the circle's rectangle. The latter is the same symbol as used in determining the ground-plan of the Great Pyramid of Cheops and in the study of Plato's *Timaeus*.

The circle's rectangle is seen as MNJK, its square as GHJK.

At this early stage the square tells us little, but its lines will shortly be revealing both the column distribution and details of the temple's ground-plan.

The acute-angled triangle ELF in the large square points its apex downwards. It indicated the circle's rectangle by its intersections with the circle's perimeter (points G and H) and indeed marks off the square on the rectangle. Part of it is of course the acute-angled triangle in square GHJK.

The division we must aim for is the splitting of this square 3 × 3, something which was discussed in detail in Fig. 59.

This is marked by the intersections of the acute-angled triangle with the square's diagonal cross. The two lower intersections have been ringed for clarity, but for the same reason we omit the other acute-angled triangle which marks the upper intersections.

Now square GHJK has been divided into 3 × 3 smaller squares.

The upper three squares, i.e. a third of square GHJK, comprise the rectangle GHPO.

Each of these three upper squares is subdivided similarly 3 × 3 and when the dividing lines of the resultant small squares are produced downwards through the diagram we discover that square GHJK has been—naturally—split into nine vertical strips, corresponding exactly to the column widths and spacing.

Just within the square's vertical side GK there is a space (1), then comes a column (2), space (3), and so on.

Thus nine strips mark off five spaces and four columns. Note how accurately the strips correspond to the thick bases of the columns.

The complete width of the temple of course is more than these nine strips, which comprise line KJ. The width is, as we know, equal to base-line CD. Another column was thus placed on each side of our square, 10 and 11.

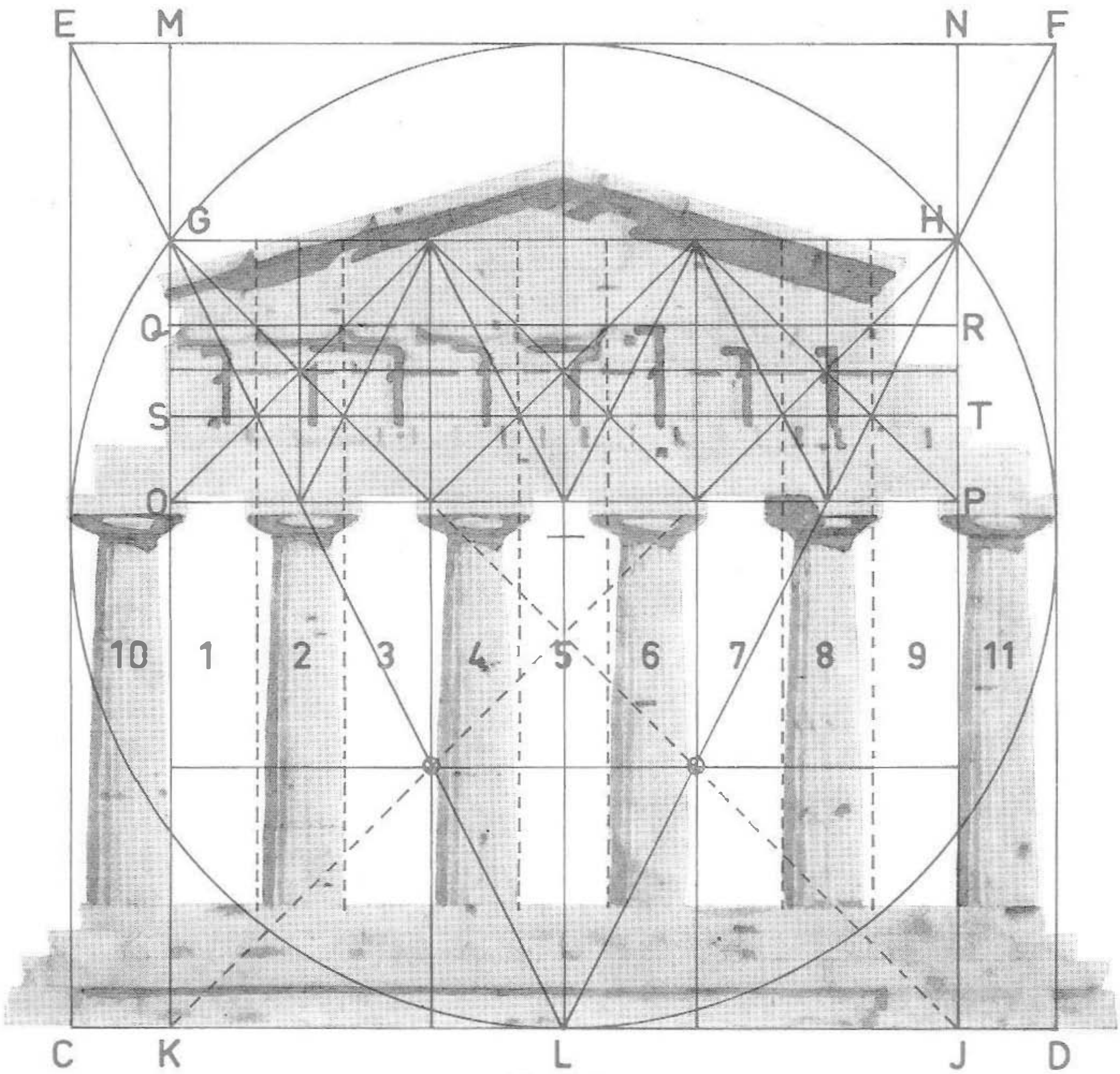


Fig. 177.

This diagram has provided two valuable details:

Observation 8: Column thickness.

Observation 9: Spacing of front colonnade.

Returning to rectangle GHPO we see that, in addition to the work of its vertical divisions, it has horizontal lines of division. These are QR and ST.

Without going into minute detail, we can see that the horizontal 3 × 3 lines of division and the squares' horizontal axis indicate parts of the frieze, and the actual decorative portion runs along the horizontal axis.

The drawing is not sufficiently detailed to reveal the appearance of the frieze but the reader can nevertheless appreciate that the same guiding lines have been used in the frieze as in the column spacing: the projecting and recessed areas alternate at the same rate as the column-space-column arrangement.

Observation 10: Indication of frieze decoration and its sub-division.

These two facade diagrams have provided us with ten valuable factors in the structure. One, with the sacred cut three times diminishing one inside the other, was solely concerned with the temple's erection; the other, with symbol "U" and the 3-part division, was applied in horizontal projection the principal aim of which was column spacing.

Our attention must swing now from the facade to the ground-plan but first let us note one factor about the column spacing.

We saw earlier that square GHJK cut the two outer columns off from the diagram, leaving four frontal columns within the square on the circle's rectangle. The two outside columns are actually the most important in the front colonnade since they are the initial columns in the cloister running the full length, on each side, of the temple.

But note their placing. Hard against line NJ but not touching line FD.

In other words, if we were to construct a companion square GHJK to the right or left of the original, the outer column would fit in as the first column in the new square. As opposed to the present square containing four columns and five spaces, the adjacent square would have five columns and four spaces. Which in turn emphasises that the outer column on each side matches the other four columns precisely, it has not been brought out flush with the outer basic square (FD).

In jumping from the facade to the ground-plan it is essential that a geometric link be retained. Such a link in the Temple of Ceres is the square on the circle's rectangle, GHJK.

Instead, as in the facade, of dividing this square into nine smaller squares, the procedure is reversed. It is repeated in

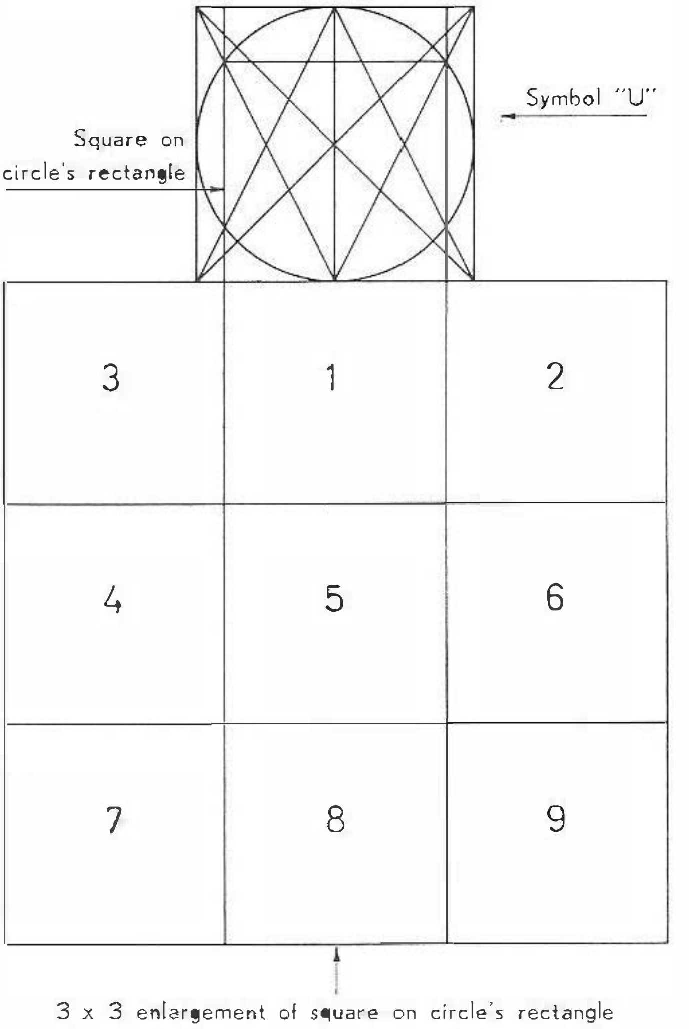


Fig. 178.

full 3 × 3 times, producing a large new square which is to form the basis of planning for the ground-plan.

We see this sketched in *Fig. 178.* Uppermost in the diagram we have symbol "U" with the circle's rectangle and its square. This of course is the same symbol as in Fig. 177.

The small square in the symbol is in fact the one we know as GHJK. It is repeated first as square 1, then to the right as square 2, the left as square 3, and so on up to 9. This gives us our basic square for projection of the ground-plan.

We have the first diagrammatical plan of the lay-out in *Fig. 179* and we note first of all that the temple's ground-plan is in the form of a rectangle.

On the extreme outer edge of the temple we see the outside steps leading up to

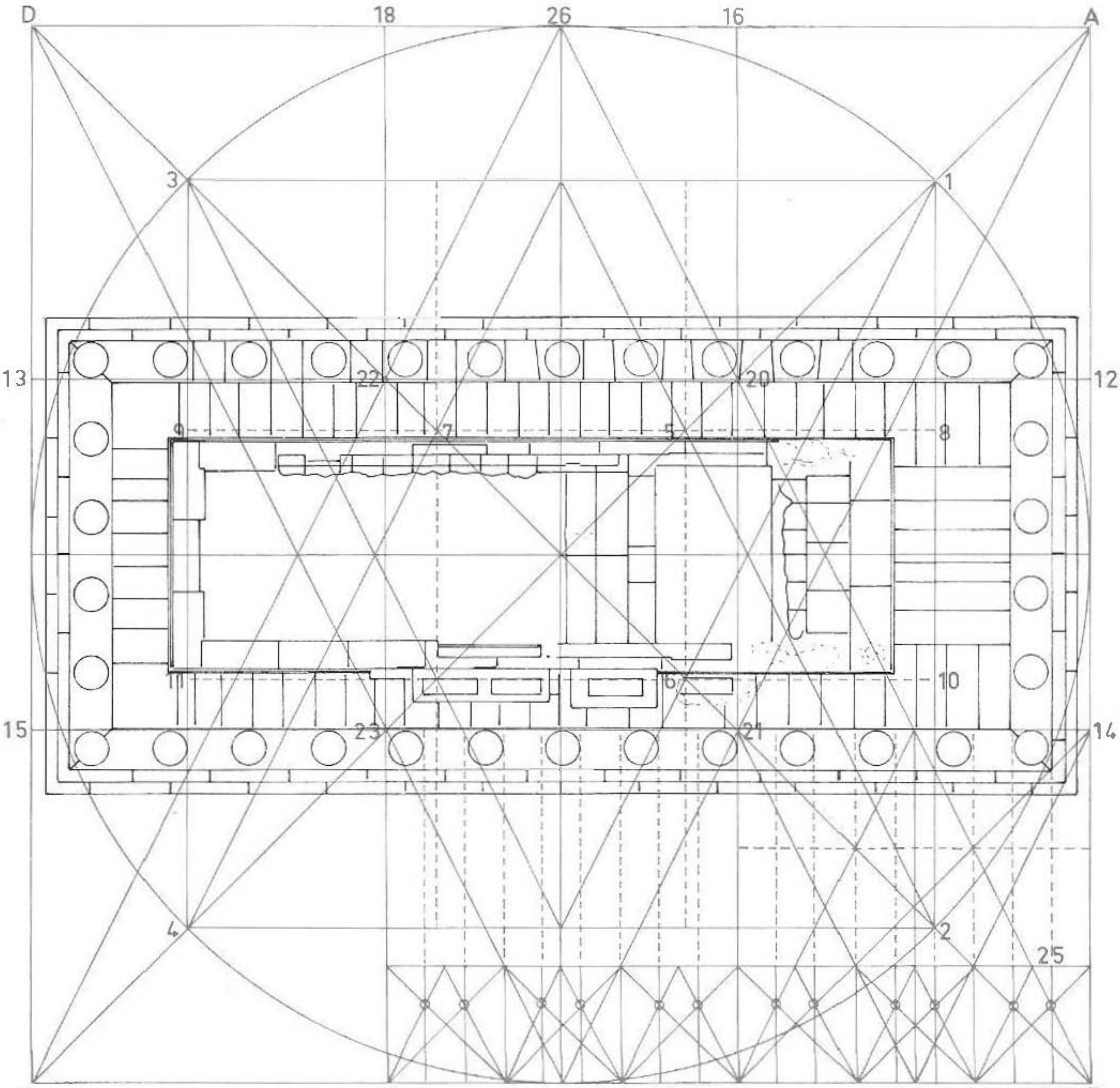


Fig. 179.

the building, then comes the characteristic colonnade, followed by the cloister which stretches across the width of the building until it meets the wall surrounding the inner temple.

Outside the rectangle of the temple we place our new basic square (shown in Fig. 178) and we are certain that this chosen dimension is correct since the side-length of one of the nine squares matches the familiar pattern of column spacing along the facade.

This length is seen at 12-14, and repeated from 14 to B and from 12 to A. Line AB is thus the correct side-length in the main square ABCD. The 3 × 3 division lines are seen as 12-13 and 14-15 horizontally, and 16-17 and 18-19 vertically.

Since line 12-14 was intentionally measured to coincide with the frontal colonnade, we can of course register the fact that lines 12-13 and 14-15 run along the inner edge of the two long rows of side columns and mark the outer edge of the cloister.

Observation 11: Indication of outer edge of cloister.

One point may already have occurred to the reader: that the basic square is slightly larger than the temple's length. This might lead one to presume that the plan was incorrect. But this is not the case.

If we compare the ground-plan with the earlier illustrations of the facade we notice that there are three steps on the facade while only two are shown on the ground-plan. Here is a somewhat illogical point of difference between two drawings reputedly produced by the same man. But we can readily ascertain whether the fault lies in the drawing of the facade or the ground-plan.

An excellent photograph of this temple is contained in A. W. Lawrence's book, *Greek Architecture*, Penguin Books Ltd., Middlesex. The picture is reproduced in Fig. 175.

In spite of the crumbling surface of the stonework we can quite clearly see three steps, particularly in the left-hand corner. Soil and dust threaten to remove some of the steps from sight—but we know they are there!

It is amusing to record how a knowledge of ancient geometry can detect errors in a drawing otherwise confirmed only by a photograph or inspection on site. Providing of course that geometric familiarity is properly applied.

Thus we see that our drawing of the ground-plan ought to show three outer steps instead of two.

To return to our inspection of the drawing, we can establish that three times the length of the old facade square GHJK provides the total length of the temple.

Observation 12: Three times the length of the facade square equals temple's total length.

We recall how this square accurately indicated the placing and spacing of the facade colonnade, having four columns and five spaces. We also remember that moving the square either to right or left provided a new arrangement of five columns and four spaces.

This is what we see done along the *temple's length. We start with square* 14-B-17-21. This has the same grouping of columns-spaces as the original facade square.

We divide this square 3 × 3 and the resultant squares are also subjected to the same division, i.e. the same process as applied to the frieze.

We see that the 3 × 3 sub-division matches perfectly the temple's column-space grouping, and in square 21-17-19-23 we see that it applies here, too. We have here the logical continuation of five columns and four spaces, whereas in square 23-19-C-15 we switch back to four columns and five spaces.

The whole of the temple's length is now divided in 27 parts (3 × 9) and we start and finish with a space so that the long colonnade has 13 columns and 14 spaces.

Observation 13: The 13 columns of temple's length indicated by 3 × 3 division.

If we now revert to the first square 14-B-17-21 and consider point 14 where the temple's outer step lies, we see that a 3-part division of the space before the first column provides the depth of the three steps. Line B-24 (which equals of course the total depth of the three steps) indicates the depth of the individual step when divided in three.

Observation 14: Individual spacing between the columns indicates total depth of stepping, and division by 3 provides dimensions of individual steps.

We return now to a reflection of our large basic square to see whether it will reveal further information. We enter the vertical and diagonal crosses and the circle.

The original architect of the temple of Ceres probably drew in all his guide-lines from the start, but we shall enter them only as required.

Inside this construction we draw the half-size square, not as part of the sacred cut but linking the points of intersection of the diagonal cross with the circle. This procedure was demonstrated very early in the book and is identical to symbol "H".

We see the square as 1-2-3-4. Look first at the side-length 3-4, how it cuts across the inner temple end wall and thus indicates the end of the inner sanctum.

It may seem peculiar that this line runs *through* the wall instead of marking the outer or inner edge. But I think the reason is strictly functional, bearing in mind the wall's purpose as the entrance to the inner temple. There was a door in the wall and since the wall at this point is more than a meter thick it is highly likely that the door itself was hung on the wall as indicated by line 3-4.

Observation 15: Indication of one end of the inner temple and marking of doorway.

If we divide square 1-2-3-4 by three in the usual way (diagonals and acute-angled triangles) we see how the horizontal lines of division coincide with the width of the inner temple including the thick walls, i.e. they indicate the inner boundary of the cloister whereas the same lines in the large square marked the outer dimensions of the cloister. The lines are 8-9 and 10-11. The latter matches the ground-plan better than 8-9. On this (the lower) side of the inner temple there are indications of what were probably small chambers, and line 10-11 matches this part precisely when the wall thickness of about 15-20 cm is included.

I do not think an error was committed here, neither in the drawing nor measuring of the plan. The reason for the seeming gap between 8-9 and the outer wall of the sanctum is I believe that the architect was obliged to equate theory with practical requirements.

The small rooms at and to the left of point 6 in the diagram naturally had a specific and perhaps ritualistic purpose in the structure, otherwise they would not have been placed precisely at this point.

Since they must go there, space must be allotted to them and to the cloister. We can see that where the largest room juts into the cloister there is a space of only 70 cm to the nearest column. If the inner temple had been expanded on each side by the 20-cm "discrepancy" the passage at this point would have been a mere 50 cm wide: too narrow for a normal adult to pass without twisting sideways, at any rate an awkward width.

The architect therefore chose to pass line 10-11 through the inner side of these rooms and to let the wall of the inner temple suit accordingly. Since the opposite side (8-9) has no such extra room projections he was obliged for the sake of symmetry to keep that wall, too, within the line of plan.

I think it was this strictly functional reason that forced the planner to depart slightly from his geometric diagram, and in the next diagram we shall see that this trifling deviation affects not the width of the inner temple but only the wall thickness. The temple width is decided by a different set of factors.

We note that the outer side of the inner temple seems to have had a form of beading or ornamental fluting (indicated on the plan by closely drawn lines). It would appear to run along the bottom of the wall, but since the entire inner temple has gone measurement was probably so difficult that the width of this beading was hard to establish.

I have been unable to discover this precisely but it is not impossible that in fact the ornamental beading had at one time extended so far out towards the colonnade that it filled the distance between the inner temple's outer wall and line 8-9 (10-11). For example, we saw earlier how our geometric diagram could reveal details missing from the dimensional drawing.

Observation 16: The division of the half-size square indicates the width of the cloister and the outer width of the inner temple.

The inner temple was presumably divided into several chambers. In the middle of the sanctum particularly there appears to have been an extremely thick wall of approx. 3 m. Whether it was a wall or a marking of some sort on the floor is impossible to tell, either from photographs or drawings, but it is interesting to note the near coincidence with the edge of this "wall" of the main vertical axis 26-27. This we merely note as an interesting point.

For the same reason as before, we shall move from this diagram to a new one to avoid a confusion of lines.

Fig. 180 is the same basic square, named this time BCED. The circle, too, is of course the same as previously. We still bear in mind the missing third step around the outside.

In Fig. 179 we had a 3-part division of the square. In this new diagram we shall work with the sacred cut, which we construct from each of the four corners. Again we have symbol "M".

The two horizontal sacred cuts JR and MH we discover coincide exactly with the line which marks the edge of the main temple platform, i.e. just before the first of the three steps.

Thus the two horizontal sacred cuts indicate the width of the actual temple at the base of the columns. This is not the same as the original square drawn in the facade plan; there is in fact a difference of 30 cm between line RH in Fig. 180 and line OP in Fig. 176.

Observation 17: Width of the temple at the top of the steps is determined by the horizontal sacred cuts in the large square.

The vertical sacred cut FP marks the inner temple's interior at the opposite end from the entrance. Thus we have found the constructive plan for both end walls.

Observation 18: Inner temple's interior depth indicated by vertical sacred cut.

The combination of all four sacred cuts creates the usual small square at its centre, GNKO. In this we again construct the four sacred cuts.

We see now how the two horizontal cuts mark the width of the inner temple in the same manner as the corresponding lines in the large square indicated the temple's width at the base of the columns. The lines are 1-T and 2-Z_2.

The width of the inner temple equals therefore the distance between points 1 and 2.

Observation 19: Width of inner temple indicated by sacred cut in square formed by larger sacred cut.

Whereas in the previous diagram the vertical axis indicated the inner edge of a transverse wall or floor-marking in the inner temple, we see here how the vertical sacred cut XV marks the other side of this wall or floor lay-out.

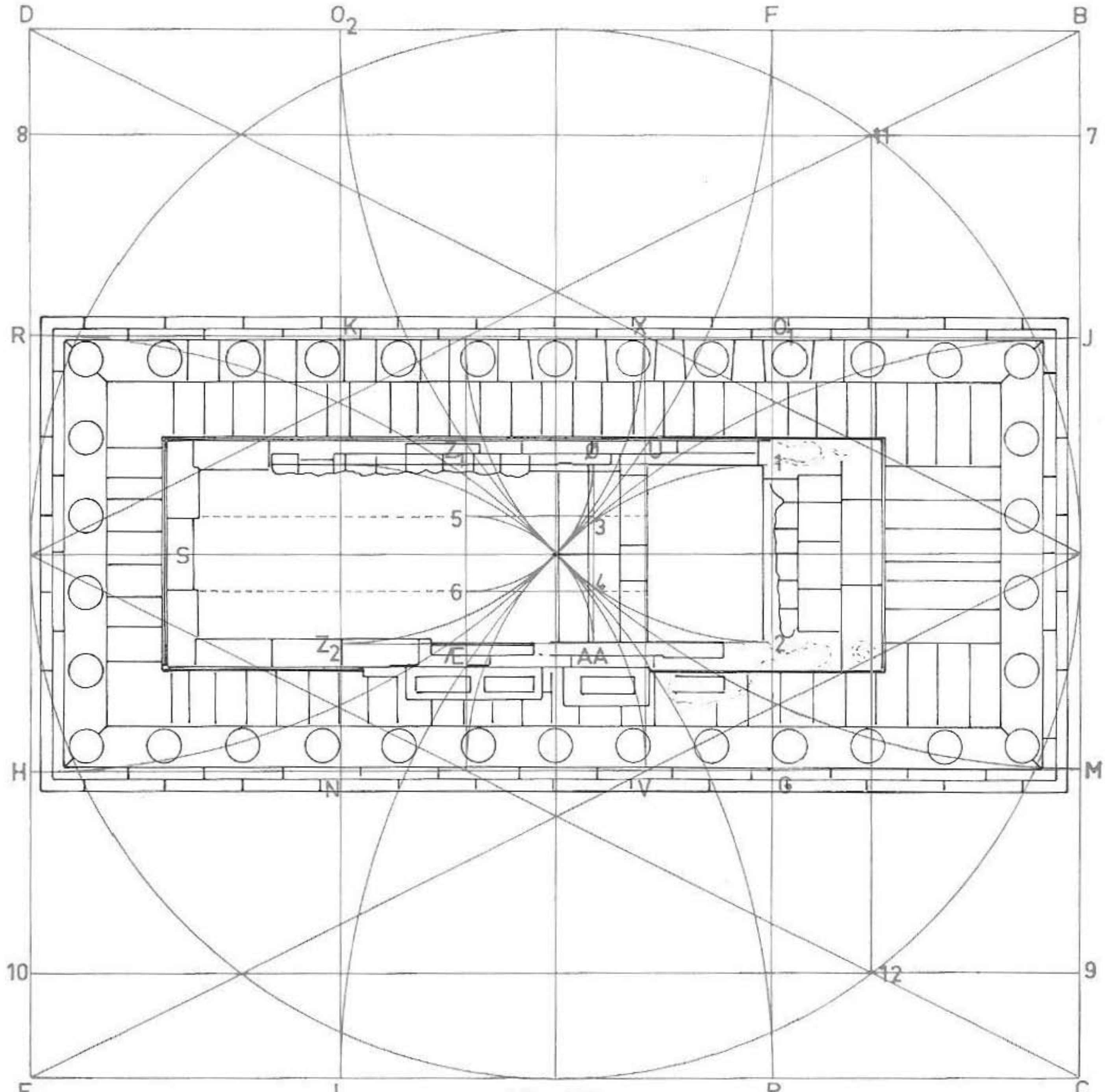


Fig. 180.

There is an interesting point here. The sacred cut in the large square formed the inner temple's end wall and the same vertical cut in the smaller square forms the edge of an apparent wall or marking on the floor. Thus the inner temple has a small chamber or area bounded on all sides by sacred cuts.

Observation 20: The vertical sacred cut in the small square, combined with previous observations, indicates the thickness of transverse wall in inner temple.

For a third time we construct the sacred cut combination, this time within the small square UYÆZ_1, in other words the central square which is a product of square GNKO, which again resulted from the sacred cuts in the large square.

Again the horizontal sacred cuts contain vital information. The two lines in question, 3-5 and 4-6, have in the diagram been produced as a broken line along the length of the inner temple, and we see how they mark exactly the width of the entrance to the inner temple.

Observation 21: Width of the inner temple entrance door indicated by the horizontal sacred cuts in the smallest square, produced by sacred cuts three times within each other.

We have completed our study of the information provided by the sacred cut, and return to the large square BCED.

In it we construct the circle's rectangle 7-8-9-10 and, as with the facade, we complete the largest square possible within this rectangle, 8-10-12-11.

We see how the vertical side 11-12 of this rectangle runs right through the inner temple's end wall, indicating fairly closely the external length of the inner temple.

The reason for its running *through* the wall instead of flush with it may have been the same as the reason for the line running through the entrance wall. It is conceivable that a small vestry door lay on this line and held some significance in religious ritual. But if there was in fact no second entrance in this wall, I am afraid an alternative reason for this line's apparent displacement escapes me. I cannot see why it should be wrongly placed when the rest of the architectural plan fits so well.

It is also possible of course that the total length of the inner temple is determined by yet another line, which is not drawn into our diagram. Another possibility is that there may have been a small deviation during building from the desired dimension and that executed. The difference is in the region of 35 cm.

As with one or two other factors, we shall record this as a point of interest and not as a precise observation.

To summarise our survey of this ancient temple: We have registered 21 precise observations, each giving a clear indication of the temple's constructive lay-out. All of these are exact dimensions, permitting us to produce a complete structural drawing of the entire temple, based on any square.

The drawing can be prepared without any form of measurement outside the geometric lines we have followed.

The architect of the Temple of Ceres at Paestum thus based his plan on the square surrounding the facade and, applying the sacred cut, three-part division and the circle's rectangle, able to lay down all the dimensions of the front elevation.

In switching to the ground-plan he took the square we worked with in the facade (square on the circle's rectangle) and by multiplying it 3 × 3 instead of dividing he achieved a large basic square, and proceeded to work out the details as in the facade. It is interesting to note that the whole task of planning this apparently complicated temple structure depends on selection of the appropriate basic square —and applying ancient geometric principles in the proper way.

I think there can hardly be doubt concerning the accuracy of the system. Selection of the wrong basic square prevents any of the dimensions from fitting into the diagram.

We have seen why the Temple of Ceres has six columns in its front colonnade and thirteen along each side. Other column ratios will be discussed later.

18

The Theseum

THE NEXT temple we shall investigate is the Temple of Hephaistos, known also as the Theseum, in Athens. It lies at the heart of Greece's ancient cultural centre.

It has been extremely well preserved and is indeed one of the finest examples of Greek architecture from the period 459—444 B.C. (see *Fig. 181*).

Externally the Theseum shares many features with the Temple of Ceres. Like the latter, it has six columns on the facade and thirteen along each side. Compared with its width the temple is much lower than the Ceres building. This indicates that a different factor was selected in the basic square of origin when determining the height.

The temple is of the traditional variety, with steps leading up from the outside.

The facade width of the Hephaistos temple is stated to be 13.708 m, its length is 31.77 m. Both of these dimensions are somewhat greater than the corresponding measurements of the Temple of Ceres.

The dimensions, extracts from A. W. Lawrence's book, *Greek Architecture*, give no indication whether they apply to the lower step or the edge of the platform, at the head of the steps, on which the actual temple stands.

The drawings on which we shall base our analysis are copies of originals held by the Academy of Fine Arts (Kunstakademiet) in Copenhagen, and executed in accordance with measurements taken at the site.

They were photographically copied and reduced to a suitable size for our purpose.

We begin our examination in the usual way: on the temple's facade. We have to locate first of all the basic square on which all the building's remaining pro-

Fig. 181.

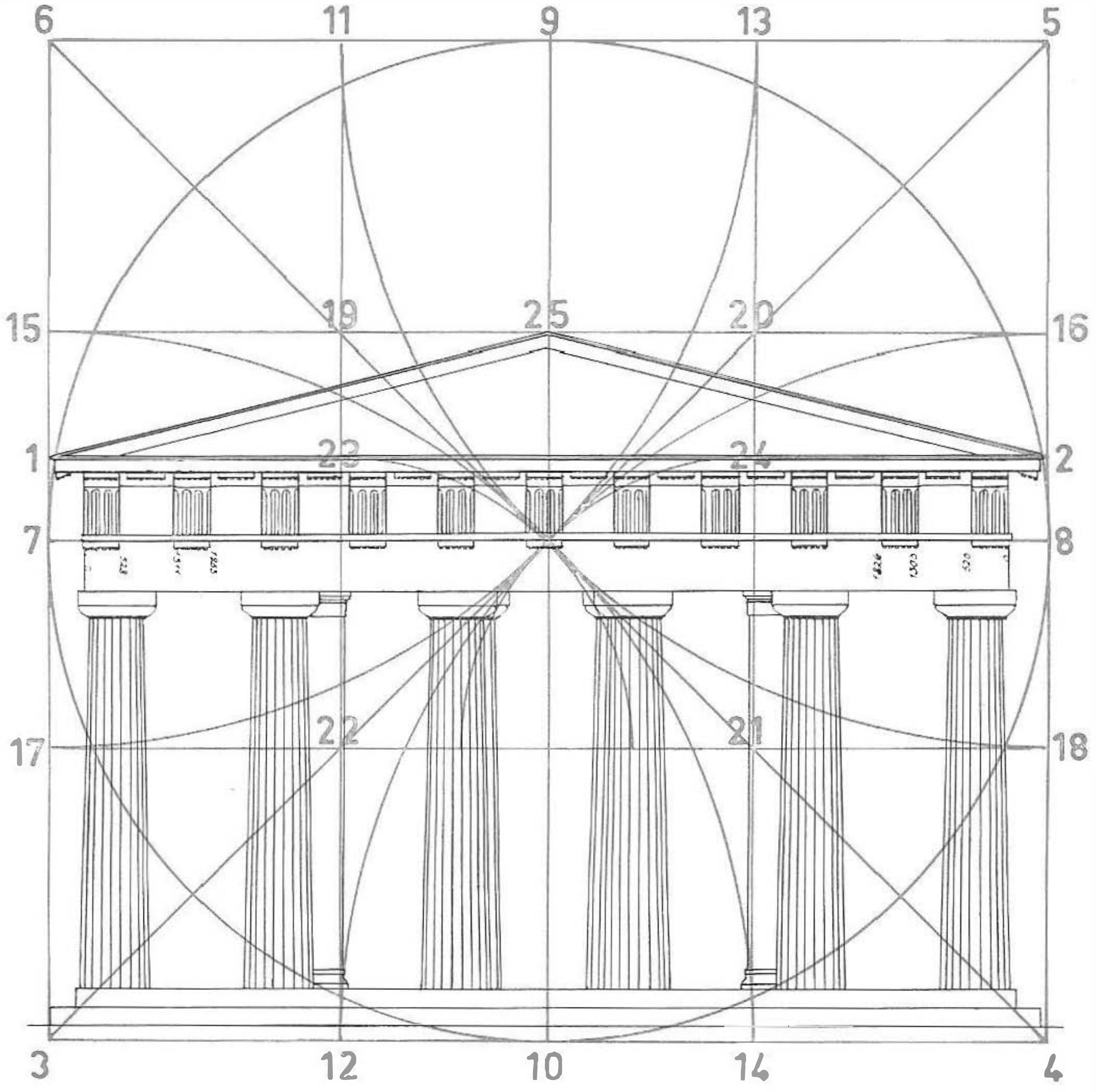


Fig. 182.

portions are founded. Once again we find it to be flush with the outer, horizontal extremities of the front face. We construct the square on a line drawn horizontally through the widest part of the roof. This line is seen in *Fig. 182* as 1-2. The complete basic square is 3-4-5-6.

We may already have noted that the base-line 3-4 has been moved one step lower than the foundation line of the drawing. Once again the problem of whether to trust implicitly every line of a drawing, or whether to delve a little deeper and find out exactly what exists in loco.

The draughtsman perhaps regarded the outside steps as an unimportant detail, not giving a thought to whether he should include two or one. But these steps are in fact a fairly vital part of the structural plan since they indicate the lower extreme of the basic square and therefore the point from which vertical measurements are made.

If one ignores something as apparently innocent as a missing step, it is impossible to get any of our familiar symbols to fit the temple plan.

But the Theseum has not one but two steps. Already our analysis exposes a seeming error in "official" material. We shall examine further this fault.

We saw in Fig. 181 a photograph of the Theseum by the Greek photographer Polykrates. It illustrates how well the temple has stood up to Time. All the columns and the building's roof and inner sanctum are there.

The part of the temple we see is the eastern end which, with its shallow stairway, rests on the original rocky plateau. But we cannot see sufficiently clearly how many individual steps there are.

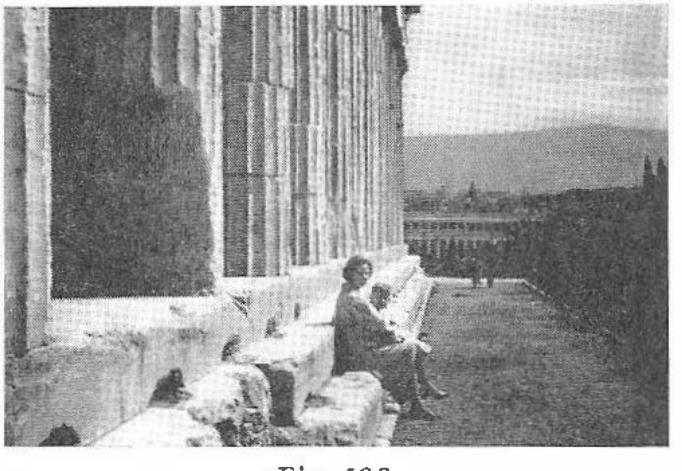


Fig. 183.

Fig. 183 shows a photograph taken by the author during a visit to Greece to clear up just such points of doubt. The analytical drawing made me suspect that there were two steps (in addition to the actual temple platform) instead of one. Prior to my inspection I had been unable to obtain a clear photograph on account of the shrubbery around the temple.

The actual analytical drawing was made several years before my Greek trip —which proves again that ancient geometry, properly applied, can supply information and trace faults which may otherwise pass unnoticed.

Now that we have established the existence of a second step, we can return to our analysis of Fig. 182. Entry of the vertical and diagonal crosses indicate already the first observation. The horizontal axis 7-8 cuts through the facade level with the bottom of the frieze.

Observation 1: Indication of lower edge of temple frieze.

We now enter the same symbol in our square as in the corresponding diagram of our last analysis, i.e. symbol "M" with its four sacred cuts.

These are seen at 11-12, 13-14, 15-16 and 17-18.

The lines of this symbol provide us with an indication of the temple's height, this being level with the upper horizontal sacred cut (15-16). This explains the difference in height between this building and the Temple of Ceres. Whereas here the line marks the apex of the sloping roof, in the preceding analysis it marked the top of the vertical wall (thus the bottom level of the roof).

Symbol "M" of course produces a smaller square at its centre, called here 19-20-21-22.

The upper horizontal sacred cut in this new square (line 23-24) marks the top of the temple's vertical wall. The roof pitch is therefore decided by the distance between the upper sacred cut in the basic square and the corresponding line in the inner square. The pitch can be drawn in as 1-25-2.

Thus we can record:

Observation 2: Indication of temple's height.

Observation 3: Indication of lower edge of roof.

Observation 4: Roof pitch.

We can from this analytical drawing of the facade record one or two details not

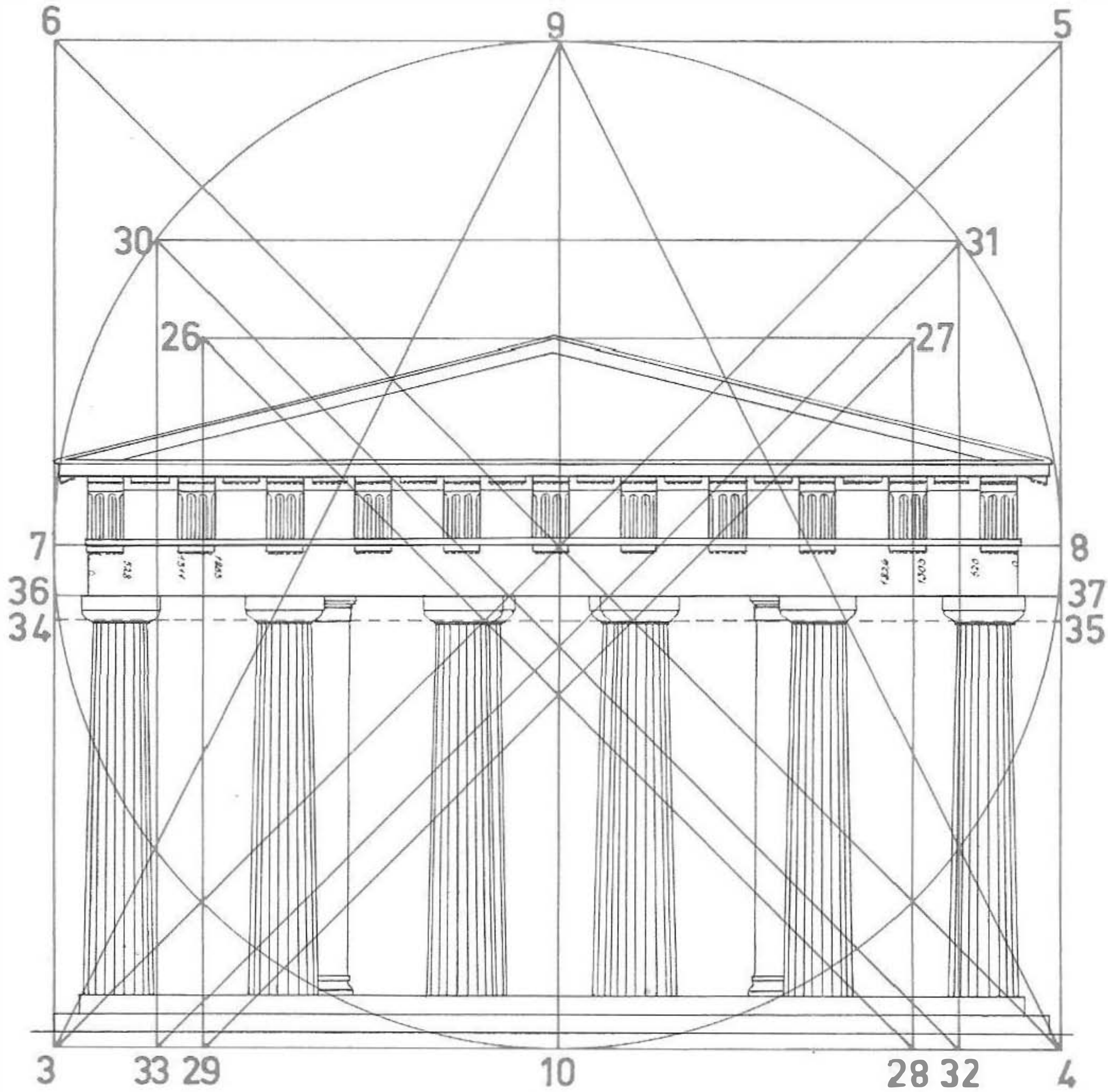


Fig. 184.

strictly concerning the front of the temple.

We can make out, behind the second column from each side, what appears to be another column. In fact these are the cornerstones of the inner temple.

We see how the two vertical sacred cuts in the basic square (11-12 and 13-14) most decisively mark the placing of these corners.

Observation 5: Indication of net width of inner temple.

Thus we see how our first analytical inspection of the facade of the Theseum, and the application of symbol "M", have revealed five important structural details of the temple's plan.

We cannot, however, apparently derive further information from this first diagram so we turn to a fresh drawing, *Fig. 184*.

This is the same basic square 3-4-5-6, and the same numbering of the vertical and diagonal crosses.

This analysis is based on symbol "H".

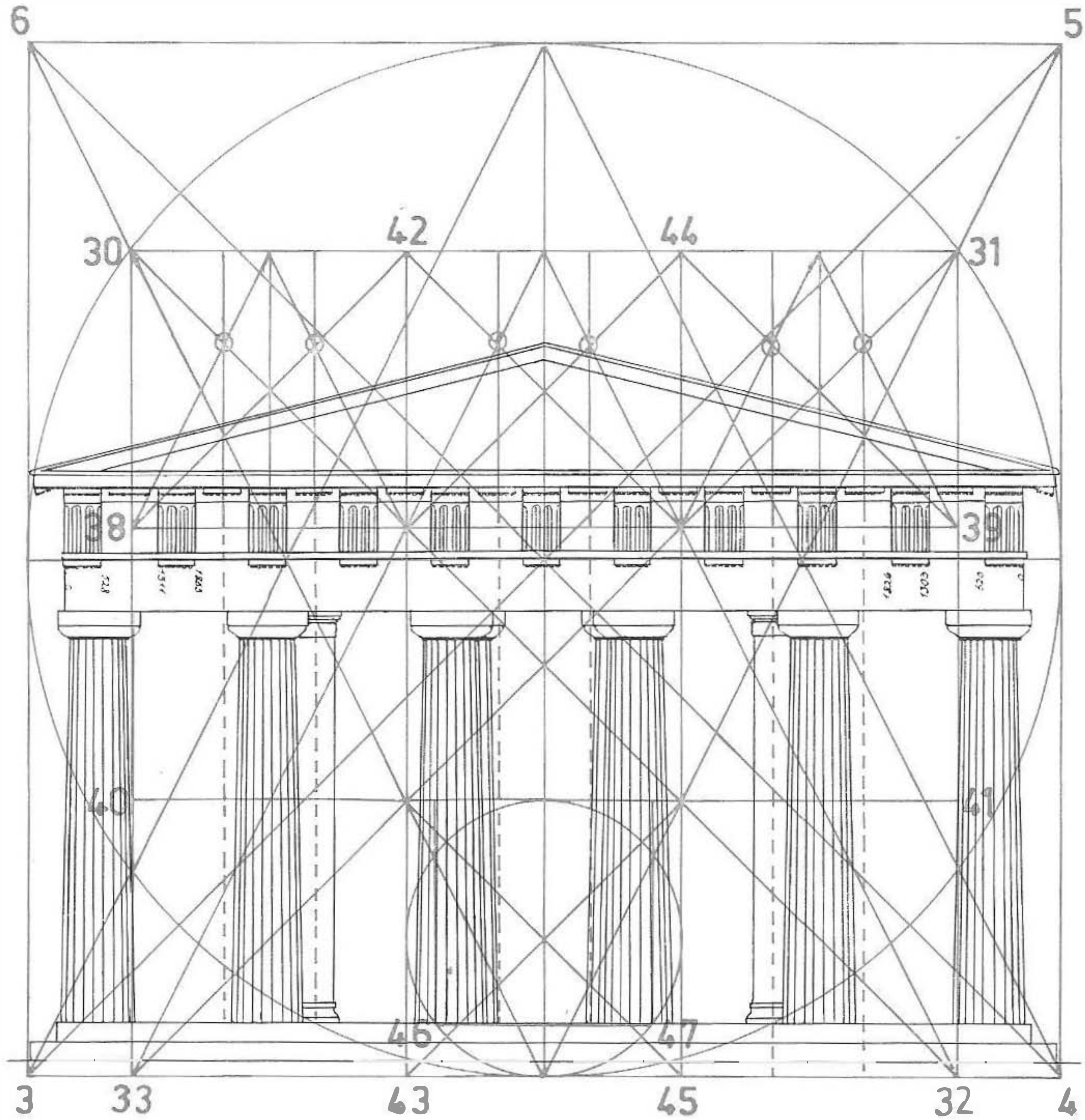


Fig. 185.

We now construct the half-size square 26-27-28-29 and place it on the centre of the base-line. In other words, we have placed symbol "N" upon our existing drawing.

The diagonals of this square are entered, and we go on to construct (as in the Temple of Ceres) symbol "U", i.e. the square on the circle's rectangle. This is 30-31-32-33, and here again we enter the diagonals.

Now we have three squares placed one inside the other and—as we shall see—it is the interplay between their diagonals that promises to reveal further structural information.

Let us, to simplify matters, call the basic square A, the square on the circle's rectangle (30-31-32-33) B, and the basic square's half-size version (26-27-28-29) C.

We see how A's diagonals at their intersection mark the bottom of the frieze (alternatively, the upper edge of the main supporting beam), a point which was established in our study of Fig. 182.

The same set of diagonals intersect those of square C and in so doing indicate the top edging of the columns. This is shown as a broken line 34-35.

The diagonals of A and B also intersect of course, and their job is to mark the underside of the supporting beam (and therefore the top of the column capital). This is line 36-37.

We can now make the following additional observations:

Observation 6: Thickness of supporting beam.

Observation 7: Thickness of capital.

Observation 8: Indication of top of columns.

Of the vital information necessary to the structural height of the temple we now lack only the marking of the temple floor (and thus the base of the columns). But since this is available only from the symbol that indicates the column spacing, it is best we move over to *Fig. 185.*

This fresh drawing has the same basic square 3-4-5-6 as in the two previous diagrams.

Inside the basic square we again place symbol "U" which, we recall, was used in the preceding figure. The reason for moving to a fresh diagram, however, was purely technical. There would otherwise have been too many lines to permit a clear examination.

The point of this next part of our analysis is to isolate the vertical lines of division in the building's facade, i.e. principally the column spacing. It is intriguing to discover that precisely the same symbol is applied here as with the Temple of Ceres. Here again then a remarkable sweep of similarity between the old building in southern Italy and the Theseum in Athens.

We have the square on the circle's rectangle as 30-31-32-33 which is divided by our usual method into 3 × 3 smaller squares. The dividing lines are 38-39, 40-41, 42-43 and 44-45.

It is still worth remembering that we use no measurement of any kind to divide this square. It is purely a combination of the diagonals at their intersection with the acute-angled triangles: a geometric construction.

The three upper squares of these nine (making up the area 30-31-38-39) are in their turn split into three. These latest lines are produced, broken, down through the diagram towards the bottom of the square on the circle's rectangle, which is now split into nine vertical strips. Here again the division which provides for four columns and five spaces.

We will note that in this instance the columns do not completely fill their respective strips. In other words, the columns of the Theseum are slimmer, compared with the temple's other dimensions, than the columns of Ceres.

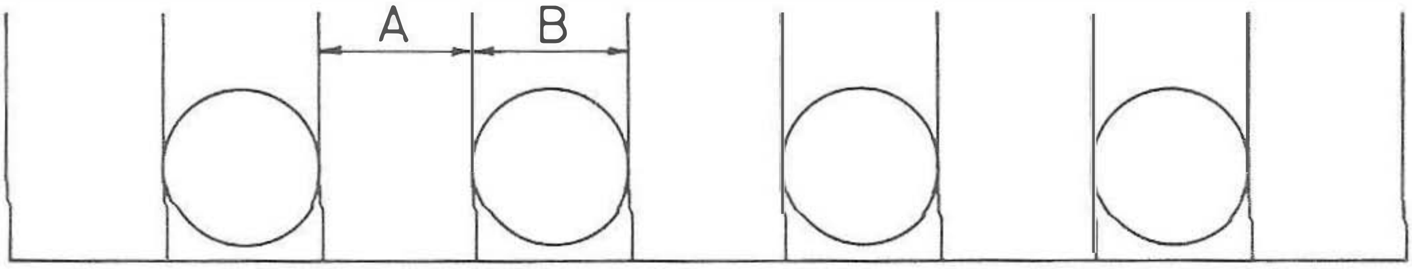
It would appear that in this latest temple the nine strips are intended to mark the width of the column capital, since the column itself is narrower, both at its base and head.

Consequently the space between each pair of columns is greater than the actual column—some of the width normally occupied (e.g. in the Temple of Ceres) by the column has been allotted to the spacing.

We have an illustration of this in *Fig. 186* in which the two column arrangements are shown in horizontal section. The difference is immediately apparent. Whereas A and B in the Temple of Ceres are equal, in the Theseum their proportion is almost 2:1.

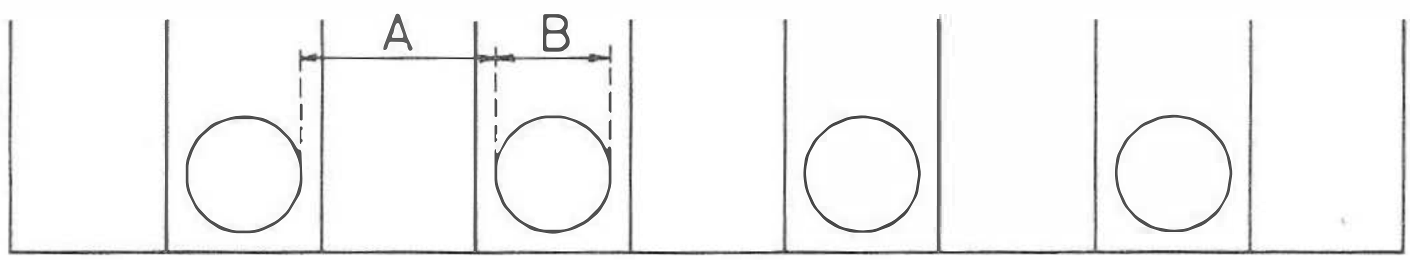
In fact Fig. 186 is not strictly accurate.

Column spacing Temple of Ceres



Space (A) equals diameter of column (B)

Column spacing The Theseum



Space (A) considerably greater than column (B)

Fig. 186.

The difference in reality is hardly as great as shown. But the slight exaggeration was intended more clearly to illustrate the point.

We see therefore that in spite of having slimmer columns and a definite difference between column and spacing, the temple facade nevertheless derives its lines of division from the same symbol in both temples. It is merely a question of the architect selecting one of a number of choices within a certain predetermined framework.

To revert to Fig. 185 we see the division by three is emphasised in the frieze. It is evident that approximately $\frac{1}{3}$ of the width of each rectangular strip is occupied by a decorative area. When we divide the rectangles again into three strips we find that the two dividing lines coincide with the vertical lines of the ornamental portion. But since the diagram is rather small for our purpose we shall not (indeed cannot) go into detail. We note the fact merely as an interesting point. The column spacing however is noted as

Observation 9: Placing and spacing of frontal columns.

The previously mentioned factor of the facade, the marking of the temple floor and therefore of the column bases, is to be found in the small square on the baseline defined by 43-45.

In this square we enter symbol "U" but instead of positioning it as in the main basic square, we turn it upside-down. We now see how line 46-47 of this symbol's small square (the square on the circle's rectangle) marks the level of the temple floor and thus the column bases.

Observation 10: Indication of floor level and column bases.

This rounds off the information we require regarding the facade. We know sufficient about the respective heights and widths to be able to reconstruct the front

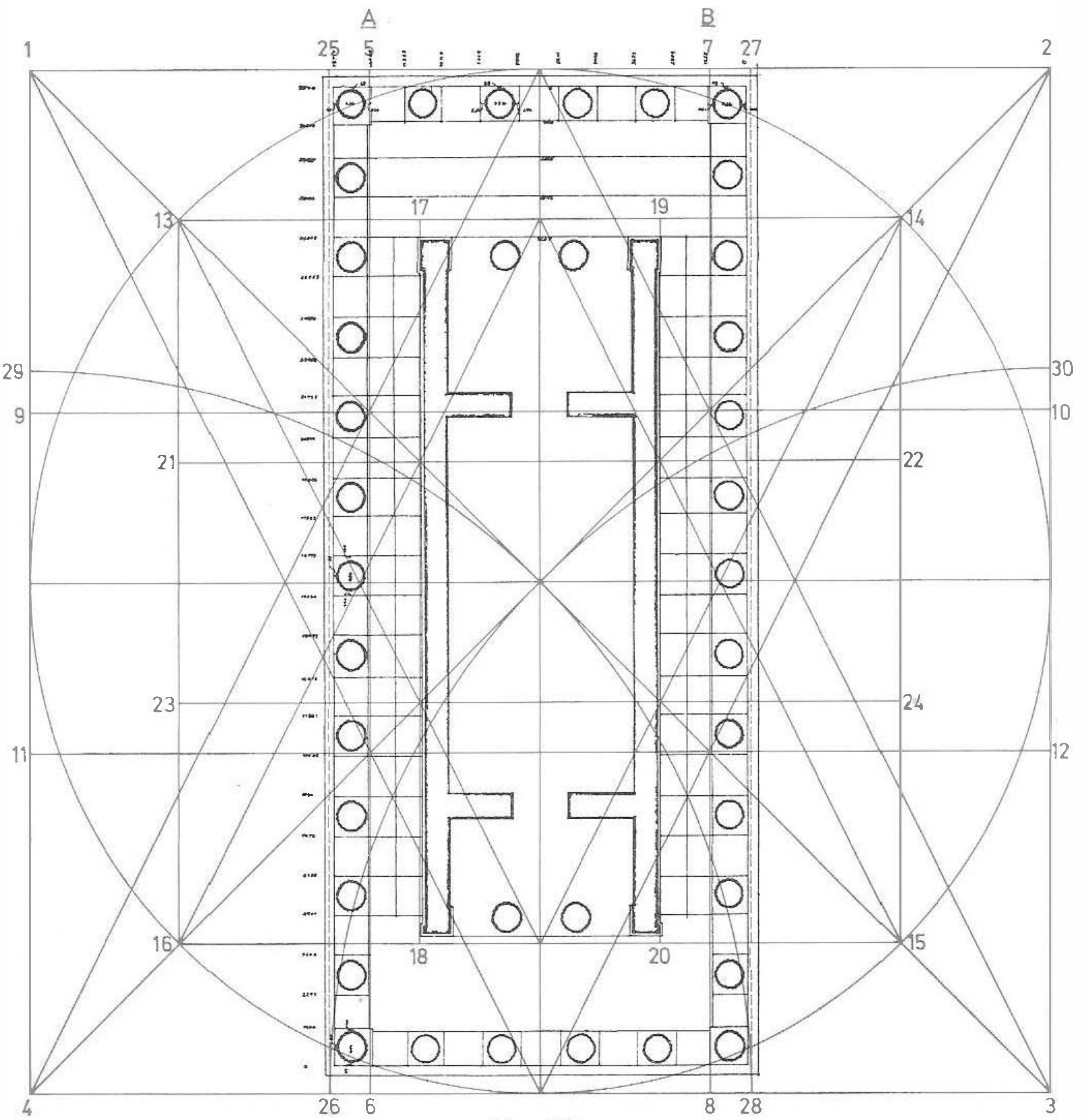


Fig. 187.

of the building, and can therefore turn our attention confidently to the ground-plan.

We recall from the Temple of Ceres that the large basic square of origin from which we traced the ground-plan was obtained by multiplying the square on the circle's rectangle 3 × 3.

The procedure is identical with the Theseum. Again a vivid point of similarity in the planning of these two ancient buildings. In *Fig. 187* we have the ground-plan drawn in upon the 3 × 3 squares in the proper proportion.

At the top of the diagram we see a line marked AB. This corresponds exactly to line 32-33 in the facade diagram, i.e. the base-line of the square on the circle's rectangle which, in the facade, provided a host of valuable information.

Line AB has been measured out to the right and left, providing line 1-2, which

Erratum: In Fig. 187 the red portion of the analysis should be elevated 1 mm in relation to the ground-plan.

has become the side in the new basic square 1-2-3-4. This is therefore 3 × 3 times as great as the square on the circle's rectangle in the facade. And the same arrangement as in the Temple of Ceres.

Normal procedure again. Enter the vertical and diagonal crosses and the two acute-angled triangles. This provides the marking of the vertical and horizontal lines of 3-part division.

The two lines that run the length of the temple are 5-6 and 7-8. They run flush with the inside of the two side colonnades. This of course is not surprising since it was the distance between these two points that formed the basis for the construction of our large main square.

But we should not forget that the point of origin was not selected at random; it was dictated by the analysis of the facade and, since an area from the temple's front elevation provides information about the ground-plan, we ought to regard this as

Observation 11: Marking of inside line of side colonnades.

We now enter in our basic square its inside circle which of course allows us to construct the square equal to half the area of our basic square. This is seen as 13-14-15-16, and is thus symbol "H".

Just as we 3 × 3 divided our large square so we do with this new square. The lines of division are 17-18, 19-20, 21-22, 23-24.

We can immediately see how the vertical lines closely define the outer wall of the inner temple. Thus the width of the cloisters are determined by the 3-part division of the large square and the same division of the small square.

This we record as

Observation 12: Marking of outer wall of inner temple and thus width of cloisters.

To be strictly correct we should perhaps have put a front elevation drawing at the end of the temple to show the thickness of the respective walls. Because in Fig. 182 we saw that the internal width of the inner temple was marked by the sacred cut in the original basic square. Combining our two pieces of information we can say:

Observation 13: Marking of internal width of inner temple in Fig. 182 combined with new marking of external width, indicates thickness of inner temple walls.

There is yet another bit of information to be obtained from the half-size square. Its base-line 15-16 marks the end wall of the inner temple.

Observation 14: Marking of external end wall of inner temple.

Our next step is to construct the sacred cut in main square 1-2-3-4. This is accomplished in the usual way, by using as the radius of our arc line 0-3 (from the centre to the lower right corner).

This half diagonal of course is equal to the side of the half-size square and is measured out along base-line 3-4 to form 3-26. Similarly along 3-2 to form 3-30.

The same procedure is repeated from point 4, providing 4-28 and 4-29.

Since we require only the vertical sacred cuts, these are entered as 25-26 and 27-28.

We see immediately how the sacred cut in the main square marks the gross width of the temple at platform level, i.e. above the approach steps. This is the same symbol as used in the Temple of Ceres for the same purpose.

We call this

Observation 15: Marking of temple's gross width.

This completes our study of the longitudinal lines of the temple. Our analysis requires to show now only the column placing and spacing, and the transverse division of the inner temple.

Since the temple has 13 columns along each side, like the Temple of Ceres, the obvious assumption is that the architect of the Theseum applied the same procedure for placing the columns, i.e. taking as his basis the spacing of the front colonnade and using it to position the side columns.

But this would require a space at each end of the 5/4/5 formation, and in the Ceres building that space is occupied by three steps.

Here in the Theseum there are only two steps. And moreover in the groundplan we can readily see that the distance from the extreme point of the facade column to the outer point of the furthest step does not equal, as one might expect, $\frac{1}{27}$ of the total temple length. And since a column spacing such as that in the Temple of Ceres necessitates that the side of the building is divided into 3 × 9 spaces, alternating between spaces and columns, the first space would *have* to be $\frac{1}{27}$ for this arrangement to apply.

Another factor indicates that the procedure in this temple is different from the preceding: the column spacing is not uniform. The first space at each end of the long side of the temple is considerable less than the others.

We see this shown in *Fig. 188* in the upper right corner of the temple. The first space (A) on the diagram measures 10 units, while the second space (B) measures 13 units. There is a difference therefore of 30 % between these two spaces, and since this is repeated at each of the other three corners we can assume that it was intended.

We see the column spacing in Fig. 188 in which the half-size square is again shown as 13-14-15-16. As in the previous diagram, it is divided 3 × 3 and we see three of the nine squares contained in rectangle 13-17-18-16. Each of these is then supplied with the vertical and diagonal crosses and the acute-angled triangles.

This produces the eight-pointed star, or the symbol we refer to as "S", the symbol applied earlier in dividing the square's area into a series of smaller squares.

We saw in Fig. 59 how this symbol was applied in dividing the square 6 × 6. Here, too, we shall use that particular division.

We may recall that the symbol contained various points of intersection through which, if we drew a horizontal or vertical line, we could obtain the appropriate subdivision of our square.

In this diagram we want a 6-part division of our three small squares, and the 6-part division has the same operative line as the 3-part. This line is 25-26 in our diagram. Each of the three squares can thus be divided into six rectangles, and simultaneously line 13-16 is split into 18 parts.

We see now that from the upper left corner of the diagram (point 13) every second 6-part dividing line carries a column, placed centrally on the line. Nine columns are thus positioned by line 13-16. Only the pair at each end of the temple remain to be positioned.

Since the corner column of the temple is determined by the frontal colonnade, the architect required in fact only to place one column (the second) at each end of the temple.

He decided here to retain the same distance between this column and no. 3 as in the placing of the first nine columns. The distance therefore between no. 2 and the corner column is rather less than the other spaces.

Thus we have achieved the last bit of vital information we needed, which we note as

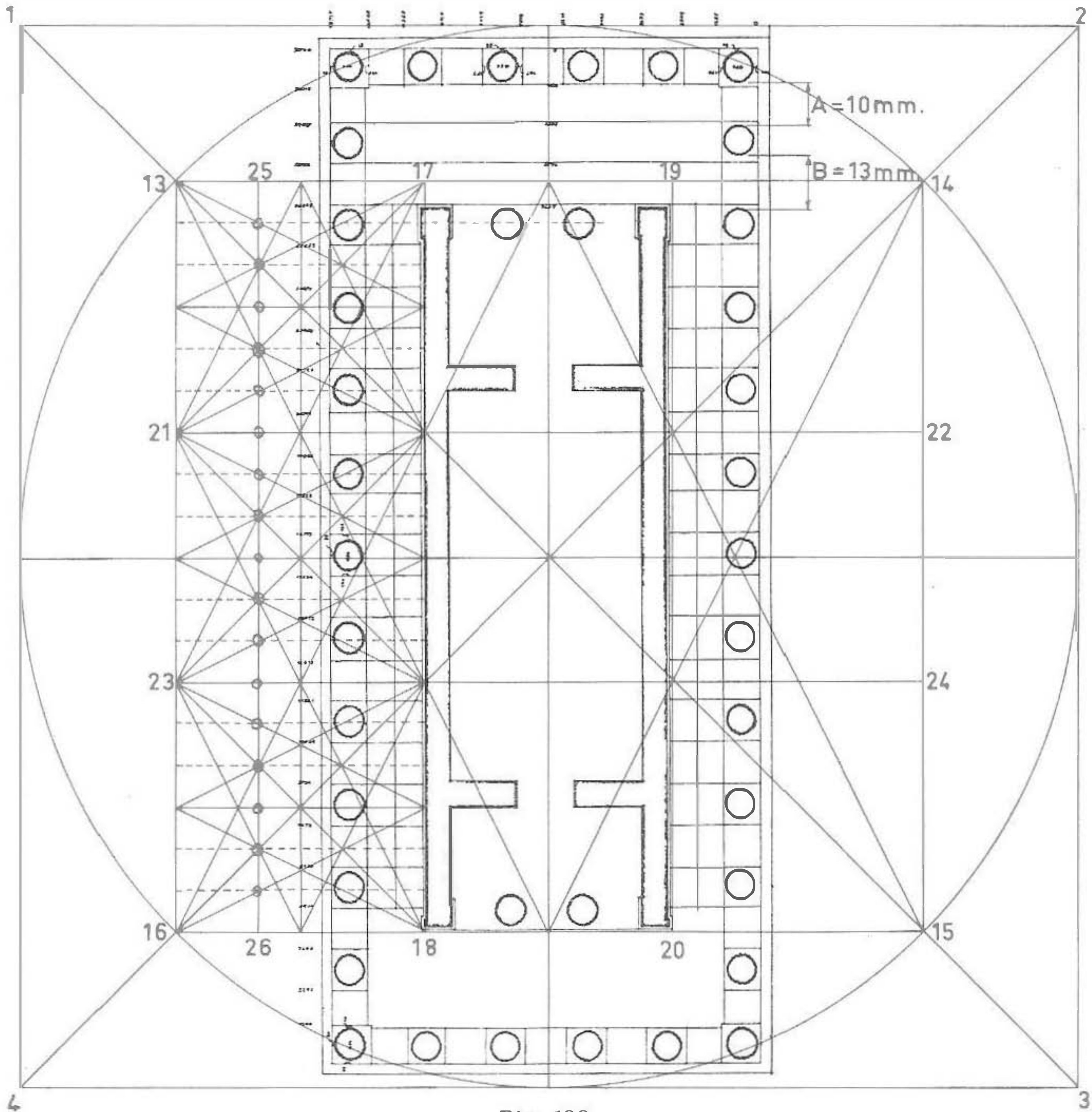


Fig. 188.

Observation 15: Spacing of side colonnade.

We shall conclude our analysis of the Theseum here. In the same way as with the Temple of Ceres we can—once we realise how to apply the appropriate aspects of ancient geometry—reconstruct the facade and ground-plan of the temple, retaining the correct dimensions, proportions and lay-out without resorting to additional illustrative or dimensional material. We have at our fingertips the necessary structural information to rebuild the Theseum.

The finished plan of such a building would of course require a mass of other, more intricate, detailed drawings. But compared with the principal features which we have uncovered, these would be no more than mere details, doubtless yielding to a really thorough survey their relationship to and origin in ancient geometry.

Temple of Poseidon

AND FROM Greece—back to Paestum in southern Italy! The reason for the backwards/forwards switch between these sites is that our analyses are becoming increasingly more intricate. I have chosen to progress from simple to more difficult plans regardless of geographical situation, rather than deal first with one area of the Mediterranean and then with the other.

At Paestum only a few hundred yards from the Temple of Ceres we find another temple, larger than either of the two previously examined. It was dedicated to the god of the sea (the Greeks called him *Poseidon*, the Romans named him *Neptune*), and is the best-preserved of Paestum's three ancient temples.

The building is 79½ feet wide and 196¾ feet long, and A. W. Lawrence places it in the period 460—470 B.C.

In many respects there is close resemblance between the Temple of Poseidon and the Theseum. Both rise two steps above the ground, both have six frontal columns, and their height/width ratios are almost identical. Proportionally speaking, Poseidon's temple is a fraction lower than the Theseum. In reality, of course, the former is nearly four times larger.

Despite the similarity the dimensions of the Temple of Poseidon are based on a different theory and plan from those of the Theseum. The Temple of Poseidon appears to have been planned by a much more skilled and able exponent in the mystery of geometry. The analysis will

Fig. 189.

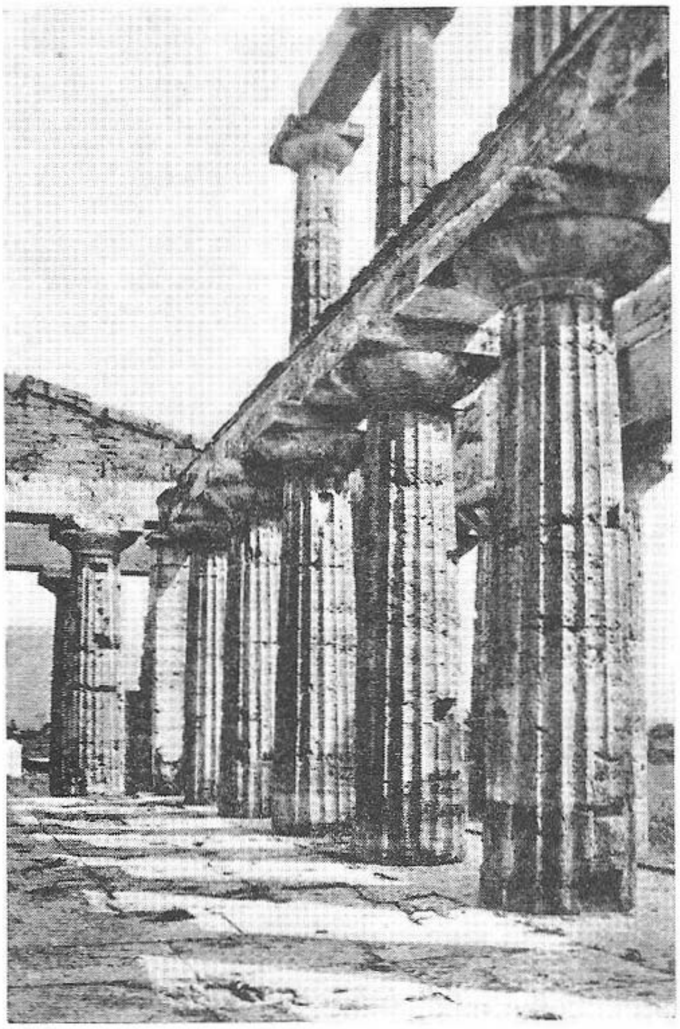


Fig. 189a.

prove to be more complicated than those of the two previous temples, although the order of application of the symbols is, generally speaking, maintained.

In the Temple of Poseidon one feature stands out immediately as differing from the earlier temples: the building has 14 columns at each side contrasting with the others' 13.

We have already analysed two procedures which, though different, each produced a 13-column arrangement. This will be a third method of division, but with 14 columns resulting.

As with the previous analyses, our study will be conducted on drawings and material kindly lent by the Academy of Fine Arts (Kunstakademiet) in Copenhagen. They were compiled in 1922 by Danish architect Buus-Jensen on the basis of on-the-spot measurements, and were part of an attempt to test the theories on ancient building design set out in Macody Lund's book, *Ad Quadratum.*

According to a report lodged with the drawings, Buus-Jensen was unable to get the theories to fit his measurements. On some points there were differences of more than one meter.

The drawings were executed on fairly thin paper enabling the author to have photostat copies made. But as these were too large to suit the present volume they were touched up in Indian ink and reduced photographically to the required format.

In *Fig. 189* we have a photograph of the Temple of Poseidon. We observe the fine state of preservation of the building's exterior.

All columns and lintels, and both pediments are more or less intact. The roof and most of the inner temple have, however, collapsed or been dismantled through the years.

In addition to the external colonnade the Temple of Poseidon (differing once again from the others) has an inner colonnade within the inner temple.

These latter columns were erected a little distance in from the inner wall, providing a cloister inside the inner temple.

Time has reduced much of the interior of the temple to ruins, but some of the inner temple wall and columns remain. *Fig. 189 a* is a view of the inner temple.

We begin our analysis of the facade with *Fig. 190.* The first step, as always, is to locate the basic square that surrounds the facade.

Experiment shows that for this analysis we require, in addition to the basic square, its double-size version. The basic square is therefore not the outer one in our diagram but that marked 1-2-3-4.

It is constructed in the recognised manner. We take the horizontal extreme of the temple's roof, apply it as the square's

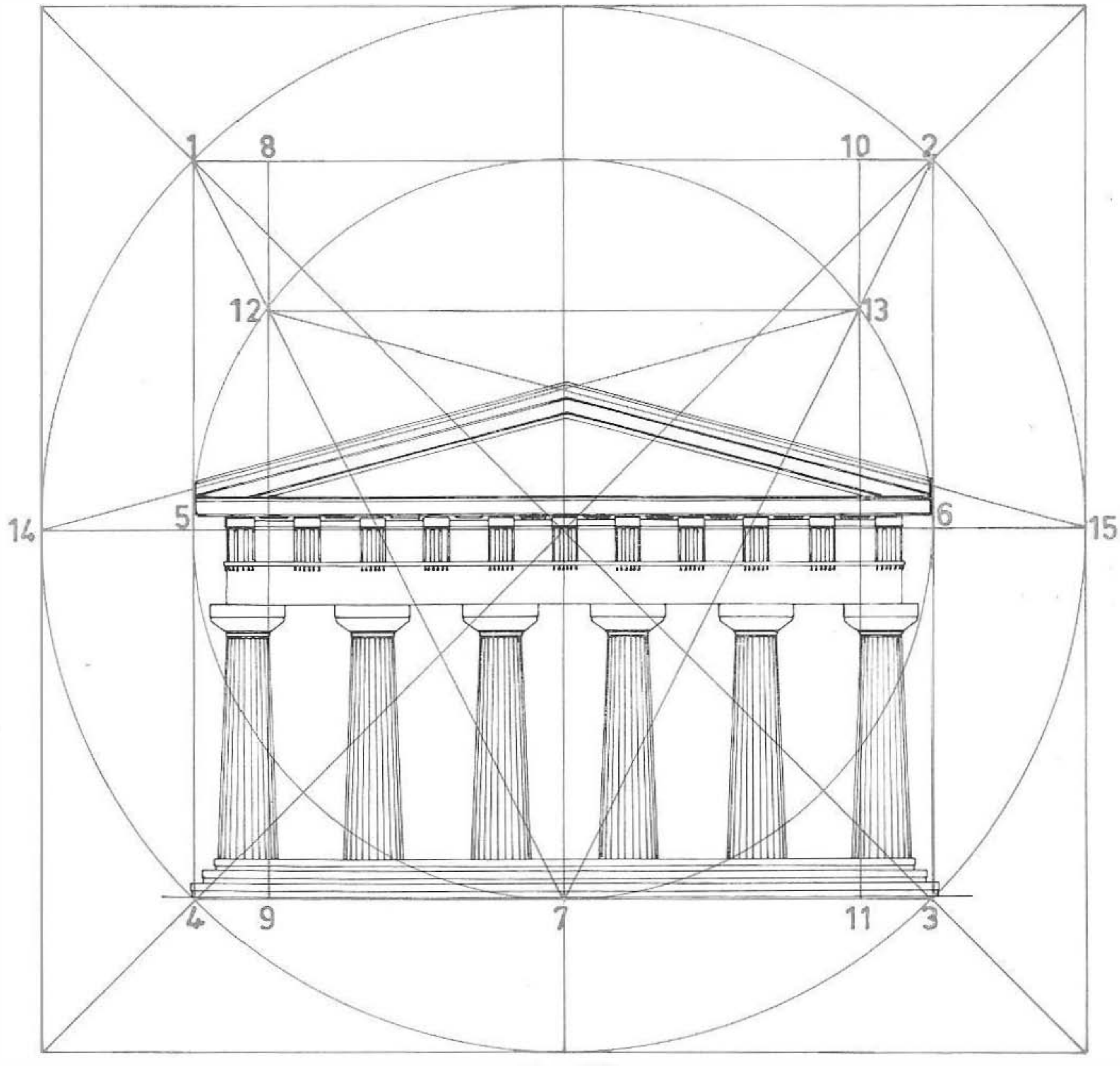


Fig. 190.

base-line 3-4, and construct the square on that line.

Still guided by tradition, we enter symbol "Q" in square 1-2-3-4, and expand this to symbol "U", showing the square on the circle's rectangle, 12-13-11-9.

Perhaps the first point we notice is the horizontal axis of our basic square, line 5-6. It coincides precisely with the uppermost horizontal of the frieze, running under the line of the roof. This may be recorded as *Observation 1.*

Square 12-13-11-9 is of course the square on the circle's rectangle, and we recall how important it was in analysing the two previous temple structures. Not only did it govern the distribution of columns, but it was also a basic factor in the lay-out of the ground-plan.

It has the same significance and plays the same part in the Temple of Poseidon —but in a slightly different way.

Look at the square's two vertical sides. We see that instead, as previously, of indicating the base of the two outer columns, these mark the tops of the columns.

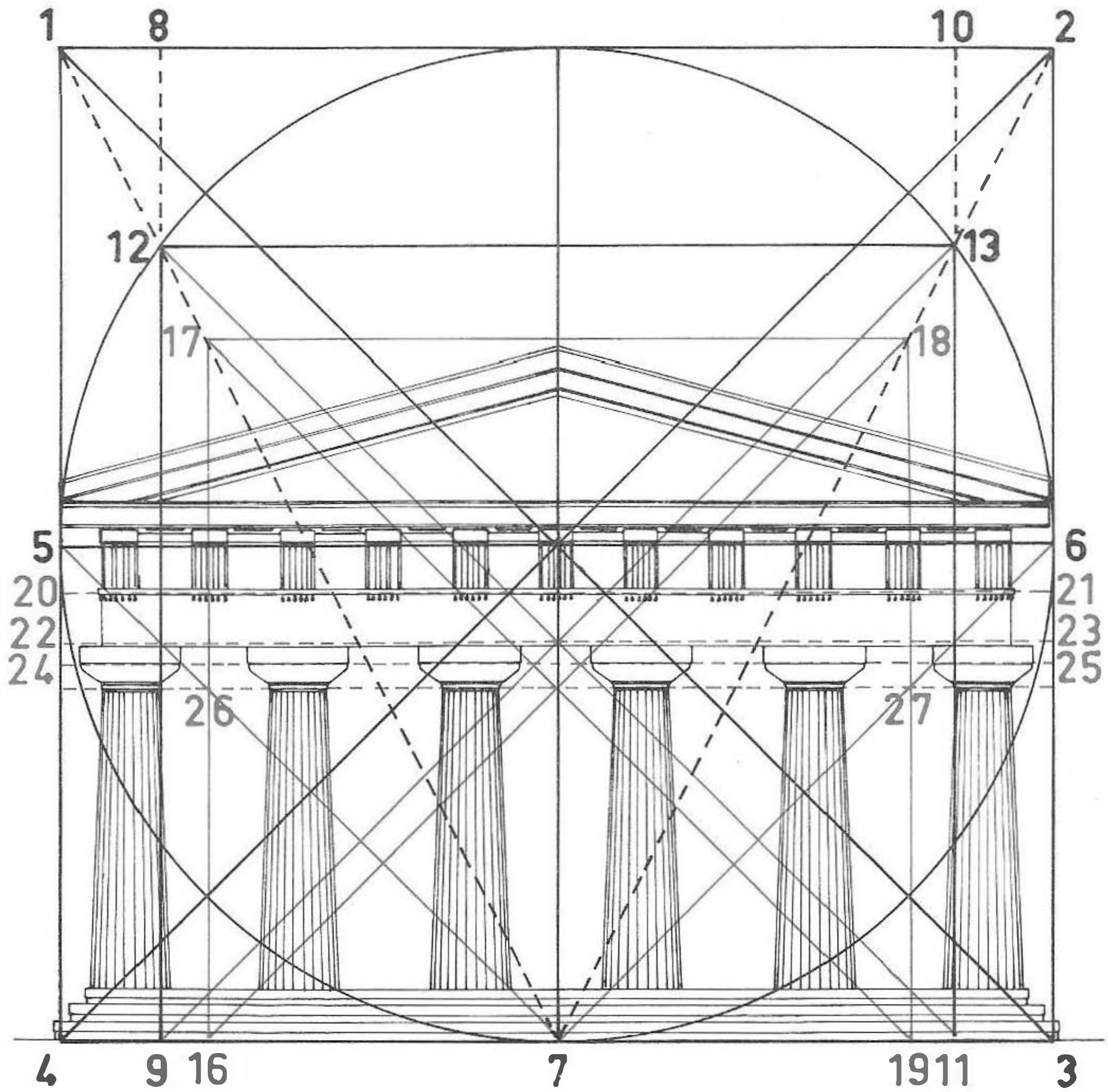


Fig. 191.

This tends to produce a rather thick column. Since the temple has a much more massive structure than the previous two examined, it is possible that the building's sheer weight led to this choice.

The whole problem of selecting column thickness, length and taper was—as far as I have been able to ascertain—determined by clearly defined rules with, however, ample opportunity for variety. We shall study this later.

We now construct in our diagram the double-size version of our basic square. The simplest way to do this is to describe the basic square's outside circle and construct a square around it. We recall from earlier experience that the square within a circle is related to the square outside the circle in the ratio 1:2.

The horizontal axis of the new large square coincides of course with the same axis of the basic square. This line is 14-15.

We can now see the part played by this line and the upper side of the square on the circle's rectangle. A line from point 14 to 13 and another from 15 to 12 indicate precisely, at their intersection, the temple's maximum height, i.e. the apex of the pediment, and at the same time mark the roof-pitch.

The above procedure, with the basic square and its double-size version, produces a temple design which is a shade lower proportionately than the traditional choice of the sacred cut in the basic square. At the same time, however, the architect achieves an attractive and unusual angle of pitch.

From this diagram we move over to *Fig. 191*. We have here again the basic square 1-2-3-4 and its internal lines. We no longer, however, require the large outer square. Instead we shall make some use of the basic square's *half-size* version.

Whereas the large outer square shared the same centre as the basic square, the half-size version is lowered to the bottom of the diagram.

The square is 16-17-18-19, but its constructive guide-lines have been partially omitted for clarity.

There is an illustration here of the departure from the plan of the Theseum. Line 17-18 is in fact the sacred cut in the basic square. This was the line in the Theseum which indicated the height of the building. Here in the Temple of Poseidon we can see that the building is rather lower than the sacred cut—proving that the architect determined the height not in this diagram but as shown in the previous figure.

We now have three squares placed within each other. The outer basic square 1-2-3-4, the intermediate square (on the circle's rectangle) 12-13-11-9, and the new inner half-size square 16-17-18-19. In fact this is the same construction used in the facade of the Theseum.

The vertical and diagonal crosses—where these lack—are entered in the three squares.

The diagonal crosses intersect just under the centre of the diagram, creating a small square consisting of two squares and two rectangles. This is a vital square, revealing several dimensions of the temple facade.

Line 20-21 indicates the lower horizontal edge of the frieze and at the same time marks the top of the beam across the columns. Line 22-23 marks the bottom of this beam and therefore the top of the column and abacus.

The smaller of the two squares indicates the depth of the abacus/capital, and we see how line 24-25 marks the junction of the two. The bottom of the capital (i.e. the neck of the column) is indicated by the horizontal axis in square 16-17-18-19.

Thus we find that all the horizontal divisions we require from the horizontal line of the roof to the neck of the columns are indicated by lines between the appropriate axes of the basic square and its half-size version.

While examining the diagonals, it is also interesting to note that the diagonal cross in the basic square indicates—at points 3 and 4—the angle of ascent of the outer steps.

To illustrate yet another point about the Temple of Poseidon we move to *Fig. 192* in which we again start with the basic square 1-2-3-4. And, as in the preceding diagram, we have the half-size square 16-17-18-19, with its horizontal axis cutting across the neck of the columns.

This latter square is divided into four quarter squares by its vertical cross. The two lower squares are individually equipped with their respective half-size versions, symbol "H", by inscribing an inside circle and an inside square. We see that the bases of these two new small squares indicate (by line 28-29) the temple's floor level, i.e. the top of the approach steps. Since the columns rest directly on the

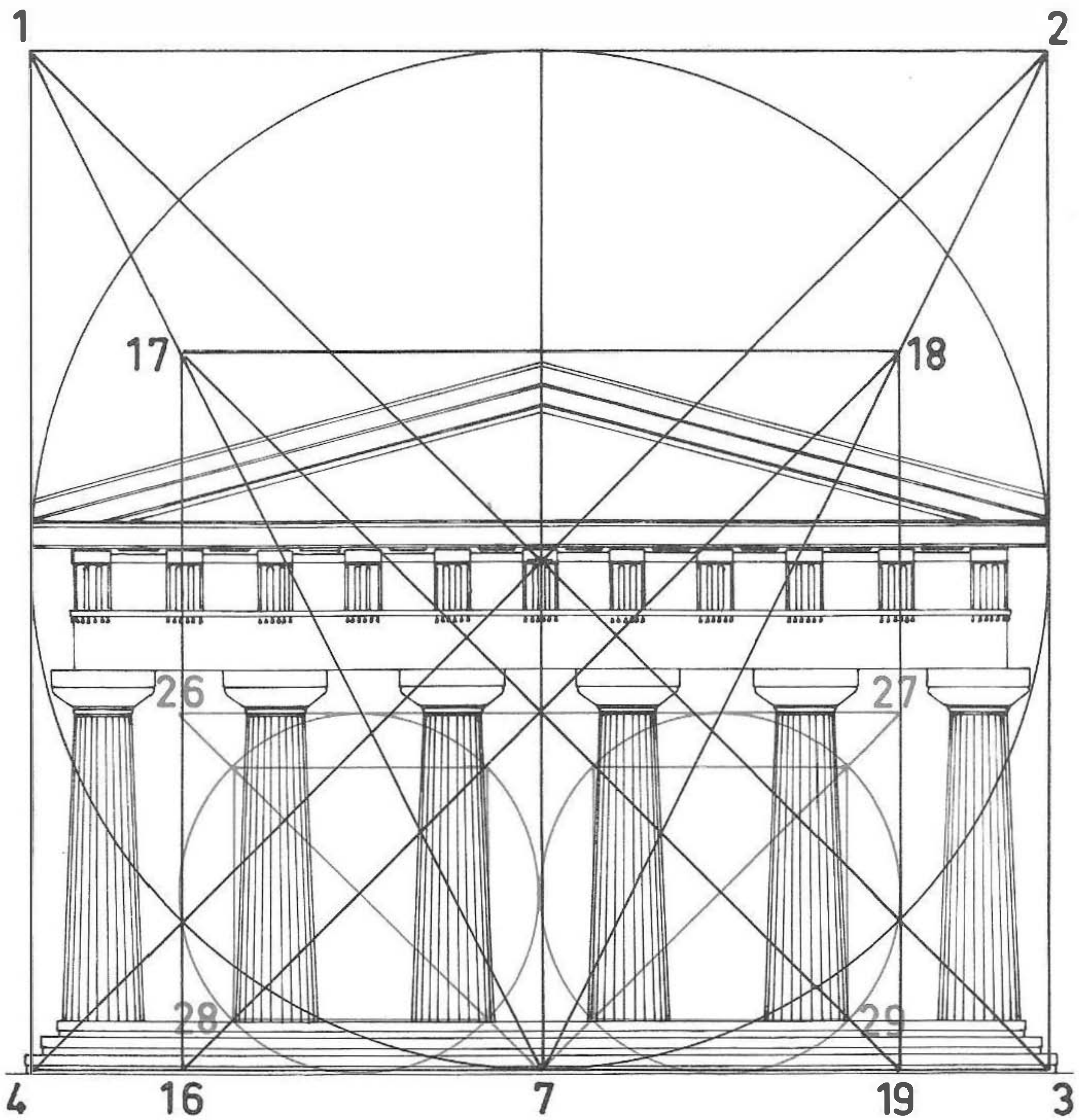


Fig. 192.

floor of the temple, this line also indicates the base of the columns.

The neck of the column, as we have already discovered, is marked by line 26-27. This diagram thus shows the length of the column from base to capital.

As we also know the depth of the capital (distance between 26-27 and 22-23) we therefore have a marking for the total height of the column.

We have traced most of the horizontal lines of the temple's facade and can recapitulate:

Fig. 190,

Observation 1: Uppermost horizontal of frieze.

Observation 2: Indication of temple's total height.

Observation 3: Indication of roof-pitch.

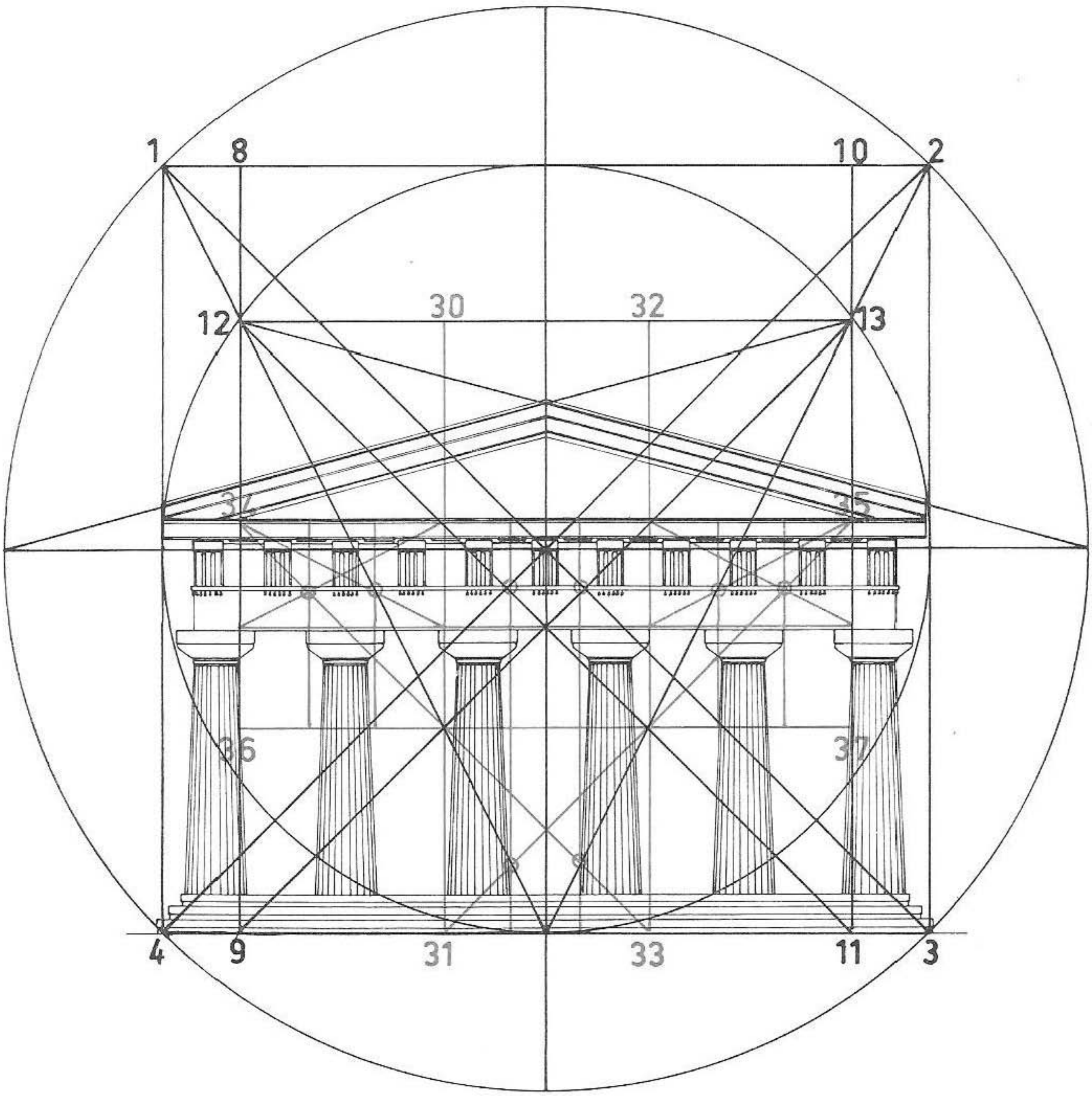


Fig. 193.

Fig. 191,

Observation 4: Bottom of frieze (and top of chief beam).

Observation 5: Lower edge of chief beam.

Observation 6: Total height of columns.

Observation 7: Depth of abacus and capital respectively.

Observation 8: Angle of ascent of approach steps.

Fig. 192,

Observation 9: Temple floor and thus column base.

These first three diagrams have yielded nine unmistakable pieces of information about the facade planning of the Temple of Poseidon. The only really vital factor missing now is the placing and spacing of the columns.

Column distribution in this temple was, as we shall see, determined by the same factors as in the two previously discussed temples. We must therefore enter symbol "U" in the basic square, i.e. the symbol comprising the square on the circle's rectangle.

This is shown in *Fig. 193.*

The square on the circle's rectangle is 12-13-11-9 and is sub-divided 3 × 3 in the usual manner by entering the acute-angled triangles.

The dividing lines are 30-31, 32-33, 34-35 and 36-37.

If we now enter the same 3-part division in the lower central square (base-line 31-33) but apply only the vertical lines, we see this particular division matches exactly the placing of the two middle columns. A space flanked on either side by a column, all three making up line 31-33.

The upper of the 3-part dividing lines in the large square is 34-35, and this marks the top edge of the transverse beams which form the horizontal line of the roof.

The three central squares contained in the area 34-35-37-36 have also been split vertically into three strips each, and we see how the individual portions of the frieze match perfectly these nine strips.

The main purpose of this diagram has been to demonstrate that the distribution of columns was executed in exactly the same manner as in the two preceding examples, i.e. by manipulation of the 3-part division of the square on the circle's rectangle after the two outer columns had been marked off by the vertical sides of the circle's rectangle.

We may add these to the existing observations:

Fig. 193,
Observation 10: Spacing of frontal columns.
Observation 11: Placing of main decorative areas of the frieze.
Observation 12: Uppermost horizontal line of roof.

Having obtained roof thickness and distribution of the columns there remain few doubtful factors in the facade. We can turn our attention to the ground-plan, and we shall see that these dimensions are also determined by the same geometric rules.

*

Passing from our analysis of the facade to the ground-plan, we require as a link the large basic square of the ground-plan in order by geometric sub-division to trace the lay-out of the temple.

As with the other temples, the ground-plan's basic square in the Temple of Poseidon is geometrically connected with the facade. The procedure is a repeat of the process seen in earlier analysis: the square on the circle's rectangle in the facade diagram (square 12-13-11-9 in Fig. 193) is laid out 3 × 3 in order to provide the ground-plan's square.

The initial analytical drawing is seen in *Fig. 194*. The first point we shall examine is the spacing and position of the columns along each side of the temple. We have already observed that these in the Temple of Poseidon number 14 as opposed to the 13 of the two preceding temples.

But first a note of caution. A very close examination of the diagram will reveal a slight error in draughtsmanship or perhaps in structure: the side-walls and end-walls of the temple are not at right angles to each other. There is an error of ½° and it was found on the drawings lent to the author by the Academy of Fine Arts (Kunstakademiet), Copenhagen.

In the following analysis I have turned my study to the side of the temple on which the error of angle is less obvious.

It was considered that the best way to illustrate column spacing, particularly in relation to the facade, was to include a front view drawing of the temple in Fig. 194. Under this facade plan is repeated the arrangement on which was based the distribution of the frontal columns. The now familiar square 12-13-11-9.

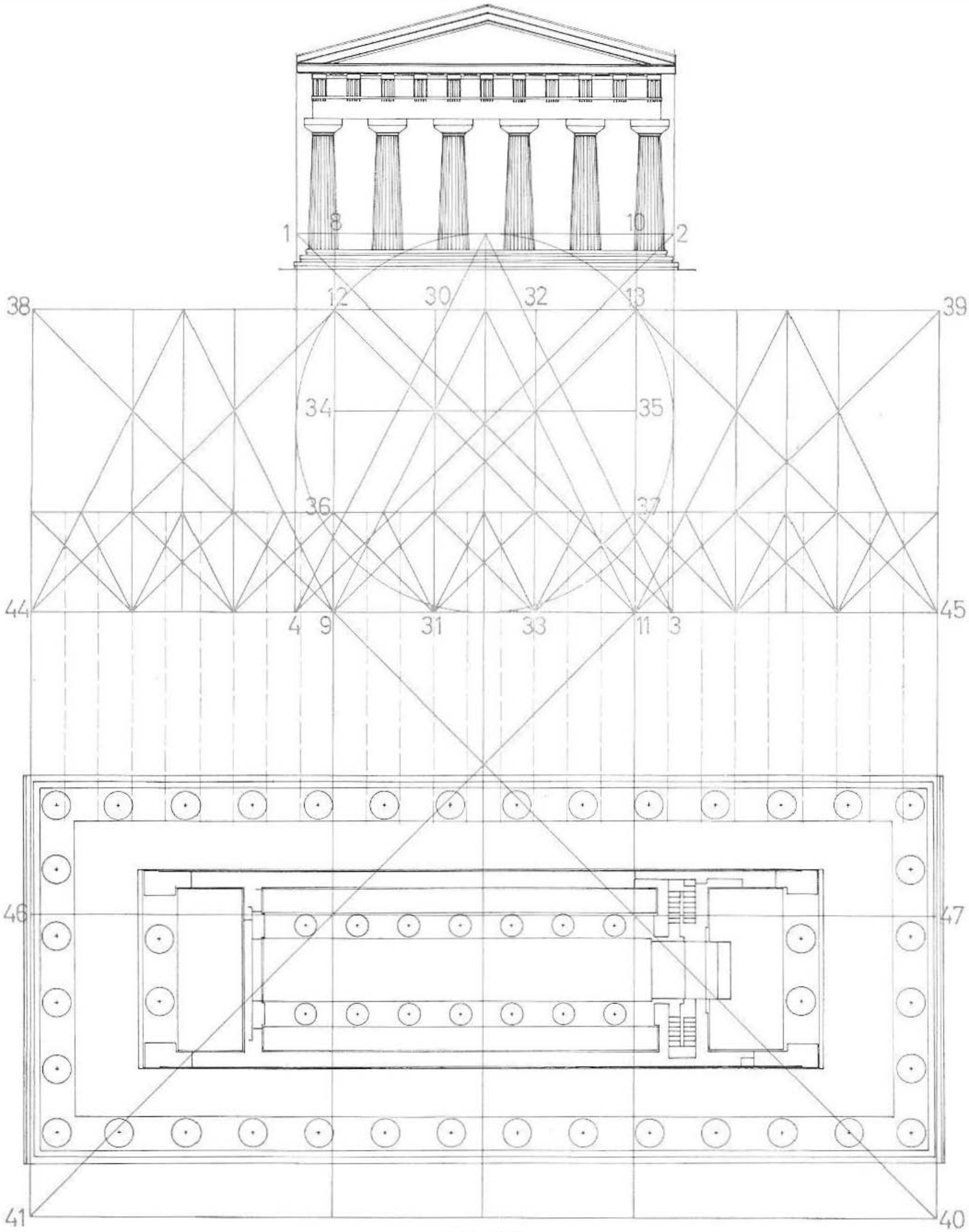


Fig. 194.

From this square we construct (3 × 3) the ground-plan's basic square 38-39-40-41.

The latter extends the full length of the temple platform + one step of the approach steps. In the previous analyses the corresponding basic square took in two steps. Apparently the Temple of Poseidon was allotted a rather longer base (in relation to width) than the two other temples.

The ground-plan of the temple has not been placed centrally upon the diagram. The reason is that our object in this particular analysis is to illustrate column spacing, and it will be to our advantage to keep the diagram as free from unnecessary lines as possible.

The new large basic square has its 3-part lines of division (the horizontals are 44-45 and 46-47); and the facade square 12-13-11-9 has been split the same way.

The division of the latter square is produced vertically downwards through the centre of the diagram and as before it provides the same distribution of columns (four columns, five spaces) as in the front elevation. This comparison illustrates that the facade and side columns have in principle the same process of distribution.

If in the same manner we divide up the two squares to the right and left of the central square (i.e. the whole area 38-39-45-44) we split the side of the temple into 27 strips.

The central square 12-13-11-9 contained four columns and five spaces, which means that the adjacent squares must have five columns and four spaces if we are to retain the process of alternation.

Thus we see that this arrangement produces 14 columns and 13 spaces, starting at each end with a column.

We saw in the Temple of Ceres that the side of that building was similarly split into 27 strips, but in that instance the architect had decided to start and finish the colonnade with a space.

This was presumably to accommodate the two approach steps. It would have been technically impossible to place the facade hard against the side of the basic square in the Temple of Ceres since the steps would otherwise have been eliminated.

Looking at the actual structure (of the Temple of Ceres) we can count 12 spaces between the columns. The spaces at each end of the colonnade can be detected only on a geometric plan such as that in Fig. 179.

The architect of the Temple of Poseidon also split the side of his building into 27 strips, but the strip at each end contains only one of the approach steps, not two.

The decision to place one step inside the basic square landed the designer in a bit of difficulty. If he was to be guided strictly by the plan, the end columns would have to stand just inside the line of the basic square—and therefore on the approach step. This would have been neither aesthetically beautiful nor structurally convenient.

The architect therefore adjusted the plan slightly and moved the end column back on to the platform, leaving room for the required step. Measurement of the drawing shows that each of the four corner columns has been drawn in a little, with less space between these and their neighbours than between the other columns.

Fig. 195 is a diagram of the column lay-out in the Temple of Ceres (Y) and the Temple of Poseidon (Z). Columns are indicated by "o" and spaces by "x".

We see that the same process of division can provide at least two different column arrangements. Later, in our study of the Parthenon, we shall see yet another variation.

Back in Fig. 194 there is one final feature which requires comment. We note that apart from the end columns the geometric distribution fits the side colonnade. But, bearing in mind the facade analysis, we would perhaps have expected some irregularity in the ground-plan.

In the facade we saw that the circle's rectangle indicated the *neck* of the column and not, as in the previous analyses,

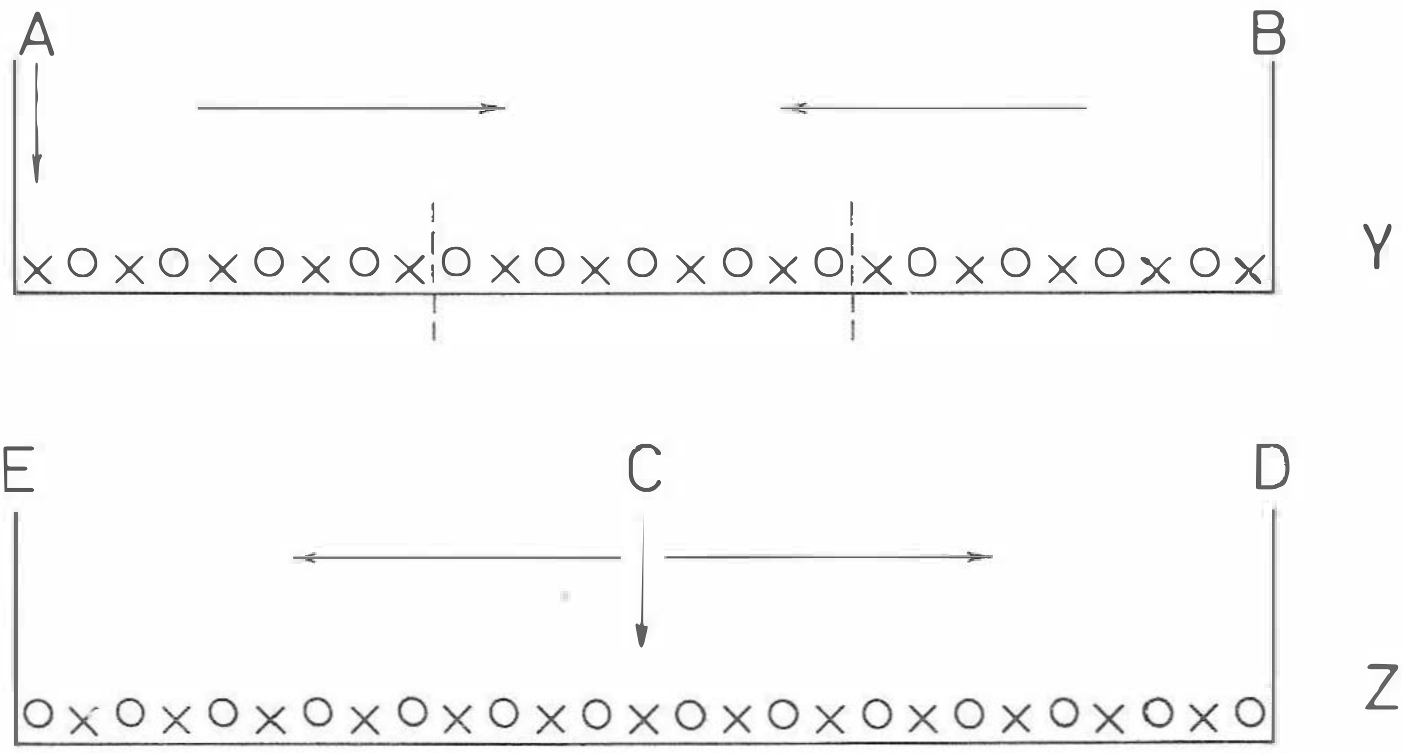


Fig. 195.

the base. This meant that the rectangle cut slightly into the body of the conical column. We might have expected to see the same thing happen in the ground-plan.

But the fact that this did not occur can be attributed only to one factor: an error in the source material. Measurement shows that the columns in Fig. 194 are in fact slimmer than the base of those in Fig. 190 but thicker than the neck.

The view of the ground-plan columns in Fig. 194 is therefore a cross-section of the real thing, presumably taken midway up the cone.

I suspect that it was not done deliberately but was the result of a drawing error or faulty measurement. Whatever the cause, this is the reason that the ground-plan columns do not extend over the guide-lines as they do in the facade.

This does not affect the principle of distribution, but it will lead to a difference between column and space as illustrated in Fig. 186.

Our analysis of the ground-plan continues in *Fig. 196* where the ground-plan has been placed in a more familiar position, i.e. centrally in the diagram. The 3 × 3 lines of division have also been entered.

Our first study is of square 12-13-11-9 which we recognise as the square on the circle's rectangle from the facade, and the square used to build up the basic unit of the ground-plan.

In this square we execute the same construction as in Fig. 193, i.e. we enter the circle's rectangle and the latter's square. This square is divided 3 × 3 and the lower trio of resultant squares similarly divided 3 × 3.

The vertical lines in the final division are produced to cut through the colonnade within the inner temple, and we see that these transverse lines of division match the inner columns.

Once again evidence of the same procedure in placing columns. The square on the circle's rectangle was used in the facade; and again in the ground-plan. Two symbols within each other, the large

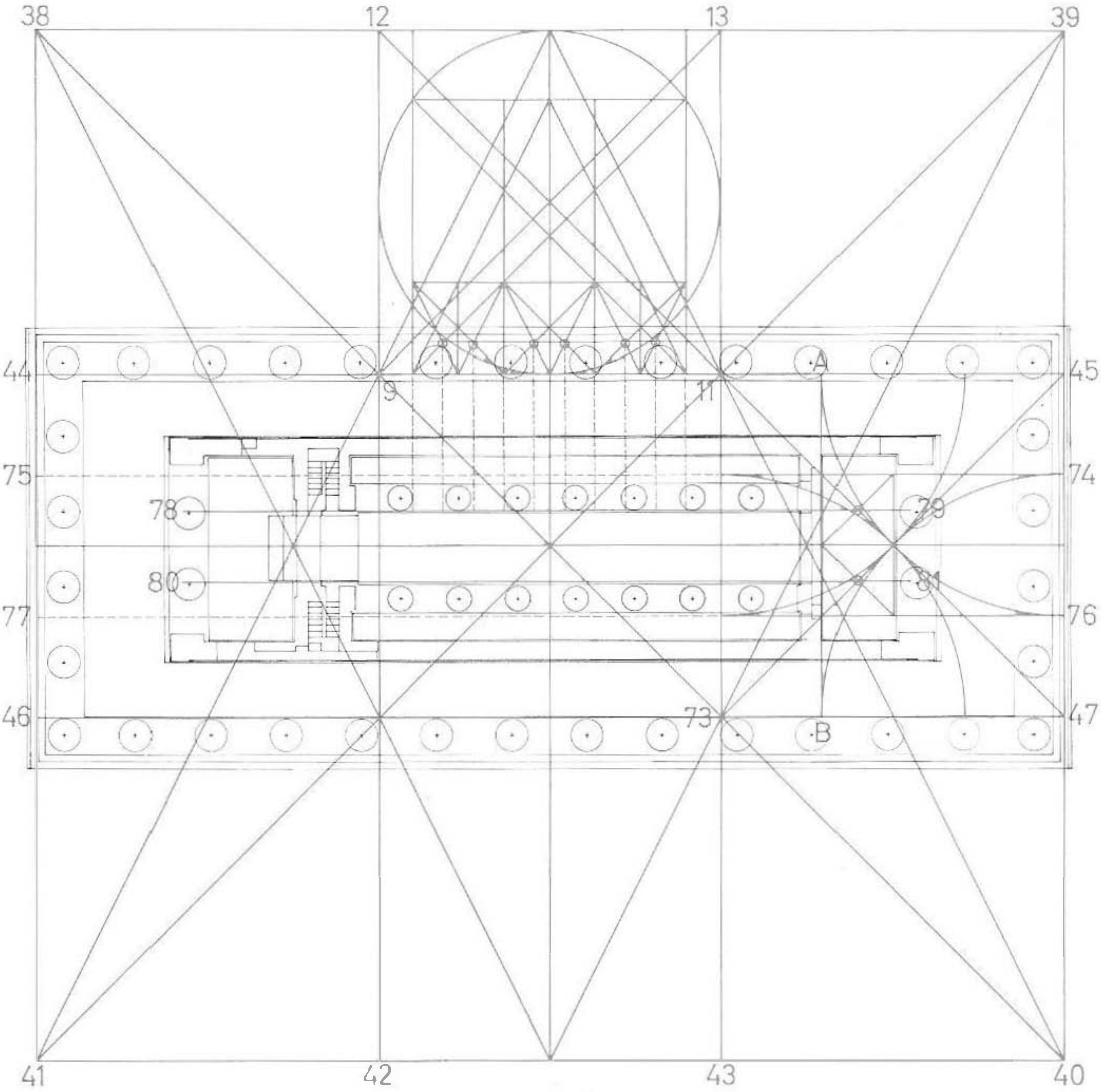


Fig. 196.

one governing the large outer columns and the smaller determining the positions of the smaller inner columns.

In accepted fashion the 3-part dividing lines 44-45 and 46-47 run flush with the inside of the large columns. But this should be no surprise since of course this was the dimension in the facade diagram that formed the basis of our ground-plan.

In square 11-45-47-73, on the right of the diagram, we enter the sacred cut from each corner. This provides two vertical and two horizontal cuts and represents symbol "M". The horizontal cuts have been produced throughout the length of the temple and are seen as 74-75 and 76-77. We can record that these indicate the edge of the inner cloister, i.e. they run flush with the inner columns.

The sacred cut has created the usual small squares at the centre of square 11-45-47-73. One side of the small square (produced to AB) indicates the edge of the back wall in the entrance porch.

If in this small square we enter the half-size version from tip to tip of the

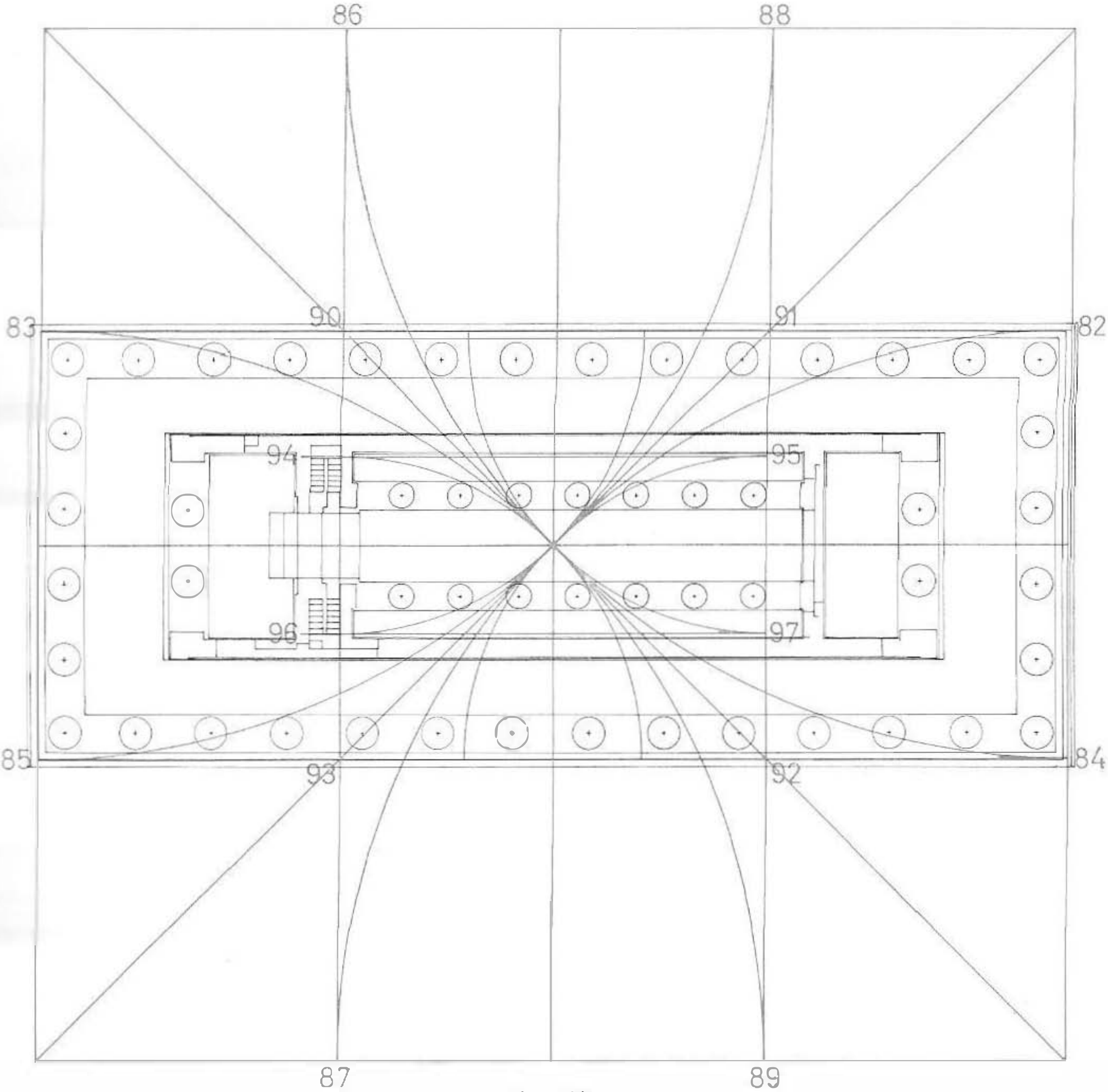


Fig. 197.

vertical cross, which is in fact the same as symbol "F", this produces a quartering of the square. Two of the dividing lines are produced through the inner temple as 78-79 and 80-81. These mark the inside edge of the inner colonnade and cut directly through the two sentinel columns outside each end of the inner temple.

The lines of the sacred cut and of its 4-part division have thus laid down the outer and inner edges of the inner colonnade, and as we already have the positions of the individual columns from square 12-13-11-9 we would be ready to go ahead with the inner temple's planning.

To avoid a clash of too many lines we shall pass on to *Fig. 197* where we naturally have as our basis the main square. Within it we enter all four sacred cuts, i.e. representing symbol "M".

The two horizontals, 82-83 and 84-85, mark the width of the temple. In so doing they apparently allow—just as with the temple's length—for one of the approach steps.

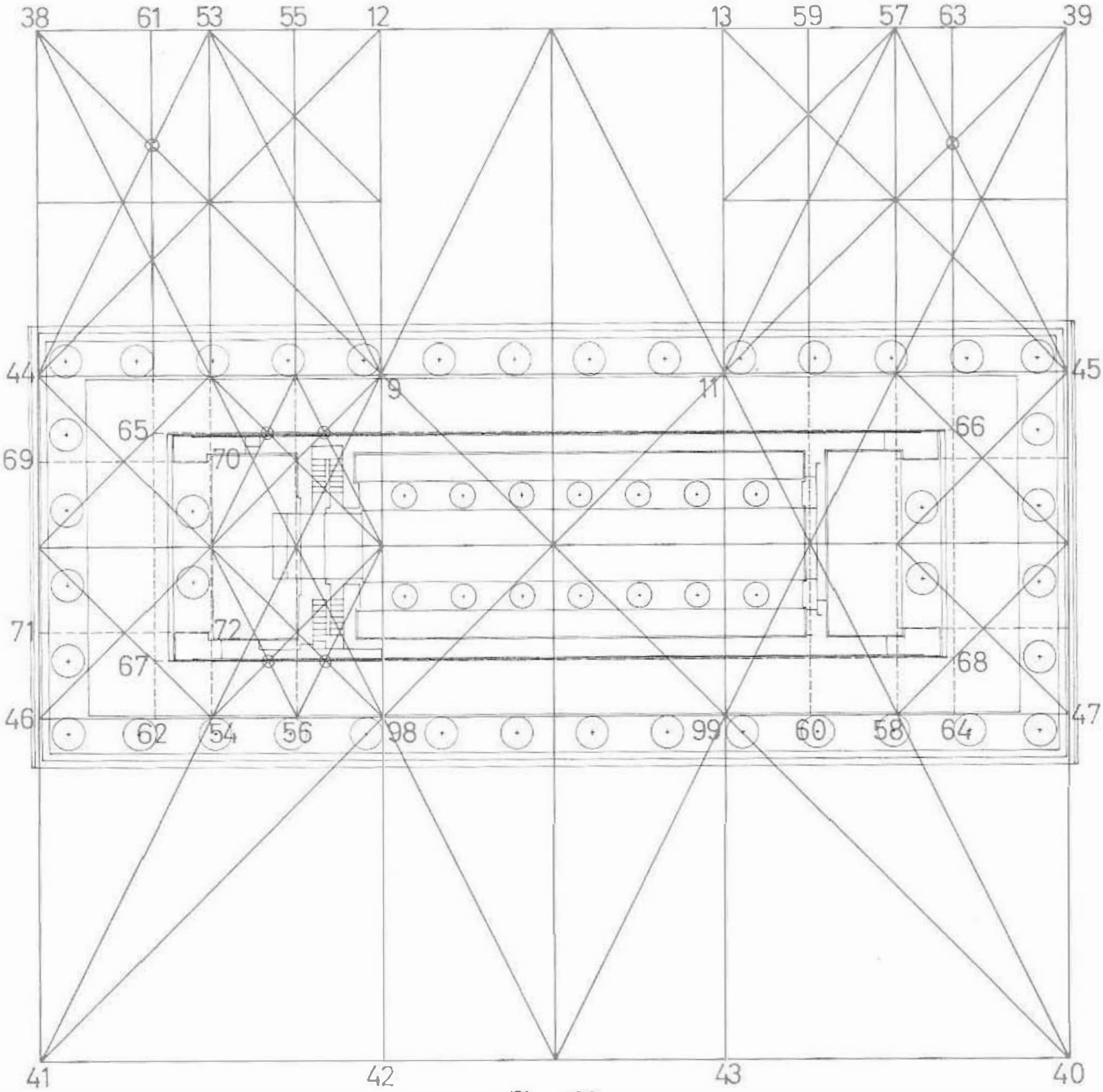


Fig. 198.

The four sacred cuts produce a central square 90-91-92-93. In this square, too, we enter symbol "M" and this provides us with another important part of the structure: the horizontals mark the inside of the inner temple wall. The lines are 94-95 and 96-97. Thus we have the width of the inner cloister.

Our analysis continues in *Fig. 198* where we find the basic square split 3 × 3.

In the upper left corner one of the nine squares is numbered 38-12-9-44. It is complete with its acute-angled triangles, which indicate the lines of 3-part division. At the same time on the right of the square we see one of the 4-part dividing lines (produced to 55-56).

If we examine the various lines of division, starting on the left, we come first to one of the 3-part lines, produced through the diagram as 61-62. It indicates the end of the inner temple, outside the two columns that stand guard at the entrance to the porch.

We come next to the square's vertical axis, produced from 53 to 54. It marks the extremity of the small porch leading to the inner temple.

The next line is the 4-part line of division 55-56; it governs the depth of the porch, towards the inner temple. Passing as it does quite deliberately through the inner temple's end-wall, it has apparently been used to position the massive door which no doubt at one time shut the inner temple off from the outside world.

We find a similar lay-out at the other end of the temple in square 13-39-45-11. The lines of 3- and 4-part division have been entered here, too, and play a corresponding role.

Lines 61-62 and 63-64, which mark the length of the inner temple, have not in our diagram occupied as accurate a placing as we might have expected. This must presumably be because the drawing does not show the two steps which lead from the outer main cloister up to the platform of the inner temple. The geometric marking thus indicates the precise length of the inner temple, inclusive of these steps.

The procedure of executing the geometric constructions in the corner squares and producing their lines downwards through the diagram instead of placing the constructions in the middle of the diagram was preferred in order to avoid too many lines clashing. It would have been too crowded had the geometric lines and lines of structure been jostling for a place in the diagram.

In square 44-9-98-46, which lies across the left end of our temple, we enter the vertical cross and produce four quarter-squares.

In the two squares nearest the centre of the diagram we now enter the lines of 3-part division. Two of these, 65-66 and 67-68, are produced through the diagram —and mark accurately the total width of the inner temple.

We can calculate the width of the outer cloister from this information. It lies between 44-45 and 65-66. Numerically we see that the cloister occupies $\frac{1}{6}$ of the distance between the two side colonnades, i.e. distance from 9 to 98. The inner temple is thus equal to $\frac{2}{3}$ of that distance.

At the entrance to the inner temple we se that the walls terminate in two solid rectangular columns. It is quite possible that this reinforcement of the wall was designed to carry a set of heavy doors, for at one time there must have been a division between the outer cloister and the inner temple.

The thickness of this column, too, is determined by square 44-9-98-46. We recall that the square was quartered by its vertical cross and that a 3-part division of these smaller squares indicated the total width of the inner temple. In the other two squares we have entered their horizontal axes (69-70 and 71-72).

These lines, in conjunction with the lines of 3-part division, mark accurately the thickness of the rectangular column. The arrangement is repeated at the other end of the temple, but without numbering.

These four analyses of the ground-plan have indicated all the principal lines of structure in the Temple of Poseidon. If we superimposed the four diagrams upon the same drawing we should be able to pick out all the temple's lines.

Let us take a look at the information obtained from our study.

Fig. 193,
Observation 1: Length of ground-plan.
Observation 2: Placing and spacing of the 14 side columns.

Fig. 194,
Observation 3: Outer edge of main cloister.
Observation 4: Placing and spacing of columns in inner temple.

Observation 5: Outer edge of colonnade in inner temple.
Observation 6: Inner edge of same colonnade.
Observation 7: Positioning of two columns at each entrance to the inner temple.
Fig. 196,
Observation 8: Total width of temple, including top approach step.
Observation 9: Internal width of inner temple.
Fig. 197,
Observation 10: Total external length of inner temple, including end columns.
Observation 11: Length of inner temple, including internal depth of porch.
Observation 12: Length of inner temple, excluding porch.
Observation 13: External width of inner temple (and thus width of main cloister).
Observation 14: Thickness of the rectangular columns terminating inner temple walls.

We have now traced almost all the main lines of the Temple of Poseidon, and have followed the architect's train of thought as he applied his knowledge of ancient geometry to the task before him. We have seen the various symbols he selected. And we are able to reproduce the structure of the temple on the basis of geometric principles.

Theoretically we could produce any size of sketch of the temple, requiring no drawing of the existing building to assist us. Our sketch would be equally as accurate as the dimensional drawings produced by Buus-Jensen in 1922.

Further examination of temple details would doubtless reveal that these, too, are based on the rules of ancient geometry.

One is perhaps tempted to ask whether a geometric diagram would not fit any old building—even buildings which have nothing whatever to do with ancient geometry or classical history.

It would certainly be possible, bearing in mind the vast variety of lines and angles open to the ancient geometer, to "hit" an isolated handful of features and dimensions in objects or buildings entirely divorced from ancient geometry. But there could be no question of complete solution. It would be absolute coincidence.

One only requires to conduct a strict geometric analysis of such a foreign building or structure to discover that it has nothing whatever to do with the subject. It would be impossible to fit the complete building into a geometric analysis.

An accurately executed geometric analysis is a devastating phenomenon. It will reveal relentlessly any discrepancy of angle that may have crept into the finishing building, or pinpoint with its razor-sharp intersections any carelessly shortened or lengthened line.

We have so far analysed three temples the dimensions for each of which have been governed by a common factor: location and determination of the facade's basic square. Failure to locate this square prevents one progressing a single step along the track of analysis. If in spite of this failure one persists in continuing, then only coincidence and lucky strikes will result.

I can say this with great feeling! For all too often I have struggled to locate the proper square of origin. Experiment followed experiment. Until finally the break-through came. The facade's basic square revealed itself. When the analysis was completed the whole picture was clear. But the right starting point was essential.

Some structures were subjected to several hundred analytical attempts—before

I discovered some fault or other in the source material.

Even the simple process of taking a photostat copy of a drawing is not the straightforward action one might expect. The paper may stretch lengthwise or crosswise, increasing the height or width or length in relation to each other. The variation need only be around 2 % to ruin the chances of a proper analysis.

The ancient architect no doubt produced a master diagram on which he had entered *all* the symbols he wished to apply to his plan. He then selected the lines required. But in a review such as ours it would be impracticable to construct a master drawing of that type; if only for reasons of legibility, we would be unable to reproduce it sufficiently large in this volume.

The Parthenon

High on the Athenian Acropolis, with a supreme view of the surrounding country and coast, stands one of Greek's finest architectural survivors from antiquity: the Parthenon.

Built about the period 447—440 B.C. the Parthenon thus enjoys the venerable age of 2400 years. We have a picture of it in *Fig. 199*.

But this was not the first temple to be erected on this magnificent site. The Parthenon we know today was built on the ruins of an earlier columned temple. Historical accounts indicate that the original (?) temple was also a building of considerable size, having almost the same length as the present Parthenon although being slightly narrower.

Archaeologists have uncovered the old foundations, which (see picture in *Fig. 200*) extend to a depth of 11 meters in places.

The previous temple on this site would appear to have resembled those examined in the past few pages, since it had six columns at the front. The present-day Parthenon, however, has eight frontal columns, and 17 on each side.

We discover from several surviving documents that the Parthenon was built by architects Callicrates and Ictinus. These documents also show that work on the temple decorations lasted until 432 B.C.

The building is said to be 228.141 feet long at the head of the approach steps, i.e. the edge of the platform on which the actual columns rest. The width is 101.361 feet. Evidently a building of no mean proportions.

Although a superficial inspection might lead the casual observer to believe that the Parthenon resembles the temples previously analysed, a more intense examination reveals this likeness to be only slight. As mentioned above, the Parthenon is an octastyle temple, and we can also see that it is much wider in relation to its length than temples to which we are accustomed. Later analysis will also show that the planning was conducted on a rather different basis from the other rectangular temples. But these deviations do not signify a departure from accepted principles. They illustrate simply one more variation on the theme of ancient geometric planning.

Fig. 199.

The ground-plan which we use in this analysis has been taken from *Feuilles de Delphes,* Topografi et Architecture Releves et Restaurations par K. Gottlob, Paris 1925.

The reproductions are exact photostat copies of the diagrams contained in that volume, and identical in size.

Oddly enough, there was no diagram in the book of the temple's facade. A number of frieze details and other decorations were shown, but a complete drawing of the front elevation was lacking.

Instead the facade drawing was taken from the German work: *Abbildungen zur algemeinen Bauzeitungen,* Vienna 1838.

Fig. 201 shows this facade. The original drawing was twice as large. It was felt that a half-size version of the drawing would be sufficient for our analysis, and moreover this particular result comes within 5 mm of the ground-plan's size.

Fig. 200.

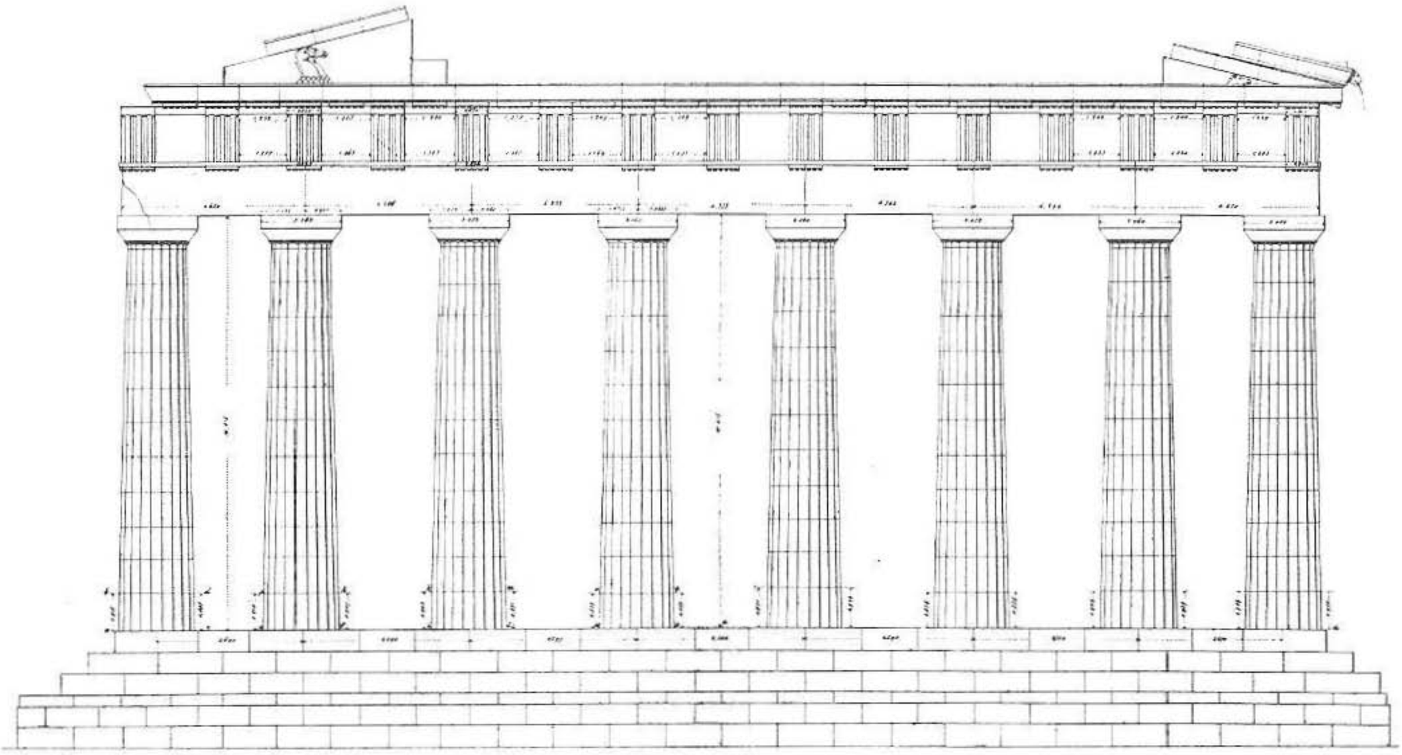


Fig. 201.

The difference in size between the facade and the ground-plan presents no problem. Indeed it is of little significance as long as we bear in mind that it exists. It does, however, mean that a direct comparison of measurement cannot be made between the two plans. There is nothing to prevent us carrying on with our analysis of the front elevation, however, and once the column placing has been discovered we can transfer the column proportions to the ground-plan, irrespective of size.

So much for a brief look at the Parthenon and the material on which we shall base our analysis. Let us now move on to our first diagram, *Fig. 202.*

It is more than important, it is vital in any such analysis to select the correct basic square. If your choice is incorrect, none of the details will fit. We have previously been aware that the basic square in the front elevation provided—through its geometric divisions and multiplications—every one of the principal dimensions of the respective temple's structure, both in the facade and in the ground-plan.

On account of the building's unusual shape (length, width and relative height) I could see plainly that the Parthenon could not be treated in the same way as the three other temples had been analysed. The approach would have to be different. But where should one start?

In addition to the points of differentiation mentioned earlier between the Parthenon and the others, one factor stands out clearly to the analyst who studies the facade of this new challenge.

The temple has of course eight columns at the front, providing seven frontal entrances to the cloister.

A close study of these eight columns and seven spaces quickly shows that the two spaces on the extreme right and left are considerably narrower than the five others. The difference is so vivid that it can be seen even without measurement. Measurement in fact shows that the two outer spaces are in reality ½ m narrower than the five remaining spaces.

A difference of that size—almost 25 %—cannot have been other than intended. There can have been no question of misplacement.

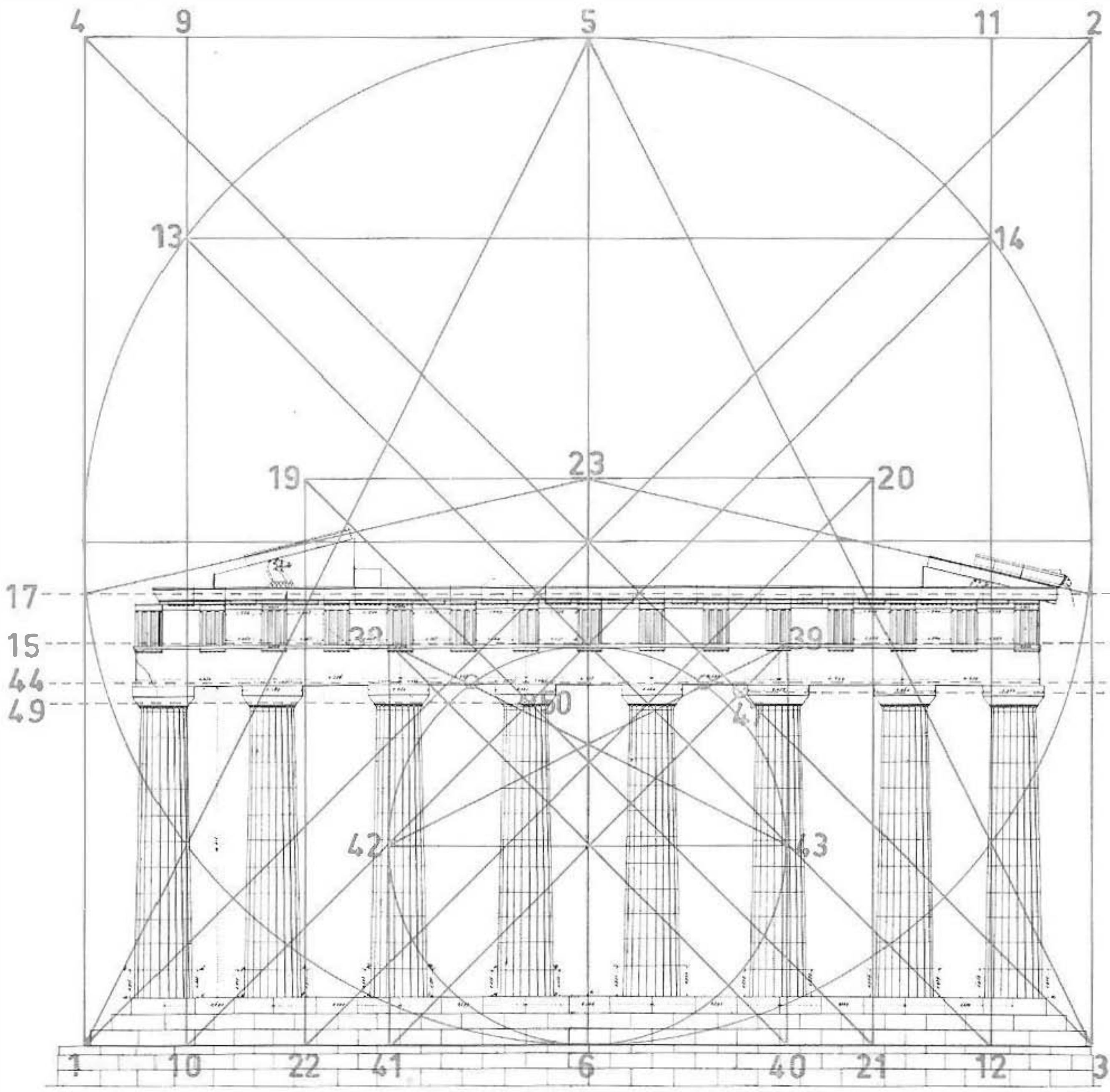


Fig. 202.

We can see then that the front colonnade is not uniformly arranged. Even if we adopted the same procedure as with the Temple of Neptune and used the circle's rectangle to slice off the outer columns, the remaining outer space would be smaller than the other divisions. Thus even the common 9-part division would not fit.

After a few unsuccessful experiments I found that the Parthenon's basic square rested on the outer step. We saw earlier how the angle of the ascending outer steps usually followed the line of the diagonals in the basic square. In this instance we start with the diagonals (determined by the steps). These are drawn as 1-2 and 3-4. We now construct our basic square, taking in the temple's three steps. The square of course is 1-4-2-3.

We enter the vertical and horizontal axes, and add the acute-angled triangle 1-5-3. Then the basic square's circle. In the normal way this marks off the position of the circle's rectangle, which we can draw in as 9-10-12-11.

These lines, as we have seen in earlier temple analysis, cut off the two outer columns. But, as we discussed above, this gives no clue at all to the column distribution.

The construction continues with the production of the square on the circle's rectangle 13-14-12-10, but since line 13-14 lies well above the upper limit of the temple it can tell us nothing. Only when we construct this square's half-size version do we glean our first bit of information.

This square is 19-20-21-22 and the first thing we may notice is that it contains in its width four columns and five spaces. Now we are on familiar ground. The difference between this temple and the others we have studied is therefore that the earlier temples applied the square on the circle's rectangle in order to determine the column distribution, whereas in the Parthenon the half-size version of this square is used for the same purpose.

This latter square has yet another purpose: its top line 19-20 marks the height of the temple. The Parthenon obviously has a lower roof (proportionately) than, for example, the Temple of Ceres or the Theseum. This is because the architects of the Parthenon took as their determinative not the square on the circle's rectangle nor the combination of sacred cuts but the square shown in Fig. 202.

So far our diagram has three squares: the basic square, the square on the circle's rectangle, and the half-size version of the latter.

The diagonals of all three of these squares have been entered, creating a series of intersections and small squares immediately below the horizontal axis of the diagram.

The centre of the upper small square indicates horizontally the top of the frieze panel, and of course simultaneously the horizontal line of the roof. This is shown by the broken line 17-18.

At the intersection of this line with the vertical sides of the basic square we find the point of origin for the roof pitch. We see how lines 17-23 and 23-18 closely follow the angle of roof in the two sections that remain.

We carry on with the construction, producing the half-size version of 19-20-21-22 and—as with the others—placing it in the centre of the base-line. This is square 38-39-40-41.

We notice how the upper line of this square marks the lower edge of the temple's frieze. The line is produced to the side of the diagram as 15-16.

In this final square we enter the vertical and diagonal crosses, and the lines of the acute-angled triangles. To avoid a confusion of lines only the upper half of these triangles has been entered. The lines are 38-43 and 39-42.

We then inscribe the square's internal circle and naturally this indicates the position of the circle's rectangle. But since it is the transverse triangles we have entered, the rectangle is thus shown horizontally. The points of intersection are marked by line 44-45, and we see that it indicates the underside of the lintel which rests above the columns.

If we follow diagonal 3-4 we see that it intersects the diagonal in our small square 39-41. This point of intersection was apparently chosen as the line of division of the capital. Above the broken line 47-48 the capital rises vertically, beneath it the capital drops diagonally.

In a similar manner two other diagonals indicate the height of the capital. The intersection of 14-10 and 19-21 marks the head of the column. This point has been shown by broken line 49-50.

We must switch now to a fresh diagram to prevent a clash of too many lines, and to examine the spacing of the frieze and colonnade. But we have already assembled quite a lot of information from one diagram. It comprised simply four squares: the basic square, the square on

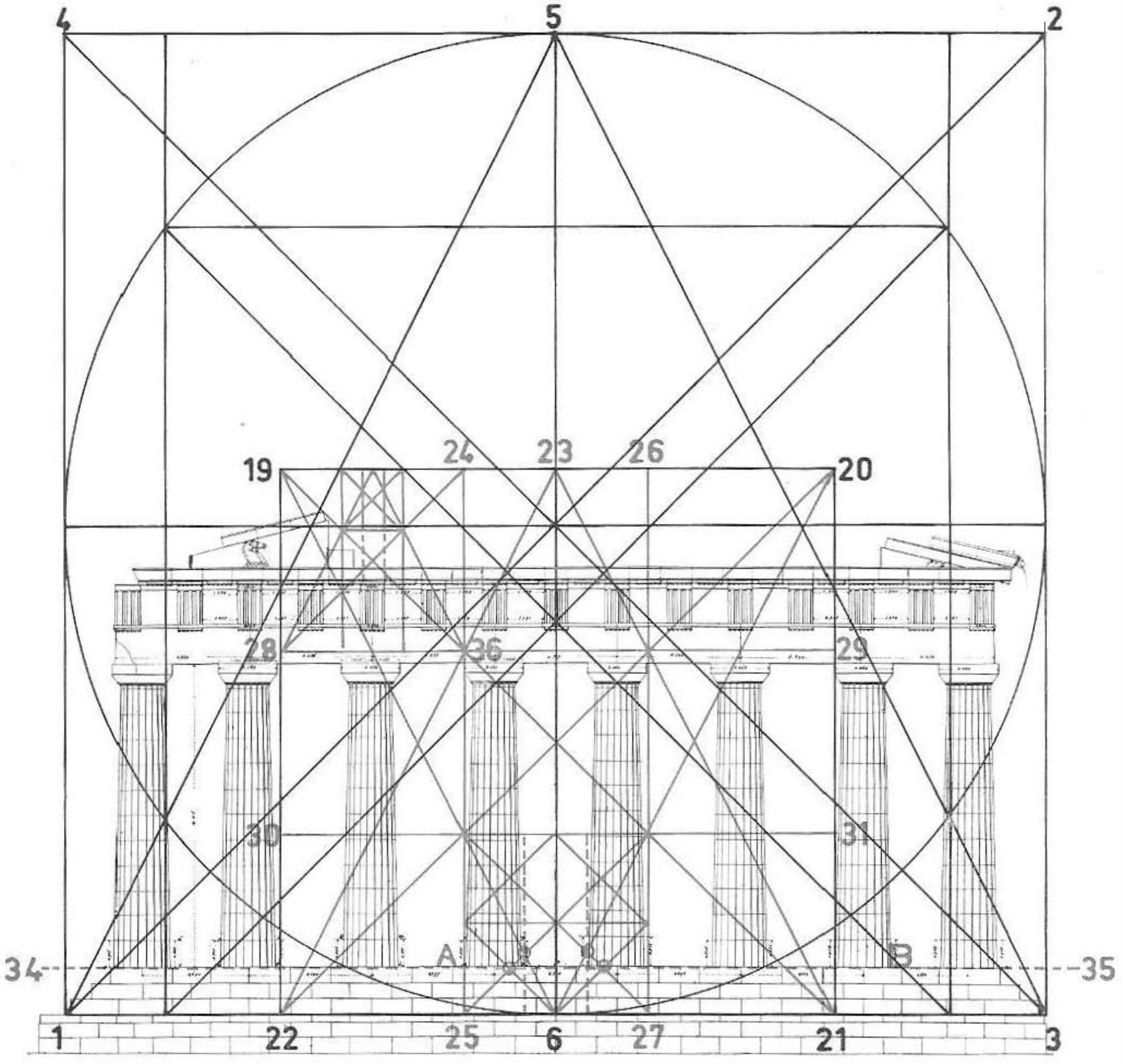


Fig. 203.

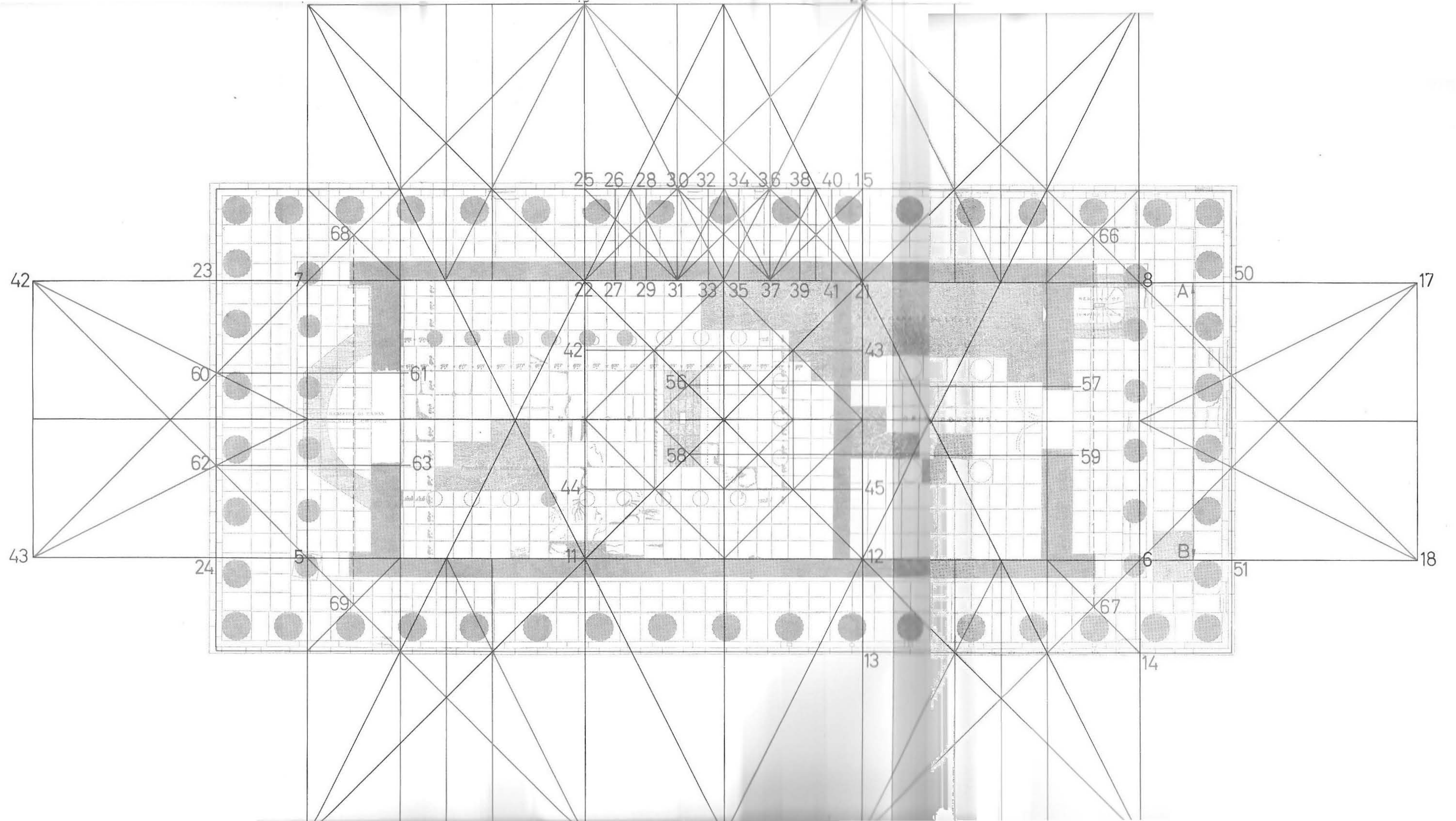
the circle's rectangle, the latter's half-size and its quarter-size.

We discovered: the temple's height, roof-pitch, roof's horizontal line and top of frieze, the bottom of the frieze and top of the lintel, lintel's bottom edge, division of the capital, and capital's height.

For a complete picture of the facade we need in effect three further facts: bottom of the columns which would also tell us where the top of the approach steps lies, column distribution, and decoration of the frieze. We find details of these in *Fig. 203* which is in essence almost exactly the same as the preceding diagram. We have the basic square, the square on the circle's rectangle and—at 19-20-21-22—the half-size version of the latter.

Instead of completing the quartet by constructing the quarter-size square, we now divide 19-20-21-22 into 3 × 3. We saw this procedure in analysis of previous temples in which the square used to decide the column placing was divided 3 × 3, and later used as a source for the ground-plan's basic square.

The lines of division are (vertically) 24-25 and 26-27, and (horizontally) 28-29


25 26 28 30 32 34 36 38 40 15
68
66
23
7
22 27 29 31 33 35 37 39 41 21
8
50
A
42
17
42
43
60
61
56
57
62
63
58
59
44
45
43
24
5
11
12
6
B
18
51
69
67
13
14

and 30-31. This splits the square into nine smaller squares.

We then take the lower central square (base-line 25-27) and execute the same 3 × 3 division within it. We see immediately how accurately this indicates the columnar placing.

The square covers exactly two columns and one space and each of these three units occupies precisely one-third of the base-line 25-27.

In the same small square we execute a 4-part division horizontally. Only the lowest of the dividing lines is entered, and produced to each side of the diagram (line 34-35). We see how this line indicates the height of the outside steps and ---of course—the horizontal line of the temple floor as well as the bottom of the columns that surround the temple.

In the upper left corner we trisect square 19-24-36-28. Then we proceed to sub-divide one of the resultant squares in a similar manner, and find that this latter division indicates the width of the rectangular decorations which make up the frieze. We can quite clearly see that the architect chose to make the width of the decorations 1/9 of the small square, while the intervening spaces were made 2/9 of the square, twice as wide as the decorations.

*

Fig. 204 is a diagram of the ground-plan. Bearing in mind the slight difference in size between the facade and the ground-plan drawings, we cannot simply transfer one of our squares (i.e. 19-20-21-22) from the front elevation to the ground-plan. Otherwise none of the plan would fit our diagram.

Instead of a direct transfer we must take the square's geometric relation to the columns and transfer this to the ground-plan. We shall then have compensated for the difference in measurement.

We see on Fig. 203 that square 19-20-21-22, which we shall be using in the ground-plan, covers exactly the width of four columns and five spaces. This distance in Fig. 203 is called AB. We must therefore transfer AB's proportion to our new diagram and stretch it across four columns and five spaces. We can then use that line as the basis of a new square representing the same geometric properties as 19-20-21-22 in the facade.

In Fig. 204 we find this line—still named AB—on the right. As agreed, we construct a new square with AB equal to its side-length. This square 6-8-17-18 will be the square of origin from which all remaining dimensions of the ground-plan will be determined.

We recall from previous experience that the square which in the facade indicated columnar distribution had to be transferred to the ground-plan and multiplied 3 × 3.

We try the same procedure here. The new large basic square is seen as 1-2-3-4.

We are immediately a little surprised to discover that the basic square is much shorter than the temple's length. Are we on the wrong track? After all, we are accustomed to the basic square equalling the length of the temple precisely.

If, however, we examine the diagram closely we discover that our basic square 1-2-3-4 in fact marks the length of the inner temple, not the outer.

Whereas we are used to seeing the square's horizontal 3-part dividing lines marking the inside of the outer colonnade, the same lines (5-6 and 7-8) indicate here the inside width of the inner temple.

We see, too, that the vertical lines of our main square (1-4 and 2-3) run flush with the outer edge of the six columns placed at each end of the inner temple entrance.

The next aspect we require to examine

is the fixing of the outer temple dimensions. This is revealed together with the determination of column spacing and column numbers.

Our basic square consists of 3 × 3 smaller squares. Let us look at the three upper squares, bounded by 1-2-8-7. We shall use the centre of the three to find the column spacing. We have called this square 19-20-21-22: the same designation given to the square which revealed the column spacing in the facade.

We divide this square 3 × 3 and see the lower horizontal dividing lines as 25-15. We see immediately that this line is identical to the outer width of the whole temple. We can fill out most of the temple's length by trisecting the square to the right and left. The same can be done on the other side of the temple.

We have now established that the temple's width is 5/9 of the width of the large basic square.

The length of the temple is determined in the same way. We place the original square (which provided the 3 × 3 basic square of this diagram) at each end of the temple. The one on the left is 7-42-43-5.

This square is divided in the usual manner 3 × 3, and one of its dividing lines 23-24 indicates the temple's length.

Since the same situation is repeated at the entrance end of the temple we can record that the total length of the temple is 11/9 of our basic square 1-2-3-4. We can see how this extra one-third square has been added to the inner temple in order to make up the perimeter of the outer temple.

Returning now to square 19-20-21-22 in which we discovered the temple's total width, we direct our attention to the three lower squares which lie within the temple. We see that—once they have been divided 3 × 3—they reveal the column spacing.

Square 19-20-21-22 covers five columns and four spaces, which arrangement we recognise from our analysis of other rectangular temples.

This provides a regular column distribution within the confines of our basic square of 27 units: 13 columns and 14 spaces. As we saw earlier this might just as easily have been 14 columns and 13 spaces.

Since the architect's choice here was the former method, it is natural that the unit immediately outside the basic square should be occupied by a column. The diagram illustrates clearly why the architect gave the Parthenon 17 columns along each side, and we can also see why the space between the last two columns is less than between the others.

As was the case in the temples already analysed we see that the designer of the Parthenon required to make a slight adjustment at each end of the side colonnades. The end column at each corner had to be moved a short distance in towards the others to avoid placing it upon the approach steps. As the diagram shows, the steps are bounded at each end of the temple by lines 23-24 and 50-51.

To stick stubbornly to the geometric plan, the architect would have had to put the column on the steps if it was to have the same interval between it and its neighbour as between the others. He had two alternatives. He could either maintain the column spacing, and extend the length of the temple slightly at each end to accommodate the steps; or keep the temple length as dictated by the geometric plan and simply pull the end columns in a little.

His choice was the second, and in a way it is the more natural. It resembles the pattern we saw established in the facade whereby the geometric figure stipulated a small variation in column spacing on the right and left. This is what the architect repeated at each side of the temple. Thus he has achieved a certain harmony between front and side elevations. One is determined by geometry, the other the result of compromise.

Of the ground-plan's details we now have the width and length of the inner temple inclusive of columns. We have, too, the width and length of the outer temple, and the peculiar distribution of columns, with an explanation of why there are 17 along each side and not some other number.

But there are several important dimensions yet to be traced. We have the inner temple's length inclusive of the columns at each end, but have not yet determined the internal width of the inner temple. This is done in a similar manner to fixing the temple's total length.

The total length was arrived at by adding $\frac{2}{9}$ to the sides of the basic square 1-2-3-4, i.e. $\frac{1}{9}$ at each side.

The end walls of the inner temple are positioned by deducting $\frac{1}{9}$ of the basic square from each end of the temple. We see that the resulting lines 52-53 and 54-55 cut straight through the walls at each end of the inner sanctum.

So far the ground-plan has yielded to either multiplication or division by *three,* but other divisors have also been applied to the ground-plan.

The next division we shall examine concerns the inner temple the length of which was of course determined by the 3×3 multiplication of the square used in the facade to place the columns. By deducting $\frac{1}{9}$ of the ground-plan's basic square from each end of the inner temple (plus columns) we arrive at the end walls and lines 52-53 and 54-55.

But, as we can see in the diagram, the side walls do not finish on these lines. They extend a little to form a niche at each end of the inner temple.

The extreme ends of the side walls are indicated by a 2-part division of the small square which revealed the column distribution of the side colonnade.

We can see that in the upper right corner square 20-2-8-21 is split vertically into three uniform rectangles, and the lower horizontal line of 3-part division has been entered in order to show the temple's width.

This construction creates three small squares on base-line 8-21. If we examine the small square on the right (at corner 8) we find that its 2-part division marks the end of the side wall with line 66-67 at one end of the temple and with 68-69 at the other end. The square which, divided in two, indicates the end of the side wall is also therefore one of the squares which, divided in three, mark the column placing and spacing at that particular corner.

Thus we see that in places the architect has deviated in the ground-plan from the dominating 3×3 formation.

Another such departure from the main theme is in the positioning of the columns within the actual inner temple. We see this in the centre of the nine squares which make up our basic square 1-2-3-4.

We enter in this central square (21-22-11-12) the half-size version, which in effect means that we divide the square 4×4 since the lines of the half-size automatically mark this division on the original square's diagonals.

We see how lines 42-43 and 44-45 run along the inside of the two colonnades, indicating between them (42-44) a width equal to half the width of 21-22-11-12.

Repeating the 4-part division a stage further, we see how lines 56-57 and 58-59 mark the width of the entrance doorway on the right of the diagram.

The entrance on the left is drawn in such a way that the ruins conceal the original width of the doorway. But—as lines 60-61 and 62-63 imply—it is possible that this doorway was, like so many other factors in the ground-plan, governed by the 9 × 9 or 3 × 3 arrangement since its width equals 1/9 of the side of the basic square 1-2-3-4.

This would seem to complete our examination of the dimensions both in the facade and ground-plan. And here—as in the other temples—we have seen that ancient geometric principles were followed religiously.

All the temple's dimensions are derived from a single line of origin in the facade, and are strictly bound in geometric proportion to that choice.

*

We have seen once again that ancient geometry is definitely not a general, loosely knit system to be fitted uniformly throughout a building. It is on the contrary a system of clearly defined principles which provide a designer/architect with ample opportunity for development without requiring to depart from accepted regulations.

To conduct an analysis of a building it is thus vital that one is absolutely familiar with the rules of the system so as to be aware of the scope available.

The next factor of importance is location of the correct point of origin on which to base the analysis, for it should be borne in mind that the basic line or square dictates every other dimension and proportion within the temple or building. If the proper foundation is not selected, it will later prove impossible to account for the details of your analysis and to match up the lines of your diagram with the lines of the building. And there is no point whatever in attempting to transfer directly an analysis of one temple to an entirely different building. The details simply will not match.

The certainty and accuracy of the ancient geometric system is so great that if, in attempting to fit it to an ancient building or temple, it does not coincide with that building's plan, then it is the user's knowledge of the system which is imperfect—not the system.

On the other hand the system is flexible enough to provide a wealth of variations for determining the plan of a particular temple in terms of ancient geometry; it is not possible—as I have seen tried with other systems—to lift a diagram ad lib from one temple and fit it directly in another's plan.

My choice of temples for analysis purposes was made not on the basis of their geometric form but according to the written and drawn material available to me. For it had to be extremely accurate. The quality required to be first-class. Without some sort of guarantee that this was the case, an analysis of course would have been hopeless.

Of the satisfactory material available to me, not one temple has failed to surrender to the principles of ancient geometric analysis although perhaps after some hard searching. It was a matter of pinpointing the architect's thoughts. On the other hand, I have been obliged to reject several drawings which failed to meet the standard of accuracy required in such analyses. When I later obtained more accurate drawings and dimensions of the suspect temples it invariably proved that rejection of the originals had been justified. And the more accurate material then succumbed to geometric study.

It naturally provided a quiet sense of satisfaction any time the rigidity of the geometric system revealed errors—perhaps tiny—in a particular drawing. This applied particularly to the pitch of temple roofs. And of course any error in the facade plan, however slight, would be multiplied threefold in construction of the ground-plan.

Plans and proportions of temple columns

We have spent some time on geometric analyses of four columned temples of antiquity, examining both front elevation and ground-plan. All four buildings had their own special characteristics and varied in height and width. The general dimensions differed and column distribution, too, was peculiar to the individual temple. In spite of these variations ancient geometry revealed a factor common to all four in planning and determining their dimensions: the application of geometric symbols.

Our study has already hinted that in the planning of these temples there was a fairly standard procedure in the use of the symbols, e.g. the same symbol determined the same factors in the various buildings.

Each symbol of course comprised horizontal, vertical and diagonal lines and it was in the flow of these lines that the designer found his freedom. Even though he worked in accordance with the system's established principles he was able himself to decide, for example, which of the horizontal lines to take as an indication of the temple's height. But this freedom of choice aside it would appear that he was bound to acknowledge and comply with the ancient, traditional rules concerning the symbols' use and order of application. Only the odd individual on occasion stretched the rules to suit his purposes, but never so much that he broke with the ancient symbols. He may, however, have dared to apply them in a different order from the established.

The four temples we have studied in the past chapter must be regarded as fair examples of all such antique buildings. Space does not allow a deeper or more extensive temple inspection. I may assure my reader, however, that I have—in addition to these four—conducted analyses of a similar nature on many other temples of antiquity and each—with reservations on the quality of available material and drawings—has been "broken" by ancient geometry. The hardest task was finding the original basic square.

Tracing this square can take considerable time but once located it is simply a matter of meditating upon the lines of the temple in order to assess which of the symbols to apply first.

The foregoing analyses laid bare the principal dimensions, lines and proportions of the four temples so clearly and logically that we can quickly recognise and remember which symbols were used in which order in a particular temple, and which lines were used for what purpose. This would in effect mean that a temple brother would be able quickly and accurately to recall the procedure applied in any particular temple visited without requiring a mass of written notes. In turn, this would mean that if he himself was faced with the job of planning a temple, he would probably try to avoid using exactly the same lines as used in other buildings so as not consciously to design a stone-for-stone copy of anything previously built.

This is why we may find several temples resembling each other perhaps to the point of mild confusion—yet without two being exactly alike.

To our unpractised ears it may sound an involved process to retain in one's mind the various symbols and their application to the appropriate temples. But skill in the system makes this art no more difficult than today remembering various mathematical formulae or equations. Or even telephone numbers. Remembering a telephone number is precisely the same process: calling to mind a certain series of digits, i.e. our number symbols.

There are thousands of details in the work of planning a temple and few of these have been illustrated in our analyses, but the purpose of this book is not to produce a detailed temple drawing. It is to show in general terms the significance of the use of ancient geometry in this particular sphere. And I believe this has been achieved.

The same guiding lines had doubtless been applied to the advanced planning of temple detail as we saw in fixing the main dimensions. One of these details worthy of special mention is the temple column.

It is a recognised fact that one of the features about ancient buildings which provided a special characteristic was the multiplicity of massive columns surrounding the temples.

We have seen that these columns were conical in shape, slimmer at the top than at the base. What we shall study is the factor that decided the column's thickness and its diameter at base and top.

In all four temple analyses we noted that the square on the circle's rectangle was the source from which the column distribution originated, and in the case of the Temple of Ceres, the Theseum and the Temple of Neptune the column distribution was revealed by trisection of the square. Only the Parthenon differed. In that instance it was the half-size version of the same square that—again by trisection—decided the placing of the columns.

Thus the square on the circle's rectangle has proved to be from a planning standpoint an extremely important factor, placing as it does with its 3 × 3 division the temple columns, and with its 3 × 3 multiplication producing the large basic square on which the temple's ground-plan

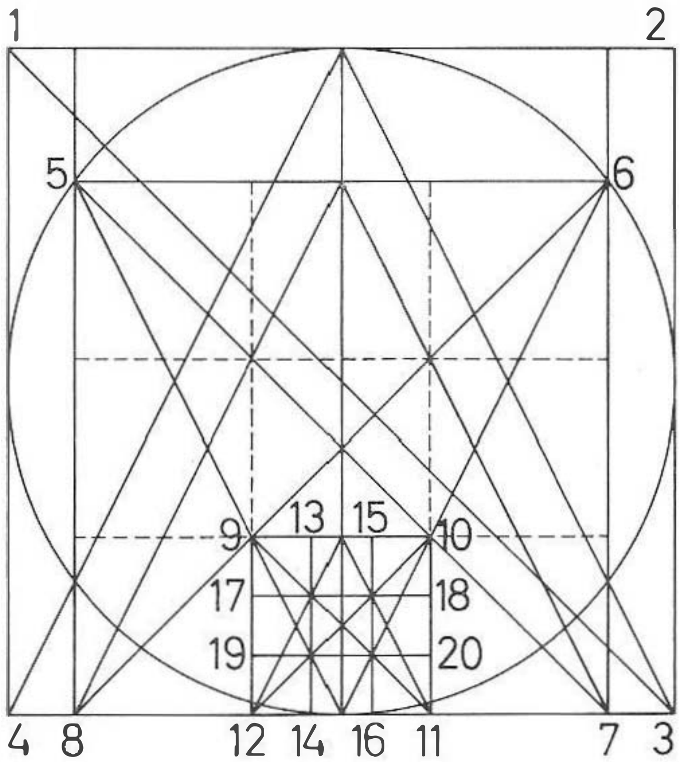


Fig. 205.

was constructed. It would therefore be a natural reaction to turn to this square in attempting to trace details of column thickness.

All the columns in the temples we have surveyed are basically round. Some are smooth, others fluted from top to bottom. But the latter, despite their ornamentation, remain in cross-section a circle.

With our knowledge of ancient geometry we know that a circle originates in or around a square, so we ought to look for the square on which our respective columns are based.

In *Fig. 205* we see symbol "U", which contains the square on the circle's rectangle, i.e. the same symbol as used to find the column distribution in three of our temples.

By the same means as in the analyses we divide this square 3 × 3. The basic square is 1-2-3-4 and the square on the circle's rectangle is 5-6-7-8.

Trisection produces nine smaller squares, one of which we again sub-divide 3 × 3. This is 9-10-11-12 and the lines of division are (vertically) 13-14 and 15-16, and (horizontally) 17-18 and 19-20.

Each of the resultant small squares is now in area equal to $\frac{1}{81}$ of the square on the circle's rectangle. As I see it, it is this small square which in each case proved to be the constructive basis of our temple columns.

We saw several examples of column thickness measured at the base of the column. In the Temple of Ceres we discovered that column distribution was regular, i.e. that the column and space are equally wide. Since small square 9-10-11-12 with its vertical lines of 3-part division consists of three rectangular areas with room for two columns and one space or vice verse, it means that the base of the column has been modelled on the circle within the smallest square. The actual regular distribution illustrates and confirms this.

The selection on the other hand may take a different course, as in the Temple of Neptune where the base of the column jutted slight over the line of the plan, suggesting that the small square may have determined the diameter of the top of the column instead of the base. An example of the base resulting from the top dimension instead of the more normal reverse procedure.

All the columns we have examined have been conical, widest at the base, narrowing towards the top. In other words, two diameters were used to fix their dimensions.

In *Fig. 206* we see various symbols used in deciding these diameters.

In example I we see how the circle within a given square determines the base, while this circle's half-size version (symbol "H") determines the width of the top. The size of the square was fixed by the already discussed division of the square on the circle's rectangle (Fig. 205), and this means naturally that both the top and bottom of the column are geometrically related to the original basic square of the temple.

This means of constructing the columns and fixing of the two diameters may have been used in several temples, without any two sets of columns being alike. For simply by varying the height of the column the architect produced a different structural appearance. The higher the column, the less angled becomes its sloping side.

Example II is basically the same as I except that the column is about $\frac{1}{5}$ shorter. We see immediately that the visual impression alters radically in spite of the identical dimensions top and bottom.

Our list of symbols provides a large number of circles of different diameter, and it is possible that a combination of these was used to fix the width of columns at top and bottom respectively.

Regardless which circle was selected, we always have a given square within or outside of which to draw the circle.

The distribution is regular if column and space are equal. In such a case the diameter of the base has been selected as the circle within the small square.

In the Theseum we saw that the distribution was irregular, the space being wider than the column. This showed that in the case of this temple the diameter of column base was selected from a circle which was less than the circle inside the small square. Symbol "U", for example, may have been drawn within the small square.

We see this done in example III in which the circle within the square on the circle's rectangle was used as the base. Symbol "H" was applied to the top of the column.

This construction produced a fairly solid column since there is little difference between top and bottom diameters. But in example IV in which symbol "U" was used for the base and the basic

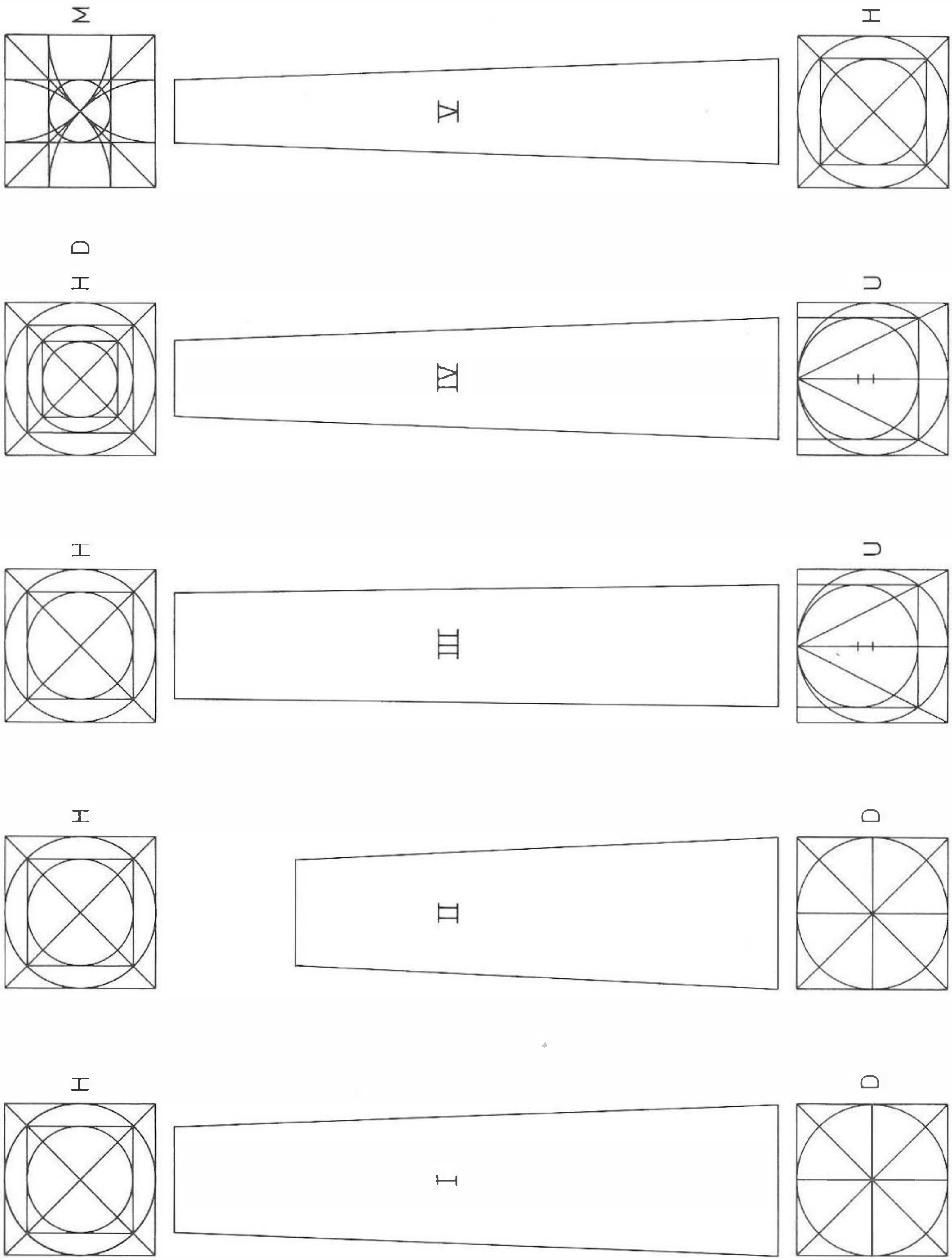


Fig. 206.

square's quarter-size used for the top, we have an apparently slim, even delicate column shape.

Example V shows yet another of the many combinations possible. Symbol "H" was used to find the diameter of the base, while "M" was applied to the diameter of the column's top. This construction, too, produced an attractive, slim column.

Proportions such as those shown in Fig. 206 are only a handful of the many possibilities afforded by our list of symbols. They have been given merely to illustrate the fact that the constructive factor in the front elevation is carried through to such detail as the column diameter. The architect must decide which combination he wants for his temple.

This (free) selection has perhaps to a certain extent been influenced by geographical position of the temples and their time of building. For on this aspect there have doubtless been a number of leading designers in each area who helped form particular styles, just as some great artists have influenced styles of painting.

That ancient designers possessed a finely developed sense of proportion and appearance concerning their creations is shown, among other things, by the fact that many temple columns curve very slightly outwards from top to bottom. This was to counter the optical illusion, in an absolutely straight-lengthed column, that the sides were concave. It also gave the column a heavier appearance.

Two examples of this are the Temple of Ceres in Paestum and the Parthenon outside Athens. But this optical aid does not alter the fact that the column, top and bottom, was firmly founded in the constructive basic square which, like all other temple proportions, was geometrically connected to the basic square in the facade. Nothing can alter the importance of this connection.

CHAPTER TWELVE

Geometry applied on the Ancient Building Site

WE HAVE witnessed the birth of ancient geometry and have followed its complex development from the most primitive stages to the point where it became a refined, useful and efficient system of calculation. And we watched as the ancient Egyptians and Greeks applied the system to their everyday lives, their architecture, their culture.

Before moving on to illustrate further examples of the system's use by civilisations nearer our own time, I think this would be an appropriate stage to show in more detail how the sketches, drawings and plans of these architects were converted to terms of measurement and dimension on the actual building site. For this side of the picture is of fundamental importance to something like ancient geometry. Lacking a convenient means by which to carry out this conversion process, from plan to site, the system could never have enjoyed the enormous popularity (within confined circles) which it did.

We are aware from, among other things, ancient Egyptian papyri that the Nile-dwellers possessed large and small units of linear measure. This was the subject of an earlier chapter.

Neighbouring nations similarly had their own units of measure—establishment of a standard measure becomes more and more essential as national and international trade expands. Standard units of measure develop to fill a need.

Measurement of smaller items such as textiles, timber, etc. was probably met by a system of small units. But land-surveying may have demanded a separate and considerably larger standard. Even though two countries or geographical areas may not have shared the same standard of measure, they were doubtless able to trade satisfactorily. They simply reckoned one system against the other and bartered (or ignored) the difference.

Exactly the same procedure exists to this day. Each and every day of life some business or commercial representative somewhere has to convert the traditional British system of inches, feet, yards, etc. to its metric equivalent. And similarly there is constant calculation from centimeters and meters into inches and feet. This difference in our two standards of measure has never presented any hindrance to British or Continental trade. Irritation, perhaps, but difficulty, no!

If on the other hand the dimensions and plans of ancient architecture were based on a purely numerical system founded on units of measure, the differences in respective standards would doubtless prove too much of a problem and would prevent any geometric system from spreading —as in fact ancient geometry did—from country to country, civilisation to civilisation, down through the centuries.

Although no doubt each ancient society had its own highly developed system of measurement and setting out dimensions, I do not think this was applied to the monumental construction work with which we associate antiquity today. To this end they had developed an infinitely more flexible, more adaptable system, namely ancient geometry. For the latter enabled its user to operate with dimensions not normally available to a more conventional system of measure. Not even our own well-developed system today can meet some of the requirements fulfilled by ancient geometry. Even with the aid of our smallest units of measure.

Consider, for example, the task of constructing a square exactly twice the area of a given square. An extremely elementary problem to solve for the ancient geometrician. He simply took as the side of his double-size square the diagonal of the original square. Already our own system of calculation falls down, for if the original square measures 1 unit along each side its diagonal becomes $\sqrt{2}$, and this is an irrational number, i.e. it ends in an infinite decimal fraction 1.41421 ......, never expressible as an integer or finite fraction, never bearing a clean-cut relation to our side of 1 unit regardless into how many minute divisions we break that unit.

From earliest times the art of building was the source from which every other form of art derived; other forms of art played a subsidiary role such as the decoration of a particular building. Always the building itself was the principal work of art, providing a motive and place for lesser skills. And the main work was not planned according to a simple system of equal divisions.

The real and original work of art, the constructive plan, was probably executed by the master architect himself—guided, as we have seen, by the rules and principles of ancient geometry.

We can picture the plan, drawn on a suitably large or small piece of clay, papyrus, slate, sand-tray or similar material —completely devoid of any scale or standard measure. Depending purely on proportions.

Perhaps the site for the new building was selected before the plan was drawn, perhaps not. The process of transferring the details of the drawing to the site need not start until building (or rather site-work) had actually commenced.

The size of the new temple was no doubt determined by requirements, wishes or area of land available, and the first factor to be established was the proposed length of the building.

The size of the site may have influenced the choice in certain circumstances, but in any event we can imagine a particular length marked out by two wooden stakes or pegs.

If we imagine that the object is to build a Grecian-style temple and see in our minds the architect's drawing, perhaps ½ meter across the facade, then the problem is simply to find the conversion ratio between the dimensions of the drawing and the size of the actual ground-plan. For such a ratio will determine the building's remaining dimensions.

To pinpoint this ratio our architect friend follows a simple procedure: he cuts a stick or length of wood exactly to fit the base of the circle's rectangle in his drawing. This done, he fashions a second stick precisely three times as long as the first. We recall that a temple's length was normally three times the width of the circle's rectangle (or its square).

If his drawing, across the base of the circle's rectangle, measured (say) 40 cm then that would be the length of his first measuring stick. The second, three times as long, would thus measure 1 meter 20 cm.

Off he goes now to the building site to measure the distance between the two pegs which he earlier stuck in the ground.

If the long measuring stick fits 27 times into the length of the site, then the temple's length (in our present-day standard of measure) will be $27 \times 1.20 = 32.4$ m.

Our architect/surveyor has no concept of this actual measure. Neither does it interest him. He has obtained the information he needs: that his drawing and the length of the site are in the ratio 1:27. Each measurement on the drawing can now be multiplied 27 times in order to fit the actual temple plan.

Had the measuring stick divided only 15 times into the length of the site (i.e. the distance between the two pegs), the ratio would have been 1:15 and so on.

If division showed that the site was in fact $26\frac{3}{4}$ and not 27, our architect would have taken the obvious and in the circumstances logical step of moving one of the pegs backwards or forwards a little to provide a convenient ratio.

With this single piece of real measurement the master is now able to cut sticks to fit every dimension in the building. He simply instructs that the length of each respective stick be multiplied 15 or 27 (or whatever) times on the site. No further measurement is applied. None is necessary.

We have all read or heard at some time of the "string" method used by the ancients—particularly the Egyptians—in planning and building temples, palaces, etc. But apart from the fact that they could set a right-angle by means of their strings, we know little of the purpose of this material.

Wall murals and reliefs from ancient Egypt often portray the use of string techniques. But the pictures by their very immobility say little of the method.

Let us imagine the situation of our architect. He stretches a string first of all between the two original pegs in the ground. With this string as its vertical axis, he then constructs—with strings—the basic square of the ground-plan.

Once this is completed he has no difficulty in building up the rest of the particular symbol chosen as the basis for his design, since all the lines—apart from the circle—are straight. All he needs is a sufficient supply of string!

The two diagonals in the square automatically indicate the centre of his diagram, and from this point it is a simple matter—with a length of string as radius—to walk round the square marking off those arcs of the circle he requires as points of intersection.

With outstretched strings the architect has marked off every one of the temples dimensions, he has in fact reproduced his original drawing—only 27 times its size. If the foundations have been prepared (and no geometry is required for these) he can now instruct his team of workers to begin building the temple.

The conversion ratio of drawing to site (1:27) is no longer required in effect. Merely for purposes of checking perhaps. For once the strings have been set out, all dimensions and angles are there on the ground for easy reference.

The actual erection and structure of the temple was guided by the lines of the facade, and I suspect that the conversion ratio for the front elevation was required not at the building site but at the stone quarry, where the facing stones were cut out and dressed ready for building.

The lines of the symbol would be set out in string in the appropriate proportions (1:27) and slowly, as the stone mason completed his work on each slab of rock, the blocks would be laid into position within the symbol. And the facade would be built up completely—*lying flat upon the ground.*

The same procedure is likely to have been applied to the temple walls.

Thus the required blocks of stone could be hewn and dressed at the quarry and, as required, transported to the site for erection. In this way the stone masons were able to work on undisturbed, as long as the latest row of blocks were left in position until superseded by the row on top.

If the building site itself was surrounded by trees or other buildings, which prevented the laying out of the ground-plan's basic square, another convenient area was used to lay out the plan. The dimensions were then transferred by strings to the site, the original length of string (between the two original pegs) in this case being the starting-point for the actual temple measurements.

This is one highly feasible theory of how the ancient architect's drawing was transferred to the building site. The size of the drawing was unimportant, as was the size of the completed building. Measurement was made in accordance with the principle of comparison, the guiding light at all times being the basic square and its inner guide-lines. If the building was large, the lines and dimensions were suitable proportioned. If the building was a small one, its dimensions were reduced accordingly. But in either case the respective lengths and heights were in the same proportion in reality as on the master's drawing.

Another relevant problem concerning ancient building technique, one that has caused many a puzzled modern mind to squirm at the thought, is how the primitive construction workers were able to move and manipulate the massive slabs of stone used in their temples and similar buildings. How was it possible without aids such as cranes, winches, etc.?

This is not a question that is directly related to ancient geometry as such but is nevertheless one of immense interest, and a query often posed.

As opposed to the realms of ancient geometry, it is not possible to *prove* how the ancients tackled the problem of handling these great blocks of stone. Theory and guesswork are the only course open to us. But by a simple process of eliminating the possible alternatives we can approach a method which in its application appears not only to be a possible solution but may very well be the most probable solution to the mystery.

People accustomed to working with massive, bulky or dense weights frequently express their incredulity at the common theory that the answer lies simply in numbers; that it is merely a question of employing a sufficiently large work force. Immediately the factor must be considered, in the case of a heavy beam of stone, of how much space there is around the beam for each worker to apply his strength. And the theory that the beam could be lifted by laying ropes underneath and having several teams of workers "in layers" sounds to me to have been dreamed up by a man who has never faced the ordeal in practice of organising the elevation of a heavy load.

Imagine, for example, a stone lintel 6 m long, 1 m wide and 1 m high. This is a common size in classical architecture, often to be found as the architrave of a temple, resting immediately above the columns.

Let us imagine that this lintel is of granite with a specific gravity of 2.8. It would thus weigh 16,800 kg or roughly 17 tons.

A full-grown man can lift around 100 kg, and if we picture the stone surrounded by a team of men, each individual would require ½ m to give him room to lift. Shoulder to shoulder in this way, there is room for 12 men along each side and two at each end: a total of 28 men.

Each of the 28 men can lift the equivalent of 100 kg = total 2,800 kg. These

28 cannot therefore lift the beam—they cannot even move it! According to the theory of applying more men to the task, we would require 168 men on the job. In other words, they would have to stand in six layers around the stone in order to lift it off the ground.

Let us assume that we manage to work lengths of timber under the stone to give these 168 men a grip of the load. This would tremendously increase the burden for they would be no mere toothpicks. To reach the six rows of men on each side of the beam we would require at least 3 m of timber each side plus 1 m to slip under the stone, which means that each of these wooden "planks" would have to be 7 m long, and would have to withstand a vertical stress at the centre of approx. 1½ tons without cracking.

But what have we achieved at the end of this effort? We have raised the stone beam from the ground—but how do we raise it 8—10 meters into the air to a precise place in our building?

This cannot be done by multiplying our work-force. Neither can it be done by hauling on ropes, for steel hawsers of the type used today simply did not exist. And in any event there quite plainly was no room on a building site for such numbers of workers.

No, the solution to this problem must be sought in quite another direction, from an entirely different approach. The foregoing theory of mass labour would in reality prove impossible to operate—and we have applied it only to relatively minor loads of 16—17 tons. We are well aware that ancient engineers dealt with much greater loads than these. For example, the huge beams above the King's Chamber of the Great Pyramid (see Chapter Seven) each weigh about 70 tons.

As I see it, the engineer of the distant past was much more subtle in his approach than to experiment with brute force. Instead he utilised the load's own weight as the lifting force. A paradox? Hardly. Consider the following method by which *one single man* could complete the task shown to be almost beyond the capabilities of 168 men—and even to do it with greater ease.

In *Fig. 207* we see the same beam as discussed earlier: 6 meters long, 1 m wide and 1 m high, with a total weight of 16.8 tons. The beam is resting on the ground and the object is to raise it two meters above its present position.

The first thing our stone-mason does is to find the middle of the beam, under which he digs a small trench about 10 cm (4 inches) deep.

He then places two specially cut stones with a triangular section (*Fig. 208*) side by side in the trench, as in *Fig. 209*.

His next step is to dig away the earth from under the beam to a depth of about 10 cm, working outwards towards the ends. This leaves the beam balancing upon the two sharpened edges of the stones underneath, *Fig. 210*.

Perhaps he finds it necessary to prepare a simple foundation under these two stones, a foundation of flat slabs in order to distribute the pressure from the beam upon the ground. But this is a purely

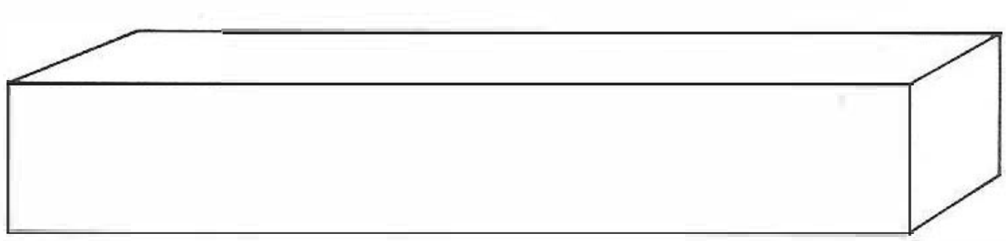


Fig. 207.

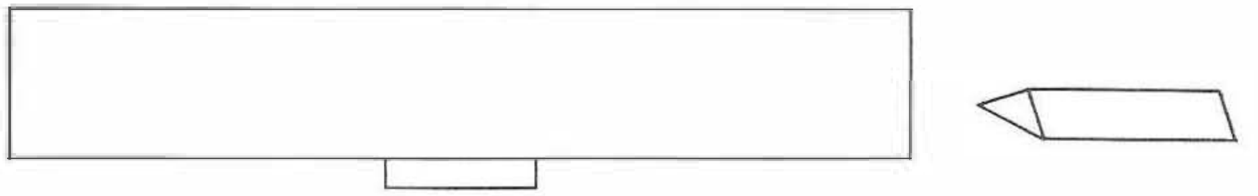


Fig. 208.

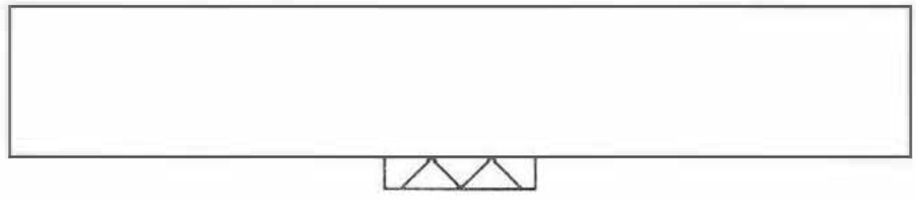


Fig. 209.

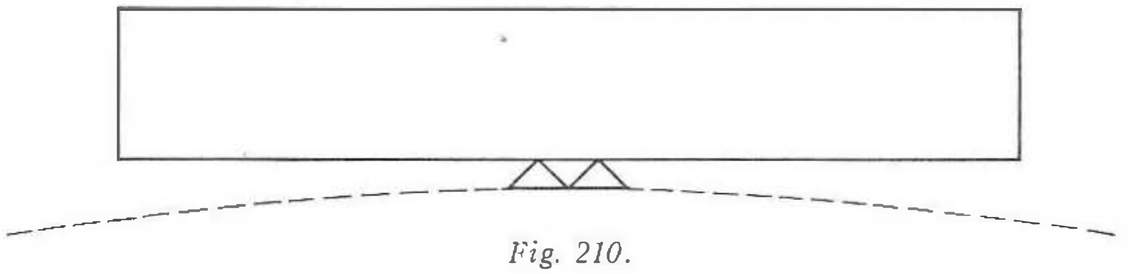


Fig. 210.

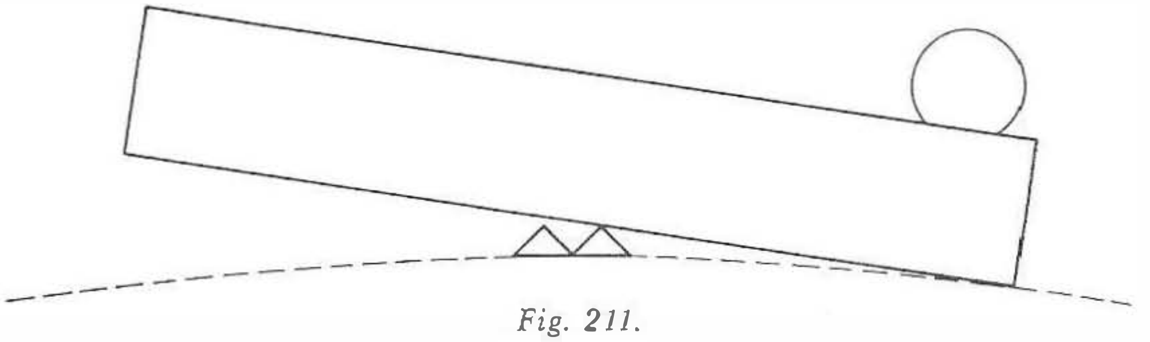


Fig. 211.

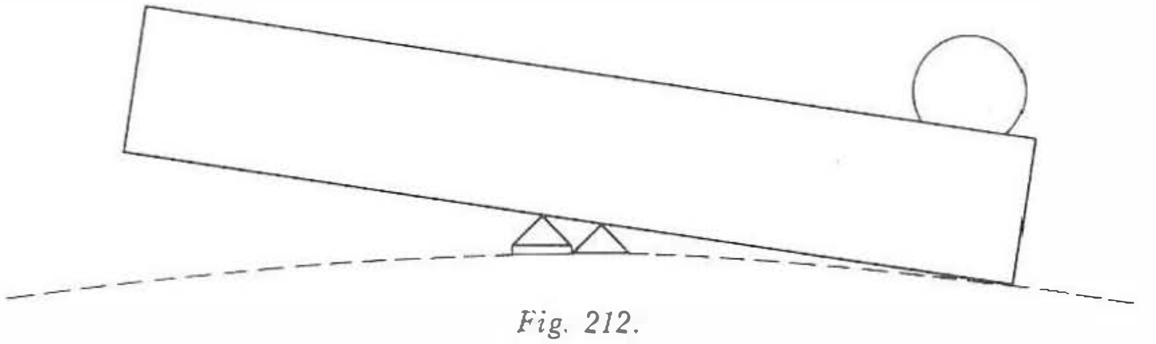


Fig. 212.

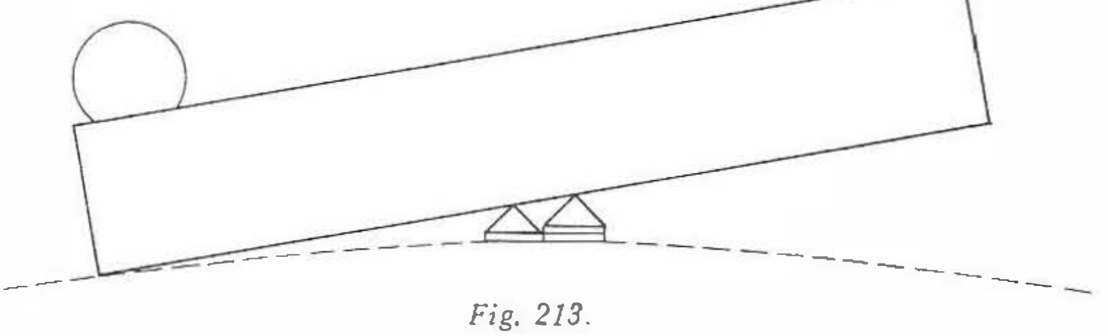


Fig. 213.

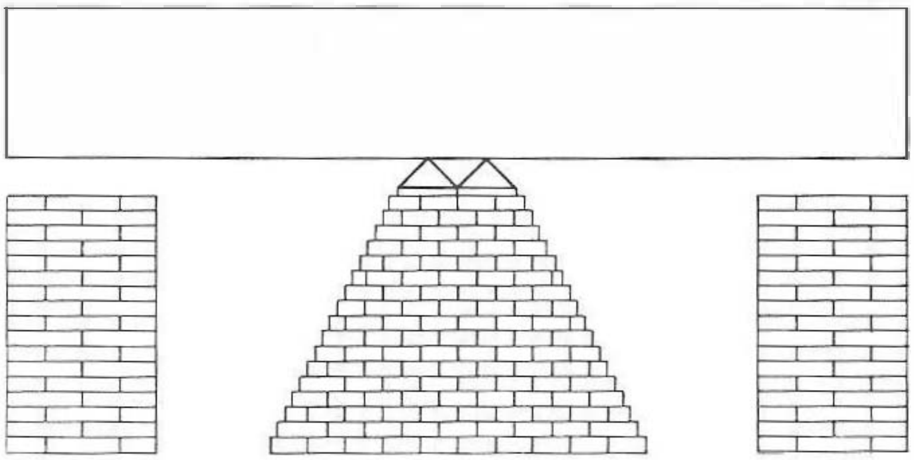


Fig. 214.

practical matter, and would depend a lot on the carrying capacity and density of the actual soil/rock surface.

The placing of this 17-ton beam in the manner described, without the application of excessive force, is something we may surely take as a feasible project. The most difficult part really would be to find the beam's vertical axis, for the beam might well be slightly heavier at one end than at the other. But the craftsman's experience and ability would no doubt serve him in good stead here.

Now he begins to lift the beam: He places a suitably heavy rock on top of the beam at one end. If it is the right weight, it tips the beam slightly—removing the weight from one of the underlying stones, *Fig. 211.*

This stone is removed, a suitable flat slab placed underneath, and the stone replaced under the beam, *Fig. 212.*

The balance-rock is moved to the other end of the beam, tipping the beam in the other direction—removing its weight from the hitherto undisturbed triangular stone. The beam has been raised the first few centimeters of the way to its goal. The same procedure of forming a foundation of slabs is followed, *Fig. 213.*

Now the mason moves with a regular rhythm: shift the rock, tip the beam, raise the underlying fulcrum, shift the rock, tip the beam . . . . ad infinitum! All the while broadening the foundation on which the beam rests and—most important—building up a temporary column of stone or timber under each end of the finely balanced beam to prevent it tipping too far to either side and crashing out of control. We see the position illustrated in *Fig. 214.*

In this way the giant lintel can be raised by a single man. Whether it is 6 m long and weighs 17 tons or is 12 m long and weighs 34 tons is virtually immaterial. Once the monster is placed on its seesaw the task becomes one of routine.

If two men work on the project, one engaged solely in moving the balance-rock backwards and forwards to weigh the appropriate end of the beam down, the work tempo can be increased considerably. And if a third man is employed to help to build up the supports, it is amazing how rapidly such a massive beam can be raised to great heights.

Imagine the foregoing procedure applied on the site of a Greek temple. The beam has to be placed above three columns. The method is as follows: Only two of the columns are erected. The third (central) for practical reasons must wait. The actual raising of the beam will be carried out as described above. We see it, nearing the desired height, in *Fig. 215* in which we have a bird's eye view. It is at

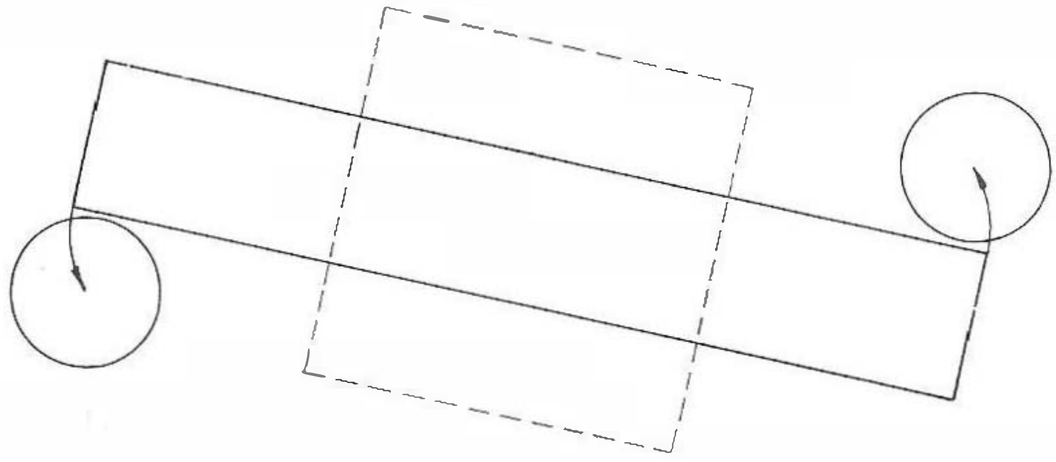


Fig. 215.

an angle to the two standing columns. Its central point, and fulcrum are where the third column will ultimately be placed.

When the beam has been raised a few centimeters *above* the height of the columns, the final stone/fulcrum is replaced by another shape of stone, with a gently curved upper edge, *Fig. 216*.

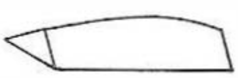


Fig. 216.

Once this stone is in position and the beam rests upon it, the giant can be swung like the needle of a compass into position above the two waiting columns. Granted that the starting point at ground level was accurate, this manoeuvre can be carried out with complete precision.

The beam is now more or less in position, but still a little higher than its intended resting place. Wedges are jammed under each end. The fulcrum and its foundations may now be removed, *Fig. 217* (see next page).

All the foundation material, stone slabs and debris may now be cleared off the site, and the third and central column erected. Once this is done the wedges can be knocked out—and the 17-ton beam is in its proper place.

I am convinced that this must have been the procedure for handling, raising and placing such giant lumps of stone, and I believe it was kept a traditional secret by those circles engaged in stonework and stone-masonry; just as geometricians retained their particular "trade" secrets.

When large blocks had to be positioned in a building or other structure they were transported to the building site. But when the actual lift was ready to be performed all outsiders (non-initiates) were banished from the area, and only a handful of select workers and assistants stayed behind to help the master mason set the big blocks in their proper places.

I feel sure that this system of engineering, or an adaptation of it, was employed at various sites throughout the world. It is a most likely explanation, for example, of the raising of the cross-beams at Stonehenge in the south of England.

In conclusion I am bound to say that I believe the problem of *transport* from the quarry to the building site was one which represented greater headaches to the ancient engineer than the actual process of erection. Since the journey was probably executed by laying planks along the trail and then pushing or dragging the blocks over rollers or logs, it would appear to have been an extremely awkward arrangement. In its application of

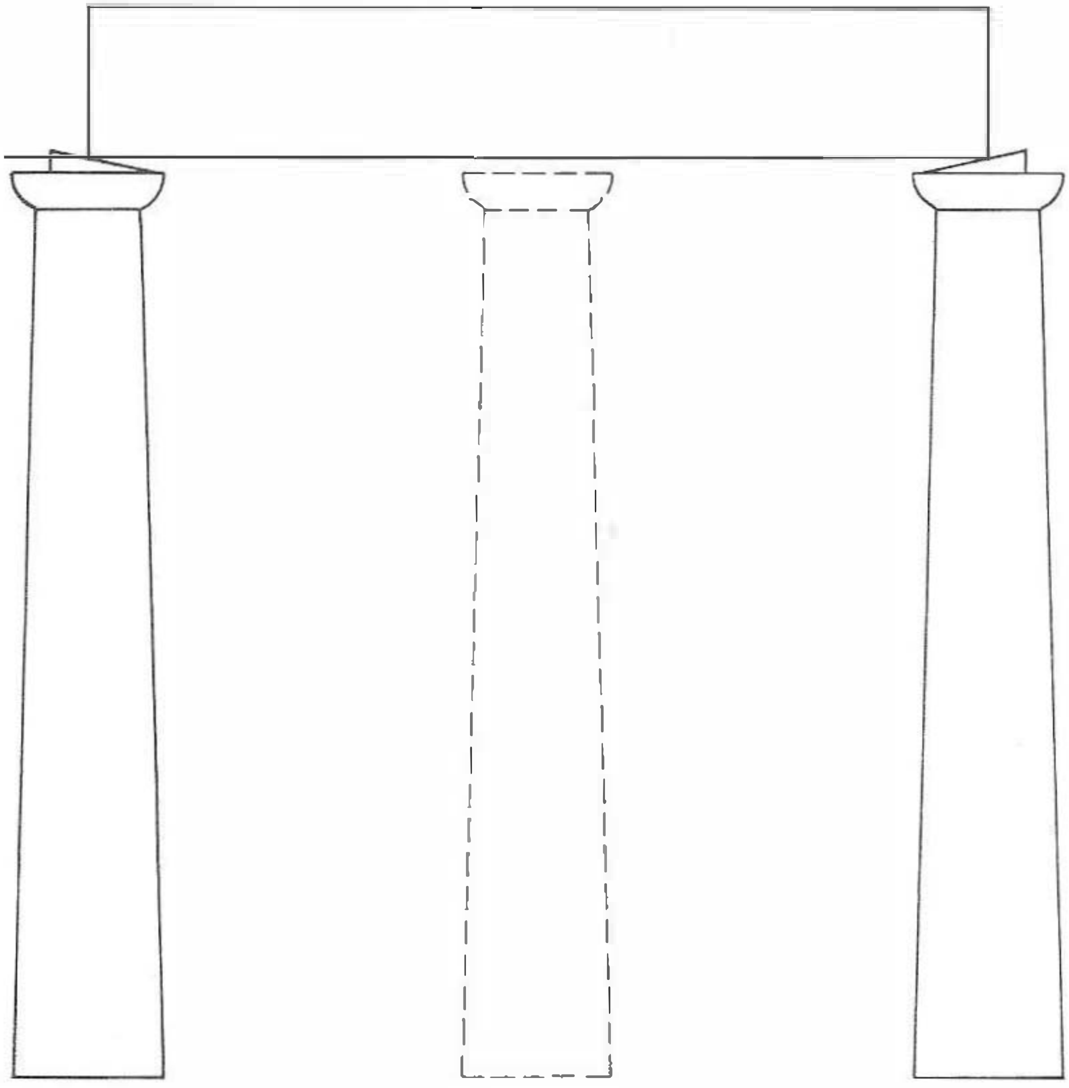


Fig. 217.

labour, quite the opposite of the efficient and simple job of raising the block above the ground. Imagine the need for brute strength. Men heaving and hauling endlessly on ropes. Manhandling the monster towards the building site.

Perhaps it was this part of the operation that required a mass of slaves? And history has mistakenly interpreted the slave armies as being necessary to the lifting process? When in fact they were not even on the site!

CHAPTER THIRTEEN

Triumphal Arches of the Early Christian Era

SCATTERED THROUGHOUT southern Europe, usually in or near major towns, are a number of decorative monuments known under the general term of Triumphal Arches. Their usual purpose was to celebrate some historical event.

Archaeological research and history have placed the construction period of these arches somewhere between 100 B.C. and 350 A.D. They represent therefore an architectural link between the temples of antiquity and the churches and cathedrals of the Middle Ages. Most characteristic of these monuments include:

	Built ca.
Arch of Augustus at Rimini ...	27 B.C.
Arch of Augustus at Susa	8 B.C.
Arch of Titus at Rome	70 A.D.
Arch of Trajan at Benevento ..	114 A.D.
Arch of Hadrian at Athens	120 A.D.
Arch of Septimius Severus at Rome	207 A.D.

Not far from the Arch of Titus stands perhaps one of the finest and best-known triumphal arches in the world: *the Arch of Constantine,* at the head of Via di S. Gregorio, one of the main approaches to the Colosseum. We shall be concentrating our study on this particular sample of Early Christian arches, built about 312 A.D.

Considering their respective dates of construction, we would expect to find ancient geometry used as a factor in the design of these arches. And we need not be disappointed in the least.

In *Fig. 218* we have a photograph of the Arch of Constantine. The picture is one which appeared in *Klassisk Kunst* by Leo Hjortsø and Ada Bruun. It is the illustration I found most suitable for purposes of analysis as it is almost directly at right-angles to the camera.

For such an analysis as we intend executing a photograph has—in the geometer's eye—both good and bad points.

The former are self-evident. There is no danger of incorrect measurements or drawings creeping in. The proportions should reproduce exactly as they appear in reality. The camera, as they say, does not lie.

But the bad points are also worth bearing in mind. It is rarely possible to obtain a photograph taken at *exactly* 90° to the subject, and it is normal procedure that such photographs are taken from ground-level which—again thanks to perspective—means that the photographic angle of the lower portion of the building is different from the angle at the top. Depending on the relative height of the subject this can well produce a distorted picture geometrically, and one which is hopeless in terms of accurate geometric analysis.

Fig. 218.

But these points notwithstanding I always prefer to conduct analyses on the basis of photographic material.

One of the first features one notices about the Arch of Constantine is its incredibly ornate face. It is covered with vast areas of relief and carving. There is in fact no single area of the facade which has escaped the sculptor's chisel.

This is in almost direct contrast with Greek temple buildings which certainly bore occasional areas of relievo or a decorative frieze, but which were principally in proportions and detail pure and unadorned. The ornamental areas were restricted to one or two clearly defined parts of the structure.

In our analysis of this arch we shall be tracing not only the main dimensions and plan of the building but also the lines which determine the areas of ornamental work into which the facade is divided.

Historical references help us to pinpoint the approximate date of construction of the arch at 312 A.D. It was erected in memory of Emperor Constantine's victory over Maxentius. The reliefs and sculptured areas reputedly depict scenes from and in glorification of this event.

The first part of our analysis, as usual, must be to locate the constructive square of origin, the basic square.

We see the first analytical drawing in *Fig. 219* in which, after a number of false starts, I discovered that the centre of the basic square lies in the prominent frieze which runs through the upper half of the Arch.

If we take the centre of the frieze as our point of origin and measure the distance to the building's base, we can then use this line as the radius of a circle. And it is the square described around this circle that becomes our basic square. It is seen as 1-2-3-4.

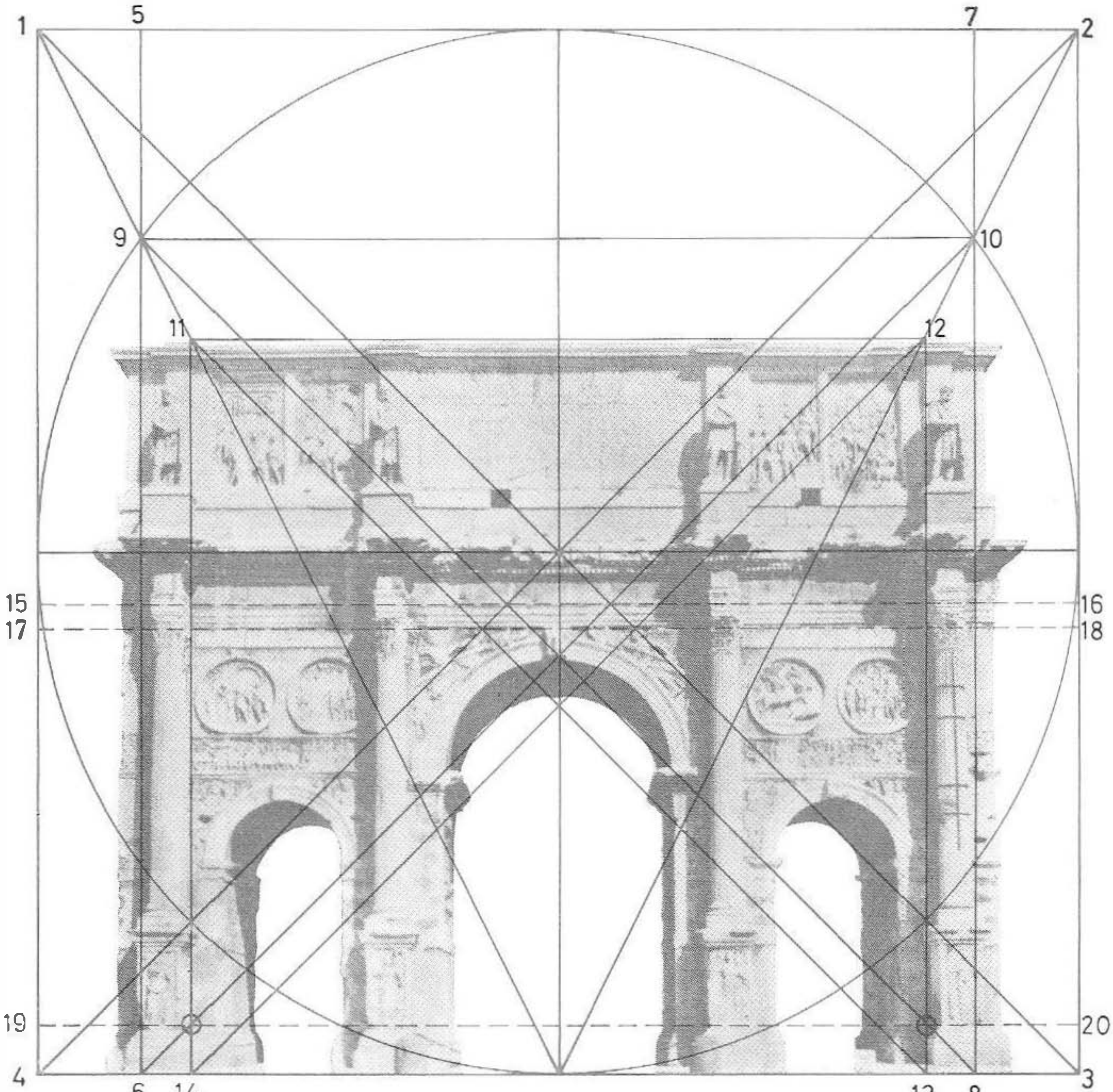


Fig. 219.

The immediate impression is that this square has nothing in common with the Arch, being wider and higher than the latter. But we shall in fact shortly see that this very square and various divisions within it indicate almost every dimension in the building and its ornamentation.

We have the basic square and its inscribed circle. The vertical and diagonal crosses naturally intersect at our point of origin, the centre of the upper frieze. The downward pointing acute-angled triangle is entered to mark the location of the circle's rectangle and the latter's square. The rectangle is seen as 5-6-7-8.

These two lines (5-6 and 7-8) illustrate the first marking of the facade. The face of the building bears four columns, two of which are placed on the extreme right and left respectively. These outer columns (indeed all four) have a square base, a cylindrical body, and end with a square top above the main frieze. On each column stands a statue. Note how the sides of the circle's rectangle indicate the outer edges of the square base and top of the outside columns.

Our next step is to enter the half-size version of the basic square: 11-12-13-14. Its vertical sides mark the inside of the same two columns. And the upper horizontal marks the top of the building (11-12).

Apart from these three important points of structure, this particular drawing also has something to tell us of the Arch's ornamentation.

Line 9-10 completes the square on the circle's rectangle. Thus we have within each other the three squares which proved of great structural importance in earlier temple building: the outer basic square, the square on the circle's rectangle, and the half-size version of the former. All share the same base-line.

The diagonal crosses of all three squares are entered.

As we recall from earlier analyses, these crosses form a small square immediately below the diagram's horizontal axis. The points at which the basic square's diagonals intersect the diagonals of the other two squares provide us with two interesting lines. These two lines—produced through the points of intersection—indicate the height of the frieze which runs horizontally across the building under the main frieze. The lines are 15-16 and 17-18.

One of the characteristics of the Arch of Constantine is the series of four columns which appear to carry the weight of the Arch's superstructure. The lower portion of these columns, as mentioned earlier, is square. The base proper is therefore also square. But the base is much higher than one might expect.

The base is marked by broken line 19-20, which is located by the intersections of the diagonals of the square on the circle's rectangle with the vertical sides of the basic square's half-size version. These points of intersection are circled for easy recognition.

As usual, for reasons of clarity we switch to another diagram before executing further figures. *Fig. 220* has the same basic square as the previous diagram. The inscribed circle is also there.

The intersections of the circle and the diagonals of the square indicate the location of another half-size version of the main square, i.e. 21-22-23-24.

We see how the base of this new square (23-24) marks the division in the four columns between the square base and the actual cylinder form.

According to the dictates of ancient geometry, without any measurement, we divide square 21-22-23-24 into 16 smaller squares or 32 small triangles.

Apart from the vertical cross, we see the lines of division in 25-26, 27-28, 29-30 and 31-32.

Two of the small squares have been further divided into eight triangles each by their vertical and diagonal crosses. Let us look at the right-hand of these two squares, whose vertical axis is 35-36. We note that this axis cuts between the two circular reliefs, each of which occupies ¼ of the square. The other part of the vertical cross, i.e. the square's horizontal axis, coincides with the lower edge of the smooth frieze immediately above the circular reliefs.

Producing the vertical axis (35-36) and its counterpart on the other side of the facade upwards, we find that they cut between the two decorative panels above the main frieze.

Producing the two lines downwards, we find that they form the centre-line of the two gateways on either side of the main arch.

Lines 25-26 and 27-28 mark the outer line of the two inner columns in the same way as the verticals of the circle's rectan-

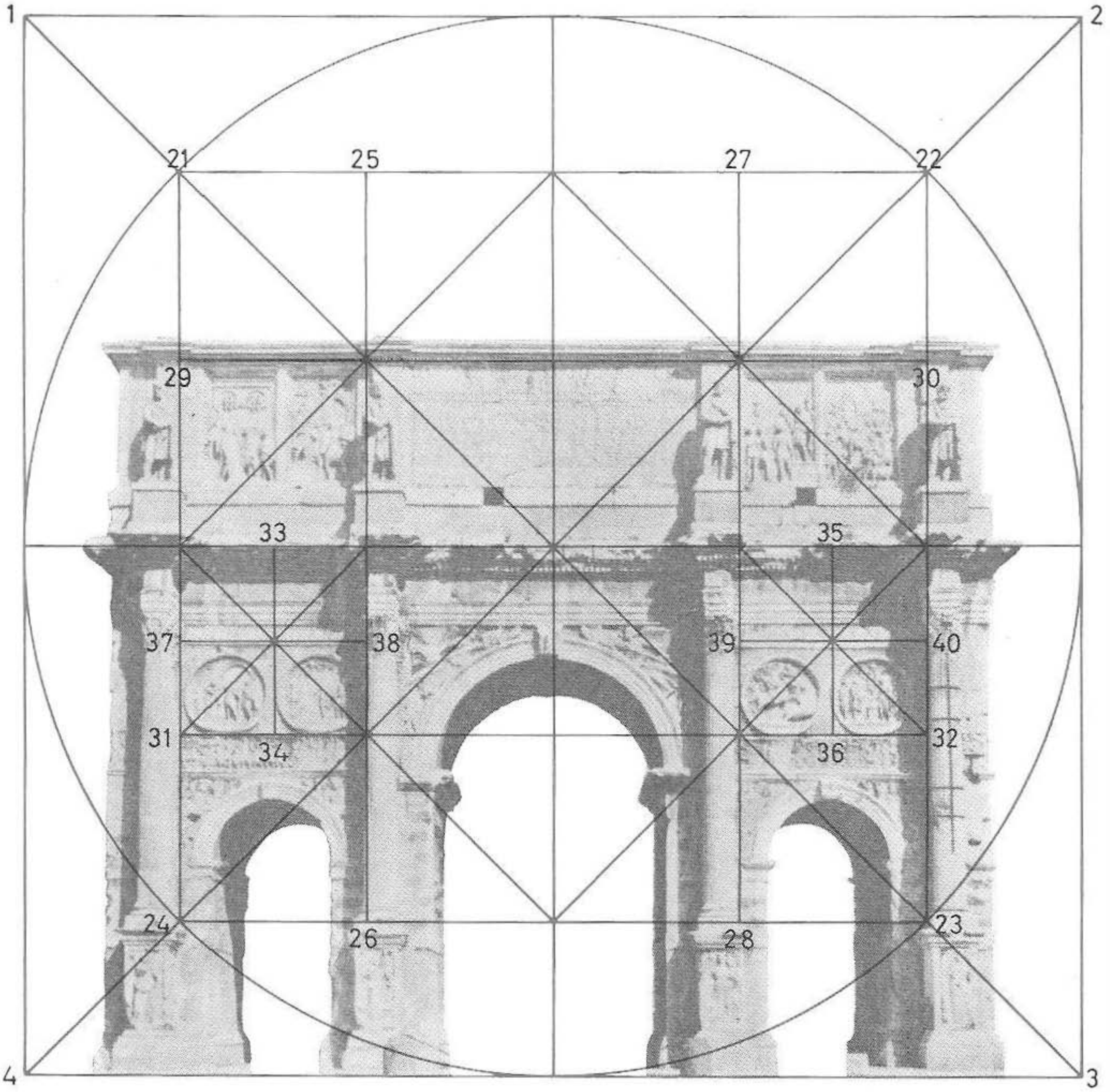


Fig. 220.

gle marked the inside of the two outer columns.

Horizontal 29-30 indicates the height of the four pieces of sculpture atop the columns, passing just over their heads.

Thus we see that in our analysis so far scarcely a single line has been without purpose. Every one has been applied either as a line of the main structure or to mark some of the building's characteristic ornamentation.

Since we are still on the look-out for one or two vital lines of construction not immediately evident on our present diagram, we move on to *Fig. 221.*

Once again our basic square is 1-2-3-4. This time it is to the sacred cut that we turn, entering all four: 37-38 and 39-40 (vertically) and 41-42 and 43-44 (horizontally).

The four sacred cuts of course create a new square in the centre of the diagram. This is divided by a procedure applied also in the previous diagram: into 16 small squares.

We see how verticals 45-46 and 48-49,

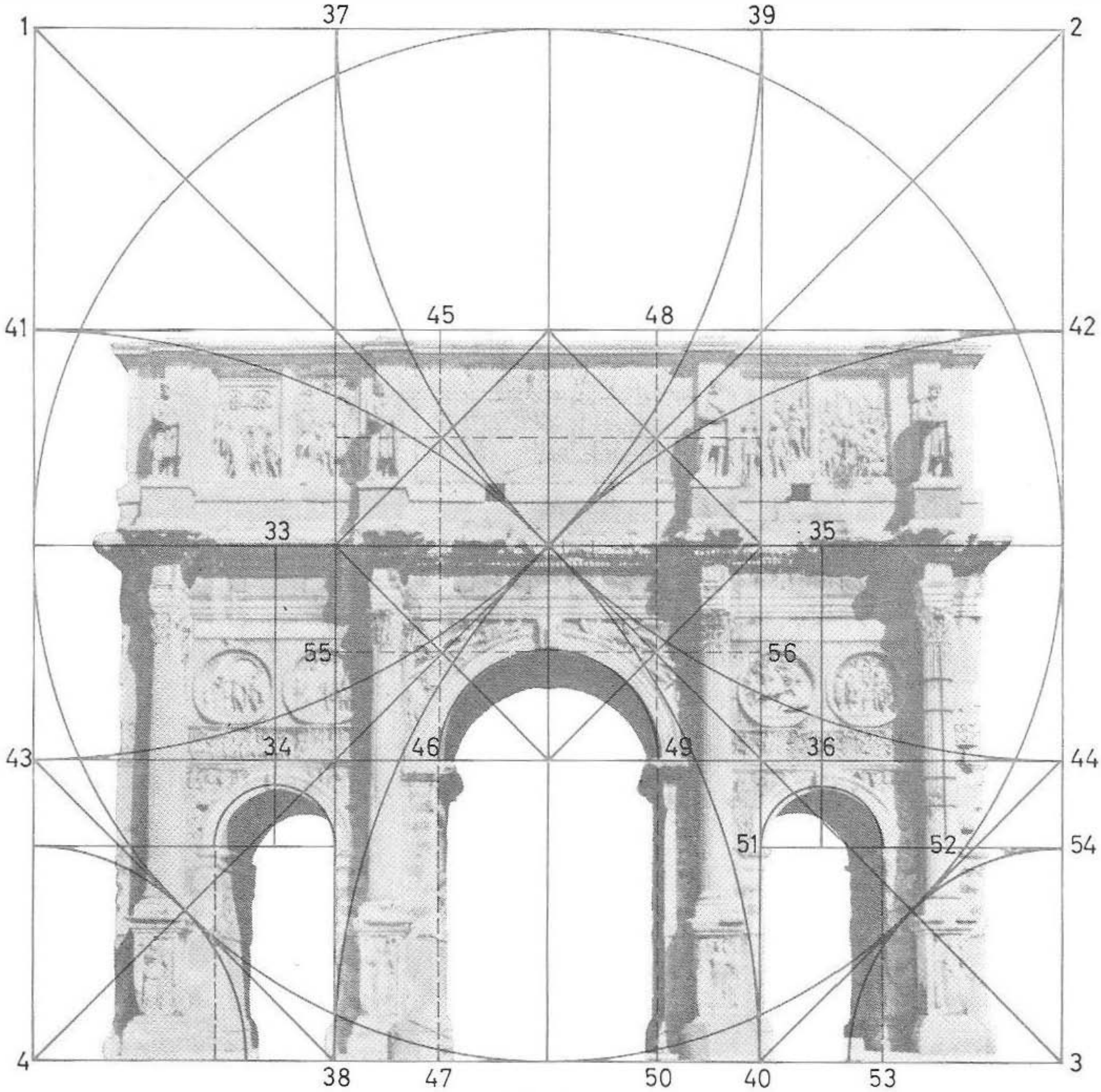


Fig. 221.

produced to points 47 and 50 respectively, indicate the width of the main gateway.

Simultaneously we note how the lowest of the three horizontal dividing lines (55-56) indicates the height of the main gateway.

The base-line of the small central square (created by the four sacred cuts) carries the centre of the circle used to indicate the curve of the main arch. This line, which is of course the same as the lower horizontal sacred cut in the main square, also marks the bottom of the frieze at each side of the building, and the ledge on which the main arch itself rests. To illustrate this arrangement clearly the curve of the archway is shown in the diagram.

The design of the two small gateways must be seen in conjunction with the previous analysis. We recall that line 35-36 indicated the centre-line (vertical) of the small gate. This line (and its counterpart in the other gate, 33-34) has been entered in Fig. 221. The square from which it was created has been omitted. The rest of the planning for this archway is as follows:

The sacred cut divides the basic square into nine quadratic areas: five squares and four rectangles.

Four of the squares are in the four corners of the diagram, the fifth in the centre. We are interested in the square in the lower right corner of the basic square, i.e. three corners of which are numbered 44-3-40.

At the point where the upper horizontal sacred cut of this square (51-54) intersects line 35-36 (produced) we have the centre of the circle on which the two side arches were created.

Just as the *lower* horizontal sacred cut in the basic square indicated the ledge of the main archway, so the *upper* horizontal sacred cut in the two small squares indicate the corresponding projecting ledge in the two side arches.

Thus we see that the same factor, the sacred cut, was responsible for the curve of all three arches.

The curve of the side arch, at its intersection with 51-54, indicates the width of the side gateway, one side of which is indicated by the vertical sacred cut (39-40) in the basic square. The other side has been shown by broken line 52-53.

This more or less rounds off our search for the main dimensions in the Arch of Constantine. Once again we have satisfied ourselves that they stem from one single proportion, i.e. the size of the basic square. Once this has been located it is virtually a matter of routine to the exponent of ancient geometry to trace the remaining dimensions. And it is interesting in this case to note how closely the analysis just completed corresponds to analytical diagrams of temple-planning in Greece.

Simply because space (or the lack of it) so dictates, the analysis of the Arch of Constantine in Rome must serve as a representative for other triumphal arches of its period. It is perhaps the one best-known to the world and the one most often the subject of discussion.

But I can assure my reader that I have applied the strict rules of ancient geometry to several others arches of that period, and have traced an overwhelming number of similar points between their planning and the stipulations of geometry.